# PERFORMANCE EVALUATION OF CONCATENATED CHANNEL CODES AND SPACE TIME CODES UNDER FADING CHANNELS

***A THESIS***

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*Submitted by*

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## *CERTIFICATE*

This is to certify that the thesis entitled "Performance evaluation of concatenated channel codes and space time codes under fading channels" being submitted by Ms. Surbhi Sharma, to the Department of Electronics and Communication Engineering, Thapar University, Patiala, Punjab for the award of Doctor of Philosophy in Electronics and Communication Engineering, is a bona-fide research work carried out by her under the supervision and guidance of the undersigned. Her thesis has reached the standard of fulfilling of requirements of the regulations related to degree. The contents in this thesis have not been in part or full submitted to any other university for award of any degree or diploma.

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# *ABSTARCT*

The use of multiple antennas at the transmitter and/or the receiver provides high data rate and diversity in many different forms. This can be useful to meet the ever growing demand of high data rate wireless services, provided we concatenate the multiple antenna systems with channels codes to efficiently exploit the advantages of the these system. Low density parity check (LDPC) codes are linear block codes constructed by sparse parity check matrices. These codes are powerful in terms of error performance and, especially, have low decoding complexity. LDPC codes when employed with multiple antenna system yields astonishing performance close to the Shannon theoretical limits.

In the first part of the thesis, LDPC coded systems that employ single transmit and multiple receive antennas, i.e., single-input multiple-output (SIMO) systems are studied. In particular Low Density parity check codes are combined with optimum diversity combining (LDPC-OC) scheme that works efficiently in the presence of interferer for both additive white Gaussian noise (AWGN) and frequency-flat Rayleigh fading channels. The analytical bounds on the bit error rate (BER) performance are derived for the under loaded case (when the number of interferers are less than or equal to number of antenna array elements) and over loaded case (when the number of interferers are greater than antenna array elements. In particular, the analytical bounds are derived for the special case of single interferer (under loaded case) and for large number of interferers (overloaded case). For the proposed systems, the results obtained with analytical bounds are verified with the help of simulation results. It is demonstrated that these bounds can be efficiently used to evaluate the error performance.

The second part of the thesis studies LDPC coded systems that employ multiple transmit and multiple receive antennas, i.e., multiple-input multiple-output (MIMO) systems. Particularly the proposed system is based on the concatenation of LDPC codes with space time block codes (LDPC-STBC).

Two scenarios for the proposed LDPC-STBC system have been considered (i) Concatenated LDPC-STBC system without antenna selection (ii) Concatenated LDPC-STBC system with antenna selection. The concatenation is done such that all code words in one subset/sub-code are associated with an identical pair wise error probability. For both scenarios, first we obtained the results using Monte Carlo simulation for various modulation schemes in various fading environment. We also derived the analytical tight bound for both scenarios using Chernoff bounds. It is shown that on the quasi-static fading channel full diversity can be achieved and an increase in coding gain results from the concatenation. For the concatenated LDPC-STBC system with antenna selection, it is analytically shown that the diversity gain does not change. We can exploit the full diversity advantage promised by the MIMO system that uses all available antenna elements, provided that the space-time code employed has full spatial diversity. It is also shown that the coding gain with antenna selection deteriorates significantly compared to the full-complexity one. When the simulation results are compared with the analytical results, an SNR gap of less than 2 dB between the waterfall positions of the tight bounds and the simulation results is evident.

**Dedicated to**
**To my husband, Bankesh**
**and lovely sons**
**Ritvik and Saatvik**

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***(Surbhi Sharma)***

**Content**

**CHAPTER 3** ***(Background on Error Correcting codes )***



**CHAPTER 4** ***( Performance analysis of LDPC codes in SIMO system)***

## ACRONYMS

| | |
|---|---|
| A/D | Analog to digital converters |
| APP | A post-priori probability |
| ASE | Analytical spectral efficiency |
| AST | Associated space-time codes |
| AWGN | Additive White Gaussian Noise |
| BCH | Bose, Ray-Chaudhuri and Hocquenghem codes |
| BCJR | Bahl, Cocke, Jelinek and Raviv algorithm |
| BEC | Binary erasure channels |
| BER | Bit Error Rate |
| BER | Bit-error probability |
| CC | Convolution codes |
| CCI | Co-Channel Interferers |
| CDF | Cumulative distribution function |
| CEP | Conditional error probability |
| CSI | Channel state information |
| dB | Decibel |
| DE | Density evolution |
| DPSK | Differential phase shift keying |
| DVB-s | Digital video broad-caste standard |
| EGC | Equal Gain combining |
| EXIT | Extrinsic information transfer |
| FEC | forward error correction |
| FER | Frame error rate |
| GA | Gaussian approximation |
| GCQ | Gauss–Chebyshev quadrature |
| I.I.D | Independent and Identically distributed |
| ISI | Inter symbol interference |
| LDPC | Low density parity check codes |
| LDPC-OC | Low density parity check codes-optimum combined |
| LFSR | Linear feedback shift registers |

| | |
|---|---|
| LLR | Log likelihood ratio |
| LOS | Line of Sight |
| LTE | Long term evolution |
| MAP | Maximum a priori |
| MGF | Moment generating function |
| MIMO | Multiple Input and Multiple Output |
| MISO | Multiple Input Single Output |
| ML | Maximum- likelihood |
| M-PSK | M-ary phase shift keying |
| MQAM | Multilevel Quadrature Amplitude Modulation |
| MRC | Maximum Ratio Combining |
| NLOS | Non Line Of Sight |
| OC | Optimum Combining |
| OSTBC | Orthogonal space time block codes |
| PDF | Probability density function |
| PEP | Pair- wise error probability |
| PSK | Phase shift keying |
| QAM | Quadrature Amplitude Modulation |
| QOSTBC | Quasi-orthogonal space time block codes |
| QPSK | Quadrature phase shift keying |
| RF | Radio frequency |
| RS | Reed-Solomon codes |
| SC | Selection Combining |
| SEP | Symbol-error probability |
| SIMO | Single Input and Multiple Output |
| SINR | Signal-To-Interference Plus Noise |
| SNR | Signal to Noise Ratio |
| STBC | Space time block codes |
| STTC | Space time trellis codes |
| TCM | Trellis coded modulation |
| TD | Turbo decoder |

| | |
|---|---|
| TSTC | Turbo space–time codes |
| UC-OC | Uncoded-optimum combined |
| V-Blast | Vertical Bell Laboratories Layered Space-Time |
| VLSI | Very large scale integration |
| WiMAX | Worldwide Interoperability for Microwave Access |
| WLAN | Wireless local area network |

| | |
|---|---|
| $\cong$ | Approximately equal to |
| $\gg$ | Much greater than |
| $\ll$ | Much less than |
| $\leq$ | Less than equal to |
| $\geq$ | Greater than equal to |
| R | Code rate for error correcting codes |
| K | Number of uncoded bits for error correcting codes |
| $P_b$ | Bit error probability |
| $P_e$ | Probability of error |
| E(R) | Channel reliability |
| $\text{argmax}(p_{X/Y}(x/y))$ | Maximize the probability of transmitted symbol x given that y is received |
| $\sigma^2$ | Variance |
| $H_p$ | Parity check matrix for error correcting codes |
| E | Erasure probability of the binary erasure channel |
| $\|x\|$ | Magnitude of x |
| L | Number of interferers |
| $M_R$ | Number of receive antenna |
| $M_T$ | Number of transmit antenna |
| $L_r$ | Number of selected receive antenna |
| $L_t$ | Number of selected transmit antenna |
| $\otimes$ | Convolution |
| H | MIMO channel matrix |
| X | Transmitted signal |
| Y | Received signal |
| N | AWGN noise |
| $x^T$ | The transpose of vector x |
| $x^H$ | The Hermitian (conjugate transpose) of vector x |

| | |
|---|---|
| $arg\max_{x}[f(x)]$ | The value of x that maximizes the function f(x) |
| $arg\min_{x}[f(x)]$ | The value of x that minimizes the function f(x) |
| Q(x) | Gaussian Q-function |
| E[X] | The expected value of random variable X |
| $\|x\|_F$ | The Frobenius norm of the matrix A (i.e., sum of absolute value squares of all the entries of A) |
| det ( A ) | The determinant of matrix A |
| trace ( A ) | The trace of matrix A |
| Γ | Gamma function |
| $I_K$ | Identity matrix |
| P(X/yj ) | A posteriori probability |
| N0/2 | Noise power spectral density |
| $P_s$ | Power of a signal |
| P | Power of interfere |
| $\hat{x}$ | Estimated decoded vector |
| $N_v$ | Number of variable nodes related to LDPC codes |
| $M_c$ | Number of check nodes related to LDPC codes |
| *A* | the scalar *a* |
| ***a*** | the vector ***a*** |
| ***A*** | the matrix ***A*** |
| [***A***]*k,l* | the (*k, l*)th element of ***A*** |

***The classic Shannon-Hartley law suggests that the achievable channel capacity increases logarithmically with the transmit power. By contrast, the MIMO capacity increases linearly with the number of transmit antennas, provided that the number of receive antennas is equal to the number of transmit antennas. With the further provision that the total transmit power is increased proportionately to the number of transmit antennas, a linear capacity increase is achieved upon increasing the transmit power, which justifies the spectacular success of MIMOs…………….....***

# *Chapter I*

# *INTRODUCTION*

## 1.1 Introduction to multiple antennas

In recent years, wireless communication systems have become more and more popular in various operating environments, such as mobiles, laptops, security systems, and other household and business establishments. The next generation of wireless communication systems will offer remote connectivity to internet, high speed multimedia services, high-definition video streaming, internet telephony, online gaming, voice over Internet Protocol (VoIP) and IPTV etc These services require a significant increase in the capacity of wireless systems, however, at the same time, the design of faster, more reliable, and spectrally efficient wireless communication systems is becoming ever more difficult. The performance of a wireless system is subjected to various adverse conditions, such as limited transmission power, scarcity of the electromagnetic spectrum, signal attenuation and fluctuation in wireless links or channels. In order to utilize the available resources efficiently one has to look for various techniques for designing spectrally efficient systems. A common and well understood method for improving the quality and reliability of wireless links or for combating the deleterious effects of fading is to employ multiple antennas at the transmitter and/or the receiver end, known as spatial diversity. The development of multi antenna technology provides a solution to overcome the diverse effect of physical channel and improve the capacity of the system. Transmit/receive spatial diversity uses multiple separate, collocated antennas for transmit and receive functions. The use of multiple antennas at both transmit and receive end, result in a Multi Input Multi Output System [1]. All these techniques require some sort of pre or post processing to recover the desired message.

Receive diversity (known as a SIMO system) uses multiple receive antenna elements to improve the reliability of reception in a fading environment, providing both array gain and diversity gain. In receive diversity, the post processing is carried out by combining independent fading signals associated with multiple receive antennas. Depending on the sophistication of the system, the signals can be added directly (equal gain combining (EGC)) or weighted and added coherently (maximum ratio combining (MRC)) [2] to increase the signal to noise ratio (SNR) at the output of receiver. MRC outperforms various combining techniques in all fading channels, but performance of maximal ratio combining for space diversity reception degrades in the presence of multiple co-channel interference (CCI) sources. In this scenario, antenna arrays, with optimum combining [3], have been shown to combat both the multipath fading of the desired signal and CCI, subsequently increasing the performance of wireless communication systems. With optimum combining, the signals received by several antenna elements are weighted and combined to maximize the output signal-to interference-plus-noise ratio (SINR) [4]. Optimum combining achieves a larger output SINR than MRC, and is thus highly desirable in an interfering environment. MRC is the optimum linear combining technique for coherent reception; with independent fading at each antenna element in the presence of spatially white Gaussian noise [5].

*Transmit diversity*, uses multiple transmit antennas and a single receive antenna, creating a Multiple Input Single Output (MISO) system but requiring an appropriate pre-processing or coding of the transmitted signals before transmission. For transmit diversity techniques, Tarokh *et al.* proposed space–time trellis coding (STTC) [6]–[12] by jointly designing the channel coding, modulation, transmit diversity, and the optional receiver diversity scheme. STTC performs extremely well at the cost of relatively high decoding complexity. In addressing the issue of decoding complexity, Alamouti [13] discovered a remarkable scheme for transmission using two transmit and single receive antenna. A simple scalar decoding algorithm was also proposed by Alamouti that can be generalized to an arbitrary number of receive antennas. This scheme is significantly less complex than STTC coding using two transmitter antennas, although there is a loss in performance as Alamouti schemes does not provide any coding gain, and the SNR required for a given Bit Error Rate (BER) is greater as compared to STTC [14]. Despite the associated performance penalty, Almouti's scheme is appealing in terms of its simplicity and performance. This proposal motivated Tarokh *et al.* [14], [15] to generalize Almouti's scheme to an arbitrary number of transmit antennas, leading to the concept of space-time block codes (STBC). The challenge of transmit-diversity systems is that the signals transmitted from different antennas interfere with each other at the receiver and transmit diversity systems provide no array gain.

MIMO systems also require pre processing of the signals, and depending on the pre processing techniques employed, they are subdivided into three main categories, viz pre-coding [16] or beam forming, spatial multiplexing [17] and diversity coding [1].

In pre-coding, the signals from each transmitter are pre processed in such a way that the signal power is maximized at the receiver input. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher signal-to-noise ratios [17]. Diversity coding is used on the transmitter side when the transmitter has no knowledge of the channel. Diversity coding exploits the independent fading in the multiple antenna links to enhance signal diversity. As there is no channel knowledge in diversity coding, therefore there is no beam forming or array gain from diversity coding.

Researchers have also shown that by employing multiple antennas at the transmitter and receiver, known as MIMO system, the capacity of the system can be drastically increased compared to a single input and single output system, at a reduced power and bandwidth [18]. In rich scattering environments with high SNR, the capacity of MIMO systems increases linearly with numbers of transmit and receive antennas. In addition, the *spatial diversity* provided by MIMO systems offers an effective way of combating the signal fluctuation due to the fading effects in wireless channels. Thus the major advantages of MIMO systems are the substantial increase in channel capacity, which immediately translates to higher data throughputs, significant improvement in data transmission reliability, i.e. very low bit error rates. These advantages are achievable without any expansion in the required bandwidth or increase in the transmit power.  Nowadays, use of multiple transmit and multiple receive antennas (MIMO) is seen as the key enabling technology for wireless delivery of broad band data on the physical layer in many wireless standards, such as, IEEE 802.11n (WLAN) [19-20], IEEE 802.16e/m (WiMAX) and 3GPP Long Term Evolution (LTE)[21-24] and reconfigurable architecture [25]. The success of the VBLAST-based wireless system manifests significant capacity potential in a MIMO system, which not only answers the demand for ubiquitous multimedia access, but also deeply attracts researchers' interests into developing a more close-to-capacity MIMO system [26].

After worldwide researchers' efforts, people have found that multi antenna based communication systems concatenated with carefully designed channel codes are capable of supporting data rates near their corresponding channel capacities [27]. The use of Forward Error Correcting (FEC) codes (popularly known as channel codes) with MIMO systems has also become an active area of research. Channel coding with added spatial dimensions increases the reliability of the wireless link and helps in achieving high code rates at unpredictable spectral efficiencies. There are various channel codes (Convolution Code, Turbo Code, Low Density Parity Check Code), which can be concatenated with the MIMO system. LDPC codes have recently been proposed as the outer code, which drive either space-time codes or direct transmission as the inner code in a concatenated coding scheme for multi-input multi-output (MIMO) systems [28]. The desirable characteristic of LDPC codes, other than their near-capacity performance, is that the complexity of the decoder only grows linearly with the block length of the code. As reported in the literature, performance within a fractional (dB) of the capacity can be easily achieved at a block length of more than hundred thousand in AWGN channels [29]. On the other hand, the length of the outer code is usually limited to a couple of thousands, for practical reasons. For example, in the IEEE 801.11n standard, the length of the LDPC code is around 2000. Thus amongst all channel codes, LDPC codes are a broad class of powerful error correcting codes, which yields astonishing performance close to the Shannon theoretical limits. So in this research, LDPC codes are considered as the outer codes in a concatenated system.

LDPC codes were discovered by Gallager in 1963 [30], and were relatively ignored for about three decades until rediscovered by Richardson [31]. New approaches have developed with the re-introduction of LDPC codes with an iterative decoding structure which ensures that codes with good performance are being designed. The LDPC codes can be represented by a sparse parity check matrix, which alternatively can be represented by a bipartite graph. While performing decoding on graphs, messages are passed from bit nodes to check nodes and from check nodes to bit nodes iteratively, which is often referred to as a message passing algorithm. Iterative processing has achieved near–capacity on a single antenna on a Gaussian and Rayleigh fading channel [32]. Thus the iterative techniques can be used to exploit the performance limits in the SIMO and MIMO system. In an attempt to approach the capacity and other performance parameters of SIMO channel, [33] proposes an iterative decoding scheme with LDPC codes. However the effect of interference of equal/unequal power sources at the receiver end has not been considered.

LPDC codes are beneficial for lower delay and lower BER as they are flexible in terms of code rate and other parameters. Due to good hamming distance properties of LDPC codes, these codes clearly do not show error floors. Another advantage of LDPC codes is the ability to implement fully parallel decoders, thanks to the mechanism of their decoding algorithm. The parallel iterative decoding algorithms of LDPC codes are easily implementable in VLSI technology.

### 1.2 Research motivation

To achieve some targets, communication engineers try their best to utilize limited resources, and find the optimum solution that meets the demand of a given application as well as ensuring that the solutions provided are economical. So there is need of trade-off among targets and demand.

Some common objectives of any communication system can be as follows

(i) At the lowest signal power, the proposed system is able to achieve a lowest error probability.

(ii) To achieve the highest spectral efficiency.

(iii) As the spectrum is very limited, one of the objectives is to achieve the highest data rate for a given bandwidth

(iv) To reduce the complexity of the overall system.

As discussed earlier, to achieve the low BER, LDPC codes that perform close to the Shannon limit are very useful. For attaining the highest spectrum efficiency, a simple spatial diversity technique, which does not involve any loss of bandwidth, is constituted by the employment of multiple antennas at the receiver. Alamouti scheme avoid the more complex joint detection of multiple symbols, transmitted by multiple transmit antennas, thereby reducing the complexity of the overall system. One of the implementation issues of MIMO systems is the increased hardware complexity and cost. A popular approach being employed to address the issue is a technique called the antenna selection. It has been shown that antenna selection maintains the same diversity order as the full antenna system, which makes antenna selection even more attractive.

Therefore in this contribution, a prospective approach is used which provides further research advances in the field of concatenated coded-MIMO systems with antenna selection. Hence in the generalized idea for a proposed system, a coded system is mapped to symbols by a modulation block. Since different spectrally efficient modulation techniques are used, we are sending a number of bits (called symbols) over one signalling interval. These symbols are then transmitted over a SIMO/MIMO channel through single/multiple antennas. When using multiple antennas there are many propagation paths, and if the antennas are sufficiently far apart then the channel and its fading coefficients are considered to be independent. At the receiver, the likelihood ratios (or the soft estimations) of the coded bits are computed by the demodulation block. Evaluating the performance of the systems under message passing decoding is carried out by considering the whole sequence of symbols transmitted. In particular, a number of communication systems (SIMO and MIMO) are investigated with LDPC codes for different modulations over different fading channels. Besides this, the concept of antenna selection has also been investigated with the proposed coded MIMO system. The aim of choosing this popular scenario of wireless communication is to provide the most economical and best solution to this congested environment with so much interference. We tried to find some solutions that define the objectives of our thesis as given below.

### 1.3 Objectives of thesis

Motivated by the success of LDPC codes and iterative decoding, this thesis proposes a communication system based on concatenated LDPC codes with Space-Time Block Codes with antenna selection in various fading channels. In designing the communication system, the main objectives of the design are given as follows

1. To evaluate the performance of various concatenated codes for SIMO systems in a multi-path fading environment.
2. To propose a new concatenated coding scheme, and to compare its performance with existing concatenated schemes based on parameters like BER.
3. To evaluate the performance of various concatenation schemes for MIMO systems using various modulation techniques.
4. To evaluate the performance of the proposed concatenated coding scheme with antenna selection.

### 1.4 Contribution of the thesis

Based on the above proposed objectives, the prime objective of the thesis is to determine the performance of concatenated LDPC-STBC coded communication systems in various fading channels. The performance evaluation is carried out on the basis of bit error rate, the most common criterion for the performance evaluation of a communication system. Different band-efficient modulation schemes are used to manipulate the constraints, such as data rate and bandwidth. Typically, the modulation function tries to combat the bad effect of wireless channels in different ways. Various modulation techniques have been analysed for line of sight (LOS) and non line of sight (NLOS) paths. To validate the computer simulation analysis, analytical bounds have been derived for both the SIMO system and the MIMO system. The main contributions of the thesis are listed below.

**Contribution 1: Performance evaluation of LDPC codes with optimum combining in the presence of interferers.**

In this contribution, we have evaluated the performance of LDPC codes for a SIMO system using an optimum combining diversity technique. First we evaluated the performance of the uncoded system with optimum combining over flat fading channels, and analysed the system in the presence of single interferer.

The work has been extended for the LDPC coded system with optimum combining with multiple interferers and multiple receive antennas. Our main contribution in this chapter is the derivation of closed form expression for the BER performance of the LDPC coded-optimum combined system. The analytical results are derived for two different scenarios (i) *Number of Interferers are Less Than or Equal to the Number of Array Elements (under-loaded case).*(ii) *Number of Interferers are Greater Than the Number of Array Elements (overloaded case).* For the first scenario, special case for the single interferer has been considered and the second scenario has been extended for the large number of interferer. The BER analysis done over frequency-flat Rayleigh fading channels has been compared with simulated results to validate the derived analysis.

**Contribution 2: Performance evaluation of concatenated LDPC-STBC codes in various fading channels**

Recent advances in error control coding have shown that irregular LDPC codes can achieve exceptionally good performance with single antenna system over a wide class of channels [34]. In particular, they have demonstrated reliable transmission at signal-to-noise ratio (SNR) extremely close to the Shannon limit on the additive white Gaussian (AWGN) channel in which efforts have also been made to incorporate LDPC codes with multiple antenna systems. Significant performance improvement has been demonstrated using LDPC codes of quasi- regular structure for space-time wireless transmission. With a relatively small number of transmit antennas, it is shown that LDPC codes of quasi-regular construction are able to achieve higher coding gain than previously proposed space-time trellis codes, turbo codes, and convolution codes in quasi-static fading channels [35].

In this thesis, we have analysed a concatenated LDPC-STBC system in various fading channels for different modulation techniques. In the proposed system, the coded data is mapped to the same signal constellation and transmitted over multiple antennas. This chapter investigates the coded modulated system with message passing decoding algorithm. The tight bound on the bit error probability for the proposed system is derived. The upper bounds obtained for additive white Gaussian noise (AWGN) with Rayleigh fading channels, are applicable to any constellation and mapping. The tightness of the bounds is verified by simulation results of the LDPC-STBC coded system. The bounds obtained are useful to benchmark the performance.

**Contribution 3: Simulation of concatenated LDPC-STBC codes with antenna selection**

All advantages of a MIMO system are achieved at the cost of increased overall complexity. A separate RF chain consisting of low noise amplifiers, down converters and analog to digital converters, is required at every receive antenna employed, thereby also increasing the implementation cost. Antenna selection [36] is one of the attractive approach to mitigate this requirement. So we have evaluated the performance of proposed LDPC-STBC system with antenna selection in various fading channels.

**Contribution 4: To obtain the tight upper bounds on concatenated LDPC-STBC codes with antenna selection**

Till now, the work reported in the literature for concatenation of LDPC and STBC with antenna selection uses computer simulations. There is no previous work based on the performance analysis of a concatenated LDPC-STBC scheme with receive antenna selection. In this contribution our aim was to obtain tight bounds on the concatenated LDPC-STBC scheme with receive antenna selection. We have compared the analytical, as well as simulation, results that validate our results for the derived upper bounds.

## 1.5 Thesis Organization

The organization of the thesis is as follows:

*Chapter I* presents the introduction to multiple antenna systems. In addition, the need for concatenation of channel codes and space-time codes is also discussed. It also includes information on the motivation, major contributions and organization of the thesis.

*Chapter II* covers a comprehensive literature survey of channel codes implemented with various diversity combining schemes at the receiver end. The study is further extended for the concatenated channel codes with multiple antenna system at both transmitter and receiver end. As discussed in a section (1.4), antenna selection is an important approach to reduce hardware complexity. Therefore a brief survey related to implementation of channel codes with antenna selection has also been discussed. Finally, a brief summery that targets towards the objective of thesis is also included at the end of each section.

*Chapter III* presents a brief background on error correction coding and discusses the encoding and decoding algorithms for LDPC codes. The technique of density evolution is also presented.

*Chapter IV* presents and compares the performance of the proposed LPDC-OC scheme in the presence of interferers. Our main contribution in this chapter is the derivation of probability of error of LDPC-OC system in the presence of external interference. Derivation is extended for overloaded and under loaded cases when the number of interferers is greater than the number of receive antennas and vice versa. Both analytical and simulation analysis has been carried out and the results are compared for both analyses.

*Chapter V* presents a system model of the proposed concatenated LDPC-STBC scheme. The main part of this chapter is the analysis of concatenation of the LDPC-STBC scheme in the various fading environments. For line of sight (LOS) and non line of sight (NLOS) paths, the proposed concatenated scheme is analysed for different modulation techniques. The derived tight upper bounds are useful to benchmark the proposed system's performance,

*Chapter VI* considers the constraints on increasing the overall system complexity. As the number of antennas at the receiver end increases, the need of more RF chains required also increases. Therefore to reduce the overall system complexity, a concept of antenna selection is used along with LDPC codes. A norm-based selection is used to select the receive antennas. We mainly derived tight bounds for the LDPC-STBC system with receive antenna selection. Results derived in chapter V (without antenna selection) are compared with the results obtained with antenna selection. Also Monte Carlo simulations have been carried out and results obtained through simulations are compared with the analytical results.

Finally, the conclusions drawn, recommendations and suggestions for the future scope of the work are given in *chapter VII*.

# *Chapter II*

# *LITERATURE SURVEY*

In this chapter we present a comprehensive literature review of various performance related issues of concatenated channel coding with diversity combining schemes in multiple antenna systems in different fading environments. In section 2.1, various diversity schemes in SIMO systems with/without channel coding have been discussed. Section 2.2 extends the literature survey for the same work in MIMO systems. In section 2.3, different antenna selection techniques in MIMO systems are studied and described.

## 2.1 Forward error correcting codes in spatial diversity system

In 1996, Foschini [37] predicted spectral efficiencies in wireless systems of tens or even hundreds of bits/s/Hz under the right circumstances. He suggested that such high spectral efficiency could be achieved by the use of arrays of multiple antenna elements at both ends of a wireless link, a technique now known as *multiple-input multiple-output*, and MIMO, transmission. In September 1998, Lucent issued a press release [38] entitled, “Bell Labs scientists shatter limit on fixed wireless transmission,” claiming that its researchers had found a method of achieving spectral efficiencies greatly in excess of those predicted by the Shannon bound [39]. Despite the stir created by Lucent’s announcement, in reality MIMO systems were not a sudden break-through discovery, but developed out of the concept of *diversity*. Diversity techniques are important methods for improving the transmission reliability of digital communication systems by combating the effect of fading.

Diversity reception (which is regarded as a SIMO system) uses multiple receive antenna elements to improve the reliability of reception in a fading environment. By applying diversity combining techniques, multiple copies of the same transmitted signals are received by multiple antennas. A combiner combines these multiple copies of the signal to improve the noise performance of the system. The analysis of the combiner is generally performed in terms of SNR. The success of diversity schemes depends on the degree to which the signals are uncorrelated. If these multiple signal copies have a low or no correlation of fading coefficients, the effect of fading is significantly reduced. The various types of diversity techniques with multiple antenna systems are discussed next.

The study of diversity reception system was carried out in 1950 by Van Wambeckt [40] to determine the characteristics, limitations and relative merits of space, polarization and frequency diversity systems. In [41], the same author extended the study with experimental work to determine the effects of fading on ordinary and diversity receiving systems. The results were expressed statistically in terms of the non usable circuit time and also in terms of the number of fades per minute below the least usable level.

Real expansion of the diversity started with the classical paper of Brennan [42], in which he provided analyses of three types of diversity combining systems in practical use. These are: selection combining (SC), maximal-ratio combining (MRC), and equal-gain combining (EGC) systems. The effects of various departures from ideal conditions, such as non-Rayleigh fading and partially coherent signal or noise voltages, were considered. Also in this paper the author derived the simple expressions for the mean signal-to-noise power ratios at the output of various combiners.

W. Lee [43] described the mutual coupling effects on an M-branch maximum ratio combiner with correlated signal fading in a mobile radio environment. The results in the mobile radio case showed that the average power delivered to the receiver reduces as the antenna spacing of an antenna array decreases (i.e. the mutual coupling increases). The mutual coupling effects on an M-branch MRC were considered in terms of average power, average SNR and cumulative distribution. In [44], the author extended the same work with a large number of receive antennas in a mobile radio environment to produce an insight into the effects of mutual coupling in an antenna array with many elements. In [45], *Lee* presented the mobile radio performance for a two-branch equal-gain combining receiver with correlated signals. With growing interest in wireless personal communication, analytical expressions for the bit-error rate (BER) and outage probability for the different diversity combiners are required to design effective signaling techniques.

Tellambura et al. [46] provided the analytical solution of M-ary Phase Shift Keying (M-PSK) for selection diversity and maximum ratio diversity schemes. The expression was derived for both Rayleigh and Rician Fading environments. The results showed that diversity reception (especially MRC) provided significant improvements in terms of both the BER and outage probability. In fact, using two-branch MRC diversity, 16-ary PSK outperformed BPSK without diversity. Furthermore, increasing the number of diversity branches reduced the dependency of the average BER and outage probability on the fading environment.

The same work for quadrature amplitude modulation (QAM) was further extended by Chang-Joo et al. [47]. In particular, the closed form expression for the symbol error probability of QAM with L-fold diversity in Rayleigh fading channels has been derived. The derivation was mainly extended for maximum ratio combining and selection combining techniques. The results obtained showed that the probability of error decrease with the increase in order of diversity and incremental diversity gain per additional branch decreases as the number of branches becomes larger.

Annamalai et al. [48] derived the Symbol error rate (SER) for maximal ratio combining (MRC) and equal gain combining (EGC) with multilevel quadrature amplitude modulation (MQAM) on an arbitrary Nakagami fading channel. The results obtained were sufficiently general to allow for arbitrary fading parameters, as well as dissimilar mean signal strengths across the diversity branches.

To reduce the equipment complexity and to achieve cost effective solutions, Adachi et al. [49] proposed a simplified switching diversity technique to be used in a digital FM land mobile radio. Switching diversity techniques are a kind of selection combining diversity technique that adopts a simple method which receives two radio frequency (RF) signals periodically by switching two antenna branches at a moderately higher rate than the bit rate. The feasibility of this periodic switching diversity technique was verified by laboratory simulation and field tests. The results obtained show that average BER performance can be improved by increase in switching rate.

Winters [50] studied and analysed switched diversity with feedback for Differential Phase Shift Keying (DPSK) mobile radio systems in the case of a base-to-mobile path only. The effect on the bit error rate with number of antennas, threshold level at the mobile and the feedback response time of the system were determined. Results showed a significant decrease in bit error rate and a corresponding increase in channel capacity compared to a non diversity system. Although the channel capacity of the switched diversity with feedback system was less than that of some other systems, the system is shown to be much simpler and less costly to implement.

All combining techniques discussed above improve the SNR at the output of the combiner. Interference at each antenna element was assumed to be independent. However, in practice the some interfering signal is present at all receiving antennas. Therefore, Winters [51] proposed optimum combining techniques which not only combat fading of the desired signal (as with maximal ratio combining) but that also reduces the power of the interfering signals at the output of the receiver. This technique was designed to achieve a higher output SINR. In the absence of interference, OC performance was same as that of MRC [5]. For analytical results, the author [52] used Moment Generating Function (MGF) based approach to find the bit error rate and Monte Carlo method to obtain the computer simulation results. Results were also extended for multiple interferers. In addition, upper bounds on the bit error rate (BER) of optimum combining in wireless systems with multiple co-channel interferers for M-PSK techniques were derived. These bounds allow the calculation of the outage probability in shadow fading environment.

Performance evaluation of optimum combining in terms of outage probability and BER with Rayleigh fading and co-channel interference has been derived by Shah et al. [4]**.** The paper considered binary phase shift keying (BPSK) modulation in a flat Rayleigh fading environment when the number of interferes is not less than the number of antenna elements. Closed form expressions were derived for the outage probability and the average probability of the error using hyper-geometric functions.

Villier [53] analysed the performance of optimum combining in the presence of multiple equal power interferers and noise when the number of interferers is less than the number of antenna elements. The desired signal and interferers were subjected to flat Rayleigh fading. An approximate expression of the probability density function (pdf) of the output signal-to-interference-plus-noise ratio (SINR), cumulative distribution function (CDF) of the SINR and the bit-error rate (BER) of some binary modulations were derived. In typical cellular wireless scenarios, the signal of interest needs to compete with multiple interfering signals whose power is close to that of the desired signal (as well as numerous interfering signals whose power is much less than that of the desired signal). When the number of interference sources is larger than the number of degrees of freedom of the array, the array is unable to cancel all of the interfering signals. However, a significant increase in capacity can be achieved with a moderate increase in SINR at the output of the antenna array.

In [5], Shah et al. compared the performance of maximal ratio combining with optimum combining for space diversity reception in digital cellular mobile radio systems with multiple co-channel interference (CCI) sources. Analytical expressions were derived for the probability density function of the array output signal-to-interference ratio (SIR), the outage probability, and the average probability of bit error with maximal ratio combining. All results were obtained under the assumption of equi powered CCI sources. For MRC, the results hold for an arbitrary number of interference sources, while for OC the number of sources were larger than or equal to the number of antennas. In this case, the array degrees of freedom was not sufficient to null the interference sources and thus fading of the interference has a significant impact on the performance of the combiner.

Aalo [54] studied the effect of co-channel interference on the performance of digital mobile radio systems in a Rayleigh fading environment. The average bit error rate (BER) of an antenna array system with an optimum combining scheme that maximizes the output signal-to-interference-plus-noise ratio was analysed. BER expressions which are easy to evaluate numerically, were derived for coherent binary phase-shift keying schemes in an environment with co-channel interference and noise. In this case, one and two interferers were considered.

The case of multiple interferers is more challenging. The conventional way for deriving the expression of symbol-error probability (SEP) or bit-error probability (BEP) starts with deriving the probability density function (pdf) of the SINR conditioned on channel realisations of the desired signal and the interference. For an exact conditional SEP, the Gaussian assumption is necessary for the interference plus noise. The unconditional SEP is obtained by averaging first over the desired signal channel, and then averaging over the interference channel. In [55], Lao et al. took a different approach by performing the analysis directly on the decision statistic rather than on the SINR. For BPSK, this approach allows exact BEP analysis and it also requires averaging over the fading of the interference.

Along with the diversity techniques that provide the diversity gain, channel coding is an important technique that exploits the full information transfer potential of the channel. The main goal of *channel coding* is to protect the message against channel noise. Therefore diversity reception and channel coding are useful means for combating multipath fading and channel noise. In mobile radio environments, to design spectrally efficient systems, the former provides the diversity gain and the latter provides the coding gain. *V.k. jain et al.* [56] investigated the combined effect of forward error correcting (FEC) codes and use of parallel cancellation technique for interference cancellation. For MRC diversity techniques, bit error rate in various fading environment has been analyzed in [57].

To achieve both gains, Adachi and Suda [58] considered a combination of the effects of multiple-branch selection diversity reception on channel coding gain for BCH-coded QPSK land mobile radios. They found that the diversity reception affects the optimum code rate and coding gain from a power and spectrum efficiencies points of view. Results showed that for a decoded BER of $10^{-3}$, the power efficiency can be maximised at a code rate of around 0.4, when diversity reception is used.

In [59], the authors experimentally evaluated the concatenated channel coding scheme with combined post detection diversity for $\pi/4$ shift QDPSK signal transmitted over a Rayleigh fading channel. For concatenated channel coding, RS codes were considered as the outer codes and BCH is considered as the inner codes. Two branch post detection diversity was achieved using MRC and selection combining techniques. The effect of co-channel interference was also considered. The total BER was well approximated by the sum of the individual BER's due to AWGN, delay spread and CCI, since they are independent causes of errors.

In [60], Rasmussen et al. compared two equal gain combining strategies, interleaved code combining and averaged diversity combining, along with trellis coded modulation (TCM). The performance analysis of the interleaved code combining system was recognised as a special case of multiple TCM coding. A modification of the branch labelling of the error state diagram for TCM codes was suggested in order to accommodate the performance evaluation of trellis coded averaged diversity receivers over slowly fading channels.

Al-Semari and Fuja [61] derived tight upper bounds on the pair wise error probability for all three combining schemes, namely SC, MRC and EGC, in the case of a TCM system. Using the transfer function approach, upper bounds were expressed in product form to permit bounding of the bit error rate (BER). In each case, it was assumed that the diversity branches are independent with perfect recovery of CSI. These proposed new bounds were compared with Chernoff bounds and computer simulation results. The results indicate that the joint use of simple coding and diversity results in a substantial improvement in Rayleigh fading over the use of separate, more complex codes (without diversity) or a higher degree of diversity (without coding).

Ramesh Annavajjala et al. [62] analytically developed the performance analysis of coded, coherent communication systems on independent and identically distributed Nakagami-fading channels with selection combining (SC) diversity. Using the pair wise error probability expressions, the union bound-based bit-error performance of trellis-coded modulation schemes was dervied. Finally, an exact expression for the channel cutoff rate for the aforementioned coded system, as a function of the MGF of the SNR at the output of the combiner, was also derived**.**

John Di Choi et al. [63] derived an analytical expression for the asymptotic ($M \to \infty$) performance of M-ary frequency-shift keying (M-FSK) in multiple frequency-nonselective, slowly fading channels, with RS coding, side information, and diversity reception. Using these derived expressions, the minimum SNR required for arbitrarily reliable communication over Nakagami fading channel with RS coding was determined, for a broad range of parameters specifying the code rate, fading severity, and diversity order.

About sixty years ago, Shannon characterised channel coding as a means to achieve channel capacity. Since then, both theoretical and practical coding techniques have constantly been developed to achieve Shannon limit. Nowadays, techniques for practically closing the gap to channel capacity exist for several simple channels. This has been made possible by the invention of powerful coding methods such as Low-Density Parity-Check

(LDPC) codes. Since the introduction of LDPC codes, a huge amount of resources in the scientific community have moved towards the investigation of iterative detection and decoding techniques. These codes are also employed with diversity to achieve both diversity gain and coding gain.

Using a similar approach, Gounai et al. [64] employed irregular LDPC codes with a spatial diversity system, as the LDPC codes can achieve good performance approaching the Shannon limit. They derived the SNR thresholds of regular LDPC codes and irregular LDPC codes with low diversity order as the optimum degree distributions of an irregular LDPC code depends largely on the diversity order. With high diversity order, the optimum degree distribution depends both on the diversity order and the combining scheme.

Selection diversity combining in conjunction with LDPC was employed by Tan et al. in [65]. The probability of error of this scheme over independent and identically distributed (i.i.d) and independent and non identically distributed (i.n.i.d) flat Rayleigh fading channels was derived using the extrinsic information transfer (EXIT) chart and Gaussian approximation (GA) methods. The derived expression was compared with simulated results. It was shown that as the diversity order increases, the BER performance improves. However the diversity gain decreases as the number of diversity branches increases.

In [66], Tan et al. extended the work of [65] to selection combining cascaded with maximum-ratio combining (SC/MRC) combined with LDPC codes. The bit error rate (BER) expressions of uncoded SC/MRC and SC/MRC with LDPC codes over an independent and identically distributed Rayleigh-fading channel was derived based on the Gaussian approximation approach. These expressions were also applicable to independent selection combining and maximum-ratio combining as special cases. These expressions were able to achieve a significant reduction in computational time with a reasonable accuracy for analysing the BER performance as compared to simulations and Density evolution (DE).

Li et al. [67] proposed LDPC codes combined with cascaded combining (SC/MRC and MRC/SC) which was able to achieve excellent bit-error rate (BER) performance over multipath fading channels. The average output signal-to-noise ratio and the uncoded BER expressions of cascade combining and over i.i.d Nakagami-$m$ fading channels were derived.

The BER expressions of cascade combining with LDPC codes over i.i.d. Nakagami-$m$ fading channels were derived based on the GA approach. These expressions include SC and MRC as special cases. The expressions obtained from the GA analysis provided better theoretical framework for the understanding of the system's performance and allowed more efficient computing of the BER and SNR thresholds compared to DE and simulations.

*In summary, with regards to the SIMO system, various researchers have worked on different receive diversity schemes such as equal gain diversity, selection diversity, switch diversity and maximum ratio diversity. As these techniques only provide diversity gain, and no coding gain, investigators employed channel coding with the different diversity schemes discussed in this section [58-67]. All these techniques neglect the effect of interference. The optimum diversity combining techniques include the effect of interference, but until now no work is reported in the literature that integrates the work of optimum combining with channel codes. In this thesis, the performance evaluation of LDPC coded optimum combining system has been taken as one of the objective.*

## 2.2 Concatenated channel codes with MIMO system

For multiple antenna systems, different transmit diversity techniques have been introduced to get benefit of antenna diversity in the down-link. The Multi-Input and Single-Output system, known as the MISO system, places the diversity burden on the base station. The use of transmit diversity at base stations in a cellular or wireless local area network has attracted special interest; this is primarily because of performance increase possible without adding extra antennas, power consumption or significant complexity to the users device. In addition, the cost of the extra transmit antennas at the base station can be shared among all users. The first form of transmit diversity was antenna hopping. In a system using antenna hopping, two or more transmit antennas were used interchangeably to achieve a diversity effect. For instance, in a burst or packet-based system with coding across the bursts, every other burst can be transmitted via the first antenna and the remaining bursts through the second antenna. Antenna hopping attracted some attention during the early 1990's as a comparatively inexpensive way of achieving a transmit diversity gain in systems such as GSM. More recently there has been strong interest in *systematic* transmission techniques that can use multiple transmit antennas in an *optimal* manner [68-71]. The basic scheme proposed by Almouti [13] of a simple (2 Tx X 1 Rx) scheme attracted the interest of researchers because it achieves full diversity gain with a simple maximum-likelihood decoding algorithm. This was later generalised by Tarokh et al. [14] for varying the number of transmit and receive antennas. Almouti's scheme does not provide any coding gain, but the low decoder complexity of STBC allows concatenation with powerful outer error correcting codes for providing such coding gains.

Researchers have already been working on the concatenated codes. The idea of concatenated codes was first conceived by *David Forney* [72] as a solution to the problem of finding a code that has exponential decreasing error probability with increased block length. The concatenating coding was regularly applied in the area of deep space communication where a concatenation of convolution code and Reed-Solomon codes was used. Since then, these concatenated codes became a popular construction both within and outside of the space sector. It is still notably used today for satellite communication, such as the digital video broadcasting standard [73].

In some cases, it is demonstrated that an STBC used in conjunction with an outer channel code (block codes such as RS, LDPC, convolution and turbo codes) can be superior, in terms of performance, to an STTC code, even at a lower complexity [74]. For high data rate transmission over wireless fading channels, space-time block codes provide the maximal possible diversity advantage for multiple transmit antenna systems with a very simple decoding algorithm. To achieve a significant coding gain, space-time block codes have been concatenated with an outer code.

Bauch et al. [75] proposed the concatenation of turbo-TCM with space-time block codes. For the encoding of turbo TCM, two schemes, namely Binary Turbo Code with Gray Mapping and Turbo-TCM with Unger-bock Codes, were used. The overall decoding complexity of proposed system was determined by the trellis complexity of the outer code. Bauch showed that the concatenation of space-time block codes with two Turbo TCM schemes provide significant coding gain in addition to diversity gain.

Gulati et al. [76] proposed concatenation schemes based on recursive realisations of space–time codes. The concatenation scheme is based on serial concatenation of convolution codes, where the inner code is a space–time code. The main advantage of this scheme was that in addition to providing marginally improved performance to other more complex concatenated schemes on quasi-static fading channels, this scheme provides significant gains on time varying channels and independent fading channels. In general, the performance of the proposed system is improved with increase in the length, whereas the performance of other schemes such as concatenation with an outer convolution code, Reed–Solomon code or that of a simple space–time code will deteriorate with increase in length. Hence, the proposed scheme will offer a greater advantage as the length of the code words increases.

The LDPC codes have exponential decreasing error probability with increased block length. This advantage is taken into account by Gulati et al. in [77], where three different concatenated space-time coding schemes; (a) direct transmission of random-like codes, (b) concatenated scheme with O-STBC as the inner code, (c) concatenated scheme with STTC as the inner code, have been discussed over quasi-static channels from a capacity perspective. The constrained modulation outage capacities for these schemes were computed.

Futaki et al. [78] analysed LDPC coded MIMO systems with iterative turbo decoding (MIMO-LDPC-TD). The simulation results showed that the MIMO-LDPC-TD can achieve a good error rate performance with reduced decoding complexity. It was also shown that in a flat Rayleigh fading channel, particularly in fast flat Rayleigh fading channels, LDPC-STBC schemes provided a better frame error rate (FER) than turbo-STBC schemes. But the FER of the LDPC-STBC was worse than that of the Turbo-STBC in a quasi-static Rayleigh fading channel. Furthermore, it was shown that the FER of the LDPC-STBC without a channel interleaver (CI) is better than that of the Turbo-STBC with CI in a fast fading channel.

In [79], the author extended the work of the turbo LDPC-STBC and proposed a new MIMO-LDPC system with iterative turbo decoding employing two LDPC encoders and decoders. It was shown that the MIMO-LDPC-TD (Turbo decoder) can achieve good performance with reduced decoding complexity at each decoder on flat Rayleigh fading channels, particularly on a slow fading channel. It was also shown that the MIMO-LDPC and the MIMO-LDPC-TD can achieve better error rate performance than the MIMO-Turbo on a fast Rayleigh fading channel.

Ohhashi and Ohtsuki [80] analysed the error rate performance of a LDPC-STBC concatenation scheme using the density evolution method. They also optimised irregular LDPC codes for LDPC-STBC by using density evolution. The density evolution method is basically a analytical scheme which computes the average probability of incorrect messages by analysing the probability density function (pdf) of the messages passed from the bit nodes and the check nodes in each iteration of the decoding algorithm. The proposed scheme was implemented with Almouti's two transmit and one receive antenna and was then extended for two receive antennas. Both analytical threshold and simulation results have been obtained for the regular as well as irregular LDPC-STBC concatenation scheme. It was shown that the irregular LDPC-STBC scheme performs better than the regular LDPC scheme.

For multiple antennas, Li et al. [35] examined the performance of short binary and non-binary LDPC codes in flat Rayleigh fading channels with multiple transmit and receive antennas. For small array transmit antenna's, the simulation results demonstrated that the multilevel LDPC codes achieve good performance over MIMO channels, and they perform better than either binary LDPC codes or space-time trellis codes over quasi-static MIMO channels. For large transmit arrays, work has also been extended for D-blast layered architecture using LDPC codes as the component code of each layer. It was concluded that quasi-regular LDPC codes are well suited for high bit rate space-time wireless transmission with short frames. However, irregular codes have a greater potential to achieve channel capacity, especially with large frames

For the joint symbol decoding invoked transmit diversity scheme, Guo and Hanzo [81] made use of the pearls belief propagation algorithm for constructing a belief network which was employed for developing a Bayesian network-based joint detection aided transmit diversity. In general, a Bayesian network consists of a set of random variables.

These random variables are represented by the nodes of the network called *parent nodes* and *child nodes*. These nodes are often referred to as evidence nodes or observation nodes. Through these nodes *a posteriori* information corresponding to the rest of the nodes in the network is transferred. Similarly, LDPC codes may be characterised by variable and check nodes corresponding to which a Bayesian network can be constructed. Based on the posterior probability, a tentative hard decision can be made and the resultant codeword is checked by the LPDC code's parity check matrix. When communicating over uncorrelated channels, the proposed scheme is benchmarked over the original bit by a bit detection scheme. Results showed the BER comparison for joint detection and bit by bit detection schemes for 4-QAM, 16-QAM and 8-PSK modulation techniques. In addition, decoding complexity comparisons were presented for both detection schemes

Hou et al. [82] designed a receiver structure for a serial concatenated MIMO-LDPC system. In the proposed scheme, Almouti based STBC was used as the inner coder and LDPC was used as the outer coder. The suggested system combines the demapper and decoder in an iterative manner, and demapper transfer curves were well aligned with the decoder characteristics. Extrinsic information transfer (EXIT) chart analysis was applied to study the convergence behaviour of the proposed schemes. Results showed that EXIT charts match very well with the simulated decoding trajectories, and they helped in explaining the impact of different mappers and different demappers

Zhang and Lee [83] presented a closed-form random coding bound that can be used to predict the performance of a more practical system in which the Turbo or LDPC code was used as the outer code (in place of the random block code) to derive the inner space-time modulation block. SNR vs BER results have been obtained for union bounds and compared with simulation results of the LDPC system. It was shown that the random coding bound shows a good prediction of the water fall position and provided an upper bound to the simulation results.

In [84], the authors extended their work of [83] and analytically found the union bound for concatenated LDPC/Turbo codes and space-time block codes over a fast fading

Rayleigh channel for PSK modulation techniques in a combinatorial manner. The concatenation was done such that all code words in one subset/sub-code are associated with an identical pair-wise error probability. When the simulation results are compared with the analytical results, an SNR gap of less than 1.0 dB between the waterfall positions of the union bound and the simulation results is evident. The error floor appears almost flat, since at the high SNR region the union bound is mainly determined by the code words with the smallest Hamming distance.

For the *Q*-ary Modulation Systems [85-86], Zhang et al. mapped LDPC code words onto space-time modulation blocks and referred to them as associated space-time (AST) code-words. They also derived union bounds for the LDPC-AST system. Simulation results and analytical results have been compared for different channel scenarios and modulation techniques. The upper bounds were found to be useful and act as benchmark for the practical performance of iterative detection and decoding algorithms. Over quasi-static Rayleigh and Rician fading, the author further derived the bounds of the LDPC-OSTBC scheme [87]. For determining the upper bounds, the Fano-Gallager bounding technique was used that divides the range of the fading gain into two disjoint regions. The proposed approach seems to overcome the excessive codeword multiplicity problem in conventional union bounds, and works well for different channels and coded modulation scenarios.

In [88], Lanxun and Weizhen analysed the performance of ABBA-LDPC and OSTBC-LDPC schemes in a quasi-static fading channel. In [89], this work was extended with a QOSTBC-LDPC scheme. For both schemes [88-89] , BER results have been obtained using Monte Carlo simulations. It was shown that as the number of transmit antennas increases, the BER results improved remarkably.

For multiple access multi-user MIMO systems, Wu and Lee [90] considered a LDPC coded modulation scheme in MIMO multiple-access systems. For the proposed system analysis, the combinatorial union bounding techniques derived in [84] have been used. The union upper bound on maximum-likelihood (ML) decoding error probability for turbo-like or LDPC codes provided a performance prediction of the proposed transmission system, although the ML decoding was usually prohibitively complex for long block codes. Union upper bounds on ML detection, using the distance distribution of the outer LDPC codes, have been derived with closed-form expressions for specific SNRs.

*In summary, for the MIMO systems, various researchers have worked with different concatenated channel coding and space-time coding techniques. Various channel coding techniques such as BCH, R-S, convolution and turbo have been concatenated with space-time codes in order to secure the advantages of both diversity gain and coding gain. Amongst all coding techniques, LDPC coding techniques that perform near the Shannon limit, in concatenation with STBC scheme is one that area in which researchers are working. In this thesis, the performance evaluation of the concatenation of LDPC-STBC codes has been taken as one of the objectives.*

## 2.3 Concatenated channel codes with MIMO system with antenna selection

The advantages of MIMO systems can mostly be explained in terms of spatial diversity and spatial multiplexing gains. All these advantages of a MIMO system are achieved at the cost of increased overall complexity of the system. A separate RF chain consisting of low noise amplifiers, down converters and analog to digital converters is required at every receive antenna employed, thereby increasing the implementation cost also. In order to reduce the overall cost, the transmitter/receiver link uses all available antennas, while the receiver/transmitter link chooses the best $L_r/L_t$ out of $M_R/M_t$ antennas (popularly known as Receive/Transmit antenna selection). In terms of "best", we use only those antennas that maximise the capacity. Therefore antenna subset selection is an attractive solution to the complexity issue, and furthermore greatly improves the throughput/reliability trade-off [91]. In such subset selections, the number of RF chains is smaller than the actual number of antenna elements. The RF chains are connected to the "best" antenna elements, where "best" depends on the channel state. In many scenarios, judicious antenna selection may incur little or no loss in system performance, while significantly reducing system cost (compared to full-complexity systems with the same number of antenna elements). Therefore in this thesis, the concatenated LDPC-STBC system performance is optimised by receive antenna selection (RAS). A brief overview of the work done in this area is presented next.

For MIMO systems, the concept of antenna selection was first introduced by Molisch and Win in 2001 [35]. In their paper, the upper bounds were derived to increase the overall capacity of the system with antenna selection. They compared the analytical results with computer simulations. It was shown that selecting the best '$L_r$' receive antennas gave almost the same capacity as the full-complexity system. A significant increase in capacity is only achieved with $L_r$ transceiver chains, plus an RF switch is required instead of full $M_R$ transceiver chains. It was shown that the results of this paper can serve as a guideline for designing reduced-complexity MIMO cellular systems for third and fourth generation communications.

Since space-time block codes maximise diversity and reduce the complexity of the system over fading channels, Gore et al. [92] suggested a novel transmit antenna selection diversity with space-time code over the optimal antenna pair that provide additional diversity gain. They presented an optimal selection rule that quantified the performance in terms of gain in the average SNR. The Monte Carlo simulation had been conducted to investigate to the effect of antenna selection on the probability of symbol error.

Chen et al. [93] derived the exact probability of bit error rate of Alamouti scheme based transmit antenna selection on a flat Rayleigh fading channel with BPSK modulation. The proposed scheme achieved full diversity at a high SNR. The exact expression obtained was compared with approximate results, as well as with the simulation results. Based on the analysis, it was proven that this scheme achieves a full diversity order asymptotically, as if all the transmit antennas were used. With a specified diversity order as the design target, this scheme can dramatically decrease the number of receive antennas at the mobile set by deploying most of the antennas required at the base station.

Bahceci et al. [94] studied the multiple antenna transmission system with antenna selection. The main approach was to determine the pairwise error probability (PEP) for full rank, and rank deficient space-time block codes with antenna selection. Diversity and coding gains for the computed PEP were determined by computing the upper bounds on the considered system. It was shown that the diversity gain of rank deficient system with antenna selection deteriorates when compared with the full rank space-time block codes. Several simulation results and analytical results were presented that validate the upper bound results.

Aydin and Altunbas [95] simulated the frame error rate performance of a concatenated LDPC and STC scheme with transmit antenna selection over quasi-static and fast Rayleigh fading channels. It was shown that transmit antenna selection provides diversity gain over quasi-static fading channels and coding gain over fast fading channels. It is also concluded that diversity gain and coding gain over the fast fading channel is improved by LDPC coding iteration.

For the quasi-static fading channel, Ghrayeb et al. [96] analysed the performance of MIMO systems with receive antenna selection The basic idea was that, for a given number of receive antennas, the receiver uses the best out of the available antennas. In selecting the best antenna, a pragmatic selection criterion that is optimal in the sense of achieving the maximum signal-to-noise ratio (SNR) at the receiver was used. The work mainly deals with the impact of antenna selection on diversity and coding gain. It was shown that over quasi-static fading channel while diversity gain is maintained but the coding gain is reduced by the value upper bounded by 10 log ($M_R/L_r$), where $M_R$ is number of receive antenna and $L_r$ is number of selected antenna.

For the concatenation of channel codes with receive antenna selection, Hamouda and Ghrayeb [97] studied the performance of a serial concatenated scheme comprising a convolution code (CC) and a space-time block code (STBC), separated by an interleaver for receive antenna selection. Performance bounds were derived for the CC-STBC scheme. Simulation results for $M_T = 2$ and $L_t = 2$ with receive antenna selection provides a coding gain approximately 2 dB higher than the corresponding $M_T = 2$ and $L_t = 1$ case. They examined the impact of performing antenna selection at the receiver on the diversity order and coding gain of the system. Results were also compared for quasi-static vs. fast fading channels in which the fast fading channel provided a much larger diversity order than the block fading case for the same space diversity order (same number of transmit and receive antennas). This large diversity order is due to the time diversity delivered by the channel interleaving process which dominates the overall system diversity.

For the frequency selective fading channel, the work of [97] was extended by Gucluoglu et al. [98] in which antenna selection was based on maximising the SNR at the output. It was assumed that perfect channel state information is available at the receiver end. Utilising the results of [96], which considered frequency flat fading channels, it was demonstrated that by applying antenna selection one can still achieve full available diversity, determined by the number of antennas and number of ISI taps. To derive these results, it was assumed that underlying code is full-rank for the frequency-selective channel, otherwise, the diversity order is reduced and it becomes a function of the number of antennas selected.

Zeng and Ghrayeb [99] extended the work of [97] and derived upper bounds for the BER of a CC-STBC concatenated scheme with receive antenna selection. The results showed that for a CC-STBC concatenated scheme with receive antenna selection the diversity order is maintained with a loss in the coding gain upper bounded by $10 \log_{10} (M_R / L_r)$ dB, where $M_R$ is the number of available receiving antennas and $L_r$ denotes the selected antennas. The authors also derived a tighter upper bound on the BER for the Alamouti scheme when the receiver uses the best antenna, i.e., $L_r = 1$. These upper bounds can be extended in a straightforward manner to other types of outer codes and fading channels, including fast, block, and slow fading channels. Finally, the simulation results that validate the analysis were discussed.

The concatenated LDPC-STBC scheme with adaptive modulation was investigated by Zhang et al. [100] for transmit as well as receive antenna selection. In this scheme the antenna selection is based on the criterion of maximising the average spectral efficiency (ASE) of the whole system for a given bit error rate constraint. It was also assumed that the channel state information is perfectly fed back to transmitter by adaptive control signalling. Simulation results showed a trade-off between the ASE and BER under the MIMO environment.

*In summary, antenna selection is very powerful technique to reduce the overall complexity of the system. Various researchers have worked on antenna selection techniques with/without concatenated channel codes and space-time MIMO system. Till now, the work reported in the literature for concatenated schemes include convolution and space-time block coded systems* *with antenna selection. Therefore in order to optimise the concatenated system's performance, we extended the work of concatenated LDPC-STBC system with antenna selection, which defines the fourth objective of our thesis.*

*Everybody believes in the exponential law of errors:*

*The experimenters, because they think it can be proved by mathematics;*

*And the mathematicians, because they believe it has been established by observation.*

-Whittaker and Robinson

# Chapter III

# BACKGROUND ON ERROR CONTROL CODING

*The objective of this chapter is to introduce the basic concepts of error control coding in general and LDPC codes in particular. These concepts serve as background material for subsequent chapters. The maximum likelihood (ML) decoding and the sum-product decoding algorithms are presented for general codes and LDPC codes respectively. Issues regarding the implementation of LPDC codes such as node complexity, number of iterations required and routing complexity are discussed briefly.*

In the present day scenario design, development and deployment of the communication systems aim at efficient utilisation of available resources for transferring the information reliably between a sender and a recipient. Practically, during the transfer some amount of unreliability is tolerated to achieve a pre-defined level of consumption of limited resources. The primary resources such as time, space, power, and bandwidth of the electro-magnetic radiation are used to convey information. Furthermore for the effective utilisation of available resources, knowledge of the mechanisms that may cause interference in a given transmission scenario must be available in designing and analysing a communication system. While analysing the performance of communication system, the most significant and unpredictable parameter is channel, as the degradation of a signal propagating from a transmitter to a receiver is strongly dependent on their locations relative to the external environment. A common property of all communication channels is that the received signal contains the *additive white Gaussian noise (AWGN) noise*, which fundamentally limits the rate of communication. Along with AWGN noise, other major impairment parameters associated with channel are multi-path fading, inter-symbol interference, attenuation of signal, interference between different wireless carriers, and spreading of radiated electromagnetic power in space. Keeping all these aspects in mind, for reliable communication, the major concern of the designer is to control and reproduce the data at receiver end. Hence, a fundamental step to implement an advanced communication systems is its analysis and design in presence of channel impairments. The generalized system model of communication system having all system blocks is explained in the next section

### 3.1 A Framework of communication system

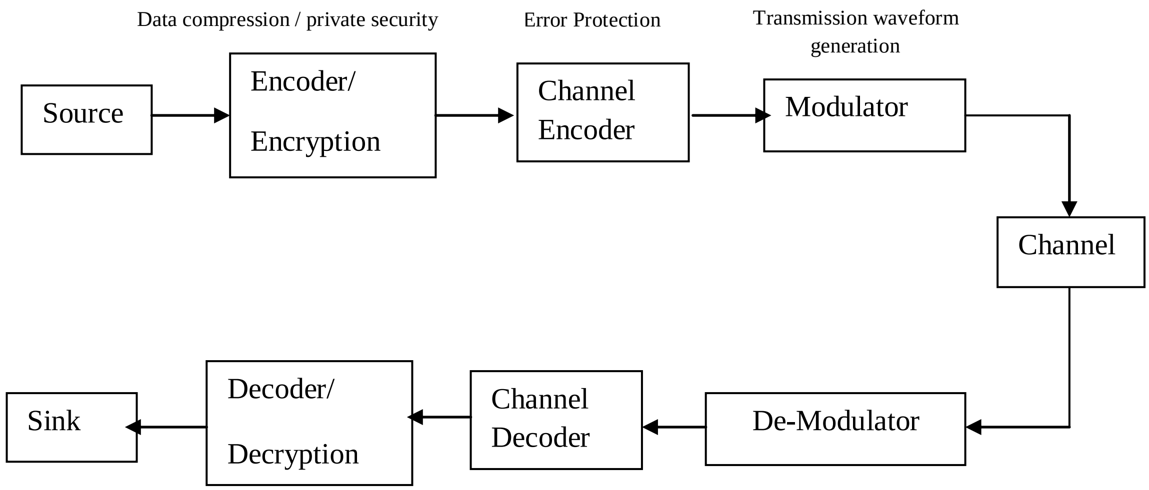


**Figure 3.1: Block Diagram of communication system**

A digital communication embodies functionality to perform physical action on information. A generalised framework of communication system is shown in Figure 3.1. From the source side, data is encoded, modulated and transmitted over the channel. At the sink side, data is re-processed to get the original form back. All the associated blocks have mathematical descriptions to control their performance.

The source is the data to be sent that can be digital or analogue (A/D converters are required) data. It can be acquired from any audio/video signal, computer file or may be any random data. The data to be processed is further forwarded to encoder block

The encoder can be a source encoder which further compresses the data or an encrypter which scrambles or hides the data so that unintended listener is unable to recognise the information.

The channel coder is the first step towards the error correction/detection step. It basically adds the redundant bits to the information signal so that any errors introduced in the information signal can be corrected at the receiver end. After encoding, the information/message bits are modulated and transmitted through single/multiple antennas. The modulator transforms the FEC encoded bits to the symbols for transmission. As choosing an appropriate modulation format is a difficult task, one should consider the following points while selecting a modulation:

The modulation technique should

- be energy efficient; i.e., provide low bit error rate (BER) at low signal to noise ratio (SNR)
- be bandwidth efficient.
- perform well in multipath fading and time varying channels.
- have low out of band radiation.
- have low carrier to co-channel interference ratio.

At the receiver side, the signal is received, de-modulated and decoded. Finally the signal is transformed to the original sequence.

The major thrust of this research work is mainly dedicated to implementation of error correction techniques with multiple antenna systems. Considering the brief aspects of the communication system, the main question revolves around the reliable transmission of information at a desired rate. The answer to this question was given by the Shannon [39] in his landmark paper in which it was demonstrated that errors introduced by a noisy channel or any storage medium can be reduced to any desired level without sacrificing the rate of information transmission. Shannon's work was extended by a large number of researchers by development of efficient error correcting codes [101] which achieve the Shannon limit

### 3.2 The Channel coding theorem

When one considers the error performance of digital communication systems, the coding theorem given by the Shannon is one of the most general forms of the error performance evaluation and needs to be understood [39].

[Theorem 1] *The channel capacity of a discrete memoryless channel with input* $\boldsymbol{x}$ *and output* $\boldsymbol{y}$ *is defined as the maximum mutual information between* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *where the maximum is taken over all possible input distributions bits* $p_x(x)$

$$C = \max_{p_X(x)} I(\boldsymbol{x};\boldsymbol{y}) \text{ bits per symbol,} \tag{3.1}$$

For the band limited AWGN channel, this channel capacity is computed as follows

$$C = B \log\left(1 + \frac{E_b}{N_0 B}\right) , \tag{3.2}$$

where B is bandwidth of the channel and $E_b$ is the bit energy of the received signal, and $N_0$ is the power spectral density of noise. Recall that the code rate R for any error correcting code is given by $R = k/N_c$ where k is the number of uncoded bits and $N_c$ is coded bits.

[Theorem 2] The channel coding theorem states that

*If the transmission rate R fits the condition R < C, then for an arbitrary value of probability of error e>0, there exists a code that makes the error probability of the transmission less than e. If R > C then there is no guarantee of reliable transmission; that is, the arbitrary value of e is not bounded by zero and it may be exceeded.*

The coding theorem proposed by Shannon was proved, assuming the use of optimum decoder. The optimal decoder is used to minimise the probability of error is based on the MAP/ML (maximum *a posteriori* probability/maximum likelihood) criterion,

For the MAP criterion, assume we transmit a codeword over a channel with input vector $\boldsymbol{x}$. The transmitter transmits the codeword over the channel with probability $p_X(\boldsymbol{x})$ and let $\boldsymbol{y}$ denote the observation at output with probability $p_Y(\boldsymbol{y})$. To minimise the probability of error, the MAP decoder chooses the codeword $\boldsymbol{x}$ that maximises $p(\boldsymbol{x}/\boldsymbol{y})$ i.e., the decoded codeword $\hat{\boldsymbol{x}}$ is chosen according to the rule [102]

$$\hat{\boldsymbol{x}} = \operatorname{argmax}_{x \in X} p(\boldsymbol{x}/\boldsymbol{y}) \tag{3.3}$$

The probability $p(\boldsymbol{x}/\boldsymbol{y})$ is called "a posteriori probability" (APP) for $\boldsymbol{x}$, so a decoder that selects the codeword with maximum a posteriori probability is a maximum a posteriori decoder.

The ML decoder, on the other hand, always chooses the codeword that is most likely to have produced $\boldsymbol{y}$ i.e. for a given received vector $\boldsymbol{y}$ , the ML decoder will choose the codeword $\boldsymbol{x}$ that maximises the probability $p(\boldsymbol{y}/\boldsymbol{x})$.The ML decoder returns the decoded codeword $\hat{\boldsymbol{x}}$ according to the rule

$$\hat{\boldsymbol{x}} = \text{argmax}_{x \in X}\, p(\boldsymbol{y}/\boldsymbol{x}) \tag{3.4}$$

The probabilities $p(\boldsymbol{x}/\boldsymbol{y})$ and $p(\boldsymbol{y}/\boldsymbol{x})$ in (3.3) and (3.4) are closely related by Bayes' rule and given as

$$p(\boldsymbol{y}/\boldsymbol{x}) = p(\boldsymbol{y},\boldsymbol{x})/p(\boldsymbol{x}) = p(\boldsymbol{x}/\boldsymbol{y})p(\boldsymbol{y})/p(\boldsymbol{x}) \tag{3.5}$$

If each codeword in (3.5) is equally likely to have been sent, then $p(\boldsymbol{x}/\boldsymbol{y}) = p(\boldsymbol{y}/\boldsymbol{x})$ , so the MAP and ML decoding will return identical results. The key step in the MAP and ML decoding process is to compute a post-priori (APP) probabilities $p(\boldsymbol{y}/\boldsymbol{x})$ i.e., find the distribution of $\boldsymbol{x}$ given the observation $\boldsymbol{y}$. Therefore using the optimal decoders, for every considered codeword, $2^k$ (where k is the information bits), one needs to compute the APP probabilities. Hence the computational complexity is very high in optimal decoder case. So, decoders have to use different algorithms to reduce computational complexity, such decoders are called suboptimal decoders. Suboptimal decoders use iterative decoding algorithms having low computation complexity but performance equivalent to optimal decoders. Iterative decoding algorithms find their application in digital broadcasting, deep space and mobile communication based applications. Moreover iterative algorithms provide less complex solutions with self-contained treatment of iterative error correction techniques.

### 3.3 Practical error correcting codes and their computational complexities

There are two main algorithmic complexities associated with the use of error correcting codes: encoding complexities at the transmitter side and decoding complexities at the receiver side. The former task is usually easy to perform efficiently, since the construction of a code always works on some encoding procedure. But on the decoder side, it is not easy to perform the task as noise is added in the received signal. Since the implementation of codes is based on some encoding rule, the decoding algorithm uses the information from such rules to detect the codeword correctly and to reduce the necessary computation.

There are mainly two types of codes that are implemented practically: Linear Block codes and Convolution codes. Linear block codes are a class of parity check codes that can be characterised by the notation ($N_c$, k) where $N_c$ is the number of coded bits and k is the number of uncoded bits. The binary cyclic codes are a subclass of linear block codes. The encoding of such codes is implemented with linear feedback shift registers (LFSR) and decoding is performed by syndrome calculation method accomplished with LFSR. The decoding is often hard decision decoding and soft decision decoding. The soft decision decoding is more complex as the length of code increases; and observes the following steps: 1) calculate the syndrome; 2) locate the error pattern; 3) perform the modulo-2 addition on the error patterns. After performing these steps, the corrected vectors will be recovered.

A class of powerful block codes is BCH [103] codes, which allow multiple error correction and perform both hard and soft decision decoding. When comparing soft and hard decision decoding for BCH codes, it is shown that for a fixed code rate, hard decision decoded BCH codes of length 8* $N_c$ perform better than soft decision decoded BCH codes of length $N_c$. The one subclass of BCH codes that is particularly useful in non-binary cases is Reed – Solomon (RS) codes. The RS codes achieve the largest possible code minimum distance for any linear block code with the same encoder input and output block length. RS codes are particularly useful in burst error correction (i.e., they are effective for the channels that have memory) [104-105]. The RS codes are cyclic codes and the encoding and decoding procedure is analogues to the BCH codes. Some soft decoding algorithms have been developed for RS codes. However the complexity of these codes is still high as the length of code increases [106-107]. Another type of linear block code is LDPC codes that are based on the sparse parity check matrix. A detailed discussion of LDPC codes will be given in the next section. A comparative analysis of LDPC codes with RS codes is very well presented in [108-109].

Convolution code [110] is an important class of error correcting codes which is described by three integers: $N_c$, $k$ and $K$, where the ratio $k/ N_c$ is the code rate and the parameter $K$ is known as constraint length and represents the number of $k$-tuple stages. The encoding function of block codes maps blocks of data into (longer) blocks of data while the convolution codes map the streams (continuous and infinite) of data into (more) streams of data. The mapping is realised by sending the input streams over the linear filter. The name “convolution codes” stems from the fact that the filtering operation can be expressed as a convolution. The operation of a convolution encoder may be represented by using a state transition diagram, trellis structure or tree structure.

For the decoding of convolution codes, two approaches can be considered. The first approach is implemented using the Viterbi algorithm. This approach makes it possible to guarantee a minimum error probability for the decoded sequence as well as for the information blocks. The second approach that works at the level of information block is MAP (*maximum a posteriori*) criterion. This approach guarantees a minimum error probability for the information blocks only. The disadvantage is its complexity compared to the Viterbi algorithm, while its advantage is that it makes it possible to associate reliability of information to each decoded information block. The performance of convolution codes approaches the Shannon limit when the memory of the shift register increases and computational complexity per bit of the Viterbi decoder increases exponentially with memory length.

The idea of concatenation was conceived by Forney et al. [111] to achieve a larger coding gain. Various researchers worked on different concatenation schemes with serial and parallel implementation [112-114]. Turbo codes can be thought of as a refinement of a concatenated encoding structure plus an iterative algorithm for decoding. Turbo codes are class of error correcting codes that come closer to approaching the Shannon limit very closely than any other class of error correcting codes, and achieve remarkable performance with a relatively low complexity of encoding and decoding algorithm.

The turbo encoder implements a random interleaver between two component convolution codes in parallel or serial configuration. At the decoder side, the input bits to the individual decoder are decoded separately using soft output Viterbi algorithm or MAP/BCJR algorithm. The concept behind turbo decoding is to pass the soft decision from the output of one decoder to the input of the other decoder and to iterate the process several times so as to produce reliable information. In the convolution codes, the complexity of trellis decoding is approximately proportional to the “size” of the trellis, where the size is the number of transitions (block length) times the number of states. On the other hand, in a turbo code, decoding is performed on every component code instead of the overall code. Hence, the decoding complexity per symbol is constant and the overall complexity is low.

When comparing turbo codes with LDPC codes [115] (which also perform near to the Shannon limit), both coding techniques implement iterative decoded algorithms using alternating exchanges of soft-decision information. The main differences are: if the code length is sufficiently large, the BER performance of LDPC codes is extremely good. For the block length of $N_c$ = 10,000, the BER performance curve is only 0.1 dB away from the Shannon limit. But the larger block lengths lead to significant decoding delays, and considerable encoding and decoding complexity.

On the other hand, turbo codes are well suited to intermediate block or constraint length applications. But at high BER, the error floor effect is present. For both turbo and LDPC codes, the original iterative decoding methods are more complex than their logarithmic versions. Simplified variants of the logarithmic decoding algorithms lead to even lower complexity decoding algorithms, usually performed by applying max or min functions. As might be expected, however, the trade-off is some level of degradation in the corresponding BER performance.

### 3.4 Low Density Parity Check Codes

Gallager codes, now widely known as low-density parity check (LDPC) codes [30], are linear block codes that are constructed by designing a sparse parity check matrix $\boldsymbol{H_p}$; that is, for the binary case, a matrix that contains relatively few '1's spread among many '0's. Gallager's original paper, apart from various LDPC code constructions, also presented an iterative method of decoding the codes, which was capable of achieving excellent performance. However, the complexity of the iterative decoding algorithm was beyond the capabilities of the electronic processors available then, which is why the codes were forgotten until 1996, even in spite of an attempt by Tanner in 1981 [116] to revive interest in them. The first construction method proposed for the design of the sparse parity check matrix $\boldsymbol{H_p}$ associated with these codes involves the use of a fixed number of '1's per row and per column of that matrix. In this case, LDPC codes are said to be regular. However, the number of '1's per row and column can be varied, leading to the so-called irregular LDPC codes.

### 3.4.1 Representation of LDPC code

As their name suggests, LDPC codes are block codes with a parity check matrix that contains only a small number of non-zero entries. The sparseness of LDPC codes is essential, as, in the iterative decoding, complexity increases with increase in code length. LDPC codes are very much similar to other block codes. When compared to other classical block codes, in LDPC codes a suitable parity check matrix is constructed first and then a suitable encoder is determined for the code afterwards. On the decoding side, classical codes are generally decoded with ML decoding. These codes can also work well with iterative decoding algorithms if and only if they are represented by a sparse parity check matrix. Practically it is not possible to find the parity check matrix that is most appropriate for classical block codes. On the other hand, LDPC codes are decoded iteratively using a graphical representation of the parity check matrix known as a Tanner graph. For the graphical construction of a Tanner graph, consider the following parity check matrix of rate ½, length 10, irregular LDPC codes.

$$\boldsymbol{H_p} = \begin{bmatrix} 1&1&1&1&0&1&1&0&0&0 \\ 0&0&1&1&1&1&1&1&0&0 \\ 0&1&0&1&0&1&0&1&1&1 \\ 1&0&1&0&1&0&0&1&1&1 \\ 1&1&0&0&1&0&1&0&1&1 \end{bmatrix} \tag{3.6}$$

In this matrix, the number of 1’s in each row is 6 and number of 1’s in each column is 3. The given matrix is not sparse because the code is still short. This code can be represented in terms of a bipartite graph as shown in Figure 3.2

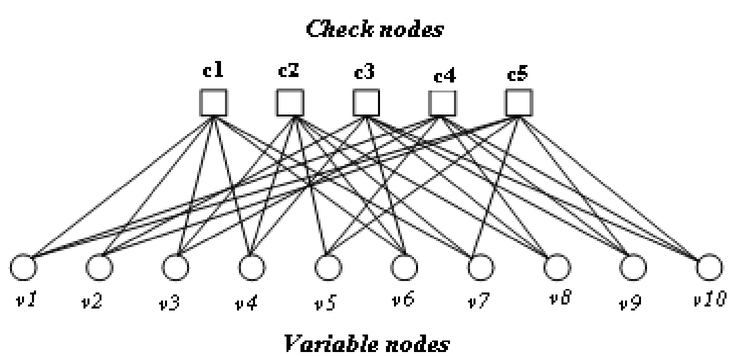


**Figure 3.2: Bipartite graph of regular LDPC codes of length 10 and rate ½ [102]**

As shown in figure 3.2, LDPC codes consist of two kinds of nodes: a variable node $v$ which corresponds to coded bits of a codeword and corresponds to the number of columns in parity matrix $\boldsymbol{H_p}$, while the check node $c$ corresponds to the parity check constraint satisfied by variable nodes. It also corresponds to number of rows in $\boldsymbol{H_p}$. Edges connect the variable nodes to a check node according to the parity check matrix $\boldsymbol{H_p}$. A node of degree $i$ is said to have a degree $i$ if $i$ branches depart from it and connect to $i$ different nodes of other kinds. Thus the degree of a variable node $d_v$ is number of checks it participates in, while the degree of a check node $d_c$ is the number of variable nodes that are connected to check .If all the variable nodes have a same degree and so do the check nodes, then the LDPC codes are known as regular LDPC codes otherwise, they are irregular LPDC codes. For irregular LDPC codes, the degree of variable nodes and check nodes are chosen according to distribution and are often presented as a degree distribution pair. The variable and check degree distribution is denoted by

$$\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1} \text{ and } \rho(x) = \sum_{j=2}^{d_c} \rho_j x^{j-1}, \quad (3.7)$$

where $\lambda_i(\rho_j)$ is the fraction of edges emanating from variable node of degree $i$ (check node of degree $j$). If the graph has $t$ branches and the corresponding parity check matrix has $t$ non zero entries, the number of variable nodes, $n_i$of degree $i$ is

$$n_i = \frac{t\,\lambda_i}{i} \quad (3.8)$$

And the number of check node, $m_j$ of degree $j$ is

$$m_j = \frac{t\,\rho_j}{j} \quad (3.9)$$

Therefore the total number of variable nodes $N_v$ is given by

$$N_v = \sum_i n_i \quad (3.10)$$

And the total number of check nodes $M_c$ is given by

$$M_c = \sum_j m_j \quad (3.11)$$

A *cycle* in a Tanner graph is a sequence of connected nodes which starts and ends at the same node in the graph and which contains other nodes no more than once. The length of a cycle is the number of edges it contains, and the *girth* of a graph is the size of its smallest cycle.

### 3.4.2 Encoding of LDPC codes

LDPC codes are linear block codes; they can be encoded using an $(N_c \times k)$ generator matrix $\boldsymbol{G}$. In particular, a codeword $\boldsymbol{c}$ can be obtained as $\boldsymbol{c} = \boldsymbol{x}.\boldsymbol{G}$ where the vector $\boldsymbol{x} = [x_1, x_2, \ldots \ldots x_k]$ is the message to be communicated. The given code is considered to be symmetric since the first $k$ bits contain message bits. For systematic codes, the generator matrix contains the $(k \times k)$ identity matrix $\boldsymbol{I}_k$ as its first $k$ columns and is represented as

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{I}_k \\ \ldots \\ \boldsymbol{P} \end{pmatrix}, \tag{3.12}$$

where **P** is $(M_c - k) \times k$ matrix that generates the parity bits of the codeword. Hence, the encode vector $\boldsymbol{c} = \boldsymbol{x}.\boldsymbol{G}$ is in the form $(x_1, x_2, \ldots \ldots x_k, p_1, p_2 \ldots \ldots \ldots .. p_{M_c - K})^T$. The parity check matrix $\mathbf{H_p}$ in a systematic form, generated by **G** is defined as $\boldsymbol{H_p} = (\boldsymbol{P} \vdots \boldsymbol{I_{n-k}})$. The encoded code word is consider to be valid if $\mathbf{H_p}.\mathbf{C}$=0.

Although $\boldsymbol{H_p}$is sparse [117-118] by definition, the $\boldsymbol{P}$ of **G** matrix is, in general, dense. This means that, in order to store $\boldsymbol{G}$ (leaving the identity matrix), $k(N_c - k) = N_c^2\left(\frac{1}{R} - 1\right)$ bits of memory are needed, where R is the code rate. In the case of LDPC codes, $M_c$ is very large and typically ranges from $10^3$ to $10^4$. This means that for storing the $\boldsymbol{G}$ matrix, at least $10^6$ bits of memory is required. Thus encoding becomes very complex.

### 3.4.3 Decoding Algorithm

Most classical block codes employ ML or MAP detection in which errors can be corrected by comparing the received vector with every codeword and the most likely codeword is guaranteed to return. Practically it is possible only when the numbers of message bits are small. However, for LDPC codes, Gallager introduced iterative decoding algorithms which exploit the sparseness of the parity check matrix of code. The class of iterative algorithms used to decode LDPC codes are collectively termed as *message passing algorithms* [119] since their operation can be explained by passing the messages along edges of tanner graph. Another message passing algorithm which is based on hard decisions and low complexity is popularly known as the *bit flipping algorithm* [120]. The third algorithm which is most commonly used is the *Sum product algorithm/belief propagation decoding algorithm* [121-123]. In this algorithm, messages are probabilities (or their log likelihood ratios) that represent a level of belief about the value of codeword bits. This is the most accurate algorithm based on the iterative exchange of real valued reliabilities of codeword bits. The brief overview of these decoding algorithms is presented next.

#### *3.4.3.1 Message passing algorithm*

A message passing algorithm is most appropriately used on binary erasure channels (BEC). These channels have a well defined property that either the bit will be correctly received or completely erased with some probability 'ε'. Each erased bit can be determined with the help of a check node, if it is the only erased bit in its parity check equation. The following steps are taken into consideration when messages are passed along the tanner graph.

- A variable or bit node sends the same outgoing message to each of its connected check nodes. This message is labelled as $L_i$ for $i^{th}$ bit node, and declares the value of the bit as 1 or 0, if the value is known or 'e', if the bit is erased.
- If the check node receives the 'e' (erased) message, it calculates the value of the unknown bit by choosing the value that satisfies the parity check equations.
- The check node sends back different messages to each of the connected bit nodes. The message $L_{c\to v(j,i)}$ passed from the $j^{th}$ check node to the $i^{th}$ bit node declares the value 0/1 or 'e' as determined by the check node.
- If the bit node receives the erased message, then once again, the process of passing the messages to the check node and back to the bit node starts. This process is repeated until all the bit values are known or the maximum number of iterations has passed and the decoder halts.

*3.4.3.2 Bit flipping algorithm*

A bit flipping algorithm is based on the principle that an incorrect codeword bit will be contained in a large number of parity check equations, each of which will be able to calculate the correct value of that bit. This algorithm is also known as the hard decision message passing algorithm. The following steps are observed for the algorithm

- A bit node sends the message to the check node in a similar fashion as discussed in the message passing decoder.
- Then, the $j^{th}$ check node determines its decision on the $i^{th}$ bit node by assuming that the $i^{th}$ bit has been erased and choosing the value 0 or 1 that satisfies the parity check equations. The value determined by the $j^{th}$ check node is completely independent of the value for the $i^{th}$ bit just received by it.
- The check node is said to be creating extrinsic information about the $i^{th}$ bit.
- At the bit node, all the extrinsic information about a bit is compared with information received from the channel to determine the most likely bit value.
- If the majority of messages received by a bit node are different from its received value, then the bit node flips its current value.
- The bit flipping algorithm immediately stops whenever a valid codeword has been found by check of parity check equations being satisfied, i.e., $\hat{\boldsymbol{c}}\boldsymbol{H}_{\boldsymbol{p}}^{\boldsymbol{T}} = 0$, otherwise, the process is repeated until the maximum number of iterations and the decoder halts.

***3.4.3.3 Sum product algorithm/belief propagation***

The sum product algorithm of LDPC decoding is best understood as iterative message passing algorithm that passes the message between variable and check nodes along each edge. The Sum product algorithm is a soft decision message passing algorithm similar to the bit flipping algorithm which accepts an initial hard decision on the received bit as input whereas the sum product accepts the probability for each received bit as an input. These received bit probabilities are also called *a priori* probabilities because they were known to the decoder in advance before the LDPC decoder was operated.

For a binary variable $x$, it is easy to find $p(x = 1)$ given $p(x = 0)$, since $p(x = 1) = 1 - p(x = 0)$. Therefore we need to store only one probability value for $x$. Log likelihood ratios are used to represent the metrics for a binary variable by a single value and are given as

$$L(x) = log \frac{p(x=0)}{p\ (x=1)} \tag{3.13}$$

The sign of $L(x)$ provides a hard decision on $x$ and the magnitude $|L(x)|$ is the reliability of the decision. Translating from LLR back to probabilities is given as

$$p(x = 1) = \frac{e^{-L(x)}}{1+ e^{-L(x)}}\ and\ p(x = 0) = \frac{e^{L(x)}}{1+ e^{L(x)}} \tag{3.14}$$

The benefit of logarithmic representation of a probability is that, when the probability needs to be multiplied, LLR only needs to be added. This reduces the complexity of the sum product decoder. For a transmission over a Rayleigh fading channel, $k$ information bits are encoded into an LPDC binary codeword of length $M_c$. It is assumed that a BPSK modulation is used and an additive white Gaussian channel is present. If $\boldsymbol{X} = (x_1, x_2 \ldots . x_{M_c})$ is the transmitted code word after modulation and $\boldsymbol{R} = (r_1, r_2\ \ldots, r_{M_c})$ is the received code word then

$$r_d = c_d\, x_d + \ n_d$$

where $d$=1,2…. $M_c$ and $n_d$ is a Gaussian distributed noise sample with zero mean, variance $\sigma_n^2$. and $c_d$ is the normalized Rayleigh fading factor with mean zero and variance 1 and having pdf

$$p(c_d) = 2c_d\, exp(-{c_d}^2). \tag{3.15}$$

For the uncorrelated Rayleigh fading channel, the conditional pdf [124] of the $d^{th}$ output $r_d$ is

$$p(r_d/n_d\ , c_d\ ) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(r_d - n_d . c_d)^2}{2\sigma_n^2}\right) \tag{3.16}$$

Initial message $q$ passed from the bit to check node in terms of LLR is

$$q = log\frac{p(r_d/x_d=1)}{p(r_d/x_d=-1)} = \frac{2}{\sigma_n^2} r_d . c_d \tag{3.17}$$

As shown in Figure 3.3, the output message at a given variable node that is passed to the check node is the summation of the initial message $q$ (it can be the sample from the channel or correspond to a priori information on the corresponding bit) and the incoming edge LLRs ${L_{c\rightarrow v,l}}^{p-1}(x_d)$ for $l = 1,2, \dots . . d_{v-1}$ along each of variable node edges. (Initially the entire message coming from the check node is set to zero and after the first iteration, all messages coming into variable nodes from check nodes are considered independent).

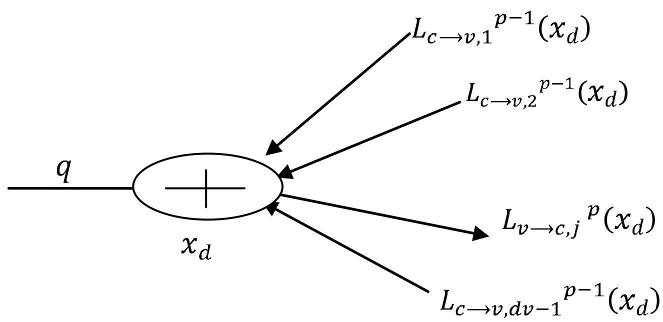


**Figure 3.3: Variable node computation**

It is assumed that initially all the incoming LLR at variable node $x_d$ is equal to zero. i.e.

$$q_{(ch\rightarrow v,l)}{}^{0}(x_d) = 0, \qquad where\ l = 1,2, \dots . d_v - 1 \tag{3.18}$$

Thus the $j^{th}$ outgoing LLR of a given variable node $x_d$ at $p^{th}$ iteration as shown in figure 3.4 is given as

$${L_{v\rightarrow c,j}}^{p}(x_d) = q + \sum_{l=1}^{d_{v-1}} {L_{c\rightarrow v,l}}^{p-1}(x_d) \tag{3.19}$$

The output message computed from (3.19) is transferred to the check node $c$. The $j^{th}$ output message at the check node is given by (3.20) and the equivalent representation is shown in Figure 3.4 [30].

$${L_{c\rightarrow v,l}}^{p}(r_d) = 2tanh^{-1} \prod_{j=1}^{dc-1} \tanh\left({L_{v\rightarrow C,J}}^{p}(r_d)\right) \tag{3.20}$$

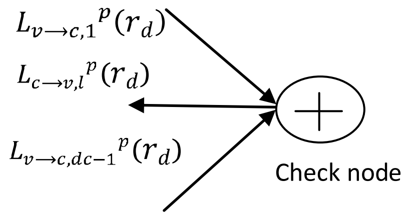


**Figure 3.4: Check node computation**

The computed check node message from (3.20) is now transferred to the variable nodes.

*Compute the final reliability value.*

At the end of the decoding process each variable node computes an output reliability value as follows:

$$L(x_d) = q + \sum_{l=0}^{d_v-1} L_{c\to v,l}^{p}(x_d) \tag{3.21}$$

Hard decision on a bit is obtained as follows

$$\hat{x}(r) = 1 \qquad if\ L(x_d) < 0$$

$$= 0 \qquad if\ L(x_d) < 1 \tag{3.22}$$

**3.5 Density evolution**

Density evolution (DE) is analytical technique which has been used to understand the limits of performance of LDPC decoders [125]. Ideally, for a given code and decoder, we would like to know for which level of channel noise the decoder will be able to correct the errors. Thus it introduces the idea of a channel threshold, above which the code performs well and below which the probability of error is negligible. Consider a generic iterative decoding algorithm, with transmitted vector $\boldsymbol{x}_d$. The receiver observes the vector $\boldsymbol{r}_d$ and generates the first message vector $\boldsymbol{m_0}$. The iterative process begins in which the message vector $m_p$ at $p^{th}$ iteration depends upon the previously obtained message set $m_{p-1}$ and the received vector $\boldsymbol{r_d}$. Assume the process continues up to the maximum $l$ number of iterations and $\hat{a}$ is computed as a function of the last message set and the initial input. The performance of the iteratively decoded codeword (ensemble) can be tracked by the evolution of the conditional joint pdf of all possible transmitted code words. Since in LDPC codes the codeword length is very high, this implies that LDPC codes are required to compute the evolution of the conditional joint pdf of about $10^4$-$10^5$ random variables. Therefore the performance of LDPC codes is computable via the density evolution method.

Thus the density evolution method refers to the evolution of pdf' s of messages being passed around in iterative decoding where the messages are modelled as random variables. Such knowledge of pdfs allows the prediction of condition (e.g. SNR) under which the decoder bit error probability will converge to zero. The derivation of the density evolution requires a number of properties of iterative ensembles. These are as follows. 1) symmetric distribution property, i.e., the LLR output by the iterative decoder is symmetric, if the channel is symmetric; 2) transmission of all zero code-words, i.e., using symmetric distributions, the iterative decoding performance can be shown to be independent of the codeword transmitted; 3) cycle free graphs, i.e., if there are no cycles in the Tanner graphs of length $2l$ or less we can assume that the iterative decoder messages are independent for up to $l$ iterations, which makes density evolution possible.

### 3.5.1 DE on binary erasure channel for irregular LDPC codes

In the message passing algorithm, a check or bit node either receives the bit values 0 or 1 if bits are known or 'e' if bit is erased. Let $q_{0l}$ to be the probability at iteration $l$ that a check-to-bit message is an $e$ and $p_{0l}$ to be the probability that at iteration $l$ that a bit-to-check message is an $e$ (i.e. $p_{0l}$ is the probability that a codeword bit remains erased at iteration $l$) [126]. It is assumed firstly that the channel is memoryless, so that none of the original bit probabilities were correlated, and secondly that there are no cycles in the Tanner graphs of length $2l$ or less, as a cycle will cause the messages to become correlated. With this assumption, the probability that such incoming messages are given by the product of probabilities of $(1 - p_{0l})$ . The probability that one or more incoming messages received as $e$ is given by

$$q_{0l} = 1 - (1 - p_{0l})^{d_c - 1}. \quad (3.23)$$

While passing the messages from check node to bit node, at the $l^{th}$ iteration, the probability that message received from the channel is an erasure 'ε' and all other incoming messages are also erasures with probability $q_{0l-1}$. If all the messages are considered to be independent, then the probability that all the messages received to the bit node are erasures is given by

$$p_{0l} = \varepsilon\,(q_{0l} - 1)^{d_v - 1}$$
$$or\ \ p_{0l} = \varepsilon\,(1 - (1 - p_{0l})^{d_c - 1})^{d_v - 1} \tag{3.24}$$

As in the irregular LDPC codes, the weights of rows and columns are not fixed, i.e., the number of ones in a given parity check matrix is not fixed. Therefore the density evolution of irregular LDPC codes is characterised by degree distribution pair $\lambda(x)$ and $\rho(x)$ already described in section 3.4. To find the average probability that a check to bit node message is in error, let the probability that an edge is connected to degree $d_c$ of check node is $\rho_{d_c}$. Taking the average over all the edges, the average probability is given by

$$q_{0l} = \sum_{\rho} \rho_j \big(1 - (1 - p_{0l})^{j-1}\big)$$
$$= 1 - \textstyle\sum_{\rho} \rho_j (1 - p_{0l})^{j-1} \tag{3.25}$$

Using the function as given below

$$\rho(x) = \textstyle\sum_{j=2}^{d_c} \rho_j x^{j-1} \tag{3.26}$$

$q_{0l}$ can be re-written as

$$q_{0l} = 1 - \rho(1 - p_{0l}) \tag{3.27}$$

Similarly, the probability that the edge is connected to degree $d_v$ of bit node is $\lambda\, d_v$. The probability that a bit to check node message is an erasure 'ε' is given by

$$p_{0l} = \varepsilon \textstyle\sum_i \lambda_i (q_{0l-1})^{i-1} \tag{3.28}$$

Using the function

$$\lambda(x) = \textstyle\sum_{i=2}^{d_v} \lambda_i x^{i-1} \tag{3.29}$$

Finally substituting the value of $q_{0l-1}$ , $p_{0l}$ can be re-written as

$$p_{0l} = \varepsilon\lambda(1 - \rho(1 - p_{0l-1}) \tag{3.30}$$

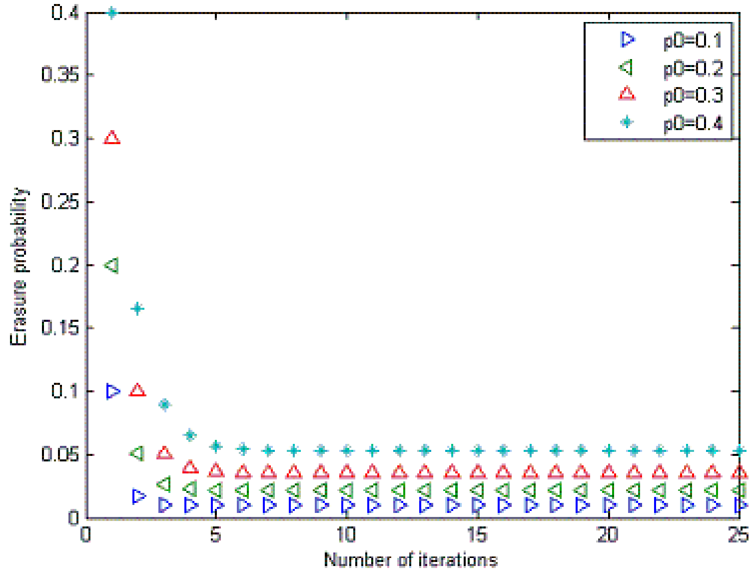


**Figure 3.5: Erasure probability for different values of $\varepsilon$**

We apply the recursion for different values of $\varepsilon$ and Figure (3.5) shows the evolution of erasure probability with number of iterations. It is shown that for the perfect chosen degree distribution, the erasure probability may converge to zero.

### *3.5.2 Density evolution on binary input memory less channel*

For the binary input memoryless channel, the messages passed from bit node to check node or vice versa are represented by taking the log likelihood ratio (LLR) of the probabilities that the received bit is 0 or 1. To find the evolution of the pdf, we define a probability density function associated with the message passing decoder.

Let $p_{0l}$ be the pdf for the variable to check node at the $l^{th}$ iteration, $q_{0l}$ be the pdf for the check to variable node at the $l^{th}$ iteration and $q$ be the initial pdf for the LLR of the received signal. It is assumed that all incoming messages are independent and the Tanner graph is cycle free. Thus, the outgoing LLR of a given variable node $x_d$ is given as defined in (3.21) and re-written as

$$L_{v\rightarrow c,j}{}^{p}(x_d) = \underbrace{q}_{\text{I}} + \underbrace{\sum_{l=1}^{d_{v-1}} L_{c\rightarrow v,l}{}^{p-1}(x_d)}_{\text{II}}$$

Since the incoming messages are independent, the pdf of a random variable formed from the summation given in II can be obtained by convolution [127] of the pdf of the $d_v - 1$ incoming message from the check node and the pdf of messages coming from the channel

$$P_v(x_d) = q \otimes P_c(x_d)^{\otimes\, d_{v-1}}, \tag{3.31}$$

where $P_c(x_d)^{\otimes\, d_{v-1}}$ is the convolution of $d_{v-1}$ copies of $P_c(x_d)$. Averaging over the bit degree distribution $\lambda(x)$

$$P_v(x_d) = q(x) \otimes \sum_i \lambda_i P_c(x_l)^{\otimes\, i-1}\ . \tag{3.32}$$

To calculate the probability of error, integrate the pdf of variable node from $-\infty$ to 0

$$P_e = \int_{-\infty}^{0} P_v(x_d). \tag{3.33}$$

## 3.6 Implementation issues related to LDPC codes

There are mainly three issues related with the implementation of LDPC codes.

- Sparseness of the parity check matrix that is directly related to the complexity of Tanner nodes.
- Complexity related to passing the message through the node referred to as routing complexity.
- The number of iterations required to successfully decode a given codeword.

A Tanner graph successfully represents the parity check matrix in a graphical form. The *node complexity* of a Tanner graph will depend upon the type of algorithm implemented to decode a given message. For example, the simple message passing and bit flipping algorithm discussed in section (3.4.3.4 and 3.4.3.5) requires only XOR operations at bit and check nodes. This implies reduced implementation complexity for the both decoders. For the sum product algorithm, the log likelihood ratios reduce the multiplication evaluation at bit node to sum, but a check node still requires calculation of exponential and logarithmic functions.

*Routing complexity* of a given decoder will depend upon the number of Tanner graph edges and parallel/serial implementation. A Tanner graph will have $N_v d_v$ edges where $N_v$ is number of bit nodes and $d_v$ is their degree. Therefore it requires $n_k N_v d_v$ wires where $n_k$ is the number of bits used to represent each message. With the serial implementation of a message passing decoder, all the individual check and bit nodes compute one at a time so the decoder requires less wires to be routed but additional storage memory is required to store all the check and bit nodes. In parallel implementation, all the bit node processing is done in one step and all check node processing is done at another. Therefore there is less delay in the processing but more wires/routing connections are required at once, thereby increasing the routing complexity. Such a type of implementation is more suitable for the small graphs. To attain a valid codeword that satisfies all parity check equations, sometimes a large *number of iterations* is required. If the noise and fading is less then the decoder will take less iteration to decode the message successfully. The sum product algorithm generally requires up to 1000 iterations to attain the original codeword. The effective implementation of the stopping criterion also reduces the number of iterations required to decode a message [128-130].

### 3.7 Conclusion

In this chapter, a brief introduction to error correcting codes has been presented. In the first part of the chapter, the framework of the coded communication system has been discussed. This framework will serve as base model for our next chapters. In the subsequent section, we have discussed the forward error correcting (FEC) codes used practically and presented the associated computational complexities of these codes.

As LDPC codes perform near to the Shannon limit, a detailed discussion on encoding-decoding algorithms of LDPC codes has been presented. Mainly three decoding algorithms: message passing, bit flipping and sum product algorithms, have been discussed in detail. It is found the sum product algorithm (SPA) based on logarithmic representation is less complex of the three. Density evolution (DE) which gives the limits of LDPC decoders has been discussed for different channel scenarios. Hence in this entire thesis, LDPC codes with SPA have been chosen for decoding of LDPC codes. The idea of implementation complexities related to LDPC codes that have been discussed in the last part of this chapter. Keeping all these aspects, the implementation of LDPC codes with a MIMO system has been considered in this research work.

*Never discard information prematurely that may be useful in making a decision until all decisions related to that information have been completed.*

*-Andrew Viterbi*

# *Chapter IV*

## *PERFORMANCE ANALYSIS OF LDPC CODES IN SIMO SYSTEM*

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*The main motivation in this chapter is performance analysis of low density parity check coded-optimum combined (LDPC-OC) system in the presence of co-channel interferers. For analysis of LDPC-OC system two cases are investigated (i) when the number of interferes are less than or equal to the number of receive antenna array elements (ii) when number of interferers greater than the number of array elements. For both cases the closed form expressions of the bit error rate in terms of upper bounds are derived. The analytical and simulation results are presented and conclusions are drawn in the last section of the chapter.*

In wireless communication, frequency reuse is necessary to increase spectral efficiency, since spectrum is very costly and limitedly available. In such a system user data may be corrupted by the signals from nearby users, resulting in co-channel interference (CCI). Hence co-channel interference is major impediment to high-capacity transmission in power and bandwidth-limited wireless communication systems. Another limiting factor associated with wireless system is channel fading. Different diversity techniques are used to combat fading. The term spatial diversity refers to multi antenna system, whose primary goal is to mitigate multipath fading that arises in scattering environment. It has been proved that maximal ratio combining (MRC) provides the best performance amongst various combining techniques. It maximizes the instantaneous received SNR [2] but without considering the effect of co-channel interference. In other words, MRC treats interference from other users as Gaussian noise. Therefore, MRC is optimal in systems with no interference.

In the presence of interference, a multi-antenna receiver can apply Optimum Combining (OC) technique. This combining technique is a well-known method in space diversity reception that combines and weights the received signals to combat both fading and co-channel interference [5]. This method maximizes the SINR at the output of the combiner and reduces the power of interference by exploiting the correlation of the interference across the multiple receiver antenna elements. The optimum combiner is an ML receiver when the transmitted signals of interfering users as well as the additive noise are Gaussian-distributed [131]. Unlike the Gaussian noise, however, CCI is generally treated as colored noise having non-zero diagonal terms in its covariance matrix [132]. Hence, the power allocation must be optimized with an equivalent channel matrix derived from the CSI of desired and interfering signals.

To implement OC, the receiver needs knowledge of both the desired user's channel as well as the interference covariance matrix that is a function of the interfering users' channels. However, MRC requires only the desired user's channel information at the receiver. Therefore, OC is a more complex diversity combining technique as compared with MRC. OC achieves higher output SINR than maximal ratio combining and is useful even when the number of interferers exceeds the number of antenna array elements. This improved SINR efficiency can manifest itself in the cellular mobile radio application as a reduction in the number of base stations and/or an increased channel capacity through greater frequency reuse [133].

As diversity system provides diversity gain only, therefore to further improve the performances at high data rates, we need coding gain. Diversity gain can be increased to certain extent as resources are limited. Therefore many researchers have employed different channel coding techniques with different diversity techniques. In chapter 2, literature survey [58-67] based on these techniques has been already been presented. LDPC codes are emerged as powerful iterative codes with low decoding complexity and have effective performance in various fading channels. In this chapter we investigate the performance of low density parity check codes with optimum combined system (LDPC-OC) in the presence of co-channel interference. The system model used for analysis of LDPC-OC system is discussed next.

### 4.1 System Model of LPDC coded-optimum combined system

We consider the LDPC-OC system shown in figure 4.1 in which $M_R$ receive antenna array elements have been used to improve the system performance in the presence of co-channel interference. The system employs BPSK modulation and the channel is characterized by the flat Rayleigh fading.

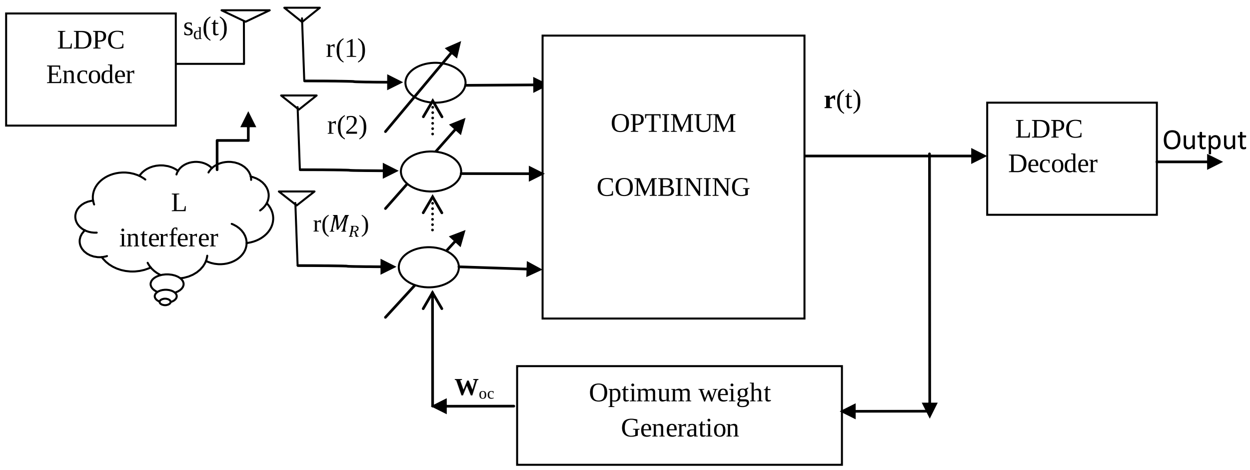


**Figure 4.1: System Model of LPDC coded-optimum combined system**

At the transmitter side, LDPC coded bits are transmitted over channel. At the receiver side, $M_R$ receive antenna array elements operates in the presence of $L$ co-channel interferers. Further, it is assumed that all the interferers have equal power. The received signal vector at the antenna element consists of desired signal, interference signal and noise, and is given as

$$\boldsymbol{r}(t) = \sqrt{P_s}\, s_d(t)\boldsymbol{c_d} + \sum_{k=1}^{L} \sqrt{P_k}\, s_k(t)\boldsymbol{c_k} + \boldsymbol{n}, \tag{4.1}$$

where $s_d(t)$ and $s_k(t)$ are the desired and $k^{th}$ interfering signals respectively. $\boldsymbol{c_d}$ and $\boldsymbol{c_k}$ are channel gains for the desired and $k^{th}$ interfering signals respectively and are assumed to be mutually independent $N$-dimensional complex Gaussian vector having mean power $P_s$ and $P_k$ respectively. **n**(t) is AWGN vector, each element of which has zero mean and variance $\sigma_n^2$, where $\sigma_n^2 = \frac{1}{2R*E_b/N_0}$. R is code rate, $E_b$ is the bit energy of desired signal, $N_0$ is the noise power spectral density.

Let,

$\bar{\gamma}_s = \frac{P_s}{\sigma_n^2}$ is average SNR of desired signal per antenna.

$\bar{\gamma}_k = \frac{P_k}{\sigma_n^2}$ is the average SNR of interference per antenna of $k^{th}$ interfering user.

$\gamma_s = \frac{\boldsymbol{c}_d\, \boldsymbol{c}_d{}^H}{\sigma_n^2}$ is the SNR of desired signal at the output of combiner.

The pdf of the $\gamma_s$ is given by [54]

$$p_{\gamma_s}(\gamma_s) = \frac{1}{\bar{\gamma}_s{}^{M_R}\Gamma(M_R)} \bar{\gamma}_s{}^{M_R-1} exp\left(\frac{\gamma_s}{\bar{\gamma}_s}\right), \tag{4.2}$$

for $\gamma_s \geq 0$ and $\Gamma(.)$ is the Gamma function.

$\gamma_k = \frac{\boldsymbol{c}_k \boldsymbol{c}_k{}^H}{\sigma_n^2}$ is the SNR of $k^{th}$ interference at the output of combiner ,

The pdf of the $\gamma_k$ is given by

$$p_{\gamma_k}(\gamma_k) = \frac{1}{\bar{\gamma}_k{}^{M_R}\Gamma(M)} \bar{\gamma}_k{}^{M_R-1} exp\left(\frac{\gamma_k}{\bar{\gamma}_k}\right), \tag{4.3}$$

for $\gamma_k \geq 0, \quad k = 1,2, \ldots\ldots L.$

Denoting the $\boldsymbol{w}$ as weight vector, the output of the combiner can be written as

$$z = \boldsymbol{w}^H \boldsymbol{r}(t) = \sqrt{P_s}\, \boldsymbol{w}^H\, \boldsymbol{s}_d(t)\boldsymbol{c_d} + \sum_{k=1}^{L} \sqrt{P_k}\, \boldsymbol{w}^H \boldsymbol{s_k}(t)\boldsymbol{c_k} + \boldsymbol{w^H n}, \tag{4.4}$$

$(.)^{\boldsymbol{H}}$ denotes the Hermitian transpose.

The SINR $\gamma_{OC}$ at the output of combiner using (4.4) is given as

$$\gamma_{OC} = \frac{P_s \boldsymbol{s_d(t)}\, \boldsymbol{w_{oc}}^H \boldsymbol{c_d}^H \boldsymbol{c_d}\, \boldsymbol{w_{OC}}}{P_k \boldsymbol{s_k(t)} \boldsymbol{w_{oc}}^H \sum_{k=1}^{L} \boldsymbol{c_k}^H \boldsymbol{c_k} \boldsymbol{w_{OC}} + \boldsymbol{w_{OC}}^{\boldsymbol{H}}} \quad . \tag{4.5}$$

In (4.5) $\boldsymbol{w_{OC}}$ is the optimal weight vector calculated using minimum mean square error technique, given by [5]

$$\boldsymbol{w}_{OC} = \boldsymbol{R_{ni}^{-1} c_d}\,, \tag{4.6}$$

where $\boldsymbol{R_{ni}}$ is noise plus interference covariance matrix given by

$$\boldsymbol{R_{ni}} = E\left\{\left[\sum_{k=1}^{L} \sqrt{P_k}\boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)\right]\left[\sum_{k=1}^{L} \sqrt{P_k}\boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)\right]^H\right\}. \tag{4.7}$$

The SINR $\gamma_{OC}$ can equivalently written as [2]

$$\gamma_{oc} = P_s \boldsymbol{c_d}^H\, \boldsymbol{R_{ni}}^{-1} \boldsymbol{c_d}, \tag{4.8}$$

The channel gain $\boldsymbol{c_d}$ and entries of covariance matrix $\boldsymbol{R_{ni}}$ are random variables; the SINR $\gamma_{oc}$ is also a random variable.

In the given analysis, all interferers are assumed to be of equal power, therefore the covariance matrix $\boldsymbol{R_{ni}}$ can also written as [2]

$$\boldsymbol{R_{ni}} = \sum_{k=1}^{L} \boldsymbol{c}_k\, \boldsymbol{c}_k{}^H + \sigma_n^2 \boldsymbol{I}, \tag{4.9}$$

where $\boldsymbol{I}$ is the identity matrix of dimension $LxL$. $\boldsymbol{R_{ni}}$ can be diagonalized by a unitary transformation [53]

$$\boldsymbol{R_{ni}} = \boldsymbol{V^H} diag\{\lambda_1, \lambda_2 \ldots\ldots \lambda_{M_R}\}\boldsymbol{V}, \tag{4.10}$$

where $\lambda_1, \lambda_2 \ldots \ldots . \lambda_N$ are the eigen-values of $\boldsymbol{R_{ni}}$ and the vector $\boldsymbol{v} = \boldsymbol{V}\boldsymbol{c_d} = [\boldsymbol{v_{d1}}, \boldsymbol{v_{d1}}, \ldots \boldsymbol{v_{dN}}]^{\boldsymbol{T}}$ retains the properties of $\boldsymbol{c_d}$**.** It should be pointed out that, it is usually very difficult to evaluate the eigen-values and their corresponding pdf. For special case L=1 of single interferer, eigen-values are given by [54]

$$\lambda_1 = s_d + \sigma_n^2, \qquad \lambda_i = \sigma_n^2, \quad i = 2,3, \ldots \ldots L$$

### 4.2. Upper bounds of LDPC-Optimum Combining Receivers

In this section, we evaluate the performance of LDPC coded– optimum combined system in terms of bit error rate for two scenarios

1. *In the first case* we have considered the LDPC-OC system when the number of interferes are less than or equal to the number of receive antenna array elements i.e. $L \leq M_R$, in independent identically distributed Rayleigh fading.
2. *The Second case p*rovides the analysis of LDPC-OC system when number of interferers are greater than the number of array elements $L > M_R$, in independent identically distributed Rayleigh fading.

   The power of all interferers is c*onsidered to be* equal in both cases.

Optimum Combining (OC) with Low Density Parity-check Codes (LDPC) as an outer code improves the performance of overall system (LDPC-OC) by providing additional coding gain. The followings steps are used to derive the upper bounds of the considered system.

**Step 1**: The pdf of optimum combiner output SINR conditioned on interference power is found using moment generating function (MGF) approach.

**Step 2**: The unconditional pdf $p(\gamma_{oc})$ of SINR of optimum combining scheme is derived using the conditional pdf of step1.

**Step 3**: Transform the pdf $p(\gamma_{oc})$ of SINR of optimum combining into channel gain $c_d$. (To make the channel LLR unconditional as the channel LLR is always conditioned on channel gain).

**Step 4**: Find the unconditional pdf of channel LLR.

**Step 5**: Find the pdf of variable node by convolving the pdf of channel LLR (calculated in step 4) with the pdf of check node

**Step 6**: Find the probability of bit error $P_e$ of overall LDPC-OC system

### 4.2.1. When the number of interferers are less than or equal to the number of array elements (CASE I)

***Step 1: Evaluate the conditional pdf of optimum combiner output SINR using moment generating function (MGF)***

As the number of interferers is less than the number of array elements and we are considering the special case for the single interferer. The pdf of SNR of interferer at the output of combiner (eq. 4.3, putting k=1) can be written as

$$p_{\gamma_1}(\gamma_1) = \frac{1}{\bar{\gamma}_1{}^{M_R}\Gamma(M_R)}\bar{\gamma}_1{}^{M_R-1} exp\left(\frac{\gamma_1}{\bar{\gamma}_1}\right) \quad (4.11)$$

The moment generating function (MGF) of $\gamma_{oc}$ conditioned on $\gamma_1$ is given as [54]

$$\psi_{\gamma_{OC}}(s|\gamma_1) = \left(\frac{\frac{\gamma_1+1}{\bar{\gamma}_s}}{\frac{\gamma_1+1}{\bar{\gamma}_s}-s}\right)\left(\frac{\frac{1}{\bar{\gamma}_s}}{\frac{1}{\bar{\gamma}_s}-s}\right)^{M_R-1} \quad (4.12)$$

MGF can be associated with the pdf using the following equation

$$p(\gamma_{oc}|\gamma_1) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \psi_{\gamma_{OC}}(t|\gamma_1)e^{-t\gamma_{oc}}dt \quad (4.13)$$

Putting the value of (4.12) into (4.13), the pdf conditioned on $\gamma_1$ is evaluated as

$$p(\gamma_{oc}|\gamma_1) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \left(\frac{\frac{\gamma_1+1}{\bar{\gamma}_s}}{\frac{\gamma_1+1}{\bar{\gamma}_s}-t}\right)\left(\frac{\frac{1}{\bar{\gamma}_s}}{\frac{1}{\bar{\gamma}_s}-t}\right)^{M_R-1} e^{-t\gamma_{oc}}dt$$

$$= \frac{1}{2\pi i}\left(\frac{\gamma_1+1}{\bar{\gamma}_s}\right)\left(\frac{1}{\bar{\gamma}_s}\right)^{M_R-1}\int_{-\infty}^{\infty}\left(\frac{1}{\frac{\gamma_1+1}{\bar{\gamma}_s}-t}\right)\left(\frac{1}{\frac{1}{\bar{\gamma}_s}-t}\right)^{M_R-1} e^{-t\gamma_{oc}}dt \quad (4.14)$$

Putting $t = ix \; and \; dt = i\,dx$ in (4.14), gives

$$= \frac{1}{2\pi i}\left(\frac{\gamma_1+1}{\bar{\gamma}_s}\right)\left(\frac{1}{\bar{\gamma}_s}\right)^{M_R-1}\int_{-\infty}^{\infty}\left(\frac{1}{\frac{\gamma_1+1}{\bar{\gamma}_s}-ix}\right)\left(\frac{1}{\frac{1}{\bar{\gamma}_s}-ix}\right)^{M_R-1} e^{-ix\gamma_{oc}} \; i\,dx \quad (4.15)$$

Then solving the above equation using the integral [134 equation no. 3.384 (7)] given as

$$\int_{-\infty}^{\infty}(\beta-ix)^{-u}\;(\gamma-ix)^{-v}\;e^{-ipx}dx = \frac{2\pi}{\Gamma(u+v)}\;e^{-\beta p}\;p^{u+v-1} *_1 F_1(u;u+v;(\beta-\gamma)p) \quad (4.16)$$

We get the pdf of output of combiner conditioned on single interferer SNR as

$$p(\gamma_{oc}|\gamma_1) = \frac{(1+\gamma_1)\gamma_{oc}{}^{(M_R-1)}}{\bar{\gamma}_s{}^{M_R}\,\Gamma(M_R-1)} e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}\,{}_1F_1\left[1; M_R-1; -\frac{\gamma_1\,\gamma_{oc}}{\bar{\gamma}_s}\right], \tag{4.17}$$

where ${}_1F_1[.]$ represents the hypergeometric function.

***Step 2: To find the unconditional pdf of SINR at the output of OC***

The unconditional pdf of SINR at the output of optimum combiner is found by integrating conditional pdf over the pdf of $\gamma_1$ can be evaluated as

$$p(\gamma_{oc}) = \int_0^\infty \mathrm{p}(\gamma_{oc}|\gamma_1)\ \mathrm{p}(\gamma_1)d\gamma_1 \tag{4.18}$$

Substituting the values from (4.11) and (4.17) into (4.18)

$$p(\gamma_{oc}) = \int_0^\infty \frac{(1+\gamma_1)\gamma_{oc}{}^{(M_R-1)}}{\bar{\gamma}_s{}^{M_R}\,\Gamma(M_R)} e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}\,{}_1F_1\left[1; M_R - 1; -\frac{\gamma_1\,\gamma_{oc}}{\bar{\gamma}_s}\right]\ \frac{1}{\bar{\gamma}_1{}^{M_R}\Gamma(M_R)}\bar{\gamma}_1{}^{M_R-1} exp\left(\frac{\gamma_1}{\bar{\gamma}_1}\right)d\gamma_1$$

$$= \frac{\gamma_{oc}{}^{(M_R-1)}e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}}{\bar{\gamma}_s{}^{M_R}\,\bar{\gamma}_1{}^{M_R}\Gamma(M_R)\Gamma(M_R)}\int_0^\infty (1+\gamma_1)\bar{\gamma}_1{}^{M_R-1}\,{}_1F_1\left[1; M_R-1; -\frac{\gamma_1\,\gamma_{oc}}{\bar{\gamma}_s}\right]\ \ exp\left(\frac{\gamma_1}{\bar{\gamma}_1}\right)d\gamma_1 \tag{4.19}$$

Putting $\frac{\gamma_1}{\bar{\gamma}_1} = y\ or\ \gamma_1 = y\,\bar{\gamma}_1, \Gamma(M_R) = (M_R-1)!$ into (4.19) and solving, gives

$$= \frac{\gamma_{oc}{}^{(M_R-1)}e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}}{\bar{\gamma}_s{}^{M_R}\,\bar{\gamma}_1{}^{M_R}(M_R-1)!(M_R-1)!}\int_0^\infty (1+\bar{\gamma}_1 y)\bar{\gamma}_1{}^{M_R-1}\,{}_1F_1\left(1\colon M_R; -\frac{\bar{\gamma}_1\gamma_{oc}y}{\bar{\gamma}_s}\right)e^{-y}\,dy \tag{4.20}$$

Using the identity [53]

$$K_{M_R-1}(\bar{\gamma}_s,\gamma_{oc}) = \frac{1}{(M_R-1)!\bar{\gamma}_s{}^{M_R}} e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}\gamma_{oc}{}^{(M_R-1)} \tag{4.21}$$

$$\frac{1}{(M_R-1)!}\,{}_1F_1(1\colon M_R; z) = \frac{1}{z^{M_R-1}}\left[e^z - \sum_{n=0}^{M_R-2}\frac{z^n}{n!}\right] \tag{4.22}$$

Putting the values of (4.21) and (4.22) in (4.20) and integrating the above equation leads to the exact unconditional density function

$$p(\gamma_{oc}) = \frac{K_{M_R-1}(\overline{\gamma}_s, \gamma_{oc})}{\left(1+\frac{\overline{\gamma}_1}{\overline{\gamma}_s}\gamma_{oc}\right)} \left[1 + M_R \bar{\gamma}_1 \frac{1+\frac{(M_R-1)}{M_R}\frac{\overline{\gamma}_1}{\overline{\gamma}_s}\gamma_{oc}}{1+\frac{\overline{\gamma}_1}{\overline{\gamma}_s}\gamma_{oc}}\right] \tag{4.23}$$

For $M_R$>>1 then above equation becomes

$$p(\gamma_{oc}) = \frac{K_{M_R-1}(\overline{\gamma}_s, \gamma_{oc})}{\left(1+\frac{\overline{\gamma}_1}{\overline{\gamma}_s}\gamma_{oc}\right)} (1 + M_R \bar{\gamma}_1) \tag{4.24}$$

***Step 3: Transformation of pdf of SINR of optimum combining in terms of channel gain***

The conditional pdf of uncorrelated Rayleigh fading channel is always conditioned on channel gain. The SINR of optimum combining $\gamma_{oc}$ can be written in terms channel gain $c_d$ using relationship

$$\gamma_{\mathrm{oc}} = \frac{\overline{\gamma_s}\, {c_d}^2}{\sigma_n^2} \tag{4.25}$$

Transforming the pdf of $\gamma_{oc}$ into channel gain using the transformation of RV's , we get pdf of $c_d$ as

$$p(c_d) = \frac{2c_d * \overline{\gamma}_s}{\sigma_n^2} * \frac{1}{(M_R-1)!\, {\overline{\gamma}_s}^{M_R}} \frac{\left(\frac{\overline{\gamma}_s {c_d}^2}{\sigma_n^2}\right)^{M_R-1} e^{\frac{\overline{\gamma}_s {c_d}^2}{\sigma_n^2 \overline{\gamma}_s}}}{\left(1+\overline{\gamma}_1 \frac{{c_d}^2}{\sigma_n^2}\right)} (1 + M_R \overline{\gamma}_1), \tag{4.26}$$

which further simplifies to

$$p(c_d) = \frac{(1+M_R\overline{\gamma}_1)\, 2{c_d}^{2M_R-1} \exp\left(-\frac{{c_d}^2}{\sigma_n^2}\right)}{(M_R-1)!\, \sigma_n^{2M_R}\left(1+\frac{\overline{\gamma}_1}{\sigma_n^2}{c_d}^2\right)} \tag{4.27}$$

***Step 4: Finding the pdf of channel LLR***

The optimum combined signal $\mathbf{r}(t)$ at the output of combiner is fed to the LDPC decoder. The decoder, decodes the received signal by message passing algorithm. The general idea of the message-passing decoding algorithm is to pass messages in a cycle, where each cycle has two phases. In the first phase, messages are passed from the variable node $v$ of degree $j$ to the check node $c$ of degree $i$ of the bipartite graph. In the second phase, messages are passed back from the check node $c$ to the variable node $v$. The LLR of variable node that passes the message to check node on edge $e$ is summation of LLR of check node message received on all incoming edges except $e$ and initial channel LLR. This is also equivalent to the convolution of their pdfs.

To find the initial channel LLR, consider LDPC coded symbol $s_d$ mapped into the signal point $z$ = (1-2$s_d$). For the uncorrelated Rayleigh fading channel with normalized fading factor $c_d$, the conditional pdf [81] of desired signal available at the input of LDPC decoder is

$$p(s_d/z\,, c_d) = \frac{1}{\sqrt{2\pi\sigma_n^2}}\; exp\left(-\frac{(s_d - z.c_d)^2}{2\sigma_n^2}\right) \tag{4.28}$$

Further it is assumed that full channel state information (CSI) is available at the receiver side, the block length of LDPC codes is infinite and all zero code-words are sent i.e $z$=1. The initial channel message $q$ passed from the bit to check node is as explained in chapter 3 (eq 3.17) is re-produced as

$$q = \frac{2}{\sigma_n^2} s_d c_d$$

Using the transformation of random variable approach, the density function yields in terms of $q$

$$\mathrm{p}(q|c_d) = \frac{\sigma_\mathrm{n}}{2c_d\sqrt{2\pi}}\exp\left(-\frac{\left(\mathrm{q}-2{c_d}^2/\sigma_\mathrm{n}^2\right)^2}{8{c_d}^2/\sigma_\mathrm{n}^2}\right) \tag{4.29}$$

with mean $2{c_d}^2/\sigma_\mathrm{n}^2$, variance $4{c_d}^2/\sigma_\mathrm{n}^2$ , The unconditional pdf of channel LLR is calculated by averaging the equation (4.29) over the pdf of channel gain found in step 3

$$\mathrm{p_0(q)} = \int_0^\infty \mathrm{p}(\mathrm{q}|c_d,)\,\mathrm{p}(c_d)\mathrm{d}c_d \tag{4.30}$$

Solving for $\mathrm{p_0(q)}$, we have

$$\mathrm{p_0(q)} = \int_0^\infty \frac{\sigma_\mathrm{n}}{2c_d\sqrt{2\pi}}\exp\left(-\frac{(\mathrm{q}-2{c_d}^2/\sigma_\mathrm{n}^2)^2}{8\,{c_d}^2/\sigma_\mathrm{n}^2}\right)\frac{(1+M_R\bar{\gamma}_1)\;2{c_d}^{2M_R-1}\exp\left(-\frac{{c_d}^2}{\sigma_n^2}\right)}{(M_R-1)!\,{\sigma_n}^{2M_R}\left(1+\frac{\bar{\gamma}_1}{\sigma_n^2}{c_d}^2\right)}\mathrm{d}c_d$$

$$= \frac{(1+M_R\bar{\gamma}_1)}{\sqrt{2\pi}(M_R-1)!{\sigma_n}^{2M_R-1}}\int_0^\infty \frac{1}{c_d}\,exp\left(-\frac{(q-2{c_d}^2/\sigma_n^2)^2}{8{c_d}^2/\sigma_n^2}\right)\frac{{c_d}^{2M_R-1}\,exp\left(-\frac{{c_d}^2}{\sigma_n^2}\right)}{\left(1+\frac{\bar{\gamma}_1}{\sigma_n^2}{c_d}^2\right)}dc_d \tag{4.31}$$

Expanding the squared term of exponential in (4.31), we get

$$= \frac{(1+M_R\overline{\gamma}_1)}{\sqrt{2\pi}(M_R-1)!\sigma_n^{2M_R-1}} \int_0^\infty exp\left(-\frac{q^2\sigma_n^2}{8.c_d{}^2} - \frac{c_d{}^2}{2\sigma_n^2} + \frac{q}{2}\right) \frac{c_d{}^{2M_R-1}\, exp\left(-\frac{c_d{}^2}{\sigma_n^2}\right)}{\left(1+\frac{\overline{\gamma}_1}{\sigma_n^2}c_d{}^2\right)} dc_d \qquad (4.32)$$

Substituting $3/2\sigma_n{}^2 = a$ and $\frac{q^2\sigma_n{}^2}{8} = b$ and solving (4.32), we get unconditional pdf of channel LLR as given below

$$p_o\,(q) = \frac{(1 + M_R\overline{\gamma_1})exp\;(q/2)}{\sqrt{2\pi}(M_R - 1)!\,\sigma_n{}^{2M_R-1}}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{\;M_R-1}} - \frac{\overline{\gamma_1}}{\sigma_n{}^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{\;M_R}}\right]\left[\frac{1}{2}\sqrt{\frac{\pi}{a}}exp\left(-\sigma_n\sqrt{\frac{a}{2}}|q| + \frac{q}{2}\right)\right]$$

(4.33)

***Step 5: To find the pdf of variable node***

The pdf of variable node $p_v(q)$ is found by convolving the pdf of channel LLR $p_0(q)$ with the pdf of check node $p_c(q)$

$$p_v(q) = p_0(q) \otimes p_c(q) \qquad (4.34)$$

$$p_v(q) = \int_{-\infty}^{\infty} p_c(q-\tau)\, p_0(\tau)d\tau \qquad (4.35)$$

The pdf of the check node is given by [124]

$$p_c(q) = \frac{1}{\sqrt{4\pi m_c}}\exp\left(-\frac{(q-m_c)^2}{4m_c}\right) \qquad (4.36)$$

where $m_c$ is the mean of the check node $c$ of degree $i$.

It is very difficult to update these pdf's at all $l$ iterations. To simplify we assume that all messages involved in the decoding process have a symmetric Gaussian distribution in which only the mean value needs to be updated iteratively At the ($l$ +1)th iteration, the mean of the check node $c$ is updated by as [124]

$$m_c^{l+1} = 4\left[\mathrm{erfc}^{-1}\left(\sum_{i=2}^{d_c}\rho_i\left(1-\left(1-2P_b^l\right)\right)^{i-1}\right)\right]^2, \qquad (4.37)$$

where erfc (.) is complementary error function. $d_c$ is the number of check nodes for given irregular LDPC codes, $\rho_i$ is the fraction of edge emanating from check node of degree $i$ and $P_b^l$ is the bit error probability at $l^{th}$ iteration. The pdf of the variable node is calculated by convolving the pdf of channel LLR (4.33) with the pdf of check node (4.36) and using the integral in [134, equation 3.322(2)], Solving we get,

$$p_v(q) = \frac{(1+M_R\overline{\gamma_1})}{4\sqrt{2a}(M_R-1)!\,{\sigma_n}^{2M_R-1}}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{\,M_R-1}} - \frac{\overline{\gamma_1}}{{\sigma_n}^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{\,M_R}}\right]$$

$$exp\left[\frac{{\sigma_n}^2 a m_c}{2} - \frac{m_c}{4}\right]\left[exp\left[\left(\sigma_n\sqrt{\frac{a}{2}}+\frac{1}{2}\right)q\right]\left[1-\emptyset\left(\frac{q}{2\sqrt{m_c}}+\sigma_n\sqrt{\frac{am_c}{2}}\right)\right]+\right.$$

$$\left. exp\left[\left(-\sigma_n\sqrt{\frac{a}{2}}+\frac{1}{2}\right)q\right]\left[1-\emptyset\left(-\frac{q}{2\sqrt{m_c}}+\sigma_n\sqrt{\frac{am_c}{2}}\right)\right]\right] \quad (4.38)$$

***Step 6: To find the probability of bit error of overall LDPC-OC system***

To obtain the probability of bit error $P_e$ of overall LDPC-OC system, we integrate the pdf of variable node given by (4.38) from -∞ to 0,

$$P_e = \int_{-\infty}^{0} p_v(q).dq \quad (4.39)$$

$$P_e = \int_{-\infty}^{0}\frac{(1+M_R\overline{\gamma}_1)}{4\sqrt{2}(M_R-1)!\sigma_n^{2M_R-1}}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial b^{M_R-1}} - \frac{\overline{\gamma}_1}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial b^{M_R}}\right]\left[\sqrt{\frac{1}{b}}exp\left(\frac{\sigma_n^2 b m_c}{2}-\right.\right.$$

$$\left.\left.\frac{m_c}{4}\right)\right]\left\{\left(exp(\sigma_n\sqrt{\frac{b}{2}}+\frac{1}{2})q\right)\left[1-\Phi\left(\frac{q}{2m_c}+\sigma_n\sqrt{\frac{bm_c}{2}}\right)\right]+\left(exp(-\sigma_n\sqrt{\frac{b}{2}}+\frac{1}{2})q\right)\left[1-\right.\right.$$

$$\left.\left.\Phi\left(-\frac{q}{2m_c}+\sigma_n\sqrt{\frac{bm_c}{2}}\right)\right]\right\}dq \quad (4.40)$$

After solving (4.40), we get

$$P_e = \frac{(1+M_R\bar{\gamma}_1)}{4\sqrt{2}(M_R-1)!\,\sigma_n^{2M_R-1}}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{M_R-1}} - \frac{\bar{\gamma}_1}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{M_R}}\right]$$

$$\left[\frac{4}{\sqrt{a}-2\sigma_n^2 a^{\frac{3}{2}}}erfc\left(\sigma_n\sqrt{\frac{am_c}{2}}\right)\exp\left(\frac{\sigma_n^2 a m_c}{2}-\frac{m_c}{4}\right)+\frac{8\sigma_n}{\sqrt{2}(2\sigma_n^2 a-1)}erfc\left(\sqrt{\frac{m_c}{2}}\right)\right] \quad (4.41)$$

Using the similar arguments for updation of mean of check nodes, at ($l$ +1) th iteration, averaging (4.41) over all the bit node degrees $j$, we get

$$P_e =$$

$$\frac{(1+M_R\overline{\gamma}_1)}{4\sqrt{2}(M_R-1)!\sigma_n^{2M_R-1}}\sum_{j=2}^{d_v}\lambda_k\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{M_R-1}}-\right.$$

$$\left.\frac{\overline{\gamma}_1}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{M_R}}\right]\left[\frac{4}{\sqrt{a}-2\sigma_n^2a^{\frac{3}{2}}}erfc\left(\sigma_n\sqrt{\frac{am_c^{l+1}(j-1)}{2}}\right)\exp\left(\frac{\sigma_n^2a(j-1)m_c^{l+1}}{2}-\frac{(j-1)m_c^{l+1}}{4}\right)+\right.$$

$$\left.\frac{8\sigma_n}{\sqrt{2}(2\sigma_n^2a-1)}erfc\left(\sqrt{\frac{(j-1)m_c^{l+1}}{2}}\right)\right] \quad (4.42),$$

Because at ($l$+1) th iteration mean of variable node $v$ of degree $j$ is $m_v^{l+1}=(j-1)m_c^{l+1}$.

### 4.2.2. When the number of interferers are greater than the number of array elements (CASE II)

In the second case, when the number of interferers are greater than the number of array elements, the unconditional pdf of SINR of optimum combining scheme is available in literature [4] and therefore we directly move to step 3. The unconditional pdf in overloaded case is given by

$$\mathrm{p}_{\gamma_{oc}}(\gamma_{oc})=\frac{\Gamma(\mathrm{L}+1)}{\Gamma(M_R)\Gamma(\mathrm{L}+1-M_R)}\left(\frac{\mathrm{P_s}}{\mathrm{P_k}}\right)^{\mathrm{L}+1-M_R}\frac{{\gamma_{oc}}^{M_R-1}}{\left(\frac{\mathrm{P_s}}{\mathrm{P_k}}+\gamma_{oc}\right)^{\mathrm{L}+1}},$$

***Step 3: Transformation of pdf of SINR of optimum combining in terms of channel gain***

Considering optimum diversity combining, the density function of maximum SIR $\gamma_{oc}$ at the array output of uncoded system is given by [5]

$$\mathrm{p}_{\gamma_{oc}}(\gamma_{oc})=\frac{\Gamma(\mathrm{L}+1)}{\Gamma(M_R)\Gamma(\mathrm{L}+1-M_R)}\left(\frac{\mathrm{P_s}}{\mathrm{P_k}}\right)^{\mathrm{L}+1-M_R}\frac{{\gamma_{oc}}^{M_R-1}}{\left(\frac{\mathrm{P_s}}{\mathrm{P_k}}+\gamma_{oc}\right)^{\mathrm{L}+1}}, \quad (4.43)$$

where $\gamma_{oc}\geq 0$ , $1\leq M_R\leq$ L. Since the system is considered as interference limited, therefore it is also assumed that degree of freedom of system is insufficient to suppress all the interferers. Transforming the pdf of $\gamma_{oc}$ which is in equation (4.43) into channel gain $c_d$ using the similar approach used in (4.26), we get

$$p(c_d) = \frac{2c_d\,\Gamma(L+1)p_k^{M_R}}{\sigma_n^{2M_R}\Gamma(M_R)\Gamma(L+1-M_R)} \frac{{c_d}^{2(M_R-1)}}{\left(1+\frac{P_k}{\sigma_n^2}{c_d}^2\right)^{L+1}} \tag{4.44}$$

***Step 4: Find the pdf of channel LLR***

Using the similar approach of under loaded case explained in step 4, conditional probability density function of initial channel message conditioned on channel gain is given by (4.29). The unconditional pdf of channel LLR is calculated by averaging the equation (4.29) over the channel gain (4.44) using (4.30), we get

$$p_0(q) = \int_0^{\infty} \frac{\sigma_n}{2c_d\sqrt{2\pi}} \exp\left(-\frac{\left(q-2{c_d}^2/\sigma_n^2\right)^2}{8{c_d}^2/\sigma_n^2}\right) \frac{2c_d\,\Gamma(L+1)p_k^{M_R}}{\sigma_n^{2M_R}\Gamma(M_R)\Gamma(L+1-M_R)} \frac{{c_d}^{2(M_R-1)}}{\left(1+\frac{P_k}{\sigma_n^2}{c_d}^2\right)^{L+1}}\, dc_d \tag{4.45}$$

Solving (4.45), we get unconditional pdf of channel LLR as given below

$$p_0(q) = \frac{p_k^{M_R}\Gamma(L+1)}{\sigma_n^{2M_R-1} 2\sqrt{2}\,\Gamma(M_R)\Gamma(L+1-M_R)}$$
$$\left\{(-1)^{M_R-1} \frac{\partial^{M_R-1}}{\partial a^{M_R-1}}\left[\frac{1}{\sqrt{a}}\exp\left(-\sqrt{\frac{a}{2}}\sigma_n\,|q| + \frac{q}{2}\right)\right] - \right.$$
$$\left.\frac{P_k L}{\sigma_n^2}(-1)^{M_R} \frac{\partial^{M_R}}{\partial a^{M_R}}\left[\frac{1}{\sqrt{a}}\exp\left(-\sqrt{\frac{a}{2}}\sigma_n\,|q| + \frac{q}{2}\right)\right]\right\} \tag{4.46}$$

***Step 5: To find the pdf of variable node***

The pdf of the variable node is again calculated by convolving the pdf of check node (4.36) with the pdf of channel LLR (4.46), solving we get,

$$p_v(q) = \frac{p_k^{M_R}\Gamma(L+1)}{4\sqrt{2}\,\sigma_n^{2M_R-1}\,\Gamma(M_R)\Gamma(L+1-M_R)}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{M_R-1}} - \frac{P_k L}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{M_R}}\right]$$
$$\frac{1}{\sqrt{a}} exp\left[\frac{a\sigma_n^2 m_c}{2} - \frac{m_c}{4}\right]\left[exp\left[\left(\sigma_n\sqrt{\frac{a}{2}} + \frac{1}{2}\right)q\right]\left[1 - \emptyset\left(\frac{q}{2\sqrt{m_c}} + \sigma_n\sqrt{\frac{a m_c}{2}}\right)\right] + exp\left[\left(-\sigma_n\sqrt{\frac{a}{2}} + \right.\right.\right.$$
$$\left.\left.\left.\frac{1}{2}\right)q\right]\left[1 - \emptyset\left(-\frac{q}{2\sqrt{m_c}} + \sigma_n\sqrt{\frac{a m_c}{2}}\right)\right]\right] \tag{4.47}$$

***Step 6: To find the probability of bit error of overall LDPC-OC system***

To obtain the probability of bit error $P_e$ of overall LDPC-OC system, integrate the pdf of variable node given by (4.49) from -∞ to 0

$$P_e = \int_{-\infty}^{0} \frac{p_k^{M_R}\Gamma(L+1)}{4\sqrt{2}\,\sigma_n^{2M_R-1}\Gamma(M_R)\Gamma(L+1-M_R)}\left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial b^{M_R-1}} - \frac{P_k L}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial b^{M_R}}\right]\left\{\left[\frac{1}{\sqrt{b}}exp\left(\frac{b\sigma_n^2 m_c}{2}-\frac{m_c}{4}\right)\right]\left[exp\left(\sqrt{\frac{b}{2}}\sigma_n\, q+\frac{q}{2}\right)erfc\left(\frac{q}{2\sqrt{m_c}} + \sqrt{\frac{bm_c}{2}}\sigma_n\right)\right]\right\}dq \quad (4.48)$$

After solving (4.48), we get

$$P_e = \frac{p_k^{M_R}\Gamma(L+1)}{4\sqrt{2}\,\sigma_n^{2M_R-1}\Gamma(M_R)\Gamma(L+1-M_R)} \left[(-1)^{M_R-1}\frac{\partial^{M_R-1}}{\partial a^{M_R-1}} - \frac{P_k L}{\sigma_n^2}(-1)^{M_R}\frac{\partial^{M_R}}{\partial a^{M_R}}\right]\left[\frac{1}{\sqrt{a}}\exp\left(\frac{a\sigma_n^2 m_c}{2}-\frac{m_c}{4}\right)\right] \left[\frac{1}{\sigma_n\sqrt{\frac{a}{2}}+\frac{1}{2}} - \frac{2}{1+\sigma_n\sqrt{2a}}\left[\operatorname{erfc}\left(\sigma_n\sqrt{\frac{am_c}{2}}\right) - \exp\left(-\frac{a\sigma_n^2 m_c}{2}+\frac{m_c}{4}\right)\operatorname{erfc}\left(\frac{\sqrt{m_c}}{2}\right)\right]\right] \quad (4.49)$$

Using the similar arguments as discussed in case 1 step 6 for updation of mean of check nodes, at ($l$ +1) th iteration, averaging (4.49) over all the bit node degrees $j$ is

$$P_e = \frac{p_k^{M_R}\Gamma(L+1)}{4\sqrt{2}\,\sigma_n^{2M_R-1}\Gamma(M_R)\Gamma(L+1-M_R)}$$

$$\sum_{j=2}^{d_v} \lambda_j \left\{ \left[ (-1)^{M_R-1} \frac{\partial^{M_R-1}}{\partial a^{M_R-1}} - \right.\right.$$

$$\left. \frac{P_k L}{\sigma_n^2} (-1)^{M_R} \frac{\partial^{M_R}}{\partial a} \right] \left[ \frac{1}{\sqrt{a}} exp\left( \frac{a\sigma_n^2 (j-1) m_c^{l+1}}{2} - \frac{(j-1) m_c^{l+1}}{4} \right) \right] \left[ \frac{1}{\sigma_n \sqrt{\frac{a}{2}+\frac{1}{2}}} - \right.$$

$$\frac{2}{1+\sigma_n\sqrt{2a}} \left[ erfc\left( \sigma_n \sqrt{\frac{a(j-1)m_c^{l+1}}{2}} \right) - \right.$$

$$\left.\left.\left. exp\left( -\frac{a\sigma_n^2 (j-1) m_c^{l+1}}{2} + \frac{(j-1) m_c^{l+1}}{4} \right) erfc\left( \frac{\sqrt{(j-1) m_c^{l+1}}}{2} \right) \right] \right] \right\} \quad (4.50)$$

Because at ($l$+1) th iteration mean of variable node $v$ of degree $j$ is $m_v^{l+1} = (j-1)m_c^{l+1}$.

## 4.3 Results and discussion

### 4.3.1 Analytical details and results

In this section, the analytical results for the LDPC-OC system have been presented. To evaluate the performance, the system has been considered as under-loaded (i.e. when the receivers are less than or equal to the number of interferers) and overloaded (i.e. when the number of interferers are greater than the number of array elements). For both cases the number of receive antennas $M_R$ are taken as 3, 4, 5 and 6. For the under-loaded case the effect of only single interferer is considered .For the over-loaded case, number of interferers can be varying. In our case we have taken the number of interferers '*L*' = 18. For the irregular LDPC coded system, rate ½ (3, 6) has been considered with degree distribution $\lambda(x)=0.3321x+0.3307x^2+0.3372x^5$ and $\rho(x)=0.982x^5 + 0.018x^6$.

#### *4.3.1.1 Under-loaded case (when the number of interferes are less than or equal to the number of receive antenna array elements)*

*For the under-loaded case*, figure 4.2 and figure 4.3 show the BER results for the Uncoded – optimum combined (UC-OC) system and LDPC-OC system for the varying receive antennas in the presence of single interferer. For both cases, it is shown that significant diversity gain is achieved when the number of receive antenna increases. In table 4.2, for the LDPC coded system, it is shown that if we increase the diversity order from 3 to 6, diversity gain is improved by 5.5 dB at BER of $10^{-4}$.

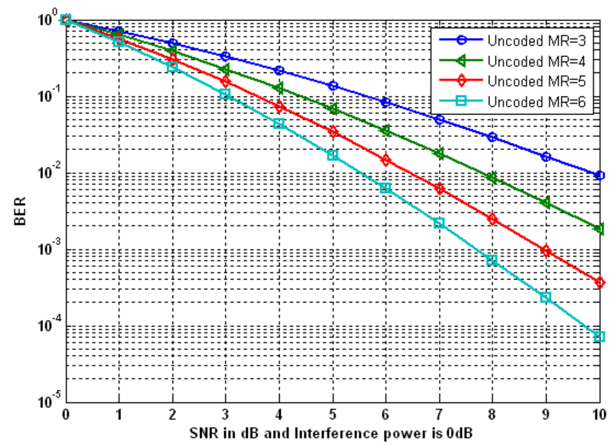


**Figure 4.2: Analytical BER of uncoded-OC system having number of receive antennas $M_R$ =3, 4, 5, 6 and single interferer (under-loaded case)**

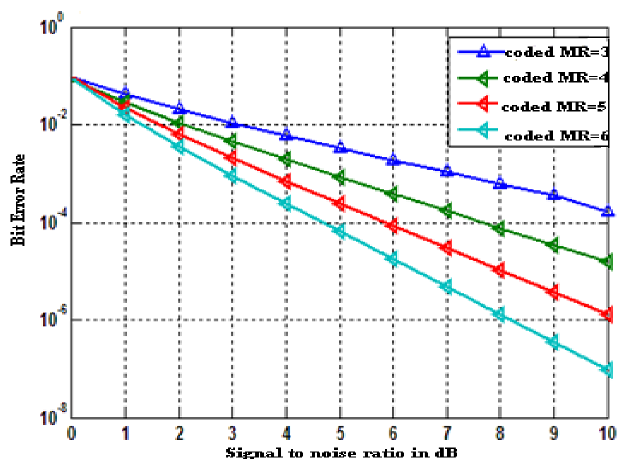


**Figure 4.3: Analytical BER of LDPC-OC system having number of receive antennas $M_R$ =3, 4, 5, 6 and single interferer (under-loaded case).**

**Table 4.1: SNR required for LDPC-OC system at BER of $10^{-4}$ with single interferer (under-loaded case)**

| SNR required in dB | |
|---|---|
| **Number of receive antenna** | **Signal to Noise ratio** |

| | |
|---|---|
| $M_R$ =3 | 10.1 |
| $M_R$ =4 | 7.5 |
| $M_R$ =5 | 5.9 |
| $M_R$ =6 | 4.6 |

Figure 4.4 shows the comparison of bit error rate for UC-OC system [54] with bit error rate of LDPC-OC system obtained from (4.42) for number of receive antenna 5 and 6. Table 4.2 shows that at BER $10^{-2}$, coding gain of LDPC-OC system has an improvement of 4.7 dB over UC-OC system when the numbers of antennas are 5 and an improvement of 4.3 dB when number of antenna's are 6. Therefore it is shown that the performance of LDPC-OC is much better than the performance of UC-OC system.

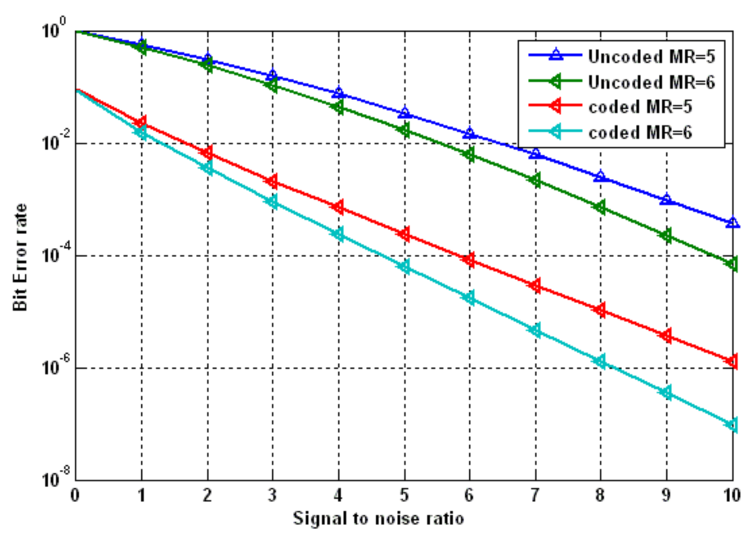


**Figure 4.4: Analytical comparison BER of UC-OC and LDPC-OC having number of receive antennas $M_R$ =5, 6(under-loaded case).**

**Table 4.2: SNR required for UC-OC and LDPC-OC at BER of $10^{-2}$ with single interferer(under-loaded case)**

| SNR required in dB | | |
|---|---|---|
| **Number of receive antennas** | **Uncoded-optimum combined** | **LDPC coded-optimum combined** |
| 5 | 6.3 | 1.6 |
| 6 | 5.5 | 1.2 |

***4.3.1.2 Overloaded case (when the number of interferes are greater than the number of receive antenna array elements)***

*For the over-loaded case*, figure 4.5 and figure 4.6 shows the BER results for the UC-OC system and LDPC-OC system for the varying receive antenna with 'L'=18. For both cases, it shown that significant diversity gain is achieved .

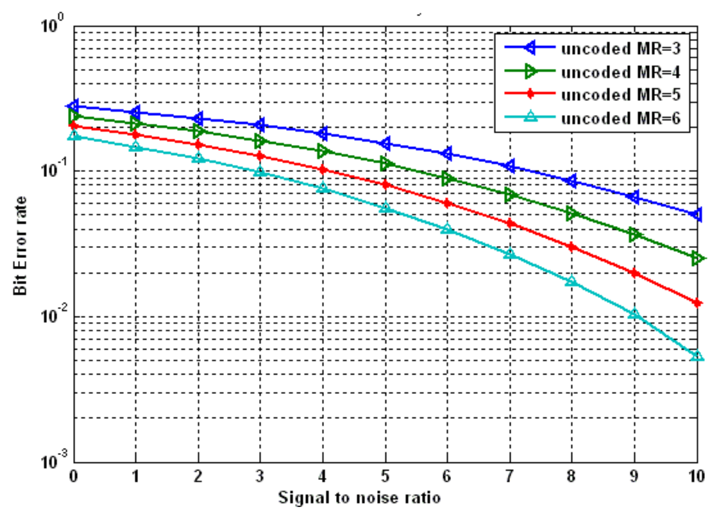


**Figure 4.5: Analytical BER of UC-OC having number of receive antennas with $M_R$ =3, 4, 5, and 6(over-loaded case).**

when the number of received antenna increases. In table 4.3, for the LDPC coded system, it is shown that if we increase the diversity order from 3 to 6, diversity gain is improved 4.4 dB at BER of $10^{-4}$.

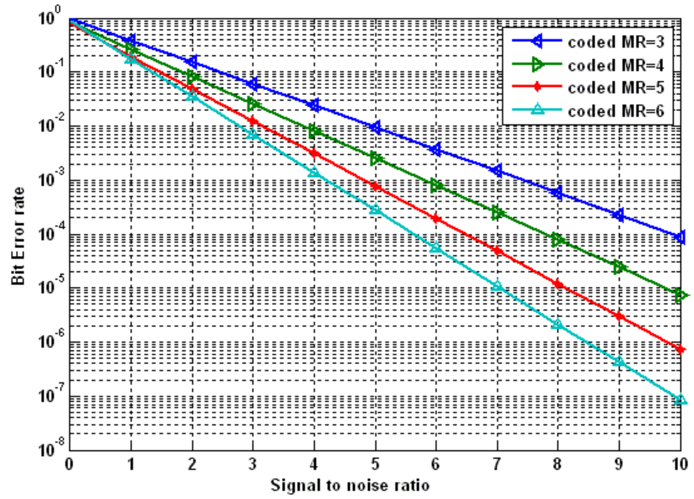


**Figure 4.6: Analytical BER of LDPC-OC system having number of receive antennas $M_R$ =3, 4, 5 and 6 with 'L'=18(over-loaded case)**

**Table 4.3: SNR required for LDPC-OC system at BER of $10^{-4}$ with 'L'=18 (over-loaded case)**

| **SNR required in dB** | |
|---|---|
| **Number of receive antenna** | **Signal to Noise ratio** |
| $M_R$ =3 | 10 |
| $M_R$ =4 | 7.8 |
| $M_R$ =5 | 6.5 |
| $M_R$ =6 | 5.6 |

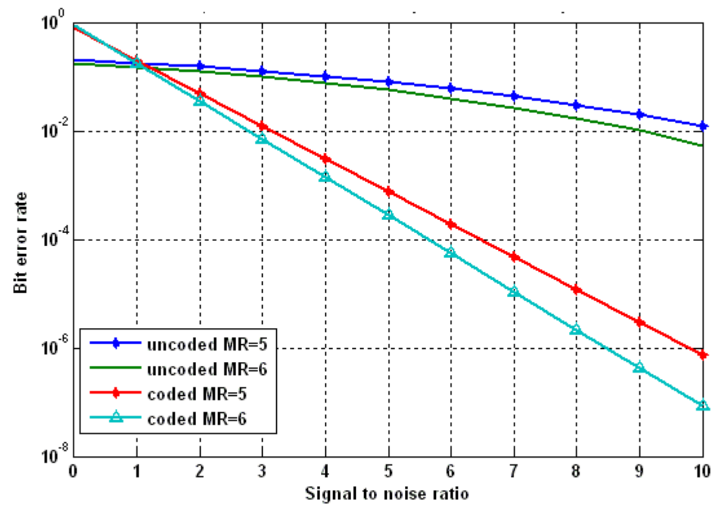


**Figure 4.7: Analytical comparison BER of OC and LDPC-OC having number of receive antennas $M_R$ =5(over-loaded case)**

**Table 4.4: SNR required of UC-OC and LDPC-OC at BER of $10^{-2}$ with 'L'= 18 (over-loaded case)**

| **SNR required in dB** | | |
|---|---|---|
| **Number of receive antennas** | **Uncoded-optimum combined** | **LDPC coded-optimum combined** |
| 5 | 10 | 3 |
| 6 | 9 | 2.8 |

Figure 4.7 shows the comparison of bit error rate obtained for UC-OC system with bit error rate of LDPC-OC system obtained from (4.50) for number of receive antenna 5 and 6. Table 4.4 shows that at BER $10^{-2}$, coding gain of LDPC-OC system has an improvement of 7 dB over UC-OC system when the number of antenna's are 5 and an improvement of 6.2 dB when number of antenna's are 6 . Therefore it is shown that the performance of LDPC-OC is much better than the performance of UC-OC system.

### 4.3.2. Simulation details and results

The performance of the proposed system is evaluated for *irregular* LDPC codes with the mean column weight of 3. Soft decoding with density evolution method determines the threshold above which the code performs well. The focus is to use Monte Carlo simulations to demonstrate the BER performance of LDPC-optimum combined (LDPC-OC) system. The BER is potted for different number of receive antennas ($M_R$) where $M_R$ = 2, 3 and 4. The channel is modeled as flat Rayleigh faded with complex Gaussian distribution with mean 'm' = 0 and covariance matrix $\boldsymbol{R}$ and is denoted as $CN(m, \boldsymbol{R})$. The noise is considered white. For finding BER of BPSK, a frame consisting of 200 coded bits is sent from transmitter with all the bits having random values 0 and 1. The results below have been obtained from 1000 independent iterations of each such frame.

At the receiver end, coded signal is received at the input of optimum combiner. Then recovered signal is sent to the LDPC decoder. After decoding, uncoded signal which is taken as reference signal and decoded signal is compared and BER is evaluated. Figure 4.8 shows the BER performance of the uncoded-optimum combined (UC-OC) system with and without the presence of interferer for $M_{R\,=}$ 2, 3 and 4. It is shown that without interferer $M_R$ =4 provide the best result in terms of coding gain and diversity gain. As the number of antenna decreases, the slope of curve decreases thereby showing decrease in diversity gain. Figure 4.9 shows the normalize plot of LDPC-OC system. It is again shown that with the increase in number of receive antennas, normalized diversity gain improves as the slope of curve increases. It is also clear from figure 4.9 that a significant coding gain improvement is obtained with LDPC coded system for $M_R$ = 3 and 4 as compared to UC-OC system. In figure 4.10 and figure 4.11 results of UC-OC system and LDPC-OC system are compared for $M_R$ = 3 and $M_R$ =4 respectively. Table 4.5 shows values of BER at SNR = -9 dB. It is shown that there is a great improvement in bit error rate with LDPC-OC scheme for $M_R$ =3 and $M_R$ = 4.

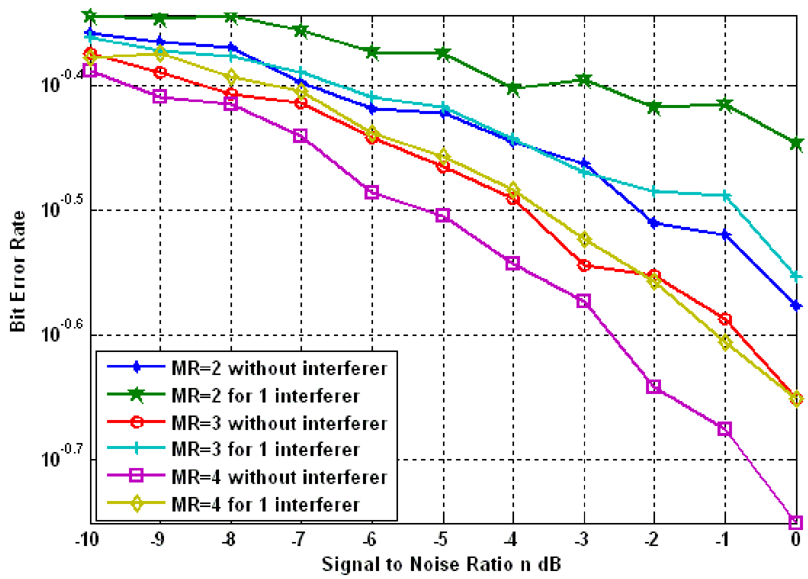


**Figure 4.8: BER of uncoded-optimum combined system in the presence of Single interferer**

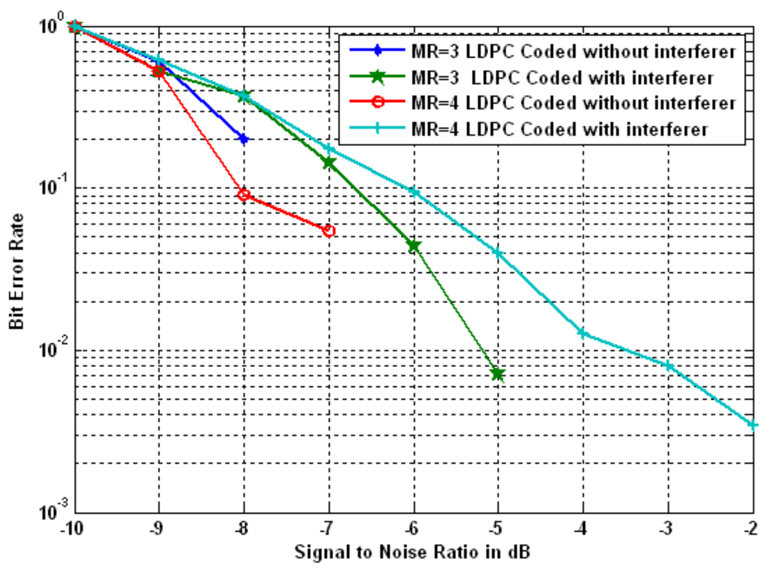


**Figure 4.9: BER of normalized LDPC-optimum combined in the presence of Single interferer**

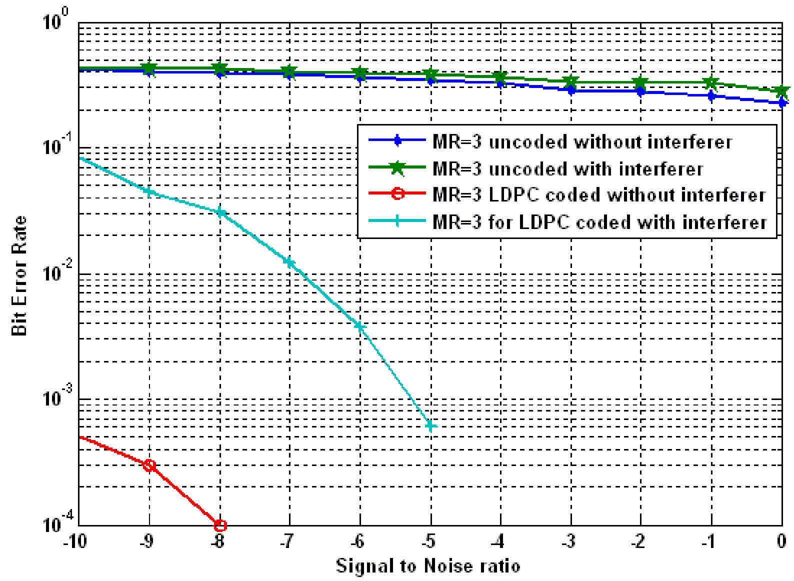


**Figure 4.10: BER of LDPC-optimum combined and uncoded-optimum combined system in the presence of single interfere with single Tx antenna and three Rx antenna**

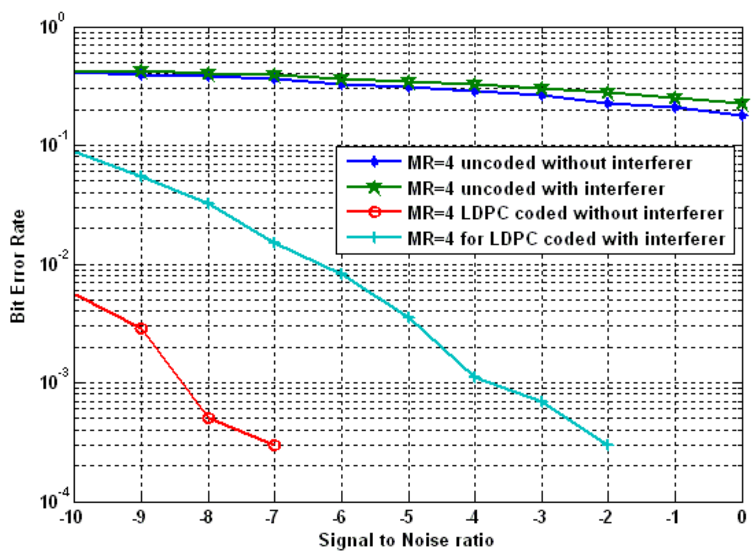


**Figure 4.11: BER of LDPC-optimum combined and uncoded-optimum combined system in the presence of single interfere with single Tx antenna and 4 Rx antenna**

**Table 4.5: Simulation based BER performance of uncoded and LDPC coded system**

| At SNR = -9 dB with single interferer | | | | |
|---|---|---|---|---|
| **Number of receive antennas** | **Uncoded** | | **LDPC coded** | |
| | **Without interferer** | **With interferer** | **Without interferer** | **With Interferer** |
| 3 | $4 * 10^{-1}$ | $4.2 * 10^{-1}$ | $3 * 10^{-4}$ | $4.5 * 10^{-2}$ |
| 4 | $3.8 * 10^{-1}$ | $4.1 * 10^{-1}$ | $3 * 10^{-3}$ | $5 * 10^{-2}$ |

## 4.4 Conclusion

In this chapter, we have proposed and presented the analysis of low density parity check codes with optimum combining that reduces the effect of co-channel interference. Hence improves the overall SINR for the given system. Based on the optimal weight selection, the received signal is combined in such a way that it not only suppress the interfering signal but also combat the multipath fading. It has been observed through the simulation as well as analytical results that the proposed LDPC-OC system reduces the bit error rate for a given SNR as compared to the uncoded-OC system and able it to transmit the desired signal even when the co-channel interference is very large *(overloaded case)*. The results obtained shows that simulated LDPC-OC system provides an accurate fit with analytical results obtained *(under-loaded case)*.

*It is a capital mistake to theorize before you have all the evidence. It biases the judgment.*

*-Sir Arthur Conan Doyle*

# *Chapter V*

## *PERFORMANCE ANALYSIS OF LDPC CODES IN MIMO SYSTEM*

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*This chapter extends the analysis of previous chapter of single transmit multiple receive antenna case to the multiple transmit and multiple receive antennas case. In this analysis, concatenated system of low density parity check codes with space time block code is considered. Tight upper bounds in terms of bit error probability have been derived for the proposed LDPC-STBC system. While analyzing the performance of the proposed system, it is assumed that perfect channel state information is available at the receiver. The tightness of the bound is verified by simulation results and is useful to benchmark the error performance of the proposed concatenated system employing sum-product decoding. Finally analytical and simulation results have been discussed and important conclusions are drawn.*

The explosion of wireless telecommunication technology has marked the end of second millennium. The numbers of internet and mobile users have increased exponentially. With latest high speed services available on both applications, the user of both has augmented expectations in terms of quality of service and capacity. To increase the capacity of system, a novel solution suggested is the use of multiple element arrays at both transmitter and receiver. Wireless systems with multiple antenna elements at both edges are referred as MIMO system. There are some general concepts of signal processing used with MIMO system commonly called space time coding/space time processing. Data transmission in space time (ST) coding system is carried out in two dimensions, the space dimension and the time dimension, as the acronym ST suggests. The space dimension is spanned by multiple transmit-antennas while the time dimension is spanned by multiple time intervals over which multiple blocks are transmitted. Multiple antenna system became popular a decade ago as a result of fundamental work of Wittneben [135], Telatar [136], Foschini [2], Foschini and Gans [27]. But the real breakthrough came with the work of Tarokh *et al.*[6] in which two code design criteria namely rank criterion and determinant criterion have been proposed for flat fading channels with coherent receivers. The idea of proposed high-performance space-time trellis codes is similar to idea of convolution coding in which there is a basic trellis structure that determines the coded symbols to be transmitted from the different antenna elements. However, these codes suffer from rather high decoding complexity. In the same year, Alamouti proposed space-time block coding (STBC) scheme for two transmit and multiple receive antennas [13] with maximum likelihood (ML) decoder having very low complexity. Inspired by this work, Tarokh *et al.* generalized Alamouti's code to multiple transmits antennas using the theories of orthogonal and amicable designs [14]. The codes developed in [14] are known as orthogonal STBCs (OSTBCs) and are able to provide high performance at very low decoding complexity. Some other designs of OSTBCs have also been recently developed in [137-139]. Among many class of codes, OSTBC's have many desirable properties such as OSTBC yield an orthogonal mixing matrix irrespective of MIMO channel matrix, enables great performance analysis even for non separable constellation. It also includes the celebrated 2x1 Alamouti code, which is implemented in many wireless communication standards. It has already been discussed in previous chapters that use of channel codes with MIMO system support high data rates. Further, to improve the quality of service that leads to improved link reliability, the channel coding is one of the effective way of achieving it. While the space time coding is two dimensional, channel coding relies on one dimension block of transmission with single antenna. Space time block codes proposed by Alamouti provide only the diversity gain and the channel coding provides the coding gain. Therefore to take the advantage of both gains, a concatenated system is proposed. In the previous chapter, we have evaluated the performance of LDPC codes in the SIMO system. In this chapter, we extend the analysis of LDPC codes to STBC based MIMO system. The system model for the proposed concatenated LDPC-STBC system is discussed in the next section.

### 5.1 System Model of concatenated LDPC-STBC system

Consider a LDPC coded MIMO system with $M_T$ transmit antennas and $M_R$ receive antennas that is illustrated in Fig 5.1. Assume the information bit stream $\boldsymbol{u}$ of $k_i$ information bits are encoded into LDPC binary codeword, $\boldsymbol{c}=(c_1\, c_2\, c_3 \ldots\ldots c_l)$ of length $l$. The modulator adopts a constellation of size $2^{k_b}$ where $k_b$ depends on the modulation (*e.g. for BPSK*, $k_b=2$). Thus the length of sequences after modulation is $l/k_b$ symbols and given by $\boldsymbol{s} = (s_1\, s_2\, s_3 \ldots\ldots s_{l/k_b})$. Each group of modulated symbol $\boldsymbol{s}$ is fed to STBC encoder at each time index $t$ and generates a codeword $\boldsymbol{U}$ of size $[M_T \times D]$. The outer codeword $\boldsymbol{c}$ is mapped into space time block $\boldsymbol{U} = \left[\boldsymbol{U}[1]\boldsymbol{U}\,[2] \ldots\ldots \boldsymbol{U}[D]\right]$, where $D$ is the frame length which is transmitted over $M_T$ antennas.

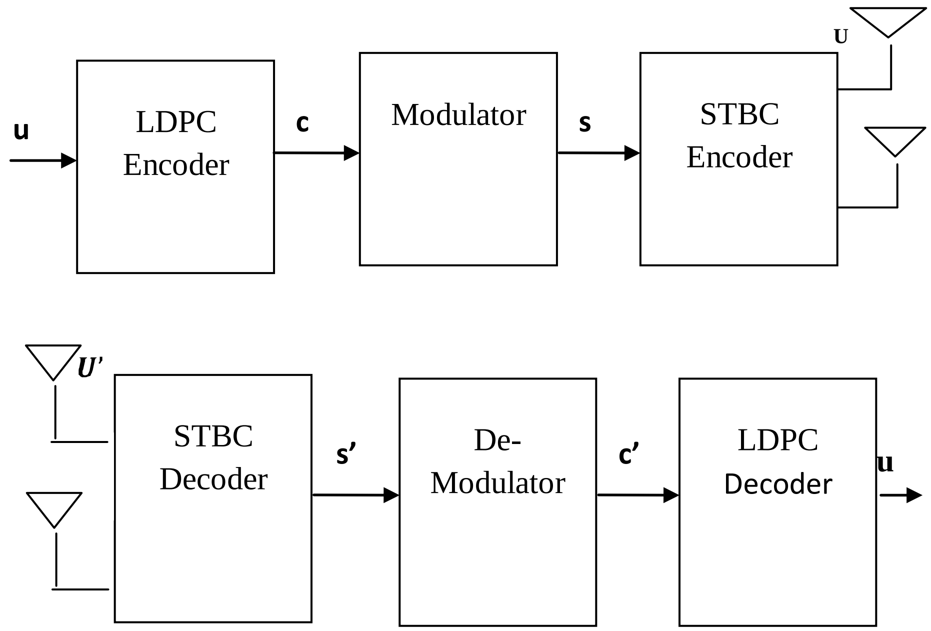


**Figure 5.1: System model of concatenated LDPC-STBC system**

The channel is assumed to be IID Gaussian with quasi-static flat fading with each entry of channel matrix characterized as $CN(0, \boldsymbol{I})$. The received signal $\boldsymbol{u}_j'$ at $j^{th}$ antenna can be represented as

$$\boldsymbol{u}_{j,k}' = \sqrt{\frac{E_s}{N_0 M_T}} \sum_{i=1}^{M_T} \boldsymbol{h}_{ij} \boldsymbol{u}_{i,k}, + \boldsymbol{n}_{j,k} \ , \tag{5.1}$$

where $k = 1,2 \dots \dots D,\ j = 1,2 \dots \dots\ M_R$ and $\boldsymbol{h}_{i,j}$ denotes the complex Gaussian channel coefficients between $i^{th}$ transmit and $j^{th}$receive antennas. $\boldsymbol{u}_{i,k}$ is the transmitted signal from antenna $i$ for $k^{th}$ frame and $\boldsymbol{u}_{j,k}'$is the received signal from antenna $j$ for $k^{th}$ frame.

The input-output relationship can be written in the matrix form as

$$\boldsymbol{U}' = \sqrt{\frac{E_s}{N_0 M_T}}\ \boldsymbol{H}\ \boldsymbol{U} + \boldsymbol{N}, \tag{5.2}$$

where $\boldsymbol{U}' = [\ \boldsymbol{u}'[\boldsymbol{1}]\boldsymbol{u}'[\boldsymbol{2}] \dots \dots \dots \boldsymbol{u}'[\boldsymbol{D}]]$ and $\boldsymbol{N} = [\boldsymbol{n}[\boldsymbol{1}]\ \ \boldsymbol{n}[\boldsymbol{2}] \dots \dots \dots \boldsymbol{n}[\boldsymbol{D}]]$ are the matrices of size $[M_R\ \times\ D]$. $E_s$ is the total average energy available at the transmitter over a symbol period, $\boldsymbol{H}$ is the $[M_R\ \times\ M_T]$ channel transfer function and **N** is the ZMCSCG distributed noise with covariance matrix $[\boldsymbol{N}_0\ \boldsymbol{I}_{M_R}]$.

## 5.2 Pair wise error probability of concatenated system with upper bounds

Assume that receiver has perfect channel state information available; the optimal decision rule minimizing the probability of error is given as [140]

$$\hat{\boldsymbol{U}} = \arg\min_U \boldsymbol{P}(\boldsymbol{U}/\boldsymbol{U}', \boldsymbol{H})). \tag{5.3}$$

For a given transmiited matrix and set of channel coeffiecnts, elements of received martix are jointly Gaussian since noise is white and Gaussian. As, the likelihood function is proportional to the squared Euclidean distance between the received matrix and transmitted matrix multiplied by channel coofficient matrix, the optimal decoding rule results in

$$\hat{\boldsymbol{U}} = \boldsymbol{arg}\min_U \left\| \boldsymbol{U}' - \sqrt{\frac{E_s}{N_0 M_T}} \boldsymbol{U}\boldsymbol{H} \right\|^2, \tag{5.4}$$

where $\| . \|^2$ denotes the Frobenius norm of matrix. Clearly , the resulting decision rule can also be written as

$$\hat{\boldsymbol{U}} = \boldsymbol{arg}\min_U \sum_{k=1}^{D} \sum_{j=1}^{M_R} \left| \boldsymbol{u'}_j(\boldsymbol{k}) - \sqrt{\frac{E_s}{N_0 M_T}} \sum_{i=1}^{M_T} \boldsymbol{h}_{i,j} \boldsymbol{u}_i(\boldsymbol{k}) \right|^2 \tag{5.5}$$

Assume that two space-time block code-words are given by $\boldsymbol{U}1$ and $\boldsymbol{U2}$. The pairwise error probability $P(\boldsymbol{U}1 \to \boldsymbol{U}2)$ is the probability that the received signal vector is closer to the erroneous codeword $\mathbf{U}2$ given that the codeword $\mathbf{U1}$ is transmitted. Conditioned on the instantaneous channel realization, the pair wise error probability is given by [141]

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) = P\left( \left\| \boldsymbol{U}' - \sqrt{\frac{E_s}{N_0 M_T}} \boldsymbol{U_1}\boldsymbol{H} \right\|^2 > \left\| \boldsymbol{U}' - \sqrt{\frac{E_s}{N_0 M_T}} \boldsymbol{U_2}\boldsymbol{H} \right\|^2 \right), \tag{5.6}$$

where $\boldsymbol{U'} = \sqrt{\frac{E_s}{N_0 M_T}} \boldsymbol{U_1}\boldsymbol{H} + N$. This can equivalently written as

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) = P\left( \|\boldsymbol{N}\|^2 > \left\| \boldsymbol{N} - \sqrt{\frac{\boldsymbol{E_s}}{\boldsymbol{N_0 M_T}}} \boldsymbol{X} \right\|^2 \right), \tag{5.7}$$

where $\boldsymbol{X} = (\boldsymbol{U_2} - \boldsymbol{U_1})\boldsymbol{H}.$

Writing the argument of the probability expression, we have

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) = P\left( \sum_{t=1}^{l} \sum_{J=1}^{M_R} \left| \boldsymbol{n_j}(\boldsymbol{t}) \right|^2 > \sum_{t=1}^{l} \sum_{J=1}^{M_R} \left| \boldsymbol{n_j}(\boldsymbol{t}) - \sqrt{\frac{E_s}{N_0 M_T}} \boldsymbol{x_j}(\boldsymbol{t}) \right|^2 \right). \tag{5.8}$$

Where $\boldsymbol{x_j}(\boldsymbol{t})$ is the (*t*, *j*)th element of matrix $\boldsymbol{X}$ . Solving the R.H.S of (5.8) and cancelling the common terms, we get

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) = P\left(\sum_{t=1}^{l}\sum_{J=1}^{M_R} 2\,Re\{(\boldsymbol{x_j}(\boldsymbol{t}))^*\,\boldsymbol{n_j}(\boldsymbol{t})\} > \sum_{t=1}^{l}\sum_{J=1}^{M_R}\sqrt{\frac{E_s}{N_0 M_T}}\left|\boldsymbol{x_j}(\boldsymbol{t})\right|^2\right) \quad (5.9)$$

Assuming the noise is Gaussian with zero mean and ½ variance, the L.H.S of (5.9) is Gaussian random variable with mean zero and variance $2\|\boldsymbol{X}\|^2$. Therefore we obtain [141]

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) = Q\sqrt{(\rho/2M_T)\|\boldsymbol{X}\|^2}\,, \quad (5.10)$$

where $\rho = \frac{\mathrm{E_s}}{\mathrm{N_0}}$.

Writing the argument of the probability expression, the PEP between $(\boldsymbol{U_1}, \boldsymbol{U_2})$ can be given as [140, eq 6.5]

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) = Q\left(\sqrt{\frac{E_s\,\sum_{t=1}^{l}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\boldsymbol{h}_{i,j}^{2}(t)\left(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\right)^2}{2*M_{T*}N_0}}\right) \quad (5.11)$$

where $\sum_{t=1}^{l}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\boldsymbol{h}_{t,i,j}^{2}\left(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\right)^2 = \|\boldsymbol{H}(\boldsymbol{U_1}-\boldsymbol{U_2})\|_F^2$ .

The upper bound on PEP can be found using Chernoff bound given by

$$Q(x) \leq \frac{1}{2}\,e^{\frac{-x^2}{2}} \quad (5.12)$$

Applying the chernoff bound to (5.11)

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) \leq \frac{1}{2}e^{-\left(\frac{E_s\,\sum_{t=1}^{l}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\boldsymbol{h}_{t,i,j}^{2}\left(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\right)^2}{4M_T N_0}\right)} \quad (5.13)$$

Simplifying (5.13) we get

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) \leq \frac{1}{2}\prod_{t=1}^{l} e^{\left(\frac{-\rho}{4M_T}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\boldsymbol{h}_{t,i,j}^{2}\left(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\right)^2\right)}$$

Le$\xi_t^2 = \frac{\rho}{4M_T}\left(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\right)^2$ and $\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\boldsymbol{h}_{t,i,j}^{2} = \boldsymbol{Z}_t$

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}/\boldsymbol{H}) \leq \frac{1}{2}\prod_{t=1}^{l} e^{(-\xi_t^2 \boldsymbol{Z}_t)} \quad (5.14)$$

RVs $\mathbf{Z}_t$ are independent and have chi square distribution each with $2M_TM_R$ degree of freedom and the probability density function is given by [99]

$$f_Z(\boldsymbol{z_t}) = \frac{1}{(M_TM_R-1)!}\, e^{-\boldsymbol{z_t}}\, \boldsymbol{z_t}^{M_TM_R-1} \tag{5.15}$$

Generalized unconditional PEP for proposed system can be written as

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) = \int_0^\infty P\big(\boldsymbol{u}_{1,t} \to \boldsymbol{u}_{2,t}|H\big)\, f_Z(\boldsymbol{z_t})\; d\,\boldsymbol{z_t} \tag{5.16}$$

Putting (5.14) and (5.15) into (5.16)

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \le \int_0^\infty \frac{1}{2}\prod_{t=1}^{l} e^{-\xi_t^2\, \boldsymbol{Z_t}} \quad \frac{1}{(M_TM_R-1)!}\; e^{-\boldsymbol{z_t}}\, \boldsymbol{z_t}^{M_TM_R-1}\;\; d\,\boldsymbol{z_t} \tag{5.17}$$

$$\le \frac{1}{2(M_TM_R-1)!}\prod_{t=1}^{l}\int_0^\infty e^{-\boldsymbol{Z_t}(\xi_t^2+1)}\boldsymbol{z_t}^{M_TM_R-1}\, d\boldsymbol{z}_t \tag{5.18}$$

Put $\boldsymbol{Z}_t(\xi_t^2+1) = Q_t$ into (5.18)

$$\le \frac{1}{2(M_TM_R-1)!}\prod_{t=1}^{l}\int_0^\infty e^{-Q_t}\left(\frac{Q_t}{\xi_t^2+1}\right)^{M_TM_R-1}\; dQ_t \tag{5.19}$$

The Gamma function is defined as

$$\int_0^\infty e^{(-Q_t)}\,(Q_t)^{M_TM_R-1}dQ_t = \Gamma(M_TM_R) = (M_TM_R-1)! \tag{5.20}$$

Using Gamma function, (5.19) will be written as

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \le \frac{1}{2(M_TM_R-1)!}\prod_{t=1}^{l}\;\frac{(M_TM_R-1)!}{(\xi_t^2+1)^{M_TM_R}} \tag{5.21}$$

Putting the value of $\xi_t^2$ , (5.21) becomes

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \le \frac{1}{2}\prod_{t=1}^{l}\;\left\{\frac{1}{\left(\frac{\rho}{4M_T}\big(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\big)^2+1\right)^{M_TM_R}}\right\} \tag{5.22}$$

$$P^{STBC} = P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \le \frac{1}{2}\prod_{t=1}^{l}\;\left(\frac{\rho}{4M_T}\big(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t}\big)^2+1\right)^{-M_TM_R}$$

$$\le \frac{1}{2}\left(\frac{\rho}{4M_T}(\boldsymbol{U_1}-\boldsymbol{U_2})^2+1\right)^{-lM_TM_R} \tag{5.23}$$

Equation (5.23) represents the PEP of inner STBC of one codeword '**S**'. Since **U=[U**[1],**U** [2]…… **U** [D]] has independent '**D**' code-words, the total PEP of inner code word is given by

$$P(\boldsymbol{U} \rightarrow \boldsymbol{U'}) = \prod_{d=1}^{D} P^{STBC}(\boldsymbol{U_{1,d}} - \boldsymbol{U_{2,d}}) \tag{5.24}$$

Since all 'D' received vector symbols in the codeword may be stacked together, we may calculate the PEP of outer coder as

$$P(\boldsymbol{c} \rightarrow \boldsymbol{c'}) \leq P(\boldsymbol{U} \rightarrow \boldsymbol{U'}) =: \frac{1}{2}\prod_{d=1}^{D}\left(\frac{\rho}{4M_T}\ \left|(\boldsymbol{U_{1,d}} - \boldsymbol{U_{2,d}})\right|^2 + 1)^{-l\,M_T M_R}\right) \tag{5.25}$$

### 5.3 Results and discussion

#### 5.3.1 Analytical details and Results

We first verify the effectiveness of the derived upper bounds for the concatenated LDPC-STBC system by plotting the bit error rate with variation in SNR. The number of transmit antennas are taken a 2 and receive antennas are taken as 3 and 4. The results are plotted for SNR varying from 1 dB to 10 dB. Figure 5.2 presents the plot of analytical results for the varying number of receive antenna. As shown in table 5.1, increased diversity order requires less SNR at a given bit error rate.

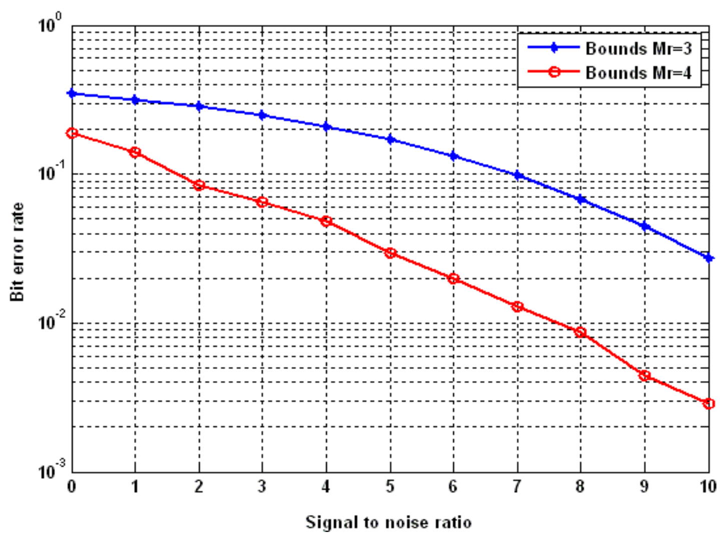


**Figure 5.2: Analytical results of concatenated LDPC-STBC scheme for varying receive antenna.**

**Table 5.1: SNR required (in dB) by concatenated LDPC-STBC system for varying receive antenna's at at BER of $10^{-4}$**

| Number of receive antennas ($M_R$) | Signal to Noise ratio(dB) |
|---|---|
| 3 | 4 |
| 4 | 2.8 |

### 5.3.2. Simulation Details and Results

In the considered model, a quasi-static MIMO fading channel model is assumed. Further it is also assumed that the channel remains static during 'D' blocks of transmitted data and changes independently from one block to another block. The results are evaluated for randomly generated LDPC (200,100) codes with Log Domain Sum-Product decoding algorithm. Distance spectrum for LDPC codes is calculated according to method given in [21]. To evaluate the BER, a block consisting of 100 frames is fed to LDPC encoder. The results have been obtained from 1000 independent iterations of each frame, for each value of SNR. The LDPC encoded signal is mapped into space and time domain creating orthogonal sequences that are transmitted from different antennas as a STBC signal. Simulations results for the proposed concatenated STBC-LDPC system have been obtained for two transmit and three receive antennas. The channel is considered as flat Rayleigh faded and Rician faded. The channels are modelled as complex Gaussian distributions with a mean $m$ and covariance matrix $\boldsymbol{R}$ i.e $CN(m, \boldsymbol{R})$. In Rayleigh faded channels $m = 0$, and for Rician faded channels the mean $m$ is sum of the power in the line-of-sight and the local-mean scattered power i.e. $\boldsymbol{R} = \boldsymbol{I}$ (Where $\boldsymbol{I}$ is identity matrix) for both Rayleigh as well as Rician faded environments. The noise considered as additive is modelled as a circular symmetric complex Gaussian random variable with mean 0 and variance 1 i.e. $CN(0, \boldsymbol{I})$. Figure 5.3 shows the plot of the bit error rate for MPSK and QAM modulation schemes in Rayleigh and Rician faded environments. A normalized comparison of LDPC-STBC system is shown in figure 5.4. It is shown that QAM techniques in rician faded environment outperform other modulation schemes. The input SNR required for various constellations of PSK and QAM modulations at a BER of 5 x $10^{-1}$ is given in Table 5.2.

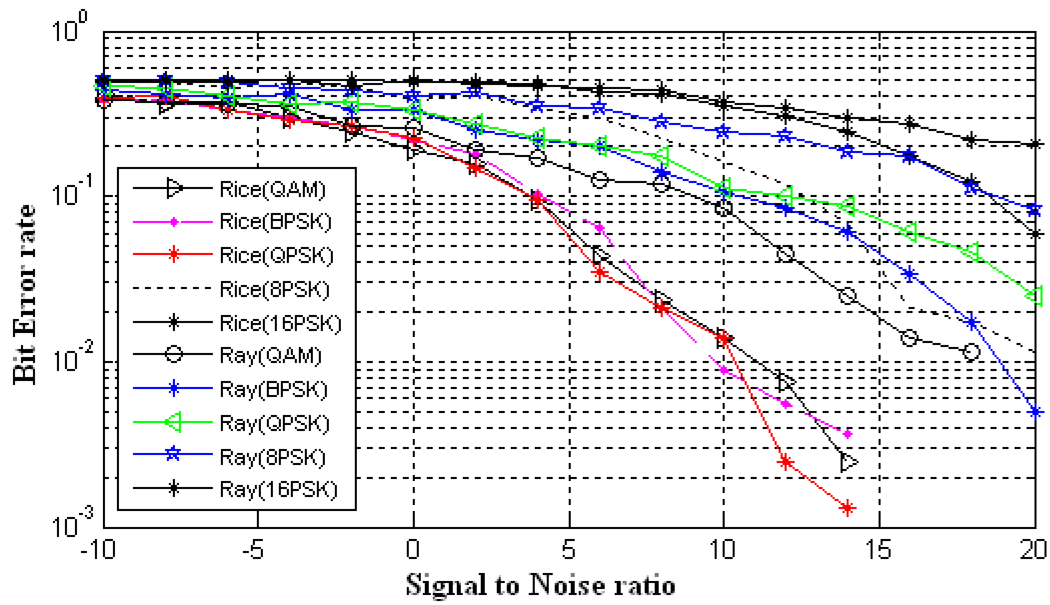


**Figure 5.3: Comparison of BER of concatenated LDPC-STBC system in Rayleigh and Rician fading for different modulation schemes**

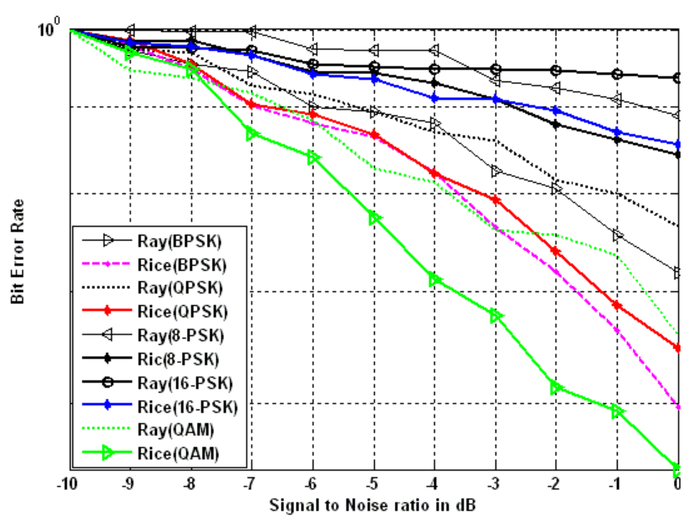


**Figure 5.4: Normalized comparison of BER of concatenated LDPC-STBC system in Rayleigh and Rician fading for different modulation schemes**

**Table 5.2: SNR required for concatenated LDPC-STBC system for different modulation techniques in NLOS and LOS environments at BER of 5x $10^{-1}$**

| Modulation scheme | SNR Required(in dB) | |
|---|---|---|
| | Rayleigh fading | Rician Fading |
| QAM | -0.9 | -5.2 |
| BPSK | 14 | -3 |
| QPSK | 13.8 | -2 |
| 8-PSK | >20 | 14.1 |
| 16-PSK | >21 | 20 |

### 5.3.3. Comparison of analytical results with upper bounds

In this section we compared derived upper bounds and simulation results of the concatenated LDPC-STBC system, assuming perfect channel estimation available at the receiver end. In figure 5.5 and table 5.3, the derived bound as in (5.25) shows close match with simulated results when we adopt $M_T$=2 and $M_R$=3 or 4. It is clear from the figure that all the curves have the same slope at high SNR, suggesting that they have same diversity order $M_TM_RL$. Analytical results using Chernoff bound closely match with the simulated results.

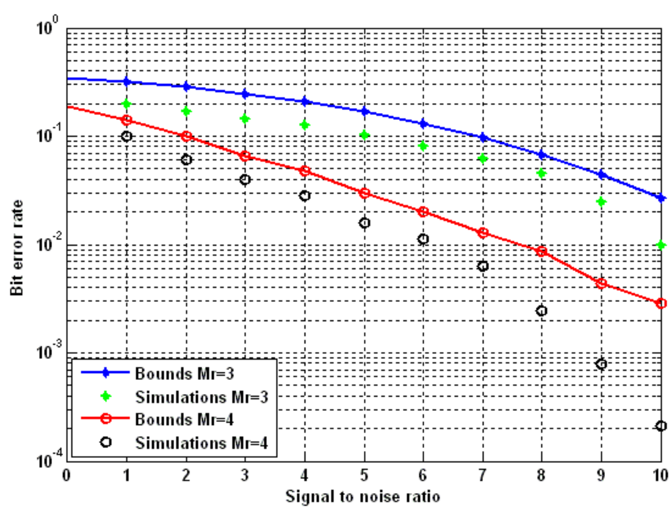


**Figure 5.5: Comparison of simulation with upper bounds of LDPC –STBC scheme with $M_T$=2, $M_R$=3and 4**

**Table 5.3 : Comparison of SNR required for concatenated LDPC-STBC system for varying receive antenna's with simulation results at BER of 1 x $10^{-1}$**

| Number of receive antenna's($M_R$) | SNR required (in dB) for concatenated LDPC-STBC system | |
|---|---|---|
| | Simulated | Analytical |
| 3 | 5 | 6.8 |
| 4 | 1 | 2 |

### 5.4 Conclusion

In this chapter, we have presented a simple serially concatenated structure consisting of LDPC and STBC codes. The proposed scheme is evaluated for various modulations in different fading environment for varying receive antennas. As the channel codes provide the coding gain, it is found that, a significant diversity gain is also achieved with the employment of STBC codes. The analytical bound for the suggested system also presented to verify our simulation results.

As the system become more complex with the increase in the deployed receive antennas, therefore the concept of antenna selection has been introduced with concatenated LDPC-STBC system which will be discussed in chapter 6. The suggested system reduces the overall complexity, achieves significant diversity gain and substantial coding gain.

***The way towards truth is strewn with errors.***

***Who does not make them, who does not touch them,***

***does not fight with them and finally doesn't obviate them***

***with his own forces, does not reach the truth.***

***-Constantin Tsatsos***

# *Chapter VI*

# *PERFORMANCE ANALYSIS OF LDPC CODES IN MIMO SYSTEM WITH ANTENNA SELECTION*

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*This chapter extends the performance analysis of concatenated LDPC-STBC system with receive antenna selection. The proposed system is equipped with multiple antennas at transmitter as well as at the receiver end. Proposed research work mainly deals with selection of best receive antennas $L_r$ from available $M_R$ receive antennas. The first part of chapter give a brief overview on the antenna selection along with various antenna selection algorithms reported in literature .The tight upper bounds for the bit error probability for the concatenated system with receive antenna selection have been derived. The tightness of the bound is verified by simulation results and is useful to benchmark the error performance of the proposed concatenated system with antenna selection. Finally analytical and simulation results have been discussed. The conclusions are given in the last section*

## 6.1 Introduction to antenna selection system

Regardless of the use as diversity or spatial multiplexing system, the main drawback of any MIMO system is the increased complexity, and thus cost. While additional antenna elements (patch or dipole antennas) are usually inexpensive, and digital signal processing becomes ever cheaper, the RF elements are expensive. Each RF element requires its own hardware (Power amplifiers, low noise amplifiers, analog to digital converter etc.) as shown in figure (6.1). Due to this reason there is now great interest in antenna selection schemes [36] by which best selected $L_t/L_r$ out of $M_T/M_R$ antennas are chosen. It often makes better sense to employ a few down conversion chains along with a cross bar switch to chose best antennas from available antennas that reduces the number of required RF chains from $M_T/M_R$ to $L_t/L_r$. Thus Antenna selection provides a new form of diversity in MIMO systems with low cost and relatively small overhead increase while keeping the most of benefits of MIMO system. In many scenarios, judicious antenna selection may incur little or no loss in system performance, while significantly reducing system cost. Although it is argued that, by Moore's Law, and deployment of new efficient design architectures (e.g. the direct-conversion technique), the cost of RF chains may be reduced to the same magnitude as antenna elements in the near future. Antenna selection is still beneficial in improving the system performance, since it is operated in the RF domain, and essentially does not interfere with any other performance-enhancement techniques. In other words, antenna selection is not proposed as an alternative, but rather as an enhancement to other advanced processing (e.g. the transmit beam forming), to achieve better performances. Antenna selection can be performed at the transmitter, at the receiver, or at both ends simultaneously.

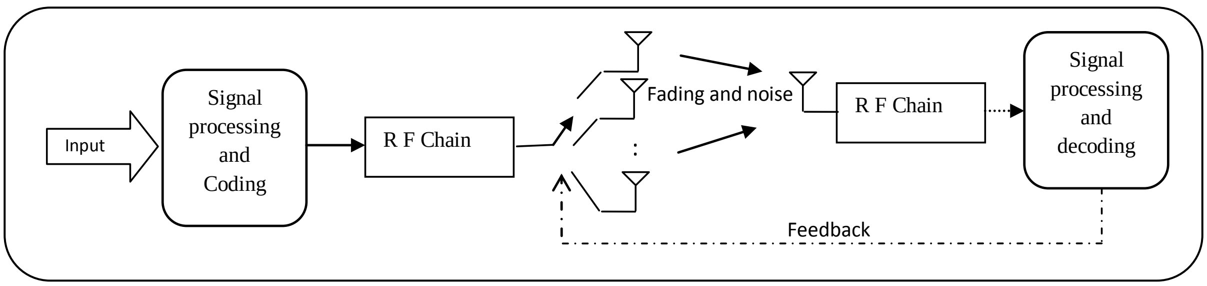


**Figure 6.1: Transmit antenna selection [142]**

Transmit antenna selection (TAS) requires a feedback path from the receiver to the transmitter (Fig. 6.1). This feedback rate is rather small, especially for single antenna selection. To study multiple transmit antenna selection; assume that there are $M_T$ antennas at the transmitter, and one antenna at the receiver. At the transmitter, we only choose best $L_t$ antennas from a subset of $M_T$ transmit antennas for transmission. For selecting best $L_t$ antennas, let the channel power gain, $\boldsymbol{h_k}$ associated with $k^{th}$ transmitting antenna.

$$\boldsymbol{h_k} = \sum_{i=1}^{M_T} \left|\boldsymbol{h_{k,i}}\right|^2, \;\; 1 \leq k \leq M_T \tag{6.1}$$

The random variables $\boldsymbol{h_k}$ are rearranged in an ascending order of magnitude and denoted by

$$\boldsymbol{h_1} \leq \boldsymbol{h_2} \leq \cdots\cdots \leq \boldsymbol{h_{M_T}} \tag{6.2}$$

The $M_T$ transmit antennas corresponding to $\{\boldsymbol{h}_{M_T} \cdot\cdot\cdot, \boldsymbol{h}_{M_T-L_t+1}\}$ will be selected for transmission. Equivalently, out of $M_T$, the subset choses $L_t$ antennas that maximize the total received power will be selected. The main advantages of an ($M_T$, $L_t$; $M_R$) TAS system is that it can achieve a full diversity order of $M_T M_R$. Along with that, the decoding complexity is the same as that of the $(M_T M_R)$ baseline system. In general, an ($M_T$, $L_t$; $M_R$) TAS system has significant SNR advantage over an ($M_T$ , $M_R$) system.

Similar to transmitter antenna selection, receive antenna selection as shown in figure (6.2) chooses best subset of the total number of $M_R$ receive antennas. The received SNR ($\gamma_{sel}$) is

$\gamma_{sel} = \max_j |\boldsymbol{h_j}|^2$,

where $h_j$ is the $j^{th}$ element of $\boldsymbol{h}$ and corresponding to the channel between the single transmit and $j^{th}$ receive antenna.

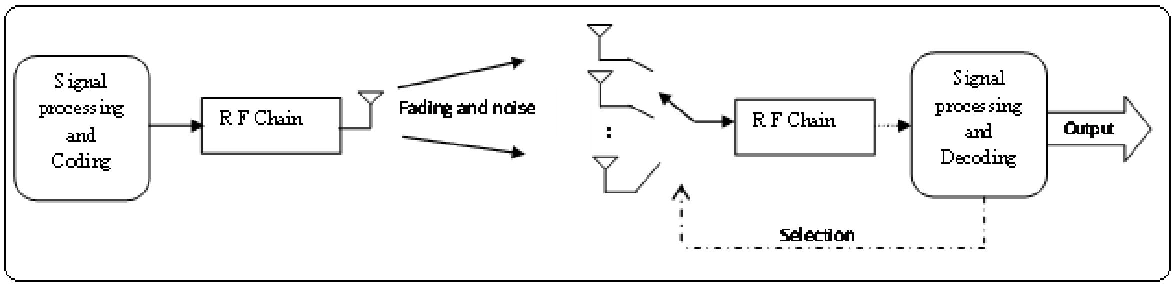


**Figure 6.2: Receive antenna selection[142]**

The antenna selection technique can be applied simultaneously both to the transmitter and receiver (Fig. 6.3). In this scenario, there are $M_T$ transmitting and $M_R$ receive antennas. The transmit and receive side will select $L_t$ and $L_r$ RF chains, respectively. Denote the overall $M_R\times M_T$ channel matrix by $\boldsymbol{H}$, and the $L_r \times L_t$ channel matrix representing the selected antennas by $\widetilde{\boldsymbol{H}}$. The SNR of the equivalent channel is proportional to the Frobenius norm of the selected channel matrix $\|\widetilde{\boldsymbol{H}}\|^2 = \sum_{ij} |\widetilde{\boldsymbol{h}}_{\boldsymbol{ij}}|^2$.Therefore, joint transmit/receive selection strategies must choose a subset of the rows and columns of $\boldsymbol{H}$ to maximize the sum of the squared magnitudes of transmit-receive channel gains. This is not an easy task; for example, successively choosing the best receivers and then the best transmitters will not necessarily result in an overall optimal choice.

In this thesis we have considered receive antenna selection (for increasing the SNR of received signal) based on the power of received signals. Suppose there are two rows of $\boldsymbol{H}$ which are identical since both have the same information, we can delete any row without losing any information.

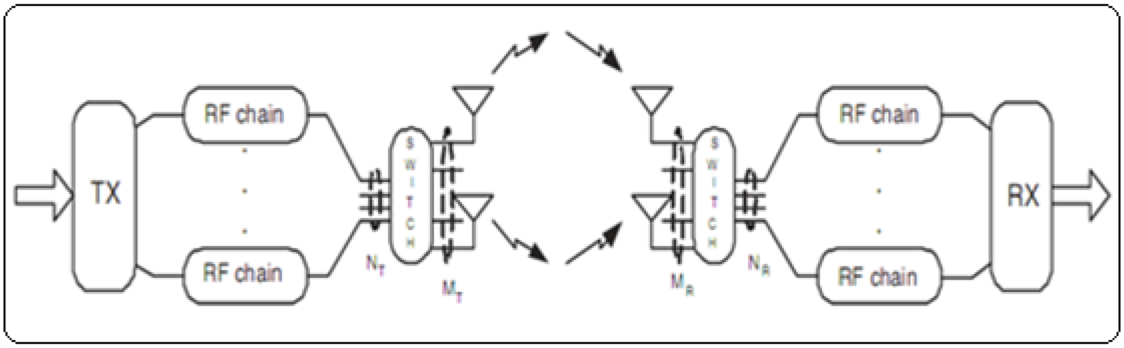


**Figure 6.3: Antenna selection in MIMO [142]**

If both have different powers, we delete the row with the lowest power. In this manner we can have the channel matrix whose rows have minimum correlation and have maximum power.

## 6.2 Antenna selection algorithm

For selecting the antennas, there are two main antenna selection criteria includes the maximization of the channel capacity and maximization of the received signal-to-noise ratio. To increase both parameters, MIMO framework allows wide range of selection algorithms that provide the full diversity order. Existing literature includes many antenna selection algorithms [143] for transmit and/or receive antennas. The following section provides a brief review on a various antenna subset selection algorithm.

- ***Capacity optimization method***

  In this method it is assumed that transmitter has no channel knowledge, hence transmitter power is distributed equally among all transmitter antennas to maximize the channel capacity. Thus the instantaneous capacity using all available transmit and receive antennas is given by

  $$C = log_2 \det(\boldsymbol{I}_{M_R} + \frac{\rho}{M_T} \boldsymbol{H}^H \boldsymbol{H}),$$

  where $\rho$ represents average signal to noise energy per receive antenna.

  Assuming antenna selection is performed at the receiver side and the receiver has the perfect channel knowledge so that the capacity of $\widetilde{\boldsymbol{H}}$ is given as [141]

  $$C(\widetilde{\boldsymbol{H}}) = log_2 \det(\boldsymbol{I}_{L_r} + \frac{\rho}{M_T} \widetilde{\boldsymbol{H}}^H . \widetilde{\boldsymbol{H}}) \tag{6.3}$$

The optimal set of receive antenna should be chosen so as to maximize the capacity of resultant system.

- ***Singular Value based***

This method of antenna selection based on the knowledge of minimum and maximum singular-values of the MIMO channel matrix. The singular value is calculated by singular value of decomposition of matrix $\boldsymbol{H}^H\boldsymbol{H}$.

$$\boldsymbol{H}^H\boldsymbol{H} = \boldsymbol{U\Sigma V}^H, \tag{6.4}$$

where **U** and $\boldsymbol{V^H}$(conjugate transpose) are the complex unitary matrices and $\boldsymbol{\Sigma}$ is the diagonal matrix with non negative real number on the diagonal.

There are two methods that use of singular value decomposition techniques for the antenna selection. The *first method* is based on the maximum of minimum singular value [144], given as

$$r = arg\max_{r\epsilon R} \min_i \lambda_i, \tag{6.5}$$

where r is one element from the set R={1,2,3….$M_T$} if selecting one antenna out of available $M_T$ antennas, $\lambda_i$ is the $i^{th}$ singular-value of sub-matrix ($\boldsymbol{H}_{L_t,M_T}^{r}{}^H \times \boldsymbol{H}_{L_t,M_T}^{r}$). The selection of r is done by searching for channel sub matrix which has minimum singular values of all subset of **H** of $(M_R \times M_T)$.

*Second method* is based on the maximum ratio of minimum singular value and maximum singular value, given as

$$r = arg\,max_{r\epsilon R} \frac{min_i\,\lambda_i}{max_i\,\lambda_i}.$$

The complexity of second method is slightly higher than first method because it requires the calculation of two singular-values for each subset combination.

- ***Norm based***

This method select the antenna based on the maximum Frobenius norm of $\widetilde{\boldsymbol{H}}$ which is the sum of square of amplitude of the elements of **H**. This method maximizes the signal to noise ratio and minimizes the instantaneous probability of error at the receiver. The norm based method is widely used because of low computational complexity [145]. This method calculates the Frobenius norm of all rows of **H** and select that subset which has maximum norm. The resulting sub channel matrix would contain $L_t/L_r$ antennas out of $M_T/M_R$ row of corresponding **H.** For receive antenna selection, Frobenius norm of the sub channel matrix is given as

$$f_{norm}^{M_R} = \sum_{t=0}^{M_T} \left\| \boldsymbol{H}_{M_R,t} \right\|_F, \tag{6.6}$$

where $\boldsymbol{H}_{M_R,t}$ is the ${M_R}^{th}$ row of channel matrix **H.** and the antenna subset is calculated as

$$r_{norm} = arg\ \max_{.} \sum_{M_R=r(1)}^{r(L_r)} f_{norm}^{M_R}$$

It is shown [145] that in the common wireless conditions norm based antenna selection method is less complex and gives better performance. In the work of concatenated LDPC-STBC system, we are using norm based antenna selection method.

### 6.3 System Model of concatenated LDPC-STBC system with antenna selection

In the proposed work, we are considering the same model developed in chapter 5. The system model consists of $M_T$ transmit antennas and $M_R$ receive antennas. The channel is assumed to be flat faded and time invariant. The signal is modeled as

$$\boldsymbol{y} = \boldsymbol{H}\,\boldsymbol{x} + \boldsymbol{n}, \tag{6.7}$$

where $\boldsymbol{y} = \left[y_1, y_2 \dots \dots \dots \dots \dots y_{M_R}\right]^T$ as $(M_R \times 1)$ vector corresponding to signal received at $M_R$ antennas and $\boldsymbol{x} = \left[s_1, s_2 \dots \dots \dots \dots \dots s_{M_T}\right]^T$ corresponds to $(M_T \times 1)$ symbol vector transmitted from $M_T$ transmit antennas, **n**= $\left[n_1, n_2 \dots \dots \dots \dots \dots n_{M_R}\right]^T$ is AWGN noise with mean zero and variance ½ per dimension and **H** is $(M_R \times M_T)$ channel matrix. The channel is considered as Rayleigh Faded i.i.d channel whose entries are i.i.d zero mean circular Gaussian random variable with variance one. The perfect channel state information is assumed to available at the receiver. Hence for a given channel $L_t$ out of $M_T$ and $L_r$ out of $M_R$ antennas are selected and used for transmission and reception respectively.

### 6.4 Upper bounds on Pair wise error probability of concatenated system with antenna selection

In this section we derive an upper bound on the pair wise error probability of concatenated coding scheme with receive antenna selection. This is the case when the receiver uses only $L_r$ antennas out of $M_R$ receiving antennas (where $1 \leq L_r \leq M_R$). Here norm based antenna selection is considered. The chosen subset is based on the maximum Frobenius norm at the receiver. We consider that the sum of the largest $L_r$ out of $M_R$ non negative numbers is always equal to and greater than the average of these $M_R$ multiplied by $L_r$, so we have [99]

$$\sum_{j=M_R-L_r+1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 \geq \frac{L_r}{M_R} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 \,. \tag{6.8}$$

When the receiver selects the best $L_r$ antennas, the conditional pair-wise probability that the receiver will select the $\boldsymbol{U_2}$ over $\boldsymbol{U_1}$ conditioned on channel $\mathbf{H}$ can be upper bounded as [140]

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) = Q\left(\sqrt{\frac{E_s L_r \sum_{t=1}^{l} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 (\boldsymbol{u}_{1,t} - \boldsymbol{u}_{2,t})^2}{2 M_T M_R N_0}}\right) \tag{6.9}$$

Applying chern-off bound given in (5.12) to (6.9)

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) \leq \frac{1}{2} e^{-\left(\frac{E_s L_r \sum_{t=1}^{l} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 (\boldsymbol{u}_{1,t} - \boldsymbol{u}_{2,t})^2}{4 M_T M_R N_0}\right)}$$

Simplifying above equation, we get

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}/\boldsymbol{H}) \leq \frac{1}{2} \prod_{t=1}^{l} e^{\left(\frac{-\rho L_r}{4 M_T M_R} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 (\boldsymbol{u}_{1,t} - \boldsymbol{u}_{2,t})^2\right)}, \tag{6.10}$$

where $\rho = \frac{E_s}{N_0}$. Putting $\zeta_t^2 = \frac{\rho L_r}{4 M_T M_R} (\boldsymbol{u}_{1,t} - \boldsymbol{u}_{2,t})^2$ and $\sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \boldsymbol{h}_{t,i,j}^2 = \boldsymbol{Z}_t$

Considering RVs $\mathbf{Z}_t$ are independent and have chi square distribution each with $2M_T M_R$ degree of freedom and the probability density function is given by (5.15). Putting the value of pdf from (5.15) into (6.10) for finding unconditional PEP, we get

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \leq \int_0^\infty \frac{1}{2} \prod_{t=1}^{l} e^{-\zeta_t^2 \boldsymbol{Z}_t} \frac{1}{(M_T M_R - 1)!} e^{-\boldsymbol{z}_t} z_t^{M_T M_R - 1} \, d\boldsymbol{z_t} \tag{6.11}$$

$$\leq \frac{1}{2(M_T M_R - 1)!} \prod_{t=1}^{l} \int_0^\infty e^{-\boldsymbol{Z}_t (\zeta_t^2 + 1)} \boldsymbol{z}_t^{M_T M_R - 1} \, d\boldsymbol{Z}_t \tag{6.12}$$

Putting $Z_t(\zeta_t^2 + 1) = Y_t$ into (6.12)

$$P(\boldsymbol{U_1} \to \boldsymbol{U_2}) \leq \frac{1}{2(M_T M_R - 1)!} \prod_{t=1}^{l} \int_0^\infty e^{-Y_t} \left(\frac{Y_t}{\zeta_t^2 + 1}\right)^{M_T M_R - 1} dY_t \tag{6.13}$$

Using the similar approach in solving (5.22), (6.13) reduces to

$$P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}) \leq \frac{1}{2} \prod_{t=1}^{l} \frac{1}{\left(\frac{\rho L_r}{4M_T M_R}(\boldsymbol{u}_{1,t}-\boldsymbol{u}_{2,t})^2+1\right)^{M_T M_R}} \tag{6.14}$$

$$P^{RAS} = P(\boldsymbol{U_1} \rightarrow \boldsymbol{U_2}) \leq \frac{1}{2} \prod_{t=1}^{l} \left(\frac{\rho L_r}{4M_T M_R}\left(\boldsymbol{u}_{1,t} - \boldsymbol{u}_{2,t}\right)^2 + 1\right)^{-M_T M_R} \tag{6.15}$$

Again assuming, since **U=[U**[1],**U** [2]…… **U** [D]] are independent then the total PEP of inner space time code word with antenna selection

$$P(\boldsymbol{U} \rightarrow \boldsymbol{U}') = \prod_{d=1}^{D} P^{RAS}\left(\boldsymbol{U_{1,d}} \rightarrow \boldsymbol{U_{2,d}}\right) \tag{6.16}$$

Since all 'D' received vector symbols in the codeword may be stacked together, we may calculated the PEP of outer coder with antenna selection as

$$P(\boldsymbol{c} \rightarrow \boldsymbol{c}') \leq P(\boldsymbol{U} \rightarrow \boldsymbol{U}') =: \frac{1}{2}\prod_{d=1}^{D}\left(\frac{\rho L_r}{4M_T M_R} \left|\left(\boldsymbol{U_{1,d}} \rightarrow \boldsymbol{U_{2,d}}\right)\right|^2 + 1)^{-lM_T M_R}\right) \tag{6.17}$$

If we set $L_r = M_R$ the bound in the above expression reduce to `the pair wise error probability for the full complexity system that is when the receiver uses all the available antenna

$$P(\boldsymbol{c} \rightarrow \boldsymbol{c}') \leq P(\boldsymbol{U} \rightarrow \boldsymbol{U}') =: \frac{1}{2}\prod_{d=1}^{D}\left(\frac{\rho}{4M_T} \left|\left(\boldsymbol{U_{1,d}} - \boldsymbol{U_{2,d}}\right)\right|^2 + 1)^{-lM_T M_R}\right) \tag{6.18}$$

By comparing equation (6.17) and (6.18) it is clear that the diversity order is maintained with antenna selection but loss in the coding gain which is upper bounded by 10 $\log_{10} M_R/L_r$ dB

## 6.5 Results and discussion

### 6.5.1 Analytical details and Results

We first verify the effectiveness of the derived upper bounds for the concatenated LDPC-STBC system with antenna selection by plotting the bit error rate with variation in SNR. The number of transmit antennas are taken a 2 and receive antennas are taken as 3 and 4 for varying selected antennas. The results are plotted for SNR varying from 1 dB to 10 dB. Figure 6.4 and figure 6.5 presents the analytical results for the bit error rate performance for varying number of selected antenna. As shown in table 6.1, increased diversity order requires less SNR at a given bit error rate.

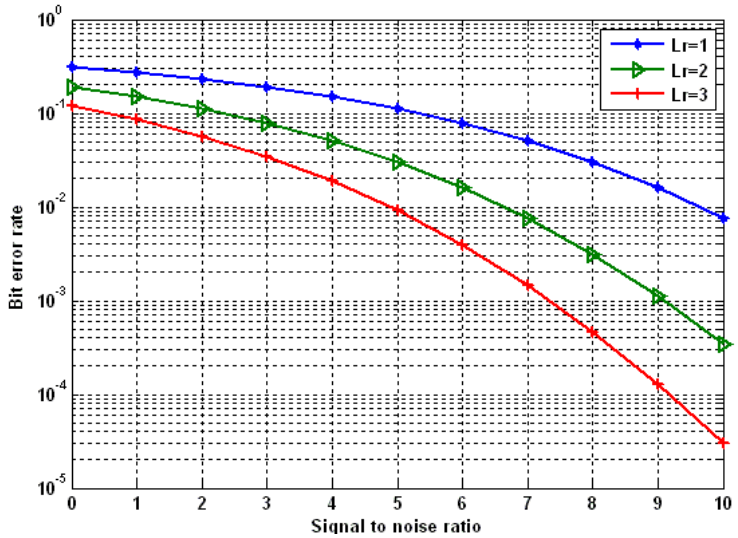


**Figure 6.4: BER for LDPC-STBC with antenna selection for $M_T$=2 $M_R$=3, $L_r$=1, 2, 3**

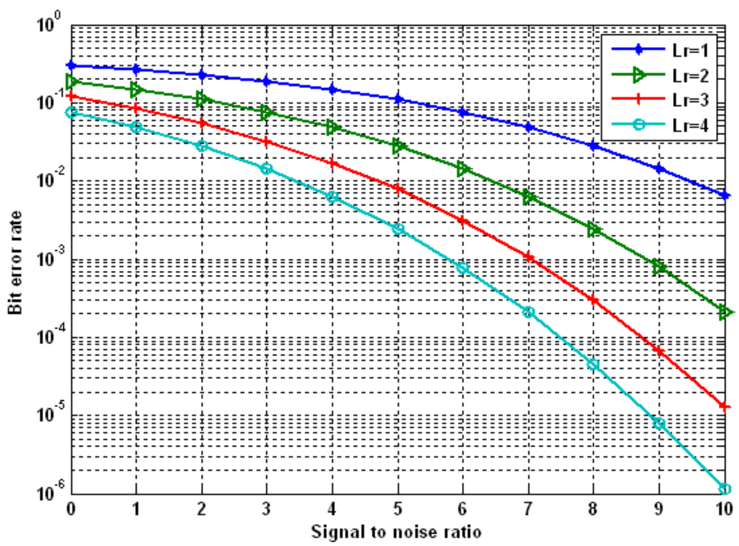


**Figure 6.5: BER for LDPC-STBC with antenna selection for $M_T$=2 $M_R$=4, $L_r$=1, 2, 3 and 4**

**Table 6.1: SNR required for concatenated LDPC-STBC system for varying selected antenna's at BER 1 x $10^{-2}$**

| Selected antennas ($L_r$) | SNR Required (in dB) ( $M_T$=2 $M_R$=3) | SNR Required (in dB) ( $M_T$=2 $M_R$=4) |
|---|---|---|
| **1** | **9.6** | **9.4** |
| **2** | **6.6** | **6.5** |
| **3** | **4.9** | **4.6** |
| **4** | **NA** | **3.4** |

### 6.5.2 Simulation Results

In this section, we have presented the simulation results of the concatenated LDPC-STBC system with antenna selection. It is assumed that perfect channel state information is available at the receiver end. For simulations a quasi-static MIMO fading channel model is assumed. Further it is also assumed that the channel remains static during 'D' blocks of transmitted data and changes independently from one block to another block. A norm based antenna selection is done at the receiver end. The simulation results are evaluated for randomly generated LDPC (200,100) codes with Log Domain Sum-Product decoding algorithm. Distance spectrum for LDPC codes is calculated according to method given in [21]. To evaluate the BER, a block consisting of 100 frames is fed to LDPC encoder. The LDPC encoded signal is mapped into space and time domain creating orthogonal sequences that are transmitted from different antennas as a STBC signal. The modulation is assumed to be BPSK. Figure 6.6 and Figure 6.7 shows the comparison of proposed system for MPSK and MQAM techniques in I.I.D Quasi static Rayleigh fading channel. Table 6.2 and Table 6.3 shows that BPSK and QAM outperform other modulation schemes.

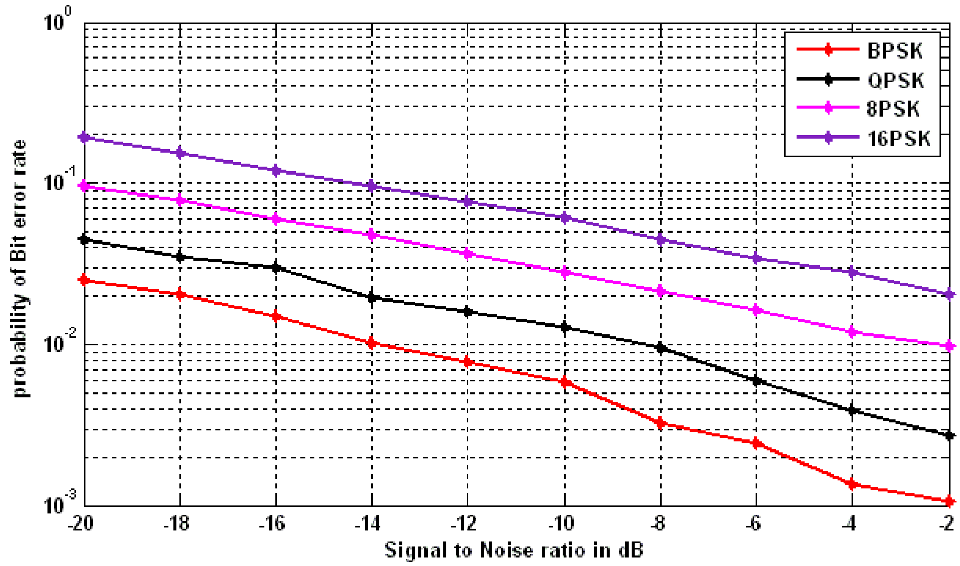


**Figure 6.6: Comparison of LDPC-STBC with antenna selection for MPSK modulation on I.I.D Quasi static Rayleigh fading channel**

**Table 6.2 : SNR required for concatenated LDPC-STBC system with antenna selection for MPSK modulation techniques with varying value of 'M' on I.I.D Quasi static Rayleigh fading channel at BER of 9 x $10^{-1}$**

| **Modulation scheme(M)** | **SNR Required (in dB)**<br>**( $M_T$=2 $M_R$=4, $L_r$= 2)** |
|---|---|
| BPSK, M=2 | -18 |
| QPSK, M=4 | -14 |
| 8-PSK, M=8 | -7.5 |
| 16-PSK, M=16 | -2 |

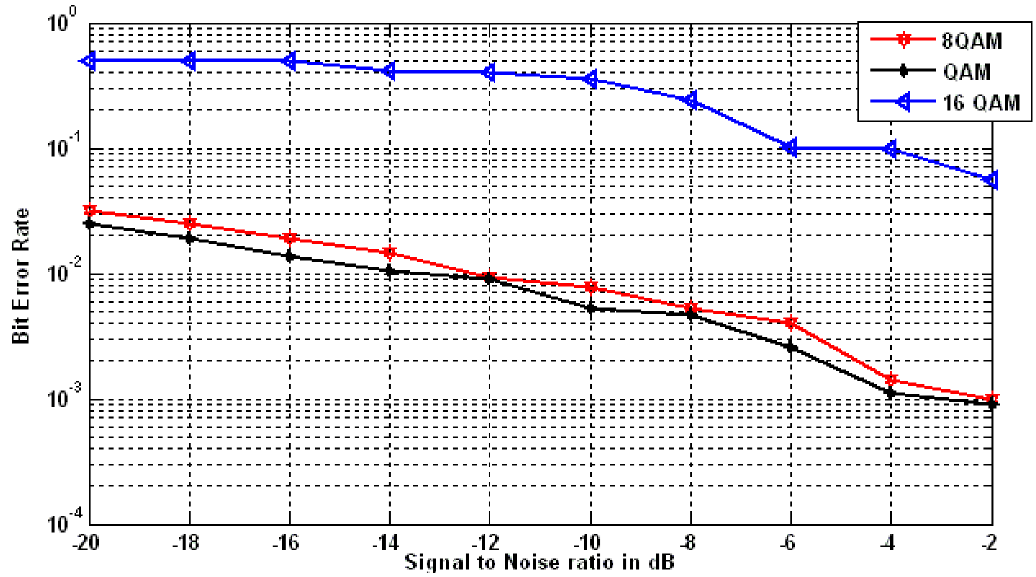


**Figure 6.7: Comparison of LDPC-STBC with antenna selection for MQAM modulation on I.I.D Quasi static Rayleigh fading channel**

**Table 6.3: SNR required for concatenated LDPC-STBC system with antenna selection for MQAM modulation techniques with varying value of 'M' on I.I.D Quasi static Rayleigh fading channel at BER of $1 \times 10^{-0.4}$**

| **Modulation scheme(M)** | **SNR Required (in dB)** **( $M_T$=2 $M_R$=4, $L_r$= 2)** |
|---|---|
| QAM, M=4 | -18 |
| 8QAM, M=8 | -16 |
| 16 QAM,, M=16 | >-2 |

### 6.5.3. Comparison of analytical results with simulation results

In this section we compare the derived upper bounds and simulated results of the concatenated LDPC-STBC system with antenna selection for BPSK modulation technique. The comparison of derived upper bounds of concatenated system and simulated results, with antenna selection is shown in figure (6.8) for the case of $M_T$=2, $M_R$ =3 and $L_r$ =1, 2 and 3. It is again clear that diversity order is maintained with antenna selection as the curves have the same slope at high SNR, which revalidates with our analytical results derived in (6.17). Moreover loss in SNR due to antenna selection for $L_r$=1 and $L_r$=2 are 5.3 dB and 3.3 dB at BER of $10^{-4}$ as compared to the case when $L_r$ = 3 i.e. no antenna selection is used. Also it is clear from the figure that gap between upper bounds and simulations gets smaller as $L_r$ approaches $M_R$ due to the fact that the approximation (6.8) becomes more accurate as $L_r$ increases. By comparing equation (6.17) and (6.18) it is clear that the diversity order is maintained with antenna selection but loss in the coding gain which is upper bounded by 10 $\log_{10} M_R/L_r$ dB. Table 6.4 provides the signal to noise ratio required for various selected antennas.

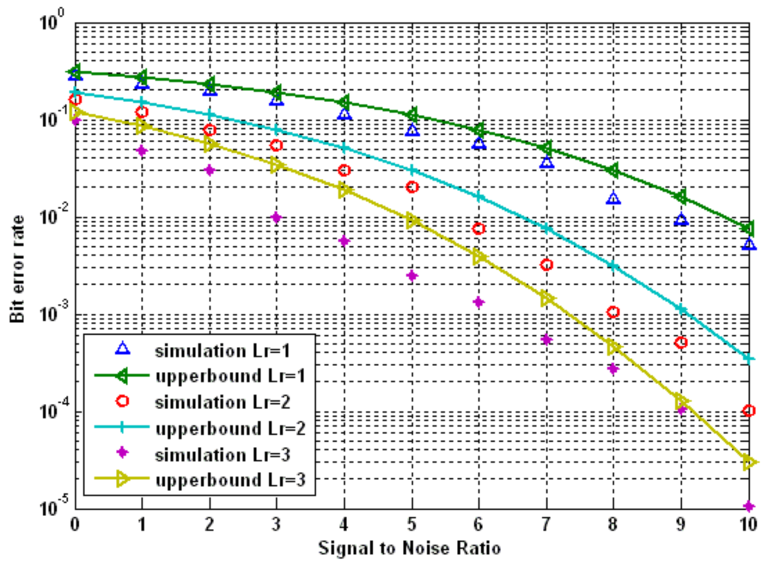


**Figure 6.8: Comparison of simulation with upper bounds of LDPC–STBC scheme with antenna selection when $M_T$=2, $M_R$=3 and Lr=1,2 and 3**

**Table 6.4: Comparison of analytical SNR required for concatenated LDPC-STBC system for varying selected antennas with simulation results**

| SNR Required in dB at BER of 1 x $10^{-2}$ | | |
|---|---|---|
| **Number of selected antenna's($L_r$)** | **Concatenated LDPC-STBC system with antenna selection $M_T$=2 $M_R$=3** | |
| | Simulated | Analytical |
| 1 | 8.9 | 9.6 |
| 2 | 5.6 | 6.6 |
| 3 | 3 | 4.7 |

## 6.6 Conclusion

We have investigated the performance of LDPC-STBC concatenated system with antenna selection in this chapter. We have considered the case when the fading is i.i.d. and receiver has the perfect CSI. Antenna selection based on received signal power is performed only at the receiver. We have shown analytically that the diversity gain does not change with antenna selection and therefore we can exploit the full diversity advantage promised by the MIMO system that uses all available antenna elements, provided that the space-time code employed has full spatial diversity. The upper bound of concatenated LDPC-STBC system, with norm based receive antenna selection, is derived using Chernoff bound. The upper bound for BER of orthogonal space time block coded system concatenated with LDPC codes, with selection has been analyzed and simulated for various modulation techniques in Rayleigh fading environment. With the proposed scheme the diversity order is maintained with antenna selection, but loss in the coding gain is upper bounded by 10 $\log_{10} M_R/L_r$ dB.

# *Chapter VII*

# *CONCLUSION AND FUTURE SCOPE*

## 7.1. Conclusion

Wireless communication using multiple-input multiple-output (MIMO) systems enables increased spectral efficiency for a given total transmit power. Increased capacity is achieved by introducing additional spatial channels that are exploited by using space-time coding. MIMO systems are a natural extension of developments in antenna array communication. The main advantages of multiple receive antenna is increase in gain and spatial diversity. MIMO systems provide a number of advantages over single-antenna-to-single-antenna communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths. Under certain environmental conditions, the power requirements associated with high spectral-efficiency communication can be significantly reduced. Although STBC scheme provides diversity gain but does not provide coding gain. Hence LDPC codes are concatenated with space time block codes.

The thesis is mainly devoted to the analysis of a bandwidth efficient communication scheme based on irregular LDPC codes in SIMO and MIMO system.

The first part research reported in this dissertation has been looked into the challenges posed by co-channel interference in SIMO system. In chapter 4, proposed LDPC-OC system was investigated for varying receive antennas in the presence of interference. The optimum combining based on optimal weight selection to improve the overall signal to interference plus noise ratio (SINR). The proposed algorithm is investigated for simulation and analytical based channels. We also extend our analysis to investigate the performance for single interferer in Independent Identically Distributed Fading and multiple interferers for under loaded and overloaded cases respectively. The tools of density evolution with iterative sum product decoder have been used to optimize irregular LDPC codes and to compute minimum operational signal-to-noise ratios for MIMO fading channels. With the assumption that full channel state information is available at receiver end, it was shown that the LDPC-OC system significantly reduces the SNR required in various fading channels even when the number of interferers is very large. Upper bound expressions for the probability of bit error are derived using hypergeometric functions are derived. Theoretical results are demonstrated by Monte Carlo simulations.

In chapter 5, we have shown that serial concatenation of low density parity check codes and space time block codes performs well in Rayleigh, Rician fading channels. We have shown on the quasi-static fading channel full diversity can be achieved and an increase in coding gain results from the concatenation. This scheme is modulated using different various modulation schemes. It is shown through simulation results that QAM achieves best bit error rate performance when compared with other schemes. The system is scalable in terms of the number of transmit as well as receive antennas.

We have derived the upper bounds for concatenated LDPC-STBC system for varying number of receive antennas. It is shown that LDPC codes provide significant coding gain and STBC provides the diversity gain. In general, the performance of the proposed codes will improve with increase in the length, as compared to other concatenated schemes (Convolution code, Reed-Solomon code). Therefore, the proposed scheme will offer higher advantage as the length of the code words increases.

In chapter 6, we investigated concatenated LDPC-STBC system based on antenna selection for a MIMO wireless system. We considered the case when the fading is independent and identically distributed, the receiver has the full CSI and antenna selection (based on received signal power) is performed only at the receiver. We have analytically shown that the diversity gain does not change and hence, we can exploit the full diversity advantage promised by the MIMO system that uses all available antenna elements, provided that the space-time code employed has full spatial diversity. For the proposed concatenated system with antenna selection, we have shown that the coding gain with antenna selection deteriorates significantly compared to the full-complexity one. Furthermore, we computed tighter upper bounds in closed form. We have also compared simulation results and observed that the results are in agreement with the theoretical analysis.

### 7.2. Future work

Chapter IV discussed the problem of co-channel interference in wireless scenario. Therefore it is shown that a optimum combining scheme with LDPC coded system achieves a best results in terms of bit error rate even. We showed in the same chapter that performance of the proposed system is optimum a) when there is single interferer b) when the number of interferers are very large, over the flat Rayleigh channel model, in which a desired signal competes with CCI. The case treated in this thesis is mainly that of equal-power interferers. In a typical cellular mobile radio signal environment, there will be several interfering signals whose power is close to that of the desired signal, and numerous interfering signals whose power is much less than that of the desired signal [3]. When the number of interferers exceeds the number of antenna elements, the array is unable to cancel every interfering signal. A moderate increase in SINR at the output of the antenna array can result in a significant increase in system capacity. This implies that the presented work can be extended for the unequal power interferers various fading channel, particularly correlated channels in which the adjoining array element can also act as one of the interferer.

Chapters 5 and 6 of this thesis studied the bandwidth-efficient LDPC coded MIMO communication systems with/without antenna selection, where the space diversity and time diversity are emphasized. Further studies can be carried out with frequency diversity techniques, i.e., using multi-carrier techniques such as orthogonal frequency division multiplexing (OFDM). These techniques have proved to have immunity to impulse noise and fading.

Another important future area of work is concatenation of LDPC with golden space time block codes which are recently developed as optimal space time block codes with full rate and full diversity gain.

The practical implementation of the bandwidth-efficient LDPC coded communication systems in hardware is also a natural topic for further studies. The hardware implementation of these systems can be implantation on the MIMO test bed or on the software defined radio (SDR) systems. Software Defined Radio (SDR) system is a useful and adaptable future-proof solution to cover both existing and emerging standards, it provides elements with re-configurability, intelligence and software programmable hardware. In addition, the emerging user requirements on reconfigurable mobile systems and networks are paving the way for the introduction of re-configurability in future mobile systems. The SDR implementation of encoder/decoder algorithms is becoming more suitable for high-speed real-time applications because SDR system uses both DSP processor and FPGA process to compute the result.

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