

**IMPLEMENTING MULTIPLICATIVE DIMENSIONAL
REDUCTION METHOD FOR PROBABILITY BASED
SEISMIC ANALYSIS OF RCC FRAMES**

A Thesis Submitted in Fulfilment of the Requirement for the Award of the Degree of

MASTER OF ENGINEERING

IN

STRUCTURAL ENGINEERING

Submitted By

SAVLEEN TAKKAR

(801624026)

Under Supervision of

Dr. SANDEEP K. SHARMA

Associate Professor
Mechanical Engineering Department

Dr. SHRUTI SHARMA

Associate Professor
Civil Engineering Department



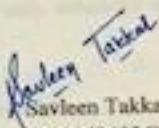
THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

CIVIL ENGINEERING DEPARTMENT
THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY
(A DEEMED TO BE UNIVERSITY), PATIALA, PUNJAB
JUNE, 2018

DECLARATION

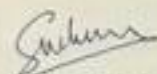
I, Savleen Takkar, hereby declare that the work presented in this thesis entitled "Implementing Multiplicative Dimensional Reduction Method For Probability Based Seismic Analysis Of RCC Frames" in fulfilment of the requirement for the award of degree of Master of Engineering (Structures) submitted at the Civil Engineering Department, Thapar Institute of Engineering & Technology (Deemed to be University), Patiala, Punjab is an authentic record of work carried out under supervision of Dr. Sandeep Kumar Sharma (Associate Professor, Department of Mechanical Engineering, TIET) and Dr. Shruti Sharma (Associate Professor, Department of Civil Engineering, TIET) from January, 2018 to June, 2018.

Date: August 24, 2018


Savleen Takkar
(801624026)

CERTIFICATE

This is to certify that the above declaration made by the student concerned is correct according to the best of our knowledge and belief. Moreover, the matter presented in this has not been submitted either in part or full to any other university or institute for the award of any other degree.



Dr. Sandeep Kumar Sharma
Associate Professor
Department of Mechanical Engineering
TIET (Deemed to be University)
Patiala, Punjab


24/8/18

Dr. Shruti Sharma
Associate Professor
Department of Civil Engineering
TIET (Deemed to be University)
Patiala, Punjab

ACKNOWLEDGEMENT

At this eventual point of my Masters, I'd like to express the deepest appreciation to my supervisor, Associate Professor, Dr. Sandeep Kumar Sharma, who possesses the attitude and substance of a genius. Without his guidance and persistent help, this dissertation wouldn't have been possible.

Secondly, I'd like to express my heartfelt gratitude to my co-supervisor, Associate Professor, Dr. Shruti Sharma for the continuous support in my thesis, for her patience, motivation, and immense knowledge.

Besides my advisors, I owe my sincere appreciation to Dr. Heaven Singh for his discussion and advices in the area of seismic analysis. His constructive feedback truly enhanced my work.

Nevertheless, a very special gratitude goes out to all my friends in the impact hub...

And finally, I'd say that words always fall short for all that our families do for us. So, herein, words aren't enough to express my thankfulness to my adorable parents, Sr. Jaswinder Singh Takkar and Mrs. Parveen Takkar, and my loving sister, Nimar, for their incessant support, encouragement and love throughout my life, and for showing me the right path always.

Savleen Takkar
ME-Structures

ABSTRACT

Likelihood of the presence of uncertainties in engineering systems is an unhidden, yet, widely accepted fact. Uncertainties are usually encountered in input variables (Loading, Material properties, Geometrical properties, etc.), in response variables (Displacements, Stresses, etc.), and in the relationships between them. All these uncertainties can be dealt with the aid of 'Reliability Analysis', thereby, providing the engineers accurate predictions of the probability of a structure performing adequately during its lifetime.

Hence, it can be ascertained that 'Uncertainty Analysis' of any structural system is an important part of engineering probabilistic analysis. Uncertainty analysis incorporates: (a) Evaluation of the statistical moments of the response, (b) Assessment of the entire probabilistic distribution of the response, and (c) Computation of the parametric sensitivity analysis of the system. The actual model of system's response is usually a high-dimensional function of input variables. Although Monte Carlo Simulation (MCS) has been standardized for the same purpose, yet, it may necessitate extra analytical efforts to achieve an acceptable level of accuracy, especially for the analysis of complex deterministic systems. Hence, development of a robust, computationally effective and easy-to-implement framework is genuinely necessitated to overcome the potential inhibitions involved in the MCS' implementation for the reliability analysis of structures.

As an effective substitute to MCS, this study proposes "Multiplicative Dimensional Reduction Method (MDRM)" to ease out the reliability analysis of structural systems. Further, this study advances 'MDRM' by combining it with the "Maximum Entropy (MaxEnt) Principle", wherein, 'Fractional Moments' and not traditional 'Integer Moments' are considered as constraints. This novel computational approach allows fairly accurate estimations of both the statistical moments and the probabilistic distribution of the response of interest. In addition, the proposed scheme provides the 'Global-Variance based Sensitivity Indices' as a by-product. Therefore, no extra computational efforts are necessitated for sensitivity analysis. The entire work is performed by integrating Microsoft Excel and MATLAB.

The efficiency and efficacy of the proposed approach for the structural reliability analysis is demonstrated through pilot study and two main studies. The pilot study is based upon the 'Minimum Tensile Reinforcement Required in Beams'. The main studies implement MDRM for probability based "Seismic Analysis of a 4—Storeyed RCC Frame", with a different pattern of input variables.

To sum up, it can be envisaged that the proposed approach is computationally affordable without compromising accuracy, as it saves lot of time, thereby yielding similar results with reasonable accuracy as provided by time consuming and computationally expensive standard technique like MCS. The results of such work have significance in future studies for the estimation of the probability of the response exceeding a safety limit and for establishing safety factors related to acceptable probabilities of structural failures.

TABLE OF CONTENTS

CHAPTER	Page No.
<i>Declaration</i>	ii
<i>Acknowledgement</i>	iii
<i>Abstract</i>	iv
<i>Table of Contents</i>	vi
<i>List of Tables</i>	ix
<i>List of Figures</i>	xi
<i>List of Notations</i>	xiii
Chapter 1 Introduction	1
1.1 Reliability Engineering.....	1
1.2 Motivation.....	1
1.3 Objectives and Research Significance.....	3
1.4 Outline of the Dissertation.....	4
Chapter 2 Literature Review	5
2.1 Uncertainty Analysis.....	5
2.2 Methods for Reliability Analysis.....	7
2.2.1 Monte Carlo Simulation (MCS).....	8
2.3 Fractional Moment and Information-Theory Concepts.....	11
2.3.1 Fractional Moment.....	11
2.3.2 Entropy-Based Probability Distribution—Information Theory Concepts.....	13
2.3.2.1 General—Self-Information.....	13
2.3.2.2 Entropy.....	14
2.3.2.3 Maximum Entropy (MaxEnt) Principle.....	15
2.4 Sensitivity Analysis.....	15
Chapter 3 Multiplicative Dimensional Reduction Method (MDRM)	18
3.1 Introduction.....	18

3.1.1	Background.....	18
3.1.1	Objective.....	20
3.1.1	Organisation.....	21
3.2	Fundamentals of Multiplicative Dimensional Reduction Method (MDRM)	21
3.2.1	Background.....	21
3.2.2	Evaluation of The Statistical Moments Of The Response.....	22
3.2.3	Entropy Based Probability Distribution.....	23
3.2.4	Computational Effort.....	26
3.2.5	Global Sensitivity Analysis.....	26
3.2.5.1	<i>Primary Sensitivity Coefficient</i>	26
3.2.5.2	<i>Total Sensitivity Coefficient</i>	28
3.3	Gauss Quadrature Scheme.....	30
3.4	MDRM Implementation.....	33
3.4.1	Pilot Study—Reliability Analysis for the “Minimum Tensile Reinforcement Required in Beams”.....	35
3.4.2	Evaluation Of The Statistical Moments of the Response.....	36
3.4.3	Computation Of The Statistical Distribution of Response.....	38
3.4.4	Evaluation Of Sensitivity Coefficients.....	44
3.5	Conclusion.....	45

Chapter 4 MDRM Implementation—Seismic Analysis Of a 4- Storeyed RCC

	Structure.....	47
4.1	Introduction.....	47
4.2	Objective.....	48
4.3	Organisation.....	48
4.4	“Main Study-1”—Reliability Analysis Of a 4-Storeyed RCC Structure With '16' Input Variables.....	48
4.4.1	Description Of RCC-Frame.....	48
4.4.2	Input Grid For MDRM.....	52
4.4.3	Statistical Moments Of The Structural Response.....	56
4.4.4	Statistical Distribution Of The Response.....	59
4.4.5	Global Sensitivity Indices Using MDRM.....	62

4.5	“Main Study-2”—Reliability Analysis Of a 4-Storeyed RCC Structure With '203' Input Variables.....	62
4.5.1	General Description.....	62
4.5.2	Input Grid For MDRM.....	75
4.5.3	Statistical Moments Of The Structural Response.....	77
4.5.4	Probabilistic Distribution Of The Structural Response.....	79
4.5.5	Global Sensitivity Indices Using MDRM.....	82
4.6	Computational Time.....	84
4.7	Conclusion.....	84
Chapter 5 Conclusions And Recommendations.....		86
5.1	Summary.....	86
5.2	Conclusions.....	86
5.3	Recommendations For Future Research.....	87
References And Bibliography.....		89

LIST OF TABLES

Sr. No.	Table Details	Page No.
Chapter 3		
	<i>Table 3.1</i> Gaussian Integration Formulation for One-Dimensional Fractional Moment Calculation (<i>Zhang and Pandey, 2013</i>).....	33
	<i>Table 3.2</i> Weights and Points of the Five-order Gaussian Quadrature Rules (<i>Davis and Rabinowitz, 1984; Zwillinger, 2011</i>).....	33
	<i>Table 3.3</i> Statistics of Random Variables associated to the Minimum Tensile Reinforcement.....	35
	<i>Table 3.4</i> Input Grid for the Response Evaluation.....	36
	<i>Table 3.5</i> Output Grid for Each Cut-Function Evaluation.....	37
	<i>Table 3.6</i> Statistical Moments of the Response.....	38
	<i>Table 3.7</i> MaxEnt Parameters for the Minimum Tensile Reinforcement.....	39
	<i>Table 3.8</i> Global Sensitivity Indices Using MDRM.....	44
Chapter 4		
	<i>Table 4.1</i> Assumed Preliminary Data Required for the Analysis of Frame.....	50
	<i>Table 4.2</i> Statistics of Random Variables Associated to the Proposed Structural System.....	51
	<i>Table 4.3 (a)</i> Input Grid for the Response Evaluation.....	53
	<i>Table 4.3 (b)</i> Input Grid for the Response Evaluation.....	54
	<i>Table 4.3 (c)</i> Input Grid for the Response Evaluation.....	55
	<i>Table 4.4 (a)</i> Output Grid for Each Cut-Function Evaluation.....	56
	<i>Table 4.4 (b)</i> Output Grid for Each Cut-Function Evaluation.....	57
	<i>Table 4.4 (c)</i> Output Grid for Each Cut-Function Evaluation.....	57
	<i>Table 4.4 (d)</i> Output Grid for Each Cut-Function Evaluation.....	58
	<i>Table 4.5</i> Statistical Moments of the Response.....	58
	<i>Table 4.6</i> MaxEnt Parameters for the Base Shear.....	60
	<i>Table 4.7</i> Global Sensitivity Indices Using MDRM.....	62

<i>Table 4.8 (a)</i> Statistics of Random Variables associated to the Proposed Structural System.....	66
<i>Table 4.8 (b)</i> Statistics of Random Variables associated to the Proposed Structural System.....	67
<i>Table 4.8 (c)</i> Statistics of Random Variables associated to the Proposed Structural System.....	68
<i>Table 4.8 (d)</i> Statistics of Random Variables associated to the Proposed Structural System.....	69
<i>Table 4.8 (e)</i> Statistics of Random Variables associated to the Proposed Structural System.....	70
<i>Table 4.8 (f)</i> Statistics of Random Variables associated to the Proposed Structural System.....	71
<i>Table 4.8 (g)</i> Statistics of Random Variables associated to the Proposed Structural System.....	72
<i>Table 4.8 (h)</i> Statistics of Random Variables associated to the Proposed Structural System.....	73
<i>Table 4.8 (i)</i> Statistics of Random Variables associated to the Proposed Structural System.....	74
<i>Table 4.9</i> Input Grid for the Response Evaluation.....	76
<i>Table 4.10</i> Output Grid for Each Cut-Function Evaluation.....	78
<i>Table 4.11</i> Statistical Moments of the Response.....	79
<i>Table 4.12</i> MaxEnt Parameters for the Base Shear.....	80
<i>Table 4.13</i> Global Sensitivity Indices Using MDRM.....	83

LIST OF FIGURES

Sr. No.	Figure Details	Page No.
Chapter 2		
	<i>Figure 2.1</i> Interference of Strength and Load Variables (<i>scanned from Christensen and Baker, 1986</i>).....	6
	<i>Figure 2.2</i> Typical work flowchart for ‘Monte Carlo Simulation (MCS)’ (<i>R.H. Crawford and S.S. Rao, 1989</i>).....	9
Chapter 3		
	<i>Figure 3.1</i> Typical work flowchart for “Multiplicative Dimensional Reduction Method (MDRM)”.....	34
	<i>Figure 3.2 (a)</i> “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ‘ $m = 2$ ’	41
	<i>Figure 3.2 (b)</i> “Probability of Exceedance (POE)” Plot for ‘ $m = 2$ ’	41
	<i>Figure 3.3 (a)</i> “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ‘ $m = 3$ ’	42
	<i>Figure 3.3 (b)</i> “Probability of Exceedance (POE)” Plot for ‘ $m = 3$ ’	42
	<i>Figure 3.4 (a)</i> “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ‘ $m = 4$ ’	43
	<i>Figure 3.4 (b)</i> “Probability of Exceedance (POE)” Plot for ‘ $m = 4$ ’	43
Chapter 4		
	<i>Figure 4.1</i> Plane Frame Structure and Its Lumped Mass Model (1).....	49
	<i>Figure 4.2</i> Plan showing the Columns and Beams at “Floor” of the Plane Frame.....	49
	<i>Figure 4.3</i> “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots.....	61
	<i>Figure 4.4</i> “Probability of Exceedance (POE)” Plot.....	61
	<i>Figure 4.5</i> Plane Frame Structure and Its Lumped Mass Model (2).....	63
	<i>Figure 4.6 (a)</i> Plan showing the Columns and Beams at “Floor-1” of the Plane Frame...64	
	<i>Figure 4.6 (b)</i> Plan showing the Columns and Beams at “Floor-2” of the Plane Frame...64	
	<i>Figure 4.6 (c)</i> Plan showing the Columns and Beams at “Floor-3” of the Plane Frame...64	
	<i>Figure 4.6 (d)</i> Plan showing the Columns and Beams at “Roof” of the Plane Frame.....65	

<i>Figure 4.7</i> “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots.....	81
<i>Figure 4.8</i> “Probability of Exceedance (POE)” Plot.....	81

LIST OF NOTATIONS

Notation	Significance
$b(x)$	-- Simulation result
$b_i(x_i)$	-- One-dimensional cut-function
b_0	-- Response Constant
COV	-- Coefficient Of Variation
$E[\]$	-- Mathematical “Expectation Operation”
$Ent[f]$	-- True Entropy
$f_s(s)$	-- Probability Density Functions (PDF) of strength variable
$f_l(l)$	-- Probability Density Functions (PDF) of load variable
$\iint f_{s,l}(s, l)$	-- Joint Probability Density Function (Joint-PDF)—(1)
$f_X(x)$	-- Joint Probability Density Function (Joint-PDF)—(2)
$f_Y(y)$	-- Probability Density Function
$\bar{f}_Y(y)$	-- Estimated Probability Density Function
$g(x)$	-- Limit state function
j^{th}	-- Gauss Quadrature Point
M_δ^Y	-- Fractional Moment
n	-- Number of Independent Input Random Variables
N	-- Total number of MCS trials—(1)
N	-- Number of Gauss Quadrature points—(2)
N_{fail}	-- Number of trials for which a structural failure is indicated by the limit state function
p_{fail}	-- Probability of failure
$P_{f,MCS}$	-- Probability Of Failure In Monte Carlo Simulation
q	-- Any Positive integer
r	-- Co-ordinates of Cut-point
S_i	-- Primary Sensitivity Coefficient

$S(\cdot)$	--	Self-Information (Measure Of Uncertainty)
S_{Ti}	--	Total Sensitivity Coefficient
$T[f, \bar{f}]$	--	Cross-Entropy
Var_Y	--	Variance of the Scalar Response
$'w_j$	--	Weights of the quadrature points
x	--	Input Random Variables' vector
X	--	Any Random Variable
Y	--	Output Response Function
y_{cr}	--	Safety threshold
z	--	Any continuous positive random variable
z_j	--	Gauss point
δ	--	Any real number (fraction, not an integer)
\mathcal{L}	--	Lagrangian Function
λ_i	--	Lagrangian Multiplier
φ_i	--	Mean Of Cut-Function
τ_i	--	Mean-Square Of Cut-Function
μ_y	--	Mean of the Output Response Function
μ_{2y}	--	Mean of the Output Response Function
σ_Y	--	Standard Deviation of the Output Scalar Response

CHAPTER 1

INTRODUCTION

1.1 RELIABILITY ENGINEERING

“Reliability Engineering” came into being as a separate discipline, at first, in the United States Of America during the mid of 20th century, specifically during the early 1950s. The theory of reliability grew owing to the complexity as well as inevitability of problems, related to the electronic systems designed for use in the “Second World War”. It was ascertained that amidst the war, more than half of the equipment, either shipped or in storage became unserviceable or inoperative, even before its utilisation. Owing to these difficulties, “Reliability” was made an imperative step to be followed at an early stage in the design, development and production of any new system.

Meanwhile, the infrastructure was being developed as well, thereby leading to the construction of large-scale and complex structures. Although the safety aspects of structures were being studied, be it the study on fatigue life of materials or the pattern of loading systems, yet the failure data was not substantial, or, say, was relatively scarce, to achieve a certain level of confidence in the field of reliability. However, Reliability is not something, that is accomplished accidentally; it has to be integrated into the system as an inherent characteristic to generate confidence and hence, success.

1.2 MOTIVATION

In today’s fiercely competitive world, every engineering system is envisaged to perform effectively over its anticipated life span; the field of “Structural Engineering” is no exception. No system is said to be reliable, unless it is analysed explicitly for “reliability”. The concept of reliability holds great prominence because an engineer must design a structure that shall perform satisfactorily both in theory and practice. Since, uncertainties are unavoidable in practicality, therefore, “Reliability Analysis” emerges as a mandatory step, which has to be applied to quantify the structural safety. In each field problem, be it a bridge design problem, design of industrial structures, or design to withstand dynamic loading, the success holds upon

satisfying the reliability requirements to a sound level. Thus, a perfect amalgam of any design method with the reliability analysis is gaining popularity in engineering practice.

Having said about the integration of the reliability with the structural systems, it can be further emphasized that reliability is an investigative concept, encompassing statistical and engineering aspects. The concept necessitates sound organisation for the recognition and elimination of the causes of failures like static failure, fatigue failure, instability, etc. In general, any structural component, be it a beam, column, or a slab, is assumed to be distinctive in conventional design. The analysis and design of the component is dependent on geometry, material properties and the loads it is supposed to bear. Although in conventional design, the nominal values of all these parameters are presumed to be unique, yet in practice, all would be random variables in nature. The values of these random parameters would move around their respective mean (or expected) values in the probabilistic analysis of structures.

The explanation lies in the fact that during the rolling process of structural members in a rolling mill, the shaping rollers won't remain as it is, but would undergo gradual wear, thereby leading to the variations in the geometry of a member being rolled. Furthermore, fabrication and assembling procedures may also bring in such variations. The loads, be it dead, live or dynamic, are also considered to be varying parameters owing to deviations in the specific weight densities of materials used as well as fluctuations in the working environment around the structure.

Furthermore, "Monte Carlo Simulation (MCS)" is one such powerful engineering tool, which is widely recognised for its numerical accuracy as far as the statistical analysis and prediction of uncertainties in engineering systems is concerned. However, the method which can approximate accurate solutions for a problem can become computationally expensive, whenever it is applied for statistical analysis of the complex deterministic systems, like Dynamic Analysis of Structures, Finite Element Analysis, etc. which take long time to get evaluated. In addition, this technique doesn't perform sensitivity analysis on its own. Some extra analytical efforts are necessitated for the global-variance based sensitivity analysis, if we're using MCS for the reliability. Thus, the computational cost involved in implementing MCS in such scenarios may prove to be deterrent, despite MCS being a versatile, well-understood and easy-to-implement method.

Thus, development of a robust, computationally effective and easy-to-implement framework is genuinely necessitated to overcome the potential inhibitions involved in the MCS'

implementation for the reliability analysis of structures; Henceforth, this becomes the main motivation behind this research investigation.

1.3 OBJECTIVES AND RESEARCH SIGNIFICANCE

The main focus of this work is to evolve and implement a computational method for both the reliability and sensitivity analyses of structures, while catering to the uncertainties in applied loads, materials and geometrical properties, etc. The research has the following specific objectives:

- ✚ Develop a robust and computationally efficient “***Multiplicative Form of Dimensional Reduction Method (M-DRM)***” to approximate the structural response and the probabilistic distribution of the same response in practical structural systems, considering uncertainties in the input parameters;
- ✚ Develop a robust and computationally efficient “***Multiplicative Form of Dimensional Reduction Method (M-DRM)***” to estimate the probabilistic distribution of the structural response, in conjunction with the ‘*Maximum Entropy Principle (MaxEnt Principle)*’, where ‘*Fractional Moments*’ are considered as constraints;
- ✚ Implement “***Multiplicative Dimensional Reduction Method (MDRM)***” to estimate the structural response and its statistical variation in the ‘*Pilot Study*’ based on the “*Minimum Required Tensile Reinforcement in Beams*”;
- ✚ Implement “***Multiplicative Dimensional Reduction Method (MDRM)***” in the “*Seismic Analysis (Response Spectrum Method) of a 4-storeyed Structure*”, subjected to input uncertainties, thereby examining the efficiency and efficacy of the developed method vis-à-vis MCS.

Moreover, Realistic predictions can be acquired from the proposed framework for the reliability analysis and this can lead to future development of experimental, computational and analytical research programmes. Therefore, a logical criteria for strength and serviceability requirements of practical problems can be developed on this very basis.

1.4 OUTLINE OF THE DISSERTATION

Chapter 2 imparts an extensive literature review on the Uncertainty Analysis and Fundamentals of Fractional Moment and Information Theory. The Monte Carlo Simulation method is also reviewed, being one of the eminent reliability methods. The fundamentals of sensitivity analysis are discussed at the end of this chapter.

Chapter 3 imparts an in-depth view to the proposed method, i.e., MDRM, for estimating the probability statistics and distribution of the structural response, coupled with the sensitivity analysis. It also presents the concept of Gauss-Quadrature scheme, fundamentals of the Maximum Entropy Principle, and the associated computational cost of MDRM. The steps necessitated to apply the proposed method are expatiated through a pilot-study on the ‘Minimum Required Tensile Reinforcement in Beams’, with the consideration of uncertain input random variables.

Chapter 4 elaborates the applicability of MDRM for the “Seismic Analysis (Response Spectrum Method) of a 4 –storeyed building” and ‘Global-Sensitivity analysis’, subjected to input uncertainties, thereby examining the efficiency and efficacy of the same. The Monte Carlo Simulation technique is also applied, for the sake of comparison with the results of MDRM.

Chapter 5 recapitulates the results and conclusions of this research, coupled with some future research recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 UNCERTAINTY ANALYSIS

For structures to perform adequately during their lifetime, Civil Engineers have to analyse and design them as precisely as they can. However, even under identical conditions, some variables such as material properties, geometrical properties, applied loads, etc. may or may not possess exactly the same contemplated values. This suggests that the engineers have to deal with these uncertainties, as the adequacy of the performance of the structural systems as well as future projects is highly relied upon these readings.

In general, the sources of uncertainties are categorised as either '*Aleatory*' or '*Epistemic*'. The word "Aleatory" is derived from the Latin word "*Alea*", which signifies rolling of dice: And, "*Episteme*" is a Greek word for "Epistemic", signifying knowledge. Hence, uncertainty incorporating the randomness of the naturally occurring phenomenon, such as the earthquake statistics (magnitude and duration), defines the '*Aleatory*', while the uncertainty associated to the inaccuracies of science, owing to the dearth of knowledge or data, is categorised as '*Epistemic*' (*Der Kiureghian and Ditlevsen, 2009*). In addition, both sources of uncertainties are involved in almost every engineering problem, but the decision is all dependant on circumstances. Say, Consider "X" is any material property (a basic random variable); for example, "Characteristic Strength of Steel". So, if there is a conversation about the strength of steel in an existing building, then, it can be ascertained that specimens can be acquired from the site for testing, thereby, informing about the strength. Since, this information is all about the data, either substantial or scarce, therefore, the uncertainty in the same must be characterized as "Epistemic". However, the same uncertainty in a future building must be considered as "Aleatory", because the building is yet to be realized, even though, the control measures must have been taken in the manufacturing process. As far as the loading systems are concerned, the assessment of the source of uncertainty for both existing and future buildings somewhat differs, as every engineer tries to realize the future scenarios by gathering as much information as possible while designing. Hence, this context categorises load patterns in the "Aleatory" format.

Having said about the uncertainties, it can be stated that, since the characterization of ‘deterministic quantities of interest’ as ‘random variables’ is allowed in the probabilistic analysis, therefore, it can be used for incorporating these uncertainties in the analysis.

Likelihood of occurrence of a previously defined event defines the probability. Hence, the probability of a structure not performing adequately at a specific time is denoted as its ‘probability of failure’, while the complement of the same is termed as ‘reliability’ (*Madsen et al., 1986*).

$$\text{Reliability} = 1 - p_{fail} \quad (2.1)$$

Where, ‘ p_{fail} ’ is the ‘probability of failure’ of a structure. Any structural component is not considered to be safe or reliable, unless the strength of the same doesn’t exceed the measure of load acting on it. As stated before, these factors are treated as random variables for reliability; hence, there could be the possibility of interference in the respective distributed values of the independent strength and load variables, as shown in Figure (2.1).

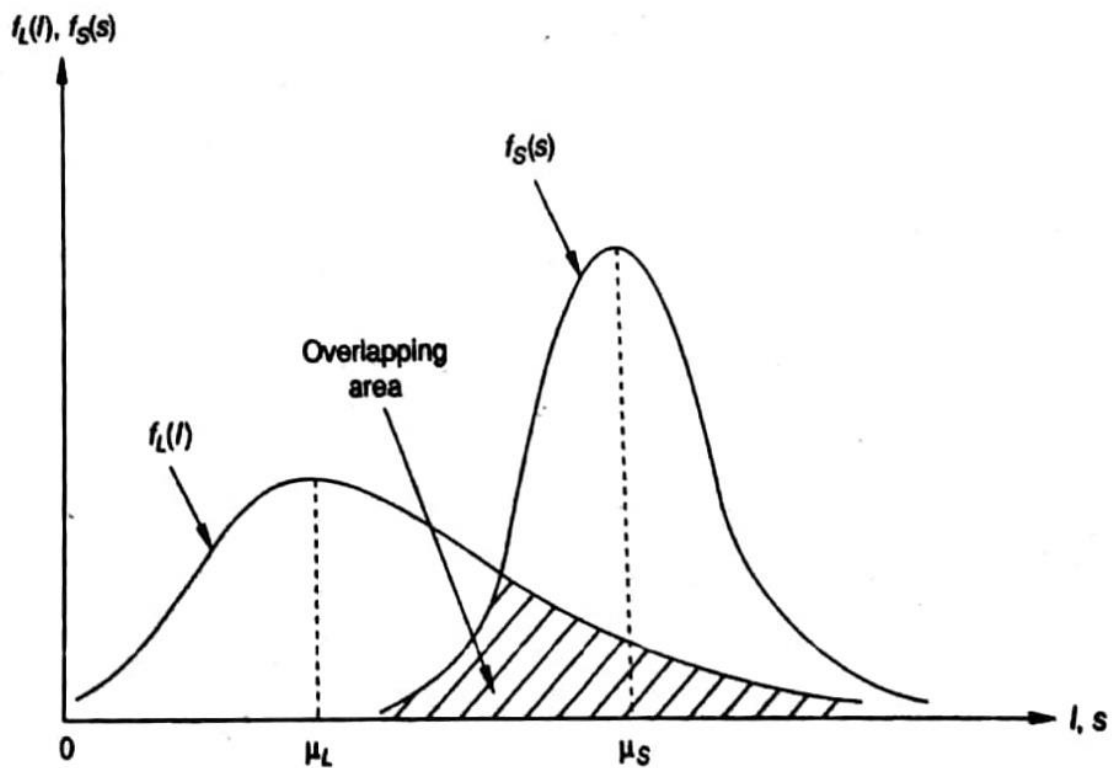


Figure 2.1 Interference of Strength and Load Variables (*Christensen and Baker, 1986*)

This overlapping of areas can be utilised for computing the probability of failure of the structural component. Say, we have the “Probability Density Functions (PDFs)” of strength and load variables, namely, $f_s(s)$ and $f_l(l)$, respectively, the reliability of a structural component can be formulated as (*Christensen and Baker, 1986*):

$$R = P(S > L) = P(S - L > 0)$$

$$R = \iint f_{S,L}(s, l) ds dl \quad (2.2)$$

where, " $\iint f_{S,L}(s, l)$ " signifies the “Joint Probability Density Function (Joint-PDF)”. The traditional methods of design, based on the concept of “Factor of Safety” are irrational, owing to the fact that the unchanged factor of safety constituting strength and load parameters might impart different probabilities of failure or reliabilities in different circumstances.

2.2 METHODS FOR RELIABILITY ANALYSIS

The main aim of the structural reliability analysis is to formulate the reliability or probability of failure of an engineering system with reference to a mathematically modelled failure criteria, meanwhile considering uncertainties, which arise in various formats. This section categorises well-established methods employed in the structural engineering domain to analyse reliability (*Nowak and Collins, 2000*):

- ✚ **Level 1 methods**, which incorporate ‘partial safety factors’ in order to ensure the adequacy of the reliability of the structure;
- ✚ **Level 2 methods**, which incorporate approximate methods, say, ‘First Order Reliability Method (FORM)’, to calculate the probability of failure on the basis of limit state function;
- ✚ **Level 3 methods**, which incorporate simulations, say ‘Monte Carlo Simulation (MCS)’, numerical integration, etc., where the probability of failure is calculated using the ‘Probability Density Function (PDF)’ of each input variable (random); The method is viewed in detail as a sub-part of this section further;
- ✚ **Level 4 methods**, where the cost and the benefits, related to the construction, maintenance, repairs, etc., of structures are also accountable. Moreover, this clause comes into picture for structures of valued economic importance, say, highway bridges, and, critical infrastructure components like nuclear powerplants, etc.

Generally, Reliability analysis proves to be a great aid to engineers to conclude if the structure has been designed effectively. Simulation analysis is considered to be the most accurate among all for the computation of the probability of failure. However, owing to its high computational cost, it can be questioned in the deterministic analysis of complex structures. Thus, it becomes imperative to develop a method, which shall allow the analysis of structures (probabilistic) within a feasible computational time.

2.2.1 Monte Carlo Simulation (MCS)

The Monte Carlo Simulation (MCS) method was first presented by “*Metropolis and Ulam (1949)*”. It is a numerical approach, an experiment performed analytically instead of being performed in a laboratory. The main originators of this method, *John Von Neumann and Stanislaw Ulam*, had named it after the casino games, typically the roulette wheel of the ‘Monte Carlo City’ situated at Monaco. The roulette wheel was the chosen one owing to its simplicity to generate random numbers. Similarly, this method necessitates the incorporation of many random (pseudo) numbers through the simulation process; Moreover, this method became widely acceptable with the evolution of computers.

As far as the structural reliability is concerned, the Monte Carlo method incorporates the construction of trial structures on a computer using generated random numbers accordingly. This further leads to the computation of percentage of structures failing in the analysis. It is an obvious statement that a large number of trials are essential to accomplish high confidence at lower levels of failure probability, thereby making the simulation technique a costly affair, though convenient.

Furthermore, the entire procedure of this method is demonstrated as a flow diagram, Figure (2.2);

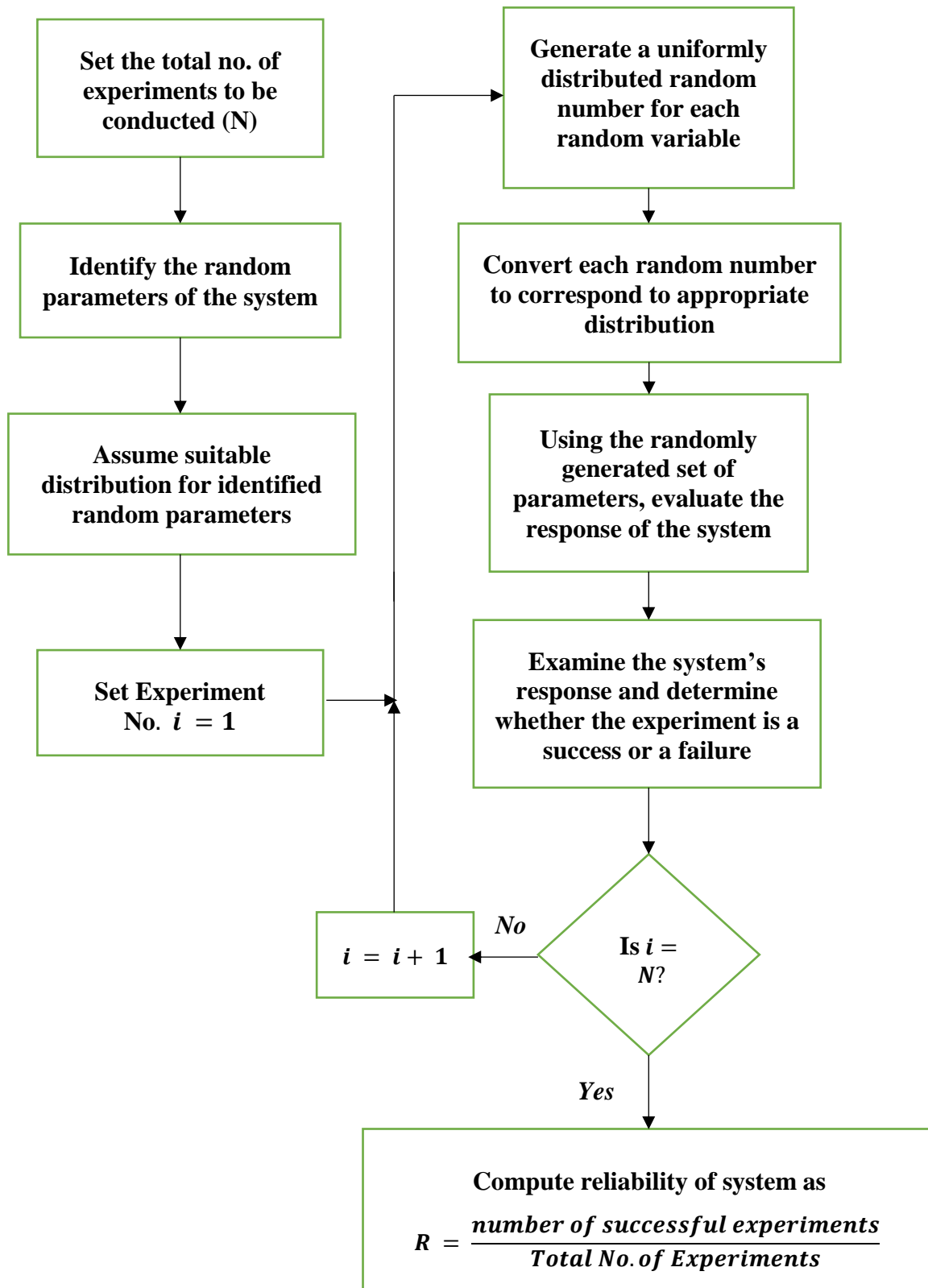


Figure 2.2 Typical work flowchart for ‘Monte Carlo Simulation (MCS)’ (R.H. Crawford and S.S. Rao, 1989)

It can be ascertained that the reliability analysis of structures incorporates the limit state function as (Nowak and Collins, 2000):

$$g(x) = y_{cr} - b(x) \begin{cases} > 0 \leftrightarrow \text{safe state} \\ = 0 \leftrightarrow \text{limit state} \\ < 0 \leftrightarrow \text{failure state} \end{cases} \quad (2.3)$$

where ' $g(x)$ ' denotes the limit state function, ' y_{cr} ' denotes the safety threshold, ' $b(x)$ ' denotes the simulation result and the input random variables' vector is represented by ' x ', i.e., $x = [x_1, x_2, \dots, x_n]$. Using Eqn. (2.2), the reliability coupled with the limit state function can be formulated as:

$$R = \int_{g(x) > 0} f_X(x) dx \quad (2.4)$$

where ' $f_X(x)$ ' denotes the "Joint Probability Density Function (Joint-PDF)" and ' $g(x) > 0$ ' signifies the safe domain. Since the limit state function isn't explicitly defined owing to high integral dimension, the Monte Carlo Approach simplifies the probability of failure practically, as follows (*Choi et al., 2007*):

$$P_{f,MCS} = N_{fail}/N \quad (2.5)$$

where ' N ' denotes the total number of MCS trials and ' N_{fail} ' denotes the number of trials for which a structural failure is indicated by the limit state function, i.e., $g(x) \leq 0$. Moreover, this simplification is accomplished by evaluating the limit state function for each variable through the deterministic code formulated for an output response function, ' $Y = b(x)$ '. In addition, the following equation evaluates the percent error incorporated in the estimation of the ' $P_{f,MCS}$ ' (*Shoorman, 1968*):

$$\% \text{ error} = 200 \sqrt{\frac{(1 - P_{f,MCS})}{N \times P_{f,MCS}}} \quad (2.6)$$

However, the following equation evaluates the efficiency of the Monte Carlo Method (*Madsen et al., 2006*):

$$COV = \sqrt{\frac{(1 - P_{f,MCS})}{N \times P_{f,MCS}}} \quad (2.7)$$

where, for the output response, the desirable *coefficient of variation* is denoted by ' COV '. According to practical considerations, the error incorporated would be lesser than that formulated above. Moreover, in practice, the measure of ' $P_{f,MCS}$ ' would be very small, indeed.

Therefore, the necessitated number of functional evaluations, i.e., the total number of MCS trials, is formulated as follows:

$$N \approx 1/(COV^2 \times P_{f,MCS}) \quad (2.8)$$

The probability of failure, in general, lies between 10^{-2} to 10^{-6} (Sudret and Kiureghian, 2002). Say, for any anonymous structure, consider a 10% 'COV' and 10^{-2} be its approximated probability of failure, then, the required number of MCS trials is 10^4 . Hence, the efficiency of MCS is relied upon the total number of required simulations, as shown in the formulations above.

2.3 FRACTIONAL MOMENT AND INFORMATION-THEORY CONCEPTS

Though integer moments do convey important characteristics about a distribution, but it can still be potentially misleading in the absence of the concerned distribution. The complete information regarding the random variable is provided by its probability distribution. The distribution associated with a particular random variable, be it in terms of 'Probability Density Function (PDF)' or 'Cumulative Distribution Function (CDF)', further augments the effective reliability analysis for an output response function. Thus, the probabilistic distribution of the structural response plays a key role in the risk analysis of engineering systems. Moreover, the distribution estimation requires fitting an analytical formulation in order to represent sampled measures precisely. Following the concept, there is an approach, which integrates fundamentals of the "Information Theory" with the "Principle of Maximum Entropy" to approximate the most unbiased probability distribution. Interestingly, to accomplish this task, the Maximum Entropy Principle (MaxEnt Principle) is coupled with the concept of "Fractional Moments", which are used as constraints in the process.

2.3.1 Fractional Moment

The Statistical Moment of any random variable, ' X ' is formulated as:

$$E[X^q] = \int_X x^q f_X(x) dx \quad (2.9)$$

where, ' q ' signifies the 'positively defined integer' and ' $f_X(x)$ ' signifies the 'continuous density function'. For ' $q = 1$ and ' $q = 2$ ', the respective statistical moments, i.e., mean and variance of a random variable can be evaluated. However, it can be stated that the above formulated statistical moment generally refers to a "Positive Integer Moment" (*Pandey, 2000*).

Thereafter, for any positive random variable ' X ', an ' δ^{th} ' order fractional moment can be formulated as (*Zhang and Pandey, 2013*):

$$E[X^\delta] = \int_X x^\delta f_X(x) dx \quad (2.10)$$

where, ' δ ' symbolizes a real number. Say, the probability distribution of ' X ' is given, then, an analytic expression for the above formulated fractional moment can be derived. As an example, ' X ' follows "Lognormal Distribution", with the probability density function as (*Walpole et al., 2007*):

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma x}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\} \quad (2.11)$$

Substituting Eqn. (2.11) in Eqn. (2.10), an analytic expression for an ' δ^{th} ' order fractional moment formulated as follows:

$$E[X^\delta] = \exp\left(\mu + \frac{\sigma^2}{2}\right) \quad (2.12)$$

Moreover, to validate the existence of a ' q^{th} ' order integer moment, the following formulation (*Piegorsch and Casella, 1985*) is the only necessary and sufficient condition, which needs to converge:

$$\int_0^\infty |x^q| f_X(x) dx = \int_0^\infty x^q f_X(x) dx < +\infty \quad (2.13)$$

Thereafter, it can be ascertained that as long as " $|\delta| \leq q$ ", where ' $q \geq 1$ ', the necessary and sufficient condition, formulated above, validates the existence of an ' δ^{th} ' order fractional moment.

Furthermore, there is an another query with reference to the concept of fractional moment, which needs to be answered; and that is, what led to the emergence of interest in the application of fractional moments with the entropy (explained further) concept? The answer lies in the practical shortcomings, usually encountered while employing traditional Maximum Entropy Principle (explained further) coupled with integer moments (*Inverardi et al., 2003*). Out of all

the practical difficulties, the thorny one is that a reasonable accuracy is achieved only when a large number of statistical moments (*order* ≥ 4) are employed to model the response distribution (*Pandey and Aiaratnam, 1996; Tagliani, 1994, 1999*), unlike the case when fractional moments are employed, wherein, a finite number of FMs are required to determine the same (*Gzyl and Tagliani, 2010*). The former case further destabilises the entropy maximization algorithm, thereby incorporating large statistical errors in the characterization of the distribution. Moreover, the inverse case also holds upon an additional problem, as a group of moments can't derive the distribution of each random variable in a precise manner. Hence, as a part of traditional method, the prior assumption regarding the distribution of each input random variable has had to be undertaken, thereby holding upon contentious and spurious inference. This is not so as far as the fractional moments are concerned, because herein, the measures of fractions typically required are less than '2', and these can be reliably approximated. Therefore, the distributions of random variables are not to be assumed in the latter case. To finalise, it can be inferred that the shortcomings encountered in the traditional entropy method can be overcome by the use of fractional moments.

2.3.2 Entropy-Based Probability Distribution—Information Theory Concepts

2.3.2.1 General—Self-Information

The “Modern Information Theory” is basically a quantitative approach to conceptualise the fundamentals of information with reference to probabilistic theory and statistics. It conceptualises the fact that the realization of any random event imparts knowledge regarding the event itself, and moreover, this knowledge or information in regard to the event varies inversely with the probability of occurrence of an event (*Pandey, 2000*).

To formulate a mathematical expression, say, consider a random event, ' Y ', which can hold upon ' n ' different states, ' Y_1, Y_2, \dots, Y_n ', with their respective probabilities of occurrence ' p_1, p_2, \dots, p_n ', meanwhile satisfying the natural constraints ' $p_i > 0$ ' and ' $\sum_{i=1}^n p_i = 1$ '; Then, the self-information in regard to a random event, ' Y_i ', is formulated as follows:

$$S(Y_i) = \log\left(\frac{1}{p_i}\right) = -\log p_i \quad (2.14)$$

The above formulated logarithmic measure for information was first introduced by (*Hartley, 1928*). However, this measure is incorporated owing to the following intuitive reasons: Any deterministic event, say, ' $p_i = 1$ ', imparts negligible information, or else, the rarer an event is, say, ' $p_i \ll 1$ ', a substantial amount of information is imparted by the realisation. Moreover, the direct relation between the self-information of an event and the associated uncertainty is clearly conveyed by Eqn. (2.14). In addition, the growth in uncertainty signifies the reduction in the probability of occurrence. To sum up, it can be ascertained that ' $S(\cdot)$ ' is reckoned as a measure of uncertainty (*Jones, 1979*).

2.3.2.2 Entropy

If the probabilities of occurrence of ' n ' different states of a random event, ' Y ', are known at a predefined level, then, is it possible to anticipate the outcome ' Y_i ' of a random experiment in advance? To answer the same, we can say that the level of difficulty encountered in this prediction depends upon the degree of uncertainty associated with ' Y ' (*Shannon, 1949*).

However, a measure of uncertainty has been defined by *Shannon (1949)*, to which he referred as “Entropy (positive and permutationally symmetric measure)”, usually accounted in thermodynamics and statistical mechanics. Furthermore, utilising Eqn. (2.14), formulated to mathematically express the concept of “Self- Information”, *Shannon's entropy* can be formulated as statistical expectation of ' $S(Y_i)$ ',

$$Ent[Y] = - \sum_{i=1}^n p_i \log p_i \quad (2.15)$$

Similar to the concept of uncertainty associated with the fundamental of self-information, the entropy holds on negligible value for an entirely certain outcome, and attains the maximum measure, when all outcomes are equally likely to occur, i.e., all are equi-probable. Moreover, Eqn. (2.15) holds on for discrete random variable. For continuous positive random variable, ' z ', with the probability density function, ' $f_z(z)$ ', the entropy is formulated as follows (*Pandey, 2000*):

$$Ent[f] = - \int_z f_z(z) \log[f_z(z)] dz \quad (2.16)$$

In addition, the axiomatic characterization of entropy and its other mathematical properties can be referred to in “*Kapur and Kesavan, (1992)*”.

2.3.2.3 Maximum Entropy (MaxEnt) Principle

The “Maximum Entropy Principle (MaxEnt) Principle” has been postulated as a rational approach by “*Jaynes (1957)*”; This approach holds upon choosing the most consistent distribution with minimum spurious information, among all probability distributions. The statement of a principle is as follows: “The most *unbiased* approximation of a probability distribution is the one which maximizes the entropy, or say, uncertainty, further bound by constraints, i.e., statistical moments of a random variable”. The chosen distribution is said to be unbiased owing to the fact that its formulation incorporates a well-ordered maximization procedure of uncertainty with reference to an unknown information. Thus, it can be ascertained that “Consistency” is a pivotal requirement in mathematical formulations. To ensure the same, “*Shore and Johnson (1980)*” postulated the following four consistence axioms:

- 1) **Uniqueness:** The final results should be invariably unique;
- 2) **Invariance:** The axes of reference shouldn't play any role in the quantification of results;
- 3) **System Independence:** Any form of information in regard to other independent systems shouldn't matter;
- 4) **Subset Independence:** Any form of information in regard to other independent subsets shouldn't matter.

However, it has been proved that the entropy maximization procedure imparts rational inferences regarding the parent distribution, thereby satisfying all four axioms, stated above. In addition to this, the mathematical formulation for the Maximum Entropy Principle with fractional moments as constraints is viewed in detail in the next chapter, meanwhile incorporating “Lagrangian Function” to approximate the probabilistic distribution.

2.4 SENSITIVITY ANALYSIS

After we apply the reliability analysis, sensitivity analysis is necessitated as a diagnostic tool to validate the performance of the structure (*Saltelli et al., 2004*). To be precise, this computational approach helps the researcher to comprehend which of the specified input random variables affects the output the most and the least. Hence, the sensitivity analysis quantifies the variation of the output response in respect of the variation of each specified input random variable. Adding to this, the structural sensitivity analysis helps in optimizing the

structural designs, thereby, formulating the structures to be cost-effective, stable and reliable throughout their lifetime. Sensitivity analysis' methods are usually classified as “*Local*” and “*Global*” (*Saltelli et al., 2008*).

The former sensitivity analysis concentrates on the output response uncertainty, by subsequently varying input random variables around their respective fixed (nominal) values (*Sudret, 2008*). Quite a large number of techniques can be used to measure response uncertainty, say, finite-difference method, differentiation, etc. Measuring the output response by partial differentiation method may save our computational time, but the precision highly relies upon the degree of linearity of the problem (*Saltelli et al., 2008*).

On the other hand, the latter approach, i.e., the “Global Sensitivity Analysis” concentrates on the output response uncertainty by varying input random variables simultaneously (considering single input variable or an array of variables). This analysis utilises the entire variation domain of the randomised variables, unlike the case in local sensitivity analysis, wherein the variation is considered locally. This is how the most sensitive parameters affecting the output response are acquired by the analyst. The “Global Sensitivity Methods” are categorised as follows (*Saltelli et al., 2000; 2008*):

✚ “*Regression-based Methods*” – These methods predict the relationship among randomised variables (say, input and output); Naturally, a straight line defines the linearity in the most precise manner and this simplest ‘straight line concept’ between two variables defines the “Linear Regression”. In addition, the ‘degree of linearity’ between every input variable and the response output is computed by the ‘correlation coefficient’. Higher value of coefficient of determination, R^2 ($0 \leq R^2 \leq 1$), defines a better relationship (*Montgomery and Runger, 2003*). Although, multiple regressions can be used now-a-days, yet the accuracy isn’t achieved, when large-scale or complex or non-linear models are encountered (*Saltelli and Sobol’, 1995*).

✚ “*Variance-based Methods*” – These methods are also known by the name “Analysis of Variance” computational techniques. By this technique, the output variance is made equivalent to the summation variance of each input randomised variable. Moreover, the concept of “Conditional Variance” of each input is brought into consideration using multi-dimensional integration (*Zhang and Pandey, 2014*). Then, simulation techniques, say, MCS, are utilised to solve correlation ratios. As

stated earlier, such techniques incorporate higher computational cost owing to large-dimensional integration processes for every sensitivity measure (*Zhang and Pandey, 2014*). This fact has necessitated the development of a feasible alternative like “Dimensional Reduction Method”, be it Additive or Multiplicative (Factorized). Chapter-3 views the mathematical formulation for global sensitivity analysis rationally.

CHAPTER 3

MULTIPLICATIVE DIMENSIONAL REDUCTION METHOD (MDRM)

3.1 INTRODUCTION

3.1.1 Background

In order to conduct reliability analysis, a function of randomised input variables is formulated to model the structural response. For example, we'd like to evaluate the strength of a slab-column connection, for which, the “punching shear-strength” is one output variable of interest. This can be formulated as a function of certain variables (input variables), such as the effective thickness of slab, compressive strength of concrete, etc. The output variable shall be designated as:

$$Y = \mathbf{b}(\mathbf{x}) \quad (3.1)$$

where ' Y ' is the desired response and ' \mathbf{x} ' denotes the vector of input random variables, i.e., $\mathbf{x} = [x_1, x_2, \dots, x_n]$. Say, the probabilistic distribution of all input variables of ' \mathbf{x} ' is known, then, the failure probability of any structure can be evaluated as (*Nowak and Collins, 2000*):

$$p_{fail} = \mathbf{p}(\mathbf{y}_{cr} - \mathbf{g}(\mathbf{x}) \leq \mathbf{0}) \quad (3.2)$$

where ' p_{fail} ' denotes the probability of failure, and, ' \mathbf{y}_{cr} ' denotes the critical threshold, where each scalar response exceeding this critical value justifies the structural failure. Further, the “limit state function” is formulated as:

$$\mathbf{g}(\mathbf{x}) = \mathbf{y}_{cr} - \mathbf{b}(\mathbf{x}) \quad (3.3)$$

Moreover, the simplified formulation for the probability of failure is explained by the following formulated integral (*Der Kiureghian, 2008*):

$$p_{fail} = \int_{\{\mathbf{g}(\mathbf{x}) \leq \mathbf{0}\}} \mathbf{f}_x(\mathbf{x}) \, d\mathbf{x} \quad (3.4)$$

where ' $\mathbf{f}_x(\mathbf{x})$ ' denotes the ‘Joint Probability Density Function (PDF)’ of the predefined vector, ' \mathbf{x} ' and, $\{\mathbf{g}(\mathbf{x}) \leq \mathbf{0}\}$ describes the failure domain. As per “*Li and Zhang (2011)*”, the above integral can be solved by:

- ✚ “Direct Integration”, but, for real-case scenarios, the “Joint Probability Density Function (Joint-PDF)” is defined implicitly and, usually, the dimension of the integral is high, equivalent to the number of uncertainties;
- ✚ “Simulations”, say, Monte Carlo Simulation (MCS), but, this technique is computationally expensive and necessitates considerable effort;
- ✚ “Approximate Methods”, say, FORM and SORM (reliability methods, based on Taylor-series expansions), but, their accuracy is unjustified owing to non-linearity of limit state function.

There is another method to perform structural reliability, by the “Method of Moment”, as it doesn’t require iterations unlike approximate methods and its computational cost is much less in comparison to simulations. However, if an ‘ N -point scheme’ is considered to compute an ‘ n -dimensional integration’, there would be ‘ N^n ’ evaluations of the output response, thereby leading to an enormous computational cost. A recent approach, namely, “High-Dimensional Model Representation (HDMR)” (*Li et al., 2001*), also known as, “Dimensional-Reduction Method (DRM)” held upon by “*Rahman and Xu (2004) and Xu and Rahman (2004)*”, can effectively deal with this problem, because herein, the evaluations of the output response would reduce to ‘ Nn ’ from ‘ N^n ’. Further, the latter technique is generally categorised into “Additive DRM (A-DRM)” and “Multiplicative DRM (M-DRM)”. By definition, a sum or product of lower order functions, orderly written in an increasing hierarchy, is used to express a multivariate function, hence, respectively known as an “Additive DRM (A-DRM)” and “Multiplicative DRM (M-DRM)”.

Furthermore, the analysis procedure necessitates two basic steps; First, both forms, i.e., A-DRM and M-DRM, compute the integer moments, say, first statistical moment corresponds to the mean and the second moment corresponds to the variance of the output response; Then, the second step requires the computation of the fractional moments, using the “Maximum Entropy Principle (MaxEnt)”, with fractional moments (*Inverardi and Tagliani, 2003*) as constraints. These fractional moments are used to determine the probabilistic distribution of the output response. In addition, both forms can perform the former step, while, research (*Zhang and Pandey, 2013*) shows that it isn’t practical to use additive form for the evaluation of fractional moments. Hence, the multiplicative form can be employed, as it simplifies the evaluation of statistical and fractional moments of the output response of interest. Moreover, the concept of the multiplicative form of dimensional reduction method, also referred to as factorized HDMR, was first put forward by “*Tunga and Demiralp (2004; 2005)*”.

Exploring the concept further, the output response function ' $Y = b(x)$ ', is computed in respect of a fixed input point (reference), namely, the “cut point”, having co-ordinates ' r ' (*Li et al., 2001*):

$$r = (r_1, r_2, \dots, r_n) \quad (3.5)$$

where, ' r_1, r_2, \dots, r_n ' represents the mean values of the respective input random variables ' x_1, x_2, \dots, x_n '.

Therefore, with all input random variables fixed, except, ' x_i ', at their respective cut points (generally, mean values), an ' i^{th} ' cut-function is determined as:

$$b_i(x_i) = b(r_1, \dots, r_{i-1}, x_i, r_{i+1}, \dots, r_n) \quad (3.6)$$

By now, it can be ascertained, that this chapter deploys a new technique, namely, “Multiplicative form of Dimensional Reduction Method (M-DRM)”, to formulate the structural response, and its probabilistic distribution, by incorporating the fundamentals of MaxEnt Principle, meanwhile deploying fractional moments as constraints; This entire formulation is held upon without simulations. Having said that, a fact can be emphasized that the proposed framework is computationally affordable as it saves a lot of time, thereby yielding similar results as provided by time consuming and computationally expensive standard techniques like Monte Carlo Simulation (MCS).

3.1.2 Objective

The main aim of this research is to evolve a general computational method for both the reliability and sensitivity analysis of structures, which can successfully deal with the potential issues cum barriers, introduced earlier. To accomplish the same, a new computational technique, namely, MDRM, is presented, to derive the statistical and fractional moments and thereby the probabilistic distribution of the output, with no aid from simulations. Although, Monte Carlo Simulation (MCS) can estimate accurate solutions for any system, be it large scale or complex models, yet, the proposed method can surmount the computational cost and hence, the efficiency of the same.

3.1.3 Organisation

Here follows the organisation of this chapter; At first, the mathematical derivation of MDRM to compute the response function is presented in “Section 3.2”. The same section also presents the usage of the derived expression to evaluate the integer moments and response distribution. It further illustrates the concept of trials required to implement the proposed method. Sensitivity analysis is depicted in sequence thereafter, using the MDRM approach. Before the implementation of MDRM, there is a need to comprehend the concept of “Gauss Quadrature Scheme”, as it has to be the first step of execution. The same scheme is demonstrated in “Section 3.3”. The steps necessitated to apply the proposed method are expatiated through a pilot-study on the “Minimum Reinforcement Required in Beams in Tension” in “Section 3.4”. In order to compare the results, MCS has also been conducted in the same section. The chapter recapitulates the conclusions in “Section 3.5”.

3.2 FUNDAMENTALS OF MULTIPLICATIVE DIMENSIONAL REDUCTION METHOD (MDRM)

3.2.1 Background

The Additive form of HDMR (A-DRM) (*Li et al., 2001; Rahman and Xu, 2004*) approximates the scalar function in an additive form as follows:

$$Y = \mathbf{b}(x) \approx \sum_{i=1}^n \mathbf{b}_i(x_i) - (n - 1)\mathbf{b}_0 \quad (3.7)$$

Or,

$$Y = \mathbf{b}(x) \approx \mathbf{b}_1(x_1) + \mathbf{b}_2(x_2) + \dots + \mathbf{b}_n(x_n) - (n - 1)\mathbf{b}_0 \quad (3.8)$$

where ' $\mathbf{b}_i(x_i)$ ' represents a predefined one-dimensional cut-function and, ' \mathbf{b}_0 ' is a response constant, computed by fixing all input random variables to their respective mean values, as:

$$\mathbf{b}_0 = \mathbf{b}(r_1, r_2, \dots, r_n) = \mathbf{Constant} \quad (3.9)$$

The same approach is followed by the multiplicative form; yet, the logarithmic transformation of the response function is derived initially, which further approximates the same in the multiplicative form as:

$$Y = \mathbf{b}(x) \approx \mathbf{b}_0^{1-n} \times \prod_{i=1}^n \mathbf{b}_i(x_i) \quad (3.10)$$

Or,

$$Y = \mathbf{b}(x) \approx \mathbf{b}_0^{1-n} \times [\mathbf{b}_1(x_1) \times \mathbf{b}_2(x_2) \times \dots \times \mathbf{b}_n(x_n)] \quad (3.11)$$

Therefore, a scalar output function is formulated in a product form in the MDRM technique accordingly. The next section shows the simplification process for both the statistical and fractional moments of the response.

3.2.2 Evaluation Of The Statistical Moments Of The Response

M-DRM's formulation technique approximates the ' q^{th} ' integer moment of the output response function as:

$$E[Y^q] = E[(\mathbf{b}(x))^q] \approx E[(\mathbf{b}_0^{1-n} \times \prod_{i=1}^n \mathbf{b}_i(x_i))^q] \quad (3.12)$$

Or,

$$E[Y^q] \approx (\mathbf{b}_0^{1-n})^q \times E[(\mathbf{b}_1(x_1))^q] \times E[(\mathbf{b}_2(x_2))^q] \times \dots \times E[(\mathbf{b}_n(x_n))^q] \quad (3.13)$$

where $E[\]$ describes the mathematical “expectation operation”, say for $q = 1$, $E[Y^q] = E[Y]$ signifies the expected or, mean value of the scalar response. In addition to this, say, all input random variables are independent, Eqn. (3.12) can be formulated as follows:

$$E[Y^q] \approx \mathbf{b}_0^{q(1-n)} \prod_{i=1}^n E[(\mathbf{b}_i(x_i))^q] \quad (3.14)$$

Thereafter, the “mean” and “mean square” of “cut function” are defined as,

$$\varphi_i = E[\mathbf{b}_i(x_i)] \quad \text{and} \quad \tau_i = E[(\mathbf{b}_i(x_i))^2] \quad (3.15)$$

Utilizing the above formulated four equations (Eqns. 3.12-3.15), the mean (μ_y) and the mean square (μ_{2y}) of the output response function can be written as follows:

$$\mu_y = E(Y) \approx \mathbf{b}_0^{1-n} \times \prod_{i=1}^n \varphi_i \quad (3.16)$$

$$\mu_{2y} = E[Y^2] \approx \mathbf{b}_0^{(2-2n)} \times \prod_{i=1}^n \tau_i$$

Thereafter, the variance (\mathbf{Var}_Y) of the scalar response Y can be formulated as:

$$\mathbf{Var}_Y = \mu_{2y} - (\mu_y)^2 \approx (\mu_y)^2 \times \left[\left(\prod_{i=1}^n \frac{\tau_i}{\varphi_i} \right) - 1 \right] \quad (3.17)$$

where further, the square root of the variance computes the standard deviation (σ_Y) of the response ' Y '. The evaluation of any ' q^{th} ' product moment, say mean, variance, etc. of the output response necessitates the computation of the ' q^{th} ' moment of all the respective cut-functions using one-dimensional integration; This numerical integration process can be optimized using the quadrature scheme, commonly referred to as "Gauss-Quadrature Scheme". Say, a ' q^{th} ' product moment of an ' i^{th} ' cut-function can be computed as a weighted sum, as:

$$E[(\mathbf{b}_i(\mathbf{x}_i))^q] = \int_{\mathbf{x}_i} [\mathbf{b}_i(\mathbf{x}_i)]^q \mathbf{f}_i(\mathbf{x}_i) \mathbf{d}\mathbf{x}_i \approx \sum_{j=1}^N \mathbf{w}_j [\mathbf{b}_i(\mathbf{x}_j)]^q \quad (3.18)$$

where ' N ' denotes the number of Gauss Quadrature points, ' \mathbf{x}_j ' denotes the co-ordinates, ' \mathbf{w}_j ' denotes the weights of the quadrature points and \mathbf{b}_i ($i = 1, 2, \dots, n$) denote the output structural response. In this process, an ' i^{th} ' cut-function is positioned at ' j^{th} ' Gauss quadrature point. This concept is demonstrated in a subsequent section through a pilot study.

3.2.3 Entropy Based Probability Distribution

The computation of the probabilistic distribution of the output response constitutes the second part of the model, after estimating its statistical moments. Here, we bring into usage the "Maximum Entropy Principle (MaxEnt Principle)" by "*Jaynes, 1957*", with fractional moments (*Inverardi and Tagliani, 2003*) as constraints, i.e., $[Y_\delta] = M_\delta^Y$, where, ' δ ' symbolizes a 'fraction', not an integer. This principle states that, 'The probability distribution, which best represents the current state of knowledge, is the one with the maximum entropy (uncertainty), in the context of available information' (*Shannon, 1949*) This concept depicts that the probabilistic distribution of any output variable can be computed using the available/already evaluated statistical moments of the same. Adding to this, the Probability Density Function (PDF), $[f_Y(\mathbf{y})]$ defines the true entropy $\{\mathbf{Ent}[f]\}$ of the output variable ' Y ', as:

$$Ent[f] = - \int_Y f_Y(y) \ln [f_Y(y)] dy \quad (3.19)$$

Furthermore, there is a function related to the MaxEnt Principle, named as ‘‘Lagrangian Function’’, formulated as:

$$\begin{aligned} \mathcal{L}[\lambda, \delta; f_Y(y)] = & - \int_Y f_Y(y) \ln [f_Y(y)] dy - (\lambda_0 - 1) \left[\int_Y f_Y(y) dy - 1 \right] - \\ & \sum_{i=1}^m \lambda_i \left[\int_Y y^{\delta_i} f_Y(y) dy - M_Y^{\delta_i} \right] \end{aligned} \quad (3.20)$$

where $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_m]^T$ symbolizes the ‘Lagrangian Multipliers’ and $\delta = [\delta_1, \delta_2, \dots, \delta_m]^T$ symbolizes the fraction incorporated with the fractional moments. To optimize the solution, the following key condition is applied:

$$\frac{\partial \mathcal{L}[\lambda, \delta; f_Y(y)]}{\partial f_Y(y)} = 0 \quad (3.21)$$

Thereafter, the estimated PDF $[\bar{f}_Y(y)]$ corresponding to the true PDF $[f_Y(y)]$ is formulated as:

$$\bar{f}_Y(y) = \exp \left(- \sum_{i=1}^m \lambda_i y^{\delta_i} \right) \quad (3.22)$$

According to the normalized condition, the integration of the PDF has to be equal to ‘one’. Thus, for $\mathbf{i} = \mathbf{0}$, $\delta = \mathbf{0}$, ‘ λ_0 ’ is formulated as:

$$\lambda_0 = \ln \left[\int_Y \exp \left(- \sum_{i=1}^m \lambda_i y^{\delta_i} \right) dy \right] \quad (3.23)$$

The above computation procedure illustrates the MaxEnt optimization procedure, with integer moments considered as constraints. However, increase in the order of the integer moments further increases the estimation error (*Pandey and Zhang, 2012*). This necessitates the usage of fractional moment constraints during the optimization procedure to compute the ‘estimated probability distribution’. Moreover, the research (*Pandey and Zhang, 2012*) shows the whole distribution can be efficiently recapitulated with just a few fractional moments, contrary to integer moments (*Zhang and Pandey, 2013*). For a positive random variable Y , the fractional moment is formulated as:

$$E[Y^\delta] = M_Y^\delta = \int_Y y^\delta f_Y(y) dy \quad (3.24)$$

where ' δ ' denotes a real number. Interestingly, a fractional moment can be evaluated using a convenient method provided by MDRM, as formulated in Eqn. (3.13),

$$\mathbf{M}_Y^\delta \approx (\mathbf{b}_0^{1-n})^\delta \times E[(\mathbf{b}_1(x_1))^\delta] \times E[(\mathbf{b}_2(x_2))^\delta] \times \dots \times E[(\mathbf{b}_n(x_n))^\delta] \quad (3.25)$$

where ' \mathbf{b}_0 ' symbolises the model response computed at the cut-point, as stated before, and each expected value $E[]$ is evaluated, as formulated in Eqn. (3.18),

$$E[(\mathbf{b}_i(x_i))^\delta] \approx \sum_{j=1}^N \mathbf{w}_j [\mathbf{b}_i(x_j)]^\delta \quad (3.26)$$

Thus, this computational approach estimates the fractions δ_i ($i = 1, 2, \dots, m$) during the entropy maximization procedure, i.e., they need not to be specified separately. Moreover, there is an alternate formulation available to evaluate the MaxEnt optimization along with fractional moments. This computational approach conceptualizes the minimization of the ‘‘Kullback-Leibler (K-L) Divergence’’, also known as ‘‘cross-entropy’’, between the true and estimated PDFs, i.e., between $[f_Y(y)]$ and $[\bar{f}_Y(y)]$ as (Kroese *et al.*, 2011):

$$T[f, \bar{f}] = \int_Y f_Y(y) \ln \left[\frac{f_Y(y)}{\bar{f}_Y(y)} \right] dy \quad (3.27)$$

$$T[f, \bar{f}] = \int_Y f_Y(y) \ln[f_Y(y)] dy - \int_Y f_Y(y) \ln[\bar{f}_Y(y)] dy \quad (3.28)$$

Using Eqns. (3.19) and (3.22) for $[Ent[f]]$ and $[\bar{f}_Y(y)]$ respectively to substitute in Eqn. (3.28) and considering Eqn. (3.24) meanwhile, the (K-L divergence) is formulated as:

$$T[f, \bar{f}] = -Ent[f] + \lambda_0 + \sum_{i=1}^m \lambda_i \mathbf{M}_Y^{\delta_i} \quad (3.29)$$

Furthermore, the Lagrangian multipliers (λ_i) and the fractional exponents (δ_i) constitute zero effect on the entropy $\{Ent[f]\}$ of the true PDF. Therefore, the above stated minimization approach is applied over the following formulation:

$$I(\lambda, \delta) = T[f, \bar{f}] + Ent[f] = \lambda_0 + \sum_{i=1}^m \lambda_i \mathbf{M}_Y^{\delta_i} \quad (3.30)$$

Thus, the MaxEnt parameters are computed by employing the following optimization:

$$\left\{ \begin{array}{l} Find: \{\delta_i\}_{i=1}^m \quad \{\lambda_i\}_{i=1}^m \\ Minimize: I(\lambda, \delta) = \ln \left[\int_Y \exp(-\sum_{i=1}^m \lambda_i y^{\delta_i}) dy \right] + \sum_{i=1}^m \lambda_i \mathbf{M}_Y^{\delta_i} \end{array} \right\} \quad (3.31)$$

This optimization is enforced in MATLAB, henceforth.

3.2.4 Computational Effort

An integration of MDRM with the Gaussian Quadrature scheme leads to an unparallel reduction in the number of evaluations of the output response function; Say, we've an output response variable, dependent on ' n ' independent input random variables. Then, an ' N – *point*' Gauss Quadrature scheme is employed upon for integration. Thus, all moments of a cut-function $b_i(x_i)$ can be computed from ' N – *evaluations*' of the output response. Moreover, ' nN ' response evaluations are necessitated for all such moments of all cut-functions. Also, the constant, namely, ' b_0 ' is also required to be evaluated, as stated previously. Hence, only $(nN + 1)$ evaluations are necessitated for the further calculation of all statistical moments and, the unbiased response distribution (probabilistic) in the M-DRM based analysis procedure. Suppose, we encounter a problem having ' 50 ' random variables; and we apply a ' 5 – *point*' Gauss-quadrature scheme. For this, we shall require ' 251 ' function evaluations.

To sum up, the “Multiplicative Dimensional Reduction Method (MDRM)” incorporates two steps; The first step evaluates all function evaluations (whatever number is required) on the basis of an input grid, to be defined beforehand. And then, the second step incorporates these function evaluations in the computation procedure of the moments and hence, the probability distribution of the response, using the entropy procedure.

3.2.5 Global Sensitivity Analysis

3.2.5.1 Primary Sensitivity Coefficient

The concept of ‘conditional variance’ is employed to look at the influence of any input random variable ' x_i ' in respect of the output scalar response ' Y ' (*Saltelli and Sobol', 1995*) as:

$$S_i = \frac{\text{Var}\{E_{-i}[Y|x_i]\}}{\text{Var}[Y]} = \frac{\text{Var}_i\{E_{-i}[Y|x_i]\}}{\text{Var}_Y} \quad (0 \leq S_i \leq 1) \quad (3.32)$$

where ' S_i ' symbolises the primary sensitivity coefficient (primary here indicates the main effect of an input random variable on the output function), and ' E_{-i} ' is the expectation operation (mean values) for all input variables except the ' i^{th} ' variable.

Further, the variance can be formulated as a subtraction of the square of the first product moment ($E[Y]$)² from the mean square $E[Y^2]$; Hence, ' S_i ' can be formulated as:

$$S_i = \frac{E_{-i}\{Y^2|x_i\} - \{E_{-i}[Y|x_i]\}^2}{Var_Y} \quad (3.33)$$

where the conditional expectations, ' $E_{-i}[Y|x_i]$ ' and ' $E_{-i}\{Y^2|x_i\}$ ', symbolize the first ($q = 1$) and the second ($q = 2$) statistical moments of the response function ' Y ', respectively.

Utilising the MDRM computational technique, the above stated conditional expectations are derived as follows:

$$\begin{aligned} E_{-i}[Y|x_i] &\approx E_{-i} \left[b_0^{(1-n)} \prod_{i=1}^n (b_i(x_i)) \right] \\ &= E \left[b_0^{(1-n)} \times b_i(x_i) \prod_{q=1, q \neq i}^n (b_q(x_q)) \right] \\ &= b_0^{(1-n)} \times E[b_i(x_i)] \times \prod_{q=1, q \neq i}^n E[b_q(x_q)] \end{aligned} \quad (3.34)$$

Utilizing Eq. (3.15), the first moment ' $E_{-i}[Y|x_i]$ ' can be further written as:

$$E_{-i}[Y|x_i] \approx b_0^{(1-n)} \times E[b_i(x_i)] \times \prod_{q=1, q \neq i}^n \varphi_q \quad (3.35)$$

Utilizing Eqns. (3.15) and (3.35), the second moment ' $E_{-i}\{Y^2|x_i\}$ ' can be formulated as:

$$E_{-i}\{Y^2|x_i\} = b_0^{(2-2n)} \times E\{[b_i(x_i)]^2\} \times \left(\prod_{q=1, q \neq i}^n \varphi_q \right)^2 \quad (3.36)$$

Or,

$$E_{-i}\{Y^2|x_i\} = b_0^{(2-2n)} \times \tau_i \times \prod_{q=1, q \neq i}^n \varphi_q^2 \quad (3.37)$$

Utilizing Eqns. (3.15) and (3.35), the square of the first moment ' $\{E_{-i}[Y|x_i]\}^2$ ' can be formulated as:

$$\{E_{-i}[Y|x_i]\}^2 = \mathbf{b}_0^{(2-2n)} \times \{E[b_i(x_i)]\}^2 \times \{\prod_{q=1, q \neq i}^n \varphi_q\}^2 \quad (3.38)$$

Or,

$$\{E_{-i}[Y|x_i]\}^2 = \mathbf{b}_0^{(2-2n)} \times \varphi_i^2 \times \prod_{q=1, q \neq i}^n \varphi_q^2 \quad (3.39)$$

Recollect the fact that, for all the cut-functions ' i ', the product of the squares of the first moments can be formulated as:

$$\prod_{i=1}^n \varphi_i^2 = \varphi_i^2 \times \prod_{q=1, q \neq i}^n \varphi_q^2 \quad (3.40)$$

Utilizing Eqns. (3.17), (3.37), and (3.39) to substitute into Eqn. (3.33) and taking into consideration Eqn. (3.40), the final form for the primary sensitivity coefficient ' S_i ' is as follows:

$$S_i \approx \frac{(\tau_i / \varphi_i^2)^{-1}}{(\prod_{i=1}^n \tau_i / \varphi_i^2)^{-1}} \quad (3.41)$$

3.2.5.2 Total Sensitivity Coefficient

The total sensitivity coefficient deals with the interactions among the specified input random variables, unlike the case of primary sensitivity coefficient. This coefficient doesn't prioritize the most influential input variables, rather it aims to identify the non-influential ones in a model (*Saltelli et al., 2008*). Here, initially, the focus is paid onto the reduction in variance, while holding all input random variables fixed at their respective mean values, except ' x_i '. This reduction in variance is denoted as ' $Var_{-i}\{E_i[Y|x_{-i}]\}$ ', where, ' x_{-i} ' defines a sub-vector of $(n - 1)$ elements, incorporating all the elements of ' x ', except ' x_{-i} '. Therefore, the remaining variance ' Var_{Ti} ' of the output response ' Y ', after holding ' x_i ' (*Saltelli et al., 2008*) is written as follows:

$$Var_{Ti} = Var_Y - Var_{-i}\{E_i[Y|x_{-i}]\} \quad (3.42)$$

Taking into consideration Eqn. (3.33), and substituting the 'Total Variance Identity' as ' $Var_Y = Var_{-i}\{E_i[Y|x_{-i}]\} + E_{-i}\{Var_i[Y|x_{-i}]\}$ ', The "Total Sensitivity Coefficient" is formulated as:

$$S_{Ti} = \frac{Var_Y - Var_{-i}\{E_i[Y|x_{-i}]\}}{Var_Y} = \frac{E_{-i}\{Var_i[Y|x_{-i}]\}}{Var_Y} \quad (3.43)$$

For the above formulation, the conditional variance ' $Var_i[Y|x_{-i}]$ ' shall be computed at first, meanwhile recollecting the condition stated for variance in the previous discussion on "Primary Sensitivity Coefficient", as:

$$Var_i[Y|x_{-i}] = E_i[Y^2|x_{-i}] - \{E_i[Y|x_{-i}]\}^2 \quad (3.44)$$

Utilising Eqn. (3.12) from the MDRM approximation procedure, the conditional expectation ' $E_i[Y|x_{-i}]$ ' is formulated as:

$$E_i[Y|x_{-i}] \approx E[b_0^{(1-n)} \times \prod_{q=1, q \neq i}^n (b_q(x_q) \times (b_i(x_i)))] \quad (3.45)$$

$$E_i[Y|x_{-i}] = b_0^{(1-n)} \times E[b_i(x_i)] \times \prod_{q=1, q \neq i}^n E[b_q(x_q)]$$

Taking into account Eqns. (3.15) and (3.45), the conditional expectation ' $E_i[Y^2|x_{-i}]$ ' can be formulated as:

$$E_i[Y^2|x_{-i}] \approx b_0^{(2-2n)} \times E[(b_i(x_i))^2] \times \prod_{q=1, q \neq i}^n E\{[b_q(x_q)]^2\} \quad (3.46)$$

Or,

$$E_i[Y^2|x_{-i}] \approx b_0^{(2-2n)} \times \tau_i \times \prod_{q=1, q \neq i}^n \tau_q^2 \quad (3.47)$$

Taking into account Eqns. (3.15) and (3.45), the conditional expectation ' $\{E_i[Y|x_{-i}]\}^2$ ' can be formulated as:

$$\{E_i[Y|x_{-i}]\}^2 \approx b_0^{(2-2n)} \times \{E[b_i(x_i)]\}^2 \times \prod_{q=1, q \neq i}^n \tau_q^2 \quad (3.48)$$

Or,

$$\{E_i[Y|x_{-i}]\}^2 \approx b_0^{(2-2n)} \times \varphi_i^2 \times \prod_{q=1, q \neq i}^n \tau_q^2 \quad (3.49)$$

Utilising Eqns. (3.47), (3.48) to substitute in Eqn. (3.44), then, the conditional variance ' $Var_i[Y|x_{-i}]$ ' is computed as:

$$\mathbf{Var}_i[\mathbf{Y}|\mathbf{x}_{-i}] \approx \mathbf{b}_0^{(2-2n)} \times \left(\prod_{q=1, q \neq i}^n \tau_q^2\right) \times (\tau_i - \varphi_i^2) \quad (3.50)$$

Further, the expectation of the conditional variance computed in the last equation, ' $\mathbf{Var}_i[\mathbf{Y}|\mathbf{x}_{-i}]$ ', is formulated as:

$$E_{-i}\{\mathbf{Var}_i[\mathbf{Y}|\mathbf{x}_{-i}]\} \approx \mathbf{b}_0^{(2-2n)} \times \left(\prod_{q=1, q \neq i}^n \tau_q^2\right) \times (\tau_i - \varphi_i^2) \quad (3.51)$$

Recollect the fact that, for all the cut-functions ' i ', the product of the mean squares can be formulated as:

$$\prod_{i=1}^n \tau_i^2 = \tau_i^2 \times \prod_{q=1, q \neq i}^n \tau_q^2 \quad (3.52)$$

Utilizing Eqns. (3.17) and (3.51), to substitute into Eqn. (3.43) and taking into consideration Eqn. (3.52), the final form for the total sensitivity coefficient ' \mathbf{S}_{Ti} ' is as follows:

$$\mathbf{S}_{Ti} \approx \frac{1 - (\varphi_i^2 / \tau_i)}{1 - \left(\prod_{i=1}^n \frac{\varphi_i^2}{\tau_i}\right)} \quad (3.53)$$

According to the definition, ' $\mathbf{S}_{Ti} \geq \mathbf{S}_i$ ', where, ' $\mathbf{S}_{Ti} = \mathbf{S}_i$ ' symbolises zero interaction among the input random variables. Hence, the difference between the two indices quantifies the interaction among the independent input random variables. Moreover, ' $\mathbf{S}_{Ti} = \mathbf{0}$ ' signifies that the ' i^{th} ' variable doesn't influence the variance of the response output; hence, being non-influential, it can be held anywhere in the distribution.

3.3 GAUSS QUADRATURE SCHEME

The "Gauss-Quadrature Scheme" helps in optimizing the numerical integration of an output function; Say, for the case of any "Lognormal Variable", the "Gauss-Hermite" integration scheme (*Zhang and Pandey, 2013*) can be employed, as shown further in Table (3.1).

For instance, the "Gauss-Hermite" quadrature scheme incorporates the approximation of the integral function in the following form (*Davis and Rabinowitz, 1984; Zwillinger, 2011*):

$$\int f_x(x) dx = \int e^{-x^2} b(x) dx \quad (3.54)$$

As per Table (3.1), the Gauss-Hermite integral is formulated as follows:

$$\int e^{-x^2} b(x) dx = \sum_{j=1}^N w_j \{b(x_j)\} \quad (3.55)$$

where, ' N ' signifies the number of evaluation/Gauss-quadrature points and w_j ($j = 1, 2, \dots, N$) signify the weights of the Hermite scheme. Moreover, the function $[b(x_j)]$ is computed on the basis of pre-defined evaluation points ' x_j ' and then, the integral is estimated as a weighted sum.

Gauss points (z_j), also referred to as “Abcissae”, and Gauss weights (w_j) for the fifth order quadrature scheme ($N = 5$) are recapitulated in Table (3.2), for various integration schemes. For orders other than '5', literature (*Davis and Rabinowitz, 1984; Zwillinger, 2011*) can be followed depending upon the required orthogonal polynomials.

The “Gauss- Hermite” scheme can be employed for the standard “Normal variable (Z)”; The probability density function PDF for such variable can be formulated as:

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \quad (3.56)$$

Thereafter, a ‘general’ normal random variable " X " gets associated with the ‘standard’ normal random variable " Z " in the following manner:

$$X = \mu + \sigma z_j \quad (3.57)$$

where ' μ ' and ' σ ' symbolize the mean value and standard deviation, respectively, of the Normal Distribution. Furthermore, the following transformation can be employed to relate the Gauss-Hermite point for $X(x_j)$ to the same for $Z(z_j)$, thereby going in-line with Eqn. (3.57),

$$x_j = \mu + \sigma z_j \quad (3.58)$$

where ' z_j ' are the Gauss points, tabulated further.

Since, the “Lognormal Distribution” is usually employed, whenever a random variable can have only positive values; therefore, transforming the PDF formulated for the normal distribution logarithmically, we get (say, X follows the lognormal distribution, if $\ln(X)$ follows the Normal one):

$$f(x) = \frac{1}{x\zeta\sqrt{2\pi}} \exp\left(-\frac{[\ln(x)-\theta]^2}{2\zeta^2}\right) \quad (3.59)$$

Where ' ζ ' symbolizes the location parameter and ' θ ' symbolizes the scale parameter in the lognormal distribution. These parameters are associated with the 'Normal Distribution' parameters, as:

$$\theta = \ln(\mu) - \left(\frac{1}{2}\zeta^2\right) \quad \text{and} \quad \zeta = \sqrt{\ln\left(1 + \frac{\sigma^2}{\mu^2}\right)} \quad (3.60)$$

Thereafter, a standard normal random variable ' Z ' is associated with the lognormal random variable ' X ', as:

$$Z = \frac{\ln(X) - \theta}{\zeta} \quad (3.61)$$

According to Eqn. (3.61), the following transformation is employed to associate the Gauss-Hermite point for $X(x_j)$ with the same for $Z(z_j)$,

$$x_j = \exp(\theta + \zeta z_j) \quad (3.62)$$

where ' z_j ' are the Gauss points, tabulated further.

Hence, for any random variable ' X ', following either normal or lognormal distributions, we can evaluate function evaluation points ' x_j ', utilizing Eqns. (3.55) to (3.62). The output response function $b(x_j)$, evaluated from each evaluation point, ' x_j ', is then multiplied with the weights, ' w_j ' (depending upon the Gaussian Quadrature). The final set of values, obtained after multiplication, are added, thereby, leading to the estimation of the integral, formulated in Eqn. (3.55).

Table 3.1 Gaussian Integration Formulation for One-Dimensional Fractional Moment Calculation (Zhang and Pandey, 2013)

DISTRIBUTION	SUPPORT DOMAIN	GAUSSIAN QUADRATURE	NUMERICAL INTEGRATION FORMULA
Uniform	[c,d]	Gauss-Legendre	$\sum_{j=1}^N w_j \left[\frac{1}{2} b \left(\frac{d-c}{2} z_j + \frac{d+c}{2} \right) \right]^q$
Normal	$(-\infty, +\infty)$	Gauss-Hermite	$\sum_{j=1}^N w_j [b(\mu + \sigma z_j)]^q$
Lognormal	$(0, +\infty)$	Gauss-Hermite	$\sum_{j=1}^N w_j [b\{\exp(\mu + \sigma z_j)\}]^q$
Exponential	$(0, +\infty)$	Gauss-Laguerre	$\sum_{j=1}^N w_j [b(z_j/\theta)]^q$
Weibull	$(0, +\infty)$	Gauss-Laguerre	$\sum_{j=1}^N w_j [b(\theta z_j^{(1/\delta)})]^q$

Legend: ' ζ' --Location parameter;
' θ' --Scale parameter;
' δ' --Shape Parameter.

Table 3.2 Weights and Points of the Five-order Gaussian Quadrature Rules (Davis and Rabinowitz, 1984; Zwillinger, 2011)

GAUSSIAN RULES	N	1	2	3	4	5
Gauss-Legendre	w_j	0.23693	0.47863	0.56889	0.47863	0.23693
	z_j	-0.90618	-0.53847	0	0.53847	0.90618
Gauss-Hermite	w_j	0.01126	0.22208	0.53333	0.22208	0.01126
	z_j	-2.85697	-1.35563	0	1.35563	2.85697
Gauss-Laguerre	w_j	0.52176	0.39867	0.07594	0.00361	0.00002
	z_j	0.26356	1.4134	3.5964	7.0858	12.641

Legend: w_j = Gauss weight;
 z_j = Gauss point;
 $N = 5$ for the Fifth-order Gauss Quadrature Rule.

3.4 MDRM IMPLEMENTATION

The following flowchart [Figure (3.1)] demonstrates the procedure to implement MDRM, wherein, the scalar output response ' Y' ' is computed analytically.

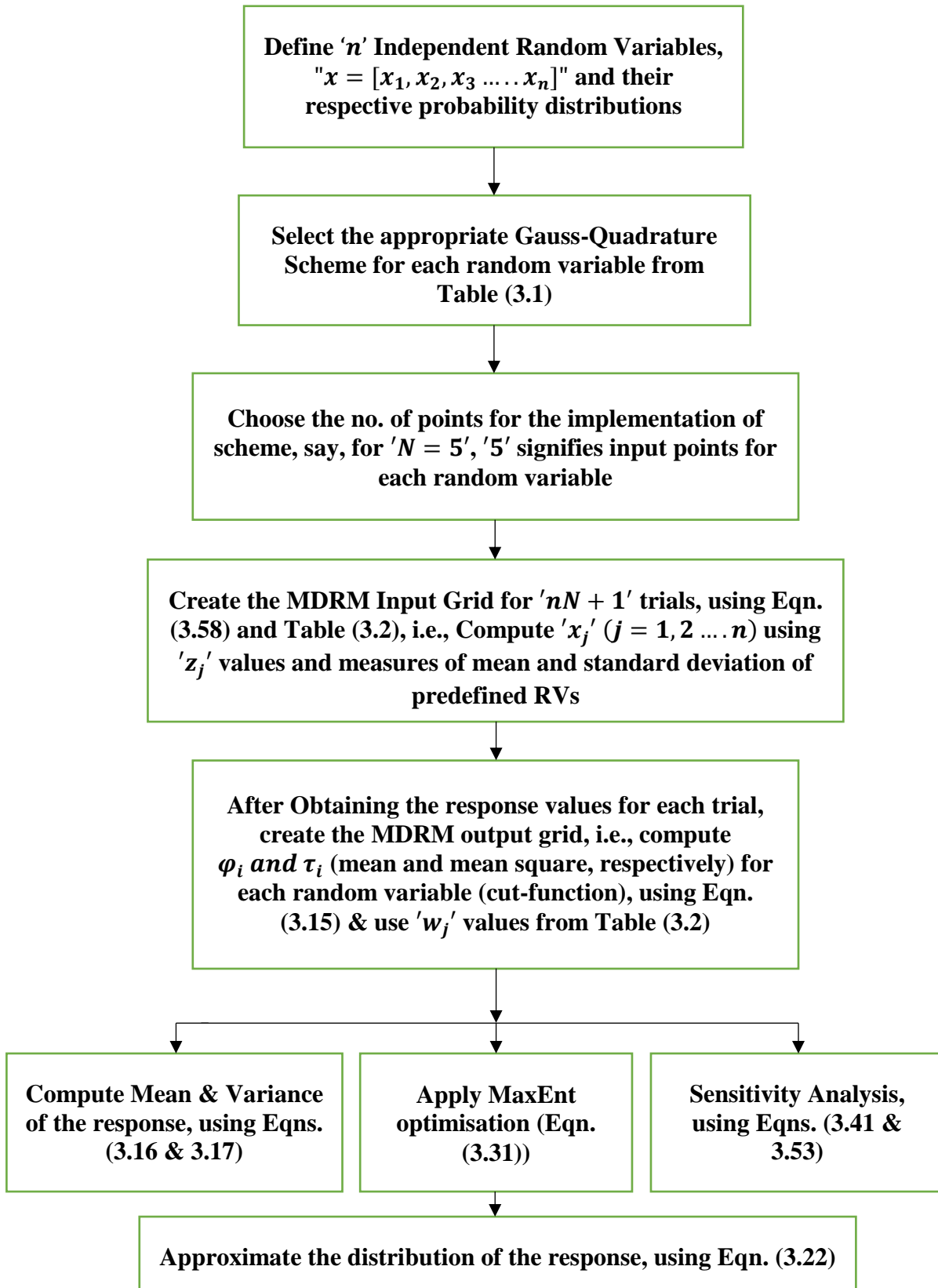


Figure 3.1 Typical work flowchart for “Multiplicative Dimensional Reduction Method (MDRM)”

3.4.1 Pilot Study—Reliability Analysis For The “Minimum Tensile Reinforcement Required In Beams”

The following pilot study on the “*Minimum Tension Reinforcement Required in Beams*” demonstrates the procedure to implement MDRM. As per the Indian Standards (*IS 456: 2000* – –“*Clause – 26.5.1.1*”), for beams, the “Minimum Area of Tensile Reinforcement required, i.e., " $A_{st_{min}}$ " is formulated as:

$$"A_{st_{min}}" = 0.85bd/f_y [f_y \text{ in MPa}] \quad (3.63)$$

where, ' b ' symbolizes the width of beam, " d " symbolizes the beam's effective depth, the characteristic strength of steel is denoted as " f_y ". For this study, '3' independent random variables are considered and all these respective input variables are assumed to follow the “Normal Probability Distribution”, Table (3.3).

Utilising the ‘Gauss-Quadrature Scheme’ at first, say for this pilot study, the “fifth-order” scheme would be followed, an input grid is tabulated to compute the response. Thereafter, the ‘Gauss Integration formula’ is used, depending upon the distributions followed by the input variables, say, in this study, all variables have been assumed to follow the ‘Normal Distribution’, then, the “Gauss-Hermite formula” would be adopted. Hence, there would be " $1 + 3 \times 5 = 16$ " response evaluations, as tabulated in Table (3.4); For each MDRM trial of a specified random variable, the remaining input random variables are held to their mean values. Out of "16" trials, "15" trials would be independent, while the last trial, i.e., the 16^{th} trial would be reserved as a mean case (all input variables are held to their means).

Table 3.3 Statistics of Random Variables associated to the Minimum Tensile Reinforcement

RANDOM VARIABLE	DISTRIBUTION	NOMINAL VALUE	MEAN VALUE	STANDARD DEVIATION	COV
f_y (MPa)	Normal	415	468.95	46.895	0.1
d (mm)	Normal	350	353.5	14.14	0.04
b (mm)	Normal	250	252.5	15.15	0.06

Table 3.4 Input Grid for the Response Evaluation

RANDOM VARIABLE	Trial No.	z_j	f_y (Mpa)	d (mm)	b (mm)	Ast_{min}
f_y	1	-2.85697	334.97	353.50	252.50	226.50
	2	-1.35563	405.38	353.50	252.50	187.16
	3	0	468.95	353.50	252.50	161.79
	4	1.35563	532.52	353.50	252.50	142.47
	5	2.85697	602.93	353.50	252.50	125.84
d	6	-2.85697	468.95	313.10	252.50	143.30
	7	-1.35563	468.95	334.33	252.50	153.01
	8	0	468.95	353.50	252.50	161.79
	9	1.35563	468.95	372.67	252.50	170.56
	10	2.85697	468.95	393.90	252.50	180.28
b	11	-2.85697	468.95	353.50	209.22	134.05
	12	-1.35563	468.95	353.50	231.96	148.63
	13	0	468.95	353.50	252.50	161.79
	14	1.35563	468.95	353.50	273.04	174.95
	15	2.85697	468.95	353.50	295.78	189.52
Fixed Mean Value	16	N/A	468.95	353.50	252.50	161.79

Legend: ' z_j ' denotes the Gauss-Hermite points

3.4.2 Evaluation Of The Statistical Moments Of The Response

The response has already been computed, as shown in the last table, for the **16** trials. Thereafter, using Eqn. (3.18), the mean φ_i and the mean square τ_i for an i^{th} cut-function are evaluated as weighted sum, as shown in Table (3.5); Then, Eqns. (3.16) and (3.17) are used from the MDRM's formulation technique, to evaluate the statistical/integer moments of the output response function, as shown in Table (3.6); Moreover, for this pilot study, the MCS technique of reliability has also been accomplished with "**1e5**" iterations in MATLAB. The results formulated further provides an upper-hand to the proposed MDRM, owing to much less number of trials in comparison to MCS method and that too with more or less the same numerical accuracy.

Table 3.5 Output Grid for Each Cut-Function Evaluation

RANDOM VARIABLE	Trial No.	w_j	Ast_{min}	$w_j * Ast_{min}$	φ_i	$w_j * Ast_{min} * Ast_{min}$	τ_i
fy	1	0.01126	226.50	2.55	163.4576	577.64	27002.8307
	2	0.22208	187.16	41.56		7779.10	
	3	0.53333	161.79	86.29		13959.90	
	4	0.22208	142.47	31.64		4507.89	
	5	0.01126	125.84	1.42		178.30	
d	6	0.01126	143.30	1.61	161.7885	231.22	26217.1251
	7	0.22208	153.01	33.98		5199.62	
	8	0.53333	161.79	86.29		13959.90	
	9	0.22208	170.56	37.88		6460.45	
	10	0.01126	180.28	2.03		365.94	
b	11	0.01126	134.05	1.51	161.7885	202.35	26269.4783
	12	0.22208	148.63	33.01		4905.77	
	13	0.53333	161.79	86.29		13959.90	
	14	0.22208	174.95	38.85		6797.02	
	15	0.01126	189.52	2.13		404.43	
Fixed Mean Value	16	N/A	161.79	N/A	N/A	N/A	N/A

Legend: ' w_j ' denotes the Gauss-Hermite weights

Table 3.6. Statistical Moments of the Response

$A_{st_{min}}$	MRDM (16 Trials)	MCS (1e5 Trials)	REL. ERROR (%)
First Moment (mm ²)	163.4600	163.4300	0.018356483
Second Moment (mm ⁴)	27143.95	27133.25	0.039435011
Std. Dev. (mm ²)	20.6000	20.5700	0.145843461
COV	0.1260	0.1259	0.127463581

Legend: M-DRM—Multiplicative Dimensional Reduction Method;
MCS—Monte Carlo Simulation;
Relative Error-- $ABS [MCS - MDRM] / MCS$;
Std. Dev.—Standard Deviation;
COV—Coefficient of Variation.

3.4.3 Computation Of The Statistical Distribution Of Response

In this section, the “MaxEnt Principle” is held upon, with fractional moments as constraints to predict the response’s probability distribution. The response " $A_{st_{min}}$ ", evaluated for "16" trials, using M-DRM’s formulation technique are integrated with the same principle. The MaxEnt parameters, i.e., " λ_i " and " δ_i " form the primary source in the approximation of the probability distribution of the response and are tabulated further. The main aim is to make the entropy converge rapidly; and for the same, the number of fractional moments generally necessitated is "4", as shown in the following Table (3.7);

Table 3.7 MaxEnt Parameters for the Minimum Tensile Reinforcement

		δ_1	δ_1	δ_2	δ_1	δ_2	δ_3	δ_1	δ_2	δ_3	δ_4
		0.9412	0.6294	0.8116	-0.4803	0.6001	-0.1372	0.3102	-0.6748	-0.762	-0.0033
	λ_0	5.9625	128.4059	703.7680316			706.050192				
	λ_1	0.0088	-22.6015	3784.411346			39.83743748				
	λ_2		6.9436	1.14772805			645.5959177				
	λ_3			-2115.754			3032.650045				
	λ_4						-995.2858726				
FRACTIONAL MOMENT	$M_X^\delta = \frac{1}{N} \sum_i^N x_i^\delta$	121.4144	24.7127	62.6330	0.0870	21.2813	0.4975	4.8536	0.0324	0.0208	0.9833
ENTROPY	$Ent[f] = \lambda_0 + \lambda_1 M_1 + \lambda_2 M_2 + \lambda_3 M_3 + \dots$	6.1219	4.4214	4.4052			4.4021				

This further gives the “Probability Density Function (*PDF*)” and “Cumulative Distribution Function (*CDF*)” plots of the minimum reinforcement required in beams as per the Indian Standards, for three entropy measures, i.e., $m = 2$, $m = 3$ and $m = 4$, in the following Figures (3.2), (3.3) and (3.4), respectively. The results themselves give an indication of the efficiency of the Multiplicative Form of Dimensional Reduction Method, with an integration of "16" trials and "3" fractional moments, as-in here, the entropy has converged rapidly. Herein, the “Probability of Exceedance (POE)” has also been traced to approximate the “probability of failure, i.e., p_{fail} ”, for the above stated entropy measures, in the respective figures; The plot for “POE” helps us observe the numerical accuracy of MDRM for almost every value of the output response and its distribution. As an instance, if we consider "**150 mm²**" as the minimum limit for the reinforcement, the associated “POE” for is approximately '**0.757**', '**0.754**' and '**0.753**' for ' $m = 2$ ', ' $m = 3$ ' and ' $m = 4$ ', respectively.

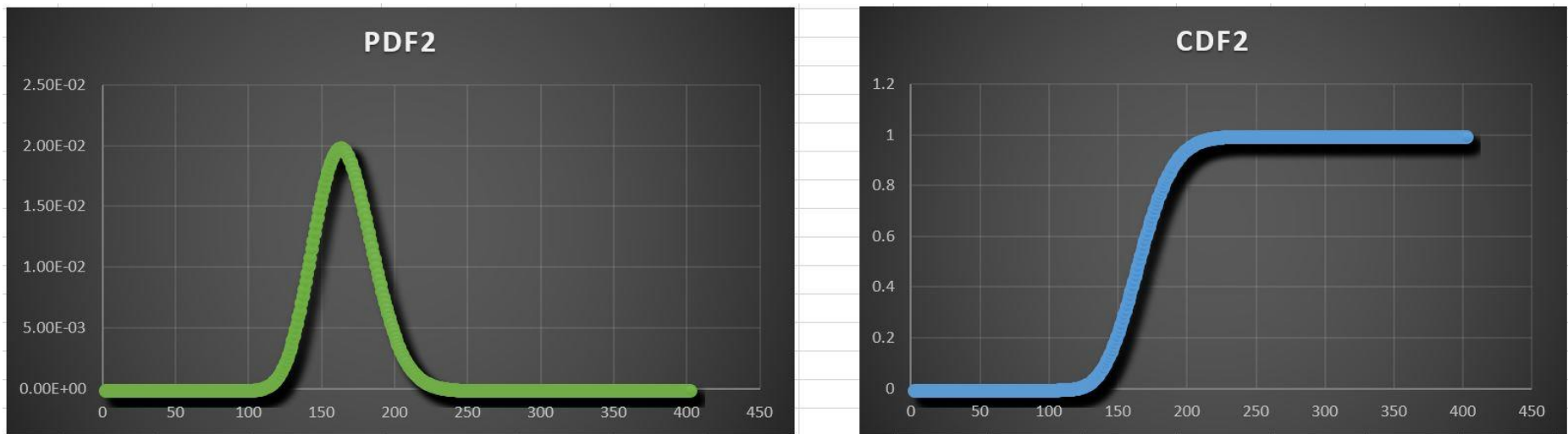


Figure 3.2(a) “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ' $m = 2$ '



Figure 3.2(b) “Probability of Exceedance (POE)” Plot for ' $m = 2$ '

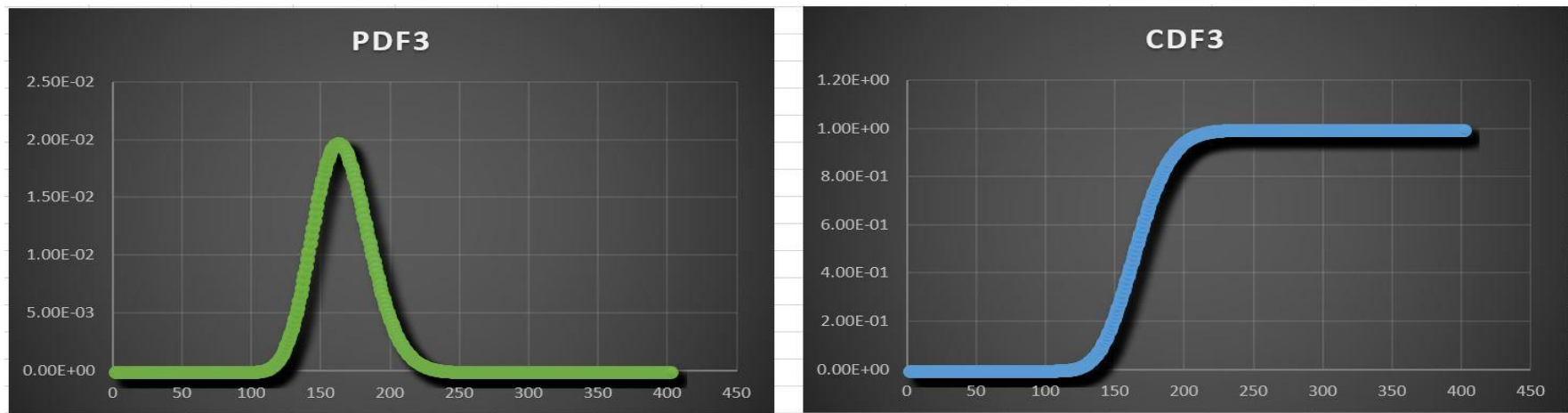


Figure 3.3(a) “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ' $m = 3$ '

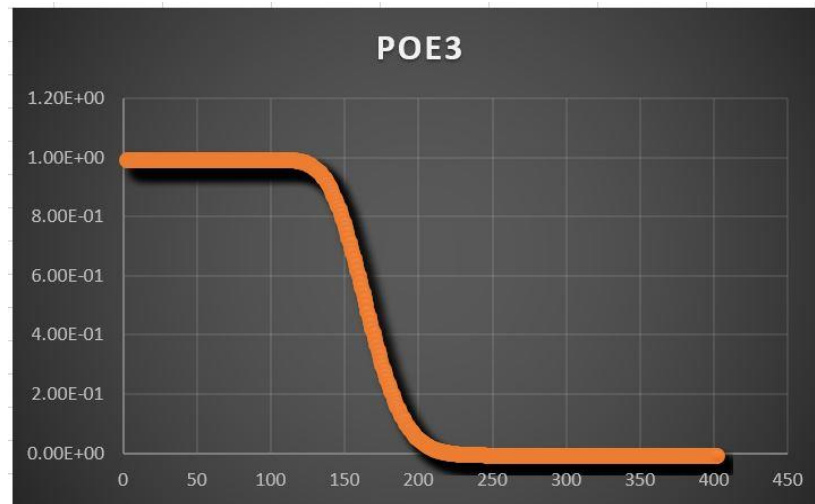


Figure 3.3(b) “Probability of Exceedance (POE)” Plot for ' $m = 3$ '

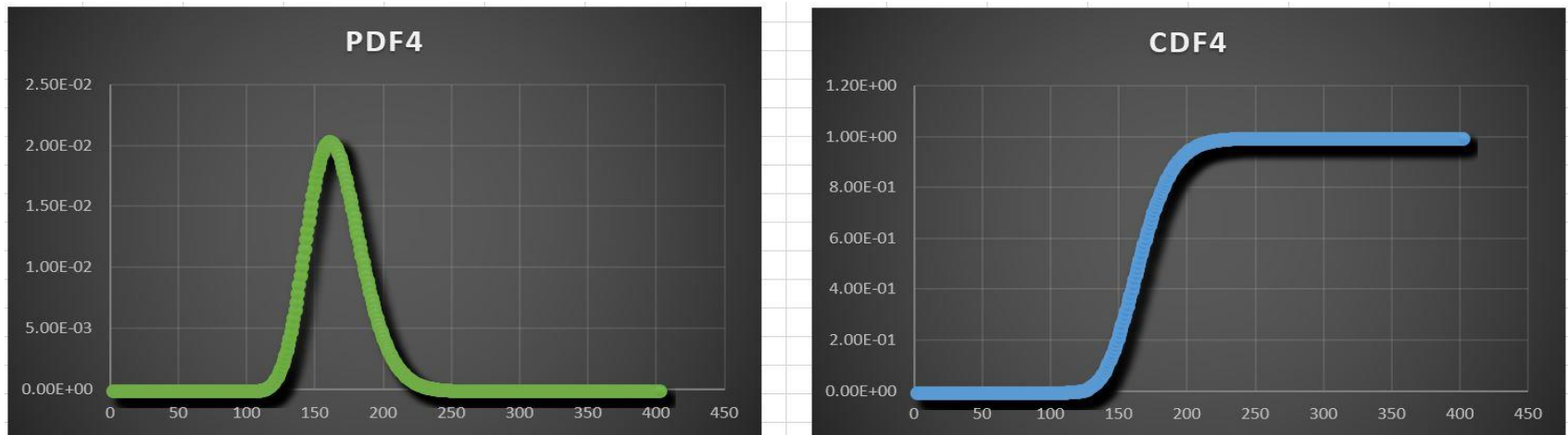


Figure 3.4(a) “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots for ' $m = 4$ '

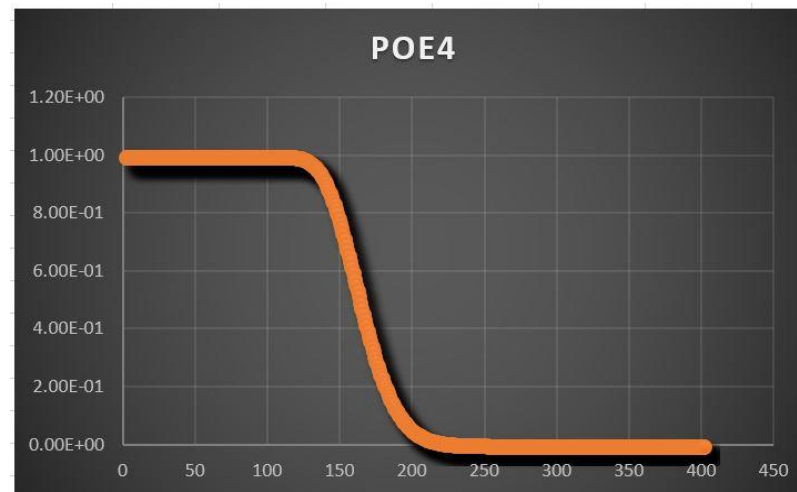


Figure 3.4(b) “Probability of Exceedance (POE)” Plot for ' $m = 4$ '

3.4.4 Evaluation Of Sensitivity Coefficients

In this section, the previously computed mean and mean square values, i.e., " φ_i " and " τ_i " of the respective cut-functions, are held upon to approximate the sensitivity coefficients, both primary and total, for each random variable, tabulated as follows;

Table 3.8 Global Sensitivity Indices Using MDRM

RANDOM VARIABLE(i)	S_i	S_{Ti}	$(S_{Ti}-S_i)\%$	R-MEASUERS from MCS
f_y	0.6711	0.6736	0.2512	0.6550
d	0.1001	0.1015	0.14	0.1006
b	0.2260	0.2287	0.27	0.2270
SUM	0.9972	1.0038	N/A	N/A

Legend: ' S_i ' denotes the 'Primary Sensitivity Coefficient';
' S_{Ti} ' denotes the 'Total Sensitivity Coefficient'.

Adding to the primary concept, it can be ascertained that no extra computational as well as conceptual efforts are necessitated for sensitivity analysis, unlike in MCS method, wherein we've to search for various other concepts to compute the same. Moreover, for this study, Sensitivity coefficients computed using MDRM's formulation technique are compared with the regression values, obtained as a part of MCS technique, in MATLAB, thereby, validating the numerical accuracy of the proposed method, as shown in the above table. It can be observed that the variance of the "Characteristic Strength of Steel" contributes to that of the minimum reinforcement required to the maximum, followed by the width and depth of the beam. Therefore, the output response, i.e., " $A_{st_{min}}$ ", is most influenced by the input variable " f_y ", owing approximately "70%" of its variance to the variance of " f_y ". Furthermore, the difference between both the sensitivity coefficients, i.e., " $S_{Ti} - S_i$ ", is really small (less than "1%"), which signifies that all three input random variables don't interact with each other. Also, the difference, " $1 - \sum_i S_i = 0.004$ ", signifies absence of interactions among the specified input random variables.

3.5 CONCLUSION

The “Multiplicative form of Dimensional Reduction Method (M-DRM)” is presented in this chapter for the reliability and sensitivity analysis of structures. The formulation technique of the same derives the equations to estimate the statistical or integer moments and the response’s probabilistic distribution. Then, the coefficients for the sensitivity analysis are formulated as a by-product, thereby, not providing any extra computational or conceptual efforts. The efficiency and flexibility of the proposed scheme lie in its numerical accuracy, achieved within a feasible time period, unlike that in MCS method. Moreover, the rules of “Gaussian Quadrature Scheme” complements MDRM, to create an input grid of independent random variables, thereby evaluating the response. An output grid is created, thereafter, to compute the mean and mean square of the respective input variables. The grids recapitulate the fact that high dimensional moment integration is simplified into a product form of low dimensional integrals. This is how, the MDRM computes the mean and variance of the structural response.

Further, the responses computed in the input grid are coupled with the “Maximum Entropy Principle (MaxEnt Principle)”. This principle utilises fractional moments as constraints. The optimisation procedure is called upon to compute these constraints, letting them not to be a priority. In addition, the MaxEnt parameters, i.e., the Lagrangian multipliers and fractional exponents are obtained using the same optimization technique. This entire process, orderly estimates the probability distribution of any specified structural response. The pilot study demonstrated in this chapter follows the entire predefined procedure to validate the numerical accuracy of the proposed scheme. The study also shows the convergence of the entropy after two fractional moments. Hence, only three iterations were sufficient to be performed to capture the distribution.

This chapter also estimates the “Probability of Failure (p_{fail})”, based upon the plot of “Probability of Exceedance (POE)”. The same plot holds upon the calculated MaxEnt parameters to compute the failure probability, thereby not incurring any extra analytical efforts. Hence, a single distribution can help the analyst to approximate the probability of failure. Furthermore, the Global Sensitivity Analysis is also carried out rigorously for the same study and the results are compared with the regression values, computed as a part of simulation technique.

The numerical accuracy of the proposed method is validated by performing Monte Carlo Simulation method for the same study. As far as the computational cost of the MDRM is

concerned, this scheme doesn't demand the leaning of any advanced software for simple structural systems. It simply necessitates the utilisation of the Microsoft Excel to a well-defined point, in order to implement the formulation. However, advanced software systems are required to implement the same, say, OpenSees software for FEA, using TCL programming. The usage of an " $N - point$ " scheme, with " n " random variables leads to " $1 + nN$ " functional evaluations, in front of " $1e5$ " evaluations in MCS technique for the same efficiency. Moreover, the latest research (*Balomenos et al., 2017*) shows that for large scale complex systems, say Finite Element Analysis, high computational cost with MCS would be unaffordable, but affordable if the proposed scheme i.e., MDRM would be used; Rather the computational time would be drastically reduced [10%] as necessitated by MCS method. Thus, the proposed method is an effective substitute for MCS technique in terms of time and computational cost.

In general, though, the MDRM requires a much feasible computational time to compute various outcomes of interest with a desirable accuracy, yet, it holds one limitation. That is, it accounts for only "Independent (uncorrelated) Input Random Variables".

CHAPTER 4

MDRM IMPLEMENTATION—SEISMIC ANALYSIS OF A 4- STOREYED RCC STRUCTURE

4.1 INTRODUCTION

The tremendous destruction of structural systems and facilities owing to the occurrence of past few earthquakes has raised some serious questions on the prevailing design and construction practices. These calamities have made professionals envisage the importance of the reliability concept, to address the disaster preparedness and mitigation practices. Moreover, the occurrence of earthquakes and their measurements, effects of vibrations and hence, the structural response have been comprehended from many years of earthquake history and meticulously documented in literature. Since then, the engineers have put in some rigorous efforts to comprehend the procedure, to counteract the dynamic effects of earthquake related calamities and further design the earthquake resistant structures in a precise and well-understood manner. These rigorous efforts have led to the several revisions of “*Indian Standard:1893*” Code of Practice on the same concept by the “*Bureau of Indian Standards, New Delhi*”. However, these codes of practice don’t contemplate the fundamentals of reliability to a desired level.

In this chapter, the “Multiplicative Form of Dimensional Reduction Method (M-DRM)” has been utilized to implement the concept of reliability in the code-based deterministic procedure, documented for the “*Seismic Analysis (Response Spectrum Method) of a 4-storeyed Regular RCC-Structure*”. The code based dynamic analysis procedure [*IS 1893 (Part – 1): 2016 (Sixth Revision)*] accounts for the linear analysis of any structure by utilising the design spectrum, thereby approximating the yielding effects. This analysis procedure accounts for the lateral forces, on the basis of the characteristics of the “Natural Vibration Modes” of the structure, which are further evaluated from the “Mass and Stiffness Distribution” over height. Moreover, “Monte Carlo Simulation (MCS)” method has also been performed to comprehend the numerical accuracy of the proposed scheme.

4.2 OBJECTIVE

The main aim of this chapter is to explore the applicability and the numerical accuracy of the proposed multiplicative dimensional reduction method, in regard to the reliability and sensitivity analysis of structural systems, subjected to the seismic analysis. Thus, the proposed scheme is implemented upon a **4-storeyed reinforced concrete structure** in this chapter, thereby accounting for the efficiency of the same in regard to a large number of independent input random variables, "16" at first and then, "203" variables.

4.3 ORGANISATION

Here follows the organisation of this chapter; At first, the proposed scheme holds upon the **4-storeyed structural system**, subjected to the code-based formulation for seismic analysis, by accounting '16' independent input variables. Meanwhile, the Monte Carlo Method is also performed to investigate the numerical accuracy of the same, in "Section 4.4"; Further, in "Section 4.5", the same system is held upon by accounting '203' such variables. The difference in the '*count*' of random variables and, that too for the same structural system is also viewed upon in the same section. "Section 4.6" provides a brief discussion on the "Computational Time", stipulated by both MCS and MDRM. The chapter recapitulates the conclusions in "Section 4.7".

4.4 "MAIN STUDY-1"—RELIABILITY ANALYSIS OF A 4-STOREYED RCC STRUCTURE WITH '16' INPUT VARIABLES

4.4.1 Description Of RCC-Frame

A two-bay, four-storey reinforced concrete frame, as shown in Figure (4.1), is chosen from *Agrawal and Shrikhande (2016 Print)* to perform the code-based dynamic analysis (response spectrum analysis) procedure and thereby, the reliability analysis using MDRM and MCS technique. Moreover, one of the transverse plane frames has also been contemplated for the justification of demonstration, by holding upon the fact that the proposed structural system is symmetric, or say regular, in elevation and plan, as shown in Figure (4.1) and Figure (4.2), respectively.

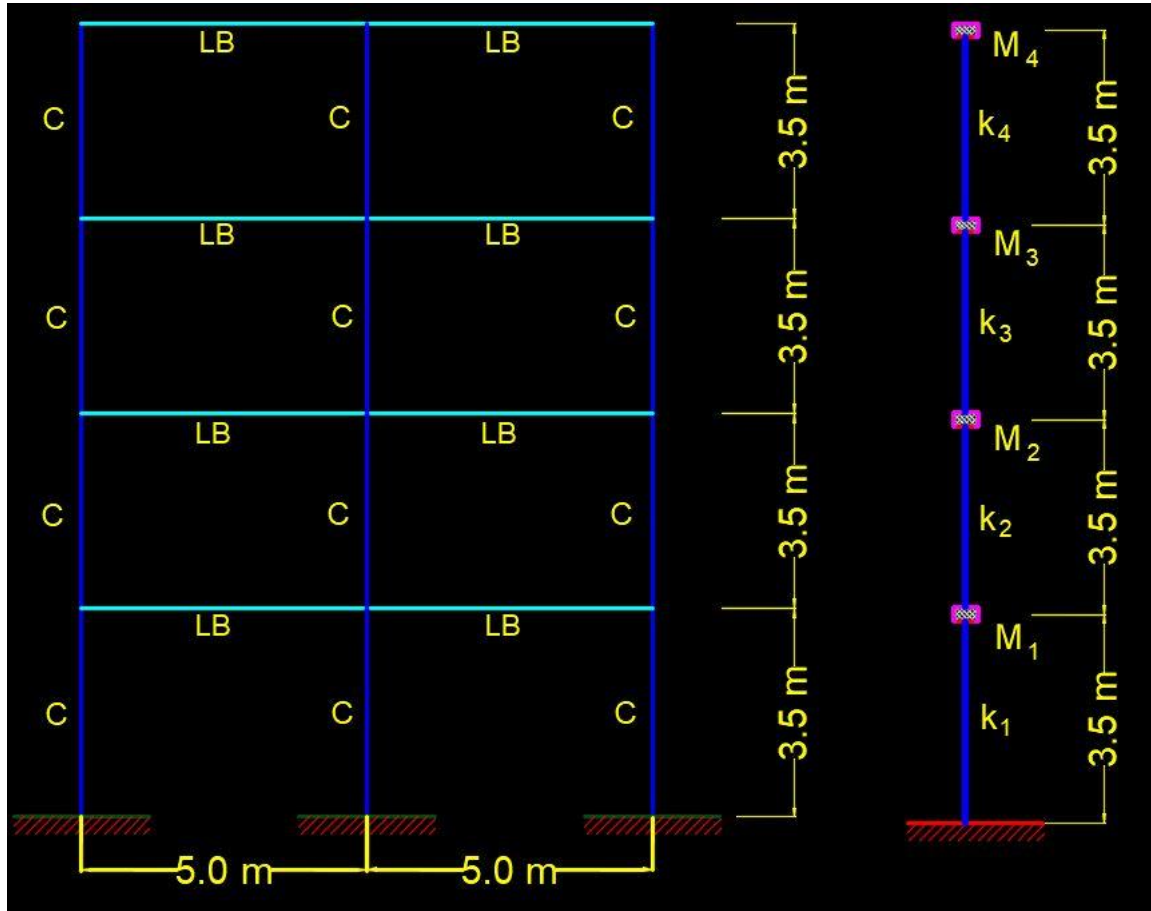


Figure 4.1 Plane Frame Structure and Its Lumped Mass Model

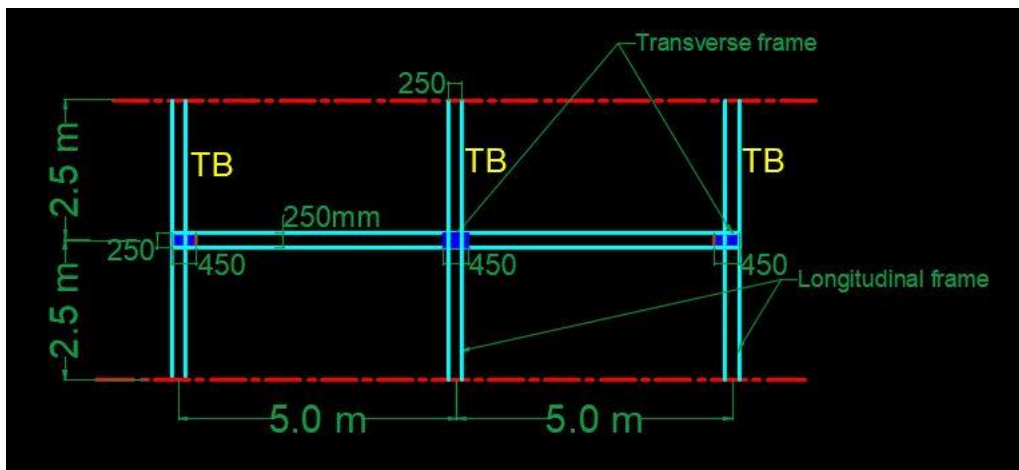


Figure 4.2 Plan showing the Columns and Beams at "Floor" of the Plane Frame

The preliminary data (nominal measures) necessitated for the analysis are assumed, as tabulated in Table (4.1); Moreover, the procedure is implemented in MATLAB.

Table 4.1 Assumed Preliminary Data Required for the Analysis of Frame

S.No	DESIGNATION	DETAIL (Nominal)
1	Type of Structure	Special RC Moment Resisting Frame
2	Seismic Zone	Fifth
3	Number of Stories	Four (G+3)
4	Floor Height	3.5 m
5	Infill Wall	250 mm Thick in Longitudinal Direction and 150 mm in Transverse Direction (including plaster)
6	Imposed Load	3.5 kN/m ²
7	Material	Concrete (M30) and Reinforcement (Fe415)
8	Size of Columns	250 mm × 450 mm
9	Size of Beams	250 mm × 450 mm in Longitudinal and 250 mm × 350 mm in Transverse Directions
10	Depth of Slab	120 mm
11	Specific Weight of Concrete	25 kN/m ³
12	Specific Weight of Infill	20 kN/m ³
13	Type of Soil	Soft Soil

Furthermore, the statistical properties of random variables are tabulated, as in Table (4.2);

Table 4.2 Statistics of Random Variables Associated to the Proposed Structural System

S.No.	PARAMETER	DESIGNATION	DISTRIBUTION	COV (%)	Mean	SD
1	fck (kN/m ²)--[1RV]	Characteristic Strength of Concrete	Normal	11.5	37200	4278
2	fy (kN/m ²)--[1RV]	Characteristic Strength of Steel	Normal	5	468950	23447.5
3	EM(kN/m ²)--[1RV]	Modulus of Masonry	Normal	20	17940000	3588000
4	ES(kN/m ²)--[1RV]	Young's Modulus of Elasticity for Steel	Normal	5	226000000	11300000
5	bc (m)--[1RV]	Coulmn Dimension (Transversely)	Normal	4	0.25125	0.01005
6	Dc(m)--[1RV]	Coulmn Dimension (Longitudinally)	Normal	4	0.45225	0.01809
7	bb_l(m)--[1RV]	Longitudinal Beam Width	Normal	4	0.2525	0.0101
8	bb_t(m)--[1RV]	Transverse Beam Width	Normal	4	0.2525	0.0101
9	db_l(m)--[1RV]	Longitudinal Beam Depth	Normal	4	0.396	0.01584
10	db_t(m)--[1RV]	Transverse Beam Depth	Normal	4	0.3465	0.01386
11	t(m)--[1RV]	Slab Thickness	Normal	12	0.1104	0.013248
12	bl(m)--[1RV]	Longitudinal Infill Wall Width	Normal	20	0.2525	0.0505
13	bt(m)--[1RV]	Transverse Infill Wall Width	Normal	20	0.1515	0.0303
14	gamma_c(kN/m ³)--[1RV]	Specific Weight of Concrete	Normal	10	26.25	2.625
15	gamma_if (kN/m ³)--[1RV]	Specific Weight of Infill	Normal	20	21	4.2
16	imp_load(kN/m ²)--[1RV]	Imposed Load	Normal	18	3.85	0.693

From the above tabulated data, it can be reckoned that a single random variable, say any material property, namely, ‘Characteristic Strength of Concrete, f_{ck} ’ is taken to be similar for all the structural components, i.e., it has been presumed that if any changes take place in this randomised variable, then, its effect would be seen in all the other components collectively, built-in using the same material; and thus, similar goes the explanation for the rest of the cases. Moreover, the considered random variables are presumed to be independent and meanwhile, following “Normal” Distribution. In totality, there are ‘**16**’ such input random variables for the reliability analysis of the proposed structural system.

4.4.2 Input Grid For MDRM

As stated earlier, the examined problem incorporates ‘**16**’ independent random variables as an input, i.e., ‘ $n = 16$ ’. Thus, the structural output, “***BASE SHEAR [SSF(1)]***”, is a product of ‘**16 cut – functions**’. Incorporating a fifth-order, i.e., ‘ $N = 5$ ’, ‘Gauss-Hermite Integration scheme’, ‘ $nN + 1 = 16 \times 5 + 1 = 81$ ’ trials are executed. The input data grid to execute MDRM is tabulated, as in Table (4.3) (similar to pilot-study, proposed in Chapter-3). The statistical analysis is executed on the basis of these tabulated results in the following section.

Table 4.3(a) Input Grid for the Response Evaluation

RV	Trial No.	z _i	1 fck (kN/m ²)	2 fy (kN/m ²)	3 EM(kN/m ²)	4 ES(kN/m ²)	5 bc (m)	6 Dc(m)	7 bb _l (m)	8 bb _t (m)	9 db _l (m)	10 db _t (m)	11 t(m)	12 bl(m)	13 bt(m)	14 gamma _c (kN/m ³)	15 gamma _{if} (kN/m ³)	16 imp_load(kN/m ²)	SSF(1)
fck	1	-2.85697	24977.88	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.44
	2	-1.35563	31400.61	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.43
	3	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	4	1.35563	42999.39	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	5	2.85697	49422.12	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.41
fy	6	-2.85697	37200.00	401961.20	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	7	-1.35563	37200.00	437163.87	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	8	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	9	1.35563	37200.00	500736.13	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	10	2.85697	37200.00	535938.80	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
EM	11	-2.85697	37200.00	468950.00	7689191.64	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.65
	12	-1.35563	37200.00	468950.00	13075999.56	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.50
	13	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	14	1.35563	37200.00	468950.00	22804000.44	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.37
	15	2.85697	37200.00	468950.00	28190808.36	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.33
ES	16	-2.85697	37200.00	468950.00	17940000.00	193716239.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	17	-1.35563	37200.00	468950.00	17940000.00	210681381.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	18	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	19	1.35563	37200.00	468950.00	17940000.00	241318619.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	20	2.85697	37200.00	468950.00	17940000.00	258283761.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
bc	21	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.22	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	330.72
	22	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.24	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	331.62
	23	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	24	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.26	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	333.23
	25	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.28	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	334.12
Dc	26	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.40	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	330.73
	27	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.43	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	331.62
	28	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	29	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.48	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	333.22
	30	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.50	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	334.11

Legend: 'z_i' denotes the Gauss-Hermite points

Table 4.3(b) Input Grid for the Response Evaluation

RV	Trial No.	z _j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	SSF(1)
			fck (kN/m ²)	fy (kN/m ²)	EM(kN/m ²)	ES(kN/m ²)	bc (m)	Dc(m)	bb_l(m)	bb_t(m)	db_l(m)	db_t(m)	t(m)	bl(m)	bt(m)	gamma_c(kN/m ³)	gamma_if (kN/m ³)	imp_load(kN/m ²)	SSF(1)
bb_l	31	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.22	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	330.82
	32	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.24	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	331.66
	33	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	34	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.27	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	333.18
	35	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.28	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	334.03
bb_t	36	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.22	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	330.31
	37	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.24	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	331.42
	38	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	39	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.27	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	333.43
	40	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.28	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	334.54
db_l	41	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.35	0.35	0.11	0.25	0.15	26.25	21.00	3.85	330.83
	42	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.37	0.35	0.11	0.25	0.15	26.25	21.00	3.85	331.67
	43	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	44	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.42	0.35	0.11	0.25	0.15	26.25	21.00	3.85	333.18
	45	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.44	0.35	0.11	0.25	0.15	26.25	21.00	3.85	334.01
db_t	46	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.31	0.11	0.25	0.15	26.25	21.00	3.85	330.31
	47	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.33	0.11	0.25	0.15	26.25	21.00	3.85	331.42
	48	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	49	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.37	0.11	0.25	0.15	26.25	21.00	3.85	333.43
	50	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.39	0.11	0.25	0.15	26.25	21.00	3.85	334.54
t	51	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.07	0.25	0.15	26.25	21.00	3.85	305.73
	52	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.04	0.25	0.15	26.25	21.00	3.85	319.76
	53	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	54	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.13	0.25	0.15	26.25	21.00	3.85	345.09
	55	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.15	0.25	0.15	26.25	21.00	3.85	359.12
bl	56	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.11	0.15	26.25	21.00	3.85	281.98
	57	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.18	0.15	26.25	21.00	3.85	308.49
	58	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	59	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.32	0.15	26.25	21.00	3.85	356.36
	60	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.40	0.15	26.25	21.00	3.85	382.87

Legend: 'z_j' denotes the Gauss-Hermite points

Table 4.3(c) Input Grid for the Response Evaluation

RV	Trial No.	z _i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
			fck (kN/m ²)	f _y (kN/m ²)	EM(kN/m ²)	ES(kN/m ²)	bc (m)	Dc(m)	bb _l (m)	bb _t (m)	db _l (m)	db _t (m)	t(m)	bl(m)	bt(m)	gamma _c (kN/m ³)	gamma _{if} (kN/m ³)	imp_load(kN/m ²)	SSF(1)
bt	61	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.06	26.25	21.00	3.85	287.07
	62	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.11	26.25	21.00	3.85	310.90
	63	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	64	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.19	26.25	21.00	3.85	353.95
	65	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.24	26.25	21.00	3.85	377.79
gamma _c	66	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	18.75	21.00	3.85	296.61
	67	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	22.69	21.00	3.85	315.43
	68	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	69	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	29.81	21.00	3.85	349.42
	70	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	33.75	21.00	3.85	368.24
gamma _{if}	71	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	9.00	3.85	236.68
	72	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	15.31	3.85	286.99
	73	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	74	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	26.69	3.85	377.87
	75	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	33.00	3.85	428.21
imp_load	76	-2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	1.87	311.97
	77	-1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	2.91	322.72
	78	0	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42
	79	1.35563	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	4.79	342.13
	80	2.85697	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	5.83	352.88
Fixed Mean Values	81	n/a	37200.00	468950.00	17940000.00	226000000.00	0.25	0.45	0.25	0.25	0.40	0.35	0.11	0.25	0.15	26.25	21.00	3.85	332.42

Legend: 'z_i' denotes the Gauss-Hermite points

4.4.3 Statistical Moments Of The Structural Response

Now, after the computation for the input grid is completed, then, the necessitated “response” obtained in the last column of input grid is utilised, further, to tabulate the output grid, as shown in Table (4.4). Following the order, similar to that followed for the output grid in pilot study, the mean and mean square measures of each cut-function are evaluated, as $\phi'_i = \sum_{j=1}^N w_j [SSF(1)]_{ij}$ and $\tau'_i = \sum_{j=1}^N w_j \{ [SSF(1)]_{ij} \}^2$, $i = 1, 2, \dots, n$, respectively, where $[SSF(1)]$ is the base shear, when i^{th} cut-function is positioned at j^{th} Gauss-Quadrature point. The output grid is tabulated on the basis of these formulated measures, as in Table (4.4); Meanwhile, the MCS technique is also performed for ' $1e5$ ' trials. Further, the numerical results computed from both the methods are then compared, as tabulated in Table (4.5).

The tabulated data shows the numerical accuracy of M-DRM with ' 81 trials' in front of MCS with ' $1e5$ trials'.

Table 4.4(a) Output Grid for Each Cut-Function Evaluation

RANDOM VARIABLE	Trial No.	SSF(1)	Weight(wj)	wj*SSF(1)	phi	wj*SSF(1)*SSF(1)	ti
fck	1	332.44	0.01126	3.743280313	332.4259203	1244.418073	110505.8875
	2	332.43	0.22208	73.82605245		24541.99396	
	3	332.42	0.53333	177.2907712		58935.40128	
	4	332.42	0.22208	73.8228855		24539.88843	
	5	332.41	0.01126	3.742930848		1244.185731	
fy	6	332.42	0.01126	3.743074802	332.4255979	1244.281436	110505.6731
	7	332.42	0.22208	73.82433854		24540.85447	
	8	332.42	0.53333	177.2907712		58935.40128	
	9	332.42	0.22208	73.82433854		24540.85447	
	10	332.42	0.01126	3.743074802		1244.281436	
EM	11	332.65	0.01126	3.745598068	332.4325885	1245.959581	110510.3231
	12	332.50	0.22208	73.8407429		24551.76203	
	13	332.42	0.53333	177.2907712		58935.40128	
	14	332.37	0.22208	73.81339744		24533.58088	
	15	332.33	0.01126	3.742078846		1243.619368	
ES	16	332.42	0.01126	3.743074802	332.4255979	1244.281436	110505.6731
	17	332.42	0.22208	73.82433854		24540.85447	
	18	332.42	0.53333	177.2907712		58935.40128	
	19	332.42	0.22208	73.82433854		24540.85447	
	20	332.42	0.01126	3.743074802		1244.281436	

Legend: ' w_j ' denotes the Gauss-Hermite weights

Table 4.4(b) Output Grid for Each Cut-Function Evaluation

RANDOM VARIABLE	Trial No.	SSF(1)	Weight(w_j)	w_j *SSF(1)	ϕ_i	w_j *SSF(1)*SSF(1)	τ_i
bc	21	330.72	0.01126	3.723958524	332.4256038	1231.604537	110506.0302
	22	331.62	0.22208	73.64543583		24422.0561	
	23	332.42	0.53333	177.2907712		58935.40128	
	24	333.23	0.22208	74.00324608		24659.9443	
	25	334.12	0.01126	3.762192167		1257.02397	
Dc	26	330.73	0.01126	3.724026714	332.4255247	1231.649642	110505.9748
	27	331.62	0.22208	73.64611053		24422.50358	
	28	332.42	0.53333	177.2907712		58935.40128	
	29	333.22	0.22208	74.00250675		24659.45158	
bb_l	30	334.11	0.01126	3.762109447	332.4256433	1256.968694	110506.0187
	31	330.82	0.01126	3.725011422		1232.301074	
	32	331.66	0.22208	73.65526924		24428.57838	
	33	332.42	0.53333	177.2907712		58935.40128	
	34	333.18	0.22208	73.99344485		24653.41264	
bb_t	35	334.03	0.01126	3.761146558	332.4256002	1256.325349	110506.2228
	36	330.31	0.01126	3.719257374		1228.496928	
	37	331.42	0.22208	73.60144198		24392.88662	
	38	332.42	0.53333	177.2907712		58935.40128	
	39	333.43	0.22208	74.04723695		24689.27098	
	40	334.54	0.01126	3.766892645		1260.166981	

Legend: ' w_j ' denotes the Gauss-Hermite weights

Table 4.4(c) Output Grid for Each Cut-Function Evaluation

RANDOM VARIABLE	Trial No.	SSF(1)	Weight(w_j)	w_j *SSF(1)	ϕ_i	w_j *SSF(1)*SSF(1)	τ_i
db_l	41	330.83	0.01126	3.72517939	332.4257473	1232.41221	110506.0824
	42	331.67	0.22208	73.65678933		24429.5867	
	43	332.42	0.53333	177.2907712		58935.40128	
	44	333.18	0.22208	73.99200963		24652.45627	
	45	334.01	0.01126	3.760997759		1256.225945	
db_t	46	330.31	0.01126	3.719257374	332.4256002	1228.496928	110506.2228
	47	331.42	0.22208	73.60144198		24392.88662	
	48	332.42	0.53333	177.2907712		58935.40128	
	49	333.43	0.22208	74.04723695		24689.27098	
	50	334.54	0.01126	3.766892645		1260.166981	
t	51	305.73	0.01126	3.442566958	332.4259579	1052.510414	110593.1985
	52	319.76	0.22208	71.01187417		22706.62047	
	53	332.42	0.53333	177.2907712		58935.40128	
	54	345.09	0.22208	76.63709687		26446.52655	
	55	359.12	0.01126	4.043648627		1452.139806	
bl	56	281.98	0.01126	3.175121681	332.4263689	895.3283916	110817.9601
	57	308.49	0.22208	68.50902701		21134.21642	
	58	332.42	0.53333	177.2907712		58935.40128	
	59	356.36	0.22208	79.14029251		28202.38607	
	60	382.87	0.01126	4.311156492		1650.627913	

Legend: ' w_j ' denotes the Gauss-Hermite weights

Table 4.4(d) Output Grid for Each Cut-Function Evaluation

RANDOM VARIABLE	Trial No.	SSF(1)	Weight(w _j)	w _j *SSF(1)	φ _i	w _j *SSF(1)*SSF(1)	τ _i
bt	61	287.07	0.01126	3.232352761	332.4261941	927.8955927	110758.1864
	62	310.90	0.22208	69.04446693		21465.86101	
	63	332.42	0.53333	177.2907712		58935.40128	
	64	353.95	0.22208	78.60469664		27821.94855	
	65	377.79	0.01126	4.253906514		1607.079985	
gamma_c	66	296.61	0.01126	3.339880009	332.4261805	990.6570579	110663.1992
	67	315.43	0.22208	70.05077657		22096.14237	
	68	332.42	0.53333	177.2907712		58935.40128	
	69	349.42	0.22208	77.59837632		27114.13909	
	70	368.24	0.01126	4.146376348		1526.859398	
gamma_if	71	236.68	0.01126	2.665020117	332.4282596	630.7577465	111631.0703
	72	286.99	0.22208	63.73408315		18290.85625	
	73	332.42	0.53333	177.2907712		58935.40128	
	74	377.87	0.22208	83.91676441		31709.39909	
	75	428.21	0.01126	4.821620683		2064.655951	
imp_load	76	311.97	0.01126	3.512757068	332.4259056	1095.866982	110557.1534
	77	322.72	0.22208	71.66873384		23128.63567	
	78	332.42	0.53333	177.2907712		58935.40128	
	79	342.13	0.22208	75.98019382		25995.0912	
	80	352.88	0.01126	3.973449648		1402.158269	
Fixed Mean Values	81	332.42	n/a	XXX	XXX	XXX	XXX

Legend: 'w_j' denotes the Gauss-Hermite weights

Table 4.5 Statistical Moments of the Response

BASE SHEAR [SSF(1)]	MRDM (81 Trials)	MCS (1e5 Trials)	REL. ERROR (%)
First Moment (kN)	332.4882	332.4742	0.004210853
Second Moment (kN ²)	112528.06	112548.67	0.018312078
Std. Dev. (kN)	44.4900	44.8300	0.758420700
COV	0.1338	0.1348	0.741839763

Legend: M-DRM—Multiplicative Dimensional Reduction Method;
MCS—Monte Carlo Simulation;
Relative Error-- $ABS [MCS - MDRM] / MCS$;
Std. Dev.—Standard Deviation;
COV—Coefficient of Variation.

4.4.4 Statistical Distribution Of The Response

In this section, The “Maximum Entropy Principle (MaxEnt Principle)” is held upon along with the fractional moments to approximate the “Probability Distribution” of the “***BASE SHEAR [SSF(I)]***” of the selected structural system. Herein, the optimization procedure is utilised, meanwhile computing the Lagrangian parameters, ' λ_i ' and ' δ_i ', to describe the approximated probability distribution. Moreover, four fractional moments, i.e., ' $m = 4$ ', are adequate to perform the analysis, as in, the entropy converges quite rapidly, as tabulated in following Table (4.6);

Furthermore, the “Probability Density Function (PDF) + Cumulative Distribution Function (CDF)” and “Probability of Exceedance (POE)”, plots for ' $m = 2$ ', ' $m = 3$ ' and ' $m = 4$ ' are shown, as in Figure (4.3) and Figure (4.4), respectively. The convergence of plots for ' $m = 3$ ' and ' $m = 4$ ' define the convergence of entropy.

Table 4.6 MaxEnt Parameters for the Base Shear

		δ1	δ1	δ2	δ1	δ2	δ3	δ1	δ2	δ3	δ4
		0.932100	0.14167685	0.99370043	0.0983162	0.4567736	0.1535166	0.6007442	-0.4281063	0.08732647	0.9695525
	λ0	5.757732701	446.3729967		704.9964874			706.241311			
	λ1	0.0048125	-226.7753583		152.4454856			4.486601969			
	λ2		0.232559663		28.72838126			373.4155143			
	λ3				-564.937349			-531.7819727			
	λ4							0.012659365			
FRACTIONAL MOMENT	$M_x^\delta = \frac{1}{N} \sum_i x_i^\delta$	224.08	2.28	320.4800	1.7700	14.18	2.44	32.71	0.08	1.66	278.54
ENTROPY	Ent[f]= λ ₀ + λ ₁ M ₁ +λ ₂ M ₂ + λ ₃ M ₃ +.....	6.8361348	4.7944607		4.845748371			4.844879471			

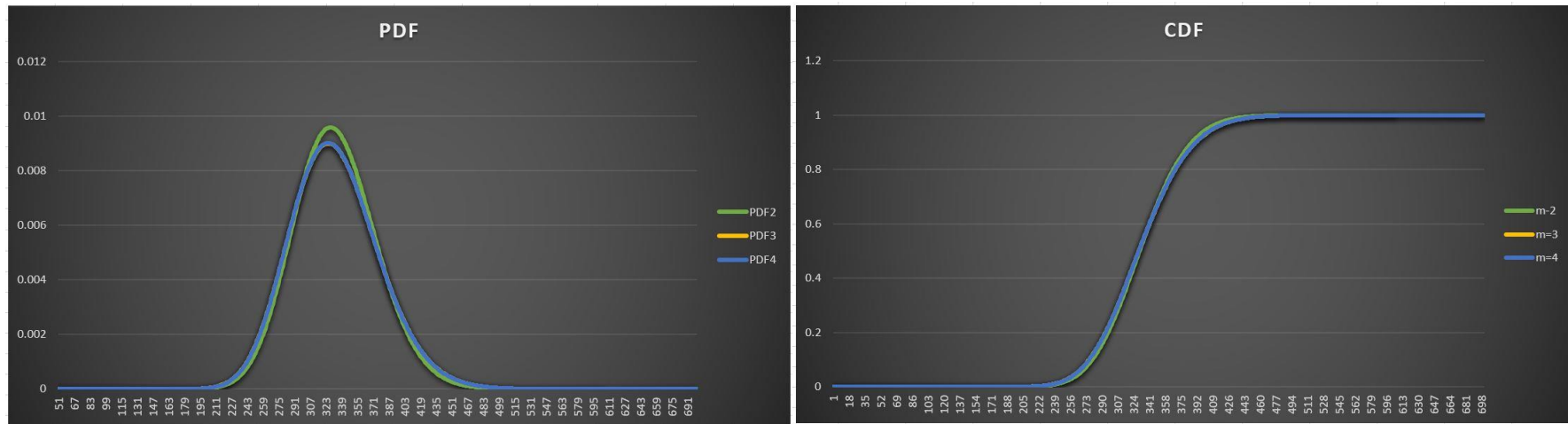


Figure 4.3 “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots

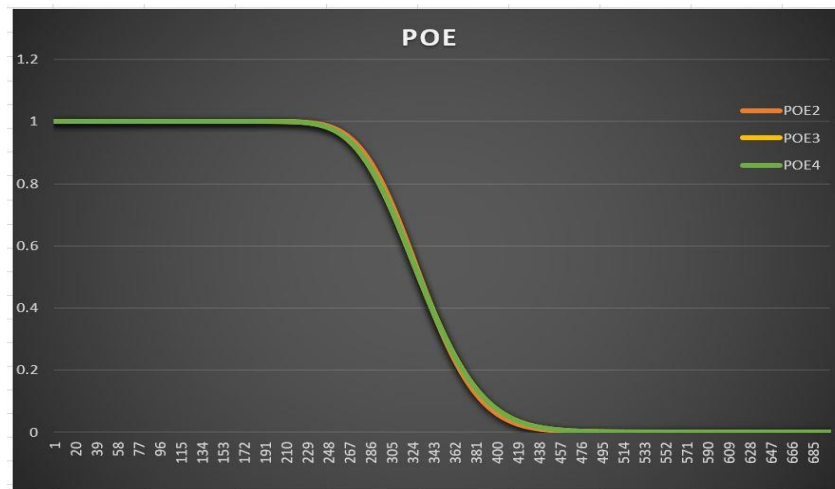


Figure 4.4 “Probability of Exceedance (POE)” Plot

4.4.5 Global Sensitivity Indices Using MDRM

As stated previously, extra computational efforts aren't necessitated to compute the sensitivity analysis' coefficients on utilising the formulation technique of MDRM. Hence, the global sensitivity indices for each of the "16" input (independent) random variables corresponding to "[SSF(1)]" are evaluated using the same. The sensitivity indices for "6" most important variables are tabulated as follows [Table (4.7)]:

Table 4.7 Global Sensitivity Indices Using MDRM

RANK	PARAMETER	S_i	S_{Ti}	$S_{Ti} - S_i$ (%)	R-MEASURES FROM MCS
1	gamma_if (kN/m3)	0.56723	0.57158	0.43518	0.5611
2	bl (m)	0.15699	0.15935	0.23633	0.1588
3	bt (m)	0.12684	0.12882	0.19788	0.1287
4	gamma_c (kN/m3)	0.07885	0.08105	0.21988	0.0797
5	t(m)	0.04355	0.04429	0.07453	0.0428
6	imp_load(kN/m2)	0.02535	0.02579	0.04423	0.0245

Legend: ' S_i ' denotes the 'Primary Sensitivity Coefficient';
' S_{Ti} ' denotes the 'Total Sensitivity Coefficient'.

From the above tabulated data, the material properties, namely "*gamma_if*" and "*gamma_c*" contribute to the response (output) to the most; Their variance contribution is around "65%", thereby making them the most influential variables for the evaluation of the namely "*base shear*" of the proposed frame. Moreover, the regression measures also depict the same trend. Adding to this, the difference between both the sensitivity coefficients, i.e., " $S_{Ti} - S_i$ ", is really small (less than "1%"), which signifies that all input random variables don't interact with each other. Also, the difference, " $1 - \sum_i S_i = 0.0055$ ", signifies absence of interactions among the specified input random variables.

4.5 "MAIN STUDY-2"—RELIABILITY ANALYSIS OF A 4-STOREYED RCC STRUCTURE WITH '203' INPUT VARIABLES

4.5.1 General Description

The same system is held upon in this section, as that chosen in "Section 4.4". The only difference lies in the fact that each structural component is randomized at its own level, i.e.,

every component is restricted to perform on its own, thereby, taking into account the practical considerations. This further can be decoded by taking the similar example, stated in the last section, i.e., of the ‘Characteristic Strength of Concrete, f_{ck} ’; It has been presumed here that any significant change in any component owing to its own material strength won’t affect the material strength of any other structural component; similar goes the explanation for the rest of the random variables. The rest would be unfolded by the sensitivity analysis, specifically for the case of random variables in regard to material properties. This can be ascertained because material is manufactured collectively for the entire structure, be it concrete, steel or masonry. This is conceptualised further in the form of elevation and floor plans, as in Figure (4.5) and Figure (4.6), respectively.

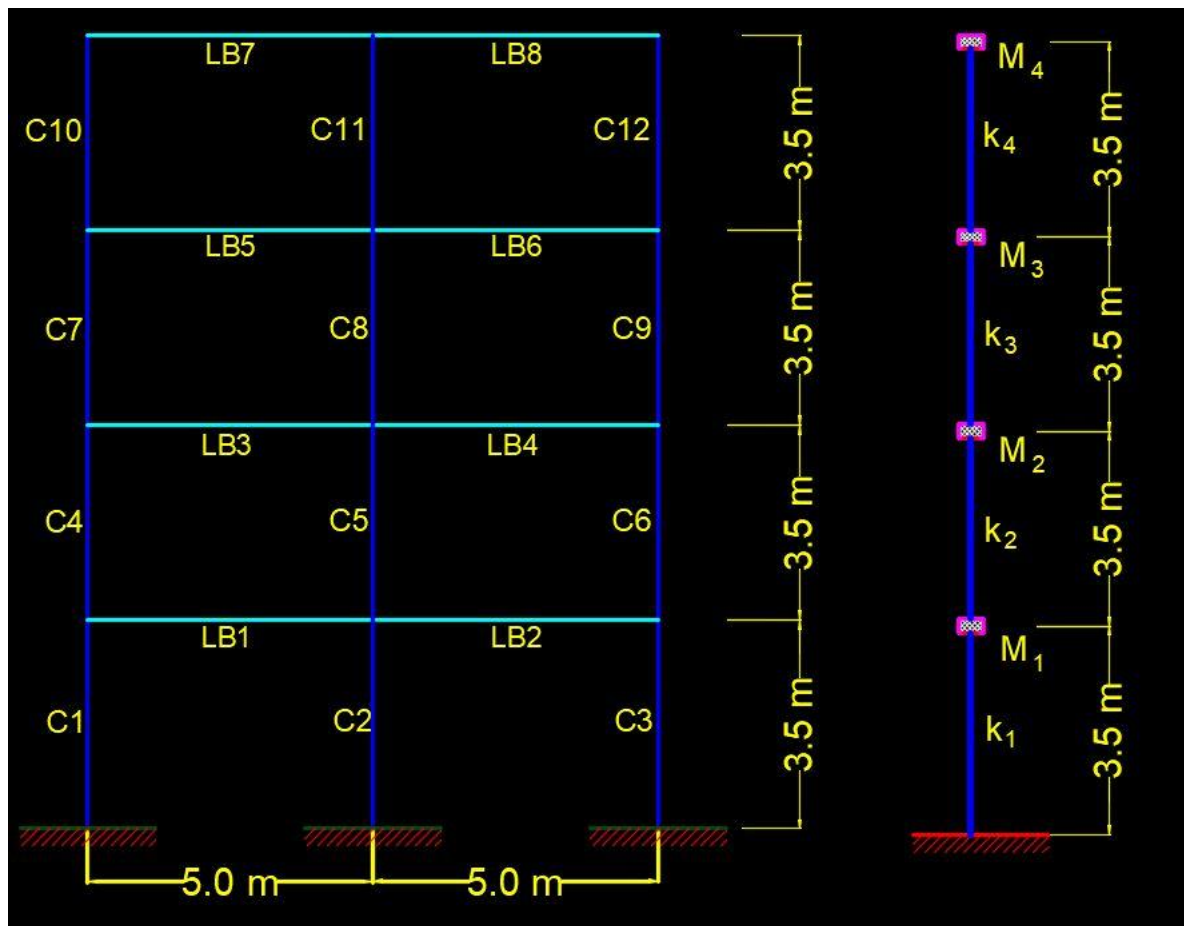


Figure 4.5 Plane Frame Structure and Its Lumped Mass Model

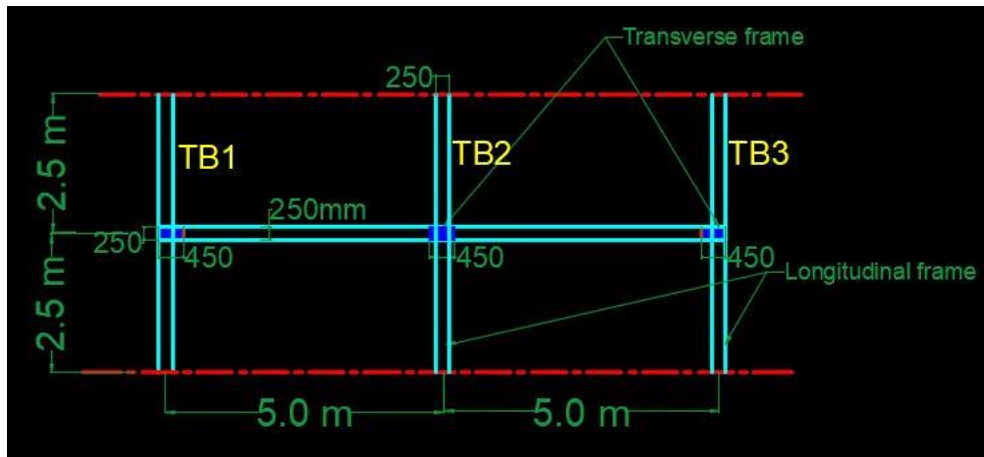


Figure 4.6(a) Plan showing the Columns and Beams at “Floor-1” of the Plane Frame

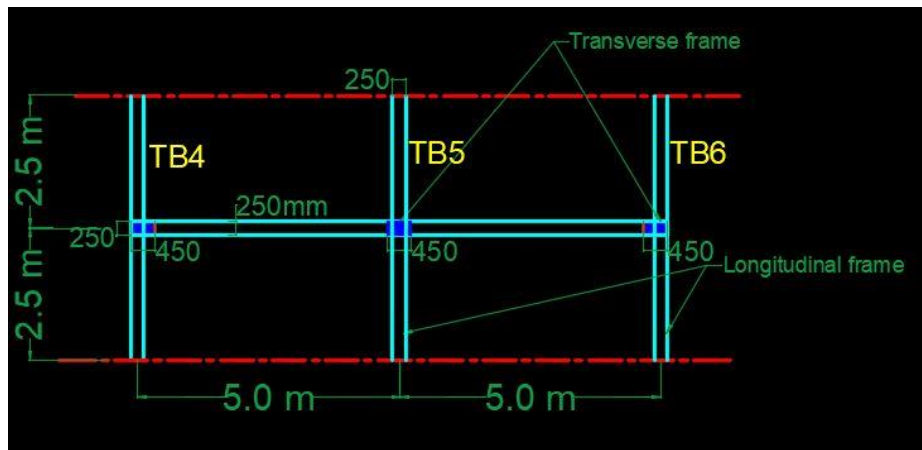


Figure 4.6(b) Plan showing the Columns and Beams at “Floor-2” of the Plane Frame

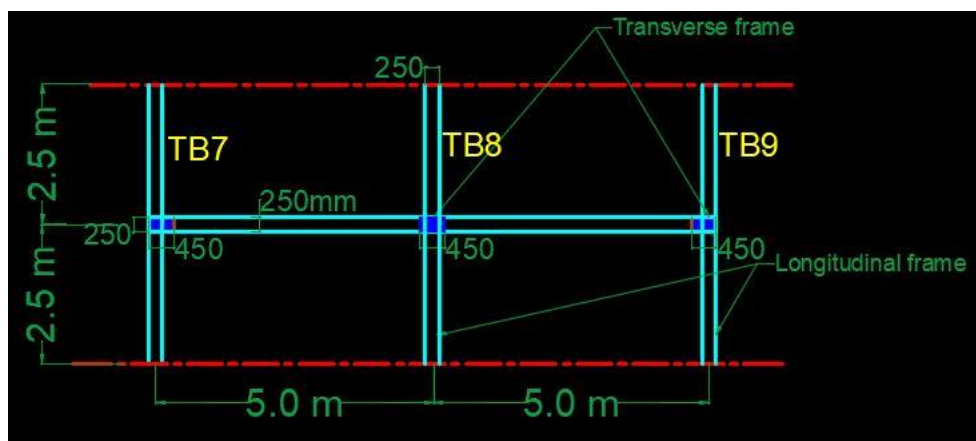


Figure 4.6(c) Plan showing the Columns and Beams at “Floor-3” of the Plane Frame

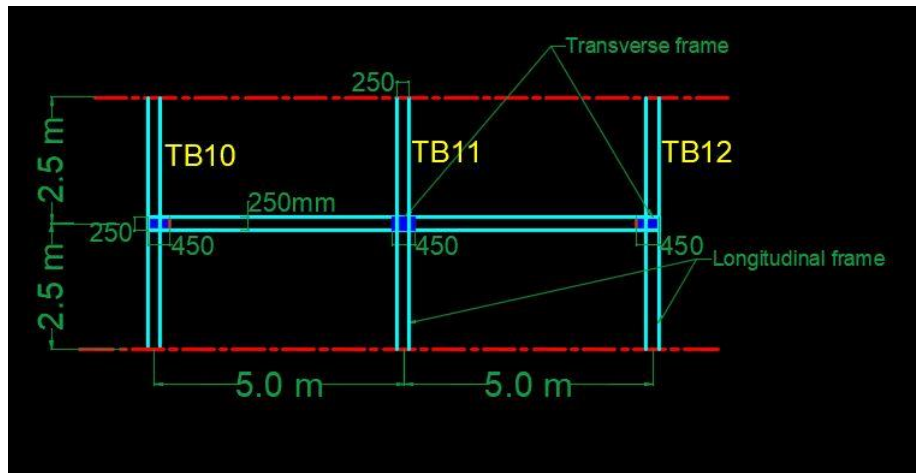


Figure 4.6(d) Plan showing the Columns and Beams at “Roof” of the Plane Frame

Furthermore, the statistical properties of random variables are tabulated, as in Table (4.8);

Table 4.8(a) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
1	Longitudinal Beam-1	fckl1 (kN/m ²)	Normal	11.5	37200	4278
2	Longitudinal Beam-2	fckl2 (kN/m ²)	Normal	11.5	37200	4278
3	Longitudinal Beam-3	fckl3 (kN/m ²)	Normal	11.5	37200	4278
4	Longitudinal Beam-4	fckl4 (kN/m ²)	Normal	11.5	37200	4278
5	Longitudinal Beam-5	fckl5 (kN/m ²)	Normal	11.5	37200	4278
6	Longitudinal Beam-6	fckl6 (kN/m ²)	Normal	11.5	37200	4278
7	Longitudinal Beam-7	fckl7 (kN/m ²)	Normal	11.5	37200	4278
8	Longitudinal Beam-8	fckl8 (kN/m ²)	Normal	11.5	37200	4278
9	Transverse Beam-1	fckt1 (kN/m ²)	Normal	11.5	37200	4278
10	Transverse Beam-2	fckt2 (kN/m ²)	Normal	11.5	37200	4278
11	Transverse Beam-3	fckt3 (kN/m ²)	Normal	11.5	37200	4278
12	Transverse Beam-4	fckt4 (kN/m ²)	Normal	11.5	37200	4278
13	Transverse Beam-5	fckt5 (kN/m ²)	Normal	11.5	37200	4278
14	Transverse Beam-6	fckt6 (kN/m ²)	Normal	11.5	37200	4278
15	Transverse Beam-7	fckt7 (kN/m ²)	Normal	11.5	37200	4278
16	Transverse Beam-8	fckt8 (kN/m ²)	Normal	11.5	37200	4278
17	Transverse Beam-9	fckt9 (kN/m ²)	Normal	11.5	37200	4278
18	Transverse Beam-10	fckt10 (kN/m ²)	Normal	11.5	37200	4278
19	Transverse Beam-11	fckt11 (kN/m ²)	Normal	11.5	37200	4278
20	Transverse Beam-12	fckt12 (kN/m ²)	Normal	11.5	37200	4278
21	Sab-1	fcks1 (kN/m ²)	Normal	11.5	37200	4278
22	Sab-2	fcks2 (kN/m ²)	Normal	11.5	37200	4278
23	Sab-3	fcks3 (kN/m ²)	Normal	11.5	37200	4278
24	Sab-4	fcks4 (kN/m ²)	Normal	11.5	37200	4278
25	Column-1	fckc1 (kN/m ²)	Normal	11.5	37200	4278
26	Column-2	fckc2 (kN/m ²)	Normal	11.5	37200	4278
27	Column-3	fckc3 (kN/m ²)	Normal	11.5	37200	4278
28	Column-4	fckc4 (kN/m ²)	Normal	11.5	37200	4278
29	Column-5	fckc5 (kN/m ²)	Normal	11.5	37200	4278

Table 4.8(b) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
30	Column-6	fckc6 (kN/m ²)	Normal	11.5	37200	4278
31	Column-7	fckc7 (kN/m ²)	Normal	11.5	37200	4278
32	Column-8	fckc8 (kN/m ²)	Normal	11.5	37200	4278
33	Column-9	fckc9 (kN/m ²)	Normal	11.5	37200	4278
34	Column-10	fckc10 (kN/m ²)	Normal	11.5	37200	4278
35	Column-11	fckc11 (kN/m ²)	Normal	11.5	37200	4278
36	Column-12	fckc12(kN/m ²)	Normal	11.5	37200	4278
37	Longitudinal Infill Wall-1	EMI1 (kN/m ²)	Normal	20	17940000	3588000
38	Longitudinal Infill Wall-2	EMI2 (kN/m ²)	Normal	20	17940000	3588000
39	Longitudinal Infill Wall-3	EMI3 (kN/m ²)	Normal	20	17940000	3588000
40	Longitudinal Infill Wall-4	EMI4 (kN/m ²)	Normal	20	17940000	3588000
41	Longitudinal Infill Wall-5	EMI5 (kN/m ²)	Normal	20	17940000	3588000
42	Longitudinal Infill Wall-6	EMI6 (kN/m ²)	Normal	20	17940000	3588000
43	Longitudinal Infill Wall-7	EMI7 (kN/m ²)	Normal	20	17940000	3588000
44	Longitudinal Infill Wall-8	EMI8 (kN/m ²)	Normal	20	17940000	3588000
45	Transverse Infill Wall-1	EMt1 (kN/m ²)	Normal	20	17940000	3588000
46	Transverse Infill Wall-2	EMt2 (kN/m ²)	Normal	20	17940000	3588000
47	Transverse Infill Wall-3	EMt 3(kN/m ²)	Normal	20	17940000	3588000
48	Transverse Infill Wall-4	EMt4 (kN/m ²)	Normal	20	17940000	3588000
49	Transverse Infill Wall-5	EMt5 (kN/m ²)	Normal	20	17940000	3588000
50	Transverse Infill Wall-6	EMt6 (kN/m ²)	Normal	20	17940000	3588000
51	Transverse Infill Wall-7	EMt7 (kN/m ²)	Normal	20	17940000	3588000
52	Transverse Infill Wall-8	EMt8 (kN/m ²)	Normal	20	17940000	3588000
53	Transverse Infill Wall-9	EMt9 (kN/m ²)	Normal	20	17940000	3588000
54	Transverse Infill Wall-10	EMt10 (kN/m ²)	Normal	20	17940000	3588000
55	Transverse Infill Wall-11	EMt11 (kN/m ²)	Normal	20	17940000	3588000
56	Transverse Infill Wall-12	EMt12 (kN/m ²)	Normal	20	17940000	3588000
57	Longitudinal Beam Width-1	bb_l1(m)	Normal	4	0.2525	0.0101
58	Longitudinal Beam Width-2	bb_l2(m)	Normal	4	0.2525	0.0101

Table 4.8(c) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
59	Longitudinal Beam Width-3	bb_l3(m)	Normal	4	0.2525	0.0101
60	Longitudinal Beam Width-4	bb_l4(m)	Normal	4	0.2525	0.0101
61	Longitudinal Beam Width-5	bb_l5(m)	Normal	4	0.2525	0.0101
62	Longitudinal Beam Width-6	bb_l6(m)	Normal	4	0.2525	0.0101
63	Longitudinal Beam Width-7	bb_l7(m)	Normal	4	0.2525	0.0101
64	Longitudinal Beam Width-8	bb_l8(m)	Normal	4	0.2525	0.0101
65	Longitudinal Beam Depth-1	db_l1(m)	Normal	4	0.396	0.01584
66	Longitudinal Beam Depth-2	db_l2(m)	Normal	4	0.396	0.01584
67	Longitudinal Beam Depth-3	db_l3(m)	Normal	4	0.396	0.01584
68	Longitudinal Beam Depth-4	db_l4(m)	Normal	4	0.396	0.01584
69	Longitudinal Beam Depth-5	db_l5(m)	Normal	4	0.396	0.01584
70	Longitudinal Beam Depth-6	db_l6(m)	Normal	4	0.396	0.01584
71	Longitudinal Beam Depth-7	db_l7(m)	Normal	4	0.396	0.01584
72	Longitudinal Beam Depth-8	db_l8(m)	Normal	4	0.396	0.01584
73	Transverse Beam Width-1	bb_t1(m)	Normal	4	0.2525	0.0101
74	Transverse Beam Width-2	bb_t2(m)	Normal	4	0.2525	0.0101
75	Transverse Beam Width-3	bb_t3(m)	Normal	4	0.2525	0.0101
76	Transverse Beam Width-4	bb_t4(m)	Normal	4	0.2525	0.0101
77	Transverse Beam Width-5	bb_t5(m)	Normal	4	0.2525	0.0101
78	Transverse Beam Width-6	bb_t6(m)	Normal	4	0.2525	0.0101
79	Transverse Beam Width-7	bb_t7(m)	Normal	4	0.2525	0.0101
80	Transverse Beam Width-8	bb_t8(m)	Normal	4	0.2525	0.0101
81	Transverse Beam Width-9	bb_t9(m)	Normal	4	0.2525	0.0101
82	Transverse Beam Width-10	bb_t10(m)	Normal	4	0.2525	0.0101
83	Transverse Beam Width-11	bb_t11(m)	Normal	4	0.2525	0.0101
84	Transverse Beam Width-12	bb_t12(m)	Normal	4	0.2525	0.0101
85	Transverse Beam Depth-1	db_t1(m)	Normal	4	0.3465	0.01386
86	Transverse Beam Depth-2	db_t2(m)	Normal	4	0.3465	0.01386
87	Transverse Beam Depth-3	db_t3(m)	Normal	4	0.3465	0.01386

Table 4.8(d) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
88	Transverse Beam Depth-4	db_t4(m)	Normal	4	0.3465	0.01386
89	Transverse Beam Depth-5	db_t5(m)	Normal	4	0.3465	0.01386
90	Transverse Beam Depth-6	db_t6(m)	Normal	4	0.3465	0.01386
91	Transverse Beam Depth-7	db_t7(m)	Normal	4	0.3465	0.01386
92	Transverse Beam Depth-8	db_t8(m)	Normal	4	0.3465	0.01386
93	Transverse Beam Depth-9	db_t9(m)	Normal	4	0.3465	0.01386
94	Transverse Beam Depth-10	db_t10(m)	Normal	4	0.3465	0.01386
95	Transverse Beam Depth-11	db_t11(m)	Normal	4	0.3465	0.01386
96	Transverse Beam Depth-12	db_t12(m)	Normal	4	0.3465	0.01386
97	Slab Thickness-1	t1 (m)	Normal	12	0.1104	0.013248
98	Slab Thickness-2	t2 (m)	Normal	12	0.1104	0.013248
99	Slab Thickness-3	t3 (m)	Normal	12	0.1104	0.013248
100	Slab Thickness-4	t4 (m)	Normal	12	0.1104	0.013248
101	Column Dim. (Transversely)-1	bc1 (m)	Normal	4	0.25125	0.01005
102	Column Dim. (Transversely)-2	bc2 (m)	Normal	4	0.25125	0.01005
103	Column Dim. (Transversely)-3	bc3 (m)	Normal	4	0.25125	0.01005
104	Column Dim. (Transversely)-4	bc4 (m)	Normal	4	0.25125	0.01005
105	Column Dim. (Transversely)-5	bc5 (m)	Normal	4	0.25125	0.01005
106	Column Dim. (Transversely)-6	bc6 (m)	Normal	4	0.25125	0.01005
107	Column Dim. (Transversely)-7	bc7 (m)	Normal	4	0.25125	0.01005
108	Column Dim. (Transversely)-8	bc8 (m)	Normal	4	0.25125	0.01005
109	Column Dim. (Transversely)-9	bc9 (m)	Normal	4	0.25125	0.01005
110	Column Dim. (Transversely)-10	bc10 (m)	Normal	4	0.25125	0.01005
111	Column Dim. (Transversely)-11	bc11 (m)	Normal	4	0.25125	0.01005
112	Column Dim. (Transversely)-12	bc12 (m)	Normal	4	0.25125	0.01005
113	Column Dim. (Longitudinally)-1	Dc1 (m)	Normal	4	0.45225	0.01809
114	Column Dim. (Longitudinally)-2	Dc2 (m)	Normal	4	0.45225	0.01809
115	Column Dim. (Longitudinally)-3	Dc3 (m)	Normal	4	0.45225	0.01809
116	Column Dim. (Longitudinally)-4	Dc4 (m)	Normal	4	0.45225	0.01809

Table 4.8(e) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
117	Column Dim. (Longitudinally)-5	Dc5 (m)	Normal	4	0.45225	0.01809
118	Column Dim. (Longitudinally)-6	Dc6 (m)	Normal	4	0.45225	0.01809
119	Column Dim. (Longitudinally)-7	Dc7 (m)	Normal	4	0.45225	0.01809
120	Column Dim. (Longitudinally)-8	Dc8 (m)	Normal	4	0.45225	0.01809
121	Column Dim. (Longitudinally)-9	Dc9 (m)	Normal	4	0.45225	0.01809
122	Column Dim. (Longitudinally)-10	Dc10 (m)	Normal	4	0.45225	0.01809
123	Column Dim. (Longitudinally)-11	Dc11 (m)	Normal	4	0.45225	0.01809
124	Column Dim. (Longitudinally)-12	Dc12 (m)	Normal	4	0.45225	0.01809
125	Long. Infill Wall Width-1	bl1 (m)	Normal	20	0.2525	0.0505
126	Long. Infill Wall Width-2	bl2 (m)	Normal	20	0.2525	0.0505
127	Long. Infill Wall Width-3	bl3 (m)	Normal	20	0.2525	0.0505
128	Long. Infill Wall Width-4	bl4 (m)	Normal	20	0.2525	0.0505
129	Long. Infill Wall Width-5	bl5 (m)	Normal	20	0.2525	0.0505
130	Long. Infill Wall Width-6	bl6 (m)	Normal	20	0.2525	0.0505
131	Long. Infill Wall Width-7	bl7 (m)	Normal	20	0.2525	0.0505
132	Long. Infill Wall Width-8	bl8 (m)	Normal	20	0.2525	0.0505
133	Trans. Infill Wall Width-1	bt1 (m)	Normal	20	0.1515	0.0303
134	Trans. Infill Wall Width-2	bt2 (m)	Normal	20	0.1515	0.0303
135	Trans. Infill Wall Width-3	bt3 (m)	Normal	20	0.1515	0.0303
136	Trans. Infill Wall Width-4	bt4 (m)	Normal	20	0.1515	0.0303
137	Trans. Infill Wall Width-5	bt5 (m)	Normal	20	0.1515	0.0303
138	Trans. Infill Wall Width-6	bt6 (m)	Normal	20	0.1515	0.0303
139	Trans. Infill Wall Width-7	bt7 (m)	Normal	20	0.1515	0.0303
140	Trans. Infill Wall Width-8	bt8 (m)	Normal	20	0.1515	0.0303
141	Trans. Infill Wall Width-9	bt9 (m)	Normal	20	0.1515	0.0303
142	Trans. Infill Wall Width-10	bt10 (m)	Normal	20	0.1515	0.0303
143	Trans. Infill Wall Width-11	bt11 (m)	Normal	20	0.1515	0.0303
144	Trans. Infill Wall Width-12	bt12 (m)	Normal	20	0.1515	0.0303

Table 4.8(f) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
145	Specific Weight Of Concrete for Long. Beam-1	gamma_cl1 (kN/m3)	Normal	10	26.25	2.625
146	Specific Weight Of Concrete for Long. Beam-2	gamma_cl2 (kN/m3)	Normal	10	26.25	2.625
147	Specific Weight Of Concrete for Long. Beam-3	gamma_cl3 (kN/m3)	Normal	10	26.25	2.625
148	Specific Weight Of Concrete for Long. Beam-4	gamma_cl4 (kN/m3)	Normal	10	26.25	2.625
149	Specific Weight Of Concrete for Long. Beam-5	gamma_cl5 (kN/m3)	Normal	10	26.25	2.625
150	Specific Weight Of Concrete for Long. Beam-6	gamma_cl6 (kN/m3)	Normal	10	26.25	2.625
151	Specific Weight Of Concrete for Long. Beam-7	gamma_cl7 (kN/m3)	Normal	10	26.25	2.625
152	Specific Weight Of Concrete for Long. Beam-8	gamma_cl8 (kN/m3)	Normal	10	26.25	2.625
153	Specific Weight Of Concrete for Trans. Beam-1	gamma_ct1 (kN/m3)	Normal	10	26.25	2.625
154	Specific Weight Of Concrete for Trans. Beam-2	gamma_ct2 (kN/m3)	Normal	10	26.25	2.625
155	Specific Weight Of Concrete for Trans. Beam-3	gamma_ct3 (kN/m3)	Normal	10	26.25	2.625
156	Specific Weight Of Concrete for Trans. Beam-4	gamma_ct4 (kN/m3)	Normal	10	26.25	2.625
157	Specific Weight Of Concrete for Trans. Beam-5	gamma_ct5 (kN/m3)	Normal	10	26.25	2.625
158	Specific Weight Of Concrete for Trans. Beam-6	gamma_ct6 (kN/m3)	Normal	10	26.25	2.625

Table 4.8(g) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
159	Specific Weight Of Concrete for Trans. Beam-7	gamma_ct7 (kN/m3)	Normal	10	26.25	2.625
160	Specific Weight Of Concrete for Trans. Beam-8	gamma_ct8 (kN/m3)	Normal	10	26.25	2.625
161	Specific Weight Of Concrete for Trans. Beam-9	gamma_ct9 (kN/m3)	Normal	10	26.25	2.625
162	Specific Weight Of Concrete for Trans. Beam-10	gamma_ct10 (kN/m3)	Normal	10	26.25	2.625
163	Specific Weight Of Concrete for Trans. Beam-11	gamma_ct11 (kN/m3)	Normal	10	26.25	2.625
164	Specific Weight Of Concrete for Trans. Beam-12	gamma_ct12 (kN/m3)	Normal	10	26.25	2.625
165	Specific Weight of Concrete for Slab-1	gamma_cs1 (kN/m3)	Normal	10	26.25	2.625
166	Specific Weight of Concrete for Slab-2	gamma_cs2 (kN/m3)	Normal	10	26.25	2.625
167	Specific Weight of Concrete for Slab-3	gamma_cs3 (kN/m3)	Normal	10	26.25	2.625
168	Specific Weight of Concrete for Slab-4	gamma_cs4 (kN/m3)	Normal	10	26.25	2.625
169	Specific Weight of Concrete for Column-1	gamma_cc1 (kN/m3)	Normal	10	26.25	2.625
170	Specific Weight of Concrete for Column-2	gamma_cc2 (kN/m3)	Normal	10	26.25	2.625
171	Specific Weight of Concrete for Column-3	gamma_cc3 (kN/m3)	Normal	10	26.25	2.625
172	Specific Weight of Concrete for Column-4	gamma_cc4 (kN/m3)	Normal	10	26.25	2.625

Table 4.8(h) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
173	Specific Weight of Concrete for Column-5	gamma_cc5 (kN/m3)	Normal	10	26.25	2.625
174	Specific Weight of Concrete for Column-6	gamma_cc6 (kN/m3)	Normal	10	26.25	2.625
175	Specific Weight of Concrete for Column-7	gamma_cc7 (kN/m3)	Normal	10	26.25	2.625
176	Specific Weight of Concrete for Column-8	gamma_cc8 (kN/m3)	Normal	10	26.25	2.625
177	Specific Weight of Concrete for Column-9	gamma_cc9 (kN/m3)	Normal	10	26.25	2.625
178	Specific Weight of Concrete for Column-10	gamma_cc10 (kN/m3)	Normal	10	26.25	2.625
179	Specific Weight of Concrete for Column-11	gamma_cc11 (kN/m3)	Normal	10	26.25	2.625
180	Specific Weight of Concrete for Column-12	gamma_cc12 (kN/m3)	Normal	10	26.25	2.625
181	Specfic Weight of Infill for Longitudinal Wall-1	gamma_ifl1 (kN/m3)	Normal	20	21	4.2
182	Specfic Weight of Infill for Longitudinal Wall-2	gamma_ifl2 (kN/m3)	Normal	20	21	4.2
183	Specfic Weight of Infill for Longitudinal Wall-3	gamma_ifl3 (kN/m3)	Normal	20	21	4.2
184	Specfic Weight of Infill for Longitudinal Wall-4	gamma_ifl4 (kN/m3)	Normal	20	21	4.2
185	Specfic Weight of Infill for Longitudinal Wall-5	gamma_ifl5 (kN/m3)	Normal	20	21	4.2
186	Specfic Weight of Infill for Longitudinal Wall-6	gamma_ifl6 (kN/m3)	Normal	20	21	4.2

Table 4.8(i) Statistics of Random Variables associated to the Proposed Structural System

S.No	STRUCTURAL COMPONENT	RV	DISTRIBUTION	COV (%)	MEAN	SD
187	Specific Weight of Infill for Longitudinal Wall-7	gamma_ifl7 (kN/m3)	Normal	20	21	4.2
188	Specific Weight of Infill for Longitudinal Wall-8	gamma_ifl8 (kN/m3)	Normal	20	21	4.2
189	Specific Weight of Infill for Transverse Wall-1	gamma_iflt1 (kN/m3)	Normal	20	21	4.2
190	Specific Weight of Infill for Transverse Wall-2	gamma_iflt2 (kN/m3)	Normal	20	21	4.2
191	Specific Weight of Infill for Transverse Wall-3	gamma_iflt3 (kN/m3)	Normal	20	21	4.2
192	Specific Weight of Infill for Transverse Wall-4	gamma_iflt4 (kN/m3)	Normal	20	21	4.2
193	Specific Weight of Infill for Transverse Wall-5	gamma_iflt5 (kN/m3)	Normal	20	21	4.2
194	Specific Weight of Infill for Transverse Wall-6	gamma_iflt6 (kN/m3)	Normal	20	21	4.2
195	Specific Weight of Infill for Transverse Wall-7	gamma_iflt7 (kN/m3)	Normal	20	21	4.2
196	Specific Weight of Infill for Transverse Wall-8	gamma_iflt8 (kN/m3)	Normal	20	21	4.2
197	Specific Weight of Infill for Transverse Wall-9	gamma_iflt9 (kN/m3)	Normal	20	21	4.2
198	Specific Weight of Infill for Transverse Wall-10	gamma_iflt10 (kN/m3)	Normal	20	21	4.2
199	Specific Weight of Infill for Transverse Wall-11	gamma_iflt11 (kN/m3)	Normal	20	21	4.2
200	Specific Weight of Infill for Transverse Wall-12	gamma_iflt12 (kN/m3)	Normal	20	21	4.2
201	Measure for Floor-1	imp_load1 (kN/m2)	Normal	18	3.85	0.693
202	Measure for Floor-2	imp_load2 (kN/m2)	Normal	18	3.85	0.693
203	Measure for Floor-3	imp_load3 (kN/m2)	Normal	18	3.85	0.693

From the above tabulated data, it can be reckoned that all random variables are made to follow the “Normal Distribution”, here again, and also, presumed to be independent. In totality, there are '203' such input random variables for the reliability analysis of the proposed structural system. However, it must be stated that the code-based procedure for the same analysis doesn't incorporate the reinforcement (steel) related properties, namely, the “Characteristic Strength of Steel, f_y ” and “Young's Modulus of Steel, E_s ”. Hence, these material properties aren't randomized in this main study, but, practically, these carry utmost importance in the field. In addition, any advanced software, like OpenSees, or parameter updating functionality in MCS can efficiently incorporate this randomization. In that case, there would have been '275' input random variables.

4.5.2 Input Grid For MDRM

As stated earlier, the examined problem incorporates '203' independent random variables as an input, i.e., ' $n = 203$ '. Thus, the structural output, “*BASE SHEAR [SSF(1)]*”, is a product of '*203 cut – functions*'. Incorporating a fifth-order, i.e., ' $N = 5$ ', ‘Gauss-Hermite Integration scheme’, ' $nN + 1 = 203 \times 5 + 1 = 1016$ ' trials are executed. The input data grid to execute MDRM is tabulated, as in Table (4.9) (similar to the first main study, proposed in the previous section). The statistical analysis is executed on the basis of these tabulated results in the following section.

Table 4.9 Input Grid for the Response Evaluation

Random Variable	Trial No.	z_j	1	2	3	...	200	201	202	203	SSF(1)
			fck1	fck2	fck3	...	gamma_ift12	imp_load1	imp_load2	imp_load3	SSF(1)
fck1	1	-2.85697	24977.88234	37200	37200.00	...	21	3.85	3.85	3.85	333.2870801
	2	-1.35563	31400.61486	37200	37200.00	...	21	3.85	3.85	3.85	332.7941146
	3	0	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737
	4	1.35563	42999.38514	37200	37200.00	...	21	3.85	3.85	3.85	332.0998436
	5	2.85697	49422.11766	37200	37200.00	...	21	3.85	3.85	3.85	331.7860366
fck2	6	-2.85697	37200	24977.88234	37200.00	...	21	3.85	3.85	3.85	333.2870801
	7	-1.35563	37200	31400.61486	37200.00	...	21	3.85	3.85	3.85	332.7941146
	8	0	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737
	9	1.35563	37200	42999.38514	37200.00	...	21	3.85	3.85	3.85	332.0998436
	10	2.85697	37200	49422.11766	37200.00	...	21	3.85	3.85	3.85	331.7860366
fck3	11	-2.85697	37200	37200	24977.88234	...	21	3.85	3.85	3.85	332.0711676
	12	-1.35563	37200	37200	31400.61486	...	21	3.85	3.85	3.85	332.2725558
	13	0	37200	37200	37200	...	21	3.85	3.85	3.85	332.4222737
	14	1.35563	37200	37200	42999.38514	...	21	3.85	3.85	3.85	332.5506046
	15	2.85697	37200	37200	49422.11766	...	21	3.85	3.85	3.85	332.6741953
...
imp_load1	1001	-2.85697	37200	37200	37200.00	...	21	1.87011979	3.85	3.85	326.6003882
	1002	-1.35563	37200	37200	37200.00	...	21	2.91054841	3.85	3.85	329.6505555
	1003	0	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737
	1004	1.35563	37200	37200	37200.00	...	21	4.78945159	3.85	3.85	335.2104683
	1005	2.85697	37200	37200	37200.00	...	21	5.82988021	3.85	3.85	338.3173262
imp_load2	1006	-2.85697	37200	37200	37200.00	...	21	3.85	1.87011979	3.85	324.7658162
	1007	-1.35563	37200	37200	37200.00	...	21	3.85	2.91054841	3.85	328.7883439
	1008	0	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737
	1009	1.35563	37200	37200	37200.00	...	21	3.85	4.78945159	3.85	336.0577307
	1010	2.85697	37200	37200	37200.00	...	21	3.85	5.82988021	3.85	340.085518
imp_load3	1011	-2.85697	37200	37200	37200.00	...	21	3.85	3.85	1.87011979	325.492744
	1012	-1.35563	37200	37200	37200.00	...	21	3.85	3.85	2.91054841	329.1319645
	1013	0	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737
	1014	1.35563	37200	37200	37200.00	...	21	3.85	3.85	4.78945159	335.7166019
	1015	2.85697	37200	37200	37200.00	...	21	3.85	3.85	5.82988021	339.3696508
FIXED MEAN VALUES	1016	N/A	37200	37200	37200.00	...	21	3.85	3.85	3.85	332.4222737

Legend: ' z_j ' denotes the Gauss-Hermite points

4.5.3 Statistical Moments Of The Structural Response

Analogous to the main study-1, the output grid is tabulated as follows [Table (4.10)]. Meanwhile, the MCS technique is also performed for '**1e5**' trials. Further, the numerical results computed from both the methods are then compared, as tabulated in Table (4.11).

Here again, The tabulated data depicts the numerical accuracy of M-DRM with '**1016 trials**' in front of MCS with '**1e5 trials**'.

Table 4.10 Output Grid for Each Cut-Function Evaluation

Random Variable	Trial No.	SSF(1)	w_j	$w_j^*SSF(1)$	ϕ_j	$w_j^*SSF(1)*SSF(1)$	τ_i
fck1	1	333.2870801	0.01126	3.752812522	1.000050752	1250.763927	1.000092108
	2	332.7941146	0.22208	73.90691697		24595.78699	
	3	332.4222737	0.53333	177.2907712		58935.40127	
	4	332.0998436	0.22208	73.75273328		24493.27119	
	5	331.7860366	0.01126	3.735910772		1239.523028	
fck2	6	333.2870801	0.01126	3.752812522	1.000050752	1250.763927	1.000092108
	7	332.7941146	0.22208	73.90691697		24595.78699	
	8	332.4222737	0.53333	177.2907712		58935.40127	
	9	332.0998436	0.22208	73.75273328		24493.27119	
	10	331.7860366	0.01126	3.735910772		1239.523028	
fck3	11	332.0711676	0.01126	3.739121347	0.999992352	1241.654391	0.999974802
	12	332.2725558	0.22208	73.7910892		24518.7538	
	13	332.4222737	0.53333	177.2907712		58935.40127	
	14	332.5506046	0.22208	73.85283826		24559.80601	
	15	332.6741953	0.01126	3.745911439		1246.168074	
...
imp_load1	1001	326.6003882	0.01126	3.677520371	1.000023486	1201.079581	1.00007529
	1002	329.6505555	0.22208	73.20879537		24133.32007	
	1003	332.4222737	0.53333	177.2907712		58935.40127	
	1004	335.2104683	0.22208	74.44354081		24954.25418	
	1005	338.3173262	0.01126	3.809453093		1288.803984	
imp_load2	1006	324.7658162	0.01126	3.656863091	1.00001125	1187.624127	1.000077558
	1007	328.7883439	0.22208	73.01731541		24007.24221	
	1008	332.4222737	0.53333	177.2907712		58935.40127	
	1009	336.0577307	0.22208	74.63170083		25080.56001	
	1010	340.0855518	0.01126	3.829362933		1302.310877	
imp_load3	1011	325.492744	0.01126	3.665048297	332.4266914	1192.946627	110512.2987
	1012	329.1319645	0.22208	73.09362668		24057.44894	
	1013	332.4222737	0.53333	177.2907712		58935.40127	
	1014	335.7166019	0.22208	74.55594294		25029.66781	
	1015	339.3696508	0.01126	3.821302268		1296.834016	
FIXED MEAN VALUES	1016	332.4222737	N/A	N/A	N/A	N/A	N/A

Legend: ' w_j ' denotes the Gauss-Hermite weights

Table 4.11 Statistical Moments of the Response

BASE SHEAR [<i>SSF</i>(1)]	MRDM (1016 Trials)	MCS (1e5 Trials)	REL. ERROR (%)
First Moment (kN)	332.4424	332.1870	0.076884405
Second Moment (kN²)	110732.30	110559.88	0.155951689
Std. Dev. (kN)	14.6414	14.5492	0.633711819
COV	0.0440	0.0438	0.556409823

Legend: M-DRM—Multiplicative Dimensional Reduction Method;
MCS—Monte Carlo Simulation;
Relative Error--*Abs* [*MCS* – *MDRM*]/*MCS*;
Std. Dev.—Standard Deviation;
COV—Coefficient of Variation.

4.5.4 Probabilistic Distribution Of The Structural Response

Here again, the “MaxEnt Principle” coupled with the concept of “Fractional Moments” is applied to approximate the probabilistic distribution of the required structural output, i.e., “*BASE SHEAR [SSF(1)]*”. Moreover, four fractional moments, i.e., ‘*m* = 4’, are adequate to perform the analysis, as in, the entropy converges quite rapidly, as tabulated in following Table (4.12); The data is tabulated, incorporating ‘3’ entropy values, i.e., ‘*m* = 2’, ‘*m* = 3’ and ‘*m* = 4’, simply because the entropy generally converges at the fourth fractional moment.

Furthermore, the “Probability Density Function (PDF) + Cumulative Distribution Function (CDF)” and “Probability of Exceedance (POE)” Plots for ‘*m* = 2’, ‘*m* = 3’ and ‘*m* = 4’ are shown, as in Figure (4.7) and Figure (4.8), respectively. The convergence of plots for ‘*m* = 3’ and ‘*m* = 4’ define the convergence of entropy.

Table 4.12 MaxEnt Parameters for the Base Shear

		δ_1	δ_2	δ_1	δ_2	δ_3	δ_1	δ_2	δ_3	δ_4
		0.298	0.6007	-0.2106	0.3668	0.4081	0.513	-0.1722	-0.0153	0.3895
	λ_0	704.2401			704.2312			705.7076		
	λ_1	-246.8475			239.9311			109.3269		
	λ_2	21.1782			-951.9445			282.2853		
	λ_3				677.0667			-181.2198		
	λ_4							-290.6287		
FRACTIONAL MOMENT	$M_X^\delta = \frac{1}{N} \sum_i x_i^\delta$	5.64235	32.71695	0.29440	8.41301	10.69293	19.66170	0.36793	0.91499	9.59828
ENTROPY	$\text{Ent}[f] = \lambda_0 + \lambda_1 M_1 + \lambda_2 M_2 + \lambda_3 M_3 + \dots$	3.0028			2.9889			2.9463		

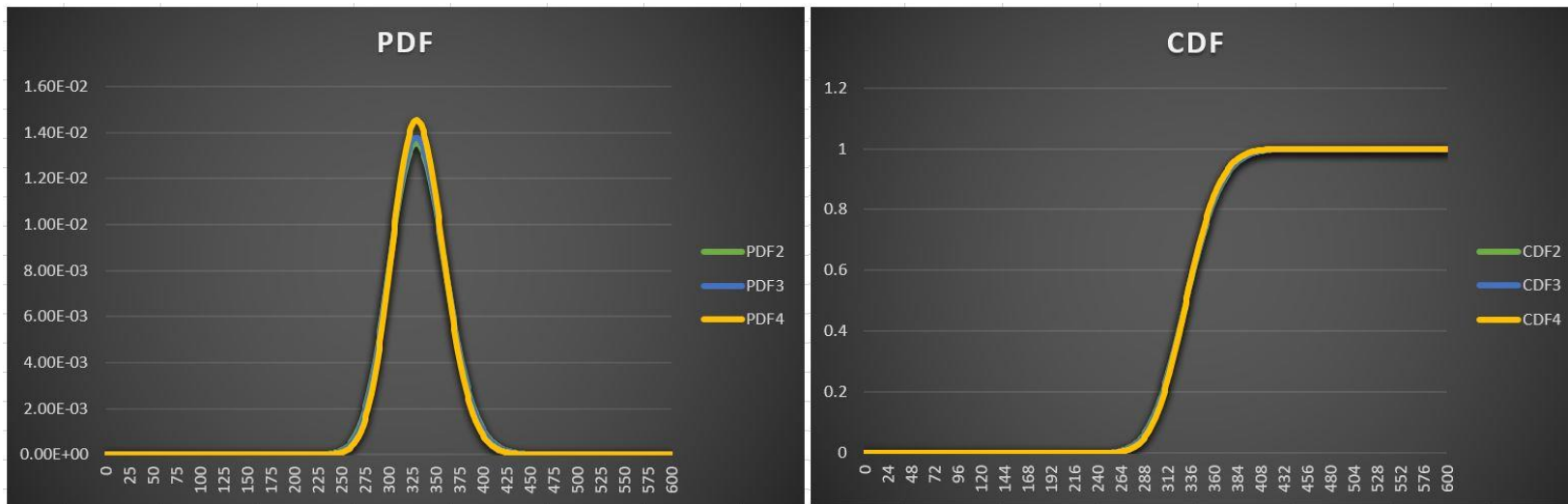


Figure 4.7 “Probability Density Function (PDF)” and “Cumulative Distribution Function (CDF)” Plots

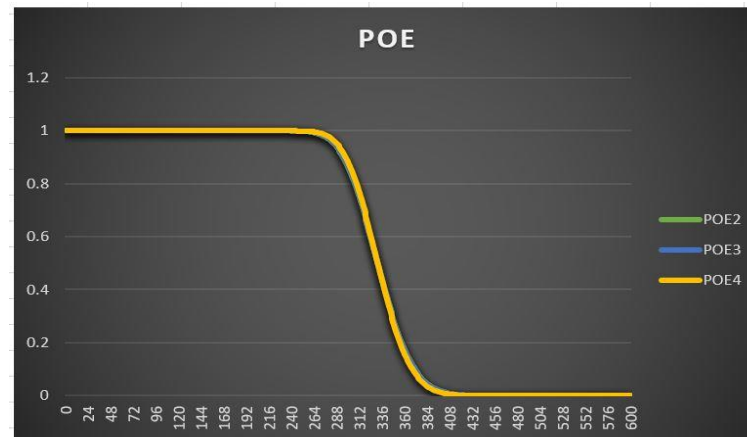


Figure 4.8 “Probability of Exceedance (POE)” Plot

4.5.5 Global Sensitivity Indices Using MDRM

As stated previously, extra computational efforts aren't necessitated to compute the sensitivity analysis' coefficients on utilising the formulation technique of MDRM. Hence, the global sensitivity indices for each of the "203" input (independent) random variables corresponding to "[SSF(1)]" are evaluated using the same. The tabulated data further summarizes the most influential structural components, for various respective parameters, Table (4.13).

Moreover, the sensitivity indices for "52" input random variables are "*negative*", namely, "*gamma_cs*", "*gamma_ifl*", "*bl*", "*imp_load*", etc., thereby, ascertaining their least influence on the output. However, the summation of "Primary Sensitivity Coefficients" and "Total Sensitivity Coefficients" are respectively, "0.9998" and "1.0002"; And this summation measure includes the measures of the latter informed parameters. Furthermore, it should be ascertained that the difference between both the sensitivity coefficients, i.e., " $S_{Ti} - S_i$ ", is has come out to be really small (less than "1%"), which signifies that all input random variables don't interact with each other (though not tabulated). Also, the difference, " $1 - \sum_i S_i = 0.0002$ ", signifies absence of interactions among the specified input random variables.

Table 4.13 Global Sensitivity Indices Using MDRM

S. No.	MAIN PARAMETER	SUB-RANDOM VARIABLE	MOST INFLUENTIAL STRUCTURAL COMPONENT	MAXIMUM SENSITIVITY INDEX
1	Characteristic Strength of Concrete (fck)	fckc	All columns (1-12) are equally influential	0.0723
		fckl	Longitudinal Beams--7 and 8	0.0722
		fckt	All Transverse Beams (1-12) are equally influential	0.0723
		fcks	All Slabs (1-4) are equally influential	0.0723
2	Specific Weight for Concrete (gamma_c)	gamma_cc	Columns--10, 11 and 12	0.0720
		gamma_cl	longitudinal Beams--1 and 2	0.0707
		gamma_ct	Transverse Beams--1, 2 and 3	0.0711
3	Specific Weight for Infill (gamma_if)	gamma_if	Infill Wall (Transverse Direction)--10, 11 and 12	0.0411
4	Modulus Of Masonry (EM)	EMl	Infill Wall (Longitudinal Direction)--7 and 8	0.0670
		EMt	All Infill Walls (Transverse) (1-12) are equally influential	0.0723
5	Column Geometry	bc	Columns--1, 2 and 3	0.0723
		Dc	Columns--3, 10, 11 and 12	0.0722
6	Longitudinal Beam Geometry	bb_l	Beams--7 and 8	0.0719
		db_l	Beams--7 and 8	0.0715
7	Transverse Beam Geometry	bb_t	Beams--1, 2, 3, 10, 11 and 12	0.0721
		db_t	Beams--1, 2, 3, 10, 11 and 12	0.0721
8	Infill Wall Width (b)	bt	Transverse Infill Walls--10, 11 and 12	0.0411

4.6 COMPUTATIONAL TIME

The main advantage of the proposed scheme MDRM is that it saves a substantial amount of time. For the response spectrum analysis of a '4 – storeyed structural system', be it for '16' or '203' random variables, '1e5' iterations as a technique to perform simulations takes hours of time period on a personal laptop (it includes development of code for the structural system on MATLAB, randomization and the running time period of code; however, it doesn't incorporate the development of MATLAB code to model the probability distribution of the structural response). As stated previously, MDRM is performed on 'Microsoft Excel'. That's why, MDRM approximation takes nearly half the time taken by that of Monte Carlo Method, even though it incorporates the development of MATLAB code to model the probability distribution of the structural response. Otherwise, this time would reduce to less than 10% of that taken by MCS, if performed using advanced software systems, say, for example, OpenSees FEA software can be utilised to perform the same dynamic analysis.

4.7 CONCLUSION

The chapter has presented the proficiency and robustness of the proposed MDRM (Multiplicative Dimensional Reduction Method) for the response spectrum analysis of a "4 – storeyed frame" under two cases. First, the frame is considered for "16" input variables, and then, the same is considered for "203" input random variables. Each and every step of the proposed scheme detailed in the previous chapter is followed rigorously. The Monte Carlo Simulation scheme is also performed meanwhile to validate the numerical accuracy of the Multiplicative-Dimensional Reduction Method.

Meanwhile, the computational cost of both the schemes is also held upon. The global sensitivity indices of all independent random variables to the response variance are evaluated as well, and this analysis forms a by-product of the proposed MDRM. Hence, additional computational or conceptual efforts aren't necessitated to compute the sensitivity indices, which predict the influence of input variables over the output. Thus, it can be ascertained that the integration of MDRM with the Maximum Entropy (MaxEnt) Principle proves to be a viable approach for any required analysis. To sum up, a fact can be emphasized that the proposed framework is computationally affordable as it saves a lot of time, thereby yielding similar results as provided

by time consuming and computationally expensive standard techniques like Monte Carlo Simulation (MCS). This becomes the main takeaway of this whole study.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this work is to impart an in-depth view of the “Multiplicative Dimensional Reduction Method (MDRM)” to perform the reliability and sensitivity analyses, with a great numerical accuracy. Several discussions have been made in this study for the same purpose, as recapitulated below:

5.1 SUMMARY

Chapter 3 bespoke the logic and formulated the mathematical equations of the proposed approach, i.e., MDRM. The “Theory of Probability” was revisited herein. The latter said theory was contemplated in detail so as to engross the concept of Multiplicative-DRM into the structural systems. This helped in evaluating the statistical moments and the probabilistic distribution of the structural output (response) and the global sensitivity indices of the input random variables in the implemented pilot study. The pilot study clearly demonstrated the procedural steps of the proposed scheme.

Chapter 4 presented the numerical applicability of the proposed MDRM to the "*Seismic (Response Spectrum) Analysis of a 4 – storeyed structural frame*", meanwhile accounting the codal procedure for the same. The Monte Carlo Simulation technique was performed meanwhile to validate the results. MS Excel and MATLAB were accounted for the same. In totality, two cases for the same frame were analysed using both the techniques. The first one incorporated "16" random variables as an input and the second one randomised "203" parameters for the same structural response, i.e., "*Base Shear*".

5.2 CONCLUSIONS

This research has comprehended a general computational framework for the “Reliability and Sensitivity Analyses” of the structural systems. Conclusions based upon the discussions and findings in this study have been grouped in two categories, as follows:

A. *“Utilisation of MDRM for Practical Systems”*

- ✚ The MDRM, being proficient, well-understood and computationally effective, can be contemplated as a viable approach for the reliability and sensitivity analyses of large or small-scale structural systems,
- ✚ MDRM can be held upon for the complete probabilistic analysis, since it computes the integer moments and the reliability distribution of the structural response of interest, meanwhile accounting for the global sensitivity indices, thereby not necessitating additional computational efforts for the same, and,
- ✚ MDRM coupled with the ‘Maximum Entropy Principle (MaxEnt Principle)’ (Fractional Moments as Constraints) can impart the numerically accurate probabilistic distribution of the required response.

B. *“Findings in regard to the Investigated Structural Systems”*

- ✚ The efficacy of MDRM makes it a credible substitute for MCS.
- ✚ As far as the pilot study is concerned, the variance of the material property plays an eminent role to the variance of the system’s response of interest, in comparison to the geometric uncertainties,
- ✚ The first main study incorporating "16" randomised uncertainties witness a significant role of the material properties over imposed load and geometric uncertainties, with "*Base Shear*", as a structural response of interest, and,
- ✚ The last study with "203" randomised variables witnesses mixed roles of all over the output response. Moreover, the same study also witnesses negative sensitivity indices (though not tabulated) of nearly "50" random variables. The latter fact recommends to comprehend an in-depth concept of the “Global Sensitivity Analysis” to avoid such spurious information, so as to have a clear idea of the influential parameters.

5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

The potential of the proposed scheme can be held out with some future research for practical structural problems, thereby taking into consideration some of the following practical aspects:

- ✚ The proposed framework accounts for uncorrelated or say, independent input variables; And this can be considered as one of the main limitations of the scheme, as the former account might not be always the case,
- ✚ In this study, almost every input distribution has been chosen from the literature, along with the statistical data of every randomised variable. But, this fact can't be overlooked that the input probability distributions can significantly influence the structural output. Hence, it is recommended that future analyses of practical problems should be held upon, accounting input parameters with their better representative statistical distributions,
- ✚ In this research, the main studies were based upon a regular-type structural frame. However, further analyses should take into account irregularities in buildings, to contemplate potential torsional effects, as far as the dynamic analyses of the structural systems are concerned, and,
- ✚ Global Sensitivity Analysis should be further investigated for the proposed framework, so as to not account for negative sensitivity indices in complex structural systems.

REFERENCES AND BIBLIOGRAPHY

- Agarwal, P., and Shrikhande, M., (2016)—*Eathquake Resistant Design of Structures*, PHI Learning Pvt Ltd., New Delhi, India;
- Balomenos, G. P., Genikomsou, A. S., Polak, M. A., and Pandey, M. D., (2017)—Probabilistic Analysis of Reinforced Concrete Flat Slabs, *International Concrete*, Vol. 321(2), 1-16;
- Choi, S-K, Grandhi, R. V, and Canfield, R. A., (2007)—*Reliability-based Structural Design*, Springer, London, UK;
- Christensen, P. T., and Baker, M. J., (1986)—*Structural Reliability Theory and its Applications*, Springer-Verlag, Berlin;
- Crawford, R. H., and Rao, S. S., (1989)—Probabilistic Analysis of Function Generating Mechanisms, *ASME J. of Mech., Trans., and Auto in Design*, Vol. 111, 479-481;
- Davis, P. J., and Rabinowitz, P., (1984)—*Methods of numerical integration*, Academic Press Inc., London, UK;
- Der Kiureghian, A., (2008)—Analysis of structural reliability under parameter uncertainties, *Probabilistic Engineering Mechanics*, (4), 351-358;
- Der Kiureghian, A., and Ditlevsen, O., (2009)—Aleatory or Epistemic? Does it matter?, *Structural Safety*, 31(2), 105-112;
- Gzyl, H., and Tagliani, A., (2010)—Hausdorff moment problems and fractional moments, *Applied Mathematics and Computation*, 216 (11), 3319 – 3328;
- Hartley, R. V., (1928)—*Transmission of information*, Bell System Technology, (7), 535-563;
- Inverardi, P. N., Pontuale, G., Petri, A., and Tagliani, A., (2003)—Hausdorff moment problems via fractional moments, *Applied Mathematics and Computation*, 144(1), 61-74;
- Inverardi, P. N., and Tagliani, A., (2003)—Maximum entropy density estimation from fractional moments, *Communication and Statistics- Theory and Methods*, 32(2), 327-0345;
- IS 456:2000—Indian Standard Plain and Reinforced Concrete Code of Practice (Fourth Revision)*, Bureau of Indian Standards, New Delhi, India;

IS 1893 (Part-1):2016—Criteria for Earthquake Resistant Design of Structures (Sixth Revision), Bureau of Indian Standards, New Delhi, India;

Jaynes, E., (1957)—*Information theory and statistical mechanics-Physical Review*, 108 (2), 171-190;

Jones, D., (1979)—*Elementary Information Theory*, Oxford University Press, New York;

Kapur, J., Kesavan, H., (1992)—*Entropy Optimization Principles with Applications*, Academic Press Inc., New York;

Li, Genyuan, Rosenthal, C., and Rabitz, H., (2001)—High dimensional model representations, *The Journal of Physical Chemistry, American Chemical Society*, 105(33), 7765-7777;

Li, Gang, and Zhang, K., (2011)—A combined reliability analysis approach with dimension reduction method and maximum entropy method, *Structural and Multidisciplinary Optimization*, 43(1), 121-134;

Madsen, H. O., Krenk, S., and Lind, N. C., (1986)—*Methods of Structural Safety*, Prentice Hall, Inc., Englewood Cliffs, NJ, USA;

Madsen, H. O., Krenk, S., and Lind, N. C., (2006)—*Methods of Structural Safety*, Dover Publications, Mineola, New York;

Metropolis, N., and Ulam, S., (1949)—The Monte Carlo Method, *Journal of the American Statistical Association*, 44(247), 335-341;

Montgomery, D. C., and Runger, G. C., (2003)—*Applied Statistics and Probability for Engineers*, 3rd Ed., John Wiley and Sons Inc., NY, USA;

Nowak, A. S., and Collins, K. R., (2000)—*Reliability of Structures*, McGraw-Hill, New-York, USA;

Pandey, M., Ariaratnam, S., (1996)—Crossing rate analysis of non-gaussian response of linear systems, *ASCE Journal of Engineering Mechanics*, 122 (6), 507 – 511;

Pandey, M. D., (2000)—Direct Estimation of Quantile Functions using the Maximum-Entropy Principle, Structural Safety, 22(1), 61-79;

- Pandey, M. D., and Zhang, X. (2012)—System reliability analysis of the robotic manipulator with random joint clearances, *Mechanism and Machine Theory*, 58, 137-152;
- Piegorsch, W., and Casella, G., (1985)—The existence of the first negative moment, *The American Statistician*, 31(9), 60-62;
- Rahman, S., Xu, H., (2004)—A univariate dimension-reduction method for multi-dimensional integration in stochastic mechanics, *Probabilistic Engineering Mechanics*, 19 (4), 393 – 408;
- Saltelli, A., and Sobol', I. M., (1995)—About the use of rank transformation in sensitivity analysis of model output, *Reliability Engineering & System Safety*, 50(3), 225-239;
- Saltelli, A., Chan K. and Scott E., (2000)—*Sensitivity analysis*, John Wiley and Sons Inc., NY, USA;
- Saltelli, A., Tarantola, S., Campolongo, F., and Ratto, M., (2004)—*Sensitivity analysis in practice: a guide to assessing scientific models*, John Wiley and Sons Ltd, Chichester, West Sussex, UK;
- Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S., (2008)—*Global sensitivity analysis: the primer*, 1st Ed., John Wiley and Sons Ltd., Chichester, West Sussex, UK;
- Shannon, C., (1949)—*The Mathematical Theory of Communication*, University of Illinois Press, Urbana;
- Shooman, M. L., (1968)—*Probabilistic Reliability: An Engineering Approach*, McGraw Hill, New York;
- Shore, J. E., and Johnson, R. W., (1980)—Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy, *IEEE Trans. on Information Theory*, 26(1), 26-37;
- Sudret, B., and Der Kiureghian, A., (2002)—Comparison of Finite Element Reliability Methods, *Probabilistic Engineering Mechanics*, 17(4), 337-348;
- Sudret, B., (2008)—Global sensitivity analysis using polynomial chaos expansion, *Reliability Engineering & System Safety*, 93(7), 964-979;

- Tagliani, A., (1994)—Maximum entropy in the hamburger moments problem, *Journal of Mathematical Physics*, 35 (9), 5087 – 5096;
- Tagliani, A., (1999)—Hausdorff moment problem and maximum entropy: A unified approach, *Applied Mathematics and Computation*, 105 (2-3), 291 – 305;
- Tunga, M.A., and Demiralp, M., (2004)—A factorized high dimensional model representation on the partitioned random discrete data, *Applied Numerical Analysis and Computational Mathematics*, 1(1), 231-241;
- Tunga, M.A., and Demiralp, M., (2005)—A factorized high dimensional model representation on the nodes of a finite hyper-prismatic regular grid, *Applied Mathematics and Computation*, 164(3), 865-883;
- Walpole, R. E., Myers, R. H., Myers, S. L., Ye, K., (2007)—*Probability and Statistics for Engineers and Scientists*, Pearson Education, Upper Saddle River, New Jersey;
- Xu, H., and Rahman, S. (2004)—A generalized dimension-reduction method for multidimensional integration in stochastic mechanics, *International Journal for Numerical Methods in Integration*, 61(12), 1992-2019;
- Zhang, X. (2013)—Efficient Computational Methods for Structural Reliability and Global Sensitivity Analyses, *PhD Thesis*, Department of Civil Engineering, University of Waterloo, Waterloo, ON, Canada;
- Zhang, X., and Pandey, M. D. (2013)—Structural Reliability Analysis based on the concepts of entropy, fractional moment and dimensional reduction method, *Structural Safety*, 43(4), 28-40;
- Zhang, X., and Pandey, M. D. (2014)—An effective approximation for variance-based global sensitivity analysis, *Reliability Engineering and System Safety*, 121(17), 164-174;
- Zwillinger, D., (2011)—*CRC standard mathematical tables and formulae*, CRC Press, Taylor & Francis Group, Boca Raton, FL, USA.

