TOTAL TIME MINIMIZATION
SOLID TRANSPORTATION PROBLEM

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Submitted by

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DEDICATED

TO

GOD, MY PARENTS

AND

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CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled "**Total Time Minimization Solid Transportation Problem**" in partial fulfilment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ABSTRACT

The problem of minimizing total cost of transportation problem is one of many well-structured problems in operations research that has been studied in detail. The transportation problem is one of the subclass linear programming problems which are similar in that they all require a linear function of a set of variables is optimized while the variables themselves satisfy a number of linear constraints for which simple and practical computational procedures have been developed.

The time minimization transportation problem is one in which a time is associated with each shipping route and the objective is to minimize the maximum time to transport all supply to the destinations. The problem become more complex in case of solid transportation problem .In the present work The total time solid transportation problem is considered in which the objective is to minimize the time of active transportation routes.

The present thesis contain three chapters. Chapter 1 is introducing in nature in which literature related to the work is also presented. Time minimizing solid transportation problem (Bhatia *et.al*.1976) has been reviewed in chapter 2. In chapter 3,a total time minimization solid transportation problem is formulated and procedure to find it solution has been developed.

CERTIFICATE

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ABSTRACT

Contents

CONTENTS

Page no.

Chapter - 1

INTRODUCTION

The problem of minimizing total cost of transportation problem is one of many well-structured problems in operations research that has been studied in detail. The transportation problem is one of the subclass linear programming problems which are similar in that they all require a linear function of a set of variables is optimized while the variables themselves satisfy a number of linear constraints for which simple and practical computational procedures have been developed.

The time minimizing transportation problem is one in which a time is associated with each shipping route and the objective is to minimize the maximum time to transport all supply to the destinations in place of minimizing cost.

In the present work a total time minimization solid transportation problem has been considered in which the objective is to minimize the total transportation time of active transportation route .

1.1. CLASSICAL TRANSPOORTATION PROBLEM

A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

Minimize $$z=\sum_{i,j} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i, a_i > 0, i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, b_j > 0, j = 1,2,\ldots,n$$

$$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$

a_i is the quantity of the product available at origin i

b_j is the quantity of the product required at destination j

c_{ij} is the cost of shipping one unit from origin i to destination j

The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing of total cost.

1.2 TIME MINIMIZING TRANSPORTATION PROBLEM (Bhatia *et.al.* 1975)

In a time minimizing transportation problem, the time of transporting goods from m origins to n destinations is minimized, satisfying certain conditions in respect of availabilities and source requirement at the destination.

Time minimization transportation problem is formulated as

Minimize $$\max \sum_{i=1}^{m} \sum_{j=1}^{n} [t_{ij} / x_{ij} > 0]$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

$$x_{ij} \ge 0, \text{ for } (i = 1,2.....m; \ j = 1,2,....n)$$

where

a_i = the amount available at the i^{th} origin.

b_j = the demand of the j^{th} destination.

The time of transportation remains independent of the amount of commodity sent as long as $x_{ij} > 0$. It is assumed that

(1) The carriers have sufficient capacity to carry goods from an origin to a destination in a single trip.
(2) They start simultaneously from the respective origin.

The time minimizing-transportation problems are of importance when it is required to transport perishable goods. Sometimes there may exists emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation.

Some methods for minimizing the time of transportation have been established. In such situations rather than minimizing the cost ,the objective is to minimize the maximum time to transport all supply to destinations satisfying certain conditions in respect of availabilities at sources and requirements at the destinations.

For any given feasible solution, X=$\left[x_{ij}\right]$ satisfying the above constraints, the time of transportation is the maximum of t_{ij} 's among the cells in which there are positive allocations, corresponding to the solution $X = \left[x_{ij}\right]$, the time of transportation is

$$Z = \max \sum_{i=1}^{m} \sum_{j=1}^{n} [t_{ij} / x_{ij} > 0]$$

The aim is to minimize this time of transportation.

This time of transportation independent of the amount of commodity sent so long as $x_{ij} > 0$ .it is assume that (i) the carriers have sufficient capacity to carry goods from an origin to destination in a single trip (ii) they start simultaneously from their respective origins.

Thus, the basic difference between cost minimizing transportation problem and time minimizing problem is that whereas the cost of transportation changes with variations in the quantity of the commodity, the time involved remains unchanged, irrespective of the quantity of the commodity involved in the occupied cells in the time minimizing transportation problem.

1.3 TOTAL TIME MINIMIZATION TRANSPORTATION PROBLEM (Nikolic 2007)

In a total time minimizing transportation problem, the total time of transporting goods from *m* origins to *n* destinations is minimized, satisfying certain conditions in respect of availabilities and source requirement at the destination.

Total time minimization transportation problem is formulated as

Minimize $$T(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} h_{ij}$$

$$h_{ij} = \begin{cases} 1 & if\ X_{ij} > 0 \\ 0 & if\ X_{ij} = 0 \end{cases}$$

Subject to
$$\sum_{j=1}^{n} x_{ij} = a_i$$
$$\sum_{i=1}^{m} x_{ij} = b_j$$
$$x_{ij} \geq 0,\ \text{for}\,(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$$

where

h_{ij} is auxiliary function show active and non- active transportation routes (activities)

a_i = the amount available at the i^{th} origin.

b_j = the demand of the j^{th} destination.

This kind of total transportation time is in a military problem where is of primary importance to analyze the total time of all means of transportation which may be exposed to danger of the enemy attacks.

1.4 TIME MINIMIZING SOLID TRANSPORTATION PROBLEM (Bhatia *et.al.*1976)

The traditional transportation problem is well-known optimization problem in operational research, in which two kinds of constraints are taken into consideration, i.e. source constraints & destination constraints. But in the real system, we always deal with other constraints and destination constraints, such as product type constraints or transportation made constraints. For such case, the transportation problem turns into solid transportation problem.

Halley (1962) described the solution of a linear multi-index transportation problem where there are three indices.

Solid transportation problem formulated as:

Suppose i =1, 2,…, m are the origins.

j = 1, 2, …, n are the destinations.

k = 1, 2, …, p are the various type of commodities to be transported in a three dimensional transportation problem.

Let x_{ijk} is amount of type k^{th} goods sent from the i^{th} factory to the j^{th} destination , t_{ijk} is the time of transporting type k^{th} of goods from the i^{th} source to the j^{th} destination, A_{jk} the requirement at the j^{th} destination of type k^{th} of goods, B_{ki} the availability of type k^{th} of goods at the i^{th} factory, E_{ij} the total amount of goods to be sent from the i^{th} factory to the j^{th}

$$\text{Minimize } \left[\underset{(j.i,k)}{Max} \; t_{ijk} / x_{ijk} > 0 \right]$$

Subject to

$$\sum_{i=1}^{n} x_{ijk} = A_{jk'} \; \sum_{j=1}^{m} x_{ijk} = B_{ki'} \; \sum_{k=1}^{p} x_{ijk} = E_{ij'}$$

Where

$$\sum_{j=1}^{m} A_{jk} = \sum_{i=1}^{p} B_{ki} , \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{m} E_{ij} \; , \sum_{i=1}^{n} E_{ij} = \sum_{k=1}^{p} A_{jk}$$

and

$$\sum_{j=1}^{m} \; \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p} \; \sum_{i=1}^{n} B_{ki} = \sum_{i=1}^{n} \; \sum_{j=1}^{m} E_{ij} \; , x_{ijk} \geq 0,$$

This constraint shows k^{th} type of commodities received by all destinations is equal to k^{th} type of commodity supplied from all origins, Different types of commodities supplied by the i^{th} source is equal to amount of commodity received by all destination from the i^{th} source, amount of commodity supplied from all origins to j^{th} destination is equal to different types of commodities received by j^{th} destination, amount of commodities received by all destination of different types of commodities is equal to amount of commodities supplied from all origins to all destination is equal to different types of commodities supplied from all origins.

The transportation starts simultaneously and the time of transportation does not depend on the amount of product transported.

1.5 LITERATUERE SURVEY:

The present work deals with multi-index time minimization transportation problem. The multi-index transportation problem is extension of classical transportation problem where there are three indices. The method of solution is an extension of the modi-method.

There are different types of transportation problems and the simplest of them is now standard in the literature was first presented by Hitchcock (1941). It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of profits, etc. from the investigation ;the entire existing objectives in single objective transportation model are represented by quantitative information. This may cause the negligence of some crucial points which cannot be described by quantitative data.

Later independently, by Koopman (1947) .Koopman began to spearhead research on the potentialities of linear programs for the study of the problems in economics. His historic paper “optimum utilization of the transportation potations systems” was based on his war time experience. Because of this and the work done earlier by Hitchcock, the classical case is often referred as the Hitchcock-Koopman’s transportation problem. Kantorovich (1942) publishes the paper on a continuous version of the problem and later with Gavurin, an applied study of the capacitated transportation problem (Kantorovich and Gavurin 1949).

The time minimizing transportation problem has been studied by Hammer (1969),Garfinkel and Rao(1971) and Szwarc (1971).Hammer (1969) and Szwarc(1971) used labeling techniques to solve the problem (I). Garfunkel and Rao(1971) solved the problem by introducing a sufficiently large cost M on certain routes. Sometimes there may exist emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation. Some methods for minimizing the time of transportation have been established. Several methods for minimizing the time of transportation are also developed. Then Bhatia *et.al.*(1975) developed a technique for minimizing time in a transportation problem. The procedure involved finite number of iterations and is based on moving from one basic feasible solution to another till the last solution is arrived at. The algorithm given by them consists of determination of an initial basic feasible solution which can be found by the methods applicable in the case of the common cost minimizing transportation problem and finding an adjacent better basic feasible solution, the procedure is repeated until no better adjacent basic feasible solution can be found. This repeated procedure deals with the determination of a cell not in the basis which, when introduced ,will either reduce the time of transportation or reduce the allocation in at least one of the cells belongs to Q,where Q Is the set of cell with positive allocations and corresponding time equal to the time of transportation. Ramakrishnan (1977) developed another method of achieving a minimum time of transportation which is very different from other existing methods. Sharma (1977) proposed a survey to provide an up-to-date account of the theoretical and computational aspects of various special cases and extensions of the transportation problem. Prakash *et al*. (1985) presented the transshipment problem with the objective to minimize the duration of transportation and to find optimal routes transportation from origins to destination with transshipment has been studied. Natarajan and Pandian (2011) presented a new blocking method for finding an optimal solution to bottleneck transportation problem, which is very different from other existing methods. They also proposed a method for finding all efficient solutions of a bottleneck-cost transportation problem. Namely, blocking zero point method is proposed which is based on zero point method. Both the proposed methods provide the necessary decision support to decision makers while they are handling time oriented logistic problems.

Nikolic (2007) proposed that the total transportation time problem regarding the time of the active transportation routes. If the multiple optimal solution exist, it is possible to include other criteria as second level of criteria and find the corresponding solutions. Furthermore, if there is multiple solution, again, the third objective can be optimized in lexicographic order. Preetvanti *et al*. (2003) presented multiple objective time transportation problem with additional restrictions (MOTTP). This MOTTP with non-linear bottleneck objective function is related to

Lexicographic Multiple Time Transportation Problem With Additional Restrictions, which is solved by a lexicographic primal code. An algorithm and its underlying theory is developed to determine an initial efficient basic solution of this MOTTP. The developed algorithm is supported by a real life example of minimizing the shipping time, loading/unloading time congestion time for transporting one from source *i* to destination *j* for a steel industry.

Haley (1962) considered the Multi-Index transportation problem in which there are three indices and presented an algorithm to solve the problem, the method of solution is an extension of Modi-method .Haley (1963) developed the theoretical concepts to justifying the methods and an extension of the necessary conditions laid down by Schell (1955). he also described the application of the technique to two special transportation problem and showed that the 'Three axial sums' problem of Schell (1955) can be written as 'Three planar sums'. Halley (1965) laid down a set of necessary conditions for a feasible solution to exist. He also proved that these conditions are sufficient. Smith (1973) gave further necessary conditions for the existence of a solution to the Multi-index transportation problem.

Bhatia *et.al.* (1976) consider the time minimizing transportation problem and proposed a procedure to find its solution which has been review in chapter2.

1.6 PRESENT WORK

The present thesis contain three chapters. Chapter 1 is introducing in nature in which literature relative to the work is also presented. Time minimizing solid transportation problem (Bhatia *et.al*.1976) has been reviewed in chapter 2. In chapter 3, a total time minimization solid transportation problem has been formulated and procedure to find it solution has been developed.

Chapter-2

TIME MINIMIZATION SOLID TRANSPORTATION PROBLEM

INTRODUCTION

An ordinary transportation problem can be written in the form of a two-dimensional table for $i=1,2,...,n;\ j=1,2,...,m,$ as shown in Fig.1. Each cell of this table represents one of the y_{ij}'s. When these y_{ij}'s are summed along the rows of the table they must equal b_i and when they are summed down the columns they must equal a_j.

$j \rightarrow$ $i \downarrow$	1	2	$\cdots$	m	
1	y_{11}	y_{12}	$\cdots$	y_{1m}	b_1
2	y_{21}	y_{22}	$\cdots$	y_{2m}	b_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
n	y_{n1}	y_{n2}	$\cdots$	y_{nm}	b_n
	a_1	a_2	$\cdots$	a_m	

Fig.1

An extension of the transportation type of problem was stated by Halley (1962) and may be thought of as a block in which the layers in all directions from restricted transportation problem. The solid problem can be set out as a three-dimensional block for $i=1,2,...,n;\ j=1,2,...,m;\ k=1,2,...,p.$ Each cell of this block represents one of the x_{ijk}'s. When these are summed along the rows (for constant j and k) they equal A_{jk}. When they are summed along the columns (for constant k and i) they equal B_{ki} When they are summed down the heights (for constant i and j) they equals E_{ij}. The arrangement of x_{ijk}'s and the boundary conditions are shown in Fig.2 and Fig.3.

The solid transportation problem can be described as minimizing the time of moving a set of p different commodities $(k=1,2,...,p)$ from n origin $(i=1,2,...,n)$ to m destinations

$(j = 1, 2, \ldots, m)$. The equation then give rise to the conditions on the amount of various type of combination that are available and required. Alternatively, the same set of restrictions arise when a single commodity has to be removed by different methods e.g. road, rail, sea, canal, air etc. Similarly the use of intermediate depots may require the use of the time minimizing solid transportation problem formulation.

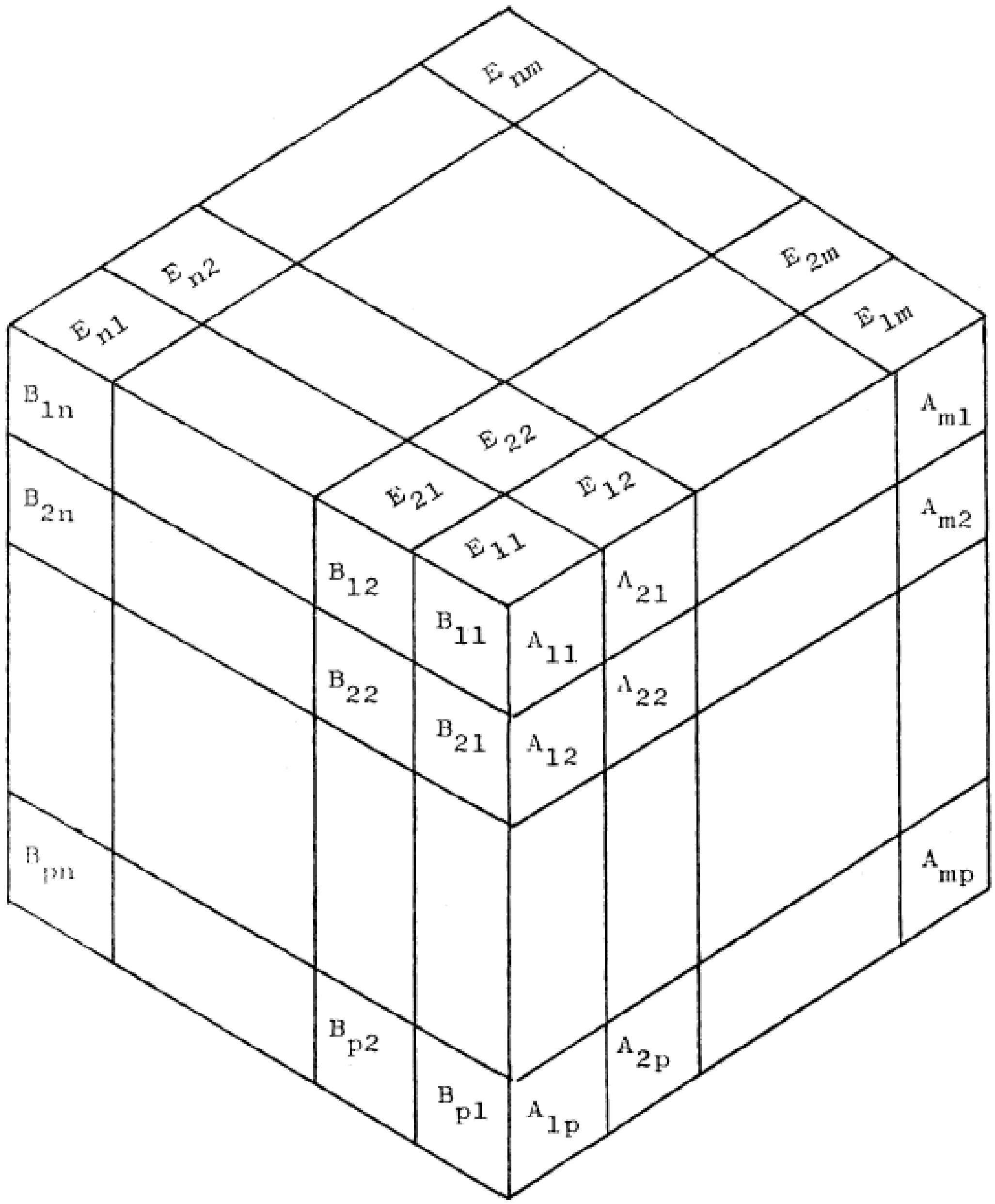


Fig. 2

A special type of problem where the method can be used is the capacitated problem (each variable has an upper bound).

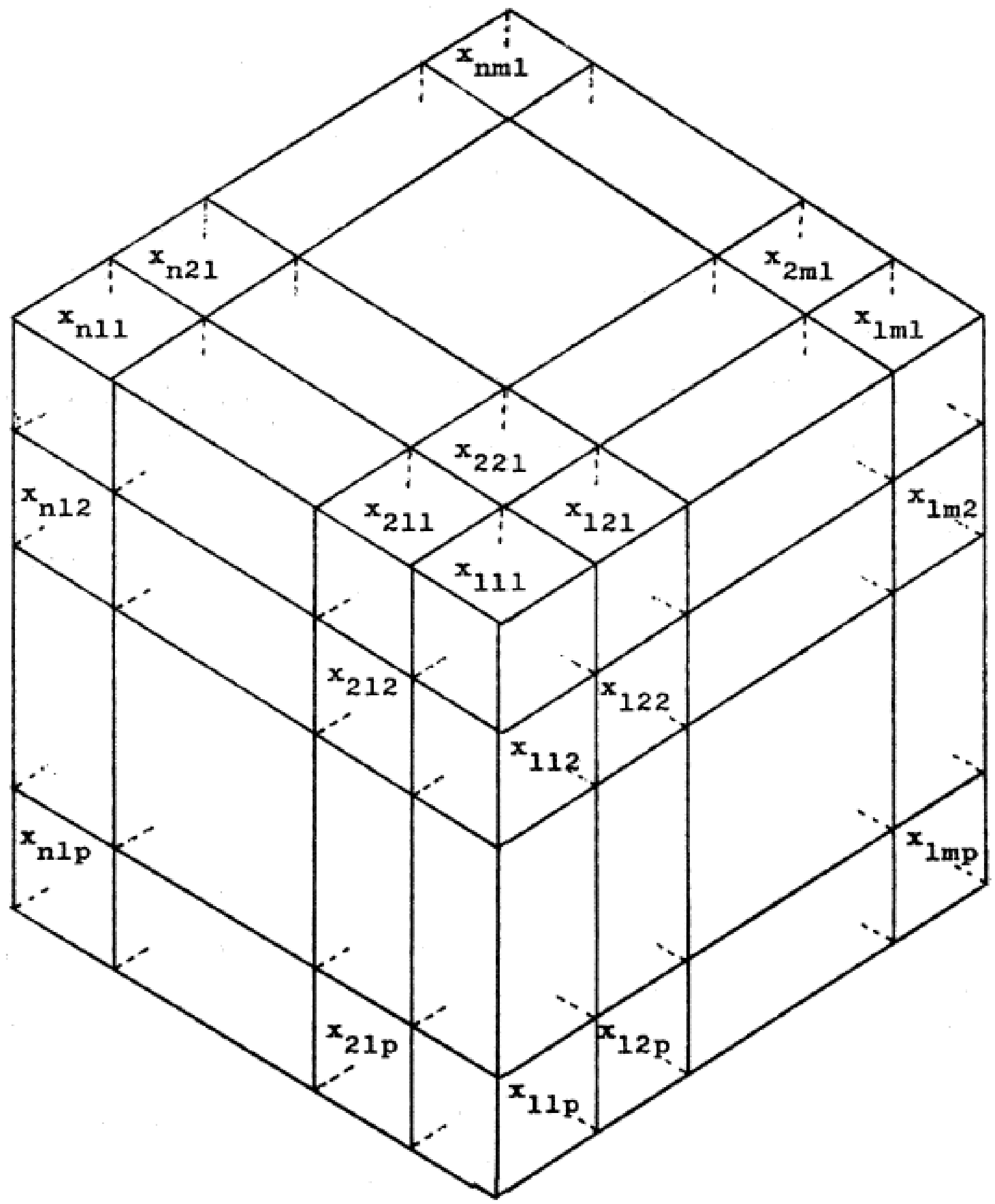


Fig. 3

2.1 FORMULATION OF TIME MINIMIZING SOLID TRANSPORTATION PROBLEM (TMSTP) (Bhatia *et.al*. 1976)

Mathematically the time minimization solid transportation is stated as:

Minimize $\left[\underset{(j.i,k)}{Max}\ t_{ijk} / x_{ijk} > 0\right]$ (2.1)

Subject to

$$\sum_{i=1}^{n} x_{ijk} = A_{jk'}\ \sum_{j=1}^{m} x_{ijk} = B_{ki'}\ \sum_{k=1}^{p} x_{ijk} = E_{ij'}$$

Where

$$\sum_{j=1}^{m} A_{jk} = \sum_{i=1}^{p} B_{ki'}\ \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{m} E_{ij'}\ \sum_{i=1}^{n} E_{ij} = \sum_{k=1}^{p} A_{jk}$$

And

$$\sum_{j=1}^{m}\ \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\ \sum_{i=1}^{n} B_{ki} = \sum_{i=1}^{n}\ \sum_{j=1}^{m} E_{ij'}\ \ x_{ijk} \geq 0,$$

Here $i = 1, 2, ..., m$ are the origins,

$j = 1, 2, ..., n$ are the destinations,

$k = 1, 2, ..., p$ are the various type of commodities,

x_{ijk} is amount of type k^{th} goods sent from the i^{th} factory to the j^{th} destination ,

t_{ijk} is the time of transporting type k^{th} of goods from the i^{th} source to the j^{th} destination,

A_{jk} the requirement at the j^{th} destination of type k^{th} of goods,

B_{ki} the availability of type k^{th} of goods at the i^{th} factory,

E_{ij} the total amount of goods to be sent from the i^{th} factory to the j^{th} destination.

It may be noted that this time of transportation is independent of the amount of the commodity sent so long as $x_{ijk} > 0$. It is assumed, that all the carriers start simultaneously. For a given feasible solution $X = \left[x_{ijk}\right]$, the time of transportation is $\left[\underset{(i,j,k)}{Max}\ t_{ijk} / x_{ijk} > 0\right]$ and thus an optional feasible solution is one, for which this time of transportation is minimum. An imbalanced solid transportation problem can always be converted into a balanced one as described by Halley (1963).

2.2 Theoretical Development

1. A better basic feasible solution: Let $X^1 = \left[x_{ijk}^1 \right]$ and $X^2 = \left[x_{ijk}^2 \right]$ be two basic feasible solutions of problem (2.1) . Let

$$M^1 = \left[(i,j,k) / x_{ijk}^1 > 0 \right],$$
$$M^2 = \left[(i,j,k) / x_{ijk}^2 > 0 \right],$$
$$T^1 = \underset{(i,j,k)\in M^1}{Max} t_{ijk},$$
$$T^2 = \underset{(i,j,k)\in M^2}{Max} t_{ijk},$$
$$Q^1 = \left[(i,j,k) / t_{ijk} = T^1, \ (i,j,k) \in M^1 \right],$$
$$Q^2 = \left[(i,j,k) / t_{ijk} = T^2, \ (i,j,k) \in M^2 \right],$$
$$P^1 = \sum_{(i,j,k)\in Q^1} x_{ijk},$$
$$P^2 = \sum_{(i,j,k)\in Q^2} x_{ijk}.$$

Then X^2 is said to be better than X^1 (if either i) $T^2 < T^1$ of ii) $T^2 = T^1, P^2 < P^1$

Optimal Solution: A feasible solution, for which $\left[\underset{(i,j,k)}{Max} \ t_{ijk} / x_{ijk} > 0 \right]$ is minimum, is said to be an optimal solution of problem (2.1).

Theorem I: There exists a basic feasible solution which is optimal for the time minimizing solid transportation problem .

Proof: Let $\left[t_{\alpha\beta\gamma} \right]$ be the set of the t_{ijk}'s corresponding to the cells of positive allocation in the feasible solution $X = \left[x_{ijk} \right]$. Let $\left[t_{\lambda\mu\nu} \right]$ be the set of t_{ijk}'s corresponding to those positive x_{ijk}'s of X which form a basic feasible solution. Essentially then $\left[t_{\lambda\mu\nu} \right] \subseteqq \left[t_{\alpha\beta\gamma} \right]$. Therefore, maximum t_{ijk} in $\left[t_{\lambda\mu\nu} \right]$ is less than or equal to the maximum of t_{ijk} in $\left[t_{\alpha\beta\gamma} \right]$. Thus, if there exists a feasible solution with time of transportation T, then there also exists a basic feasible solution is with time of transportation T', where $T' \leq T$. In case the feasible solution is optimal results $T' = T$.

Time minimizing solid transportation problem

Hence, there always exists a basic feasible solution which is optimal to problem (2.1).

Local optimal solution: A basic feasible solution $X = \left[x_{ijk} \right]$ is said to be locally optimal for a time minimizing solid transportation problem, if there does not exists any adjacent basic feasible solution better than X .

Theorem II: A locally optimal solution is also globally optimal.

Proof: Let $X^1 = \left[x_{ijk}^1 \right]$ be a locally optimal solution of the time minimizing solid transportation problem. If X^1 is not globally optimal let $X^2 = \left[x_{ijk}^2 \right]$ be a solution of (2.1), which is better than X^1. Therefore, either $T^2 < T^1$ or $T^2 = T^1$, $P^2 < P^1$. Consider a time minimizing solid transportation problem in which the time associated with cells (i, j, k) are defined as follows:

$$C_{ijk} = 1, \text{ if } t_{ijk} = T^1;$$

$$C_{ijk} = 0, \text{ if } t_{ijk} < T^1;$$

$$C_{ijk} = \infty, \text{ if } t_{ijk} < T^1.$$

Then C^1 are the time yielded by solution X^1

$$= \sum_{(i,j,k)\in M^1} C_{ijk} x_{ijk}^2 = \sum_{(i,j,k)\in Q^2} x_{ijk}^2 = P^2$$

$C^2 =$ the time yielded by solution $X^2 = \sum_{(i,j,k)\in M^2} C_{ijk} x_{ijk'}^2$ if $T^2 < T^1$, the time associated with each cell $\in M^2$ are zero , so that $C^2 = 0$. if $T^2 = T^1, P^2 < P^1$, then

$$C^2 = \sum_{(i,j,k)\in M^2} C_{ijk} x_{ijk}^2 = \sum_{(i,j,k)\in Q^2} x_{ijk}^2 \; p^2$$

Where $0 < P^2 < P^1$.

Therefore, in both the cases $C^2 < C^1$.

Thus X^1 is not the optimal solution for the time minimizing solid transportation problem. Hence there exists an adjacent basic feasible solution, say X^3, for the time minimizing solid transportation problem, than X^1, for which $C^3 < C^1$, where C^3 are the time yielded by the solution X^3.

Let T^3 be the time of transportation corresponding to the solution X^3. Now $T^3 > T^1$ because, if $T^3 > T^1$, $C^3 = \infty$ contradicts the fact that $C^3 < C^1$. Thus $T^3 \leq T^1$. if $T^3 < T^1$, X^3 is a better basic feasible solution adjacent to X^1 for the problem (2.1). If $T^3 = T^1$,

$$C^3 = \sum_{(i,j,k)\in M^3} C_{ijk} x_{ijk}^2 = \sum_{(i,j,k)\in Q^3} x_{ijk}^2 = P^3.$$

Since $C^3 < C^1$, it implies $P^3 < P^1$ and thus X^3 is a better basic feasible solution adjacent to X^1 for the problem (2.1). Hence X^3 is a better basic feasible solution adjacent to X^1 for the time minimizing solid transportation problem. This contradicts the fact that X^1 is locally optimal for problem (2.1). Therefore, our assumption that X^1 is not globally optimal is wrong. Hence a locally optimal solution of time minimizing solid transportation problem is also globally optimal.

2.3 Algorithm

In solving a time minimizing solid transportation problem, the following are the three main steps:

1. Determination of an initial basic feasible solution consisting in a number of $mnp-(m-1)(n-1)(p-1)$ non-negative variables. It can be obtained by a procedure similar to the North West corner rule. Halley (1963) has developed an alternative approach to find an initial basic solution by adding artificial variables. This approach also indicates, whether the problem has a solution or not.
2. Finding an adjacent better basic feasible solution.
3. Repetition of step 2 till no better adjacent basic feasible solution can be found.

Step 2 deals with finding a non-basic cell which, when introduced into the basis, either reduces the time of transportation or reduces the quantity in the pipe line p. To achieve this, time C_{ijk} are associated with cells (i, j, k) as follows:

$$C_{ijk} = 1, \text{ if } t_{ijk} = T^1,$$

$$C_{ijk} = 0, \text{ if } t_{ijk} < T^1,$$

$$C_{ijk} = \infty, \text{if } t_{ijk} < T^1,$$

Where T^1 is the time of transportation for the initial basic feasible solution X^1.

Taking X^1 as initial basic feasible solution for the time minimizing solid transportation problem with time defined as above, p^1being the time yielded by the solution X^1, find shadow time u_{jk}, v_{ki}, w_{ij} and evaluate $\Delta_{rst} = C_{rst} - (u_{st} + v_{tr} + w_{rs})$

for all empty cells [Modi Method]. Determine the set

$$S = [(r,s,t) / \Delta_{rst} < 0, \quad (r.s.t) \notin B^1],$$

Where B^1 is the set of basic cell of S, for which t_{rst} is the least. Again shadow time and Δ_{rst} are evaluated for this new better basic feasible solution for the time minimizing solid transportation problem and it is repeated till a stage is reached when $S = \phi.$ At this stage the optimal solution for the time minimizing solid transportation problem is reached. If the optimal time thus obtained is non-zero, it implies that it is not possible to have a basic feasible solution yielded lesser time of transportation. In this case ,a new basic feasible solution at time T^1 will be obtained ,for which time $\leq p^1$, which implies that this new basic feasible solution is better in the sense that the quantity in the pipe line p at the same time T^1 is reduced .If the optimal time minimizing solid transportation problem are zero, then a basic feasible solution for problem (2.1) has been obtained with lesser time , say T^2.[This basic feasible solution need not be necessarily adjacent to an earlier solution giving time T^1.] The basic feasible solution X^2, yielding time T^2 for problem (2.1) is again tested for improvement, if possible, by defining time at time T^2as was done previously.

2.4 Convergence

The procedure is bound to converge because of the following reasons:

1. It involves movement from one basic solution to another which is finite in number.
2. No basic feasible solution is repeated because an improvement is made at every step.

2.5 Remarks

1. Degeneracy can be handled as explained by Halley (1962).

2. The technique presented in this paper is useful for dealing with transportation of heterogeneous goods when the total quantity to be sent from the i^{th} source to the j^{th} destination is fixed and is equal say to E_{ij}. If one tries to solve this problem by breaking it into ordinary transportation problem, one for each good, then E_{ij} 's may not be satisfied.

3. It may be noticed that in the formulation of the problem, no two types of goods are to be transported in the same carrier.

4. It is assumed that there is no preferential item i.e. all the items are of equal importance.

5. If the quantity in the pipe line, p, is not to be taken care of, one can move towards minimum time of transportation at a faster rate by defining time associated with cells (i, j, k) at any time T yielded by a basic feasible solution X as follows :

$$C_{ijk} = \begin{cases} 0 & \text{if} \quad t_{ijk} < T, \\ 1 & \text{if} \quad t_{ijk} \geq T. \end{cases}$$

   If the minimum time is zero, a basic feasible solution with lesser time is obtained, and if the minimum time is positive, the time cannot be improved further.

6. A manufacturer has n factories (i) producing p type of goods (k). These goods are to be transported to m different customers (j). Let t_{ijk} be the time of transporting the k^{th} type of goods from the i^{th} factory to the j^{th} customer. Then it is required to determine a transportation schedule in such a way that the time of transportation is minimum.

2.6 NUMERICAL EXAMPLE

Let there be three origins $(i = 1, 2, 3)$ producing three types of products $(k = 1, 2, 3)$ to be supplied to three different destinations $(j = 1, 2, 3)$. The following table gives the data-time t_{ijk} at the top left corners of the respective cells, B_{ki} at the right extreme of rows, A_{jk} at the bottom of columns and E_{ij} in the $(i, j)^{th}$ cell consisting of cells $(i, j, 1)$, $(i, j, 2)$, $(i, j, 3)$ $i = 1, 2, 3;$ $j = 1, 2, 3.$

	$j=1$	$j=2$	$j=3$	
$i=1$	6 7 6 $E_{11}=10$	5 6 10 $E_{12}=6$	7 3 11 $E_{13}=9$	$B_{11}=6$ $B_{21}=9$ $B_{31}=10$
$i=2$	11 8 13 $E_{21}=21$	9 15 12 $E_{22}=10$	13 1 8 $E_{23}=14$	$B_{12}=13$ $B_{22}=15$ $B_{32}=17$
$i=3$	5 6 7 $E_{31}=22$	8 9 7 $E_{32}=13$	10 6 12 $E_{33}=12$	$B_{13}=15$ $B_{23}=14$ $B_{33}=18$
	$A_{11}=15$ $A_{12}=18$ $A_{13}=20$	$A_{21}=8$ $A_{22}=12$ $A_{23}=9$	$A_{31}=11$ $A_{32}=8$ $A_{33}=16$	

Table -2.1

Using the North-West corner rule, an initial basic feasible solution is obtained as shown in Table 2.2,2.3,2.4.

Table – 2.2

Destination $j \rightarrow$ *Origin* $i \downarrow$	1	2	3
1	8 (6)	5	7
2	11 (9)	9 (4)	13
3	5	8 (4)	10 (11)

Table -2.3

Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3
1	7 (1)	6	3 (8)
2	8 (12)	15 (3)	7
3	5 (5)	9 (9)	6

Table -2.4

Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3
1	6 (3)	10(6)	11 (1)
2	13	12 (3)	8 (14)
3	7 (17)	7 (0)	12 (1)

(in the following figures 0 or 1 at the top right corner of a cell is the cost associated with that cell.)

An initial basic feasible solution X^1 giving $T^1 = 15$, $P^1 = 3$ is given in table 2.5 below . Thus defining

$$C_{ijk} = \begin{bmatrix} 1 & IF & t_{ijk} = 15 \\ 0 & IF & t_{ijk} < 15 \\ \infty & IF & t_{ijk} > 15 \end{bmatrix}$$ entered in table 2.5, solve the cost minimizing transportation problem.

Shadow time $u_{jk'}$ $v_{ki'}$ and w_{ij} are also shown in the table.

$$S = \left[(r,s,t)/\Delta_{rst} < 0,\ (r,s,t) \notin B_1\right]$$
$$= \left[(3,1,1),\ (2,3,2)\right], \quad t_{311} = 5\ , \quad t_{232} = 7$$

(where B_1 is the set of basic cells) , therefore (3,1,1) enters the basis. The new basis feasible solution X^2 is given in table 2.6.

The time yielded by this solution are zero, so the time improves. The solution X^2 gives $T^2 = 12$, $p^2 = 4$. Again define

$$C_{ijk} = \begin{bmatrix} 1 & IF & t_{ijk} = 12 \\ 0 & IF & t_{ijk} < 12 \\ \infty & IF & t_{ijk} > 12 \end{bmatrix}$$ entered in table 2.6

		$j=1$	$j=2$	$j=3$	
$i=1$		8 0 **6**	5 0	7 0	$v_{11}=0$
		7 0 **1**	6 0	3 0 **8**	$v_{21}=0$
		6 0 **3**	10 0 **6**	11 0 **1**	$v_{31}=0$
		$w_{11}=0$	$w_{12}=0$	$w_{13}=0$	
$i=2$		11 0 **9**	9 0 **4**	13 0	$v_{12}=0$
		8 0 **12**	15 1 **3**	1 0	$v_{22}=0$
		13 0	12 0 **3**	8 0 **14**	$v_{32}=-1$
		$w_{21}=0$	$w_{22}=1$	$w_{23}=1$	
$i=3$		5 0	6 0 **4**	10 0 **11**	$v_{13}=1$
		6 0 **5**	9 0 **9**	6 0	$v_{23}=0$
		7 0 **17**	7 0 **0**	12 0 **1**	$v_{33}=0$
		$w_{31}=0$	$w_{32}=0$	$w_{33}=0$	
		$u_{11}=0$	$u_{21}=-1$	$u_{31}=0$	
		$u_{12}=0$	$u_{22}=0$	$u_{32}=0$	
		$u_{13}=0$	$u_{22}=0$	$u_{33}=0$	

Table-2.5

		$j=1$	$j=2$	$j=3$	
$i=1$		8 0 **6**	5 0	7 0	$v_{11}=0$
		7 0 **1**	6 0	3 0 **8**	$v_{21}=0$
		$w_{11}=0$; 6 0 **3**	$w_{12}=0$; 10 0 **6**	$w_{13}=0$; 11 0 **1**	$v_{31}=0$
$i=2$		11 0 **6**	9 0 **7**	13 ∞	$v_{12}=-1$
		8 0 **15**	15 ∞	7 0	$v_{22}=-1$
		$w_{21}=1$; 13 ∞	$w_{22}=1$; 12 1 **3**	$w_{23}=0$; 8 0 **14**	$v_{32}=0$
$i=3$		5 0 **3**	6 0 **4**	10 0 **11**	$v_{13}=0$
		6 0 **2**	9 0 **9**	6 0	$v_{23}=0$
		$w_{31}=0$; 7 0 **17**	$w_{32}=0$; 7 0 **0**	$w_{33}=1$; 12 1 **1**	$v_{33}=0$
		$u_{11}=0$, $u_{12}=0$, $u_{13}=0$	$u_{21}=0$, $u_{22}=0$, $u_{22}=0$	$u_{31}=-1$, $u_{32}=0$, $u_{33}=0$	

Table-2.6

$S=[(3,3,2)]$, Thus (3,3,2) enters the basis. The next solution X^3 is given in table2.7.

	$j=1$	$j=2$	$j=3$	
$i=1$	8 0 **6** 7 0 **2** 6 0 **2** $w_{11}=0$	5 0 6 0 10 0 **6** $w_{12}=0$	7 0 3 0 **7** 11 0 **2** $w_{13}=0$	$v_{11}=0$ $v_{21}=0$ $v_{31}=0$
$i=2$	11 0 8 0 **15** 13 ∞ $w_{21}=1$	9 0 **7** 15 ∞ 12 1 **3** $w_{22}=1$	13 ∞ 7 0 8 0 **14** $w_{23}=0$	$v_{12}=-1$ $v_{22}=-1$ $v_{32}=0$
$i=3$	5 0 **3** 6 0 1 7 0 **18** $w_{31}=0$	8 0 **1** 9 0 12 7 0 **0** $w_{32}=0$	10 0 **11** 6 0 1 12 1 $w_{33}=0$	$v_{13}=0$ $v_{23}=0$ $v_{33}=0$
	$u=0$ $u_{12}=0$ $u_{13}=0$	$u_{21}=0$ $u_{22}=0$ $u_{23}=0$	$u_{31}=0$ $u_{32}=0$ $u_{33}=0$	

Table -2.7

This solution gives $T^3=12,\ p^3=3.$ Also $S=\phi,$ thus the process terminates.

Hence minimum time of transportation is 12 with amount in the pipe line at this time is 3.

Chapter - 3

TOTAL TIME MINIMIZING SOLID TRANSPORTATION

3.1 INTRODUCTION

To minimize the total transportation cost is considered by many research workers in detail, however the transportation time is also relevant in a variety of real transportation problem. There are some variants of transportation problem such as minimize the total transportation time of active transportation route.

Keeping this and the problem review in chapter - 2 in view multi index total time minimization transportation problem has been considered in which the total time is to be minimized along the active transportation routes and formulated as below.

3.2 FORMULATION OF PROBLEM

The Multi-index total time minimization transportation problem in which there are m origins, n destinations and p type of commodities to be transported can be formulated as below.

$$\text{Minimize } T(x) = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} t_{ijk} h_{ijk} \tag{3.1}$$

$$h_{ijk} = \begin{cases} 1 & if\ x_{ijk} > 0 \\ 0 & if\ x_{ijk} = 0 \end{cases} \tag{3.2}$$

Subject to

$$\sum_{i=1}^{n} x_{ijk} = A_{jk}, \quad \sum_{j=1}^{m} x_{ijk} = B_{ki}, \quad \sum_{k=1}^{p} x_{ijk} = E_{ij},$$

Where

$$\sum_{j=1}^{m} A_{jk} = \sum_{i=1}^{p} B_{ki}, \quad \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{m} E_{ij}, \quad \sum_{i=1}^{n} E_{ij} = \sum_{k=1}^{p} A_{jk}$$

And

$$\sum_{j=1}^{m} \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p} \sum_{i=1}^{n} B_{ki} = \sum_{i=1}^{n} \sum_{j=1}^{m} E_{ij}, \quad x_{ijk} \geq 0,$$

Here $i=1,2,\ldots,m$ are the origins,

$j = 1,2,\ldots,n$ are destinations,

$k = 1, 2,\ldots,p$ are the various type of commodities.

h_{ijk} is auxiliary function show active and non -active transportation routes (activities),

x_{ijk} is amount of type k^{th} goods sent from the i^{th} factory to the j^{th} destination ,

t_{ijk} is the time of transporting type k^{th} of goods from the i^{th} source to the j^{th} destination,

A_{jk} the requirement at the j^{th} destination of type k^{th} of goods,

B_{ki} the availability of type k^{th} of goods at the i^{th} factory ,

E_{ij} the total amount of goods to be sent from the i^{th} factory to the j^{th} destination.

This constraints show that k^{th} type of commodities received by all destination is equal to k^{th} type of commodity supplied from all origins and different types of commodities supplied by the i^{th} source is equal to amount of commodity received by all destination from the i^{th} source. The amount of commodity supplied from all origins to j^{th} destination is equal to different types of commodities received by j^{th} destination and the amount of commodities received by all destination of different types of commodities is equal to amount of commodities supplied from all origins to all destination is equal to different types of commodities supplied from all origins.

It has been assumed that the transportation starts simultaneously and the time of transportation does not depend on the amount of product transported.

3.3 SOLUTION PROCEDURE

In the solution procedure we make the use of the combination of approaches given by Halley (1962) and Nicolic (2007).

Let $X^{(D)}$ and $X^{(D+1)}$ are two basis neighboring feasible solutions, where $X_{ijk}^{(D)}$ is entering basis variable and $X_{isk}^{(D)}$ is leaving basis variable for $X^{(D)}$:

$X^{(D)}$ contain : $X_{ijk}^{(D)} = 0$ and $X_{isk}^{(D)} > 0$

$X^{(D+1)}$ contain : $X_{ijk}^{(D+1)} > 0$ and $X_{isk}^{(D+1)} = 0$

there is : $X_{ijk}^{(D+1)} = X_{isk}^{(D)}$

In moving from $X^{(D)}$ *to* $X^{(D+1)}$ the total transportation time *T*(x) of problem (3.1) will be changed with the following values:

$$q_{ijk}^{(D)} = t_{ijk} - t_{isk}. \tag{3.3}$$

The characteristics q_{ijk} are the change of transportation time in problem (3.1).

Then the solution $X^{(D+1)}$ has:

$$T^{(D+1)} = T^{(D)} + (t_{ijk}^{(D)} - t_{isk}^{(D)}). \tag{3.4}$$

Clearly, the total time $T^{(D+1)}$ is determined by values $q_{ijk}^{(D)}$ as:

$$T^{(D+1)} = \begin{cases} > T^{(D)} & if\ q_{ijk}^{(D)} > 0 \\ = T^{(D)} & if\ q_{ijk}^{(D)} = 0 \\ < T^{(D)} & if\ q_{ijk}^{(D)} < 0 \end{cases} \tag{3.5}$$

3.4 ALGORITHM

Step 0: Find basic feasible solution $X^{(1)}$ set number of iteration $D = 1$.

Step 1: Determine the indicators $h_{ijk}^{(D)}$ of active transportation routes $X_{ijk}^{(D)} > 0$, and the total time $T^{(D)} = T\left(X^{(D)}\right)$

$$h_{ijk} = \begin{cases} 1 & if\ X_{ijk}{}^{(D)} > 0 \\ 0 & if\ X_{ijk}{}^{(D)} = 0 \end{cases}$$

$$T^{(D)} = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} t_{ijk} h_{ijk}{}^{(D)}$$

Step 2: Using equation (3.3),determine the characteristic $q_{ijk}{}^{(D)}$ for all non -basic variable $X_{ijk}{}^{(D)} = 0$ using (3.3) .Use the changing path of the basic solution (as in a stepping – stone method) and corresponding leaving basic variable, e.g. $X_{isk}{}^{(D)} > 0$ become $X_{isk}{}^{(D)} = 0$, if entering variable would be $X_{ijk}{}^{(D+1)} > 0$.

(If there is more then one path to calculate $q_{ijk}{}^{(D)}$ *then select that minimum* $q_{ijk}{}^{(D)}$ *regarding* $q_{ijk}{}^{(D)} < 0$ *which minimize the total time simultaneously for various types of commodities)*

Step 3: Check the optimality of the total time (3.1), using (3.5). If all $q_{ijk}{}^{(D)} \geq 0$, the optimal solution X^* is found. Stop otherwise go to step 4.

Step4: Determine next basic solution, using the entering variable X_{ijk} with minimum $q_{ijk}{}^{(D)}$, regarding $q_{ijk}{}^{(D)} < 0$.If this minimum $q_{ijk}{}^{(D)}$ minimize the total time simultaneously for various types of commodities, then set $D = D+1$ & go to step 1.

Otherwise, consider next minimum $q_{ijk}{}^{(D)}$ regarding $q_{ijk}{}^{(D)} < 0$ and go to step 1.

3.5 NUMERICAL EXAMPLE

Consider a $3 \times 3 \times 3$ multi-index total time transportation problem.

Minimize $T(x) = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} t_{ijk} h_{ijk}$ (3.1)

$$h_{ijk} = \begin{cases} 1 & if\ X_{ijk} > 0 \\ 0 & if\ X_{ijk} = 0 \end{cases} \quad (3.2) \qquad 1$$

Subject to $\sum_{i=1}^{n} x_{ijk} = A_{jk},\ \sum_{j=1}^{m} x_{ijk} = B_{ki},\ \sum_{k=1}^{p} x_{ijk} = E_{ij},$

where $\sum_{j=1}^{m} A_{jk} = \sum_{i=1}^{p} B_{ki}, \ \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{m} E_{ij}, \ \sum_{i=1}^{n} E_{ij} = \sum_{k=1}^{p} A_{jk}$

and $\sum_{j=1}^{m} \ \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p} \ \sum_{i=1}^{n} B_{ki} = \sum_{i=1}^{n} \ \sum_{j=1}^{m} E_{ij}, \ \ x_{ijk} \geq 0,$

Table 3.1 gives the data-time t_{ijk} at the top left corners of the respective cells, B_{ki} at the right extreme of rows, A_{jk} at the bottom of columns and E_{ij} in the $(i, j)^{th}$ cell consisting of cells $(i, j, 1), (i, j, 2), (i, j, 3)$ $i = 1, 2, 3;$ $j = 1, 2, 3.$

Table 3.1

	$j=1$	$j=2$	$j=3$	
$i=1$	6 7 6 $E_{11}=10$	5 6 10 $E_{12}=6$	7 3 11 $E_{13}=9$	$B_{11}=6$ $B_{21}=9$ $B_{31}=10$
$i=2$	11 8 13 $E_{21}=21$	9 15 12 $E_{22}=10$	13 7 8 $E_{23}=14$	$B_{12}=13$ $B_{22}=15$ $B_{32}=17$
$i=3$	5 6 7 $E_{31}=22$	8 9 7 $E_{32}=13$	10 6 12 $E_{33}=12$	$B_{13}=15$ $B_{23}=14$ $B_{33}=18$
	$A_{11}=15$ $A_{12}=18$ $A_{13}=20$	$A_{21}=8$ $A_{22}=12$ $A_{23}=9$	$A_{31}=11$ $A_{32}=8$ $A_{33}=16$	

Using the North-West corner rule, an initial basic feasible solution is obtained as shown in Table 3.2, 3.3, 3.4.

Table – 3.2

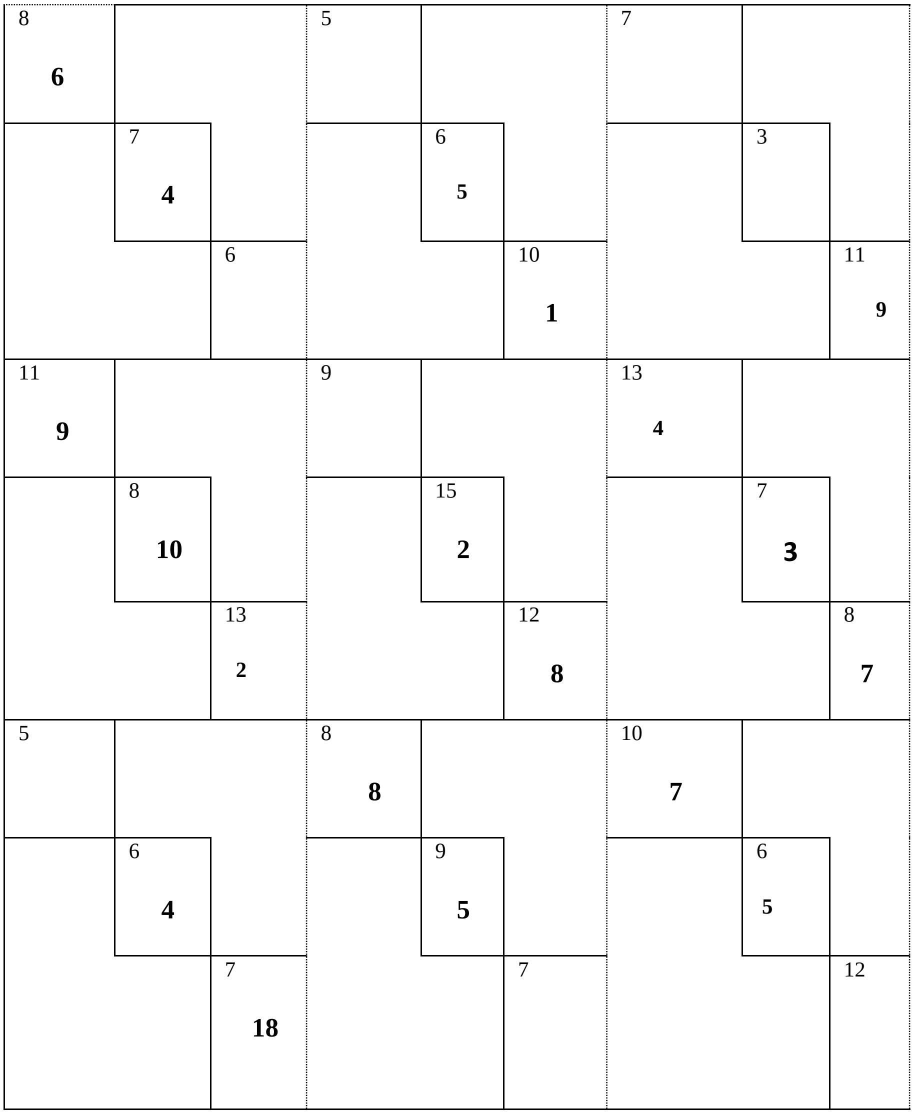
Destination j → *Origin i* ↓	1	2	3
1	8 (6)	5	7
2	11 (9)	9	13(4)
3	5	8 (8)	10 (7)

Table – 3.3

Destination j → *Origin i* ↓	1	2	3
1	7 (4)	6(5)	3
2	8 (10)	15 (2)	7(3)
3	5 (4)	9 (5)	6 (5)

Table – 3.4

Destination j → *Origin i* ↓	1	2	3
1	6	10(1)	11 (9)
2	13(2)	12 (8)	8 (7)
3	7 (18)	7	12



The value of total time, $T^{(1)} = 175$

Applying Step 2, the values q_{ijk} are calculated, for all $i, j, k \notin B$ which are given in Table 3.5.

Table – 3.5

ijk	121	131	132	113	221	311	323	333
q_{ijk}	-8	-6	-4	-4	-4	-5	-5	5

In Table 3.5, it is observed that

$$Min\{q_{ijk}, q_{ijk} < 0, i, j, k \notin B\} = -8 \ at\,(1,2,1)\,cell.$$

Therefore, the variable to enter the basis is x_{121} and the new solution is given in Table 3.6, 3.7, 3.8.

Table – 3.6

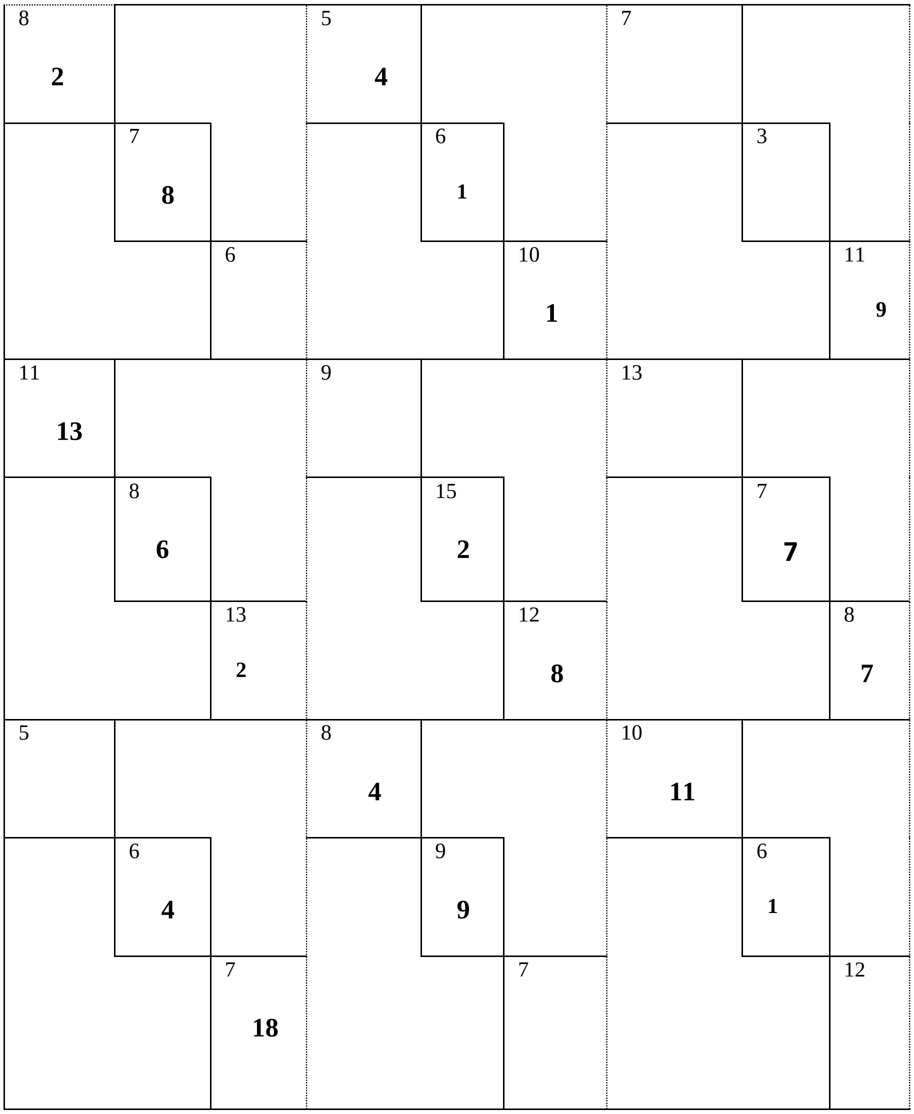
Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3
1	8 (2)	5(4)	7
2	11 (13)	9	13
3	5	8 (4)	10 (11)

Table – 3.7

Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3
1	7 (8)	6(1)	3
2	8 (6)	15 (2)	7(7)
3	5 (4)	9 (9)	6 (1)

Table – 3.8

Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3
1	6	10(1)	11 (9)
2	13(2)	12 (8)	8 (7)
3	7 (18)	7	12



The value of total time, $T^{(2)} = 167$.

Applying Step 2, the following results are obtained as shown in Table 3.9.

Table – 3.9

ijk	131	132	113	221	231	311	323	333
q_{ijk}	2	-3	-4	4	8	-3	-5	5

In Table 3.10, it is observed that

$$Min\{q_{ijk}, q_{ijk} < 0, i, j, k \notin B\} = -5 \; at\,(3,2,3)\,cell.$$

Therefore, the variable to enter the basis is x_{323} and the new solution is given in Table 3.10, 3.11, 3.12.

Table – 3.10

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	8 (2)	5(4)	7
2	11 (13)	9	13
3	5	8 (4)	10 (11)

Table – 3.11

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	7 (8)	6(1)	3
2	8	15 (8)	7(7)
3	5 (10)	9 (3)	6 (1)

Table – 3.12

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	6	10 (1)	11 (9)
2	13(8)	12 (2)	8 (7)
3	7 (12)	7 (6)	12

8 **2**			5 **4**			7		
	7 **8**			6 **1**			3	
		6			10 **1**			11 **9**
11 **13**			9			13		
	8			15 **8**			7 **7**	
		13 **8**			12 **2**			8 **7**
5			8 **4**			10 **11**		
	6 **10**			9 **3**			6 **1**	
		7 **12**			7 **6**			12

The value of total time, $T^{(3)} = 166$.

Applying Step 2, the following results are obtained as shown in Table 3.13.

Table – 3.13

ijk	131	132	113	221	231	311	212	333
q_{ijk}	2	-3	-4	4	8	-3	1	5

In Table 3.13, it is observed that

$$Min\{q_{ijk}, q_{ijk} < 0, i, j, k \notin B\} = -4 \ at\ (1,1,3)\, cell.$$

Therefore, the variable to enter the basis is x_{113} and the new solution is given in Table 3.14, 3.15, 3.16.

Table – 3.14

Destination j → *Origin i* ↓	1	2	3
1	8 (2)	5(4)	7
2	11 (13)	9	13
3	5	8 (4)	10 (11)

Table – 3.15

Destination j → *Origin i* ↓	1	2	3
1	7 (7)	6(2)	3
2	8	15 (8)	7(7)
3	5 (11)	9 (2)	6 (1)

Table – 3.16

Destination j → *Origin i* ↓	1	2	3
1	6 (1)	10	11 (9)
2	13(8)	12 (2)	8 (7)
3	7 (11)	7 (7)	12

8 **2**			5 **4**			7		
	7 **8**			6 **1**			3	
		6			10 **1**			11 **9**
11 **13**			9			13		
	8			15 **8**			7 **7**	
		13 **8**			12 **2**			8 **7**
5			8 **4**			10 **11**		
	6 **10**			9 **3**			6 **1**	
		7 **12**			7 **6**			12

The value of total time, $T^{(4)} = 162$.

Applying Step 2, the following results are obtained as shown in Table 3.17

Table – 3.17

ijk	131	132	123	221	231	311	212	333
q_{ijk}	2	-3	4	4	8	-3	1	5

In Table 3.17, it is observed that

$$Min\left\{q_{ijk}, q_{ijk} < 0, i, j, k \notin B\right\} = -3\ at\,(3,1,1)\,cell.$$

Therefore, the variable to enter the basis is x_{311} and the new solution is given in Table 3.18, 3.19, 3.20.

Table – 3.18

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	8	5(6)	7
2	11 (13)	9	13
3	5 (2)	8 (2)	10 (11)

Table – 3.19

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	7 (9)	6(0)	3
2	8	15 (8)	7(7)
3	5 (9)	9 (4)	6 (1)

Table – 3.20

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	6 (1)	10	11 (9)
2	13(8)	12 (2)	8 (7)
3	7 (11)	7 (7)	12

8			5 **6**			7		
	7 **9**			6 **0**			3	
		6 **1**			10			11 **9**
11 **13**			9			13		
	8			15 **8**			7 **7**	
		13 **8**			12 **2**			8 **7**
5 **2**			8 **2**			10 **11**		
	6 **9**			9 **4**			6 **1**	
		7 **11**			7 **7**			12

The value of total time, $T^{(5)} = 159$.

Applying Step 2, the following results are obtained as shown in Table 3.21.

Table – 3.21

ijk	131	132	123	221	231	111	212	333
q_{ijk}	2	-3	4	4	8	3	1	5

In Table 3.21, it is observed that

$$Min\{q_{ijk}, q_{ijk} < 0, i, j, k \notin B\} = -3 \ at \ (1,3,2) \ cell.$$

Therefore, the variable to enter the basis is x_{132} and the new solution is given in Table 3.22, 3.23, 3.24.

Table – 3.22

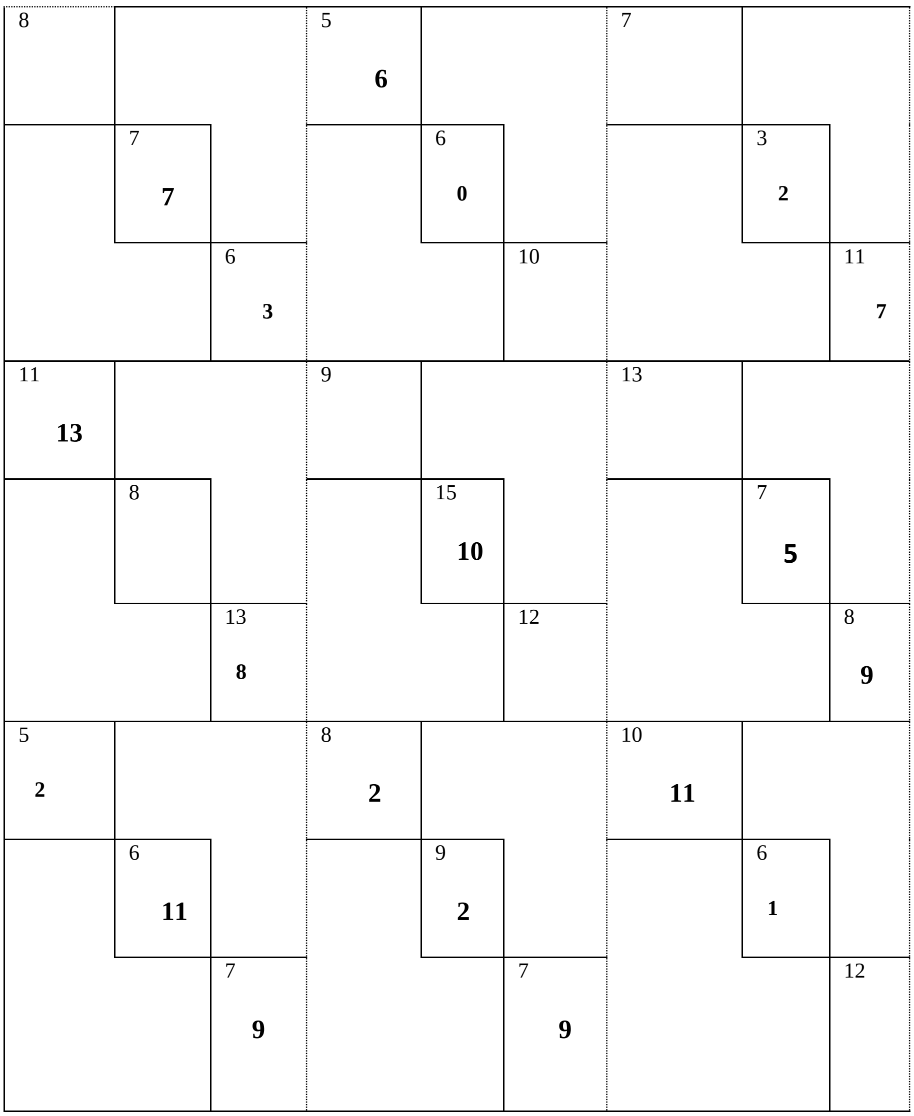
Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	8	5(6)	7
2	11 (13)	9	13
3	5 (2)	8 (2)	10 (11)

Table – 3.23

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	7 (7)	6(0)	3 (2)
2	8	15 (10)	7(5)
3	5 (11)	9 (2)	6 (1)

Table – 3.24

Destination j $\rightarrow$ *Origin i* $\downarrow$	1	2	3
1	6 (3)	10	11 (7)
2	13(8)	12	8 (9)
3	7 (9)	7 (9)	12



The value of total time, $T^{(6)} = 150$.

Applying Step 2, the following results are obtained as shown in Table 3.25.

Table – 3.25

ijk	131	223	123	221	231	111	212	333
q_{ijk}	2	9	4	4	3	3	1	5

Here all $q_{ijk} \geq 0$ for non basic cells.

Hence the optimal time is 150.

3.6 CONCLUSION

A total time minimization solid transportation problem is formulated and a procedure has been developed to find its solution.

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