

DISSERTATION

On

**Modeling and Experimental Validation of Droplet Size in Gas Metal Arc
Welding Process**

Submitted in partial fulfillment of the requirement for the award of degree

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Master of Engineering

IN

CAD/CAM Engineering

Submitted by

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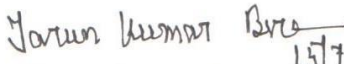
DECLARATION

I hereby declare that work done in the thesis report entitled, "Modeling and Experimental Validation of Droplet Size in Gas Metal Arc Welding Process" submitted towards partial fulfillment of requirement for award of Master of Engineering degree in CAD/CAM Engineering in Mechanical Engineering Department of Thapar University, Patiala, is an authentic record of work carried out by me under the supervision and guidance of Dr. Tarun Kumar Bera, Associate Professor and Dr. Anirban Bhattacharya, Assistant Professor of Mechanical Engineering Department, IIT Patna.

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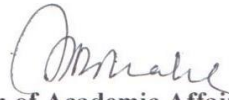
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ABSTRACT

The dynamics of capillary jet instability is a complex but interesting problem because of its uses, if we observe *gas metal arc welding* (GMAW) process, it's about the jet of molten steel interacting with both electric field and magnetic field. Since GMAW was introduced in the early of last century, it has become one of the most popular welding methods. Its efficiency and flexibility is great; different modes of transfer can be employed for different uses. There are about four ways to analyze mass transfer happening in GMAW; these methods help us to optimize droplet size of GMAW process according to the users. Numerical solution to this problem takes a long computational time, so the objective of this work was to model GMAW through bond graph in order to save time and increase the frame-work for multi-domain modeling in bond graph. Two Theories are prevalent in analyzing GMAW drop size, *static Force balance method* (SFBM) and *pinch instability theory* (PIT). Both of the theories have their advantages and disadvantages as they have completely different underlying phenomenon. SFBM predicts the radius a little bit higher than the experimental values; however PIT needs the exact nature of disturbance. Pinch instability theory is used here to model GMAW metal transfer dynamics in bond graph. By this approach, break-up time is the function of the axial displacement of molten metal, amplitude and wave number of disturbances. Process is also simulated in commercial software package FLUENT™ and the results are validated with experimental and bond graph results.

Key words: Droplet size, Capillary jet instability, GMAW, Globular metal transfer, Bond Graph, Fluent Simulation, Hydraulics

NOMENCLATURE

a	Initial Radius
$\hat{a}$	Perturbed radius
A	Initial surface area
$\hat{A}$	Perturbed surface area
$\vec{A}$	Vector potential
C_p	Specific heat
$\vec{E}$	Electric Field
F_{em}	Electromagnetic force
F_T	Detaching force
F_g	Gravitation force
g	Gravitational acceleration constant
H, h	Enthalpy
k	Thermal conductivity
K_b	Boltzmann's constant
k_c	Curvature
I	Welding current
I_d	Current in molten drop
m	Wave number
P_1	Pressure at plane P1
P	Pressure
P_{atm}	Atmospheric Pressure
$\dot{Q}_r$	Radial volume flow rate
$\dot{Q}_z$	Axial volume flow rate
r	Radius
r_d	Radius of the drop
R_a	Radiation loss
$\vec{u}$	Velocity vector
u_r	Radial velocity vector
u_z	Axial velocity vector
z	Axial Co-ordinate
α	Disturbance amplitude
β	Liquid fraction
γ	Surface tension
ζ	Representative Streamline
μ_0	Magnetic permeability constant
φ	Scalar potential
ρ	Density (dynamic phase)

ρ_g	Density of gas phase (assumed air)
ρ_l	Density of liquid phase
ω	In viscid growth rate of the disturbance
θ	Arc covered angle

SUBSCRIPTS

d	Drop
em	Electromagnetic force
g	Gas Phase
l	Liquid phase
r	Radial direction
metal	Property of metal
z	Axial direction

LIST OF ACRONYMS

CFD	Computational fluid dynamics
GMAW	Gas metal arc welding
PIT	Pinch instability theory
MHD	Magnetohydrodynamic
SFBM	Static force balance method
VOF	Volume of Fluid

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Since Leonardo da Vinci observed how a jet breaks and forms a droplet and then retracts after droplet is detached. This topic has been worked on regularly in scientific community, from Lord Rayleigh to Jens Eggers. As our interest and understanding of the subject began to expand, we started to see the beauty, the complexity and amount of daily life systems this phenomenon was involved in. We in the scientific community haven't solved the problem fully but we have understood the problem in a very good manner. That knowledge serves as a framework as we model some of the more advance processes with the existing theory, like inkjet printing, droplet transfer of gas metal arc welding (GMAW), jet propulsion and many more.

1.1 Background and Motivation

Before Lord Rayleigh published his pioneering study on instability of jets [1], Savart and few others have already done some work but Lord Rayleigh's study proved them wrong. In an unrelated paper, Lord Rayleigh introduced the idea of spark based photography which was perfect for finding the experimental results for capillary jet instability, which unfortunately displayed great non linearity effects. A non linear analysis was not performed by Rayleigh. Hence, the governing equations did not justify the result. The non linear theory in one dimension and three dimension, Navier Stoke equation was given by Eggers in [2, 3]. Since then there has been a great increase in the application of the problem. It is used in wide range of applications from diesel engine technology to nuclear fission. One of its applications is in GMAW, where Eager suggests in [4] that pinch instability method can be easily used to approximate the size of welding droplet. The application of the theory to GMAW was given utmost importance and a good deal of effort was also provided on modeling the general instability growth.

The governing equations of capillary jet instability exhibit singularity at the time of detachment, which makes the exact shape and size of the droplet, after the detachment non-deterministic [5] and since the equation becomes singular, the time of detachment also cannot be ascertained. This is the most-non linear system which we come across in daily life.

1.2 Simulation and modeling in FLUENT™

Fluent uses its Graphics user interface to model the system; it requires the user to put boundary conditions and then it solves the system with the solvers integrated in the software. Solvers are designed to provide solution and to represent it visually too. Fluent has robust solvers; in our case, GEO RECONSTRUCT tracks the fluid gas interface perfectly. This software is vast in its applications, although, there are some disadvantages. Those disadvantages can be addressed by User Defined Functions (UDF), but they are difficult to use because of complexity and lack of literature.

1.2.1 Capillary jet instability

The simulation was aimed to see the behavior of jet induced by gravity from a source of water, with in viscid conditions. Simulation domain and preprocessing setup is fairly easy. The surface tension does most of the work. In the later stages of the experiment when the instabilities grow and there is a steep interface the GEO RECONSTRUCT solver is required, to give accurate interface between water and air. Various works like [6] have been published regarding the analysis of numerical parameters like courant number in the GEO-RECONSTRUCT (geometrical reconstructing).

1.2.2 Application to GMAW process

Since the introduction of gas metal arc processes in 1940, this process has been welcomed with open hands. This type of welding is considered best in some cases and is used highly in building purposes. Since the GMAW welding cost for an appropriate frame is large; research started in GMAW area, which has escalated quite nicely and steadily across the years. For the mass transfer in GMAW, we have four different theories. The two most prevalent are described below:

- Static Force Balance Model theory

According to this theory the center of the droplet is taken and force balance is applied there. The resultant of Electromagnetic, drag, momentum flux and surface tension force is examined. Surface tension and drag are called attaching forces, on the other hand the electromagnetic force and gravity are called detaching forces, according to this theory when attaching forces become equal to detaching forces, break up of droplet takes place.

- Pinch instability theory

Pinch instability theory is a modification of Rayleigh-Plateau instability where the radial pressure applied is function of surface tension and current both, although literature gives two

locations of the force application [6], but due to the location of lower plane we do not consider that force. Only one plane has feasible chances of propagating the instability. We assume that place as the effort application point of neck where the derivative of the surface equation is 0.

1.3 Modeling and simulation in bond graph

Bond graph is an efficient way of modeling systems, by the method of bond graph contribution of every single element is apparent in the final differential equation governing the system. This method uses the power balance at different points to chalk out the state space relations for the system and codes them for solution in a C program. System is solved by a solver included in Symbols 200™. Once the modeling theory of any domain is given, that domain can easily be modeled with very less mathematical information (by using the behavior of elements). The robust solver handles the state space dynamic equation quite well and gives us instantaneous results. Bond Graph can easily model mechanical, electrical, thermal, electromagnetic, hydraulic and chemical systems; although its use in mechatronic systems is very widespread, but its use in other domains is also increasing with the increase in research in this field.

Bond Graph technique was developed by Prof. H.M. Paynter, in 1959 at MIT. The basic premise of bond graph modeling is the power balance at the assumed/actual junction points of the system and the similarity of the elements which are used in different systems. Their nomenclature is often different but we can easily see the similarity in application. A short example of a dissipater can explain the things very nicely. A dissipater is essentially a heat sink where heat is dissipated in all directions, in an electric circuit resistor is considered as a dissipater. Resistor gives heat according to the flow of current i.e. I^2R and voltage drop of IR similarly if we look at the damper used in the mechanical domain, it serves the same purpose as resistance in electric circuit. The force is converted to heat energy; moreover the force absorption is also proportional to the flow of mechanical system i.e. velocity.

Bond graph theory works on power conservation at every point of the system; hence it is worth noting that power directions also play an important role. In the next step power is disassociated into two independent variables i.e. effort and flow such that their product is always power. The product of effort and power variable has invariably the units of power.

1.2.3 Elements in Bond Graph

Bond graph is made by I,C,R,SE,SF,TF,GY elements and '0','1' junctions. These Elements are defined in the last sentence give us the information of flow, effort and direction of power. These elements are characterized in following way

- Single Port Passive elements
- Single port active/source elements
- Multiple port conversion elements
- Junction Elements
- Single Port elements

These elements if they are passive take one variable from the bond graph and return the other; on the other hand source elements just give one variable to the graph. We will illustrate the behavior by figures and equations.

Compliance Element

This element stores the energy but does not dissipate it, which is definitive of a mechanical spring. We have a hydraulic system; hence the compliance denotes stretch ability or compressive nature of the fluid. Hydraulic compliance is defined by ratio of the volume flow rate and the rate of pressure variation. Equation 1.1 is Hydraulic transformation of the final value.

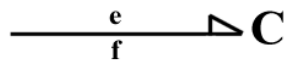


Fig. 1.1 Bond Graph of resistive Element

Here, arrow is denoting power direction as we can see that a spring or stretching fluid will take power from circuit and store it.

$$e = K \cdot x$$

$$P = C \int_t^{\infty} \dot{Q} dt \quad (1.1)$$

Inertial Element

It is defined as pressure difference required for unit volume flow rate change. It takes effort as input and gives flow to the system. A mass in linear mechanical system does the same thing. Equation 2.3 is Hydraulic transformation of the final value.

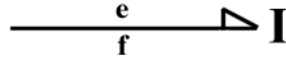


Fig. 1.2 Bond Graph of resistive Element

$$\begin{aligned}
 F &= m\ddot{x} \\
 \Rightarrow \frac{df}{dt} &= m \\
 \Rightarrow Q &= \frac{1}{I} \int_t^\infty P dt
 \end{aligned} \tag{1.2}$$

Resistances

This element can perform functions of both abovementioned elements, they can take in either effort or flow and return the opposite, but in the hydraulic theory pressure dissipation is very difficult thing to calculate, Hence only two type of resistances are used called Bernoulli and Drag resistances. Equation 1.3 gives the value of Bernoulli's resistance. This resistance is modulated.

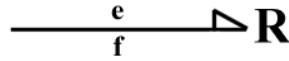


Fig. 1.3 Bond graph of resistive element

$$\begin{aligned}
 e &= Rf \\
 \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(I)^2} - \frac{1}{A(0)^2} \right) \dot{Q}^2 \right\}
 \end{aligned} \tag{1.3}$$

Source of effort

This element gives effort to the system and extracts '0' flow. In hydraulic system Constant pressure pump is SE.



Fig. 1.4 Bond graph of source of effort element

Source of flow

Element that give us the flow are called source of flow elements. A constant volume flow rate pump is source of flow in hydraulic domain.



Fig. 1.5 Bond Graph of Source of flow Element

Following table shows the corresponding elements and their symbols in various domains:

Table 1.1 Effort and Flow variables in different domains

S No.	Element	Mechanical Equivalence	Electrical Equivalence	Hydraulic Equivalence	Symbols
1	Inertia	Mass (m) (linear Systems) Moment of inertia (angular systems)	Inductor	Inertance	I
2	Capacitance	Mechanical Spring	Capacitance	1/(Hydraulic Compliance)	C
3	Resistances	Damper	Resistance	Bernoulli and Drag resistances	R
4	Source of Effort	Load/Force applied	Constant Voltage source	Pressure source	SE
5	Source of flow	Velocity source (Cam)	Constant Current source	Volume flow rate source.	SF
6	Transformer	_____same_____	_____same_____	_____same_____	TF
7	Gyrator	_____same_____	_____same_____	_____same_____	GY

Basic two-port Elements

‘TF’ and ‘GY’, called transformer and gyrator respectively, do not create or destroy energy; they just redistribute the effort and flow variables. Transformer varies the effort or flow variable to a multiple of previous effort or flow. In gyrator flow is amplified as effort in other bond. Difference of behavior is shown in the Fig. 1.6.

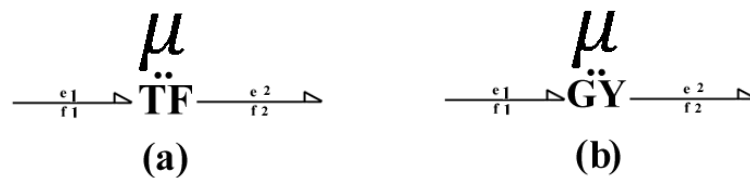


Fig. 1.6 (a) Transformer and (b) gyrator elements

Difference in transformer is evident in set of equations 1.4 and 1.5, former belong to the transformer.

$$\begin{aligned} e_2 &= \mu e_1 \\ f_2 &= \mu f_1 \end{aligned} \quad (1.4)$$

$$\begin{aligned}
e_2 &= \mu f_1 \\
f_2 &= \mu e_1
\end{aligned}
\tag{1.5}$$

Junction Elements

‘1’ Junction represents flow equalizing junction. So if 3 branches marked 1, 2 and 3 on ‘1’ junction in that case

$$\begin{aligned}
e_1 + e_2 + e_3 &= 0 \\
f_1 &= f_2 = f_3
\end{aligned}
\tag{1.6}$$

‘0’ Junction represent effort equalizing junction. In the last scenario equations convert to

$$\begin{aligned}
f_1 + f_2 + f_3 &= 0 \\
e_1 &= e_2 = e_3
\end{aligned}
\tag{1.7}$$

1.4 Bond graph model of capillary jet

Governing equations in both radial and axial directions is modeled and then the resultant flow was added with geometric assumption. The disturbance is given as a sinusoid with amplitude growth is used; viscous slowing is not modeled as the model is in viscid.

1.5 Bond graph of GMAW process

Bond Graph of simple capillary jet instability is modified with new expressions to describe the GMAW process. The bond graph is made completely in hydraulic domain, attempts for multi domain bond graph was made, but the literature involving calculation of inductances for spherical volume was not very clear, after that a attempt towards making a model of SFBM was done. In SFBM there were a large number of assumptions, and most importantly rate of necking and growth of assumed spherical droplet was user defined.

In the end Capillary jet instability was modeled and modified for GMAW process, although the bond graph was not multi domain bond graph, but a framework is devised for all necking related problems.

1.6 Objective of the thesis

- To simulate capillary jet instability and GMAW problem in Fluent™
- To determine the appropriate theoretical model for capillary jet instability in fluent and the flow characteristics

- To append the capillary jet instability using bond graph to make the instability usable in GMAW process and subsequently to make a bond graph of GMAW process
- To compare simulation, experiment and bond graph results for validation

1.6 Organization of the thesis

This thesis is organized as discussed below:

Chapter 1 deals with the introduction to the thesis

Chapter 2 lists the literature reviewed and the topics which are reviewed from existing publications and utilized for the work.

Chapter 3 lists the models, techniques and boundary conditions used for the simulation of capillary jet instability and GMAW process.

Chapter 4 deals with the bond graph modeling of both capillary jet instability and GMAW process.

Chapter 5 gives us conclusions of complete thesis, comparison of results, effects and scope for the future work.

2.1 Introduction

The literature which was studied and subsequently became the foundation for this study is reviewed in this chapter. This chapter will be characterized on the basis of the type of problems involved in the body of work; however fluent modeling and solver theory is not included in this chapter as they are discussed alongside the simulation method in Chapter 3. So the literature review is structured in a particular way which helps us to formulate the problems and link them to each other. This chapter will be characterized in four parts

- Formulation and specifications of capillary jet instability problem
- GMAW analysis methods and simulation setup techniques
- Capillary jet instability in GMAW process
- Introduction to bond graph modeling

2.1.1 Formulation and specifications of capillary jet instability problem

Lord Rayleigh first wrote [1] in which he argued how disturbances cause the thin cylindrical jet to disintegrate into droplets. This is completely the result of surface tension, when a cylindrical jet is unperturbed the amount of surface area is in an unstable equilibrium, once any disturbance causes it to reduce. It reduces more and then disintegrates into droplets. In other analysis we take the governing equations from [2] and perform linear stability analysis to see how the disturbance does propagate in the cylindrical fluid column.

Realizing the potential and applicability of this problem, a lot of research was directed in this way. Internal dynamics in the drop was explained by Eggers [3]. Rayleigh [7] illustrated the advantages of high speed photography using spark. These images established the existence of large singularities and non-linearity; however the Rayleigh in [1] has done linear stability analysis, so the non-linearity effects were not represented in the theoretical work. This is one of the most complex phenomena we experience daily in our life. The complete modeling of the singularities in one dimension Navier Stokes equation was done by Eggers in 1994 [2] and description of complete internal dynamics was also illustrated by Eggers in 2005 [3].

We will do the linear stability analysis by method illustrated in [1] and find out what is the main parameters we are concerned with.

- **Linear Stability analysis with Rayleigh Work principle**

Since we are perturbing the base state, and then that perturbation grows because liquid tends to be in the shape where it has lowest surface energy, as we are considering only surface tension. Here we analyze change in area due to the perturbation, and see when the system remains unstable and breaks into droplets or when it is neutrally stable. Change in Area from the base state will be an accurate measure of system energy as we are considering energy because of surface tension.

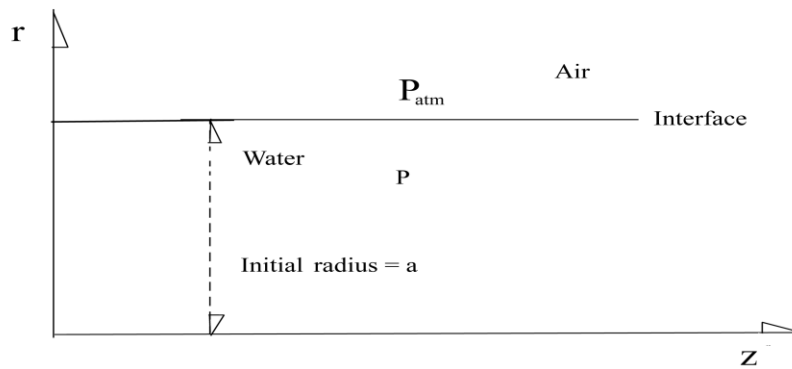


Fig. 2.1 Base State

In Fig. 2.1, cylinder surface area in unperturbed state is $A = \int_0^\lambda \int_0^{2\pi} r d\theta dz$

For a cylinder

$$A = 2\pi a\lambda$$

$$\Rightarrow V = \int_0^\lambda \int_0^{2\pi} \int_0^a r dr d\theta dz$$

$$\Rightarrow V = \pi a^2 \lambda \quad (2.1)$$

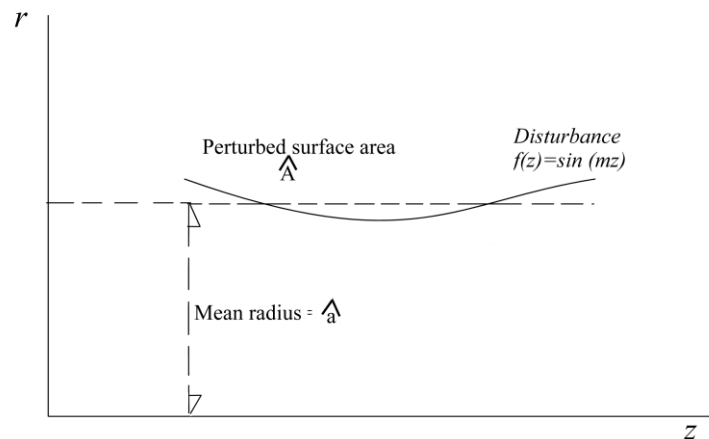


Fig. 2.2 Perturbed State of jet

In the perturbed state surface area can be calculated by

According to the figure given above $(ds)^2 = (dz)^2 + (dr)^2$

$$\begin{aligned}\Rightarrow \left(\frac{ds}{dz}\right)^2 &= 1 + \left(\frac{dr}{dz}\right)^2 \\ \Rightarrow \frac{ds}{dz} &= \left(1 + \left(\frac{dr}{dz}\right)^2\right)^{\frac{1}{2}}\end{aligned}\quad (2.2)$$

Radius can be written as $r = a(1 - f(z))$. Here ‘ a ’ is taken as the initial amplitude of the disturbance $f(z)$. $f(z)$ is mostly denoted by $\sin(mz)$, where m is the wave number of the disturbance causing instability. $r = a(1 - \sin(mz))$ is substituted in Eq. 2.2.

$$\frac{ds}{dz} = 1 + m^2 a^2 \sin^2(mz)$$

Surface area in perturbed state is

$$\Rightarrow \hat{A} = \int_0^\lambda \int_0^{2\pi} r d\theta \frac{ds}{dz} dz = \int_0^\lambda \int_0^{2\pi} (\hat{a} + a \cos(kz)) (1 + m^2 a^2 \sin^2(mz)) d\theta dz \quad (2.4)$$

As the amplitude of disturbance is taken as a small quantity, we can ignore higher powers of a , eliminating terms with 3rd power of a and above we get the changed surface area given by $\hat{A}$.

$$\hat{A} = 2\pi(\hat{a}\lambda) + \frac{\hat{a}^2 a^2 m^2 \lambda}{4} \quad (2.5)$$

New volume of perturbed state is calculated by

$$\begin{aligned}\hat{V} &= \int_0^\lambda \int_0^{2\pi} \int_0^r r dr d\theta dz \\ \Rightarrow \hat{V} &= \frac{\int_0^\lambda \int_0^{2\pi} (\hat{a} + a \cos(mz))^2 d\theta dz}{2} \\ \Rightarrow \frac{\hat{V}}{\lambda} &= \pi(\hat{a}^2) + \frac{a^2}{2}\end{aligned}\quad (2.6)$$

Since the volume will be constant in both cases we set $\frac{\hat{V}}{\lambda} = \frac{V}{\lambda}$

$$\hat{a} = \sqrt{1 - \frac{a^2}{2a^2}} \text{ using binomial expansion}$$

$$\Rightarrow \hat{a} = \left(1 - \frac{a^2}{4a^2}\right) \quad (2.7)$$

Using 2.7 in 2.4, we get

$$\frac{\hat{A}}{\lambda} = 2\pi\hat{a} + \frac{\hat{a}\alpha^2\pi}{2} = 2\pi\left(a - \frac{\alpha^2}{4a}\right) + \frac{\alpha^2 m^2 \pi}{2} = 2\pi a - \frac{2\pi\alpha^2}{4a^2} + \frac{\pi\alpha^2 m^2}{2a} + \alpha^4(\dots\dots) \quad (2.8)$$

Truncating 4th power of α

$$\begin{aligned} \hat{A} &= \alpha^2 \pi (1 - a^2 m^2) \\ \frac{\hat{A}}{\lambda} &= \left(\frac{A}{\lambda} - \frac{\hat{A}}{\lambda}\right) = \left(\frac{\alpha^2 \pi}{2a}\right) (1 - m^2 a^2) \end{aligned} \quad (2.9)$$

If Eq. 2.9 is negative, the instability will grow as the surface area has started decreasing which in turn will affect the unstable equilibrium of energy. If $ma < 1$ system is stable, whereas if it $ma > 1$ the disturbance will grow.

Growth of the amplitude of the disturbance in an in viscid is given in [8]. The expression for amplitude is $\alpha = e^{\omega t} \alpha$

Although in viscous flow, the expression for amplitude is $\alpha = e^{-i\omega t} \alpha$, but the imaginary (-i) component shows the viscous slowing. The in viscid growth rate of disturbance is given by

$$\omega = \frac{\gamma(1-(ka^2))^{\frac{1}{2}}}{\rho_w a^3 \ln\left(1+\frac{1}{ka}\right)} \quad (2.10)$$

This equation is derived from the one dimension Navier Stokes equation assuming the long wave property and by using geometrical boundary conditions and dispersion relation of a cylindrical jet.

2.1.2 GMAW analysis methods and simulation setup techniques

In this part the models are discussed that can be used to analyze the metal transfer process in GMAW. Two basic theories which come out are the Static Force Balance Model (SFBM) [6] and pinch instability theory (PIT) [9], although both the theories utilize different phenomenon to structure the problem [10]. One is being based on the Rayleigh-Plateau instability and other on the basis of force balance. The forces in the force balance method are derived in [11,15].

For the numerical evaluation of the GMAW process two things are important. One is magnetohydrodynamic DC arc and second is modeling of heat transfer. Governing and

boundary conditions of plasma arc are illustrated in [12]; they were appended with heat transfer equations and discussion of mass transfer by eager in [13]. The literature involved in simulation of a GMAW metal transfer process is more than abundant, the processes have been studied with radiation and metal vapors. Still the deterministic nature of the solution is not guaranteed [5].

The forces in globular metal transfer are derived by pendant shape assumption. Basic relations and the assumed pendant shape are shown in the figure below:

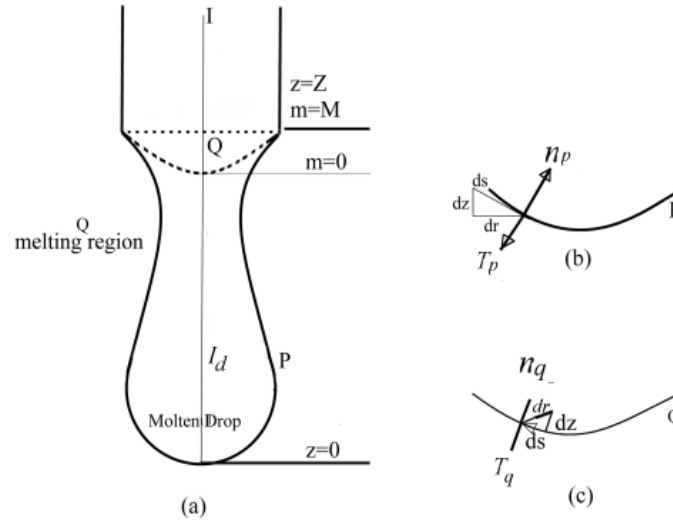


Fig. 2.3 Pendant Shape approximation

This Figure is modified with some simplifications to Figure below which describes both pinch instability theory and SFBM forces. Since the arc starts from plane 1, the disturbance is also applied on plane 1. The chamfered conical area depicted as Q can be modeled as a disc, both physically and mathematically. The figure is divided in three parts as shown in the figure below. One (I) being the conical melting shape presented as a disc (both of them have same Lorentz force in z direction).

Second (II) is the part from which electron emission is not taking place, here the force is independent from the current path, third (III) part is the bottom conducting zone. Addition of these three gives us the total Lorentz force in z direction given in Eq. 2.11.

F_{em} With simplifications (as shown in figure below, is calculated in [11] by Amson.

$$F_{em} = \left(\frac{\mu_0 I_d^2}{4\pi}\right) \left[\ln(\delta \sin(\theta)) - \frac{1}{4} - \frac{1}{(1-\cos \theta)} + \frac{2}{(1-\cos \theta)^2} \times \ln \frac{(1+\cos \theta)}{2} \right] \quad (2.11)$$

Calculation of pinch force at P_1 is given in [9]. It is calculated for the expression as given in [6]

$$P_1 = \frac{\mu_0 I^2}{8\pi^2 r_d^2} + \frac{4\gamma}{2r_d} \quad (2.12)$$

For simulation of the problem Fluent 12.0 Theory Manual was referred to in general [15].

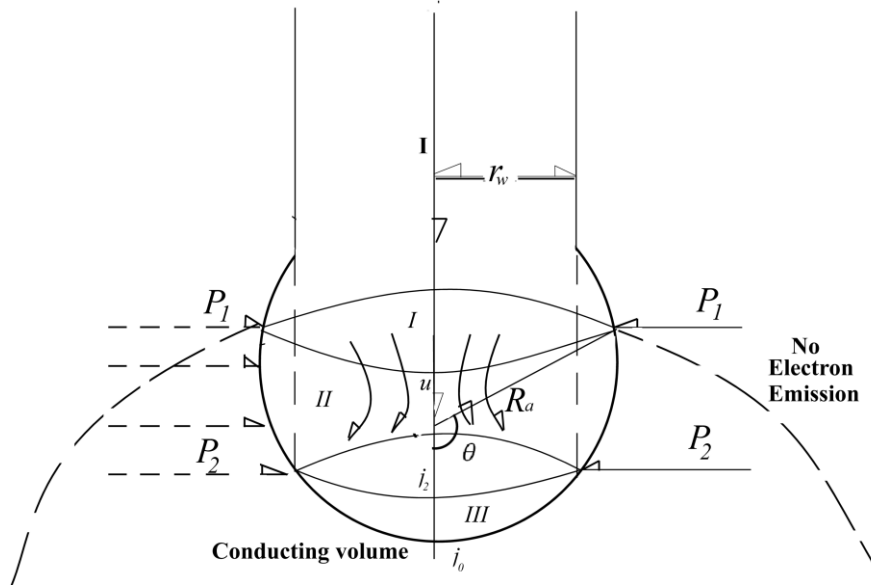


Fig. 2.4 Simplified pendant shape [5]

2.1.3 Capillary jet instability in GMAW process

Eager in [10] discusses both ways available to model the GMAW process. In this paper Eager suggested that SFBM predicts a larger size than the experimental value. On the other hand pinch instability theory can predict the size accurately, however the singularity encountered while calculating time of detachment is a point of worry in the latter approach. The pinch force acting on P_1 calculated in [6] is used as the pinch force in bond graph modeling in Section 4.2.

2.1.4 Introduction to bond graph modeling

Alvin Toffler is beautifully quoted in [16] as “One of the most highly developed skill in contemporary western civilization is dissection; the split up of the problems into their

smallest possible components. We are good at it. So good, we often forget to put the pieces back together again.”

Bond graph is a modeling technique which tries to put the pieces back together and try to create a modeling technique that can be used to model the system of any domain with reasonable efficacy and flexibility.

Basically the objective of bond graph is to apply power conservation at every point in the system and to derive the corresponding governing equations with help of mathematical behavior of elements. Once the governing equations are derived they are solved numerically with traditional numeric integration methods. The concept of causality enables us to use this tool for fault identification.

Bond graph can easily be made into signal flow graph, which in turn can give us transfer functions. This is the main reason why bond graph is so extensively used in mechatronics systems.

Modeling of hydraulic system and converting Navier Stoke equation into hydraulic power circuit equation is well documented in [16]; although it was done so with the assumption of a representative streamline (ζ) in the direction of flow. This one variable is the physical manifestation of one dimensional Navier Stoke equation used.

However, that particular approach is not that useful for modeling complex flows. If two assumptions, i.e. fluid is inside the rigid boundary and an average streamline can be treated as the representative of the flow, are approximately valid; than the standard definitions given in the section 1.2 apply; but outside that sphere modeling hydraulic systems in one dimension is not always accurate. Even when the assumptions are valid there is lot of complex behavior involved. In [17] the bond graph of boundary flow in a pipe segment is modeled, governing equations used are still one dimensional. In [18] a complete formulation has been presented to model CFD systems in bond graph by discretization and then using power balance on a control volume. Complete process of nodal solutions and interpolation functions has been defined by IC fields.

2.2 Literature gaps

There is very little literature available about Rayleigh-plateau instability. The analysis of capillary jet instability and its uses in GMAW process is a very new area of research. There are discontinuities in the analytical model of capillary jet instability too. There are still some non modeled singularities in the analytical solution. This problem is generally

analyzed in non-dimensional analysis; absolute axial and radial volume flow rates in droplet of GMAW process are very much challenging.

3.1 Modeling of capillary jet simulation in Fluent

3.1.1 Governing equations for capillary jet instability problem

An axis-symmetric mesh is created, with minimum orthogonal quality value of 0.89. It is modeled for jet break-up with volume of fluid (VOF) technique using water<liquid> as a secondary phase coming out of the velocity inlet with negligible velocity and body force acting on it. Analysis has been done with quite a few assumptions so as to compare with the bond graph result which has been developed for in viscid flow. Governing equations are derived from the one dimensional approximation of Navier-Stokes equation for the capillary jet instability [1].

- **Equation of continuity**

$$\frac{du_r}{dr} + \frac{\partial u_z}{\partial z} = \frac{u_r}{r} \quad (3.1)$$

As the liquid is considered as in-viscid, all viscosity terms vanish and following form of momentum equations are solved, moreover, no energy equation is required as the thermal behavior is not considered in water droplet detachment analysis.

- **Momentum Equations**

$$\begin{aligned} \frac{Du_r}{Dt} &= -\left(\frac{1}{\rho}\right)\left(\frac{\partial P}{\partial r}\right) \\ \frac{Du_r}{Dt} &= -\left(\frac{1}{\rho}\right)\left(\frac{\partial P}{\partial r}\right) - g \end{aligned} \quad (3.2)$$

Surface tension is cause of varying interface and hence, for all the limitations; problem has to be formulated in explicit formulation with a varying and very small time step. Courant number at a constant time step increases as the curvature of the interface increases.

The main aim of this simulation is to find the expression of axial displacement (z) with respect to time, although at the centerline velocity equation is singular, but for a certain set of boundary conditions, a non deterministic expression can be formed by regression analysis between z and time. Moreover, as we all know droplet detachment is a very complex problem for which a perfect solution is still not available, a preliminary bond graph model is also made and simulation can be used to verify bond graph results.

3.1.2 Fluent modules used

VOF multiphase method is used with two phases where water is assigned as the secondary phase. K-epsilon method is used for turbulence analysis with scalable wall function. Description and governing equations of abovementioned models is given in Section 3.2.

3.1.3 Domain description and boundary conditions

- **Domain**

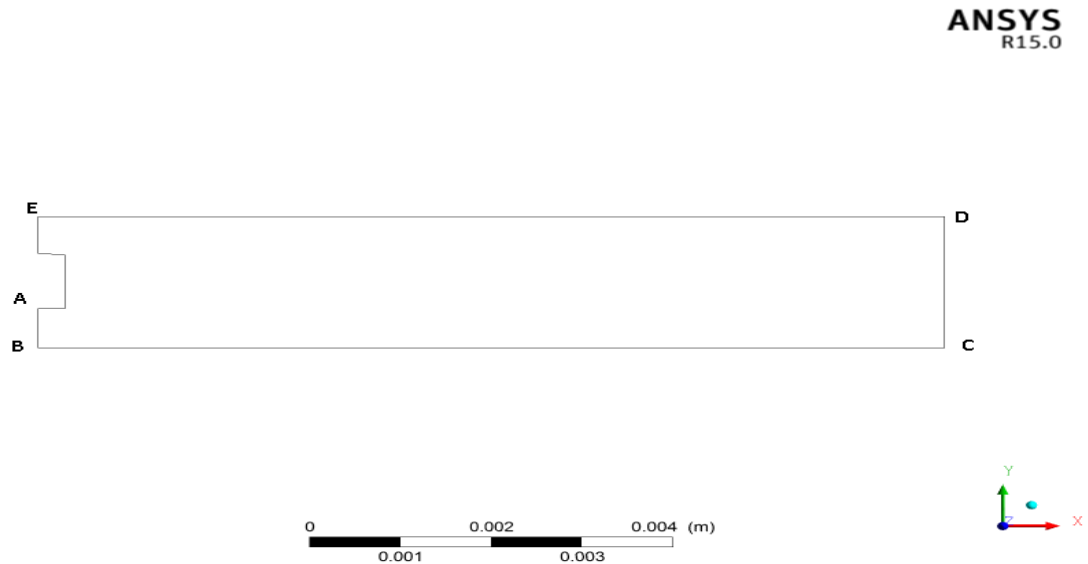


Fig. 3.1 Fluid flow axis symmetric domain for capillary jet instability

- **Boundary conditions**

As the simulation is only dealing with the flow of the fluid, no energy changes have been considered. So, the boundary conditions are only the momentum boundary conditions. Only two boundary conditions are required and these are given in the Table 3.1.

Table 3.1 Momentum boundary conditions

Sl. No.	Boundary name and location	Value	Dimension
1	Velocity Inlet, AB	0 m/s	0.6 mm
2	Walls, BC,CD,DE and EA	No slip condition	AB=10mm CD= 2mm

In the in viscid simulation Courant number is highly sensitive to the time step, hence constant monitoring of the Courant number is involved. As the curvature of the water-air interface gets complicated courant number increase with this effect, to counter this smaller

time step is taken. Courant number ideally should be around '0.5'. Once it crosses the value of '1', it increases rapidly. At 250 solutions are declared diverging by fluent.

3.1.4 Results

As we are only interested in the volume fraction of the water, results as volume fraction plot are given below:

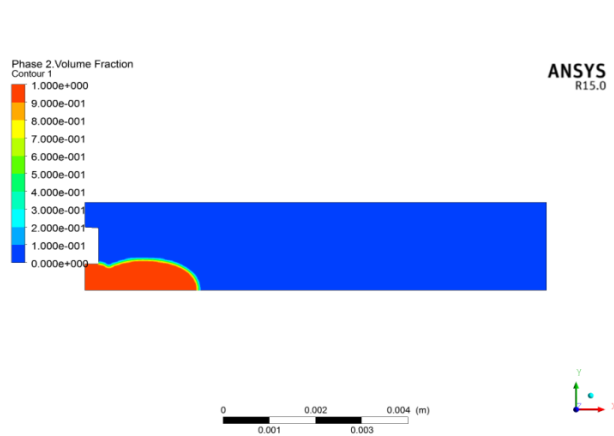


Fig. 3.2 Volume fraction (water) at 25.58 ms

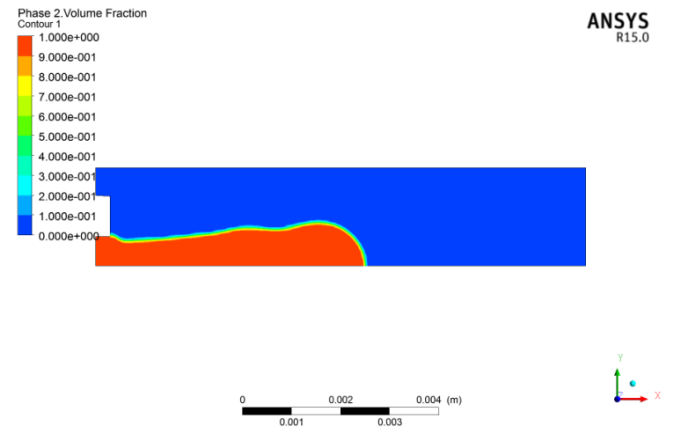


Fig. 3.3 Volume fraction (water) at 72.08 ms

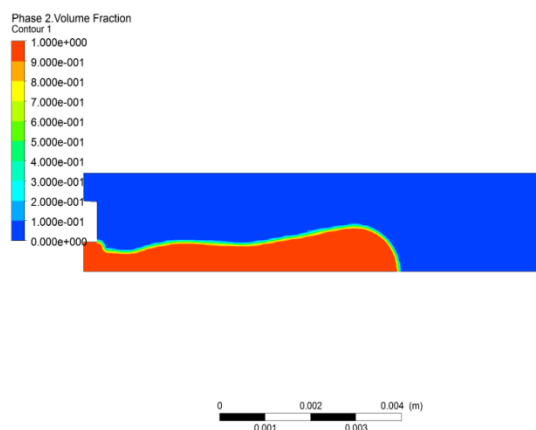


Fig. 3.4 Volume fraction (water) at 81.58 ms

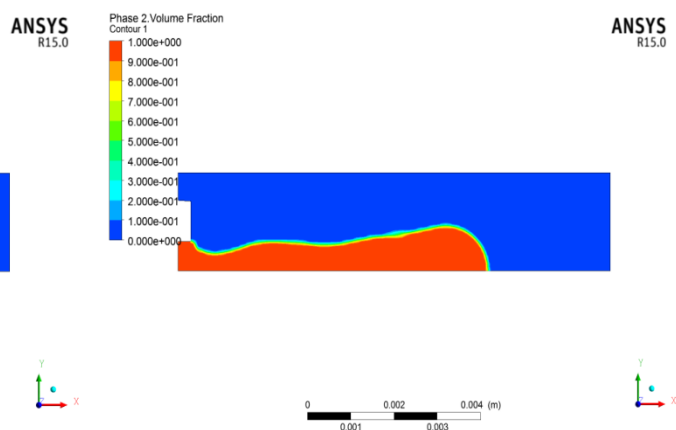


Fig. 3.5 Volume fraction (water) at 82.08 ms

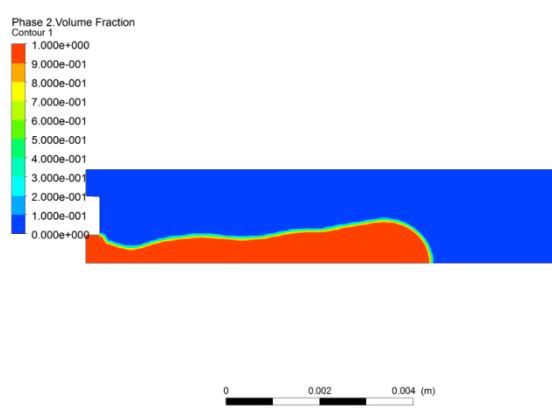


Fig. 3.6 Volume fraction (water) at 84.08 ms

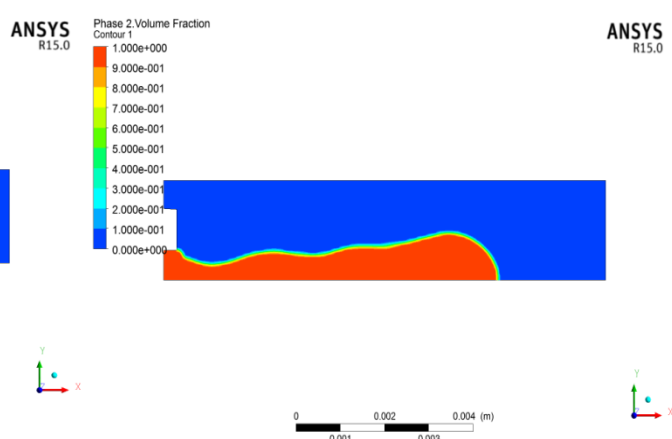


Fig. 3.7 Volume fraction (water) at 85.58 ms

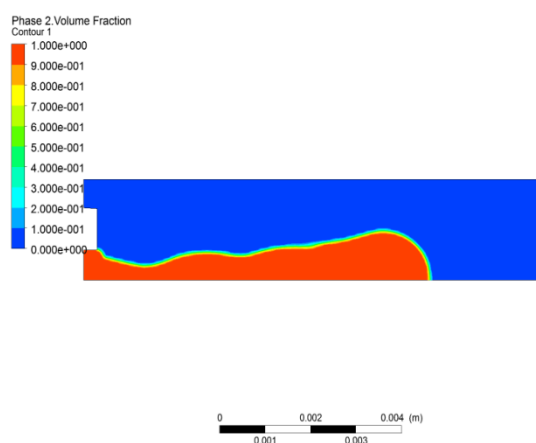


Fig. 3.8 Volume fraction (water) at 87.58 ms

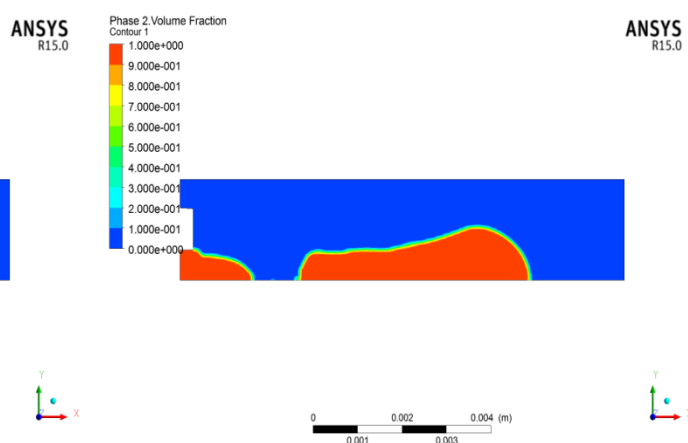


Fig. 3.9 Volume fraction (water) at 90.46 ms

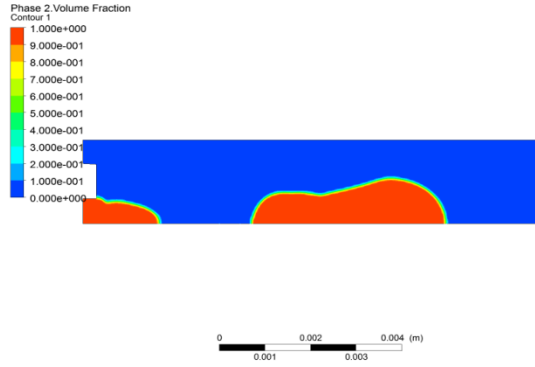


Fig. 3.10 Volume fraction (water) at 91.08 ms

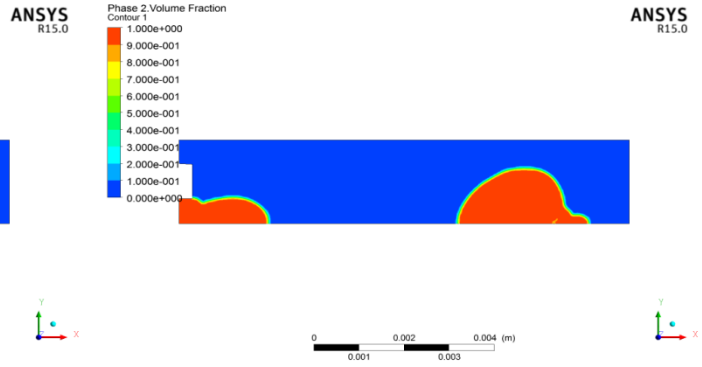


Fig. 3.11 Volume fraction (water) at 97.46 ms

Although, when we compare results with bond graph results, the results are in good agreement as we can see the disturbance wave, which can be plotted by taking a small disturbance as compared to initial radius of 0.6 mm. The output radius is close to 2 mm, exact volumes are not calculated as they have little consequence.

The main thing to be noted here is that, when the video of the simulation is made, a lot of oscillations can be seen and this particular aspect is not accounted for the calculation of the axial co-ordinates. So, there can be two theories for the difference in times of detachment in bond graph and fluent.

One reason can be that oscillations are not accounted for in the expression of z co-ordinate. It can be done by including a hydraulic compliance, but the limitation is the volume of the neck portion is not very easy to compute.

3.2 Modeling of GMAW globular transfer in Fluent™

3.2.1 Modeling techniques used [15,20]

- **Multiphase model-VOF (volume of fluid) method**

VOF method is used as we have two or more immiscible fluids with well defined boundaries. Suppose we have a system consisting of a gas and liquid, using VOF method we have to handle two mass balance equations.

$$\frac{\partial \rho_g}{\partial t} + \nabla(\rho_g \cdot v_g) = 0 \quad \text{and} \quad \frac{\partial \rho_l}{\partial t} + \nabla(\rho_l \cdot v_l) = 0 \quad (3.3)$$

where the subscripts 'g' and 'l' describe the gas phase (the gaseous phase) and liquid phase (water in our case), respectively. Every immiscible liquid is denoted by a phase and any

number of phases can be defined. Interaction between different phases can be provided. For instance, surface tension between any two phases with the option of wall adhesion can be provided in VOF method.

VOF method can be characterized into two different methods.

- **Implicit**

Here, volume fraction is calculated iteratively with momentum equations. It is the faster way to analyze systems as large time steps are acceptable, but diffusion of interface and poor convergence wherever surface tension is involved, makes its use limited.

- **Explicit**

It captures very sharp interfaces due to the Geo-Reconstruct scheme, although it takes a long computational time as the time step has to be small to keep the courant number below 1. Solution diverges as the courant number reaches the value of 250. It does not converge easily for compressible systems and poor meshes especially with very low orthogonal quality. 3 phase VOF is used where argon and molten steel are the secondary phases. Explicit analysis is used with average time step size of 1 microsecond in above mentioned analysis. Body Force can also be provided in the VOF multiphase toolbox only. Geo-Reconstruct does not fill cells with cell volume fraction, they fix a minimum volume and if the volume fraction is greater than a given value than a complete volume cell is shown. In other case an empty face is shown in the contour [19].

While Using Solidification and melting VOF method is preferred to model coupling between phase changing material and medium, as the interface is clearly between melting/solidifying fluids in detail. VOF method is generally used in simulating sloshing, waves, jet break-up and interface tracking problem involving any two immiscible fluids.

- **Solidification and Melting**

Fluent uses the Enthalpy-porosity approach to simulate solidification and melting; liquid solid mushy zone is treated as the porosity zone. Value of porosity of a cell is the liquid fraction in a particular cell. Moreover its value can be accessed in a user defined function as C_POR (cell, thread).

Enthalpy-Porosity formulation has enthalpy instead of temperature as primary dependent variable in energy equation. Enthalpy of a material is evaluated as sum of sensible enthalpy: h , latent heat: ΔH

$$H = h + \Delta H$$

$$\text{Where } h = h_{ref} + \int_{T_{ref}}^T C_p dT \quad (3.4)$$

Liquid fraction, denoted by β is described by Eq. 3.5; this saves us from writing additional user defined functions (UDF's) to model the melting process. This approach like most others has its advantages and disadvantages; best advantage is that too many user defined functions are not required. Liquid fraction is calculated as

$$\beta = \frac{T - T_{solidus}}{T_{liquidus} - T_{solidus}} \quad (3.5)$$

$$\Rightarrow \beta = 0 \text{ for } T < T_{solidus}$$

$$\Rightarrow \beta = 1 \text{ for } T > T_{liquidus}$$

Now we can calculate latent heat, $\Delta H = \beta L$. The energy equation is re-written in the terms of enthalpy for each every phase having heat source

$$\frac{\partial(\rho H)}{\partial t} + \nabla \cdot (\rho v H) = \nabla \cdot (k \nabla T) + S_h \quad (3.6)$$

where S_h is energy source, due to the coupling with VOF method, Geo-Reconstruct scheme chalks out the interface, moving wall is not necessary for melting or solidification in fluent. A small region can be adapted at boundary where all the boundary conditions can be patched; Hence, need of deforming wall is eliminated.

Momentum source depends upon the porosity/liquid fraction of the cell, when fluid solidifies β goes to zero and velocity of that cell goes to zero, whereas any cell with $\beta=1$ will have velocity that is equal to specified boundary condition value.

Mushy Zone parameter is indicative of the slope of the damping curve. Its value varies between $10^4 \sim 10^7$. A high value of parameter means high damping of velocity and hence, high rate of solidifying. On the other hand, too high value will tend to oscillate the solution as solidification will occur with even minor temperature perturbations. In this case, temperatures remain well outside the solidification/melting temperature, so it is of little consequence.

Solution for liquid fraction between two time steps n and $n+1$ is approached by following iterations.

$$\text{where } \beta_{(n+1)} = \beta_n + \alpha_\beta \Delta\beta \quad \Delta\beta = \text{Predicted liquid fraction change} \quad (3.7)$$

$$\alpha_\beta = \text{Specified constant(Liquid fraction update)}$$

- **Magneto hydrodynamics (MHD)**

It is an add-on module which is available in fluent. MHD has been described in Section 2. Modeling of these systems is possible in fluent using MHD toolbox. This module has two options magnetic induction and electric potential method. [20]

We use the Electric Potential method to model plasma arc, which is assumed to be in local thermodynamic equilibrium, i.e. while the properties of arc vary spatially and in time domain, but the rate of change is comparatively small and we can assume a thermodynamic equilibrium in the vicinity of the particular point in study. This means a plasma arc where temperature is varying with radiation loss can be said to be in local thermodynamic equilibrium.

In other words, emission at any point can be defined by Kirchhoff's law of radiation. It is very important if we want to take radiation into account. Joule heat is calculated by the electric potential method. So, the ionized plasma temperature is accurately calculated.

- **Electric Potential Method**

As we know in MHD Electric field has causal precedence for both current density (j) and magnetic field. To distinguish causal characteristics, causal diagrams for electromechanical and MHD systems are given respectively in Fig.3.12 [21].

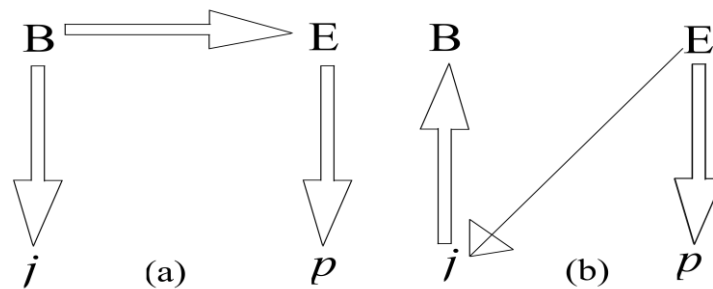


Fig. 3.12 Difference of causal precedence in electromechanical and magnetohydrodynamic systems

To calculate the current density we solve the potential equation using ohm's law

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \quad (3.8)$$

By definition of Ohm's Law: $j = \sigma(\vec{E} + \vec{u} \times \vec{B}) = \sigma(-\nabla\varphi + \vec{u} \times \vec{B})$

where $\vec{j} = \text{Current Density}$, $\varphi = \text{Scalar Potential}$, $\vec{A} = \text{Vector Potential}$,

$\vec{u} = \text{Velocity Vector}$, $\vec{B} = \text{Magnetic Field Vector}$

Inside an ideal conductor of electricity, $\nabla \cdot \vec{j} = 0$. The scalar electric potential equation is given by

$$\nabla^2\varphi = \nabla \cdot (\vec{u} \times \vec{B}) \quad (3.9)$$

Boundary condition is put in RHS of equation above. After this we have equation in single variable (scalar Potential). After calculating $\vec{j}$ we can use it to find Lorentz force and joule heat respectively by the formulae given as

$$\vec{F} = \vec{j} \times \vec{B} \quad \vec{F} = \text{Lorentz force} \quad (3.10)$$

Analytical expression for the above force is derived in Section 2.1.4. Joule Heat can be given by the following expression

$$Q = \frac{1}{\sigma} (\vec{j} \cdot \vec{j}) \quad Q = \text{Joule Heat (W/m}^3\text{)} \quad (3.11)$$

We will be using the Electric potential method to structure the DC arc and in the modeling of full GMAW setup.

3.2.2 Governing MHD equations for a DC arc

Governing Equations are written for the MHD system, which simulate the plasma arc. As we know ohm's law is written as $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$, we can write $\vec{E}$ in terms of scalar (φ) and vector potential ($\vec{A}$)

$$\Rightarrow \vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$$

As we know

$$\vec{B} = \nabla \times \vec{A} = \mu\vec{H}$$

$$\Rightarrow \nabla \cdot E = -\nabla^2 \phi - \frac{\partial(\nabla \cdot A)}{\partial t} \quad (3.12)$$

Second term in RHS of above equation is essentially zero as divergence of vector potential is zero. Rearranging ohm's law and substituting the value of E in Ohm's law.

$$\Rightarrow \nabla \cdot E = \nabla \cdot \left(\frac{j}{\sigma} - \vec{u} \times \vec{B} \right)$$

By current conservation divergence of j will be 0 and by putting that value in equation

$$\text{above } \Rightarrow \nabla \cdot E = -(\vec{u} \times \vec{B})$$

Using the identity

$$\begin{aligned} \nabla \cdot (\vec{u} \times \vec{B}) &= \vec{B} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{B}) \\ \Rightarrow \nabla^2 \phi &= \vec{B} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{B}) \end{aligned} \quad (3.13)$$

Taking curl of equation $B = \mu H$

$$\Rightarrow \nabla B = \mu \nabla H$$

We know $B = \nabla \times A$ and $\nabla \times H = j$

$$\begin{aligned} \Rightarrow \nabla \times (\nabla \times \vec{A}) &= \mu \vec{j} \\ \Rightarrow \nabla \times (\nabla \times \vec{A}) &= \nabla(\nabla \cdot A) - \nabla^2 A \quad \text{since } (\nabla \cdot A) = 0 \end{aligned}$$

$$\Rightarrow \nabla^2 A = -\mu \vec{j} \quad (3.14)$$

3.2.3 Governing equation in Cartesian Co-ordinates

- **Equation of Continuity**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (3.15)$$

- **Momentum Equation**

$$\frac{\partial(\rho \vec{u})}{\partial t} + \rho \nabla(\rho \vec{u}) = -\nabla p + \mu \nabla^2 \vec{u} + \vec{j} \times B \quad (3.16)$$

- **Energy Equation**

$$\partial(\rho h)/\partial t + v \cdot \nabla(\rho h) = \nabla \cdot (k \nabla T / C_p) + \vec{j}^2 / \sigma + (5K_b)/2e(\vec{j}) \cdot \nabla(\rho T) - R_a \quad (3.17)$$

where k , k_b , e , R_a are thermal conductivity, boltzmann's constant, electrode charge and radiation loss constant, R_a is assumed to be 0 in our case as we are primarily concerned about mode dynamics of metal transfer not the exact thermal configuration of the arc. Since our model is axis symmetric, we will write these equations in cylindrical co-ordinate. These equations have been derived in full in [3]. Fluent cannot emulate a 3D simulation in 2D without axis symmetric model, in general 2D format the depth of '1m' is automatically assumed hence the simple 2D simulation cannot be assumed representative of even one frame of revolution. This is because a simple 2D domain is assumed to be '1m' deep.

3.2.4 Governing Equations in cylindrical co-ordinates

- **Equation of Mass balance**

$$\frac{1}{r} \frac{\partial(\rho r u_r)}{\partial r} + \frac{\partial(\rho u_z)}{\partial z} = 0 \quad (3.18)$$

- **Conservation of Radial and Axial Momentum**

$$\begin{aligned} \frac{1}{r} \frac{\partial(\rho r u_r^2)}{\partial r} + \frac{\partial(\rho u_r u_z)}{\partial z} \\ = -\frac{\partial P}{\partial r} + \left[\frac{2}{r} \left(\frac{\partial(\mu r \frac{\partial u_r}{\partial r})}{\partial r} \right) - \mu \left(\frac{2u_r}{r^2} \right) + \frac{\partial(\mu(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}))}{\partial z} \right] \\ - \vec{J}_r \vec{B}_\theta \\ \frac{\partial(\rho r u_z^2)}{\partial r} + \frac{\partial(\rho u_z^2)}{\partial z} = \\ -\frac{\partial P}{\partial r} + \left[\frac{2}{r} \left(\frac{\partial(\mu r(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}))}{\partial r} \right) + \mu \left(\frac{2u_r}{r^2} \right) \right] + \vec{J}_r \vec{B}_\theta \end{aligned} \quad (3.19)$$

- **Conservation of thermal energy**

$$\begin{aligned} \left(\frac{1}{r} \right) \left(\frac{\partial(\rho r u_r h)}{\partial r} \right) + \frac{\partial(\rho u_z h)}{\partial z} \\ = \frac{1}{r} \frac{\partial \left(\frac{k r}{C_p} \frac{\partial h}{\partial r} \right)}{\partial r} + \frac{\partial \left(\frac{k}{C_p} \frac{\partial h}{\partial z} \right)}{\partial z} + \frac{\vec{J}_z^2 + \vec{J}_r^2}{\sigma} + \frac{5k_b}{2e} \left(\left(\frac{J_r}{C_p} \right) \frac{\partial h}{\partial r} + \left(\frac{J_z}{C_p} \right) \frac{\partial h}{\partial z} \right) - R_a \end{aligned}$$

(3.20)

where k , k_b , e , R_a are thermal conductivity, boltzmann's constant, electrone charge and Radiation loss constant R_a . Radiation is ignored in this model.

- **Conservation of charge**

$$\left(\frac{1}{r}\right) \frac{\partial \left(\sigma_e \frac{\partial \phi}{\partial r}\right)}{\partial r} + \frac{\partial \left(\sigma_e \frac{\partial \phi}{\partial z}\right)}{\partial z} = 0 \quad (3.21)$$

$$\vec{J} = \frac{I}{\pi r^2} \quad (3.22)$$

$$\nabla \cdot \vec{A} = 0 \quad (3.23)$$

Domain design

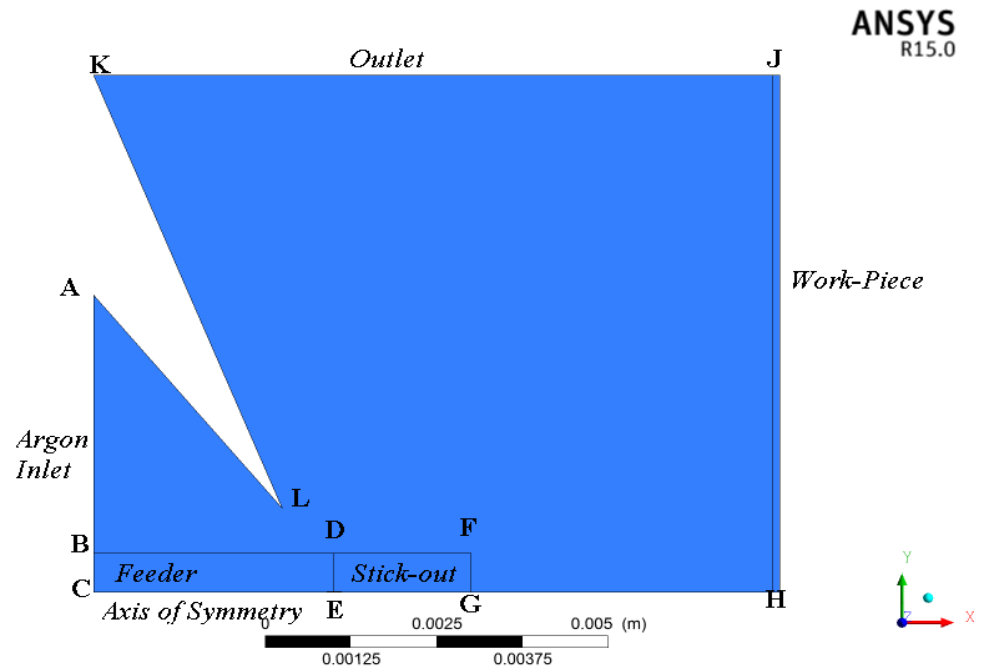


Fig. 3.13 GMAW Domain

Figure 3.13 above shows the domain of computation used in GMAW setup. GH is the arc length which is 5mm and electrode radius is 0.6 mm. Volume flow rate for the shielded

plasma arc is 12 L/min. AB is 5 mm and we compute the inlet velocity of argon. Complete domain is 10 by 10 mm.

- **Meshing**

A quadrilateral Dominant mesh with minimum face size of 0.5 microns was structured in ICEM CFD™. Mesh had minimum orthogonal quality of 0.81 and maximum aspect ratio of 2.81. Hence, the Quality of the mesh can be characterized as good.

3.2.5 Boundary Conditions

Boundary conditions in different domains of selected surfaces are given below according to the Fig. 4.1. Nomenclature according to which boundary conditions are described is described in the Table 3.2.

- **Boundary Nomenclature**

Table 3.2 Boundary Nomenclature

Edge	Boundary Condition	Name of the Selection
AB	Inlet, inlet type-velocity inlet	Velocity Inlet
BC	Feeder, solid	Feeder
FG	Electrode Base	Electrode Base
AL and KL	Walls	Wall
JK	Outlet, Outlet type-Outflow	Outlet
JH	Work-Piece	Work Piece
CH	Axis of Symmetry(Axis)	Axis

- **Inlet and Outlet Flow and Momentum conditions**

Table 3.3 Momentum boundary Conditions

Boundary	Specifications	Value
Inlet (5mm)	Velocity Inlet- Perpendicular to edge, Argon	3.1 m/s
Outlet	Pressure-Outlet	Default

- **Description of Flow and Momentum Boundary Conditions**

No slip Condition is used for the wall function where direction of the velocity vector is perpendicular to the inlet surface; hence an additional mass balance equation has to be followed i.e. $\frac{\partial(\rho u_z)}{\partial z} = 0$.

Wall Adhesion is provided and a surface tension coefficient of 0.006 is provided between argon and other phases/Walls. Consequently argon establishes inert gas shielded arc space shown in figure below.

This shape of shielded plasma arc enclosed by shielding gas shown below is established during every simulation of GMAW. It can be achieved in the way described above or a UDF can be written which defines the parabolic profile of velocity which enters the domain parallel to electrode wall.

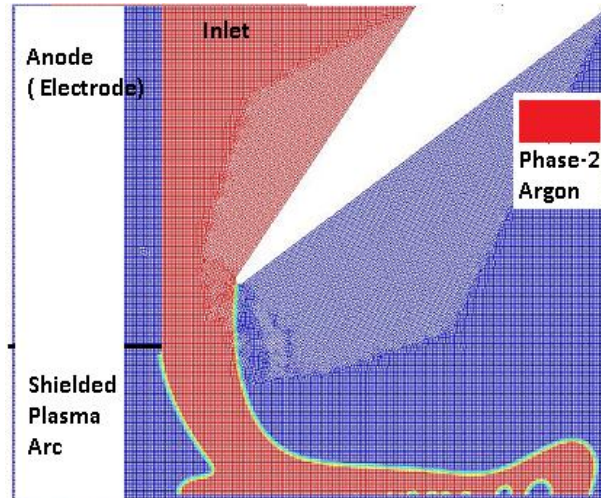


Fig. 3.14 Volume fraction of argon in a stable 3mm arc

- **Thermal Boundary Conditions**

Table 3.4 Thermal boundary Conditions

Named Selection	Boundary Condition
Electrode Base	Enthalpy of anode (Electrode) i.e. h_{metal}
Cylindrical Surface	h_{metal}
Edge BC	h_{metal}
Velocity Inlet	h_{argon} at constant temperature 300K
Outlet	Constant Temperature 300 K ($\frac{\partial h}{\partial r} = 0$)
Work-Piece	h_{metal}

- **Description of thermal boundary conditions**

Enthalpy inside and on the boundary is assigned a value of 753 J/Kg-K corresponding to reference temperature 298.15K. One of the advantages of using Solidification-Melting toolbox is the enthalpy-porosity formation. It calculates the change in the enthalpy with temperature. Inlet is fixed at 300K and in terms of enthalpy it will be argon's enthalpy at 300K. In the Work-Piece heat transfer is not modeled as penetration and solidification is not important for analysis of metal transfer, although the work-piece wall adjacent to domain is defined as conducting walls so that it does not affect the configuration of arc. Outflow as the inlet is also fixed at temperature of 300K.

- **Magneto Hydrodynamic boundary conditions**

Table 3.5 MHD boundary Conditions

Named Selection	Boundary Condition
Electrode Base	Scalar Potential- φ_c
	$\frac{\partial \vec{A}}{\partial n} = 0$
Velocity Inlet, Outlet Work-Piece	$\frac{\partial \varphi}{\partial z} = 0$ and $\frac{\partial \varphi}{\partial r} = 0$ respectively
	Zero vector flux $\frac{\partial \vec{A}}{\partial z} = 0$

- **Description of Magneto Hydrodynamic boundary conditions**

Edge BC of the electrode is considered as an iso-potential line, scalar potential is 0 in on the edge. After this equation of charge continuity is solved on the electrode. One assumption which has been emphasized by [3] is that the conductivity of the electrode is considered to be far much larger than that of the plasma. In that case the slope of the potential is quite steep in the areas of electrode and work-piece. This statement is validated in Figure 3.15 and 3.16. In which the work-piece in geometry is made longer, purposely to see the change of potential in Work-Piece.

In special reference to fluent, named selection base is set at constant scalar potential is set to '22 V' and according to the boundary condition $\nabla \cdot \vec{A} = 0$. Hence the shadow region of the wall can be declared as a perfect conducting surface, lastly the Cylindrical surface is patched a initial value calculated by the formula $\vec{J} = \frac{I}{\pi r_e^2}$. 'I' is the welding current supplied.

3.2.6 Results

- **Temperature contour Validation according to [12]**

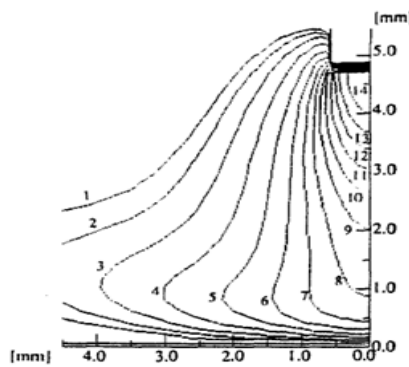


Fig. 3.15 Temperature contour of a DC arc [12]

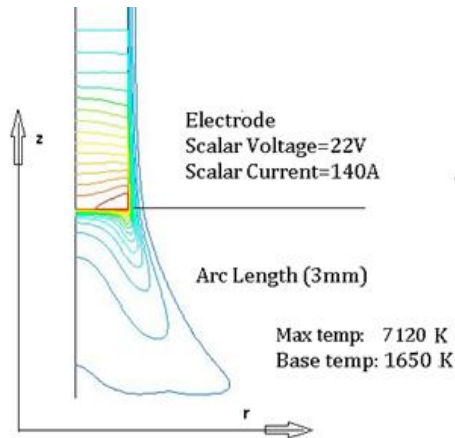


Fig. 3.16 Arc contours of 3mm arc

- Temperature contours of plasma arc in 23V 130A

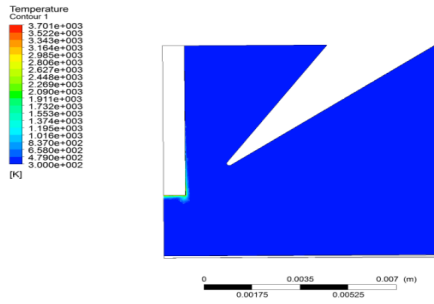


Fig. 3.17 Temperature contour at 0.013 ms

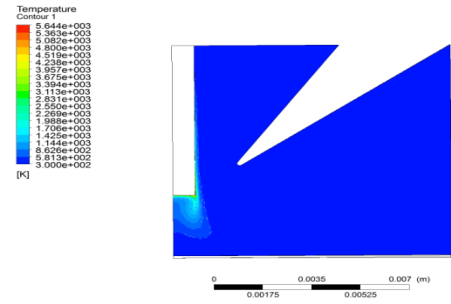


Fig. 3.18 Temperature contour at 0.765 ms

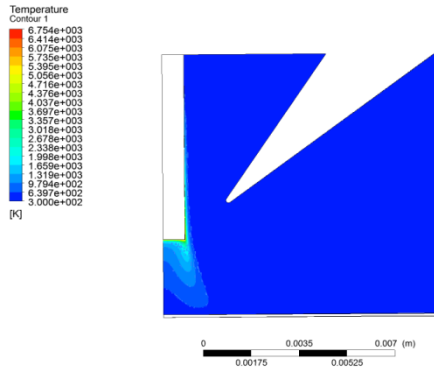


Fig. 3.19 Temperature contour at 1.30 ms

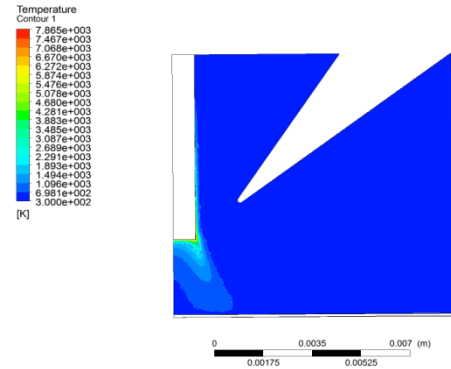


Fig. 3.20 Temperature contour at 2.20 ms

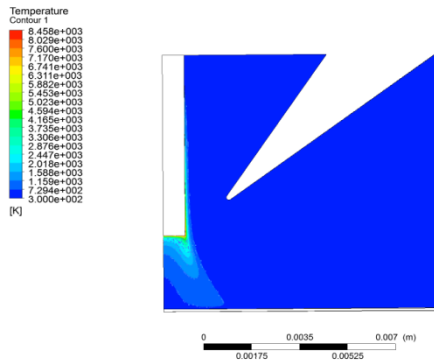


Fig. 3.21 Temperature contour at 2.56 ms

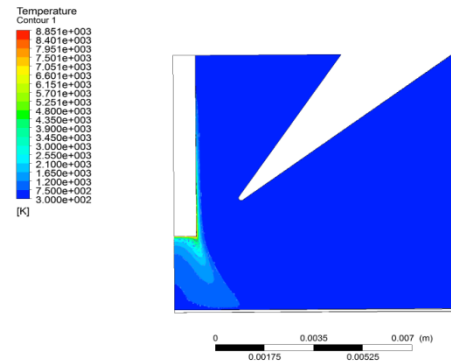


Fig. 3.22 Temperature contour at 3.51 ms

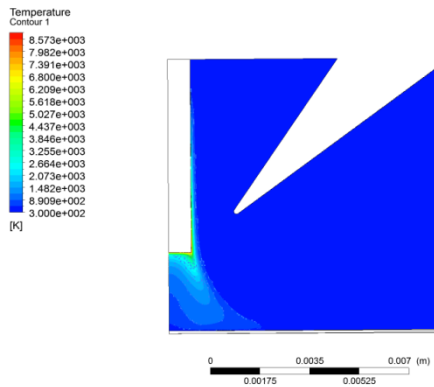


Fig. 3.23 Temperature contour at 4 ms

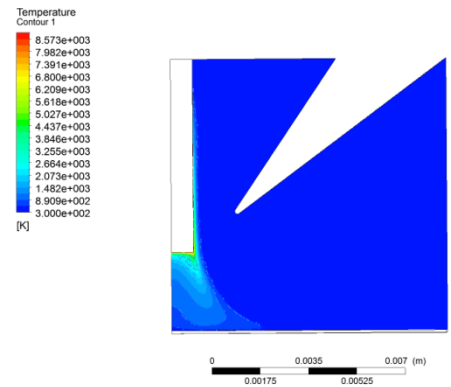


Fig. 3.24 Temperature contour at 4.5 ms

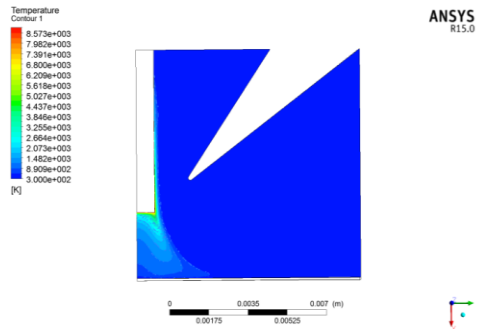


Fig. 3.25 Temperature contour at 5.04 ms

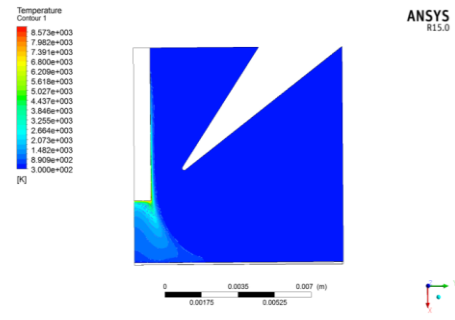


Fig. 3.26 Temperature contour at 5.35 ms

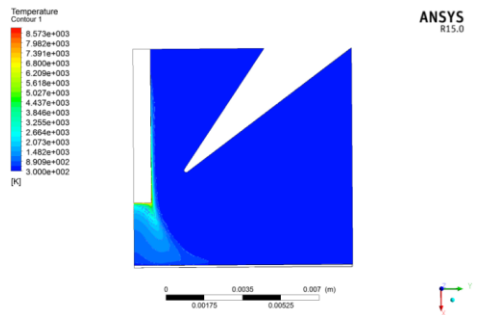


Fig. 3.27 Temperature contour at 6 ms

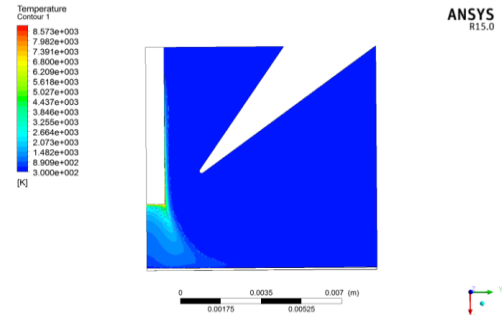


Fig. 3.28 Temperature contour at 6.5 ms

In the above mentioned results the temperature increases to 8871 K and then it settles for around 8600 K in steady state. The residual line of energy becomes steady after a short time if only the arc is stable and has reached its steady state. However if the same current is increased to 180 the maximum temperature reaches 12000K. A plot of spatial distribution of scalar potential is presented below:

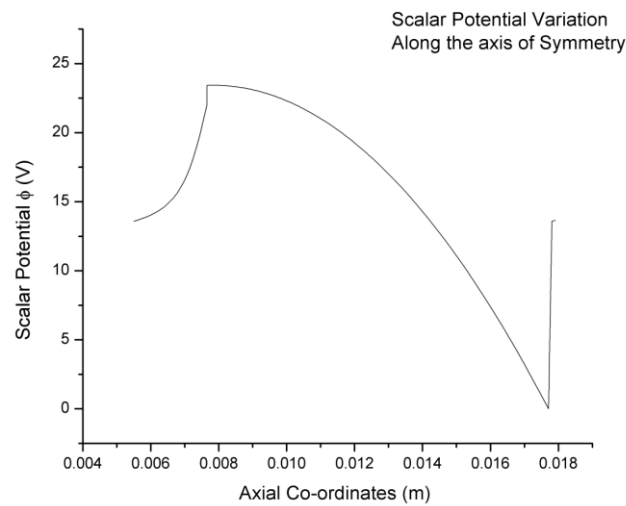


Fig. 3.29 Spatial variation of scalar Potential along Axis of Symmetry

This plot is in agreement of the boundary condition that is illustrated in the description of the magneto hydrodynamic boundary conditions. Here, we can see that the potential rises from 0 to 23V in the electrode and in very small spatial co-ordinate domain. Then as we discussed it drops slowly in the plasma as its resistivity and diffusivity is different. In the work piece the wall is at potential 0, then we see a sharp spike in the voltage which takes it value to around 10V.

- **Volume fraction of Molten Steel in 5mm GMAW arc**

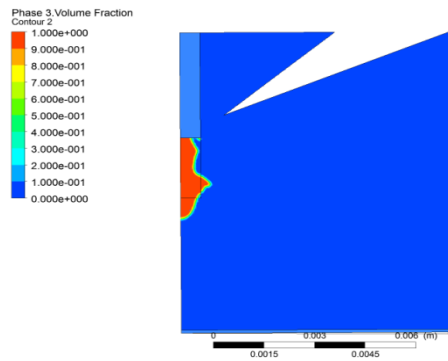


Fig. 3.30 Volume fraction at 176.08 ms

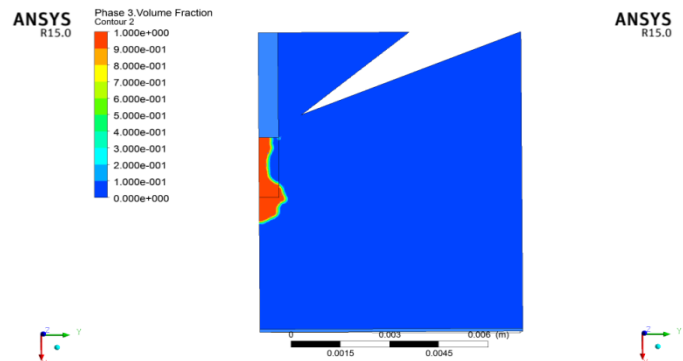


Fig. 3.31 Volume fraction at 198.4 ms

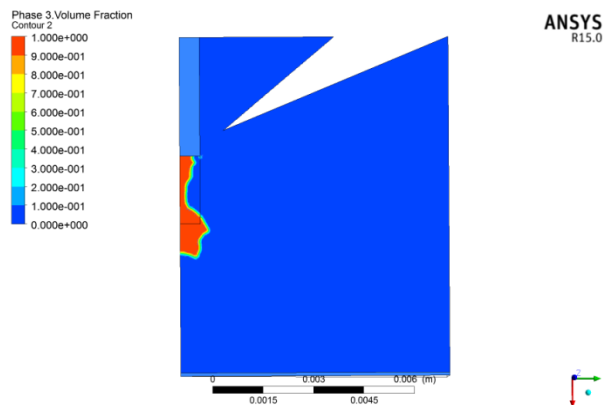


Fig. 3.32 Volume fraction at 201.08 ms

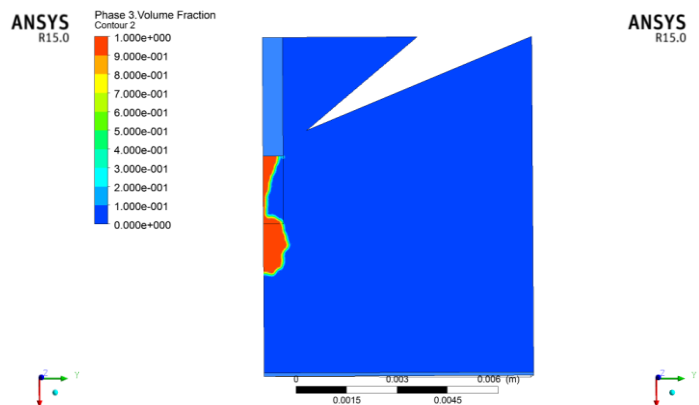


Fig. 3.33 Volume fraction at 201.08 ms

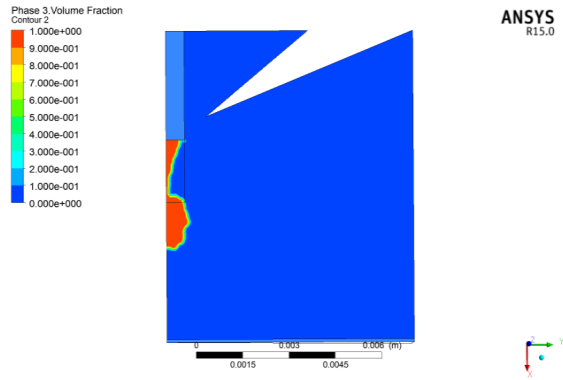


Fig. 3.34 Volume fraction at 202.58 ms

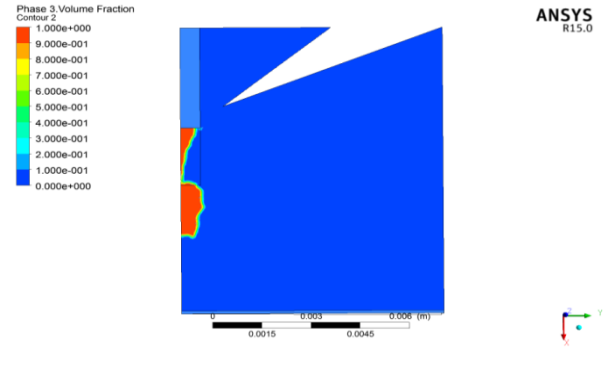


Fig. 3.35 Volume fraction at 204.78 ms

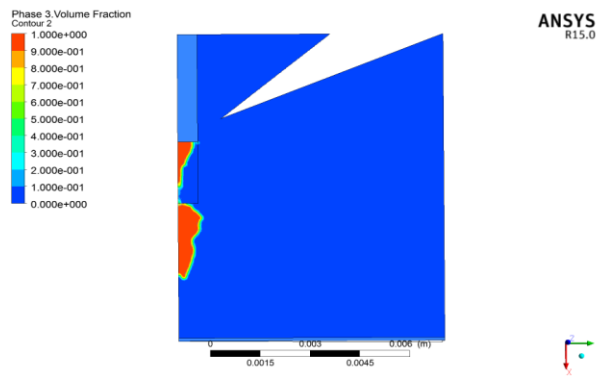


Fig. 3.36 Volume fraction at 205.08 ms

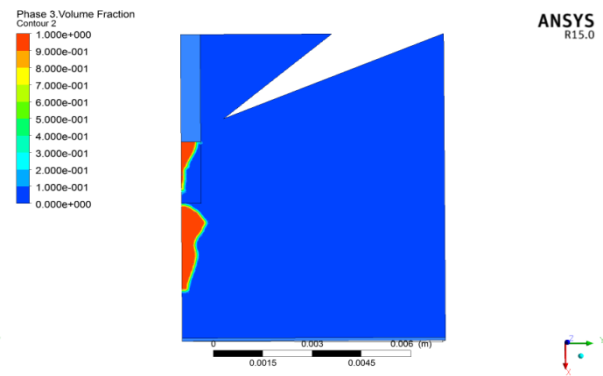


Fig. 3.37 Volume fraction at 206.48 ms

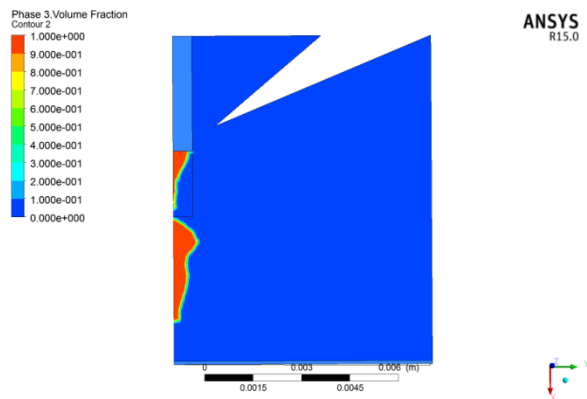


Fig. 3.38 Volume fraction at 207.08 ms

Geometry of this 5mm arc GMAW setup is dependent on the experimental pictures in Fig. 4.11. This indicates that the arc length is more than 3mm. When the length of arc is increased we see a more uniform melting and drop formation, although it still is not a perfect sphere. Globular transfer tends to a large amount of splatter. This can be illustrated by the Figure 3.39. Here we can see some satellite droplets still attached to the stick-out volume. When these droplets fall since their size is small any force that is not perpendicular to the

electrode base will take them away from the normal impact point. Hence the splatter can be justified by the non-perpendicular nature of Lorentz force at some instant; this phenomenon can be induced by applying magnetic field.

A UDF was not created to measure the volume of the detached drop, but after approximation we can say that radius of output droplet is around 1.3 mm, whereas the experimental drop was 1.48mm in radius. The uniform transfer of metal into a drop and its growth is characteristic of globular transfer mode. The setups described in the temperature contour have the arc length as 3mm, in that case droplet detaches and it does not look like a sphere and the instability frequency is very high. That GMAW setup can be defined as one in the transition zone.

4.1 Modeling of capillary jet instability

The capillary jet instability is modeled on the basis of Rayleigh work principle [1]. Our objective is to formulate the disturbing wave and its growth in the cylindrical column and we will study both radial and axial parameters.

Basic approach here is to manipulate the governing equations into a particular form described in [16], where any term of Navier Stokes equation can be defined as a product of hydraulic effort variable i.e. pressure and flow variable (volume flow rate). After establishing the corresponding momentum and continuity equation in hydraulic power circuit form, we proceed to derive the inertance and resistances from the Navier Stokes equation. Last step after evaluating the parameters is to model the growth of the initial disturbance and analyze different parameters which might affect the drop detachment. Capillary jet problem is a very complex problem which still is not understood completely due to development of singularities at detachment and hence, the time of detachment and shape after detachment cannot be deterministically predicted; so, a sometimes when disturbing signal is stochastic, a probability density function in a certain domain of time is defined to implicate the probable times of detachment.

4.1.1 Assumptions

- Axial displacement is found out by interpolating the time required to reach certain points and are given in Table 4.1. This data set produces a parabola which can be approximated by $z = 2t^2$.

Table 4.1 Data points from simulation

S. No.	Displacement (m)	Time (s)
1	0.0003	0.00296
2	0.0005	0.00400
3	0.0007	0.00528
4	0.0009	0.00600
5	0.0012	0.00732
6	0.0015	0.01024

- Fluid flow domain is in-viscid, any term with viscosity vanishes.

- Fluid flow is laminar, so that an average streamline can be used to define the general behavior of the flow. Inability to model turbulence and presence of singularities prevent the modeling of internal drop dynamics.
- Long wavelength approximation is used because the wavelength is usually quite large with respect to initial jet radius, denoted as ‘a’.

4.1.2 Governing equations and their conversion to hydraulic power circuit

- **Momentum equations**

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) = - \frac{\partial P}{\partial r}$$

$$\Rightarrow \frac{Du_r}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial r} \quad (4.1)$$

Similarly,

$$\rho \left(\frac{\partial (ru_z)}{\partial t} + u_z \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) = - \frac{\partial P}{\partial z}$$

$$\Rightarrow \frac{Du_z}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial z} \quad (4.2)$$

- **Equation of continuity**

$$\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} = - \frac{u_r}{r} \quad (4.3)$$

Equation 4.1 is integrated for the volume on convective derivative and surface integral on radial pressure distribution; both sides are multiplied with u_r in order to manipulate it to hydraulic power circuit equation.

$$\iiint_0^r u_r \left(\frac{Du_r}{Dt} \right) dv = - \frac{1}{\rho} \oint u_r \left(\frac{\partial P}{\partial r} \right) dS$$

where $\dot{Q} = \iint \vec{u} \cdot d\vec{S}$ holds for both radial and axial component. $\frac{\partial P}{\partial r}$ is assumed to be $P_2 - P_1$ according to Fig. 4.1 and assumption 4.

$$\dot{Q} \int_0^l \rho \left\{ \frac{1}{A(\zeta)} \ddot{Q} + \frac{\dot{Q}^2}{2} \frac{\partial((1/A(\zeta))^2)}{\partial \zeta} \right\} d\zeta = P_2 \dot{Q} - P_1 \dot{Q} \quad (4.4)$$

where P_1 is the radial pressure applied by surface tension, P_2 being the pressure of the output of the hydraulic circuit. Equation 4.4 can also be written as

$$P_2 \dot{Q} - P_1 \dot{Q} = \left\{ \left(\int_0^l \rho \left(\frac{1}{A(\zeta)} \right) d\zeta \right) \ddot{Q} \right\} \dot{Q} + \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}^2 \right\} \dot{Q} \quad (4.5)$$

Equation 4.5 defines the hydraulic circuit in a very explicit way. We can deduct from it, the value of inertance and resistance. Resistance discussed is modulated resistance, discussed as a Bernoulli resistance in section 1.2.3.

4.1.3 Growth of the disturbance

When the disturbance grows in vicinity of other fluid, it tries to vary the growth of disturbance; usually it gives the expression of the growth of the disturbance by the equation $\epsilon = \epsilon_0 e^{-i\omega t}$ where ϵ_0 denotes the amplitude of initial disturbance and ω is the rate of propagation. Since, our case is in-viscid, imaginary terms are associated with viscous slowing. Hence, while using the long wavelength approximation, we expand $e^{-i\omega t}$ and ignore the imaginary part.

In other approach, we can try to find ω with complete in-viscid boundary conditions. It can be seen in the approach stated above that $-i$ is factor of the viscous growth, hence we assume that the in-viscid growth rate as $e^{\omega t}$, where ω is computed by the latter approach [8].

$$\omega = \frac{\gamma(1-(ka^2))^{\frac{1}{2}}}{\rho_w a^3 \ln\left(1+\frac{1}{ka}\right)} \quad (4.6)$$

After all these considerations, we take amplitude of the disturbance equal to initial radius 'a'. Radius at any instant can be defined as $r = a(1 - e^{\omega t} \sin(mz))$ where 'm' is the wave number of the disturbance, moreover disturbance is taken as sinusoidal of product of axial co-ordinate 'z' and wave number 'm'.

4.1.4 Calculation of inertance and resistance expressions

From Eq. 4.2 and 4.4, we can state the inertance I as

$$I = \int_0^l \frac{1}{A(\zeta)} d\zeta \quad \text{Where } \zeta \text{ is the assumed to be representative streamline of the flow.}$$

- **Radial inertance**

$$I_r = \rho \int_0^l \left(\frac{1}{2\pi r d\zeta} \right) d\zeta \quad (4.7)$$

It is evident from Eq. 4.7 that radial inertance does not depend on generalized streamline co-ordinate ζ . Hence, for any amount of axial length radial inertance is given as $I_r = \frac{\rho}{2\pi r}$.

- **Axial inertance**

I) Axial inertance

$$I = \left(\frac{\rho}{\pi r^2} \right) \int_0^l d\zeta \quad (4.8)$$

Since, we are integrating over a relatively small length of jet per unit time step, we assume cross sectional area can be assumed constant over a time step. Integral of generalized stream function ζ is the z co-ordinate. This gives us the value for axial inertance.

$$I_z = \frac{\rho z}{\pi r^2} \quad (4.9)$$

II) Axial and radial resistance

Both resistances are Bernoulli's resistance where the pressure drop is related to change in cross section.

$$R_r = \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_r^2 \right\} \quad (4.10)$$

$$R_z = \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_z^2 \right\} \quad (4.11)$$

4.1.5 Explanation of the bond graph model

Se1 is indicative of the radial pressure on jet because of surface tension

$$\Delta P = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \gamma k_c \quad (4.12)$$

ΔP is the pressure difference between the interface adjacent convex neck surfaces. R_1 and R_2 Represent axial and radial curvatures, respectively. Curvature is adapted from [5], where pinch force is $\gamma(k_c)$, k_c being the curvature.

$$k_c = 1/(f(z)(1 + \dot{f}(z)^2))^{\frac{1}{2}} - \ddot{f}(z)/(1 + \dot{f}(z)^2)^{\frac{3}{2}} \quad (4.13)$$

Effort in axial direction is because of the gravity which can be given as $\rho g z$. Regression of the data in Table 4.1 produces a fairly accurate polynomial of 2nd degree. Hence, we can approximate z with the expression $\frac{1}{2}at^2$.

Bond graph is made of the neck part of the falling drop, as we know the singularities present in the governing equations limit us from knowing the exact detachment time and size/shape of the drop. Hence, to avoid those singularities output is taken in a flow activated C element. In that way we are only concerned with amount of flow entering the drop. We can easily find radius of a corresponding spherical drop. Bond graph is shown in Fig. 4.2.

Transformer between bond 9 and 11 has the modulus of u_1 . This is quite significant in the bond graph as we can visualize that radial flow perpendicular to the curved neck can be separated into two components. Radial component decreases the neck radius and builds up radial pressure which can counter P_{atm} . However, the axial component of radial flow can be

summed up with axial flow. This is made possible by assuming the curvature of the neck and thus finding out the value of angle at which surface tension is acting with respect to horizontal axis. Figure 4.1 describes the theory and is helpful to find out the angle.

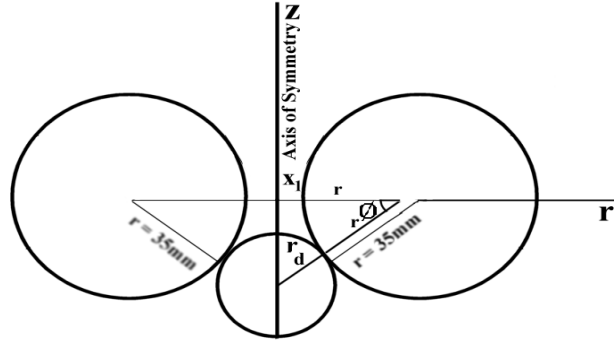


Fig. 4.1 Angle approximation from curvature

We have assumed that if the curvature of neck surface is drawn into a full circle, the radius will be 35 mm. On the basis of that we can conclude that

$$\cos \phi = \frac{r+x}{r+r_1} \quad (4.14)$$

where $r_1 = \delta a$ and the value of x is assumed to be 0.5mm, for the specific case of initial radius of 0.6 mm. If the value of δ is taken as 2.5, $\phi = 0.002$ rad, $\sin \phi = 2 \times 10^{-3} = u_1$. Similarly, we can see that complete axial component is going into the output and hence, $u_2 = 1$.

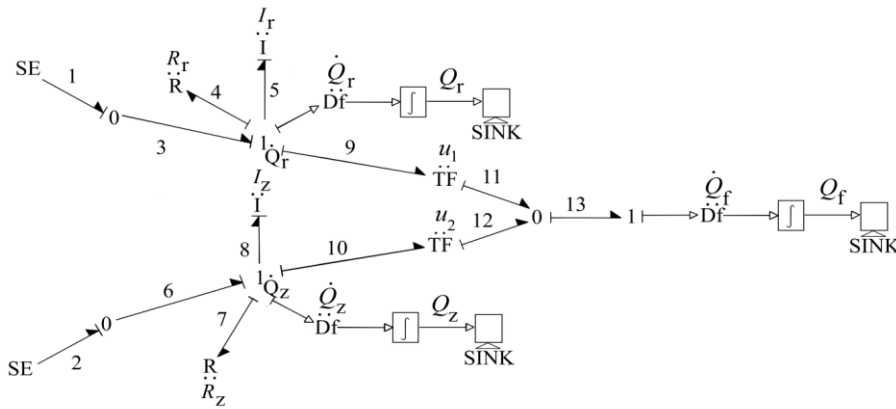


Fig. 4.2 graph of capillary jet instability

4.1.6 Expressions and parameters

The expressions used in bond graph model are given in Table 4.2.

Table 4.2 Expressions used in Symbols 200™

Sl. No.	Symbol	Name of property	Expression
1	Se1	Radial effort	$\Delta P = \gamma(k_c)$
2	Se2	Axial effort	$SE2 = \rho g z$
3	I_r	Radial inertance	$\frac{\rho}{2\pi r}$
4	I_z	Axial Inertance	$I_z = \frac{\rho z}{\pi r^2}$
5	R_r	Radial Resistance	$R_r = \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_r^2 \right\}$
6	R_z	Axial Resistance	$R_z = \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_z^2 \right\}$
7	ω	In viscid growth rate	$\omega = \frac{\gamma(1 - (ka^2))^{\frac{1}{2}}}{\rho_w a^3 \ln \left(1 + \frac{1}{ka} \right)}$
8	r	Radius	$r = a(1 - e^{\omega t} \sin(mz))$
9	k_c	Curvature	$k_c = 1/(f(z)(1 + \dot{f}(z)^2))^{\frac{1}{2}} - \ddot{f}(z)/(1 + \dot{f}(z)^2)^{\frac{3}{2}}$
10	z	Axial co-ordinate	$\frac{1}{2}gt^2$

In Table 4.1 disturbance is taken as a sin wave with amplitude equal to initial radius ‘a’. Disturbance is a function of both wave number and axial co-ordinate. However, the axial co-ordinates are approximated by regression analysis of the data obtained by simulation. Analytical expressions for axial co-ordinates are available but they are derived by non-dimensional analysis. Same thing in dimensional analysis is very difficult to infer. Simulation parameters are given in Table 4.3.

Table 4.3 Simulation parameters

Parameter	Values	Units
a	0.0006	m
g	4	$\frac{m}{s^2}$
m	0.69	-
γ	0.0728	N/m
ρ	1000	kg/m ³

4.1.7 Results

- **Radial and axial volume flow rate**

Radial volume flow rate increases with time, then a steep drop at the time low radius. Although by the time curve goes below 0, the radius is already 0. However axial flow rate is more like a sinusoid. It increases at first and then decreases with same curvature. Axial flow rate in fluent of the same experiment is comparable to the bond graph results.

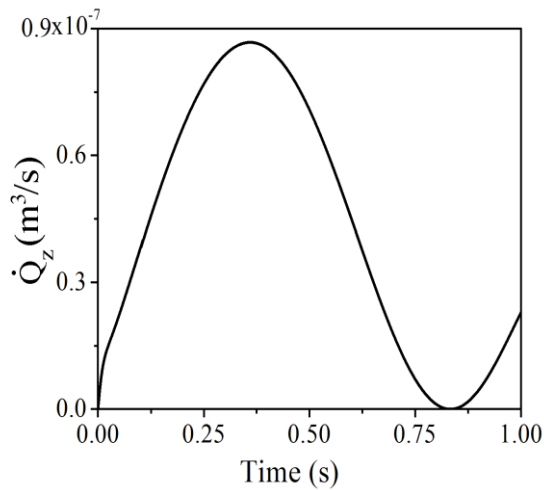


Fig. 4.3 Radial volume flow rate

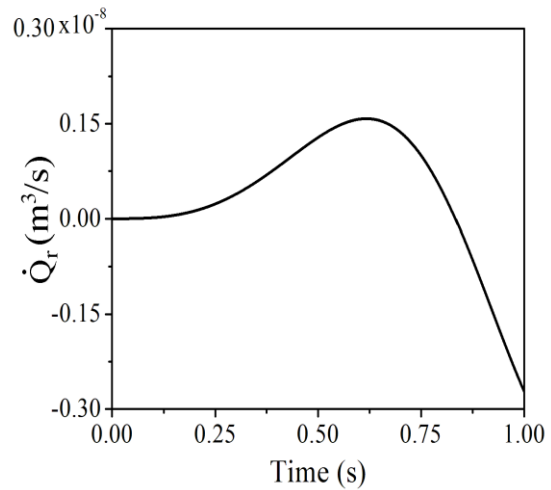


Fig. 4.4 Axial volume flow rate

- **Radius**

Radius and its variation depends on two variables i.e. wave number and axial coordinate z . The following graph plots the radius variation due to different wave numbers. We have found out previously in linear stability analysis that wave number 0.69 grows most rapidly and usually causes the detachment. However in low wave numbers like 0.2, 0.3 the variation of radius is very sluggish; hence its use in maintaining cylindrical columns for printing in DoD printers is widespread.

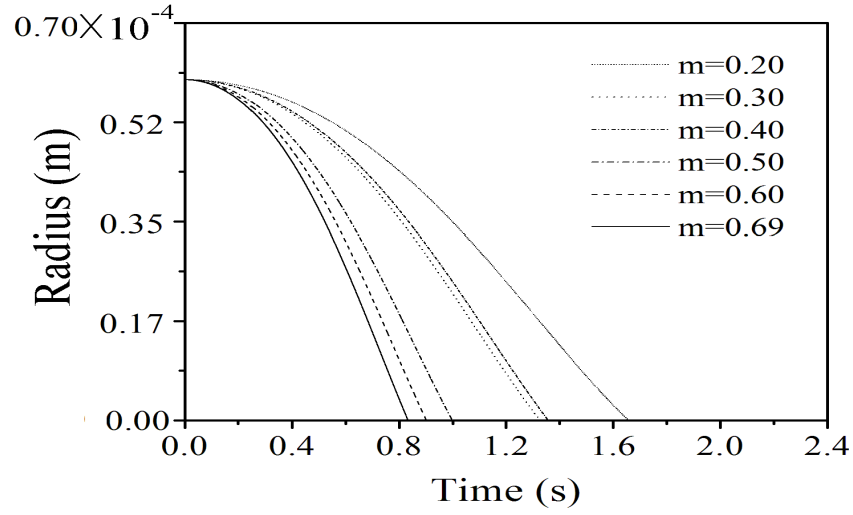


Fig. 4.5 Plot of radius for various wave numbers

- **Final drop Volume**

An integrator at the end of the combined flow gives us the volume of the drop, although it is not correct to consider the drop spherical. If it is spherical drop, the radius can always be found out. This application of capillary jet instability does not consider the formation of satellite droplets. Hence the graph below is the graph of output volume or Q_f .

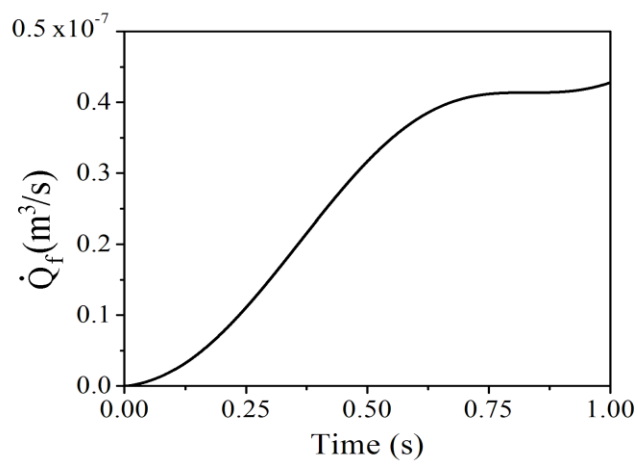


Fig. 4.6 Final volume plot (Q_f)

4.2 Modeling of GMAW process

Eager in [10] discusses both approaches to the analysis of metal transfer. In static force balance model the predicted drop size is larger than the experimental size. Although the pinch instability theory (PIT) can predict the droplet size correctly but frequency of the disturbance is very hard to find out. Windowed FFT of the spatial potential variation is required to find the frequency of disturbance; moreover arguments for deterministic nature of droplet size are discussed in [6]. Correct prediction of droplet size and detachment time can at best be expressed as a probability density function. So, the question on the deterministic nature of the GMAW metal transfer is legit.

Bond graph of SFBM model involves numerous user inputs, for instance rate of the change of cross section and the simultaneous position of the point monitored, has to be provided by user. Literature is not conducive to find out rate of change of cross section and axial position in SFBM.

On the other hand pinch instability theory (PIT) has its own problems as the exact frequency of disturbance is always difficult to figure out. This bond graph has been created by extending the contents of section 4.1 and modifying it to the process of GMAW pinch instability theory.

4.2.1 Assumptions

- Heat transfer between arc and the electrode is not considered; hence we can essentially assume the electrode as a cylindrical liquid column of certain radius.
- Flow is still considered in viscid and most importantly laminar, so that the average streamline assumption remains valid.
- The radius of neck curvature is assumed to be 35mm as calculated in Section 4.1. Since pinch instability theory is used, Lorentz force is applied as derived in Eq. 2.11. An auxiliary plot is included to see how much SFBM is in agreement with result of pinch instability method with variation of axial pressure.
- Source of effort driving the flow in axial direction, is derived from the Lorentz, drag and gravity forces acting on the electrode as mentioned in Section 2.3.

4.2.2 Manipulation of capillary jet instability bond graph

The radial effort is modified according to the Eq. 2.12 discussed in [5]. Radial effort thus; can be defined by the equation

$$P = \frac{\mu_0 I^2}{2\pi^2 \delta^2 a^2} + \frac{4\gamma}{\delta a}. \quad (4.15)$$

Average pressure is derived in [6]. Basically it can be said that axial pressure is resultant of the sum of surface tension, drag, gravity and Lorentz force. Lorentz force is calculated in the paper cited above as

$$F_{em} = \left(\frac{\mu_0 I_d^2}{4\pi} \right) \left[\frac{1}{2} + \frac{1}{(1-\cos \theta)} + \frac{2}{(1-\cos \theta)^2} \times \ln \frac{(1+\cos \theta)}{2} \right] \quad (4.16)$$

$$F_\gamma = 2\pi\gamma(r)$$

$$F_d = 6\pi r u_z$$

$$F_g = \rho Q_f g$$

Resultant force is $F_T = F_\gamma + F_d - F_{em} - \rho Q_f g$ here we can see that force (F_T) per unit area is the input SE in z direction.

Although we are solving the problem with pinch instability theory, still we evaluate the variation of the axial pressure to see if the detachment criterion of SFBM is met at the same time when pinch instability detaches the droplet.

Radial component which contribute to the complete flow is derived as in section 4.1 with help of Fig. 4.1, with 35 mm neck curvature assumption.

4.2.3 Bond graph model and its expressions and parameters

Same bond graph is used as in Section 4.1. Only the expressions (Table 4.4) would be different.

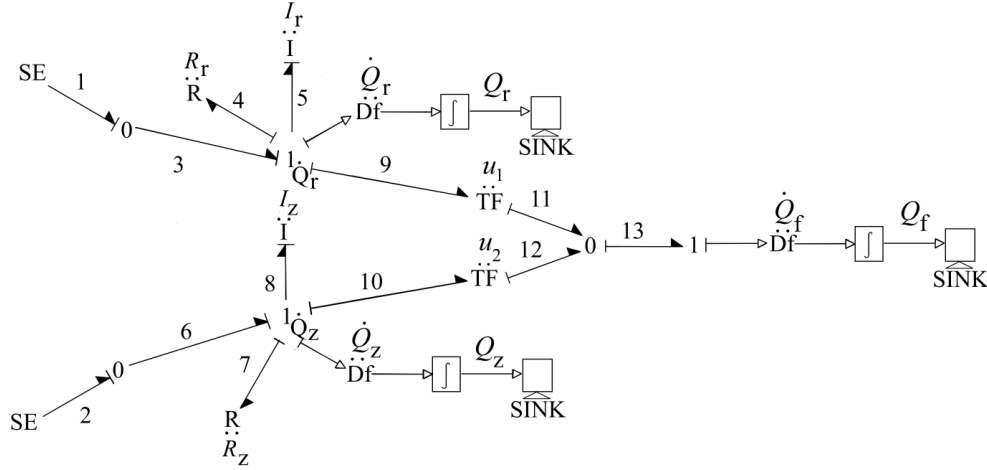


Fig. 4.7 Bond graph model for GMAW metal transfer

Fig. 4.1

Table 4.4 Modified expressions for GMAW

S. No.	Symbol	Name of property	Expression
1	SE1	Radial effort	$P = \frac{\mu_0 I^2}{2\pi^2 \delta^2 a^2} + \frac{4\gamma}{\delta a}$
2	SE2	Axial effort	$F_\gamma + F_d - F_{em} - \rho Q_f g$
3	I_r	Radial inertance	$\frac{\rho}{2\pi r}$
4	I_z	Axial Inertance	$I_z = \frac{\rho z}{\pi r^2}$
5	R_r	Radial Resistance	$R_r = \left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_r^2 \right\}$
6	R_z	Axial Resistance	$\left\{ \left(\frac{\rho}{2} \right) \left(\frac{1}{A(l)^2} - \frac{1}{A(0)^2} \right) \dot{Q}_z^2 \right\}$
7	ω	In viscid growth rate	$\omega = \frac{\gamma(1 - (ka^2))^{\frac{1}{2}}}{\rho_w a^3 \ln \left(1 + \frac{1}{ka} \right)}$
8	r	Radius	$r = a(1 - e^{\omega t} \sin(\omega z))$
9	k_c	Curvature	$k_c = 1/(f(z)(1 + \dot{f}(z)^2))^{\frac{1}{2}} - \ddot{f}(z)/(1 + \dot{f}(z)^2)^{\frac{3}{2}}$

10	z	Axial co-ordinate	$\frac{1}{2}gt^2$
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We see here expressions have changed only for radial and axial pressure, this is partly because some of the expressions, for instance curvature of the neck is independent of the material or process parameters, however others depend on some material property such as density. Hence their value must change.

For the value of expression 10, a simple interpolation is done, although the shape of the graph was approximately the same as described by the data in Table 4.1. But value for g is approximately $\frac{(10)m}{s^2}$.

- **Simulation Parameters**

Table 4.5 GMAW simulation parameters

S. No.	Name	Value
1	a	0.0006 m
2	G	$10 \frac{m}{s^2}$
3	M	20
4	γ	1.6 n/m
5	P	$7200 Kg/m^3$
6	I	130 A
7	Δ	2.5
8	H	0.001
9	μ_0	15.56×10^{-7}

4.2.4 Results

- **Radial and Axial Flow**

Radial and axial flows have similar behavior as in Section 4.1. Only significant difference is that z axis is taken in opposite direction as compared to Section 4.1 as exact SFBM detaching force is also monitored and denoted as F_T .

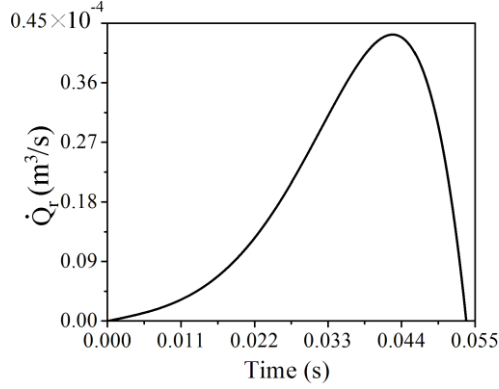


Fig. 4.8 Radial flow in GMAW electrode

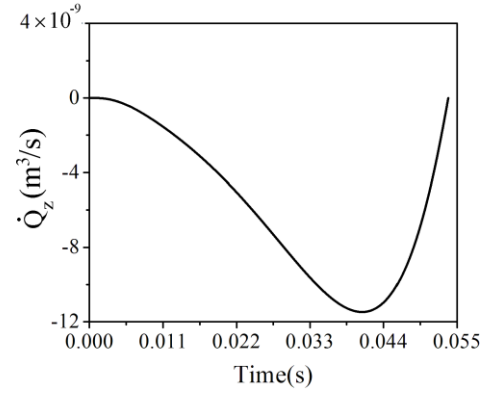


Fig. 4.9 Axial flow in GMAW electrode

4.2.5 Radius

Since we have modeled it according to pinch instability theory, only thing affecting the radius size can be the wave number of the disturbance. The way to compute the frequency of a given system is to see after how much time the singularity is encountered in the simulation. That time indicates the detachment of droplet.

This approach gives us a two pronged effect, on one hand we see the graph as it goes singular simultaneously we observe the F_r graph. If they vary at exactly the same time, it should be safe to say that that the simulation is correct. In the Figure 4.8, plot of the radius is given for two wave numbers 20 and 50, whereas in Figure 4.9 we show the detaching force.

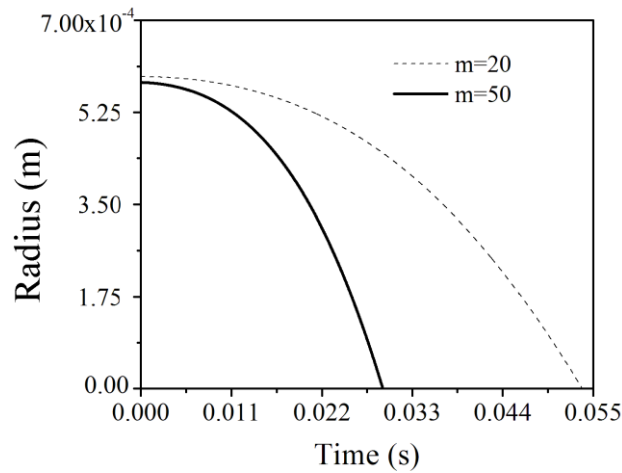


Fig. 4.10 Radius for different wave numbers

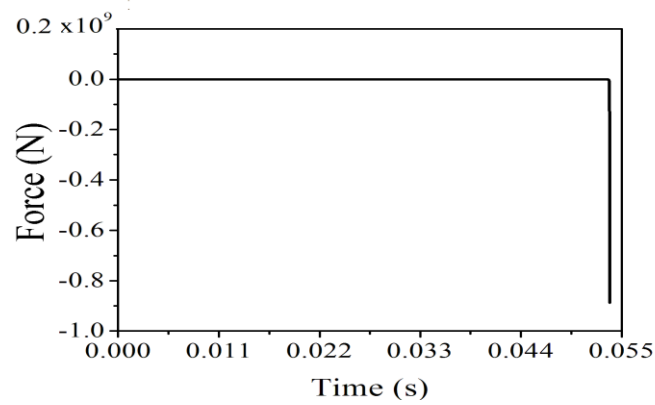


Fig. 4.11 Detaching force corresponding to $m=50$

4.2.6 Model output

Assumption of the drop to be spherical drop can be an acceptable process in capillary jet instability and globular transfer; although in GMAW it can never be done without knowing the mode of metal transfer. Output droplet can be non-spherical in other modes than globular. Output volume is plotted against time in the graph given in Fig. 4.10.

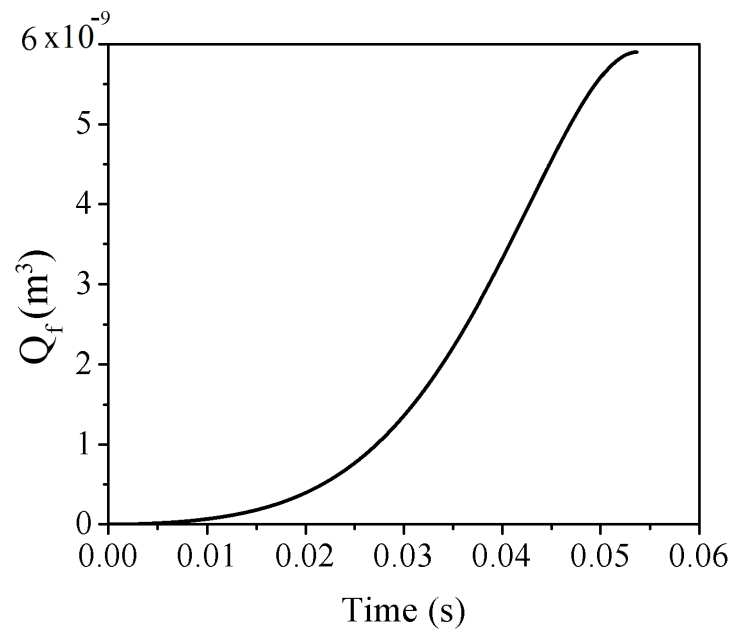


Fig. 4.12 Volume vs. time in GMAW model

4.3 Comparison of experimental results and simulation (Bond graph and FLUENT)

In Fig. 4.11, the experimental picture is shown. The high speed photography was performed on GMAW process. The process parameters were 0.6mm radius electrode, 23 V scalar

Potential and 130 A welding current. With the help of image analysis (through Matlab) of the picture shown in Fig. 4.11, the radius of the drop was found to be 1.38 mm. The modeling of the actual GMAW process is done through Fluent. The detaching globule has a radius of approximate 1.26 mm. It is an approximate value as UDF to calculate detached volume was not compiled in fluent for this purpose. In Fig 4.13, it is observed that the radius is approximately 1.21 mm if the output volume is assumed as spherical. Despite the assumptions, the model output is sufficiently close to experimental output.

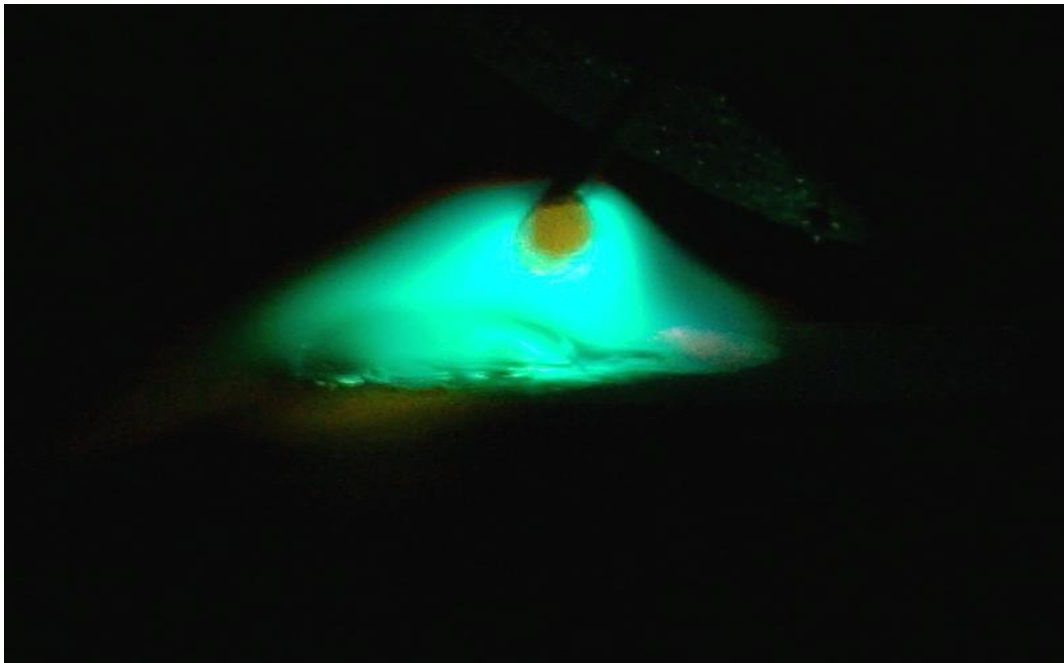
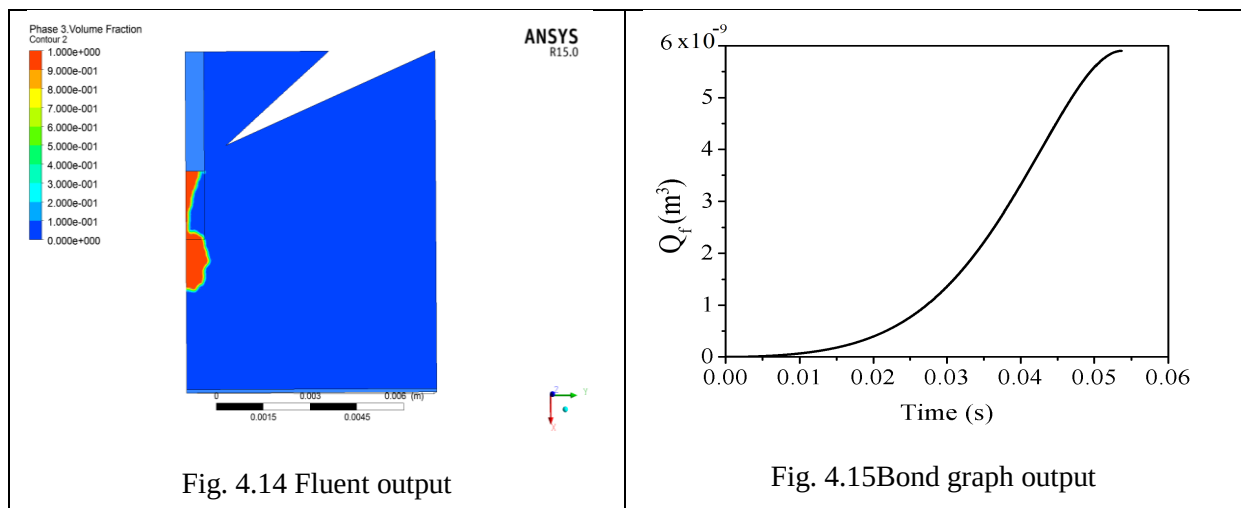


Fig. 4.13 Experimental Results



5.1 Conclusions

5.1.1 Capillary jet instability problem

- The in-viscid simulation in the section 3.1.3, we get the final volume of the droplet approximately as $3.5 \times 10^{-8} \text{ m}^3$. Volume of drop computed via simulation isn't ideal as we can see that because of wall adhesion water has stuck out of the orifice, although in the process we see different wave patterns of disturbances; one wave has amplitude reduction and other has amplitude growth at the same time. These both seem to superpose. Once the instability reduces the radius of the neck to a certain value after that, the droplet detaches rather quickly.
- In the axial direction, motion is very complex, the oscillations in z direction observed in the simulation of capillary jet instability were very random and hence, the approximation of the flow velocity by regression analysis of first few points may be only valid for the cases where oscillations are evened out intentionally. Although, when looking from distance, oscillations are less apparent and so, the flow appears to accelerate at first and then lower its velocity at the time of the break-up.
- Amplitude and wave number of disturbance is something that will be an essence of this simulation. In this case, the bond graph is too simple, but is made for single long wavelength disturbance. We can see that the radius approaching zero can be varied by varying the wave number of disturbances.

5.1.2 GMAW model

- The static force balance method theory and pinch instability theory both predict the droplet detachment breakup at same time.
- Similar to the capillary jet instability model, the frequency of droplet formation is dependent on the wave number of the instability. Theoretically the result depends upon the nature of signal; stochastic result is given for stochastic disturbances and deterministic result is given for fully determined disturbances.

5.2 Scope for future work

- C coding can be written in Symbols 200 handlers which detect the wave numbers of the input noise via windowed Fourier transform and then superpose the radii having different wave numbers.
- CFD analysis can be performed for the capillary jet instability problem with help from IC fields according to the theory in [18].
- Comparing simulation results of radial and axial flow with experimental value can be done.
- Singularity modeling can be done to eliminate the singularities in bond graph model of GMAW process.
- If we can model heat transfer domain, magnetohydrodynamic and hydraulic domain in a single model, the system will be easily observable and controllable.

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