RELIABILITY AND MAINTENANCE SCHEDULING ACTIONS FOR SERIES - PARALLEL SYSTEM

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Reliability and Maintenance Scheduling Actions for Series-Parallel System"** in partial fulfillment of the requirement for the award of degree of Master of Science, School of Mathematics (SOM), Thapar Institute of Engineering & Technology (TIET) (Deemed to be University), Patiala is an authentic record of my own carried out under the supervision of Dr. Harish Garg, Assistant Professor, SOM, TIET Patiala.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

(Dr. Harish Garg)
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Date: October 24, 2019

Abstract

The purpose of this work is to addresses some optimization models to process the performance of the series-parallel system. In it, system performance has been carried in the form of a reliability-cost optimization problem. The contradictory nature between the objectives is determined by establishing their fuzzy goals and consequently, the optimization problem under attention becomes a fuzzy programming problem. In a fuzzy multi-objective optimization problem, an optimal solution that concurrently optimizes all the goals is rarely possible. In such circumstances, one frequently measures to seek the most reliable potential solution in the appearance of rough, or imprecise erudition which is as close to the decision maker's (DM's) expectations. Exploration for such a pleasant solution necessitates working the multi-objective fuzzy optimization problem in an interactive way wherein the DM is originally required to define his or her choices. Based on given choices, the dilemma is answered and the DM is provided with a reasonable solution. If the DM is happy with this answer the problem stops there, unless, invited to alter their choices in the knowledge of the earlier received outcomes. This iterative procedure is resumed till a satisfying solution is arrived which is close to DM's expectations.

Apart from this, we also addressed some preventive maintenance (PM) schedule and their action namely (1a)-, (1b)- and (2p)- rather than only the replacement of the system. The optimal policy corresponding to each PM's actions is computed based on the optimal PM interval. For it, a multi-objective optimization model by considering the system availability and maintenance cost per unit time under the PM and corrective maintenance. Based on their optimal period, periodic PM scheduling is proposed and their corresponding PM actions, to save the money and increase the reliability of the system.

The objective of this work is to investigate the appearance of some series-parallel

systems more firmly by expressing an optimization model.

The present dissertation is composed into four chapters which are concisely compiled as follows:

A concise description of the similar work of various authors in the evaluation of reliability and maintainability is exhibited in the first chapter. In **Chapter 2**, the basic and preliminaries linked to the reliability are furnished.

Chapter 3 offers a new interactive multi-objective reliability-cost optimization model of a series-parallel system. The contradictory nature within the objectives is resolved by defining the fuzzy region of satisfaction. Further, by addressing the choices of the decision-maker towards the given objectives, the estimated bi-objective optimization model is molded into the single objective optimization model. Finally, the received problem is solved with the help of the PSO algorithm. Based on the reported parameters and the optimal cost and reliability of the system, a decision-maker can choose a good one according to his need to arrive at the desired goal.

In **Chapter 4**, we perform a periodic maintenance scheduling actions of a series-parallel system by examining the three main actions, namely (1a), (1b) and (2p)-maintenance for a multi-components system based on maintenance cost. The overall preventive maintenance (PM) interval of the system is considered as the minimum among them. Finally, the behavior of the system is scrutinized based on the reliability achievement and the effect of the PM actions is founded based on maximizing the maintenance-benefit analysis. The approach has been explained through a numerical case study.

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Chapter 1

Introduction

In the modern era of global struggle and quicker delivery times, it has become crucial for all production practices to work adequately during their expected life span. Any damaging outcomes of irresponsible behavior of such devices or systems lead to the desire for reliability analysis. Therefore, in modern years, the meaning of reliability theory has been increasing hugely with the change of recent technology to make good outcomes with high quality and design highly reliable systems. The scientific theory of reliability took shape and increased as a result of the demands of developing technology and in particular out of the experiments with multiple military systems during World War II. It was reported that during World War II nearly 60 percent of the airborne equipment shipped was destroyed on arrival and 50 percent of the spare parts and equipment in storage became unserviceable before they were used. Nowadays, researchers are spending more awareness to the real-life problem of enhancing the performance as well as the profit margin of an industrial system. Therefore, in recent years, system reliability displays an important problem in assessing the completion of an engineering system and when it is low, efforts are wanted for each subsystem/unit of a system by overcoming their likelihood failures. For this comprehensive awareness of failure operation of the system as well as its components are required so that proper maintenance policies may be employed for raising its performance.

The purpose of this work is to examine the performance of some series-parallel systems more strictly by formulating an optimization model. Concise discussion on some issues related to reliability, maintainability, and optimization of a system have been studied and are addressed section-wise hereafter.

1.1 Review of Literature

In this section, a brief literature review regarding reliability/ availability evaluation and optimization is furnished. Reliability is a popular idea that has been marked for years as an excellent quality of a person. The application of the reliability analysis for the systems was treated almost half-century back by the researchers [1]. It has ever been considered as a valuable agent for risk analysis, production availability studies and design of systems. Gupta and Sharma [2] formed a mathematical model to assess the availability and Mean Time To Failure (MTTF) of a two-unit cold standby system. Dinesh-Kumar and Birolini [3] assessed the estimated expression for system availability of a repairable series-parallel system. In current times, a diversity of methods are accessible in the books for examining the reliability and availability of the repairable systems. Some of them are optimization, Monte Carlo simulation, fault tree analysis (FTA), etc. [4–7]. El-Damcese and Tamraz [8] has examined the reliability and availability of the parallel repairable system with degradation facility and common-cause failures by using Markov and supplementary variable techniques.

Huang et al. [9] gave a posbist FTA, based on posbist reliability theory, for predicting and diagnosing failures and estimating the reliability and safety of systems. Gupta et al. [10], Rani et al. [11] proposed a rule to compute reliability, availability, and MTBF of the industrial system. Garg [12] exhibited the reliability analysis of the repairable industrial system using vague theory. Leung et al. [13] proposed a model for a two–dissimilar–component cold standby repairable system with one repairman and repair priority. Garg et al. [14] presented a bi-objective reliability optimization problem for series-parallel system. Sharma et al. [15] discussed a basic structure that makes use of both qualitative and quantitative procedures for risk and reliability analysis of the system. Garg et al. [16] presented multi-objective reliability optimization problem under interval environment. Taghipour et al. [17] intended a method for investigating the statistical maintenance data for a general infusion pump from a Canadian General Hospital with censoring and missing information. Garg et al. [18] analyzed the performance of the system using Weibull distribution. Niwas and Garg [19] examined the reliability by using the cost-free warranty policy.

Reliability availability and maintainability of the equipment also perform an essential role in managing both the quantity and quality of the products. They propose at estimating and predicting the probability of the failure and optimizing the operational control related to the requirement of the failures, i.e. maintenance policies. Ireson et al. [20] and Blischke and Murthy [21] reported concurrent engineering in detail and presented examples of different industries. In literature, there is a deficiency of field failure data of production lines. Blischke and Murthy [22] reported 26 problems on reliability and maintainability and statistical techniques illustrated. Liberopoulos and Tsarouhas [23] presented a case study of chipitas food processing system, based on the simplified assuming that the failure and repair times of the workstations of the lines have exponential distributions. Dinesh-Kumar et al. [24] exhibited an optimization model for optimizing the reliability, maintainability, and supportability under performance-based logistics using goal programming. Upadhyay and Gupta [25] presented the posterior analysis of the modified two-parameter Weibull distribution.

Bellman and Zadeh [26] encouraged the construction of fuzzy optimization by providing the aggregation operators, which combine fuzzy goals and fuzzy decision space. After this pioneering work, a great number of articles dealing with fuzzy optimization problems have come out. Zimmermann [27] initiated the application of fuzzy theory to optimization by solving theoretical, fuzzy, linear programming problems. Huang [28] presented a fuzzy multi-objective optimization decision-making problem on the series-system reliability with reliability and cost objectives. Huang et al. [29] formed an effective multi-objective optimization method named as intelligent interactive multi-objective Optimization Method. Mahapatra and Roy [30] offered a fuzzy multi-objective mathematical programming technique based on generalized fuzzy set and they implemented it in multi-objective reliability optimization models. Apart from these, diverse methods have been offered by the various researchers in which both kinds of uncertainties; optimization and ambiguity are examined [31–38].

1.2 Objective of the Thesis

By keeping the motivation from the reliability and maintainability issues addressed in the literature to solve the reliability optimization problems, this thesis discusses some reliability and maintenance scheduling actions for series-parallel systems. In the present thesis, the reliability optimization model has been composed to design the reliable components of a system to maximize their profit and length. For it, some bi-objective reliability-cost optimization model and by considering the preventive maintenance scheduling and actions as compared to corrective maintenance actions are discussed intensely. The obtained optimization model is solved with one of the evolutionary algorithm named as particle swarm optimization. Based on the final reported parameters and the optimal cost or reliability of the system, a decision-maker can choose a desirable one according to his need to reach the desired goal.

1.3 Structure of the Thesis

The present dissertation is composed into four chapters which are concisely compiled as follows:

A concise description of the similar work of various authors in the evaluation of reliability and maintainability is exhibited in the present one. In **Chapter 2**, the basic and preliminaries linked to the reliability are furnished.

Chapter 3 offers a new interactive multi-objective reliability-cost optimization model of a series-parallel system. The contradictory nature within the objectives is resolved by defining the fuzzy region of satisfaction. Further, by addressing the choices of the decision-maker towards the given objectives, the estimated bi-objective optimization model is molded into the single objective optimization model. Finally, the received problem is solved with the help of the PSO algorithm. Based on the reported parameters and the optimal cost and reliability of the system, a decision-maker can choose a good one according to his need to arrive at the desired goal.

In **Chapter 4**, we perform a periodic maintenance scheduling actions of a series-parallel system by examining the three main actions, namely (1a), (1b) and (2p)-maintenance for a multi-components system based on maintenance cost. The overall preventive maintenance (PM) interval of the system is considered as the minimum among them. Finally, the behavior of the system is scrutinized based on the reliability achievement and the effect of the PM actions is founded based on maximizing the maintenance-benefit analysis. The approach has been explained through a numerical case study.

Chapter 2

Preliminaries

This chapter describes basic definition related to reliability and maintainability, which represents the main aspects of reliability engineering.

2.1 Reliability Aspects

The purpose of reliability engineering is to promote methods and tools to assess and demonstrate reliability, maintainability, availability, and safety of systems and their components/equipment, as well as to support design/production engineers in building in these characteristics.

2.1.1 Reliability

Reliability is a characteristic of an item(component or system), expressed by the probability that the item (component/system) will perform its required function under given conditions for a stated time interval [39].

Mathematically,if we define continuous random variable T to be the time to failure of the component/system; $T \geq 0$, then the basic reliability function $R(t)$, is defined for time to failure of the system (or subsystem) as

$$R(t) = Pr(T \geq t) = 1 - \int_0^t f(u)\ du \tag{2.1}$$

where $R(t) \geq 0$, $R(0) = 1$, and $\lim_{t \to \infty} R(t) = 0$ and $f(t)$ failure probability density function.

In addition to the probability function, there is another function, called the failure rate or hazard rate function, is often used in reliability. It provides an instantaneous (at time t) rate of failure. The conditional probability of a failure in the time interval from t to $t + \delta t$ given that the system has survived to time t is

$$Pr\{t \leq T \leq t + \delta t \mid T \geq t\} = \frac{R(t) - R(t + \delta t)}{R(t)} \tag{2.2}$$

then $\frac{R(t) - R(t + \delta t)}{R(t)\delta t}$ is the conditional probability of failure per unit of time (failure rate). The rule of conditional probability therefore dictates that:

$$\lambda(t) = \frac{-dR(t)}{dt} \cdot \frac{1}{R(t)} = \frac{f(t)}{R(t)} \tag{2.3}$$

then $\lambda(t)$ is known as the instantaneous hazard rate or failure rate function. Based on these hazard rate function, the reliability function can be derived as

$$R(t) = \exp\left[-\int_0^t \lambda(u)\, du\right] \tag{2.4}$$

The mean time to failure(MTTF) of the system is defined as

$$MTTF = \int_0^\infty R(t)dt \tag{2.5}$$

2.1.2 Maintainability

Maintainability refers to the measures taken during the development, design, and installation of a manufactured product that reduce required maintenance, manhours, tools, logistic cost, skill levels, and facilities, and ensure that the product meets the requirements for its intended use [40]. The key maintainability figures of merit are the mean time to repair (MTTR) and a limit for the maximum repair time. To quantify the repair time, let T be the continuous random variable representing the time to repair a failed unit, having a probability density function of $h(t)$, then the cumulative distribution function $M(t)$ is defined below [40]

$$\mathrm{P(T \leqslant t)} = M(t) = \int_0^t h(u)du \tag{2.6}$$

The MTTR is defined as:

$$\mathrm{MTTR} = \int_0^\infty t\ h(t)dt = \int_0^\infty (1 - \mathrm{M}(t))dt \tag{2.7}$$

For an exponential distribution, maintainability of the system (M_s) with constant repair rate parameter μ_s is defined as

$$M_s(t) = 1 - e^{(-t/\mathrm{MTTR})} = 1 - e^{-\mu_s t} \tag{2.8}$$

2.1.3 Availability

Availability is defined as the probability of a product or system working satisfactorily at any given point of time when used under the given conditions of use [40]. At any given time t, the system will be operational if the following conditions are met:

$$A_s(t) = \frac{\mu_s}{\lambda_s + \mu_s} + \frac{\lambda_s}{\lambda_s + \mu_s} e^{-(\lambda_s + \mu_s)t} \tag{2.9}$$

where, λ_s and μ_s are respectively the failure and repair rates of the system.

The steady state availability, A, of a system is the limit of the instantaneous availability function as time approaches to infinity (∞) and is represented by

$$A = \lim_{t \to \infty} A_s(t) = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{2.10}$$

where, MTTR and MTBF are the mean time to repair and the mean time between failures of the system/component respectively.

2.2 Particle Swarm Optimization

Particle swarm optimization (PSO) was developed by Kennedy and Eberhart in 1995 [41, 42], based on the swarm behavior such as fish and bird schooling in nature.

2.2.1 PSO Algorithm

A PSO algorithm [41, 42] maintains a swarm of particles, where each particle represents a potential solution. In analogy with evolutionary computation paradigms, a swarm is similar to a population, while a particle is similar to an individual. The particle swarm process is stochastic in nature in which velocity vector is used to update the current position of each particle in the swarm. The velocity vector is updated and is based on the "memory" gained by each particle, conceptually resembling an autobiographical memory, as well as the knowledge gained by the swarm as a whole. Thus, the position of each particle in the swarm is adjusted according to its own experience and that of its neighbors. Numerically, let x_k^i and v_k^i respectively be the position and velocity of i^{th} particle in the search space at k^{th} iteration then the position of this particle at $(k+1)^{th}$ iteration is updated through the equation,

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{2.11}$$

and are illustrated in Fig. 2.1.

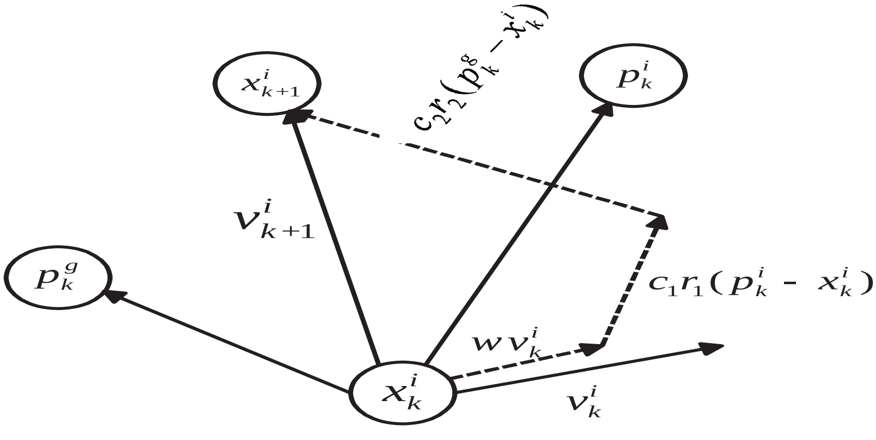

Figure 2.1: PSO position and velocity update movement.

Initial position x_0^i of the i^{th} particles randomly taken $x_0^i \sim U(x_{\min}, x_{\max})$ from a uniform distribution in the range [$x_{\min}$, $x_{\max}$], where $x_{\min}$ and x_{max} represent the design variables' lower and upper bounds respectively. v_{k+1}^i is the updated velocity vector of i^{th} particle at $(k+1)^{th}$ iteration. It is the velocity vector that drives the optimization process, and reflects both the experimental knowledge of the particle and socially exchanged information from the particle's neighborhood. The experimental knowledge of a particle is generally referred to as the cognitive component, which is proportional to the distance of the particle from its own best position (referred to as the particle's personal best position) found. The socially exchanged information is referred to as the social component of the velocity equation. The velocity vector of each particle is calculated through the equation,

$$v_{k+1}^i = \underbrace{w \cdot v_k^i}_{\text{inertia}} + \underbrace{c_1 \cdot r_1 \cdot (p_k^i - x_k^i)}_{\text{personal influence}} + \underbrace{c_2 \cdot r_2 \cdot (p_k^g - x_k^i)}_{\text{social influence}} \tag{2.12}$$

where v_k^i is the velocity vector at k^{th} iteration, r_1 and r_2 represent random numbers between 0 and 1; p_k^i represents the best ever position of i^{th} particle, and p_k^g corresponds to the global best position in the swarm up to k^{th} iteration. The movement of the i^{th} particle during k^{th} iteration to $(k+1)^{th}$ iteration is depicted in Fig. 2.1.

In equation (2.12), the first term $w\dot{v}_k^i$ is also referred as the inertia component and serves as a memory of the previous flight direction i.e. movement in the immediate past.

This memory term can be seen as a momentum which prevents the particle from drastically changing direction, and to the bias towards the current direction. The second term $c_1 r_1 (p_k^i - x_k^i)$ in equation (2.12) quantifies the performance of the i^{th} particle related to the past performance and is called cognitive component. In a series, the cognitive component resembles individual memory of the position that was best for the particle. The effect of this term is that particles are drawn back to their own best positions,resembling the tendency of individuals to return to situations or places that most satisfy in the past. The third term $c_2 r_2 (p_k^g - x_k^i)$ named as social component quantifies the performance of i^{th} particle relative to the group of particles, or neighbors. The effect of the social component is that each particle is also drawn towards the best position found by the particle's neighborhood.

2.2.2 Parametric aspects of PSO

The basic PSO is influenced by a number of control parameters [41–44], namely the dimension of the problem, number of particles, acceleration coefficient, inertia weight, number of iterations , and random value which scale the contribution of the cognitive and social components. Additionally, if velocity clamping or constriction is used, the maximum velocity and constriction coefficient also influence the performance of the PSO. This section discusses these parameters below:

- **Swarm size:** i.e the number of particles in the swarm: Mostly, larger is the number of swarm, greater is the initial diversity of the swarm - provided that good uniform initialization scheme is used to initialize the particles. A large swarm allows larger parts of the search space to be covered per iteration. However, having more particles increases the per iteration computational complexity, and degrades to a parallel random search. However, sometimes is also the case that more particles may lead to fewer iterations to reach a good solution, compared to smaller swarms.

- **Number of iterations:** The number of iterations to reach a good solution is also problem-dependent. Too few iterations may terminate the search prematurely. A too large number of iterations have the consequence of unnecessary added computational complexity (provided that the number of iterations is the only stopping condition).

- **Acceleration coefficient:** The acceleration coefficient, c_1 and c_2 called cognitive and social parameters respectively, together with the random vectors r_1 and r_2, control the stochastic influence of the cognitive and social components on the overall velocity of a particle. The constant c_1 expresses how much confidence a particle has in itself, while c_2 expresses how much confidence particle has in its neighbors. With $c_1 = c_2 = 0$, particles keep flying at their current speed until they hit a boundary of the search space (assuming no inertia). If $c_1 > 0$ and $c_2 = 0$, all particles are independent hill-climbers. Each particle finds the best position in its neighborhood by replacing the current best positions if the new position is better. On the other hand, if $c_1 = 0$ and $c_2 > 0$, the entire swarm is attracted to a single point, p_g.

- **Inertia weight:** The parameter w is called the inertia weight which plays an important role in the PSO convergence behavior since it is employed to control the exploration abilities of the swarm. A dynamic variation of inertia weight is proposed by Shi and Eberhart [43] in which w is linearly decreasing with each algorithmic iteration as shown in Eq. (2.13).

$$w_{k+1} = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{k_{\max}} \right) k \tag{2.13}$$

The essential steps of the particle swarm optimization can be summarized as the pseudo code given in Algorithm 1.

Algorithm 1 Pseudo code of Particle swarm optimization (PSO)

1: Objective function: $f(\mathbf{x}), \quad \mathbf{x} = (x_1, x_2, \ldots, x_K)$;
2: Initialize particle position and velocity for each particle and set $k = 1$.
3: Initialize the particle's best known position to its initial position i.e. $p_k^i = x_k^i$.
4: **repeat**
5: Update the best known position (p_k^i) for each particle as
  (a) Calculate fitness value
  (b) If the fitness value is better than the best fitness value (pbest) in history.
  (c) Set current value as the new pbest.
6: Update the swarm's best known position (p_k^g)
7: Calculate particle velocity according to the velocity equation (2.12).
8: Update particle position according to the position equation (2.11).
9: **until** requirements are met.

2.3 Parameter Setting

In all the experiments throughout the thesis, the values of the common parameters used in each algorithm such as population size and total evaluation number are chosen to be the same, and are set to be randomly as $20 \times D$ and 1000 respectively where D is the dimension of the problem. The method has been implemented in Matlab and the program has been run on a T6400 @ 2GHz Intel Core(TM) 2 Duo processor with 2GB of Random Access Memory. In order to eliminate stochastic discrepancy, 30 independent runs have been made that involves 30 different initial trial solutions. The termination criterion has been set either limited to a maximum number of generations or to the order of relative error equal to 10^{-6}, whichever is achieved first. The other specific parameters of algorithms are given below:

GA Settings: In all experiments, we employed a real coded GA having evaluation, fitness scaling, crossover, mutation units. Roulette wheel selection criterion is employed to choose better fitted chromosomes. One-point crossover with the rate of 0.85 are employed and random point mutation with rate 0.01 are used in the present analysis for the reproduction of new solution.

PSO Settings: Cognitive and social components (c_1 and c_2 in (2.12)) are constants that can be used to change the weighing between personal and population experience, respectively. In our experiments cognitive and social components were both set to 1.49. Inertia weight (w), which determines how the previous velocity of the particle influences the velocity in the next iteration, was defined as linearly decreasing from initial weight $w_{\max} = 0.9$ to final weight $w_{\min} = 0.4$ with the relation given in equation (2.13) [43, 44].

Chapter 3

Interactive Multi-objective Reliability-Cost optimization model of series-parallel system

Multi-objective optimization is a design methodology that optimizes a collection of objective functions systematically and simultaneously. It has been intensively used in engineering applications for design automation over multiple conflicting design objectives. In this chapter, we offer an approach to solving the reliability-cost problem of some series-parallel system. The various preferences of the decision making towards the assessing of their decision are considered and hence provided their corresponding solution. The impreciseness in the data is managed with the help of the fuzzy optimization techniques by defining the membership functions for each objective.

3.1 Introduction

To remain competitive and to provide timely and accurate services the companies are viewing reliability and maintainability issues as a part of a corporate quest to improve the quality of the products/processes and services delivered. A company cannot choose a rapid response plan if its systems are unavailable and unreliable. In a classical optimization model, the system and element lifetimes are expected to be random variables and the system performance such as system reliability is estimated by using the probability theory. Regrettably, this premise is not relevant in a wide range of situations. In such situations, the traditional reliability theory does not constantly provide beneficial knowledge to the practitioners due to the weakness of being able to handle only quantitative information [45–48]. The sequences obtained are therefore not much useful. This is primarily because there is a meaningful impact on subjective information. It has, therefore, become inevitable to consider subjective information along with quantitative databases to arrive at useful results in analysis. The use of the fuzzy set theory is an extra way to examine subjective information or uncertainties [49]. Due to incomplete and uncertain input information, mathematical models of such problems are acquired in the fuzzy environment and the optimization problem under consideration becomes a fuzzy programming problem.

In reliability optimization problems, it is often needed to minimize or maximize several objectives subject to several constraints. For instance, a planner is needed to minimize the system cost while concurrently maximizing system reliability. Therefore, multi-objective functions display a critical perspective of the reliability design of the engineering systems. Fuzzy set as suggested by many researchers is used to account for the uncertainty included in the available information i.e. in the extracted data. Fuzzy goals corresponding to each objective are set by united the results of their system behavior analysis for the optimization of system performance. For this, a fuzzy bi-objective optimization problem with decision variables in the form of a crisp range is expressed.

In these approaches, the parameters (coefficients) involved are treated as fuzzy parameters, since in tradition they are often known to experts as fuzzy numerical data rather than crisp data. Bellman and Zadeh [26] inspired the development of fuzzy optimization by providing the aggregation operators, which combine fuzzy goals and fuzzy decision space. After this pioneering work, a great number of articles dealing with fuzzy optimization problems have come out. Zimmermann [27] received the application of fuzzy theory to optimization by solving theoretical, fuzzy, linear programming problems. Huang [28] presented a fuzzy multi-objective optimization decision-making problem of series system reliability with two or more objectives. Apart from these, numerous approaches have been proposed by the various researchers in which both kinds of uncertainties; vagueness and ambiguity are considered [28–38, 50–52].

In the light of reviewed literature, an interactive fuzzy multi-objective reliability optimization problem (MOROP) has been introduced in this chapter, based on past failure and repair data, for optimization of system performance. For this, we establish a reliability-cost problem of some series-parallel system. The various choices of the decision making towards the assessing of their decision are taken and hence provided their corresponding solution. The presented approach is illustrated through a case study of one system of a urea fertilizer plant.

3.2 Reliability Optimization Model

The problem of system reliability may be set as a typical non-linear programming problem with non-linear cost-functions in a diverse fuzzy environment.

3.2.1 Formulation of Reliability-cost Model

Reliability is one of the vital attributes of performance in arriving at the optimal design of a system. In practical, consider the reliability problem of a system consists of n components. Each component has reliability R_i for the i^{th} components for $i = 1, 2, \ldots, n$. Then the system reliability is written in the form of reliability of each components as

$$R_s(R_1, R_2, \cdots, R_n) = \begin{cases} \prod_{i=1}^{n} R_i & \text{for series system} \\ 1 - \prod_{i=1}^{n} (1 - R_i) & \text{for parallel system} \\ \text{or combination of series and parallel system} \end{cases} \tag{3.1}$$

According to Aggarwal and Gupta [53] the cost of reliability is monotonically increasing function of reliability. Based on Tillman et al. [54, 55] the i^{th} components reliability cost is $C_i(R_i)$. Hence the system cost is given by

$$C_s(R_1, R_2, \cdots, R_n) = \sum_{i=1}^{n} C_i(R_i) \tag{3.2}$$

where $C_i(R_i) = a_i \log\left(\dfrac{1}{1 - R_i}\right) + b_i$, $C_i > 0$, a_i and b_i are the constants associated with the system cost which depends on the intrinsic property of the system/component for $i = 1, 2, \ldots, n$.

In reliability optimization problems, it is usually wanted to minimize or maximize several objectives subject to several constraints. To compare the characteristics of the model, a proper form of optimization model by holding the system reliability and cost as an objective is

$$
\begin{aligned}
Maximize \quad &: \quad R_s(R_1, R_2, \cdots, R_n) = \begin{cases} \prod_{i=1}^{n} R_i & \text{for series system} \\ 1 - \prod_{i=1}^{n} (1 - R_i) & \text{for parallel system} \\ \text{or combination of series and parallel system} \end{cases} \qquad (3.3) \\
Minimize \quad &: \quad C_s(R_1, R_2, \cdots, R_n) = \sum_{i=1}^{n} \left\{ a_i \log \left(\frac{1}{1 - R_i} \right) + b_i \right\} \\
subject\ \ to \quad & \quad R_{i,\min} \leqslant R_i \leqslant 1, \quad R_{s,\min} \leqslant R_s \leqslant 1 \quad \text{for} \quad i = 1, 2, \ldots, n
\end{aligned}
$$

3.2.2 System Reliability Optimization Model in Fuzzy Environment

Conventional optimization methods assume that all study information of an optimization model is exactly familiar. However, in the modern world, we usually face the circumstances that we have to decide ambiguity due to the appearance of unfinished or imprecise information. Thus, in a realistic touch, the creation of the functions in the optimization problem (3.3) is not clear. To get the model more manageable and versatile to the process, the model (3.3) can be interpreted with fuzzy numbers as.

$$
\begin{aligned}
\widetilde{Max} \quad &: \quad \{R_s, -C_s\} \qquad (3.4) \\
subject\ \ to \quad & \quad R_{i,\min} \leqslant R_i \leqslant 1, \quad R_{s,\min} \leqslant R_s \leqslant 1 \quad \text{for i=1,2,\ldots, n}
\end{aligned}
$$

The symbol $\widetilde{Max}$ denotes a relaxed or fuzzy version of 'Max'

3.3 Interactive Methods for Solving Multi-Objective Reliability optimization Problem

In usual, a reliability optimization problem is resolved with the hypothesis that the coefficients or cost of components are designated in a specific way. In modern life, it is not likely to get proper information due to the occurrence of uncertainties. Thus, due to this unfinished and uncertain information, mathematical representations are viewed under the fuzzy conditions.

In a fuzzy multi-objective optimization problem, an optimal solution that concurrently optimizes all the goals is rarely possible. In such circumstances, one frequently measures to seek the most reliable potential solution in the appearance of rough, or imprecise erudition which is as close to the decision maker's (DM's) expectations. Exploration for such a pleasant solution necessitates working the multi-objective fuzzy optimization problem in an interactive way wherein the DM is originally required to define his or her choices. Based on given choices, the dilemma is answered and the DM is provided with a reasonable solution. If the DM is happy with this answer the problem stops there, unless, invited to alter their choices in the knowledge of the earlier received outcomes. This iterative procedure is resumed till a satisfying solution is arrived which is close to DM's expectations.

The detail of the computational method for solving the fuzzy multi-objective reliability optimization problem (MOROP) iteratively is described in the following steps:

Step 1: **Find the ideal and anti-ideal values of each objective function:** Solve the optimization problem (3.3) as a single objective non-linear programming problem, for obtaining the ideal solutions R_i^*, by taking one objective at a time under a given set of constraints. Based on these ideal solutions, the pay-off matrix is formulated as

$$\begin{bmatrix} R_s(R_1^*) & C_s(R_1^*) \\ R_s(R_2^*) & C_s(R_2^*) \end{bmatrix}$$

From this matrix, the lower and upper bound for each objective is calculated as

$$R_s^l = \min\{R_s(R_1^*), R_s(R_2^*)\} \quad ; \quad R_s^u = \max\{R_s(R_1^*), R_s(R_2^*)\}$$

and

$$C_s^l = \min\{C_s(R_1^*), C_s(R_2^*)\} \quad ; \quad C_s^u = \max\{C_s(R_1^*), C_s(R_2^*)\}$$

Step 2: **Establishing the fuzzy goals towards the objective functions:** Let $\widetilde{f}_1$ and $\widetilde{f}_2$ be the fuzzy region of satisfaction of system reliability (R_s) and system cost (C_s) respectively and μ_{R_s} and μ_{C_s} be their corresponding membership functions. Then the fuzzy objective stated by a designer can be quantified by eliciting corresponding linear membership functions, using the minimal and maximal feasible values of each objectives as obtained during the Step 1, and are defined as

For maximization goal (R_s)

$$\mu_{R_s}(x) = \begin{cases} 1, & R_s(x) \geqslant R_s^u \\ \frac{R_s(x) - R_s^l}{R_s^u - R_s^l}, & R_s^l \leqslant R_s(x) \leqslant R_s^u \\ 0, & R_s(x) \leqslant R_s^l \end{cases} \tag{3.5}$$

Here, $\mu_{R_s}(x)$ is strictly monotonically increasing function of $R_s(x)$.

For minimization goal (C_s)

$$\mu_{C_s}(x) = \begin{cases} 1, & C_s(x) \leqslant C_s^l \\ \frac{C_s^u - C_s(x)}{C_s^u - C_s^l}, & C_s^l \leqslant C_s(x) \leqslant C_s^u \\ 0, & C_s(x) \geqslant C_s^u \end{cases} \tag{3.6}$$

Here, $\mu_{C_s}(x)$ is strictly monotonically decreasing function of $C_s(x)$.

Step 3: **Equivalent single optimization problem:** Using the achieved objectives' membership functions of R_s and C_s, it is very important to choose the aggregation operator. Owing to the importance of each objective in terms of weight vector w_1 and w_2, called objective weights, Huang [28] defined the problem as

$$\mu_D = \left(1 \wedge \frac{\mu_{R_s}}{w_1}\right) \wedge \left(1 \wedge \frac{\mu_{C_s}}{w_2}\right) \tag{3.7}$$

Thus, by considering the DM/system expert preferences as $W = [w_1, w_2]$, problem is formulated as a single objective optimization problem and is given as:

$$\begin{aligned} &Maximize \quad : \quad \mu_D(x) = \min\left(\min\left(1, \frac{\mu_{R_s}(x)}{w_1}\right), \min\left(1, \frac{\mu_{C_s}(x)}{w_2}\right)\right) \\ &Subject \;\; to \quad x_k^l \leq x_k \leq x_k^u, \quad ; \quad k = 1, 2, \ldots, K, \\ &\qquad\qquad\qquad w_t \in [0, 1], \qquad\quad t = 1, 2 \end{aligned} \tag{3.8}$$

where w_t represents the t^{th} objective weight suggested by DM, x is the vector of decision variables, x_k^l and x_k^u are the lower and upper bounds of decision vector x_k, respectively. The obtained optimization problem is solved with PSO algorithm.

Step 4: **Adjusting the preference parameters:** If the DM is satisfied by the solution obtained in Step 3 then the approach stops successfully. Otherwise update the key preference parameters W=$[w_1, w_2]$ to meet the DM's choice and the method again go back to Step 3. The process is repeated until DM is satisfied.

3.4 System Description

The field study is carried in the National Fertilizer Limited plant situated at Panipat, near Delhi, having urea production capacity ranging from 1500–2000 metric tons per day. A fertilizer industry is a large, complex and repairable engineering unit which is a combination of ammonia and urea plant. The raw materials for urea production are ammonia and carbon dioxide. The urea plant is composed of urea synthesis, urea decomposition, urea crystallization, and urea prilling system.

Among this system, the most important part is the prilling system. In this system urea crystals are melted, sprayed through distributors and fall in urea prilling tower against the ascending air allowing getting prilled on the way. The prilled urea is collected at the bottom of the urea prilling system and sent to the bagging section.

3.4.1 Prilling and Carbon Recovery Unit

In brief, the various components in series configuration and their associated subsystems of this unit are described as follows.

- Subsystem (U_j) consists of four units viz- cyclone (U_1), screw conveyor (U_2), meltor (U_3) and strainer (U_4). Failure of any one unit causes complete failure of the system.
- Subsystem (V), consists of eleven distributors operating simultaneously with one standby. Failure of any one unit does not affect the system availability. Complete failure take place only when more than one failure occurs.
- the belt conveyor (W), consists of one unit and is employed for carrying the product to trommer. Failure of belt conveyor leads to huge accumulation of urea, blocking the flow path of prilled urea. Hence failure of subsystem (W) would affect the working of the system.

The systematic diagram of the working components of the system is shown in Fig. 3.1.

3.4.2 Mathematical Model and parameter setting of the system

Let R_i and C_i be the reliability and cost of the component of the system while R_s and C_s be the overall system reliability and cost. Then, the MOROP for the examined system is

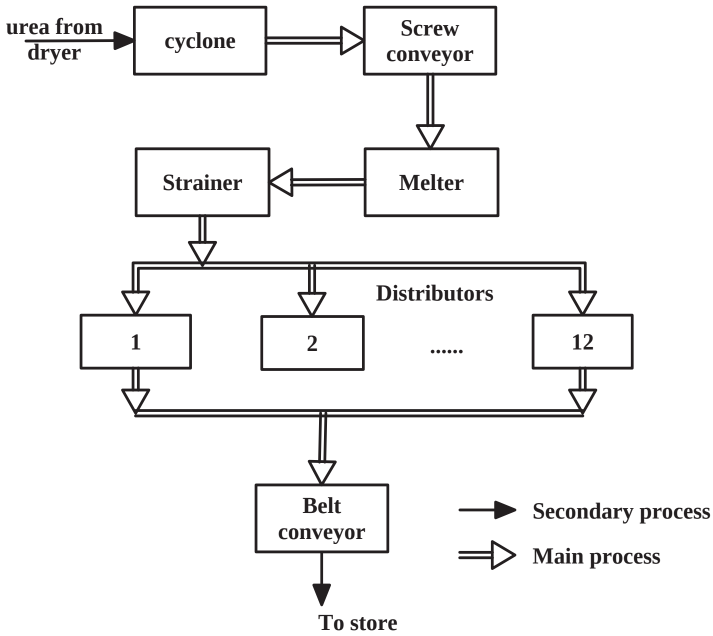

Figure 3.1: Systematic diagram of Prilling and Carbon Recovery Unit

expressed by taking systems reliability and cost as an objective and crisp failure rates (λ_i) and repair times (τ_i) as decision variables under the considered ±15% uncertainty level or support towards the data. The model may be expressed mathematically as

$$\begin{aligned}
\text{Maximize } R_s &= \exp(-\lambda_s t) \\
\text{Minimize } C_s &= \sum_{i=1}^{17} \left\{ a_i \log\left(\frac{1}{1-\exp(\lambda_i t)} \right) + b_i \right\} \\
\text{subject to } & (1-s)x_k \le x_k \le (1+s)x_k \\
& k = 1, 2, \ldots, 34 \\
& x = [\lambda_1, \lambda_2, \ldots, \lambda_{17}, \tau_1, \tau_2, \ldots, \tau_{17}]^T \\
& \lambda_5 = \lambda_6 = \ldots = \lambda_{16} \\
& \tau_5 = \tau_6 = \ldots = \tau_{16} \\
& s = 0.15 (\text{considered uncertainty level})
\end{aligned}$$

where λ_s is the system failure rate expressed as

$$\lambda_s = \sum_{i=1}^{4} \lambda_i + \prod_{i=5}^{16} \lambda_i \left[\sum_{i=5}^{16} \prod_{\substack{j=5 \\ i \neq j}}^{16} \tau_j \right] + \lambda_{17} \tag{3.9}$$

The information linked to the main components of the system, in the form of failure rate (λ_i' s) and repair time (τ_i's), is collected and are given in Table 3.1.

Table 3.1: Input Data for Prilling and Carbon Recovery Unit

Component→	Cyclone ($i = 1$)	Screw conveyor ($i = 2$)	Meltor ($i = 3$)	Strainer ($i = 4$)	Distributors ($i = 5, ..., 16$)	Belt conveyor ($i = 17$)
Failure rate $\lambda_i(\text{hrs}^{-1})$	0.0035	0.0105	0.00131	0.0051	0.0027	0.0104
Repair time $\tau_i(\text{hrs})$	1.5	2.0	1.35	1.45	1.0	2.5

Randomly taken different values for the parameters a_i and b_i are tabulated in Table 3.2.

Table 3.2: Values of a_i's and b_i's for Prilling and Carbon Recovery Unit

Component→	Cyclone ($i = 1$)	Screw Conveyor ($i = 2$)	Meltor ($i = 3$)	Strainer ($i = 4$)	Distributors ($i = 5, ..., 16$)	Belt Conveyor ($i = 17$)
a_i	7.5	10	8.75	6.54	3.53	5.5
b_i	50	70	65	50	30	50

3.4.3 Solution procedure

The steps of the proposed method are executed here to find the optimal solution of the model described in the above section.

Step 1: Solve the above model for R_s and C_s independently and hence after obtaining their optimal solution, a pay off matrix is formulated as

$$\begin{array}{cc} R_s & C_s \end{array}$$
$$\begin{bmatrix} 0.72674108 & 930.103195 \\ 0.73484183 & 906.239993 \end{bmatrix}$$

Based on this payoff matrix, we get the ideal values of the objective functions are $R_s^l = 0.72674108$, $R_s^u = 0.73484183$, $C_s^l = 906.239993$ and $C_s^u = 930.103195$.

Step 2: By using these ideal values, a linear membership function corresponding to R_s and C_s are constructed as

$$\mu_{R_s}(x) = \begin{cases} 1, & R_s(x) \geqslant 0.73484183 \\ \frac{R_s(x)-0.72674108}{0.73484183-0.72674108}, & 0.72674108 \leqslant R_s(x) \leqslant 0.73484183 \\ 0, & R_s(x) \leqslant 0.72674108 \end{cases} \quad (3.10)$$

and

$$\mu_{C_s}(x) = \begin{cases} 1, & C_s(x) \leqslant 906.239993 \\ \frac{930.103195 - C_s(x)}{930.103195 - 906.239993}, & 906.239993 \leqslant C_s(x) \leqslant 930.103195 \\ 0, & C_s(x) \geqslant 930.103195 \end{cases} \quad (3.11)$$

Step 3: By using fuzzy satisfaction degrees given in Eqs. (3.10), (3.11) and the importance factor towards each objective, the following equivalent crisp optimization model is formulated as

$$\begin{aligned} \text{Minimize} \quad : \quad = \quad & \min\left(\min\left(1, \frac{\alpha_1(x)}{w_1}\right), \min\left(1, \frac{\alpha_2(x)}{w_2}\right)\right) \\ \text{subject to} \quad & \alpha_1(x) = \mu_{R_s}(x); \alpha_2(x) = \mu_{C_s}(x) \\ & (1-s)x_k \leq x_k \leq (1+s)x_k \\ & k = 1, 2, \ldots, 34 \\ & x = [\lambda_1, \lambda_2, \ldots, \lambda_{17}, \tau_1, \tau_2, \ldots, \tau_{17}]^T \\ & \lambda_5 = \lambda_6 = \ldots = \lambda_{16} \\ & \tau_5 = \tau_6 = \ldots = \tau_{16} \\ & s = 0.15(\text{considered uncertainty level}) \end{aligned} \quad (3.12)$$

Step 4: PSO has been used to solve the equation (3.12) with the initial preference of the weight towards the objectives as $W1 = [1\ 1]$. Since in the first iteration DM does not want the preference towards the objectives i.e, they pay equal attention towards each objective. Hence the weight vector is taken as 1 for each objective. Results corresponding to first iteration are shown in Table 3.3 under the iteration I column, which is: $(R_s, C_s) = (0.73515995, 909.333116)$ with membership value $(\mu_{R_s}, \mu_{C_s}) = (1.00, 0.87038)$. Not satisfied with this outcome or willing to know other possible optimal solutions, keeping this result in view, let DM is decided to give more importance on reliability objective than cost objective and gave a preference weight vector 1 corresponding to reliability and 0.5 to cost i.e W2=[1 0.5]. In other words, DM wants to pay two times more attention to the reliability objective than the cost objective. Outcome of this iteration is: $(R_s, C_s) = (0.73874358, 915.249094)$ with membership value $(\mu_{R_s}, \mu_{C_s}) = (1, 0.62246)$. The process is repeated until DM is fully satisfied. In this way, DM obtained a different possible solution for different satisfaction levels. The process is stopped after V iteration (it may continue further till DM is satisfied). Outcome of iteration V is: $(R_s, C_s) = (0.73076532, 908.937000)$ with membership value $(\mu_{R_s}, \mu_{C_s}) = (0.49683, 0.88698)$. This result shows that 49.683% achievement for reliability and 88.698% for the cost of respective fuzzy goals. A comparison of the results listed in the table with the results obtained by GA. Thus for different preferences suggested by DMs, optimum values of systems' reliability and cost are achieved. The optimum design parameters of design variables corresponding to optimum values are also summarized in Table 3.3.

3.5 Conclusion

In this chapter, we have introduced an approach for solving the reliability-cost optimization problem under a distinct fuzzy environment. In the estimated model, the reliability and cost both recognized as an objective concurrently to optimize the production of the series-parallel system. A contradictory nature between the objectives is fixed by setting the fuzzy region of satisfaction in terms of linear membership functions. Since reliability decision is usually presented in the earliest stage of system design and the information at this stage is incomplete and imprecise, it is essential to rely on the wisdom of DMs and experts. Based on their choices in the form of weights towards the objectives along with the achieved objective membership functions, a MOROP is reformulated as a crisp optimization model. Eventually, based on the different choices of the experts, towards the ratings of the objectives, a model is determined with the PSO and GA algorithms' iteratively. The achieved optimal failure rate and repair times of the system will also support the DM's to prepare the repair policy. Furthermore, based on the reported parameters and the optimal cost and reliability of the system, a decision-maker can choose a desirable one according to his need to reach the desired goal.

Table 3.3: Solution of the Reliability Optimization Problem of Prilling and Carbon Recovery Unit

Iteration		I		II		III		IV		V	
		DM's specifications									
	w_1	1		1.0		0.8		0.2		0.5	
	w_2	1		0.5		0.2		0.8		1.0	
		Optimal solutions									
		GA	PSO	GA	PSO	GA	PSO	GA	PSO	GA	PSO
	μ_{R_s}	0.89836501	1	1	1	1	1	0.30343729	0.33043175	0.76236645	0.49683581
	μ_{C_s}	0.84388590	0.87038105	0.66231054	0.62246889	0.45202323	0.54777748	0.80550901	0.83076796	0.86331950	0.88698052
	R_s	0.73401851	0.73515995	0.73504385	0.73874358	0.74208385	0.74609841	0.72919915	0.72941782	0.73291682	0.73076582
	C_s	909.965376	909.333116	914.298345	915.249094	919.316473	917.031471	910.881171	910.278412	909.501628	908.937000
		Decisons variables corresponding to main components of the system									
Cyclone	λ_i	0.00383864	0.00402419	0.00346138	0.00309557	0.00313603	0.00321235	0.00342517	0.00389626	0.00396206	0.00402500
	τ_i	1.72239805	1.65735947	1.39354434	1.52287456	1.57598926	1.70677541	1.47997715	1.33345304	1.38763749	1.27500000
Screw conveyor	λ_i	0.01081259	0.01056475	0.01001962	0.00965047	0.01014786	0.00920898	0.01067895	0.01130749	0.01140976	0.01207500
	τ_i	2.15992312	2.26868575	1.80571304	2.23738527	1.71434659	2.16249796	1.74572794	2.16410617	1.95936347	1.70000000
Meltor	λ_i	0.00147316	0.00149069	0.00116191	0.00122187	0.00142269	0.00131407	0.00149837	0.00134386	0.00148765	0.00150650
	τ_i	1.29539596	1.55195752	1.52278438	1.50935279	1.22012463	1.41049030	1.27103504	1.18178729	1.35420941	1.14750000
Strainer	λ_i	0.00531248	0.00565568	0.00506771	0.00521986	0.00566511	0.00559143	0.00447018	0.00562367	0.00527566	0.00491972
	τ_i	1.53413042	1.33489413	1.60721157	1.58299586	1.53420780	1.59419958	1.43362065	1.40471074	1.43047767	1.23250000
Distributors	λ_i	0.00308255	0.00310191	0.00298851	0.00295963	0.00258511	0.00281316	0.00308159	0.00305671	0.00308215	0.00310500
	τ_i	0.86076815	1.11660468	0.97243514	0.89969987	0.89862251	1.03822924	0.98088386	0.99996760	1.06832319	0.85000000
Belt conveyor	λ_i	0.00948524	0.00903141	0.01107190	0.01109267	0.00945762	0.00996295	0.01150817	0.00937957	0.00893717	0.00884000
	τ_i	2.55353519	2.60284494	2.54068251	2.17901956	2.68707552	2.15273268	2.41597052	2.39040962	2.76655273	2.12500000

Chapter 4

Periodic Preventive maintenance scheduling actions of series - parallel system

In this chapter, we offer periodic maintenance scheduling actions of a series-parallel system by studying the three main actions, namely (1a), (1b) and (2p)-maintenance for a multi-components system based on maintenance cost. The overall preventive maintenance (PM) interval of the system is grasped as the minimum among them. Finally, the behavior of the system is observed based on the reliability performance and the effect of the PM actions is built based on maximizing the maintenance-benefit analysis. The approach has been explained through a numerical case study.

4.1 Introduction

Today's, an industrial system becomes so complex due to the large demand of the system and their internal structures and hence, as a result, it is difficult to maintain the system performance within a regular interval of time. Furthermore, the unfortunate consequences of low availability and high maintenance cost of such systems led to the request for high reliability, high maintainability, and low mean time to support. To improve the performance of the system, there is a need to opt for proper maintenance action to maintain their life and increase their profit. Therefore, this leads to the study of the impact of the maintenance and its scheduling is the most important task of the systems.

In many systems, there are mainly two strategies appropriated for the maintenance -

corrective (unplanned) and preventive (planned) maintenance. In unplanned maintenance, no maintenance action is taken out until the component is shut-down. However, in planned maintenance, a system is reviewed at a regular interval of time by delivering the task like painting, waxing, repair, replacement, etc. By doing the PM actions, the length of the component increases while in corrective maintenance (CM), the cost of the component is very high. To scrutinize the outcomes of the maintenance of a multi-component system by combining the following three main aspects of the PM actions.

- **(1a)-maintenance (mechanical service):** This type of action highlights on managing a system on the normal operating condition. It simply raises the deteriorated environment so that it can harmonize the sub-components to a more vigorous condition by lubricating, cleaning the dust, etc.

- **(1b)-maintenance (repair):** This type of activity is mainly utilized for some subcomponents which are valuable and/or obtaining them is not easy. It commonly includes the actions of (1a) and sewing/replacing for some simple parts such as springs, seals, belts, etc.

- **(2p)-maintenance (replacement):** This type of work is to substitute the subcomponent/ subsystem with a new one. It is usually adopted for the key subcomponents to withdraw dangerous damage that occurred.

Thus, this chapter intends to examine the impact of these PM actions on series-parallel systems. The interval of determining the PM schedule can be obtained either by numerical analysis or by optimization technique. To examine these thoroughly, we express a bi-objective optimization model for each subcomponent of the system by monitoring the maintenance cost and the availability of the system as an objective. The formulated model has been solved with the help of the PSO algorithm. Based on the obtained interval length of each component, the PM action (1a, 1b or 2p) has been chosen with an improvement factor for the system to raise the reliability of the system. The implementation of the PM action onto each component of the system is selected based on their maintenance-benefit analysis at each stage. The complete demonstration of the process is explained through a numerical example.

4.2 Maintenance planning and Scheduling

In this section, we express an optimization model to define the optimal length of the scheduling to save the money and increase the life of the component as well as the system.

4.2.1 Assumptions

The following are the assumptions made during executing the process.

i) The failure rate follows Weibull distribution and repair time is constant for each component of the system.

ii) Unpredictable failure of the system is determined by 1c-maintenance scheduling.

iii) The improvement factors and time are given.

iv) CM time is larger than PM time.

v) Dynamic behavior of the system reliability about PM action is given.

4.2.2 PM scheduling model

To balance the availability and the cost of the system, a decision-maker forever mark for a good solution to raise the performance of the system. For it, we create an availability-cost optimization model based on PM scheduling, whose mathematical structures are given as follows.

The availability of the system under CM and PM, denoted by A_c and A_p respectively, are given as

$$A_c = \frac{M}{M + m_c} \tag{4.1}$$

and

$$A_p = \frac{\overline{T}}{\overline{T} + m_p R(T) + m_c[1 - R(T)]} \tag{4.2}$$

where M is MTBF, m_c, m_p are mean repair time for CM and PM respectively. $\overline{T}$ is mean time between replacement. By setting $\alpha = \frac{\overline{T}}{M}$, $\mu = \frac{m_c}{M}$, $\gamma = \frac{m_p}{m_c}$, the Eqs. (4.1), (4.2) becomes

$$A_c = \frac{1}{1 + \mu} \tag{4.3}$$

and

$$A_p = \frac{\alpha}{\alpha + \mu[1 - k_1 R(T)]} \tag{4.4}$$

where $k_1 = 1 - \gamma$.

On the other hand, the maintenance cost under CM and PM, denoted by C_c and C_p respectively are as follows.

$$C_c = \frac{c_c m_c}{M + m_c} = \frac{c_c \mu}{1 + \mu} \tag{4.5}$$

and

$$\begin{aligned} C_p &= \frac{m_c c_c[1 - R(T)] + m_p c_p R(T)}{\overline{T} + m_p R(T) + m_c[1 - R(T)]} \\ &= \frac{\mu c_c[1 - (1 - \delta\gamma)R(T)]}{\alpha + \mu[1 - (1 - \gamma)R(T)]} \end{aligned} \tag{4.6}$$

where c_c, c_p represent the maintenance cost per unit time for CM and PM respectively; $\delta = \frac{c_p}{c_c}$.

For PM action, we should have

$$C_p - C_c < 0 \qquad ; \qquad A_p - A_c > 0 \tag{4.7}$$

By using Eqs. (4.3), (4.4), (4.5), (4.6), we simplify Eq. (4.7) and hence we obtain

$$\alpha > 1 - k_2 R(T) \qquad \text{and} \qquad \alpha > 1 - k_1 R(T) \tag{4.8}$$

where $k_2 = (1 - \delta\gamma) + \mu\gamma(1 - \delta)$.

As the factor $\gamma, \delta \in [0, 1]$ and hence $\mu\gamma(1 - \delta) <<< 1 - \delta\gamma$. Thus, k_2 can be approximately as

$$k_2 \cong 1 - \delta\gamma \tag{4.9}$$

Further, $\delta\gamma < \gamma$ and hence $k_2 \geq k_1$.

Thus, in view of Eq. (4.9), the Eq.(4.6) becomes

$$C_p = c_c \left[\frac{1 - k_2 R(T)}{\alpha/\mu + 1 - k_1 R(T)} \right] \tag{4.10}$$

For a system, consider n_p & n_c be the number of components replaced through PM and CM for a long operating time. Thus, the total number of replaced component is

$$N = n_p + n_c \tag{4.11}$$

Let $R(x)$ be the reliability of the component and T be the time of replacement of the component. Thus, $R(T)$ represent the reliability of the component that survive until the next PM. Thus, the value of n_p and n_c becomes

$$n_p = R(T) \cdot N, \tag{4.12}$$

and

$$n_c = [1 - R(T)] \cdot N \tag{4.13}$$

Consequently, the life cycle cost (LCC) of the system is given by

$$\begin{aligned} LCC &= n_p c_p + n_c c_c \\ &= c_p R(T) \cdot N + c_c [1 - R(T)] \cdot N \\ &= c_p R(T) \cdot \frac{T_0}{\int_0^T R(x)dx} + c_c [1 - R(T)] \cdot \frac{T_0}{\int_0^T R(x)dx} \end{aligned} \tag{4.14}$$

where $N = \frac{T_0}{\int_0^T R(x)dx}$ and $\int_0^T R(x)dx$ represent the mean time between replacements.

The minimum value of T corresponding to LCC is determined by

$$\begin{aligned} & \frac{d(LCC}{dt} = 0 \\ \Rightarrow \quad & \frac{T_0 \left[\int_0^T R(x)dx R'(T)(c_p - c_c) - R^2(T)(c_p - c_c) - R(T)c_c \right]}{\left[\int_0^T R(x)dx \right]^2} = 0 \\ \Rightarrow \quad & R(T) + h(T) \int_0^T R(x)dx = \frac{c_c}{c_c - c_p} \\ \Rightarrow \quad & R(T) + h(T) \int_0^T R(x)dx = \frac{1}{1 - \delta} \end{aligned} \tag{4.15}$$

where $h(\cdot)$ is the hazard rate function. Here, Eq. (4.15) gives the miminum value of T say T_a for which LCC cost is minimum. Thus, during PM scheduling, the value of T (PM interval schedule) should be less than T_a (i.e., $T \leq T_a$). Thus, during PM scheduling, Eq. (4.15) becomes

$$R(T) + h(T) \int_0^T R(x)dx \leq \frac{1}{1 - \delta} \tag{4.16}$$

4.2.3 Optimization model

During the PM schedule, the principal task is to minimize the maintenance cost per unit time and concurrently increase the availability of the system. Thus, it will drive us to formulate a bi-objective optimization model under the constraint as stated in the above section i.e.,

$$
\begin{aligned}
\text{Maximize} \quad & A_p = \frac{\alpha}{\alpha + \mu[1 - k_1 R(T)]} \\
\text{Minimize} \quad & C_0 = \frac{C_c}{c_c} = \frac{1 - k_2 R(T)}{\alpha/\mu + 1 - k_1 R(T)} \\
\text{subject to} \quad & \alpha > 1 - k_1 R(T) \\
& \alpha > 1 - k_2 R(T) \\
& 0 \leq \alpha \leq 1 \\
& A_p \geq \min\{A_i\} \\
& R(T) + h(T) \int_0^T R(x)dx \leq \frac{1}{1-\delta}
\end{aligned}
$$

The above optimization model is converted into single-objective model as

$$
\begin{aligned}
\text{Maximize} \quad & \frac{A_p}{C_0} \\
\text{subject to} \quad & \alpha > 1 - k_1 R(T) \\
& \alpha > 1 - k_2 R(T) \\
& 0 \leq \alpha \leq 1 \\
& A_p \geq \min\{A_{pi}\} \\
& R(T) + h(T) \int_0^T R(x)dx \leq \frac{1}{1-\delta}
\end{aligned}
\tag{4.17}
$$

Since model is nonlinear in nature, so we solve it with the help of PSO algorithm, whose brief description is given in Section 2.2 of Chapter 2.

4.3 Reliability under PM

For complex systems, the reliability can often be achieved through a PM either by 1a, 1b or 2p actions. Such an action can overcome the effect of aging or wear out and have a notable impact on the life of the system. Let m_1 be the improvement factor of (1a)-maintenance which is a ration of the survival part life to the original life. Let T^* be the optimal interval obtained after solving the model (4.17) and $R_{0,j}$ be the initial reliability of the j^{th} stage and $R_{v,j}$ be the reliability of the survival part. Then, the reliability at the j^{th} stage is defined as

$$R_j(t) = R_{0,j} R_{v,j}(t) \tag{4.18}$$

By considering the PM interval T^*, the expression of $R_{v,j}(t)$ becomes

$$R_{v,j}(t) = R\left(\frac{t-(j-1)T^*}{m_1}\right); \quad (j-1)T^* \leq t \leq jT^* \tag{4.19}$$

Next, we evaluate the effect of the various PM actions on $R_{0,j}$ and $R_{v,j}$ to model the reliability as follows:

i) **(1a)-maintenance:** Under this action, the failed component is not improved and hence by Eq. (4.19), the reliability of surviving part is computed, whereas the initial reliability $R_{0,j}$ is set as

$$R_{0,j} = R_{0,j-1} \cdot R(T^*) \tag{4.20}$$

$$R_{f,j-1} = R_{0,j} \tag{4.21}$$

where $R_{0,j-1}$, $R_{f,j-1}$ is initial and final reliabilities of the system on $(j-1)^{th}$ stage.

ii) **(1b)-maintenance:** Under this action, the failed component is recovered with an improvement factor $m_2 \in [0,1]$ and hence $R_{0,j}$ is computed as

$$R_{0,j} = R_{f,j-1} + m_2(R_0 - R_{f,j-1}) \tag{4.22}$$

Here, R_0 is initial reliability of the system.

iii) **(2p)-maintenance:** Under this action, system is replaced with a new one with initial reliability R_0 and final reliability is computed as

$$R_{v,j}(t) = R(t-(j-1)T^*) \tag{4.23}$$

Based on the optimal replacement time (T^*) of each component of system, obtained from the solution of model (4.17), the optimal replacement time of the system is found as $T = \min\{T^*\}$, while PM action namely (1a) and (1b) are taken when $T^* > T$. Further, PM schedule has been done when reliability at the next stage is less than threshold reliability $R_{\min}$ i.e., $R(2T) \leq R_{\min}$. Finally, the maintenance action has been taken based on the maintenance-benefit analysis at j^{th} stage computed by

$$B_{i,k} = \frac{\int_{t_j}^{\infty} R_{i,j+1}(x)dx - \int_{t_j}^{\infty} R_{i,j}(x)dx}{C_{i,k}} \tag{4.24}$$

where $k = 1, 2, 3$ represents the PM actions (1a, 1b, 2p) and i be the component, $C_{i,k}$ be the maintenance cost.

4.4 Case study

To demonstrate the above procedure, we take a numerical case study of the press unit of the paper mill.

4.4.1 System Description

The main function of this unit is to reduce the moisture content of the paper by pressing the pulp under the rolls received from forming unit of machine. The system consists of synthetic belt (felt), upper and bottom rollers as main components. The unit receives wet paper sheet from forming unit on to the synthetic belt, which is further, carried through press rolls thereby reducing the moisture content to almost 50-60%. The system consists of the following subsystems defined as:

- **Synthetic Felt(A):** It consists of a single belt only. Its failure causes the complete failure of the system.

- **Upper Rollers(B):** This subsystem consist of bearing, bending and rubber wear arranged in series configuration.

- **Lower Rollers(C):** It also has bearing, bending and rubber wear arranged in series configuration.

The block diagram of the working components of the system is shown in Fig. 4.1.

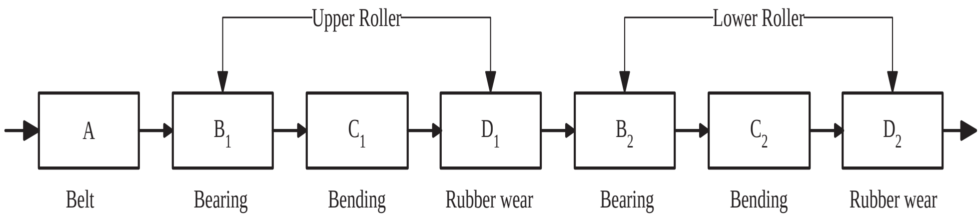

Figure 4.1: Block diagram of the Press unit

4.4.2 Scheduling action of the system

The reliability of the system is computed as

$$R_s(t) = R_C(t) \prod_{u=1}^{4} R_u(t) \tag{4.25}$$

Here $u = 1, 2, 3, 4$ corresponding to the component felt, roller bearing, bending and rubber wear, $R_C(t) = \exp(-0.0002t)$ represent the reliability of the surplus part of the system.

The input data related to the component of the system are summarized in Table 4.1, where β, θ represent the shape and scale parameters of Weibull distribution. Based on the

Table 4.1: Input Data for the Main Components of Press Unit

Components	Failure data		Repair time
	Weibull distribution		
	Scale (θ)	Shape (β)	m_c hrs
Felt ($i = 1$)	537	1.40	5.0
Bearing ($i = 2, 5$)	370	1.70	3.0
Bending ($i = 3, 6$)	426	1.51	3.5
Rubber Wear ($i = 4, 7$)	252	1.54	4.0

given information, the model (4.17) is formulated for each component of the system and the PSO algorithm has been used to optimize them. The results corresponding to them have been summarized in Table 4.2 associated with their availability and cost. From this table it is observed that

i) The PM interval of the given components of the system is 515, 73, 157, 86 hrs, respectively.

ii) The cost of the each component under CM and PM environment has been summarized in the table. It is clearly seen that the cost of the component decreases after implementing the PM scheduling.

Table 4.2: Optimal PM scheduling parameter of the components

Component	Decision Parameters	MTTF (hrs)	Cost		Availability	
			With CM	With PM	With CM	With PM
Felt	$\theta = 537$, $\beta = 1.40$, $\gamma = 0.4$, $\mu = 0.0102$ $\delta = 0.6$	489.434	0.010112	0.009641	0.989887	0.989505
Roller bearing	$\theta = 370$, $\beta = 1.70$, $\gamma = 0.2$, $\mu = 0.0091$ $\delta = 0.4$	330.130	0.009005	0.005378	0.990994	0.991621
Roller bending	$\theta = 426$, $\beta = 1.51$, $\gamma = 0.29$, $\mu = 0.0091$ $\delta = 0.6$	384.264	0.009026	0.007727	0.990973	0.991061
Rubber wear	$\theta = 252$, $\beta = 1.54$, $\gamma = 0.4$, $\mu = 0.0176$ $\delta = 0.4$	226.804	0.017330	0.014321	0.982669	0.980180

iii) It is also observed that after incorporating the PM scheduling, the availability of the system during PM increases as compared to CM actions.

To address the PM action on to the systems, Eq. (4.24) has been implemented for improving the performance of the system. For it, the cost of (1a)-and (1b)- are set to be 30% and 60% respectively of (2p)-maintenance. The initial reliability is set as 0.99999 for all components and the length of the system is expected to be 500 hrs. The minimum reliability is set to be $\{0.8, 0.6, 0.8, 0.8\}$ to access whether the PM action needs to be conducted or not, while the improvement factor are set to be $m_1 = m_2 = \{0.82, 0.75, 0.90, 0.80\}$. The PM interval for the whole system is set to be $T = \min\{515, 73, 157, 86\} = 73hrs$. Based on this information, the impact of each PM action onto the components are noticed and represented in Fig. 4.2. It is clearly seen that the impact of the different PM actions on the reliability of the system as well as each component. Thus, it also shows the significance of the (1a)-, (1b)- and (2p)- maintenance actions rather than only (2p)-action. This investigation will help the analyst to study the impact of each component on the final functioning of the system and hence scheduling the maintenance at a regular interval of time will increase the life of the system.

On the other hand, the simultaneously impact of the considered PM action, namely 1a-, 1b- and 2p-, on to each component and the whole system is depicted graphically in Fig. 4.3. From this figure and the PM scheduling action, it is noticed that after some time $T = 73hrs$, the first component needs to undergo (2p)- maintenance schedule

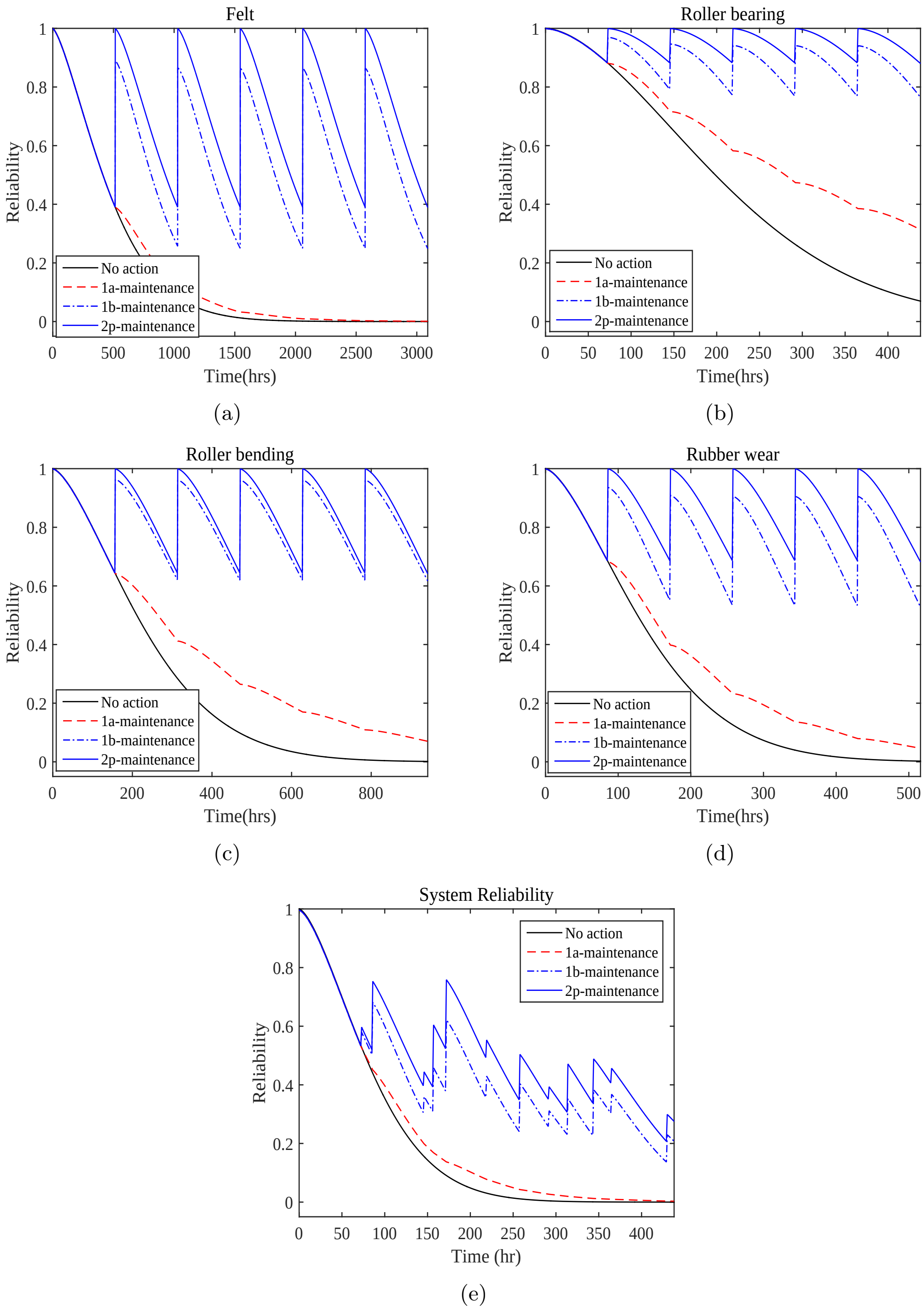

Figure 4.2: Effect of (1a)-, (1b)- and (2p)-maintenance actions on each component and system

while the second, i.e., roller bearing needs to be performed no maintenance. Similarly at $T = 73hrs$, the third and fourth component, namely roller bending and rubber wear needs to be executed (2p)-maintenance action. This task is represented as (3,0,3,3) after the first scheduling action period. Similarly, we can analyze their action at the periodic interval of time and the complete actions are summarized in Table 4.3. Here, in this table, 0 represents no action.

Table 4.3: PM schedule and action of the system

Stage	Time (hrs)	PM actions			
$j = 1$	73	3	0	3	3
$j = 2$	146	3	0	3	3
$j = 3$	219	3	0	3	3
$j = 4$	292	3	0	3	3
$j = 5$	365	3	0	3	3
$j = 6$	438	3	0	3	3

4.5 Conclusion

In this chapter, we study the impact of the periodic PM by considering the three actions, 1a, 1b and 2p on to the performance of the system. To maintain the length of the system, the optimal PM time is calculated by formulating and solving the bi-objective optimization problem with availability and cost as an objective. The PSO algorithm has been used to obtain the optimal value of the function, due to the non-linearity of the objective function. The whole PM interval of the system is to obtain as a minimum among the component PM time. Further, the scheduling of the PM program is acquired based on the maintenance-benefit analysis of the system reliability and their corresponding actions have been done, until the expected life of the system meets their requirement. The detailed impact of the PM action on each component as well as the whole system will help the system analyst to take necessary maintenance actions rather than only replacement, to reduce the maintenance cost.

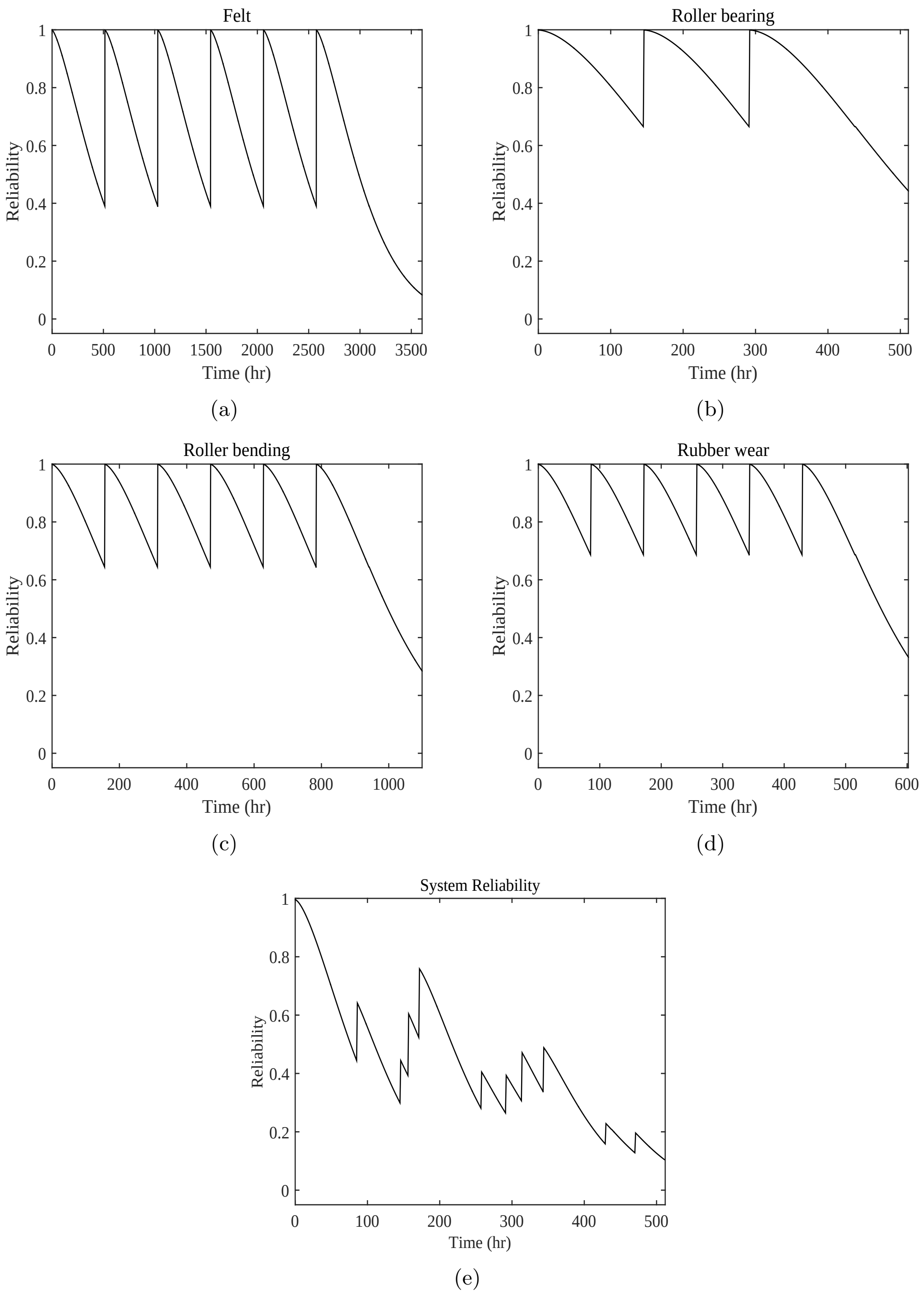

Figure 4.3: Performance of the components and system after implementing PM actions

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