PROJECT REPORT

ON

CHARM MESON SPECTROSCOPY

Submitted by

Manpreet Kaur

M.Sc. Final Year (Physics)

(3000804014)

Under the guidance of

Dr. Alka Upadhyay

School of Physics and Material Science

THAPAR UNIVERSITY

PATIALA

CERTIFICATE

This is to certify that the report entitled **"Charm Meson Spectroscopy"** submitted by Ms. **Manpreet Kaur** of M.Sc. (Physics), Thapar University, Patiala, was carried out by her under our supervision and guidance of **Dr. Alka Upadhyay**. I have not submitted this material for credit towards any other degree at Thapar University, Patiala or any other university.

Dr. Alka Upadhyay

Assistant professor,

School of Physics and Material Science,

Thapar University, Patiala,

Head of department:

Dr. O. P. Pandey

Professor,

School of Physics and Material Science

Thapar University, Patiala.

Dr. R. K. Sharma 23/7

Dean of Academic Affairs

Thapar University, Patiala.

ACKNOWLEDGEMENT

I express my deep sense of gratitude and respect to my guide ***Dr. Alka Upadhyay*****, Assistant Professor, School of Physics and Material Science, Thapar University, Patiala,** for her keen interest and valuable guidance, strong motivation and constant encouragement during the course of the work. I thank her from the bottom of my heart for introducing me to Particle Physics. I thank her for her great patience, constructive criticism and myriad useful suggestions apart from invaluable guidance to me. I am sure that the knowledge gained through my association with my supervisor shall go a long way in helping me to realize my goals in life.

I am also thankful to my colleague Ms. Sandeep Kaur and all classmates for their help and support at every stage during the project report work.

Finally, I would like to express my deepest gratitude to my **parents and family,** without whom I am nothing, to provide me great opportunities, everlasting support, big encouragement and lots of love.

Manpreet Kaur
(Manpreet Kaur)

Date: 15 July, 2010

Place: Thapar University, Patiala.

NOTATIONS AND ABBREVIATIONS

Units:

The basic units in physics are length, mass and time and the SI system expresses these in metres, kilograms and seconds. The units commonly used in high energy physics are summaries in the given below table:

Quantity	**High energy unit**	**Value in SI unit**
Length	1 fm	10^{-15} m
Energy	1 GeV= 10^{9}eV	1.602 x 10^{-10} J
Mass, E/c^2	1 GeV/c^2	1.78 x 10^{-27} kg
$\hbar = h/(2\Pi)$	6.588 x 10^{-25} GeV s	1.055 x 10^{-34} J s
c	2.998 x 10^{23} fm s^{-1}	2.988 x 10^{8} m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162 x 10^{-26} J m

Natural units, $\hbar$ = c =1	
Mass	1 GeV
Length	1 GeV^{-1} = 0.1975 fm
Time	1 GeV^{-1} = 6.59 x 10^{-25} s

Relations between energy units:

1MeV = 10^{6} eV

1 GeV = 10^{3} MeV = 10^{9} eV

1 TeV = 10^{3} GeV= 10^{12} eV

The unit of length is the femto-metre (fm) or fermi, where 1 fm = 10^{-15} m. The commonly used unit of energy is the GeV, convenient because it is typical of the mass-energy mc^2 of strongly interacting particles.

In calculations, the quantities ħ = h/(2Π) and c occur frequently, sometimes to high powers and it is advantageous to use units in which we set ħ = c = 1. Having chosen these two units, we are still at liberty to specify one more unit, e.g. the unit of energy and the common choice, as indicated above, is the GeV. With c = 1, this is also the mass unit. As shown in table, the unit of length will then be 1 GeV^{-1} = 0.197 fm, while corresponding unit of time is 1 GeV^{-1} = 6.59 x 10^{-25} s.

Abbreviations:

ATLAS - **A** **T**oroidal **L**HC **A**pparatu**S**

CERN - European Organization for Nuclear Research

CESR – Cornell Electron positron Storage Ring

DESY - Deutches Elektronen SYnchrotron

DORIS - **Do**ppel-**Ri**ng-**S**peicher (in English as “Double-Ring Storage”)

HERA - **H**adron-**E**lektron-**R**ing-**A**nlage

LEP - Large Electron-Positron collider

LHC - Large Hadron Collider

PEP - Positron Electron Project

PETRA - **P**ositron-**E**lektron-**T**andem-**R**ing-**A**nlage

SPEAR - Space Plasma Exploration by Active Radar

SLAC - Stanford Linear Accelerator Center

MOTIVATION

The driving motivation behind particle physics experiments is the desire to uncover the true nature of fundamental forces and particles. Our current standard model is believed to be an effective theory, which has a deeper underlying theory reachable in the next generation of experiments. In the electroweak sector, where great successes of the past decades have predicted and verified the unification of the electromagnetic and weak nuclear forces, precision measurements at the CERN large electron positron collider (LEP) and the Fermi-lab proton-antiproton collider (Tevatron) demand that there be either a light Higgs particle with a mass less than about 200 GeV or a physical system mimicking its interactions. At the same time, the requirement that the theory be stable even with a Higgs, as well as the observation of cold dark matter in the universe, compellingly point to new physics at the Tera-scale.

In the flavor sector, a decade of increasingly precise measurements of the properties of heavy quarks has shown remarkable agreement with the standard model predictions, and we are now moving into an era of precise investigations of the neutrino sector and the physics behind it had a very significant role in cosmology. There are some hints that this new physics might be accessible to upcoming experiments. In the strong sector, the theory of quantum chromo-dynamics (QCD) has been successfully used to predict the behavior of quarks and gluons at high energies observed at the HERA collider at DESY Hamburg as well as LEP and the Tevatron, but the lower energy regime, where they are bound into particles such as protons and neutrons, remains theoretically and experimentally challenging and requires further investigation.

The ATLAS experiment is one of the two general-purpose detectors at the LHC at CERN, the European Laboratory for Particle Physics. When the proton beams produced by the LHC interact in the center of the detector, a variety of different particles with a broad range of energies may be produced. It is designed to measure the broadest possible range of signals. This is intended to ensure that whatever form any new physical processes or particles might take, it will be able to detect them and measure their properties.

ABSTRACT

Heavy quark physics gives a unique opportunity to test the prediction of QCD and standard model. Within the framework of heavy quark effective theory and chiral perturbation theory, masses of heavy mesons are studied. The approximate symmetries of Quantum Chromo-Dynamics in the infinite heavy quark $(Q = c)$ mass limit $(m_Q \to 0)$ and in the chiral limit for the light quarks $(m_q \to 0, q = u, d, s)$ can be used together to build up an effective chiral Lagrangian for heavy and light mesons describing strong interactions among effective meson fields. Expanding about a symmetry limit (Λ~1GeV) provides us with a means for describing non-perturbative effects by a series of low energy parameters (effective coupling) which can be determined from experimental data.

In our thesis, we use the well-determined masses of the ground states, $J^P = 0^-(D^0, D^+, D_S^+)$, $1^-(D^{*0}, D^{*+}, D_S^{*+})$ and the strange first excited states, $J^P = 0^+(D_0^0, D_0^+, D_{0S}^+)$, $1^+(D_1^{0'}, D_1^{+'}, D_{1S}^+)$ to predict the masses of the non-strange first excited state in the framework of heavy hadron chiral perturbation theory.

A mass formula is developed for the charm mesons and comprehensive analysis is done for the low lying charm meson states including the *O(1/m$_C$)* corrections, The key differences is that we have constrain the values of the parameters *g,g' ,h* to the values that are determined from the decays. Furthermore, we have imposed the requirement that m_s = 90 MeV and have required the parameters determining the tree-level hyperfine structure to be in a range determined by the well-established states. This approach points to values for the masses of those states that are smaller than the experimental determinations.

Contents

Chapter-I

Introduction

Chapter-II

Heavy quark effective field theory

Chapter-III

Charm meson in Heavy hadron chiral perturbation theory

Chapter-IV

Chapter-I

Introduction

1.1 High energy physics:

To introduce briefly subatomic-particle and their detection before treating particle detectors in detail, then we conclude with two examples that illustrate how a variety of detectors work together in typical high-energy-physics experiment. The experimental study of subatomic particles and their interaction has revealed an unexpected layer of substructure underlying the atomic nucleus has shed light on the evolution of the universe in the earliest moments following the Big-Bang. This field of research is commonly referred to as elementary-particle physics or (because of highly energetic particle beams employed) high energy physics. [1]

"**Particle Physics** is a branch of physics that studies the elementary constituents of matter and radiation, and the interactions between them. It is also called **High Energy Physics**, because many elementary particles do not occur under normal circumstances in nature, but can be created and detected during energetic collisions of other particles."[2]

Protons, electrons, neutrons, neutrinos and even quarks are often featured in news of scientific discoveries. All of these and a whole "zoo" of others are tiny sub-atomic particles too small to be seen even in microscopes. While molecules and atoms are the basic elements of familiar substances that we can see and feel, we have to "look" within atoms in order to learn about the "elementary" subatomic particles and to understand the nature of our Universe. The study of particles at high energy is called Particle Physics, Elementary Particle Physics or High Energy Physics (HEP).

Atoms were postulated long ago by the Greek philosopher Democritus, and until the beginning of the 20th century, atoms were thought to be the fundamental indivisible building blocks of all forms of matter. Protons, neutrons and electrons came to be regarded as the fundamental particles of nature when we learned in the 1900's through the experiments of

Rutherford and others that atoms consist of mostly empty space with electrons surrounding a dense central nucleus made up of protons and neutrons as shown in fig 1.1

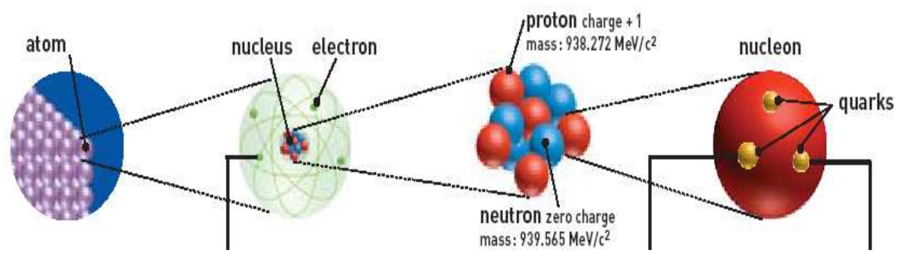


Fig 1.1 Quarks present inside the atom

The science of particle physics surged forward with the invention of particle accelerators that could accelerate protons or electrons to high energies and smash them into nuclei to the surprise of scientists, a whole host of new particles were produced in these collisions [3].

1.2 Standard model:

An enormously successful theory devised to explain how subatomic particles interact with each other [4]. The Standard model (SM) classifies particles and describes their interactions [5]. All known particle physics phenomena are extremely well described within the Standard Model of elementary particles and their fundamental interactions. The SM provides a very elegant theoretical framework.

We understand by elementary particles the point-like constituents of matter with unknown substructure up to the present limits of $10^{-18} - 10^{-19}$m. These are of two types, the basic building blocks of matter themselves known as matter particles and the intermediate interaction particles.

The first ones are fermions of spin s = $\frac{1}{2}$ and are classified into leptons and quarks. The known **leptons** are: the electron, e^-, the muon, μ^- and the tau, τ^- with electric charge Q = −1 (all charges are given in units of the elementary charge e) and their corresponding neutrinos ν_e, ν_μ

and ν_τ with Q = 0. The known **quarks** are of six different flavors: u, d, s, c, b and t and have fractional charge Q = $\frac{2}{3}$, $\frac{-1}{3}$, $\frac{-1}{3}$, $\frac{2}{3}$, $\frac{-1}{3}$ and $\frac{2}{3}$ respectively.

The quarks have an additional quantum number i.e. the color which for them can be of three types, generically denoted as q_i, i = 1, 2, 3. We know that color is not seen in nature and therefore the elementary quarks must be confined into the experimentally observed matter particles, the hadrons. These colorless composite particles are classified into baryons and mesons. The baryons are fermions and made of three quarks i.e. qqq. For example, the proton consist of p ~ uud, and the neutron consist of n ~ ddu. The mesons are bosons and made of one quark and one anti-quark. For example, the pions consist of $\pi^+ \sim \bar{u}$ d and $\pi^- \sim \bar{d}$u [6]. The six leptons, six quarks have three families of matter or three generations and the force carriers are shown in fig 1.2

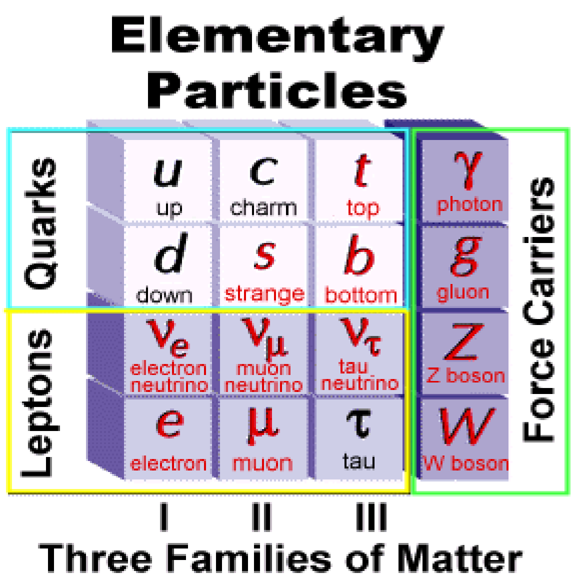


Fig 1.2 Six leptons, six quarks and force carriers

The standard model also describes three fundamental interactions known as the strong, electromagnetic and weak forces. The mediators of the strong force are gluons. The mediator of the electromagnetic force is the photon and the massive $W^{\pm}$ and Z^0 bosons mediate the weak force. Only quarks interact via the strong force and only charged particles interact electromagnetically. All particles interact via the weak force. The fourth fundamental force, gravity is much weaker than the other three mentioned and is not described by the standard model [5].

1.3 Quark Model:

In 1963 Gell-Mann and Zweig proposed a model that explained the spectrum of strongly interacting particles (i.e. hadrons) in terms of elementary constituents called quarks. The quark model was developed to account for the regularities observed in the hadron spectrum, with hadrons interpreted as bound states of localized but essentially non-interacting quarks. It provides us a simple picture of internal structure of hadrons and an effective way to describe their dynamics at high energy. Much of the success of the model lies in the circumstance that to a reasonably good approximation; we can regard quarks as free or weakly interacting particles (except for the confining mechanism). Mesons were expected to be quark-anti-quark bound states. Baryons were interpreted as bound states of three quarks. The quark constituents of the baryons are assigned to have spin ½ from the observed spins of low-lying baryons.

The low-lying baryons were interpreted in the quark model as symmetric states of space, spin and $SU(3)_f$ flavor degrees of freedom. However, Fermi-Dirac statistics requires a total anti-symmetry of the wave function. The resolution of this dilemma comes through the introduction of color degree of freedom. The baryon wave functions are totally anti-symmetric in the color degree of freedom. Of course, the introduction of another degree of freedom would lead to a proliferation of states, so the color degree of freedom had to be supplemented by a requirement that only color singlet states exist in nature. Hence proton would be a bound state of 'uud' quarks and neutron would be a bound state of 'udd' quarks which makes them color singlet. This model had great success in predicting new hadronic states and in explaining the strength of electromagnetic and weak interaction transitions among different hadrons. In particular, it naturally incorporates the most important symmetry relations among hadrons.

Once quark structure of hadrons got some acceptance, it became natural to look for the dynamics obeyed by the quark system responsible for the composition of hadrons as well as for hadronic reactions. In order to get experimental information on quark dynamics, the most sensible way is to probe inside of hadrons (e.g., proton) by applying a beam of structure-less particles such as leptons. We need much higher energies and larger momentum transfers for the study of hadronic structure to have higher resolutions. The electromagnetic form factors are key ingredients to the understanding of the internal structure of composite particles like the nucleon, since they contain the information about the distributions of charges and currents. The knowledge of hadron form factors, especially for the nucleons and the pions, represent an important source of information about their electromagnetic structure. By varying the momentum transfer, large as well as small distances can be explored, allowing one to learn about hadronic physics. De-Broglie wavelength of an electron becomes much shorter than the size of a typical nucleus at sufficiently high energies in GeV range. In such cases, the scattering result is dominated by the charge distributions within individual nucleons. The primary interest of scattering at these energies shifts to the structure of nucleon rather than that of nucleus.

The quarks are classified as "light" or "heavy" depending on their entries in the mass matrix m of QCD Lagrangian equation. These masses are "running" as well: they depend on the scale μ at which they are determined. The masses of the lightest quarks (u and d), $m_{u,d} < 10$MeV (estimated at a renormalization scale μ~1 GeV) are very small compared to typical hadron masses of order 1 GeV, such as those of the ρ meson or the nucleon. The strange quark mass, m ≈ (100 -150) MeV is an order of magnitude larger than $m_{u,d}$ but still counted as "small" on hadronic scales. The charm quark mass $m_c \approx (1.1\text{-}1.4)$ GeV takes an intermediate position while the b and t quarks $m_b \approx (4.1\text{-}4.4)$ GeV, $m_t = (174 \pm 5)$ GeV fall into the "heavy" category. These different quark masses set a hierarchy of scales, each of which is governed by distinct physics phenomena [7].

1.4 Quarks and Leptons:

Quarks:

Hadrons are composed of quarks, of which (like the leptons) only six types are known [1]. Quarks are the fundamental building blocks of nature [8]. The name of quark began from

1964, when Murray Gell-Mann and George Zweig suggested that hundreds of the particles known at the time could be explained as combinations of just three fundamental particles. Gell-Mann chose the name **"quarks,"** pronounced "kworks" for these three particles, a nonsense word used by James Joyce in the novel *Finnegan's Wake:* "Three quarks for Muster Mark!" [9]

Gell-Mann received the 1969 Nobel Prize for his work in classifying elementary particles. These are designated as up and down, charm and strange, top and bottom [1]. They combine to form larger particles, such as protons and neutrons [8]. The list of quarks is given in the table 1.1

Table 1.1

Flavor		**Mass (GeV/c^2)**	**Electric Charge (e)**
u	Up	0.004	+2/3
d	Down	0.008	-1/3
c	Charm	1.5	+2/3
s	Strange	0.15	-1/3
t	Top	176	+2/3
b	Bottom	4.7	-1/3

Quarks are also grouped into three generations:

$$\binom{\mathbf{u}}{\mathbf{d}},\binom{\mathbf{c}}{\mathbf{s}},\binom{\mathbf{t}}{\mathbf{b}}$$

The quarks come in pairs with the members of a pair differing in the electric charge by one unit. The up, charm and top quark have charge $\frac{+2}{3}$e. The down, strange and bottom quarks have charge $\frac{-1}{3}$e. For each type of quark, there exists corresponding anti-quark with opposite electric charge. Quarks are bound together into hadrons by the strong force. [1]

Leptons:

Leptons are defined as particles which do not participate in the strong interaction, i.e. they interact only via the weak force, electromagnetic force, and gravity. There are only six types of leptons: the (negatively charged) electron, muon and tau and their corresponding neutrinos: the electron neutrino, muon neutrino and tau neutrino. The three charged leptons all have charge -1e while the neutrinos are neutral. For each type of lepton, there exists a corresponding antiparticle. Leptons and anti-leptons have equal mass, spin, lifetime and electric charges equal in magnitude but (for charged leptons) opposite in sign. Leptons are grouped into three generations i.e.

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

that matched the charged leptons with corresponding neutrinos.

Leptons and quarks each have one quantum of spin. Experimental evidence from particle accelerators suggests that all leptons are truly elementary, i.e. they are point-like and cannot be subdivided any further.

1.5 Hadrons and Gluons:

Hadrons:

Hadrons are defined as particles which do interact through the strong force. Hadrons are made of quarks. Thus, hadrons are susceptible to all four basic forces of nature. Hadrons can be further subdivided by their spin:

(i) Hadrons with half-integer spins ($\frac{1}{2},\frac{3}{2},...$) are fermions and are called **baryons**. Baryons are massive particles which are made up of three quarks [10].

B = qqq

This class of particles includes the proton and neutron. The lightest baryon is the proton, which is also the only hadron stable in free space. The structure of proton is given in fig 1.3. The neutron although is stable inside the nucleus [11]. The structure of neutron is given in fig 1.4

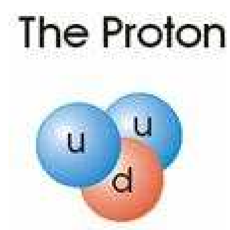


Fig 1.3 Structure of proton

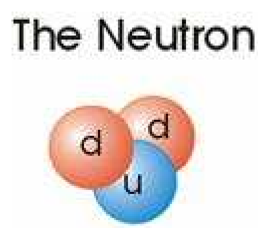


Fig 1.4 Structure of neutron

Other baryons are the lambda, sigma, xi and omega particles. Besides charge and spin (1/2 for the baryons), the two other quantum numbers are assigned to these particles: baryon number (B=1) and strangeness (S) and they can be equal to -1 times the number of strange quarks included.

(ii) Hadrons with integer spins (0, 1, 2,...) are bosons and referred to as **mesons**. Mesons are intermediate mass particles which are made up of a quark and anti-quark pair i.e. [10].

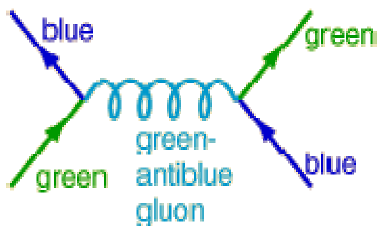
M = q$\bar{q}$

This class of particles includes pions (Π^+, Π^0) as shown in fig 1.5. Other mesons are Π^-, K^+, K^- and η

Fig 1.5 Structure of pion (Π^+, Π^0*)*

Gluons:

Gluons are the elementary particle that mediates or carries the strong forces [12]. Gluons transmit “color” force between quarks [13] or Gluons are the exchange particles for the color force between quarks. Gluon interactions are often represented by a Feynman diagram. Note that the gluon generates a color change for the quarks. The gluon exchange picture there converts a blue quark to a green one and vice-versa as shown in fig 1.6



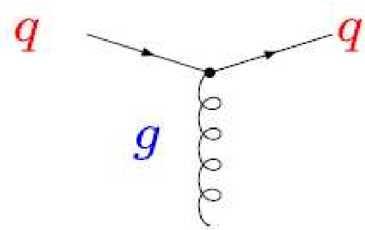
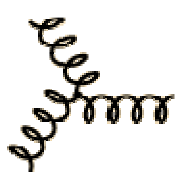


Fig 1.6 Feynman diagram for an interaction between quarks via a gluon

The gluons are in fact considered to be bi-colored i.e. it carrying a unit of color and a unit of anti-color as shown in the fig 1.6. The range of the strong force is limited by the fact that the gluons interact with each other as well as with quarks. These properties contrast them with photons, which are mass-less and of infinite range. The gluons do carry the "color charge".

Within their range of about a fermi (10^{-15}), the gluons can interact with each other, and can produce virtual quark- anti-quark pairs. The property of interaction with each other is very different from the other exchange particles and raises the possibility of gluon collections referred to as "glueballs" [14]. Because gluons carry color and the property associated with the strong interaction that they can interact with each other. Possible Feynman diagrams for those interactions [12] are shown in fig 1.7



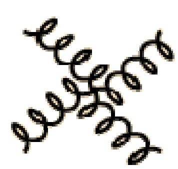


Fig 1.7 Gluon-gluon interaction at 3-point and 4-point

1.6 Fundamental interactions:

Fundamental interactions are the way that the simplest particles in the universe interact with each other. An interaction is a fundamental when it cannot be described in terms of the other interactions. There are only four known types of interactions in the whole of physics:

1) Electromagnetic interaction
2) Strong interaction
3) Weak interaction
4) Gravitational interaction

1) Electromagnetic interaction:

This type of interaction occurs between any two particles that have electric charge. Photons are the carrier particles of electromagnetic interaction as shown in fig 1.8 . The force is

$$F_{coulomb} = \frac{ke^2}{r^2}$$

and the constant 'ke^2' has the units of energy times distance.

It is the force that holds electrons and protons together in atoms, and which hold atoms together to make molecules. It is 100 times weaker than strong interaction at short distances.

A dimensionless constant which characterizes the electromagnetic force is

$$\alpha = \frac{2\pi ke^2}{hc} = \frac{1}{137}$$

This coupling constant is also called the "fine structure constant".

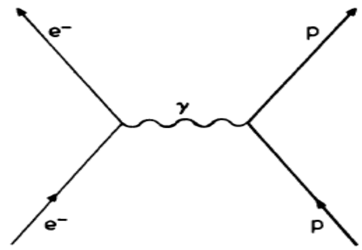


Fig 1.8 Interaction via photon

2) Strong interaction:

Strong interaction also called strong force. It holds quarks and gluons together to form protons, neutrons and other particles. The word "strong" is used since the strong interaction is the strongest of the four fundamental forces. Its strength is 100times the strength of electromagnetic force, some 10^5 times greater than weak force, and about 10^{39} times greater than gravitation. The strong interaction is also the force that binds proton and neutrons together. The strong force is mediated by gluons, acting upon quarks, anti-quarks and gluons themselves. The residue strong interaction between the quarks that make up the protons and neutrons. This is detailed in the theory of Quantum chromo-dynamics (QCD).

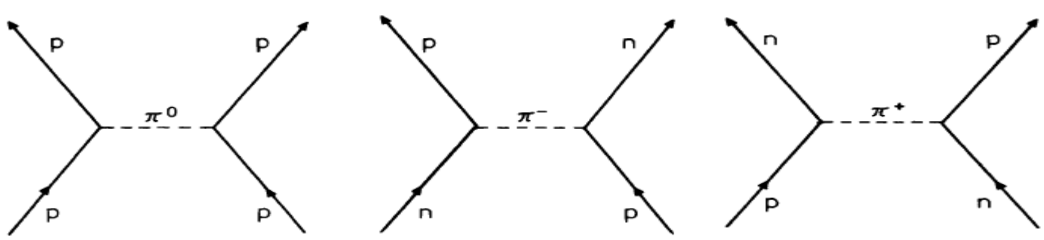


Fig 1.9 Exchanging neutral and charged pions

The strong force between nucleons is consistent with a strong force coupling constant of about 1:

$$\alpha_s \approx 1$$

Analysis of the coupling constant with quantum chromo-dynamics gives an expression for the diminishing coupling constant. The coupling decreases logarithmically, a phenomenon known as asymptotic freedom. The coupling decreases approximately as

$$\alpha_s(k^2) = \frac{g_s^2(k^2)}{4\pi} = \frac{1}{\beta_0 ln\frac{k^2}{\Lambda^2}}$$

where β_0 is a constant. In non-Abelian gauge theories, the beta function can be negative. Conversely, the coupling increases with decreasing energy. This means that the coupling becomes large at low energies.

3) Weak interaction:

It is also called weak force. It is due to exchange of heavy $W^{\pm}$ and Z^0 bosons. Its most familiar effect is emission of electrons or positrons by neutrons in atomic nuclei and the associated radioactivity. The word 'weak' derives from the fact that the typically field strength is 10^{-11} the strength of electromagnetic force and some 10^{-13} that of the strong force, when forces are compared between particles interacting in more than one way.

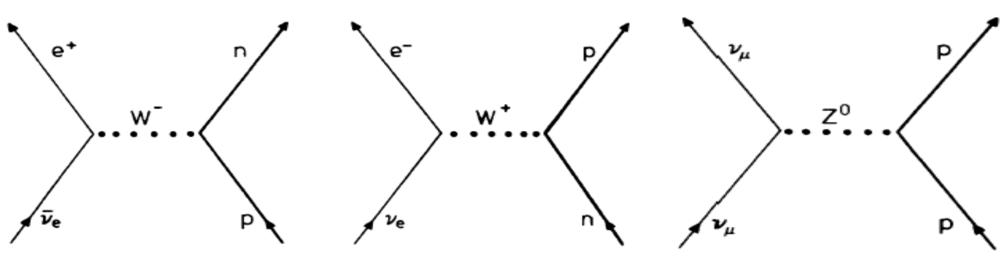


Fig 1.10 Exchanging heavy boson

The weak coupling constant is given as

$$\alpha_W \approx 10^{-6} - 10^{-7}$$

4) Gravitational interaction:

Gravitation or gravity is the natural phenomenon by which objects with mass attract with one another. Gravitational interactions occur between any two objects that have same energy. Mass is just one possible form of this energy. (Photons are mass-less, but they experience gravitational forces). The force is

$$F_{gravity} = \frac{Gm_1m_2}{r^2}$$

and the constant 'Gm_1m_2' has the units of energy times distance.

Gravitational interactions between fundamental particles are extremely weak, at least thirty orders of magnitude smaller than the weak interaction. Hence, gravitational effects can be ignored in particle physics. The strength of the gravity force relative to the electromagnetic force is given by:

$$\frac{F_{gravity}}{F_{electric}} = \frac{Gm_pm_p}{ke^2} = 8.1\text{X } 10^{-37}$$

Using the electro-magnetic coupling constant of 1/137, then leads to a gravitational coupling constant [15]

$$\alpha_g = 5.9 \times 10^{-39}$$

The four fundamental interactions summarized in table 1.2

Table 1.2

Interaction	**Strength**	**Range**	**Gauge Boson**	**Energy (mc^2)**	**Spin**	**Charge(e)**
Strong	1	10^{-15}	8 gluons	0	1	0
EM	α= 1/137	∞	Photon	0	1	0
Weak	10^{-13}	10^{-18}	$W^{\pm}$, Z^0	81, 93 GeV	1	±1, 0
Gravity	10^{-38}	∞	Graviton	0	2	0

1.7 Unification:

For decades, physicists have been striving for the unification of the four forces into one universal force as it is not quite satisfactory to have four different theories to account for these four forces. The electromagnetic interaction of particles is explained by a well established modern theory of Quantum Electrodynamics (QED). The weak interaction had its own theory but these two have now been combined as the Electroweak Theory in the Standard Model. The strong interaction between quarks and gluons has another theory called Quantum Chromo-dynamics (QCD), where the equivalent of electric charge is named "color". And Einstein's General Theory of Relativity explains how the gravity we know is a manifestation of the basic geometry of space-time.

Just as Maxwell showed that electricity and magnetism were manifestations of the same basic phenomenon of electromagnetism, the Electroweak theory, which in 1979 won the Nobel Prize for Glashow, Salam and Weinberg, succeeds in unifying the Weak and Electromagnetic interactions into what is called the Electroweak force. When we noted the intrinsic strengths of the four different interactions in the Table 1.2, we omitted to say that these strengths could depend on the "temperature" or energy level of the interaction. Although these strengths are quite different at present temperatures (e.g., at 300K or equivalent energy of about 1/40 eV), the weak interaction depends strongly on the energy and in collisions at near 1000 GeV, it gets just as strong as the electromagnetic interaction. The Electroweak theory of the Standard Model explains all this. The basic equations are symmetric in the way that the two interactions occur and in fact the masses of all the quanta are zero. However, as the temperatures drops, the symmetry is broken and the quanta split up into four different gauge bosons of different masses: the W^+ and W^- (both 80 GeV), the Z^0 (91 GeV) and the photon g with zero mass. At "room temperature", the massive W and Z do not play an important part. But at very high energies of 300 GeV or more, the difference between the zero mass photon and the heavier W and Z bosons is erased and they all act equally strongly. In 1983, W boson and in 1984, Z boson were observed at the CERN laboratory in Geneva, in high energy collisions of protons with antiprotons. They had the predicted masses. The Standard Model was on its way.

There is however one piece of evidence yet to be found. We mentioned above that the basic symmetry of the electroweak theory is broken as the temperature drops and the forces separate in strength as the bosons gain mass. All the forces of nature should be capable of being described by a single theory but only at high energies should the behavior of the force combine this is called unification. Before the unification point,the forces are indistinguishable and have symmetry. After the unification point, the forces act differently and the symmetry is broken [3].

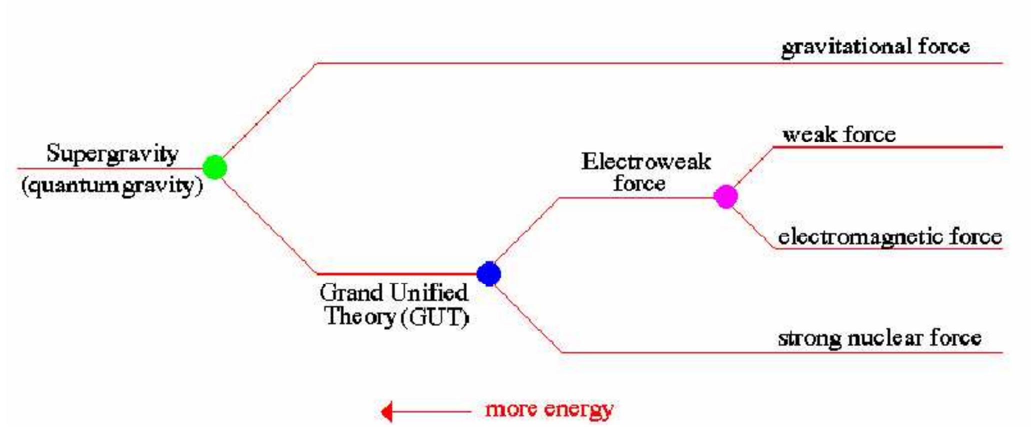


Fig 1.11 Unification of all forces

1.8 Beyond the Standard Model:

The theory of strong interaction has been added to that of the electroweak interaction to make a composite picture called standard that describes the structure of matter down to 10^{-18}m. It includes all the known constituents of matter - six leptons, six quarks and the three strongest of the four forces that govern their behavior. As its name suggests, the standard model has been a considerable success, and its founder received over twenty noble prizes over the years for their work.

But the standard model contains too many loose ends. To begin with, important elements of the model have to be inserted arbitrarily. Instead of telling us the values of 18 basic quantities, such as the masses of the leptons and quarks, the model requires us to measures them ourselves; indeed, the essential fact that there are exactly three generations of leptons and quarks comes from experiment, not theory. The strong force that binds nucleons into nuclei and is mediated by meson exchange is the external manifestations of the color force between quarks in the nucleons that is mediated by gluon exchange, but nobody has been able to actually derive the details of the strong hadrons' force from color quark force.

The next step is to weave together the electroweak and color interactions into a **grand unified theory** (GUT) that reveals the exact relationship between leptons and quarks. Among other things, a valid GUT should explain why the electron, lepton and proton (composite of quarks) have electric charges of the same magnitudes. In order to do this, proposed GUTs require the existence of a lepton-quark interaction that would eventually cause protons to decay with a half-life of 10^{30} to 10^{33} years, which means that today's matter is inherently unstable. Experiments show that the proton half-life is at least 10^{32} years, so the question of ultimate proton stability has no answer as yet [13].

Even a GUTs is incomplete because it would not include space-time and therefore gravity. It is hypothesized that a ``Theory of Everything'' (TOE) will bring together all the fundamental forces, matter and curved space-time under one unifying picture. For cosmology, this will be the single force that controlled the Universe at the time of formation. The current approach to the search for a TOE is to attempt to uncover some fundamental symmetry, perhaps symmetry of symmetries. There should be predictions from a TOE, such as the existence of the Higgs particle, the origin of mass in the Universe [8].

The mass of the Higgs particle (Higgs bosons) cannot be predicted from standard model, but it is thought to be substantial, perhaps as much as $1TeV/c^2$, a thousand times the proton mass. Finding the Higgs boson would be a major step in validating the standard model, knowing its mass and behavior would help to tie up loose ends in the model. Looking for the Higgs boson is one of the motivations for building particle accelerators more powerful than the existing one, which are inadequate for this search [11].

1.9 Accelerators and detectors:

Accelerators:

Particle accelerators are scientific instruments used to accelerate elementary particles to very high energies. They are of paramount importance for the study of elementary particle physics because the fundamental structure of matter is most clearly revealed by observing reactions of elementary particles at the highest possible energies. Historically, the development of elementary particle physics has been strongly coupled to advances in the physics and technology of particle accelerators. The first modern particle accelerators were developed in the 1930s and led to fundamental discoveries in nuclear physics. From 1930 to 1990, the energies attainable in particle accelerators have increased at an exponential rate, with an average doubling time of about two years. The consequence has been enormous progress in our understanding of the fundamental forces and constituents of matter.

Astronomy, cosmology, and space travel have expanded our frontiers outward to the limits of the universe and to its earliest moments. In the opposite direction, atomic, nuclear, and particle physics have pushed inward toward the ultimate constituents of matter. Both frontiers offer thrilling adventure and great triumphs. Both call for impressively large and expensive machines. And, surprisingly, the discoveries on the innermost scale shed light also on the grandest event of cosmology: the Big Bang, a veritable cauldron of elementary particles. To explore them deep inside, atoms are bombarded with beams of particles brought to high energy in an accelerator: the higher the projectile's energy, the deeper it can probe into an atom and its nucleus. More spectacularly, as a consequence of relativity, a collision with enough energy can also create new particles. The required energy is the equivalent of the total mass created [16].

Progress in our understanding of Nature has come through the interplay between theory and experiment. In particle physics, experiment now depends primarily on the great particle accelerators and ingeneous and complex particle detectors, which have been built, beginning in the early 1930s with the Cockroft–Walton linear accelerator at Cambridge, UK, and Lawrence's cyclotron at Berkeley, USA. The Cambridge machine accelerated protons to 0.7 MeV; the first Berkeley cyclotron accelerated protons to 1.2 MeV. For a time after 1945 important results were obtained using cosmic radiation as a source of high energy particles, events being detected in photographic emulsion, but in the 1950s new accelerators provided beams of particles of increasingly high energies. The TEVATRON at Fermi-lab is where the top quark was discovered. The physics of the top quark is as yet little explored. It makes only a brief appearance in our text, though it is an essential part of the pattern of the Standard Model. The upgraded LEP2 at CERN is able to create $W^+ W^-$ pairs, and will allow detailed studies of the weak interaction. The most ambitious machine likely to be built in the immediate future is the Large Hadron Collider (LHC) at CERN. It is expected that with this machine it will be possible to observe the Higgs boson, if such a particle exists. The Higgs boson is an essential component of the Standard Model. It is also widely believed that the physics of Super-symmetry, will become apparent at the energies upto 14 TeV, which will be available at the LHC [17].

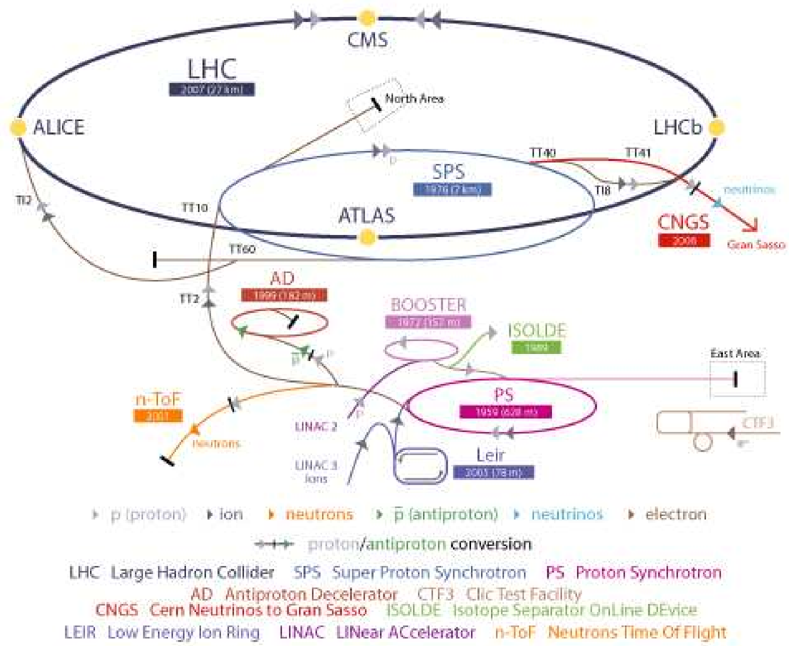


Fig 1.12 CERN Accelerator Complex

Particle detection:

As we know that the up and down quarks are the constituents of proton and neutron. Together with the electron they constitute all the visible matter around us. The heavier quarks and charged leptons all decay into the lighter ones via interactions. So they are not freely occurring in nature. But they can be produced in laboratory or cosmic ray experiments. The muon and the strange quark were discovered in cosmic ray experiments in the late forties, the latter in the form of K meson. Next to come were the neutrinos. Although practically mass-less and stable the neutrinos are hard to detect because they interact only weakly with matter. The ν_e-

was discovered in atomic reactor experiment in 1956, for which Reines got the Nobel Prize in 1995. The ν_μ was discovered in the Brookhaven proton synchrotron in 1962, for which Lederman and Steinberger got the Nobel Prize in 1988. The first cosmic ray observation of neutrino came in 1965, when the ν_μ was detected in the Kolar Gold Field experiment. The rest of the particles have all been discovered during the last 25 years, thanks to the advent of the electron-positron and the antiproton-proton colliders.

Table 1.3

Machine	Location	Beam	Energy (GeV)	Radius	Highlight
SPEAR	Stanford	e^+e^-	$3+3$		c, τ
DORIS	Hamburg	"	$5+5$		b
CESR	Cornell	"	$8+8$	125 m	"
PEP	Stanford	"	$18+18$		–
PETRA	Hamburg	"	$22+22$	300 m	g
TRISTAN	Japan	"	$30+30$		–
S$\bar{\text{P}}$PS	CERN	$\bar{p}p$	$300+300$	1 Km	W, Z
TEVATRON	Fermilab	"	$1000+1000$		t
SLC	Stanford	e^+e^-	$50+50$	–	Z
LEP-I	CERN	"	"	5 Km	Z
(LEP-II)		"	$100+100$	"	W
HERA	Hamburg	ep	$30+800$	1 Km	–
LHC (LEP Tunnel)	CERN	pp	$7,000+7,000$	5 Km	Higgs? SUSY?
NLC	?	e^+e^-	$500+500$?	–	"

First came, the windfall of the seventies with a quick succession of discoveries mainly at the e^+e^- colliders: charm quark (1974), Tau lepton (1975), bottom quark (1977) and the gluon (1979). This was followed by the discovery of W and Z bosons (1983) and finally the top quark

(1995) at the $\bar{p}$p colliders. In view of the crucial role played by the colliders in these discoveries a brief discussion on them is in order. First came, the Stanford e^+-e^- collider (SPEAR) with energy similar to that of the fixed-target proton syncrotron at Brookhaven. In fact, the charm quark was simultaneously discovered at both these machines in 1974, for which Richter and Ting got the 1975 Nobel Prize. The tau lepton was also discovered at SPEAR the following year, for which Pearl got the Nobel Prize in 1995. The bottom quark was discovered in the fixed-target proton synchrotron at Fermi-lab in 1977; and soon followed by the study of its detailed properties at the e^+-e^- colliders at Hamburg (DORIS) and Cornel (CESR). Then the construction of a more energetic e^+-e^- collider (PETRA) at Hamburg resulted in the discovery of gluon in 1979 [18] as shown in table 1.3.

1.10 Symmetries and Symmetry Breaking:

The special unitary groups SU(n) are encountered repeatedly in particle physics theories. It is the group of n X n unitary matrices with unit determinant:

$$U^{\dagger}U = UU^{\dagger} = 1 \text{ and det. } U = 1.$$

It is SU(2) in iso-spin invariance; SU(3) in the 'eight-fold way'; the standard gauge model of strong and electroweak interactions uses SU(3) X SU(2) X U(1); the simplest grand unification group is SU(5). In this, we concentrate on groups SU(2) and SU(3) [19].

SU(2) symmetry:

Iso-spin refers to the SU(2) flavor sub-group for the two lightest quark flavors u and d whereas hypercharge is proportional to the net number of strange anti-quarks minus strange quarks or strangeness of a hadron. Iso-spin and hypercharge are both approximately conserved [16].

There are three group parameters. We write the 2 X 2 unitary uni-modular matrices as

$$U(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \exp\{i\varepsilon_a \sigma_a\}$$

where the σ_as are 2 X 2 trace-less hermitian matrices. We choose the basis to be the standard Pauli matrices.

The generators defined by

$$J_i = \frac{1}{2}\sigma_i \qquad \text{with } i=1,2,3,\ldots\ldots$$

will give the commutation relation

$[J_a, J_b] = i\varepsilon_{abc}J_c$ where ε_{abc} is the totally anti-symmetric symbol and ε_{123}=1 [19]

SU(3) Symmetry:

The symmetry group SU(3) is figured prominently in elementary particle physics. There are two important and distinct SU(3) symmetries that are relevant for the strong interactions: SU(3) color symmetry of the quark and gluon dynamics and SU(3) flavor symmetry of light quarks. Each of these symmetries refers to an underlying threefold symmetry in strong interaction physics. Mathematically, SU(3) is the group of special unitary 3 X 3 matrices.

It is conventional to define the generators of SU(3) in terms of the eight Gell-Mann matrices λ^a

$$T^a \equiv \frac{1}{2}\lambda^a$$

The generators T^a of SU(3) satisfy the commutation relation

$$[T^a, T^b] \equiv T^a T^b - T^b T^a = i \sum_{c=1}^{8} f^{abc}\, T^c$$

Where f^{abc}, the structure constant of SU(3) are real numbers.

All the higher dimensional representation of SU(3) can be obtained as products of the fundamental 3 and anti-fundamental $\bar{3}$ representations. The product of 3 and $\bar{3}$ representation yield the eight dimensional representations. The 8 of SU(3) is called adjoint representation of

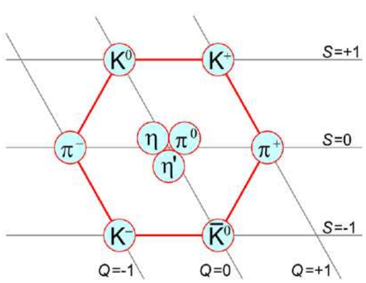
SU(3). The SU(3) generators T^a, a=1,2……8 form an eight dimensional adjoint representation of SU(3). In general, every SU(3) representation exhibits three-fold symmetry in the (T^3, T^8) plane.

The SU(3) color group is the exact symmetry of the standard model, which accounts for the strong interaction of quarks and gluons. The theory of strong interaction is called quantum chromo-dynamics (QCD). Quarks occur in the fundamental three-dimensional representation of SU(3) color. In QCD color charge is conserved in the interactions of quarks, anti-quarks and gluons. A quark and an anti-quark couple to a colored gluon, so a gluon in the eight dimensional adjoint representation carries both color in the fundamental representation and anti-color in the $\overline{3}$ representation.

The SU(3) flavor group is an approximate symmetry of QCD resulting from the universality of quark-gluon couplings. All quark flavors with a given color couple to gluons in precisely the same that is gluons are flavor-blind. The light quarks up, down and strange occur in the fundamental three-dimensional representation of SU(3) flavor. SU(3) flavor symmetry is not an exact symmetry because the masses of the u, d and s quarks are not same and so the quark flavors are distinguishable. Nevertheless, the mass difference of the u, d and s quarks are all small compared to the sale at which the QCD coupling constant becomes large, so neglecting the mass splitting of the three light quarks is a good approximation.

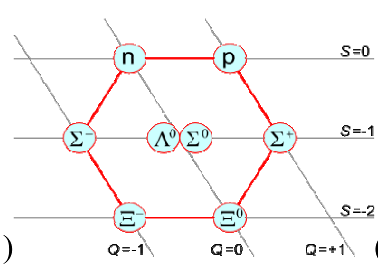
SU(3) flavor symmetry is a useful approximate symmetry in QCD because hadrons containing light quarks and anti-quarks of different flavors have similar properties. Colorless hadrons either mesons and baryons can be organized into SU(3) flavor multiplet [16]. Nine states are formed and these form an SU(3) octet and singlet. Symbolically this is written as

$$3 \otimes \overline{3} = 8 \oplus 1$$

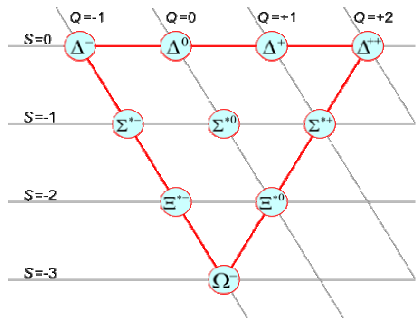


(i)

)



(ii)



(iii)

Fig 1.13 (i) Mesons of spin 0 (pseudo-scalar) form a nonet and (ii) Baryons of spin 1/2 form a nonet (iii) Baryons of spin 3/2 form a decuplet.

Breaking of symmetry:

Symmetry breaking in physics describes a phenomenon where (infinitesimally) small fluctuations acting on a system crossing a critical point decide a system's fate, by determining which branch of a bifurcation is taken. For an outside observer unaware of the fluctuations (the "noise"), the choice will appear arbitrary. This process is called symmetry "breaking" [20]. It is the phenomenon in a medium that the original symmetry group of the medium must be lowered (broken) to the symmetry group of the phenomenon by the action of some cause. In this sense, symmetry breaking is what "creates the phenomenon". Generally, the breaking of certain symmetry does not imply that no symmetry is present, but rather that the situation where this symmetry is broken is characterized by a lower symmetry than the situation where this symmetry is not broken.

There are two different types of symmetry breaking of the laws: "**explicit**" and "**spontaneous**", the case of spontaneous symmetry breaking being the more interesting from a physical as well as a philosophical point of view.

1) Explicit symmetry breaking:

Explicit symmetry breaking indicates a situation where the dynamical equations are not manifestly invariant under the symmetry group considered. This means, in the Lagrangian

(Hamiltonian) formulation, that the Lagrangian (Hamiltonian) of the system contains one or more terms explicitly breaking the symmetry.

2) Spontaneous symmetry breaking:

Spontaneous symmetry breaking (SSB) occurs in a situation where the given symmetry of equations of motion, solutions exist which are not invariant under the action of this symmetry without any explicit asymmetric input [21].

1.11 Quantum Chromo-dynamics:

Quantum Chromo-dynamics (QCD) was proposed in the 1970s as a theory of the strong interactions [22]. It is the gauge field theory that describes the interactions of colored fermionic fields (quarks) and non-Abelian colored gauge fields (gluons). It is the theory of strong interactions and one of the components of the SU(3) X SU(2) X U(1) Standard Model. The (local) gauge transformations form the group SU(3)(color group) with the quarks and gluons belonging to the fundamental (triplet) and adjoint (octet) representations of the color group. The gluons mediate the interaction between the quarks, and in addition, due to the non-Abelian character of QCD, the gluons also interact with each other. The strength of interaction between colored objects is scale dependent. The renormalized QCD coupling constant is small at high energy i.e. asymptotic freedom [23]. Asymptotic freedom is a characteristic property of quantum chromo-dynamics (QCD), the component of the Standard Model that describes the strong interactions [16]. In this domain the fundamental fields of QCD, quarks and gluons, can be probed experimentally in a direct way. The strong coupling α_s is a running coupling constant with the property that:

$$\alpha_s(q^2) \rightarrow 0, \text{ as } q^2 \rightarrow \infty$$

The explicit dependence of α_s on the momentum transfer q^2 can be worked and upto $O(\alpha_s^2)$ is given as:

$$\alpha_s (q^2) = \frac{2\Pi}{b_0 \ln\left(\frac{q^2}{\Lambda^2}\right)+(\frac{b_1}{b_0}) \ln\left(\frac{q^2}{\Lambda^2}\right)}$$

Asymptotic freedom of the theory allows us to calculate those processes for which q^2 is sufficiently large, so that perturbation theory is applicable [24] and high-precision theoretical predictions using perturbation theory can be confirmed.

On the other hand, the attraction between two colored objects grows with increasing distance, that leads to the confinement of quarks and gluons, so that only colorless particles (SU(3) singlet) manifest themselves as asymptotic states. At low energy, the color degrees of freedom are not directly observable. In this case, the hadrons dynamics is equivalent to an effective field theory of colorless hadrons [23].

Chapter-II

Heavy Quark Effective Theory

2.1 Effective Field Theory:

An important idea that is implicit in all descriptions of physical phenomena is that of an effective theory. The basic premise of effective theories is that dynamics at low energies (or large distances) does not depend on the details of the dynamics at high energies (or short distances). As a result, low energy physics can be described using an effective Lagrangian that contains only a few degrees of freedom, ignoring additional degrees of freedom present at higher energies.

It describes low energy physics in terms of a few parameters. These low energy parameters can be computed in terms of parameters of high energy theory. This computation can be done explicitly when the high energy theory is weakly coupled. If the high energy theory is strongly coupled, in QCD, one usually treats the low energy parameters as free parameters that are fit to experiment. There are some vestiges of the high energy interactions in the symmetry constraints on the low energy theory, and in small corrections to low energy dynamics. Thus high precision low energy experiments can be used to probe high energy dynamics, and provide an alternative to high energy experiments [25].

2.2 Heavy Quark Symmetry and Chiral Symmetry:

Heavy quark symmetry:

A considerable amount of progress has been made toward the understanding of low energy hadronic physics. For hadrons containing a heavy quark Q, the QCD dynamics of the heavy quark possesses an additional symmetry in the limit $M_Q \to \infty$**.** As $M_Q >> \Lambda_{QCD}$(which is 1GeV in our case), the *QCD* dynamics of the heavy quark is independent of the heavy-quark mass and spin. Consequently, a new flavor and spin symmetry will emerge in the sector of hadrons containing one heavy quark. This is known as the heavy quark symmetry [26].

The strong interactions of systems containing heavy quarks are easier to understand than those of systems containing only light quarks. The reason for this is asymptotic freedom. Here the effective coupling constant of QCD becomes weak in processes with large momentum transfer, corresponding to interactions at short-distance scales [27]. The effective coupling constant is given by

$$\alpha_s(Q^2) = g^2_{eff}(Q^2)/4\pi = \frac{12\pi}{(33-2n_f)\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}$$

decreases at large Q^2, i.e., the strong interaction become weak at short distances. At large distances (small Q^2), on the other hand, the coupling constant becomes strong, leading to non-perturbative phenomena such as the confinement of quarks and gluons on a length scale $R_{had} \sim 1/\Lambda_{QCD} \sim 1$ fm, which determines the typical size of hadrons. Roughly speaking, $\Lambda_{QCD} \sim 0.2$ GeV is the energy scale that separates the regions of large and small coupling constant. When the mass of a quark Q is much larger than this scale, $m_Q >> \Lambda_{QCD}$, Q is called a heavy quark [28]. The quarks of the standard model fall naturally into two classes: u, d and s are light quarks, whereas c, b and t are heavy quarks. For heavy quarks, the effective coupling constant $\alpha_s(m_Q)$ is small, implying that on length scales comparable to Compton wavelength $\lambda_Q \sim 1/m_Q$ the strong interactions are perturbative and much like the electromagnetic interactions. This is the reason why charmonium type systems are much easy to handle than other mesons [29].

For systems composed of a heavy quark and other light constituents, things are more complicated. The size of such systems is determined by R_{had}, and the typical momenta exchanged between the heavy and light constituents are of order Λ_{QCD}. The heavy quark is surrounded by a most complicated strongly interacting cloud by light quarks, anti-quarks and gluons. This cloud is sometimes referred as the "brown muck".

In this case, it is the fact that $m_Q >> \Lambda_{QCD}$, $\lambda_Q << R_{had}$, that the Compton wavelength of the heavy quark is much smaller than the size of the hadron, which leads to simplifications. To resolve the quantum numbers of the heavy quark would require a hard probe with $Q^2 \geq (m_Q)^2$. The soft gluons which couple to the "brown muck" can only resolve distances much larger than λ_Q. Therefore, the light degrees of freedom are blind to the flavor (mass) and spin orientation of the heavy quark. They only experience its color field, which extends over large distances because of confinement [30].

Hence, even if we replace the heavy quark with other heavier flavor, the collection of light quark and gluons inside the mesons won't be able to find the difference. They will just see a triplet color source sitting at the centre in both cases. This gives a **SU(N_f) symmetry** to theory, where N_f corresponds to number of heavy quarks in the theory. Since spins interact only through relativistic effects, the chromo-magnetic effect tends to zero as $m_Q \rightarrow \infty$. Thus, heavy quark spin decouples from remaining part meson leading to additional **SU(2) spin symmetry**.

The combination of above two symmetries leads to SU($2N_f$) spin-flavor symmetry. Consequence of these two symmetries is that the configuration of light quark and soft gluons (brown muck) remain same in hadronic systems which differ only in the quantum numbers of heavy quark as $m_Q \rightarrow \infty$.

The heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinite. These corrections are of order Λ_{QCD}/m_Q .The condition $m_Q >> \Lambda_{QCD}$ is necessary and sufficient for a system containing a heavy quark to be closed to the symmetry limit.

Chiral symmetry:

Chiral symmetry is an internal symmetry of right and left handed spinors. It has importance in low energy hadronic physics, since its spontaneous breaking generates Goldstone bosons with negative parity, zero spin, unit iso-spin and zero baryon number called pions. Thus a broken approximate chiral symmetry entails the existence of pions where u and d quarks have small but non-zero masses whereby spontaneous breaking of a symmetry is expressed as the non-vanishing of the vacuum when operated by the charge Q. A spontaneously broken symmetry relates processes with different numbers of Goldstone bosons.

As the masses of light quarks are small compared to Λ_{QCD}, in the chiral limit, their masses will go to zero and making the masses of heavy quarks, m_c, m_b and m_t to be infinity.

$$m_q \to 0, q = u, d, s$$

In this limit QCD Lagrangian L_{QCD} becomes invariant under the following group of (space-time independent) transformations which act on the three flavor indices (u, d, s):

$$\begin{aligned} q = q_L + q_R \to q' = g_R q_R + g_L q_L \\ q_R = \frac{1}{2}(1+\gamma_5)q, \end{aligned} \tag{2.1}$$

$$\begin{aligned} q_L = \frac{1}{2}(1-\gamma_5)q \\ g_I g_I^{\dagger} = 1, \det g_I = 1; I = L, R \end{aligned} \tag{2.2}$$

The above group of transformations (2.1) and (2.2) is $SU(3)_R \times SU(3)_L$ and the resulting symmetry of the QCD Lagrangian is called chiral symmetry of QCD [31].

2.3 Chiral Perturbation Theory:

Chiral perturbation theory provides a systematic method for discussing the consequences of the global flavor symmetries of QCD at low energies by means of an effective field theory. The effective Lagrangian is expressed in terms of those hadronic degrees of freedom which, at low energies, show up as observable asymptotic states [32]. At low energies, it implies that the $SU(3)_L \times SU(3)_R \times U(1)_V$ of the Lagrangian is spontaneously broken down to $SU(3)_V \times U(1)_V$ at a scale $\Lambda_{QCD} \sim 1$ GeV, so there are generation of eight Goldstone bosons(π's, K's and η). The interactions of the Goldstone bosons at energies much smaller than Λ_{QCD} can be calculated using an effective Lagrangian. The light quarks u, d, and s can be treated as approximately mass-less and having masses very small as compared to Λ_{QCD}, whereas the heavy quarks c, b and t are those whose masses are large as compared to this energy scale. The heavy quarks are integrated out of the low energy effective theory [33]. The non-vanishing masses of the light pseudo-scalars in the "real" world are related to the explicit symmetry breaking in QCD due to the light quark masses [32].

It is based on a non-proved theorem that states that apart from causality and unitarity, the contents of a quantum field theory is dictated by the symmetries it possesses. The idea is thus to replace the quarks and gluons of QCD by the pseudo-scalar mesons and write down the most general Lagrangian involving these particles that the same symmetries as the QCD Lagrangian. The ChPT generating functional admits an expansion in powers of external momenta and quark masses. Although ChPT is not a renormalizable theory the results can be rendered finite order by order in the expansion. The prize to pay is that new terms (with unknown constants) have to be included in the Lagrangian at each order in the expansion [34].

2.4 Renormalization:

Renormalization is a technique required for extracting meaningful predictions from a quantum field theory. The description of particles and their interactions involves various physical parameters, such as the masses and charges of particles. These parameters are not known a priori and must be measured. Quantum effects can shift the values of these parameters and the measured physical value will include the quantum modifications. These notions form the basis of the renormalization program. When considered more carefully, these concepts can be used to show that the predictions of many theories are independent of any new physics that might appear at very high energies, such as the issue of whether or not the divergences found in perturbation theory exist. Renormalization theory also leads to the idea of a "running" charge, that is, one that depends on the energy scale at which it is measured.

Renormalization refers to the process by which the sum of all contributions to a physical parameter is adjusted so that the total is equal to the experimentally measured value of this parameter and all predictions are expressed in terms of the experimental value [16].

2.5 An Effective theory for Heavy Mesons:

In the particle physics, it is often the case that the effects of a very heavy particle become irrelevant at low energies. It is then useful to construct a low energy effective theory in which this heavy particle no longer appears. Eventually, this effective theory will be easier to deal with, than the full theory. When we proceed to higher energies its limitations become apparent and a new effective theory can provide an adequate level of description for higher energies.

The symmetries of heavy and light quarks are exploited to formulate heavy-quark effective theory (HQET) theory that describes the low energy interaction among heavy mesons. This theory provides a simple description of the processes where a heavy quark interacts with a light degrees of freedom by the exchange of soft gluons.

Clearly, Λ_{QCD} is the scale of the hadronic physics of our interest and m_Q is the mass of the heavy quark which is much larger than this energy scale. However, a subtlety arises since one wants to describe the properties and decays of hadrons which contain a heavy quark. Hence it is not possible to remove the heavy quark completely from the effective theory. What is possible, however, is to integrate out the "small components" in the full heavy-quark spinors, which describes fluctuations around the mass shell [30].

Effective Lagrangian:

Since the heavy quark constitute most of the mass of heavy meson and the velocity of heavy quark is going to be that of meson itself. Hence, we can write

$$p_Q^{\mu} = m_Q v^{\mu} + k^{\mu} \qquad (2.3)$$

where v^{μ} is four velocity and k^{μ} is residual momentum of heavy quark which is of order of Λ_{QCD}. We note that changes in heavy quark velocity vanish as $\Lambda_{QCD}/ m_Q \rightarrow 0$. Now we will introduce a large and small component field h_v and H_v as below,

$$h_v(x) = e^{(im_Q v.x)} P_+ Q(x), \qquad H_v(x) = e^{(im_Q v.x)} P_- Q(x) \qquad (2.4)$$

where $P_{\pm} = \dfrac{(1 \pm v)}{2}$ are the projection operators. Here we have used the relation $v.v = 1$. The exponential pre-factors in the field definitions is a rescaling precisely defined to get m_Q out of leading term in Lagrangian. From above equations we get,

$$Q(x) = e^{(-im_Q v.x)}[h_v(x) + H_v(x)] \qquad (2.5)$$

From the definitions of projection operators, we can see that the large and small components fields h_v and H_v satisfy the constraints

$$\begin{aligned} v h_v &= e^{(im_Q v.x)} v \frac{(1+v)}{2} Q \\ &= e^{(im_Q v.x)} \frac{(1+v)}{2} Q \\ &= h_v \end{aligned}$$

and similarly, for $v H_v = - H_v$

Now, we see these fields to rewrite the QCD Lagrangian as follows

$$L_{QCD} = L_q + L_Q \qquad (2.6)$$

where q corresponds to light and Q corresponds to heavy quarks.

$$\begin{aligned} L_Q &= \overline{Q}(i\not{D} - m_Q)Q \\ &= e^{im_Q v.x}[\overline{h_v} + \overline{H_v}](i\not{D} - m_Q)e^{-im_Q v.x}[h_v + H_v] \\ &= [\overline{h_v} + \overline{H_v}](i\not{D} - m_Q)[h_v + H_v] + [\overline{h_v} + \overline{H_v}](m_Q)[h_v + H_v] \\ &= \overline{h_v} i\not{D} h_v + \overline{H_v}(i\not{D} - 2m_Q)H_v + \overline{h_v}(i\not{D} - 2m_Q)H_v + \overline{H_v} i\not{D} h_v \end{aligned}$$

using the standard γ matrix relations we obtain the relation,

$$\frac{1+\not{v}}{2}\gamma_\mu \frac{1+\not{v}}{2} = v^\mu \frac{1+\not{v}}{2} \qquad (2.7)$$

With this relation and using the fact that v $h_v = h_v$ and v $H_v = - H_v$ we get,

$$L_Q = \overline{h_v}(iv.D)h_v - \overline{H_v}(iv.D + 2m_Q)H_v + \overline{h_v}(i\not{D}_\perp)H_v + \overline{H_v}(i\not{D}_\perp h_v)$$

where $D_\perp^\mu = D^\mu + v^\mu(v.D)$ and is orthogonal to heavy quark velocity i.e. $v.D_\perp = 0$. From the above equation we can see that the h_v field corresponds to mass-less degree of freedom while H_v represents fluctuations with mass $2m_Q$ which corresponds to pair creation. These are the heavy degrees of freedom that will be eliminated in the process of constructing effective theory. Using the above Lagrangian, we get the equation of motion as

$$\frac{\partial L}{\partial H} = (iv.D + 2m_Q)H_v = i\not{D}_\perp h_v = 0 \quad (2.8)$$

By solving this equation we get,

$$H_v = \frac{1}{2m_Q + iv.D} i\not{D}_\perp h_v = 0 \quad (2.9)$$

which tells that the small component field is indeed goes as$1/m_Q$. Substituting this into the Lagrangian we obtain

$$L_{eff} = \overline{h_v}(iv.D)h_v + \overline{h_v} i\not{D}_\perp \frac{1}{2m_Q + iv.D} i\not{D}_\perp h_v \quad (2.10)$$

Let us rewrite this non-local Lagrangian in powers of $1/m_Q$ as below.

$$L_{eff} = \overline{h_v}(iv.D)h_v + \frac{1}{2m_Q}\overline{h_v} i\not{D}_\perp \frac{1}{1 + \frac{iv.D}{2m_Q}} i\not{D}_\perp h_v$$

$$= \overline{h_v}(iv.D)h_v + \frac{1}{2m_Q}\sum_{n=0}^{\infty}\overline{h_v} i\not{D}_\perp \left(\frac{iv.D}{2m_Q}\right)^n i\not{D}_\perp h_v$$

Using the identity

$$\frac{1+\not{v}}{2}\gamma_\mu \frac{1-\not{v}}{2}\gamma_\nu \frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}(g_{\mu\nu} - v_\mu v_\nu - i\sigma_{\mu\nu})\frac{1+\not{v}}{2}, \quad (2.11)$$

Where $[iD^\mu, iD^\nu] = ig_\alpha G^{\mu\nu}$ we write the Lagrangian as

$$L_{eff}=\overline{h_v}v.Dh_v+\frac{1}{2m_Q}\overline{h_v}(iD_{\perp})^2h_v+\frac{g_\alpha}{4m_Q}h_v\sigma_{\mu\nu}G^{\mu\nu}h_v+O(1/(m_Q^2) \quad (2.12)$$

In the limit $m_Q\rightarrow\infty$ the Lagrangian becomes

$$L_\infty=\overline{h_v}iv.Dh_v \quad (2.13)$$

It is obvious from above Lagrangian that it does has spin-flavor symmetry as $m_Q\rightarrow\infty$ (because it does not have both Dirac matrices and mass term). But if consider the Lagrangian given by equation (2.12), which has two additional terms, breaks this symmetry. The first term in the Lagrangian arising from the off-shell motion of the heavy quark, breaks the flavor symmetry while the second term arising from chromo-magnetic interaction of heavy quark spin with the gluon field, breaks the spin symmetry. If we include hard gluon exchange, then the coefficient in the Lagrangian changes and it looks like [29]:

$$L=\overline{h_v}(iv.D)h_v+\frac{a_1}{2m_Q}\overline{h_v}(iD)^2h_v+\frac{g_\alpha a_2}{4m_Q}4m_Qh_vG^{\alpha\beta}h_v.$$

Chapter-III

Charm Meson in HHχPT

3.1 Charm mesons:

Heavy hadrons contain a heavy quark and light quarks and/or anti-quarks and gluons. All the degrees of freedom other than the heavy quark are referred to as the light degrees of freedom *l*. For example, a heavy $Q\bar{q}$ meson has an anti-quark $\bar{q}$, gluons and an arbitrary number of $q\bar{q}$ pairs as the light degrees of freedom. Although the light degrees of freedom are some complicated mixture of the anti-quark $\bar{q}$, gluons and $q\bar{q}$ pairs, they must have the quantum numbers of a single anti-quark $\bar{q}$. The total angular momentum of the hadron **J** is a conserved operator. We have also seen that the spin of the heavy quark $\mathbf{S_Q}$ is conserved in the $m_Q \rightarrow \infty$ limit. Therefore, the spin of the light degrees of freedom $\mathbf{S}_l$ is defined by

$$\mathbf{S}_l \equiv \mathbf{J} - \mathbf{S_Q}$$

is also conserved in the heavy quark limit. The light degrees of freedom in a hadron are quite complicated and include super-positions of states with different particle numbers. Nevertheless, the total spin of the light degrees of freedom is a good quantum number in heavy hadrons. We will define the quantum numbers j, s_Q and s_l as the Eigen values $\mathbf{J}^2 = j(j+1)$, $\mathbf{S_Q}^2 = s_Q(s_Q+1)$, $\mathbf{S}_l^2 = s_l(s_l+1)$. Heavy hadrons come in doublets (unless $s_l = 0$) containing states with total spin $j_\pm = s_l \pm ½$ obtained by combining the spin of the light degrees of freedom with the spin of heavy quark $s_Q = ½$. These doublets are degenerate in the $m_Q \rightarrow \infty$ limit. If $s_l = 0$, there is only a single j = ½ hadron.

Mesons containing a heavy quark Q are made up of a heavy quark and light anti-quark $\bar{q}$ (plus gluons and $q\bar{q}$pairs). The ground states mesons are composed of a heavy quark with $s_Q = \frac{1}{2}$ and light degrees of freedom with $s_l = \frac{1}{2}$, forming a multiplet of hadrons with spin $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$ and the negative parity, since quarks and anti-quarks have opposite intrinsic parity. These states are the D and D^* mesons if Q is a charm quark. The field operators which annihilate these heavy quark mesons with velocity *v* are denoted by $P_v^{(Q)}$ and $P^*_{v\mu}{}^{(Q)}$ respectively. The light anti-quark can be either a $\bar{u}$, $\bar{d}$ or $\bar{s}$ quark, so each of these heavy meson fields form a $\bar{3}$ representation of the light quark flavor group $SU(3)_V$. The SU(3) weight diagram for the $\bar{3}$ representation is shown as in fig 3.1 [35]:

D_s^+, D_s^{*+}

$c\bar{s}$

D^0, D^{*0} $\qquad\qquad$ D^+, D^{*+}

$c\bar{u}$ $\qquad\qquad$ $c\bar{d}$

Fig 3.1 Flavor SU(3) weight diagram for the spin-0 pseudo-scalar and spin-1 vector $c\bar{q}$ mesons. The vertical direction is hypercharge and horizontal direction is I_3, the third component of iso-spin.

Due to the heavy quark spin symmetry, hadrons may be classified according to the light fields angular momentum and parity j^P which are conserved quantum numbers. In other words, we can switch off the heavy quark spin using the super-flavor symmetry, and then the hadrons' momentum and parity will be j^P [36]. Mesons containing a quark or anti-quark i.e. $Q\bar{q}$ ($\bar{q} = \bar{u}, \bar{d}, \bar{s}$). The ground-state (S-wave) meson has $j^P = \frac{1}{2}^+$; the excited P-wave mesons have $j^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$. When we switch the heavy quark spin on, each of these mesons becomes a degenerate doublet. Its components are transformed into each other by the heavy quark spin symmetry operations. We have the ground-state doublet 0^-, 1^-, and the excited P-wave doublets 0^+, 1^+ and 1^+, 2^+.

For Q = c, when the $s_l = \frac{1}{2}^+$ mesons form multiplets of spin 0^+ and 1^+ states named D_0 and D_1. When the $s_l = \frac{1}{2}^-$ mesons form multiplets of spin 0^- and 1^- states named D and D^* [35]. The bound states of a c quark with a light anti-quark, the $J^P = 0^-$ D mesons and $J^P = 1^-$ D* mesons as shown in table 3.1 and fig 3.2 [37]:

D^0 and $D^{*0} = c\bar{u}$,

D^+ and $D^{*+} = c\bar{d}$, $\quad$ D_S and $D^*_S = c\bar{s}$

Table 3.1

S_Q=1/2	S_l=1/2- Ground state		S_l =1/2+ LL excited state	
J^P	0^-	1^-	0^+	1^+
General states	D	D^*	D_0	$D_1^{'}$
$c\bar{u}$	$\mathbf{D^0} = 1864.84$ ± 0.17MeV (m_{H_1})	$\mathbf{D^{*0}} = 2006.97$ ± 0.19MeV $(m^*_{H_1})$	$\mathbf{D}_0^0 = ?$ (m_{S_1})	$\mathbf{D}_1^{0'} = 2422.3$ ± 1.3MeV $(m^*_{S_1})$
$c\bar{d}$	$\mathbf{D}^+ = 1869$ ± 0.20MeV (m_{H_2})	$\mathbf{D}^{*+} = 2010.27$ ± 0.17MeV $(m^*_{H_2})$	$\mathbf{D}_0^+ = ?$ (m_{S_2})	$\mathbf{D_1}^{+'} = ?$ $(m^*_{S_2})$
$c\bar{s}$	$\mathbf{D}_S^+ = 1968.49$ ± 0.34MeV (m_{H_3})	$\mathbf{D}_S^{*+} = 2112.3$ ± 0.5 MeV $(m^*_{H_3})$	$\mathbf{D}_{0S}^+ = 2317.8$ ± 0.6MeV (m_{S_3})	$\mathbf{D}_{1S}^+ = 2535.35 \pm 0.34$ ± 0.5MeV $(m^*_{S_3})$
	Pseudo scalars	Vector	scalars	Axial vector

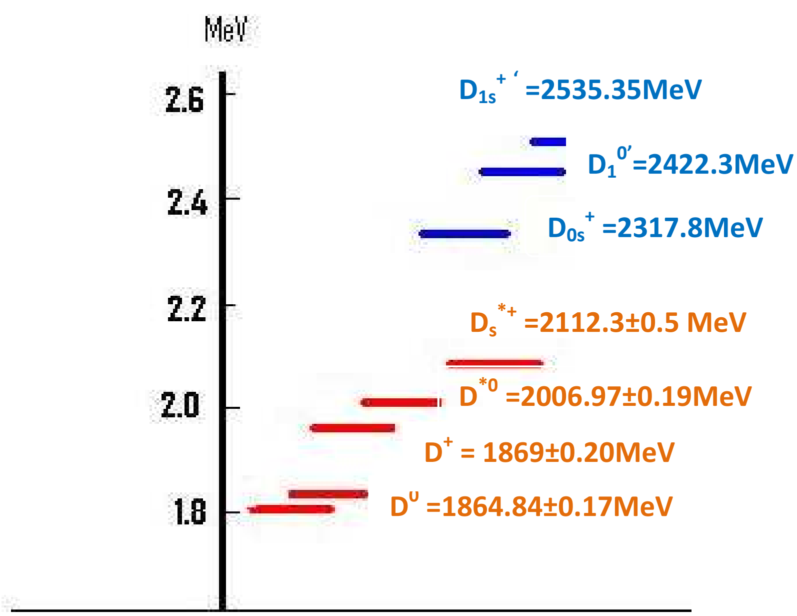


Fig 3.2 Spectroscopy of charm mesons

3.2 The effective chiral Lagrangian for heavy mesons:

The quark contribution to the QCD Lagrangian

$$L_{quarks} = \bar{q}(i\not{D} - m_q)q + \bar{Q}(i\not{D} - m_Q)Q$$

separates naturally into two pieces: the first contribution comes from light quarks (q = u, d, s) whereas the second one is due to heavy quarks (Q = c, b, t). Each of the two exhibits a distinct symmetry. The light-quark sector has an approximate $SU(3)_L$ X $SU(3)_R$ flavor chiral symmetry because the current masses are all very small on the typical hadron energy scale. The symmetry is spontaneously broken to the usual vector SU(3) and the chiral symmetry is reflected in the presence of eight Goldstone bosons: the pions, kaons and η [38].

The lowest lying mesons which contain a single heavy quark Q are pseudo-scalar and vector mesons with spin of the light degrees of freedom s_l = ½ . Degenerate spin zero and spin one mesons result when the spin of the light degrees of freedom is combined with the spin of the heavy quark s_Q = ½. Since the pseudo-scalar and vector mesons are degenerate in the heavy quark mass limit and the spin of the heavy quark is conserved by low energy strong interactions [39].

In HHχPT, the ground state doublet ($J^P = 0^-, 1^-$) consisting of pseudo-scalar and vector meson is represented by the field

$$H_a = \frac{1+\not{v}}{2}(H^{\mu}{}_a \gamma_\mu - H_a \gamma_5)$$

whereas the lowest lying excited state($J^P = 0^+, 1^+$) consisting of scalar and axial vector meson is represented by the field

$$S_a = \frac{1+\not{v}}{2}(S_a^\mu \gamma_\mu \gamma_5 - S_a)$$

The kinetic term of these above two fields are included in:

$$L_v^{kinetic} = -Tr\left[\overline{H_a}(iv.D_{ba} - \delta_H \delta_{ab})H_b\right] - Tr\left[\overline{S_a}(iv.D_{ba} - \delta_S \delta_{ab})S_b\right]$$

δ_H and δ_S are the residual masses of the H and S fields, respectively and D_{ba} is chirally covariant derivative. In the theory with only H fields one is free to set δ_H =0. Once S fields are added to the theory, there is another dimension-full quantity, δ_H - δ_S , which does not vanish as $m_q \to 0$ and $m_Q \to \infty$.

The fields have axial couplings to the pseudo-Goldstone bosons

$$L_v^{axial} = gTr\left[\overline{H_a}H_b \not{A}_{ba}\gamma_5\right] + g^{'}Tr\left[\overline{S_a}S_b \not{A}_{ba}\gamma_5\right] + hTr\left[\overline{H_a}S_b \not{A}_{ba}\gamma_5 + h.c\right]$$

where g, g' and h are dimensionless constants to be determined from the experiments. The mass term in the Lagrangian in terms of symmetry breaking and symmetry conserving parameters is written as

$$\begin{aligned} L_v^{mass} = & -\frac{\Delta_H}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu} H_a \sigma_{\mu\nu}\right] + \frac{\Delta_S}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu} S_a \sigma_{\mu\nu}\right] + a_H Tr\left[\overline{H_a} H_b\right] m_{ba}^{\xi} - \\ & a_S Tr\left[\overline{S_a} S_b\right] m_{ba}^{\xi} - \sigma_S Tr\left[\overline{S_a} S_a\right] m_{bb}^{\xi} + \sigma_H Tr\left[\overline{H_a} H_a\right] m_{bb}^{\xi} - \frac{\Delta_H^{(a)}}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu} H_b \sigma_{\mu\nu}\right] m_{ba}^{\xi} \\ & + \frac{\Delta_S^{(a)}}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu} S_b \sigma_{\mu\nu}\right] m_{ba}^{\xi} - \frac{\Delta_H^{(\sigma)}}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu} H_a \sigma_{\mu\nu}\right] m_{bb}^{\xi} + \frac{\Delta_S^{(\sigma)}}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu} S_a \sigma_{\mu\nu}\right] m_{bb}^{\xi} \end{aligned}$$

where $m_{ba}^{\xi} = \frac{1}{2}(\xi m_q \xi + \xi^+ m_q \xi^+)_{ba}$

Δ_H and Δ_S are the spin-flavor symmetry violating terms and first two terms are responsible for the hyperfine mass splitting. Where *a* and *σ* are functions of $1/m_Q$ which starts at O(1). The next four terms contains which are of $O(m_q)$ and preserve heavy-quark spin-symmetry. The term proportional to *a* results in $SU(3)_V$-violating mass splitting amongst the H_a^{μ} mesons. The term proportional to *σ* leads to a single contribution to the masses which depends linearly on the light quark masses and is the heavy meson analog of the pion-nucleon sigma term. Finally, the remaining last terms contain which are of $O(m_q)$ and violate heavy-quark spin symmetry.

The fields H_a describes the mesons made up by the heavy quark Q and the light anti-quark $\overline{q_a}$ (a=1,2,3). This H_a field transforms as a doublet under heavy quark spin symmetry and as $\overline{3}$ under flavor $SU(3)_V$. m_q is the diagonal light quark mass matrix.

$$m_q = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}$$

In order to construct invariant derivative couplings, one need covariant derivatives or gauge fields. This is provided by the vector and axial current

$$V^\mu = \frac{1}{2}\left(\xi\partial^\mu\xi^+ + \xi^+\partial^\mu\xi\right) \qquad A^\mu = \frac{1}{2}\left(\xi\partial^\mu\xi^+ - \xi^+\partial^\mu\xi\right)$$

Under SU(3) ⊗ SU(3) chiral symmetry these field transforms as

$$V^\mu \to UV^\mu U^+ + U\partial^\mu U^+$$

$$A^\mu \to UA^\mu U^+$$

$$\xi \to L\xi U^+$$

where

$$\xi = e^{i\pi/f} \quad , \quad \Sigma = \xi^2 = e^{2i\pi/f}$$

and f ≈132 MeV is the pion decay constant.

The pseudo-scalar Goldstone boson are incorporated in a 3 × 3 unitary matrix $\Sigma(x) \in SU(3)$ transforming under $SU(3)_L \otimes SU(3)_R$ as

$$\Sigma \to g_L \Sigma g_R^+$$

The meson octet is introduced via the exponential representation

$$\Sigma = \exp(\frac{2iM}{f})$$

where M is a 3 × 3 hermitian, traceless matrix:

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}} \end{bmatrix}$$

An effective chiral symmetric Lagrangian describing low momentum interactions of heavy mesons with the pseudo Goldstone bosons of the 0^- octet can be written as [40]:

$$L = \frac{f^2}{8} Tr\left[\partial^\mu \Sigma \partial_\mu \Sigma^+\right] + \lambda_0 Tr\left[m_q \Sigma + m_q \Sigma^+\right] - iTr\left[\overline{H}_a v_\mu \partial^\mu H_a\right] + \frac{i}{2} Tr\left[\overline{H}_a v_\mu H_b (\xi^+ \partial^\mu \xi + \xi \partial^\mu \xi^+)_{ba}\right]$$
$$+ i\frac{g}{2} Tr\left[\overline{H}_a H_b \gamma^\mu \gamma_5 (\xi^+ \partial^\mu \xi - \xi \partial^\mu \xi^+)_{ba}\right] + \lambda_1 Tr\left[\overline{H}_a H_b (\xi \partial^\mu \xi + \xi^+ \partial^\mu \xi^+)_{ba}\right] +$$
$$\lambda_1' Tr\left[\overline{H}_a H_a (m_q \Sigma + m_q \Sigma^+)_{bb}\right] + \frac{\lambda_2}{m_Q} Tr\left[\overline{H}_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu}\right] + -----$$

3.3 Mass formula for heavy mesons

The residual masses for the ground state and excited state charm meson is given by:

$$m^0_{R_a} = \delta_R + \frac{n_J}{4}(\Delta_R + \Delta^\sigma_R \overline{m} + \Delta^{(a)}_R m_a) + \sigma_R \overline{m} + a_R m_a$$
$$+ \frac{g^2_R}{f^2} c^{R_a} K_1 + \frac{h^2_R}{f^2} c^{R_a} K_2$$

where *R* is an index that labels the ground state (*H*) and excited state (*S*), each of the ground and excited states having members corresponding to $J = 0, 1$, with $n_0 = -3, n_1 = 1$, the index *a* labels the light flavour and runs over *u, d, s*, the functions K_1 and K_2 are the chiral loop functions, and the *cRa* are coefficients listed in [41] and $g_H = g, g_S = g'$ in the notation therein.

At tree level these residual masses for all 12 states of D-meson is written as below:

$$m_{H_a}^0 = \delta_H - \frac{3}{4}\Delta_H + \sigma_H \overline{m} + a_H m_a - \frac{3}{4}\Delta_H^{(\sigma)}\overline{m} - \frac{3}{4}\Delta_H^{(\sigma)} m_a$$

$$m_{H_a^*}^0 = \delta_H + \frac{1}{4}\Delta_H + \sigma_H \overline{m} + a_H m_a + \frac{1}{4}\Delta_H^{(\sigma)}\overline{m} + \frac{1}{4}\Delta_H^{(\sigma)} m_a$$

$$m_{S_a}^0 = \delta_S - \frac{3}{4}\Delta_S + \sigma_S \overline{m} + a_S m_a - \frac{3}{4}\Delta_S^{(\sigma)}\overline{m} - \frac{3}{4}\Delta_S^{(\sigma)} m_a$$

$$m_{S_a}^0 = \delta_S + \frac{1}{4}\Delta_S + \sigma_S \overline{m} + a_S m_a + \frac{1}{4}\Delta_S^{(\sigma)}\overline{m} + \frac{1}{4}\Delta_S^{(\sigma)} m_a$$

where $m_a = (m_u, m_d, m_s)$, $m_q = diag(m_u, m_d, m_s)$ and $\overline{m} = m_u + m_d + m_s$

The residual masses are defined to be the difference between the real masses and an arbitrarily chosen reference mass of $O(m_Q)$. here the asterisk denotes the spin-1 member of the heavy quark doublet. The loop corrections to the masses are shown in fig 3.3. Light lines represent the H fields and heavy lines represent the S fields.

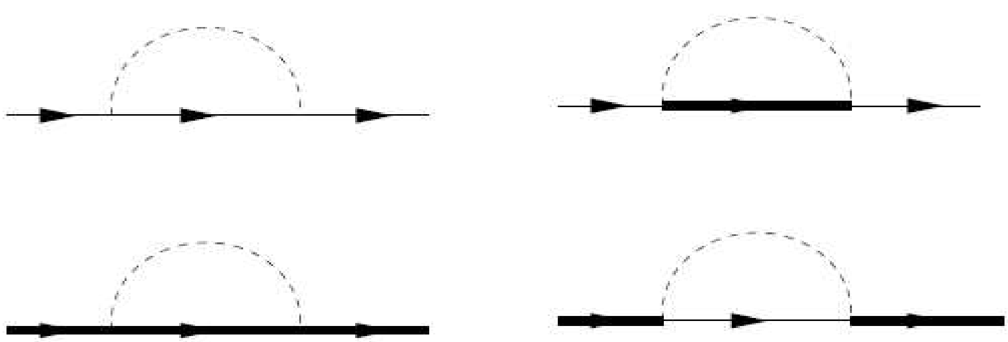


Fig 3.3 One-loop energy diagrams for the H and S fields. H fields are light lines, S fields are thick lines and goldstone bosons are dashed lines.

We have defined the function K_1 and K_2 as

$$
\begin{aligned}
K_1(\eta, M) &= \frac{1}{16\pi^2}\left[(-2\eta^3 + 3M^2\eta)\ln(\frac{M^2}{\mu^2}) + 2\eta(\eta^2 - M^2)F(\frac{\eta}{M}) + 4\eta^3 - 5\eta M^2\right] \\
K_2(\eta, M) &= \frac{1}{16\pi^2}\left[(-2\eta^3 + M^2\eta)\ln(\frac{M^2}{\mu^2}) + 2\eta^3 F(\frac{\eta}{M}) + 4\eta^3 - \eta M^2\right]
\end{aligned} \tag{3.1}
$$

Where

$$
F(x) = 2\frac{\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - Tan^{-1}(\frac{x}{\sqrt{1-x^2}})\right] \qquad |x| < 1
$$

$$
F(x) = -2\frac{\sqrt{x^2-1}}{x}\ln(x + \sqrt{x^2-1}) \qquad |x| > 1 \tag{3.2}
$$

The function $K_1(\eta, M)$ appears whenever the virtual heavy meson inside the loop is in the same doublet as the external heavy meson while $K_2(\eta, M)$ appears when the virtual heavy meson is from the opposite parity doublet.

In the limit, $M \ll \eta$ the eq. (3.1) and (3.2) become

$$
\begin{aligned}
K_1(\eta, M) &= \frac{1}{16\pi^2}\left[-2\eta^3 \ln(\frac{4\eta^2}{\mu^2}) + 3\eta M^2 \ln(\frac{4\eta^2}{\mu^2}) + \frac{3M^4}{4\eta}\ln(\frac{M^2}{4\eta^2}) + ...\right] \\
K_2(\eta, M) &= \frac{1}{16\pi^2}\left[-2\eta^3 \ln(\frac{4\eta^2}{\mu^2}) + \eta M^2 \ln(\frac{4\eta^2}{\mu^2}) - \frac{M^4}{4\eta}\ln(\frac{M^2}{4\eta^2}) + ...\right]
\end{aligned} \tag{3.3}
$$

In these equations, we have dropped polynomials of η, M. The functions $K_1(\eta, M)$ and $K_2(\eta, M)$ have well-defined $M/\eta \to 0$, limits. Furthermore, the dependence on M is analytic when $\frac{M}{\eta} \to 0$, so in this limit the S fields can be integrated out and their effect on the chiral corrections can be absorbed into local counter-terms as expected. This limit is not relevant to the real world as $\eta \ll M$. In the opposite limit, $\eta = 0$, which is relevant for loops in which external and virtual heavy mesons are the same. The eq.(3.3) become

$$K_1(\eta,M) = -\frac{M^3}{8\pi} + \frac{3}{16\pi^2}\eta M^2 \ln(\frac{4\eta^2}{\mu^2}) + O(n^3)$$

$$K_2(\eta,M) = \frac{1}{16\pi^2}\eta M^2 \ln(\frac{4\eta^2}{\mu^2}) + O(\eta^3)$$

Including the one loop diagrams and the one loop masses formulae are given below [42].

$$m_{H_1} = m^0_{H_1} + \frac{g^2}{f^2}\left[\frac{3}{2}K_1(m^0_{H_1^*} - m^0_{H_1}, m_\pi) + \frac{1}{6}K_1(m^0_{H_1^*} - m^0_{H_1}, m_\eta) + K_1(m^0_{H_1^*} - m^0_{H_1}, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m^0_{S_1} - m^0_{H_1}, m_\pi) + \frac{1}{6}K_2(m^0_{S_1} - m^0_{H_1}, m_\eta) + K_1(m^0_{S_3} - m^0_{H_1}, m_K)\right].$$

$$m_{H_3} = m^0_{H_3} + \frac{g^2}{f^2}\left[2K_1(m^0_{H_1^*} - m^0_{H_3}, m_K) + \frac{2}{3}K_1(m^0_{H_3^*} - m^0_{H_3}, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m^0_{S_1} - m^0_{H_3}, m_K) + \frac{2}{3}K_2(m^0_{S_1} - m^0_{H_1}, m_\eta)\right].$$

$$m_{H_1^*} = m^0_{H_1^*} + \frac{g^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1(m^0_{H_1} - m^0_{H_1^*}, m_\pi) + \frac{1}{6}K_1(m^0_{H_1} - m^0_{H_1^*}, m_\eta) + K_1(m^0_{H_3} - m^0_{H_1^*}, m_K)\right]$$
$$+\frac{g^2}{f^2}\frac{2}{3}\left[\frac{3}{2}K_1(0, m_\pi) + \frac{1}{6}K_1(0, m_\eta) + K_1(m^0_{H_3^*} - m^0_{H_1^*}, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m^0_{S_1^*} - m^0_{H_1^*}, m_\pi) + \frac{1}{6}K_2(m^0_{S_1^*} - m^0_{H_1^*}, m_\eta) + K_1(m^0_{S_3^*} - m^0_{H_1^*}, m_K)\right]$$

$$m_{H_3^*} = m^0_{H_3^*} + \frac{g^2}{f^2}\frac{1}{3}\left[2K_1(m^0_{H_1} - m^0_{H_3^*}, m_K) + \frac{2}{3}K_1(m^0_{H_3} - m^0_{H_3^*}, m_\eta)\right]$$
$$+\frac{g^2}{f^2}\frac{2}{3}\left[2K_1(m^0_{H_1} - m^0_{H_3^*}, m_K) + \frac{2}{3}K_1(0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m^0_{S_1^*} - m^0_{H_3^*}, m_K) + \frac{2}{3}K_2(m^0_{S_3^*} - m^0_{H_3^*}, m_\eta)\right]$$

$$m_{S_1} = m^0_{S_1} + \frac{g'^2}{f^2}\left[\frac{3}{2}K_1(m^0_{S_1^*} - m^0_{S_1}, m_\pi) + \frac{1}{6}K_1(m^0_{S_1^*} - m^0_{S_1}, m_\eta) + K_1(m^0_{S_3^*} - m^0_{S_1}, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m^0_{H_1} - m^0_{S_1}, m_\pi) + \frac{1}{6}K_2(m^0_{H_1} - m^0_{S_1}, m_\eta) + K_2(m^0_{H_3} - m^0_{S_1}, m_K)\right]$$

$$m_{S_3} = m_{S_3}^0 + \frac{g'^2}{f^2}\left[2K_1(m_{S_1^*}^0 - m_{S_3}^0, m_K) + \frac{2}{3}K_1(m_{S_3^*}^0 - m_{S_3}^0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m_{H_1}^0 - m_{S_3}^0, m_K) + \frac{2}{3}K_2(m_{H_3}^0 - m_{S_3}^0, m_\eta)\right]$$

$$m_{S_1^*} = m_{S_1^*}^0 + \frac{g'^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1(m_{S_1}^0 - m_{S_1^*}^0, m_\pi) + \frac{1}{6}K_1(m_{S_1}^0 - m_{S_1^*}^0, m_\eta) + K_1(m_{S_3}^0 - m_{S_1^*}^0, m_K)\right]$$
$$+\frac{g'^2}{f^2}\frac{2}{3}\left[\frac{3}{2}K_1(0, m_K) + \frac{1}{6}K_1(0, m_\eta) + K_1(m_{S_3^*}^0 - m_{S_1^*}^0, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m_{H_1^*}^0 - m_{S_1^*}^0, m_\pi) + \frac{1}{6}K_2(m_{H_3^*}^0 - m_{S_1^*}^0, m_\eta) + K_2(m_{H_3^*}^0 - m_{S_1^*}^0, m_K)\right]$$

$$m_{S_3^*} = m_{S_3^*}^0 + \frac{g'^2}{f^2}\frac{1}{3}\left[2K_1(m_{S_1}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_1(m_{S_3}^0 - m_{S_3^*}^0, m_\eta)\right]$$
$$+\frac{g'^2}{f^2}\frac{2}{3}\left[2K_1(m_{S_1^*}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_1(0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m_{H_1^*}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_2(m_{H_3^*}^0 - m_{S_3^*}^0, m_\eta)\right]$$

From experimental data, the residual masses for $j^P = \frac{1}{2}^-$ and $j^P = \frac{1}{2}^+$ were calculated and they are:

$m_{H_1} = -106.1 MeV$, $m_{H_3} = -4.75 MeV$, $m_{H_1^*} = 35.4 MeV$, $m_{H_3^*} = 139.1 MeV$,

$m_{S_1} = 335.0 MeV$, $m_{S_3} = 344.4 MeV$, $m_{S_1^*} = 465.0 MeV$, $m_{S_3^*} = 486.3 MeV$

where we have used $(m_{H_1} + 3m_{H_1^*})/4$ as reference mass. Using the one loop calculations, we can fit these observed masses to get fundamental parameters such strong coupling constant (q), coupling of pions to heavy meson (f), the mass and hyperfine splitting parameters etc. If we use these mass values in tree level expressions, we get the following values:

$$\delta_S + \sigma_S \overline{m} = 431.6 \pm 26.09 MeV$$
$$\delta_H + \sigma_H \overline{m} = -4.8 \pm 0.65 MeV$$
$$\Delta_H + \Delta_H^{(\sigma)} \overline{m} = 141.3 \pm 1.17 MeV$$
$$\Delta_S + \Delta_S^{(\sigma)} \overline{m} = 129.4 \pm 49.72 MeV$$

$a_H = 1.2 \pm 0.01$, $a_S = 0.21 \pm 0.29$, $\Delta_H^{(a)} = 0.028 \pm 0.02$, $\Delta_S^{(a)} = 0.14 \pm 0.55$

If we use the one loop mass formulae, then we will have 11 parameters: $g, g', h, a_H, a_S, \Delta_H^{(a)}, \Delta_S^{(a)}$ $, \delta_H + \sigma_H \overline{m}, \delta_S + \sigma_S \overline{m}$, $\Delta_H + \Delta_H^{(\sigma)} \overline{m}$ and $\Delta_S + \Delta_S^{(\sigma)} \overline{m}$. The parameters $\sigma_H, \sigma_S, \Delta_H^{(\sigma)}$ and $\Delta_S^{(\sigma)}$ cannot be separately determined because they always appear in linear combination with the parameters δ_H, δ_S, Δ_H and Δ_S respectively. Hence we have absorbed the contribution of the parameters $\sigma_H, \sigma_S, \Delta_H^{(\sigma)}$ and $\Delta_S^{(\sigma)}$ into the measured values of δ_H, δ_S, Δ_H and Δ_S respectively. Now we have only eight mass data (and equations) for 11 parameters, which gives different possibility for the parameters. One such fit is given below.

$g = 0.27 \pm 0.06$, $h = 0.69 \pm 0.09$, $g' = 0.0$, $\delta_H = 261 \pm 0.74 MeV$, $\delta_S = 368.1 \pm 2.93 MeV$,

$\Delta_H = 629.8 \pm 1.79 MeV$, $\Delta_S = 672.4 \pm 8.5 MeV$, $a_H = 0.6929 \pm 0.01$, $a_S = -7.77 \pm 0.03$,

$\Delta_H^{(a)} = -2.13 \pm 0.03$, $\Delta_S^{(a)} = 0.04 \pm 0.03$

HQET PARAMETERS:

Heavy quark symmetry can be used to obtain relations between hadron masses. The hadron mass in the effective theory is m_H - m_Q , since the heavy quark mass m_Q has been subtracted from all energies. At order m_Q, all heavy hadrons containing Q are degenerate, and have the same mass m_Q. At the order of unity, the hadron masses get the contribution

$$\frac{1}{2}\left\langle H^{(Q)} \middle| \mathrm{H}_0 \middle| H^{(Q)} \right\rangle \equiv \overline{\Lambda} \tag{3.4}$$

Where H_0 is the order $1/m_Q^0$ terms in the HQET Hamiltonian obtained from the Lagrangian term $\overline{Q}_v(iv.D)Q_v$, as well as the terms involving light quarks and gluons. In this section, the hadron states $\left|H^{(Q)}\right\rangle$ are in the effective theory with $v = v_r = (1,0)$. Here $\overline{\Lambda}$ is a parameter of HQET and has the same value for all particles in a spin- flavor multiplet. The values will be denoted by $\overline{\Lambda}$ for the D and D^*. In the SU(3) limit, $\overline{\Lambda}$ does not depend on the light quark flavor. If SU(3) breaking is included, $\overline{\Lambda}$ is different for the $D_{u,d}$ and D_s mesons, and will be donated by $\overline{\Lambda}_{u,d}$ and $\overline{\Lambda}_S$ respectively.

At order $1/m_Q$, there is an additional contribution to the hadron masses given by the expectation values of the $1/m_Q$ correction to the Hamiltonian:

$$\mathrm{H}_1 = -L_1 = \overline{Q}_v \frac{D_\perp^2}{2m_Q} Q_v + a(\mu) g \overline{Q}_v \frac{\sigma_{\alpha\beta}}{4m_Q} Q_v. \qquad (3.5)$$

The matrix element of the terms in Eq.(3.5) define two non-perturtubative parameters, λ_1 and λ_2

$$\begin{aligned} 2\lambda_1 &= -\left\langle H^{(Q)} \left| \overline{Q}_{v_r} D_\perp^2 Q_{v_r} \right| H^{(Q)} \right\rangle, \\ 16(S_Q.S_l)\lambda_2(m_Q) &= a(\mu) \left\langle H^{(Q)} \left| \overline{Q}_{v_r} g \sigma_{\alpha\beta} G^{\alpha\beta} Q_{v_r} \right| H^{(Q)} \right\rangle. \end{aligned} \qquad (3.6)$$

Where $a(\mu) = \left[\frac{\alpha_S(m_Q)}{\alpha_S(\mu)} \right]^{9/(33-2N_q)}$

and $\mathrm{N_q}$ is the number of light quarks flavor

Here λ_1 is independent of m_Q, and λ_2 depends on m_q through the logarithmic m_Q dependence of $a(\mu)$; $\lambda_{1,2}$ have the same value for all states in a given spin-flavor multiplet and are expected to be of the order of Λ_{QCD}^2.

The native expectation that the heavy quark kinetic energy is positive suggests that λ_1 should be negative. The λ_2 matrix element transforms like $S_Q.S_l$ under the spin symmetry, since that is the transformation property of $\overline{Q}_{v_r} \sigma_{\alpha\beta} G^{\alpha\beta} Q_{v_r}$. Only the two upper components of Q_{v_r} are non zero, since $\gamma^0 Q_{v_r} = Q_{v_r}$ and $\overline{Q}_{v_r} \sigma_{\alpha\beta} G^{\alpha\beta} Q_{v_r}$, reduces to the matrix elements of $\overline{Q}_{v_r}\sigma.BQ_{v_r}$, where B is the chromo-dynamics field . The operator $\overline{Q}_{v_r}\sigma Q_{v_r}$ is the heavy quark spin, and the matrix element of B in the hadron must be proportional to the spin of the light degree of freedom, by rotational invariance and time–reversal invariance , so that the chromo-dynamics operator contribution is proportional to $S_Q.S_l$. Using $S_Q.S_l = (J^2 - S_Q^2 - S_l^2)/2$, one finds that

$$
\begin{aligned}
m_D &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2(m_c)}{2m_c}, \\
m_{D^*} &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{\lambda_2(m_c)}{2m_c}
\end{aligned}
\qquad (3.7)
$$

The average mass of a heavy quark spin symmetry multiplet, e.g. $(3m_{p^*} + m_p)/4$ For the meson multiplet, does not depend on λ_2. The magnetic interaction λ_2 is responsible for the $D^* - D$ splitting.

Eq (3.7) gives the meson mass relation

$$4\lambda_2 \approx m_{D^*}^2 - m_D^2 \approx 0.55 GeV^2 \qquad (3.8)$$

Upto correction of the order $1/m_c$ and ignoring the weak m_q dependence of λ_2 .Similarly, one finds that

$$
\begin{aligned}
\overline{\Lambda}_s - \overline{\Lambda}_{u,d} &= m_{D_S} - m_{D_d} = 99 \pm 1 MeV \\
\overline{\Lambda}_\Lambda - \overline{\Lambda}_{u,d} &= m_{\Lambda_c} - m_D = 416 \pm 1 MeV
\end{aligned}
\qquad (3.9)
$$

The parameter λ_1 and λ_2 are non-peturbative parameter of QCD and have not been computed from first principles, it might appear that very little has been gained by using eq. (3.7) for the hadron masses in terms of $\overline{\Lambda}$, λ_1 and λ_2. However, the same hadronic matrix elements also occur in other quantities, such as form factors. One can then use the value of $\overline{\Lambda}$, λ_1 and λ_2 obtained by fitting to the hadron masses to compute the form factors, without making any model dependent assumptions [35].

3.4 HQET Formula

The commonly used definitions of the quark masses for heavy quarks are the pole mass, the potential model mass used in γ and φ spectroscopy, and HQET mass. Physical quantity such as hadron masses can in principle be computed in HQET in terms of the HQET mass m_Q. Computation cannot be done analytically in practice because of non-perturbative effects in QCD, which prevent the extraction of quark mass from QCD Lagrangian. Nevertheless for heavy quarks it is possible to parameterize the non-perturbative affects to a given order in $1/m_Q$ expansion in terms of a few unknown constants that can be obtained from the experiments. B and D meson masses in the heavy quark effective theory are given in terms of a single non-perturbative parameter $\overline{\Lambda}$.

In general, the mass of a hadron H_Q containing a heavy quark Q obey an expansion of the form

$$m_X = m_Q + \overline{\Lambda} + \frac{\Delta m^2}{2m_Q} + O(\frac{1}{m_Q})$$

where X is the hadron, it can be either a ground state (H) or an excited state(S), m_Q is the mass of the heavy quark.

X = H, S

where as $$\Delta m^2 = -\lambda_1 + 2[J(J+1) - \frac{3}{2}]\lambda_2$$

J is the total spin of meson. The two parameters λ_1 and λ_2 are non perturbative parameters of QCD. Here λ_1 is independent of m_Q and λ_2 depends on m_Q logarithmically. λ_1 and λ_2 have same values for all states in a given spin-flavor multiplet and are expected to be of the order of Λ_{QCD}^2. $\overline{\Lambda}$, λ_1 and λ_2 characterize the properties of light constituents. The term $-\lambda_1/m_Q$ arises from kinetic energy of the heavy quark inside hadron (meson). The magnetic interaction λ_2 describes the interaction of the heavy quark spin with the gluon field and responsible for D*- D splitting. Thus the formula for masses can be written in terms of HQET parameters $\overline{\Lambda}$, λ_1 and λ_2 (from eq. 3.8 and 3.9) as

$$m_X^{(Q)} = m_Q + \overline{\Lambda} - \frac{\lambda_1^X}{2m_Q} + n_j \frac{\lambda_2^X}{2m_Q}$$

where $n_j = +1$ for J=1, $n_j = -3$ for J=0

In particular for D mesons, the masses for the spin state (0, 1) can be written as:

$$m_D = m_c + \overline{\Lambda} - \frac{\lambda_1^D}{2m_c} - 3\frac{\lambda_2^D}{m_c}; \qquad m_{D^*} = m_c + \overline{\Lambda} - \frac{\lambda_1^D}{2m_c} + \frac{\lambda_2^D}{m_c}$$

where $\overline{m}_H^{(Q)} = (3m_{H^*}^{(Q)} + m_H^{(Q)})/4$ and $\overline{m}_S^{(Q)} = (3m_{S^*}^{(Q)} + m_S^{(Q)})/4$

The difference between spin averaged masses of the $J^P = \frac{1}{2}^-$ and $J^P = \frac{1}{2}^+$ mesons is given by

$$\overline{m}_S^{(Q)} - \overline{m}_H^{(Q)} = \overline{\Lambda}^S - \overline{\Lambda}^H - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_1^H}{2m_Q} \qquad (3.10)$$

It leads to the formulas for the splitting of the even and odd-parity states

$$\overline{m}_S^{(b)} - \overline{m}_H^{(b)} = \overline{m}_S^{(c)} - \overline{m}_H^{(c)} - (\lambda_1^S - \lambda_1^H)(\frac{1}{2m_c} - \frac{1}{2m_b}) \qquad (3.11)$$

This eq. (3.11) represents the standard relation between the charm meson and bottom meson. Since $\overline{\Lambda}$ is a parameter of HQET and has the same value for all particles in a spin-flavor multiplet. Hence ($\overline{\Lambda}^S - \overline{\Lambda}^H$) is same for D and B mesons.

Using the experimental values for the masses of the ground state and the excited state D meson we can calculate specifically their spin-averaged masses of the $j^p=1/2^-$ and $½^+$ meson as [42].

$$\begin{aligned}
\overline{m}_{H_1}^c &= (3m_{H_1^*}^{(c)} + m_{H_1}^{(c)})/4 = [3(35.4) - 106.1]/4 = 0.025 \\
\overline{m}_{H_3}^c &= (3m_{H_3^*}^{(c)} + m_{H_3}^{(c)})/4 = [3(139.1) - 4.75]/4 = 103.06 \\
\overline{m}_{S_1}^c &= (3m_{S_1^*}^{(c)} + m_{S_1}^{(c)})/4 = [3(465.0) + 335.0]/4 = 432.50 \\
\overline{m}_{S_3}^c &= (3m_{S_3^*}^{(c)} + m_{S_3}^{(c)})/4 = [3(486.0) + 344.4]/4 = 450.80
\end{aligned} \qquad (3.12)$$

3.5 Spectroscopic implications:

The spin-flavor symmetry leads to many interesting relations between the properties of hadrons containing a heavy quark. The most direct consequences concern the spectroscopy of such states. In the $m_Q \to \infty$ limit, the spin of the heavy quark and the total angular momentum j of the light degrees of freedom are separately conserved by the strong interactions. Because of heavy-quark symmetry, the dynamics is independent of the spin and mass of the heavy quark. Hadronic states can thus be classified by the quantum numbers (flavor, spin, parity etc.) of the light degrees of freedom. The spin symmetry predicts that for fixed $j \neq 0$, there is a doublet of degenerate states with total spin $J = j \pm 1/2$. The flavor symmetry relates the properties of states with different heavy-quark flavor.

Consider as an example, the ground state mesons containing a heavy quark. In this case the light degrees of freedom have the quantum numbers of a light anti-quark and the degenerate states are pseudo-scalar ($J = 0$) and vector ($J = 1$) mesons. In the charm system, one knows experimentally:

$$m_{D^*} - m_D \approx 142 \text{ MeV}, \qquad m_{D_s^*} - m_{D_s} \approx 142 \text{ MeV} \qquad (3.13)$$

These mass splitting are in fact reasonably small. To be more specific, at order $1/m_Q$ one expects hyperfine corrections to resolve the degeneracy. This leads to refined prediction

$$m_{D^*}^2 - m_D^2 \approx \text{constant}$$

The data is compatible with this:

$$m_{D^*}^2 - m_D^2 \approx 0.55 \text{ GeV}^2 \qquad (3.14)$$

The spin symmetry also predicts that $m_{D_s^*} - m_{D_s} =$ const. but this constant could in principle be different from that for non-strange mesons, since the flavor quantum numbers of the light degrees of freedom are different in both cases. Experimentally, however,

$$m_{D_s^*} - m_{D_s} \approx m_{D^*} - m_D$$

indicating that to first approximation hyperfine corrections are independent of the flavor of the "brown muck".

One can also study excited meson states, in which the light constituents carry orbital angular momentum. It is tempting to interpret D_1(2420) with $J^P = 1^+$ and D_2(2420) with $J^P = 2^+$ as the spin doublet corresponding to j = 3/2. In fact, the small mass difference

$$m_{D_2^*} - m_{D_1} \approx 35 \text{ MeV}$$

supports this assertion. One then expects

$$m_{D_2^*} - m_{D_1} \approx 0.17 \text{ GeV}^2 \qquad (3.15)$$

The fact that this mass splitting is smaller than for the ground state mesons in (3.14) is not unexpected. For instance, in the non-relativistic constituent quark model the light anti-quark in these excited mesons is in a p-wave state and its wave function at the location of the heavy quark vanishes. Hence, in this model hyperfine corrections are strongly suppressed systems.

A typical prediction of the flavor symmetry is that the "excitation energies" for the states with different quantum numbers of the light degrees of freedom are approximately the same in the charm system. For instance, one expects

$$m_{D_s} - m_D \approx 100 \text{ MeV}, \qquad m_{D_1} - m_D \approx 557 \text{ MeV}, \qquad m_{D_2^*} - m_D \approx 593 \text{ MeV} \qquad (3.16)$$

There are many more applications of heavy-quark symmetry to the strong interactions of hadron containing a heavy quark [30].

Chapter IV

Summary and Conclusion

Within the framework of heavy quark effective theory and chiral perturbation theory, masses of even and odd parity charm mesons are studied. Heavy quark and chiral symmetries are exploited to formulate a theory that can explain the experimental data to a very precise extent. In this work, we have analyzed the masses of lowest-lying even-parity excited states in the open charm system. We have used the framework of HHχPT including the $O(1/m_C)$ corrections, namely the framework employed in Mehen paper.

Our main motivation is to find the masses of the non-strange excited states from the observed experimental values of all the ground states and excited strange mesons only. In the present work, we employ the expressions presented in the comprehensive work of Mehen and Springer.

Mass formulae is developed for ground state $J^P = 0^-$ and 1^- and first excited state, $J^P = 0^+$ and 1^+ charmed meson up to one loop chiral corrections. Heavy quark symmetry and chiral symmetry is used to obtain the relation between heavy meson masses. There are in total 12 equations for each states, and imposition on chiral symmetry ($m_u = m_d$) bring the number to just 8 equations in terms of symmetry breaking and conserving variables. The equations are thus fitted with the specific ranges to the variables constrained by the experimental data. The key differences is that we are now constraining the values of the parameters *g, g', h* to the values that are determined from the decays. Specifically we restricted them within a range of 0 to 1. Furthermore we have imposed the requirement that $m_s \sim 130$ MeV and m_u, $m_d \sim 4$ MeV at our energy scale of 1GeV, and have required the parameters determining the tree-level hyperfine structure to be in a range determined by the well-established states.

From this fit, we obtain for the non-strange 0^+ state the mass of 2155.7 MeV and for 1^+ 2395.1 MeV. We find that the masses for the non-strange states that we determine can be lower than the numbers obtained by the Belle Collaboration. These values (ranges) will be useful for experimentalists, who are looking for these states to tell them where to look for. These states have masses far below the theoretical predictions compared to the corresponding D_u states. There are also some contravesial issues about newly discovered states D_s = 2317 MeV and D_s = 2460 MeV. Also this heavy quark formalism can be used to analyse the decays of heavy meson accurately than any other existing theories.

REFERENCES:

[1]. Daniel M. Kaplan and Kenneth S. Nelson, http://arXiv:physics/9805026v2 12 Jun, 2002

[2]. http://burro.astr.cwru.edu/stu/advanced/extras_particle-phys.html

[3]. Nari Mistry, Laboratory for Elementary Particle Physics, Cornell University.

[4]. http://www.daviddarling.info/encyclopedia/S/standard_model.html

[5]. Edward Eric Johnson, Measurements of charm meson Production in 10.5 GeV electron positron annihilation

[6]. M. Herrero, arXiv: hep-ph/9812242v1 3Dec 1998

[7]. M Gell-Mann, R Oakes, and B Renner, Phys. Rev. 175 , 2195 (1968)

[8]. http://abyss.uoregon.edu/~js/ast123/lectures/lec07.html

[9]. http://particle-adventure.org/quark naming.html

[10]. http://hperphysics.phy-astr.gsu.edu/HBASE/Particles/hadron.html

[11]. Arthur Beiser, Concepts of modern physics (6th edition) Tata-McGraw Hill

[12]. www.encyclopedia.com/doc/1E1-gluon.html

[13]. www.newton.dep.anl.gov/askasci/phy99/phy99211.htm

[14]. http://hyperphysics.phy-astr.gsu.edu/hbase/particles/expar.html

[15]. http://hyperphysics.phy-astr.gsu.edu/HBASE/forces/funfor.html

[16]. John S. Rigden, Building Blocks of matter, A Supplement to the MacMillian Encyclopedia of Physics.

[17]. W. N. Cottingham and D. A. Greenwood, An introduction to the standard model of particle physics (2nd edition)

[18]. D. P. Roy, http://arXiv.org/abs/hep-ph/9912523v1

[19]. Ta-Pei Chung and Ling Fong Li, Gauge theory of Elementary Particle Physics.

[20]. en.wikipedia.org/wiki/**Symmetry_breaking**

[21]. http://plato.stanford.edu/entries/symmetry-breaking/

[22]. Olga Lakhina, Study of meson properties in quark model.

[23]. http://people.web.psi.ch/markushin/papers html/pip prop/nod8.html

[24]. J.D. Bjorken, Phys. Rev.179,1547 (1969); J.D. Bjorken and E.A. Paschos, Phys. Rev.185,1975 (1969).

[25]. http://prr.hec.gov.pk/chapters/2505-1.pdf

[26]. Aneesh V. Manohar, Effective field theories, arXiv:hep-ph/9606222v 4 June 1996

[27]. http://psroc.phys.ntu.edu.tw/cjp/download.php?d=1&pid=894

[28]. www.fuw.edu.pl/~lfqcd/yan.pdf

[29]. B. Ananthanarayan, A. Upadhyay, K. Shivraj, Masses and decays of charmed mesons in heavy hadron chiral perturbation theory.

[30]. Shivaraj K, Meson masses in Heavy Hadron Chiral Perturbation Theory.

[31]. M. Neubert, Physics Reports 245(1994) 259-395

[32]. S.L. Adler and R.F. Dashen, Current Algebras, W.A. Benjamin, Mass(1968).

[33]. Stefan Scherer, Introduction to Chiral Perturbation Theory,arXiv:hep-ph/0210398v1 29 Oct 2002

[34]. Aneesh V. Manohar, Chiral Perturbation Theory, arXiv:hep-ph/9305298 v1 21 May 1993

[35]. F. Cornet, Chiral Perturbation Theory

[36]. Aneesh V. Manohar, Mark B. Wise, Heavy Quark Physics (2000)

[37]. A. G. Grozin, arXiv: hep-ph/9908366 v1 16 Aug 1999

[38]. Howard Georgi, Heavy Quark Effective Theory

[39]. T. M. Yan et.al, Heavy quark symmetry and chiral dynamics, Phys. Rev. D v 46 1 August 1992

[40]. Elizabeth Jenkins, Heavy meson masses in Chiral Perturbation Theory with Heavy Quark symmetry, arXiv: hep-ph/9212295 v1 21 Dec 1992

[41]. A. Upadhyay, PLB 651(2007) 124-128, Lowest-lying even parity excited states in the open charm system.

[42]. Thomas Mehen and Roxanne P Springer, Even and Odd Parity Charmed Meson Masses in HHChPT, arXiv: hep-ph/0503134 v2 15 Jul 2005.