Heavy Quark Effective Theory For Mesons

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The supervision of

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Dedicated

To

My Parents And My Brother

CERTIFICATE

This is to certify that Mr. Satwinder Singh, Roll No. 300904015 has worked on this dissertation report as a partial fulfillment for award of the degree of MASTERS OF SCIENCE in physics. I certify that the matter embodied in this report is of candidate's own record and not submitted to any other university in any part or full form for the award of such a degree.

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Units And Physical Constants

1. Units

Quantity	High Energy Units	Value in SI Units
Length	1 fm	$10^{-15}m$
Energy	1GeV=10^9eV	1.602×10^{-10}J
Mass,E/c^2	1GeV/c^2	1.78×10^{-27}kg
ħ=$h/2\pi$	6.588×10^{-25}GeV	1.055×10^{-34} Js
C	2.988×10^{23}fms^{-1}	2.998×10^{8}m s^{-1}
ħc	0.1975GeV fm	3.162×10^{-26}Jm

Natural Units	**ħ =c=1**
MassMc^2/c^2	1 GeV
Lengthħc$/Mc^2$	1GeV^{-1}=0.1975fm
Timeħc$/Mc^3$	1GeV^{-1}=6.59×10^{-25}s

2. Constants

Constant	Symbol	Value
Electron Charge	E	1.602..× 10^{-19}C
Electron Mass	m_e	0.511…MeV
Proton Mass	m_p	938.3…MeV
Strong Coupling Constant	G	1
Fine Structure Constant	A	$1/137.05$
Planck’s Constant (Reduced)	Ћ	6.582…×10^{-22}MeV

ABSTRACT

Symmetries of heavy and light quarks are exploited to formulate a theory that describes the low energy interaction among heavy mesons. Within the framework of heavy quark effective theory and chiral perturbation theory, masses of even and odd parity charmed mesons are studied. Mass formulae is used[T. Mehen and R. P. Springer, Phys. Rev. D72 (2005) 034006] for ground state $J^P = 0^-$ and 1^- and first excited state, $J^P = 0^+$ and 1^+ charmed meson up to one loop chiral corrections. For this we formulated the Lagrangian and applied the two symmetries i.e. chiral symmetry and the heavy quark symmetry. These symmetries are used to obtain the relation between heavy meson masses in terms of the symmetry conserving(σ_s, σ_H, a_s and a_H) and symmetry breaking parameters(Δ_H,Δ_s,Δ_H^σ,$\Delta_H^{(a)}$,Δ_S^σ,$\Delta_S^{(a)}$). Some parameters cannot be calculated separately; hence their values have been absorbed in other parameters. With the resulting eight equations in eight variables, these have been fitted properly to get the desired masses of those charmed mesons which were ambiguous at the various experiments. We have performed a constrain fit to the charmed meson spectrum for specific values of couplings(g, g', and h).We worked for energy range ~1GeV. The masses of u and d quark are taken to be equal to each other having value ~ 4MeV while mass of s quark has been taken to be ~ 90 MeV. We also worked for fittings to get close values for the mass splittings.

CONTENTS:

Chapter 1

Introduction

1.1 Elementary Particle Physics:

The world is full of diverse objects. It is natural curiosity of man to ask with other questions that:

- Are the things around us made up of elementary (fundamental) particles?
- If yes, then how are they classified?
- How do they interact with each other?

And many other questions as well.

The branch of physics where we try to find answers of all these and related questions is called Elementary Particle Physics, It is also known as High Energy Physics(due to the experiments being performed at much higher energies than the normal ones). The past two decades have seen a remarkable progress in the physics of elementary particles. This could have been possible by a new generation of particle accelerators and detectors. We collect huge amount of experimental data from these huge machines. Before proceeding further let us have a look over some of these huge machines.

1.2 Accelerators and Detectors:

Accelerators: Accelerators are the machines that accelerate the elementary particle to very high energies. Higher energies are quite necessary to achieve because at lower energies we can see only few elementary particles and their interactions. At higher energies so many particles are produced which are not seen at lower energies and along with that we see new phenomenon and interactions also.

Detectors: when the particles accelerated by the accelerators are made to have collisions, enormous amount of elementary particles are produced with different energies and travelling in various directions. We need to detect these particles by some means and calculate their energies, momentum, masses etc. in order to understand the phenomenon involved during the collisions. This work is done by the detectors. Some of these are discussed below:

BABAR: BABAR is a High Energy Physics experiment located at SLAC National Accelerator Laboratory, near Stanford University, in California. It was built at SLAC to study the millions of B mesons produced by the PEP-II storage ring. The BaBar Collaboration consists of approximately 600 physicists and engineers from 75 institutions in 10 countries. The goal of the experiment is to study the violation of charge and parity (CP) symmetry in the decays of B mesons. This violation manifests itself as different behaviour between particles and anti-particles and is the first step to explain the absence of anti-particles in everyday life.

To study CP violation the BABAR experiment exploits the 9.1 GeV electron beam and the 3 GeV positron beam of the PEP-II accelerator. The two beams collide in the centre of the experiment, producing Y(4S) mesons which decay into equal numbers of B and anti-B mesons[1].The 2008 Nobel Prize in Physics was awarded to Kobayashi, Maskawa, and Nambu for their work on symmetry breaking and CP violation. Understanding the origins of CP violation is one of the primary goals of the experimental programs of both the BaBar and Belle experiments[2].

CLEO: CLEO-c is a dedicated program of charm physics at the Cornell Electron Storage Ring (CESR-where electron & positron beams from opposite side with energy 8GeV each collide with each other). They are making the weak physics more clear behind both charm and bottom quark decays by helping to disentangle it from the confounding strong-interaction dynamics. In particular, its measurements of D and D_s meson decays to leptonic and semileptonic final states are crucial tests of the Lattice QCD techniques used to compute important heavy quark processes[3].

CDF: CDF (Collider Detector at Fermilab) is one of two detectors at the Tevatron at Fermilab, located outside Chicago. The Tevatron collides proton and anti-proton beams with a combined energy of about 2 TeV and has a rich physics program, including bottom physics, top physics, W measurements and searches for the Higgs [4].

The Tevatron accelerates protons and antiprotons close to the speed of light, and then makes them collide head-on inside the CDF detector. The CDF detector is used to study the products of such collisions; by doing this we try to reconstruct what happened in the collision and ultimately try to figure out how matter is put together and what forces nature uses to create the world around us [5].

FOCUS: FOCUS, aka E831, is a heavy-flavor photo production experiment located in the Wide Band Area of Fermilab [6]. Its analysis includes:

- High precision studies of charm semileptonic decays
- Studies of hadronic charm decays (branching ratios and Daltiz analyses)
- Lifetime measurements of all charm particles
- Searches for mixing, CP/CPT violation, rare and forbidden decays
- Spectroscopy of excited charm mesons and baryons.

From such accelerators and detectors we now have a huge amount of experimental data. Therefore we need a framework or theory which can account for these experimental data. Physicists have developed a theory called Standard Model which explains the fundamental particles, their properties, their interactions, their dynamics etc. It is discussed in the next section.

1.3 Standard Model:

The standard model (SM) of particle physics is a theory concerning the electromagnetic, weak, and strong nuclear interactions, which explains the dynamics of the known subatomic particles. Developed throughout the early and middle 20th century, the current formulation was finalized in the mid1970's based upon experimental confirmation of the existence of quarks. It is a model which very beautifully explains almost all the experimental data collected from various collider experiments.

Still, the Standard Model (SM) cannot be considered as a complete theory of fundamental interactions because it does not incorporate the physics of general relativity, such as gravitation and dark energy. It also does not correctly account for neutrino oscillations (and their non-zero masses). Although the standard model is theoretically self-consistent, it has several unnatural properties giving rise to puzzles like the strong CP problem and the hierarchy problem.

Standard model assumes that all the things are made up of two types of particles namely quarks and leptons. And they interact with each other via interchange of Bosonic particles called gauge bosons. The quarks and leptons are the fundamental particles of the standard model. They are fermions having spin quantum number which is half integer and obey Fermi-Dirac statics.

(i) Quarks:

Quarks come in 6 flavors: up (u), down (d), strange (s), charm (c), bottom (b) and top (t).

The quarks can be arranged in three generations of doublets, the properties of the generation being similar, but the masses of which are successively heavier:

$$\binom{u}{d}\binom{c}{s}\binom{t}{b}$$

u, c and t have charge $= +2\ /\ 3$e While d, s and b quarks have charge $= -1/3$e.

Also the quarks carry an additional charge known as color charge. Quarks come in three colors blue, red and green.

(ii) Leptons:

The leptons come in two types (charged and neutral) and in three flavors e, μ and τ and the neutral leptons or neutrinos denoted by the symbols ν. The leptons can also be arranged in three generational doublets as shown below:

$$\binom{\nu_e}{e}\binom{\nu_\mu}{\mu}\binom{\nu_\tau}{\tau}$$

Here all the neutrinos have charge zero and all the other leptons have charge$= -1$e.

One of the most interesting difference between leptons and quarks is that leptons can be observed as free particles individually but it is not so for quarks. Quarks are always found in bound states.

The interaction between various particles occurs due to exchange of gauge bosons, the following table summarises basic properties of these bosons:

Name	Symbol	Charge(e)	Spin	Mass (GeV/c^2)	Interaction mediated
Photon	γ	0	1	0	Electro-magnetic
W Bosons	$W^{\pm}$	± 1	1	80.4	Weak
Z boson	Z	0	1	91.2	Weak

Table 1.1Gauge Bosons and their Properties

(iii) Hadrons & Gluons:

Quarks are not found in free-state, rather they are found in bound state. These bound states are called hadrons. Is has been seen till now that the quarks are either in a bound state containing three quarks or in a bound state containing one quark and one antiquark.

Baryons: The bound states with three quarks are known as Baryons. All the baryons are having spin half integral ($1/2$, $3/2$, etc.), hence are called fermions and obey F-D statistics. Common examples of baryons are proton (uud), neutron (udd), sigma particles, xi particles, lambda particle, omega particle (sss) etc.

B = qqq

Mesons: The bound states with quark-antiquark combination are called MESONS. All the mesons have integral spin (0, 1, 2, etc.), i.e. they are Bosonic in nature and follow the Bose-Einstein statistics. Some common examples of mesons are pi-mesons, K-mesons, eta-meson, J/Ψ, rho-meson etc.

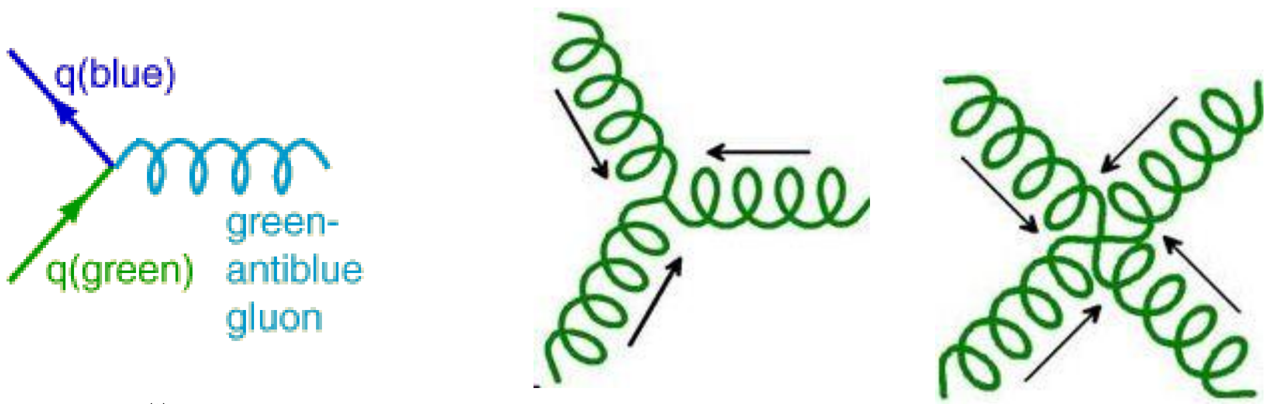
M = $q\overline{q}$

If we take three flavours of quarks, then the quarks lie in the fundamental representation, 3 (called the triplet) of flavour SU(3). The antiquarks lie in the complex conjugate representation $\overline{3}$. The nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet). The notation for this decomposition is $3 \otimes \overline{3} = 8 \oplus 1$.

If the flavour symmetry were exact, then all nine mesons would have the same mass. The physical content of the theory includes consideration of the symmetry breaking induced by the quark mass differences, and considerations of mixing between various multiplets.

Gluons: Gluons are the elementary particle that mediates or carries the strong forces. Gluons transmit "color" force between quarks. The gluon generates a color change for the quarks as shown in the fig. Here in fig.1.1(i) a green quark gets changed into a blue quark by emitting a green–anti-blue gluon. Here it is important to mention that Gluons are different from the other bosons as they carry an extra degree of freedom called color. Gluons come in 8 colors. Due to this property of carrying color, the gluons can interact with other gluons also.

The fig.1.1(ii) and fig 1.1(iii) given below shows a three gluon and a four gluon interaction respectively.



(i) (ii) (iii)

Fig 1.1 (i) A green quark changing to blue quark by exchange of gluon (ii) Three gluon Interaction (iii) Four gluon Interaction

(iv) Fundamental Interactions:

Each particle in the universe interacts with the other particles around it. For example earth moves around sun due to gravitational force between earth and the sun. An interaction is said to be fundamental if it cannot be defined in terms of the other ones. So far there are fundamental four forces known to us:

(a) Gravitational interaction:

It is the force due to which any particle that has mass interacts with all other particle attractively. It is the very first force known to man, but unfortunately it is not describable in the standard model. We do not still have a quantum theory of gravity. Newton was first to give a good classical theory for gravitation, later Einstein modified it and gave general theory of relativity to describe it.

The gravitational force between two particles with masses m_1 and $m_2$is given as:

$$F_{gravity} = \frac{gm_1m_2}{r^2}$$

Where r is the distance between the masses. It is the weakest foe but interestingly its range is infinite.

(b) Electromagnetic interaction:

Any particles having electric charge can interact electromagnetically. It is the force that holds the protons and electrons together inside the atom. The quantum theory that explains the interactions of electrically charged particles like electrons is Quantum Electrodynamics (QED). Photons are the carrier particles of electromagnetic interaction.

It is given as: $$F_{em} = \frac{ke^2}{r^2}$$

A dimensionless constant which characterizes the electromagnetic force is

$$\alpha = \frac{2\pi ke^2}{hc} = \frac{1}{137}$$

This coupling constant is also called the "fine structure constant".

(c) Strong interaction:

It holds quarks and gluons together to form protons, neutrons and other particles. The word “strong” is used since the strong interaction is the strongest of the four fundamental forces. Its strength is100 times the strength of electromagnetic force, some 10^5times greater than the weak force, and about 10^{39} times greater than gravitation. The strong interaction is also the force that binds proton and neutrons together. The strong force is mediated by gluons, acting upon quarks, anti-quarks and gluons themselves. Analysis of the coupling constant with quantum chromo-dynamics(QCD) gives an expression for the diminishing coupling constant. The coupling decreases logarithmically, a phenomenon known as asymptotic freedom. The coupling decreases approximately as

$$\alpha_s(k^2) = \frac{{g_s}^2(k^2)}{4\pi} = \frac{1}{\beta_0 \ln\frac{k^2}{\Lambda^2}}$$

Where β_0 is a constant. In non-Abelian gauge theories, the beta function can be negative. Conversely, the coupling increases with decreasing energy. This means that the coupling becomes large at low energies.

(d) Weak interaction:

It is due to exchange of heavy $W^{\pm}$ and Z^0 bosons. Its most familiar effect is emission of electrons or positrons by neutrons in atomic nuclei and the associated radioactivity. The word 'weak' derives from the fact that the typically field strength is 10^{-11} the strength of electromagnetic force and some10^{-13} that of the strong force, when forces are compared between particles interacting in more than one way.

All these interactions are summarised in tabular form given below:

Interaction	**Current Theory**	**Mediators**	**Relative Strength**	**Range (m)**
Strong	QCD	Gluons	10^{38}	10^{-15}
Electromagnetic	QED	Photons	10^{36}	∞
Weak	Weak Theory	W and Z bosons	10^{25}	10^{-18}
Gravitation (Not included in SM)	General Theory of Rel.	Graviton (Not in SM)	1	∞

Table 1.2 Four Interactions and Their Properties

(v) Unification:

As mentioned above, till now, we know about four types of fundamental forces. To explain these forces, we have individual theories. For example for gravitation we have Einstein's theory of general relativity, for electromagnetic force we have QED, for Strong force there is QCD etc. but physicists find it unsatisfactory to have individual theories for all these forces. Scientists believe that all these forces are just the manifestation of a single force. There is a single force that behaves differently at different energy scales. It is supposed that immediately after the Big-Bang the equilibrium temperature was so much high that all these forces were unified and one could see only one force. As the time passed, the temperature of the universe got down, when the temperature of universe got cooled down to less than 100 GeV, the symmetry was spontaneously broken and the forces got separated out.

Based on such concepts, the physicists are trying to unify the forces into a single force. The first unification was done by Maxwell, who showed that electricity and magnetic force are the manifestation of the same phenomenon called ELECTROMAGNETISM. In 1979 for Glashow, Salam and Weinberg won the Nobel Prize for succeeding in unifying the Weak and Electromagnetic interactions into what is called the Electroweak force and consequently we got another theory called Quantum Electroweak Theory. Quantum electroweak theory presents a unified description of two of the four fundamental forces of nature: electromagnetic and the weak nuclear force. These two forces appear very different at low energies; The EW theory models them as two different aspects of the same force. At present theories are proposed for further unification like GUT(Grand Unification Theory), that unites the electroweak and strong force together. Unfortunately we still are not able to unite gravitation into unified model. If in future we have a quantum theory of gravitation, we may unite gravitation also with other forces.The unification is accomplished under an $SU(3)\otimes SU(2)\otimes U(1)$ gauge groups.

1.4 Symmetries and Groups

In simple language a shape is said to be symmetric when it becomes exactly like another if you flip, slide or turn it correspondingly we have reflection, translation, rotation symmetries etc.

Any type of transformation which when applied on the system does not change it, is called symmetry transformation.

In particle physics the symmetries are of great importance. It was not until 1917 that the dynamical implications of symmetry were completely understood. In that year Emmy Noether published her famous theorem relating symmetries and conservation laws:

NOETHER'S THEOREM: SYMMETRIES ↔CONSERVATION LAWS [7]

That is for each symmetry there exists a conservation law; conversely, every conservation law reveals an underlying symmetry. If a system is invariant under translations in space, then momentum is conserved; if it is symmetrical under rotations about a point, then angular momentum is conserved. Similarly, the invariance of electrodynamics under gauge transformations leads to conservation of charge.

There is another very interesting fact about symmetries. The set of symmetry transformations obey the properties of the group like closure property, identity, associativity and also other properties of the groups. Practically, all the continuous groups appearing in particle physics are Lie Groups.

Lie algebra and Lie Groups: Lie algebras were introduced to study the concept of infinitesimal transformations. The term Lie algebra was introduced by Herman Weyl in 1930's. Lie Groups are those continuous groups whose parameter space is locally Euclidean. By Locally Euclidean, we mean that the parameter space within the infinitesimal vicinity of any element of the continuous group is taken as finite dimensional Euclidean space.

Generally the continuous Lie groups appearing in particle physics are U(n), SU(n), O(n) and SO(n) group. U(n) stands for the unitary group of dimension one. A set of all n×n matrices form a unitary group. Unitary matrices are those for which the inverse is equal to the transpose conjugate $U^{-1} = \overline{U}^{*}$. For n=1, U(1) is Abelian, for n>1, U(n) is non Abelian. When the unitary matrices of U(n) group have their determinant equal to 1, then they are called special unitary SU(n) matrices.

SU(2) : In particle physics, SU(2) group is very important group as it can describe both spin and isospin. We can therefore have SU(2) spin and SU(2) isospin groups. The nucleons proton and neutron can be considered as the member of SU(2) group, both being spin $(1/2)$ particles having isospin value = $1/2$. This symmetry is broken when we consider isotopic spin which has value $+1/2$ for proton while $-1/2$ for neutron.

The elements (matrices) of SU(2) group have their determinant = +1. The generators of SU(n) group must be traceless Hermitian matrices of order n. also the maximum number of traceless Hermitian matrices of order n are n^2-1. Therefore SU(n) group shall have n^2-1 traceless Hermitian matrices as its generators. Hence SU(2) group has 2^2-1= 3 generators. We write the 2×2 unitary uni-modular matrices as

$$\mathrm{U}(\varepsilon_1.\varepsilon_2,\varepsilon_3) = \exp\{\mathrm{i}\varepsilon_\mathrm{a}\sigma_\mathrm{a}\}$$

Where the $\sigma_a's$ are 2×2 traceless Hermitian matrices. We chose the basis to be the standard Pauli matrices.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The generators defined by $J_i = {}^{\sigma_i}/_2$ will give the commutation relation

$$[J_a, J_b] = i\varepsilon_{abc}J_c$$

SU(3):- Basically the group SU(3) originated as an extension of SU(2) isospin group in which Λ particle was included to p and n to form a fundamental triplet. When Λ and K particles were discovered they were found to be produced normally but to decay with long life time. It was postulated that these new particles possessed a new additive quantum number, strangeness S, which is conserved in strong interactions but violated in weak interactions. Strangeness is associated with a U(1) symmetry. There is a linear relation among S,Q and the generator $T_3$and the relation is

$$\mathrm{Q} = \mathrm{T}_3 + \frac{\mathrm{Y}}{2} \qquad \text{with} \qquad \mathrm{Y=B+S}$$

This is called Gell Mann-Nishijima relation. Here B is baryonic number while Y is hypercharge. Isospin and hypercharge are approximately conserved but their linear combination i.e. electric charge is always conserved [8].

The fundamental triplet is (u, d, s) and the SU(3) group that uses ((u, d, s) as the fundamental triplet is called SU(3) flavor. With the introduction of color, (R,G,B) also became a fundamental triplet and we have SU(3) color group.

SU(3) group has 3^2-1=8 generators. We write the 3×3 unitary matrices as

$$U(\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots \dots .., \varepsilon_8) = \exp\{i\varepsilon_a \lambda_a\}$$

The λ's are 3×3 traceless Hermitian matrices, which may be chosen to have the form

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

And satisfy the commutation relation

$$\left[\frac{\lambda_a}{2} . \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$$

$f_{abc}$is totally anti-symmetric.

SU(3) flavor symmetry is a very useful approximate symmetry in QCD because hadrons containing light quarks and antiquarks of different flavors have similar properties. Hadrons either mesons or baryons can be organized into SU(3) flavor multiplets. Nine states are formed and these form an SU(3) octet and singlet. Symbolically it can written as

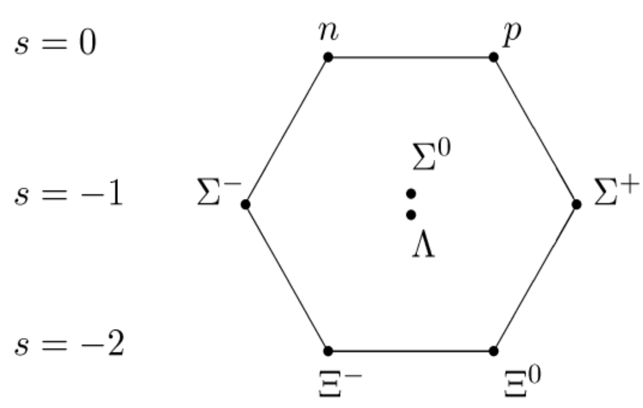


Fig. 1.2 Spin (1/2) Baryon Octet

$$3 \otimes \overline{3} = 8 \oplus 1$$

For example ground state (L=0), spin ½ baryons have similar properties and form an octet as shown in fig. 1.2. Similarly spin(3/2) baryons form a decuplet and mesons with spin 0 form an octet as shown in fig. below:

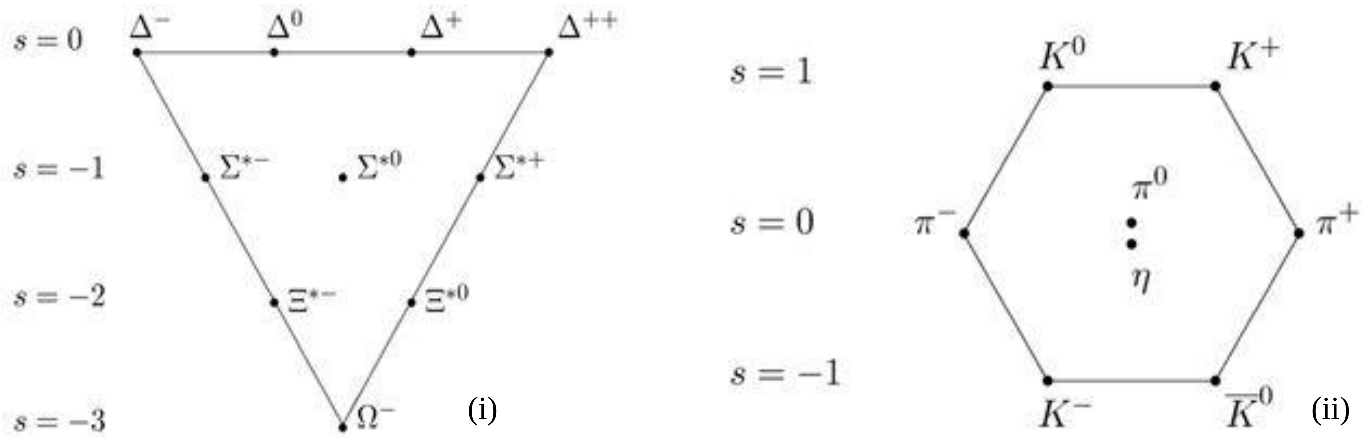


Fig. 1.3 (i) Baryon spin (3/2) Decuplet (ii) Meson spin 0 Octet

1.5 Symmetry breaking:-

Symmetry can be exact, approximate, or broken. Exact means unconditionally valid; approximate means valid under certain conditions; broken can mean different things, depending on the object considered and its context. Generally, the breaking of certain symmetry does not imply that no symmetry is present, but rather that the situation where this symmetry is broken is characterized by a lower symmetry than the situation where this symmetry is not broken. Symmetry breakings are classified as:

(i) Spontaneous symmetry breaking

(ii) Explicit symmetry breaking

(i) Spontaneous symmetry breaking:-

It is possible that the vacuum or the ground state of a physical system has less symmetry than the Lagrangian density. This possibility is called the Spontaneous Symmetry Breaking(SSB). The phenomenon of SSB is known to occur in several branches of physics. Usually, systems exhibiting SSB have an infinite number of degrees of freedom. Since field theories are intrinsically theories with infinite number of degrees of freedom, it might be that the theory of fundamental constituents of matter is a spontaneously broken one. In fact, SSB plays a pivotal role in the standard model in the unification of the weak and the electromagnetic interactions.

To understand it, let us consider a system described by a Lagrangian density L which is invariant under a continuous group G of transformations. We examine the ground state of the system. If the system possesses a unique ground state, invariant under G, we have a situation where symmetry properties of L and the vacuum(ground) state are same. Such a theory is the normal one. On the other hand, it might happen that the system has several ground states which transform into each other under the transformation in G. then, if for some reason, one of the ground states is singled out as the physical ground state of the system (the others being unphysical), the symmetry is lost and the theory is said to be spontaneously broken[9].It is only the spontaneous breaking of the continuous symmetry that we get a massless scalar field. It was proved by J. Goldstone that the spontaneous breaking of a continuous global symmetry always generates one or more massless scalar bosons and such bosons are called Goldstone bosons[10].

(ii) Explicit symmetry breaking :-

Explicit symmetry breaking indicates a situation where the dynamical equations are not manifestly invariant under the symmetry group considered. This means, in the Lagrangian (Hamiltonian) formulation, that the Lagrangian (Hamiltonian) of the system contains one or more terms explicitly breaking the symmetry. Such terms can have different origins:

(a) Symmetry-breaking terms may be introduced into the theory by hand on the basis of theoretical/experimental results, as in the case of the quantum field theory of the weak interactions, which is expressly constructed in a way that manifestly violates mirror symmetry or parity. The underlying result in this case is parity non-conservation in the case of the weak interaction, first predicted in the famous (Nobel-prize winning) 1956 paper by T. D. Lee and C.N. Yang.

(b) Symmetry-breaking terms may appear in the theory because of quantum-mechanical effects. One reason for the presence of such terms — known as “anomalies” — is that in passing from the classical to the quantum level, because of possible operator ordering ambiguities for composite quantities such as Noether charges and currents, it may be that the classical symmetry algebra (generated through the Poisson bracket structure) is no longer realized in terms of the commutation relations of the Noether charges. Moreover, the use of a “regulator” (or “cut-off”) required in the renormalization procedure to achieve actual calculations may itself be a source of anomalies. It may violate a symmetry of the theory, and traces of this symmetry breaking may remain even after the regulator is removed at the end of the calculations.

(c) Finally, symmetry-breaking terms may appear because of non-renormalizable effects. Physicists now have good reasons for viewing current renormalizable field theories as effective field theories, that is low-energy approximations to a deeper theory (each effective theory explicitly referring only to those particles that are of importance at the range of energies considered). The effects of non-renormalizable interactions (due to the heavy particles not included in the theory) are small and can therefore be ignored at the low-energy regime. It may then happen that the coarse-grained description thus obtained possesses more symmetries than the deeper theory. That is, the effective Lagrangian obeys symmetries that are not symmetries of the underlying theory. These “accidental” symmetries, as Weinberg has called them, may then be violated by the non-renormalizable terms arising from higher mass scales and suppressed in the effective Lagrangian [11].

Chapter 2

Heavy Quark Effective Field Theory (HQET)

2.1 Effective Field Theories:

An effective field theory(EFT) is an approximate theory (usually a quantum field theory) that includes appropriate degrees of freedom to describe physical phenomena occurring at a chosen length or energy scale, while ignoring degrees of freedom at shorter distances (or, equivalently, at higher energies). EFTs are approximations by their very nature. Once the relevant degrees of freedom for the problem at hand have been established, the corresponding EFT is usually treated perturbatively [12].

We have many EFT's in physics for example, BCS theory for superconductors in an EFT, Fermi's theory of weak interactions is an EFT, For hadrons containing one heavy quark (such as the bottom or charm), an effective field theory which expands in powers of the quark mass, called the heavy-quark effective theory (HQET) has been found useful, For hadrons containing two heavy quarks, an effective field theory which expands in powers of the relative velocity of the heavy quarks, called non-relativistic QCD (NRQCD) [13], has been found useful, For hadron reactions with light energetic (collinear) particles, the interactions with low-energetic (soft) degrees of freedom are described by the soft-collinear effective theory (SCET) [14].

Effective field theory (EFT) is a very powerful tool in quantum field theory [15]. It provides a systematic formalism for the analysis of multi-scale problems. This is particularly important in QCD, where the value of the running coupling $\alpha_s(\mu)$can change significantly between different energy scales. EFT greatly simplifies the practical calculations in the field theory. Consider a quantum field theory with a characteristic energy scale E_0, and suppose we are interested in the physics at some lower scale $E \ll E_0$. Of course, most systems have several characteristic scales, but we can consider them one at a time. Effective field theory is a general method for analyzing this situation.

Effective field theories have become a widely used tool in modern elementary particle physics. An effective theory treatment is convenient if the problem under consideration involves very disparate mass scales such that the physics that is to be described happens at much lower energies than the scale set by some heavy particles in the theory. In such a case it is useful to switch to an effective theory in which the heavy degrees of freedom do not appear explicitly; they only reappear in the effective theory as higher dimensional operators. The scale of the coupling constants is set by the large mass and thus these contributions are small, if the scale of the physics described with the help of the effective theory is small compared to this large mass. Here, It is important to keep in mind that an effective theory is always valid only in a limited region of scales, a natural cut-off is given by the mass of the particle which has been removed by switching from the full to the effective theory.

In the present report we will be discussing about HQET, in this case the large scale is the mass $m_Q$of the heavy quark, and the leading term of this expansion is the static limit. This effective theory (Heavy Quark Effective Theory, HQET) is a powerful tool, allowing for numerous purely QCD based calculations. Before moving to HQET a little knowledge about Heavy quark symmetry and Chiral symmetry is necessary, so in next section these symmetries are discussed.

2.2 (i) Heavy Quark Symmetry:-

We will introduce the ideas of heavy quark symmetry and the heavy quark limit, which exploit the simplification of certain aspects of QCD for infinite quark mass, $m_Q \rightarrow \infty$. We will see that while these ideas are extra ordinarily simple from a physical point of view, they are of enormous practical utility in the study of the phenomenology of bottom and charmed hadrons. One reason for this is the existence not just of an interesting new limit of QCD, but of a systematic expansion about this limit. The technology of this expansion is the Heavy Quark Effective Theory (HQET).Heavy Quark Effective Theory (HQET) or, more generally, the expansion in inverse powers of the heavy quark mass $\mathrm{m_Q}$, has become a generally accepted and widely used tool in heavy quark physics [16]. It allows one to use heavy quark symmetry to make accurate predictions of the properties and behavior of heavy hadrons in which the theoretical errors are under control [17].

There are several reasons why the strong interactions of systems containing heavy quarks are easier to understand than those of systems containing only light quarks. The first is asymptotic freedom, the fact that the effective coupling constant of QCD becomes weak in processes with large momentum transfer, corresponding to interactions at short-distance scales. At large distances, on the other hand, the coupling becomes strong leading to non-perturbed phenomena such as the confinement of quarks and gluons on a length scale $R_{\text{had}} \sim 1/\Lambda_{QCD} \sim 1$fm,which determines the size of hadrons. Roughly speaking, $\Lambda_{QCD} \sim 0.2$GeV is the energy scale that separates the regions of large and small coupling constant .When the mass of a quark Q is much larger than this scale; it is called a heavy quark.

For heavy quarks, the effective coupling constant $\alpha_s(m_Q)$ small, implying that on length scales comparable to the Compton wavelength $\lambda_Q \sim 1/m_Q$the strong interactions are perturbative and similar to the electromagnetic interactions. In fact, the Quarkonium systems $\overline{(\text{Q}}\text{Q})$, whose size is of order $\lambda_Q/\alpha_s\ (m_Q) \ll R_{\text{had}}$, are very much hydrogen-like.

Systems composed of a heavy quark and light constituents are more complicated, however. The size of such systems is determined by R_{had} , and the typical momenta exchanged between the heavy and light constituents are of order Λ_{QCD}. The heavy quark is surrounded by a most complicated, strongly interacting cloud of light quarks, antiquarks, and gluons. In this case it is the fact that $\lambda_Q << R_{\text{had}}$, i.e. that the Compton wavelength of the heavy quark is much smaller than the size of the hadron, which leads to simplifications. To resolve the quantum numbers of the heavy quark would require a hard probe; the soft gluons exchanged between the heavy quark and the light constituents can only resolve distances much larger than λ_Q. Therefore, the light degrees of freedom are blind to the flavor (mass) and spin orientation of the heavy quark. They experience only its color field, which extends over large distances because of confinement. In the rest frame of the heavy quark, it is in fact only the electric color field that is important; relativistic effects such as color magnetism vanish as $m_Q \to \infty$.

That the heavy-quark mass becomes irrelevant can be seen as follows: As $m_Q \to \infty$,the heavy quark and the hadron that contains it have the same velocity. It may be noted that the velocity of the heavy quark plays a very important role. In the work of K. Hagiwara, A.D. Martin, and M.F. Wade[18], one can see the importance played by the velocity of the heavy quarks as; indeed hidden in the analysis is a peaking about a velocity super selection rule[19]. It is to the credit of Isgur and Wise[20] that they were able to extract the physical, model independent implication, namely, that for infinitely heavy quarks, velocity is the only important parameter. In the rest frame of the hadron, the heavy quark is at rest, too. The wave function of the light constituents follows from a solution of the field equations of QCD subject to the boundary condition of a static triplet source of color at the location of the heavy quark. This boundary condition is independent of m_Q, and so is the solution for the configuration of the light constituents.

It follows that, in the limit $m_Q \to \infty$, hadronic systems which differ only in the flavor or spin quantum numbers of the heavy quark have the same configuration of their light degrees of freedom . Although this observation still does not allow us to calculate what this configuration is, it provides relations between the properties of such particles as the heavy mesons B, D, B^* and D^*, or the heavy baryons Λ_b and Λ_c (to the extent that corrections to the infinite quark-mass limit are small in these systems). These relations result from some approximate symmetry of the effective strong interactions of heavy quarks at low energies. The configuration of light degrees of freedom in a hadron containing a single heavy quark with velocity v does not change if this quark is replaced by another heavy quark with different flavor or spin, but with the same velocity. Both heavy quarks lead to the same static color field. For N_h heavy-quark flavors, there is thus an SU $(2N_h)$ spin–flavor symmetry group, under which the effective strong interactions are invariant. These symmetries are in close correspondence to familiar properties of atoms: The flavor symmetry is analogous to the fact that different isotopes have the same chemistry, since to a good approximation the wave function of the electrons is independent of the mass of the nucleus. The electrons only see the total nuclear charge. The spin symmetry is analogous to the fact that the hyperfine levels in atoms are nearly degenerate. The nuclear spin decouples in the limit $\mathrm{m_e}/\mathrm{m_n} \to 0$.

Heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinitely heavy. In many respects, it is complementary to chiral symmetry, which arises in the opposite limit of small quark masses. However, whereas chiral symmetry is a symmetry of the QCD Lagrangian in the limit of vanishing quark masses, heavy-quark symmetry is not a symmetry of the Lagrangian (not even an approximate one), but rather a symmetry of an effective theory, which is a good approximation of QCD in a certain kinematic region. It is realized only in systems in which a heavy quark interacts predominantly by the exchange of soft gluons. In such systems the heavy quark is almost on shell; its momentum fluctuates around the mass shell by an amount of order Λ_{QCD}. The corresponding fluctuations in the velocity of the heavy quark vanish as $\Lambda_{QCD}/m_Q \rightarrow 0$. The velocity becomes a conserved quantity and is no longer a dynamical degree of freedom. Nevertheless, results derived on the basis of heavy-quark symmetry are model-independent consequences of QCD in a well-defined limit. The symmetry-breaking corrections can, at least in principle, be studied in a systematic way.

A convenient framework for analyzing these corrections is provided by the HQET.

(ii) Chiral symmetry:

Chiral symmetry is an internal symmetry of right and left handed spinors. It has importance in low energy hadronic physics, since its spontaneous breaking generates Goldstone bosons with negative parity, zero spin, unit isospin and zero baryon number called pions. Thus a broken approximate chiral symmetry entails the existence of pions where u and d quarks have small but non-zero masses whereby spontaneous breaking of a symmetry is expressed as the non-vanishing of the vacuum when operated by the charge Q. A spontaneously broken symmetry relates processes with different numbers of Goldstone bosons.

As the masses of light quarks are small compared to Λ_{QCD} , in the chiral limit, their masses will go to zero and making the masses of heavy quarks, m_c, m_b and $m_t$to be infinity.

$m_q \rightarrow 0$, where q can be u, d or s.

2.3 Chiral Perturbation Theory:

Quantum chromo-dynamics (QCD) , the theory of strong interactions is formulated for quark and gluon degrees of freedom and its fundamental coupling constant is the strong coupling constantα_s . at large momenta this coupling constant becomes small due to asymptotic freedom. When the energies under consideration are small enough for the t, b and c quark degrees to have frozen out, striking features of the spectrum include the fact that some of the mesons are very light compared to the baryons. Here approximate chiral symmetries are spontaneously broken.

The members of the pseudo-scalar octet are the Goldstone bosons associated with these spontaneously broken symmetries [21].

The QCD Hamiltonian contains a quark mass term which breaks the symmetry. The vector and axial currents are not exactly conserved. Since the masses of u, d and s are, however, small, the divergence of the currents which generate SU(3) x SU(3) approximately vanishes. Accordingly the low energy theorems of SU(3) X SU(3) should be approximately valid in the real world. The subgroup SU(2) x SU(2) is an exact symmetry if m, = md = 0. Since the masses of u and d are tiny, the low energy theorems associated with this subgroup should show even smaller deviations. The deviations from chiral symmetry may be studied by treating the quark mass term in the Hamiltonian as a perturbation, with massless QCD as the unperturbed system (chiral perturbation theory) [22].

Chiral perturbation theory is the effective low-energy theory of the strong interactions and is an expansion of the Green functions of its currents associated with the near masslessness of the three lightest quarks, and with the spontaneously broken approximate axial vector symmetries, in powers of momentum and quark masses.

2.4 The Heavy Quark effective theory:

The effects of a heavy particle become irrelevant at low energies. It is then useful to construct a low energy effective theory, in which this heavy particle no longer appears. Eventually this effective theory will be easier to deal with than the full theory.

To remove the degrees of freedom of the heavy particle we will involve three steps:

1. At low energies the heavy particle does not appear as an external state hence it is possible to identify the heavy particle fields and then integrate them out in the generating functional of the Green functions of the theory[22].
2. However, although the action of the full theory is usually a local one, what results after this first step is a non-local effective action. The non-locality is related to the fact that in the full theory the heavy particle with mass M can appear in virtual processes and propagate over a short but finite distance $\Delta x \sim 1/M$. Thus, a second step is required to obtain a local effective

Lagrangian: the non-local effective action is rewritten as an infinite series of local terms using the OPE. Roughly speaking, this corresponds to an expansion in powers of $1/M$.

It is in this step that the short-and long-distance physics is disentangled. The long-distance physics corresponds to interactions at low energies and is the same in the full and the effective theory. But short-distance effects arising from quantum corrections involving large virtual momenta (of order M) are not reproduced in the effective theory, once the heavy particle has been integrated out.

3. In a third step, they have to be added in a perturbative way using renormalization-group techniques. These short-distance effects lead to are normalization of the coefficients of the local operators in the effective Lagrangian.

The HQET gives a simplified description of the processes in which the heavy quark interacts with the light degrees of freedom predominantly by the exchange of the soft gluons. Clearly $m_Q$is the high energy scale in this case, and Λ_{QCD} is the scale of the hadronic physics, in which we are interested.

The situation is illustrated as shown in the fig.2.1

At short distances i.e. for energy scale larger than the heavy quark mass, the physics is perturbative and described by ordinary QCD. For mass scale much below the heavy quark mass, the physics is complicated and non-perturbative because of confinement.

Our goal is to obtain a simplified description in this region using an effective field theory. To separate short-and long-distance effects, we introduce a separation scale μ such that $\Lambda_{QCD} \ll \mu \ll m_Q$.The HQET will be constructed in such a way that it is identical to QCD in the long-distance region, i.e. for scales below μ. In the short-distance region, the effective theory is incomplete, however, since some high-momentum

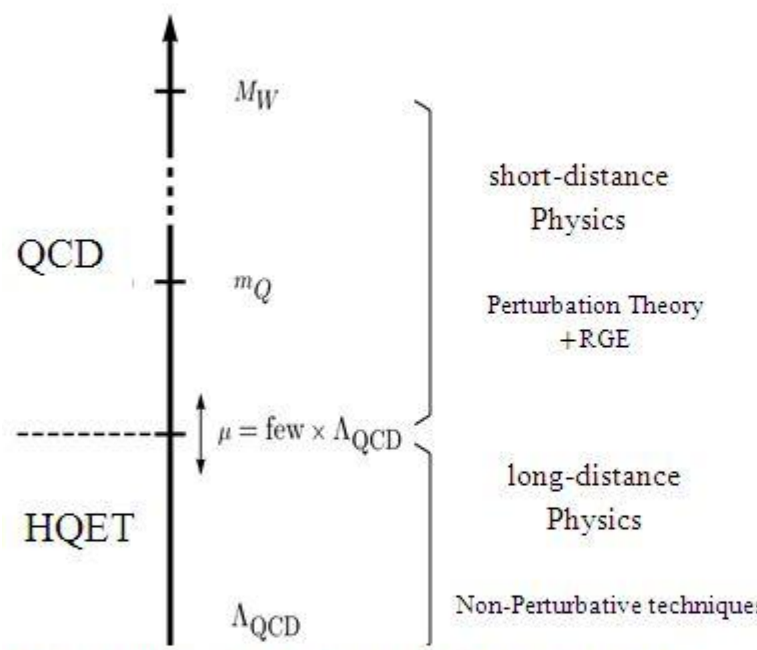


Fig 2.1 Philosophy of Heavy Quark Effective Theory

modes have been integrated out from the full theory. The fact that the physics must be independent of the arbitrary scale μ allows us to derive renormalization-group equations, which can be employed to deal with the short-distance effects in an efficient way.

Compared with most effective theories, in which the degrees of freedom of a heavy particle are removed completely from the low-energy theory, the HQET is special in that its purpose is to describe the properties and decays of hadrons which do contain a heavy quark. Hence, it is not possible to remove the heavy quark completely from the effective theory. What is possible is to integrate out the "small components" in the full heavy-quark spinor, which describe the fluctuations around the mass shell.

The starting point in the construction of the low-energy effective theory is the observation that a very heavy quark bound inside a hadron moves more or less with the hadron's velocity v. Georgi [23] describes the situation rather vividly. If the bound state, made up of a light quark system and one heavy quark, is moving with velocity v^{μ} and is almost on shell.

Then the 4−momentum of the bound state is

$$P_Q{}^{\mu} = m_Q v^{\mu} + k^{\mu}$$

Where the components of the so-called residual momentum k are much smaller than m_Q. Note that v is a four-velocity. Interactions of the heavy quark with light degrees of freedom change the residual momentum by an amount of order $\Delta k \sim \Lambda_{QCD}$, but the corresponding changes in the heavy-quark velocity vanish as $\Lambda_{QCD}/m_Q \to 0$. In this situation, it is appropriate to Introduce large-and small-component fields, h_v and H_v, by

$$h_v(x) = e^{im_Q v.x} P_+ Q(x)\,,\; H_v(x) = e^{im_Q v.x} P_- Q(x)$$

Where P_+ and P_- are projection operators defined as

$$P_{\pm} = \frac{1 \pm \not{v}}{2}$$

By solving we get

$$Q(x) = e^{-im_Q v.x}(H_v(x) + h_v(x))$$

The projection of the new fields satisfy $\gamma^\mu h_v = h_v$ and $\gamma^\mu H_v = -H_v$. In the rest frame i.e.$v^\mu = (1,0,0,0)$, h_v corresponds to the upper two components of Q while H_v corresponds to the lower ones. Whereas h_v annihilates a heavy quark with velocity v, H_v creates a heavy antiquark with velocity v.

In terms of new fields, the QCD Lagrangian for a heavy quark takes the form

$$L_Q = \overline{Q}\left(i\gamma^\mu D - m_Q\right)Q$$

From this an effective Lagrangian is obtained which on simplification and on applying heavy quark limit $m_Q \to \infty$, becomes

$$L_\infty = \overline{h_v} iv.Dh_v$$

Which is called the effective Lagrangian of the HQET [24]. We will learn more about effective Lagrangian and its various terms with their significance in the next chapter.

Chapter 3

Heavy Mesons & Their Masses

3.1 Heavy Mesons:

In Standard Model the quarks u, d and s can be considered as light quarks while the quarks c, b and t as the heavier ones. In the theory of heavy mesons (e.g. HQET) we consider the mesons containing one light quark and one heavy quark because these type of systems contain new symmetries (like heavy quark symmetry) at a suitably chosen energy scale. These new symmetries make our calculations very easy. For example, due to heavy quark symmetry, the light degrees of freedom become blind to the heavy quark's spin, i.e. the spin of heavy quark becomes irrelevant or un-affecting factor. Hence we can ignore the heavy quark spin & consequently the calculations become easy. Top quark do not form bound states. So we are left with charm and bottom quarks as heavier quarks to form bound state.

D-mesons:- The mesons containing one charm quark and one light anti quark that may be $\bar{u}, \bar{d}$ or $\bar{s}$ are called D-mesons.

B-Mesons:- The mesons containing one bottom quark and one light anti quark that may be $\bar{u}, \bar{d}$ or $\bar{s}$ are called B-mesons.

The different states of the mesons are represented by the quantum number J^{P}, in which J represents the total angular momentum of the system and P represents the parity of the system. Spins angular momentum and orbital angular momentum of the quarks inside the meson couple to give the total angular momentum J = 0, 1, or any other value. We denote the ground states of D-mesons 0^- and 1^- with D and D^*, while the first exited states 0^+ and 1^+ with D_0 and D_1. The different D-mesons with u, d or s anti-quark are differentiated from each other by putting additional subscripts and superscripts to the corresponding D sign for the meson. As mentioned in previous chapter, in the heavy quark limit the coupling of heavy quark spin to the light degrees of freedom vanishes, in such a case the angular momentum and parity of the light degrees of freedom j^pcan be used to classify the heavy meson states. The spectrum consists of the degenerate heavy meson doublets with definite j^p. The $J^P = 0^-$ and 1^- heavy mesons are members of the $j^p = \frac{1}{2}^-$ ground state doublet. The lowest lying excited states, $J^P = 0^+$ and 1^+ heavy mesons are the members of the $j^p = \frac{1}{2}^+$ doublet. There is also an excited doublet of heavy mesons with $j^p = \frac{3}{2}^+$, whose members have $J^P = 1^+$ and 2^+. The $j^p = \frac{3}{2}^+$ mesons decay to the ground state by D-wave pion emission typically have width $\Gamma \sim 20$ MeV, and therefore have well measured masses. The hyperfine splitting of all these heavy quark doublets are suppressed by $1/m_Q$. Where m_Q is the heavy quark mass.The masses of different D mesons with possible uncertainties in their masses are shown in the following table:

$\boldsymbol{s_Q = \left(\frac{1}{2}\right)}$	$\boldsymbol{s_1 = \left(\frac{1}{2}\right)^-}$ ground state		$\boldsymbol{s_2 = \left(\frac{1}{2}\right)^+}$ LL excited state	
$\mathbf{J^P}$	0^-	1^-	0^+	1^+
General states	D	D^*	D_0	D_1
c$\bar{\text{u}}$	$D^0 = 1864.83 \pm 0.14$MeV	$D^{0*} = 2006.96 \pm 0.16$ MeV	${D_0}^0$ =?	${D_1}^0 = 2422.0 \pm 0.6$MeV
c$\bar{\text{d}}$	$D^+ = 1869 \pm 0.16$ MeV	D^{*+} =2010.25 ± 0.14MeV	${D_0}^+ = ?$	${D_1}^+ = ?$
c$\bar{\text{s}}$	${D_s}^+ = 1968.47 \pm 0.33$ MeV	${D_s}^{*+} = 2112.3 \pm 0.5$ MeV	${D_{0s}}^+ = 2317.8 \pm 0.6$ MeV	${D_{1s}}^+ = 2535.29 \pm 0.20 \pm 0.5$MeV
	Pseudo scalars	Vector	Scalars	Axial vector

Table 3.1 D-mesons and their masses.

The values of these masses have been taken from PDG 2010[25]

The spectrum of the $J^P = 0^+$ and 1^+ charmed meson presents a number of puzzles for theory. Before their discovery, quark model and lattice calculations predicted that the masses of the $J^P = 0^+$ and 1^+ charmed strange mesons would be significantly higher than observed.

Masses for some stats in the table are not shown here (the question marks), this is because there is much ambiguity in their masses. The reasons why these ambiguities arise are discussed in the next section.

3.2 Why ambiguity in masses?

When we measure the masses of particles for example of a meson at experiments, they do not come as we expect. We got different mass values than given by the theories. We got ambiguous masses. But why this ambiguity arises? The answer lies in self-energy diagrams. Suppose we see a D-meson at an experiment & measured its mass. For us in initial stage we had a meson, it traveled a distance and got observed. But here we have not considered the fact "what happened to the meson during short journey?". If the meson has sufficient energy, it is possible for it to emit another light particle like pion, kaon or eta etc. from itself & gets changed to another meson, which after very short time reabsorbs the emitted particle and gets changed to original (initial) meson. The diagrams showing such exchanges are called self-energy diagrams. Also mesons are made up of quarks, which have fractional electric charges. So there exist Columbic interactions between them, it will contribute to the mass. The interaction of quarks with light degrees of freedom influences the mass of the meson. Also the quarks have spin, the spins of constituent quark can interact in many ways, this again contributes to the mass of the meson. If we take all such interactions into account, we can have a good agreement between theory and experiments. For such interactions we write an interaction Lagrangian. This Lagrangian contain various terms which are discussed in next section. These terms represent the various contributions to the mass and mass splitting. Till now we talked only about the exchange of one particle only, but we can have much complex picture with exchange of more than one particle. While deriving mass formula for final stage particle, we have to consider all possibilities. More the terms are included in Lagrangian, more we will be close to the accurate value. But we encounter many problems while tackling with higher order terms in the formula. Higher order terms many times make our calculations infinite or indeterminate, sometimes the terms go uncontrollable, we cannot solve them. This is not a favorable case. In such case we try to remove the infinities in our calculations. The process of removing infinities of a theory is called "Renormalization" discussed in next section. In our studies we can apply approximations to the Lagrangian terms for example we can assume mass of the heavy quark $m_Q \to \infty$, while mass of the light quark $m_q \to 0$. With these conditions, the terms containing m_Q in the denominator and m_q in the numerator become = 0 and calculations become somewhat controllable.

3.3 Renormalization:

As mentioned in previous section, when our calculations become uncontrollable, they give infinite or indeterminate results. Renormalization is any of a collection of techniques used to treat these infinities arising in calculated quantities. Renormalization was first developed in quantum electrodynamics (QED) to make sense of infinite integrals in perturbation theory. Initially viewed as a suspicious provisional procedure by some of its originators, renormalization eventually was embraced as an important and self-consistent tool in several fields of physics and mathematics. While calculations, when we integrate over all possible values, some integrals become divergent. These divergent integrals lead to infinite results. We here mention two types of divergences.

(i) Ultraviolet Divergence: An ultraviolet divergence can be described as a divergence which comes from

- The region in the integral where all particles in the loop have large energies and momenta.
- Very short wavelengths and high frequencies fluctuations of the fields, in the path integral for the field.
- Very short proper-time between particle emission and absorption, if the loop is thought of as a sum over particle paths [26].

(ii) Infrared Divergence: An infrared divergence is a situation in which an integral, for example a Feynman diagram, diverges because of contributions of objects with very small energy approaching zero, or, equivalently, because of physical phenomena at very long distances [27]. The theory without massless particles does not have any infrared divergence at all. QED is a theory with infrared divergences because it includes photon, which is massless. In a theory where we take the approximation of taking small masses to be approaching to zero may contain infrared divergences.

3.4 The Lagrangian for Heavy Mesons:

The lowest lying mesons with one heavy quark are the pseudo scalar and the vector mesons with spin of the light degrees of freedom = ½. When this spin ½ of the light degrees of freedom couples with the spin half of the heavy quark, then we get spin zero and spin one degenerate doublet of mesons.

In HHχPT, the ground state doublet ($J^P = 0^- \; and \; 1^-$) is represented by the field

$$H_a = \frac{1+v}{2}(H_a^{\mu}\gamma_{\mu} - H_a\gamma_5),$$

Here a is the SU(3) index. In charm mesons sector H_a consists of the D^0, D^+, D_s^+ pseudo-scalar mesons and H_a^{μ} are the D^{*0}, D^{*+}, D_s^{*+} vector mesons.

The lowest lying excited states are the $J^P = 0^+ \; and \; 1^+$ i.e. $j^P = \frac{1}{2}^+$ doublet. It is represented by the fields

$$S_a = \frac{1+v}{2}\left(S_a^{\mu}\gamma_{\mu}\gamma_5 - S_a\right),$$

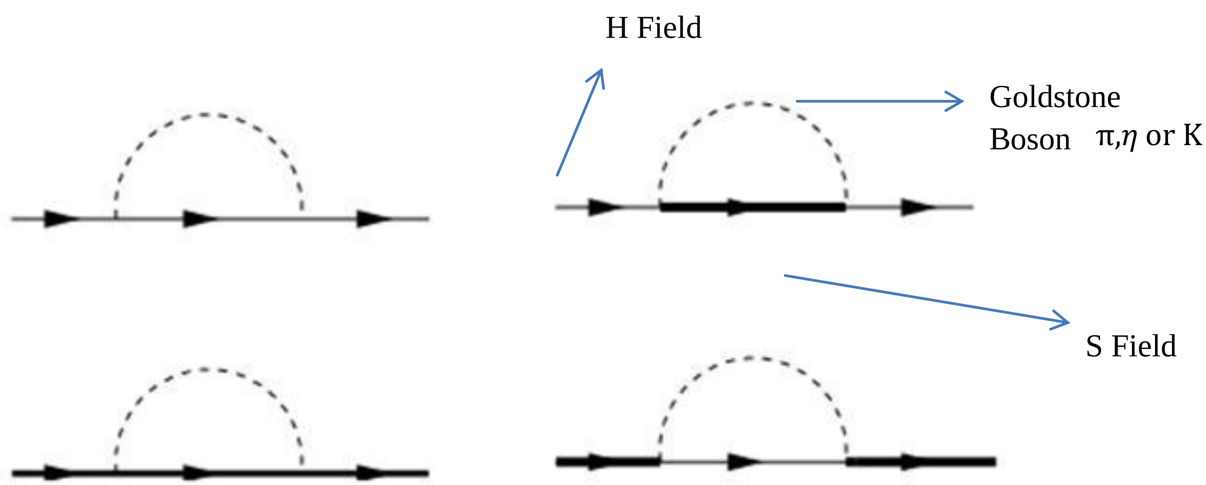
The total Lagrangian involves various terms like kinetic term, axial term, mass term, terms preserving symmetries(like heavy quark spin symmetry), symmetry violating terms etc.

The kinetic term of these above fields are included in :

$$L_v^{kinetic} = -Tr\left[\overline{H_a}(iv.D_{ba} - \delta_H\delta_{ab})H_b\right] + Tr\left[\overline{S_a}(iv.D_{ba} - \delta_S\delta_{ab})S_b\right]$$

Where δ_H and δ_S are the residual masses of the H and the S fields respectively. And D_{ba} is the charily covariant derivative. In the theory with only H fields one is free to set $\delta_H = 0$. Since the only dimensionful parameters entering the loops in the theory are hyperfine splittings and the meson masses, the UV divergences vanish in the $m_q \to 0$ and $m_Q \to \infty$ limit. Divergences in loop corrections are cancelled by counter terms which are O(m_q) or O(m_Q). Once the S fields are included to the theory, there is another dimensionful quantity, $\delta_S - \delta_H$, which does not vanish as $m_Q \to \infty$ and $m_q \to 0$. The H self-energy diagrams with virtual S fields give a UV divergent contribution which survives in the $m_q \to 0$ and $m_Q \to \infty$ limit. Such a divergence must be canceled by a mass counter term which respects the SU(3) and the heavy-quark spin symmetry and the only available counterterm is $\delta_H Tr\overline{H_a}H_a$. However, after one-loop divergences are canceled one is free to define the finite part of δ_H for convenience.

The fields have axial couplings to the pseudo-Goldstone boson, which means, as earlier mentioned that the fields convert to each other through the exchange of Goldstone Bosons, it is shown in the diagram given below:



The Goldstone Boson octet is given by

$$\Pi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}} \end{bmatrix}$$

These axial couplings are included in the term given below:

$$L_v^{axial} = gTr\left[\overline{H_a}H_b \not{A}_{ba}\gamma_5\right] + g'Tr\left[\overline{S_a}S_b \not{A}_{ba}\gamma_5\right] + hTr\left[\overline{H_a}S_b \not{A}_{ba} + h.c.\right],$$

Where g, g' and h are the dimensionless constants to be determined from experiment.

The other terms in the Lagrangian are the higher order terms. Higher dimensional operators of the chiral Lagrangian which break heavy quark spin-flavor symmetry and chiral symmetry involve factors of $1/m_Q$ or the insertion of the light quark mass matrix M=diag(m_u, m_d, m_s). For the calculation of the heavy meson masses, it is useful to classify the symmetry-violating operators by the number of insertions of the quark mass matrix and whether or not they violate the heavy quark spin symmetry. Operators which respect heavy quark spin symmetry have coefficients which start at O(1) in the $1/m_Q$ expansion, whereas operators which violate heavy quark spin symmetry have coefficient which start at O($1/m_Q$). Counter terms to the one loop calculations of the heavy meson masses are proportional to the power of the light quark masses. Thus term with up to two insertion of M are needed for the one-loop calculation.

The singlet mass term m_H is a function of $1/m_Q$, but is irrelevant for the calculation of meson mass splittings. There is only one other term with no insertion of the light quark mass matrix,

$$L_v^{\$} = -\frac{\Delta}{8} Tr\overline{H_v}\sigma^{\mu\nu}H_v\sigma_{\mu\nu} = -\Delta Tr\overline{H_v}S_{Q\,v}^{\alpha}H_vS_{lv\alpha}$$

This term violates the heavy quark spin symmetry and is responsible for the hyperfine ($P^* - P$) mass splitting at the leading order. The parameter Δ is a function of $1/m_Q$ which starts at linear order in $1/m_Q$, because violations of heavy quark spin symmetry are suppressed by at least one power of $1/m_Q$. The values of $-S_Q.S_l = \overrightarrow{S_Q}.\overrightarrow{S_l} = \frac{1}{2}\left(\vec{S}_T{}^2 - \vec{S}_Q{}^2 - \vec{S}_l{}^2\right)$ equals -3/4 for pseudo scalar mesons P and ¼ for the vector mesons P^*. Thus, this operator results in a hyperfine splitting($P^* - P$) = Δ at tree level.

The terms in the chiral Lagrangian which are proportional to the light quark masses and which respect the heavy quark spin symmetry are given by

$$L_v^M = a\,Tr\overline{H_v}H_vM_\xi + \sigma\,TrM_\xi\,Tr\overline{H_v}H_v$$

Where a and σ are the functions of $1/m_Q$ which start at O(1). The term proportional to a results in $\mathrm{SU(3)_v}$ violating mass splittings amongst the $P_a^{(*)}$ mesins. The term proportional to σleads to a singlet contribution to the masses which depends linearly on the light quark masses, and is the heavy meson analog of the pion-nucleon sigma term.

Chiral Lagrangian terms with one insertion of the light quark mass matrix which violate the heavy quark spin symmetry are

$$L_v^{M\$} = -\frac{1}{8}\Delta^{(a)} Tr\overline{H_v}\sigma^{\mu\nu}H_v\sigma_{\mu\nu}\, M_\xi - \frac{1}{8}\Delta^{(\sigma)} Tr M_\xi Tr\overline{H_v}\sigma^{\mu\nu}H_v\sigma_{\mu\nu}$$

These two terms produce hyperfine splittings proportional to a light quark mass. The term proportional to $\Delta^{(a)}$ leads to the light quark flavor-dependent hyperfine splittings, whereas the term proportional to the $\Delta^{(\sigma)}$ yields a flavor singlet contribution to the hyperfine splittings which is linear in light quark masses[28].

Keeping all these factors in mind the mass term in the Lagrangian in terms of symmetry breaking and symmetry conserving parameters can be written as

$$L_v^{mass} = -\frac{\Delta_H}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu}H_a\sigma_{\mu\nu}\right] + \frac{\Delta_S}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu}S_a\sigma_{\mu\nu}\right] + a_H Tr\left[\overline{H_a}H_b\right]m_{ba}^{\xi} -$$
$$a_S Tr\left[\overline{S_a}S_b\right]m_{ba}^{\xi} - \sigma_S Tr\left[\overline{S_a}S_a\right]m_{bb}^{\xi} + \sigma_H Tr\left[\overline{H_a}H_a\right]m_{bb}^{\xi} - \frac{\Delta_H{}^{(a)}}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu}H_b\sigma_{\mu\nu}\right]m_{ba}^{\xi}$$
$$+ \frac{\Delta_S{}^{(a)}}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu}S_b\sigma_{\mu\nu}\right]m_{ba}^{\xi} - \frac{\Delta_H{}^{(\sigma)}}{8} Tr\left[\overline{H_a}\sigma^{\mu\nu}H_a\sigma_{\mu\nu}\right]m_{bb}^{\xi} + \frac{\Delta_S{}^{(\sigma)}}{8} Tr\left[\overline{S_a}\sigma^{\mu\nu}S_a\sigma_{\mu\nu}\right]m_{bb}^{\xi}$$

Where $m_{ba}^{\xi} = \frac{1}{2}\left(\xi m_q \xi + \xi^+ m_q \xi^+\right)_{ba}$

Δ_H and Δ_s are spin flavor symmetry violating terms and the first two terms are responsible for the hyperfine mass splitting. The next four terms contains terms which are of O(m_q) and preserve heavy quark spin symmetry. The terms proportional to a results in$SU(3)_V$ violating mass splitting amongst H_a^{μ} mesons. The term proportional to σ leads to a single contribution to the masses which depends linearly on the light quark masses. Finally, the remaining last terms contain which are of O(m_q) and violate heavy quark spin symmetry [29].

3.5 Mass formula for heavy hadrons:

Here we talk in terms of residual masses. The residual masses are defined to be the difference between the real mass and an arbitrarily chosen reference mass of O(m_q).

At tree level the residual masses can be given as

$$m_{H_a}^0 = \delta_H - \frac{3}{4}\Delta_H + \sigma_H\overline{m} + a_H m_a - \frac{3}{4}\Delta_H^{(\sigma)}\overline{m} - \frac{3}{4}\Delta_H^{(\sigma)} m_a$$

$$m_{H_a^*}^0 = \delta_H + \frac{1}{4}\Delta_H + \sigma_H\overline{m} + a_H m_a + \frac{1}{4}\Delta_H^{(\sigma)}\overline{m} + \frac{1}{4}\Delta_H^{(\sigma)} m_a$$

$$m_{S_a}^0 = \delta_S - \frac{3}{4}\Delta_H + \sigma_S\overline{m} + a_H m_a - \frac{3}{4}\Delta_S^{(\sigma)}\overline{m} - \frac{3}{4}\Delta_S^{(\sigma)} m_a$$

$$m_{S_a^*}^0 = \delta_S + \frac{1}{4}\Delta_S + \sigma_S\overline{m} + a_S m_a + \frac{1}{4}\Delta_S^{(\sigma)}\overline{m} + \frac{1}{4}\Delta_S^{(\sigma)} m_a$$

Where $m_a = (m_u, m_d, m_s)$and $\overline{m} = m_u + m_d + m_s$. The asterisk is denoting the spin-1 member of the heavy quark doublet. In the isospin limit $m_u = m_d$.

These are the residual masses without considering any exchange of the Goldstone Bosons. If we consider the exchange of these Bosons then the one loop calculations for the heavy mesons exchanging pion, kaon and eta are written with $1/m_Q$correction. We write our results in terms of the functions K_1 , K_2 and F(x).

$$K_1(\eta, M) = \frac{1}{16\,\pi^2}\left[(-2\eta^3 + 3M^2\eta)ln\left(\frac{M^2}{\mu^2}\right) + 2\eta(\eta^2 - M^2)F\left(\frac{\eta}{M}\right) + 4\eta^3 - 5\eta M^2\right]$$

$$K_2(\eta, M) = \frac{1}{16\,\pi^2}\left[(-2\eta^3 + M^2\eta)ln\left(\frac{M^2}{\mu^2}\right) + 2\eta^3 F\left(\frac{\eta}{M}\right) + 4\eta^3 - \eta M^2\right]$$

Where

$$F(x) = 2\frac{\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - Tan^{-}\left(\frac{x}{\sqrt{1-x^2}}\right)\right] \qquad |x| < 1$$

$$F(x) = -2\frac{\sqrt{x^2-1}}{x}\, ln\left(x + \sqrt{x^2} - 1\right) |x| > 1$$

The function $K_1(,M)$ appears whenever the virtual heavy meson inside the loop is is the same doublet as the external heavy meson, while $K_2(\eta, M)$ appears when the virtual heavy meson is from the opposite parity doublet.

If the limit M<<η these functions become

$$K_1(\eta, M) = \frac{1}{16\pi^2}\left[-2\eta^3 ln\left(\frac{4\eta^2}{\mu^2}\right) + 3M^2 ln\left(\frac{4\eta^2}{\mu^2}\right) + \frac{3M^4}{4\,\eta} ln\left(\frac{M^2}{4\eta^2}\right) + \cdots\right]$$

$$K_2(\eta, M) = \frac{1}{16\pi^2}\left[-2\eta^3 ln\left(\frac{4\eta^2}{\mu^2}\right) + \eta\, M^2 ln\left(\frac{4\eta^2}{\mu^2}\right) - \frac{M^4}{4\,\eta} ln\left(\frac{M^2}{4\eta^2}\right) + \cdots\right]$$

In these equations we have dropped polynomials of η, M. The functions K_1(η, M) andK_2(η,M) have well-defined M → 0 limits. Furthermore, the dependence on M is analytic when M/η → 0, so in this limit the S fields can be integrated out and their effect on the chiral corrections can be absorbed into local counter-terms as expected. This limit is not relevant to the real world as $\eta \sim M$. In the opposite limit, η = 0, which is relevant for loopsin which external and virtual heavy mesons are the same,

$$K_1(\eta, M) = -\frac{M^3}{8\pi} + \frac{3}{16\pi^2}\eta M^2 ln\left(\frac{4\,\eta^2}{\mu^2}\right) + O(\eta^3)$$

$$K_2(\eta, M) = -\frac{1}{16\pi^2}\eta M^2 ln\left(\frac{4\,\eta^2}{\mu^2}\right) + O(\eta^3)$$

Including loop terms we get:

$$m_{H_1} = m^0_{H_1} + \frac{g^2}{f^2}\left[\frac{3}{2}K_1\left(m^0_{H_1^*} - m^0_{H_1}, m_\pi\right) + \frac{1}{6}K_1\left(m^0_{H_1^*} - m^0_{H_1}, m_\eta\right) + K_1\left(m^0_{H_3^*} - m^0_{H_1}, m_K\right)\right]$$
$$+ \frac{h^2}{f^2}\left[\frac{3}{2}K_2\left(m^0_{S_1} - m^0_{H_1}, m_\pi\right) + \frac{1}{6}K_2\left(m^0_{S_1} - m^0_{H_1}, m_\eta\right) + K_2\left(m^0_{S_3} - m^0_{H_1}, m_K\right)\right]$$

$$m_{H_3} = m^0_{H_3} + \frac{g^2}{f^2}\left[2K_1\left(m^0_{H_1^*} - m^0_{H_3}, m_K\right) + \frac{2}{3}\left(m^0_{H_3^*} - m^0_{H_3}, m_\eta\right)\right] + \frac{h^2}{f^2}\left[2K_2\left(m^0_{S_1} - m^0_{H_3}, m_K\right) + \frac{2}{3}\left(m^0_{S_3} - m^0_{H_3}, m_\eta\right)\right]$$

$$m_{H_1^*} = m_{H_1^*}^0 + \frac{g^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1\left(m_{H_1}^0 - m_{H_1^*}^0, m_\pi\right) + \frac{1}{6}K_1\left(m_{H_1}^0 - m_{H_1^*}^0, m_\eta\right) + K_1\left(m_{H_3}^0 - m_{H_1^*}^0, m_K\right)\right]$$
$$+ \frac{h^2}{f^2}\left[\frac{3}{2}K_2\left(m_{S_1^*}^0 - m_{H_1^*}^0, m_\pi\right) + \frac{1}{6}K_2\left(m_{S_1^*}^0 - m_{H_1^*}^0, m_\eta\right) + K_2\left(m_{S_3^*}^0 - m_{H_1^*}^0, m_K\right)\right]$$

$$m_{H_3^*} = m_{H_3^*}^0 + \frac{g^2}{f^2}\frac{1}{3}\left[2K_1\left(m_{H_1}^0 - m_{H_3^*}^0, m_K\right) + \frac{2}{3}K_1\left(m_{H_3}^0 - m_{H_3^*}^0, m_\eta\right)\right] + \frac{g^2}{f^2}\frac{2}{3}\left[2K_1\left(m_{H_1^*}^0 - m_{H_3^*}^0, m_K\right) + \frac{2}{3}K_1\left(0, m_\eta\right)\right]$$
$$+ \frac{h^2}{f^2}\left[2K_2\left(m_{S_1^*}^0 - m_{H_3^*}^0, m_K\right) + \frac{2}{3}K_2\left(m_{S_3^*}^0 - m_{H_3^*}^0, m_\eta\right)\right]$$

$$m_{S_1} = m_{S_1}^0 + \frac{g'^2}{f^2}\left[\frac{3}{2}K_1\left(m_{S_1^*}^0 - m_{S_1}^0, m_\pi\right) + \frac{1}{6}K_1\left(m_{S_1^*}^0 - m_{S_1}^0, m_\eta\right) + K_1\left(m_{S_3^*}^0 - m_{S_1}^0, m_K\right)\right]$$
$$+ \frac{h^2}{f^2}\left[\frac{3}{2}K_2\left(m_{H_1}^0 - m_{S_1}^0, m_\pi\right) + \frac{1}{6}K_2\left(m_{H_1}^0 - m_{S_1}^0, m_\eta\right) + K_2\left(m_{H_3}^0 - m_{S_1}^0, m_K\right)\right]$$

$$m_{S_3} = m_{S_3}^0 + \frac{g'^2}{f^2}\left[2K_1\left(m_{S_1^*}^0 - m_{S_3}^0, m_K\right) + \frac{2}{3}\left(m_{S_3^*}^0 - m_{S_3}^0, m_\eta\right)\right] + \frac{h^2}{f^2}\left[2K_2\left(m_{H_1}^0 - m_{S_3}^0, m_K\right) + \frac{2}{3}\left(m_{H_3}^0 - m_{S_3}^0, m_\eta\right)\right]$$

$$m_{S_1^*} = m_{S_1^*}^0 + \frac{g'^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1\left(m_{S_1}^0 - m_{S_1^*}^0, m_\pi\right) + \frac{1}{6}K_1\left(m_{S_1}^0 - m_{S_1^*}^0, m_\eta\right) + K_1\left(m_{S_3}^0 - m_{S_1^*}^0, m_K\right)\right]$$
$$+ \frac{g'^2}{f^2}\left[\frac{3}{2}K_1\left(0, m_\pi\right) + \frac{1}{6}K_1\left(0, m_\eta\right) + K_1\left(m_{S_3^*}^0 - m_{S_1^*}^0, m_K\right)\right]$$
$$+ \frac{h^2}{f^2}\left[\frac{3}{2}K_2\left(m_{H_1^*}^0 - m_{S_1^*}^0, m_\pi\right) + \frac{1}{6}K_2\left(m_{H_1^*}^0 - m_{S_1^*}^0, m_\eta\right) + K_2\left(m_{H_3^*}^0 - m_{S_1^*}^0, m_K\right)\right]$$

$$m_{S_3^*} = m_{S_3^*}^0 + \frac{g^2}{f^2}\frac{1}{3}\left[2K_1\left(m_{S_1}^0 - m_{S_3^*}^0, m_K\right) + \frac{2}{3}K_1\left(m_{S_3}^0 - m_{S_3^*}^0, m_\eta\right)\right] + \frac{g'^2}{f^2}\frac{2}{3}\left[2K_1\left(m_{S_1^*}^0 - m_{S_3^*}^0, m_K\right) + \frac{2}{3}K_1\left(0, m_\eta\right)\right]$$
$$+ \frac{h^2}{f^2}\left[2K_2\left(m_{H_1^*}^0 - m_{S_3^*}^0, m_K\right) + \frac{2}{3}K_2\left(m_{H_3^*}^0 - m_{S_3^*}^0, m_\eta\right)\right]$$

We work in the isospin limit, in where the masses H_1 and H_2, for instance are identical. Hence we obtain above eight masses. To determine the experimental values of m_{H_1} and $m_{H_1^*}$, we average the masses of the two known isospin states. We will measure all the masses relative to the non-strange spin averaged H mass, so $\left(m_{H_1} + {}^3/_4\, m_{H_1^*}\right) = 0$. The experimentally measured residual masses are:

$$m_{H_1} = -106.1MeV \quad m_{H_3} = -4.75MeV \quad m_{H_1^*} = 35.4MeV \quad m_{H_3^*} = 139.1MeV$$

$$m_{S_1} = 335.0MeV \quad m_{S_3} = 344.4MeV \quad m_{S_1^*} = 465.0MeV \quad m_{S_3^*} = 486.3MeV$$

If we try to fit these masses using one loop corrections then we see that calculations depend on eleven parameters g, g', h,$a_H, a_S, \Delta_H^{(a)}, \Delta_S^{(a)}$, $\delta_H + \sigma_H\overline{m}, \delta_S + \sigma_S\overline{m}, \Delta_H + \Delta_H^{(\sigma)}\overline{m},$

$\Delta_S + \Delta_S^{(\sigma)}\overline{m}$. The parametersσ_H ,σ_S, $\Delta_H^{(\sigma)}$,$\Delta_S^{(\sigma)}$ cannot be separately determined because they always appear in linear combination with the parameters δ_H,δ_S, Δ_H, Δ_S respectively. So the contribution of these four parameters is absorbed into measured values of δ_H, δ_S, Δ_H and Δ_S respectively[29].

In our work we try to fit the values of residual masses given above by varying the values of various parameters. Also the values of the mass splittings are available at various experiments; we also try to fit these mass splittings over a range of the parameters. We also studied the effect of variation in the values of the parameters on various splittings using a graphical method. All these fittings, graphs and their analysis is discussed in the next chapter.

Chapter 4

Results, Analysis & Summary

Recalling from previous chapter we know that there were 11 parameters on which the mass or mass splittings depend. Out of these some cannot be determined separately and we absorbed their contribution into other parameters. At last we remained with eight parameters. In our work we tried to vary these 8 parameters to fit the experimental values of the masses and mass splittings. The values or their possible range of some parameters is available from various experiments. For example possible ranges of different coupling constants $g, g' and\ h$, are determined from various decay processes. An analysis of D^* decay using one loop calculation without explicit excited states yields $g = 0.27^{+0.06}_{-0.03}$ [30]. From the widths of the non-strange resonances observed by Belle we have extracted $h = 0.69 \pm 0.009$[31]. Both the coupling constants are of the order of unity. Taking an indication from here, in our work we vary these vary their values over the range 0-1 so as to include all possible values.

We used $f = 120\ MeV$ extracted in ref[30] using the one loop formulae for pion and kaon decay constants. We set $m_u = m_d = 4\ MeV$ and $m_s = 90\ MeV$. Other parameters are unknown. We use Mathematica 5.1 as a programming language to fit he values. The results for fitting of masses are given below:

$$g = 0.01 \quad g' = 0.01\ ; \quad h = 0.05 \quad \delta_H = 4 \quad \delta_S = 432; \quad \Delta_H = 147\ \Delta_S = 128$$

$$a_H = 1.1 \qquad a_S = 0.21 \quad \Delta_H^a = -0.04 \quad \Delta_S^a = 0.14$$

Below is given the table of masses calculated in our work and observed experimentally. All the observed masses are taken from Particle Data Group [26]. D_0^0 mass has also been measured at Belle[32] and is equal to 2308$\pm$36 similarly other masses are also available but our results are matching more closely with that given in PDG.

And the fitted masses are given as

S. No.	State	Calculated Residual Mass	Calculated Real Mass (MeV)	Experimental Real Mass (MeV)	Mesonic State
1	m_{H_1}	-105.971	1867.039	1867.21	$D^{0,+}$
2	m_{H_3}	-5.27028	1967.739	1968.47$\pm$0.33	$D_s^{\pm}$
3	m_{S_1}	329.722	2314.087	2318$\pm$29	D_0^0
4	m_{S_3}	341.077	2314.551	2317.8$\pm$0.6	D_{s0}^+
5	$m_{H_1^*}$	44.4608	2017.2387	2008.60	$D^{*0,+}$
6	$m_{H_3^*}$	136.485	2109.495	2112.1	D_s^{*+}
7	$m_{S_1^*}$	460.509	2433.519	2438$\pm$31	D_1^0
8	$m_{S_3^*}$	483.281	2456.291	2459.5$\pm$0.6	D_{s1}^+

Once we have idea about possible values of parameters for masses, in the neighborhood of these values we tried to see graphically that how much the value of splitting alters for a particular parameter if we keep other parameters fixed. From these graphs we could extract the approximate range of the parameters which are giving the valid results for the mass splittings. The brief analysis of these plots is given here:

1. Spin splitting and mass splitting both are showing negligible variation with parameter δ_H (Residual mass for H field)as well as δ_S (the residual mass for S field).E.g. spin splitting ($1^- - 0^-$) is having almost constant value 145.085$\pm$0.002 for wide variation in δ_H.Similar results are coming forδ_S . So in our fittings we keep both these parameters as fixed.
2. The second parameter Δ_H (which is spin symmetry breaking term) shows a linear variation with spin splitting(SS)($0^- - 1^-, 0^+ - 1^+$). There is negligible effect of these two parameters on the mass splittings(MS) ($0^-(s) - 0^-(u), 1^-(s) - 1^-(u)$ etc.). The plots of SS for H-fields are shown. Similar variation is seen in case of S-fields, when we vary the parameter Δ_S. The term Δ_H (Δ_S)which is responsible for violation of heavy quark spin flavor symmetry, and is responsible for hyperfine (P^* -P) mass splitting at leading order. The term with Δ_H (Δ_S) due to coupling between

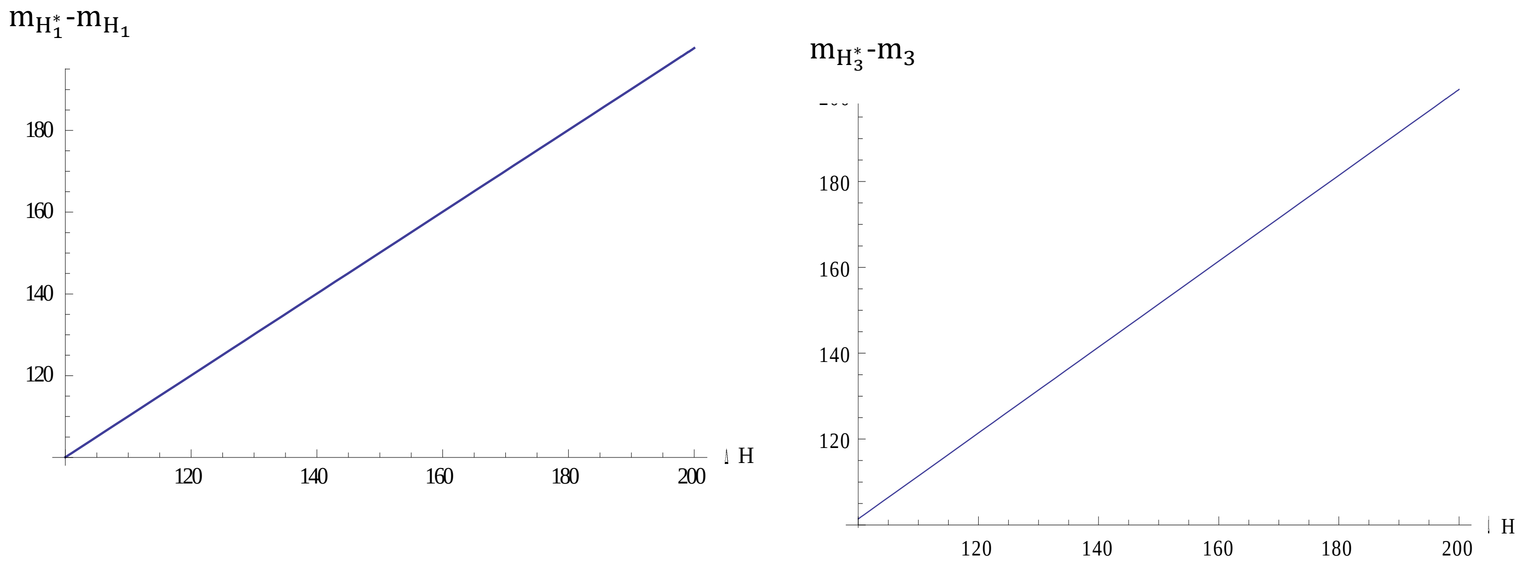


Fig 4.1 Variation of spin splitting with Δ_H for (i) $m_{H_1^*}$-m_{H_1} (ii) $m_{H_3^*}$-m_3

light degrees of freedom and heavy quark gives a factor of -3/4 for pseudo scalar mesons but it gives a factor of $1/4$for vector mesons. This implies that P^*-P= Δ at tree level.Therefore the graph for$m_{H_1^*} - m_{H_1}$and $m_{H_3^*} - m_{H_3}$shows approximately zero splitting (spin) for $\Delta_H = 0$).

The possible range for Δ_H (Δ_S) is between 130-150. The best suited value for Δ_H and Δ_S are approximately ~ 140 MeV.

3. The next term is Δ_H^a and it corresponds to heavy quark spin flavor symmetry violating term.

Let us first see the plots given below:

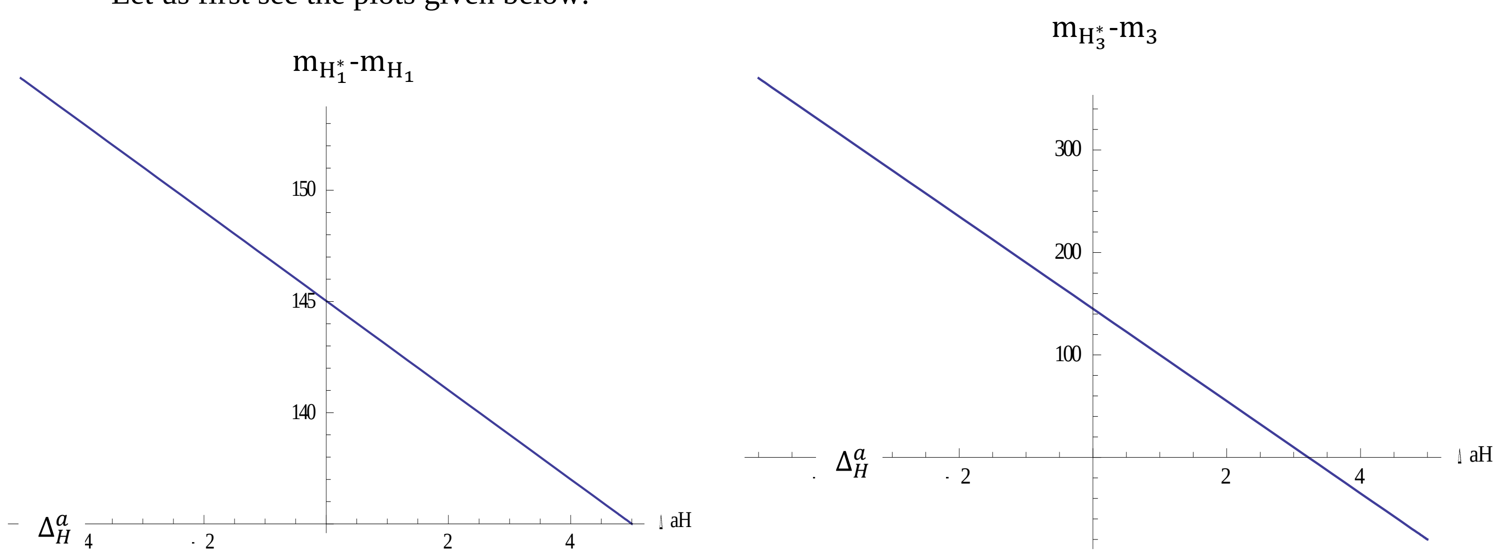


Fig 4.2 Variation of spin splitting with Δ_H^a for (i) $m_{H_1^*}$-m_{H_1} (ii) $m_{H_3^*}$-m_3

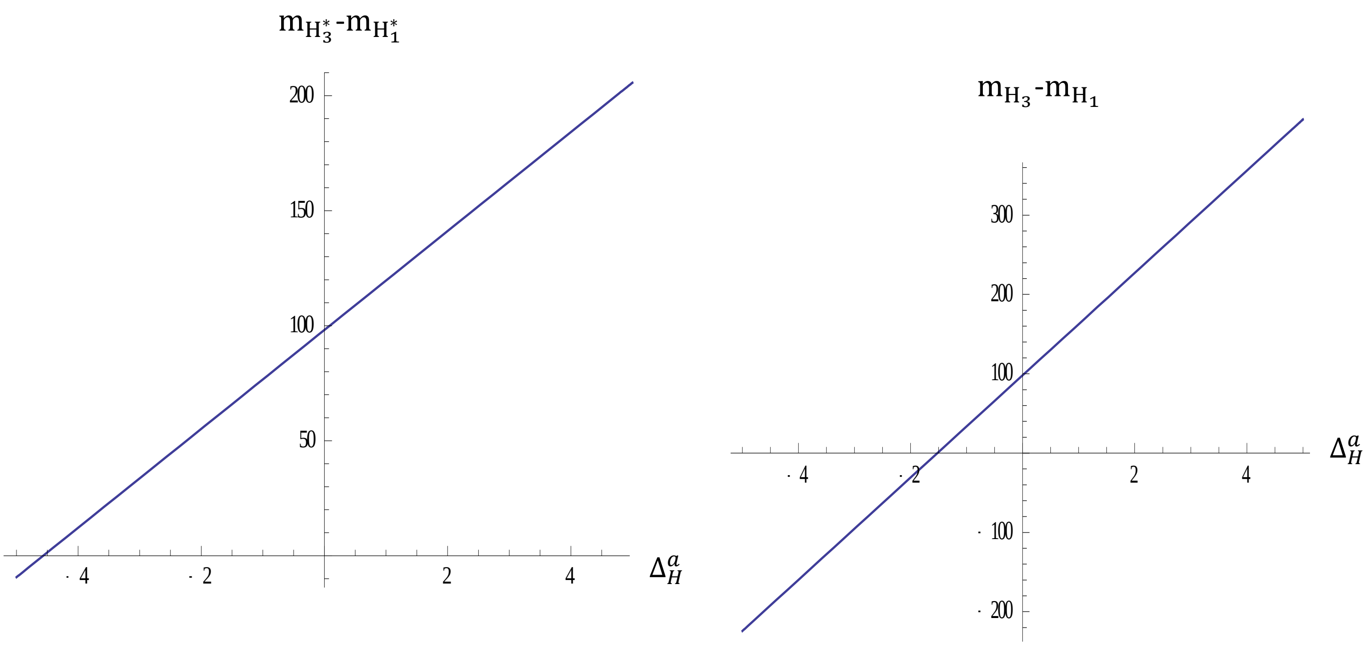


Fig 4.3 Variation of mass splitting with Δ_H^a for (i) $m_{H_3^*}$-$m_{H_1^*}$(ii)m_{H_3}-m_{H_1}

The variation is a straight line for both spin splittings and mass splitting. The terms containing Δ_H^a leads to light quark flavor dependent hyperfine splitting. These graphs show that with the change in Δ_H^a the spin splitting in hadrons with u or d quark does not vary very much while the splitting for the hadrons containing 's' quark change rapidly . This is very obvious, because more is the mass difference between light & heavy degrees of freedom, more is the heavy quark symmetry (Spin & Flavor symmetry)consequently lesser is the effect of change in spin of heavy quark. The effect can directly be seen on these plots.

Similar type of behaviour is seen for the parameter Δ_S^a for corresponding plots.

4. The term a_H has a linear graph with mass splittings only with sufficient variation as shown in plots below. The terms proportional to a_H results in$SU(3)_V$violating mass splittings amongst the vector mesons. The graphs for the term a_H shows approximately linear variation with spin splittings between range 100-200, the range in which our splitting lies. This is due to reason the terms including a_H respects spin

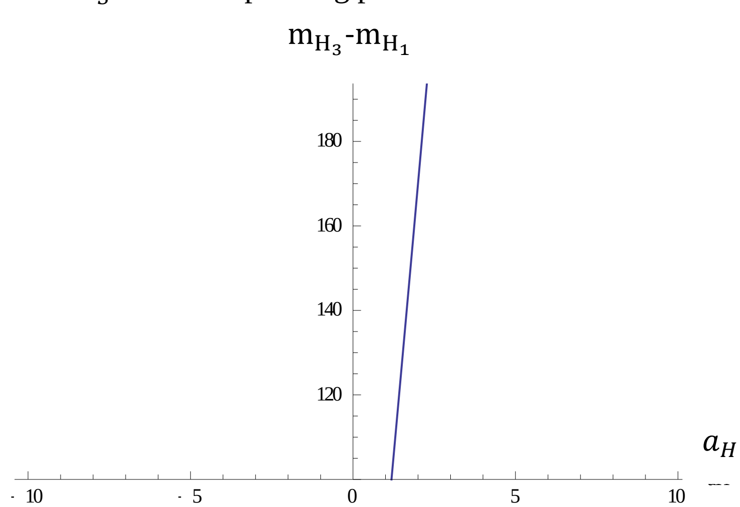


Fig 4.4 (i) Variation of spin splitting with a_H for m_{H_3}-m_{H_1}

symmetry.

We analysed these graphs and extracted the range of parameters for which the splitting curves were falling in the experimentally measured values regions.

Finally having idea about approximate ranges for all parameters, we tried to fit the splitting in these ranges and the results for fitting of mass splittings are given as:

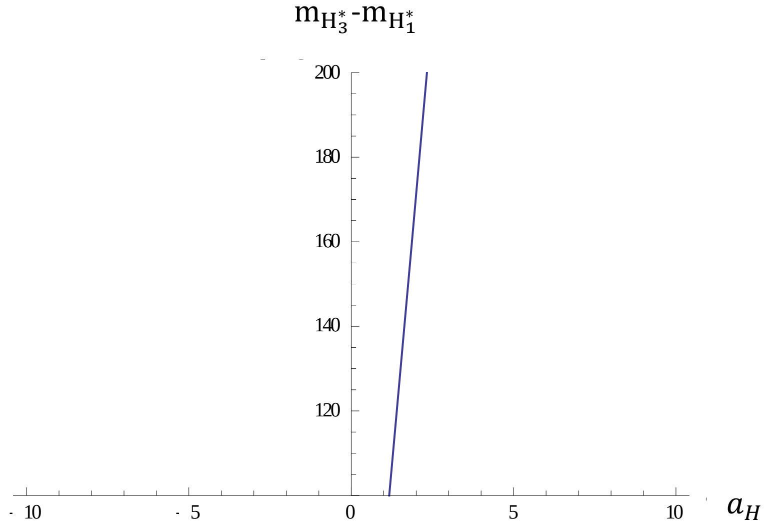


Fig 4.4 (ii) Variation of spin splitting with a_H for $m_{H_3^*}$-$m_{H_1^*}$

$g = 0.01$ $g' = 0.01$; $h = 0.01$ $\delta_H = 4$ $\delta_S = 432$; $\Delta_H = 144$ $\Delta_S = 126$ $a_H = 1.1$

$a_S = 0.2$ $\Delta_H^a = -0.03$ $\Delta_S^a = 0.14$

Analysis for different spin splitting for D_s and D_u mesons in $0^+, 1^+$, and$0^-, 1^-$states:-

The calculated value for spin splitting for D_s and $D_u$are as follows:

1. $D_s^* - D_s(1^- - 0^-)$ = 141.374 MeV
2. $D_s^* - D_s(1^+ - 0^+)$ = 138.675MeV
3. $D_u^* - D_u(1^- - 0^-)$ = 143.912MeV
4. $D_u^* - D_u((1^+ - 0^+)$=126.466MeV

The values given by particle data group are :-

1. $D_s^* - D_s(1^- - 0^-)$ = 143.8MeV
2. $D_s^* - D_s(1^+ - 0^+)$ = 141.9MeV
3. $D_u^* - D_u(1^- - 0^-)$ =142.1MeV
4. $D_u^* - D_u(1^+ - 0^+)$ =130MeV

The calculated mass splittings:

1. $D_s^* - D_u^*(1^- - 1^-)$ = 94.0396MeV
2. $D_s - D_u(0^- - 0^-)$ = 96.578MeV
3. $D_s^* - D_u^*(1^+ - 1^+)$ = 20.3321MeV
4. $D_s - D_u(0^+ - 0^+)$ = 8.1228MeV

The values from experiments are :-

1. $D_s^* - D_u^*(1^- - 1^-)$ = 105.4MeV
2. $D_s - D_u(0^- - 0^-)$ = 98.88MeV
3. $D_s^* - D_u^*(1^+ - 1^+)$ = 21.3MeV
4. $D_s - D_u(0^+ - 0^+)$ = 9.4MeV

Conclusion:

Various approaches have been used to study heavy-light mesonic sector in literature. Specifically effective field theories exploit the symmetries of hadrons(mesons) to make model independent predictions when the dynamics of these hadrons are too hard to solve explicitly. For example, the properties of a hadron containing a very heavy quark are insensitive to the orientation of the heavy quark spin. This heavy quark spin symmetry can be used to make predictions for the production and decay of heavy mesons and quarkonia at collider experiments.

In our work heavy hadron chiral perturbation theory is applied to study the ground state and the low lying excited states of charm mesons. Where we have used the mass formula already formulated by T. Mehen and Springer [29]. The mass formula has been developed for the one loop correction. The corrections depend on a number of unknown or poorly determined coupling constants. The Unknown parameters are either in term of symmetry violating or symmetry conserving parameter. Our approach is to use some of the parameters from the literature and range of values of coupling constants, and fit the masses for one loop contributions. Here the masses for the non-strange D neutral and charged mesons in 0^+ and mass for charged D meson in 1^+ can be predicted. From our fittings the masses of non-strange D mesons in 0^+state are coming out to be ~2314.087. In our calculations we are approximating masses of u and d quark equal, hence this calculated mass is the average of the masses of both mesons(D_0^0 and D_0^+). The mass of D meson in 1^+ state i.e. D_1^+ from our calculations is coming to be ~2433.519. The parameter set for these masses is

$g = 0.01$ $g' = 0.01$; $h = 0.01$ $\delta_H = 4$ $\delta_S = 432$; $\Delta_H = 144$ $\Delta_S = 126$ $a_H = 1.1$

$a_S = 0.2$ $\Delta_H^a = -0.03$ $\Delta_S^a = 0.14$

Although Heavy Quark Effective Theory was given long before 1992 by Isgur and Wise but there was not a considerable progress in this theory in past time. It was because we lacked the experimental data. But now days from various experiments we are getting enormous amount of data. These data indicate that the predictions of these theories are in good agreement with experimental data.

We are working on the models that can predict the masses of D mesons (or use the observed masses to do reverse calculation for predicting fundamental parameters). Hence if we have reliable data on the any of the parameters, quoted in the one loop mass relations, from other experiments or theoretical models, we would be able to fix the remaining parameters in the theory. Also these relations will be useful in predicting the masses of a few B mesons which have not been observed at the experiments yet. From the Heavy quark theory we have the relation

$$\frac{m_{H^*}^{(b)} - m_H^{(b)}}{m_{H^*}^{(c)} - m_H^{(c)}} = \frac{m_{S^*}^{(b)} - m_S^{(b)}}{m_{S^*}^{(c)} - m_S^{(c)}} = \frac{m_c}{m_b}$$

up to O($1/m_Q$) corrections. Thus, all the hyperfine splittings in the bottom sector are related to those in the charm sector by a universal factor. Hopefully we can also extend the concept for the baryons (having three quarks).

The work in this paper provides further stimulus for better experimental measurements of charmed meson masses as well as discovery of their bottom strange counterparts. Once better data becomes available, it would be interesting to test models of chiral symmetry breaking which make specific predictions for the coupling constants and HQET parameter appearing in the mass proportionate terms of HHχPT Lagrangian.

References:

[1] http://www-public.slac.stanford.edu/babar

[2] http://www.slac.stanford.edu/BFROOT/BABAR.html

[3] http://www.lepp.cornell.edu/Research/EPP/CLEO/#The_CLEO_c_Detector

[4] http://www.lepp.cornell.edu/Research/EPP/CDF/WebHome.html

[5] http://www-cdf.fnal.gov/public/index.html

[6] http://www-focus.fnal.gov/

[7] D. Griffiths, Intro. To Elementary Particles, 105

[8] Ta-Pei Cheng and Ling-Fang Li, Gauge theory of elementary particle physics, 115

[9] M.P. Khanna, Intro. To Particle Physics

[10] D.C. Joshi, Intro. To Quantum Electrodynamics and Particle Physics

[11] http://plato.stanford.edu/entries/symmetry-breaking/

[12] http://www.sciencedirect.com/science/referenceworks/9780125126663

[13] Eric Braaten, hep-ph/9702225v1

[14] www.gk-eichtheorien.physik.uni-mainz.de/Dateien/Bullay.pdf

[15] J. Polchinski, hep-th/9210046v2 8 Jun 1999

[16] Some recent reviews include: N. Isgur and M. Wise: contribution to Heavy Flavors, A. Buras and M. Lindner (eds.) (World Scientific, Singapore, 1992); M. Neubert, Phys. Rep. bf 245 (1995) 259; Int. J. Mod. Phys. A11 (1996) 4173; T. Mannel, J. Phys. G 21 (1995) 1007

[17] Matthias Neubert, hep-ph/10512222 v1

[18] K. Hagiwara, A.D. Martin, and M.F. Wade, Phys. Lett. B228 (1989), 144

[19] F. Hussain, J.G. Korner and G. Thompson, Ann. Phys. 206 (1991), 334

[20] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989), 113

[21] www.ias.ac.in/pramana/v61/p911/fulltext.pdf

[22] J. Gasser and H. Leutwyler, Annals of Physics 158, (1984), 142-210

[23] H. Georgi, Phys. Lett. B240 (1990) 447

[24] M. Neubert, (1996), hep-ph/9610385v1

[25] K. Nakamura, et. al. (Particle Data Group), Journal of Physics G37, (2010) 075021

[26] http://en.wikipedia.org/wiki/Ultraviolet_divergence

[27] http://en.wikipedia.org/wiki/Infrared_divergence

[28] E. Jenkins, hep-ph/9212295v1 21 Dec 1992

[29] T. Mehen and R. P. Springer1, Phys. Rev. D72 (2005) 034006

[30] I. W. Stewart, Nucl. Phys. B 529, (1998), 62

[31] T. Mehen and R. P. Springer, Phys. Rev. D 70, (2004) 074014

[32] K. Abe et al. [Belle Collaboration], Phys. Rev. D 69, (2004) 112002