Review of Application of Optimization Technique in Electrical Engineering

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Submitted By

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Dedicated to God, My Parents and My Supervisor

Declaration

I hereby certify that the work, which is being presented in the thesis, entitled **"Review of Application of Optimization Technique in Electrical Engineering"** in partial fulfillment of the requirements for the award of the degree of **Masters of Science** in **Mathematics and Computing** and submitted to the School of Mathematics, Thapar Institute of Engineering and Technology, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Sanjeev Kumar,** Assistance Professor and **Dr. S.K. Aggarwal,** Assistance Professor and other research work which is duly listed in the reference section.

The matter presented in this thesis has not been submitted elsewhere for the award of any other degree or diploma from any institution.

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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Shabnam Tayal

Abstract

Power Generation scheduling involves the allocation of the load selection of units to be placed in operation and between these operating units. In this integer programming is used. Formulation of economic dispatch problem as an integer programming is taken into account the discontinuous input-output characteristics and start-up cost of generators. For solving this new method i.e., gomory-cut method used for integer programming. By mathematical optimization problem,so many optimizing problem have been solved. Such problem as the economic dispatch, in many of the facts; allocation pollution dispatch ; maximum interchange ;hydrothermal unit commitment and dispatch ; generation transmission and distribution expansion planning; maintenance scheduling and substation switching, var scheduling have been formulated and solved. Modern mathematical optimization techniques, such as linear, nonlinear, quadratic and dynamic programming and their many combinations and extension have been exploited. Now, some of the formulation and solutions to these problem are explained in this are reviewed.

Contents

Chapter 1

Introduction

1.1 Optimization Problem

A finite dimensional optimization problem can be described is called Mathematical programming problem

$$max \quad g(y)$$

subject to

$$u_i y \geq 0$$
$$v_j y = 0$$
$$y \varepsilon R^n$$

1. OBJECTIVE FUNCTION : It is linear function of the decisions variables explaining the objective of the decision-maker. The most typical form of objective function are: maximize g(y) or minimize g(y).

2. DECISION VARIABLES : These are physical and economic quantities whose numerical values indicate the solution of the linear programming problem. These variables are under the control of the decision-maker and could have an impact on the solution to the problem under consideration among these variables should be linear.

3. CONSTRAINTS : These are equations which is made of of practical limitations. The mathematical form of the constraints are : g(x)$\geq$c or g(x) $\leq$ c or g(x) = c

4. FEASIBLE SOLUTION: It is defined as that solution which satisfies all the constraints and is non- negative is known as a feasible solution
.

5. OPTIMAL SOLUTION : At a point, when the objective function is maximized or minimized that solution is called optimal solution.

1.2 Linear Programming

The optimization problem based on the nature of constraints functions and objective functions. If all functions g and u_i, (i =1,2,....,m) are linear functions of the decision variables y, then it is called Linear Optimization Problem or Linear Programming Problem(LPP). LPP can be solved by Graphical method, Algebra method, Simplex method, Big-m method, Two Phase method and so on.

$$max \quad y_1 - 2y_2$$

subject to

$$y_1 + y_2 \leq 4$$
$$-2y_1 + y_2 \leq 3$$
$$y_1, y_2 \geq 0.$$

1.3 Non-linear Programming

If either the objective function g or at least one of the constraints is non linear function of the decision variables y, then it is called nonlinear optimization problem or non-linear programming problem. For Example:

$$max \quad z = 25y_1 + 30y_2 - 0.30y_2^2$$

subject to

$$y_1 + 0.2y_1^2 + 3y_2 + 0.35y_2^2 \leq 1200$$
$$y_1 + y_2 \leq 6000$$
$$y_1, y_2 \geq 0.$$

1.4 Integer Programming

Integer Programming is one of the type of Linear Programming.

1.4.1 Canonical Form of Integer Programming

An integer linear program in canonical form is expressed as:

$$max \quad c^T x$$

subject to

$$Ax \leq b$$
$$x \geq 0$$

and x ε Zn
and an ILP in standard form is expressed as

$$max \quad c^T x$$

subject to

$$Ax + s = b,$$

$$s \geq 0,$$

and x ε Z
where c,b are vectors and A is a matrix, where all entries are integers.

An integer programming problem is a mathematical optimization or feasibility program in which all of the variables are restricted to be integers.

Use of Integer program(gamory cut method) in real life.
How the economic scheduling problem may be formulated as an integer linear program. An integer programming problem is a mathematical optimization or feasibility program in which all of the variables are restricted to be integers.

1.4.2 Method used for solving Integer Programming:

Integer Programming can be Solved by two methods:
(1) Branch-Bound Method
(2) Gomory-Cut Method

1.4.2.1 Branch-Bound Method

The first Branch-Bound algorithm was developed in 1960 by A.Land and G.Doig for the general mixed and pure ILP problem. Later, in 1965, E.Balas developed the additive algorithm for solving ILP problems with pure binary variables.The additive algorithms computations were so simple that it was hailed as a possible breakthrough in the solution of general ILP.Unfortunately, it failed to produce the desired computational advantages. Moreover, the algorithm, which initially appeared unrelated to the Branch-Bound technique,was shown to be but special case of the general Land and Doig algorithm.

Steps for Branch-bound algorithm Assuming a maximization problem, set an initial lower bound z = $-\infty$ on the optimum objective value of ILP. Set $i = 0$.
Step 1: Bounding
Set LP, the next subproblem to examined. Solve LP, and attempt to fathom it under three conditions:

(a) The optimal z-value of LP, cannot yield a better objective value then the current lower bound.
(b) LP yields a better feasible integer solution then the current lower bound.

(c) LP has no feasible solution.

Two cases will arise.

(a) If LP is bounded and better solution is found, update the lower bound. If all subproblems has been bounded, Stop; the optimum ILP is associated with current finite lower bound. If no finite lower bound exist then the problem has no solution.Else set i=i+1, and repeat step 1.

(b) If LP is not fathomed, go to step 2 for branching.

Step 2 : Branching

Select one of the integer variable x_j ,whose optimum value x* in the LP is not integer. Eliminate the region

$$[x_j*] \leq x_j \leq [x_j*] + 1$$

(where [v] defines the largest integer $\leq$ v) by creating two LP subproblems that corresponds to

$$x_j \leq [x_j*] \quad and \quad x_j \geq [x_j*] + 1$$

set i=i+1, and go to step 1.

The given steps apply to maximization problems. For minimization, we replace the lower bound with an upper bound (whose initial value is z = ∞).

The Branch-Bound algorithm can be extended directly to mixed problems(in which some of the variables are integer). If a variable is continuous, we simply never select it as a branching variable . A feasible subproblem provides a new bound on the objective value if the values of the discrete variables are integers and the objective value is improved relative to the current bound.

1.4.2.2 GOMORY-CUT METHOD

Using the simplex method to solve a linear program produces a set of equations of the form

$$x_i + \sum a_{i,j} x_j = b_i$$

where x_i is a basic variable and the $x_j' s$ are the nonbasic variables. Rewrite this equation so that the integer parts are on the left side and the fractional parts are on the right side:

$$x_i + \sum \lfloor a_{i,j} \rfloor x_j - \lfloor b_i \rfloor = b_i - \lfloor b_i \rfloor - \sum (a_{i,j} - \lfloor a_{i,j} \rfloor) x_j$$

For any integer point in the feasible region the right side of this equation is less than 1 and the left side is an integer, therefore the common value must be less than or equal to 0. So the inequality

$$b_i - \lfloor b_i \rfloor - \sum (a_{i,j} - \lfloor a_{i,j} \rfloor) x_j \leq 0$$

must hold for any integer point in the feasible region. Furthermore, nonbasic variables are equal to 0s in any basic solution and if xi is not an integer for the basic solution x,

$$b_i - \lfloor b_i \rfloor - \sum (a_{i,j} - \lfloor a_{i,j} \rfloor) x_j = b_i - \lfloor b_i \rfloor \geq 0$$

So the inequality above excludes the basic feasible solution and thus is a cut with the desired properties. Introducing a new slack variable x_k for this inequality, a new constraint is added to the linear program, namely

$$x_k + \sum(\lfloor a_{i,j} \rfloor - a_{i,j})x_j = \lfloor b_j \rfloor - b_i, x_k \geq 0$$

x_k an integer.

After that add this gomory cut to last obtain optimal table and apply dual simplex method till all values are not become integers repeat the process from start on the last obtain table.

Let the given LPP be

$$max\ z = c^T x$$

subject to

$$Ax = b$$

$$x \geq 0$$

x_J integer for j $\in J_1 \subset$ j = $\{\}1, 2, ..., n\}$

1.4.3 Use of Integer Progrmming

Now use this integer programming in power system problem.
Power system problem is of type :

(1) Unit Commitment problem

(2) Economic Low Dispatch system

(1) Unit Commitment system : In unit power system; we can determine that according to requirement of electricity how much units should be switched on and off.

(2) Economic Low Dispatch power system : In a power system ; we can able to determine that how much allocation of generation must be given to different units for economic objection.

1.5 Brief Survey of the Literature

Optimization establish a very important branch of modern applied mathematics. So many problem are present in the areas of engineering , operation research, management science, computer science, financial engineering and economics can be formulated as optimization problems. There are three major factors by which optimization is identified are theory, algorithms and applications. In real life because of various algorithms there is proper use of optimization.
Though the presence of optimization technique can be possible during the time of Isac Newton, L.

Euler, J.L.Lagrange and A.L.Cauchy, for linear Programming by G.B. Dantzig there was a development of simplex method in the mid 40's, mathematical optimization started in the way today we understand it. Due to H. W. Kuhn and A. W. Tucker in 1951, there is another major development which gave necessary and sufficient conditions for nonlinear programming problems [1]. These conditions are known as Karush-Kuhn-Tucker Conditions. In the area unconstrained optimization the algorithm development in nonlinear optimization is done due to the pioneer work by W. C. Davidon, R. Flecther And M. J. D. Powell in late 50's. Now in this, a real life application in electric engineering field is taken up. That is economic dispatch problem. Economic dispatch is defined as the process of allocating generation levels to the generating units. The Economic Scheduling of electric generators include both the selection of the units to operate at any time and the allocation of power demands among these operating units. By calculating the minimum cost for a day by choosing present section performance and dispatching methods gave an idea for further plans to minimize the cost of electric energy.

In this economic dispatch problem has been extensively investigated. By considering incremental cost of power from generators and in this effect of transmission losses in the form of penalty factor is also present. Then mathematical theory of this portion of scheduling problem is developed. Now days, By using the priority list which ranks the generators in the order in which they are to be started not by analytical methods the selection of generators to operate during time interval is done. This list is made on the basis of average cost of energy in the each unit and other individual considerations such as type of steam cycles and locations etc. To minimize cost schedule, include the use of digital computers by investigation of generator start-up and shutdown to search over the many possible combinations of generators and start-up rules. A direct algorithm for was given by R.E. Gomory on the solution of integer programming for the analytical solution of generator scheduling problem that is Gomory cut method.

[2] paper will describe that how economic scheduling problem is converted into integer programming. In which all values are whole numbers.

ALBERT M. SASSON AND HYDE M. MERILL proposed important power system planning and operation problems and formulate them as optimization problems. Such problems are formulated and solved. The large number of paper available is a current measure of current immense activity in this area.

Economic operation of power system is met by meeting the load demand through optimal scheduling of power generation. Main form of optimal power flow problem is to minimize the cost of fuel. In economic dispatch problem the control variables are real power generation of different generator. Optimal real power scheduling will make sure the economic benefits and decrease the release of polluting gases. Economic Dispatch problem main is to met the total demand and losses in optimal scheduling of real power generation from committed units while satisfying the constraints. Economic dispatch is large scale problem i highly non-linear constrained optimization problem by satisfying all constraints while achieving minimum cost. The Objective Function of minimizing the cost has multiple optimal solutions. There is always a demand for new methods for highly objective functions. Further new methods are expected to produce accurate results for Economic Dispatch Problem.

In optimal load flow problem basically refers to the problem of solving network equations.

Mathematically, the problem can be stated as the solution of many number of equations. Economic dispatch is one of the major problem which is explained. More problem dealt with are optimal load flow formulations to the economic dispatch, var scheduling and allocation, pollution dispatch, and maximum interchange problems. In addition, optimization applications in the areas of hydrothermal unit commitment and dispatch; substation switching; maintenance scheduling; generation, transmission and distribution expansion planning and capacitor optimization are explained.

Physical constraints description is imposed by various formulations and solution techniques and methods are given. The initial application so called B constant formulation [3]. The goal is to schedule the power generation units to minimize fuel consumption cost. It is based on a model that solves unit commitment problems. To determine the most economic scheduling of generating units, this can be done by utilizing forward dynamic programming. The model is consists of four generating units. The results obtained is the applications of forward dynamic programming method offer substantial reduction in fuel consumption cost. Within a 24-hour period, the fuel consumption cost has been reduced from dollar 116,326 to 102,181. Means saving of fuel cost is about 12.16 percent. The shows the importance of applying modeling schedule programs to the operation of power generation units. Quantity of fuel is less, power and pollution to.

By Lagrangian method, for real power generation economic schedule is obtained and by gradient method allocation of reactive power generation[4]. In this total production cost is minimized till alternate real and reactive power requirements for economic system operation are computed.
In [5], a unified approach to economic dispatching problems and load-flow, minimum-loss are explained. To coincide with the minimum of a function of the power system equations, a load-flow solution ie represent. To solve the load-flow problem, an unconstrained minimization method, developed by Fletcher-Powell is used. This indicates the presence of solution by any method. Several constrained minimization techniques that solve the minimum-loss and economic dispatching problems are investigated.
By means of linear programming techniques, the feasibility of solving optimal load flow problems in [6]. In 1969,a method by Powell for constrained minimization has explained a better algorithm than previous method. Powell's method and its accompanying Fletcher-Powell method, which performs the actual minimizations, were presented. As required by the nonlinear programming approach the power system optimal load flow problem and its equations represented.
The convergence that posses by newton method by Jacobian matrix has led to use Hessian matrix for faster convergence [7]. The hessian load flow minimize the objective function by some additional terms so that this method solve both normal and optimal load problems simply and quickly.

In [8], the main large-scale control problems in electric power system operation,frequency and change power control, from optimal allocation of generation and system stability and dynamic security, transmission resources to network state estimation, are explained and the method to solve them at that time are present. Most important work is applications of decomposition by physical decoupling and relaxation of decentralized controls, of coordination and hierarchies, of large-scale system analysis areas open to further development.

In [9], The Dinney-Tinney work on Optimal load flow problem is very powerful. In the system by exact optimal contingency constraints, formulation and solution gives optimal and secure system.

To made a distinction between spinning reserve and regulating margin a coordination technique is made which determines the least expensive generation schedule required to maintain a total specified power system regulating margin. The effect on operating cost of applying the regulating margin restraint is also investigated in [10] .

Chapter 2

Review : Power Generation Scheduling

Power Generation Scheduling by integer programming

2.1 Introduction

In this a paper is reviewed ”Power Generation Scheduling by Integer programming- Development of Theory” . In this, the economic scheduling of electric generator is taken. It include the selection of unit to operate at any time and power as per requirement to each unit. This paper explain how the economic scheduling problem is converted into the integer programming All values are must be whole integers. This economic dispatch problem includes the following :

1. The discontinuous power output characteristics of the generators, that is it has either no power output, or a minimum output.

2. The cost of on and off of each unit.

3. The dispatch of load by incremental costs.

Steam-turbine generating units are include minimum three types of non-linearities in the dollar-per-hour input to power output characteristics. First one is cost is incurred when the turbine is started; second one is the output is constrained to lie between the highest and lowest ratings; and last is input-output relation is complicated by the presence of valve loops. So to formulate this, an example is used.

Suppose a power system with two generators. The startup costs, minimum outputs costs, and incremental costs are given in given in the following table:

Such characteristics has been found to given a lower cost schedule than connected straight line segments when the effects of valve loops are considered.

(a) The on-off indicator for generator i in time period will be:
y_{jt} = 1 if generator is scheduled to operate
y_{jt} = 0 otherwise
(b) The start up indicator for generator i in period t will be:
w_{jt} = 1 if generator is scheduled to start
w_{jt} = 0 otherwise
(c) The shutdown indicator for generator i in period t will be:

Table 2.1: Generator Cost Charcterstics

Symbol	Name	Generator 1	Generator 2
U_j	Maximum Rating	80 mw	120 mw
L_j	Minimum Rating	20 mw	30 mw
c_jL	Cost at minimum rating	$\$60/hr$	$85/hr
c_jl	Ist incremental cost step	$2.3/mw-hr	$2.0/mw-hr
$\triangle\ _il$	Power available at ci_j	30mw	20mw
ci_2	2nd incremental Step	$3.0/mw-hr	$2.8/mw-hr
$\triangle$i$_2$	Power available at ci$_2$	30mw	70mw
ci_u	Start-up cost	$20	$30
ci_d	Shutdown cost	$10	$15

Table 2.2: Capacity Requirement

P_t	Estimated load	50mw	100mw
	Spinning reserve	15mw	20mw
S_t	Total Requirement	65mw	120mw

v_{jt} = 1 if generator is scheduled to start
v_{jt} = 0 otherwise
(d)The output from generator i at incremental cost step j in period t will be :
x_{jkt} = mw
The solution must satisfy the following requirements:
1. All nonnegative variables are present.
2. Assume all variables have integer values.
3. The total capacity required for each period must have less than or equal to operating units capacity,R_t.

$$80y_{11} + 120y_{21} \geq 65$$
$$80y_{12} + 120y_{21} \geq 120$$

4. From each unit, the sum of the power output must be equal to estimated load,D_t.

$$20y_{11} + x_{111} + x_{121} + 30y_{21} + x_{211} + x_{221} = 50$$

$$20y_{12} + x_{112} + x_{122} + 30y_{22} + x_{212} + x_{222} = 100$$

5.At each and every incremental cost step, the power output from each generator must be less than, or equal to, the maximum available capacity, and if the generator is not operating then it must be zero.

$$30y_{11} \geq x_{111}, 30y_{12} \geq x_{112}$$
$$30y_{11} \geq x_{121}, 30y_{12} \geq x_{122}$$
$$20y_{21} \geq x_{211}, 20y_{22} \geq x_{212}$$
$$70y_{21} \geq x_{221}, 70y_{22} \geq x_{222}$$

6. The start indicator for that period, v_{it} must be 1, when generator started.The shutdown indicator for that period, w_{it} must be 1 when generator is off(shutdown).

$$y_{11} - y_{10} = w_{11} - v_{11}$$

$$y_{21} - y_{20} = w_{21} - v_{21}$$

$$y_{12} - y_{11} = w_{12} - v_{12}$$

$$y_{22} - y_{20} = w_{22} - v_{22}$$

7. For each generator, the on-off indicator must be not greater than 1, in each period.

$$y_{11} \leq 1, y_{21} \leq 1$$

$$y_{12} \leq 1, y_{22} \leq 1$$

8. The object is to minimize the cost function for each time period, where the cost function is the sum of the costs of stopping, starting, producing power at minimum output, and producing power above the minimum output. Let the cost be represented by the symbol z.

$$z = 20w_{11} + 30w_{21} + 10v_{11} + 15v_{21} + 60y_{11} + 85y_{12} + 2.3x_{111} + 3.0x_{121}$$

$$+2.0x_{211} + 2.8x_{221} + 20w_{12} + 30w_{22} + 10v_{12} + 15v_{22} + 60y_{12} + 85y_{22}$$

$$+2.3x_{112} + 3.0x_{122} + 2.0x_{212} + 2.8x_{222}$$

This competes the formulation of above problem. The generator scheduling example does not has start up costs which vary with length of the time generator has been shut down. Additional equations should be also in noticed, specially generator requirements, such as keeping one unit at the station on one line, could be entered as further constraint without affecting the method of solution.

2.2 Method of Solution

Firstly, the economic scheduling of generators is mathematically solved by assuming that all variables have integer values.The problem, without the integer requirement is a linear programming problem is solvable by dual method or simplex method. Then in 1958 to solve linear program with additional integer requirements was presented by R. E. Gomory. The derivation of additional constraint from already present in the problem is the key to his method.This addition of the constraint will allow all of the integer solutions to the original problem, but eliminate the non integer solutions.

To solve generator scheduling problem gomory hand calculations led to discovery of a more general algorithm and original integer algorithm which is named eucledian escalator method. Referred to the dual eucledian method and is identical to the method presented by gomory in 1960.So first write this formulated equations in standard form.

In standard form:

$$80y_{11} + 120y_{21} - S_1 = 65$$

$$80y_{12} + 120y_{22} - S_2 = 120$$

$$30y_{11} - x_{111} - S_3 = 0$$
$$30y_{12} - x_{112} - S_4 = 0$$
$$30y_{11} - x_{121} - S_5 = 0$$
$$30y_{12} - x_{122} - S_6 = 0$$
$$20y_{21} - x_{211} - S_7 = 0$$
$$20y_{22} - x_{212} - S_8 = 0$$
$$70y_{21} - x_{221} - S_9 = 0$$
$$70y_{22} - x_{222} - S_{10} = 0$$
$$0 = y_{11} - y_{10} - w_{11} + v_{11}$$
$$0 = y_{21} - y_{20} - w_{21} + v_{21}$$
$$0 = y_{12} - y_{11} - w_{21} + v_{12}$$
$$0 = y_{22} - y_{21} - w_{22} + v_{22}$$
$$y_{11} + S_{11} = 1$$
$$y_{21} + S_{12} = 1$$
$$y_{12} + S_{13} = 1$$
$$y_{22} + S_{14} = 1$$
$$20y_{11} + x_{111} + x_{121} + 30y_{21} + x_{211} + x_{221} + S_{15} = 50$$
$$20y_{12} + x_{112} + x_{122} + 30y_{22} + x_{212} + x_{222} + S_{16} = 100$$

Table 2.3: Initial table

	P_0	x_{111}	x_{121}	x_{211}	x_{221}	y_{11}	y_{21}	w_{11}	w_{21}	x_{112}	x_{121}	x_{212}	x_{222}	y_{12}	x_{21}	w_{12}	w_{22}
z	25	-2.3	-3.0	-2.0	-2.8	-60	-85	-30	-45	-2.3	-3.0	-2.0	-2.8	-50	-70	-30	-5
s_1	-50	-1	-1	-1	-1	-20	-30	0	0	0	0	0	0	0	0	0	0
s_2	0	1	0	0	0	-30	0	0	0	0	0	0	0	0	0	0	0
s_3	0	0	1	0	0	-30	0	0	0	0	0	0	0	0	0	0	0
s_4	0	0	0	1	0	0	-20	0	0	0	0	0	0	0	0	0	0
s_5	0	0	0	0	1	0	-70	0	0	0	0	0	0	0	0	0	0
s_6	-65	0	0	0	0	-80	-120	0	0	0	0	0	0	0	0	0	0
v_{12}	1	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0
v_{22}	1	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0
s_7	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
s_8	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
s_9	-100	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-20	-30	0	0
s_{10}	0	0	0	0	0	0	0	0	0	1	0	0	0	-30	0	0	0
s_{11}	0	0	0	0	0	0	0	0	0	0	1	0	0	-30	0	0	0
s_{12}	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-20	0	0
s_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	0	0
s_{14}	-120	0	0	0	0	0	0	0	0	0	0	0	0	-80	-120	0	0
v_{13}	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	-1	0
v_{22}	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	-1
s_{15}	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
s_{16}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

Table 2.4: Final Table

	P_0	s_{10}	s_{10}	v_{12}	x_{121}	s_1	x_{221}	w_{11}	w_{21}	x_{111}	v_{22}	s_{12}	x_{122}	s_6	s_{16}	w_{12}	w_{21}
z	399	-1	-0.5	-20	-1	-2	-0.8	-30	-45	-0.3	-25	-0.8	-0.2	-2.8	-5	-10	-20
s_1	30	30	0	-30	0	0	0	0	0	1	0	0	0	0	0	30	0
s_2	30	30	0	-30	1	0	0	0	0	0	0	0	0	0	0	30	0
s_4	20	20	0	-20	-1	1	-1	0	0	-1	-50	0	0	0	50	20	50
s_5	70	0	0	0	0	0	1	0	0	0	-70	0	0	0	70	0	70
s_6	135	80	0	-80	0	0	0	0	0	0	-120	0	0	0	120	80	120
v_{11}	0	-1	0	1	0	0	0	-1	0	0	0	0	0	0	0	-1	0
v_{21}	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	-1	0	-1
s_7	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	0
s_8	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	-1
x_{211}	0	-20	0	20	1	-1	1	0	0	1	30	0	0	0	-30	-20	-30
y_{11}	1	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	1	0
y_{21}	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	1
s_{11}	30	30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
s_{13}	70	50	1	0	0	0	0	0	0	0	0	1	-1	1	120	0	0
s_{14}	80	80	0	0	0	0	0	0	0	0	0	0	0	0	120	0	0
x_{112}	30	30	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{112}	20	0	0	0	0	0	0	0	0	0	0	1	0	0	20	0	0
x_{212}	0	-50	-1	0	0	0	0	0	0	0	0	-1	1	-1	-50	0	0
y_{12}	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
y_{22}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

2.3 Conclusion

It presents that how the economic scheduling problem is formulated as an integer linear program taking into account the generators, discontinuous input-output characteristics and start-up costs. Gomory-Cut Method for integer programs, linear programs with whole number answers required is successfully applied to the solution of the integer linear programming.

Chapter 3

Some Applications of Optimization Techniques to Power System Problems

3.1 Introduction

Optimization technique have been successfully used in so many fields.These different Fields are:
1. Mechanics
2. Economic and Finance
3. Electrical Engineering
4. Civil Engineering
5. Operation Research
6. Control Engineering
7. Geophysics
8. Molecular Modeling

From all these above fields Electrical engineering is the most important field. Some application in electric field are discussed as below:

1. Economic Dispatch
2. Var Optimization:
3. Pollution Dispatch
4. Maximal interchange
5. Hydrothermal Dispatch and Unit Commitment
6. Optimal Switching
7. Generation Expansion Planning
8. Distribution Design and Operation
9. Capacitor Optimization

Optimal load flow is one of the important problems in which so many factors are present by which there is so much effect on minimizing cost these are described below:
(i) Var Optimization: Var Optimization is defined as the process of optimally balancing voltage levels and reactive power by reducing system losses, peak demand or energy consumption or combination of three.
By Optimizing different cost function many different different problems can be formulated as Op-

timal load flow problems. By observing voltage and network constraints,optimizing control of reactive resources by minimizing reactive resources is another problem. To solve this problem Non-Linear Programming is used.
To handle the size of problem and for the transmission of real-time data here uses a decomposition approach in multi-area multicomputer optimized control of voltage in interconnected power system.

(ii) Pollution Dispatch: The transformation of energy from coal, natural gas, nuclear sources or oil are involved in the generation of power from thermal plants. Due to emissions from power plants in this transformation various form of pollution take place. In this so many type of particulates are of concern like Oxides of sulphur and nitrogen and solid particulates. To minimizing either emission or concentration of pollutant is encourage to some work towards dispatching power at different given locations. In first case, emission curves are instead of generation cost curves. and in second Pollutant dispersion curve is used.

(iii) Maximum interchange : For normal operation and so many other cost benefits utilities are interconnected sometime in emergencies to help each other, like in loss of large generating units.The power capacity that can be interchanged between two utilities does not depend only upon the capacities of tie lines but also on non-tie lines specially when transmission contingencies can occur. Interesting to be noted that for utility for interchange of maximum capacity in ahead of time.

(iv) Hydrothermal Dispatch and Unit Commitment : To solve this big problem of deciding which units should be online and their time period of operating means during day time or week for a mixed and thermal and hydro generation power system is the objective function. Most important is to minimize the cost during the time period subject to operating and physical constraints. This big problem is itself a part of boarder problem which is itself defined as a long term operating policies for fuel and water management in the face of forecast on future load demand s and water availability.

(v) Optimal Switching : Low-voltage and intermediate substations link the distribution networks of local loads and transmission system of high voltage. For good in performance and protection purposes, switches are given that make it possible to alter the configuration of the substation. So many changes are done to isolate devices, under normal circumstances that will undergo maintenance. Like in emergency, if any equipment has get damaged and overloads, in the system or we can say locally also then there is a need for configuration changes.The important factor to avoid intermediate configuration is the sequence of changing switching which are unacceptable from the point of view of personal safety and any damage equipment. This may lead to switching strategy in many steps. Now, an Optimization problem is set up for minimizing the number of switching operations. On reducing intermediary steps is the disruption of service to loads is the constraining factor.

For solving substation switching sequence problem, some references have reported. In [11], as a mathematical logic problem the problem is formulated. A heuristic report has been explained in [12], to decrease the storage and solution time. In [13], a mathematical programming problem is set up that is: initial arrangements and desired final conditions are given, in which to do is minimizing the penalty function. Penalty function is which penalizes unnecessary switching operation.

(vi) Generating Plant Maintenance Scheduling: Generating plant scheduling problem that are optimal is a part of a more general scheduling problem that include the unit commitment problem, fuel utilization scheduling and hydro and pumped hydro scheduling. It is a separate problem. Usually each year 3000 dollars/MW are spent for plant maintenance, for 10000 MW of capacity around 30 millions dollars per year are spent.

Significantly, optimization techniques are employed differently. Usually, for one periods of one to a few weeks for maintenance once or twice a year, a unit must be not responding or say that out of service. Within some recommended time period for each unit this must be done. Limited resources are available for maintenance (megawatts, parts, manpower). Various units may be maintained with some constraints on the sequence. When each and every unit is maintained, these constraints are defined as as linear inequality or equality in terms of decision variables are define.

In this two classes of approach are discussed. The first one is to try to optimize a reliability criterion and the another one is to minimize cost, subject to level of reliability.
In each week, the reliability criterion includes the reserve capacity. One approach is to seek to level the reserve capacity over a year, the problem can be formulated as minimizing the loss of load probability. In case of the power problem, the dimensionality makes the problem difficult. To obtain suboptimal schedule in reasonable time these methods have all use of clever heuristic strategies.
An suboptimal techniques these methods all have the disadvantages 1) Even when one schedule exist, they may fail to find a schedule satisfying problem and 2) they do not optimize the necessary criteria. By proper choice of a maintenance schedule One minimum cost approach appears to minimize annual production costs. Simultaneously, the hydro scheduling problem can also handle by this method. Other method are explained are also discussed.

Annual fuel cost are compatibly insensitive to maintenance schedules. That maintenance are very sensitive to schedule explained by Dapazo and Merill. Because of unnecessary work maintaining a unit too soon costs money. Because of decrease in availability of unit and because of accelerating deterioration of the machinery.

(vii) Generation Expansion Planning: The generation expansion planning problem can be defined as very easily, decide a time frame over many years what and when a new generating capacity to add and where it should be add. The objective function is in terms of operating and capital costs is chosen of least expensive expansion program by satisfying all constraints. The constraints have that capacity which satisfy all the demands and there are limits on various types of generation that can be added.

(viii) Distribution Design And Operation: Distribution system are the networks. These are supply local loads from high voltage substations. Than Transmission Planning , Distribution Planning are seems to be more tractable. Distribution systems are tend to be less extensively geographically, realistic planning can be done over limited time, and reliability criteria are not as stringent. In this networks are subject to be less severe extremes of operating conditions.
Mesh, radial or radially operated mesh networks are designed by seeing all the characters like conductor current capacity ,which is related to conductor size and cost , and voltage drop which is allowable is along a distribution line, which is related to conductor size and its electrical characteristics. Some factors are opposite to each other like at incremental cost better voltage regulation

is obtained. An Optimization problem is designed : minimizing the capital expenditure subject to regulation, meeting the load constraints and current carrying capacity and the most important factor is cost of transmission losses. As such Conductor size is greater, these loses will decrease.
A Branch and Bound technique is used in [14] to radial distribution systems. This technique takes advantage of the special structure of such systems. Voltage regulations constraints are explicitly included.

(ix) Capacitor Optimization: Related long range planning problem allocation of var resources so much work has done. Adequate capacitor compensation helps keeps voltage up , important in contingency conditions. System losses reduces by proper use of capacitors.

In [15], the number of units at a bus being considered , sum of capacitor unit admittances is the cost function. A complete non-linear formulation is made , for each of a number of contigency the problem is solved. A contigency gradient technique is used to minimize the objective function when constraints are incorporated. The number of units at each bus us rounded to integer value and optimization is repeated..

3.2 Optimal Load Flow Problem

A power system is a mode of generating plants,transmission lines and loads.through three sources power enters in a transmission system:
1. Neighboring System Through tie line
2. Hydro Plants
3. Thermal Plants

In real time the power demand at various load buses or nodes in the system is assumed. Power loss in the transmission system is also assumed. And sum of load demands P_D is equal to the losses P_L has to be equal to the generation.

The term "Power" used to represent the real power P(megawatts).
Reactive effect introduced by the capacitance and inductance of transmission lines capacitive and inductive loads and generation is a sinusoidal-steady state. And there is a reactive power Q.

As we all know that Kirchoff Law is generally written in terms of Complex power nodal voltage V, δ, in the nodal form. Similarly, in power system complex power nodal injections and load nodes to complex power flows through transmission lines also written in terms of V, δ. Then the final equations obtained in the form of V, δ are called Network Equations.

The Problem of solving these Network Equations is referred to the term "Load Flow".

For each node, two piece of data is given P, V for generation nodes and P, Q, for load nodes. A single generation node given is treated differently and is given V, δ. For sinusoidal equations δ is referred to zero and to balance the effect of losses no P is allowed to pass this quantity. This node is known as slack node. Network equations are written For each injection data quantity resulting in a system of equal number of algebraic equations and nodal voltage variables. The load flow problem, mathematically can be written in the form of n equations

$g(y) = 0, y = (y_1, ..., y_n)^t$

The y or voltage variables are known as state variables of the system. If once these state variables are known then all other electric quantities like reactive generation, slack-bus generation, and power flows in transmission lines, can be calculated easily.

By varying input data, different load solutions can be obtained. Optimal load flow is used to refer to load flow solutions or an operating state where some power system quantity is optimized, subject to constraints on the problem variable and on some functions of these variables.

So many problems can be defined by different choices of objective functions and constraints.

3.2.1 Economic Dispatch

The problem, Economic dispatch is of minimizing the cost of fuel at thermal plants, by considering that hydro generation is defined previously and the network configuration is known. Which thermal units are on-line are also known. In this problem the constraints generally found in trading off complexity for solution speed. Under economic dispatch in today time many power system are operated with calculations made on-line in every few minutes. For generating units, control signals are sent to generating station to adjust their output power in accordance with optimization results, under normal circumstances. In interconnected systems, by a load frequency control process optimization results are adjusted which has the task of total power interchange with neighboring utilities from deviating from present levels and preventing frequency .

In 1960's, application of a computerized economic dispatch problem dates is the first application. After 15 years of theoretical work and semi manual and manual economic dispatch operation.

A representation of the physical constraints imposed by so many formulations will be given as the solution and methods techniques are presented. So called B constant formulation is the initial application. Sum of thermal fuel costs as a function of generation real power P is the objective function. Real Power Balance Equation is a single equality constraint. Inequality constraints place limits on P.
Mathematical Formulation is given by:

$$min \quad h(y)$$

subject to

$$g(y) = 0$$
$$\tilde{y} \leq y \leq \bar{y}$$

$$where \quad y = (y_1, ..., y_n)^t$$

The solution of these problem is solved by using Kuhn-Tucker optimality conditions

$$\frac{\partial h(y)}{\partial y} + \lambda \frac{\partial g(y)}{\partial y} = 0$$

$$g(y) = 0$$

y within limits.

It is necessary to have an analytic expression to solve these conditions for the transmission losses term in g(y) in terms of generation power y. A taylor's series expansion of P_L is given

$$P_L = C_0 + y^t C_1 + y^t C_2 y$$

where C_0 scalar, C_1 vector, and C_2 matrix are B-constants. All these constants have to approximate the behavior of network at different values of y.
Limitation on the dispatch problem imposes by the use of B constants. Dopezo et al. introduce the

new idea of an exact loss formula where losses are explained as function of Complex nodal voltages and injections at all buses. For online applications, if on-line load flow or state estimation are available then we can say that these quantities could be known. An economic dispatch problem is solved With online approach of loss formula would be calculated at any time. For off line application, the state is establish through a load flow computation and at every iteration this step is repeated again to solve Kuhn-Tucker equations. To introduce the effect of network, the use of loss formula further limits the scope of the problem in that constraint on other system quantities can not be found out. Now the further work introduce the network equation directly as equality constraints. Inequality constraints can be set up for voltage magnitude, flows in transmission lines and real and reactive generation and so on. Then the problem is:

$$min \quad h(y)$$

subject to

$$g(y) = 0$$

$$j(y) \geq 0$$

$$where \quad y = (y_1, ...y_n)^t$$

In this problem formulation, it is important that directly all network equations appear as equality constraints or as part of $h(y)$ in order for the that all y variables are interrelated for model to complete.
This is the standard nonlinear programming problem. To change the problem from constrained minimization to unconstrained minimization problem so many penalty methods are present. The Fiacco McCormick SUMT, Lootsma, and Zangwill transformations were work together with the Fletcher-Powell unconstrained minimization technique. The first require an initial state vector y that produces no inequality constraint violations. In general it is difficult to to achieve. The Zangwill penalty function is of the type

$$H(y) = h(y) + g^t(x)S_1 g(y) + j^t(y)S_2 j(y)$$

where H(x) is sequentially minimized with an increasing value for the diagonal S matrices, each time. When constraint is violated only j(y) term is present. In, this the Zangwill transformation is used instead of the powell extension to :

$$H(y) = h(y) + (g(y) + p_1)^t S_1 (g(y) + p_1) + (j(y) + p_2)^t S_2 (j(y) + p_2)$$

Transformer tap ratios of phase shifting and voltages magnitude transformer were introduced as additional y variables.Instead of removing the term set value of j(y) equal to zero then in the powell transformation violated inequality constraints become feasible. In maintaining the unconstrained function continuous there is a effect of this variation.

The Fletcher-Powell technique was referred to steepest decent techniques. With the former technique, minimization of the inverse Hessian matrix of the unconstrained function is iteratively and for the correction of each step

$$\Delta y = -J \Delta_y h(y)$$

such that as

$$\Delta y \longrightarrow 0, J \longrightarrow G^{-1}$$

The memory gradient technique is used , calculating corrections from

$$\Delta y = \beta \Delta_y h(y) + \alpha \Delta \hat{y}$$

where $\Delta \hat{y}$ is the Δy of the previous iteration.

In fletcher powell method a one dimensional search is referred to calculate β for each iteration. In memory gradient approach , tho dimensional search is made. The motivation for using other methods instead of the Fletcher-Powell comes from the need to store J,a full matrix. sparsity properties of the hessian matrix are used in hessian matrix are used in calculating Δ y from the optimally ordered Gaussian triangularization solution of

$$G\Delta y = -\Delta_y f(y)$$

Followed by one dimensional search.

In power system , there are strong coupling between real power flows and voltage angles and between reactive power flows and voltage magnitudes. However ,P,δ and Q,V variable sets are weakly coupled. For solving the economic dispatch decomposition techniques are suggested. Firstly, the minimization of losses is with voltage magnitude as variables and reactive constraints. Then it is followed by minimization of thermal fuel cost with voltage angles as variables and real power constraints, by keeping the voltage magnitude fixed at the level found in the first problem. By Zangwill-Flectcher-Powell process both problems can be solved.

Now, Two set of variables are defined : the state variables of the load flow problem, y variables, and the load flow data control variables, v variables. The network equations relate y and v variables. The problem can now be stated as follows:

$$min \quad f(y, v) \qquad (y = (y_1, ...y_n)^t)$$

subject to

$$g_1(y) = 0$$
$$j(y, v) \geq 0$$
$$\tilde{v} \leq v \leq \bar{v}$$
$$g_2(y, v) = 0$$

An F(y,v) function is defined h(y,v) by penalizing with violated functional inequality constraints and n equations g(y,v) are defined by joining the k equality constraints $g_1 = 0$ and the n-k network equations $g_2(y, v) = 0$. The Kuhn-tucker optimality conditions are following:

$$g(y, v) = 0$$
$$\frac{\partial H}{\partial y} + (\lambda^t \frac{\partial g}{\partial y})^t = 0$$
$$\frac{\partial H}{\partial v} + (\lambda^t \frac{\partial g}{\partial v})^t = 0$$

.

The equations first set is solved for y by choosing the feasible value of v. This is called load flow problem. To solve the problem load flow if newton method is used with triangular factorization techniques in this triangularized Jacobian is used using equation second set is used. The third group describes the reduced gradient of H wi th respect to v variables. To calculate corrections for v variables gradient is used:

$$\Delta v = \alpha \Delta_v H.$$

According to the Kuhn-Tucker optimality conditions, violations of inequality constraints on v variables due to above correction Δ_v should be prevented by setting v at its limit. Now, new iteration is performed and continue this until $\Delta_v H = 0$ for all off limit v variables.

A strategy is formed where the violated y variable is replaced with a v feasible variable. By holding it at the limit thus enabling direct controls to be placed on the first. For this strictly functional equational constraints, with the addition of new slack variable δ they are transformed to equality constraints:

$$j(y, v) - \delta = 0$$

$$\delta \geq 0$$

Thus equality constraints are formed the part of set of $g(y, v) = 0$ and δ is added among the y variables. Transformed constraints are added only when they are maintained and violated in the problem unless the dual variable λ_i of a constraint sign changes, indicating that the process of minimizing wants to move forward into the region where that inequality satisfied.

The choice of β is made by directing Δv onto Δy by assuming that the move should be along g(y,v)=0

$$(\frac{\partial g}{\partial y})^t \Delta y + (\frac{\partial g}{\partial v})^t \Delta_v = 0.$$

The scalar β is chosen because such that y and v remains in limits.

The main reduced gradient approach is found in different different variations. In order to decrease the size of the problem form the large number of functional constraints, with the solution of the original network equations the method used is started. Functional inequality constraints are then investigated to determine the violated ones. At this point, an auxiliary problem is first posed :

$$min \quad h(y, v)$$

subject to : real power balance equations

$$j(y, v) + \Delta = 0$$

$$\tilde{y} \leq y \leq \bar{y}$$

$$\tilde{v} \leq v \leq \bar{v}$$

Load dispatch problem is slightly converted by a quadratic approximation of the power balance equation and functional constraints should be linearized. In auxillary problem, instead of network equations power balance equations is taken and violated functional inequality constraint is present.

The problem formulations which has some security and contingency related constraints, in addition to the usual constraints on real and reactive generation, voltage magnitude and transformer tap ratios. Here, Contingency constraint refer to the prevention of overloads in transmission facilities and prevention of overload some time after the contingency.

In this spinning reserve constraint are involved.

: Spinning Reserve is that quantity of extra generation which is produced in short period of time from generating units which are already online. There are units on spinning reserve amount and also to its rate of change or thermal gradient.

The reduced gradient approach is used for the decomposed problem of sequentially minimizing cost and transmission losses taking into study the natural decoupling of P, δ and Q,V is presented.

Now, the three references are explained. In this each reference has discussed each contributes and each contribute has different formulation variation to the problem of economic dispatch. The main problem of minimizing thermal fuel cost subject to the real power balance equation is taken here with a equality constraint on a desired total regulating margin. There is difference between spinning reserve and regulating margin. By P-δ linearized decoupled load flow equations, limits are imposed on transmission system power flow.Now these all are added to the main problem formulation where all constraints are linear and the cost function is non-linear.To solve this particularly structured problem technique used is The Rosen gradient. Now, the original problem is taken by the use of loss formula for the calculation of

$$(\frac{\partial P_L}{\partial y})^t = (\frac{\partial P_L}{\partial v})^t . \frac{\partial v}{\partial y}$$

the $\frac{\partial v}{\partial y}$ is referred to the Jacobian of the network equations. After the load flow solution, solving for $\frac{\partial P_L}{\partial v}$ gives the required numerical values.

The compatibility is expanded to all generation of real and reactive power, and transformers taps, and control variables. Inequality constraints on functional variables and state constraints are regarded or we can say considered incrementally with respect to y variables, which are expressed in terms of increments of v variables through Jacobian relationship of network equations. Then the problem formulation becomes:

$$min \quad f(\Delta v)$$

subject to

$$v_1(\Delta v) = 0$$

$$g(\Delta v) = 0$$

$$\tilde{\Delta v} \leq \Delta v \leq \bar{\Delta v}.$$

To obtain the current conditions and accurately linearize constraints the solution strategy assumes the presence of state estimator. According to matrix inversion process the Lagrangian conditions are confirmed and to determine when to release control variables and functional constraints from their limits the Kuhn-Tucker dual variables are used.

3.3 Conclusion

Economic dispatch problem is the important problem in optimal load flow problem. Economic dispatch problem can be solved using various optimization techniques such as nonlinear, linear, integer and quadratic and dynamic programming. The problem formulation of economic load dispatch using various optimization techniques have been researched.

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