An Inverse Hyperbolic Framework for Deterministic and Uncertain Laminated Composite Structures for Free Vibration Response

A Dissertation Submitted

in partial fulfillment of the requirements

for the degree of

Master of Engineering

in

CAD/CAM Engineering

by

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July 2019

CERTIFICATE

This is to certify that the work done in this thesis report title "**An Inverse Hyperbolic Framework for Deterministic and Uncertain Laminated Composite Structures for Free Vibration Response**" submitted in partial fulfillment of requirement for the award of Master of Engineering degree in CAD/CAM Engineering in the Mechanical Engineering department of Thapar University, Patiala, is an authentic record of work carried out by me under the guidance of Dr. Neeraj Grover and Dr. Rajendra Kumar, Mechanical Engineering department, Thapar Institute of Engineering and Technology, Patiala. The matter embodied in this report has not been submitted in any part or full to any other university or institute for the award of any degree.

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Acknowledgement

I take this opportunity to express my sincerest regards and gratitude to **Dr. Neeraj Grover and Dr. Rajendra Kumar** for guiding me through my research project. The experience of learning under their guidance was truly a lifetime opportunity.

I would also like to extend my warm regards to the Mechanical Engineering Department which has provided tremendous support throughout the course duration.

I take great pleasure in acknowledging the support of my fellow course mates with whom I got to learn so many things.

At last but not the least I would like to thank my family for their unconditional and tireless support.

Jasjit Singh

Abstract

In the present work, free vibration response of laminated composite spherical and cylindrical shells is carried out with and without material uncertainties. A non-polynomial framework particularly inverse hyperbolic shear deformation theory (IHSDT) is implemented to model the structural behavior of composite shells. The governing equations are obtained by utilizing the principle of virtual work. The linear structural kinematics and Generalised Hooke's law are also employed to include the material behavior. The coupled differential equations in explicit form are obtained. The coupling of these equations is reduced by assuming the cross ply configuration of composite structures. Further, these explicit coupled differential equations are solved in the exact manner for simply supported boundary conditions by employing a series solution. An Eigen value problem is obtained for the free vibration response. Further, in order to consider the effect of material uncertainties, Monte Carlo simulation (MCS) is implemented. The implementation of MCS enables to predict the second order statistics (mean and variance) of the free vibration response. An in-house MATLAB code is developed based on the developed formulation for deterministic as well as uncertain response. Various numerical studies are carried out to validate the present development. The effect of various parameters such as lamination sequence, material anisotropy, span-thickness ratio, radii of curvature are observed for doubly curved spherical and cylindrical shells. The effect of uncertain material properties on the free vibration response of these structure is also analyzed. It is concluded that IHSDT can be accurately and efficiently applied for the vibration analysis of doubly curved spherical and cylindrical shells. Further, the consideration of uncertain material properties for the optimum design of composite structures for vibration analysis is essential.

Keywords: Laminated Composite Shells; Free Vibration; Uncertainty Quantification; Navier Solution

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Nomenclature

Symbols	Description
a	Length of composite shell
b	Width of composite shell
h	Thickness of composite shell
$\left[\overline{Q_{ij}}\right]$	Reduced transformation stiffness matrix
$\left[\overline{R}\right]$	Stiffness matrix
$[M]$	Mass matrix
ω	Natural frequency
u	Displacement along x axis
v	Displacement along y axis
w	Displacement along z axis
$\emptyset_1, \emptyset_2$	Shear rotation in x and y plane
R_1	Radius of curvature along x axis
R_2	Radius of curvature along y axis
σ	Stress vector
ε	Strain vector
ρ	Density
N,M,P	Resultant stresses, moments and higher order moments respectively

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Chapter 1

Introduction

1. 1. Composites

A composite material is made up of two or more than two materials which differ from each other in terms of physical and chemical properties. The materials are mixed together to form a material which takes advantage of the properties of the each of the constituent material, thus offering better physical and chemical properties. Unlike mixtures, the constituent materials remain distinguishable and they do not blend in each other. The composite materials are in use from ancient times but their recent application in field of mechanical structures have paved way for some unique design possibilities, which were earlier hard to achieve. Another advantage of using a composite material is that the flexibility it offers for its design as it can be designed for plethora of design needs. The structure of a composite consists of two main constituent materials, which are classified as matrix and reinforcement. The matrix is used to maintain the reinforcement in its desired position while the reinforcement provides basic structural strength and other properties to the entire structural safety. The composites can be classified in terms of the type of reinforcement used.

1.2. Types of Composites

The composites are formed from various types of reinforcement materials. Below mentioned are some of the composites based on the type of reinforcement they use:

a) Particulate reinforced composites
b) Fiber reinforced composites
c) Structural composites

a) The reinforcement material in the form of particles is used in particulate reinforced composites. Infused in the matrices, these particles impart strength to the entire composite structure. Common examples of particulate reinforced composites include concrete (gravel and sand as reinforcement and cement as matrices), and ceramic and metal matrix composites.

b) The matrices bind thin and long fibres in a fibre reinforced composites. These long fibres can significantly improve the structures integrity, by enhancing mechanical properties. However their fabrication requires more attention as they are not that easy to manufacture as compared to particulate reinforced composites. The fibre reinforced plastics and fibre reinforced glass are some of the widely used fibre reinforced composites.

c) The structural composites are formed when layers of materials are stacked over each other and bonded together by matrices. In general cases, these materials include laminates made of fibre reinforced composites or particulate reinforced materials. The resulting composite structure offers enhanced quality in terms of strength and stiffness. This composite has become highly preferred material of choice in aerospace, automotive, and robotics industry. Common examples include carbon-fibre/epoxy composites and Kevlar/epoxy composites.

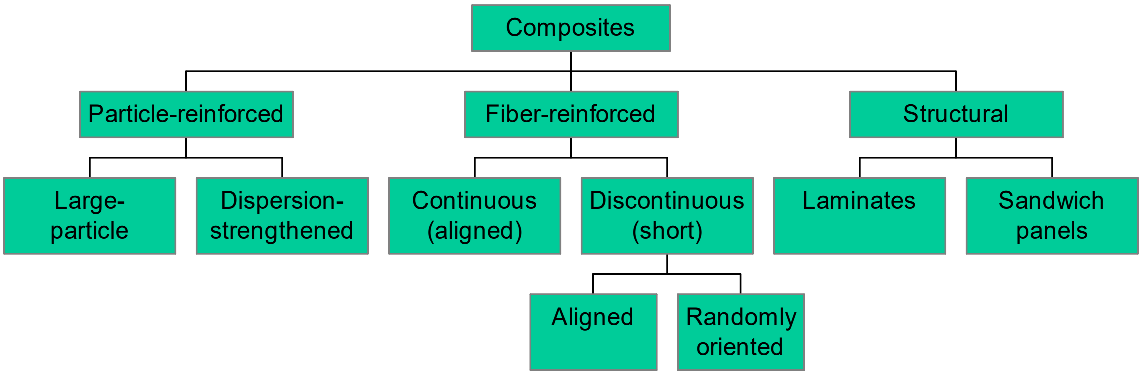

Fig. 1.1. Types of composites

Further classification of the composites is based on type of matrix they utilize. Based on most commonly used matrix materials, there are three composites:

i. Metal matrix composite (MMC)
ii. Ceramic matrix composite (CMC)
iii. Polymer matrix composite (PMC)

i. Metal Matrix Composites

The existing properties of metals are outclassed by the metal matrix composites, which specifically improve certain properties of the metal, such as coefficient of thermal expansion, thermal and heat conductivity, tensile strength etc. This opens a whole new horizon for the applications of these composites in the field of thermal components of the modern machinery and other avenues that require higher strength to weight ratios.

ii. Ceramic Matrix Composites

Exploiting the higher strength and stiffness of the ceramics, the composites formed with this matrix yield high hardness and offer unmatched surface wear resistance. These composites are also immune to corrosive effects.

iii. Polymer Matrix Composites

Due to their natural resistance towards corrosion and easy manufacturability, the polymer matrix composites have found their application in almost every industry. Their combination with carbon fibre reinforced composites have given rise to whole new composite material that offers significant improvement over its conventional metallic counter parts.

1.3. Advantages Offered by Composites

- They offer higher yield and tensile strength as compared conventional metals
- Composites can be fabricated for higher torsional stiffness.
- The effect of fatigue on composites is significantly less.
- Composites can offer better strength to weight ratios, which in some cases are even enough to replace metals like aluminum and steel.
- They are more stable in terms vibration response and contribute to noise free operations.
- The flexibility offered by composites is unparalleled, as they can be manufactured for any desired material properties. .
- Composites often exist for longer periods of time.
- Longer life time improves the life cycle cost of the composites.
- Corrosion resistance is one of the most significant property of the composites.
- Due to their versatility in terms of manufacturability, composites can be used to produce readily available decorative structures.

1.4. Applications of Composites

- Aerospace industry
- Robotics Industry
- Automobile Industry
- Electronics Industry
- Telecommunication Industry
- Defense and Arms Industry

From the frames of the aircrafts, to the bulletproof vest of a soldier, the composites have come a long way. The need of the 21st century for sturdier and lighter material has given birth to unconventional composite materials. While their applicability was relevant from centuries, it is only now that their true potential has started to unfold. Because of these composites, many of the modern luxuries are made possible.

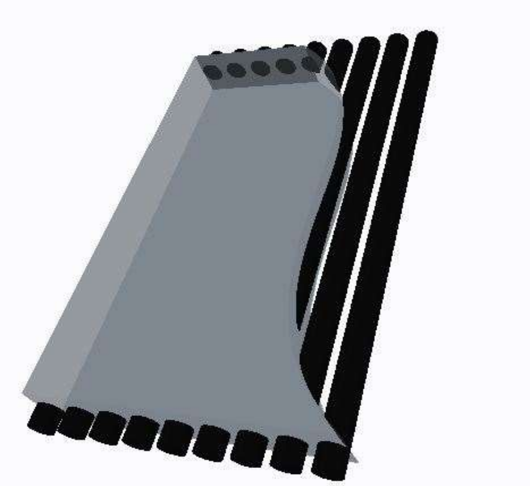

Fig. 1.2. Carbon epoxy fiber composite

Fig. 1.3. Ply wood laminated composite

1.5. Beams, Plates and Shells

A mechanical system is made up of number of components, such as beams, plates and shells. A beam is structural unit in a mechanical system which is used to bear the weight of other components. It is used to ensure the functional rigidity of the structure. Beams vary across various cross sectional shapes and different types of boundary conditions. On the basis of cross sectional shapes beams can be classified as:

- Rectangular section
- T- section
- I- section
- Triangular section
- Circular section

Beam can also have tapered cross section. While tapered beams are not so commonly used, they offer some special advantages in certain fields. One of the recent application of tapered beams is in the field of micro mechanical equipments, where their capability to reduce the effects of pull-in parameters have been found to be commendable. Beams can be arranged in a variety of boundary conditions, depending upon the structural needs of the mechanical design. Beams in these boundary conditions can be classified as follows:

- Cantilever beam
- Simply supported beam
- Overhanging beam
- Continuous beam
- Fixed-fixed beam

In addition to beams, a mechanical structure consists of plates and shells, which are structures whose thickness is small as compared to its length and breadth. While plates are generally planar, the shells are curved along length and breadth and can be classified as curved plates. Depending number of axis of rotation, shells can be of two types.

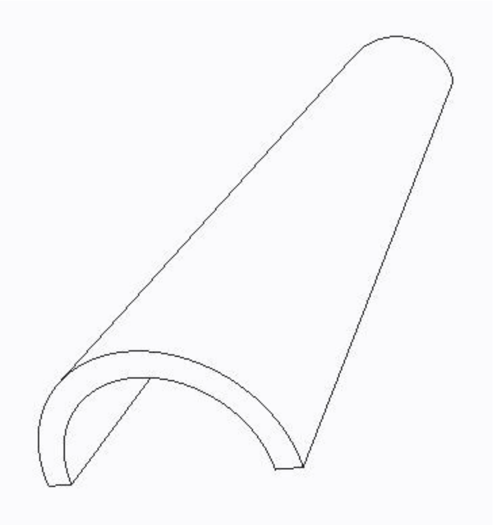

Fig. 1.4. Cylindrical shell

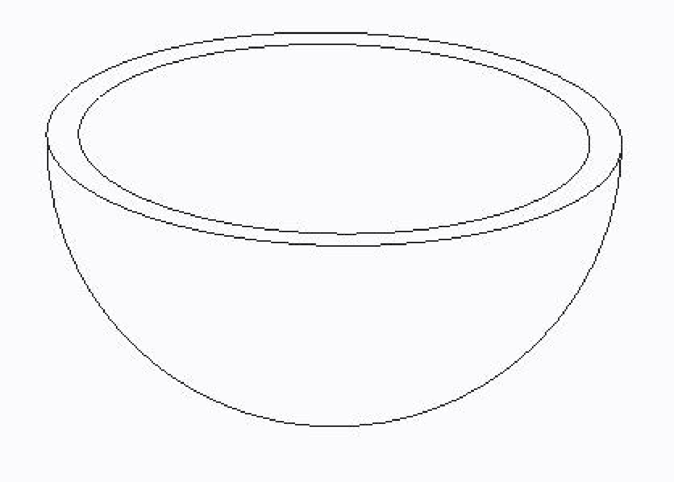

Fig. 1.5. Spherical shell

The cylindrical shells have one axis of rotation and spherical shells have two axis of rotation. Almost every mechanical system is made up of these structural components. The mechanics of these components are also linked. The Kirchhoff-Love theory for plates is similar to Euler-Bernoulli beam theory for beams. Similarly other theories such as Mindlin-Reissner theory of plates are somewhat similar to first order theory of beams. Shells and plates play key role in structural integration of a machine. They are used as boiler casings, auto mobile chassis (which is a combination of beams, plates and shells).

1.6. Composite Structures

Rapidly changing design needs and the effort to build better performance machinery has led to use of composite materials in structural components. Offering greater advantages over structures made of traditional materials, the composite structures are being specifically designed for certain design needs. Even though the manufacturing cost of composite structure is comparatively high, the use of composite materials is gaining momentum. The use of laminated composite in manufacturing of mechanical components is quiet relevant nowadays. The performance of the part created from laminated composite material largely depends upon the number layers and the orientation of the layers of the composite material. These parameters can be altered to achieve desired design needs. Thus structural components such as shells and plates are significantly benefited from introduction of laminates of composite materials. The laminated composite structures find their applications in wide variety of industries. Some of them include aerospace industry, robotics industry and automobile industry.

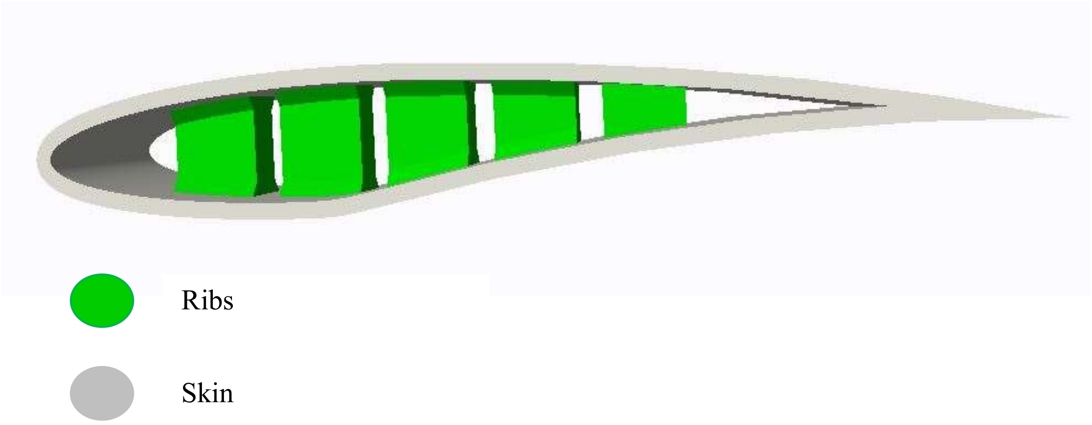

Fig. 1.6. Composite Wing

While each composite structure is crucial, there are still some scenarios where one of the structure may offer advantages over other. For instance in case of bridges, the shell structures are preferred over plates as they offer more rigidity and hold well to stresses. Similarly, where parts are need to designed more economically, plate structures can be utilized as they are easier to design and manufacture. Some applications may require combinations of these structures. Aircraft wings are a perfect example of combination of various composite structures. The ribs and the skin of the wing are made of up laminated composite materials, where the ribs act as a composite beams and skin act as a composite shell surrounding the ribs. It is interesting to know that same material can be used for both the shell and beam part of the wing, with only difference being the number of layers and their orientation in each structure. On contrary to the wings made up of other materials, these composite wings are lighter and are more rigid.

1.7. Vibration Analysis

Over the years, design analysis have improved. With various modes failure being indentified, the design analysis process has become an essential part of product design. One such mode of failure is the failure due to vibration. The failure under the influence of vibrations is caused by the forces generated by the vibration in a structure. The constant exposure to these forces lead to generation of cracks and faults in the structure, which further compromise the integrity of the structure leading to its failure. The structures can be subjected to free or forced vibration. The example of Tacoma Narrow bridge is a popular example of failure due to free vibration. The suspension bridge failed under the vibrations created by winds, blowing at the speed of 65 km/hr. Similar situations can occur in mechanical machines, where both free and forced vibrations are the concerned fields. The vibration analysis for free vibration of a structure is done to obtain the natural frequency of the structure, which is then used in design process to avoid any event of resonance. For forced vibrations, the response of structure for a particular vibration frequency is determined and the results are utilized to make the structure less prone to that particular range of vibration frequencies.

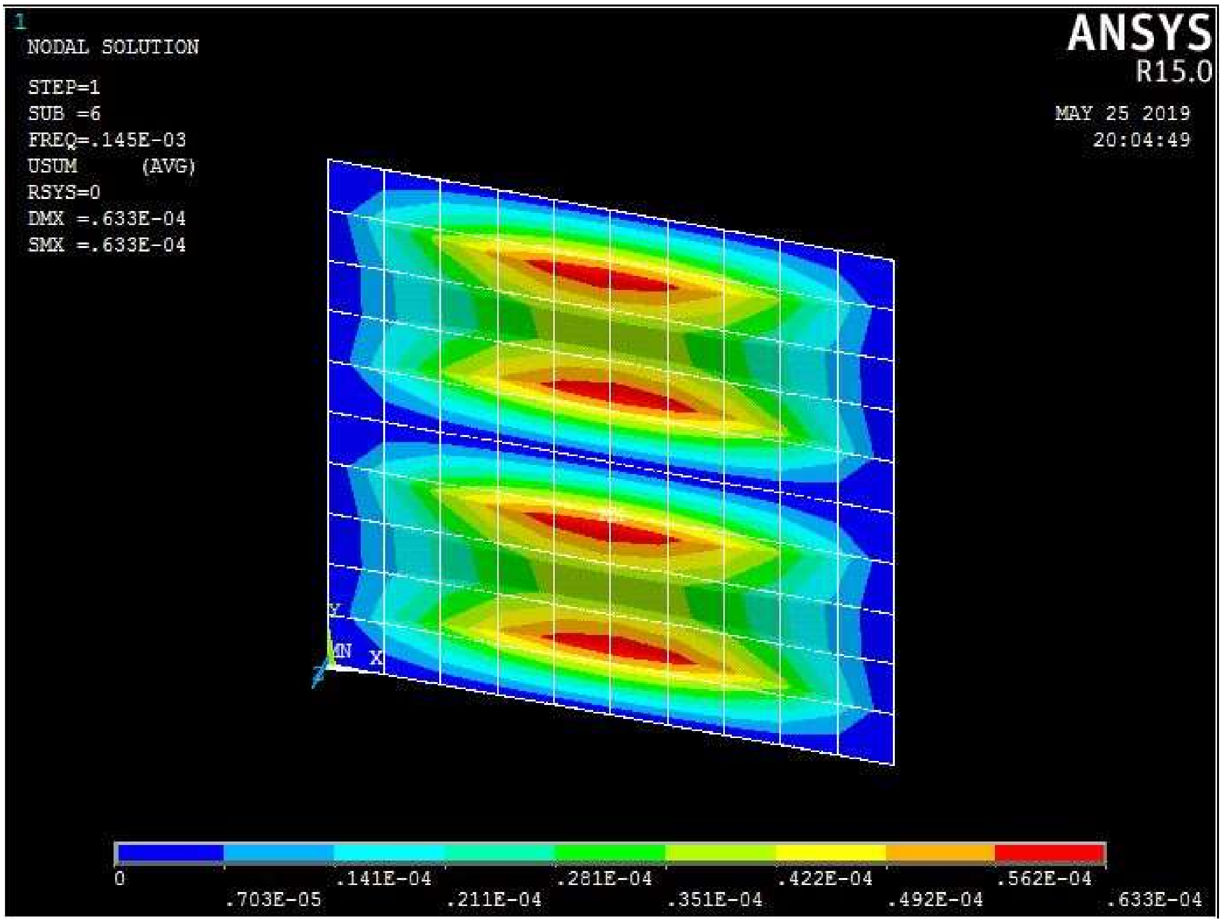

Fig. 1.7. Modal analysis of laminated composite

1.8. Vibration Analysis with Random Parameters

The manufacturing process involves series of complex steps that have to be performed with certain accuracy. However, there is always a degree of randomization in the accuracy of manufacturing process, which may lead to non uniformity in various parameters. The deterministic results do not serve well, especially when it comes to commercialized production, which requires critical risk assessment, based on realistic results. The raw material, that is used for production also exist with random properties, which is also a major cause of uncertain product quality. Along with geometric and material uncertainties, the structures are also subjected to varying load conditions which can be classified as uncertain load parameters. The presence of these uncertainties is a challenge to the designers. The entire process of designing becomes complex, as it has to accommodate the effect of randomization of the design parameters. The effect of these uncertainties is quiet visible on the vibration analysis of the mechanical and civil structures. For the case of composite structures, there are multiple uncertain material properties. This is due to the fact that the composite structures generally exhibit anisotropic and orthotropic material layout.

While geometric uncertainties can be dealt with better and more accurate manufacturing techniques, the material uncertainties require a different approach. There are multiple techniques which have been developed, some of them are the following:

- Monte Carlo Simulation: This is one of the most common method of estimating the effects of randomization. This method involves the following steps :
  a) Establish a domain of inputs
  b) Randomize the inputs using probability distribution
  c) Solve to obtain deterministic results utilizing generated random inputs
  d) Generate the results statistically

- Perturbation Technique: The perturbation technique is based on introducing randomness at the end results of a deterministic analysis. As there are fewer results that require random distribution, this method delivers a slightly less complex computation format. While the accuracy may not be on par with the Monte Carlo simulations, the process is absolutely viable for situations where time constraints are stringent.

Chapter 2

Literature Review

2.1. Introduction

This chapter deals with the existing state of the art in the broad area of vibration analysis of laminated composite structures with and without material uncertainties. The review of literature is focused on the modelling approaches of the laminated composite structures, analysis techniques and uncertainty quantification techniques. Based upon the review performed, the observations and gaps in the existing literature are identified and hence the objectives and scope of the present work is defined.

2.2. Vibration Analysis of Laminated Composite Beams

Two of most common beam theories that govern the beams behavior towards load are the Euler-Bernoulli beam theory and Timoshenko beam theory. Introduced in the year 1750, the Euler-Bernoulli beam theory is widely used for its simplicity and its accurate results. Timoshenko beam theory is well suited for thick beams. The higher order equation presented by the Timoshenko beam theory takes into the account of shear stresses acting on the beam. For composite beams, which are generally subjected to shear stresses, this theory proves to be very helpful. In recent studies, these theories have played quiet an important role.

Jaehong Lee (2000) worked on vibration analysis of laminated composite beams. The beams were subjected to delamination. Applying Hamilton's principle and using Finite Element Method (FEM), it was found that the layer wise approach for vibration analysis was sufficient.

The effect of vibration on a beam with variable cross section was studied by Chaudhari and Maiti (2000). The modeling of vibrations of transverse nature were discussed for a beam of variable cross section. The effect was studied for a beam with a crack and without a crack.

The effect of vibration on a composite beam, with multiple delaminations was studied by Lee et al. (2002). The composite beam was subjected to axial compressions. By determining the effect of multiple delamination on the natural frequency and buckling load on the composite beam, it was concluded that amount of delaminations and their position greatly affects the beams response towards vibrations.

For vibration analysis of composite beams, the use of dynamic field transfer matrix was made in the work of Ellakany et al. (2004). By collaborating this method with analog beam methods, a complex but significantly accurate method for dynamic analysis composite beams was presented.

In an attempt to develop a more convenient method of obtaining response of beams of tapered or variable cross section towards vibration, the Differential Quadrature Method (DQM) was used by Malekzadeh and Shahpari (2005) on various plates of variable cross sections. When the results were compared with the existing results, it was found that the DQM was pretty accurate and it can also be used as computational tool for vibration analysis of tapered beams.

An analytical model was developed by Singh et al. (2006). The model was developed for low order, for a non uniform composite beam. The model made use of Euler-Bernoulli beam equations and was found to be capable of fairly determining the effect of vibrations on the composite beam.

A dynamic stiffness matrix method was presented by Banerjee et al. (2008). This matrix was developed to investigate the effects of vibration on a composite beam which undergo bending and show torsional motion. By solving the differential equations, based on Hamilton's principle in closed analytical form and by applying Wittrick-Williams algorithm, natural frequencies and mode shapes were obtained. The developed method proved to be applicable for the problems of composite wings.

For delaminated beams, two models were proposed by Ramtekkar (2009). The contact-interface model was found to be good for representing actual dynamics of the delaminated beams, whereas the unconstrained-interface model proved to be slightly less accurate, especially at higher modes.

Following polyspectral approach, Nichols and Murphy (2010), worked on detecting the delamination in a composite beam. By determining the non linearity of the beam using a simple model and by examining this non linearity using Volterra series solution, the effect on polyspectra was interpreted in terms of size and depth of delamination.

Vibration analysis of composite beams for large amplitude was studied by Gunda et al. (2011), using Rayleigh Ritz method. The clamped symmetric as well as asymmetric orientations were studied. The non linear harmonic radian frequency is presented in simple closed form. Improving the closed form solutions by comparing and correcting it with existing results of harmonic motion, it was concluded that the composite beams exhibit different frequency patterns when compared to beams made up of traditional material.

Another simple method of vibration analysis was presented by Qibo Mao and Stanislaw Pietrzko (2012). Adomian Decomposition Method (ADM) was used to analyze vibration response of a variable width tapered beam. By making use of this method, dimensionless natural frequencies were obtained along with mode shapes. This method converges fast and provide accurate results.

Beams with variable circular cross section were studied by Abdelghany et al. (2015). The Differential Transformation Method (DTM) was used. Also the differential equations were also solved in MATLAB. Although DTM is proved to be suitable for circular tapered beam, its application in other cross sectional geometries is not restricted.

The vibration analysis of a variable width tapered beam was conducted by Godara and Joglekar (2017). The work was done for fixed-free and fixed-fixed micro beam. The results of the study provided the advantages of a non prismatic beams over prismatic beam. Also by making use of DTM, the natural frequencies and mode shapes were obtained.

2.3. Vibration Analysis of Composite Shells and Plates

The ongoing research on behavior of composite shells and plates towards vibration, can be traced back to the earlier works of J. N. Reddy (1982). The work was centered around shear deformable finite element that could be used for predicting the bending effects in an anisotropic shell. This work differed from other researches as it was specifically designed for anisotropic materials instead of orthotropic materials. While most composite materials posses anisotropy, this research turned out to be a very reliable method of determining the behavior of the composite shell structures. Utilizing computational resources (IBM 370/158 computer), the research was conducted on a spherical shell and results were presented. The result showed presence of transverse shear effects, which must be accounted for better prediction of the results. While the research was able to provide tremendous insight, the use of higher order deformation theory and further extension on the basis of non linearity was pended due to lack of results.

A similar research was conducted by, for orthotropic layered composite structures. Presenting improvement over existing theories, the higher order theory was developed by Reddy and Liu (1985), which was capable of plotting the transverse shear strains. The results for natural frequencies and bending were determined by considering the use of Navier solutions. The methodology was quiet successful in representing the effects of strains with great accuracy.

Working on the laminated composite shells, the effects of stresses due to thermal loading were analyzed by Savoia and Reddy (1995). Addressing one of the most common problems with laminated composites, the efforts were made to identify and determine the behaviorally response by considering a both cross ply and antisymmetric angle ply laminated composites in a three dimensional quasi static environment. The simply supported boundary condition was considered and solutions obtained were by following the Navier like approach.

The non linear response of composite shells towards vibration have been a matter of concern for modern design engineers. The basic vibration response depends upon the stiffness of the material, but at times with larger amplitude, the deformed shape of structure may result in varying stiffness across the entire cross section. This is where the complexity starts weigh in and the results may not always be accurate. Using Rayleigh Ritz method, Singh (2000) conducted a research on linear and non linear vibration of composite shells. The governing equations were achieved using first order shear deformation theory. Due to the use of first order shear deformation theory (FSDT), the introduction of the shear correction factor was required. The results obtained were in the form of displacements, for a trapezoidal shaped plate and a shallow shell. The research demonstrates the effective utilization of Rayleigh Ritz method, and proves to be accurate in determining the random effects of the vibrations on a composite structure.

Another approach can be seen in the works of Ferreira et al. (2003), which followed the third order theory of Reddy and analyzed the composite plates using multiquadric radial basis function. The use of higher order shear deformation theory (HSDT) was adopted to establish precise results for transverse shear stress and contribute to the methods involving no shear correction factors. The composite plates and sandwich plates were considered for this research and the results obtained for both of them were compared with the results obtained by FSDT. One of the major findings of the research were the kinematics shown by the two different theories. The kinematics shown by higher order shear deformation theory were far more accurate than the one shown by first order shear deformation theory. Due to this, the results obtained by HSDT were comparatively better than that obtained by FSDT.

The use of poly-harmonic splines was first seen in the works of Ferreira (2004), for the analysis of composite plate structures. Avoiding the reliance on any shape parameter, which is generally required when analysis is carried out using radial basis function, the formulation presented made excellent use of poly-harmonic or thin plate splines radial basis function. The centre point being the only parameter of concern for the RBF splines, makes the entire formulation less computationally demanding and offer simpler method to use. The research followed the layer wise theory and successfully achieved accurate results without any use of shear correction factor. The results obtained were in the comparison with the ones obtained with the first order shear deformation theory and significant progress for sandwich plates were noted.

The mechanical systems such as boilers, fuselage of a plane, and other pressure vessels generally have a shape factor of a cylinder. The use of laminated composites in these mechanical structures, help in improving their performance in terms of strength and weight. The use of laminated composites however, demands the use of non traditional methods for determination of stress distribution across the structure. The anisotropic and orthotropic nature of the material is the primary cause of this complexity, which possess even more tedious problems when it comes to arrangement of laminated composites in a circular fashion. An attempt was made by Santos et al. (2005), to formulate a method for analysis of the laminated composites in the aforementioned arrangement. The work provided a model based on finite element in a cylindrical coordinate system, which was used for vibration, bending and buckling analysis. The most redeeming feature of the research was the simple semi analytical approach that made the entire process computationally efficient. This was achieved by expanding the equations of motion, based on elasticity theory with Fourier series.

The established higher order theories for the plates have been used in various research works for determining the effects the vibration on plate. There is multitude of approaches that have been followed. One such research presented by Ferreira et al. (2006), was based on meshless approach for static and free vibration analysis of the composite shells. This approach was based on multiquadric radial basis function, which was initially proposed by Kansa. Analytical results for multiple arrangements of spherical and cylindrical composite shells proved the applicability and accuracy of the presented formulation.

Ferreira et al. (2006) followed another mathematical approach, which involved the use of collocation method in tandem with first and third order shear deformation theory for free vibration analysis of the composite plates. A mesh less approach was followed based on Petrov-Galerkin formulation. The results yielded stood in good agreement with the existing results. By not using any nodal evaluation the method presented was found to be less demanding on the computational resources. The control of the performance of the method shifts from nodal elements to the number of collocation points and their positions. Also the mutliquadric functions played an important role in the approximation purposes.

The effect of flaws in geometric structure can have a significant effect on the vibration response of composite plates and shells. Considering a cylindrical shell, Jansen (2007) presented two models based on analytical solution. The effects of imperfections in geometry and amplitude were considered and worked upon. The shell used was made of glass epoxy composite, thus posing an anisotropic structure with three layer angle ply arrangement. Various scenarios of imperfections and varying amplitude were considered the established results were found to be in good agreement with earlier presented results. As the method presented accounts for the effects of imperfection that may occur during manufacturing process, the research addresses the problems faced by design engineers during pre manufacturing stage in very intuitive way.

The work on non linear response of composite shells towards vibration, continued and another important research work was presented by Amabili (2011). The use of Amabili-Reddy higher order deformation theory with Lagrange energy equations yielded the governing equations. This forced vibration research maintained the dampening effects for modal results. The considered cylindrical shell is excited with a radial vibrations. This was the first research attempt at addressing the effects of non linear forced vibrations on composite shells, whose major findings were presented by comparing it with other studies on the non-linear vibrations. The results established the position of the research in terms of its applicability in various arrangements, including its ability to predict accurate results for thin laminates.

Working on the lines of inverse hyperbolic shear deformation theory (IHSDT), Grover et al. (2013) conducted a study on a C^0 continuous isoparameteric biquadratic-quadrilateral serendipity element, for vibration analysis of composite plates. Using finite element solutions along with Navier type solutions, the effect of free vibration on the composite beam was analyzed. The study concluded that with same computational capability, the inverse hyperbolic shear deformation theory can be used to obtain more accurate results. For higher modes of composite plates, this method is found to be very suitable.

2.4. Vibration Analysis of Composite Shells and Plates with Uncertain Material Properties

In a realistic situation, there are uncertainties that affect the overall analysis process. The result generated differ over a range of input values, which can cause poor assessment of failure modes. Material properties are one of the most common factors where uncertainties occur quiet often. This randomness cannot be avoided and have to be taken into account by the means of mathematical formulation. To assess the effects of the uncertainties in material properties, Singh et al. (2001) presented an approach, to predict the results for composite plates, which were subjected to free vibrations. The higher order shear deformation theory was used for deriving dynamics of the system and random material properties were obtained by utilizing perturbation technique of the first order. The conclusion was summed up by establishing some important findings, which were validated by result comparison with Monte Carlo simulation (MCS) results of the similar problem.

The flexibility of fabrication offered by the composite structures, has lead to its use in various shapes and sizes. The aerospace industry and marine industry are one of the biggest beneficiaries of the composite structures. The performance of an aeronautical machine significantly rely upon its ability to reduce drag, which is achieved by exploiting the aerodynamic shapes and introducing them to the framework design. While there exist countless combinations of various shapes that can be used to affect the magnitude of the drag, the most commonly used shape is of a conical cylinder. This particular shape poses a challenge when the composite materials are used instead of conventional materials. The works of Tripathy et al. (2007) address the free vibration response of conical shells with uncertain material properties. The methodology followed was quiet similar to the earlier methods, which incorporated the use of perturbation technique on a composite structure governed by higher order shear deformation theory. They validated the results with the existing results and a Monte Carlo simulation, the conclusion was presented with noteworthy points for conical composite shells.

Based on perturbation technique, Pandit et al. (2010) developed a technique for vibration analysis of laminated composite plates, which used the higher order zig-zag theory. The random material properties were introduced using perturbation technique. The finite element modeling was considered for preparation of composite plates, which proved to generate accurate results. The entire research was aimed at improving existing higher order zig-zag theory, while considering the presence of uncertainties in material properties.

Apart from perturbation technique and Monte Carlo simulation, there are other methods of determining the effects of uncertainties. The spectral approach is one of the lesser known method in this field, and its combined use with fuzzy variable is even a newer approach. Adhikari and Khodaparast (2014) introduced this method for its application in the field of finite element analysis. This approach was based around using fuzzy variables. The results were compared with numerical simulations and good agreement was achieved. However, the research still insists on further exploration in this field for improved computational efficiency.

During the manufacturing of the composite shells (which generally involve laminates of the composite materials), some uncertainties are encountered, in the form of geometrical alignment of layers and material properties of the composite materials. Due to these uncertainties it becomes difficult to predict the actual factor of safety required, and the design ends up being either poorly designed or being over engineered. Such a problem requires a formulation that can derive the closest required factor of safety, by taking consideration of all these uncertainties. Dey et al. (2015) conducted on research on the similar problem. A spherical shell was taken into consideration and stochastic free vibration analysis based on Kriging model was conducted for it.

When compared with direct Monte Carlo approach, it was found that the Kriging model can handle more number of input variables, which made the entire process efficient. The results obtained by Kriging model were more accurate than the ones based on Monte Carlo simulations.

The Isogeometric stochastic analysis is another method of assessing the effect of uncertainty over a composite structure. This method was presented by Hien and Noh (2017), and it revolves around introduction of randomness at various locations in a given volume of the material. In other words, the magnitude of uncertainties vary at different positions in the structure. This research followed a stochastic approach for vibration analysis of the laminated composites. The perturbation technique is used in tandem with isogeometric analysis for determination of natural frequencies. For its validation, the Monte Carlo simulation were taken into account and the results obtained were found to be in satisfactory.

The studies conducted so far dealt with the macroscopic aspect of the composite structures. But, at microscopic level, there have not been much progress in terms of reliability studies. The probabilistic research at microscopic level was conducted by Omairey et al. (2018), in order to decipher the causes of failure in composite structures. The choice of composite used were the fiber reinforced plastics and for determining the uncertainties, representative elementary volume homogenization is used. Various material parameters have been examined. In conclusion it was established that uncertainties at microscopic level have significant effect on the overall stiffness of the structure, whose proper determined can prove to be quiet beneficial.

2.5. Observations from the Literature Review

The literature review is performed focused on the vibration analysis of composite structures with and without material uncertainties. It is observed that the composite structures are modelled in the framework of classical theory, first order shear deformation theory, and higher order shear deformation theories in equivalent single layer approaches. The classical theory does not consider transverse shear effects which are significant in the laminated structures. The FSDT considers these effects however, it requires shear correction factor. The higher order shear deformation theories consider these effects without the use of shear correction factor. Further, the higher order theories are also modelled in polynomial (Taylor's series) and non-polynomial theories. The vibration analysis of laminated structures is carried out using various analytical and approximate approaches. There exist a few studies which have considered the material uncertainty associated with the composite materials and studied their influence on the response of composite structures. On the basis of the literature review conducted, the following gaps are identified.

- Vibration analysis of the laminated composite shells is performed on the basis of classical, first order and higher order theories. However, a recently developed inverse hyperbolic shear deformation theory in non-polynomial framework is not implemented to study the vibration of laminated composite shells.
- The consideration of uncertain material properties

2.6. Objectives

On the basis of literature review performed and identified gaps, the objective of the present work is defined. The objective of the present work is to determine the free vibration response of deterministic and uncertain laminated composite structures in the framework of inverse hyperbolic shear deformation theory. The detailed objectives are as follows:

- Mathematical modelling of spherical and cylindrical shells structures using IHSDT.
- Free vibration behavior of deterministic laminated composite shells and plates.
- Free vibration behavior of uncertain laminated composite shells and plates.

2.7. Summary

A detailed literature review is carried out focused on vibration behavior of composite structures with and without material uncertainties. The observation are made through the review of the literature and the gaps are identified. The objective of the present work is framed in the view of these observations and gaps. The review also assisted in understanding of various techniques for modelling, deterministic analysis and uncertainty analysis.

Chapter 3

Mathematical Formulation

3.1. Introduction

In this chapter, mathematical formulation for the vibration analysis of laminated composite shells is presented. The governing equations are developed using the principle of virtual work. The inverse hyperbolic shear deformation theory is used to express the displacement field. The linear structural kinematics is assumed along with Generalised Hooke's law for composite materials. The explicit differential equations are obtained and solved using Navier solution for simply supported conditions. Further, the mathematical approach to implement the Monte Carlo simulation is discussed.

3.2. Governing Equations for Vibration Analysis

A spherical composite shell with R_1 and R_2 as radius of curvature is considered. The shell has its dimensions as a×b×h and is represented in Fig. 3.1. The effects of free vibration can be ascertained by utilizing constitutive relations with displacement field and strain-displacement relations.

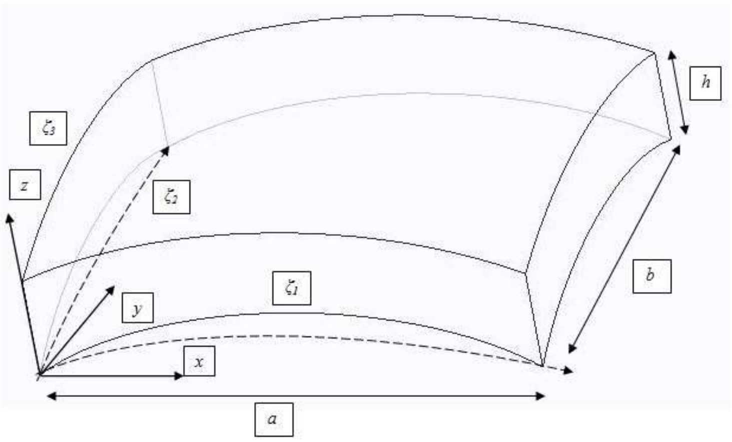

Fig. 3.1. Composite shell

3.2.1. Constitutive relations

The constitutive relations consist of properties that the structural material possess. The laminated composite shell is made by stacking the layers of orthotropic material over each other. For an orthotropic material, the Hooke's law states that:

$$[\sigma] = \left[\overline{Q_{ij}}\right]\{\varepsilon\} \tag{1a}$$

Where

$$[\sigma] = [\sigma_1 \quad \sigma_2 \quad \sigma_4 \quad \sigma_5 \quad \sigma_6] \tag{1b}$$

$$\{\varepsilon\} = [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_4 \quad \varepsilon_5 \quad \varepsilon_6] \tag{1c}$$

Where $\left[\overline{Q_{ij}}\right]$ is the matrix of reduced transformation stiffness matrix, which depend upon material properties and the angular placement of the reinforcement, which is denoted by θ. This angular placement is taken with respect to the coordinate system of structure.

3.2.2. Displacement field

The realistic behavior of shells, towards the free vibration and shear deformation is determined with assistance of inverse hyperbolic shear deformation theory. The theory is well suited for precise determination shear deformation effects and for free vibration it is represented by the following equations:

$$\begin{aligned}
\bar{u}(\zeta_1,\zeta_2,\zeta_3) &= \left(1+\frac{\zeta_3}{R_1}\right)u + \zeta_3\left(\Omega\phi_1 - \frac{\partial w}{a_1\partial\zeta_1}\right) + f(\zeta_3)\phi_1 \\
\bar{v}(\zeta_1,\zeta_2,\zeta_3) &= \left(1+\frac{\zeta_3}{R_2}\right)v + \zeta_3\left(\Omega\phi_2 - \frac{\partial w}{a_2\partial\zeta_2}\right) + f(\zeta_3)\phi_2 \\
\bar{w}(\zeta_1,\zeta_2,\zeta_3) &= w
\end{aligned} \tag{2}$$

Here, the displacement on the middle plane is given by u , v and w along $(\zeta_1, \zeta_2, \zeta_3)$, while ϕ_1 and ϕ_2 represent the shear deformation. The function $f(\zeta_3) = \sinh^{-1}\left(\frac{rz}{h}\right)$ where r =3 (Grover et al., 2013). These equations are incorporated in an attempt to mitigate the use of shear correction factor, which is required to consider the effects of shear on the end layers of the shell structure. As lesser input parameters are required, the entire process becomes simpler.

3.2.3. Strain-displacement relations

For the above presented displacement field, the stress-strain placement is presented as follows:

$$\varepsilon_1 = \frac{1}{A_1}\left(\frac{\partial \bar{u}}{\partial \zeta_1} + \frac{1}{a_2}\frac{\partial a_1}{\partial \zeta_2}\bar{v} + \frac{a_1}{R_1}\bar{w}\right) \tag{3a}$$

$$\varepsilon_2 = \frac{1}{A_2}\left(\frac{\partial \bar{v}}{\partial \zeta_2} + \frac{1}{a_1}\frac{\partial a_2}{\partial \zeta_1}\bar{u} + \frac{a_2}{R_2}\bar{w}\right) \tag{3b}$$

$$\varepsilon_4 = \frac{1}{A_2}\frac{\partial \bar{w}}{\partial \zeta_2} + A_2\frac{\partial}{\partial \zeta_3}\left(\frac{\bar{v}}{A_2}\right) \tag{3c}$$

$$\varepsilon_5 = \frac{1}{A_1}\frac{\partial \bar{w}}{\partial \zeta_1} + A_1\frac{\partial}{\partial \zeta_3}\left(\frac{\bar{u}}{A_1}\right) \tag{3d}$$

$$\varepsilon_6 = \frac{A_2}{A_1}\left[\frac{\partial}{\partial \zeta_1}\left(\frac{\bar{v}}{A_2}\right)\right] + \frac{A_1}{A_2}\left[\frac{\partial}{\partial \zeta_2}\left(\frac{\bar{u}}{A_1}\right)\right] \tag{3e}$$

Where

$$A_1 = \left(1 + \frac{\zeta_3}{R_1}\right)a_1 \tag{3f}$$

$$A_2 = \left(1 + \frac{\zeta_3}{R_2}\right)a_2 \tag{3g}$$

Certain assumptions are made for shallow cross ply laminated structure. These assumptions are as follows:

$$\frac{\partial a_1}{\partial \zeta_1}=0 \tag{3h}$$

$$\frac{\partial a_2}{\partial \zeta_2}=0 \tag{3i}$$

$$\frac{\zeta_3}{R_1}=\frac{\zeta_3}{R_2}=1 \tag{3j}$$

3.2.4. Principle of virtual work

The principle of virtual work, in its dynamic form is used to derive the governing differential equation. The principle of virtual work can be stated as follows:

$$\int_0^T \left(\delta U-\delta T\right)dt=0$$

$$\delta U=\int_\Omega \left\{ \int_{-h/2}^{h/2} \left[\sigma_{11}\delta\varepsilon_{11}+\sigma_{22}\delta\varepsilon_{22}+\sigma_{44}\delta\varepsilon_{44}+\sigma_{55}\delta\varepsilon_{55}+\sigma_{66}\delta\varepsilon_{66}\right]d\zeta_{33}\right\}d\zeta_{11}d\zeta_{22} \tag{4a}$$

$$\delta T=\int_\Omega \left(\int_{-h/2}^{h/2} \rho[\delta u+\ddot{v}\delta v+\ddot{w}\delta w]d\zeta_{33}\right)d\zeta_{11}d\zeta_{22} \tag{4b}$$

The governing differential equations are divulged in the form of resultant stresses and moments, which can be derived with the utilization of fundamental lemma of variational calculus. These can be expressed in the following equations.

$$\frac{\partial N_1}{\partial x_1}+\frac{\partial N_6}{\partial x_2}=0 \tag{5}$$

$$\frac{\partial N_2}{\partial x_2}+\frac{\partial N_6}{\partial x_1}=0 \tag{6}$$

$$-\frac{N_1}{R_1}-\frac{N_2}{R_2}+\frac{\partial^2 M_1}{\partial x_1^2}+\frac{\partial^2 M_2}{\partial x_2^2}+2\frac{\partial^2 M_6}{\partial x_1\partial x_2}=q+\overline{N}_{xx}\frac{\partial^2 w_0}{\partial {x_1}^2}+2\overline{N}_{xy}\frac{\partial^2 w_0}{\partial x_1\partial x_2}+\overline{N}_{yy}\frac{\partial w_0}{\partial x_1} \tag{7}$$

$$\frac{\partial P_1}{\partial x_1}+\frac{\partial P_6}{\partial x_2}+\Omega\frac{\partial M_1}{\partial x_1}+\Omega\frac{\partial M_6}{\partial x_2}-Q_1-K_1=0 \tag{8}$$

$$\frac{\partial P_2}{\partial x_2}+\frac{\partial P_6}{\partial x_1}+\Omega\frac{\partial M_2}{\partial x_2}+\Omega\frac{\partial M_6}{\partial x_1}-Q_2-K_2=0 \tag{9}$$

The resultant stresses are given by:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial u}{\partial x_1}+\frac{w}{R_1} \\ \frac{\partial v}{\partial x_2}+\frac{w}{R_2} \\ \frac{\partial v}{\partial x_1}+\frac{\partial u}{\partial x_2} \end{Bmatrix}+\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}\begin{Bmatrix} \Omega\frac{\partial \phi_1}{\partial x_1}-\frac{\partial^2 w}{\partial x_1^2} \\ \Omega\frac{\partial \phi_2}{\partial x_2}-\frac{\partial^2 w}{\partial x_2^2} \\ \Omega\frac{\partial \phi_1}{\partial x_2}+\Omega\frac{\partial \phi_2}{\partial x_1}-2\frac{\partial^2 w}{\partial x_1 \partial x_2} \end{Bmatrix}+\begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial \phi_1}{\partial x_1} \\ \frac{\partial \phi_2}{\partial x_2} \\ \frac{\partial \phi_2}{\partial x_1}+\frac{\partial \phi_1}{\partial x_2} \end{Bmatrix} \tag{10}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix}=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial u}{\partial x_1}+\frac{w}{R_1} \\ \frac{\partial v}{\partial x_2}+\frac{w}{R_2} \\ \frac{\partial v}{\partial x_1}+\frac{\partial u}{\partial x_2} \end{Bmatrix}+\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}\begin{Bmatrix} \Omega\frac{\partial \phi_1}{\partial x_1}-\frac{\partial^2 w}{\partial x_1^2} \\ \Omega\frac{\partial \phi_2}{\partial x_2}-\frac{\partial^2 w}{\partial x_2^2} \\ \Omega\frac{\partial \phi_1}{\partial x_2}+\Omega\frac{\partial \phi_2}{\partial x_1}-2\frac{\partial^2 w}{\partial x_1 \partial x_2} \end{Bmatrix}+\begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial \phi_1}{\partial x_1} \\ \frac{\partial \phi_2}{\partial x_2} \\ \frac{\partial \phi_2}{\partial x_1}+\frac{\partial \phi_1}{\partial x_2} \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix}=\begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial u}{\partial x_1}+\frac{w}{R_1} \\ \frac{\partial v}{\partial x_2}+\frac{w}{R_2} \\ \frac{\partial v}{\partial x_1}+\frac{\partial u}{\partial x_2} \end{Bmatrix}+\begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix}\begin{Bmatrix} \Omega\frac{\partial \phi_1}{\partial x_1}-\frac{\partial^2 w}{\partial x_1^2} \\ \Omega\frac{\partial \phi_2}{\partial x_2}-\frac{\partial^2 w}{\partial x_2^2} \\ \Omega\frac{\partial \phi_1}{\partial x_2}+\Omega\frac{\partial \phi_2}{\partial x_1}-2\frac{\partial^2 w}{\partial x_1 \partial x_2} \end{Bmatrix}+\begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{bmatrix}\begin{Bmatrix} \frac{\partial \phi_1}{\partial x_1} \\ \frac{\partial \phi_2}{\partial x_2} \\ \frac{\partial \phi_2}{\partial x_1}+\frac{\partial \phi_1}{\partial x_2} \end{Bmatrix} \tag{12}$$

$$\begin{bmatrix} Q_2 & K_2 \\ Q_1 & K_1 \end{bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix} \sigma_2 \\ \sigma_1 \end{Bmatrix}\left[1 \quad g'(\zeta_3)\right]d\zeta_3 \tag{13}$$

Resultant inertia terms are given by the following equation:

$$\left[I_0 \quad I_1 \quad I_2 \quad I_3 \quad I_4 \quad I_5\right]=\int_{-h/2}^{h/2}\rho^k\left[1 \quad \zeta_3 \quad \zeta_3{}^2 \quad f(\zeta_3) \quad \zeta_3 f(\zeta_3) \quad (f(\zeta_3))^2\right]d\zeta_3 \tag{14}$$

For further simplification, eqs. (5)-(9) can be presented in the form of displacements and rotations. This is can be done by expressing them using the following the integrals:

$$\begin{bmatrix} A_{ij} & B_{ij} & D_{ij} & E_{ij} & F_{ij} & H_{ij} \end{bmatrix} = \int_{-h/2}^{h/2} \left[\overline{Q_{ij}^{(k)}}\right] \begin{bmatrix} 1 & \zeta_3 & \zeta_3^{\,2} & g(\zeta_3) & \zeta_3 g(\zeta_3) & (g(\zeta_3))^2 \end{bmatrix} d\zeta_3 \tag{15}$$

Where *i,j* = 1,2,4,5 and 6.

$$\begin{bmatrix} K_{ij} & L_{ij} \end{bmatrix} = \int_{-h/2}^{h/2} \left[\overline{Q_{ij}^{(k)}}\right]_{2\times 2} \begin{bmatrix} g'(\zeta_3) & (g'(\zeta_3))^2 \end{bmatrix} d\zeta_3 \tag{16}$$

Where *i,j* = 4,5.

The substitution of the resultant stresses, moments and higher order moments in the Eqs. 5-9 yield the explicit form of coupled differential equations. These differential equations for cross ply lamination sequence are as follows:

$$\begin{aligned}
&A_{11}\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{1}{R_1}\frac{\partial w}{\partial x_1}\right)+B_{11}\left(\Omega\frac{\partial^2 \phi_1}{\partial x_1^2}-\frac{\partial^3 w}{\partial x_1^3}\right)+E_{11}\left(\frac{\partial^2 \phi_1}{\partial x_1^2}\right)+E_{12}\left(\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}\right)+A_{12}\left(\frac{\partial^2 v}{\partial x_1 \partial x_2}+\frac{1}{R_2}\frac{\partial w}{\partial x_1}\right)+\\
&B_{12}\left(\Omega\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}-\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right)+A_{66}\left(\frac{\partial^2 v}{\partial x_1 \partial x_2}+\frac{\partial^2 u}{\partial x_2^2}\right)+B_{66}\left(\Omega\left(\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}+\frac{\partial^2 \phi_1}{\partial x_2^2}\right)-2\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right)+\\
&E_{66}\left(\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}+\frac{\partial^2 \phi_1}{\partial x_2^2}\right)=0
\end{aligned} \tag{17}$$

$$\begin{aligned}
&A_{21}\left(\frac{\partial^2 u}{\partial x_1 \partial x_2}+\frac{1}{R_1}\frac{\partial w}{\partial x_2}\right)+B_{21}\left(\Omega\frac{\partial^2 \phi_1}{\partial x_1 \partial x_2}-\frac{\partial^3 w}{\partial x_1^2 \partial x_2}\right)+E_{21}\left(\frac{\partial^2 \phi_2}{\partial x_2^2}\right)+A_{22}\left(\frac{\partial^2 v}{\partial x_2^2}+\frac{1}{R_2}\frac{\partial w}{\partial x_2}\right)+\\
&B_{22}\left(\Omega\frac{\partial^2 \phi_2}{\partial x_2^2}-\frac{\partial^3 w}{\partial x_1 \partial x_2^3}\right)+E_{22}\left(\frac{\partial^2 \phi_2}{\partial x_2^2}\right)+A_{66}\left(\frac{\partial^2 v}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)+B_{66}\left(\Omega\left(\frac{\partial^2 \phi_1}{\partial x_1 \partial x_2}+\frac{\partial^2 \phi_2}{\partial x_2^2}\right)-2\frac{\partial^3 w}{\partial x_2 \partial x_1^2}\right)+\\
&E_{66}\left(\frac{\partial^2 \phi_1}{\partial x_1 \partial x_2}+\frac{\partial^2 \phi_2}{\partial x_1^2}\right)=0
\end{aligned} \tag{18}$$

$$\begin{aligned}
&-\frac{1}{R_1}\left[A_{11}\left(\frac{\partial u}{\partial x_1}+\frac{w}{R_1}\right)+B_{11}\left(\Omega\frac{\partial \phi_1}{\partial x_1}-\frac{\partial^2 w}{\partial x_1^2}\right)+E_{11}\left(\frac{\partial \phi_1}{\partial x_1}\right)+A_{12}\left(\frac{\partial v}{\partial x_2}+\frac{w}{R_2}\right)+B_{12}\left(\Omega\frac{\partial \phi_2}{\partial x_2}-\frac{\partial^2 w}{\partial x_2^2}\right)+E_{12}\left(\frac{\partial \phi_2}{\partial x_2}\right)\right]\\
&-\frac{N_2}{R_2}+\frac{\partial^2 M_1}{\partial x_1^2}+\frac{\partial^2 M_2}{\partial x_2^2}+2\frac{\partial^2 M_6}{\partial x_1 \partial x_2}=0
\end{aligned} \tag{19}$$

$$\begin{aligned} &E_{11}\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{1}{R_1}\frac{\partial w}{\partial x_1}\right)+F_{11}\left(\Omega\frac{\partial^2 \phi_1}{\partial x_2^2}-\frac{\partial^3 w}{\partial x_1^3}\right)+H_{11}\left(\frac{\partial^2 \phi_1}{\partial x_1^2}\right)+E_{12}\left(\frac{\partial^2 v}{\partial x_1 \partial x_2}+\frac{1}{R_2}\frac{\partial w}{\partial x_1}\right) \\ &+F_{12}\left(\Omega\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}-\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right)+H_{12}\left(\frac{\partial^2 \phi_2}{\partial x_1 \partial x_2}\right)+\frac{\partial P_6}{\partial x_2}+\Omega\left(\frac{\partial M_1}{\partial x_1}+\frac{\partial M_6}{\partial x_2}\right)-Q_1-K_1=0 \end{aligned} \tag{20}$$

$$\begin{aligned} &E_{21}\left(\frac{\partial^2 u}{\partial x_1 \partial x_2}+\frac{1}{R_1}\frac{\partial w}{\partial x_2}\right)+F_{21}\left(\Omega\frac{\partial^2 \phi_1}{\partial x_1 \partial x_2}-\frac{\partial^3 w}{\partial x_1^2 \partial x_2}\right)+H_{21}\left(\frac{\partial^2 \phi_2}{\partial x_2^2}\right)+E_{22}\left(\frac{\partial^2 v}{\partial x_2^2}+\frac{1}{R_2}\frac{\partial w}{\partial x_2}\right) \\ &+F_{22}\left(\Omega\frac{\partial^2 \phi_2}{\partial x_2^2}-\frac{\partial^3 w}{\partial x_2^3}\right)+H_{22}\left(\frac{\partial^2 \phi_2}{\partial x_2^2}\right)+\frac{\partial P_6}{\partial x_1}+\Omega\left(\frac{\partial M_2}{\partial x_2}+\frac{\partial M_6}{\partial x_1}\right)-Q_2-K_2=0 \end{aligned} \tag{21}$$

The coupled differential equations mentioned in Eqs. 17-21 are solved to obtain the response of the structure.

3.3. Solution Methodology

In order to obtain results based on IHSDT, and to validate its capability in determining the effects of free vibration on a simply supported composite shell, the Navier type closed form solution is considered. The above mentioned solution is proven to be precise and does not encounter any errors. This makes Navier solution one of the most preferred closed form solution, however limiting its applicability for simply supported boundary conditions. The boundary conditions that are specified as follows:

$$u = w = \phi_1 = N_1 = M_1 = 0 \qquad \text{at } 0, a \tag{22}$$

$$v = w = \phi_2 = N_2 = M_2 = 0 \qquad \text{at } 0, b$$

For presented boundary conditions, assumptions made are stated as follows:

$$u = U_{mn} \cos \alpha x_1 \sin \beta x_2 \tag{23}$$

$$v = V_{mn} \sin \alpha x_1 \cos \beta x_2 \tag{24}$$

$$w = W_{mn} \sin \alpha x_1 \sin \beta x_2 \tag{25}$$

$$\phi_1 = X_{mn} \cos\alpha x_1 \sin\beta x_2 \tag{26}$$

$$\phi_2 = Y_{mn} \sin\alpha x_1 \cos\beta x_2 \tag{27}$$

where $\alpha = \dfrac{m\pi}{a}$ (28)

$$\beta = \frac{n\pi}{b} \tag{29}$$

The substitution of the above mentioned solution in the governing differential equations, yield a set of homogeneous equations, which is presented in the form of eigen-value problem. The eigen values can be referred for natural frequencies, while eigen vectors can be referred for determining the mode shapes. The eigen value is expressed in following equation.

$$\left[\overline{R}\right] - \omega^2 [M]\{\Delta\} = 0 \tag{30}$$

Where $\left[\overline{R}\right]$ and $[M]$ are the stiffness and mass matrix respectively.

3.3.1 Vibration of uncertain laminated composites

In the present work, Monte Carlo simulation is implemented to include the effect of uncertain material properties on the free vibration response. In this method, random samples of the uncertain material properties parameters are generated. These distribution of the random samples may be uniform or Gaussian (normal). The governing equation (Eq. (30)) is solved for each sample to get the response of individual samples. The statistical estimation (mean and variance) of the natural frequency is then obtained considering the response due to all random samples. For example, if there are *n* samples for the analysis, the mean and variance of natural frequency are obtained as:

$$E(q) = \frac{1}{n}\sum_{k=1}^{n} q(k) \tag{31}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{k=1}^{n} q^2(k) - n.E^2(q) \tag{32}$$

It is clear that number of samples strongly influence the statistical response characteristics of the output random variables. It requires quite large number of samples to accurately estimate the uncertain response of a structural system associated with large scale uncertainties. However, this method is quite useful and frequently used for the validation purposes.

3.4. Summary

The mathematical formulation for vibration analysis of laminated composite shells structures with and without material properties is developed in the framework of IHSDT. The IHSDT, linear strain-displacement relations, Generalised Hooke's law are implemented in principle of virtual work and a Navier type closed form solution is employed to solve the coupled differential equations. The eigen value problem to examine the free vibration response is yielded and the same is extended for the MCS approach to include the effect of material uncertainties.

Chapter 4

Results and Discussion

4.1. Introduction

In this chapter, the numerical results and discussion is presented for the formulation developed in IHSDT framework. A MATLAB code is developed for the derived analytical formulation. The considered area of concern include spherical and cylindrical shallow shells, thick shells, shallow plates and thick plates, in cross ply arrangement. The material properties selected for the composite structures are presented in the table below.

Table 4.1 Material Properties of the composite shells

MM 1			MM 2		
S. No	Material Property	Value	S. No	Material Property	Value
1	E_{11}	25(Gpa)	1	E_{11}	Open(Gpa)
2	E_{22}	1(Gpa)	2	E_{22}	1(Gpa)
3	G_{12}	0.5(Gpa)	3	G_{12}	0.6(Gpa)
4	G_{13}	0.5(Gpa)	4	G_{13}	0.6(Gpa)
5	G_{23}	0.2(Gpa)	5	G_{23}	0.5(Gpa)
6	ν_{12}	0.25	6	ν_{12}	0.25
7	ν_{21}	0.01	7	ν_{21}	0.01
8	ρ	1(kg/m^3)	8	ρ	1(kg/m^3)

Various examples of spherical and cylindrical shells and plates are considered to examine the free vibration response. The effects of various parameters such as radius of curvature, span-thickness ratio, material anisotropy and lamination sequence are observed on the natural frequency. Further, the uncertainty quantification of free vibration response of these structures is obtained in the framework of Monte Carlo simulation.

4.2. Free Vibration Analysis in Deterministic Framework

The analysis is carried out for the free vibration response considering the material properties of the composites to be deterministic in nature. Further, both symmetric and anti-symmetric laminates are examined for the free vibration case. The results are obtained in the non-dimensional form $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$. The symmetric lamination sequence is considered as 3 layered [0/90/0] and four layered [0/90/90/0]. All the layers are of equal thickness and constituted of MM1. All the edges of the shells are assumed to be simply supported. The obtained results in the framework of IHSDT for doubly curved spherical shells are listed in Table 4.2 and Table 4.3 for [0/90/0] and [0/90/90/0] respectively along with the results presented by Ferreira et al. (2006) and Reddy and Liu (1985). It should be noted that Reddy and Liu (1985) presented the closed form solutions based on HSDT and Ferreira et al. (2006) presented the radial basis method for the Reddy's HSDT. For the radial basis function, the comparison have been made with results obtained by the square grid layout, with 17 nodes on each side. The obtained results are in agreement with the existing results. Moreover, it is also observed that natural frequency increases with increase in span-thickness (*a*/*h*) ratio and natural frequency decreases with increase in radius of curvature.

Table 4.1. Non-dimensional fundamental frequencies $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ of [0/90/0] cross ply laminated spherical shells

a/*h*	Method	*R*/*a*					
		5	10	20	50	100	10^9
	Reddy and Liu (1985)	12.060	11.860	11.810	11.790	11.790	11.790
10	Ferreira et al. (2006)	12.063	11.861	11.810	11.796	11.794	11.793
	Present (IHSDT)	11.894	11.693	11.642	11.628	11.626	11.625
	Reddy and Liu (1985)	31.020	20.350	16.620	15.420	15.240	15.170
100	Ferreira et al. (2006)	31.216	20.394	16.595	15.361	15.176	15.115
	Present (IHSDT)	31.015	20.347	16.618	15.411	15.230	15.170

Further, the analysis is carried out for four layered symmetric cylindrical shells with lamination sequence [0/90/90/0] and [0/90/0]. The radii of curvature are taken as $R_2 / b = \infty$ while R_1 / a is varied from as 5 to 10^9. The present results along with the existing results are indicated in Table 4.3 and Table 4.4 which shows a good agreement. The comparison of Table 4.2 and Table 4.3 reveal that the natural frequency of cylindrical shells is lower than the corresponding spherical shells.

Table 4.2. Non-dimensional fundamental frequencies $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ of [0/90/90/0] cross ply laminated spherical shells

a/h	Method	R/a					
		5	10	20	50	100	10^9
	Reddy and Liu (1985)	12.040	11.840	11.790	11.780	11.780	11.780
10	Ferreira et al. (2006)	12.051	11.848	11.796	11.782	11.780	11.779
	Present (IHSDT)	11.935	11.734	11.682	11.668	11.666	11.665
	Reddy and Liu (1985)	31.100	20.380	16.630	15.420	15.230	15.170
100	Ferreira et al. (2006)	31.296	20.422	16.595	15.359	15.170	15.109
	Present (IHSDT)	31.100	20.380	16.628	15.412	15.230	15.169

Table 4.4 Non-dimensional fundamental frequencies $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ of [0/90/90/0] cross ply laminated cylindrical shells

a/h	Method	R_1/a					
		5	10	20	50	100	10^9
	Reddy and Liu (1985)	11.830	11.790	11.780	11.780	11.780	11.780
10	Ferreira et al. (2006)	11.837	11.793	11.782	11.779	11.779	11.779
	Present (IHSDT)	11.681	11.639	11.629	11.626	11.625	11.625
	Reddy and Liu (1985)	20.360	16.630	15.550	15.230	15.190	15.170
100	Ferreira et al. (2006)	20.503	16.628	15.502	15.178	15.126	15.108
	Present (IHSDT)	20.359	16.624	15.546	15.230	15.185	15.169

Table 4.5 Non-dimensional $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ fundamental frequencies of [0/90/0] cross ply laminated cylindrical shells

a/h	Method	R/a					
		5	10	20	50	100	10^9

	Reddy and Liu (1985)	11.850	11.800	11.790	11.790	11.790	11.790
10	Ferreira et al. (2006)	11.851	11.807	11.796	11.793	11.793	11.792
	Present (IHSDT)	11.681	11.639	11.629	11.626	11.625	11.625
	Reddy and Liu (1985)	20.330	16.620	15.550	15.240	15.190	15.170
100	Ferreira et al. (2006)	20.535	16.641	15.510	15.178	15.130	15.115
	Present (IHSDT)	20.326	16.614	15.544	15.230	15.185	15.170

The effect of material anisotropy on the free vibration response of symmetric laminated composite spherical shell ($R_1/a = R_2/b = 5$) is also examined for thick and thin shells. The four layered [0/90/90/0] doubly curved shells wit material properties MM2 is considered. It is observed that the natural frequency increases with increase in anisotropy and as the thickness of the shell decreases, the natural frequency increases.

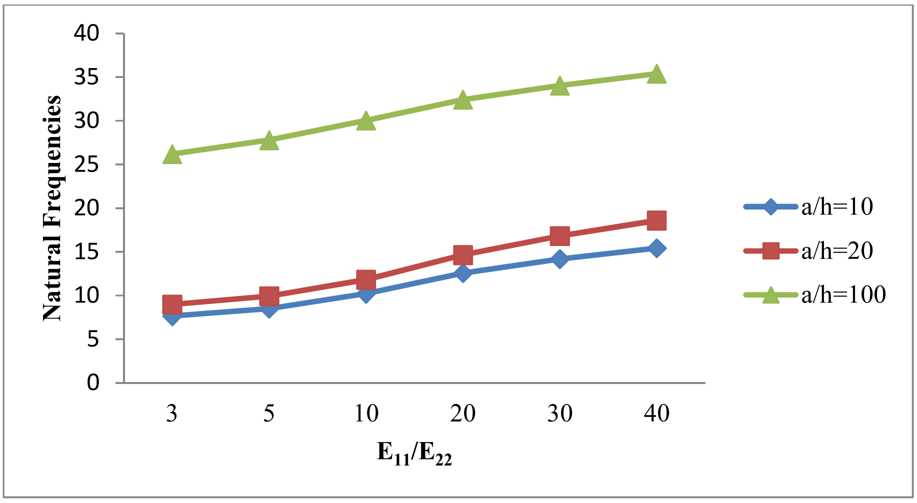

Fig. 4.1. Effect of anisotropy on spherical composite shell [0/90/90/0], MM 2, R/a=5

Further, the anti-symmetric [0/90] doubly curved shells with span-thickness ratio as 10 and 100 are considered for the free vibration response. The non-dimensional results are compared with the existing results in Table 4.5 and a good agreement is observed.

The anti-symmetric cylindrical shells with a/h = 10 and 100 are evaluated for the free vibration response for R/a ranging from 5 to 10^9 and the results are presented in Table 4.6 and

Table 4.7. It is observed that natural frequency decreases with increase in radius of curvature. Also, the thick shells possess lower natural frequency as compared to the thin shells.

Table 4.6 Non-dimensional fundamental frequencies $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ of [0/90] cross ply laminated spherical shells

a/h	Method	R/a					
		5	10	20	50	100	10^9
	Reddy and Liu (1985)	9.337	9.068	8.999	8.980	8.977	8.976
10	Ferreira et al. (2006)	9.342	9.069	8.999	8.979	8.977	8.976
	Present (IHSDT)	9.388	9.122	9.054	9.035	9.032	9.031
	Reddy and Liu (1985)	28.840	16.710	11.840	10.060	9.784	9.688
100	Ferreira et al. (2006)	29.049	16.762	11.806	9.982	9.692	9.590
	Present (IHSDT)	28.839	16.712	11.844	10.065	9.784	9.688

Table 4.7 Non-dimensional fundamental frequencies $\bar{\omega} = \omega\left(a^2 / h\right)\sqrt{\rho / E_{22}}$ of [0/90] cross ply laminated cylindrical shells

a/h	Method	R/a					
		5	10	20	50	100	10^9
	Reddy and Liu (1985)	9.023	8.979	8.972	8.973	8.975	8.976
10	Ferreira et al. (2006)	9.094	9.013	8.989	8.980	8.960	8.975
	Present (IHSDT)	9.150	9.070	9.046	9.036	9.033	9.031
	Reddy and Liu (1985)	16.690	11.840	10.270	9.783	9.712	9.688
100	Ferreira et al. (2006)	16.852	11.825	10.195	9.702	9.620	9.598
	Present (IHSDT)	16.703	11.844	10.271	9.784	9.713	9.688

The Fig. 4.2 and Fig. 4.3 show the graph for the free vibration studies, conducted for spherical composite shells, [0/90/90/0]. Both the graphs signify a general observation that concludes the inverse proportionality between thickness of the shell and its natural frequency. From the graphical representations, it is observed that as the thickness of the shell decreases, its corresponding natural frequency increases. However, the increment in number of layers also show same effect, but not to that extent as shown by thickness reduction. The effect of anisotropy is shown, where it can be seen that with the increase in anisotropy of the material, the natural frequency of the structure also increases.

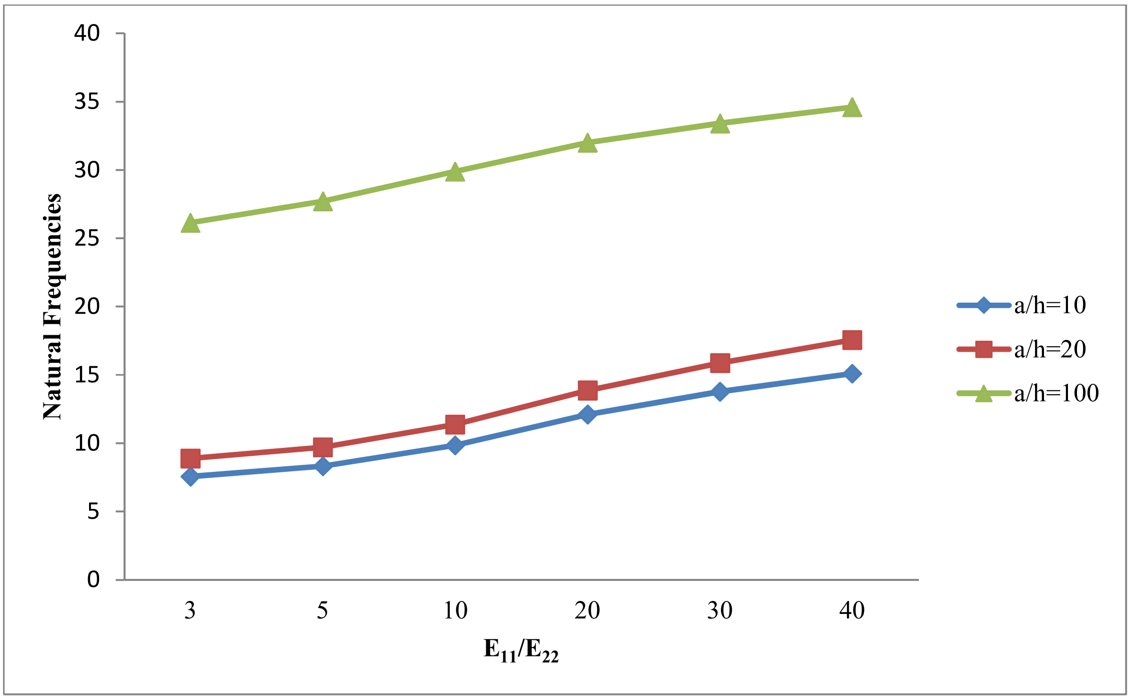

Fig. 4.2. Effect of anisotropy on spherical composite shell [0/90/0/90], MM 2, R/a=5

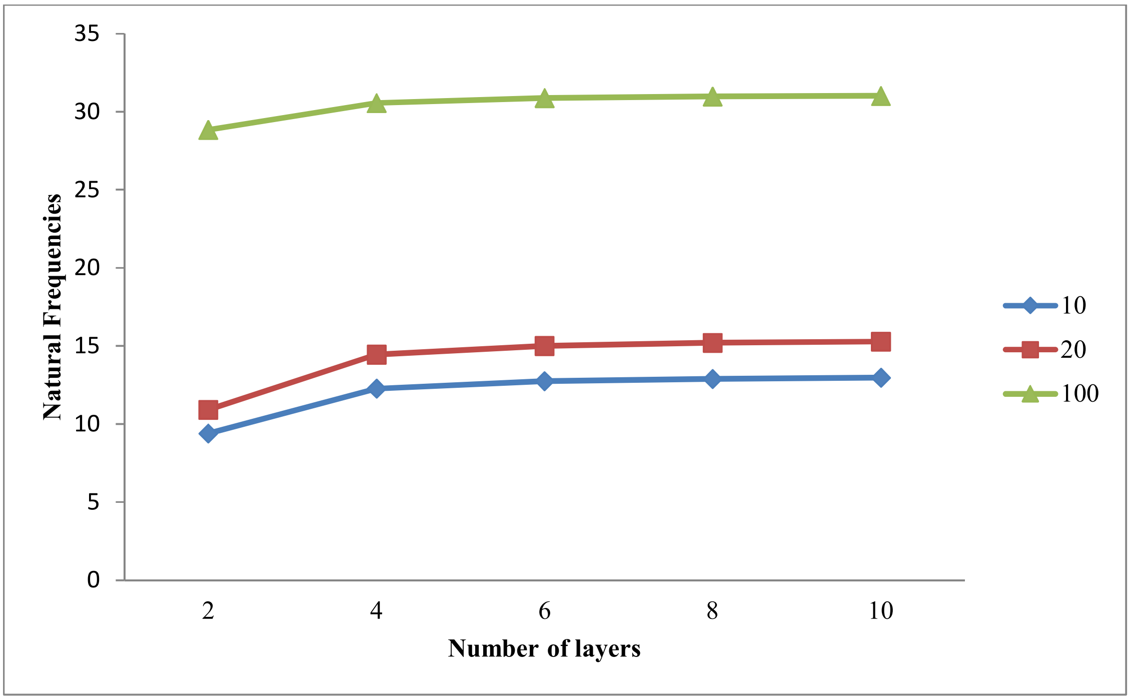

Fig. 4.3. Effect of number of layers on spherical laminated composite shells, MM 1, R/a=5

4.3. Free Vibration Analysis with Uncertain Material Properties

In this section, the material properties of the composite are considered to be uncertain and the effect of these uncertainties on the free vibration response of laminated composite shells is quantified. The material randomness is considered as individual randomness in each property (E_{11}, E_{22}, G_{12}, G_{23}, G_{13}, and v_{12}) and simultaneous variability (all properties considered random). The extent of variability is chosen as 5% to 20%. In MCS, the study is carried out to decide the number of samples for the random input parameter. The random input parameter is considered as E_{11}. The number of samples are considered in the range of 1000 to 10000. After observing the convergence of the results as indicated in Table 4.7, the number of samples to be used were decided as 10000 for further analysis.

For validation of the results with uncertain material properties, a laminated composite square plate with lamination sequence [0/90/90/0] is considered. The material property E_{11} is considered random. The SD/mean ω^2 is obtained and the results are listed in Table 4.8 along with the results given by Singh et al. (2001), Pandit et al. (2010) and Grover et al. (2014). The comparison of the results conclude the validation of the results.

Table 4.8 Effect of sampling size on SD/Mean ω^2 [0/90/90/0] spherical shells with random material property (E_{11})

a/h	N	SD/mean			
		0.05	0.10	0.15	0.20
10	1000	0.0242	0.0513	0.0757	0.1105
	2000	0.0239	0.0470	0.0842	0.0966
	4000	0.0238	0.0527	0.0707	0.0902
	6000	0.0243	0.0440	0.0668	0.1145
	8000	0.0238	0.0443	0.0670	0.0678
	10000	0.0243	0.0534	0.0915	0.1445

Table 4.9 SD/Mean of Fundamental Natural Frequency of Symmetric Cross-ply Plate [0/90/90/0] with Randomness in E11

SD/Mean, E_{11}	Present	Grover et al. (2014)	Singh et al. (2001)	Pandit et al. (2010)
0.05	0.0251	0.0254	0.0257	0.0256
0.1	0.0506	0.0507	0.0514	0.0512
0.15	0.0851	0.0761	0.0772	0.0768
0.2	0.1175	0.1015	0.1033	0.1024

The symmetric laminated composite spherical shells with lamination sequence [0/90/90/0] are considered for the vibration analysis with uncertain material properties. The radii of curvature is varied from 5 to 10^9. The numerical results in the form of SD/mean ω^2 are presented in Table 4.9 and Table 4.10 for a/h =10 and 100 respectively. It is observed that variability in E_{11} influences the frequency response dominantly while the variability in Poisson ratio least influences the vibration response. Further, 5% simultaneous randomness in all properties influences the vibration response by 2.81% to 2.91% for a/h =10 and 3.6% to 4.58% for a/h =100 for the range of curvatures considered. Thus, the response of thin shells is influenced more due to uncertainties present in the system as compared to thick shells

Table 4.10 Vibration of symmetric spherical shells a/h = 10 [0/90/90/0] with uncertain material properties

Parameter	Sample size	R/a						
		5	10	20	50	100	1000	10^9
E_{11}	0.05	0.0243	0.0253	0.0252	0.0252	0.0256	0.0252	0.0256
	0.10	0.0534	0.0505	0.0506	0.0507	0.0511	0.0508	0.0531
	0.15	0.0915	0.0715	0.0898	0.0852	0.0723	0.0901	0.0895
	0.20	0.1445	0.1093	0.1206	0.1176	0.1106	0.1210	0.1118
E_{22}	0.05	0.0027	0.0028	0.0029	0.0029	0.0029	0.0029	0.0029
	0.10	0.0054	0.0056	0.0057	0.0057	0.0057	0.0058	0.0059
	0.15	0.0084	0.0088	0.0085	0.0089	0.0085	0.0087	0.0089
	0.20	0.0112	0.0116	0.0114	0.0117	0.0114	0.0117	0.0117
G_{12}	0.05	0.0053	0.0036	0.0031	0.0030	0.0030	0.0029	0.0029
	0.10	0.0106	0.0071	0.0062	0.0059	0.0058	0.0059	0.0059
	0.15	0.0160	0.0104	0.0093	0.0089	0.0089	0.0088	0.0087
	0.20	0.0215	0.0139	0.0122	0.0119	0.0117	0.0119	0.0122
G_{13}	0.05	0.0054	0.0056	0.0056	0.0056	0.0057	0.0056	0.0056
	0.10	0.0106	0.0120	0.0117	0.0112	0.0112	0.0118	0.0112
	0.15	0.0149	0.0164	0.0163	0.0160	0.0157	0.0163	0.0160
	0.20	0.0215	0.0193	0.0200	0.0198	0.0227	0.0201	0.0198
G_{23}	0.05	0.0123	0.0130	0.0132	0.0132	0.0131	0.0131	0.0133
	0.10	0.0254	0.0257	0.0250	0.0286	0.0258	0.0274	0.0255
	0.15	0.0396	0.0447	0.0408	0.0538	0.0401	0.0455	0.0334
	0.20	0.0544	0.0580	0.0671	0.0912	0.0483	0.0531	0.0357
ν_{12}	0.05	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
	0.10	0.0016	0.0016	0.0016	0.0017	0.0016	0.0017	0.0017
	0.15	0.0024	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025
	0.20	0.0032	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033
All	0.05	0.0281	0.0291	0.0259	0.0291	0.0294	0.0295	0.0291
	0.10	0.0567	0.0587	0.0596	0.0588	0.0592	0.0598	0.0588
	0.15	0.0950	0.0961	0.0871	0.0990	0.0971	0.0874	0.0991
	0.20	0.1311	0.1367	0.1316	0.1358	0.1379	0.1319	0.1359

Table 4.11 Vibration of symmetric spherical shells a/h = 100 [0/90/90/0] with uncertain material properties

Parameter	Sample size	R/a						
		5	10	20	50	100	1000	10^9
E_{11}	0.05	0.0126	0.0254	0.0363	0.0417	0.0430	0.0433	0.0433
	0.10	0.0254	0.0507	0.0729	0.0837	0.0812	0.0817	0.0817
	0.15	0.0411	0.0814	0.1179	0.1362	0.1179	0.1187	0.1187
	0.20	0.0561	0.1095	0.1586	0.1835	0.1257	0.1264	0.1264
E_{22}	0.05	0.0007	0.0014	0.0020	0.0022	0.0020	0.0020	0.0020
	0.10	0.0014	0.0028	0.0039	0.0044	0.0040	0.0040	0.0040
	0.15	0.0021	0.0042	0.0059	0.0067	0.0060	0.0060	0.0060
	0.20	0.0028	0.0056	0.0079	0.0090	0.0083	0.0083	0.0083
G_{12}	0.05	0.0351	0.0208	0.0093	0.0041	0.0029	0.0026	0.0026
	0.10	0.0714	0.0421	0.0189	0.0082	0.0058	0.0052	0.0052
	0.15	0.1005	0.0608	0.0278	0.0123	0.0086	0.0078	0.0078
	0.20	0.1522	0.0876	0.0385	0.0165	0.0118	0.0107	0.0106
G_{13}	0.05	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
	0.10	0.0000	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002
	0.15	0.0001	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003
	0.20	0.0001	0.0002	0.0003	0.0004	0.0004	0.0004	0.0004
G_{23}	0.05	0.0001	0.0002	0.0003	0.0003	0.0002	0.0003	0.0003
	0.10	0.0002	0.0004	0.0005	0.0006	0.0005	0.0005	0.0006
	0.15	0.0004	0.0007	0.0010	0.0014	0.0010	0.0010	0.0010
	0.20	0.0007	0.0015	0.0020	0.0022	0.0014	0.0014	0.0014
ν_{12}	0.05	0.0004	0.0007	0.0010	0.0011	0.0006	0.0006	0.0006
	0.10	0.0008	0.0016	0.0021	0.0023	0.0013	0.0013	0.0013
	0.15	0.0013	0.0024	0.0032	0.0035	0.0019	0.0019	0.0019
	0.20	0.0018	0.0033	0.0044	0.0048	0.0025	0.0025	0.0025
All	0.05	0.0360	0.0348	0.0403	0.0435	0.0456	0.0458	0.0458
	0.10	0.0703	0.0679	0.0784	0.0846	0.0935	0.0940	0.0940
	0.15	0.1101	0.1001	0.1118	0.1191	0.1308	0.1315	0.1315
	0.20	0.1238	0.1161	0.1319	0.1415	0.1527	0.1533	0.1533

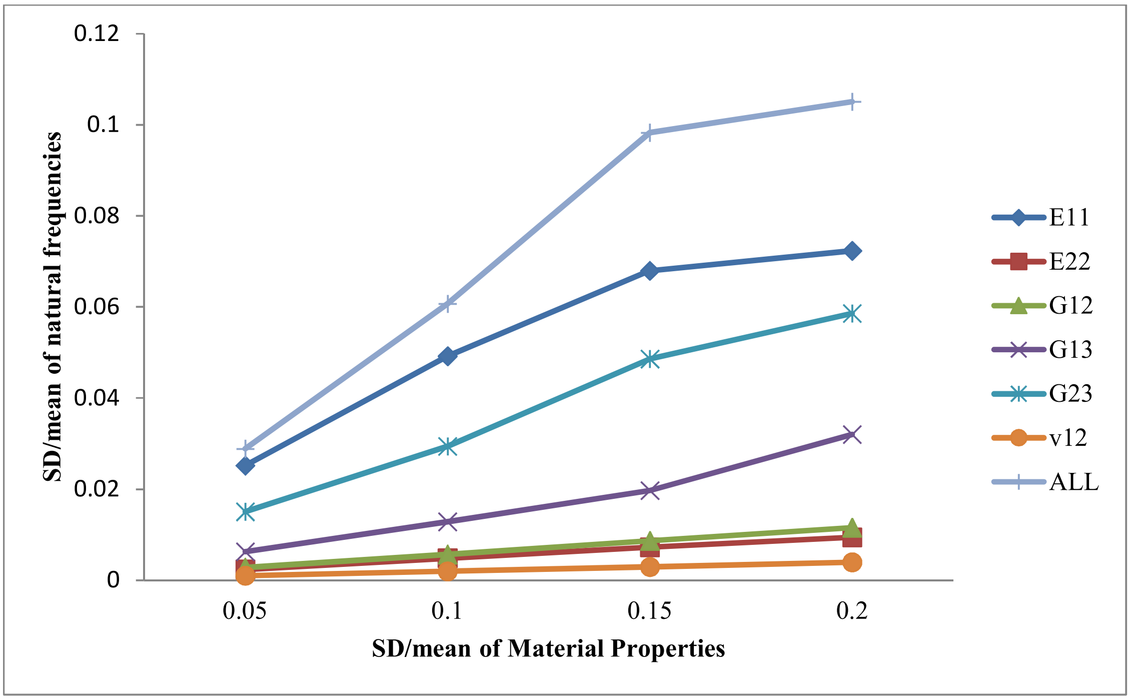

Fig. 4.4. Natural frequency with uncertain material properties [0/90/90/0] cylindrical shells, R/a=5, a/h=10

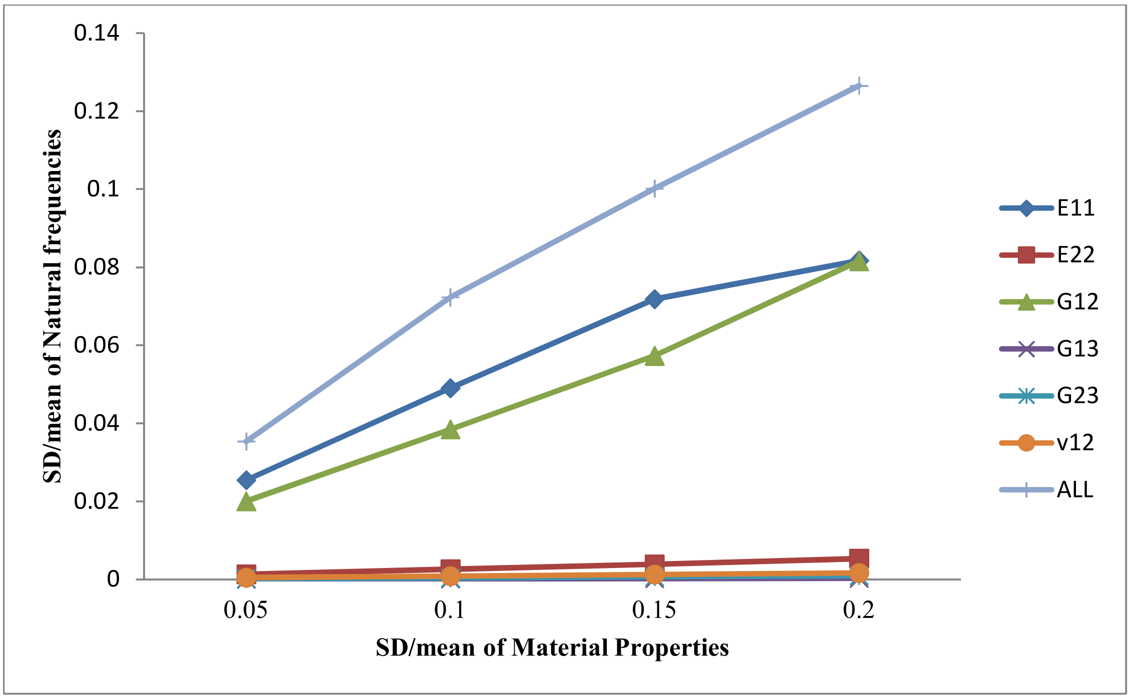

Fig. 4.5. Natural frequency with uncertain material properties [0/90/90/0] cylindrical shells, R/a=5, a/h=100

The Monte Carlo simulations for vibration response of cylindrical shells, with [0/90/90/0] arrangement are shown in the Fig. 4.5 and Fig. 4.6. The Monte Carlo simulations revealed that certain random material properties, when varied individually, had little to no effect on the natural frequencies of the structure. While, other uncertain material properties caused significant effect on vibration response of the shells. And coupled together, these uncertain material properties showed tremendous variation from the results obtained by deterministic studies.

Table 4.12 Vibration of anti symmetric spherical shells a/h = 10 [0/90] with uncertain material properties

Parameter	Sample size	R/a						
		5	10	20	50	100	1000	10^9
E_{11}	0.05	0.0231	0.0244	0.0247	0.0246	0.0246	0.0246	0.0246
	0.10	0.0445	0.0470	0.0476	0.0495	0.0496	0.0496	0.0496
	0.15	0.0653	0.0689	0.0699	0.0791	0.0791	0.0791	0.0792
	0.20	0.0744	0.0781	0.0791	0.1059	0.1060	0.1060	0.1060
E_{22}	0.05	0.0091	0.0096	0.0098	0.0108	0.0109	0.0109	0.0109
	0.10	0.0184	0.0192	0.0194	0.0216	0.0216	0.0216	0.0216
	0.15	0.0276	0.0291	0.0295	0.0306	0.0307	0.0307	0.0307
	0.20	0.0377	0.0398	0.0403	0.0420	0.0420	0.0421	0.0421
G_{12}	0.05	0.0094	0.0072	0.0066	0.0062	0.0062	0.0061	0.0061
	0.10	0.0184	0.0141	0.0130	0.0125	0.0124	0.0124	0.0124
	0.15	0.0274	0.0211	0.0194	0.0186	0.0185	0.0185	0.0185
	0.20	0.0382	0.0292	0.0268	0.0252	0.0252	0.0251	0.0251
G_{13}	0.05	0.0038	0.0040	0.0041	0.0042	0.0042	0.0042	0.0042
	0.10	0.0073	0.0077	0.0078	0.0095	0.0095	0.0095	0.0095
	0.15	0.0109	0.0114	0.0116	0.0122	0.0122	0.0122	0.0122
	0.20	0.0113	0.0118	0.0120	0.0130	0.0130	0.0130	0.0130
G_{23}	0.05	0.0008	0.0009	0.0009	0.0022	0.0022	0.0024	0.0022
	0.10	0.0017	0.0018	0.0018	0.0043	0.0043	0.0043	0.0043
	0.15	0.0027	0.0029	0.0029	0.0067	0.0067	0.0067	0.0067
	0.20	0.0036	0.0038	0.0039	0.0101	0.0101	0.0101	0.0101
ν_{12}	0.05	0.0004	0.0005	0.0005	0.0008	0.0008	0.0008	0.0008
	0.10	0.0009	0.0010	0.0010	0.0017	0.0017	0.0017	0.0017
	0.15	0.0014	0.0014	0.0015	0.0026	0.0026	0.0026	0.0026
	0.20	0.0018	0.0019	0.0019	0.0036	0.0036	0.0036	0.0036
All	0.05	0.0313	0.0322	0.0325	0.0295	0.0295	0.0295	0.0295
	0.10	0.0645	0.0663	0.0669	0.0583	0.0583	0.0583	0.0583
	0.15	0.0905	0.0934	0.0943	0.0823	0.0823	0.0823	0.0823
	0.20	0.1077	0.1101	0.1108	0.0985	0.0986	0.0986	0.0986

Table 4.13 Vibration of anti symmetric spherical shells a/h = 100 [0/90] with uncertain material properties

Parameter	Sample size	R/a						
		5	10	20	50	100	1000	10^9
E_{11}	0.05	0.0058	0.0112	0.0191	0.0252	0.0265	0.0269	0.0269
	0.10	0.0119	0.0226	0.0384	0.0506	0.0532	0.0541	0.0541
	0.15	0.0201	0.0364	0.0611	0.0806	0.0847	0.0861	0.0862
	0.20	0.0287	0.0497	0.0820	0.1077	0.1132	0.1151	0.1151
E_{22}	0.05	0.0016	0.0046	0.0088	0.0119	0.0125	0.0128	0.0128
	0.10	0.0033	0.0092	0.0175	0.0237	0.0249	0.0254	0.0254
	0.15	0.0049	0.0135	0.0254	0.0339	0.0357	0.0363	0.0363
	0.20	0.0066	0.0182	0.0345	0.0463	0.0487	0.0496	0.0496
G_{12}	0.05	0.0417	0.0322	0.0191	0.0098	0.0079	0.0073	0.0073
	0.10	0.0850	0.0653	0.0386	0.0019	0.0160	0.0146	0.0146
	0.15	0.1183	0.0924	0.0559	0.0292	0.0238	0.0218	0.0218
	0.20	0.1833	0.1385	0.0799	0.0401	0.0323	0.0295	0.0295
G_{13}	0.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.10	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
	0.15	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
	0.20	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
G_{23}	0.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.20	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
ν_{12}	0.05	0.0002	0.0004	0.0007	0.0008	0.0010	0.0010	0.0010
	0.10	0.0004	0.0010	0.0016	0.0020	0.0020	0.0020	0.0020
	0.15	0.0006	0.0015	0.0024	0.0030	0.0031	0.0031	0.0031
	0.20	0.0009	0.0020	0.0033	0.0041	0.0042	0.0043	0.0043
All	0.05	0.0411	0.0325	0.0297	0.0324	0.0332	0.0334	0.0334
	0.10	0.0806	0.0642	0.0588	0.0639	0.0654	0.0666	0.0660
	0.15	0.1313	0.0988	0.0858	0.0909	0.0927	0.0933	0.0933
	0.20	0.1442	0.1124	0.1007	0.1082	0.1106	0.1114	0.1115

The vibration response of doubly curved antisymmetric, [0/90] composite shell, is represented in the table 4.11 and table 4.12 for uncertain material properties. The a/h ratio is taken as 10 and 100 respectively. From the data presented, it is noted that, with randomness in all the material properties, there was higher deviation from the exact values of the natural frequency. It is also observed that some properties have very little effect on the natural frequencies of the structure. While some properties showed increasing levels of deviation as the curvature of the structure reduced, there were some which showed the opposite trend. The thick shells showed more deviation in terms of natural frequencies as compared to thin shells.

4.4. Summary

This chapter showed some insights over the vibration analysis of composite shells, both in deterministic and uncertainty framework. Through comparison with the existing results from the available literature, good agreement was achieved. For practical applications, the material uncertainties must be considered design analysis, which has been thoroughly examined in this chapter. Though the exact solutions were accurate, it is still recommended that the randomness in material properties must be considered. Some of the other major findings include the effects of thickness and number of layers, over natural frequencies of the composite structure.

Chapter 5

Conclusion

5.1. Concluding Remarks

This research was aimed at developing an analytical approach for determination of natural frequencies of a laminated composite shells. The utilization of Inverse Hyperbolic Shear Deformation theory assisted the formulation to trace the realistic effects of free vibration on a composite shell. Solution methodology based on Navier solutions, helped achieve accurate results for the simply supported boundary conditions. The effect of material uncertainty has also been a point of consideration of this research. The material uncertainty impact over vibration analysis was calculated using Monte Carlo simulations, which presented the results for varying natural frequencies.

Multiple analysis have been performed, with various structural layout and material properties. The comparison with the available data show that presented methodology is pretty accurate, and can deliver results in an efficient way. The Monte Carlo simulations took comparatively more time as compared to some other methods, but the accuracy achieved by MCS is unmatched and thus is a viable way of addressing the material uncertainty problems. The MCS is also advantageous if the availability of a good computation resource is not an issue. The study also registered some noteworthy trends, which are mentioned below:

- The thickness of the composite shell affects the natural frequency in a dominant way. There exists an inverse proportionality relation between the thickness of the shell and the natural frequency.
- Number of layers in the composite also affect the natural frequencies of the entire structure.
- The curvature of the shell is another parameter that plays an important role in the vibration response. With decrease in shell curvature, it was noted that the natural frequencies also decreases.
- Thick shells were registered with lower values of natural frequencies when compared with the thin shells.

- All uncertain material properties do not have same impact over the vibration response of the composite shell structure. However, all uncertain material properties combined together produced maximum variation in the exact results for the natural frequencies.

5.2. Scope for the Future Work

The same research methodology can be combined with finite element modeling, to achieve results for other boundary conditions. For uncertainty studies, other uncertain parameters such as geometric uncertainty and load uncertainty can be included for the research. Some other methods of determining the uncertainty, such as perturbation techniques can also be explored.

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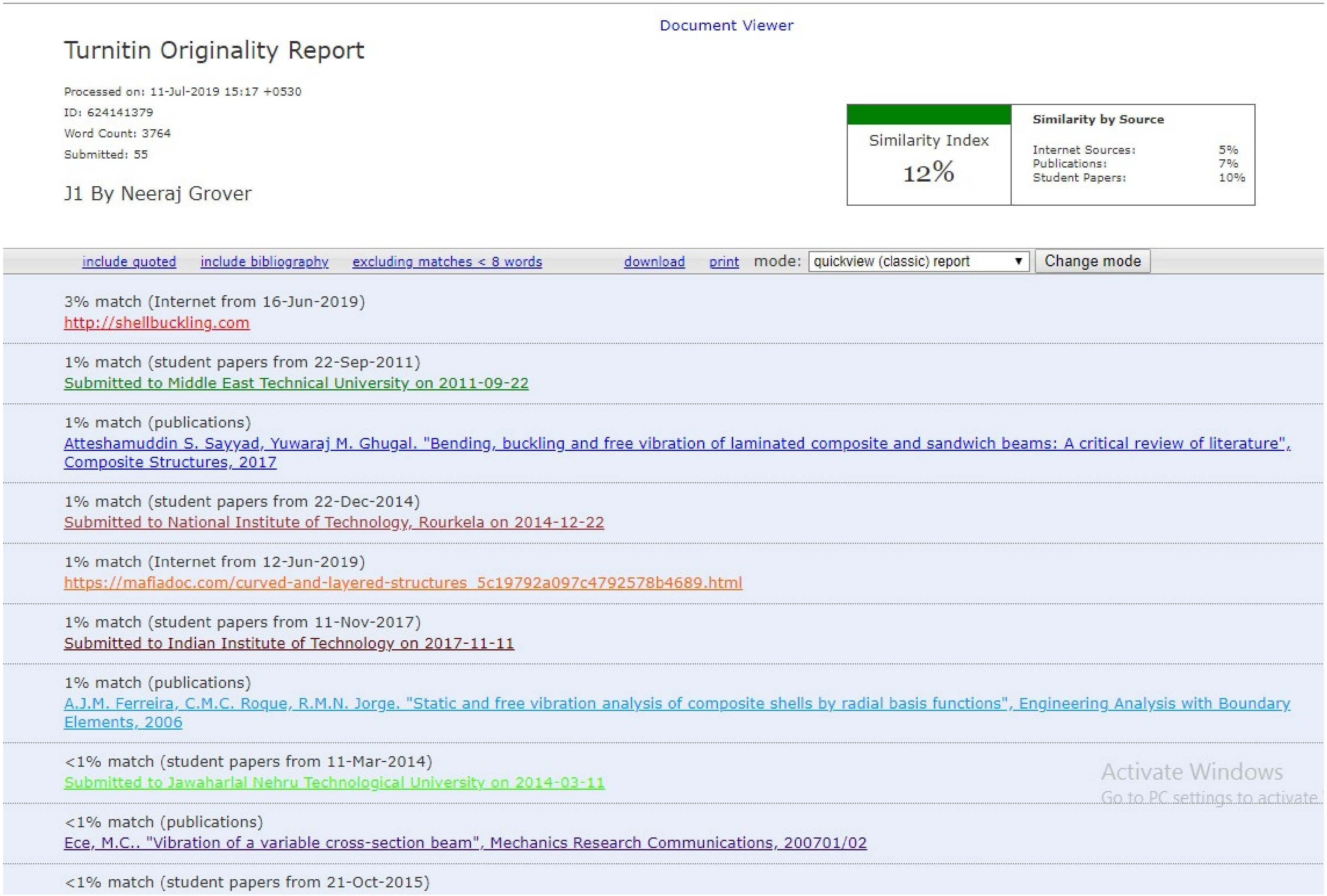

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