ANALYSIS AND DESIGN OF ADAPTIVE VOLTERRA FILTERS FOR SYSTEM IDENTIFICATION IN TIME-VARYING ENVIRONMENT

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CERTIFICATE

I, **Amrita Rai**, hereby declare that the thesis entitled, **"Analysis and Design of Adaptive Volterra Filters for System Identification in Time-Varying Environment,"** submitted to Thapar University, Patiala, in partial fulfillment of the requirement for the award of the Degree of **Doctor of Philosophy** in **the Electronics and Communication Engineering** is a record of original and independent research work done by me during 2009-2015. This thesis has been conducted under the guidance of **Dr. Amit Kumar Kohli**, Associate Professor, Electronics and Communication Engineering Department, Thapar University. It has not formed the basis for the award of any Degree/Diploma/Associate-ship/Fellowship or other similar title to any candidate of any university.

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This is to certify that above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

To satisfy the ever-increasing demand for the nonlinear system identification in various fields of engineering, as well as to tackle the system nonlinearities in the presence of non-Gaussian noise, interest has peaked in the adaptive nonlinear signal processing techniques. The nonlinear systems are conventionally modeled using the polynomial paradigms, whose output signals can be related to the input signals through the truncated Volterra series expansion. The nonlinear Volterra filter exhibits property that it depends linearly on the coefficients of the filter itself; therefore the principles of optimum linear filter theory can be naturally extended to the optimum nonlinear Volterra filter theory. These nonlinear filters are attractive because these may be able to approximate a large class of nonlinear systems with great parsimony in the use of coefficients.

The nonlinear adaptive filtering techniques for the system identification (based on the Volterra model) are widely used for the identification of nonlinearities in the domain of communication and signal processing applications. We first present the variable forgetting factor (VFF) least squares (LS) algorithm for the polynomial channel paradigm, which provides improved tracking performance under the nonstationary environment. The main focus is on updating VFF, when each time-varying fading channel is considered to be a first-order Markov process. It may be inferred from the simulation results that in addition to efficient tracking under the frequency-selective fading wireless channels, the incorporation of proposed numeric variable forgetting factor (NVFF) in LS algorithm reduces the computational complexity. Subsequently, the improved tracking capability of a numeric variable forgetting factor recursive least squares (NVFF-RLS) algorithm is presented for the first-order and second-order time-varying Volterra systems under the nonstationary environment. The nonlinear system tracking problem is converted into a state estimation problem of the time-variant system. The time-varying Volterra kernels are governed by the

first-order Gauss-Markov stochastic difference equation, upon which the state-space representation of this system is built. In comparison to the conventional fixed forgetting factor recursive least squares algorithm, the NVFF-RLS algorithm provides better channel estimation as well as channel tracking performance in terms of the minimum mean square error (MMSE) for the first-order and second-order Volterra systems. The NVFF-RLS algorithm is adapted to the time-varying signal by using the updating prediction error criterion, which accounts for the nonstationarity of the observed signal. The demonstrated simulation results manifest that the proposed method has good adaptability in the time-varying environment, and it also reduces the computational complexity.

However, an important issue in the system identification is the effect of measurement noise on the parameter estimation results. This measurement noise is usually considered to be a white Gaussian stochastic process with finite second-order statistics, which makes the mean squared error an appropriate metric for the estimation error. But, the non-Gaussian statistical signal processing plays an important role when signal and / or noise deviates from the ideal Gaussian model. The stable distributions are among the most significant non-Gaussian models. We next present the adaptive polynomial filtering using the generalized variable step-size least mean p^{th} power (GVSS-LMP) algorithm for the nonlinear Volterra system identification, under the α-stable impulsive noise environment. Due to the lack of finite second-order statistics of the impulse noise, we espouse the minimum error dispersion (MED) criterion as an appropriate metric for the estimation error, instead of the conventional minimum mean square error criterion. For the convergence of LMP algorithm, the adaptive weights are updated by adjusting $p \geq 1$ in the presence of impulsive noise characterized by $1 < \alpha < 2$.

In many practical applications, the auto-correlation matrix of input signal has the larger eigenvalue spread in the case of nonlinear Volterra filter than in the case of linear finite impulse response filter. In such cases, the time-varying step-size is an appropriate option to mitigate the adverse effects of eigenvalue spread on the convergence of LMP adaptive algorithm. Therefore, the generalized variable step-size updating criterion is proposed in combination with the LMP algorithm, to identify the slowly time-varying Volterra kernels, under the non-Gaussian α-stable impulsive noise scenario. The simulation results are presented to demonstrate that the proposed GVSS-LMP algorithm is more robust to the impulsive noise in comparison to the conventional techniques, when the input signal is correlated or uncorrelated Gaussian sequence, while keeping $1 < p < \alpha < 2$. It also exhibits flexible design to tackle the slowly time-varying nonlinear system identification problem.

The adaptive nonlinear signal processing has also found applications in the field of audio and speech signal processing. The nonlinearity of amplifiers and / or loudspeakers gives rise to the nonlinear echo in the acoustic systems, which severely deteriorates the quality of audio and speech communication systems. Further, we present a nonlinear acoustic echo cancellation algorithm, which includes two distinct modules in cascade. The first module is a polynomial Volterra filter, which is an equivalent paradigm for a loudspeaker with the nonlinear distortion. The second module in the presented cascaded structure is a linear tapped-delay-line (finite impulse response) filter, which is analogous to the impulse response of the acoustic path. In the proposed adaptive structure, the adaptive nonlinear filter in the first module tackles the nonlinear constituents of the Volterra model, which uses the conventional fixed step-size normalized least mean square (FSS-NLMS) algorithm. However, the adaptive linear filter in the second module deals with the linear constituents of the Volterra model as well as the linear impulse response of the acoustic path, in which the generalized variable step-size normalized least mean square (GVSS-NLMS) algorithm is incorporated to suppress the adverse effects of nonstationarity / distortion. Computer simulation results demonstrate that the presented GVSS-NLMS algorithm based approach outperforms the FSS-NLMS algorithm based Volterra filtering, as far as the convergence and tracking characteristics are concerned. In simulation of the real-time environment and appropriate parameter setting for the third-order polynomial model, it provides approximately 5 dB performance advantage over the conventional nonlinear filtering approach in the tracking mode, in terms of the reduction in mean squared error. Moreover, the presented adaptive technique exhibits lower computational complexity than the conventional FSS-NLMS based polynomial Volterra filtering used for the acoustic echo cancellation.

Based on the aforementioned research work, which mainly emphasis on the adaptive nonlinear system identification problem, it is apparent that the adaptive nonlinear filtering is an exciting and challenging area with a wide variety of applications; and potential breakthroughs with great impact on practical applications are expected in the near future.

LIST OF PUBLICATIONS

1. Amrita Rai and Amit Kumar Kohli, “Volterra Filtering Scheme using Generalized Variable Step-size NLMS Algorithm for Nonlinear Acoustic Echo Cancellation,” *Acta Acustica united with Acustica* , vol. 101, no. 4, pp. 821 – 828, July 2015. (SCI Indexed by Thomson Reuters; Impact Factor = 0.679).
2. A. Rai and A.K. Kohli, “Adaptive Polynomial Filtering using Generalized Variable Step-Size Least Mean p^{th} Power (LMP) Algorithm,” *Springer, Circuits, Systems, and Signal Processing*, vol. 33, no. 12, pp. 3931 – 3947, December 2014. (SCI Indexed by Thomson Reuters; Impact Factor = 1.118).
3. A. K. Kohli and A. Rai, “Numeric variable forgetting factor RLS algorithm for second-order Volterra filtering,” *Springer, Circuits, Systems, and Signal Processing,* vol. 32, no. 1, pp. 223 – 232, February 2013. (SCI Indexed by Thomson Reuters; Impact Factor = 1.118)

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ACRONYMS AND ABBREVIATIONS

AEC	:	Acoustic Echo Canceller
ANC	:	Active Noise Control
ARMA	:	Autoregressive Moving Average
AVSS-LMP	:	Aboulnasr's Variable Step-Size LMP
BLAST	:	Bell Laboratories Layered Space-Time
BLMS	:	Block Least Mean Square
D-BLAST	:	Diagonal-BLAST
DC	:	Direct Current
DFE	:	Decision Feedback Equalizers
DFT	:	Discrete Fourier Transform
DFF	:	Dynamic Forgetting Factor
EKF	:	Extended Kalman Filtering
ERLE	:	Echo Return Loss Enhancement
FFF	:	Fixed Forgetting Factor
FFF-RLS	:	Fixed Forgetting Factor RLS
FIR	:	Finite Impulse Response
FLOM	:	Fractional Lower Order Moments
FSS-NLMS	:	Fixed Step-Size Normalized Least Mean Square
FSS-LMS	:	Fixed Step-Size LMS
FS-LMS	:	Filtered-s Least Mean Square
FX-LMS	:	Filtered-x Least Mean Square
FRLS	:	Fast Recursive Least Squares
G-LMS	:	Two-Step LMS
GSM	:	Global System for Mobile Communication

GVSS	:	Generalized Variable Step-Size
GVSS-LMP	:	Generalized Variable Step-Size LMP
GVSS-NLMS	:	Generalized Variable Step-Size NLMS
H-BLAST	:	Horizontal-BLAST
KVSS-LMP	:	Kwong’s Variable Step-Size LMP
KF	:	Kalman Filtering
LS	:	Least Squares
LMP	:	Least Mean p^{th} Power
LMS	:	Least Mean Square
LMF	:	Least Mean Forth
LU	:	Lower Upper Triangular Matrix
MA	:	Moving Average
MED	:	Minimum Error Dispersion
MG-LMS	:	Modified Two-Step LMS
MIMO	:	Multi-Input Multi-Output
MSE	:	Mean Squared Error
MMSE	:	Minimum Mean Square Error
NLMS	:	Normalized LMS
NLAEC	:	Nonlinear Acoustic Echo canceller
NARMAX	:	Nonlinear Autoregressive moving average with exogenous inputs
NVFF	:	Numeric Variable Forgetting Factor
NVFF-RLS	:	Numeric Variable Forgetting Factor Recursive Least Squares
QAM	:	Quadrature Amplitude Modulation
QR-RLS	:	QR Decomposition Based RLS
RLS	:	Recursive Least Squares

SS	:	Step-Size
STBC	:	Space-Time Block-Coded
STTC	:	Space-Time Trellis-Coded
SOVFs	:	Second Order Volterra Filters
SNR	:	Signal to Noise Ratio
SVSS	:	Stochastic-Gradient Variable Step-Size
SVSS-LMS	:	Stochastic-Gradient Variable Step-Size LMS
TDNNs	:	Time Delay Neural Networks
TVVS	:	Time-Varying Volterra System
TV	:	Time-Varying
VFF-RTLS	:	Variable Forgetting Factor Recursive Total Least Squares
V-BLAST	:	Vertical-BLAST
VFF	:	Variable Forgetting Factor
VSS	:	Variable Step-Size
VSS-LMS	:	Variable Step-Size LMS
VFX-LMS	:	Volterra Filtered-x LMS

CHAPTER 1

INTRODUCTION BASED ON LITERATURE REVIEW

Nonlinear-system-identification is an imperative topic that plays a pivotal function in various real-time problems in the field of signal processing and communication systems [1]. The nonlinear-systems exhibit specific degrees of nonlinearity, and those do not possess the superposition property. However, it is difficult to ascertain a mathematical paradigm for an unknown nonlinear-system through an “input-output” connection, in which the linear-system identification techniques [2] provide misleading results. The theory as well as applications of the nonlinear-system classification are as immense and diverse as the polynomial-systems themselves. Among several types of nonlinear-system models, the Volterra-system [3] is one of the most frequently used paradigms due to its roots in the Taylor-series expansion of the nonlinear-functions with memory. The Volterra-series corresponds to a nonlinear-system as a set of multi-dimensional convolutions [3]. It is used to model a range of nonlinear-systems such as auditory systems [4], mechanical systems [5], [6], electromechanical loudspeakers [7]. The Volterra-system is also recognized as the polynomial system, and it has found applications in the different fields of engineering, such as communication systems [8], [9], speech processing systems [10], image processing systems [11], and biomedical engineering [12]. The common applications of such a Volterra-series model comprise system prediction due to the random excitation and nonlinear amendment. For either of these applications, it is needed to implement the Volterra paradigm of the underlying nonlinear-system, which is achieved using a discrete-time signal processing algorithm termed as the “Volterra-filter”.

The Volterra-system paradigm is tremendously trendy in the nonlinear-filtering [13]. It has formed an identity of its own in the field of adaptive-filtering structures. Its two major types include a) adaptive-filters utilizing curtailed Volterra-series delineation of the nonlinear-systems; b) adaptive-filters incorporating the recursive nonlinear difference-equations to relate the inputs-signals with output-signals of underlying systems. A group of different nonlinear-systems comprising the Volterra-filtering structures can be estimated with random-accuracy using the bilinear-paradigms with predetermined number of filter-weights [14], [15], which are extremely striking from an implementation point of view. But, the problems associated with stability of such recursive-filters are quite large in case of nonlinear-configurations than for the linear-configurations [16], [17]. Unlike the case of linear-configurations, which are exclusively categorized by the systems' unit impulse response functions, it's not possible to determine a combined structure to describe the different nonlinear-systems. However, difficulties are substantially high in case of nonlinear-filtering configurations. An extremely pervasive-type of nonlinearity is the saturation-type nonlinear characteristic, which is a major cause of system instability in the field of electronics and communication technology. Therefore from stability point of view, the adaptive-filters utilizing truncated Volterra-series are preferable for the nonlinear-system-identification. Moreover, no general scheme exists to obtain the Volterra-filter coefficients for every type of nonlinear-filtering configurations. But, these filter-coefficients can only be determined for the system, which possesses finite and known order. In some circumstances, even the noise without finite second-order statistics adversely affects the performance of conventional nonlinear-system-identification methods. This is mostly true for impulsive-noise, which arises more commonly in the applications, such as seismology, geophysics, astrophysics, biomedicine, communications and underwater acoustics [18], [19], [20]. In such scenarios, alternative schemes need to be identified to classify the nonlinear-systems working under the impulsive-noise environment.

Although the linear-filters are beneficial in a large number of applications and comparatively straightforward from theoretical as well as implementation point of view, yet there are aforementioned real-time situations that need the nonlinear processing of signals engrossed, which necessitates the usage of nonlinear-filtering configurations equipped with the polynomial-paradigms of nonlinearities. However, drawbacks of Volterra-series elaboration are akin to those of Taylor-series elaboration – no expansion performs well when there is sudden abruptness in the system depiction. Moreover, the computational-complexity of implementing filters using the Volterra-system is reasonably high. But, the Volterra-series elaboration can be used for the representation of a large group of the nonlinear-filtering configurations, and it is also remarkable in the adaptive-filtering implementations because this expansion is the linear arrangement of nonlinear-functions of input-signals. And most of the real-time applications of polynomial-filtering configurations incorporating Volterra-series elaborations engross the low order paradigms.

The analysis of nonlinear-systems proposed by Wiener in [21] involves the white Gaussian input-signals and G-functionals to describe the nonlinear-filtering configuration behaviour. Subsequently, numerous researchers have explored and incorporated Volterra-series elaboration and corresponding depictions for the estimation/prediction as well as time-invariant nonlinear-system-identification in [22] – [32]. Under practical conditions in the time-varying environment, the adaptive-filter estimates the required response-signal using a second-order or a third-order curtailed Volterra-series elaboration, in which the error-signals (difference between the desired-signal and estimated-signal) may be expressed as a linear grouping of input-signals to every filter-coefficient. It makes the notional performance investigation of such filters comparatively simple extension of the linear-filtering applications. The least mean square (LMS) filtering configuration [33] updates the filter coefficient-vector at every instant of time using the steepest-descent procedure, which attempts to reduce the mean squared error to minimum. The adaptive Volterra-filtering configurations with TV convergence parameter values are given in [34]. The adaptive procedure engaged in these filtering configurations is the variant of sign algorithm [35], which is much easier to realize than least mean square algorithm. Further, the adaptive Volterra-filtering configurations based on the distributed arithmetic-realizations have been provided in [36], [37]. A gradient adaptive quadratic-filtering algorithm employing an LU-decomposition of the quadratic-kernel matrices is proposed in [38]. In case of adaptive linear-filters, the eigenvalues of auto-correlation matrix of the input-signal vector manage the convergence-rate through the step-size (SS). Generally, the larger eigenvalue-spread (ratio of the largest eigenvalue to smallest eigenvalue) leads to lower convergence speed, which appears to be troublesome in case of nonlinear-filtering configurations because eigenvalue-spreads are generally very immense.

However, recursive least squares (RLS) algorithms can improve convergence-mode behaviour, which is less reliant on the statistical properties of input-signals. This exponentially weighted RLS algorithm provides accurate key for the optimization-problem, in case of SOVFs, which depends on the value of forgetting-factor. This factor also manages the memory-length of adaptive-filtering configuration. Its computational-complexity can be reduced using the matrix-inversion-lemma. An efficient approximate RLS adaptive solution is presented in [39]. However, these approximations consider that input-signal to adaptive-filtering configuration is Gaussian. The underlying system fails to perform well when the input-signal corresponds to the non-Gaussian distributions [40]. It suffers due to the meagre numerical-characteristics possessed by fast-RLS algorithm. But, the polynomial-filtering [3], [41]–[46] is still an exciting and demanding field with a broad range of purposes, which are based on the Volterra-series and Wiener-series paradigms.

1.1 Volterra and Wiener-Series based Nonlinear-Systems

Nonlinear-system paradigms are categorized into nonparametric and parametric types. The nonparametric nonlinear paradigms include Volterra-series and Wiener-series paradigms [45]. The Volterra-model articulates the input-output association of a nonlinear-configuration in terms of the kernel's multi-dimensional operations playing a function similar to impulse response in the linear time-invariant system hypothesis. In continuous time-domain, the output-signal $y(t)$ in response to input-signal $x(t)$ is represented as

$$y(t)=\sum_{n=1}^{\infty}\int_{-\infty}^{+\infty}...\int_{-\infty}^{+\infty}h_n(\tau_1,\ .....\ ,\tau_n)x(t-\tau_1)\ ...\ x(t-\tau_n)d\tau_1...\ d\tau_n \qquad (1.1)$$

Here, kernel's value $h_n(\tau_1,\ .....\ ,\tau_n)$ weighs n-fold products of the input-sequences at the time-lags $\tau_1,\ .....\ ,\tau_n$, reminiscent of the multi-dimensional convolution-integration. It is noteworthy that the $n=1$ term in Volterra-series is the conventional linear convolution-integral. These kernel-functions are also suitable to represent this input-output association in frequency-domain. This frequency-domain paradigm includes multi-dimensional Fourier-transformations of the kernel-functions. Moreover, Wiener-series paradigm is an orthogonal description of the Volterra-series, which is devised to facilitate the analytical investigations, where input-signal is assumed to be Gaussian stochastic-process. However in parametric nonlinear-configuration paradigms, input-output detail is represented in terms of the mathematical function calculated by using the restricted number of parameters. Let us consider a quadratic-configuration described by the following equation

$$y(t)=ax(t)+bx^2(t) \qquad (1.2)$$

This system is entirely expressed in terms of two real valued numbers a and b. The output-signal is nonlinear corresponding to the input-signal, but it is linear corresponding to system-parameters. This condition can be checked by differentiating the output-signal with respect to these parameters. If equating to null, the partial-derivative of output-signal with respect to parameters leads to the linear-configuration equations (involving nonlinear-operations of the known input-signals), then this paradigm is the linear-in-parameters paradigm. These parameters can be "fit" to the observed input-output statistics by the straightforward linear techniques (such as LS method). The simplicity of connection between the output-signals and parameters is what makes the linear-in-parameters paradigms much desirable from the realistic applications viewpoint. Therefore, parametric paradigms can be sought as a particular case of nonparametric paradigms. If Volterra-series is curtailed to the N^{th}-order $\left(1 \le n \le N\right)$, then the output-signal is expressed as

$$y(t) = \sum_{n=1}^{N} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} h_n\left(\tau_1, \ \dots\dots \ , \tau_n\right) x\left(t-\tau_1\right) \ \dots \ x\left(t-\tau_n\right) d\tau_1 \dots \ d\tau_n \tag{1.3}$$

However, the causal, discrete-time, finite-memory, finite order Volterra-series paradigm can be represented as

$$y(t') = \sum_{n=1}^{N} \sum_{\tau_1=0}^{M} \dots \sum_{\tau_n=0}^{M} h_n\left(\tau_1, \ \dots\dots \ , \tau_n\right) x\left(t'-\tau_1\right) \ \dots \ x\left(t'-\tau_n\right) \tag{1.4}$$

with $t' = \dots\dots, -2, -1, 0, +1, +2, \dots\dots$

Here, the input-signal $x\left(t'\right)$ ought to be sufficiently rich in order to exclusively establish the underlying nonlinear-configuration. Moreover, the input-signals should fully excite/probe the underlying configuration. The input-signal must have adequate temporal variability to classify the dynamics of concerned configuration. Once a nonlinear paradigm is chosen, a criterion/cost-function is utilized to measure its fitness. The system-recognition is a procedure of estimating that paradigm, which optimizes this criterion. The most frequent cost-function is the summation of squared errors between the estimated-signal generated at the output of model and real-time measured output. The system-recognition problem includes determination of the kernel-function in Eq. (1.3). The output-signal is linear with respect to kernels. The squared error criterion-function is given as

$$J\left(x, y, h\right) = \int_{-\infty}^{+\infty} \left[y(t) - \sum_{n=1}^{N} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} h_n\left(\tau_1, \ \dots\dots \ , \tau_n\right) \prod_{k=1}^{n} x\left(t-\tau_k\right) d\tau_k \right]^2 dt \tag{1.5}$$

The linear-in-parameter paradigms are largely tractable among the available nonlinear-system depictions because squared error criterion-function is quadratic in terms of parameter values. Therefore, system-parameters that reduce squared error to the minimum can be obtained by the solution of linear-system equations. In general, main difficulty encountered in the linear-in-parameter paradigm identification is a large parameter-space. Here, LS solution can be significantly vulnerable to the additive noise in measurements.

However, nonlinear-system-identification includes a sequential method to update the parameter-value approximations, as the fresh input-signal and output-signal samples become accessible. A simple method for designing the adaptive algorithms is to improve the gradient-descent optimization techniques by swapping the actual gradient with the instantaneous estimate (local-gradient based on the fresh input-signal and output-signal sample values). Such adaptive procedures are analogous in type as well as character to the well-known LMS algorithm. However, the convergence-rate investigation for adaptive nonlinear-filtering algorithms is quite demanding than in its linear-filtering counterparts. With an appropriate input signal-sequence, if various system-modes are excited to altering degrees (unlike linear FIR-filtering system recognition, in which the white-noise input signal-sequences uniformly stimulate every system-mode), then these irregular stimulations can result in low convergence-rate in the system-identification procedures. Moreover, convergence-mode and tracking-mode performances of many nonlinear-system-identification procedures have not been meticulously investigated so far. Noteworthy exceptions comprise a few particular types of the linear-in-parameters paradigms.

1.2 Nonlinear-System-Identification using Volterra-Filters

In case of system-identification, the paradigm usually takes the form of linear or nonlinear (e.g., autoregressive moving average (ARMA)) difference-equation, where the significant terms and filter tap-coefficients are to be predicted/estimated. The nonlinear-systems can also be expressed by a functional-expansion (e.g., Volterra-series), where the Volterra-kernels are to be predicted/estimated, or by a parallel-cascade combination, where the static- and dynamic-elements / parameters are to be predicted/estimated. In case of nonlinear-system modelling, the discrete-time Volterra-series has been addressed in [47], in which the kernel estimation is performed using the parallel-cascade method. However in the time-series investigation, a non-Fourier sinusoidal-series scheme has been stressed. The frequencies, amplitude and phase angles are estimated by an orthogonal procedure. A search of the candidate sinusoidal is performed until the given mean squared error criterion is assured. Panicker *et al*. proposed the parallel-cascade realizations of the curtailed Volterra-filtering configurations in [48] with arbitrary but significant order of nonlinearities, which uses parallel as well as multiplicative groupings of the lower order Volterra-filters. It is a systematic approach to approximate the high order Volterra-configurations in the parallel-cascade formation utilizing less number of branches and a limiting mean squared error in the output-signals produced through these configurations. Pearson *et al*. consider a particular type of the second-order Volterra-models in [49], which focuses on the influence of structural restrictions and non-Gaussian input-sequences on the model identification problem.

However, system-identification with noisy input-output measurements has been particularly addressed in [50] through the optimization of mean squared error criterion, especially in case of adaptive-filtering. Here, the mean squared error is known to provide paradigms that approximate the conditional expectation of target-output. When the input-signal is also contaminated by noise, this method provides biased estimate of the paradigm parameters under the severity of bias-term, which is dependent on noise-power. Ozertem *et al*. [50] have proposed the application of main idea behind the error-whitening criterion to the unbiased recognition of second-order Volterra-series paradigms of the nonlinear dynamic-systems. However, this technique is not applicable to the higher order Volterra-paradigms. A discrete system with memory, inserted between the source and modulator, is designed in [8] with a goal to provide an equivalent channel with a distortionless linear part and no nonlinearities up to a given order. This compensator design is based on a Volterra-series channel-paradigm, and on the theory of p^{th} - order inverse systems. The existence of Volterra-filter equalization and the approximation of output error bound of the resulting system are reported in [51], when a p^{th}- order Volterra-equalizer instead of an exact Volterra-equalizer is connected in cascade with a nonlinear-system. The concept of local stability for a Volterra-system is introduced, and the algorithmic formulae of a p^{th}- order inverse equalizer via multi-dimension z-transform are presented. These Volterra-equalizers designed using the p^{th}- inverse method require a reduced number of order of the nonlinearity and kernel lengths, as compared with the fixed point method; such that the output errors tend to zero when an order p tends to infinity. However, the nonlinear intersymbol-interference is usually inevitable in the telecommunication and digital-storage channels. The decision-feedback equalizer alleviates these nonlinear effects by involving the adequate nonlinear feedback-filtering procedures. In [52], Tsimbinos *et al*. have proposed DFE with the nonlinear feedback-filter based on the discrete-time Volterra-series, in which the error-propagation, error-probability, stability and error-recovery-time results for the N^{th} -order nonlinear-channels are derived. Specifically, the travelling-wave-tube amplifiers or the solid-state power-amplifiers are incorporated in the satellite nonlinear-channels [53]. For higher than 4-QAM constellations, nonlinear characteristics of power-amplifier stage in the transmitter typically provide significant amplitude and phase-distortions to the transmitted-signal. In order to obtain higher data-throughput on the fixed bandwidth satellite nonlinear-channels, using higher order modulations, Volterra-model based nonlinear-channel equalizers are used. Particularly, the lower order Volterra-series based adaptive-filters are used for this application, especially if rectangular pulse-shaping is considered. This facilitates us to speed up the convergence-time of adaptive algorithm. However, the equalization of satellite communication using the Volterra-filtering provides similar results in comparison to the complex-bilinear recurrent neural network [54] and multilayer perceptron-type neural networks [55], as far as mean squared error and bit error rate criterion are concerned. The MMSE solution for Volterra-filtering configurations is presented in [44], which is based on the consideration that filter input-signal is normally-distributed. The iterative factorization method is used to develop a sub-type of Volterra-systems, which can significantly reduce computational-burden of filter operation. The efficacy of the Volterra-filter is illustrated by using it in the analysis of nonlinear drift-oscillatory behaviour of the moored-vessels subject to the arbitrary sea-waves.

The functioning of genetic-optimization in some nonlinear-systems in case of the active noise control based on Volterra-filtering is discussed in [56]. Though standard filtered-x algorithm converges to local minima, but the genetic algorithms can handle this problem ably. Additionally, this set of algorithms does not necessitate the identification of secondary-paths. However, applicability of genetic algorithm to the problem of parameter-optimization for adaptive FIR-filtering, combining the genetic and LMS algorithms is another innovative approach [57]. LMS algorithm computes the filter-coefficients and genetic algorithm searches the optimal step-size adaptively. As SS affects the stability and performance of adaptive algorithm, therefore it is necessary to apply a scheme to control it [58].

The MMSE problem is explicitly solved in [59] for the tandem connection of two SOVF systems. The optimum-solutions for a simplified linear-version of MMSE equalizer are also detailed. This simple equalizer is tested when incorporated in the nonlinear ultra-wideband receiver at the front end. The LMS compensator with a training-sequence has been used to test the performance of presented equalizer. The simulation results depict that LMS-equalizer is only capable to achieve the presented MMSE equalizer after a long training, which is not advisable in any communication system. An adaptive algorithm is presented in [60] to classify third-order frequency-domain Volterra-filtering coefficients, which is related to the discrete-time Fourier-transform of time-domain Volterra-filter coefficient-vector. This technique rests upon the block-LMS procedure based on overlap-and-save scheme. Previously suggested approaches [61], [62] to predict/estimate the frequency-domain Volterra-filtering coefficients minimize the frequency-domain output MSE. The estimated frequency-domain Volterra-filtering coefficients are found to be different from time-domain coefficients, and this is due to the periodicity-property of DFT. But, adaptive algorithm in [60] estimates the frequency-domain coefficients by minimizing the time-domain output MSE. However, the multistep-size frequency-domain adaptive-filter is competent to track both stationary- as well as nonstationary-signals [63]. It proceeds by employing a set of three SS values and a knowledge-based tactic to choose the optimum set of filter-coefficients, and thus the optimum SS value iteratively. The main advantage of the multistep-size algorithm over the conventional LMS is that better performance is always achieved without the knowledge of the input-signal characteristics, such as signal power, degree of nonstationarity, SNR and stability bounds. Based on analysis of nonlinear SOVFs, and utilizing the continuous signal's discrete-time cosine-transform domain second-order polynomial nonlinear-filter for the communication nonlinear-channels, a structure of nonlinear adaptive equalizer is proposed in [64], in which the adaptive algorithm is realized using NLMS approach. A flexible and efficient sub-band adaptive SOVF structure for the nonlinear-system-identification is discussed in [65], in which the filter-bank schemes along with adaptive-filtering algorithms are used. The presented configuration can attain equivalent system-identification performance compared with that of a fullband SOVF structure at low computational-complexity.

Volterra-filtering is a traditional tool for the equalization of nonlinear-channel and system modelling, noise excision, echo suppression, signal estimation or detection etc. But, computational-burden in Volterra-filtering exponentially increases with the degree of nonlinearity. The work presented by Bernardini in [66] is the contribution to well-organized computation of Volterra-filtering coefficients with generic-order nonlinearity existing in a range of communication applications. This method is based on the understanding of M^{th}-order single-dimension Volterra-filters in terms of the M-dimensional linear convolution, and it espouses a multi-dimensional fast-convolution technique. Moreover, a fast-convolution based on conventional multi-dimensional fast-Fourier-transformation in case of Volterra-filtering is outperformed by the direct-computational method. This scheme is beneficial because of the usage of typical multi-dimensional fast-Fourier-transform, which exploits the symmetries of signals encountered in the computation of Volterra-filtering algorithms, and which makes it better than the direct-computational method.

Further, adaptive procedures for the nonlinear-system-identification in short-time Fourier-transform domain are detailed in [67]. This adaptive technique encompasses the parallel grouping of the linear components, characterized by the cross-band filtering between sub-bands and quadratic-component, which is modelled using the multiplicative cross-components. The paradigm parameters are adaptively updated by least mean square algorithm. Avargel *et al*. [67] have also derived precise expressions for transient-state as well as steady-state MSE in the frequency-bins for the white normally-distributed input-signals. It is also shown that estimation of the nonlinear components enhances the mean squared error performance only if power-ratio of nonlinear-to-linear constituents is comparatively large. Further, if the number of cross-band filtering components increases, then low steady-state mean squared error can be attained at the cost of alleviated convergence-rate. However, literature in [68] is concerned with the identification of nonlinear-systems represented by Volterra-expansions and driven by stationary, zero-mean Gaussian-inputs, with arbitrary spectra that are not essentially white. The methods for the computation of Volterra-kernels both in the time- and frequency-domain are developed based on the cross-cumulant information. The derived kernels are optimal in the mean squared error sense for the noncausal systems, for which the order recursive procedures based on minimum mean square error reduction are also derived in [69]. Special emphasis is focused on the output response of a linear-filter driven by the higher order white-noise, which is concerned with the identification of second-order Volterra-systems using the general stationary input-signals. The Gaussian assumption is not always realistic. In contrast to existing methods [70], the cumulants rather than the moments are employed for the second-order nonparametric Volterra-system identification. This approach demonstrates that the cross-cumulant information between the input and the output leads to Fredholm integral equations of the second kind in the frequency-domain [69], [70].

Next, the identification of TV nonlinear-system based on a single-realization of the system input–output is considered in [71]. To enable the identification, the system's time-variation is estimated by the weighted sum of the known basis-sequences. Usage of wavelet-packet basis-sequences enhances the flexibility of underlying model, which allows a suitable-basis for selection. A basis selection method is presented using the best-basis algorithm to choose a minimum entropy wavelet-packet basis. Selecting individual sequences using the multiple hypotheses testing procedure attains a particular level of assurance that the final model encloses only significant sequences. This work can find applications in the field of biosignal processing and geophysics, which includes wavelet domain nonlinear-filtering for the magnetic resonance imaging denoising [72] and nonlinear adaptive methods for the classification of synthetic aperture radar images [73]. However, the parametric-complexity complicates design methods based on this paradigm. Such drawback for system-classification is explored in [74] using the fixed pole expansion scheme within the Volterra-paradigm configuration. The fixed pole expansion technique incorporates orthonormal basis-functions derived from the fixed-real of fixed-complex pole-positions to expand the Volterra-filtering kernels, and it reduces the number of predicted/estimated parameters. Subsequently, the gradient-descent algorithm that adaptively chooses the pole-positions is also developed.

The third-order filtering configuration with memory is recognized by using a novel implementation of LMS algorithm for the Volterra-filtering kernel approximation in [75]. The precision of presented algorithm is appreciated by the comparison of approximated third-order Volterra-coefficient with real coefficient. It is shown that adaptive algorithm exhibits adequate convergence under the diverse noisy conditions. The annexe of least mean square algorithm realization to high order Volterra-filtering is also promising, and it includes a few minor changes. However, the correlation properties of input-vector determine convergence-speed of LMS algorithm for the adaptive Volterra-filters. These are found to be optimum, when the nonlinear input-components are uncorrelated. Moreover, the correlation properties for the SOVFs depict that when input-signal is whitened, the nonlinear terms automatically become uncorrelated [76].

In particular, nonlinear adaptive Volterra-filtering configurations have been incorporated to model the nonlinear-channels encountered in the satellite-communication applications [77], [78]. The nonlinearity is basically due to the application of high power-amplifier employed in the transmission. When dealing with land-mobile satellite-communication systems, the channels are time-variant and can be modelled by a general M^{th}-order Markovian-paradigm to explain these variations [79]. Hence, to take into account the effects of amplifier's nonlinearity and channel time-variations, the equivalent baseband-channel is modelled by using the TV Volterra-filter. Besbes *et al.* [80] have analyzed the behaviour and parameter tracking capabilities of the adaptive Volterra-filters, governed by a generic LMS algorithm. In literature, the convergence analysis of adaptive Volterra-filters is usually carried out for the small step-size (SS). However from a realistic viewpoint, to maximize convergence-rate or to find out the critical SS, one requires a theory that is applicable for a wide adaptation SS range. The purpose of the research work in [80] is to give a technique tailored for finite alphabet input case. This situation is commonly faced in different digital transmission systems. The presented analysis, corresponding to the large SS case, is derived without any independence assumption. Gutierrez *et al*. [81] have evaluated the performance of Volterra-equalization structures and maximum-likelihood sequence decoding receivers for the compensation of signal distortion in the nonlinear satellite-communication receivers. Additionally, the performance of receiver with the fractionally-spaced-equalizer followed by the Volterra-equalizer is also presented. Here, tuning of the equalizer weight-vector is assumed including the multiple-step least mean square algorithm that enhances convergence-mode statistical properties. Due to minimization of the mean squared error, the nonlinear-equalizers can limit the noise-signal in a direction orthogonal to the direction of a decision-error. It is found that a substantial improvement in the mean squared error does not essentially translate to commensurate improvement in the error-probability. The fractionally-spaced Volterra-equalizer provides enhanced performance relative to Volterra-equalizer, if the receiver filter is not matched to the encountered channel.

By considering the Volterra-filter as a pseudo-linear operator, adaptive algorithms for iteratively determining sets of filter-weights that satisfy the MMSE criterion have been introduced in [82]. In this category of algorithms, the method of least mean square is an important means of filter adaptation. Due to the varied output-response of the Volterra-filter's processing-stages to least mean square adaptation, it is usually desirable to implement a parallel adaptation scheme, in which different SS values are incorporated to adapt the different sets of weights at the various processing-stages. The calculation of bounds for each SS to guarantee the mean square convergence is not commonly tractable. This results in ad hoc methods for the design of LMS step-sizes in case of any typical application. If Volterra-filtering is articulated as a series of constrained optimization-problems, starting with a most favourable linear-filter, and then proceeding to the higher order filtering configurations while maintaining the lower order filter-weights fixed, then a partial-decoupling of the filter-kernels is achieved. This in turn leads to the development of parallel adaptive algorithms, whose SS values are restricted not by the inverse of power in whole Volterra observation-vector, but these are restricted only by the power in parts of observation-vector of the same or lower order. This permits us to devise a set of weights that allows lower order weights to converge more swiftly. This also provides opportunity to design a modular-filtering configuration, in which the higher order processing stages can be added to a conventional filter without involving recomputation of lower order weights. An amendment of Volterra-filter, in which the filter-weight vector of a specified order is optimized independently, is given in [82]. Using this approach, the MMSE filtering problem as the succession of constrained optimization-problems is solved. It produces a partially-decoupled normal-equation for the Volterra-filtering algorithm. From this normal-equation, it is possible to devise an adaptation routine that utilizes the philosophy of partial-decoupling, which is analogous to Volterra-filtering LMS algorithm, but with noteworthy structural changes that permit a simple derivation of bounds on the algorithm's SS values, subsequently these bounds are dependent on the respective diagonal-blocks of the Volterra auto-correlation matrix. It brings into being a trustworthy set of design strategy that permits speedy convergence of the lower order weight-vector.

The NLMS algorithm is a significant alternative of the traditional LMS procedure for the adaptive linear-filtering. It exhibits a lot of benefits over the LMS algorithm, including higher convergence-rate and a regular TV choice of SS parameter that affects the stability as well as steady-state mean squared error. A supporting FSS that is frequently introduced in the NLMS algorithm has the benefit that its stability region is independent of the signal-statistics. Kalluri *et al*. [83] have generalized the NLMS algorithm by presenting a category of nonlinear NLMS-type algorithms, which are appropriate for a wide variety of nonlinear-filter configurations. A conventional nonlinear NLMS-type algorithm by choosing an optimum time-varying SS is acquired in [83] that minimizes the next-step mean squared error at each iteration of the traditional nonlinear LMS-type algorithm. A dimensionless supplementary SS, whose stability-range is independent of the signal-statistics, is also introduced. The stability-region can be obtained empirically for any known nonlinear-filter type. Under similar conditions, the results signify that NLMS-type algorithm, in general, converges faster than its LMS-type counterparts.

A reduced complexity version can be attained by employing the singular-value decomposition to the coefficient-matrix in order to achieve an approximation based on its most significant eigenvectors. The research work presented in [49], [84], [85] constrain the filter configuration to offer convergence to a distinctive solution. Though these techniques can efficiently elevate complexity of the operational structure, but the output of underlying system turns out to be a nonlinear-function of the filter-coefficients. Therefore, the estimation of filter-coefficients develops into a nonlinear-estimation problem about the global optimum value. Moreover, the stability as well as convergence performance of adaptive algorithm can not be established by the parallel-cascade structural design. In [86], a new adaptive joint-process-filtering configuration using the pipelined feedforward second-order Volterra structural design to diminish the computational-burden is presented. This design is composed of two sub-sections: a nonlinear sub-section performing a nonlinear-mapping from an input-space to the intermediate-space using the feedforward SOVF; and a linear-combining structure performing the linear-mapping from the intermediate-space to an output-space. Sicuranza *et al*. [36] suggest the possibility of exploiting memory-oriented configurations that properly match the characteristics of distributed-arithmetic, for designing the adaptive nonlinear-filters, which are expressed by using the truncated discrete-time Volterra-series. As in such configurations, the filter-coefficients do not emerge in an explicit-form, it is essential to alter the general LMS adaptation algorithms. Therefore, using the matrix-notations, two different adaptation algorithms are devised for the conventional nonlinear-operator of order-k in case of Volterra-expansion in [36].

Various authors have shown the utility of SOVF updating using the LMS-type method for a wide domain of applications. Some investigations of the performance of these configurations have been given by Koh and Powers in [44], who have suggested the usage of optimal LS second-order filtering procedures for the Gaussian input-signals. They have also explored the stability limits along with MSE (misadjustment) outcomes for second-order LMS-type filters, once more using Gaussian input-signals. Dokic *et al*. [87] observe the effects of bias or DC-component, on characteristics of SOVF using LMS algorithm. Without any DC-component, the filter can itself generate a biased output-signal. It is shown that the inclusion of a DC-component consistently deteriorates the performance of adaptive systems, in terms of the convergence-rate and steady-state error. Dissimilar to customary LMS, second-order configuration converges non-uniformly, even in the case of white input-signals. However in a number of cases, the DC-term in fact brings into being an elementary change in the gross behaviour of the underlying system.

The block-LMS (BLMS) algorithm is presented in [88], which is a generalization of the well-known LMS (stochastic gradient with a constant gain). The BLMS, which averages the instantaneous gradient over a fixed length measurement block, is characterized by the slow adaptation characteristics (inversely proportional to the block length). However, it is in particular suitable for those cases, where the measurement rate is higher than the available computational-power. Such algorithms can serve in the polynomial identification problems, where the large dimension implies high computational demands, even in the relatively simple algorithms. An example of the usage of BLMS algorithm in the identification of nonlinear-systems is presented by Im and Powers in [60], where the third-order Volterra-system is recognized by estimating its frequency-domain coefficients. However, the problem of time-varying systems is not considered here, although constant gains are employed (keeping the adaptive algorithm “alive” for potential tracking). It is well-known that while stability is preserved under the small gains, good tracking is provided by the large gains. But the VSS-LMS, in which the algorithm’s gain is kept proportional to the square of output estimation-error, is another approach that allows good tracking. The design of adaptive algorithms for the tracking of smoothly TV systems is explored in [89]. A criterion for appraising the tracking potential of an algorithm in such situations was proposed in a previous work given in [33]. The range of credibility of this criterion is demonstrated to be much wider than the expected. On contrary, the multistep algorithms (presented in the Soviet-literature) are generalized and systematically explored; they are found to show substantial improvements over the traditional (one-step) procedures for the tracking.

In [87], authors demonstrate that zeroth-order component deteriorates the stability-bound and also enhances the steady-state mean squared error to some extent. But, no quantitative assessment of mean squared error deterioration is discussed. In general, the earlier works assumed lossless conditions, when zeroth-order component is eliminated in the SOVF paradigm. Without zeroth-order component, the knowledge of required output-mean is stringent requirement. Adaptive zeroth-order component is beneficial, if the required output-mean is unknown TV. Sayadi *et al.* explores the steady-state characteristics of LMS adaptive SOVF with the zeroth-order expression for the Gaussian input-signals in [90], in which least mean square procedure based updating equations are presented. Subsequently, steady-state characteristics of recursions are investigated for the random-walk paradigm for an unidentified system parameter values, and steady-state excess mean squared error characteristics are also appraised. The theoretical performance results are illustrated to be in adequate agreement with the simulation outcomes, particularly for the low values of SS. The least mean square convergence-mode and tracking-mode properties are analyzed in [90], when utilized for the classification of nonstationary linear-filtering configurations. The mean squared error is revealed to engage the tradeoff between the gradient-error (elevating with SS) and lag-error (alleviating with SS). This investigation describes zeroth-order component adaptation for SOVF model. Its key contribution comprises of the evaluation of steady-state mean squared error related to the filter's quadratic-component and zeroth-order component.

Different time-varying SS procedures have been suggested to improve the characteristics of least mean square algorithm. Testing with such algorithms reflects that their statistical properties are highly susceptible to noise. However, it is recognized that final excess mean squared error is directly related to SS, while its convergence-time rises as the SS value falls. The innate drawback of least mean square algorithm also necessitates the settlement between the opposing primary needs of fast convergence-speed and the low level misadjustment required in many of the adaptive-filtering applications. Consequently, different authors have continuously gazed for substitute means to enhance its characteristics. A famous scheme is to incorporate a time-variant SS in the conventional least mean square weight updating recursion [91]–[94]. It is based on the usage of high SS, when adaptive algorithm is faraway from the optimum-solution, and therefore accelerating the convergence-rate. When this algorithm reaches close to optimal-state, low SS values are utilized to attain the low-rank of misadjustment, and hence attaining superior overall system performance. It can be achieved by adjusting SS in agreement with a criterion that can give an estimated measure of adaptation-process. Various criteria are employed: squared instantaneous-error [91], sign alterations of succeeding samples of gradient [93], attempting to decrease squared error at every instant of time [92], or cross-correlation of inputs and error [94]. The performance of recently proposed variable step-size (VSS) LMS algorithms is quite perceptive to noise [63], [95]. Its beneficial characteristics over the conventional least mean square algorithm is usually achieved only under high SNR environment. It is intuitively apparent by observing that the criteria controlling SS updating of such adaptive procedures have been directly calculated from instantaneous-error, which is corrupted by noise. As measurement-noise is inevitable in every real-time system, therefore efficacy of adaptive algorithm is reviewed by its performance under noisy environment. A robust VSS-LMS-type algorithm, exhibiting high convergence-rate at early states of adaptation while assuring lower misadjustment, is presented in [96]. Its performance is not affected by existing uncorrelated disturbance signals. An approximate investigation of convergence-mode and steady-state performance for the zero-mean stationary Gaussian-inputs and for nonstationary optimal weight-vector is also given. The simulation outcomes comparing the presented algorithm to the conventional VSS algorithms [97] evidently specify its better performance under the stationary as well as nonstationary environments.

In a nonstationary situation, the optimal value of SS parameter of least mean square algorithm strikes equilibrium between the amount of lag-noise and gradient-noise [98]. However under real-time scenario, the optimal value of SS parameter cannot be calculated *a priori* due to unknown channel parameter values. Additionally, a fixed step-size may not respond to the time-variant channel parameter values, leading to the poor tracking-mode performance. For dealing with this problem, a number of VSS-LMS algorithms have been designed in last two decades. Two major classes of these adaptive algorithms, which utilize the gradient-descent scheme to adjust SS value, are Mathews' algorithm [92] and Benveniste's algorithm [99], [100], focusing on a general SS value. One benefit of Mathews' algorithm over Benveniste's algorithm is its straightforwardness. However, this causes deterioration in the performance, when the measurement-noise is high relative to the time-variations in noise at the filter-output due to the plant's TV impulse response. In such case, the adaptation parameter employed to update the LMS algorithm SS parameters is required to be relatively small for the MSE to converge close to its optimal value. As a result, convergence speed of the SS parameters gets reduced. However, Benveniste's algorithm supersedes Mathews' algorithm by smoothing the gradient-vector estimates, and it is computationally more complex than Mathews' algorithm. There are some variations of the basic algorithms of Mathews as well as Benveniste. One of them employs multiples of FSS to update the different Volterra-filter tap-weights, leading to the multiple step-size LMS algorithm [92], [101]. This exhibits better tracking characteristics as compared with using a common SS value when categorizing a two-path communication channel [101]. It also surmounts the problem of compensating nonexistent tap-weights of the communication channel [102]. One more field, where the multiple SS algorithm is effectual, is in acoustic echo excision/suppression [103]. Yet another variant of VSS-LMS algorithm is there, when the linear update recursion of SS is replaced by the multiplicative update recursive procedure [101]. The gradient adaptive step-size LMS algorithms (main family of VSS-LMS algorithms) are revisited in [104]. A generalization to a group of the studied algorithms is presented. It is illustrated that this results in a fresh class of VSS-LMS algorithms with reduced complexity, but with no apparent loss in the performance. Two novel gradient-based VSS-LMS algorithms are given in [105], based on a concise assessment of the weaknesses of earlier VSS-LMS algorithms in the high measurement-noise scenarios. The first algorithm is intended for those applications, where the measurement-noise signal is statistically stationary and the second is primarily for the statistically nonstationary noisy environment. These algorithms are also verified through simulation to attain both a high convergence-rate and a low steady-state excess MSE, and to outperform existing VSS-LMS adaptive algorithms. Statistical investigations of LMS adaptive algorithm with uncorrelated Gaussian-data is described in [95]. Exact mathematical expressions for the steady-state MSE and the performance deterioration due to the weight-vector misadjustment are also derived. Essential and adequate conditions for the convergence of algorithm to an optimum (Wiener) solution within a finite-variance are investigated. It is established that the adaptive SS coefficient, which controls the convergence-rate of algorithm, must be restricted to an interval notably smaller than the domain generally stated in literature. The outcome of this work, therefore, places fundamental limitations on the mean squared error performance and convergence-rate of LMS algorithm. In addition, the characteristics of this algorithm are studied when a power-of-two quantizer algorithm is used, and the finite word-length effects are also considered. Two CMOS implementations of the variable step-size, power-of-two quantizer algorithm are presented to demonstrate that the performance gains are attainable with only a modest increase in the circuit complexity.

A novel LMS-type adaptive-filtering procedure with a VSS is introduced in [91]. The SS rises or falls as the MSE increases or decreases, permitting this filter to chase variations in the system as well as to yield a small steady-state error. These results reduce to well-known ones, when specialized to the constant SS case. Kohli *et al*. [106] have proposed a modified version of the two-step LMS-type adaptive algorithm encouraged by the innovative ideas of Gazor [107], which describes the nonstationary adaptation properties of this tailored two-step LMS-type (MG-LMS) algorithm for TV system-identification problem. It guarantees the stable behaviour during convergence-mode as well as the enhanced tracking performance in the smoothly TV environments. The estimated weight increment-vector is utilized for the prediction of weight-vector for the subsequent iteration. The presented amendment involves the usage of a control-parameter to scale the estimated weight increment-vector in addition to a smoothing-parameter used in the two-step LMS-type (G-LMS) algorithm, which controls the initial oscillatory-behaviour of adaptive algorithm. This investigation focuses largely on the effects of these parameters on lag-misadjustment in the tracking-process. The mathematical investigation for a nonstationary case, where the plant-coefficients are considered to follow a first-order Markov-process, illustrates that MG-LMS algorithm gives less lag-misadjustment than traditional LMS and G-LMS algorithms. Subsequently, stability criterion impresses upper-bound on the value of control-parameter. The outcomes clearly manifest that lag-misadjustment diminishes with increasing values of the smoothing-parameter and control-parameters under acceptable limits.

It is revealed in [108] that how the usage of a newly introduced algebra, known as V-vector algebra, can directly result in the realization of Volterra-filters of some order in the form of a multi-channel filter-bank. Each channel in this scheme is modelled as FIR-filter. The channels are hierarchically organized as per the number of filter-coefficients. In this way, it is also likely to develop models exhibiting reduced complexity by eliminating the less significant channels. This paradigm is further used to devise competent adaptation algorithms in the framework of nonlinear active noise control (ANC). Particularly, it is depicted that how the affine-projection algorithms employed in linear case can be extended to a Volterra-filter of some order. The derivation of filtered-x affine-projection algorithms for the nonlinear ANC systems is simply acquired using the elements of “V-vector” algebra. Such algorithms can economically replace the conventional least mean square and NLMS algorithms, which are frequently incorporated in this field, particularly when, a reduced-complexity multi-channel configuration can be exploited. The research work in [109] gives an investigation of transient-state and steady-state characteristics of diverse filtered-x affine-projection algorithms. The algorithms appropriate for the single-channel and for the multi-channel ANC systems are treated within a unified structure. However very soft assumptions are posed on ANC system paradigm, which are only needed to have a linear dependence on the output from filter-coefficients. Hence, this analysis is applicable not only to the linear FIR models, but also to the nonlinear Volterra-filters, i.e., polynomial-filters as well as other nonlinear-filter structures. The convergence study explored in [109] relies on the energy preservation arguments. This does not apply the independence theory, nor does it inflict any restriction to signal distributions. It is demonstrated that filtered-x affine-projection procedures always offer the biased approximation of MMSE solution. Nevertheless, in numerous cases, the bias-term is small and hence these algorithms can be beneficially applied to ANC.

A filter based on the second-order Volterra-series and on an IIR-filtering configuration is suggested in [110]. Such filters are capable to model more than second-order nonlinearities for the nonlinear-systems, where these nonlinearities are found to possess harmonically related association. The computational-complexity of this filter is undoubtedly less than the adaptive SOVF and third-order FIR Volterra-filters. Monin *et al*. [111] have presented a technique based on Volterra-type processing to identify various independent sources on single carrier-frequency and to decide number of sources. The usage of infinite impulse response Volterra-filter to design an appropriate discrimination test is governed by the requirement of higher order moments in such kind of nonlinear problem. This is only through the higher order moments that the multi-sources are perceptible. The usage of IIR-filtering before detection is governed by the need to obtain an accurate optimum-filter, as a function of the accumulated-data, with a known Volterra degree and state-space dimension. The latter property is important because FIR-filters do not optimize the performance for a known state-space dimension with memory, as these do not permit state information feedback. The optimum IIR-filtering is infinite-dimensional, if no constraint is imposed on state-space dimension. It is also illustrated that entire estimation and detection task can be performed in a finite recursive way, for phase-modulation. If frequency-modulation is considered, part of the *a priori* higher order moments has to be determined off-time by an appropriate Monte-Carlo procedure using the stochastic particles.

However for the specific-type of polynomial-systems, where the higher order nonlinearities are produced through recursion, the recursive second-order polynomial-filter provides enhanced performance over nonrecursive Volterra-filtering configuration. In [112], an alternative solution for the higher order Volterra-filtering configurations is suggested by introducing recursive second-order IIR-filtering. Due to recursive nonlinear feedback in the recursive SOVF, it has various applications for modelling particular groups of recursively generated higher order nonlinearities. It is utilized to minimize the number of filtering-coefficients in comparison to the nonrecursive nonlinear-filters. Volterra-filters as well as higher order statistics are important tools for the nonlinear analysis, processing and modelling. Nowak [113] has developed penalized LS estimation method for the Volterra-filtering and higher order statistics, and it is depicted that the penalized least squares estimation is a very effective way to incorporate prior information of the problem at hand without directly constraining this estimation scheme, which produces much reliable estimates. However, blind single-input-single-output Volterra-system identification from a second-order statistics-domain into a third-order statistics-domain is proposed in [114]. The full-sized Volterra-system with finite order and memory is stimulated by unobservable independent identically-distributed stationary random-sequences. It is well-known that blind identifiability is impossible in the second-order statistics-domain. Although these concluding remarks are also true in the higher order statistics-domain, yet it is demonstrated that under sufficient conditions, a large set of sparse Volterra-systems can be recognized blindly in the third-order moment-domain than in its second-order counterpart. Usual approach for the blind-deconvolution of single-input multiple-output Volterra FIR systems is expressed in [115]. These nonlinear-systems can be blindly equalized incorporating just linear FIR-filters. This scheme needs that Volterra-kernels satisfy a particular coprimeness condition. Here, input-signal possesses a minimum persistence-of-excitation order. In wireless communication systems, the functioning of amplifiers near saturation is usually needed for the efficiency reasons, leading to a nonlinearly distorted signal at amplifier output [116]. A famous model for the related baseband equivalent nonlinear-channel is a curtailed Volterra-series. The bandpass nature of wireless channels and the statistical characteristics of phase-shift keyed signals are exploited, and it is manifested that different components in Volterra-series are white as well as mutually uncorrelated. These results are useful when considering blind equalization schemes for this category of systems. But, it is also noteworthy that the instability is a major issue related to IIR adaptive-filtering due to the nonlinear phase. Therefore, our major focus is on the FIR adaptive nonlinear-filtering and its applications.

It has usually been found more necessary in the engineering of data-transmission systems to consider the impulsive-noise rather than the Gaussian-noise. The exclusive characteristic of impulse-noise is that the amplitudes are relatively much larger at the low frequencies in the case of impulse-noise as compared to the Gaussian white-noise [18], [117]. Conventional polynomial-filtering theory, based on the linear grouping of polynomial-terms, is capable to estimate significant categories of the nonlinear-systems. But, this linear blend of polynomial-terms results in meagre performance under the noisy environments characterized by heavily-tailed distributions. Barner *et al*. have extended the weighted median-filter to the group of polynomial weighted median-filters [118]. The weighted median-sample selection procedure is obviously elaborated to a polynomial-sample case, which provides a filtering-structure that exploits higher order statistical characteristics of the measured-samples, while concurrently being immune to outliers. This polynomial weighted median-filtering category is well inspired by a study of cross-component as well as square-component statistics, which rests upon the assumption that measured samples follow Laplacian-statistics. An important shortcoming of this filter is that its output is constantly constrained to a sample in window. However, a group of polynomial weighted myriad-filters [119] is presented as nonlinear-filtering structure with higher statistical competence for the bell-shaped impulsive-distributions, and those come into view in real-time as a consequence of natural phenomena [120], [121], akin to $\alpha-$stable distributions [122], [123]. Therefore, polynomial weighted myriad-filter is found to be suitable for the impulsive-noise environments, due to optimality of weighted myriad-filter. But, the computational-complexity and stability related issues limit its utility in practice.

The polynomial-filters efficiently address a lot of applications, in which the linear techniques are considered to be suboptimal, like equalization of the nonlinear-channels [9], [124] and also nonlinearity-compensation in the echo excision [125]. The simplified Volterra-filters [126] are also incorporated for AEC in GSM receivers [127]. This is also well-known that a number of optical-transformations are explained by the quadratic-term of Volterra-series [128], which result due to quadratic-relation between the optical-intensity and optical-field. The Volterra-filters are also effectual in ANC of nonlinear noise-processes [129]. The polynomial-system paradigms are also used in many communication applications. Such applications comprise modelling of the highly distorted reference-channels [130], nonlinear transmission-amplifiers [131], and the nonlinear bandpass-channels [8] in the digital transmission-systems. In image processing, the polynomial-filters are employed to perform image-enhancement [11], edge-extraction [132], edge-enhancement [133], and nonlinear prediction [134] issues. The Volterra-filtering is profitably applied for the improvement of noisy-images of curvatures [135]. Many practical problems are efficiently presented with SOVFs, those are bounded to embrace only first-order component and second-order quadratic-components. However, second-order filters are used to effectively detail problems, like the optimum-signal detection in the presence of Gaussian-noise and texture-discrimination. In restrictive second-order case, polynomial-nature of Volterra-filtering structures results in pitiable performance under the environment characterized by the heavily-tailed distributions. Undoubtedly, the quadratic-components dwelling in second-order kernels of filter intensify the influence of outliers. The harmful effects of outliers are still more prominent, if filter order is considered beyond two. However, a memoryless polynomial-filter design is presented in [136], in which the polynomial-filter is separated into a linearization-filter and a finite impulse response filter. The concept of linear LMS filter is extended to derive a memoryless polynomial least mean square adaptive-filter. It provides an effective solution to the orbit object trajectory [136], [137] as well as tracking in the field of aeronautics and aerospace engineering.

Various signal processing problems, like plant-modelling, noise-cancelling, channel-equalization can be expressed using the polynomial transfer-function, whose output-signal is contaminated by the additive independent zero-mean disturbance. This transfer-function of plant can be established preferably in an iterative way. This may be done by reducing a specific statistical measure of error to the minimum. Generally minimization is conducted in the mean squared sense, i.e., one reduces the expected value of squared error to the minimum, which generally appears in research archives, in the framework of investigation of the Gaussian-processes. A family of steepest-descent procedures for the adaptive-filtering has been designed in [138], which allows error reduction in the mean-fourth and mean-sixth sense. Throughout adaptation, the weight-vector experiences exponential relaxation towards its optimum-solution. Time-constant values are also derived in [138], and astonishingly these found to be proportional to the time-constant values that would have been attained if steepest-descent LMS algorithm of Widrow-Hoff had been employed. Moreover, behaviour of the least mean fourth (LMF) procedure is of exceptional interest. In comparison of proposed algorithm with least mean square adaptive algorithm, these are set to exhibit precisely equivalent time-constant values for the filter-coefficient relaxation-process. The LMF procedure exhibits a significantly low weight noise than least mean square algorithm. This is likely that the minimum mean fourth error procedure can do relatively superior job of LS approximation than the MSE algorithm. This interesting notion has connotations for all types of adaptive schemes, whether these are based on steepest-descent or otherwise.

A multi-channel controller based on Volterra-filters is detailed in [112]. A filtered-x affine-projection algorithm is devised for uniform quadratic-filters. The presented algorithm can also be elaborated to the higher order Volterra-kernels, and it involves the linear controllers as a specific case. A scheme to improve the performance of secondary-path modelling for ANC systems is provided in [139]. This scheme can further alleviate the distortion introduced by primary-noise, which also provides faster convergence and higher modelling accuracy. A new nonlinear adaptive procedure called the filtered-s LMS (FS-LMS) procedure for the multi-channel active control of nonlinear noise-processes is proposed in [140]. A low complexity FS-LMS procedure utilizing filter-bank technique is also recommended. It is depicted that this scheme supersedes the standard filtered-x LMS and second-order Volterra filtered-x LMS (VFX-LMS) algorithm in controlling such noise-processes. The computational-complexity investigation depicts that this scheme includes less number of computations in comparison to the second-order VFX-LMS procedure. Reddy *et al*. [141] attempt to design accurate realization of some nonlinear active noise control (ANC) algorithms, i.e., FS-LMS and VFX-LMS. The idea of reutilizing a part of computations performed for the first-sample while calculating the next-sample, for a block length of two-samples, is exploited to incorporate the fast and accurate versions of the FS-LMS and VFX-LMS algorithms, which are computationally competent. The latest research has illustrated that the linear ANC systems can be effectively employed to decrease the wideband- and narrowband-noise [129]. Particularly, linear active noise control systems are competent in diminishing the lower frequency noise-components. Under a few situations, the noise originating from TV systems can be nonlinear and deterministic noise-processes rather than random, white or tonal noise-processes. The principal noise-component at the cancelling point can possess polynomial-distortion. Further, secondary-path estimate in active noise control system, which indicates the transfer-function in-between secondary-source (secondary-speaker) and error-microphone. And it can possess non-minimum phase, and therefore violation of causality restriction is said to occur. Under such conditions, linear active noise control system undergoes deterioration. In [129], a realization of VFX-LMS algorithm based on a multi-channel configuration is detailed in case of feedforward ANC. The arithmetical simulation outcomes demonstrate that the devised algorithm accomplishes performance modification over the conventional FX-LMS algorithm under following scenarios: 1) Reference-noise is the nonlinear noise-process, and simultaneously, secondary-path approximation is of non-minimal phase; 2) Primary-path possesses nonlinear characteristics. Additionally, this VFX-LMS algorithm may also be incorporated as appropriate option in that case, where conventional FX-LMS algorithm fails to work well. The reference-channel and error-channel of active noise controllers can be saturated in practice [142], if noise-level crosses the dynamical-range of electronics appliances. The nonlinear saturation deteriorates the characteristics of active noise control systems that utilize linear-filtering using FX- LMS algorithm. Authors derive a bilinear FX-LMS procedure for the nonlinear-filtering to resolve drawbacks of signal-saturation and some nonlinear-distortions that happen in active noise control systems employed for real-time applications. However, performance of such bilinear adaptive-filtering configurations is appraised in terms of the convergence-rate, residual-noise in the steady-state and for arbitrary filtering memory-spans. The simulation results confirm that nonlinear adaptive-filtering with related bilinear FX-LMS algorithm is much efficient in eliminating the saturation-effects in active noise controllers than the linear-filtering as well as nonlinear Volterra-filtering with FX-LMS algorithm.

Generalized inverse approach is used to derive two new fast adaptive recursive least squares algorithms in [143]: an exact and stable initialization algorithm for the prewindowed signal cases. An analysis of the fast RLS algorithm is also presented. The partitioning techniques are directly applied to the signal matrix rather than the covariance-matrix. This method has better numerical properties than the conventional fast algorithms, principally when the Kalman gain is concerned. The presented relations can be used to derive other variations of the fast RLS algorithms. These can also be used to develop new “rescue” schemes. Finally, a numerical analysis illustrates some pitfalls of the fast RLS algorithms. Further, the one-to-one correspondences between the recursive least squares and Kalman-filter variables are exploited in [144] to formulate extended forms of recursive least squares algorithm. Two particular types of the extended RLS algorithm are investigated: one associated to a system-identification problem and other pertaining to the tracking of chirped sinusoidal-signals in the presence of additive noise. The presented results clearly indicate the tracking superiority of the extended-RLS algorithms as compared to the conventional recursive least squares and least mean square adaptive algorithms. Nonlinear-filtering schemes are widely utilized for the recognition of nonlinearities in diverse applications. Budura has investigated the performance of Volterra-estimator by considering the nonlinear-system-identification application in [145]. The Volterra-estimation parameters are compared to the linear-estimation. For a nonlinear-estimator based on SOVF using RLS algorithm, a new configuration is presented. The practical results illustrate that Volterra-identifier has superior performances than linear-identifier, in which different degrees of nonlinearities for the nonlinear-systems are assumed. In this scenario, QR-RLS procedure for nonlinear-filter is proposed in [146]. This procedure is based exclusively on the Givens-rotations. Therefore, this algorithm exhibits numerically stability and agreeable parallel-realizations. The computational-burden of this procedure is comparable to fast transversal Volterra-filtering configurations. This method is based on the curtailed second-order Volterra-series paradigm. And this may be simply extended to different kinds of polynomial-nonlinearities. This procedure is devised by transforming a nonlinear-filtering configuration into a similar multi-channel linear-filtering configuration with the arbitrary number of tap-coefficients in every channel. These multi-channel algorithms were unavailable previously even for any adaptive linear-filtering application. The origin of this procedure rests upon the channel-decomposition approach, which includes processing of channel in chronological way at each iteration. It excludes matrix processing, and results in a scalar realization. The related outcomes specify that this algorithm retains high convergence-rate of RLS Volterra-filters and exhibits numerically stability.

A fast-RLS adaptive nonlinear-filtering model using the second-order Volterra-series expansion is provided in [147]. This structure uses the concepts of fast-RLS multi-channel filters. It possesses computational-complexity of $O\left(N^3\right)$ multiplications, where $N-1$ denotes memory-length in number of samples of nonlinear-system paradigm. The theoretical performance investigation of its steady-state behaviour in stationary as well as nonstationary environments is explored. This study manifests that if input-signal is zero-mean normally-distributed and the adaptive-filtering is operational under stationary conditions, then steady-state excess MSE generated because of the coefficient noise-vector is independent of the characteristics of input-signals. The steady-state behaviour predicted/estimated by this investigation is in agreement with practical outcomes. A class of parallel Volterra-filters to reduce the computational-complexity is presented in [148]. However in [149], a partially-decoupled variation of recursive least squares algorithm is considered. This is based on the constrained-optimization of cumulative filter-error utilizing higher order sets of filter weight-vectors to improve the performance of lower order sets of filter weight-vectors. From this constrained-optimization, a recursive algorithm is designed, whose structure strongly looks like the conventional Volterra recursive least squares algorithm. But, it exhibits structural differences that arise from eliminating the dependence of lower order filter-weight sets on the higher order filter-weight sets. The resultant algorithm, while sub-optimal, needs less computational endeavour than the fully-coupled version. It converges to steady-state in same duration, without a significant deterioration in performance. Griffith *et al*. [149] have developed a partially-decoupled deterministic normal-equation for Volterra-filters that is similar to the partially-decoupled stochastic normal-equation designed using the constrained-optimization of sets of filter-weights. Using this tailored normal-equation, a partially-decoupled Volterra recursive least squares algorithm has been developed in a fashion akin to the derivation of a partially-decoupled Volterra least mean square algorithm. A benefit of the partially-decoupled recursive least squares algorithm is that it needs fewer computations per iteration in execution than the conventional RLS algorithm. The partially-decoupled algorithm also converges in mean square sense in nearly the same number of iterations as the fully-coupled adaptive algorithm. This is particularly correct, if the observed-signal is Gaussian, and the second-order partially-decoupled and fully-coupled filters are similar. A novel realization of third-order RLS Volterra-filtering based on decomposition of input-signal vector is proposed in [150]. Its characteristics are evaluated for the typical nonlinear-system-identification application. Comparisons, relying on filtering error, are made in every case using the linear identifier, such that the presented nonlinear identifier outperforms the linear one. In this research work, Budura *et al*. [150] have suggested new realization of third-order RLS Volterra-filter using the decomposition of input-signal vectors and symmetric kernels.

The RLS algorithm is extensively utilized in adaptive-filtering, self-tuning control appliances and parameter-identification. The conventional RLS is popular for its high-quality convergence-mode features and low mean squared error under stationary conditions. Moreover, recursive least squares algorithm using a fixed forgetting-factor cannot give acceptable performance in TV conditions. A lot of efforts have been directed towards the progress of a modified RLS algorithms. The fundamental principle of such algorithms is either to vary the forgetting-factor or to perturb the covariance-matrix. Although these algorithms can provide excellent tracking capability, yet their performances are severely deteriorated under the low SNR conditions. In [151], a novel control procedure for the forgetting-factor in recursive least squares procedure is introduced. It is based on a TV-equation of gradient of MSE rather than gradient of instant squared error. This recursive least squares procedure is revealed to possess fast tracking ability and low MSE. Particularly, this scheme can decrease the effects of disturbance/noise and makes appropriate changes in the value of forgetting-factor, which are proportional to the model-variations. In [152], a control mechanism for VFF of the RLS adaptive algorithm is introduced. This control method is principally a gradient-based method, in which the gradient is governed from a modified MSE analysis of recursive least squares algorithm. This mean squared error study exploits the correlation of inverse of the correlation-matrix with itself, which produces enhanced theoretical results, mainly in the transient-state and steady-state. This investigation yields a dynamic-equation of MSE that can be used to get a dynamical-equation of the gradient of MSE to manage forgetting-factor. The dynamic-equation can yield a positive-gradient when error is high and a negative-gradient when the error reaches to its steady-state. In comparison to other VFF algorithms, this control algorithm bestows fast tracking and small mean squared model error for the different values of SNRs.

The performance of recursive least squares algorithm is derived by the forgetting-factor. This parameter results in a compromise between 1) tracking capabilities and 2) misadjustment as well as stability. In [153], simulation outcomes specify high-quality performance and robustness of VFF-RLS algorithm. The convergence and stability of VFF-RLS algorithm are better than the conventional RLS algorithm. Finally, this RLS algorithm is incorporated in the channel-equalization under the low SNR as well as high SNR conditions [154]. The research work in [155] explains a variable forgetting-factor recursive total least squares (VFF-RTLS) algorithm to recursively calculate complete LS solution for the FIR filter. The forgetting-factor of VFF-RTLS procedure is tuned by minimizing the criterion-function (Rayleigh quotient). The VFF-RTLS adaptive procedure gives not only speedy tracking ability, but also low mean squared deviation. Based on modified precision to approximate FIR of unidentified system and flexibility to change, VFF-RTLS algorithm can be employed widely in the engineering applications. The idea of VFF is incorporated into the fast-RLS algorithms in [156]. The prewindowed as well as growing memory covariance algorithms are proposed in the transversal and lattice structures. The forgetting-factor adaptation methods, which enhance tracking performance over the standard RLS algorithms, are also recommended. The bias introduced by the usage of VFF is investigated. In [157], VFF linear LS procedure is explored to modify the tracking ability of channel estimator. A linear channel paradigm with respect to time-change details TV channel more precisely than a standard stationary channel-paradigm. To alleviate estimation-error generated because of the paradigm disparity, an improved VFF is incorporated into the suggested algorithm. In comparison to existing adaptive methods, the exponentially windowed RLS procedure with an optimum forgetting-factor and linear LS procedure, the presented scheme makes an incredible enhancement under the fast fading-channels. The effects of channel characteristics like SNR and fading-rate are also studied. A scheme for estimating TV spectra of the nonstationary signals using recursive least squares algorithm with VFFs is explained in [158]. The VFF is tuned to a nonstationary signal by the extended prediction-error criterion, which accounts for the nonstationarity of observed-signal. This scheme has high-quality adaptability in the nonstationary condition and low variance in the stationary condition. The viability of this method is illustrated with both simulation and experimental-data. Next, a novel dynamic forgetting-factor (DFF) for recursive least squares algorithm is given in [159]. Better performance in terms of the convergence-rate and tracking of noisy chirp sinusoidal signals is attained. The control of forgetting-factor in DFF-RLS algorithm rests upon the gradient of inverse correlation-matrix. As compared to the gradient of MSE algorithm, the presented scheme gives fast tracking and small MSE, even at the low SNRs. The algorithm for updating the linear-weights and quadratic-weights of SOVF is discussed in [39], which utilizes a stochastic estimate in the LS recursion-equation for the quadratic-weight matrix. This leads to a simple structure, including the predicted joint-process estimation-error and the same Kalman-gain matrix, utilized for updating the linear filter-coefficients. A Kalman-filter implementation of SOVF is compared to least mean square implementation. The KF operation is found to converge to the unidentified system parameters significantly faster than least mean square implementation. The Kalman-filtering can be applied to the problem of fixing tap-gains of transversal equalizers to minimize the mean squared distortion, as in [160]. Under noisy environment without prior information regarding channel, this adaptive-filtering algorithm results in higher convergence-rate than other procedures, and its convergence-rate depends only on the number of tap-coefficients. Starting with the vector observation model in [161], the robust Bayesian estimates of desired-vector are made for the following two different situations: 1) The state is Gaussian-distributed and the observation-error is (heavy-tailed) non-Gaussian; 2) The state is heavy-tailed non-Gaussian distributed and the observation-error is Gaussian. The bounds with respect to wide symmetric non-Gaussian families are devised for the error covariance-matrix of such estimates. The “one-step” robust estimates are then utilized to obtain robust estimates for the Kalman-filter setup. Monte-Carlo simulation outcomes show the robustness of presented estimation procedure, which might be called a robustified KF. The research work in [162] assumes the application of different Bayesian schemes to the joint recursive-approximation of dynamical-state and TV measurement-noise parameters in linear state-space paradigms. The presented adaptive KF scheme rests upon constructing the separate estimation procedure for the joint posteriori distribution of states and noise-parameters on every instant of time independently. The outcome is the recursive adaptive procedure, where the states are approximated with KF. The sufficient-statistics of noise-variance are approximated using the fixed-point iteration.

For state-space models, Chandrasekhar-factorization schemes, which were suggested in the control-theory and signal processing in early 1970's, is a fast option to KF. It can be efficiently implemented, if the state-space paradigm is time-invariant. The potential of this method derives from the information that it avoids the requirement for solving the conventional Riccati-equation. The main idea of research work in [163] is to utilize the Chandrasekhar-equations in the multi-channel first-order and SOVF. The presented scheme is an annexe of outcomes reported for MA and ARMA linear-filtering. By properly defining an extended state-vector and related matrices, a time-invariant state-space paradigm is achieved from the multi-channel depiction. Hence, it is feasible to utilize Chandrasekhar-factorization schemes to calculate KF blending-factor. It is assumed that nonlinear SOVF may be transformed into a multi-channel input linear-filter. An adaptive algorithm for SOVF based on the multi-channel Chandrasekhar fast equations is devised.

However, the Kalman-filter is an optimum estimator/predictor, which can be categorized as an algorithm for calculating the conditional-mean and covariance of probability-distribution of the state of a linear TV finite-dimensional stochastic-system (with the uncorrelated Gaussian-process and measurement-noise). The extended Kalman-filtering (EKF) extends the principles for design of a linear KF to a nonlinear problem. The major computational-burden in the standard hybrid EKF algorithm is in the estimation of state and error-covariance propagations. This error-covariance propagation includes the calculation of Jacobian, using recent estimates at each instant. Two EKFs are devised in [164] by using the measurements obtained from three sensors, single encoder, one rate-gyro and a tilt-sensor, that allow five possible combinations, to estimate the states of a pendulum test-model. These two types of EKFs are compared with a linear KF in order to reckon the modification in estimation accuracy at the high swing-angle values. Ultimately, it is demonstrated that aforementioned estimation methods can be incorporated in an efficient way to attain the required robotic tracking-control. Particularly, the discrete-time estimator based controllers neither necessitate any hardware modifications nor noteworthy increase in computational-power. Numerically proficient tracking-controller for the polynomic nonlinear-systems by a joint Volterra-series and EKF approach is proposed in [164]. The designed state-estimation method is incorporated for the state-estimation in a pendulum paradigm. The tracking-control of a pendulum is achieved by using the internal-model rule as well as feedback linearization method. It is shown that this approach is numerically competent control technique.

The research work in [165] proposes an EKF for the curtailed Volterra-systems that utilize parallel-cascade configurations. In this, the parallel-cascade configurations incorporate high order Volterra-systems as parallel connections of the multiplicative groupings of the lower order systems. A computationally efficient version of the extended Kalman-filter designed by employing a block-diagonal approximation of the covariance-matrix of the state error-vector is also proposed. Practical comparisons demonstrating relatively small performance deterioration for the approximate system over the extended Kalman-filter are also comprised in [165]. The renowned standard Kalman-filtering provides optimum-solution, but these need synchronous observations, a perfect system paradigm and precise stochastic-noise characteristics. Therefore, the performance of KF with imperfect knowledge and asynchronous sensors' observations can get deteriorated or diverged. To trim down the effects of noise-variance uncertainty, adaptive fading EKF and adaptive unscented KF are presented to get rid of this limitation. However, the received information-signals to estimate nodes from the multi-sensors have different communication time-delays and sampling-rates. In [166], the standard KF has been improved to be practicable for the state-approximation in plants with different communication time-delays in their sensors. The decentralized multi-sensor combination is utilized to approximate states in the presence of multi-rate sensors. The viability and efficiency of the proposed schemes are expressed through simulation investigations on the uninterrupted stirred tank reactor standard problem. However in [167], the optimum filtering equations with delays are presented for an Ito-Volterra process, proceeding from the discontinuous observations of Ito-Volterra type. This presented filter can be considerably simplified for a dynamic-system state, driven by a differential-equation. The filtering problems are considered with single time-delay, multiple-delays and a set of delays of the continuum power. Further, an optimum-filter for a continuous dynamic-system with continuous, multi-rate and randomly sampled measurements is considered in [168]. Using the optimal-filtering theory for Ito-Volterra systems with discontinuous measurements, the optimal-filter for the linear state-space model with continuous-time and discrete-time measurements is rigorously derived, and a number of known results are recovered, involving the Kalman-Bucy and Jazwinski-filters. It allows one to simultaneously handle analog and sampled measurements without approximations, and it is particularly convenient in case of multi-rate and arbitrarily sampled measurements, often encountered in the networked data acquisition.

Most of nonlinear-system-identification schemes derived from Volterra-paradigm consider that underlying system is time-invariant. In [169], the identification technique for the time-varying Volterra-systems (TVVSs) is explored. The system-identification process is transformed to the state estimation problem of a dynamical-system. However, TV Volterra-system recognition schemes can be categorized into four major groups: 1) Time-variations of TVVS are considered to be smooth, so that the time-invariant schemes may be functional over the little segments of information; 2) Multiple implementations of input-output are needed for the identification of the underlying time-varying system; 3) Select *a priori* described basis-functions, which are time-dependent, and TVVS may be recognized by LS scheme; 4) Adaptive algorithms like LMS and RLS procedures. Each of the aforementioned schemes has assured limitations. For example, 1) and 4) are not appropriate for usage in the framework of fast TV systems. 2) needs multiple configurations, which are not always accessible. 3) requires to choose a few suitable basis. In [169], the TV kernels are considered to follow a Gauss-Markov stochastic difference-equation. Further, TV Volterra-system is reformulated into a state-space paradigm. In this scenario, state transition-matrix, noise correlation-matrix of process-noise and variance of measurement-noise are considered to be unidentified. Subsequently, schemes to approximate these unknown parameters are suggested. With the aim of obtain unknown parameters, input-output auto-correlation is formed for exploiting the dependence of correlation-matrix of kernel-vector on them. Elements of correlation-matrix of Volterra-kernels are obtained by the application of LS algorithm. All other unidentified matrices can be determined afterwards. The approximated state transition-matrix and noise-covariance in KF method are used to recursively approximate TV Volterra-kernels. Such effort considers the parametric-paradigm for variations of TVVS. This paradigm is frequently utilized in the stochastic-process investigation, and it can be justified from the statistical point of view. Subsequently, a distinct characteristic of above discussed scheme is that it doesn't need the accurate information of stochastic difference-equation. All that needed is the second-order statistics of input-signals as well as output-signals of TVVSs, from which the unidentified dynamics of system can be approximated/predicted. Simulation examples in [169] illustrate the improved performance of the aforementioned Kalman-filtering based scheme in comparison to conventional adaptive identification schemes, like LMS and RLS adaptive algorithms.

1.3 Acoustic Echo Cancellation using Volterra-Filters

Linear-filters are frequently applied in various signal processing applications. As a matter of reality, these are well-understood within a standardized theory of the discrete-time linear-systems. However different physical systems possess nonlinear behaviour, and under these circumstances, the linear-filtering configurations perform inadequately. One case is problem of acoustic echo suppression, where digital-filter recognizes as close as possible the acoustic echo-path (extremely nonlinear). In such situation, improved system-recognition can be attained by using the nonlinear-filtering approach. However, the problem is to discover a nonlinear-filter configuration, which is capable to realize an excellent estimation of echo-path without substantial rise in the computational-burden. The standard Volterra-filtering configurations are appropriate for modelling these nonlinear-systems, but these polynomial-filters usually require a number of computational-resources for the real-time realization, like in under water acoustics [170]. In [171], a few low complexity nonlinear-filters are considered in order to find out a filter configuration, which is capable to attain performance close to those of Volterra-filtering configuration, which can be employed in the commercial acoustic echo cancellers. Some adaptive procedures, which work in frequency-domain, are presented to alleviate computational-burden. These methods are based on discrete-time frequency-domain paradigm, which estimates Volterra-filters using multiplicative-components. A most important drawback of this representation is its underlying supposition that observation-frame is adequately large as compared to memory-span of system. This supposition can be extremely restrictive particularly, if lengthy and TV impulse responses are considered (AEC applications). In [48], parameters of the presented paradigm are approximated off-line utilizing LS [172] criterion. It is demonstrated that the substantial drop in computational cost as well as considerable enhancement in the estimation precision can be attained over time-domain Volterra-paradigm, especially when the high memory-span nonlinear-systems are explored.

To enhance the convergence-rate and to alleviate the mean squared error of gradient-based adaptive procedures in the colored environment (like in AEC), a pre-filter-bank algorithm is presented in [173] by minimizing the weighted-criterion of squared errors in sub-bands. The optimum-solution achieved by minimizing this criterion is Wiener-filtering, which is independent of weight-vector. This weight-vector has a powerful effect on the behaviour of adaptive algorithm and has relationship with sub-band SS values. Specifically, the optimum weight-vector, which is derived for a random-walk TV plant in [173], depends on the spectra of input-signal and additive disturbance/noise. Without *a priori* information of these spectra, for fast starting convergence-rate and for superior tracking-mode performance under the nonstationary conditions, a straightforward VSS algorithm is employed in the pre-filter-bank algorithm in each sub-band for altering the sub-band SS values. This novel multistep-size algorithm, which is known as VSS pre-filter-bank algorithm, gets better appreciably over the conventional full-band variable step-size algorithms under colored environment. The limitation of such algorithm is increase in computational intricacy. Because filters in the filter-bank are usually narrow-band, the non-decimated output-signals of these filters are very much correlated. This correlation allows us to estimate the sub-band auto-correlation matrices using the single-rank matrices to decrease the computational-complexity of this adaptive method. This simplified version has approximately the similar performance as the true/real adaptive algorithm. However, finite-length Volterra-filters are recognized to be capable to model a large variety of realistic nonlinear-systems. An algorithm is presented in [174], which permits for diverse memory-spans of the linear as well as quadratic Volterra-kernels, while maintaining benefits of fast convolution schemes in the frequency-domain for SOVF. To achieve related adaptive implementations of suggested approach, simplification of known frequency-domain algorithms for the Volterra-filter and partitioned-block frequency-domain filter for the linear-systems are presented respectively. To exploit the benefits of adaptive frequency-domain procedures with respect to the convergence-rate, a frequency bin-wise normalization of SS is provided, which is further applied to the nonlinear AEC. The investigations verify the enhanced convergence-rate of the second-order partitioned-block frequency-domain Volterra-filter in comparison to time-domain adaptation of kernel coefficient-vector.

Nonlinear-filtering based on the Volterra-series expansion is a powerful and famous technique in the field of signal processing. However, a serious difficulty is the increased filter complexity in comparison to the linear-filtering. A proficient approximation to the SOVF, called multi-memory decomposition [10], is consisting of three linear FIR-filters and single multiplier. Hence, the number of essential filter operations is linear in term of the filter memory-length. The multi-memory decomposition coefficient determination with respect to a second-order reference-kernel is also proposed. In addition, the block-oriented and adaptive algorithms are presented, which obtain the filter-weights from input-output measurements of an unidentified system. Therefore, a linearization method for the compensation of nonlinear-distortions with a pre-processor is suggested. The pre-processor is realized on a DSP system to alleviate the nonlinear-distortion of electrodynamic loudspeakers in the practical applications. Further, two robust affine-projection sign algorithms, both of which minimize the mixed norm of l1 and l2 of the error-signal, are proposed in [175]. The proposed algorithms are shown to offer a significant improvement in the convergence speed and a significant decrease in steady-state misalignment relative to pseudo affine-projection sign algorithm. Additionally, the presented algorithms provide robust performance under impulsive-noise conditions and improved tracking of the unknown system in comparison to that provided by the pseudo-affine-projection sign and simple affine-projection sign algorithms. It has found applications in the area of system-identification and echo cancellation.

Acoustic echo suppression is a general occurrence in today's communication systems. It arises, when the auditory source and sink function in the full-duplex mode. The signal disturbance induced by the acoustic echo is disturbing for both users, and it also leads to a decrease in quality of telecommunication systems. The research work in [176] aims at the usage of adaptive-filtering schemes to diminish unnecessary echo-components, thereby improving signal reception quality. Adaptive-filters adjust their parameter values with the purpose of minimizing a function of the difference between the desired target output-signal and their output-signal. In case of the acoustic echo in communication systems [176], optimum output-signal is an echoed signal that precisely emulates the undesired echo signal. It is further utilized to negate that echo in the return-signal. Better the adaptive-filter emulates this echo, more efficient the echo suppression will be. A new method for the nonlinear AEC based on the adaptive Volterra-filtering with linear and quadratic-kernels is presented in [177], which automatically favours those diagonals contributing maximum to the output-signal of quadratic-kernel with an objective of minimizing the overall mean squared error. In particular echo suppression situations, not all coefficients are evenly significant for modelling of nonlinear-echo, but coefficients near to main-diagonal of second-order kernel portray major nonlinear-echo distortions, such that not all diagonals are required to be realized. But, it is intricate to decide an apposite number of diagonals *a priori*, as there are a large number of factors that influence this decision, like the energy of nonlinear-echo, the shape of room impulse response, or SS employed for the alteration of kernel-coefficients. This method employs adaptive scaling factors that manage the effects of each group of adjoining diagonals contributing to the quadratic-kernel output-signal. An adequate choice of these factors contributes to boost or abandon diagonals of paradigm as needed by the underlying situation. The adaptation principles for aforementioned factors are also proposed, which reduce the gradient-noise in echo cancellation. However, the future mobile multimedia-devices need higher playback levels. The general approaches to the loudspeaker compensation based on the Volterra-model enhance the sound quality only at the low playback-levels, and it may bring in more distortion at the high sound-levels. A novel Volterra-Wiener-Hammerstein paradigm is presented in [178] that is a superior match to the loudspeaker response and lends itself to having an accurate nonlinear inverse. The simulations and real-time measurements manifest that the compensation based on a new paradigm significantly alleviates the linear- and nonlinear-distortions of small loudspeakers mounted in the cell-phones. The objective as well as subjective evaluations depict significant improvements in the sound-quality.

Nonlinearities in amplifier as well as loudspeaker of speakerphones restrict the performance of linear AECs, necessitating the usage of nonlinear echo suppression schemes. The nonlinear AEC based on a Wiener-Hammerstein paradigm is presented in [179]. By modelling the true configuration of nonlinear-system, the presented canceller needs relatively small number of adaptive parameters, offering substantially lower storage and computational needs than the common nonlinear adaptive-filters. The experimental outcomes on observed loudspeaker signals point out that the presented nonlinear echo canceller offers approximately 8.4 dB enhancement in the “echo return loss enhancement” (ERLE) over the linear NLMS canceller with a few additional computations. The performance of linear AECs is restricted by the nonlinearities in signal-path. Nonlinear cone suspension and uneven magnetic-flux densities in the loudspeakers bring in nonlinear-distortion at the high cone displacement levels. Additionally, at large volume settings, the saturation-effects can arise in the loudspeaker’s power-amplifier, causing gross nonlinearities in the acoustic system that significantly impairs performance of linear AECs. Because these nonlinear phenomena are likely to come into view in the hands-free speakerphones, the adaptive nonlinear methods are needed to attain the appropriate echo cancellation. A number of schemes have been presented for the nonlinear AEC so far. However, Volterra-series based filter and neural networks are two famous nonlinear approaches. A primary disadvantage of the Volterra-filter is a large number of parameters, which are typically needed to illustrate the nonlinear-system. However, large computational load is needed to adapt such a large number of parameters. The number of parameters in a Volterra-expansion increases exponentially with the order of nonlinearity. As it is broadly accepted that the nonlinearities in loudspeaker systems are primarily of third-order and higher orders, therefore the adaptive Volterra-filter is not computationally viable in this application. While significant gains have been made recently in enhancing the competence of adaptive Volterra-filtering, these advances have been primarily in the utility of SOVF depictions, restricting their usefulness in hands-free environment. AECs using the time-delay neural networks (TDNNs) have been designed in [179]. Dissimilar to the Volterra structures, which are of the certain fixed order, TDNNs are common structures that do not need a clear characterization of the nonlinearities. The disadvantages of TDNNs involve their inclination to converge to a local minima, low convergence-rate and great computational needs. Some hybrid nonlinear/linear echo cancellers have been presented utilizing the neural networks to abandon the nonlinear-term and the linear NLMS algorithm to compensate the effects of room echo response. The echo canceller structure has the noteworthy shortcoming of requiring the second training-signal from a second microphone placed close to the loudspeaker. Therefore, a nonlinear acoustic echo canceller (NLAEC) is proposed in [179], which is comprised of a cascade of linear and memoryless nonlinear elements.

Adaptive nonlinear-filtering also plays a significant function in audio signal processing and echo control. In [180], a nonlinear-system-identification scheme is presented, in which the setup is built using adaptive SOVFs and third-order Volterra-filters with the same memory-length. The adaptation is attained using the NLMS algorithm with an appropriately selected SS parameter for the convergence as well as stability. The functions with nonlinear features are selected to verify this scheme. Performances are evaluated using the ERLE and mean squared error. Results demonstrate that Volterra-filters outperform the linear-filters in the weakly nonlinear configurations. The miniaturization of GSM handsets produces nonlinear acoustical echoes between the microphones-and-loudspeakers, when the signal-level is large. Various methods including the nonlinear cascade-filters and a bilinear-filter are suggested to compensate such acoustical echoes. A bilinear-filter is a limited nonlinear ARMA filter with exogenous inputs. Comparisons based on the standard ERLE measure, between a straightforward linear adaptive finite impulse response filter and various nonlinear-filters, are presented in [181].

A nonlinear AEC algorithm is described in [182], which is essentially aimed at the loudspeaker-distortions. The presented system is consisting of two different modules arranged in a cascaded-structure: the nonlinear-module based on the polynomial Volterra-filter models the loudspeaker; and the second-module of conventional linear-filtering classifies FIR of acoustic-path. The tracking-mode performance of the overall system paradigm is attained by an improved NMLS algorithm. The stability bounds are provided in [182], and main interest is placed on transient-behaviour of cascaded-filtering configurations. These presented systems are derived from the parallel configuration comprised of the first-order Volterra-filter and SOVF. The real problem is associated with computational-complexity and convergence-rate of adaptive algorithm. In fact, the number of filter-coefficients increases with the square of memory-length of SOVF. The ill-conditioning of input-signal auto-correlation matrix in addition to a large number of filter-coefficients results in the lower convergence-rate. The statistical characteristics are more limiting, as the nonlinear component changes speedily with acoustic-path because of the effects of convolution operation. In the situation of hands-free communication systems, the loudspeaker nonlinearities encompass powerful harmonics, in which energy relies on fundamental frequency. A novel echo suppression algorithm is investigated in [182], taking into account the loudspeaker nonlinearities. The acoustic echo cancellation in the presence of nonlinearity with memory in the channel to be identified is major concern in [183]. The mean weight behaviour of the LMS algorithm as well as MSE at the output of SOVF is derived in case of Gaussian input-data.

AECs in today’s speakerphones or video-conferencing systems depend on the supposition of a linear acoustic echo-path. But, low cost audio-equipments or constraints of moveable communication systems lead to the nonlinear-distortions, which restrict the ERLE achievable by the linear adaptation techniques. It means that an irritating nonlinearly distorted echo (“nonlinear echo”) is transmitted back to the far-end subscriber particularly at the loud speech segments. While nonlinearities with memory are of prime concern typically with the high quality audio-equipment, the memoryless nonlinearities are addressed in [184], which occur normally because of saturation-curves in case of the power-amplifier or the loudspeaker. It is applicable mainly to the mobile-equipment, where weight-constraints call for the low supply-voltages. In such cases, the echo lessening can be very much enhanced by the seventh-order polynomial-model of the saturation-curve. With normalized least mean square adaptation [176], it converges too slowly for the real-time applications. It has been shown that a cascade of the polynomial-filters and the FIR-filter can abandon such nonlinear acoustical-echoes. Another scheme employs the hard-clipping curve with least mean square adapted saturation-parameter. For both cascaded-systems, an LMS-type adaptation using a general framework is derived in [184]. To attain adequately fast convergence-rate for the real-time use, an RLS-type adaptation for the polynomial-filter is explored and experimentally established. Both schemes are compared using real-time hardware and speech-signals. It also depicts robust convergence-mode behaviour and an echo-reduction gain of up to 10 dB compared to a linear acoustic echo cancellation.

Nonlinearity of power-amplifiers or loudspeakers in the large-signal situation leads to a nonlinear-distortion of the acoustic-signal. A traditional AEC utilizing the linear adaptive-filter is unable to get rid of the nonlinear echo-components. In [185], a nonlinear echo excision scheme is explored by incorporating the nonlinear-transformation in combination with a generally used linear filter. This nonlinear-transformation is deduced from the raised-cosine function. It is exploited to equalize the nonlinearity of loudspeakers. The transformation parameters are tuned using NLMS procedure according to an unidentified nonlinear attribute of loudspeaker. This technique yields an acceptable echo annulment performance, while having small computational-complexity. In [186], a straightforward efficient nonlinear AEC method is proposed by utilizing a tunable sigmoid-function in concurrence with the traditional transversal adaptive-filtering configuration. This novel technique utilizes LMS algorithm for updating the parameters of the sigmoid-function. It also employs RLS procedure to obtain coefficient-vector of tapped-delay-line filter. The presented AEC is verified to be convergent under the soft considerations. This method provides a better echo suppression performance over the conventional Volterra-filtering approach, when echo-path experiences saturation-type nonlinear-distortion. More significantly, the suggested AEC exhibits low computational-complexity than Volterra-filtering based scheme.

A nonlinear AEC, which encompasses the nonlinear-transform realizing a variable saturation-curve and an adaptive FIR-filter, are addressed in [179]. Specifically, a three-stage cascade configuration is incorporated in [179], which has single tuning-parameter, known as, maximum-value of saturation-curve, leaving the shape of saturation-curve non-modifiable. Evidently, this paradigm with single free-parameter cannot be tuned to any random shape of a variety of nonlinear-distortions. Because of its large computational-complexity, only SOVFs or third-order Volterra-filters are usually employed, which are recognized to be competent for the small scale nonlinear-distortions. For saturation-type nonlinear-distortions, the high order Volterra-filter is required, which suffers due to over-parameterization difficulty. Moreover, the sophisticated correlation-matrix needed by the high order Volterra-filter engrosses the higher order statistics of input-signal, resulting in the high eigenvalue-spread, thereby reducing the convergence-rate of the nonlinear acoustic echo canceller. The Hammerstein-paradigm encompassing the cascade of memoryless polynomial-filter and FIR-filter is used for excision of the saturation-type nonlinear-distortion. This paradigm may be considered as a typical case of Volterra-filtering, in which cross-terms are omitted. Even though this Hammerstein-paradigm has simplified Volterra-filter, it still needs more number of filter-coefficients for the memoryless polynomial to intimately estimate the saturation-type curve. In practical applications, the severe clipping case of saturation-distortion cannot be fitted by a memoryless polynomial with a reasonable order. Recently, the raised-cosine-function based paradigm is investigated in [185], in which nonlinear feature of acoustic echo-path is estimated by the transform that is deduced from the raised-cosine-function. This paradigm is competent to be tuned to both soft-clipping as well as hard-clipping nonlinearities. It provides better performance in comparison to the Volterra-filtering approach under certain conditions [185]. Another benefit of this technique is small computational-burden, as merely two free-parameters are engaged in nonlinear-function, which need to be updated. Because of the nature of piecewise defining nonlinear-transform, it appears to be complicated to hypothetically validate the convergence characteristics of this method. Authors present a nonlinear AEC in [186] that utilizes a sigmoid-function followed by a traditional linear filter, in which parameters of the sigmoid-function and the linear transversal filter-coefficients are updated respectively, using LMS and RLS adaptive procedures. The goal in [186] is to propose computationally efficient and fast convergent echo excision technique to deal with the saturation-type nonlinear-distortion of amplifier/loudspeaker in echo-path. Algorithms for the joint adaptation of both stages and SS normalizations are presented in [187]. The higher convergence-speed of the polynomial-filter is achieved using signal orthogonalization or RLS algorithm adaptation. The adaptive SS control mechanisms are proposed using a novel system-distance measure. The experiments under adverse conditions and with real-time hardware show robust convergence characteristics with both paradigms. The investigations illustrate an echo-reduction improvement upto 10 dB at the amplitude peaks.

From the aforementioned literature review, it is apparent that the hypothesis and engineering applications of adaptive nonlinear-system-identification are as immense and diverse as the real-time nonlinear-systems themselves. The nonlinear-system-identification generally aims at high fidelity mathematical models in the presence of nonlinearities from input-output measurements conducted on the real-time configurations/structures. However, our main focus is on the identification of gaps in the field of adaptive nonlinear signal processing and its applications using the finite impulse response filters.

1.4 Statement of Problem

The presented work includes a detailed study of adaptive polynomial-filtering for the system-identification. A trendy scheme for modelling the nonlinear-systems is usage of Volterra-filtering configurations, which are striking because of their structural simplicity and flexible modelling competence. A significant characteristic of Volterra-filter is a linear relationship between the system output-signal and filter tap-coefficients, which facilitates the usage of schemes from the linear-estimation theory for the estimation of parameter values. The aim of presented work is to study and develop adaptive Volterra-filtering algorithms for the nonlinear-system-identification, under the time-varying environment, in the presence of impulse-noise and in the presence of nonlinear acoustic echo. Adaptive algorithms employed for this purpose frequently utilize the LMS algorithm because of its robustness and simple design. However, the least mean square algorithm experiences lower convergence-rate, when input-signal to adaptive-filter is found to be correlated, which is tremendously challenging when employed in case of Volterra-filter of any order. Another key limitation of the adaptive Volterra-filtering is lofty computational cost caused by a large number of paradigm parameters, particularly for the large memory systems. To expedite convergence, RLS-filtering and Kalman-filtering algorithm are engaged to update the adaptive Volterra-filter. These methods, however, significantly boost the computational-complexity of estimation-process. Therefore, our intended effort is to develop an adaptive Volterra-filtering algorithm, which requires the low computational-complexity, possesses higher convergence-rate, provides efficient tracking of the time-varying channels with minimum misadjustment, and which may be incorporated in the adaptive polynomial framework for the noise excision and also for the acoustic echo cancellation. Based on the aforementioned literature review, the presented work encompasses analysis and design of “adaptive Volterra-filters” for the system-identification in the time-varying environment.

And the problem (P), as treated in this research work, may be broken up into three main parts as follows:

P1.) Analysis and design of numeric variable forgetting-factor RLS algorithm for second-order Volterra-filtering

It is based on nonlinear-system-identification under the time-varying environment in wireless communication.

P2.) Analysis and design of adaptive polynomial-filtering using generalized variable step-size least mean p^{th} power (LMP) algorithm

It is based on nonlinear-system-identification under the α *-stable impulsive-noise environment.*

P3.) Analysis and design of Volterra-filtering scheme using generalized variable step-size NLMS algorithm for nonlinear acoustic echo cancellation

It is based on nonlinear-system-identification for the nonlinear acoustic echo cancellation.

1.5 Organisation of the Thesis

Chapter 2:- "Numeric Variable Forgetting-Factor RLS Algorithm for Second-Order Volterra-Filtering"

In this chapter, we first give details about the Gauss-Markov system model for the Volterra time-variant channel. Subsequently, we describe the numeric variable forgetting-factor recursive least squares algorithm for the SOVF, in which FIR filter-coefficients are updated using the prediction error criterion, for adaptation under the time-varying nonstationary environments. The simulation investigations are provided to compare the channel tracking performances of the numeric variable forgetting-factor RLS algorithm, the conventional fixed forgetting-factor RLS (FFF-RLS) algorithm and the Kalman-filtering (KF) algorithm, which are appraised on the basis of MMSE criterion. Finally, summary of chapter is given to illustrate the significant contributions in this chapter.

Chapter 3:- "Adaptive Polynomial-Filtering using Generalized Variable Step-Size Least Mean $\boldsymbol{p^{th}}$ ***Power (LMP) Algorithm"***

In this chapter, we first describe the slowly time-variant nonlinear Volterra-system along with the particulars of α -stable impulse-noise characteristics. We next introduce the adaptive nonlinear-system-identification method based on the minimum error dispersion criterion using the proposed variable step-size least mean p^{th} power (GVSS-LMP) algorithm, which reduces the deleterious effects of eigenvalue-spread on the performance of adaptive algorithm.

Further, the convergence-mode and tracking-mode characteristics of the presented algorithm is compared with Kwong's variable step-size LMP (KVSS-LMP) [91], [188], Aboulnasr's variable step-size LMP (AVSS-LMP) [96], [188] and stochastic-gradient variable step-size LMS (SVSS-LMS) [104] adaptive algorithms, to manifest its benefits and efficacy on the basis of simulation results. Finally, summary of chapter is given to demonstrate the significant contributions in presented research work.

Chapter 4:- "Volterra-Filtering Scheme using Generalized Variable Step-Size NLMS Algorithm for Nonlinear Acoustic Echo Cancellation"

In this chapter, we first describe the mathematical system modelling of the loudspeaker and linear acoustic-path. We next provide details of the proposed adaptive-filtering paradigm for the nonlinear-system-identification, which encompasses two different modules in cascade i.e., a polynomial Volterra-filter and a tapped-delay-line FIR-filter in cascade. Subsequently, the application of GVSS-NLMS is introduced for the adaptive acoustic echo cancellation using Volterra-filtering. Further, the simulation outcomes are also presented to reveal the efficacy of the presented scheme, by comparing it with the conventional FSS-NLMS algorithm based AECs using the polynomial-filtering approach. The second-order as well as third-order Volterra-filters are tested under stationary and nonstationary environments for the performance analysis of proposed adaptive system. Finally, summary of chapter is given to show the significant contributions in this chapter.

Chapter 5:- "Concluding Remarks and Future Scope"

We conclude the thesis with a summary of the important results and suggestions for the future work.

CHAPTER 2

NUMERIC VARIABLE FORGETTING-FACTOR RLS ALGORITHM FOR SECOND - ORDER VOLTERRA-FILTERING

2.1 Introduction

The development of nonlinear-filtering is motivated by the amount of published research and the widespread use of nonlinear digital filters. Many polynomial system models exist to represent the nonlinear-systems; all have their own advantages and limitations. Among them, the Volterra-system [3] is one of the most commonly used models because of its structural generality and the Weierstrass theorem, which states that any continuous real-valued function can be approximated by a polynomial function with an arbitrary small error. An adaptive-filter is a self-designing filter that uses a recursive algorithm (known as an adaptation algorithm or adaptive-filtering algorithm) to “design itself”. The algorithm starts from an initial guess based on the prior knowledge available to the system; it then refines the guess through successive iterations, which converges eventually to the optimal-solution in some statistical sense. In the literature, the Volterra-system is assumed to be time-invariant, which means that the Volterra-kernels do not change with time [44]. But, there are many time-varying communication channels, like mobile wireless channels and underwater acoustic channels, which need to be tracked or estimated by using the nonlinear polynomial adaptive-filtering. In this chapter, the nonlinear-channel paradigm is considered to be based on the second-order Volterra-series, which is used to describe the input-output relationship as

$$y(n) = h_0 + \sum_{k_1=0}^{M-1} h_1(n;k_1)x(n-k_1) + \sum_{k_1=0}^{M-1}\sum_{k_2=0}^{M-1} h_2(n;k_1,k_2)x(n-k_1)x(n-k_2) + e(n) \qquad (2.1)$$

with $h_1(n;k_1) = h_1(k_1;n)$ and $h_2(n;k_1,k_2) = h_2(k_1,k_2;n)$. where, h_0 is the constant zeroth-order Volterra-kernel, h_1, h_2 are the first-order and the second-order

Volterra-kernels respectively, M is the memory length, $x(n)$ is the input, and $e(n)$ is the measurement-noise with zero-mean and variance σ_e^2. The complexity of Volterra-filter is dependent upon the memory (M). In the general case, the degree of nonlinearity (K) of the Volterra-system is usually assumed to be time-invariant [89]; therefore the time-varying Volterra-system (TVVS) can be used to describe the general input-output relationship as

$$y(n)=h_0+\sum_{k=1}^{K}\sum_{m_1=0}^{M-1}\cdots\sum_{m_k=0}^{M-1}h_k\left(n;m_1,\ldots,m_k\right)\prod_{i=1}^{k}x\left(n-m_i\right)+e(n) \tag{2.2}$$

where, the aim is to identify the time-varying (TV) Volterra-kernels $h_k\left(n;m_1,\ldots,m_k\right)$ through measured $y(n)$ and $x(n)$. In our previous work [189], the channel estimation method is presented using the numeric variable forgetting-factor recursive least squares algorithm combined with the second-order polynomial time-varying channel model. When the signal experiences nonstationarity, the forgetting-factor decreases automatically to estimate the channel quickly using the extended estimation error criterion [158], [190]. However under stationary conditions, the forgetting-factor increases by increasing the memory for the accurate channel estimation. In this chapter, the NVFF-RLS algorithm is incorporated in the second-order Volterra-filter with K=2 for the nonlinear-channel estimation in a nonstationary environment.

2.2 System Paradigm

The nature of time-varying Volterra-kernel can be defined either by a stochastic-process or by a deterministic one. For the deterministic model, the periodic time-varying characteristics are reported to be appropriate. In this chapter, we choose a stochastic model for the time-varying Volterra-kernels. Since there is no knowledge about the stochastic behaviour of the TV kernels, one often finds it useful to model such a process by using the first-order Markov-model [33], [191] given by

$$\vec{h}(n+1)=\phi\vec{h}(n)+\vec{w}(n+1) \tag{2.3}$$

where, ϕ is the state transition-matrix and $\vec{w}(n)$ is the zero-mean white Gaussian process-noise vector with variance σ_w^2. After establishing the Gauss-Markov model for the time-varying kernels and the input-output relationship, the state-space equation of Volterra-system is represented by

$$\vec{h}(n) = \phi\vec{h}(n-1) + \vec{w}(n) \tag{2.4}$$

For the time-varying second-order Volterra-system as shown in Fig. 2.1, the input-output relationship is given by Eq. (2.2) with K = 2. Now, let us consider the $L \times 1$ dimensional extended filter-coefficients vector as

$$\begin{aligned}\vec{h}(n) = [&h_1(n;0), h_1(n;1), \ \ldots\ldots \ , h_1(n;M-1), h_2(n;0,0), h_2(n;0,1), \ \ldots\ldots, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ , h_2(n;0,M-1), h_2(n;1,1), \ \ldots\ldots \ , h_2(n;M-1,M-1)]^T\end{aligned}$$

where, $(.)^T$ is the matrix transpose operator. The $L \times 1$ dimensional extended input-signal vector for the SOVF with zero-mean and variance $\sigma_x^2 = 1/L$ is defined as

$$\begin{aligned}\vec{x}(n) = [&x(n), x(n-1), \ldots\ldots, x(n-M+1), \ldots\ldots, x^2(n), x(n)x(n-1), \ldots\ldots, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x(n)x(n-M+1), \ldots\ldots, x^2(n-1), \ldots\ldots, x^2(n-M+1)]^T\end{aligned}$$

Further, we can rewrite Eq. (2.2) compactly as

$$y(n) = \vec{x}^T(n)\vec{h}(n) + e(n) \tag{2.5}$$

Utilizing the fact that the Volterra-kernels are symmetrical, the value of coefficient $h_k(n;m_1,\ldots,m_k)$ is kept unchanged for any of the possible k! permutations of $m_1, m_2, \ldots, m_k$. Therefore, the Volterra-kernel remains time-invariant under different permutations of its argument. Eqs. (2.4) and (2.5) represent the time-varying Volterra-system in terms of the stochastic dynamic-system, which are governed by the Gauss-Markov model. For the mathematical analysis, the estimated Volterra-kernel vector may be represented by

$$\vec{h}'(n) = [h_1'(n;0), h_1'(n;1), \ \ldots \ , h_1'(n;M-1), h_2'(n;0,0), h_2'(n;0,1), \ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ , h_2'(n;0,M-1), h_2'(n;1,1), \ \ldots \ , h_2'(n;M-1,M-1)]^T$$

Therefore, the estimated received signal vector is denoted by

$$y'(n) = \vec{x}^T(n)\vec{h}'(n) \tag{2.6}$$

Hence, the estimation-error in the signal reception is calculated by using $v(n) = y(n) - y'(n)$ with zero-mean and variance σ_v^2.

$$v(n) = \left(\vec{h}(n) - \vec{h}'(n)\right)^T \vec{x}(n) + e(n) \tag{2.7}$$

After updating the filter-coefficients by using Eq. (2.4), the estimated error in the received signal is

$$v(n) = \left(\phi\vec{h}(n-1) - \vec{w}(n) - \vec{h}'(n)\right)^T \vec{x}(n) + e(n) \tag{2.8}$$

Under optimum conditions, it may be assumed that the estimated Volterra-kernel is given by

$$\vec{h}'(n) \simeq \phi\vec{h}(n-1) \tag{2.9}$$

Therefore, the estimated error in received signal from Eq. (2.8) results in

$$v(n) \simeq \vec{w}(n)^T \vec{x}(n) + e(n) \tag{2.10}$$

where, the variance of estimation-error in the received signal may be calculated as

$$\sigma_v^2 = \sigma_e^2\left(1 + \frac{\sigma_w^2}{\sigma_e^2}\right) \tag{2.11}$$

It is clear from the above equation that if $\frac{\sigma_w^2}{\sigma_e^2} << 1$, then $\sigma_v^2 \simeq \sigma_e^2$.

Therefore from Eq. (2.11), it is apparent that the noise introduced in signal reception due to the imperfect channel estimation is almost equivalent to the measurement-noise. Utilizing this property [189], the system-identification model based on the presented NVFF-RLS algorithm uses time-variable forgetting-factor λ (n) to update the channel state after each sample point, which is incorporated in the first-order and second-order Volterra-filter in the next section.

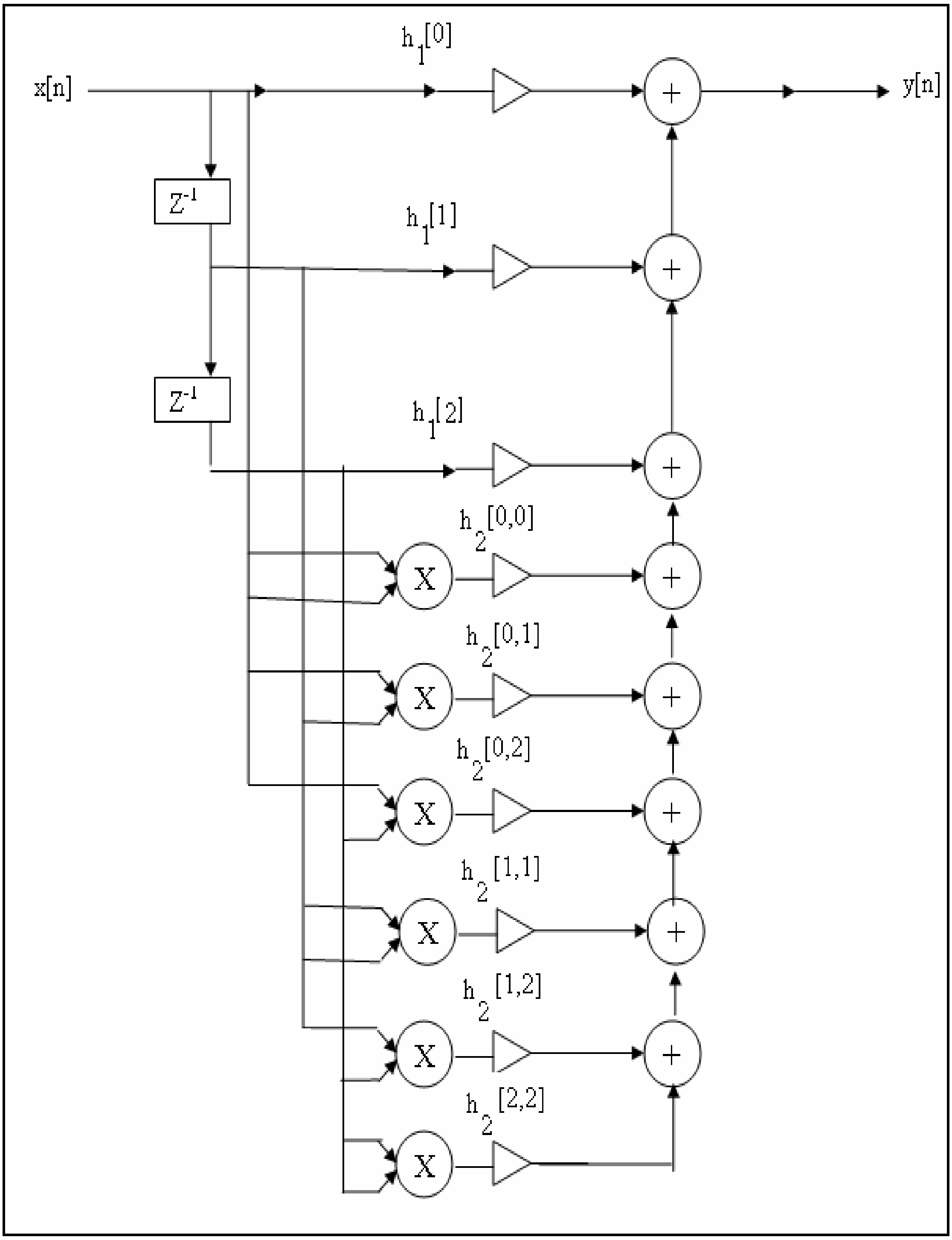

Fig. 2.1: Second-order Volterra-system with delay elements

2.3 Numeric Variable Forgetting-Factor RLS Algorithm

The technique presented in this chapter allows the variation of the forgetting-factor during the operation of RLS adaptive-filter structures. The prewindowed and the growing memory covariance algorithms in the transversal and the lattice structures were presented by Toplis and Pasupathy in [156]. Each of these algorithms simplifies to their original form, when the forgetting-factor is not undergoing any transition. The VFF algorithms allow the user to have greater flexibility in controlling the tradeoff between the lag-noise (lag-misadjustment [106]) and the channel estimation noise. This translates into improved tracking performance of the adaptive-filter operating in a nonstationary environment under the optimal parametric conditions [192].

However in the stationary case, the Volterra-kernels can be estimated with the fixed forgetting-factor with a high degree of accuracy. The estimation-error is observed to be large for the small value of the forgetting-factor, though the convergence-rate is fast due to the limited amount of available data. For the high value of the forgetting-factor, the estimation-error is less, though the convergence-rate is slow. The adaptation speed depends upon the asymptotic memory length [193] according to the following relation

$$N = 1/(1-\lambda) \tag{2.12}$$

Therefore, the forgetting-factor can also be calculated using the Eq. (2.12) as $\lambda = 1-(1/N)$. The memory lengths corresponding to $\lambda_{\max}$ and $\lambda_{\min}$ are denoted by $N_{\max}$ and $N_{\min}$ respectively. The polynomial channel model based NVFF-RLS algorithm uses forgetting-factor λ (n) to update the channel state after each sample point [189]. On the basis of the extended estimation-error criterion, the proposed algorithm for a second-order Volterra-filter is implemented as follows, such that the extended estimation-error is determined by

$$Z(n) = \frac{1}{Q}\sum_{q=0}^{Q-1} \left| v(n-q) \right|^2 \tag{2.13}$$

The value of Q must be kept smaller than the minimum asymptotic memory-length. If the process-noise σ_w^2 is very small in comparison to the measurement-noise σ_e^2, then it can be shown using Eq. (2.11) that $\sigma_v^2 \simeq \sigma_e^2$. Consequently, the extended estimation-error criterion [189] may be defined as

$$N(n) = \frac{\sigma_v^2 N_{\max}}{Z(n)} \simeq \frac{\sigma_e^2 N_{\max}}{Z(n)} \tag{2.14}$$

Therefore, the NVFF is calculated by using Eqs. (2.13) and (2.14) as

$$\lambda(n) = 1 - (N(n))^{-1} \tag{2.15}$$

In contrast to the stationary case, a smaller value of the forgetting-factor in the nonstationary environment seems to be more beneficial; which is bounded by $\lambda_{\min} < \lambda(n) < \lambda_{\max}$. Subsequently, we initialize the algorithm by setting

$$h'(0) = 0$$

$$\vec{P}(0) = \delta^{-1} I$$

$$\vec{\Pi}(n) = \vec{P}(n-1)\vec{x}(n)$$

$$\vec{k}(n) = \frac{\vec{\Pi}(n)}{\lambda(n)\sigma_e^2 + \vec{x}(n)^H \vec{\Pi}(n)} \tag{2.16}$$

$$\xi(n) = y(n) - \vec{h}'(n-1)^H \vec{x}(n)$$

$$\vec{h}'(n) = \vec{h}'(n-1) + \vec{k}(n)\xi(n)^* \tag{2.17}$$

$$\vec{P}(n) = \lambda^{-1}\vec{P}(n-1) - \lambda^{-1}\vec{k}(n)\vec{x}(n)^H \vec{P}(n-1) \tag{2.18}$$

where, $(.)^*$ and $(.)^H$ are the conjugate operator and the Hermitian transpose matrix operator. Eq. (2.17) provides the *a posteriori* estimate of $h(n)$. When the signal experiences nonstationarity, the value of the NVFF decreases to improve the tracking performance. But, the value of forgetting-factor can only vary between $\lambda_{\min}$ and $\lambda_{\max}$. The presented algorithm may also be incorporated easily in the higher order Volterra-filters to tackle the nonlinear parameter estimation.

2.4 Simulation Results

In the following simulations, we consider a second-order time-varying Volterra-system (TVVS), in which the tap-coefficients follow the Gauss-Markov model. The proposed system consists of the linear and quadratic-kernels for the first-order and second-order TVVS with memory length M = 3. We mainly investigate TVVS identification using LMS and RLS algorithms in a time-varying environment using MATLAB software. For the results presented in Fig.2.2 and Fig. 2.3, the state transition-matrix of the linear kernel is chosen to be

$$\phi = \begin{bmatrix} -1.31 & -0.43 & 2.25 \\ -0.88 & -0.53 & 2.03 \\ -0.53 & -1.43 & 2.65 \end{bmatrix} \tag{2.19}$$

Similarly, for the results presented in Fig. 2.4 and Fig. 2.5 in case of the quadratic-kernel, the state transition-matrix is chosen to be

$$\phi = \begin{bmatrix} 2.71 & -7.19 & -0.04 & 0.72 & -0.82 & 4.27 \\ 1.6 & -4.19 & -0.09 & 0.36 & -0.52 & 2.67 \\ -0.39 & 1.27 & 0.17 & -0.57 & -0.47 & 0.25 \\ 1.99 & -5.67 & -0.19 & 0.84 & -0.11 & 2.81 \\ 2.38 & -5.27 & -0.26 & 0.77 & -0.91 & 3.02 \\ 0.42 & -0.46 & -0.15 & -0.26 & -0.58 & 1.14 \end{bmatrix} \tag{2. 20}$$

However, Eqs. (2.19) and (2.20) make the state transition-matrix stable, ensuring the stationarity of $h(n)$ [169]. The presented results are based on the ensemble average of 2500 independent simulation runs for the different channel realizations. We assume averaging factor Q = 5, measurement-noise $\sigma_e^2 = 0.01$, and also consider the process-noise variance

$\sigma_w^2 << 0.01$. By invoking Eq. (2.11), it may be inferred that $\sigma_v^2 \simeq \sigma_e^2$. Therefore, we can incorporate Eq. (2.14) to calculate $N(n)$, which in turn leads to updating $\lambda(n)$ at every iteration. The performance of the adaptive channel estimators are compared on the basis of MMSE criterion, which is calculated by using the following formula

$$J(n) = E\left[\left|h(n) - h'(n)\right|^2\right] \tag{2.21}$$

where, $E(.)$ is the expectation or ensemble average operator.

$$J(n) = \frac{\sum_{j=1}^{2500}\left[\left|h(n,j) - h'(n,j)\right|^2\right]}{2500} \tag{2.22}$$

The performance of the RLS algorithm depends upon the convergence-rate, tracking, misadjustment and stability, which vary according to the value of forgetting-factor. In Fig. 2.2 and Fig. 2.4, the channel tracking performance of the first-order and second-order Volterra-systems using the NVFF-RLS algorithm is compared with the tracking performance of the conventional fixed forgetting-factor RLS algorithm and the Kalman-filtering algorithm. For this simulation, the values of $\lambda_{\max}$ and $\lambda_{\min}$ are chosen 0.975 and 0.75 respectively. For the simulation of the fixed forgetting-factor RLS algorithm, the value of λ is chosen to be 0.975 and the value of δ is fixed at 10^{-5}. The actual channel coefficient is denoted as True. These results show the lag-misadjustment between the conventional fixed forgetting-factor RLS and the proposed algorithm. It may be inferred from these simulation results that the channel tracking performance of the NVFF-RLS algorithm is better than that of the conventional RLS algorithm for the Volterra-systems, though both appear to be inferior to the Kalman-filtering algorithm based approach.

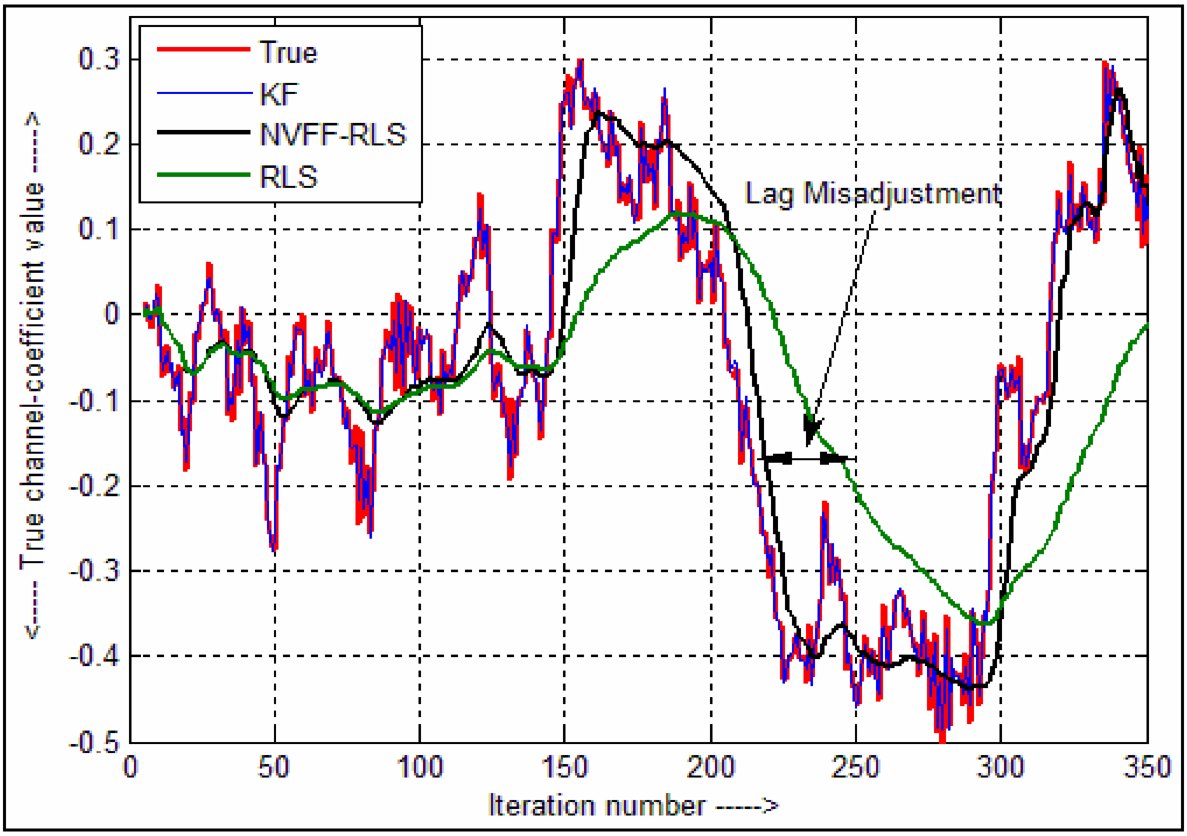

Fig. 2.2: Tracking performance of simulated algorithms for first-order Volterra-system

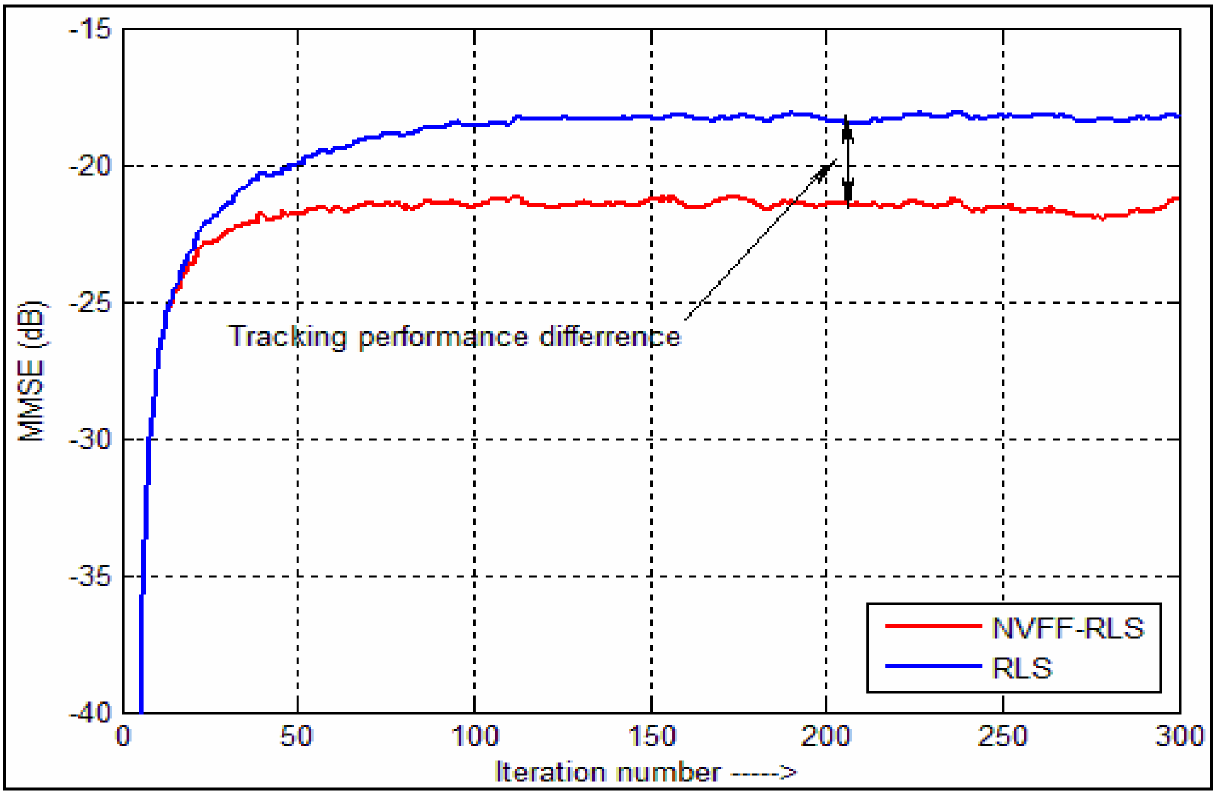

Fig. 2.3: MMSE in channel estimation for simulated algorithms for first-order Volterra-system

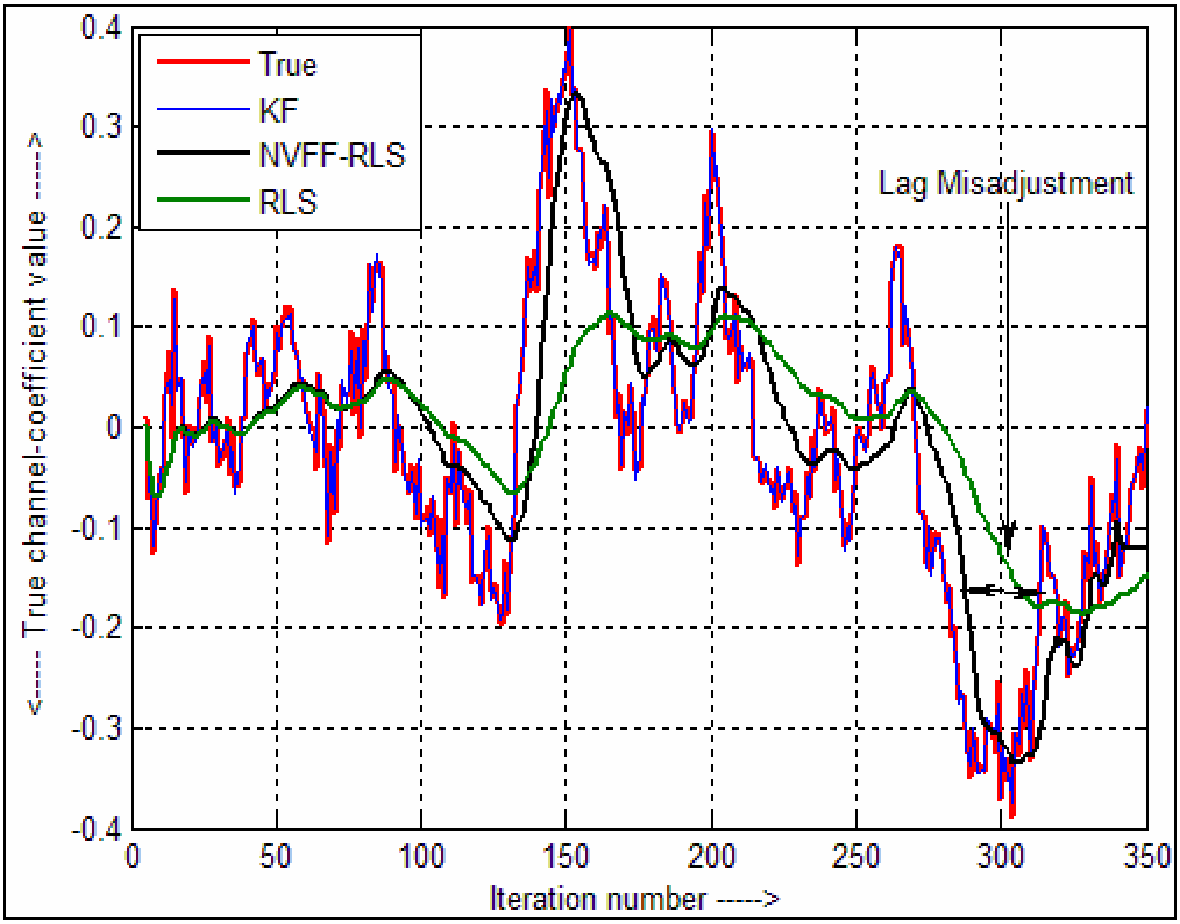

Fig. 2.4: Tracking performance of simulated algorithms for second-order Volterra-system

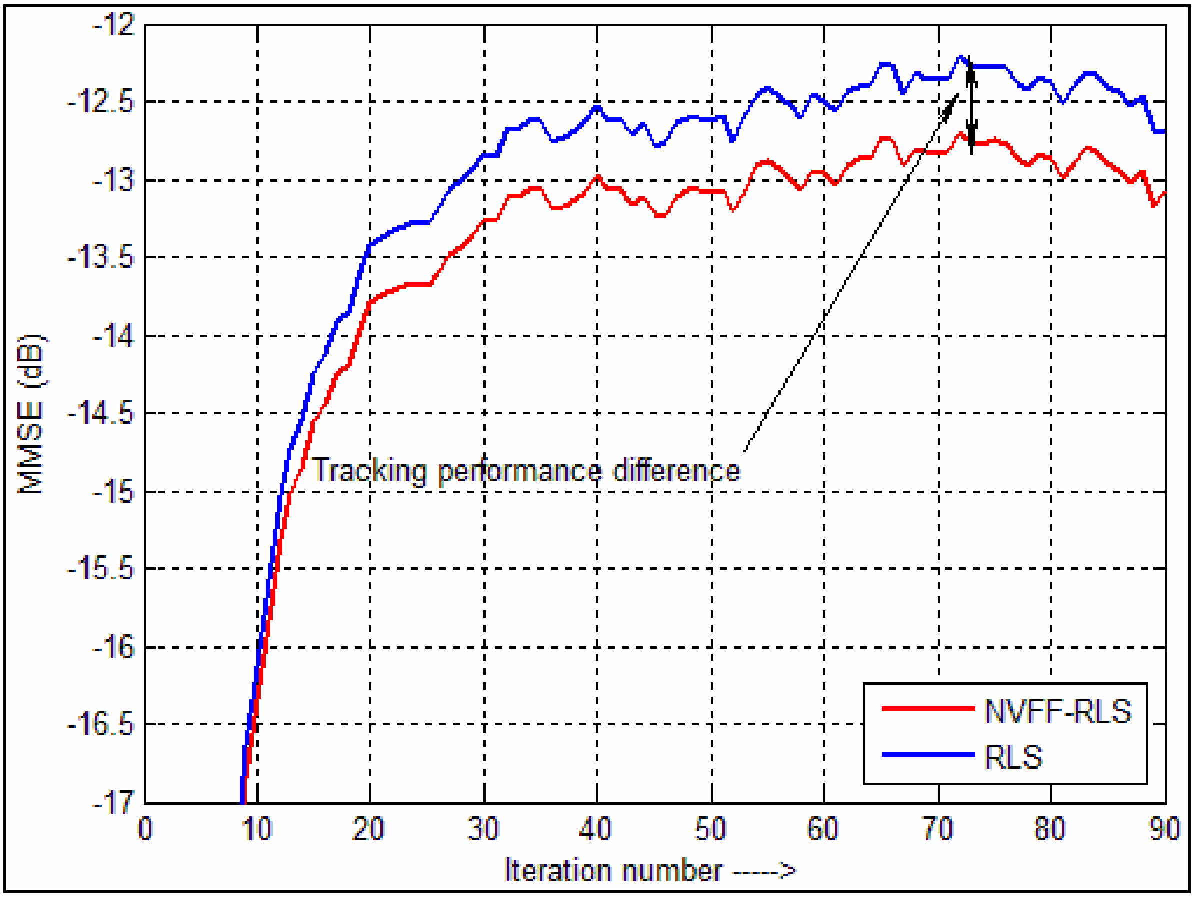

Fig. 2.5: MMSE in channel estimation for simulated algorithms for second-order Volterra-system

In Fig. 2.3 and Fig. 2.5, the MMSEs in the tracking of channel coefficients for the first-order and second-order Volterra-systems are presented respectively. Under similar time-varying environment, the NVFF-RLS algorithm performs approximately 2.5 dB better than the conventional RLS algorithm for the first-order Volterra-filtering system.

However, the tracking performance advantage is approximately 0.5 dB in the case of NVFF-RLS algorithm compared with the conventional RLS algorithm for the second-order Volterra-filtering system. This observed degradation in the performance of NVFF-RLS algorithm is due to the increased number of Volterra-kernels in the second-order systems. However, the tracking performance of the conventional RLS algorithm improves because the lag-misadjustment reduces for the second-order Volterra-filters, which is also evident from the results depicted in Fig. 2.3 and Fig. 2.5. Moreover, it is clear from the simulation results for the third-order Volterra-systems that the MMSE performance gap between the NVFF-RLS and conventional RLS algorithms is negligible.

In the above discussion, we have examined the simulations of adaptive algorithms for the first-order and second-order Volterra-systems. But with the increasing number of sources of nonlinearity, we have to look at certain adaptive algorithms or other refined adaptive algorithms that may be used to overcome the nonlinear effects. We can also apply these adaptive algorithms in the channel estimation for the latest wireless technologies like BLAST, 3G, 4G etc.

2.5 Summary of Chapter

The parameter estimation of time-varying first-order and second-order Volterra-systems is investigated using three different adaptive algorithms, namely the Kalman-filter, the fixed forgetting-factor conventional RLS algorithm, and the numeric variable forgetting-factor RLS algorithm. The Gauss-Markov model is used to define the time-variation of the kernels in a time-varying wireless environment. The parameter estimation or channel tracking performance of the NVFF-RLS algorithm supersedes the conventional fixed forgetting-factor RLS algorithm by 2.5 dB and 0.5 dB for the first-order and second-order nonlinear-systems respectively. The higher order Volterra-filters in combination with the NVFF-RLS algorithm may not prove very beneficial in tracking the time-varying wireless fading channels, but the presented nonlinear polynomial-filtering technique proves to be more computationally efficient than the available approaches of [151], [152], [153], [154], [157]. Future work includes the application of proposed adaptive polynomial-filters in the emerging fields of signal processing and communication engineering.

CHAPTER 3

ADAPTIVE POLYNOMIAL-FILTERING USING GENERALIZED VARIABLE STEP-SIZE LEAST MEAN pth POWER (LMP) ALGORITHM

3.1 Introduction

In the field of communication engineering, speech processing, image processing and biomedical engineering etc., many systems possess certain degrees of nonlinearity, which do not exhibit superposition property. Any such polynomial system [194] is also called Volterra-system [189], which is most commonly referred /used paradigm due to its roots in the Taylor series expansion of the nonlinear-functions with memory [3]. Therefore, the nonlinear-system-identification [1] is indispensable to establish a mathematical model for an unknown system through the input-output relationship. The researchers in the field of nonlinear-system-identification usually consider Volterra, Weiner [195] and Hammerstein [196] models. However, the presented research work will focus on the variable step-size adaptive nonlinear Volterra-filtering [197], due to its low computational-complexity as compared to the variable step-size adaptive Hammerstein filtering [198].

The measurement-noise is an inevitable issue in the field of nonlinear-system-identification, which is generally assumed to be a random-process with the finite-order statistics. Under such scenario, the mean squared error (MSE) appears as an appropriate metric for the estimation-error. However, the impulsive-noise [18] with the heavier distribution tail possesses approximately infinite second-order statistics, which connotes the non-Gaussian characteristics. It leads to the need of alternate methods for the nonlinear-system-identification in the presence of impulse-noise.

The Gaussian distribution is a special case of α-stable processes with $\alpha = 2$, which is characterized by the finite variance [20]. It is noteworthy that the α - stable processes in the range $1 < \alpha < 2$ are considered to be non-Gaussian with infinite variance. In [199], Stuck has discussed that a finite variance Gaussian model is appropriate over a limited range of data, while an infinite variance model is adequate in terms of matching the observed data over a wider range. Therefore, the impulse-noise occurrence may be modelled as non-Gaussian for further analysis, which favours the application of adaptive nonlinear-filtering for the noise excision and nonlinear-system-identification [161]. Under the aforementioned conditions, the cost-function based on the minimum error dispersion (MED) outperforms the conventional minimum mean square error (MMSE) based approach [122]. Moreover, it results in the development of least mean p^{th} power (LMP) adaptation algorithm, in which the cost-function is convex with respect to the filter-weights for the range $p \geq 1$. However the performance of LMP algorithm supersedes the conventional LMS algorithm, only when the value of parameter p is close to α for the range $1 < p < \alpha$. But in [188], Weng and Barner have delineated that the large eigenvalue-spread of the input-signal auto-correlation matrix has been observed in the case of Volterra-filtering, which in turn results in the slow convergence speed/rate of the LMP as well as LMS adaptive algorithms. However, the nonlinear Volterra FSS-LMS filter can encounter divergence in case of the ill-conditioned tap-input auto-correlation matrix [198].

The time-varying step-size is one of the tractable solutions to expedite the convergence process in the case of LMP algorithm. Kwong and Johnston have proposed a variable step-size LMS (KVSS-LMS) adaptive algorithm in [91] for the tracking of time-varying first-order Markovian-channels, in which the SS adjustment is controlled by the square of prediction-error. Further, Aboulnasr and Mayyas have presented a variable step-size LMS (AVSS-LMS) adaptive algorithm in [96], in which the step-size of the algorithm is adjusted according to the square of time-averaged estimate of the auto-correlation of present/instantaneous estimation-error $e(n)$ and the past estimation-error $e(n-1)$. In an alternate approach proposed by Ang and Farhang-Boroujeny in [104], the step-size of adaptive-filter is changed according to a stochastic gradient adaptive algorithm designed to reduce the squared estimation-error at each iteration, which is denoted as SVSS-LMS algorithm. All the aforementioned VSS-LMS algorithms are implemented using the linear-filtering perspective.

In this chapter, we propose adaptive nonlinear Volterra-filtering using the generalized variable step-size least mean p^{th} power (GVSS-LMP) algorithm for the slowly time-varying system-identification, in the presence of α -stable impulsive-noise. This combination of GVSS and LMP algorithm enhances the convergence-rate under the noisy environment. However, it reduces to the various VSS-LMP and VSS-LMS adaptive algorithms under the typical parametric conditions, which signifies its flexibility.

3.2 Nonlinear-System Model in Noisy Environment

3.2.1 Slowly Time-Varying Volterra-System

Among polynomial system models, the Volterra-system [3] is the preferred paradigm because its output is nonlinear with respect to the input-signals, but it is linear in terms of the kernels. Therefore, the adaptive signal processing techniques may be directly extended to the Volterra-filtering. In literature, there are many time-varying nonlinear wireless or underwater acoustic communication channels, which need to be tracked or estimated by the nonlinear polynomial adaptive-filtering. For identification of these unknown systems, we consider the configuration shown in Fig. 3.1, in which the underlying system and the adaptive nonlinear Volterra-filter are driven by the common input-signal vector $\vec{x}(n)$. In the presence of impulse-noise, the general input-output relationship of an unknown nonlinear-system can be illustrated by a truncated Volterra-series as

$$y(n)=h_0+\sum_{k=1}^{K}\sum_{m_1=0}^{M-1}\cdots\sum_{m_k=0}^{M-1}h_k(n;m_1,\ldots,m_k)\prod_{i=1}^{k}x(n-m_i)+imp(n) \qquad (3.1)$$

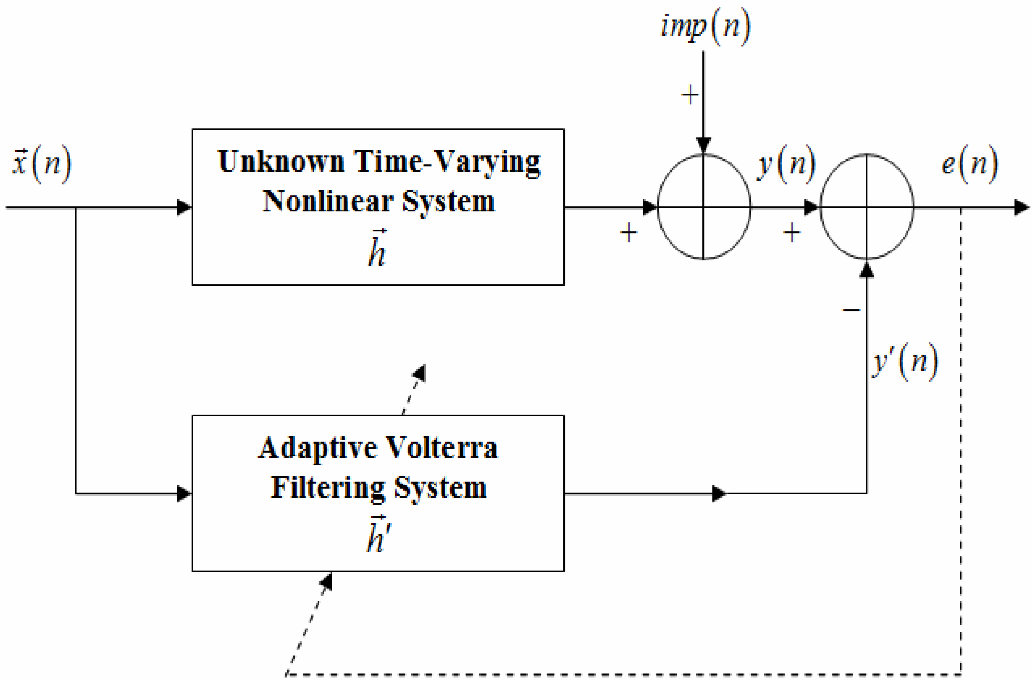

Fig. 3.1: Nonlinear slowly time-varying system-identification configuration

Typically, the second-order Volterra-series is described by the input-output relationship as

$$y(n) = h_0 + \sum_{m_1=0}^{M-1} h_1(n;m_1)x(n-m_1) + \sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1} h_2(n;m_1,m_2)x(n-m_1)x(n-m_2) + imp(n) \tag{3.2}$$

where, h_0 is the time-invariant zeroth-order Volterra-kernel, h_1, h_2 are the first-order and second-order Volterra-kernels respectively, M is the memory length, $x(n)$ is the input-signal, and $imp(n)$ is the α -stable noise with zero-mean (inevitable disturbance). The complexity of Volterra-filter is dependent upon the memory (M). In the general case, the degree of nonlinearity (K) of the Volterra-system is usually assumed to be time-invariant [89]. As the Volterra-kernels are symmetrical in nature, the value of coefficient $h_k(n;m_1,...,m_k)$ is kept unchanged for any of the possible k! permutations of $m_1, m_2, ..., m_k$. Hence, these kernels remain time-invariant under the different permutations of its argument.

In the presented work, the values of K and M are considered to be known *a priori*. For the slowly time-varying second-order Volterra-system, the input-output relationship is depicted by Eq. (3.1) with K = 2. Now, let us consider the $L\times1$ dimensional expanded filter-coefficients vector as

$$\begin{aligned}\vec{h}(n) = [&h_1(n;0), h_1(n;1),\ \ldots\ , h_1(n;M-1), h_2(n;0,0), h_2(n;0,1),\ \ldots,\\ &\ldots\ , h_2(n;0,M-1), h_2(n;1,1),\ \ldots\ , h_2(n;M-1,M-1)]^T\end{aligned} \tag{3.3}$$

where, $(.)^T$ is the matrix transpose operator. The $L\times1$ dimensional expanded input-signal vector for second-order Volterra-filter with zero-mean and variance $\sigma_x^2 = 1/L$ is denoted as

$$\begin{aligned}\vec{x}(n) = [&x(n), x(n-1), \ldots, x(n-M+1), \ldots, x^2(n), x(n)x(n-1), \ldots,\\ &\ldots\ x(n)x(n-M+1),\ \ldots\ , x^2(n-1), \ldots, x^2(n-M+1)]^T\end{aligned} \tag{3.4}$$

Further, we can express Eq. (3.2) in the vector form as

$$y(n) = \vec{h}^T(n)\vec{x}(n) + imp(n) \tag{3.5}$$

In the nonlinear-system-identification, the final goal is to identify the time-varying Volterra-kernels $h_k(n; m_1, ..., m_k)$ in Eq. (3.1) through measured $y(n)$ and $x(n)$, which follow the Random-Walk model [91], [92], [96], [104], given by $\vec{h}(n+1) = \vec{h}(n) + \vec{w}(n+1)$; where, $\vec{w}(n)$ is the zero-mean white Gaussian process-noise vector with variance $\sigma_w^2 = 0.001$ (assumed to be small for the slow time-variations). For the mathematical analysis, the adaptively estimated Volterra-kernel vector may be represented by

$$\begin{aligned}\vec{h}'(n) = [&h_1'(n;0), h_1'(n;1), \ \dots \ , h_1'(n;M-1), h_2'(n;0,0), h_2'(n;0,1), \dots\dots, \\ &\dots\dots\dots\dots\dots\dots\dots\dots , h_2'(n;0,M-1), h_2'(n;1,1), \ \dots \ , h_2'(n;M-1,M-1)]^T\end{aligned} \tag{3.6}$$

Therefore, the estimated received signal is denoted by

$$y'(n) = \vec{h}'^T(n)\vec{x}(n) \tag{3.7}$$

Hence, the output estimation-error in the signal reception is computed by using

$$e(n) = y(n) - y'(n) \tag{3.8}$$

This error-signal is fed back to the adaptive-filter (self-designing filter), which begins from an initial guess based on the prior knowledge available to the system; and then it converges eventually to the optimal-solution in some statistical sense through the successive iterations.

3.2.2 Symmetric α - Stable Noise Model

An α-stable process can be described by the following characteristic function [122], [200], as it exhibits no closed form probability density function.

$$\Phi(\Omega) = \exp\left[j\eta\Omega - \gamma|\Omega|^\alpha \{1 + j\beta \operatorname{sgn}(\Omega) S(\Omega, \alpha)\} \right] \tag{3.9}$$

where, $S(\Omega,\alpha) = \begin{bmatrix} \tan\left(\dfrac{\alpha\pi}{2}\right) & for\ \alpha \neq 1 \\ \dfrac{2}{\pi}\log|\Omega| & for\ \alpha = 1 \end{bmatrix}$, $0 < \alpha \leq 2$, $-\infty < \eta < +\infty$, $\gamma > 0$,

and $-1 \leq \beta \leq +1$. Thus, a stable distribution is completely determined by four parameters: (1) the location parameter η, (2) the index of skewness β (the distribution is symmetric about its location parameter η, when $\beta = 0$, therefore called symmetric α-stable distribution), (3) the scale parameter γ is called dispersion (the parameter $\gamma^{1/\alpha}$ plays a role similar to the standard deviation of the Gaussian distribution), and (4) α is the characteristic exponent. This shape parameter is a measure of the heaviness of the tail of distribution. The processes with small values of α are considered to be impulsive. However for the large values of α, the observed values of random variable are not far from its central location. Under typical conditions, when the value of $\alpha \to 2$ and $\beta = 0$, then $\Phi(\Omega) \to \exp\left[j\eta\Omega - \gamma |\Omega|^2 \right]$. This relevant stable distribution is Gaussian in nature. However in the presented work, the α-stable random variables do not have finite variance, but these are characterized only by the finite p^{th}-order moments for $p < \alpha$. It is noteworthy fact that all the moments of order less than α do exist and are called the fractional lower order moments (FLOM) [122], which can be derived from its dispersion and the characteristic exponent with zero location parameter as

$$E|X|^p = C(p,\alpha)\gamma^{p/\alpha} \quad \textit{for } 0 < p < \alpha$$

where, $$C(p,\alpha) = \frac{2^{p+1}\Gamma\left(\left(p+1\right)/2\right)\Gamma\left(-p/\alpha\right)}{\alpha\sqrt{\pi}\,\Gamma\left(-p/2\right)} \tag{3.10}$$

which is dependent on the values of parameters α and p, not on random variable X. In the above equation, the parameter Γ is the gamma function [122], [200].

3.3. GVSS-LMP Algorithm for Nonlinear-System-Identification

3.3.1 Least Mean p^{th} Power Adaptive Algorithm

In the field of adaptive signal processing [33], the most popular approaches for the estimation/prediction schemes are based on the MMSE criterion. The corresponding cost-function as per the Wiener theory is

$$J_{MMSE}\left(\vec{h}'\right) = E\left[\left|y(n) - \vec{h}'^{T}(n)\,\vec{x}(n)\right|^2\right] \tag{3.11}$$

where, $E[*]$ is the ensemble average operator. Using the error-signal $e(n)$, the non mean squared error criterion is discussed in the form of least mean forth (LMF) adaptive algorithm in [138], in which the cost-function is considered to be $E\left[|e(n)|^{2\bar{K}}\right]$ for $\bar{K} \geq 1$ (only integer values of $\bar{K}$). In some typical cases, the LMF algorithm with $\bar{K} > 1$ outperforms the conventional LMS algorithm by providing less noise in weights for the same speed of convergence. It has motivated the evolution of LMP algorithm for the noisy situations, in which $1 < p < \alpha < 2$. Particularly, when $\vec{h}'(n) \to \vec{h}(n)$ in the presence of α -stable noise, the residual *imp*(n) dominates in the estimation-error $e(n)$ in Eq. (3.8). Therefore, the resulting estimation-error may be assumed as approximately α -stable process, such that the FLOM [122] is

$$E\left[|e(n)|^{p-2}\right] = C(p-2,\alpha)\gamma^{((p-2)/\alpha)} = D(p,\alpha,\gamma) \tag{3.12}$$

Since the variance of α -stable noise is not finite, therefore we can utilize the MED criterion [199] i.e., the minimization of cost-function

$$J_{MED}\left(\vec{h}'\right) = E\left[\left|y(n) - \vec{h}'^{T}(n)\vec{x}(n)\right|^{p}\right] \tag{3.13}$$

It is equivalent to the minimization of p^{th} order FLOM. Unfortunately, this cost-function $J_{MED}\left(\vec{h}'\right)$ does not exhibit closed-form solution. Therefore, the stochastic gradient technique can be utilized as an alternative for the minimization of $J_{MED}\left(\vec{h}'\right)$, similar to the LMS adaptive algorithm. The basic idea is to minimize the error dispersion for each successive datum or observation as much as possible. It leads to

$$\vec{h}'(n+1) = \vec{h}'(n) - \mu(n)\nabla_h J_{MD}\left(\vec{h}'\right) \tag{3.14}$$

Akin to the steepest-descent algorithm [33],

$$\nabla_h J_{MD}\left(\vec{h}'\right) = \frac{\delta E\left[\left|y(n) - \vec{h}'^T(n)\vec{x}(n)\right|^p\right]}{\delta \vec{h}'} \tag{3.15}$$

Analogous to the stochastic gradient algorithm [122],

$$\nabla_h J_{MD}\left(\vec{h}'\right) \approx \frac{\delta\left[\left|e(n)\right|^p\right]}{\delta \vec{h}'} = p\left|e(n)\right|^{p-1}\frac{\delta\left|e(n)\right|}{\delta \vec{h}'} \tag{3.16}$$

where,
$$\begin{aligned}\left|e(n)\right| &= \operatorname{sgn}\left\{e(n)\right\}e(n)\\ &= \operatorname{sgn}\left\{e(n)\right\}\left\{y(n) - \vec{h}'^T(n)\vec{x}(n)\right\}\end{aligned} \tag{3.17}$$

Hence, $\nabla_h J_{MD}\left(\vec{h}'\right) \approx -p\left|e(n)\right|^{p-1}\operatorname{sgn}\left\{e(n)\right\}\vec{x}(n)$ (3.18)

By substituting Eq. (3.18) in Eq. (3.14), it can be shown that

$$\vec{h}'(n+1) = \vec{h}'(n) + \left\{\mu(n)\,p\right\}\left|e(n)\right|^{p-1}\operatorname{sgn}\left\{e(n)\right\}\vec{x}(n) \tag{3.19}$$

The simplified version of LMP algorithm can be represented as

$$\vec{h}'(n+1) = \vec{h}'(n) + \mu'(n)\left|e(n)\right|^{p-1}\operatorname{sgn}\left\{e(n)\right\}\vec{x}(n) \tag{3.20}$$

where, $\mu'(n) = \mu(n)\,p$ is the variable step-size (VSS), which plays a critical role in the convergence-mode of LMP algorithm [188], for the operating range $1 < p < \alpha < 2$. Although the convergence analysis of LMP algorithm is a tedious problem, yet the convergence range of variable step-size $\mu'(n)$ in Eq. (3.20) can be approximated as

$$0 < \mu'(n) < \frac{2}{D(p,\alpha,\gamma)\lambda_{Max}} \qquad \text{(loose bounded, [188])} \tag{3.21}$$

By invoking a better approximation, it can be shown that

$$0 < \mu'(n) < \frac{2}{D(p,\alpha,\gamma)tr\left(\vec{R}_{xx}\right)} < \frac{2}{3tr\left(\vec{R}_{xx}\right)} \qquad \text{(tight bounded, [63],[188])} \tag{3.22}$$

where, $tr\left(\vec{R}_{xx}\right)$ symbolizes the trace of auto-correlation matrix $\vec{R}_{xx}$ of the input-signals. The maximum value of VSS is tightly bounded by the maximum eigenvalue λ_{Max} of the matrix

$\vec{R}_{xx} = E\left[\vec{x}(n)\vec{x}^T(n)\right]$. The LMS algorithm is a special case of LMP algorithm for $p = 2$ and $\alpha = 2$ in Eq. (3.20). In the next subsection, we give details about the proposed GVSS criterion to update $\mu'(n)$ in the aforementioned iterative procedure.

3.3.2 Generalized Variable Step-Size (GVSS) Criterion

The large eigenvalue-spread in the case of Volterra-filtering necessitate the incorporation of variable step-size, in combination with the LMP adaptive algorithm, for the improved convergence-rate. Moreover, the VSS criterion is also beneficial in the tracking of slowly time-varying channels/systems. The VSS must increase or decrease as the mean squared error increases or decreases, allowing the adaptive nonlinear-filter to track changes in the underlying system and to produce a small steady-state error. It should reduce the tradeoff between misadjustment and the speed of adaptation under the slowly time-varying conditions, due to its innate capability of providing both fast tracking as well as small misadjustment. Therefore, the generalized variable step-size (GVSS) criterion is proposed to adjust the step-size under the stationary and nonstationary scenarios, which is as follows

$$\mu'(n) = \bar{\alpha}\mu'(n-1) + \bar{\gamma} J_1(n-1) + \bar{\beta} J_2(n) \tag{3.23}$$

where, $$J_1(n) = \sum_{\bar{p}=0}^{\bar{P}} \lambda_1^{\bar{p}} e(n) e(n-\bar{p}) \quad \text{with } 0 \le \lambda_1 < 1 \tag{3.24}$$

$$J_2(n) = \left[\sum_{\bar{q}=1}^{\bar{Q}} \lambda_2^{\bar{q}} e(n-\bar{q})\, \vec{x}^T(n-\bar{q})\right] \vec{x}(n) e(n) \quad \text{with } 0 \le \lambda_2 < 1 \tag{3.25}$$

where, $0 < \bar{\alpha} \le 1$, $0 \le \bar{\gamma} < 1$, and $0 \le \bar{\beta} < 1$. The parameter $\bar{\alpha}$ induces the global exponential forgetting to the VSS, the parameter $\bar{\gamma}$ controls the convergence time as well as the level of misadjustment [91], the parameter $\bar{\beta}$ adjusts the adaptive behaviour of the step-size sequence $\mu'(n)$ [92]. However, λ_1 *and* λ_2 are the local exponential forgetting-factors in Eqs. (3.24) and (3.25) respectively. For the appropriate convergence, the VSS should be bounded in the range $\mu'_{Min} \le \mu'(n) \le \mu'_{Max}$ [96]. The initial step-size is usually taken as μ'_{Max}, which ensures that the MED of algorithm remains bounded. However, μ'_{Min} is chosen to provide the minimum level of tracking ability, which is kept close to the step-size of FSS-LMS algorithm. Further in special case 1, if the values of parameter are $\bar{P}=0$ and $\bar{\beta}=0$ in Eq. (3.23), then

$$\mu'(n)=\bar{\alpha}\mu'(n-1)+\bar{\gamma}J_1(n-1) \tag{3.26}$$

$$\mu'(n)=\bar{\alpha}\mu'(n-1)+\bar{\gamma}\underline{\underline{e^2(n-1)}} \tag{3.27}$$

The underlined term in Eq. (3.27) is similar to the KVSS criterion presented in [91], (as in Eq. (A.1) of Appendix-A). Next in special case 2, if the parametric values $\bar{P}=1$ and $\bar{\beta}=0$ are in Eq. (3.23), then

$$\mu'(n)=\bar{\alpha}\mu'(n-1)+\bar{\gamma}e^2(n-1)+\{\bar{\gamma}\lambda_1\}\underline{\underline{\{e(n-1)e(n-2)\}}} \tag{3.28}$$

The underlined term in Eq. (3.28) is akin to the AVSS criterion described in [96], (as in Eq. (A.3) of Appendix-A). Subsequently in special case 3, if $\bar{\alpha}=1$, $\bar{Q}=1$ and $\bar{\gamma}=0$ are in Eq.(3.23), then

$$\mu'(n)=\mu'(n-1)+\{\bar{\beta}\lambda_2\}\underline{\underline{\left\{e(n-1)\vec{x}^T(n-1)\vec{x}(n)e(n)\right\}}} \tag{3.29}$$

The underlined term in Eq. (3.29) is analogous to the Mathews' algorithm proposed in [92]. However in special case 4, $\bar{\alpha}=1$ and $\bar{\gamma}=0$ in Eq. (3.23) results in

$$\mu'(n)=\mu'(n-1)+\{\bar{\beta}\lambda_2\}\underline{\underline{\begin{bmatrix} e(n-1)\vec{x}^T(n-1)+\lambda_2 e(n-2)\vec{x}^T(n-2)+ \\ +\ \lambda_2^2 e(n-3)\vec{x}^T(n-3)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\ \lambda_2^{\bar{Q}-1}e(n-\bar{Q})\vec{x}^T(n-\bar{Q}) \end{bmatrix}}}\vec{x}(n)e(n) \tag{3.30}$$

The underlined term in Eq. (3.30) is similar to the SVSS criterion suggested in [104], (as shown in Eq. (A.7) of Appendix-A). Therefore, the abovementioned GVSS criterion in Eq. (3.23) is incorporated in Eq. (3.20) to formulate the proposed GVSS-LMP algorithm, which is relatively computationally complex than FSS-LMP, KVSS-LMP, AVSS-LMP and SVSS-LMS algorithms.

3.4 Simulation Results

The performance evaluation of the proposed GVSS-LMP algorithm is performed by comparing it with KVSS-LMP, AVSS-LMP, and SVSS-LMS algorithm under the similar conditions, for the nonlinear-system-identification. The kernels of the unknown system (as shown in Fig. 3.1) are assumed to follow the Random Walk model for the slow time-variations in the system response (as discussed in subsection 3.2.1). As in α -stable noisy environment, the error-signal variance could be infinite, therefore the LMS algorithm based on the MMSE criterion in Eq. (3.11) seems to be an inappropriate choice in comparison with the MED criterion $J_{MED}\left(h'\right)$ in Eq. (3.12). However, the value of p in the LMP algorithm (3.20) is kept close to α for excellent results [91] in terms of the transient and steady-state behaviour, which are fixed at $\alpha = 1.75$ *and* $p = 1.6$.

The input-signal to the underlying unknown system (as shown in Fig. 3.1) may be correlated or uncorrelated Gaussian sequence $\vec{x}$. The white Gaussian input is quite apposite for the identification of kernels in the Volterra-system because it has adequate spectral representatives and sufficient amplitude variations [201]. Moreover, the Volterra-series can be expressed as G-functionals [197], which form an orthogonal set when the input is white Gaussian. However, the non-identical-independent-distributed input-signals lead to the large eigenvalue-spread of the auto-correlation matrix $\vec{R}_{xx}$ (particularly in the case of Volterra-filters), which in turn results in the slow convergence [33]. The value of minimum step-size is $\mu'_{Min} = 0.0008$ and the maximum bounded value of step-size μ'_{Max} is set by using the Eq. (3.22). The signal-to-noise ratio (SNR) is defined as the input-signal variance to the dispersion of the α -stable noise i.e., $SNR = \sigma_x^2 / \gamma$, which is kept $15dB$ for all the simulations [188]. The Volterra-kernel mean square estimation-error (performance appraisal factor) is calculated by using the following formula

$$\hat{J}(n) = E\left[\left|h(n) - h'(n)\right|^2\right] \tag{3.31}$$

As per the Monte-Carlo simulations, the performance of adaptive algorithms is compared on the basis of measured performance appraisal factor as

$$\hat{J}(n) = \sum_{j=1}^{2500}\left[\frac{\left|h(n,j) - h'(n,j)\right|^2}{2500}\right] \tag{3.32}$$

Example 1:- We consider the second-order Volterra-filter with $K = 2$ *and* $M = 3$ in the first simulation setup, with the uncorrelated Gaussian white input-sequence. Similar to the methodology opted in [96], the parameter values of the adaptive algorithms are selected to produce a comparable level of misadjustment. The values of parameters are $\lambda_1 = 0.8,\ \lambda_2 = 0.5$, $\bar{\alpha} = 0.97,\ \bar{\gamma} = 15\times10^{-5}$, $\bar{P} = \bar{Q} = 2$, $\bar{\alpha}_A = 0.97,\ \bar{\alpha}_W = 0.8,\ \bar{\rho}_W = 15\times10^{-5}$. The value of parameter $\bar{\beta}$ is varied as $\bar{\beta} = 0.00003,\ GVSS - LMP1$, $\bar{\beta} = 0.00005,\ GVSS - LMP2$, and $\bar{\beta} = 0.00015,\ GVSS - LMP3$. It is apparent from the simulation results depicted in Fig. 3.2 that the performance of GVSS-LMP algorithm improves as the value of $\bar{\beta}$ increases. The performance of $\bar{\beta} = 0.00015,\ GVSS - LMP3$ is approximately $7dB$ better than AVSS-LMP algorithm in the tracking-mode, and this proposed algorithm converges at the higher rate than other conventional algorithms.

Further, the value of $\bar{\beta} = 0.00005$ is fixed under the similar conditions. However, the values of $\bar{P}$ *and* $\bar{Q}$ are varied as $\bar{P} = \bar{Q} = 1$, $\bar{P} = \bar{Q} = 2$, $\bar{P} = \bar{Q} = 3$ in the GVSS-LMP algorithm. The simulation results demonstrated in Fig. 3.3 evidenced that $\bar{P} = \bar{Q} = 2$ is the suitable preference for GVSS-LMP algorithm, which also restricts its computational-complexity.

Subsequently, the values of $\bar{\beta} = 0.00005$ and $\bar{P} = \bar{Q} = 2$ are fixed under the similar conditions. However, the values of $\lambda_2 = 0.3,\ 0.5,\ 0.7$ are varied in the proposed GVSS-LMS algorithm. It may be inferred from the simulation results in Fig. 3.4 that the performance of presented adaptive algorithm can be improved by increasing the value of λ_2. However for $\lambda_2 > 0.75$, the observed performance advantage is marginal.

Example 2:- Now, we consider the third-order Volterra-filter with $K = 3$ *and* $M = 3$ in this simulation setup with the uncorrelated Gaussian white input-sequence. As the number of filter-weights increases in this case [33], [96], the parameter values need to be changed to maintain the value of GVSS within limits. The values of parameters are $\lambda_1 = 0.98,\ \lambda_2 = 0.5$, $\bar{\alpha} = 0.91,\ \bar{\beta} = \bar{\gamma} = 0.000025,\ \bar{P} = \bar{Q} = 2$, $\bar{\alpha}_A = 0.98,\ \bar{\alpha}_W = 0.8,\ \bar{\rho}_W = 15\times10^{-9}$. The results in Fig. 3.5 manifest that the performance advantage of GVSS-LMP algorithm is approximately $3dB$ better than the AVSS-LMP algorithm in the tracking-mode. However, the convergence-rate of both algorithms is approximately same in the initial phase. But, the significant performance degradation is observed in the case of KVSS-LMP algorithm.

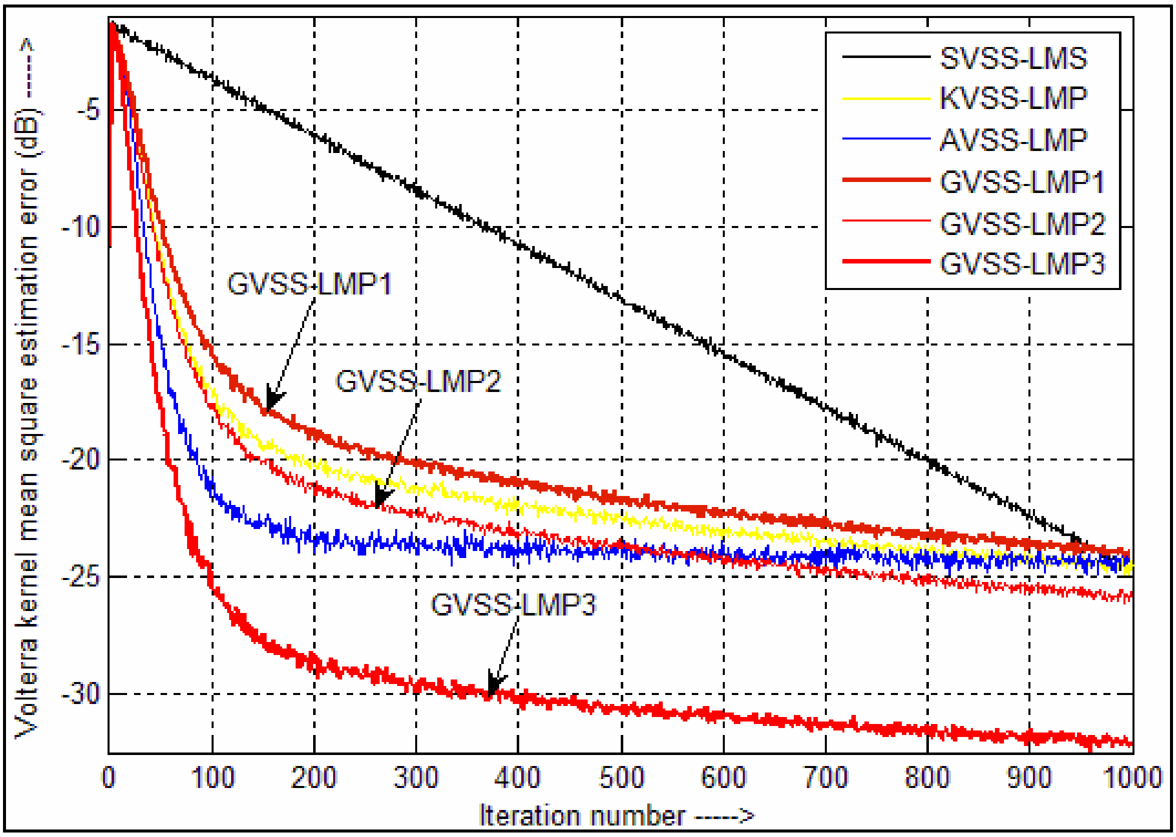

Fig. 3.2: Comparison of GVSS-LMP algorithm with conventional algorithms under varying value of $\bar{\beta}$ for the second-order Volterra-filter

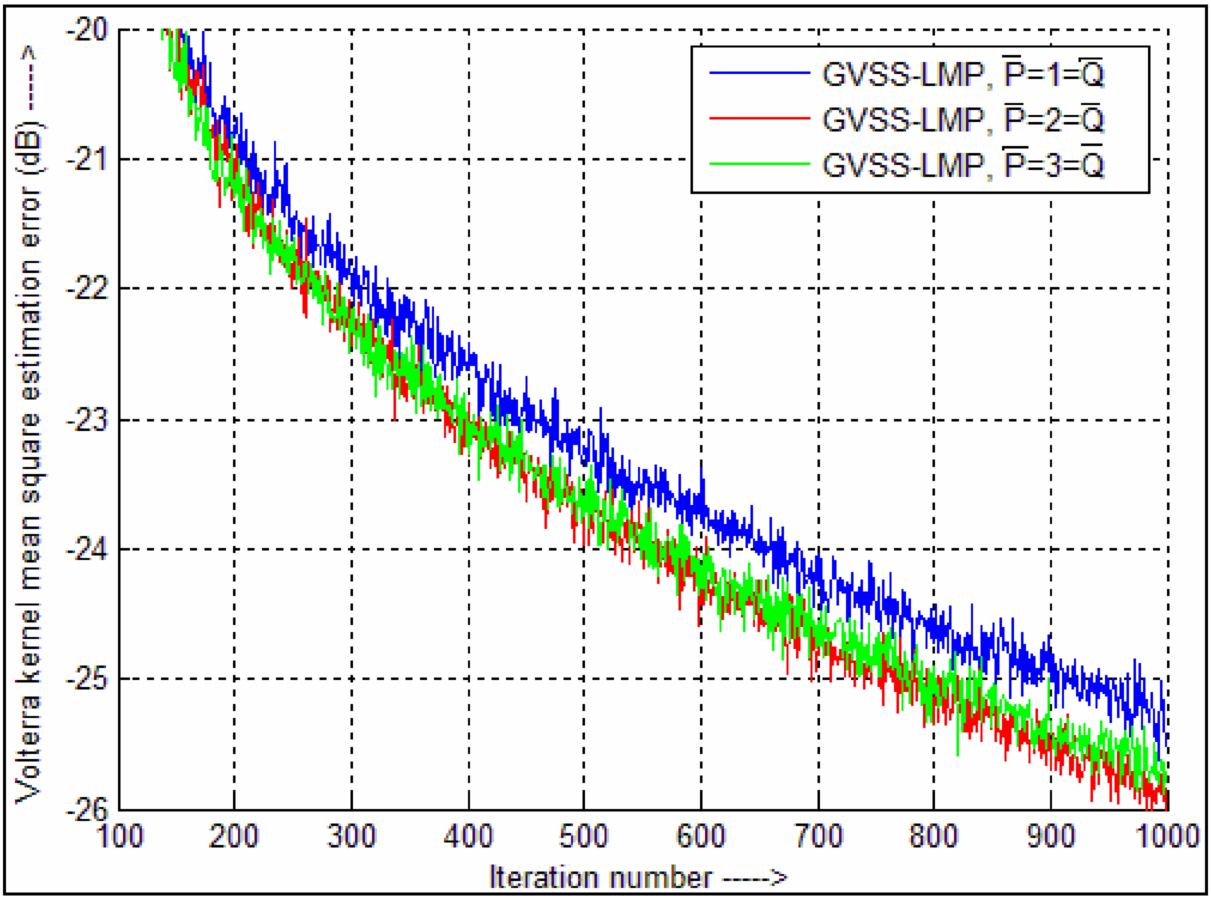

Fig. 3.3: Effects of the variation in the value of $\bar{P}$ *and* $\bar{Q}$ **on GVSS-LMP algorithm**

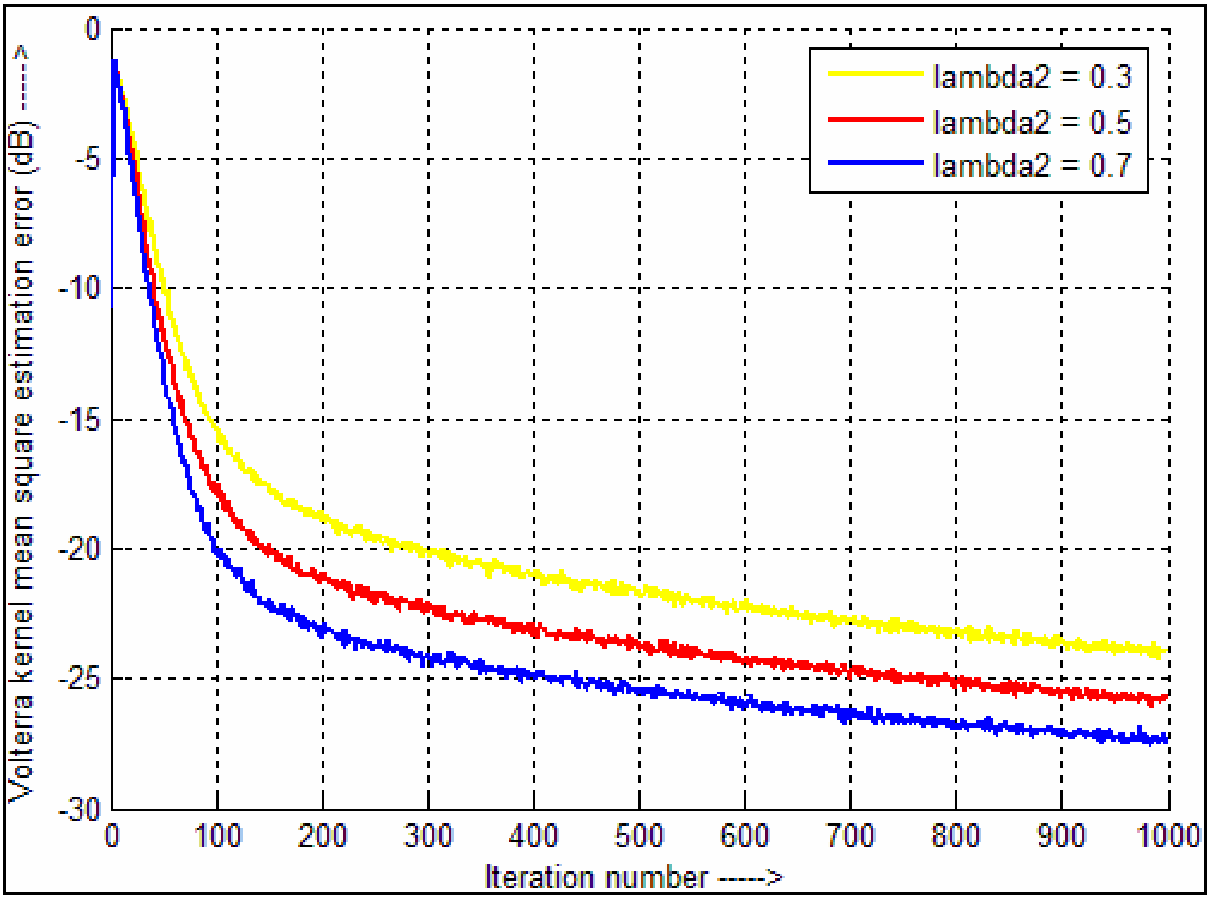

Fig. 3.4: Effects of variation in the value of λ_2 on GVSS-LMS algorithm

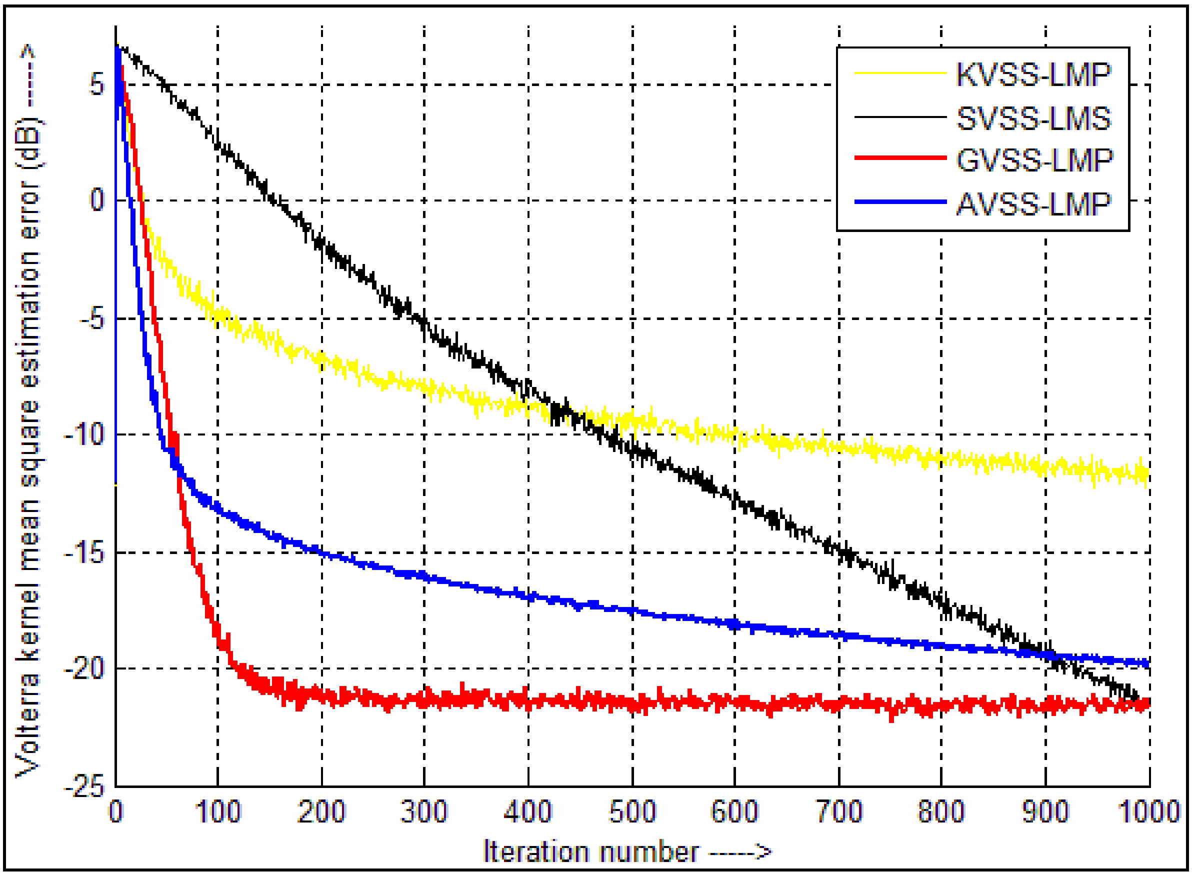

Fig. 3.5: Comparison of GVSS-LMP algorithm with conventional algorithms for the third-order Volterra-filter

Example 3:- Next, we consider the second-order Volterra-filter with $K = 2$ *and* $M = 3$ in this simulation setup, when the unknown system is excited by a correlated input-signal as $\vec{x}(n) = 0.9\vec{x}(n-1) + \vec{v}_x(n)$; where, $\vec{v}_x(n)$ is a zero-mean, uncorrelated Gaussian-noise of unity variance. This type of input-signals leads to the flattened elliptical contours, which usually cause difficulties in the convergence of stochastic gradient adaptive algorithms. The values of parameters are $\lambda_1 = 0.8,\ \lambda_2 = 0.5$, $\bar{\alpha} = 0.97,\ \bar{\gamma} = 15\times10^{-6}$, $\bar{P} = \bar{Q} = 2$, $\bar{\alpha}_A = 0.97$, $\bar{\alpha}_W = 0.8,\ \bar{\rho}_W = 15\times10^{-5}$. The value of parameter $\bar{\beta}$ is varied as $\bar{\beta} = 15\times10^{-6}, GVSS-LMP1$, $\bar{\beta} = 15\times10^{-7}, GVSS-LMP2$ and $\bar{\beta} = 15\times10^{-8}, GVSS-LMP3$.

It is observed from the results in Fig. 3.6 that the performance of GVSS-LMP algorithm improves as the value of $\bar{\beta}$ increases, but the overall performance degradation is noticed for all the algorithms. We now fix the value of parameter $\bar{\beta} = 15\times10^{-6}$ for the simulation results in Fig. 3.7, which indicate that the proposed GVSS-LMP algorithm still outperforms the conventional algorithms. The variable step-size controls the problem of eigenvalue-spread, and consequently leads to enhanced convergence-rate in the presence of impulse-noise and correlated input-signal.

On contrary to the case of uncorrelated input-signal, it may be inferred from the results presented in Fig. 3.6 and Fig. 3.7 that the gradient-misadjustment [202] is relatively more in the case of correlated input-signal. However, the convergence of GVSS-LMP algorithm is strictly dependent on the appropriate parameter tuning/setting in Eq. (3.23), while keeping the value of GVSS below μ'_{Max} in Eq. (3.22). Akin to the VSS-LMS algorithms [63], [95], the GVSS-LMP algorithm is found to be sensitive to noise disturbances in the low SNR environment.

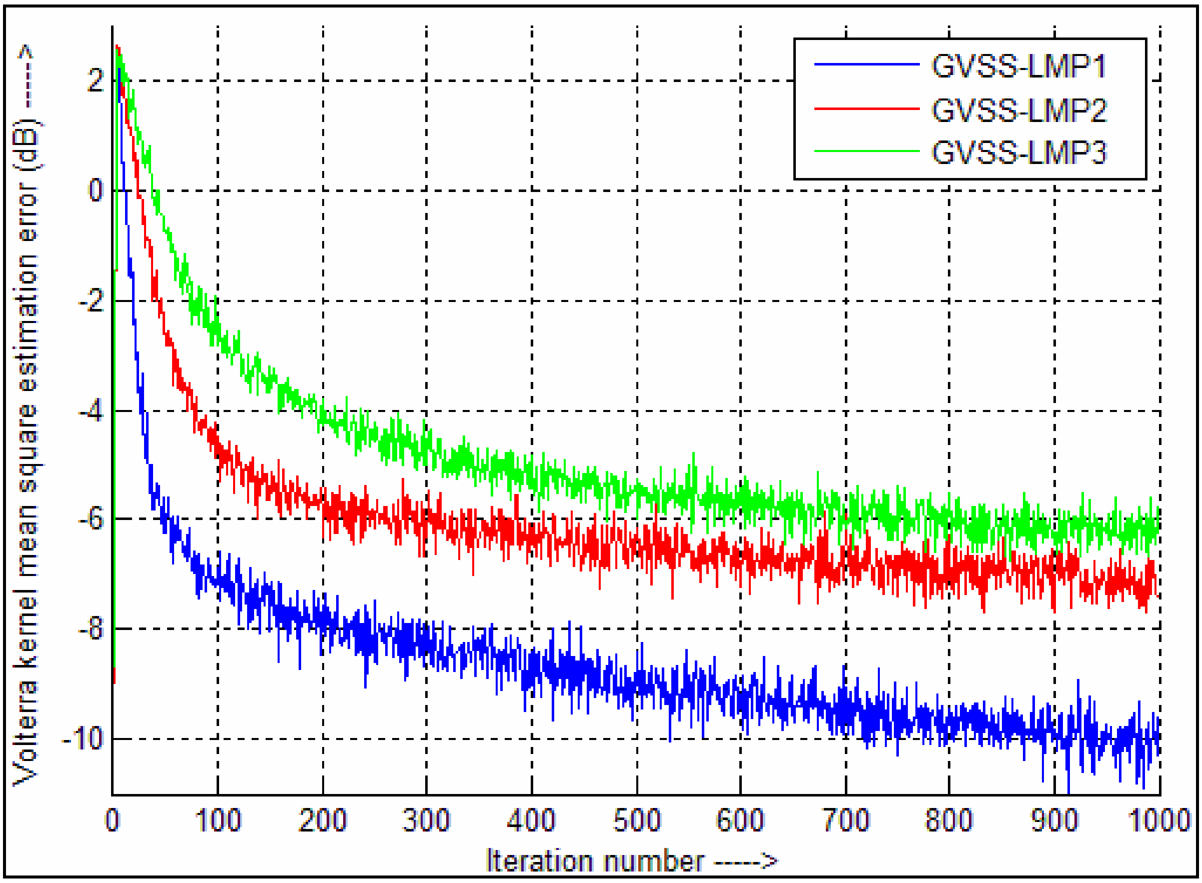

Fig. 3.6: Effects of variation in the value of $\bar{\beta}$ on GVSS-LMP algorithm for the correlated input-signal

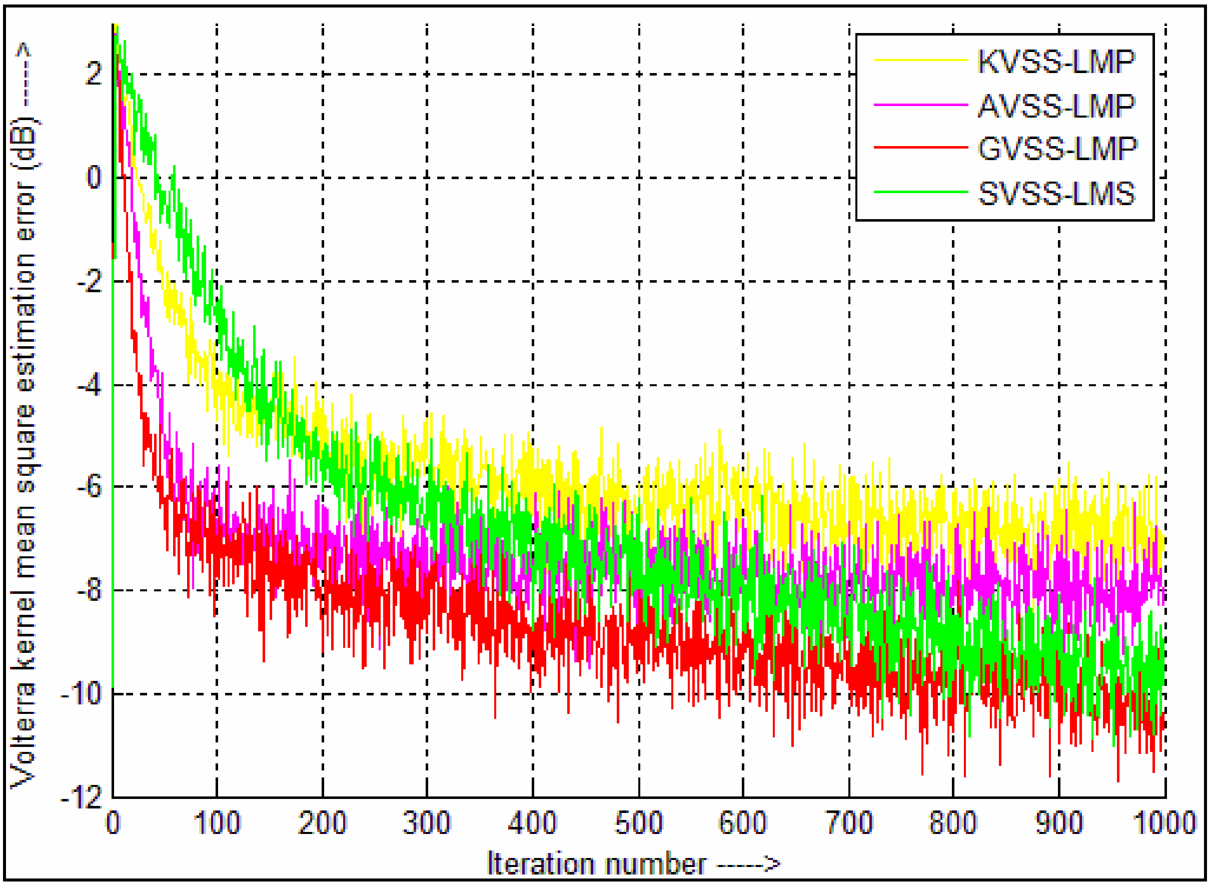

Fig. 3.7: Comparison of GVSS-LMP algorithm with conventional algorithms for the second-order Volterra-filter with correlated input-signal

3.5 Summary of Chapter

This chapter presents a generalized variable step-size least mean p^{th} power (LMP) adaptive algorithm for the α-stable noisy environment, which is based on the MED criterion. This algorithm is implemented to identify the unknown time-varying nonlinear-systems using the Volterra-filtering approach. However, the MMSE criterion is found to be a special case of MED approach. The GVSS-LMP algorithm exploits the knowledge about the previous step-size, the error auto-correlation values, the value of parameter α, the cross-correlation between error sequence and input-sequence. For excellent results, the value of parameter p is kept close to α in the range $1 < p < \alpha < 2$.

It is apparent from the simulation results that the GVSS-LMP algorithm supersedes the KVSS-LMP, AVSS-LMP and SVSS-LMS algorithms in the convergence as well as tracking-mode, when the input-signal is either correlated or uncorrelated Gaussian process. The proposed algorithm also controls the adverse effects of eigenvalue-spread of the input-signal auto-correlation matrix, by using the GVSS criterion to track the time-varying Volterra-kernels. The outperforming GVSS-LMP algorithm may find applications in the systems disturbed due to the presence of non-Gaussian impulsive measurement-noise, where the conventional FSS-LMS algorithm fails to perform well. Moreover, the different LMP algorithms with $p \neq 2$ and LMS algorithms with $p = 2$ can be derived from the GVSS-LMP algorithm by adjusting the parameters according to the requirements. Future work includes the application of proposed adaptive nonlinear Volterra-filtering technique in the emerging fields of bio-signal processing, biomedical engineering [203], nonlinearly amplified digital as well as analog communication signal processing [195] and equalization of nonlinear communication channels [53].

CHAPTER 4

VOLTERRA-FILTERING SCHEME USING GENERALIZED VARIABLE STEP-SIZE NLMS ALGORITHM FOR NONLINEAR ACOUSTIC ECHO CANCELLATION

4.1 Introduction

Nonlinear echo in acoustic systems arises due to the nonlinear characteristics of amplifiers/loudspeakers, which degrades the performance of speech and audio communication systems. These nonlinear echo components can not be cancelled by the conventional linear acoustic echo cancellers (AECs). High quality multimedia services require echo-free speech and audio transmission and reception. However, a nonlinear echo of saturation-type is commonly perceived as a major degradation factor [178]. Various nonlinear AECs have been presented in the literature [1], [33], [171], [178], [179], [181], [184], [185], [186], [194], [204], [205], but form only the tip of the iceberg in the domain of audio and speech signal processing that combat nonlinear-distortions. For mild nonlinear-distortions, the second-order Volterra-filters are found to be efficient. However, the saturation-type nonlinear-distortions require the application of higher order polynomial Volterra-filters, which encounter a high eigenvalue-spread of the input-signal vector and also the problem of over-parameterization. This leads to an alleviated rate of convergence for such nonlinear AECs [205].

The affine-projection and recursive least squares algorithms are commonly used to deal with ill-conditioned auto-correlation matrix of the input-signal, which has deleterious effects on the performance characteristics of LMS algorithm based linear AECs [206], [207]. Moreover, the saturation [208] and nonlinearities containing memory [183] have severe influence on the performance of classical linear AECs. For a loudspeaker nonlinearity with memory, the efficiency of second-order Volterra-filters (SOVFs) supersedes the linear adaptive-filters [127], [209], in which the number of coefficients grows with the square of the memory size of SOVF. It is reported in [209] that the response of SOVF is concentrated around the main section of the linear response, which corresponds to the delay introduced due to the linear acoustic-path. However, the computational-complexity can be reduced by decomposing the higher order Volterra-kernels into the product of lower order kernels [85], [182].

In this chapter, we consider a cascaded structure consisting of two modules. The first module identifies the loudspeaker parameters responsible for the nonlinearity, which is based on a polynomial Volterra paradigm. The second module identifies the parameters of the linear acoustic-path impulse response as in [182]. The normalized-LMS (NLMS) adaptive algorithm is proposed as an amicable solution in [182] to mitigate the instability problem arising due to the filter misadjustment in the transient phase. However, the nonlinear constituents vary rapidly with the acoustic-path due to the effect of their convolution, which also restricts the convergence-rate of adaptive algorithms. To enhance the convergence-rate in the presence of ill-conditioned auto-correlation matrix of the input-signal, we propose the usage of GVFF-NLMS [210] adaptive algorithm for AEC. The presented adaptive cascaded structure processes the second-order and third-order Volterra-kernels in the first module. Subsequently, the second module processes the first-order Volterra-kernels and the linear acoustic-path channel coefficients jointly, as in [182]. It reduces the computational-complexity as compared to the recursive least squares algorithm based approaches for AEC.

4.2 Nonlinear-System Model for Loudspeaker and Acoustic-Path

Let us consider a cascaded approach to model loudspeaker and acoustic-path (as shown in Fig. 4.1). The loudspeaker is assumed to be a polynomial-filter because these conventionally have memory embedded in their nonlinear part, which usually results in the harmonic distortion (based on the Taylor series expansion [3]). At the n^{th} instant of time, the input to the loudspeaker is $x(n)$. The corresponding output of loudspeaker is given as

$$\begin{aligned} x_{vf}(n) = &\sum_{\alpha_1=0}^{+\infty} g_1(\alpha_1;n)\,x(n-\alpha_1) + \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} g_2(\alpha_1,\alpha_2;n)\,x(n-\alpha_1)\,x(n-\alpha_2) \\ &+ \;.....\; + \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} ..... \sum_{\alpha_P=\alpha_{P-1}}^{+\infty} g_P(\alpha_1,\alpha_2,.....,\alpha_P;n)\,x(n-\alpha_1)\;.....\;x(n-\alpha_P) \end{aligned} \tag{4.1}$$

which is based on the causal $P^{th}-$order nonlinear Volterra-filter response. The kernel g_1 is responsible for the linear response, and the kernels $g_2, g_3, ..., g_P$ are responsible for the higher order nonlinear response. Based on audio and speech signal analysis under the realistic practical conditions, it is demonstrated in [182] that the third-order Volterra-model is a close approximation of the loudspeaker nonlinear response, such that

$$x_{eq}(n) = x_{vf}(n)\Big|_{P=3} = x_1(n) + x_2(n) + x_3(n) \tag{4.2}$$

with, $$x_1(n) = \sum_{\alpha_1=0}^{+\infty} g_1(\alpha_1;n)\,x(n-\alpha_1) = g_1(n) * x(n) \tag{4.3}$$

$$x_2(n) = \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} g_2(\alpha_1,\alpha_2;n)\,x(n-\alpha_1)\,x(n-\alpha_2) \tag{4.4}$$

$$x_3(n) = \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty}\sum_{\alpha_3=\alpha_2}^{+\infty} g_3(\alpha_1,\alpha_2,\alpha_3;n)\,x(n-\alpha_1)\,x(n-\alpha_2)\,x(n-\alpha_3) \tag{4.5}$$

where, the symbol $*$ indicates the linear convolution operator. If the impulse response of the linear acoustic-path is $h_{ac}(n)$, then it can be shown that

$$x_{23}(n) = x_2(n) + x_3(n) \tag{4.6}$$

The output of presented equivalent system model is

$$\begin{aligned} d_{eq}(n) &= h_{ac}(n) * x_1(n) + h_{ac}(n) * x_{23}(n) \\ &= h_{ac}(n) * g_1(n) * x(n) + h_{ac}(n) * x_{23}(n) \end{aligned} \tag{4.7}$$

The linear convolution of the acoustic-path impulse response with the nonlinear input $x_{23}(n)$ makes the situation more cumbersome to handle. If the actual output of microphone is $d(n)$, then for the mathematical equivalence of two models in Fig. 4.1, the stringent requirement is $d(n)-d_{eq}(n)=e_{eq}(n)\simeq 0$.

4.3. Adaptive-Filtering Paradigm for AEC using GVSS-NLMS

To design the adaptive-filtering paradigm for AEC, the Eq. (4.7) is rearranged to provide

$$d_{eq}(n)=h_{eq}(n)*x(n)+h_{ac}(n)*x_{23}(n) \tag{4.8}$$

with $h_{eq}(n)=h_{ac}(n)*g_1(n)$

The discrete-time Fourier-transform on both sides of the above equation results in

$$D_{eq}\left\{e^{j\omega}\right\}=H_{eq}\left\{e^{j\omega}\right\}\left[X\left\{e^{j\omega}\right\}+\frac{H_{ac}\left\{e^{j\omega}\right\}}{H_{eq}\left\{e^{j\omega}\right\}}X_{23}\left\{e^{j\omega}\right\}\right] \tag{4.9}$$

Let us consider that

$$\hat{H}\left\{e^{j\omega}\right\}=H_{ac}\left\{e^{j\omega}\right\}\big/H_{eq}\left\{e^{j\omega}\right\} \tag{4.10}$$

$$\hat{H}\left\{e^{j\omega}\right\}\xleftrightarrow{F}\hat{h}(n) \quad \text{(Fourier-transform pair)} \tag{4.11}$$

Taking the inverse discrete-time Fourier-transform of Eq. (4.9), to gives

$$d_{eq}(n)=h_{eq}(n)*\left[x(n)+\hat{h}(n)*x_{23}(n)\right]=h_{eq}(n)*x_{eq}(n) \tag{4.12}$$

where, $x_{eq}(n)=\delta(n)*x(n)+\hat{h}(n)*x_{23}(n)$ (4.13)

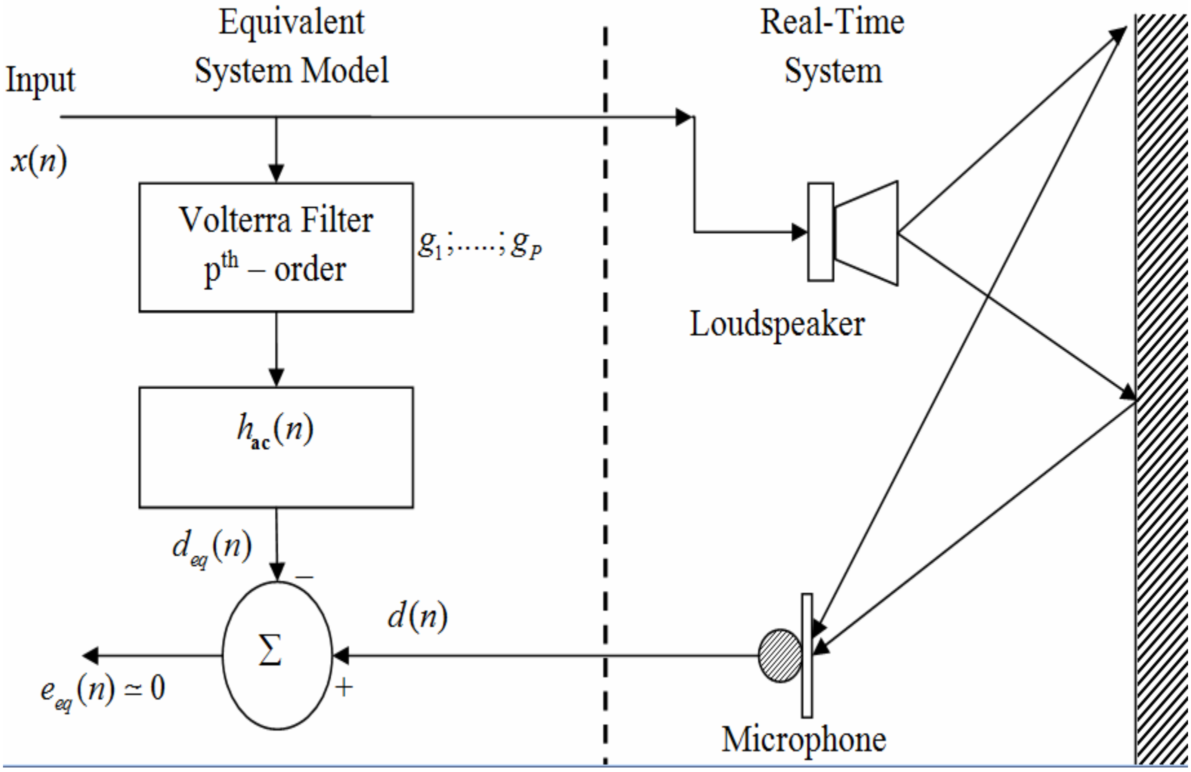

Fig. 4.1: Real-time loudspeaker, acoustic-path and microphone assembly with equivalent mathematical system model

Considering the damping nature of real-time systems [182], the upper limit of the variables α_1, α_2 and α_3 in the summations used in Eqs. (4.3) - (4.5) is kept at $L-1$. Further, it is presumed that

$$\hat{h}(n)*x_{23}(n) \simeq \sum_{\alpha_1=0}^{L-1}\sum_{\alpha_2=\alpha_1}^{L-1}\hat{g}_2(\alpha_1,\alpha_2;n)x(n-\alpha_1)x(n-\alpha_2) + \sum_{\alpha_1=0}^{L-1}\sum_{\alpha_2=\alpha_1}^{L-1}\sum_{\alpha_3=\alpha_2}^{L-1}\hat{g}_3(\alpha_1,\alpha_2,\alpha_3;n)x(n-\alpha_1)x(n-\alpha_2)x(n-\alpha_3) \tag{4.14}$$

$$x_{eq}(n) = x(n) + \sum_{\alpha_1=0}^{L-1}\sum_{\alpha_2=\alpha_1}^{L-1}\hat{g}_2(\alpha_1,\alpha_2;n)x(n-\alpha_1)x(n-\alpha_2) + \sum_{\alpha_1=0}^{L-1}\sum_{\alpha_2=\alpha_1}^{L-1}\sum_{\alpha_3=\alpha_2}^{L-1}\hat{g}_3(\alpha_1,\alpha_2,\alpha_3;n)x(n-\alpha_1)x(n-\alpha_2)x(n-\alpha_3) \tag{4.15}$$

where, δ is the impulse signal indicator, $\hat{g}_2$ and $\hat{g}_3$ are the second-order and third-order equivalent Volterra-filter kernels. The above equation is rewritten as

$$x_{eq}(n) = x(n) + x_{\hat{g}2}(n) + x_{\hat{g}3}(n) \tag{4.16}$$

where, $x_{\hat{g}2}(n) = \vec{S}_2(n)\hat{G}_2(n)$ (4.17)

with $\vec{S}_2^T(n) = \begin{bmatrix} x^2(n) & x^2(n-1) & \ldots\ldots & x^2(n-N+1) \\ x(n)x(n-1) & x(n-1)x(n-2) & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & & & \\ \ldots\ldots & & & \\ x(n)x(n-L+1) & \ldots\ldots & & \ldots\ldots \\ x^2(n-1) & \ldots\ldots & & \ldots\ldots \\ \ldots\ldots & & & \\ \ldots\ldots & & & \\ x^2(n-L+1) & \ldots\ldots & \ldots\ldots\ldots & x^2(n-L-N+2) \end{bmatrix}$ (4.18)

$$\hat{G}_2(n) = \begin{bmatrix} \hat{g}_2(0,0;n) \\ \hat{g}_2(0,1;n) \\ ..... \\ ..... \\ \hat{g}_2(0,L-1;n) \\ \hat{g}_2(1,1;n) \\ ..... \\ ..... \\ \hat{g}_2(L-1,L-1;n) \end{bmatrix} \tag{4.19}$$

where, the dimension of $\hat{G}_2(n)$ is $N_2 \times 1$ with $N_2 = L(L+1)/2$; and the dimension of $\vec{S}_2(n)$ is $N \times N_2$. Similarly, it is clear for the third-order statistics that

$$x_{\hat{g}3}(n) = \vec{S}_3(n)\hat{G}_3(n) \text{ (as in [182])} \tag{4.20}$$

where, the dimension of $\hat{G}_3(n)$ is $N_3 \times 1$ with $N_3 = L(L+1)(L+2)/6$; and the dimension of $\vec{S}_3(n)$ is $N \times N_3$. To introduce adaptive characteristics in the underlying system, the Eq. (4.16) can be modified as

$$\hat{x}_{eq}(n) = x(n) + \hat{x}_2(n) + \hat{x}_3(n) \tag{4.21}$$

with $\hat{x}_2(n) = \vec{S}_2(n)\vec{\mathbf{w}}_2(n)$ (4.22)

$$\vec{\mathbf{w}}_2(n) = \begin{bmatrix} w_2(0,0;n) \\ w_2(0,1;n) \\ ..... \\ ..... \\ w_2(0,L-1;n) \\ w_2(1,1;n) \\ ..... \\ ..... \\ w_2(L-1,L-1;n) \end{bmatrix} \tag{4.23}$$

Similarly, $\hat{x}_3(n) = \vec{S}_3(n)\vec{\mathbf{w}}_3(n)$ (4.24)

with the adaptive second-order and third-order weight vectors $\vec{\mathbf{w}}_2(n)$ and $\vec{\mathbf{w}}_3(n)$ of the

Volterra-filter respectively, as shown in Fig. 4.2 . The dimensions of $\vec{\mathbf{w}}_2(n)$ and $\vec{\mathbf{w}}_3(n)$ are $N_2 \times 1$ and $N_3 \times 1$ respectively.

The error-signal between the original received loudspeaker signal at the microphone and the output of adaptive-filtering system is

$$e(n) = d(n) - \hat{d}(n) \tag{4.25}$$

with $\hat{d}(n) = \hat{\mathbf{x}}_{eq}^T(n)\vec{\mathbf{w}}_1(n)$ (4.26)

$$\vec{\mathbf{x}}_{eq}(n) = \left[x_{eq}(n) \;\; x_{eq}(n-1) \;\; ..... \;\; x_{eq}(n-N+1) \right]^T \tag{4.27}$$

$$\hat{\mathbf{x}}_{eq}(n) = \left[\hat{x}_{eq}(n) \;\; \hat{x}_{eq}(n-1) \;\; ..... \;\; \hat{x}_{eq}(n-N+1) \right]^T \tag{4.28}$$

and the first-order adaptive weight-vector of Volterra-filter is

$$\vec{\mathbf{w}}_1(n) = \left[w_1(0;n) \;\; w_1(1;n) \;\; ..... \;\; w_1(N-1;n) \right]^T \tag{4.29}$$

As per the adaptive theory, the cost-function is defined as

$$J(n) = E\left[e^2(n) \right] \tag{4.30}$$

where, $E[.]$ is the expectation operator [211] and $(.)^T$ is the matrix transpose operator.

In the above scenario, the minimization of above cost-function leads to the weight update equations for the conventional LMS filter [182] as

$$\vec{\mathbf{w}}_1(n+1) = \vec{\mathbf{w}}_1(n) + \mu_v(n)e(n)\nabla_{\vec{\mathbf{w}}_1}\left\{ \hat{\mathbf{x}}_{eq}^T(n)\vec{\mathbf{w}}_1(n) \right\} \tag{4.31}$$

In the above equation, $\nabla_{\vec{\mathbf{w}}_1}$ is the gradient with respect to $\vec{\mathbf{w}}_1$. Its solution gives

$$\vec{\mathbf{w}}_1(n+1) = \vec{\mathbf{w}}_1(n) + \mu_v(n)e(n)\hat{\mathbf{x}}_{eq}(n) \tag{4.32}$$

where, the variable step-size $\mu_v(n)$ controls the convergence and tracking characteristics of the adaptive algorithm in the linear part of presented cascaded model.

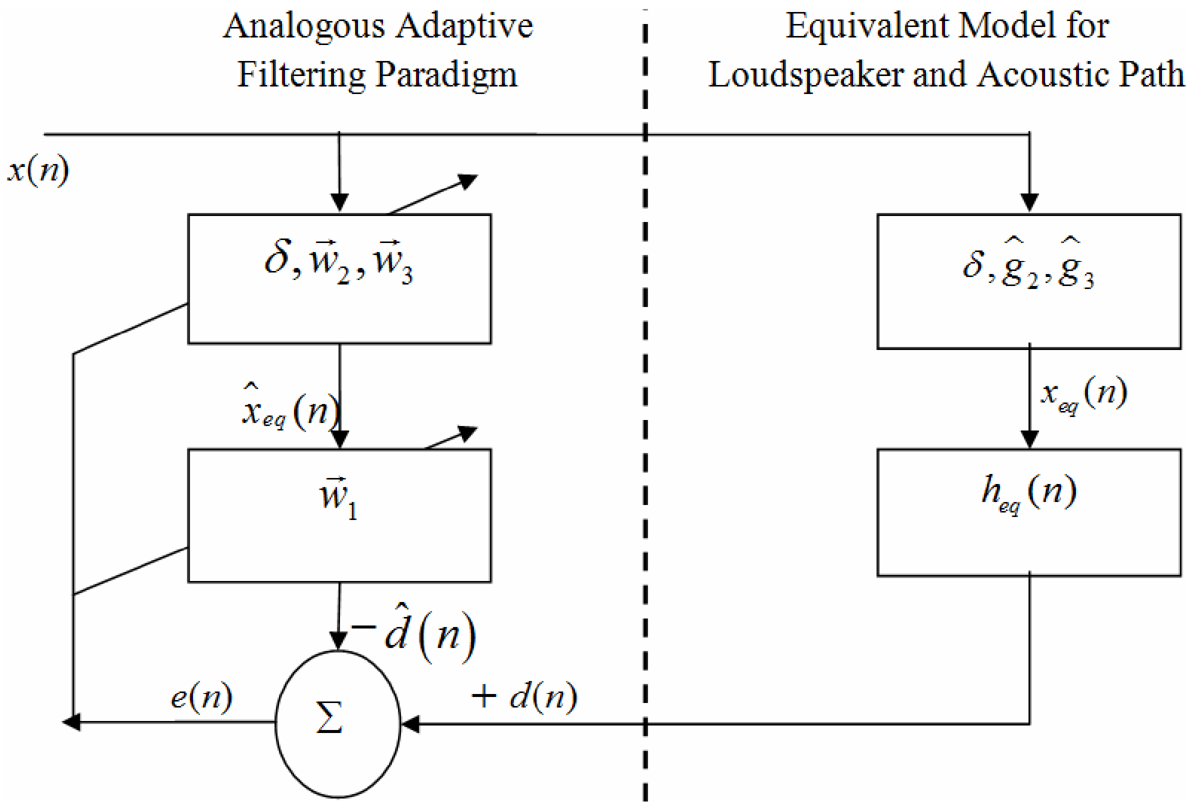

Fig. 4.2: Equivalent loudspeaker, acoustic-path and microphone assembly with adaptive system model for AEC

However, the VSS is bounded in the domain $0 < \mu_v(n) \le \frac{2}{3tr\left(\vec{R}_{xxeq}\right)}$; where the auto-correlation matrix is given as $\vec{R}_{xxeq} = E\left[\hat{\mathbf{x}}_{eq}(n)\hat{\mathbf{x}}_{eq}^T(n)\right]$. Similarly, for the second-order adaptive weight-vector [182], it is apparent that

$$\vec{\mathbf{w}}_2(n+1) = \vec{\mathbf{w}}_2(n) + \mu_s\; e(n)\nabla_{\vec{\mathbf{w}}_2}\left\{\hat{\mathbf{x}}_{eq}^T(n)\vec{\mathbf{w}}_1(n)\right\} \tag{4.33}$$

In the above equation, $\nabla_{\vec{\mathbf{w}}_2}$ is the gradient with respect to $\vec{\mathbf{w}}_2$. Its elucidation provides

$$\vec{\mathbf{w}}_2(n+1) = \vec{\mathbf{w}}_2(n) + \mu_s\; e(n)\nabla_{\vec{\mathbf{w}}_2}\left\{\vec{\mathbf{w}}_1^T(n)\left\{x(n) + \vec{S}_2(n)\vec{\mathbf{w}}_2(n) + \hat{x}_3(n)\right\}\right\} \tag{4.34}$$

$$\vec{\mathbf{w}}_2(n+1) = \vec{\mathbf{w}}_2(n) + \mu_s\; e(n)\nabla_{\vec{\mathbf{w}}_2}\left\{\vec{\mathbf{w}}_1^T(n)\vec{S}_2(n)\vec{\mathbf{w}}_2(n)\right\} \tag{4.35}$$

$$\vec{\mathbf{w}}_2(n+1) = \vec{\mathbf{w}}_2(n) + \mu_s\; e(n)\left\{\vec{S}_2^T(n)\vec{\mathbf{w}}_1(n)\right\} \tag{4.36}$$

Akin to above solution, it can be shown for the third-order adaptive weight-vector [182] that

$$\vec{\mathbf{w}}_3(n+1) = \vec{\mathbf{w}}_3(n) + \mu_s\; e(n)\left\{\vec{S}_3^T(n)\vec{\mathbf{w}}_1(n)\right\} \tag{4.37}$$

where, the fixed step-size μ_s controls the convergence and tracking characteristics of the adaptive algorithm in the nonlinear part of presented cascaded model. However, the FSS is bounded in the range $0 < \mu_s < 2$. To incorporate the stability norm in the above adaptive AEC, Guerin *et al*. [182] have suggested the usage of gradient-based adaptive NLMS algorithm [187], which results in

$$\vec{\mathbf{w}}_1(n+1) = \vec{\mathbf{w}}_1(n) + \mu_v(n)e(n)\frac{\hat{\mathbf{x}}_{eq}(n)}{\left\|\hat{\mathbf{x}}_{eq}(n)\right\|_2^2} \tag{4.38}$$

$$\vec{\mathbf{w}}_2(n+1) = \vec{\mathbf{w}}_2(n) + \mu_s\; e(n)\frac{\left\{\vec{S}_2^T(n)\vec{\mathbf{w}}_1(n)\right\}}{\left\|\vec{S}_2^T(n)\vec{\mathbf{w}}_1(n)\right\|_2^2} \tag{4.39}$$

$$\vec{\mathbf{w}}_3(n+1) = \vec{\mathbf{w}}_3(n) + \mu_s\; e(n)\frac{\left\{\vec{S}_3^T(n)\vec{\mathbf{w}}_1(n)\right\}}{\left\|\vec{S}_3^T(n)\vec{\mathbf{w}}_1(n)\right\|_2^2} \tag{4.40}$$

where, $\|.\|_2$ is the L_2 norm [33]. We now focus on the incorporation of GVSS in Eq. (4.32). The GVSS criterion is favourable in the tracking of a slowly time-varying environment [210]. The VSS must increase or decrease as the mean squared error (MSE) increases or decreases, allowing the adaptive-filter to track changes in the underlying system and thus to produce a small MSE. It also reduces the tradeoff between misadjustment and the speed of adaptation, due to its inherent capability of providing both fast tracking as well as small misadjustment. Therefore, generalized variable step-size (GVSS) criterion is proposed to fine-tune step-size under the influence of a stationary as well as nonstationary environment, which is as follows

$$\mu_v(n) = \bar{\alpha}\mu_v(n-1) + \bar{\gamma} J_1(n-1) + \bar{\beta} J_2(n) \tag{4.41}$$

where, $$J_1(n) = \sum_{\bar{p}=0}^{\bar{P}} \lambda_1^{\bar{p}} e(n) e(n-\bar{p}) \quad \text{with } 0 \le \lambda_1 < 1 \tag{4.42}$$

$$J_2(n) = \left[\sum_{\bar{q}=1}^{\bar{Q}} \lambda_2^{\bar{q}} e(n-\bar{q})\, \hat{\mathbf{x}}_{eq}^T(n-\bar{q}) \right] \hat{\mathbf{x}}_{eq}(n) e(n) \quad \text{with } 0 \le \lambda_2 < 1 \tag{4.43}$$

where, the parameter values are adjusted in the domain $0 < \bar{\alpha} \le 1$, $0 \le \bar{\gamma} < 1$ and $0 \le \bar{\beta} < 1$. The parameter $\bar{\alpha}$ stimulates the global exponential forgetting of the VSS, the parameter $\bar{\gamma}$ controls the convergence time and the level of misadjustment [210], and the parameter $\bar{\beta}$ adjusts the adaptive behaviour of step-size $\mu_v(n)$. However, λ_1 *and* λ_2 are the local exponential forgetting-factors in Eqs. (4.42) and (4.43) respectively. For the appropriate convergence, the value of VSS should be bounded in the range $\mu_{Min} \le \mu_v(n) \le \mu_{Max}$ [210]. The initial step-size is usually taken as μ_{Max} , therefore the MSE of algorithm remains bounded. However, the value of μ_{Min} is chosen to provide the minimum level of tracking ability. The advantage of cascaded system model (as depicted in Fig. 4.2) is a reduction in the number of filter-coefficients due to the segregation of linear and nonlinear coefficients in the presented adaptive method.

4.4 Simulation Results

We shall investigate the behaviour of the presented adaptive Volterra-filtering scheme using the GVSS-NLMS algorithm for the nonlinear acoustic echo cancellation. We first consider a known second-order and third-order nonlinearity to test the efficacy of the proposed technique. We next take the samples of a real-time loudspeaker signal recorded with a microphone, which is similar to the setup considered in [186]. The major appraisal factor is the echo-return-loss-enhancement (ERLE), which is used to compare the performance of the presented adaptive technique with the existing method given in [182]. Here,

$$ERLE(n)\ \ in\ dB = 10\log_{10}\left[E\left\{d^2(n)\right\}\big/E\left\{e^2(n)\right\}\right] \tag{4.44}$$

According to the Monte-Carlo simulations, the performance of adaptive algorithms is compared on the basis of measured minimum mean square error (MMSE) criterion as

$$\hat{J}(n) = \sum_{j=1}^{500} \frac{e^2(n, j)}{500} \tag{4.45}$$

The acoustic-path is modelled as a linear tapped-delay-line filter, in which the FIR filter-coefficients are Gaussian distributed (chosen at random). The number of coefficients is assumed to be equivalent to N. First, we consider the second-order nonlinear-system with Volterra-kernels as $\vec{\mathbf{g}}_s = [0.76,\ -1,\ 1,\ 0.5,\ 2,\ -1.6,\ 1.2,\ 0.8,\ 0.05]$ with $N_2 = L(L+1)/2 = 6$ and $N = L = 3$, in which $N + N_2 = 9$. In a second case, the output of the third-order Volterra-system is given as

$$\begin{aligned} x_{vf}(n) = &-0.78x(n) - 1.48x(n-1) + 1.39x(n-2) + 0.7x(n-1)x(n-2) + 1.86x(n)x(n-2) \\ &+ 3.72x(n)x(n-1) + 1.41x^2(n-2) - 1.62x^2(n-1) + 0.54x^2(n) + 0.33x(n)x(n-1)x(n-2) \\ &+ 0.15x^2(n-1)x(n-2) - 0.75x^2(n-2)x(n-1) - 1.52x^2(n)x(n-2) - 0.23x^2(n-2)x(n) \\ &- 0.12x^2(n)x(n-1) - 0.13x^2(n-1)x(n) + 0.5x^3(n-2) - 0.76x(n-2) + 0.44x^3(n) \end{aligned}$$

with $N_3 = L(L+1)(L+2)/6 = 10$, $N_2 = L(L+1)/2 = 6$ and $N = L = 3$, in which

$N + N_2 + N_3 = 19$. The values of parameters are kept at $\mu_s = 0.05$, $\lambda_1 = 0.66$, $\lambda_2 = 0.66$, $\bar{\alpha} = 0.97$, $\bar{\beta} = \bar{\gamma} = 0.8 \times 10^{-4}$ and $\bar{P} = \bar{Q} = 2$. The value of the minimum step-size is $\mu_{Min} = 0.0005$, and the maximum bounded value of step-size is $\mu_{Max} = 0.5$. The input $x(n)$ to the underlying system is a white Gaussian random sequence with unit variance and zero-mean [211].

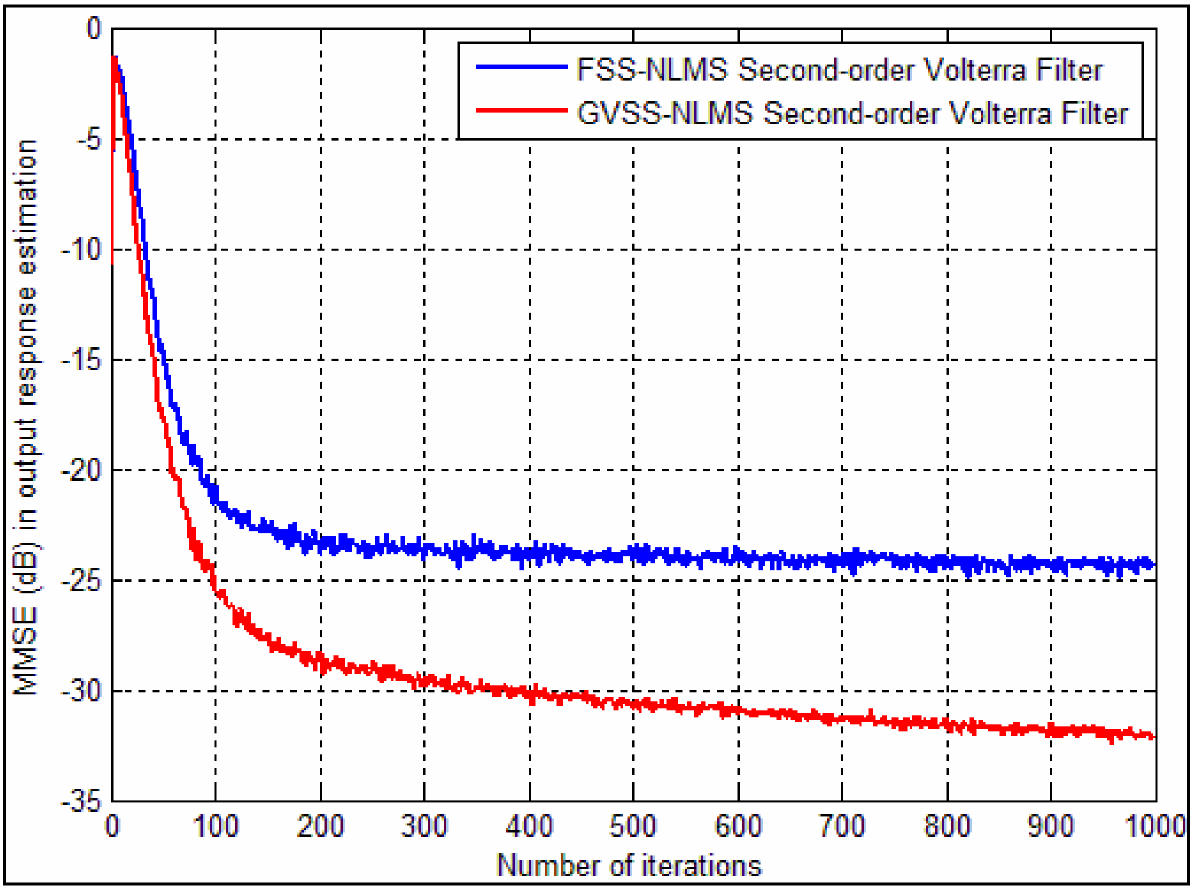

Fig. 4.3: Minimum mean square error in the estimation of output response for SOVFs

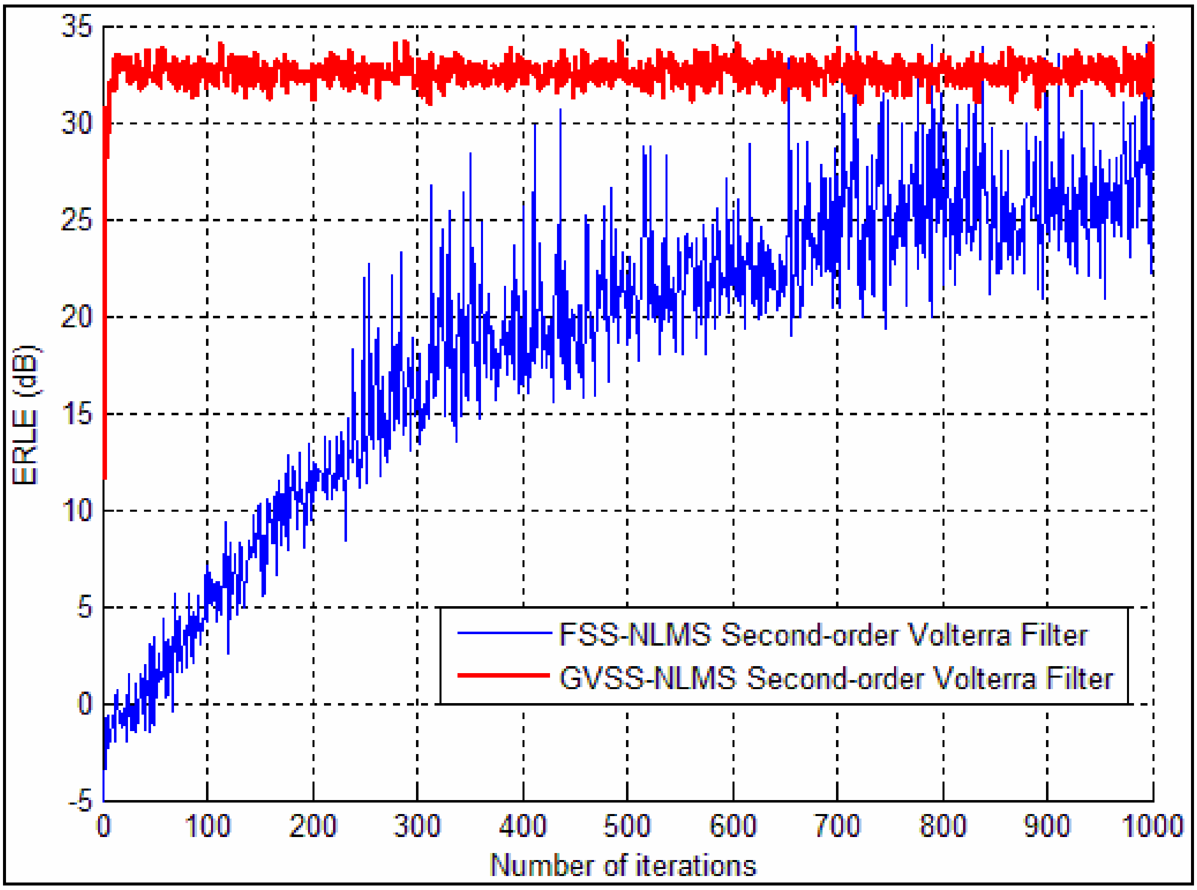

Fig. 4.4: ERLE vs number of iterations for SOVFs

It is apparent from the simulation results presented in Fig. 4.3-Fig. 4.6 that the second-order Volterra-filter exhibits higher convergence-rate and lower MMSE in comparison to the third-order Volterra-filter using the FSS-NLMS as well as GVSS-NLMS adaptive algorithm. The inferior results of the third-order Volterra-filter are due to the large number of Volterra coefficients/kernels that must be adapted. However, these results manifest that the proposed GVSS-NLMS algorithm based approach depicts a higher convergence-rate than the FSS-NLMS based scheme.

Further, the input $x(n)$ to the underlying loudspeaker-linear acoustic path- microphone assembly is a real-time sound signal. The corresponding received sound signal at the output of microphone is shown in Fig. 4.7. The acoustic-path coefficients are unknown in this case. For memory length $L=10$, the first module in the presented adaptive cascaded structure encompasses $N_2=55$ and $N_3=220$ for the nonlinear-filter. Here, the number of linear FIR filter-coefficients in the second module is $N=128$. The values of parameters are kept at $\mu_s=0.05$, $\lambda_1=0.8$, $\lambda_2=0.5$, $\bar{\alpha}=0.99$, $\bar{\beta}=\bar{\gamma}=0.8\times10^{-4}$ and $\bar{P}=\bar{Q}=2$. The value of the minimum step-size is $\mu_{Min}=0.0005$ and the maximum bounded value of the step-size is $\mu_{Max}=0.5$. Under practical conditions, the third-order Volterra-filter using GVSS-NLMS performs 5*dB* better than the FSS-NLMS based AEC as far as the ERLE is concerned (as shown in Fig. 4.8). Under similar conditions, the second-order Volterra-filter using GVSS-NLMS is compared with the third-order Volterra-filter using GVSS-NLMS. The simulation results demonstrated in Fig. 4.9 illustrate that the SOVF converges at a faster rate, but it is at the cost of higher MMSE in comparison to the third-order polynomial-filter using GVSS-NLMS.

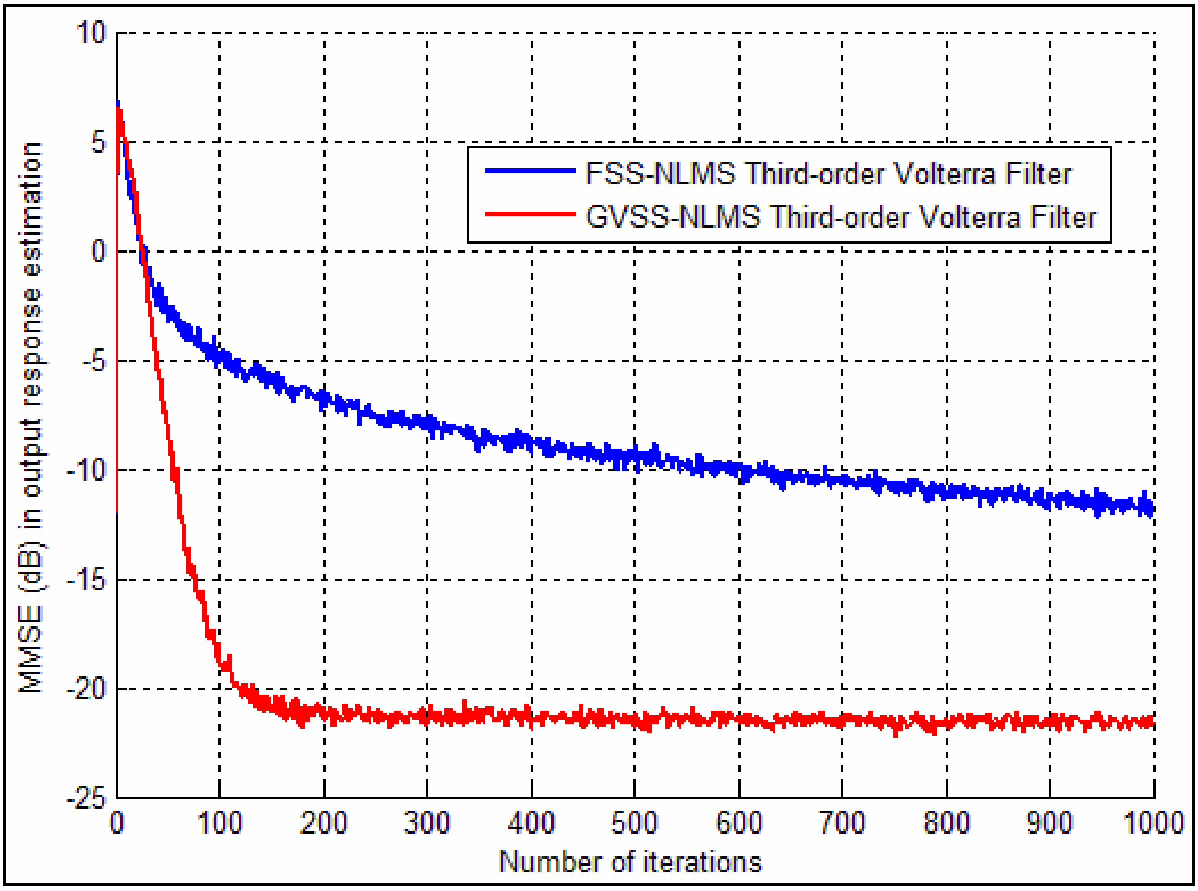

Fig. 4.5: Minimum mean square error in the estimation of output response for third-order Volterra-filters

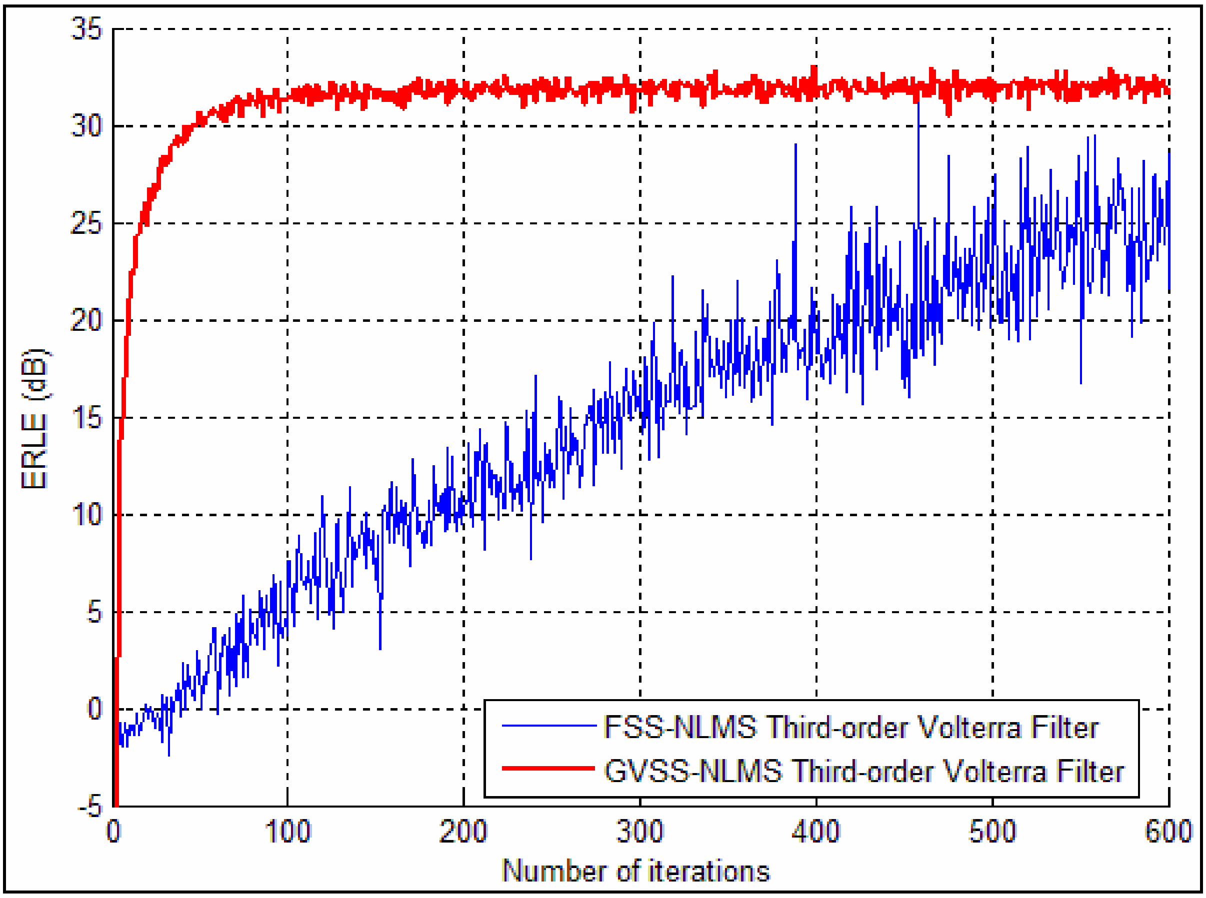

Fig. 4.6: ERLE vs number of iterations for third-order Volterra-filters

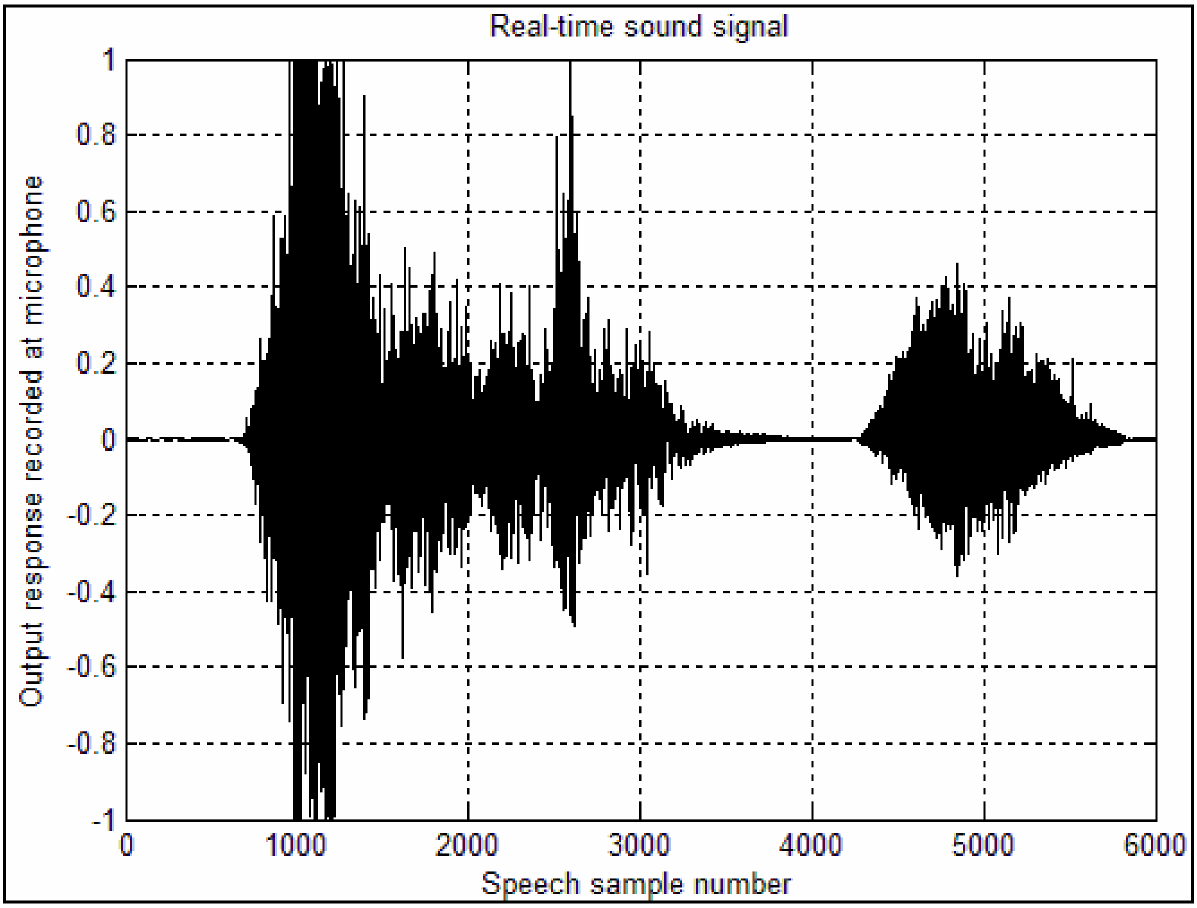

Fig. 4.7: Real-time loudspeaker acoustic-signal received at microphone

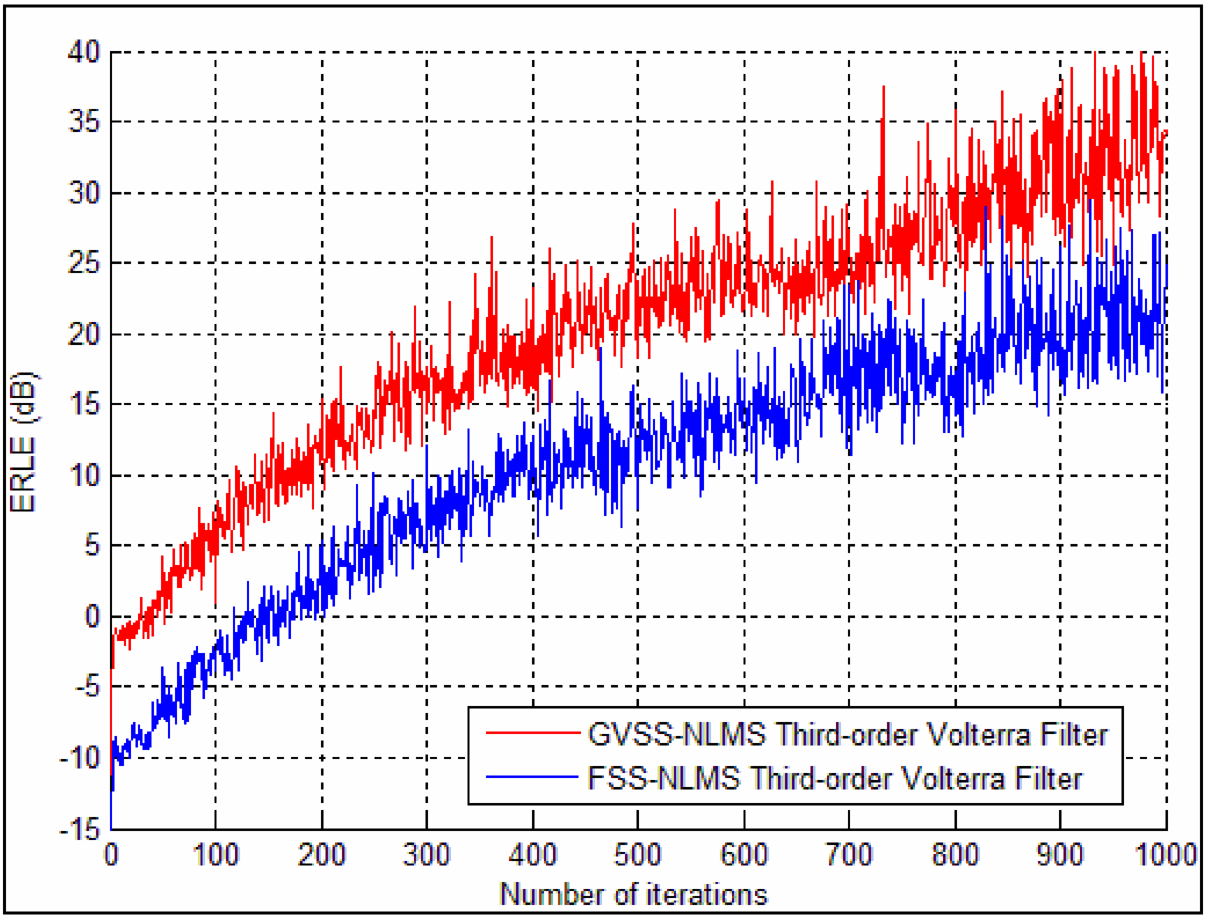

Fig. 4.8: ERLE vs number of iterations for third-order Volterra-filters for real-time acoustic-signal

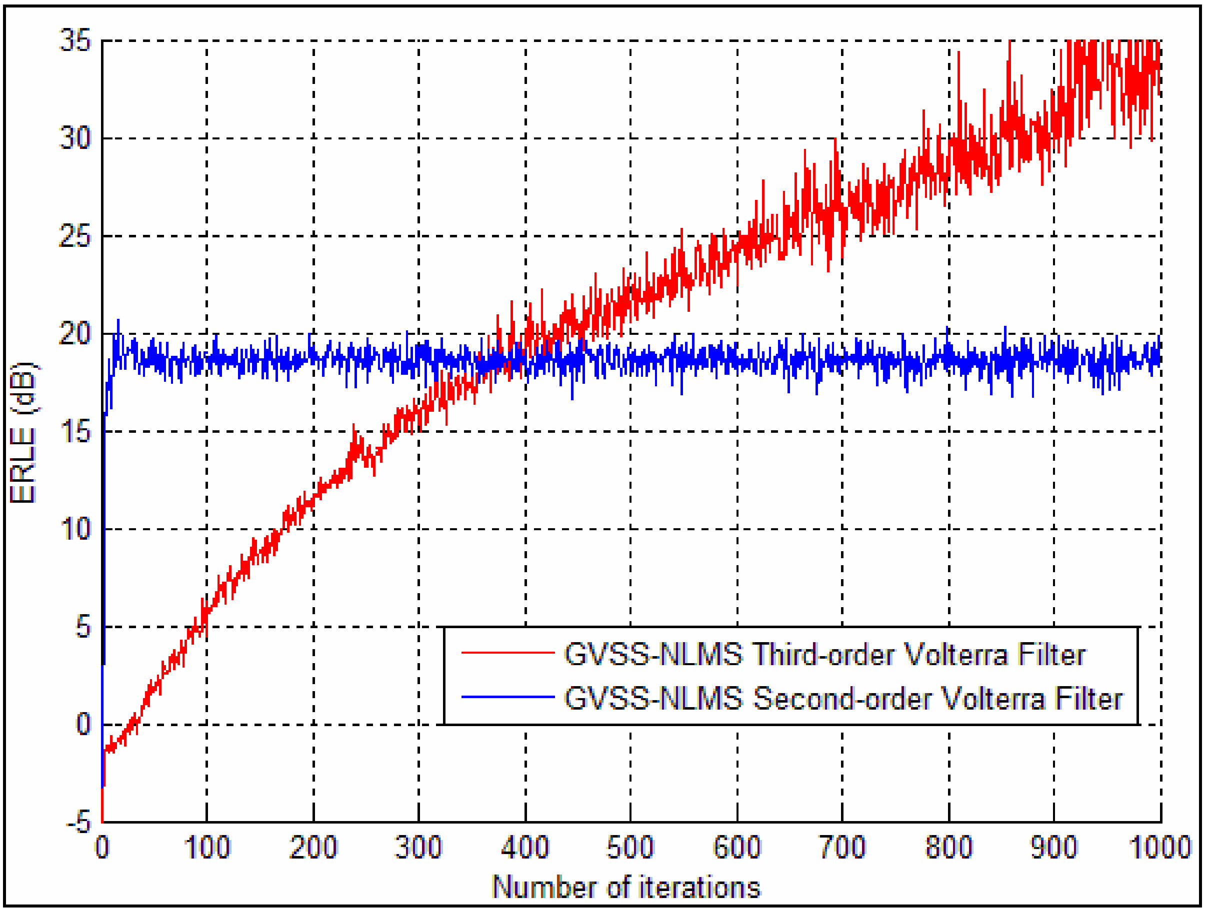

Fig. 4.9: ERLE vs number of iterations for SOVF and third-order Volterra-filters for real-time acoustic-signal

4.5 Summary of Chapter

This chapter has presented a generalized variable step-size NLMS algorithm based adaptive nonlinear acoustic echo canceller, in which the second-order and third-order Volterra-filtering techniques are incorporated. The first module of the adaptive cascaded AEC updates the higher order nonlinear components/coefficients of the Volterra-filter, and subsequently the second module updates the first-order linear components/coefficients of the Volterra-filter as well as the coefficients of the linear acoustic-path. It undoubtedly reduces the computational-complexity as compared to the conventional polynomial Volterra-filtering based AECs. The application of GVSS enhances the convergence-rate as compared to the FSS based NLMS algorithms for AECs. Moreover, the GVSS-NLMS algorithm also efficiently tracks the acoustic tap-coefficients in a slowly time-varying environment. For a known nonlinear-system scenario, the second-order Volterra-filter outperforms the third-order Volterra-filter as far as convergence-rate and MMSE in the output response estimation are concerned. This is due to a smaller number of filter-coefficients in SOVFs that need to be updated for the system-identification and echo cancellation. However under practical conditions, loudspeaker nonlinearities are modelled quite well using the third-order Volterra-filter paradigm. The simulation results connote that the third-order Volterra-filter using GVSS-NLMS performs approximately 5dB better than the FSS-NLMS algorithm based AECs, in terms of ERLE. It may be inferred that third-order adaptive Volterra-filtering is an appropriate choice for the acoustic echo cancellation, and for the estimation of the output response of the presented adaptive cascaded structure using GVSS-NLMS algorithm. Future scope includes the development of computational-complexity reduction schemes for the proposed AEC method. However, the analytical techniques required to cope with the ill-conditioned input-signal auto-correlation matrix also need to be investigated further.

CHAPTER 5

CONCLUDING REMARKS AND FUTURE SCOPE

In this research work, we have studied adaptive Volterra-filters for system-identification in the time-varying environment, which is a nonlinear signal processing technique to handle the system nonlinearities, non-Gaussian noise and acoustic echo cancellation. The principles of linear adaptive-filter theory are extended to the nonlinear adaptive-filter theory because we have used linear-in-parameters model [1] for the polynomial adaptive-filter analysis and applications. In the following, we summarize important results of our study and also give suggestions for further investigations.

5.1 Concluding Remarks

We have first considered the numeric variable forgetting-factor recursive least squares algorithm for the first-order and second-order Volterra-systems working under the time-varying channels, in which the Volterra-kernels are considered to be time-varying according to the first-order Gauss-Markov process. Under this scenario, the state-space representation of the underlying system is presented, which is well-suited for the model based adaptive algorithms, like RLS, NVFF-RLS and Kalman-filtering algorithm etc. The presented NVFF-RLS algorithm combats the nonstationarity of the signal by using the updating prediction-error criterion, which is a key feature of this channel tracking system under the time-varying wireless fading channels. Under similar conditions, it outperforms the conventional RLS algorithm in the channel tracking-mode by 2.5 dB, in terms of reduction in the MMSE in channel estimation, for the first-order Volterra-systems. The tracking performance advantage of NVFF-RLS algorithm reduces to 0.5 dB in case of SOVF configurations, in comparison to the RLS algorithm based channel tracking systems. It may be noted that the high order Volterra-filter in combination with the proposed NVFF-RLS algorithm provides channel tracking performance marginally better than the conventional RLS algorithm. However in comparison to Kalman-filtering algorithm, the computational-complexity of NVFF-RLS algorithm is significantly low. Simulation results espouse that we can achieve better tracking performance by using the NVFF-RLS algorithm than the conventional fixed forgetting-factor RLS algorithm, just at the cost of marginal boost in computational-complexity (involving NVFF calculation procedure), for the system-identification under nonstationary environment.

However the presence of measurement-noise is inevitable, which can adversely affect the performance of any adaptive channel estimation system, depending on the characteristics and statistics of the measurement-noise. The white Gaussian-noise with finite second-order statistics facilitates the use of mean squared error criterion as an appropriate metric for the estimation-error. But, the minimum error dispersion criterion is found to be suitable metric in the presence of non-Gaussian noise. The $\alpha-$stable impulse-noise (non-Gaussian) has been reported to appear in many engineering applications, which need to be investigated further. Therefore, we have next proposed the GVSS least mean p^{th} power adaptive algorithm for system-identification in the presence of $\alpha-$ stable impulse-noise, which has been incorporated in the nonlinear polynomial-filtering paradigm. Because of unavailability of the finite second-order statistics, the weights of the presented LMP adaptive algorithm are updated by keeping $p \geq 1$ in the presence of impulse-noise characterized by $1 < \alpha < 2$. In some cases, the large eigenvalue-spread of the input-signal vector degrades the tracking performance of Volterra-filter based adaptive algorithms. Under such situation, the time-varying step-size has been observed to alleviate the deleterious effects of eigenvalue-spread on the convergence-rate of LMP adaptive algorithm. The generalized variable step-size updating criterion in combination with the LMP algorithm suppresses the non-Gaussian $\alpha-$stable impulse-noise, and makes it convenient to identify the slowly time-varying Volterra-kernels. However, the conventional LMP algorithm appears to be vulnerable to the impulse-noise, and fails to perform well under the aforementioned high eigenvalue-spread conditions, as far as the convergence-rate is concerned. The simulation outcomes are presented to illustrate that the convergence-mode as well as tracking-mode performance of presented GVSS-LMP algorithm supersedes the conventional LMP, KVSS-LMP, AVSS-LMP and SVSS-LMS algorithms under the appropriate parameter settings. It is apparent from the simulation results that the parameter values in the range $1 < p < \alpha < 2$ provide best results, when p is chosen close to α. Moreover, the LMS and LMP adaptive algorithms can also be derived from GVSS-LMP algorithm by tuning the value of p. The flexible design of GVSS-LMP algorithm helps to tackle the slowly time-varying system-identification problem, which is an exclusive feature of the presented algorithm.

The VSS criterion in combination with adaptive algorithms has also found applications in the field of audio and speech signal processing, where the GVSS criterion can also be implemented. Usually, the nonlinear response of amplifiers or loudspeakers results in the nonlinear echo in the acoustic systems. To mitigate the adverse effects of echo on the quality of audio and speech signals, we further proposed the usage of GVSS-NLMS algorithm for the nonlinear echo cancellation. The two distinct modules of the acoustic echo canceller include a nonlinear polynomial Volterra-filter (equivalent to a loudspeaker with nonlinear-distortion) and a linear finite impulse response tapped-delay-line filter (equivalent to acoustic-path). However, we have presented the adaptive acoustic echo canceller, in which the first module processes the nonlinear constituents of Volterra-filter using the FSS-NLMS algorithm, and the second module processes the linear constituents of Volterra-filter as well as the linear impulse response of the acoustic-path using GVSS-NLMS adaptive algorithm. The GVSS criterion not only combats nonstationarity, but also suppresses the deleterious effects of nonlinear-distortion. The simulation outcomes are presented to connote that the convergence-mode and tracking-mode performance of GVSS-NLMS algorithm is significantly better than the FSS-NLMS algorithm based adaptive echo canceller, for the second-order as well as third-order Volterra-filters. For the third-order polynomial-model, the presented GVSS-NLMS based AEC provides approximately 5 dB performance advantage over the FSS-NLMS based AEC while dealing with the real-time acoustic-signals, in the tracking-mode, in terms of the reduction in mean squared error. Under similar real-time conditions, the second-order Volterra-filter using GVSS-NLMS algorithm converges at a faster rate, which is at a cost of lower ERLE in comparison to the third-order polynomial-filter, in the tracking-mode. However, the lower convergence-rate of the third-order Volterra-filter based AEC using GVSS-NLMS is due to the large number of Volterra-kernels, which need to be updated recursively. It may be inferred from the simulation results that the third-order Volterra-model is an apposite option for the real-time adaptive AEC, in case of the nonlinear response of loudspeaker and multipath signal propagation.

From the aforementioned discussion and results, it is evident that NVFF as well as GVSS criteria reduce the lag-misadjustment in adaptive weight-vector updating during the tracking process under the time-varying environment, which also help in combating nonstationarity, non-Gaussian noise and nonlinear echo, while tackling the system-identification problem.

5.2 Suggestions for Further Work

In this study, we have seen that the adaptive polynomial-filtering based on Volterra-model can be used for the time-varying channel estimation, non-Gaussian noise excision and acoustic echo cancellation. From a communication theorist's point of view, the backbone of modern advanced high data-rate communication systems (4G, MIMO, STBC, STTC, V-BLAST, D-BLAST and H-BLAST systems) is the accurately estimated channel state information at the receiver. Moreover, the functioning of amplifiers close to saturation is generally needed for the efficiency reasons in communication systems, which results in a nonlinearly distorted signal at the output of amplifier. Therefore, NVFF-RLS, GVSS-LMP and GVSS-NLMS adaptive algorithms can be used for the nonlinear-channel estimation, nonlinear-channel equalization and refining of nonlinearly amplified analog/digital signals using the signal processing techniques etc. This work can be extended to seismic signal processing, image processing, biomedical engineering, bio-signal processing, audio and speech signal processing applications. It is worthwhile to investigate the analytical schemes needed to deal with the ill-conditioned input-signal auto-correlation matrix. The NVFF and GVSS criteria based adaptive algorithms suggest a number of interesting avenues for further research. Another area for further research includes the development and analysis of a family of LMP algorithms. Future work includes the derivation of analytical results related to the lag-misadjustment and gradient-misadjustment for the GVSS-LMP/LMS algorithms. The lag-misadjustment in case of the NVFF-RLS adaptive algorithm needs to be investigated further. The shape of room impulse response affects the performance of AECs and ANCs, which is also an emerging issue. The variable step-size and variable forgetting-factor can be computed through the genetic algorithms and neural network techniques. These can find applications in the domain of "acoustic echo cancellation" and "active noise control" of nonlinear noise-processes.

APPENDIX - A

The literature [106], [202] of fixed-step-size (FSS-LMS) algorithm reflects a trade-off between the misadjustment and convergence-rate, which depicts that a small step-size (SS) produces small misadjustment, but at the cost of longer convergence time. Under time-varying environment, the optimum value of step-size in FSS-LMS algorithm strikes a balance between the amount of lag-noise and gradient-noise [98]. However, the optimum value of step-size can not be determined *a priori* due the unknown channel parameters. Therefore in KVSS-LMS algorithm [91], the variable step size (VSS) is attuned using

$$\mu'(n) = \bar{\alpha}\mu'(n-1) + \bar{\gamma}\,\underline{\underline{e^{2}(n-1)}} \tag{A.1}$$

In KVSS-LMS algorithm, high prediction-error causes the SS to increase in order to achieve fast tracking, while low prediction-error leads to reduction in SS to yield small misadjustment. The SS elevates or alleviates as the MSE rises or falls, which allow adaptive-filtering configuration to chase changes in TV system, as well as to reduce steady-state error. It also reduces sensitivity of misadjustment to the level of nonstationarity. This technique is heuristically sound and has resulted in various ad hoc methods, where the choice of convergence parameters rests upon the magnitude of estimation-error, polarity of the successive samples of estimation-error, measurement of the cross-correlation of estimation-error with input-samples. However, VSS-LMS algorithms are found to be sensitive to noise [95], [63] in the low signal-to-noise ratio environment because the SS update of these algorithms are directly calculated using instantaneous-error, which is couupted by noise. Further in AVSS-LMS algorithm [96], the VSS is controlled using

$$\mu'(n) = \bar{\alpha}\mu'(n-1) + \bar{\gamma}\theta^{2}(n-1) \tag{A.2}$$

$$\theta(n-1) = \bar{\alpha}_A\theta(n-2) + (1-\bar{\alpha}_A)\underline{\underline{\{e(n-1)e(n-2)\}}} \tag{A.3}$$

Here, the error auto-correlation is usually a fine gauge of proximity to the optimal-value, which eliminates the effects of uncorrelated noise-sequences on SS updating. In beginning stages of adaptive tuning, the error auto-correlation estimate is relatively high, which in turn results in a high value of SS. However, the small error auto-correlation leads to a small step-size under the optimum conditions. It results in effective adjustment of SS, while sustaining the immunity against independent noise, for the flexible control of misadjustment. The AVSS-LMS algorithm [96] shows substantial convergence-rate improvement over KVSS-LMS algorithm [91] and FSS-LMS algorithm [202] under the stationary environment for the low SNR as well as the high SNR values. However, the performance of AVSS-LMS algorithm is comparable to the FSS-LMS and KVSS-LMS adaptive algorithms under the nonstationary conditions. But in SVSS-LMS algorithm [104], the VSS is adjusted using the following recursive relation by adjusting the control-parameters $\bar{\rho}_w$ *and* $\bar{\alpha}_w$.

$$\mu'(n) = \mu'(n-1) + \bar{\rho}_w \vec{\psi}^T(n)\vec{x}(n)e(n) \tag{A.4}$$

$$\vec{\psi}(n) = \bar{\alpha}_w \vec{\psi}(n-1) + e(n-1)\vec{x}(n-1) \tag{A.5}$$

The above equation can be rewritten in expanded form as

$$\begin{aligned}\vec{\psi}^T\{n\} = {} & \bar{\alpha}_w^{\bar{Q}}\vec{\psi}^T\{n-\bar{Q}\} + \bar{\alpha}_w^{\bar{Q}-1}e(n-\bar{Q})\vec{x}^T(n-\bar{Q}) + \\ & \ldots\ldots \quad + \bar{\alpha}_w e(n-2)\vec{x}^T(n-2) + e(n-1)\vec{x}^T(n-1)\end{aligned} \tag{A.6}$$

For $0 \le \bar{\alpha}_w < 1$ and $\bar{Q} \to \textit{high value}$, the Eq. (A.6) can be approximated as

$$\begin{aligned}\vec{\psi}^T\{n\} \approx {} & \underline{\underline{\bar{\alpha}_w^{\bar{Q}-1}e(n-\bar{Q})\vec{x}^T(n-\bar{Q}) +}} \quad \ldots\ldots \\ & \underline{\underline{+\ \bar{\alpha}_w^2 e(n-3)\vec{x}^T(n-3) + \bar{\alpha}_w e(n-2)\vec{x}^T(n-2) + e(n-1)\vec{x}^T(n-1)}}\end{aligned} \tag{A.7}$$

This algorithm [104] outperforms the Mathews' algorithm [92], when both are set to track the random-walk channel under the similar conditions.

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