

“Reliability Based Design of Concrete Mixes with Partial Replacement of Cement by Silica Fume”

A Thesis submitted in partial fulfillment of the requirements for the award of the degree of

**Master of Engineering
in
Civil Engineering
(Structures)**

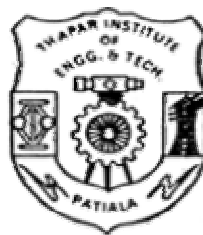
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June 2006

CERTIFICATE

This is to certify that the work presented in Thesis entitled “**Reliability Based Design of Concrete Mixes with Partial Replacement of Cement by Silica Fume**” submitted by **Mr. Rajinder Ghai** in partial fulfillment of the requirements for the award of degree of **Master of Engineering in Civil (Structures)** at **Thapar Institute of Engineering & Technology (Deemed University), Patiala**, is an authentic record of student’s own work carried out under our supervision and guidance. The matter embodied in this thesis has not been submitted anywhere for award of any other degree.

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ABSTRACT

Structural codes and standards provide the foundation of good engineering practice and a framework for addressing safety and serviceability issues in structural design. They identify natural and man-made forces that must be considered and defined magnitude of these forces. The documents on which the structural engineer places so much reliance must address the question: “how safe is safe enough?” on behalf of society as a whole. In the existing limit state design procedure, which is semi-probabilistic in nature, even though the design load is calculated statistically, research for determining the actual loading on the structure has not yielded adequate data to enable one calculate theoretical values of variations for arriving at the actual loading of the structures. Thus, at the root of the structural safety problem is the uncertain nature of

- (i) the man-made and environmental forces that act on structure,
- (ii) material strengths, and
- (iii) structural analysis procedures that at best only theoretical models.

The traditional practice of setting safety factors and revising codes based solely on experience does not work in this environment, where such trial and error approaches to managing uncertainty and safety may have unacceptable consequences. In era in which standards and public safety are being increasingly questioned in public forum, more systematic and quantitative approaches, for public safety, are essential. The probabilistic approach which accounts for the said uncertainties has, in the past two decades, been widely accepted worldwide as a new paradigm, for design of structures and evaluation of the safety of existing ones.

Reliability-based design is based on the concept that one can estimate the probability of an undesirable event such as fracture, occurring over the lifetime of a structure, despite the uncertainties involved. This design method enables to ensure the desired safety level and under specified conditions for a considered period of time.

In the present study a reliability-based design procedure for compressive strength of concrete with partial replacement of cement by silica fume has been developed

considering the strength as a random variable. The compressive strength of concrete depends upon the properties of its constituent materials viz. cement, fine aggregates, coarse aggregates and mineral admixtures such as silica fume etc. in the present investigation, a step-by-step procedure has been suggested to design the concrete mixes.

An extensive data bank on the basic variables viz. compressive strength of concrete in terms of mean, standard deviation and with-in-test coefficient of variation corresponding to the variation in parameters have been generated. The compressive strength data generated experimentally has been analysed using normal-probability distribution functions. Furthermore, based on the analysis of the generated data, partial safety factors have been computed relative to 95 per cent confidence level values.

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INTRODUCTION

1.1 GENERAL

The performance of a structure is assessed by its safety, serviceability and economy over its life cycle. However, the information about the input variables is never certain and/or complete. The uncertainties owe themselves to inherent randomness, limited information, imperfect knowledge and errors. With these uncertainties, to estimate absolute safety of a structure is impossible, due to:

- a) unpredictability of
 - i. loads on the structure during its life cycle,
 - ii. in-place material strengths, and,
 - iii. human errors.
- b) Structural idealizations in formulating the mathematical model of the structure to predict its response or behaviour, and
- c) The limitations of numerical techniques.

These factors lead to some risk with regard to the unacceptable performance of the structure. In the conventional deterministic analysis and design procedures, it is assumed that all parameters (loads, strength of materials etc) are not subjected to probabilistic variations. However, it is well known that the strengths of materials (concrete, steel etc) and the geometric properties (sectional dimensions, effective depth, diameter of bars etc) are subjected to statistical variations. Hence, to be rational in the estimation of the structural safety, the random variations of the basic parameters are to be taken into account. Since load and strength are random variables, the safety of the structure is also a statistical variable.

Code of practice for the design of reinforced concrete structures (*IS 456-2000*) is based on the concept of partial coefficients. It defines two basic limit states: ultimate limit state and the serviceability limit state. The influence of uncertainties and variability in basic variables are taken into account by characteristic values and partial coefficients. This method is “semi-probabilistic” in nature as it uses a set of predefined characteristic values, which have predetermined probability of not being achieved.

To minimize the effect of uncertainties in the design parameters, safety is ensured by taking the smallest value of load resisting capacity (R) and largest value of the load

(Q). The safety factor is then defined as; $\nu = \frac{R}{Q}$. The safety so defined is very conservative and leads to uneconomical design. An alternative method of fixing the safety is as follows:

Let ΔR and ΔQ be the admissible deviation from R and Q , respectively. For the structure to be safe

$$R > Q \quad (1.1)$$

$$R - \Delta R > Q + \Delta Q \quad (1.2)$$

$$R \left(1 - \frac{\Delta R}{R} \right) > Q \left(1 + \frac{\Delta Q}{Q} \right) \quad (1.3)$$

$$\frac{R}{Q} > \frac{\left(1 + \frac{\Delta Q}{Q} \right)}{\left(1 - \frac{\Delta R}{R} \right)} \quad (1.4)$$

Hence, the minimum value of safety factor is

$$\nu = \frac{\left(1 + \frac{\Delta Q}{Q} \right)}{\left(1 - \frac{\Delta R}{R} \right)} \quad (1.5)$$

The above definition of factors of safety not only widely varies but is also probabilistically inaccurate.

The safety factor can also be expressed as the ratio of the mean values of R and Q , which is called the central safety factor, ν_c

$$\nu_c = \frac{\text{mean value of } R}{\text{mean value of } Q} = \frac{\mu_R}{\mu_Q} \quad (1.6)$$

The drawback in this definition of safety factor can be clearly understood from the following.

Fig.1.1 shows the probability density functions $f_R(r)$ and $f_Q(q)$ of R and Q . It is observed that both distributions overlap. The shaded portion gives an indicative measure of the probability of failure of the element or the structure.

It can be easily shown that for the same value of ν_c , the value of probability of failure p_f can be different.

First, consider the case where mean action and resistance are increased by some proportion (say by k_I) as in Fig. 1.2, keeping their standard deviations constant. Thus

$$v_c = \frac{k_1 \mu_R}{k_2 \mu_R} = \frac{\mu_R}{\mu_Q} \quad (1.7)$$

It is observed from Fig.1.2 that even though v_c the same, overlap of the two curves changes, which indicates change in p_f .

Next, if the mean values of R and Q i.e. μ_R and μ_Q are kept constant (i.e. v_c is maintained constant) and dispersions in R and Q are changed as shown in Fig.1.3, it is again observed that the overlap of the two curves changes, indicating a change in the value of p_f .

Thus, the area of overlap, which provides the qualitative measure of the probability of failure, depends upon three factors:

- i. The shapes of the curves (represented by the probability distribution functions $f_Q(q)$ and $f_R(r)$).
- ii. The relative positions of two curves (represented by the means of two variables), and
- iii. The dispersion of the curves, characterized by standard deviations of two variables $\sigma_Q(q)$ and $\sigma_R(r)$.

Therefore, in order to achieve a safe design, the design variables must be so chosen that the area of overlap between the two curves lead to an acceptable probability of failure.

The best way, therefore, to define safety is by the probability of failure or reliability. According to *Freudenthal (1956)*: “Because the design of a structure embodies uncertain predictions of the performance of structural materials as well as of the expected load patterns and intensities, the concept of probability must form an integral part of any rational analysis or design; any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the structural significance of a specified condition can be evaluated”.

1.2 STRUCTURAL RELIABILITY

1.2.1 Concept

The concept of reliability has been applied to several fields and has been interpreted in many ways. Structural reliability is concerned with quantifying the performance levels of structures using the theories of probability and statistics and is always oriented towards relating element to system reliability and identifying the failure

sequence or path (*Kalaga, 1998*). Reliability, as is most commonly defined, is the probability of an element or system performing its intended function over a given period of time under the operating conditions encountered. The above definition stresses four significant parameters, viz.

- i. probability,
- ii. intended function,
- iii. time, and
- iv. operating conditions.

Because of the uncertainties involved, the reliability is the first parameter in the definition. The intended function signifies that reliability is a performance characteristic. For a structure to be reliable, it must perform certain functions satisfactorily for which it has been designed and for all time. Hence, reliability is related to time, thus bringing in the concept of life cycle of structures. The last parameter is the operating conditions, which establishes the actions or stresses that will be imposed on the structure.

The basic principle is that reliability explicitly considers the probability that the structure shall fail during its life span, and that this probability is a function of the failure probabilities of its individual elements. The American Society of Civil Engineers (ASCE) Task committees (as in *Kalaga, 1998*) have provided guidelines for the application of reliability methods to structures. The requirements include:

1. A database to describe the probability of occurrence and duration of loading.
2. Probabilistic models of strength of elements of the structure, quantifying the load resistance capacity 'R'.
3. Models to describe the loading and response.
4. Procedure for computing reliability measures such as probability of failure, safety or reliability index etc.
5. Types of limit states used in defining element/system failure.

In structural analysis and design, reliability is defined as the probability that a structure will not attain each specified limit (flexure, shear, torsion or deflection criteria) during a specified reference period (life of the structure). For convenience, the reliability R_0 is defined in terms of probability of failure, p_f which is taken as

$$R_0 = 1 - p_f \quad (1.8)$$

In case of classical reliability theory, for reliability prediction information on life characteristics of the system, operating conditions and failure distribution are needed.

However, for structural systems, it is difficult to predict the expected life or the expected failure rate or the expected time between breakdowns. So in the reliability format, it is assumed that structural failures are not due to deterioration. The structures cannot be assumed to be normally identical. The structural failures cannot be expressed in terms of relative frequency. Thus, the structural reliability differs from the classical reliability theory in many such aspects except in the probabilistic nature of the uncertainties. The probability of failure of a structure is a subjective probability. The reliability of a structure is not a unique property. It changes as the state of knowledge about the structure changes.

The acceptable probabilities for structural failures are very low viz.

- i. of the order of 10^{-3} for serviceability limit states, meaning thereby that on an average, out of 1000 normally identical structures, one may deform excessively or
- ii. of the order of 10^{-6} for ultimate limit states, which means that out of one million identical structures, one may collapse.

However, in practice structures are never identical in such large numbers and also these low probabilities have to be estimated from the statistical properties extrapolated from the available statistical data around the central values of the random variables. Thus, probabilistic methods play an important role in making rational comparisons between alternative designs.

1.2.2 Computation of Structural Reliability

Consider a simple structure with one element only. Let R be the resistance (capacity or strength) of the structure and Q the action (load or load effect, viz, bending moment, shear force etc.) on the structure. The structure is said to fail when the resistance of the structure is less than the action.

i.e.
$$p_f = P(R < Q) = P((R - Q) < 0) \quad (1.9)$$

where p_f is the probability of failure of the structure.

Let us define safety margin as a function S , represented as

$$S = R - Q \quad (1.10)$$

Let S have as mean and σ_S as standard deviation. A plot of probability distribution function $f_s(s)$ is shown in Fig.1.4.

The shaded portion of Fig.1.4 represents cases where $S < 0$. If the type of probability distribution of S is known then the probability of failure can be computed

from the value of standard deviation that exceeds zero. This is shown in Fig. 1.4 as $\beta\sigma_S$ where β is ‘safety index’ or ‘Reliability index’ defined as,

$$\beta = \frac{\mu_S}{\sigma_S} \quad (1.11)$$

which is the reciprocal of the coefficient of variation.

If the function S is normally distributed, the probability of failure is

$$p_f = \frac{1}{2} \left[\frac{(\mu_R - \mu_Q)}{\sigma_S} \right] \quad (1.12)$$

or in terms of reliability index as,

$$p_f = \frac{1}{2} - \psi(\beta) \quad (1.13)$$

where $\psi(\beta)$ is standard normal function. If β is a normal variate, then p_f can be approximated as

$$p_f = 1 \times 10^{-\beta} \quad (1.14)$$

1.2.3 Reliability-based Methods

The *Joint Committee on structural safety (1976)* has classified reliability analysis and the safety checking into three groups terming them as Level 1, 2 and 3 methods. These levels have been defined as:

Level – 1 method: A design method in which appropriate levels of structural reliability are provided on a structural element basis by the specification of a number of partial safety factors, related to some predefined characteristic value of the basic variables. The present structural design methodology (*IS: 456-2000*) with consideration of the number of limit states is nothing but Level – 1 design. The limit state is a criterion to define a particular failure or performance condition.

Level – 2 methods: A design method incorporating safety checks only at selected point(s) on the failure boundary rather than a continuous process. In these methods, certain idealizations and assumptions are used. Mean values and variances of the random variables are only required. In advanced Level – 2 reliability methods, distributions are taken care of in an approximate way. Reliability levels are defined by safety indices or equivalent “operational” or “notional” probabilities. These methods are approximate as compared to level 3 methods.

Level – 3 methods: In this method safety checking is based on exact probabilistic analysis for whole structural systems or structural elements, using full distributional approach based on failure probabilities, possibly being derived from optimization studies or assessed by other approach criteria. They are purely probabilistic methods and are exact in estimating the reliability.

In the Level – 2 methods certain idealizations and assumptions have to be made. Mean values and variances along with probability distributions are used for carrying out reliability analysis and design. Level – 2 reliability methods are more practical and are quite suitable for design (*Ranganathan, 1990*). They are suitable for calibrating codes on reliability basis. The present study uses Level – 2 methods for arriving at the design procedure for concrete mix.

1.3 AIM AND SCOPE OF THE PRESENT INVESTIGATION

It has been generally recognized that some uncertainty exists in the results of design formulae for evaluating the resistance of reinforced concrete sections. One of the major factors contributing to this uncertainty is the variability of the strengths of the constituent materials. Some previous studies have examined ways in which safe designs could be satisfactorily developed in view of these uncertainties. Two approaches have been reported in the literature (*Harr, 1987*). One approach has been to apply the techniques of error statistics to the results of full-scale tests. Another approach is to treat the material strength and the ultimate resistance of the section as random variables, where in the ultimate strength is a function of material strength with the functional relationship derived from the structural theory.

The aim of the present study is to develop a reliability-based design procedure for design of concrete mix with partial replacement of cement by silica fume by considering the materials properties (compressive strength of concrete) as random variables. The compressive strength of concrete in turn depends upon the properties of its constituent materials viz, cement, fine aggregates, coarse aggregates, silica fume etc. In the present investigation effort has been made to design the concrete mix based on the concept of confidence level for a particular reliability index.

The compressive strength data required for the probabilistic analysis has been generated in the laboratory by varying the quantities of various constituent materials. The compressive strength data thus generated has been probabilistically analyzed and a

reliability-based design procedure for design of concrete mixes has been developed. Furthermore, based on the analysis of the data thus generated, partial safety factors have been found out relative to 95 per cent confidence level values, which can be used in designing structural elements.

1.4 SILICA FUME

Silica fume is a highly reactive pozzolanic material primarily composed of silicon dioxide (SiO_2) in noncrystalline form. It is a light to dark gray or bluish green-gray powder produced by an electric arc furnace during the manufacture of silicon or ferrosilicon alloy. It has a spherical like fly ash, but it is 100 times smaller, with an average diameter size of 0.1 μm . Its specific surface area is very large compared to Portland cement (20,000 m^2/kg (by nitrogen absorption method) versus 300 m^2/kg , respectively).

1.4.1 Effect of silica fume on fresh mix concrete

Silica fume is a highly active pozzolan; it is essentially silicon dioxide in noncrystalline form, having a large specific surface area. Like other pozzolans, SiO_2 reacts to $Ca(OH)_2$ with the presence of water to form $C-S-H$. Because of the small size and the large surface area of silica fume, it absorbs a lot of water, making the water-demand for silica fume concrete very high and the setting time of the concrete very fast. In general, concrete containing silica fume has either a superplasticizer or retarder added to it. The superplasticizer increases the flow of concrete without affecting the w/c ratio. Superplasticizer propels the water from the silica fume particles to increase the workability of the concrete for a few minutes; because of this, superplasticizer also retards the mix. On the other hand, when the workability of concrete is not an issue, a retarder is used to slow down the reaction to increase the working time with concrete. Besides increasing the setting time and workability, these two chemical admixtures offer other benefits. One of the advantages is their effect on the rate of change of heat in concrete. Concrete containing silica fume releases a very high heat of hydration. As discussed by Roy (1989), the rate of hydration of silica fume is very high because of the intense reaction between the silica fume and portland cement. This high heat could cause microcracking if the concrete is subjected to wind; this is the major disadvantage of silica fume concrete. Another disadvantage of silica fume on concrete is the high viscosity,

making the concrete difficult to pump. An additional advantage of silica fume is the cohesiveness it offers the mix, causing less bleed-water. In fact, silica fume concrete does not release any bleed-water at all. This is a major advantage, as well as a disadvantage. The advantage is the elimination of segregation because no extra water exists on the top surface of the concrete. However, because water is not released, its evaporation could leave pores in the concrete if the concrete is not properly consolidated.

1.4.2 Mechanical properties of silica fume concrete

Silica fume is known for its ability to increase the strength of concrete because of its chemistry and hydration. Silica fume, when added to portland cement, reacts rapidly to form *C-S-H*. As a matter of fact, for very low *w/c* ratio concrete, the aggregate, rather than the cement paste, controls the concrete strength. Because of the higher strength of silica fume concrete, other mechanical properties of the concrete also improve.

Charif et al (1990) study the mechanical properties of silica fume concrete as part of a vast research program concerning the calculation and the reduction of concrete slab deformations. Their final results indicate concrete containing silica fume can increase the compressive strength by 100 to 150 percent. The elastic modulus can be increased by 20 to 30 percent. The tensile strength can be increased by 40 to 60 percent. The creep deformations can be reduced by 30 to 60 percent. Creep is a very important concrete property, especially for prestressed concrete. Prestressed concrete is usually designed to have higher compressive strength, thus silica fume concrete is perfect for this application. There are numerous paper published investigating creep and shrinkage of silica fume concrete.

Persson (2001) correlated laboratory and field tests of creep in HPC carried out between 1991 and 1999. For this purpose, about 400 cylinders made out of eight mix compositions of HPC were studied in the laboratory. Half the number of HPC was sealed; half of the studies were carried out on air-cured HPC. Parallel tests on strength, internal relative humidity, RH, and hydration were carried out on about 900 cubes made of HPC from the same batch as the cylinders. Fragments from the strength tests were used to observe RH. Shrinkage was studied in relation to creep. The results and analyses of the laboratory studies show a large influence of the maturity, as well as of the mix composition, of the HPC on the measured creep. The creep coefficient of HPC is smaller

than in normal concrete. In contrast, shrinkage of HPC is somewhat larger than in normal concrete.

Similar creep result is reported by *Al-Khaja (1994)* who investigated high strength concrete mixes with and without silica fume in the Arabian Gulf region. In his study, silica fume considerably reduces creep of concrete, given 1 month reduction strain of 18.5 percent for creep.

Wiegrink et al (1996) whom study the shrinkage cracking of high strength concrete, show similar shrinkage result. According to their study, silica fume concrete shows higher shrinkage and lower creep. Cracking for high-strength silica fume concrete develops much faster and is significantly wider than that of normal-strength concrete. Moreover, not only the drying shrinkage increases but also autogenous shrinkage for silica fume is very significant since this could be as high as drying shrinkage.

Kanstad et al (2000) perform a study to address the effect of silica fume on the early age crack sensitivity of HPC. Tests were performed on concrete containing silica fume ranging from 0 to 15 percent, with a w/c ratio of 0.40. They canvassed realistic temperatures, tensile strength, modulus of elasticity, and autogenous shrinkage. All tests were conducted under isothermal conditions. Data were collected for 7 days for autogenous shrinkage. They concluded an increase in silica fume content raises the tensile strength and modulus of elasticity but also causes more of autogenous shrinkage. Despite the higher drying shrinkage, increasing the curing time could reduce this.

Hindy et al (1994) report the drying shrinkage results of ready-mixed concrete containing silica fume. The concrete containing silica fume and ordinary concrete have a compressive strength of 98 MPa (14,200 psi) and 80 MPa (11,600psi) at 91 days, respectively. The effect of curing time, curing conditions, silica fume content, and w/c ratio were investigated. The drying shrinkage for both concretes decreases as the curing time increases or as the w/c ratio decreases. They also compared the drying shrinkage of small specimens with an actual-sized column made with the two concretes. From the comparison, the shrinkage measured using the small specimens overestimate the drying shrinkage of the actual-sized columns. In addition to compressive creep, an attempt has been made to study the tensile creep properties of silica fume concrete.

Koveler et al (1999) study the effect of the tensile creep behavior of concrete at early ages. They found the use of 10 percent silica fume increases the early age tensile

creep of high strength concrete in comparison with concrete without silica fume at the same w/c ratio of 0.33.

1.4.3 Durability characteristic of silica fume concrete

Silica fume improves the durability of concrete tremendously because of its denser microstructures. Many researchers *Gautefall and Havdahl (1989)* and *Gagné et al (1992)* have observed the permeability of concrete decreases when about 10 percent of silica fume is added to it. *Gagné et al (1992)* investigate the chloride-ion attack on low water-cement ratio pastes containing silica fume. As it is well established that concrete is more perceptible on chloride-ion attack when the pH of concrete is below 13. This is due to the leaching of calcium ions that facilitates the penetration of chloride ions. In their investigation they subject the specimens that consist of Type III cement, Type III cement with 6 percent silica fume, and French CPA-HPR cement with 6 percent silica fume paste, to 3 percent sodium chloride solution at a pH level of 11.5 and 13. Mercury intrusion porosimetry, X-ray diffraction, scanning electron microscopy and electron microprobe measurements are the techniques used for the study. From their result, they concluded that silica fume decrease chloride penetration at the pH level of 11.5.

Another report that shows the superior durability of silica fume concrete is by *Berke (1989)*, who investigates the effect of silica fume on the resistance of steel corrosion, erosion, and chemical attacks. In his study, concrete with embedded steel rebar are produced with varying level of silica fume from 0 to 15 percent by cement weight (0 to 30 percent for chemical testing). Other parameters in the investigation are the water-to-cement ratio and calcium nitrate. Polarization resistance and macrocell corrosion techniques are used to measure the corrosion rate in the steel rebar. The specimens are immersed in 3 percent *NaCl* solution for both methods. The United States Army Corp of Engineers method CRD-C 63-80 is used to perform the erosion test. The chemical testing is performed in 5 percent acetic acid, 1 percent sulfuric acid, 5 percent formic acid, and mixed sulfates. From his results, silica fume improves the resistance to steel corrosion, erosion, and chemical attack. Nevertheless, one major disadvantage associated with silica fume concrete is microcracks. Silica fume concrete, if improperly cured, will crack while it sets because of the temperature differential that stresses it. The temperature differential results from the heat of hydration, which is relatively high for silica fume concrete. It

causes the concrete to expand whereas the cool ambient temperature causes it to contract. This expansion and contraction could bring about enough stress for the concrete to crack.

1.5 ORGANIZATION OF THESIS

The thesis is presented in seven chapters as detailed below:

Chapter – 1 introduces the thrust area and the need for reliability-based design and the objectives of the present work.

Chapter – 2 on literature review presents the work done by various researchers in the field of statistical analysis of various parameters and reliability-based design.

Chapter – 3 details the scheme of experimentation, materials used, variables involved, concrete mixes cast incorporating variations in the parameters, techniques adopted for casting and testing of specimen. The compressive strength data generated from the experimental program has also been presented in this chapter.

Chapter – 4 deals with mathematical model developed for analysis of the compressive strength data generated. It presents the results of the probabilistic analysis carried out which subsequently have been used for development of reliability-based design procedure of concrete mix.

Chapter – 5 presents the reliability-based design procedure thus evolved for design of concrete mixes, with and without replacement of cement by silica fume.

Chapter – 6 deals with the discussions on the results obtained from the probabilistic analysis and the effects of various parameters on the design aspects. The results have been presented both in tabular form as well as in graphical form.

Chapter – 7 gives the major conclusions of the study.

References and Appendices follow in succession.

The relevant Tables and Figures, in the same order, are placed at the end of each chapter.

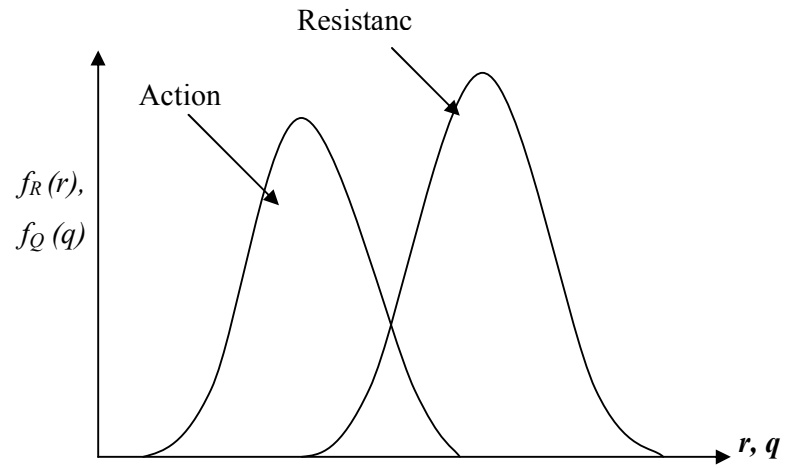


Fig. 1.1 Overlap of action and resistance distributions indicating failure probability (Ranganathan, 1990)

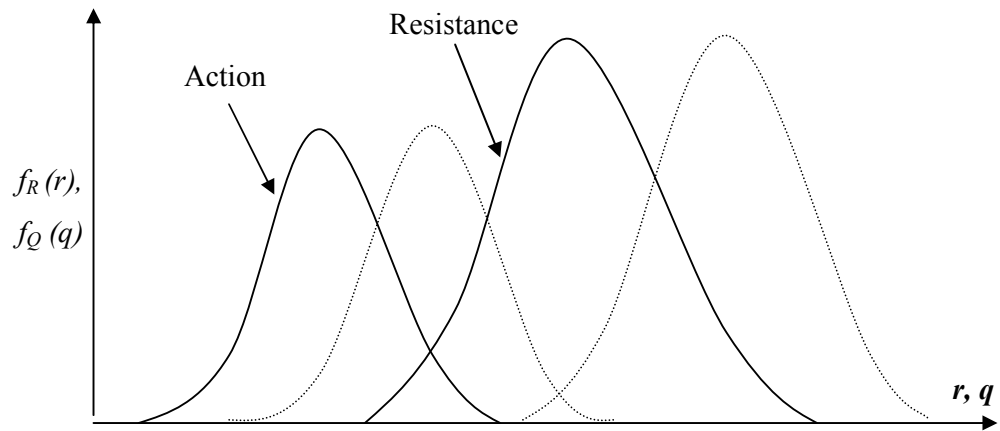


Fig. 1.2 Effect on failure probability due to proportional change in action and resistance (Ranganathan, 1990)

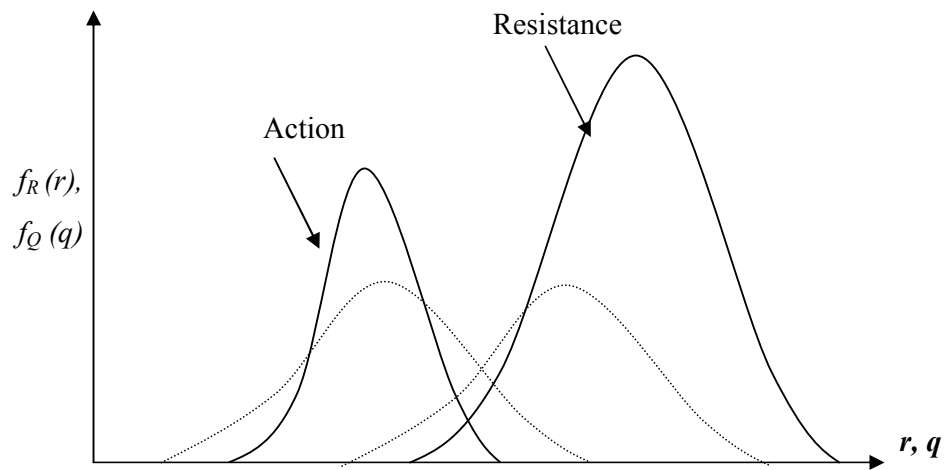


Fig. 1.3 Effect on failure due to change in dispersion of action and resistance (Ranganathan, 1990)

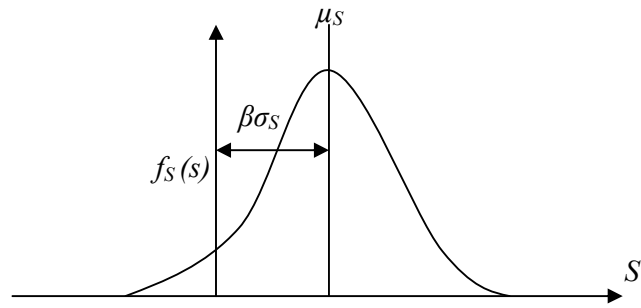


Fig. 1.4 Definition of reliability index, β (Kumar, 2002)

LITERATURE REVIEW

2.1 GENERAL

The estimation of failure probability of structures has been an active area of research for more than two decades. During this period, efficient procedures have been developed for reliability evaluation of individual limit states in large structures. The present day codes of practice for the design of reinforced concrete structures are based on the design format of partial coefficients recommended by International Standard Organizations. The partial coefficient format defines two limit states: the ultimate limit state and serviceability limit state. The influence of uncertainties and variability's for basic variables is taken into account by characteristic values and partial coefficients. This method involving use of partial coefficients is also called "semi-probabilistic" because it uses a set of predefined values which have a prescribed probability of not being obtained. "More developed" methods use a predefined set of stochastic variables directly and employ the failure probability instead of design equations in the evaluation of safety of structures.

From the foregoing it is clear that the provisions of existing codes, for the sake of being simple, do not consider explicitly the randomness of load and strength. In recent years much literature has appeared in the area of reliability based design and probability of failure of structures due to random variations of load and strength. The earliest reported work on the introduction of probabilistic concepts and procedures to problems in structural engineering dates back to 1968, when a series of papers were developed and written by members of subcommittee – E (Probabilities) of ACI committee – 348 for presentation at the 1968 ACI fall meeting at Memphis in November 1968.

In the papers presented by *Shah (1969)*, *Sexsmith and Nelson (1969)*, *Benjamin and Lind (1969)* and *Cornell (1969)*, a goal had generally been established of designing a structure and structural material (concrete) for a specified allowable failure probability. A brief review of the literature representing the work done by various researchers in the field of reliability analysis and design is presented in the following sections.

2.2 STATISTICAL DESCRIPTION OF STRENGTH OF CONCRETE

Elingwood and Ang (1974) have provided the qualitative analysis of the design uncertainties and have shown their effect on the level of risk. For purpose of analysis and design, mean and the basic variability have been estimated from the data reported in the literature. A prediction error has been assigned to the predicted mean to account for the inaccuracies in its estimation. It has been stressed that the choice of the prediction error, Δ , would reflect the confidence placed by an engineer on the accuracy of his prediction, relying perhaps on his past experience with similar situations. The prediction error with respect to compressive strength of concrete has been discussed.

The strength of reinforced concrete member may vary from the calculated or “nominal strength” due to variations in the material strengths and dimensions of the member, as well as uncertainties inherent in the equations used to compute member strengths. A review of these sources of uncertainty and their effects on the mean and coefficient of variation in the member resistances is presented below.

The variability in the compressive strength of concrete has been reported for tests on standard cylinders for concrete proportioned for a nominal compressive strength (*Elingwood and Ang, 1974*) of f'_c of 3,000 psi (21 MN/m²) and 3,456 psi (23.8 MN/m²), with coefficient of variation of 0.12. Additional data defining the mean of the strength of concrete delivered by a ready mixed concrete company during separate monthly periods has indicated an error for the measured strength of 0.07. A comparison of the strength of in-situ concrete with that determined from standard cylinders (*Bloem, 1968*) indicated that under field conditions, the strength is 10 to 21 per cent lower than the strength observed under laboratory conditions. Assuming these factors to contribute a combined uncertainty of 0.16, a prediction error has been determined to be 0.18 and corresponding uncertainty has been determined to be 0.21.

Ellingwood (1978) estimated the COV of the in-situ strength as 0.207 for average control. It has been concluded that the COV in the tensile and compressive strengths of concrete could be of the same order.

Mirza et al (1979) have derived a relationship for the coefficient of variation on in-situ compressive strength as

$$V_c = (V_{ccyl}^2 + 0.0084)^{1/2} \quad (2.3)$$

where V_{cyl} is the coefficient of variation (COV) of the cylinder tests. For average control V_{cyl} is about 15 and 12 per cent for 3000 and 5000 psi concretes, respectively, and V_c can be taken as 18 and 15 per cent.

According to *Mirza et al (1979)* the COV of the in-situ tensile strength can be taken equal to 18 per cent, which is the value assumed for the compressive strength. The coefficients of variation obtained by assuming that both the tensile and compressive strengths follow a normal distribution have been tabulated in Table 2.1.

MacGregor et al (1983) have used Monte Carlo technique for determination of probability distribution and statistics for capacities of reinforced and pre-stressed concrete members. It involved comparing the results of the concrete and methods of calculating the member's resistance to the actual test results. The bias and variability of the procedure was focused out using the concept of 'model error.'

Leon et al (1978) have compared the accurate calculation procedure to tests, to get the mean and COV of the ratio of test strength divided by calculated strength. The variability determined in this way was assumed to result from three causes

$$V_{T/C} = \sqrt{V_m^2 + V_{test}^2 + V_{spec}^2} \quad (2.4)$$

where $V_{T/C}$ is the coefficient of variation obtained directly from the comparison of the measured and calculated strengths; V_m represents the variability of the model itself, V_{test} represents the uncertainties in the measured loads due to such things as the accuracies of the gauges, errors in readings, definition of failure; and V_{spec} represents errors introduced by such things as differences between the strengths in the test specimen and in control cylinders, and variations in actual specimen dimensions from those measured.

Duval and Kadri (1998) have proposed a model to evaluate the compression strength of silica fume concrete at any time. The model is related to the water-cementitious materials and silica-cement ratios.

Kumar (2002) have used the normal and lognormal distribution for representing the concrete compressive strength and developed the mix design procedure for concrete with partial replacement of cement by fly ash. He also considered other variables as w/c ratio, zone of aggregates. The governing equation for evaluation of the design values had been given in the form

$$g(Z) = 0 \quad \text{i.e.} \\ \left(\mu_{CSrev} + \sigma_{CSrev} z_{cs} \right) - \left(\frac{1}{A} \right) \left(\mu_{wcr} + \sigma_{wcr} z_{wcr} \right)^B = 0 \quad (2.5)$$

where μ_{CSrev} and σ_{CSrev} are the revised means and standard deviations of compressive strength and μ_{wcr} and σ_{wcr} are the revised means and standard deviations of water-cementitious ratio. The values of α_1 and α_2 have been computed using *Hasofer Lind's* method for different cases using normal and lognormal distribution functions.

Bhanja and Sengupta (2002) present a paper which deals with a mathematical model development using statical methods to predict the 28-days compressive strength of silica fume concrete with water-to-cementitious ratios ranging from 0.3 to 0.42 and silica fume replacement percentage from 5 to 30 per cent. Strength results of 26 concrete mixes, on more than 300 test specimens, have been analyzed for statical modeling. The relationship between 28-days strength of SF and control concrete and silica fume replacement percentage based on test results, has been obtained as:

$$\frac{f_{SF}}{f_C} = 1.0063 + 0.0159(SF\%) + 0.0007(SF\%)^2 - 0.00003(SF\%)^3 \quad (2.6)$$

where SF% refer to silica fume replacement and f_{SF} and f_C denote the silica fume and control concretes, respectively. The values of the standard error of estimate (s) and coefficient (r) have been obtained as 0.079 and 0.842, respectively.

2.2.1 Concrete Strength in Compression

Under the current design, production, testing, and quality-control procedures the strength of concrete in a structure may differ from its specified design strength and may not be uniform throughout the structure. The major sources of variations in concrete strength are the variations in material properties and proportions of the concrete mix, the variations in mixing, transporting, placing and curing methods, the variations in testing procedures, and the variations due to concrete being in a structure rather than in control specimens.

(a) Degree of control

First and foremost, the variability of concrete strength depends on the quality control of the concreting operation. In the construction of the Skylon Tower at Niagara Falls (*Lauer and Rigby, 1966*), coefficients of variation ranging from 7 per cent to 10 per cent were achieved for field-cast laboratory-cured cylinders using exceptional quality-control methods. The coefficients of variation (*Mirza et al, 1979*) of field-cast laboratory-

cured specimens have been, in many cases, found to be 15 per cent to 20 per cent, which suggested that 20 per cent was a reasonable maximum value for average controls.

According to *Mirza et al (1979)*, based upon the data available in the literature (*Erntroy, 1960; Murdock, 1953 and ACI-214, 1965*) the average coefficient of variation can be taken as roughly constant at 10 per cent, 15 per cent and 20 per cent for strength levels below 4,000 psi for excellent, average, and poor control, respectively. For concrete with an average strength above 4,000 psi, the standard deviation remains approximately constant with values of 400, 600, and 800 psi, respectively, for the three levels of control listed previously. This is expected since the greater control required for the production of higher strength concrete contributes to smaller variability. The total variation in the concrete strength measured by control cylinders included the variation in concrete strength within a single batch. This in-batch variation may be considered as the variation in testing procedures, mixer inefficiencies, and in actual concrete strength.

The in-batch coefficients of variation of laboratory tests as reported by *Komlos (1970)* and *Ramesh and Chopra (1960)*, varied from 0.5 per cent to 8.1 per cent with an overall average value of 3.6 per cent.

American Concrete Institute Committee, ACI – 214 (1965) has recommended that the level of control for within-batch tests could be divided into three classes with corresponding coefficients of variation as 4 per cent to 5 per cent for good control, 5 per cent to 6 per cent for average control and above 6 per cent for poor control.

The *ACI committee report 214 (1988)* has subsequently revised the control standards for evaluating performance of testing program using the within-test coefficient of variation as listed in the Table 2.2.

According to the *ACI: 214 report* if within-test coefficient of variation is high (fair or poor), testing may be the reason for poor test results rather than the concrete quality.

(b) *Distribution of compressive strength of concrete specimens*

The majority of researchers including *Julian (1955)* and *Shalon and Reintz (1955)* have represented the distribution of concrete compressive strengths with a normal distribution. *Rusch et al (as reported by Mirza et al, 1979)* have performed 829 individual series of tests and found 93 per cent of these to be normally distributed. The balance was divided between positively and negatively skewed or multi-peaked distributions. At the

same time, however, *Fruedenthal (1956)*, *Julian (1955)* and *Shalon and Reintz (1955)* have suggested that the lognormal distribution gives a better fit for concrete strength in which the control is poorer than average (COV greater than 15 per cent to 20 per cent).

From the evidence available particularly from the study by *Rusch et al (as reported by Mirza et al, 1979)* it has been found reasonable to assume a normal distribution for the compression strength of concrete. In the present study both normal and lognormal distributions have been used for evolving the reliability based design procedure.

Rangantahan (1990) has presented the results of the statistical analysis of the data on various properties of concrete collected from various sources in the Table 2.3.

(c) *In-situ strength versus control strengths*

The strength of concrete in a structure tends to be somewhat lower than the strength of control cylinders or cubes moulded from the same concrete. This difference is due to the effects of different placing and curing procedures, the effects of vertical migration of water during the placing of concrete in deep members, the effects of difference in size and shape, and the effects of different stress regimes in the structure and the specimens (*Petersons, as reported by Mirza et al, 1979*). The difference in directions of casting and loading of the structure and the specimens may also influence the strength.

The average ratios of core strength to standard cylinder strength from various studies (*Bloem, 1969; Campbell and Tobin, 1967*) varied from 0.74 to 0.96 with an overall average from all studies of 0.87. *Petersons (as reported by Mirza et al, 1979)* has shown, however, that the ratio of the strength of concrete in a structure to the strength of the same concrete in a standard cylinder is not a constant and decreases as the strength level increases. *Bloem (1969)* had observed that the strengths of cores drilled from slabs were 93 per cent of the strength of push-out cylinders from the same slab. This was essentially independent of the type of cement used, the curing conditions, and the age of the concrete at testing. It was also observed that the strength of concrete cores from well-cured slabs was 90 per cent of the strength of well-cured standard cylinders moulded from the same concrete. The ratio reduced to 79 per cent for cores cut from slabs subjected to lower standards of curing nearly typical of usual field practice and cylinders from the same concrete. This suggested a reduction of approximately 12 per cent in the in-situ strength of concrete for minimum acceptable field-curing conditions.

Petersons (as reported by Mirza et al, 1979) has also concluded that the difference in in-situ strengths for minimum acceptable and good curing standards could be approximated by a factor of 0.9.

The reduction in the in-situ strength of concrete is partially offset by the requirement that the average cylinder strength must be about 700 psi to 900 psi greater than the specified strength to meet the existing design codes (*ACI Standard 318-77*). Based on this observation and on equations and data from *Allen (1970)*, *Bloem (1969)*, and *Petersons (as reported by Mirza et al, 1979)*, it has been suggested by *MacGregor (1976)* that the mean 28-day strength of concrete in a structure for minimum acceptable curing can be expressed as

$$\bar{f}_{cstr-35} = 0.675f'_c + 1,100 \leq 1.15f'_c \text{ psi} \quad (2.7)$$

in which f'_c is the design compressive strength of concrete defined in accordance with ACI standard 318-77.

(d) Effect of Volume

The in-situ strength of concrete is affected by the difference in the volumes of material under stress. The statistical theory of brittle fracture of solids (*Bolotin, 1969*) gives good estimates for the influence of size in geometrically similar specimens. According to this theory, the dependence of mean compressive strength on the volume can be represented by the expression

$$\bar{X} = \bar{X}_o \left[0.58 + 0.42 \left(\frac{v_o}{v} \right)^{1/3} \right] \quad (2.8)$$

in which X_o and v_o represent the strength and volume of a 4-inch side cube specimen. Eq. (2.8) indicates that as the volume increases, the mean strength decreases. When the volume tends to infinity, the mean strength tends to the strength of the weakest constituent element of the material and approaches $0.58 \bar{X}_o$. *Bolotin (1969)* has also suggested an alternative expression for estimating the dependence of the compressive strength variability on volume as

$$V_x = \frac{0.147 \left(\frac{v_o}{v} \right)^{1/3}}{0.58 + 0.42 \left(\frac{v_o}{v} \right)^{1/3}} \quad (2.9)$$

From Eq. (2.9) it can be observed that as the volume increases, the coefficient of variation decreases, the coefficient of variation tends to zero when volume tends to infinity.

Assuming a normal distribution, the minimum and maximum strengths have been approximated (*Mirza et al, 1979*) by the values that are three standard deviations below and above the mean, as shown in Fig. 2.1. A curve representing $0.58 \bar{X}_o$, which represents the minimum strength when volume tends to infinity, has also been plotted in the figure.

(e) Speed of loading

The observed strength of concrete is considerably affected by the rate of application of the load: the lower the rate of loading, the lower the apparent strength. This is probably due to the increase in strain with time owing to creep and micro cracking. The normal rate of loading for the standard cylinder test is approximately 35 psi/sec (*ASTM standards, 1976*) and for the cube test is 5.25 kN/sec (*IS: 516-1959*).

As reported by *Mirza et al (1979)*, *Jones and Richart* have compared the normal rate of loading for testing cylinders and observed that loading at 1 psi/sec reduces the apparent strength of concrete by approximately 12 per cent whereas loading at 1,000 psi/sec increases the strength by approximately 12 per cent. Based on standard compression cylinder tests, *Jones and Richart* have proposed a relationship between compressive strength of concrete and the rate of loading that has been rewritten in Eq. (2.8) to relate the 28 day mean strength of concrete to the nominal testing speed for cylinder tests, which has been taken as 35 psi/sec.

$$\bar{f}_{CR} = 0.89 \bar{f}_{c35} (1 + 0.08 \log R) \quad (2.10)$$

in which ($0.1 \leq R \text{ psi/sec} \leq 10,000$). Although no information is available on variability of the concrete strength due to speed effects, a very small dispersion in the strength was observed.

According to *Rusch (1960)*, concrete is capable of indefinitely withstanding stress only up to 70 per cent to 75 per cent of the strength under loads applied at 35 psi/sec.

(f) Model for in-situ concrete strength in compression

Mirza et al (1979) presented the combined effect of volume, speed of loading on the concrete compressive strength in a structure, in the following model:

$$f_{cstrR} = f'_c r_{creal} r_{in-situ} r_R \quad (2.11)$$

in which

r_{creal} = random variable relating real cylinder strength to design compressive strength;

$r_{in-situ}$ = random variable relating in-situ strength to real cylinder strength; and

r_R = random variable relating strengths at R psi/sec and 35 psi/sec.

The mean value for the in-situ compressive strength of concrete at a given rate of loading R psi/sec was found to be:

$$\bar{f}_{cstrR} = \bar{f}_{cstr35} [0.89(1 + 0.08 \log R)] \quad (2.12)$$

in which \bar{f}_{cstr35} is the mean 28 days strength of concrete in a structure at normal rate of loading.

The coefficient of variation of the in-situ compressive strength of concrete at a given rate of loading can be calculated using the model of Eq. (2.11) and has been expressed as

$$V_{cstrR}^2 = V_{creal}^2 + V_{in-situ}^2 + V_R^2 \quad (2.13)$$

The strength of concrete measured by control cylinders includes variations in the “real” concrete strength and the so-called in-test variations due to testing procedure. Thus,

$$V_{cyl}^2 = V_{real}^2 + V_{in-test}^2 \quad (2.14)$$

Using Eqns. (2.13) and (2.14) and assuming variation of concrete in a structure with respect to the compressive strength of control cylinders, $V_{in-situ}$, of 10 per cent (*Davis, 1976*) an in-test variation, $V_{in-test}$, of 4 per cent and the variation due to the rate of loading effect, V_R , to be negligible, the coefficient of variation of the in-situ strength of concrete at a given rate of loading, according to *Mirza et al (1979)*, is given by

$$V_{cstrR}^2 = V_{cyl}^2 + 0.084 \quad (2.15)$$

The in-situ strengths have also been assumed to follow a normal probability distribution as has been taken for concrete cylinder strengths.

2.3 EFFECT OF VARIATIONS ON RELIABILITY-BASED DESIGN

Costello and Chu (1969) have studied the theoretical variability in the resistance of reinforced concrete sections induced by variability in the strengths of the constituent materials. The earlier theoretical studies of the variability in the material strength have been based on “weakest-link” (*Weibull, 1955*) and “bundle” (*Daniels, 1945*) models. Both the approaches consider the material to be made of an unboundedly large collection of fibres, acting together to carry the applied load. In the former, the strength of the material is determined by the strength of the weakest fibre; there is no re-distribution of load. Once a single fibre fails, the specimen fails. In the latter approach there is a complete redistribution of the load. The specimen does not fail until the last of the fibres has failed.

Many studies (*Haugen, 1965; Misteth, 1966 and Fisz, 1963*) have indicated that the tests of strengths of nominally identical structures (or specimens) always show some scatter around a central value. The investigations have postulated that the strength of a material is distributed according to a normal (or Gaussian) law i.e., the probability density of the strength, x , is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right]; \quad -\infty < x < \infty \quad (2.16)$$

where \bar{x} and σ are mean and the standard deviation of the strength. The central limit theorem of mathematical statistics (*Fisz, 1963*) indicates that the distribution of a random variable, which is the sum (or the mean value) of other random variables, tends, to a normal distribution fairly independently of the distribution of the constituent terms, as a number of terms in the sum become unbounded. If the strength of an element is viewed as the sum of strengths of individual elements, or fibers, then it might be expected to tend toward normality. Many authors have also suggested the use of other distributions viz. lognormal (*Freudenthal, 1966*), Weibull (*Weibull, 1955*), Rosin-Rammler (*Mood and Graybill, 1963*) and beta (*Weibull, 1955*).

Costello and Chu (1969) have used the beta distribution for representing concrete and steel strengths and have found it to satisfactorily fit the available data. Using the data

available in ACI committee report (*ACI 214-57*), the authors have found out the failure probabilities of various parameters. The design resistance recommended by the ACI code is 90 per cent of the nominal value. Thus, the under-strength factor of 0.90 (*ACI, 1963*) has been seen to reduce the probability to zero, if only the effect of variability of material strengths is considered. It had been concluded that the failure probability would be higher if other factors such as variation in dimensions are also included.

Table 2.1 Statistical variations of compressive and tensile strengths (*Mirza et al, 1979*)

Property	Mean, psi	V*
Concrete normal control		
Compressive strength in structure loaded to failure in one hour	2760	0.18
$f'_c = 3000$ psi	3390	0.18
= 4000 psi	4028	0.15
= 5000 psi		
Tensile strength in structure loaded to failure in one hour	306	0.18
	339	0.18
$f'_c = 3000$ psi	366	0.18
= 4000 psi		
= 5000 psi		

* Coefficient of variation, 1 psi = 6895 Pa; 1 in = 25.4 mm.

Table 2.2 Control standards for evaluating performance of testing program using the within-test coefficient of variation (*ACI report, 1988*)

Field control testing				
Control standards for within-test coefficient of variation				
<i>Excellent</i>	<i>Very good</i>	<i>Good</i>	<i>Fair</i>	<i>Average</i>
Below 1.5	1.5 to 2.0	2.0 to 3.0	3.0 to 4.0	Above 4.0

Table 2.3 Results of statistical analysis of cube strength of concrete (*Ranganathan, 1990*)

Source	Grade	Specified strength, MPa	Mean, MPa	Standard deviation, MPa	COV, percent	Distribution	Quality control
IIT, Kanpur	M15	15	24.03	5.76	23.96	Lognormal	Nominal mix
IIT, Kanpur	M20	20	29.16	5.49	18.83	Normal	Nominal mix
IIT, Kanpur	M25	25	30.28	3.77	12.45	Normal	Designed mix
IIT, Kanpur	M35	35	42.28	5.60	13.24	Normal	Designed mix
REC, Calicut	M15	15	22.67	5.01	22.10	Lognormal	Nominal mix
IIT, Bombay	M15	15	17.56	2.69	15.33	Lognormal	Designed mix
IIT, Bombay	M20	20	26.80	4.04	15.07	Normal, Lognormal	Designed mix

Fig. 2.1 Effect of volume on mean, minimum and maximum strength (*Bolotin, 1969*)

EXPERIMENTAL PLAN AND TEST RESULTS

3.1 GENERAL

The experimental plan has been designed to generate compressive strength data for probabilistic analysis and subsequent development of reliability based design procedure. The concrete mixes have been proportioned for different values of parameters, which affect the compressive strength of concrete. The range of values of the parameters involved are given in Table 3.1

The coarse aggregates used in the study have been divided into two parts based on the percentage passing through the two consecutive sets of sieves. The detail is given in Table 3.12. The water content requirement for aggregates has been selected based upon the workability requirement. The water content variation for aggregates is also given in the same Table 3.12.

3.2 CONCRETE MIX PROPORTIONS

The proportions for various concrete mixes have been obtained by varying the water-cementitious ratio and percentage replacement of cement by silica fume. The cementitious content for each mix proportion is fixed 450 kg/m^3 . DoE method of mix design (*Gambhir, 1995*) was used for determining the proportions. Various adjustments due to variations in the specific gravity, water absorption etc. were also incorporated in the design. The mixes were designed for medium workability. The details of mix proportions for different replacement levels of cement by silica fume (0, 5 and 10 per cent) have been provided in Tables 3.2 to 3.4.

3.3 MATERIALS

Same sets of materials have been used throughout for casting of cubes for the mix proportions tabulated in Tables 3.5 to 3.11. Relevant tests in accordance with the Indian Standard codes of practice were conducted to determine the physical properties of the materials used in the study. The details of the materials along with their properties have been presented in the subsequent sections.

(a) Cement

Although all materials that go into concrete mix are essential, cement is very often the most important because it is usually the delicate link in the chain. The function of cement is first of all to bind the sand and stone together and second to fill up the voids in between sand and stone particles to form a compact mass. Although it constitutes only about 20 per cent of the volume of concrete mix, it is the active portion of binding medium and is the only scientifically controlled ingredient of concrete. Any variation in its quantity affects the compressive strength of the concrete mix.

Pozzolana Portland Cement (PPC) of 43 grade (ACC Suraksha) which is blended with 20% fly ash, has been used. The cement used was fresh and without any lumps. The cement content is 450 kg/m^3 in the present study. Cement of same brand was obtained from local market. Tests were conducted for cement as per *IS: 8112-1989*. The physical properties of cement used in the study are given in Table 3.11.

(b) Aggregates

The aggregate is the matrix or principal structure consisting of relatively inert and coarse particles. The coarse aggregate is used primarily for the purpose of providing bulk to the concrete. The most important function of fine aggregates is to assist in producing a workable and a uniform concrete mix. The fine aggregate also assists the cement paste to hold the coarse aggregate particles in suspension. This action promotes plasticity in the concrete mix and prevents segregation of the paste and coarse aggregates during its transportation. The aggregates provide about 75 per cent of the body of concrete and hence their influence is extremely important. The properties of these particles greatly affect the performance of concrete.

(c) Fine Aggregates

IS: 383 – 1970 defines the fine aggregate, as the one passing 4.75 mm IS sieve. The fine aggregate is often termed as a sand size aggregate. Locally available riverbed sand was used in the present study. The properties of the same are given in the Tables 3.5 and 3.6.

Percentage passing 600 micron sieve = 62.35

The sand conforms to grading Zone – III as per IS: 383 – 1970.

(d) Coarse aggregates

The coarse aggregate is defined, as that retained on 4.75 mm IS sieve. To increase the density of the resulting concrete mix, the coarse aggregate is frequently used in two or more sizes. Two types of aggregate with different sizes have been used in the present study. The details of the same are as below:

- i. CA – I Aggregate passing 20 mm sieve and retained on 10 mm sieve.
- ii. CA – II Aggregate passing 10 mm sieve and retained on 4.75 mm sieve.

The properties of these aggregates have been listed in Tables 3.7 to 3.10. The percentage contributions of aggregates have been taken as 67% CA – I and 33% CA – II for proportioning of the concrete mix. The coarse aggregates used were washed to remove dust and dirt and were dried to surface dry condition.

(e) Water

Generally, water that is suitable for drinking is satisfactory for use in concrete. Water from lakes and streams that contain marine life also usually is suitable. When water is obtained from sources mentioned above, no sampling is necessary. When it is suspected that water may contain sewage, mine water, or wastes from industrial plants or canneries, it should not be used in concrete unless tests indicate that it is satisfactory. Water from such sources should be avoided since the quality of the water could change due to low water or by intermittent discharge of harmful wastes into the stream.

(f) Admixtures

Water-reducing and set-retarding admixtures are permitted in order to increase the workability of the concrete and to extend the time of discharge from 60 to 90 minutes. These admixtures are permitted and often required for superstructure concrete. Chemical admixtures as defined by *ASTM C 494* are as follows:

TYPE A - Water reducing

TYPE B - Retarding

TYPE C - Accelerating

TYPE D - Water reducing and retarding

TYPE E - Water reducing and accelerating

TYPE F - Water reducing, high range

TYPE G - Water reducing high range and retarding

Conplast SP430 (Fosroc), a concrete superplasticizer based on Sulphonated Naphthalene Polymer was used as a water-reducing admixture and to improve the workability of concrete containing silica fume. Conplast SP430 has been specially formulated to give high water reductions up to 25% without loss of workability or to produce high quality concrete of reduced permeability. Conplast SP430 is non-toxic.

Conplast SP430 complies with IS: 9103:1999, ASTM C – 494 Type F, BS 5057 Part-III. The dosage of superplasticizer varied between 0.5 to 2.0 liters/100 kg of cement in plain concrete, concrete incorporating silica fume. An over dose of double the recommended amount of Conplast SP430, will result in high workability and some retardation of setting time. However the ultimate compressive strength will not be impaired. Early strength is increased up to 20% if water reduction is taken advantage of. Generally, there is improvement in strength up to 20% depending upon w/c ratio and other mix parameters. Technical data of Superplasticizer are listed in Table 3.13.

Fosroc also provided the HSWRA Structuro-100 which is able to reduce the water demand up to 40% and also specified that it required half the quantity than SP-430 for same workability and same water reduction.

(g) *Microsilica (Condensed Silica Fume)*

Silica fume, also known as microsilica or condensed silica fume, is a pozzolanic admixture. In its finely divided form and in the presence of water, it will chemically react with calcium hydroxide released by the hydration of portland cement to form compounds with cementitious properties. This light to dark grey powdery product is the result of the reduction of high purity quartz with coal in an electric arc furnace in the manufacture of silicon or ferrosilicon alloys. Silica fume rises as an oxide vapor from a furnace 2000 °C (3630 °F). It cools, condenses and is collected in cloth bags. The condensed silica fume is then processed to remove impurities and control particle size. Condensed silica fume particles are one hundred times finer than cement particles. The specific gravity of silica fume varies between 2.10 and 2.25 but can be as high as 2.55. The material is sold in

powder form or as a liquid slurry. When used in concrete it will fill the void space between cement particles resulting in a more impermeable concrete. The silica fume used in this study is provided by Elkem India Pvt. Ltd. (Microsilica Grade 920-D). The properties of silica fume according to *ASTM C 1240-99* are given in Table 3.15.

3.4 CASTING OF SPECIMENS

Thirty cubes of 150mm size were cast for each of the mixes, details of which have been given in table 3.2 to 3.4, and ten each were tested at 7 days, 28 days and 56 days of curing period. For all these mix proportions, required quantities of materials were weighed. Cement; fine aggregates and coarse aggregates were mixed dry with an accuracy of 0.5 grams to get a uniform colour. The concrete mixture was prepared by hand mixing on a watertight platform. For mixes containing silica fume due care was taken to have a uniform dispersion of silica fume. It was blended with cement before mixing it with the remaining ingredients of the concrete mix. Water with super plasticizer was added at the end and mixing was done till a uniform and homogeneous mix was achieved. The mix produced was first tested for workability (Vee-Bee consistency test) and then the cubes were cast.

All the moulds were properly oiled before casting the specimens. To facilitate proper and uniform compaction, a vibration table with fixed revolutions per minute was used for the purpose. The cubes were filled in three layers. Vibrations were stopped as soon as the cement slurry appeared on the top surface of the mould. The specimens were removed from the moulds with care after 24 hours and then cured in a curing tank at the ambient temperature for specified days. The ambient temperature for curing was $27 \pm 2^{\circ}$ C.

3.5 TESTING OF SPECIMENS

The specimens, after the fixed curing period of 7 days, 28 days and 56 days were tested for compressive strength on an automatic compression testing machine (3000kN capacity) (Plates - 3.1 and 3.2) as per procedure laid down in *IS 516:1959*. A uniform pace rate of 5.25 kN/sec was maintained through out the testing of each specimen.

3.6 RESULTS OF COMPRESSIVE STRENGTH TESTS

The results of the compressive strengths of 150 mm cubes are listed in Tables 3.18 to 3.20 for each mix in the form of means, standard deviations and within-test coefficient of variations. The within-test coefficient of variation provided in the tables has been calculated using Eq.3.1 (*ACI 214 – 1988*)

$$V_{wt} = \left(\frac{S_{wt}}{\mu} \right) \times 100 \quad (3.1)$$

where V_{wt} is within-test coefficient of variation expressed as per cent, and μ is the average strength for the class of concrete in MPa and S_{wt} is the within-test standard deviation given by Eq.3.2.

$$S_{wt} = \frac{R}{d_2} \quad (3.2)$$

where R is the average range for all tests of a class of concrete and is the difference between the highest and lowest strengths of the cubes making up a test, and d_2 is a factor based on number of cubes within the test. The value of d_2 equals 3.078 if ten cubes are used in the test and it equals 3.472 for fifteen cubes and 3.735 for twenty cubes (*ASTM STP-ISD manual, 1976*). The *ACI committee report 214 (1988)* has provided control standards for evaluating performance of testing program using the within-test coefficient of variation as listed in Table 3.14.

According to the *ACI: 214 report* if within-test coefficient of variation is high (fair or poor), testing may be the reason for poor test results rather than the concrete quality. The experimental data so generated was further probabilistically analysed and has been subsequently used in the development of reliability-based procedure for design of concrete mixes. The results of the analysis have been provided in chapter 4.

Table 3.1 Range of values of various parameters

Water-cementitious ratio	0.36, 0.38 and 0.40
Cementitious content	450 kg/m ³
Water content	Depend upon w/(c+p), 162-180 kg/m ³
% replacement of cement by silica fume	0, 5 and 10 per cent
Workability	Medium (Vee-Bee time 3-6 seconds)

Table 3.2 Detail of proportions for concrete mixes without replacement of cement by silica fume

S. No.	Mix designation	w/c ratio	Mix proportions (C:F.AGG:C.AGG)	Workability and Vee-Bee time, seconds
1.	A1	0.36	1:1.043:2.681	Medium (5.1)
2.	B1	0.38	1:1.059:2.59	Medium (4.8)
3.	C1	0.40	1:1.072:2.502	Medium (4.6)

Table 3.3 Detail of proportions for concrete mixes with 5 per cent replacement of cement by silica fume

S. No.	Mix designation	w/(c+p) ratio	Mix proportions (C:S.F:F.AGG:C.AGG)	Workability and Vee-Bee time, seconds
1.	A2	0.36	1:0.05:1.098:2.882	Medium (4.9)
2.	B2	0.38	1:0.05:1.115:2.726	Medium (4.7)
3.	C2	0.40	1:0.05:1.128:2.634	Medium (4.4)

Table 3.4 Detail of proportions for concrete mixes with 10 per cent replacement of cement by silica fume

S. No.	Mix designation	w/(c+p) ratio	Mix proportions (C:S.F:F.AGG:C.AGG)	Workability and Vee-Bee time, seconds
1.	A3	0.36	1:0.10:1.159:2.979	Medium (4.9)

2.	B3	0.38	1:0.10:1.177:2.878	Medium (4.8)
3.	C3	0.40	1:0.10:1.191:2.780	Medium (4.5)

Table 3.5 Sieve analysis of fine aggregate (riverbed sand)

Weight of sample taken = 1.0 kg

S. No.	IS sieve size	Weight retained, grams	Percentage retained	Percentage passing	Cumulative percentage retained
1.	4.75 mm	95.0	9.5	90.5	9.5
2.	2.36 mm	42.5	4.25	86.25	13.75
3.	1.18 mm	110.5	11.05	75.2	24.8
4.	600 μ m	128.5	12.85	62.35	37.65
5.	300 μ m	308.0	30.8	31.55	68.45
6.	150 μ m	281.0	28.1	3.45	96.55
7.	Pan	34.5	3.45	----	----
				$\Sigma F =$	250.7

Fineness Modulus of fine aggregate = $\Sigma F/100 = 250.7/100 = 2.507$

Table 3.6 Properties of fine aggregates

S. No.	Property	Observed value
1.	Type	Uncrushed (natural)
2.	Specific gravity (oven-dry basis)	2.68
3.	Total water absorption	1.02 per cent
4.	Moisture content	0.16 per cent
5.	Net water absorption	0.86 percent
6.	Unit mass (compact)	1.63 kg/m ³
7.	Unit mass (loose)	1.48 kg/m ³
8.	Percentage voids (compact)	20.72 per cent
9.	Percentage voids (loose)	44.65 per cent
10.	Fineness modulus	2.507
11.	Grading zone	III

Table 3.7 Sieve analysis of CA – I (20 mm) aggregates

Weight of sample taken = 3.0 kg

S. No.	Sieve No.	Weight retained	Percentage retained	Percentage passing	Cumulative percentage retained
1.	80 mm	----	0.00	100	0.00
2.	40 mm	----	0.00	100	0.00
3.	20 mm	0	0.00	100	0.00
4.	12.5 mm	2186.5	72.883	27.117	72.883
5.	10 mm	674.5	22.483	4.634	95.366
6.	4.75 mm	139.0	4.633	0.01	99.999
11.	Pan	0	0.00	----	----
				$\Sigma C =$	268.244

Fineness Modulus of Coarse aggregate (20mm) = $\Sigma C + 500/100 = 268.244 + 500/100 = 7.68$

Table 3.8 Properties of Coarse aggregates CA – I (20mm)

S. No.	Property	Observed value
1.	Type	Crushed
2.	Specific gravity	
	a) Saturated surface dry	2.825
	b) Oven-dry	2.655
3.	Total water absorption	3.645 per cent
4.	Moisture content	0.7049 per cent
5.	Net water absorption	2.94 per cent
6.	Fineness modulus	7.68

Table 3.9 Sieve analysis of CA – II (10 mm) aggregates

Weight of sample taken = 3.0 kg

S. No.	IS sieve size	Weight retained, grams	Percentage retained	Percentage passing	Cumulative percentage retained
1.	80 mm	----	0.00	100	0.00
2.	40 mm	----	0.00	100	0.00
3.	20 mm	----	0.00	100	0.00
4.	12.5 mm	555.0	18.50	81.50	18.50
5.	10 mm	890.5	29.68	51.82	48.18
6.	4.75 mm	957.5	31.92	19.9	80.1
7.	Pan	597.0	19.90	----	----
				ΣC =	146.78

Fineness Modulus of Coarse aggregate (10 mm) = $\Sigma C + 500 / 100 = 146.78 + 500 / 100 = 6.47$

Table 3.10 Properties of Coarse aggregates CA – II (10mm)

S. No.	Property	Observed value
1.	Type	Crushed
2.	Specific gravity	
	c) Saturated surface dry	2.704
	d) Oven-dry	2.635
3.	Total water absorption	1.6432 per cent
4.	Moisture content	0.806 per cent
5.	Net water absorption	0.8372 per cent
6.	Fineness modulus	6.47

Table 3.11 Physical properties of 43 grade (PPC) cement

S. No.	Property	Observed values	Value as per IS: 8112-1989
1.	Normal consistency	34 per cent	----
2.	Setting time (Vicat's apparatus)		
	a) Initial setting time	48 minutes	Not less than 30 minutes
	b) Final setting time	240 minutes	Not more than 600 minutes
3.	Fineness (% retained on 90 μ m)	3.5 per cent	Not more than 10 per cent
4.	Specific gravity (sp. gr. bottle)	3.07	----

Table 3.12 Detail of aggregates proportion

Zone	Percentage passing 20mm sieve and retained on 10mm sieve	Percentage passing 10mm sieve and retained on 4.75mm sieve	Percentage passing 4.75mm sieve and retained on 2.36mm sieve	Fineness modulus	Water content, kg/m ³
A	67	33	----	6.86	162 - 180

Table 3.13 Technical data of Superplasticizer (Conplast SP – 430)

S. No	Property	Value
1.	Colour	Dark Brown liquid
2.	Specific gravity @ 30° C	1.220 to 1.225
3.	Air entrainment	Maximum 1 per cent
4.	Chloride content	Nil to IS : 456

Table 3.14 Control standards for evaluating performance of testing program using the with-in-test coefficient of variation (*ACI report, 1988*)

Field control testing				
Control standards for with-in-test coefficient of variation				
<i>Excellent</i>	<i>Very good</i>	<i>Good</i>	<i>Fair</i>	<i>Average</i>
Below 1.5	1.5 to 2.0	2.0 to 3.0	3.0 to 4.0	Above 4.0

Table 3.15 Physical properties of silica fume (Elkem Microsilica Grade 920-D)

S. No.	Parameter	Specification	Analyzed Value
1.	Silicon dioxide (SiO ₂)	Min. 85.0 per cent	90.3
2.	Moisture content	Max. 3.0 per cent	0.6
3.	Los of ignition @ 975C	Max 6.0 per cent	2.1
4.	Carbon	Max. 2.5 per cent	0.8
5.	>45 Micron	Max. 10 per cent	0.4
6.	Bulk Density	500-700 kg/m ³	640

Table 3.16 Compressive strength data for concrete mixes without replacement of cement by silica fume

w/c ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV
0.36	34.95	1.38	2.72	45.68	1.12	1.67	50.56	1.52	1.96
0.38	32.85	1.58	3.33	41.63	1.42	2.22	46.41	1.37	1.93
0.40	29.61	1.55	3.65	37.92	1.36	2.25	43.93	1.12	1.66

Table 3.17 Compressive strength data for concrete mixes with 5% replacement of cement by silica fume

w/(c+p) ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV
0.36	34.08	1.44	2.92	52.71	1.26	1.81	62.48	1.32	1.63
0.38	31.87	1.41	3.07	48.82	1.19	1.79	56.90	1.50	1.99
0.40	31.46	1.44	3.18	43.83	1.24	1.96	52.62	1.47	2.02

Table 3.18 Compressive strength data for concrete mixes with 10% replacement of cement by silica fume

w/(c+p) ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV	Mean, MPa	S.D., MPa	With- in-test COV
0.36	32.87	1.43	3.01	60.33	1.07	1.48	69.52	1.12	1.32
0.38	30.70	1.04	2.35	55.70	1.73	2.50	63.61	1.26	1.60
0.40	30.02	1.35	3.13	50.17	1.40	2.03	59.76	1.41	1.83

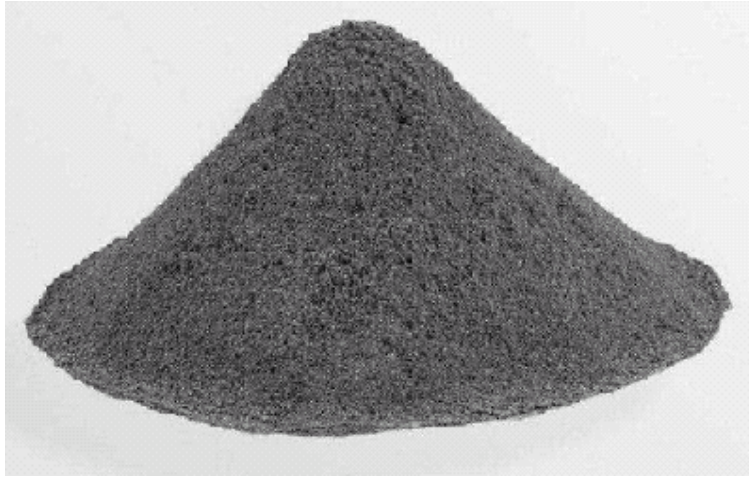


Fig. 3.1 Silica fume powder (*U.S. Deptt. of Transportation, 2005*)

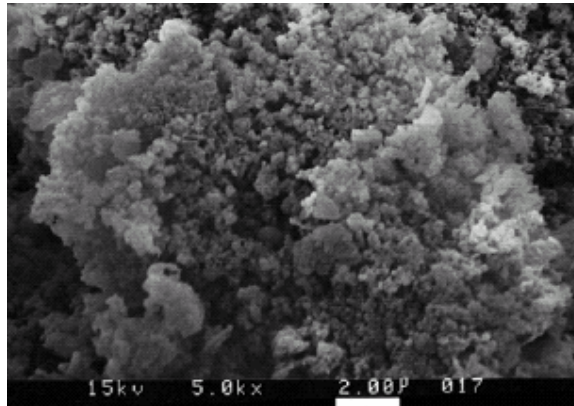


Fig. 3.2 Scanning electron microscope micrograph of silica fume particles at 20,000X (*U.S. Deptt. of Transportation, 2005*)



Plate 3.1 Photograph of Automatic Compression Testing Machine (ACTM)



Plate 3.2 Photograph of cube placed on anvil of ACTM for compressive strength test

MATHEMATICAL MODEL FOR DEVELOPMENT OF RELIABILITY - BASED DESIGN CRITERIA

4.1 GENERAL

It is generally recognized that uncertainty exists in the results of design formulae for evaluating the resistance of structural elements. One of the factors contributing to this uncertainty is the variability of the strengths of the constituent materials. The other contributing factors are the variations in loads coming on the structural elements and cross-sectional dimensions. The mathematical model used in the succeeding section, used for the development of reliability-based design procedure for concrete mixes, takes into account the variations in the water-cementitious ratio and percentage replacement of cement by silica fume.

In general the development of a reliability-based design involves the following steps:

- a) Collection and statistical analysis of the data on basic variables.
- b) Defining the probability distributions of each variable – at least in terms of mean, standard deviation and conforming probability distribution.
- c) Study of the resistance of elements and establishing their statistical characteristics.
- d) Selection of the target reliability index, β , i.e. accepted or specified level of reliability.
- e) Determination of the partial safety factors for the desired uniform reliability index, β , under all design situations within the scope of work.

The present study aims at developing a reliability-based procedure for design of concrete mixes of desired strength at a desired curing age for a particular reliability index. Once a concrete mix of desired reliability is available, the procedure for design of structural elements viz, beams, slabs etc. with a predetermined confidence level can be developed.

An extensive data bank on the basic variables viz, compressive strength of concrete in terms of mean, standard deviation and within-test coefficient of variation corresponding to variations in water-cementitious material ratio, 5 and 10 per cent replacement of cement by silica fume, curing age etc. have been generated experimentally. These are given in Tables 3.15 to 3.18 of chapter 3. This experimental

data has been used for the development of reliability-based procedure for design of concrete mixes.

4.2 STATISTICAL STUDY OF COMPRESSIVE STRENGTH

4.2.1 Basic Variables and Failure Surface

In a structural design problem, several parameters viz, geometric properties of the section, physical properties of the materials and loads coming on the structure are subjected to random variations. If the coefficient of variation of a random variable is very small, the variations may be ignored and the parameter may be considered as deterministic. Thus the parameters in any design problem, which are to be considered as random variables, are initially fixed, and those are called basic variables. An equation that is developed for a particular failure condition shall be a function of these basic variables say $X_1, X_2, X_3, \dots, \dots, X_n$.

Consider a failure function $g(X_1, X_2, X_3, \dots, \dots, X_n)$. The margin of safety, M, for this failure function can be represented by

$$M = R - S \quad (4.1)$$

where R and S represent the internal resistance and external action respectively, which are also functions of the basic variables $X_1, X_2, X_3, \dots, \dots, X_n$.

Hence,

$$M = g(X_1, X_2, X_3, \dots, \dots, X_n) \quad (4.2)$$

when this failure function is reduced to zero, i.e. $g(X_1, X_2, X_3, \dots, \dots, X_n)=0$, it is called a failure surface. A degree of safety is insured by specifying a small value for the probability of reaching a particular failure state. The magnitude assigned depends on the serviceability consequences of reaching the particular failure state. If $f_x(x)$ is the probability density function of the jointly distributed basic variables $X_1, X_2, X_3, \dots, \dots, X_n$, then the probability of failure is

$$p_f = \iiint_{g < 0} \dots \int f_x(x) dx \quad (4.3)$$

where

$$X = (X_1, X_2, \dots, \dots, X_n)$$

$$x = (x_1, x_2, \dots, \dots, x_n)$$

$$dx = (dx_1, dx_2, \dots, \dots, dx_n)$$

The multiple integral is to be evaluated over the region $g < 0$. The failure surface equation divides the space into two regions, viz. (i) safe and (ii) unsafe or failure region.

In the present study the effect of variations in water-cementitious material ratio, 5 and 10 per cent replacement of cement by silica fume and curing age etc. have been studied on the compressive strength of concrete. As the compressive strength of concrete (f_k) is directly related to the water-cementitious material ratio $\left(\frac{w}{c+p}\right)$, the failure function can be written as $g\left(f_k, \frac{w}{c+p}\right)$ and is shown in Fig. 4.1.

It is convenient to have a mathematical function (probability distribution function or PDF) to describe the random variables viz, compressive strength of concrete and water-cementitious material ratio. The choice of a distribution function has been based upon the accuracy and ease with which it can represent a given set of empirical data.

The compressive strength data generated experimentally has been analysed using normal-probability distribution function. The normal distribution is the most extensively used, as many natural phenomenon seem too governed by this distribution. These probability distribution function as used in the present study are briefly explained below.

(a) Normal distribution

If a random phenomenon (variable) arises because of several factors, which act in an additive manner to result in the phenomenon, then the model arising out of such a situation will be a normal distribution (*Ranganathan, 1990*). This distribution is known as Gaussian distribution. The PDF of normal variate is given by

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \quad (4.4)$$

where the parameter μ represents the mean and σ represents the standard of the distribution respectively. The normal distribution can designate as $N(\mu, \sigma)$. A normal distribution with parameters $\mu = 0$ and $\sigma = 1$ is called a standard normal distribution and is designated as $N(0,1)$. Thus, the PDF of standard normal variate is given by

$$f_U(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right) \quad (4.5)$$

The cumulative probability of standard normal variate, that is, $\phi(u) = F_U(u) = P(U \leq u)$ is shown along with the PDF in the Fig. 4.2 (Ranganathan, 1990).

From the Fig. 4.2 cumulative probability can be expressed as

$$\phi(u_1) = p_1 \quad (4.6)$$

Conversely, the value of u_1 at a cumulative of p_1 is given by

$$u_1 = \phi^{-1}(p_1) \quad (4.7)$$

The values of standard normal variate have been tabulated in Appendix – I.

4.2.2 Reliability Index

If the failure function is represented by Eq. (4.1) then the basic variables can be normalised using the relationship

$$Z_i = \frac{X_i - \mu_i}{\sigma_i} \quad i = 1, 2, \dots, n \quad (4.11)$$

In the z coordinate system, the failure surface is a function of z_i . Equating the failure function to zero, the failure surface equation is obtained which can be written in the normalised coordinate system. This failure surface divides the design sample space into two regions namely, safe and unsafe or failure regions.

Because of the normalisation of the basic variable, $\mu_{z_i} = 0$ and $\sigma_{z_i} = 1$ the z coordinate system has a rotational symmetry with respect to the standard deviation and the origin O will usually lie in the safe region. From the 2-D representation shown in Fig. 4.3 it is evident that as the failure surface $g(z_1, z_2)$ moves away from the origin, the reliability, $g(Z) > 0$, increases and as it moves closer to the origin, reliability decreases. Hence the position of failure surface with respect to the origin in normalized coordinate system determines the magnitude of reliability.

Hasofer and Lind (1974) defined the reliability index β as the shortest distance from the origin to failure surface in the normalized coordinate system. The point D on the failure surface in the Fig. 4.3 is called the design point. This point is called the checkpoint for the safety of structure. The reliability index β is related to the failure surface and not to the failure functions. The reliability index, $\beta = \frac{\mu_M}{\sigma_M}$ defined by *Cornell (1969)*, will coincide with the value obtained by *Hasofer and Lind* when the failure surface is a linear

function of basic variables. Thus for the Hasofer-Lind method the relation between reliability index and probability of failure as described by Eq. (4.12) can be used provided the failure function is a linear function of the normally distributed basic variables

$$p_f = 1 \times 10^{-\beta} \quad (4.12)$$

Thus, the reliability index β defined by *Cornell (1969)* can be obtained for a non linear function by expanding the function about the design point D . This means that the non linear surface is approximated by its tangent plane at the design point D . For a non-linear surface, the shortest distance of the origin to the failure surface is not unique as in case of a linear failure surface. *Shinozuka (1983)* has proved that the point D on the failure surface with minimum distance to the origin (normalized coordinate system) is the most probable failure point. Thus, the tangent plane at the design point D may be taken to approximate the value of β . If the failure is concave towards the origin, the approximation will be on the safer side, while the surface convex towards the origin it will be on unsafe side.

4.2.3 Mathematical Model

The checking of reliability of structure elements involved the evaluation of reliability index β or in other words evaluating the safety corresponding to a particular reliability index. The reliability-based design procedure is an inverse problem, wherein a design has to be achieved which shall ensure a certain level of reliability. A design procedure has been developed using the first order second moment approach using *Hasofer Lind's* method. The procedure involved the calculation of design parameters for the given target β . Different levels of reliability yield different surfaces, amounting to different designs. Hence, in the proposal reliability-based design, the design values of the variables, which will ensure that the designed have a failure surface that complies with a required reliability index β , are determined.

The limit state design methodology, stipulated in *IS: 456 – 2000*, considers a partial safety factor of 1.5 for concrete and 1.15 for steel, when assessing the strength of a structure element. These partial safety factors account for any possibility of variation in the strength of material, deviation of sectional dimensions, lack of accuracy of the calculated procedure, and risk of life and economical consequences. In the present study, independent partial safety factors for variation in strength of concrete have been determined. A step-by-step procedure for developing curves for concrete mix design and

for computing the partial safety factors taking in to account the concrete strength variation has described below.

- i. The relationship between water-cementitious material ratio and experimentally generated compressive strength (in terms of means and standard deviations) obtained for three curing ages of 7, 28 and 56 days have been shown in Figs. 4.4 to 4.6. The mathematical expressions for different cases are given below:

For concrete mixes without replacement of cement by silica fume, the relationship is presented in Fig. 4.4 and the regression equations at different curing ages are:

7 days curing	28 days curing	56 days curing
$f_k = 7.250 \left(\frac{w}{c} \right)^{-1.546}$	$f_k = 7.544 \left(\frac{w}{c} \right)^{-1.763}$	$f_k = 12.70 \left(\frac{w}{c} \right)^{-1.348}$
$R^2 = 0.9742$	$R^2 = 0.9997$	$R^2 = 0.9882$

For concrete mixes with 5 per cent replacement of cement by silica fume, the relationship presented in Fig. 4.5 and the regression equations at different curing ages are:

7 days curing	28 days curing	56 days curing
$f_k = 8.893 \left(\frac{w}{c} \right)^{-1.356}$	$f_k = 9.071 \left(\frac{w}{c} \right)^{-1.727}$	$f_k = 11.72 \left(\frac{w}{c} \right)^{-1.637}$
$R^2 = 0.8428$	$R^2 = 0.9873$	$R^2 = 0.9987$

For concrete mixes with 10 per cent replacement of cement by silica fume, the relationship presented in Fig. 4.6 and the regression equations at different curing ages are:

7 days curing	28 days curing	56 days curing
$f_k = 13.33 \left(\frac{w}{c} \right)^{-0.8769}$	$f_k = 10.34 \left(\frac{w}{c} \right)^{-1.731}$	$f_k = 15.78 \left(\frac{w}{c} \right)^{-1.448}$
$R^2 = 0.9361$	$R^2 = 0.9915$	$R^2 = 0.9929$

The relation between compressive strength and water-cementitious material ratio can be expressed in the form:

$$f_k = \left(\frac{1}{A} \right) \left(\frac{w}{c} \right)^B \quad \text{or} \quad f_k = \left(\frac{1}{A} \right) \left(\frac{w}{c+p} \right)^B \quad (4.13)$$

where A and B are the constants, w is the water content, c is the cement content and p is the amount of silica fume.

- ii. Using the with-in-test coefficient of variation for compressive strength as provided in the Table 3.18 to 3.20, the revised coefficients of variation and standard deviation have been calculated using the equation

$$\delta_{CSrev} = \left[(\delta_{CS})^2 + (\delta_{CST})^2 \right]^{1/2} \quad (4.14)$$

$$\sigma_{CSrev} = \mu_{CS} (\delta_{CSrev}) \quad (4.15)$$

where δ_{CSrev} and σ_{CSrev} are the revised values of coefficient of variation and standard deviation respectively; where δ_{CS} and μ_{CS} are the experimentally obtained values of coefficient of variation and means of compressive strengths, respectively, and δ_{CST} is the with-in-test coefficient of variation.

Consider the coefficient of variation of water-cementitious ratio at site to be 5 per cent. Thus,

$$\delta_{wcr} = 0.05 \quad (4.16)$$

The standard deviation values for water-cementitious ratio (σ_{wcr}) have been obtained by multiplying the actual values by coefficient of variation.

The values of coefficients A and B along with the means, revised coefficients of variation and standard deviations of compressive strengths for different water-cementitious material ratios and curing ages have been tabulated in the Tables from Table 4.1 to 4.3.

- iii. The governing equation of the design values has been formulated in the form (Kumar, 2002)

$$g(Z) = 0 \quad \text{i.e.} \\ \left(\mu_{CSrev} + \sigma_{CSrev} z_{cs} \right) - \left(\frac{1}{A} \right) \left(\mu_{wcr} + \sigma_{wcr} z_{wcr} \right)^B = 0 \quad (4.17)$$

where μ_{CSrev} and σ_{CSrev} are the revised means and standard deviations of compressive strength and μ_{wcr} and σ_{wcr} are the revised means and standard deviations of water-cementitious ratio. The values of α_1 and α_2 have been computed using *Hasofer Lind's* method for different cases using normal distribution function.

(a) Normal Distribution Function

The values of α_1 and α_2 have been evaluated using the equations

$$\alpha_1 = \left(\frac{-1}{K} \right) (\sigma_{CSrev} A) \quad (4.18)$$

$$\alpha_2 = \left(\frac{-1}{K} \right) (-\sigma_{wcr} B) \quad (4.19)$$

$$\alpha_1^2 + \alpha_2^2 = 1 \quad (4.20)$$

Using these values of α_1 and α_2 , the design of compressive strength and water-cementitious material ratio for varying reliability indices ($\beta = 1.0, 1.3, 2.0, 3.0$) have been obtained by Eqns. (4.21) and (4.22) as below:

$$CS^* = \mu_{CS} - \alpha_1 \beta \sigma_{CSrev} \quad (4.21)$$

$$wcr^* = \mu_{wcr} - \alpha_2 \beta \sigma_{wcr} \quad (4.22)$$

The characteristic values corresponding to 95 per cent confidence level for compressive strength have been found by using the equations as below:

$$CS^* = \mu_{CS} - 1.64 \sigma_{CSrev} \quad (4.23)$$

$$wcr^* = \mu_{wcr} - 1.64 \sigma_{wcr} \quad (4.24)$$

The design and characteristic values at different curing ages and varying reliability indices using normal probability distribution function without and with 5 and 10 per cent replacement of cement by silica fume have been provided in Tables 4.4 to 4.6.

The relationship of compressive strength with water-cementitious material ratio has been represented in the form of design curves in Figs 4.7 to 4.15. The design equations for various cases with varying reliability indices (β) can be written as below

$$f_k = A \left(\frac{w}{c} \right)^{-B} \quad \text{or} \quad f_k = A \left(\frac{w}{c + p} \right)^{-B} \quad (4.34)$$

where f_k is the design compressive strength of concrete.

The values of coefficients A and B in Eq. (4.34) have been presented in Tables 4.7 to 4.9, along with their respective regression coefficients (R^2).

4.2.4 Evaluation of Partial Safety Factors

The probabilistic procedure for design of reinforced concrete rectangular beams is based on philosophy of partial safety factors for variations in concrete strength. Partial safety factor is defined with respect to a particular value of the variable. It is defined with respect to the mean value as ratio of the design value to the mean value as given below:

$$\gamma_{ci} = \frac{x_i^*}{\mu_i} \quad (4.35)$$

where x_i^* is the design value of the variable and μ_i is the mean value. This is called central safety factor.

In the present study partial safety factor with respect to the characteristic value or the characteristic strength of concrete (the value of the strength of 150mm cubes, after 28 days of curing below which not more than 5 per cent of the test results are expected to fall) is defined as:

$$\gamma_{ci} = \frac{x_i^*}{x_{ki}} \quad (4.36)$$

where x_{ki} is the characteristic value of compressive strength.

From the design and characteristic values of compressive strength evolved in the preceding section the partial safety factors (PSF) with respect to the characteristic values for different curing ages and different replacement ratios of cement by silica fume and reliability indices have been evaluated using Eq. (4.34) and Tables 4.7 to 4.9.

The relationship between the partial safety factor and design compressive strength has been presented in Figs. 4.16 to 4.24. The values of partial safety factors for various concrete strengths, with and without replacement of cement by silica fume, for varying reliability indices, using normal probability distribution has been provided in Tables 4.13 to 4.15. The design equations for various cases with varying reliability indices can be written as below

$$\gamma_{ci} = A(f_k)^B \quad (4.37)$$

The values of A and B in eq. (4.37) have been presented in Tables 4.10 to 4.12.

Table 4.1 (a) Parameters for concrete mixes without silica fume

Curing Period	7 Days	28 Days	56 Days
Parameters	A = 0.138 B = -1.546	A = 0.133 B = -1.763	A = 0.079 B = -1.348

Table 4.1 (b) Revised compressive strength parameters for concrete mixes without silica fume

w/c ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa
0.36	34.95	4.79	1.68	45.68	2.97	1.36	50.56	3.59	1.81
0.38	32.85	5.85	1.92	41.63	4.07	1.69	46.41	3.53	1.64
0.40	29.61	6.38	1.89	37.92	4.23	1.61	43.93	3.04	1.34

Table 4.2 (a) Parameters for concrete mixes with 5% silica fume

Curing Period	7 Days	28 Days	56 Days
Parameters	A = 0.112 B = -1.356	A = 0.110 B = -1.727	A = 0.085 B = -1.637

Table 4.2 (b) Revised compressive strength parameters for concrete mixes with 5% silica fume

w/(c+p) ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa
0.36	34.08	5.14	1.75	52.71	3.00	1.58	62.48	2.67	1.67
0.38	31.87	5.39	1.72	48.82	3.02	1.48	56.90	3.30	1.88
0.40	31.46	5.57	1.75	43.83	3.44	1.51	52.62	3.45	1.81

Table 4.3 (a) Parameters for concrete mixes with 10% silica fume

Curing Period	7 Days	28 Days	56 Days
Parameters	A = 0.075 B = -0.8769	A = 0.097 B = -1.731	A = 0.063 B = -1.448

Table 4.3 (b) Revised compressive strength parameters for concrete mixes with 10% silica fume

w/(c+p) ratio	7 days curing			28 days curing			56 days curing		
	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa	Mean, MPa	Rev. COV, per cent	Rev. SD, MPa
0.36	32.87	5.29	1.74	60.33	2.31	1.39	69.52	2.08	1.45
0.38	30.70	4.12	1.27	55.70	3.99	2.22	63.61	2.55	1.62
0.40	30.02	5.48	1.64	50.17	3.45	1.73	59.76	2.99	1.78

Table 4.4 (a) Design values of 7-days compressive strength and water-cement ratio without silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>
33.28	0.356	32.77	0.355	31.60	0.352	29.93	0.348	32.20	0.353
30.93	0.376	30.35	0.375	29.01	0.372	27.09	0.369	29.70	0.374
27.72	0.395	27.16	0.394	25.83	0.391	23.95	0.386	26.51	0.392

Table 4.4 (b) Design values of 28-days compressive strength and water-cement ratio without silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>
44.33	0.357	43.92	0.356	42.97	0.354	41.62	0.351	43.46	0.355
39.94	0.376	39.43	0.375	38.24	0.373	36.55	0.369	38.85	0.374
36.32	0.396	35.83	0.395	34.71	0.392	33.11	0.388	35.29	0.393

Table 4.4 (c) Design values of 56-days compressive strength and water-cement ratio without silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>	<i>f_k</i>	<i>w/c</i>
48.75	0.358	48.20	0.357	46.93	0.356	45.12	0.354	47.58	0.357
44.77	0.378	44.28	0.377	43.14	0.375	41.50	0.373	43.73	0.376
42.59	0.397	42.19	0.396	41.26	0.394	39.92	0.391	41.74	0.395

Table 4.5 (a) Design values of 7-days compressive strength and water-cementitious ratio with 5% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>
32.33	0.357	31.81	0.356	30.58	0.354	28.83	0.351	31.21	0.355
30.16	0.377	29.64	0.376	28.44	0.374	26.73	0.370	29.06	0.375
29.71	0.396	29.18	0.395	27.96	0.393	26.21	0.389	28.58	0.394

Table 4.5 (b) Design values of 28-days compressive strength and water-cementitious ratio with 5% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>
51.13	0.356	50.66	0.355	49.55	0.352	47.97	0.348	50.12	0.353
47.35	0.375	46.90	0.374	45.87	0.371	44.40	0.366	46.40	0.372
42.32	0.395	41.87	0.393	40.82	0.390	39.31	0.384	41.36	0.391

Table 4.5 (c) Design values of 56-days compressive strength and water-cementitious ratio with 5% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>	<i>f_k</i>	<i>$w/(c+p)$</i>
60.81	0.357	60.31	0.356	59.15	0.354	57.48	0.351	59.75	0.355
55.02	0.377	54.46	0.376	53.14	0.374	51.27	0.371	53.82	0.375
50.81	0.397	50.26	0.396	49.00	0.394	47.18	0.390	49.64	0.395

Table 4.6 (a) Design values of 7-days compressive strength and water-cementitious ratio with 10% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>
31.13	0.359	30.61	0.358	29.39	0.357	27.65	0.356	30.02	0.358
29.44	0.378	29.06	0.378	28.17	0.376	26.91	0.374	28.62	0.377
28.38	0.398	27.88	0.398	26.73	0.397	25.09	0.395	27.32	0.397

Table 4.6 (b) Design values of 28-days compressive strength and water-cementitious ratio with 10% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>
58.94	0.356	58.52	0.354	57.55	0.351	56.15	0.347	58.04	0.353
53.48	0.377	52.82	0.376	51.26	0.374	49.04	0.371	52.06	0.375
48.44	0.396	47.92	0.395	46.71	0.392	44.98	0.388	47.33	0.393

Table 4.6 (c) Design values of 56-days compressive strength and water-cementitious ratio with 10% silica fume using normal distribution

<i>Reliability Index, β</i>									
<i>1.0</i>		<i>1.3</i>		<i>2.0</i>		<i>3.0</i>		<i>Ch. Value</i>	
<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>	<i>f_k</i>	<i>$w/c+p$</i>
68.07	0.358	67.64	0.357	66.63	0.356	65.18	0.354	67.15	0.356
61.99	0.378	61.51	0.377	60.37	0.376	58.76	0.373	60.95	0.376
57.98	0.398	57.44	0.397	56.19	0.396	54.41	0.393	56.83	0.396

Table 4.7 Design parameters for concrete mixes without silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
5.64	-1.73	0.979	5.24	-1.78	0.979	4.37	-1.90	0.982	3.28	-2.10	0.984	4.806	-1.83	0.981

Curing period = 28 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
6.095	-1.93	0.99	5.70	-1.98	0.99	4.84	-2.10	0.998	3.75	-2.29	0.996	5.27	-2.035	0.99

Curing period = 56 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
12.58	-1.32	0.984	12.50	-1.31	0.98	12.34	-1.29	0.98	12.16	-1.26	0.97	12.42	-1.30	0.98

Table 4.8 Design parameters for concrete mixes with 5% silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
13.85	-0.82	0.89	13.36	-0.83	0.89	12.25	-0.87	0.89	10.7	-0.94	0.91	12.23	-0.84	0.89

Curing period = 28 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
7.87	-1.82	0.986	7.57	-1.84	0.985	6.898	-1.89	0.984	5.99	-1.95	0.982	6.73	-1.82	0.984

Curing period = 56 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
10.45	-1.71	0.99	10.07	-1.73	0.99	9.21	-1.79	0.99	8.04	-1.88	0.99	8.49	-1.76	0.99

Table 4.9 Design parameters for concrete mixes with 10% silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
12.53	-0.885	0.98	12.23	-0.89	0.99	11.52	-0.91	0.99	10.48	-0.95	0.95	11.17	-0.89	0.99

Curing period = 28 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
8.99	-1.82	0.997	8.65	-1.85	0.998	7.87	-1.903	0.999	6.83	-1.99	1.0	6.786	-1.94	1.0

Curing period = 56 days

$\beta = 1.0$			$\beta = 1.3$			$\beta = 2.0$			$\beta = 3.0$			Ch. Value		
A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²	A	B	R ²
14.25	-1.52	0.994	13.76	-1.54	0.99	12.66	-1.60	0.99	11.19	-1.69	0.99	11.58	-1.58	0.99

Table 4.10 Partial safety factor coefficients for concrete without silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.28	-0.061	1.14	0.033	0.872	0.034	0.605	0.126

Curing period = 28 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.213	-0.045	1.107	-0.024	0.901	0.025	0.679	0.092

Curing period = 56 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
0.927	0.026	0.96	0.014	1.046	-0.015	1.198	-0.062

Table 4.11 Partial safety factor coefficients for concrete with 5% silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.169	-0.035	1.087	-0.019	0.917	0.020	0.719	0.075

Curing period = 28 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.09	-0.017	1.047	-0.0091	0.9524	0.0097	0.8303	0.037

Curing period = 56 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.16	-0.037	1.08	-0.016	0.924	0.017	0.743	0.064

Table 4.12 Partial safety factor coefficients for concrete with 10% silica fume (Normal distribution)

Curing period = 7 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.04	-0.0014	1.032	-0.004	0.939	0.013	0.681	0.094

Curing period = 28 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.199	-0.04	1.102	-0.0022	0.901	0.023	0.67	0.0904

Curing period = 56 days

$\beta = 1.0$		$\beta = 1.3$		$\beta = 2.0$		$\beta = 3.0$	
A	B	A	B	A	B	A	B
1.192	-0.038	1.10	-0.0202	0.910	0.0207	0.707	0.0761

Table 4.13 Partial safety factor (PSF) for concrete with 0% silica fume (Normal Distribution)

w/c ratio	After 7 days				After 28 days				After 56 days			
	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$
0.36	1.033	1.018	0.981	0.929	1.020	1.011	0.989	0.958	1.024	1.013	0.986	0.948
0.38	1.041	1.022	0.977	0.912	1.028	1.015	0.984	0.941	1.024	1.013	0.987	0.949
0.40	1.046	1.024	0.974	0.903	1.029	1.016	0.984	0.938	1.021	1.011	0.989	0.957

Table 4.14 Partial safety factor (PSF) for concrete with 5% silica fume (Normal Distribution)

w/c+p ratio	After 7 days				After 28 days				After 56 days			
	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$
0.36	1.036	1.019	0.980	0.924	1.020	1.011	0.989	0.957	1.018	1.010	0.990	0.962
0.38	1.038	1.020	0.979	0.920	1.020	1.011	0.989	0.957	1.022	1.012	0.987	0.953
0.40	1.039	1.021	0.978	0.917	1.023	1.012	0.987	0.951	1.023	1.012	0.987	0.950

Table 4.15 Partial safety factor (PSF) for concrete with 10% silica fume (Normal Distribution)

w/c+p ratio	After 7 days				After 28 days				After 56 days			
	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$	$\beta=1$	$\beta=1.3$	$\beta=2$	$\beta=3$
0.36	1.037	1.020	0.979	0.921	1.015	1.008	0.991	0.967	1.014	1.007	0.992	0.971
0.38	1.028	1.015	0.984	0.940	1.027	1.015	0.985	0.942	1.017	1.009	0.990	0.964
0.40	1.039	1.020	0.978	0.918	1.023	1.012	0.987	0.950	1.020	1.011	0.989	0.957

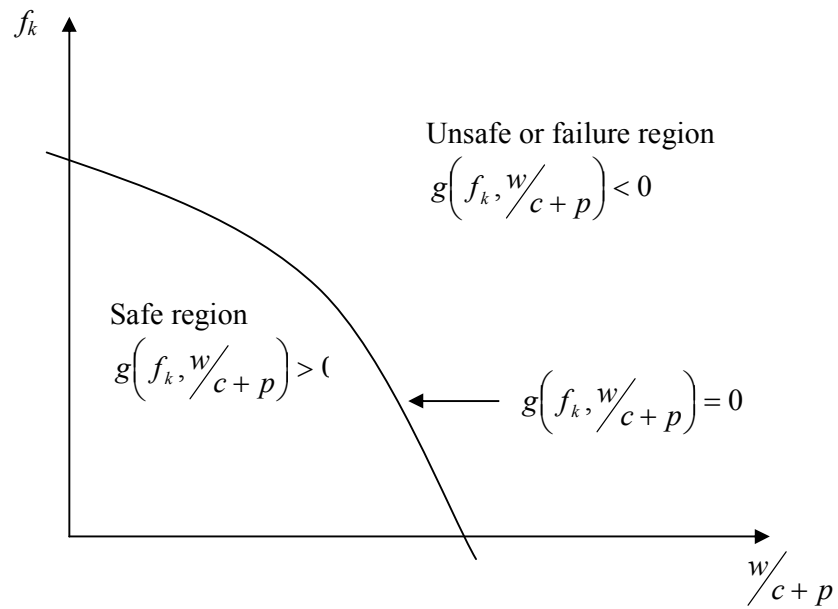


Fig. 4.1 Failure surface, failure and safe regions (Ranganathan, 1990)

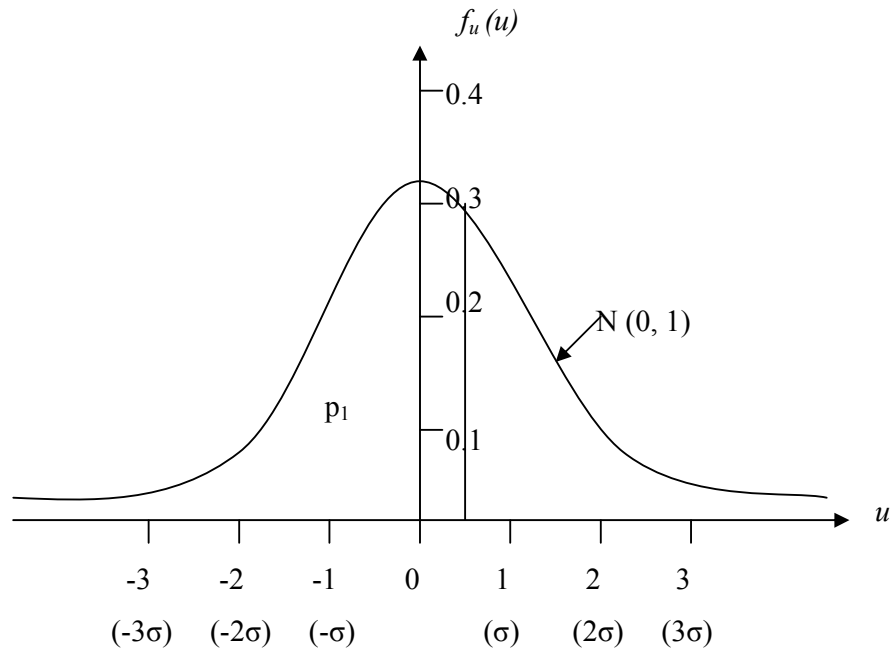


Fig. 4.2 Standard normal density function (Ranganathan, 1990)

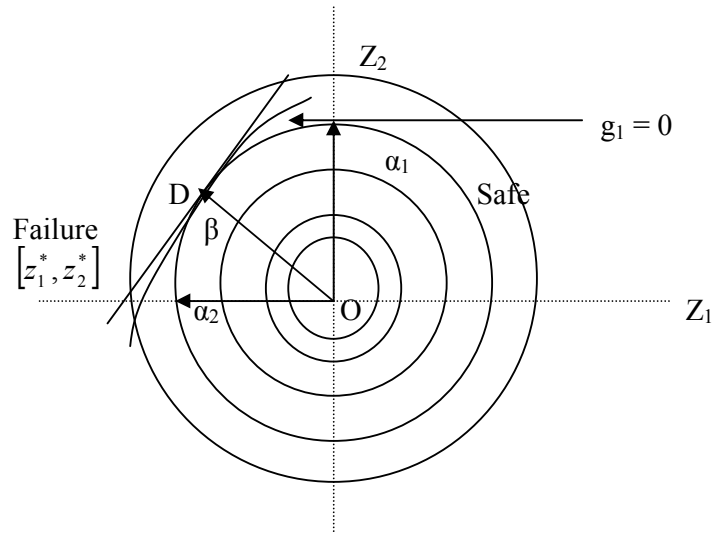


Fig. 4.3 Formulation of safety analysis in normalized coordinates (*Ranganathan, 1990*)

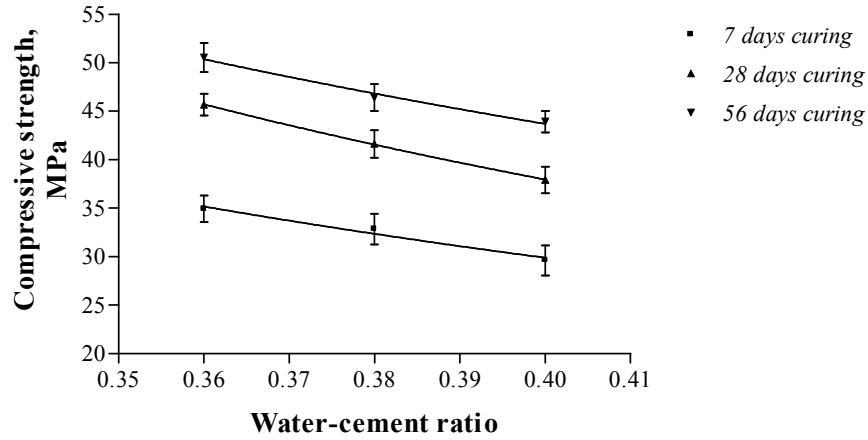


Fig. 4.4 Normalized compressive strength curves for 0% silica fume (Normal distribution)

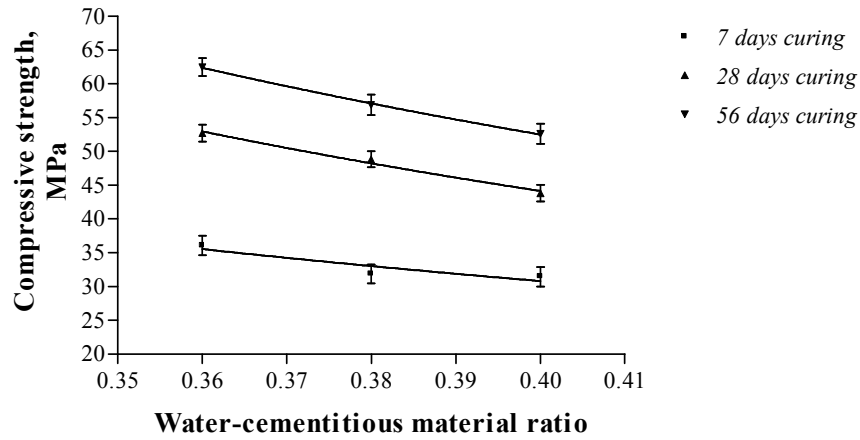


Fig. 4.5 Normalized compressive strength curves for 5% silica fume (Normal distribution)

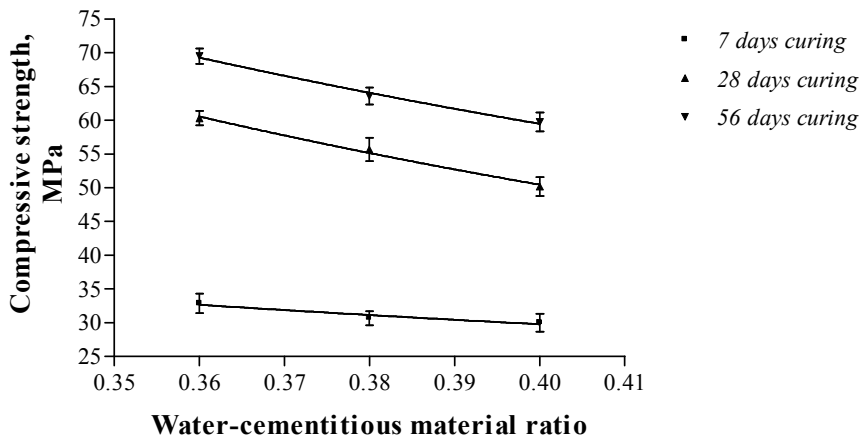


Fig. 4.6 Normalized compressive strength curves for 10% silica fume (Normal distribution)

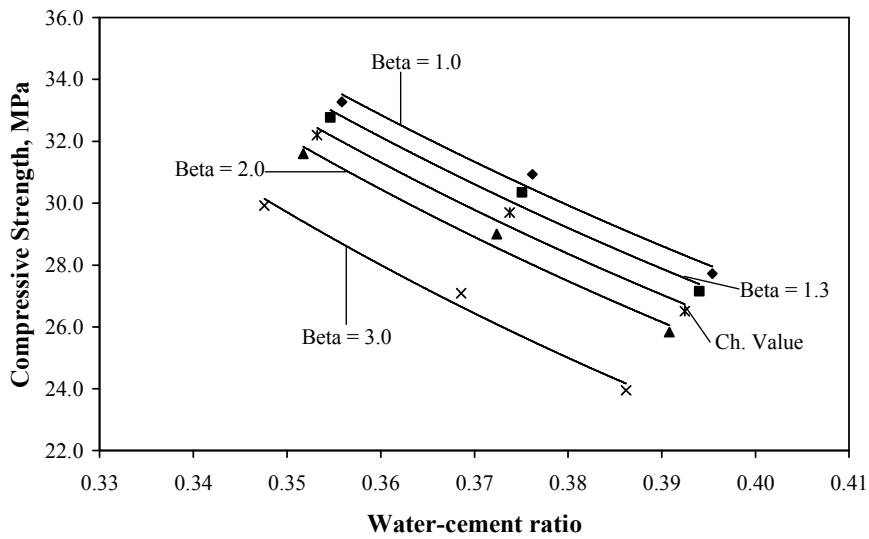


Fig 4.7 Design curves for 7 days compressive strength at various reliability indices (β) without silica fume (Normal distribution)

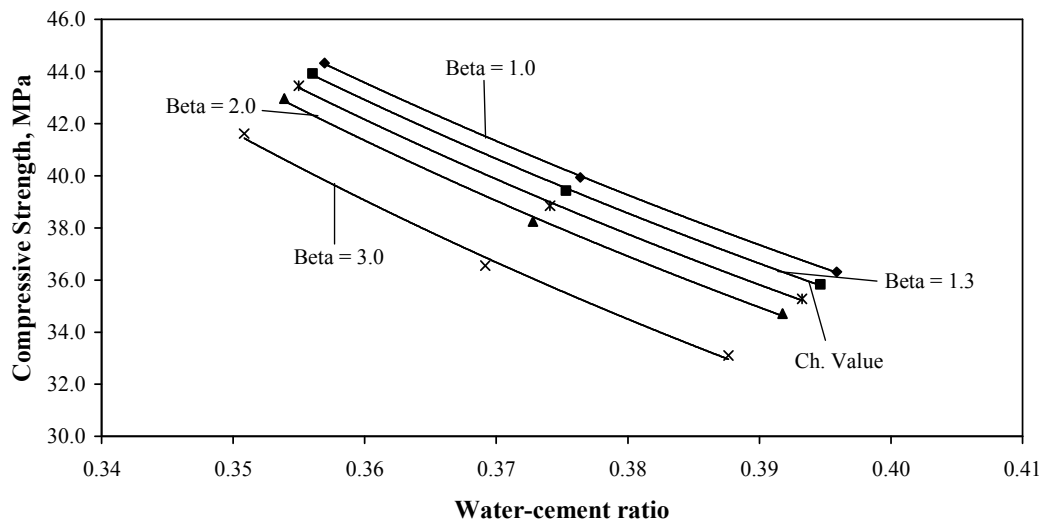


Fig 4.8 Design curves for 28 days compressive strength at various reliability indices (β) without silica fume (Normal distribution)

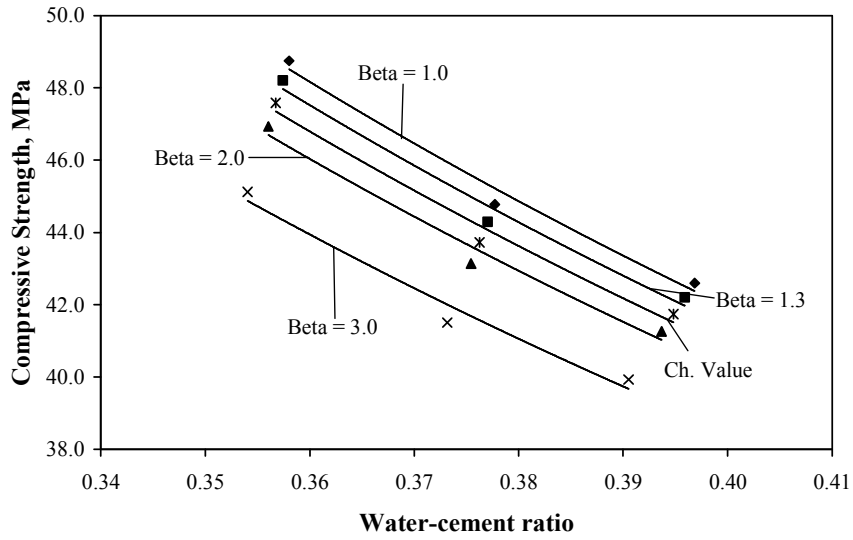


Fig 4.9 Design curves for 56 days compressive strength at various reliability indices (β) without silica fume (Normal distribution)

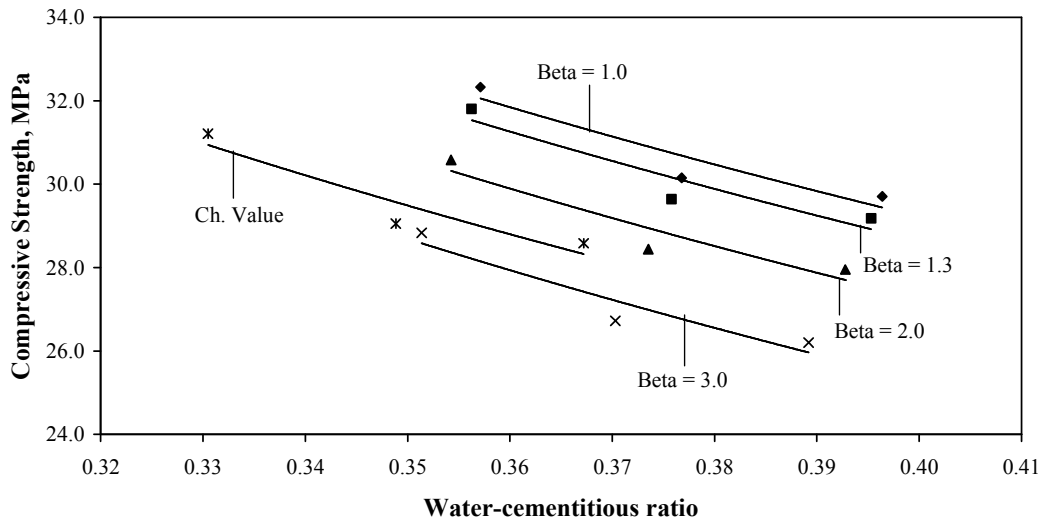


Fig 4.10 Design curves for 7 days compressive strength at various reliability indices (β) with 5% silica fume (Normal distribution)

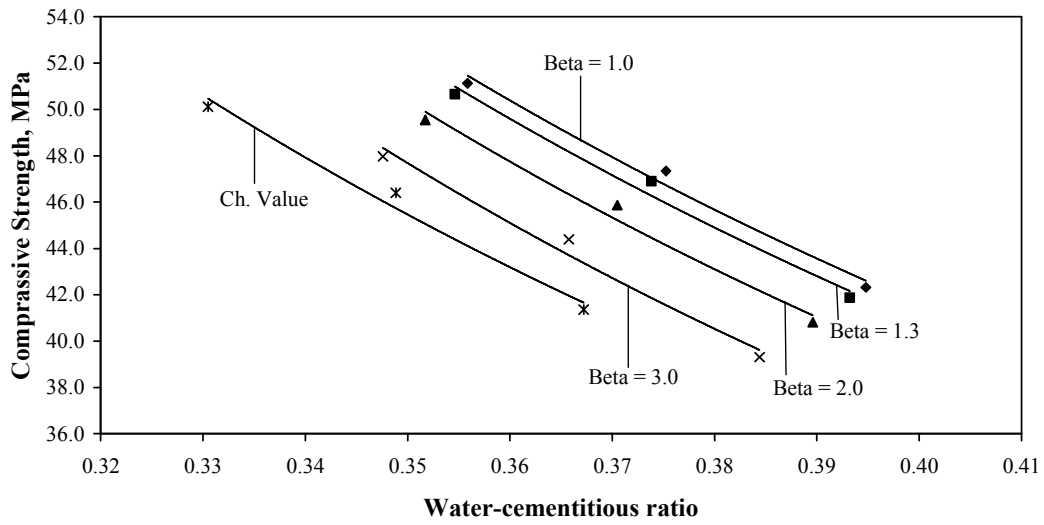


Fig 4.11 Design curves for 28 days compressive strength at various reliability indices (β) with 5% silica fume (Normal distribution)

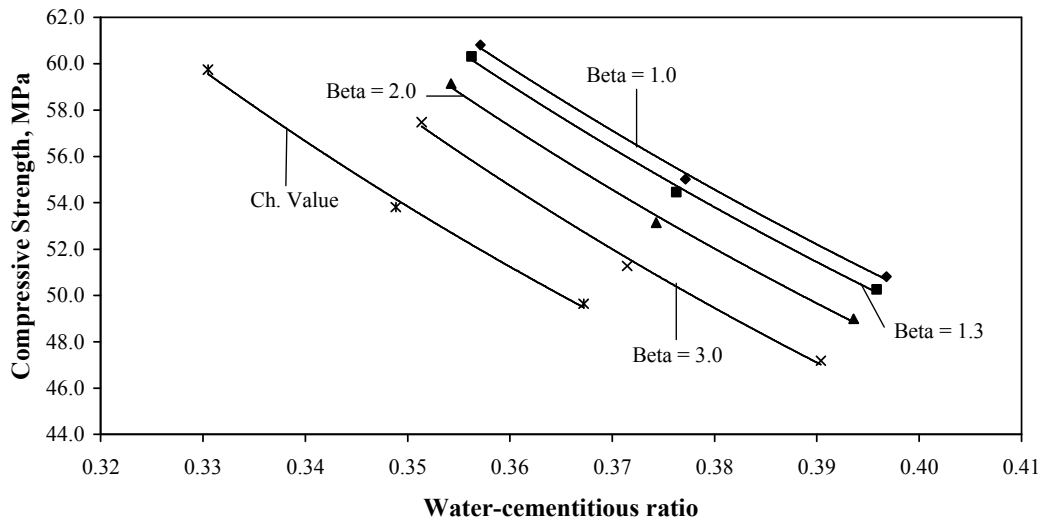


Fig 4.12 Design curves for 56 days compressive strength at various reliability indices (β) with 5% silica fume (Normal distribution)

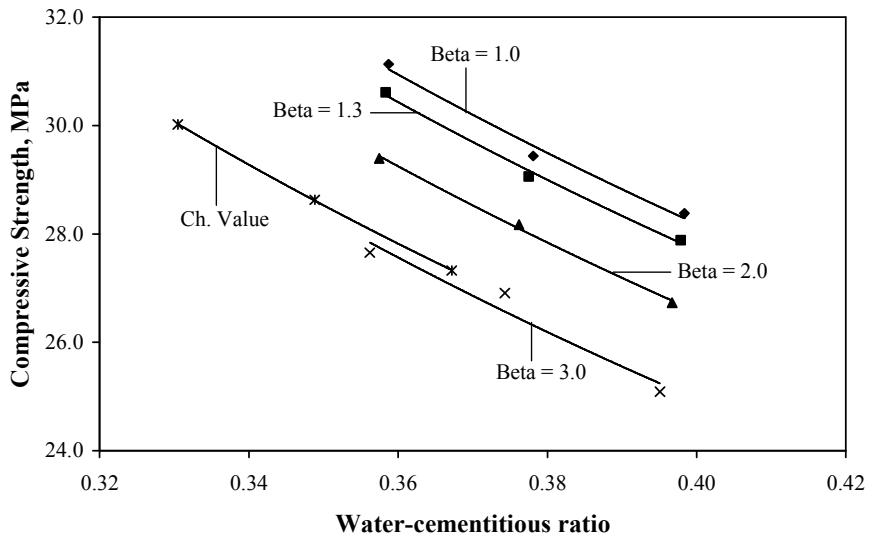


Fig 4.13 Design curves for 7 days compressive strength at various reliability indices (β) with 10% silica fume (Normal distribution)

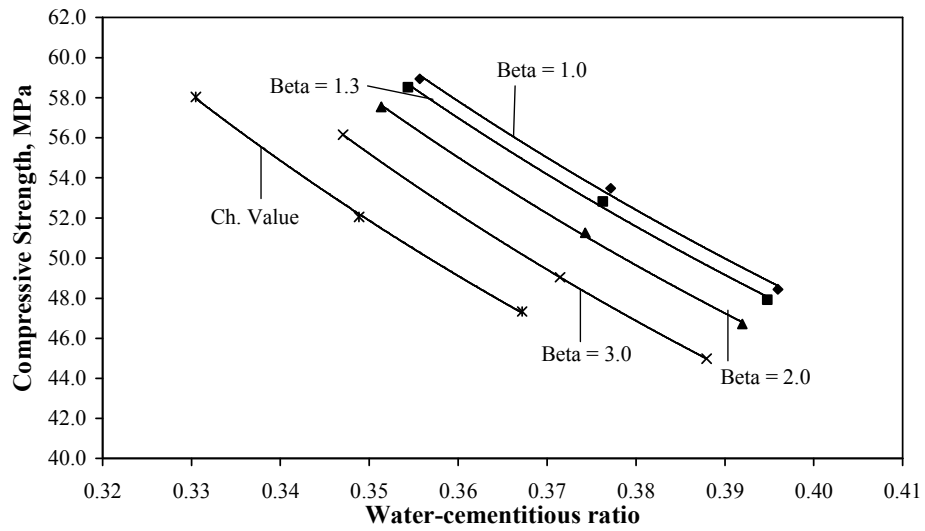


Fig 4.14 Design curves for 28 days compressive strength at various reliability indices (β) with 10% silica fume (Normal distribution)

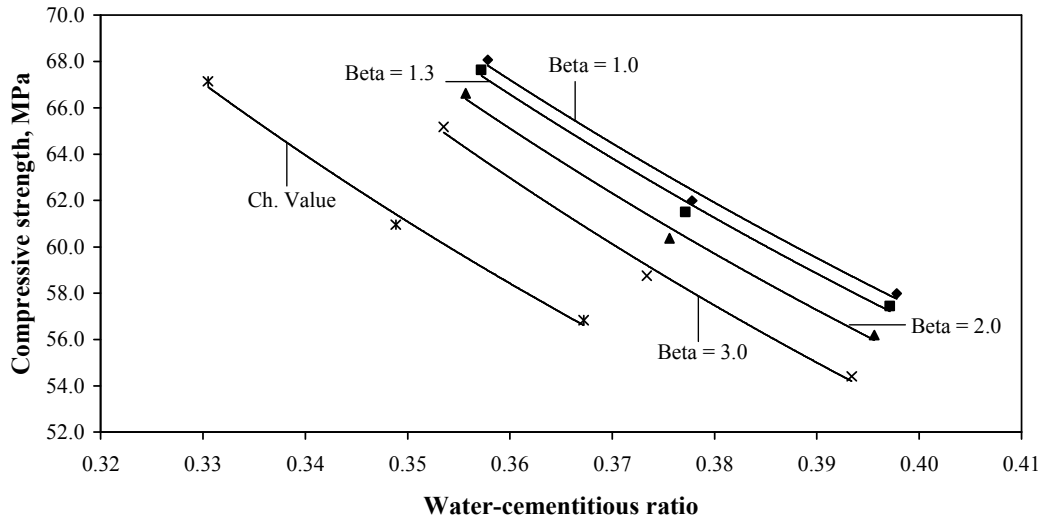


Fig 4.15 Design curves for 56 days compressive strength at various reliability indices (β) with 10% silica fume (Normal distribution)

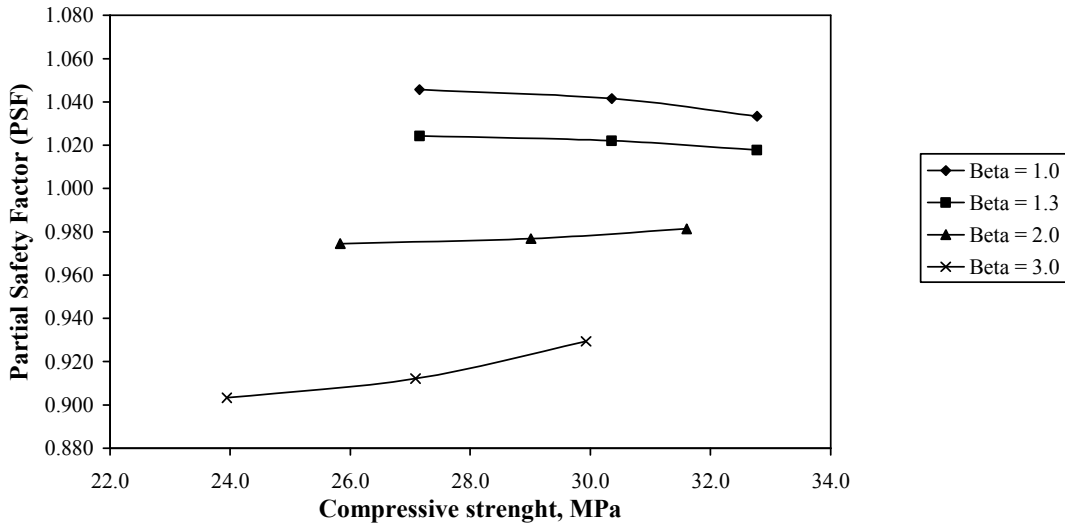


Fig. 4.16 Partial safety factors for 7 days compressive strengths at various reliability indices (β) without silica fume (Normal distribution)

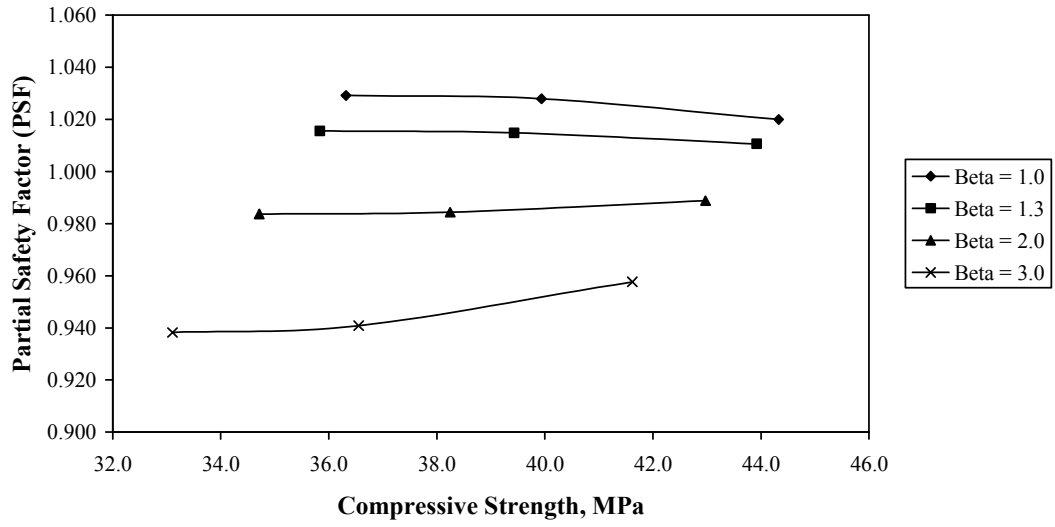


Fig. 4.17 Partial safety factors for 28 days compressive strengths at various reliability indices (β) without silica fume (Normal distribution)

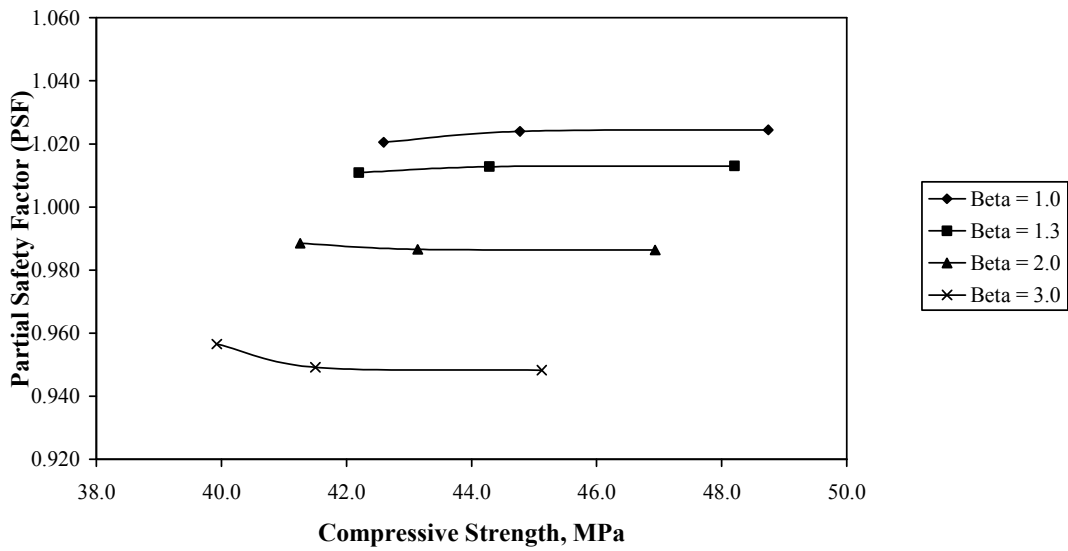


Fig. 4.18 Partial safety factors for 56 days compressive strengths at various reliability indices (β) without silica fume (Normal distribution)

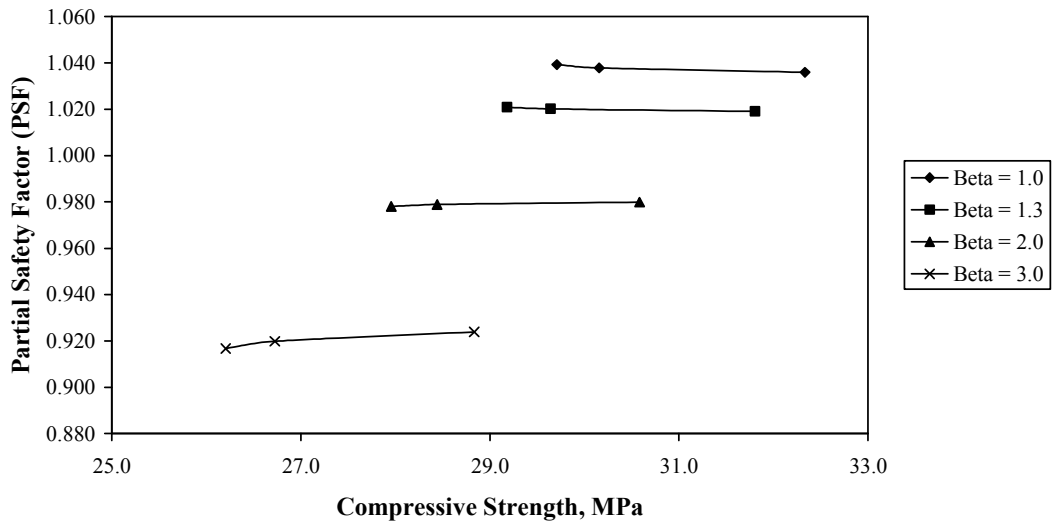


Fig. 4.19 Partial safety factors for 7 days compressive strengths at various reliability indices (β) with 5% silica fume (Normal distribution)

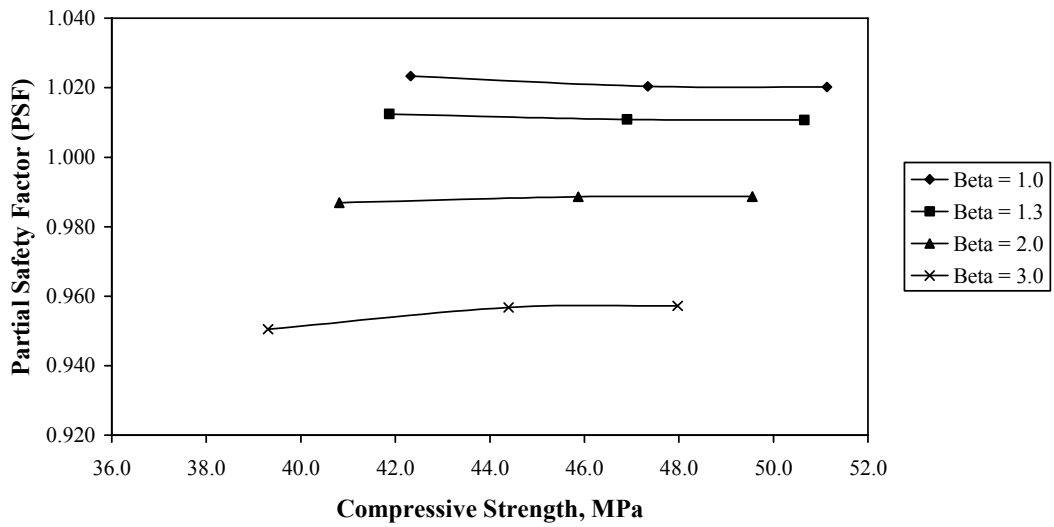


Fig. 4.20 Partial safety factors for 28 days compressive strengths at various reliability indices (β) with 5% silica fume (Normal distribution)

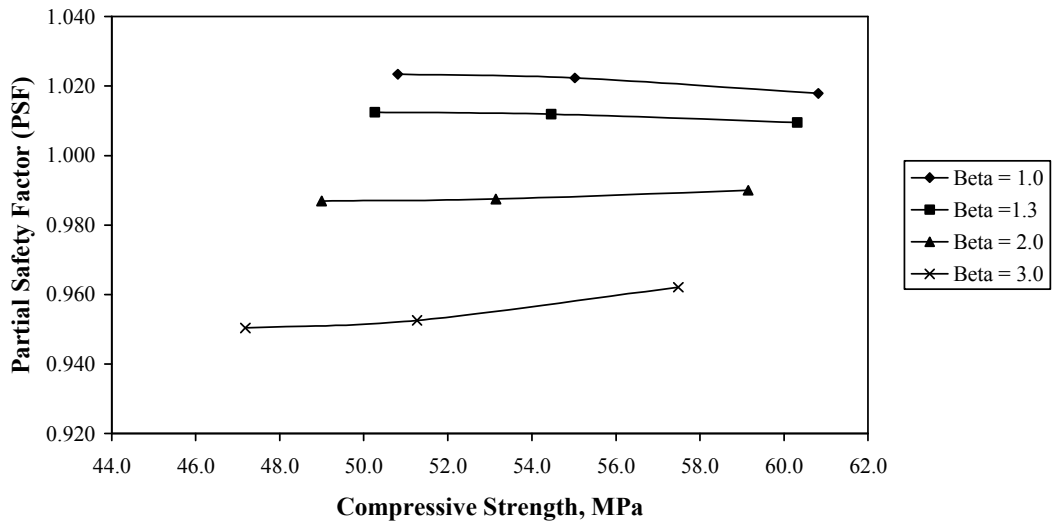


Fig. 4.21 Partial safety factors for 56 days compressive strengths at various reliability indices (β) with 5% silica fume (Normal distribution)

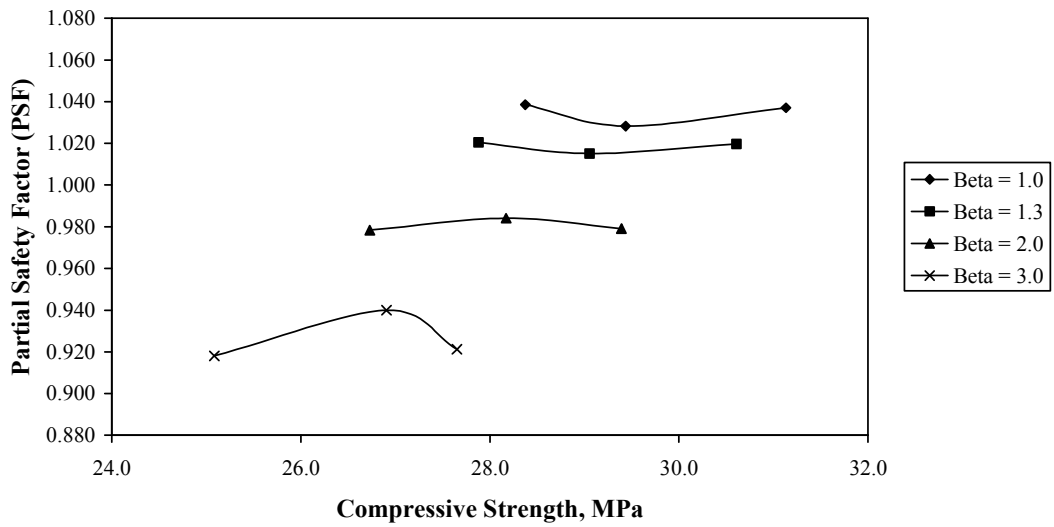


Fig. 4.22 Partial safety factors for 7 days compressive strengths at various reliability indices (β) with 10% silica fume (Normal distribution)

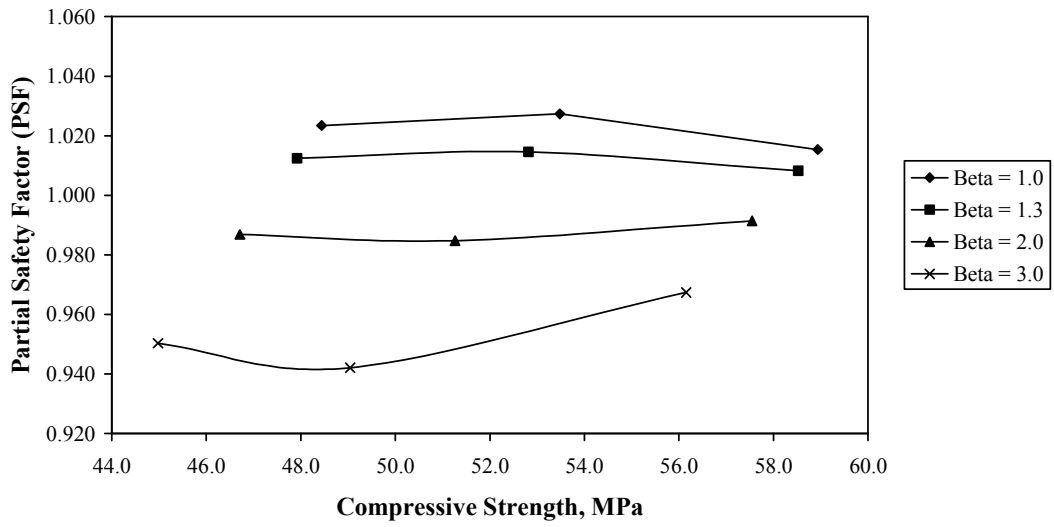


Fig. 4.23 Partial safety factors for 28 days compressive strengths at various reliability indices (β) with 10% silica fume (Normal distribution)

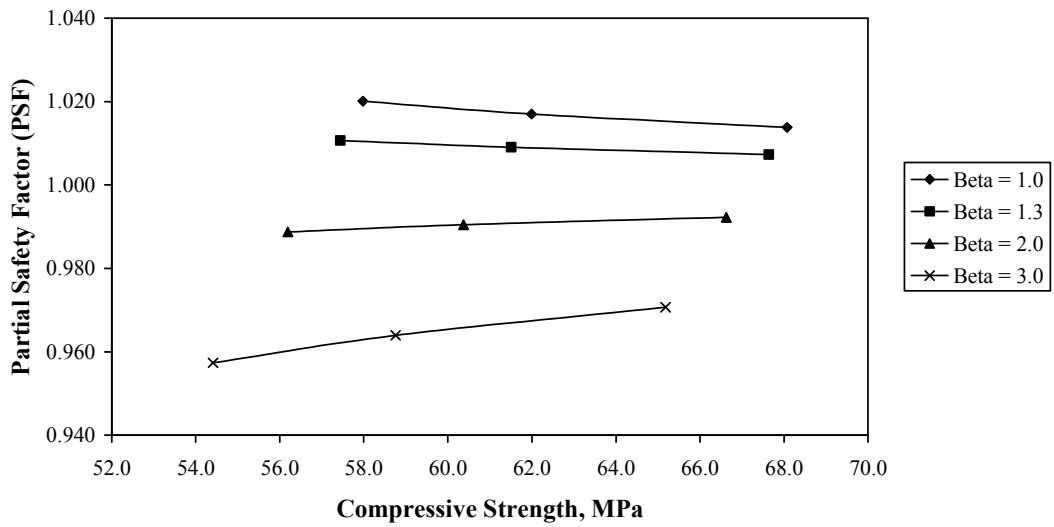


Fig. 4.24 Partial safety factors for 56 days compressive strengths at various reliability indices (β) with 10% silica fume (Normal distribution)

RELIABILITY - BASED DESIGN PROCEDURE

5.1 GENERAL

The reliability-based design procedures for concrete mixes have been proposed in this chapter. The limitation of this design procedure has been highlighted, the safety check format has been chosen, and a number of partial safety factors have been developed.

5.2 RELIABILITY - BASED PROCEDURE FOR PROPORTIONING OF CONCRETE MIXES

The process of selecting suitable ingredients for concrete and determining their relative amounts with the objective of producing concrete of required strength, durability, and workability as economically as possible is termed mix design. According to IS: 456-2000 the mix proportions shall be selected to ensure workability of fresh concrete and when concrete is hardened, it shall have the required strength, durability and surface finish.

Thus the proportioning of ingredients of concrete is governed by the required performance of concrete in two states, viz, plastic and hardened state. If the plastic concrete is not workable, it cannot be properly placed and compacted. The compressive strength of hardened concrete which is generally considered to be an index of its other properties, depends upon many factors, viz, quality and quantity of cement, water and aggregates; batching and mixing; placing, compaction and curing. The cost of concrete is related to the cost of materials required for producing a minimum mean strength called characteristic strength that is specified by the designer.

It is observed that the strength of concrete varies from batch to batch over a period of time. According to the *Indian standard recommended guidelines – 1982* (ISRG) for design of concrete mixes, the sources of variability in the strength of concrete may be considered due to following factors:

- a) Variation in the quality of constituent materials used,
- b) Variation in the mix proportions due to batching process,

- c) Variations in the quality of batching and mixing equipment available,
- d) The quality of supervision and workmanship, and
- e) Variations due to sampling and testing of concrete specimens.

The variability in the strength of concrete is taken into account by assuming that it follows the Normal or Gaussian distribution. As the cube results follow this distribution, there is always the probability that some of the results may fall below the specified strength. This factor is recognised in the *IS: 456* by introducing the concept of characteristic strength, which means that value of the strength of concrete below which not more than 5 per cent of the test results are expected to fall. Thus the ISRG procedure designs the concrete mixes corresponding the 95 per cent confidence level only. Herein a reliability-based procedure for the design of concrete mixture corresponding to a specific reliability index, which in turn is dependent upon the importance of structure, has been proposed.

5.2.1 Requirements for Reliability-based Mix Design

The various parameters controlling the design of concrete mixes are given below:

(i) Reliability Index

Based upon the importance of structure (*Seismic Design code: IS: 1893*) where concrete is to be used, the proposed reliability indices for various class of structures are given in Table 5.1

(ii) Degree of quality control at site

Quality control ensures a rational use of the available resources after testing their characteristics. In the absence of quality control at site the designer is tempted to over-design, so as to minimize the risk. Thus, it is imperative to define the degree of quality control at site in order to design the concrete mixes economically.

(iii) Desired compressive strength of concrete

The compressive strength at site is calculated based upon the degree of quality control envisaged.

(iv) Durability requirement

The durability requirement of concrete depends on environmental or exposure conditions (IS: 456-2000).

(v) Workability of concrete

It is expressed in terms of Vee-bee time (in seconds)

(vi) Particle size distribution of aggregates

Maximum nominal size and per cent of coarse aggregates as per Table 5.2

(vii) Grading zone of sand

It is based on percentage passing 600-micron IS sieve.

(viii) Specific gravity of aggregates

The specific gravity of fine and coarse aggregates used in the present study being 2.68 and 2.65 respectively, hence the proposed procedure is applicable without modification to concrete mixes using fine and coarse aggregates of the said specific gravity.

5.2.2 Step-by Step Procedure for Proportioning Concrete Mix

1. Characteristics of aggregates

The first step is to determine the properties of available coarse and fine aggregates. The maximum nominal size and the zone of coarse aggregates (Table 5.2), and grading zone of fine aggregate based upon percentage of material passing 600 micron sieve, needs to be determined.

2. Determination of target mean compressive strength of concrete

This depends upon the confidence level or the reliability index, age of curing and degree of quality control expected at site. The reliability index is fixed based upon the importance of structure as mentioned earlier.

The coefficients of variation for different degrees of control specified in Table 5.3 take into account the variations expected at site. The coefficients of variation are based on

data generated on cube strengths of the laboratory samples, which have been cast and tested under standard conditions. The coefficients of variation have been found to lie in the range 2.03 to 5.61 percent. *Kumar (2002)* assumed these variations to be log-normally distributed and based upon the number of results, which are likely to fall below the desired strength at site, factors proposed in Table 5.3 shall be used for the computation of target strength.

Depending upon the curing period chosen viz. 7 days, 28 days or 56 days, the target mean compressive strength shall be calculated by multiplying the desired strength by the factor (k) given in Table 5.3. These factors take into account the quality control envisaged at site and the number of results that can fall below the desired strength at site. Thus, the target strength is:

$$F_t = F_s \times k \quad (5.1)$$

where F_t and F_s are the target mean strength and desired strength at site, at 7 days or 28 days or 56 days of curing, respectively.

3. ***Selection of water-cementitious material ratio***

For the desired target strength, the reliability index, per cent replacement of cement by silica fume and the curing age, the water-cementitious material ratio is selected from Figs. 4.7 to 4.15 or from Tables 4.4 to 4.6 or by using the Eq. 4.34. Equation (4.34) and Tables 4.4(a), (b) and (c) represent the relationship between compressive strength and water-cementitious material ratio for 0 per cent replacement of cement by silica fume; Eq. (4.34) and Tables 4.5(a), (b) and (c) for 5 per cent replacement of cement by silica fume Equation (4.34) and Tables 4.6(a), (b) and (c) represent the relationship between compressive strength and water-cementitious material ratio for 10 per cent replacement of cement by silica.

4. ***Determination of water content***

Based on the workability required (medium) for a particular job and zone of the available coarse aggregates, the water content is selected from Table 5.2 and also calculated from the water-cementitious ratio by fixing the cementitious materials quantity up to 450 kg/m³. Use superplasticiser to maintain the required workability at lower water content than calculated. The dosage of superplasticizer can be initially fixed from Fig. 5.3

for SP-430 and from Fig.5.4 for Structuro-100 and then verified through tests conducted in the laboratory.

5. Determination of fine aggregate to cementitious material ratio

Corresponding to the w/ (c + p) ratio selected above in step-3, the fine aggregate to cementitious material ratio a_{fa} is determined for the reference curve shown in Fig. 5.1. The reference curve corresponds to the fine aggregates with 62.35 per cent material passing 600- micron sieve. The zones I, II, III and IV, represented in Fig. 5.1 are as per IS: 383-1970.

Water-cementitious material ratio is revised for the actual percentage of fine aggregate passing the 600-micron sieve, while a_{fa} is maintained at level obtained earlier. Water cementitious material ratio is then compared with the maximum free water cement ratio allowed from durability considerations (Table 5.4). The minimum of the two values is chosen for further design calculations.

6. Determination of coarse aggregate to cementitious material ratio

For the revised water-cementitious material ratio obtained in step 5, the coarse aggregate to cementitious material ratio, a_{ca} is determined from Fig 5.2.

The aggregate to cementitious material ratios are adjusted as explained below for any difference in specific gravities of fine and coarse aggregates from the reference values.

$$A_{fa} = a_{fa} \times \frac{(SG)_{fa}}{2.68} \tag{5.2}$$

$$A_{ca} = a_{ca} \times \frac{(SG)_{ca}}{2.65} \tag{5.3}$$

where $(SG)_{fa}$ and $(SG)_{ca}$ are the specific gravity values of the available fine and coarse aggregates, respectively.

7. Calculation of cement content

The cement content is obtained from the water content obtained in step-4, and revised water-cementitious material ratio obtained in step-5. The cement content so obtained is compared with the values available in Table 5.4 and adjustment from Table5.5 reproduced from IS: 456-2000, which is based on the requirements of strength and

durability. The greater of the two values is taken as the design value. But the cement content should not be more than 450 kg/m³.

8. *Calculation of quantities of materials*

The contents of cementitious material, water, fine and coarse aggregates per unit volume of concrete can be estimated based upon their values obtained in the earlier steps.

5.3 CONCRETE MIX DESIGN EXAMPLE

The proposed mix design procedure uses the relationships between water-cementitious ratio and compressive strength of concrete. The method gives proportion in terms of quantities of materials required per unit volume of concrete. The method is equally applicable to concrete mixes using 5 and 10 per cent replacement of cement by silica fume. The method is suitable for the design of normal concrete mixes having cube compressive strength as high as 55MPa, for non-air-entrained concrete and curing periods of 7, 28 and 56 days. The proposal procedure of mix proportioning detailed in pervious sections is illustrated for a mixture with following design stipulations.

5.3.1 Design Stipulations

➤ Characteristic compressive cube strength at 28 days	40MPa
➤ Reliability Index	1.3
➤ Maximum size of aggregate zone	20mm
➤ Zone of aggregate	A
➤ Degree of workability	Medium
➤ Degree of quality control	Very good
➤ Type of exposure	Mild
➤ Assumed probability distribution for strength variation	Normal
➤ Cement replacement by silica fume, per cent	10 per cent

5.3.2 Characteristics of Materials

(i) *Cement*

a) Type of cement used	Pozzolana Portland cement
b) Specific gravity of cement	3.10

c) 28 days compressive strength of 70.4mm cubes 43MPa

(ii) Aggregates

For the coarse aggregates belonging to zone-A, the fractions of coarse aggregates of types CA-I and CA-II, are 0.67 and 0.33, respectively.

a) *Specific gravity*

Coarse aggregates-I 2.61

Coarse aggregates-II 2.68

b) *Fineness modulus*

Fine aggregates 2.41

c) Water absorption

Coarse aggregates-I 1.5 per cent

Coarse aggregates-II 1.3 per cent

Fine aggregates 1.2 per cent

d) Grading of fine aggregates (IS: 383-1970)

Percentage passing 600-micron sieve 61.2 per cent

Grading zone III

5.3.3 Step by Step Procedure for Proportioning of Concrete Mix

(i) Target mean strength

For the given reliability index of 1.3 and with the presumption that the number of test results likely to fall below the desired site strength is 1 in 20, the multiplication factor k from Table 5.3 is 1.140.

The target mean compressive strength

$$F_t = 40 \times 1.140 = 45.60\text{MPa}$$

(ii) Water-cementitious material ratio

For the target mean compressive strength of 45.60MPa, the free water-cementitious, material ratio for the concrete mix corresponding to reliability index of 1.3 and without replacement of cement by silica fume, calculated from Fig. 4.8 or Table 4.7, is 0.407.

(iii) Water content

The water content for the concrete mix using coarse aggregate of zone-A and having medium workability taken from Table 5.2 is 165 kg/m³.

(iv) Fine aggregate to cementitious material ratio

The fine aggregate to cementitious material ratio obtained from Fig. 5.1 for the reference curve is

$$a_{fa} = 1.07$$

For this value of a_{fa} the water-cementitious material ratio for the percentage of material passing 600-micron sieve of 61.2 per cent is 0.385.

This value of free water-cementitious material being less than specified maximum value of 0.55 from durability considerations can be adopted for further calculations.

(v) Coarse aggregate to cementitious material ratio

For the free water-cementitious material ratio of 0.385, the coarse aggregate to cementitious material ratio as obtained from Fig. 5.5 are:

$$\text{For Coarse aggregate - I} = 1.70$$

$$\text{For Coarse aggregate -II} = 0.85$$

(vi) Revision of values of aggregate-cementitious material ratios

The revised values of the aggregate-cementitious material ratios obtained using the equations (5.2) and (5.3) are:

$$A_{fa} = (2.66 \times 1.07)/2.68 = 1.062$$

$$A_{ca-I} = (2.61 \times 1.70)/2.65 = 1.674$$

$$A_{ca-II} = (2.68 \times 0.85)/2.65 = 0.860$$

(vii) Cement content

$$\text{Cement content} \quad 165/0.385 = 428.57 \text{ kg/m}^3$$

And the maximum value of cement content is 450 kg/m³. So fix the value of cementitious content as 428.57 kg/m³ and use 1.5 per cent SP-430 or use 0.80 per cent STR-100 calculated from Fig.5.3 and 5.4.

(viii) Aggregate contents

Fine aggregate	=	1.062×428.57	=	455.14 kg/m^3
Coarse aggregate — I	=	1.674×428.57	=	717.42 kg/m^3
Coarse aggregate — II	=	0.860×428.57	=	368.57 kg/m^3

(ix) Mix proportion

Thus the trial mix proportion

Water	Cement	FA	CA-I	CA-II	SF	SP-430	STR-100
0.385	1.00	1.180	1.861	0.956	0.10	1.5%	0.80%

5.3.4 Mix design as per Indian Standard Recommended Guidelines (SP: 23-1982)

The same concrete mix of grade M40 is also proportioned using Indian Standard Recommended Guidelines procedure (ISRG) for comparison. The calculations, Tables and figures referred to have been taken from the Special Publication, (SP: 23-1982) and are provided in Appendix-II. Table 5.9 shows the comparison of quantities of materials obtained with the proposed reliability procedure and using Indian Standard Recommended Guidelines.

5.3.5 Validation

Six cubes with the proportion of materials estimated using the proposed design methodology as well as the ISRG procedure. The compressive strengths of the cubes tested after 7 and 28 days curing was:

Methodology	Mean compressive strength, MPa	
	7 days	28 days
Proposed reliability-based design procedure	37.29	58.97
ISRG	44.52	54.61

The proposed design procedure uses 26 per cent lesser cementitious materials as compared to the ISRG procedure, however the concrete mix with 10 per cent silica fume obtained less strength after 7 days of curing but obtain more strength after 28 days of curing. Thus ISRG procedure, which uses very high content of cement, is highly conservative and need revision.

Table 5.1 Reliability indices proposed for various classes of structures

S. No.	Class of structure	Reliability index
1.	Very important structures like nuclear power plants, containers of inflammable gases etc.	3.0
2.	Important structures like Hospitals, water tanks, power houses, schools, telephone exchanges, large assembly structures like cinema hall, telephone exchange etc.	2.0
3.	Multi storeyed buildings, residential complexes etc.	1.3
4.	All others	1.0

Table 5.2 Detail of aggregates proportion

Zone	Percentage passing 20mm sieve and retained on 10mm sieve	Percentage passing 10mm sieve and retained on 4.75mm sieve	Percentage passing 4.75mm sieve and retained on 2.36mm sieve	Fineness modulus	Water content, kg/m ³
A	67	33	----	6.86	162 - 180

Table 5.3 Proposed multiplication factors for evaluation of target strength (*Kumar, 2002*)

Coefficient of variation, per cent	Quality control	Multiplication factor, k, for number of test results likely to fall below the desired site strength			
		<i>1 in 10</i>	<i>1 in 20</i>	<i>1 in 50</i>	<i>1 in 100</i>
7	Excellent	1.094	1.112	1.154	1.177
8	Very good	1.108	1.140	1.178	1.204
9	Average	1.122	1.159	1.202	1.232
10	Poor	1.136	1.178	1.227	1.261

Table 5.4 Limiting value of cement content, water-cement ratio and grade of concrete for different exposure with normal weight aggregates of 20mm nominal maximum size

S. No.	Exposure	Plain concrete			Reinforced concrete		
		Minimum cement content, kg/m ³	Maximum free water-cement ratio	Minimum grade of concrete	Minimum cement content, kg/m ³	Maximum free water-cement ratio	Minimum grade of concrete
1.	Mild	220	0.60	----	300	0.55	M20
2.	Moderate	240	0.60	M15	300	0.50	M25
3.	Severe	250	0.50	M20	320	0.45	M30
4.	Very severe	260	0.45	M20	340	0.45	M35
5.	Extreme	280	0.40	M25	360	0.40	M40

Notes:

1. Cement content prescribed in the above table is irrespective of grade of cement and it is inclusive of mineral admixtures viz. fly ash, GGBS, silica fume etc. The admixtures such as fly ash or ground granulated blast furnace slag may be taken in the concrete composition with respect to the cement content and water-cement ratio if the suitability is established and as long as the maximum amounts taken into account do not exceed the limit of pozzolana and slag specified in IS: 1489 (Part – I) and IS: 455, respectively.
2. Minimum grade for plain concrete under mild exposure is not specified.

Table 5.5 Adjustment to minimum cement content for the change in aggregate size from 20mm nominal maximum size

S. No.	Nominal maximum size of aggregates, mm	Adjustment to minimum cement content given in Table 5.4, kg/m ³
1.	10	+40
2.	2	0
3.	40	-30

Table 5.6 Comparison of quantities materials

Quantities of materials, kg/m ³								Per cent	
<i>Mix design procedure</i>	<i>W/(c+p)</i>	<i>Water</i>	<i>Cement</i>	<i>FA</i>	<i>CA-I</i>	<i>CA-II</i>	<i>SF</i>	<i>SP-430</i>	<i>STR-100</i>
Design procedure (Kumar, 2002)	0.426	185	434.27	495.94	709.60	358.71	---	---	---
Proposed reliability-based design procedure with 10% SF at 56 days	0.385	165	385.71	455.14	717.43	368.57	42.86	1.5	0.80
ISRG	0.320	185	578.15	452.20	767.34	387.26	---	---	---

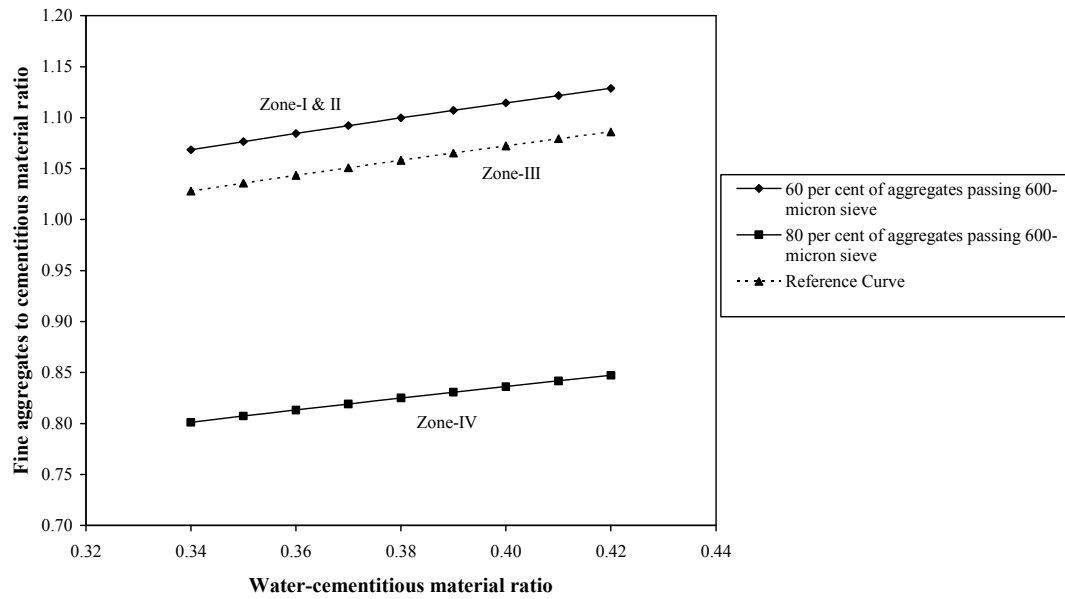


Fig. 5.1 Variation of fine aggregates to cementitious material ratio with water-cementitious material ratio for mixes with medium workability using zone-A aggregates

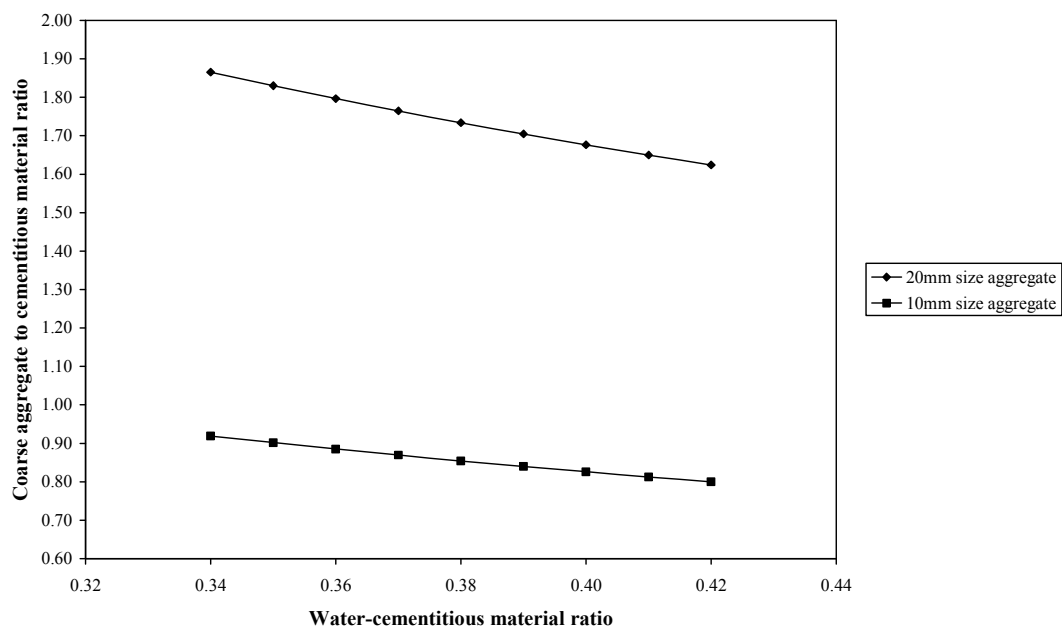


Fig. 5.2 Variation of coarse aggregates to cementitious material ratio with water-cementitious material ratio for mixes with medium workability using zone-A aggregates

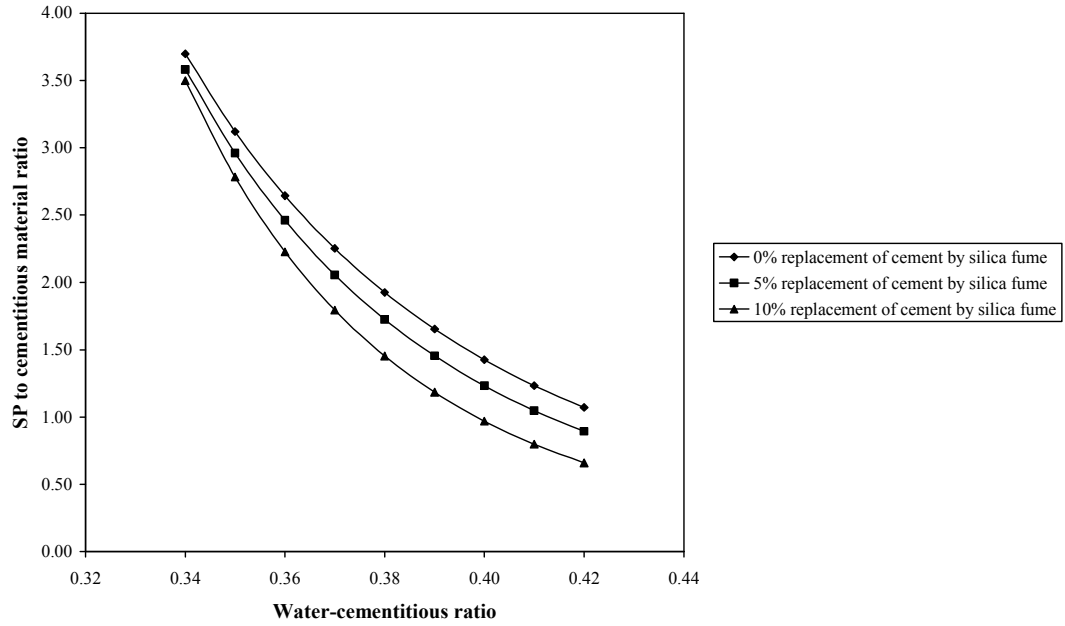


Fig. 5.3 Variation of SP dosage to cementitious material ratio with water-cementitious material ratio for mixes with medium workability (Fosroc Conplast SP-430)

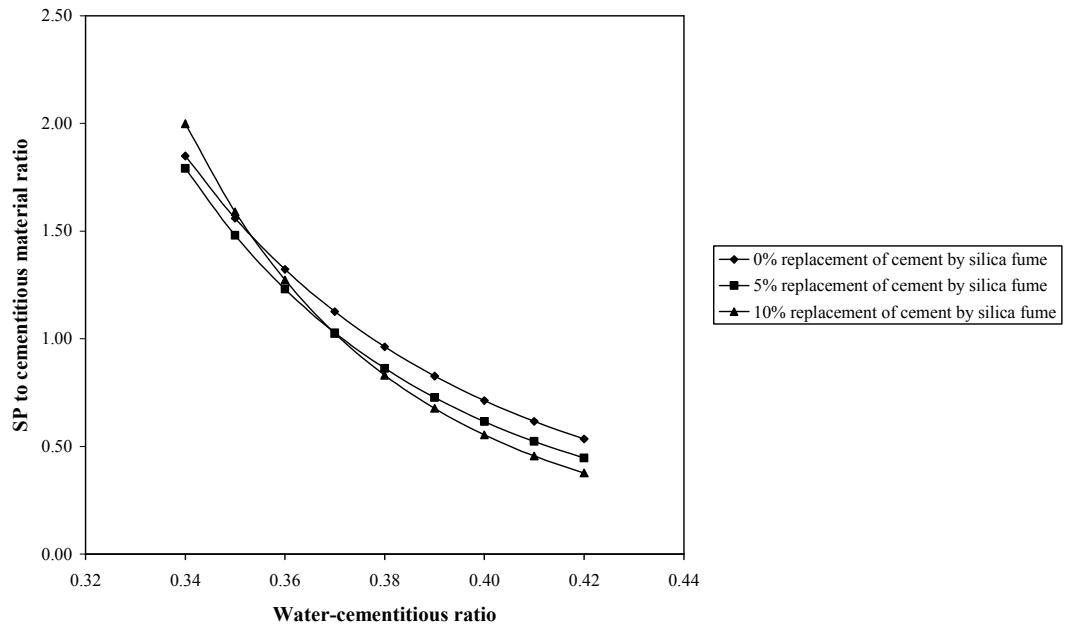


Fig. 5.4 Variation of SP dosage to cementitious material ratio with water-cementitious material ratio for mixes with medium workability (Fosroc Structuro-100)

RESULTS AND DISCUSSION

6.1 GENERAL

This chapter is divided into two sections. First section deals with the effect of variations in parameters on properties of concrete with and without silica fume. In the second section the effect of various parameters on the concrete mix proportions obtained by the proposed procedure has been discussed.

6.2 PROPERTIES OF CONCRETE

The focus in this section is on understanding the effect of various parameters on workability of fresh concrete and statistical analysis of experimentally generated compressive strength data.

6.2.1 Workability

The variations in Vee-bee time of concrete mix with water-cementitious material ratio, for different percentage replacements of cement by silica fume and different SP percentages have been presented in Tables 6.1 and 6.2 and in Figs. 5.3 and 5.4.

The concrete mixes using 5 and 10 per cent silica fume, have shown higher workability as compared to concrete mixes without silica fume, for the same water contents. However, the concrete mixes using 15 per cent silica fume have less workability as compared to mix without silica fume. For same workability, having vee-bee time between 3 to 6 seconds, the dosage of superplasticizer reduces as we increases the percentage of silica fume from 0 per cent to 10 per cent. However, the dosage is almost doubled with superplasticizer (Fosroc Conplast SP-430) as compared to superplasticizer (Fosroc Structuro-100). Thus it can be said that concrete mixes with silica fume up to 10 per cent have high workability, and this small increment in workability of silica fume

concrete mixes is due to the fact that the silica fume had greater proportion of small particles (20000 m³/kg surface area).

Duval and Kadri (1998) have also shown that the workability of silica fume concretes at low water-cementitious materials ratios with a naphthalene sulphonate superplasticizer, is not reduced for replacements up to 10 per cent silica fume.

6.2.2 Density of Concrete

The densities of the concrete mixes with and without silica fume for different $w/(c+p)$ ratios and different replacement levels are shown in Table 6.3.

From Table 6.3 it is observed that concrete mixes without silica fume using SP-430, shows higher density as compared to concrete mixes using silica fume. Also the density of concrete with 5 per cent silica fume replacement is higher than concrete with 10 per cent silica fume.

6.2.3 Statistical Properties of Compressive Strength

The statistical analysis of the experimentally generated data and its effect on various parameters is discussed below.

(a) Coefficient of variation

The frequency analysis of coefficient of variation (ratio of standard deviation to mean value) of experimentally generated compressive strength data for the concrete mixes with different percentage of silica fume at different curing ages is presented in Table 6.4. The values of the coefficient of variation have been calculated from the data given in Tables 3.18 to 3.20.

From Table 6.4 it is observed that most of the concrete mixes without silica fume have a coefficient of variation of compressive strength more than 5 per cent at 7 days curing age, but it decreases to 3 to 4 per cent as the curing age increases. It is also observed that concrete with 5 per cent silica fume have coefficient of variation 4 to 5 per cent for 7 days and less than 3 per cent for 28 and 56 days.

A similar trend is observed for concrete with 10 per cent replacement of cement by silica fume having coefficient of variation lying between 3 to 5 per cent at 7 days and nearly 3 per cent for 28 and 56 days.

(b) *With-in-test coefficient of variation*

The frequency analysis of with-in-test coefficient of variation of experimentally generated compressive strength data for the concrete mixes with different percentages of silica fume is presented in the Table 6.5. The values of with-in-test coefficient of variation have been listed in Tables 3.18 to 3.20. *The ACI committee report 214 (1988)* has suggested control standards for evaluating performance of testing program using the within-test coefficient of variation as given in Table 3.14.

From Table 6.5 it is observed that most of the concrete mixes with and without silica fume have a with-in-test coefficient of variation of compressive strength between 2 to 4 per cent. However, at 28 and 56 days curing most of the concrete mixes have a with-in test coefficient of variation lying between 1.5 to 2.0 per cent.

It is also seen that none of the concrete mixes with silica fume and very few without silica fume have a with-in-test coefficient of variation between 1.5 to 2.0 per cent at curing ages of 28 and 56 days, indicating improvement in homogeneity of hardened concrete at higher curing ages.

Thus for all the concrete mixes, with and without silica fume, the with-in-test coefficient of variation is less than permissible value of 4.0 per cent as suggested by ACI report (1988), which indicates that the testing conditions have not affected the results of compressive strengths.

6.3 COMPRESSIVE STRENGTH

The variation in normalized and design compressive strengths with water-cementitious material ratio for concrete mixes using 0, 5 and 10 per cent of silica fume at different curing ages have been presented in Figs. 6.1 to 6.9.

6.3.1 Effect of Reliability Index

In general it is observed that concrete mixes with and without silica fume designed for higher reliability index have to be proportioned using lower water-cementitious material ratios to take care of the reduction in probability of failure.

(a) At 7 days compressive strength

It is observed that normalized curve for concrete mixes obtained experimentally after 7 days of curing without silica fume gives higher strengths than the strengths given by design curves of various reliability indices. Almost same trend is observed from Figs. 6.2 and 6.3 for concrete mixes with 5 and 10 per cent silica fume.

(b) At 28 days compressive strength

After 28 days curing age it is observed from Fig. 6.4 that normalized curve obtained experimentally for concrete mixes without silica fume gives higher strengths for water-cementitious material 0.36 and more. However, the design curves for reliability index of 1.0 and 1.3 give higher strengths than normalized curves when $w/(c+p)$ ratio is less than 0.36. Similarly, the normal distribution design curves for reliability indices of 1.0 and 1.3 have higher strengths than IS curve at any $w/(c+p)$ ratio. The design curves for reliability indices of 2.0 and 3.0 show lesser strength up to $w/(c+p)$ ratio of approximately 0.37, for lesser $w/(c+p)$ values the design curves show higher strength as compared to IS curve.

From Fig. 6.5 it is observed that for concrete mixes using 5 per cent replaced silica fume the design curves with normal distribution give higher strengths for all $w/(c+p)$ ratios at 28 days. However, the design curves show strengths less than ACI curve but higher than IS curve.

From Fig. 6.6 it is observed that the normalized curve for 10 per cent replacement level of silica fume after 28 days curing gives higher strength than ACI curve at $w/(c+p)$ ratio 0.36 and less. The design curves with different reliability indices give much higher strengths than IS curve. The curves for reliability indices of 1.0 and 1.3 give same strengths at $w/(c+p)$ ratio 0.34. Thus it can be said that the design curves obtained using normal distribution give higher strengths as compared to the IS curve at 28 days curing age with 5 and 10 per cent silica fume.

(c) At 56 days compressive strength

The curves 6.7 to 6.9 show the variation of normalized and design compressive strengths with water-cementitious material ratio for concrete mixes using 0, 5 and 10 per

cent of silica fume at 56 days of curing. In general it is also observed that the normalized and the design curves for concrete mixes without silica fume after 56 days of curing give higher strength than IS curve but give less strength than ACI curve at any $w/(c+p)$ ratio. The curves for reliability indices of 1.0 and 1.3 give almost strengths for all $w/(c+p)$ ratios.

When the replacement level of silica fume is 5 per cent then the normalized curve gives higher strength than ACI curve at $w/(c+p)$ ratio 0.38 and less. It is also observed that ACI curve and the design curves for reliability indices of 1.0 and 1.3 give same strengths at $w/(c+p)$ ratio 0.35.

As the replacement level of silica fume increases to 10 per cent, the normalized and the design curves for reliability indices of 1.0 and 1.3 give higher strengths than ACI curves at all $w/(c+p)$ ratios. These curves give strengths with equal proportion and with small variation between each other. The design curves for reliability indices of 2.0 and 3.0 give strengths which lie between the ACI and IS curves, however they are closer to ACI curve. At $w/(c+p)$ ratio 0.34 the curve for reliability index 2.0 gives strength equal to the strength given by ACI curve.

6.3.2 Effect of percentage of silica fume

The effect of replacement of cement by silica fume on normalized and design compressive strengths for reliability indices (β) 1.0, 1.3, 2.0 and 3.0 with water cementitious materials ratio at different curing ages is discussed in this section and presented in Figs. 6.10 to 6.24.

(a) Normalized compressive strength

The variations of normalized compressive strength with water-cementitious material ratio at different curing ages with different replacement level of silica fume are Figs. 6.10 to 6.12. It is observed that normalized curve for 7 days curing age with 5 per cent replaced silica fume give higher strength than the strength given by curves for concrete mixes without and with 10 per cent replaced silica fume. Also the normalized curve for concrete mixes without silica fume and the curves for concrete mixes with 5 per cent silica fume give same strengths at $w/(c+p)$ ratio 0.35 and less. For $w/(c+p)$ ratios higher than 0.40, concrete with 10 per cent silica fume gives higher strength than normalized curves.

It is observed that after 28 and 56 days age of curing, the strengths of concrete mixes increase as the percentage of silica fume increases. A significant increment is observed in compressive strength of concrete with silica fume. Thus it can be said that there is large increment in compressive strength of concrete mixes in which cement is replaced by 5 and 10 per cent silica fume after 28 and 56 days.

(i) 7-days design compressive strength

It is observed from Figs. 6.13 to 6.16 that for 7 days compressive strengths at reliability indices (β) 1.0, 1.3, 2.0 and 3.0, the concrete mixes without silica fume give higher strengths for $w/(c+p)$ ratio less than 0.365. As the reliability index increases, the strength of concrete mixes without silica fume decreases with decrease in $w/(c+p)$ ratio.

Thus it can be said that at 7 days, the design compressive strengths of concrete mixes without silica fume give higher strengths for all reliability indices with $w/(c+p)$ ratio less than 0.365 and with 5 per cent silica fume replacement give higher strengths for $w/(c+p)$ ratio beyond 0.37, but the concrete mixes with 10 per cent silica fume give less strength.

(ii) 28 and 56-days design compressive strength

For 28 days compressive strength from Figs. 6.17 to 6.20 and for 56 days compressive strength from Figs. 6.21 to 6.24, it is observed for that concrete mixes with 5 and 10 per cent silica fume give higher strength than concrete mixes without silica fume at all $w/(c+p)$ ratios and at all reliability indices. The increment in concrete strength is very large for 5 and 10 per cent replacement of cement by silica fume than the concrete mixes without silica fume at 28 and 56 days curing ages. As the reliability index increases from 1.0 to 3.0, the strength of concrete mixes decreases due to higher reliability and lower probability of failure. The concrete mixes with 10 per cent silica fume give higher strength than concrete mixes with 5 per cent silica fume after 28 and 56 days curing age and at all reliability indices.

To check the higher replacement level strength, 10 cubes were cast for 28 and 56 days curing age with 15 per cent replacement of cement by silica fume. The result shows that after 28 days the strength is 35.01 and after 56 days the strength is 44.04. So, it can be seen that if the replacement level increases to 15 per cent, the strength of the concrete mixes decreases and is almost equal to the strength of concrete mixes without silica fume.

Thus, the results shows that if the replacement level is up to 10 per cent , the strength of concrete mixes increases and for higher replacement level of 15 per cent, the strength of concrete mixes decreases nearer to 0 per cent replacement level concrete strength. So, it can be said that the effective replacement level of cement by silica fume is 10 per cent.

6.4 PARTIAL SAFETY FACTORS

Partial safety factor is generally defined as the ratio of design variable to any reference value. In the present study, partial safety factors with respect to the characteristic value or the characteristic strength of concrete have been evaluated. The relationship between the partial safety factor and design compressive strength has been presented in Figs. 6.25 to 6.36. From these figures it is observed that partial safety factor decreases with increase in reliability index. It is also seen that this decrease is exponential as the reliability index increases from 1.0 to 3.0. The decrease in partial safety factor for compressive strength is insignificant as the reliability index increases from 1.0 to 1.3. However, it is of the order of 3 to 6 per cent as the reliability index increases from 2.0 to 3.0.

6.4.1 Effect of percentage of silica fume

The relationship of partial safety factor and compressive strength with different percentage replacement of cement by silica fume corresponding to reliability index β of 1.0, 1.3, 2.0 and 3.0 have been presented in Figs. 6.25 to 6.36. In general it is observed that concrete mixes designed for higher replacement of silica fume give lower partial safety factors.

(a) At 7 days compressive strength

The variation of partial safety factor with 7 days compressive strength of concrete at different curing ages has been shown in Figs. 6.25 to 6.28 it is observed that for $\beta = 1.0$ and 1.3, the partial safety factor decreases with increase in strength of concrete mix for 5 per cent replacement level, but for 10 per cent silica fume replacement, the partial safety factors does not decrease significantly and has almost a constant value. On the other hand, the partial safety factor is more when the silica fume replacement level is 10 per cent as

compared to 5 per cent. However, for $\beta = 2.0$ the trend is reverse and the partial safety factor is more for 5 per cent replacement level and increases as the strength of concrete mixes increases, whereas, for $\beta = 3.0$, the partial safety factor increases with increase in compressive strength of concrete mixes and is more for 10 per cent replacement level.

(b) At 28 days compressive strength

The variation of partial safety factor with 28 days compressive strength of concrete at different curing ages has been shown in Figs. 6.29 to 6.32 it is observed that for $\beta = 1.0$, the partial safety factor decreases as the compressive strength of concrete increases. The partial safety factor is more for 10 per cent replacement level as compared to 5 per cent replacement level. But if the strength is more than 63 MPa, the trend is reverse. For $\beta = 1.3$, the partial safety factor for 10 per cent silica fume concrete is very high as compared to 5 per cent silica fume concrete. The values of partial safety factor remain almost constant as the strength of concrete mixes increases. However the trend of partial safety factor curves are totally different for $\beta = 2.0$. The partial safety factor is more for 5 per cent replacement level, but if the strength of concrete is more than 65 MPa then the partial safety factor for concrete with 10 per cent silica fume is more. The partial safety factor for 10 per cent silica fume concrete mix varies at a higher rate as compared to 5 per cent silica fume concrete mix and almost same trend is observed for $\beta = 3.0$.

(c) At 56 days compressive strength

The variation of partial safety factor with 56 days compressive strength of concrete at different curing ages has been shown in Figs. 6.33 to 6.36 it is observed that for $\beta = 1.0$, the partial safety factor decreases as the compressive strength of concrete mixes increases at both replacement levels, but is more for 10 percent replacement. For $\beta = 1.3$ both concrete mixes have almost equal partial safety factors, with small increase observed for 10 per cent replacement level and, it decreases as the strength of concrete mix increases. For $\beta = 2.0$ and 3.0, the partial safety factor increases as the strength of concrete mixes increases. The concrete mixes have large values of partial safety factors for $\beta = 2.0$ as compared to $\beta = 3.0$ at both replacement levels, but a similar trend is observed for both reliability indices.

Thus it can be said that at different replacement levels for different reliability indices at different curing ages, a different trend is observed in the variation of partial

safety factors. This variation may be attributed to the value of characteristic strength of concrete as well.

Table 6.1 Variation of workability with water-cementitious material ratio at different replacement levels of silica fume and dosage of SP (Fosroc Conplast SP-430)

Water-cementitious material ratio	Replacement of cement by silica fume					
	0 per cent		5 per cent		10 per cent	
	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>
0.36	5.1	2.60	4.9	2.40	4.9	2.30
0.38	4.8	2.0	4.7	1.80	4.8	1.60
0.40	4.6	1.40	4.4	1.20	4.5	1.0

Table 6.2 Variation of workability with water-cementitious material ratio at different replacement levels of silica fume and dosage of SP (Fosroc Structuro-100)

Water-cementitious material ratio	Replacement of cement by silica fume					
	0 per cent		5 per cent		10 per cent	
	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>	<i>Vee-bee time, seconds</i>	<i>SP, per cent</i>
0.36	5.0	1.30	4.8	1.20	4.8	1.15
0.38	4.7	1.00	4.6	1.80	4.6	0.80
0.40	4.6	0.60	4.4	1.20	4.5	0.50

Table 6.3 Average densities of concrete mixes

Water-cementitious material ratio	Replacement of cement by silica fume		
	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>
	Average densities of concrete, kg/m ³		
0.36	2381	2357	2344
0.38	2386	2356	2336
0.40	2412	2394	2368

Table 6.4 Coefficient of variation of compressive strength for concrete mixes at different curing ages

Coefficient of variation	Number of mixes								
	<i>Curing period = 7 days</i>			<i>Curing period = 28 days</i>			<i>Curing period = 56 days</i>		
	Replacement of cement by silica fume								
	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>
<3.0	----	----	----	----	----	----	2	1	3
3.0-4.0	----	----	1	----	----	1	1	2	----
4.0-5.0	1	3	2	1	3	2	----	----	----
>5.0	2	----	----	2	----	----	----	----	----

Table 6.5 With-in-test coefficient of variation of compressive strength for concrete mixes at different curing ages

With-in-test Coefficient of variation	Number of mixes								
	<i>Curing period = 7 days</i>			<i>Curing period = 28 days</i>			<i>Curing period = 56 days</i>		
	Replacement of cement by silica fume								
	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>	<i>0 per cent</i>	<i>5 per cent</i>	<i>10 per cent</i>
<1.5	----	----	----	----	----	1	----	----	1
1.5-2.0	----	----	----	1	3	----	3	2	2
2.0-3.0	1	1	1	2	----	2	----	1	----
>3.0	2	2	2	----	----	----	2	----	----

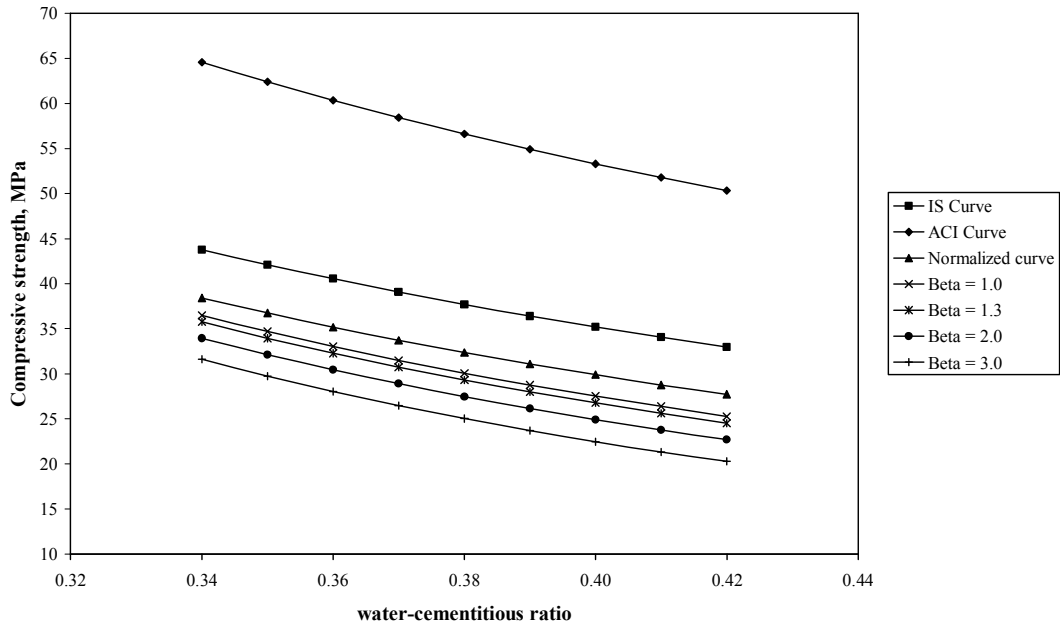


Fig. 6.1 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes without silica fume (Normal distribution)

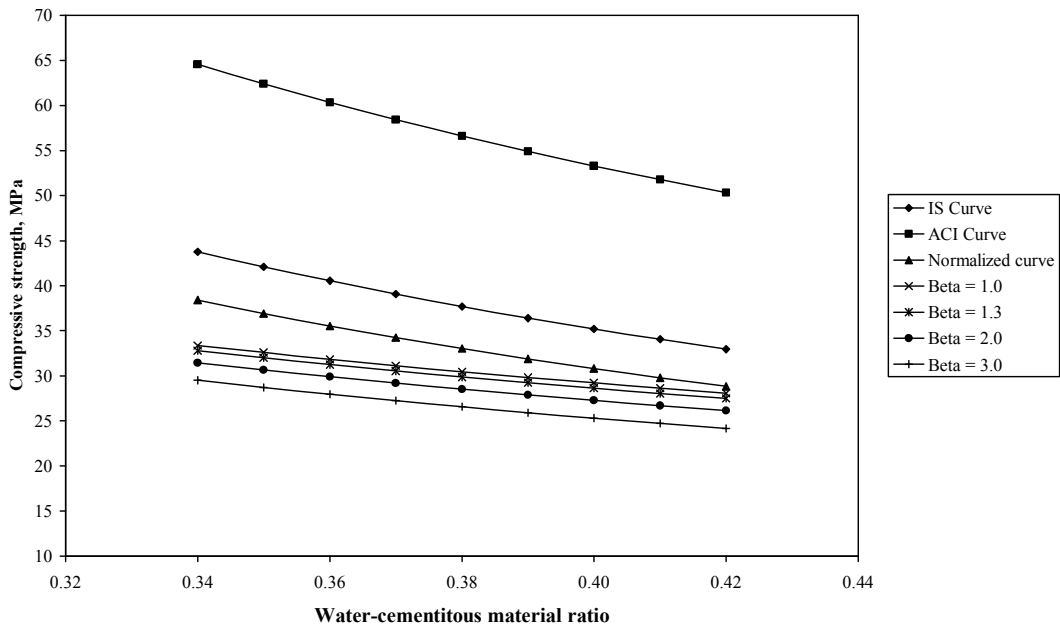


Fig. 6.2 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes with 5 percent silica fume (Normal distribution)

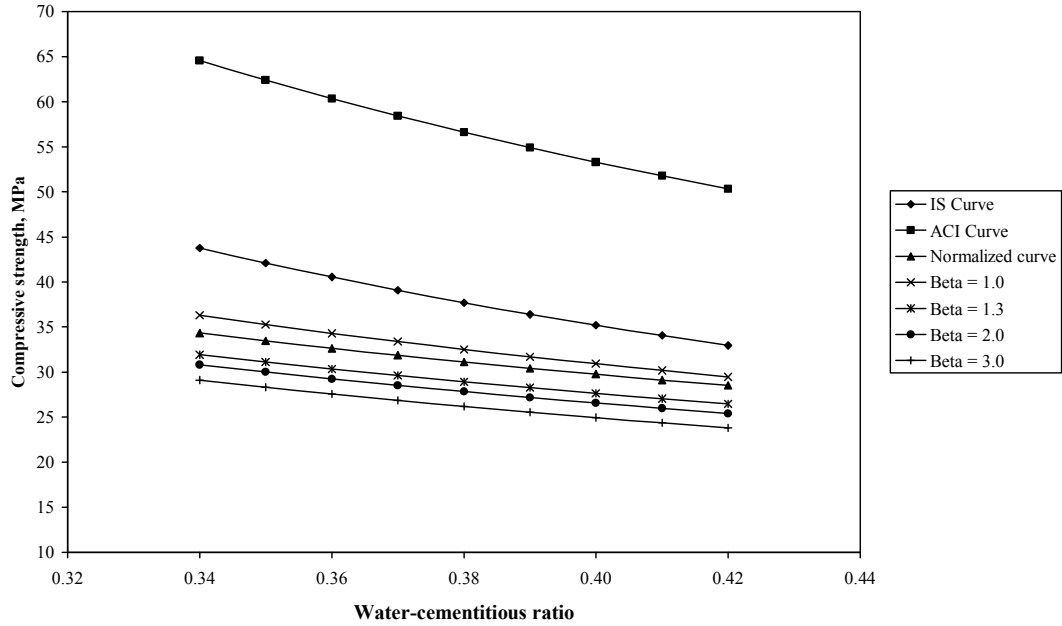


Fig. 6.3 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes with 10 per cent silica fume (Normal distribution)

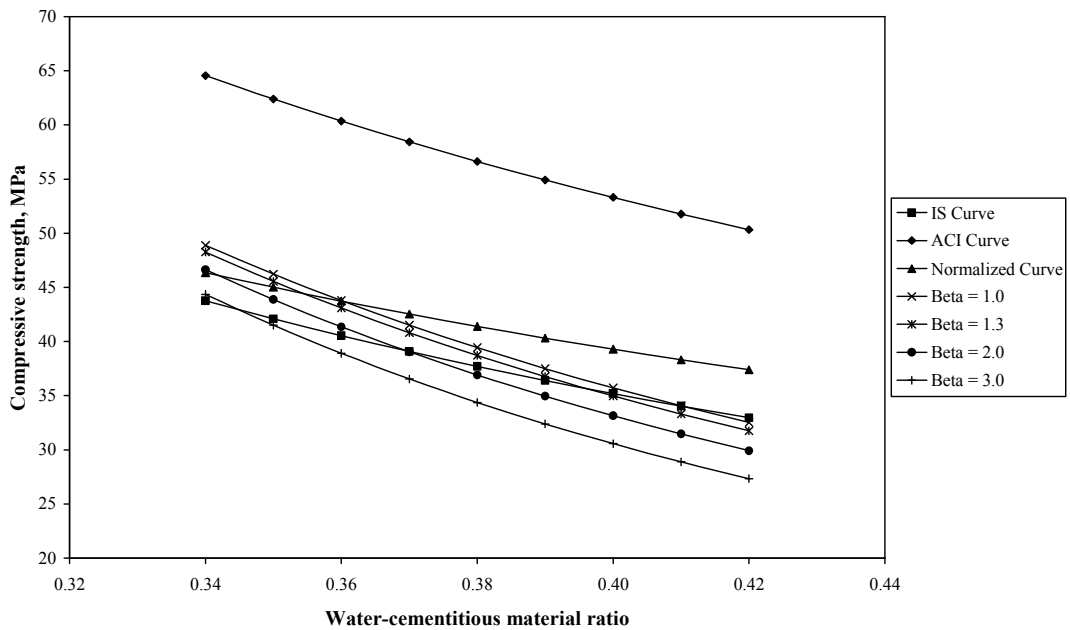


Fig. 6.4 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes without silica fume (Normal distribution)

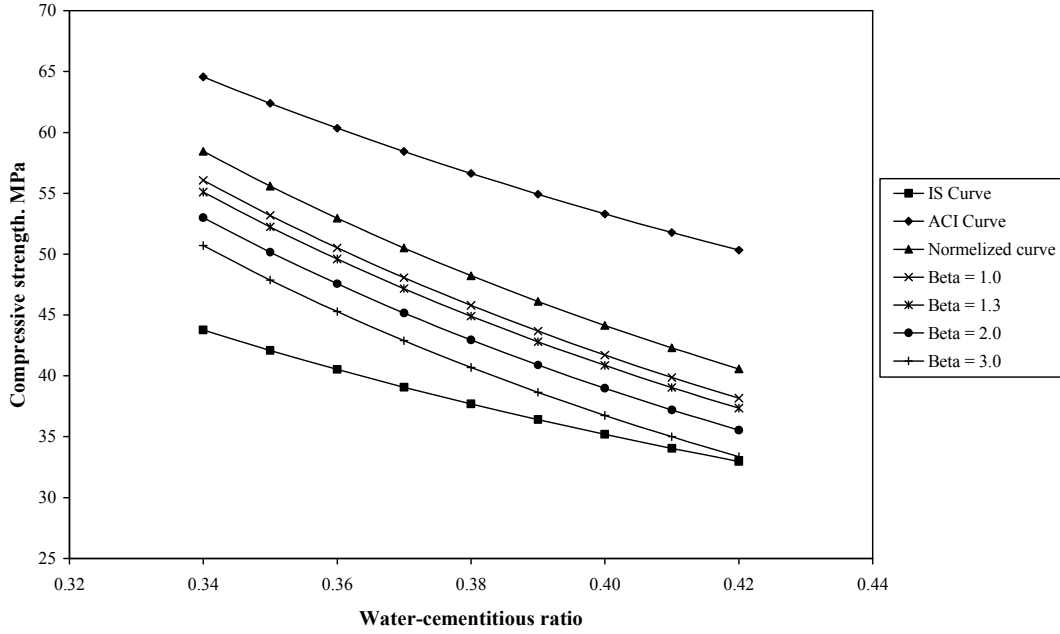


Fig. 6.5 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes with 5 per cent silica fume (Normal distribution)

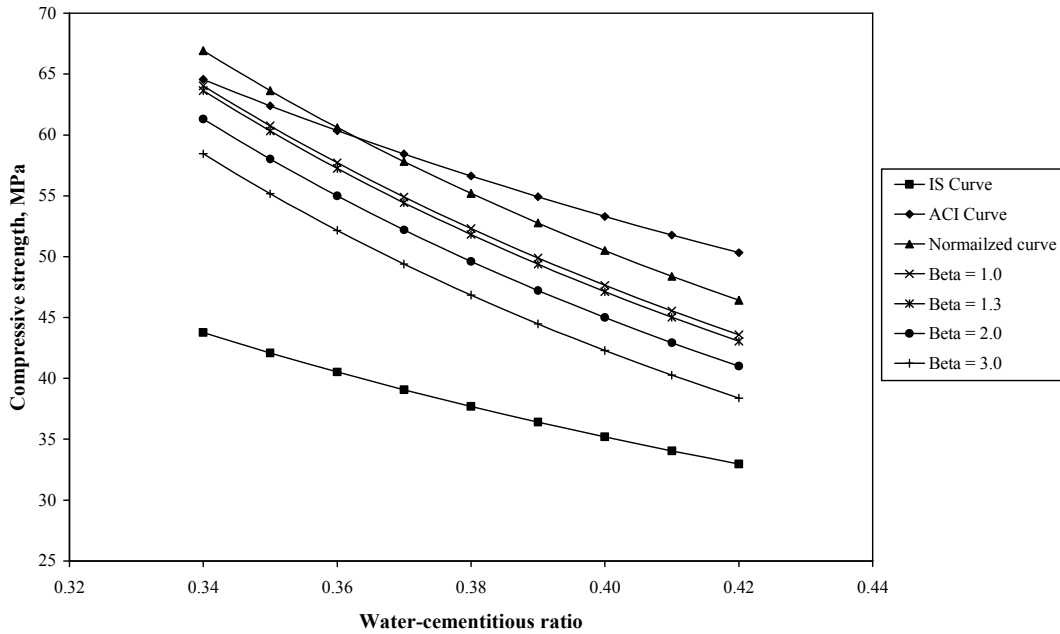


Fig. 6.6 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes with 10 per cent silica fume (Normal distribution)

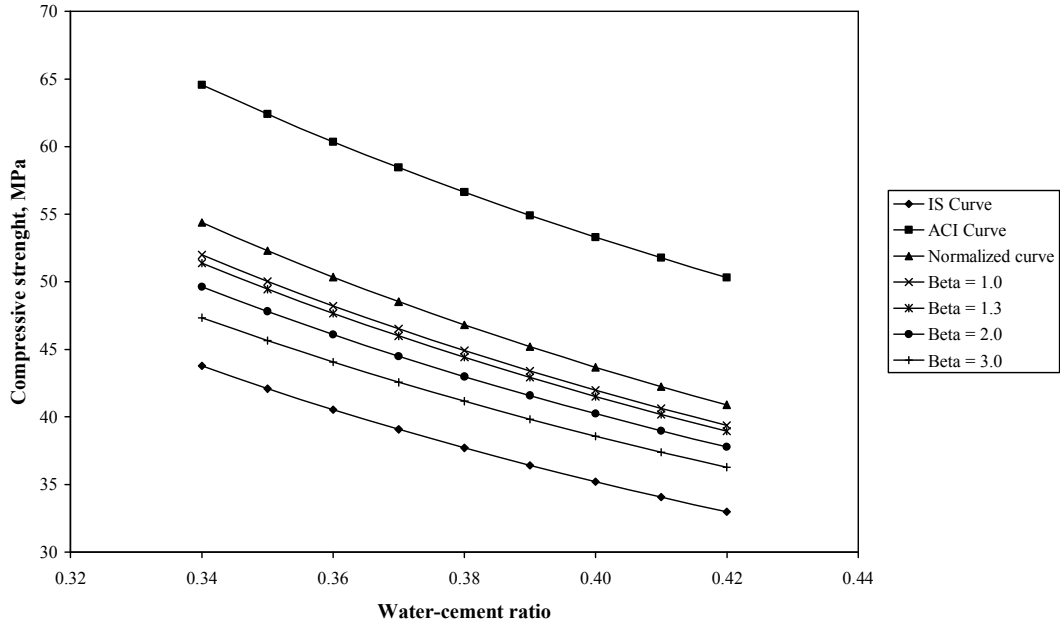


Fig. 6.7 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes without silica fume (Normal distribution)

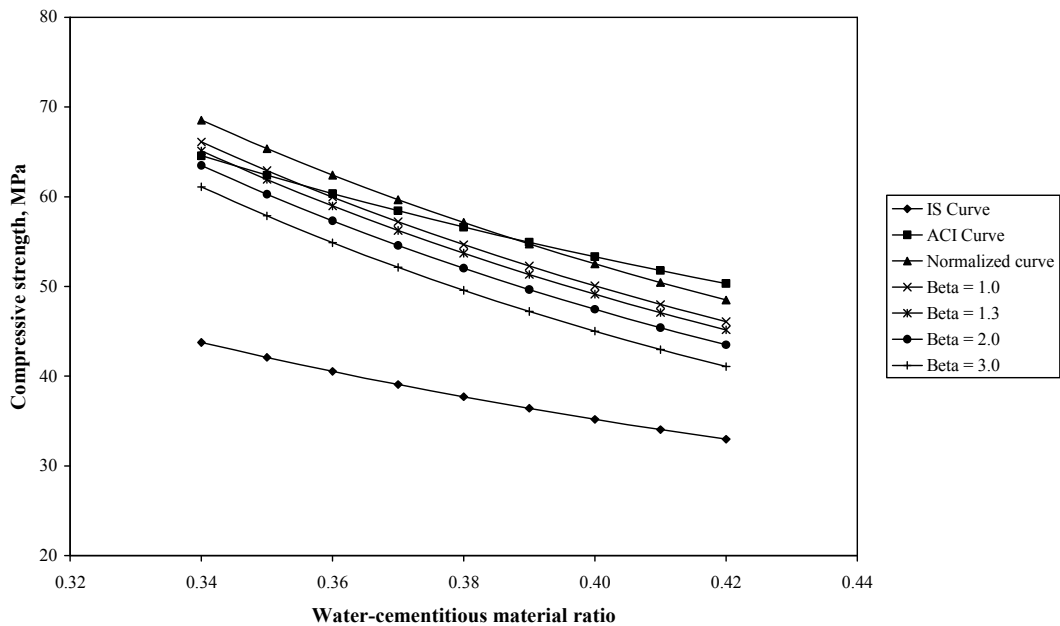


Fig. 6.8 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes with 5 per cent silica fume (Normal distribution)

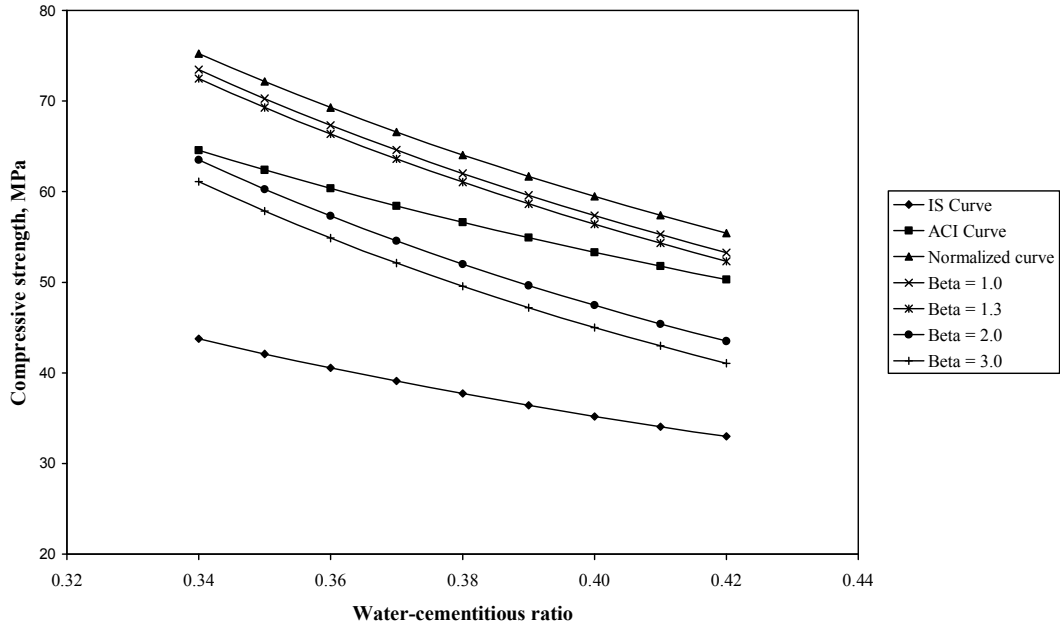


Fig. 6.9 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes with 5 per cent silica fume (Normal distribution)

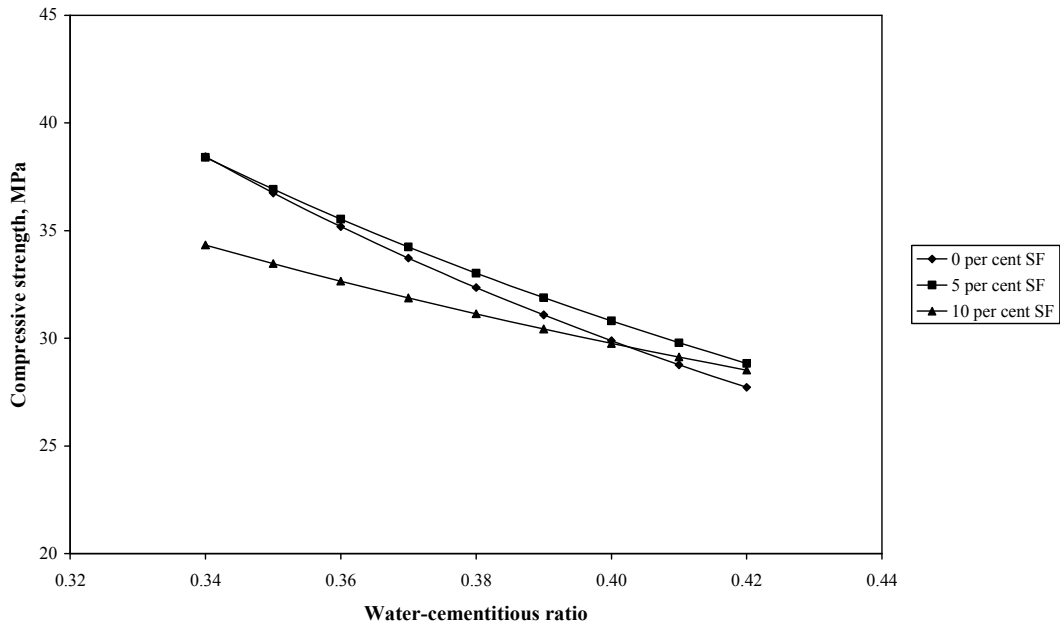


Fig. 6.10 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume (Normal distribution)

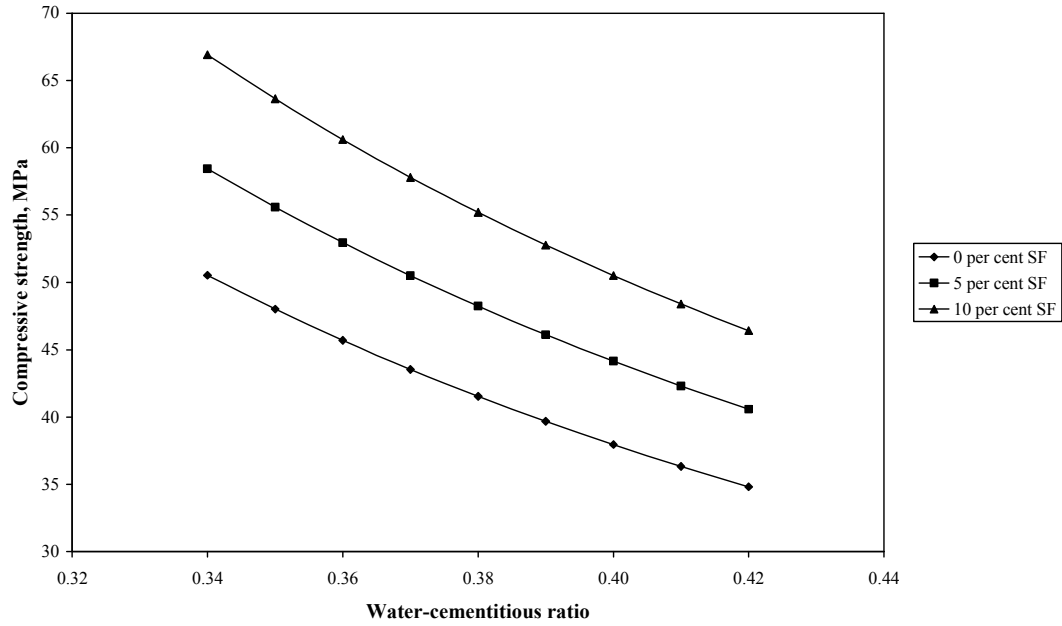


Fig. 6.11 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume (Normal distribution)

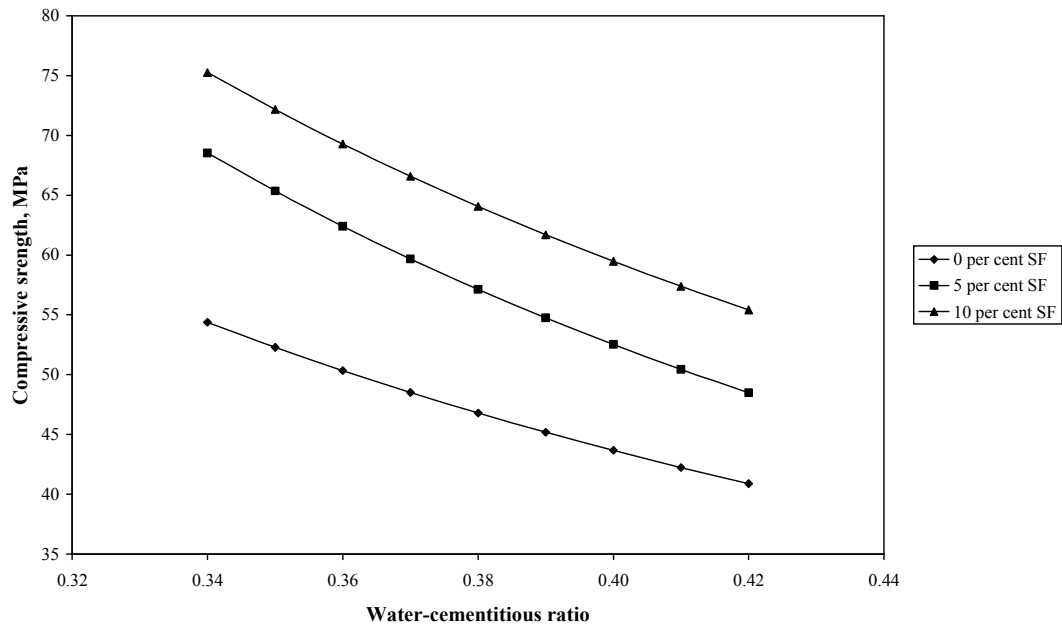


Fig. 6.12 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume (Normal distribution)

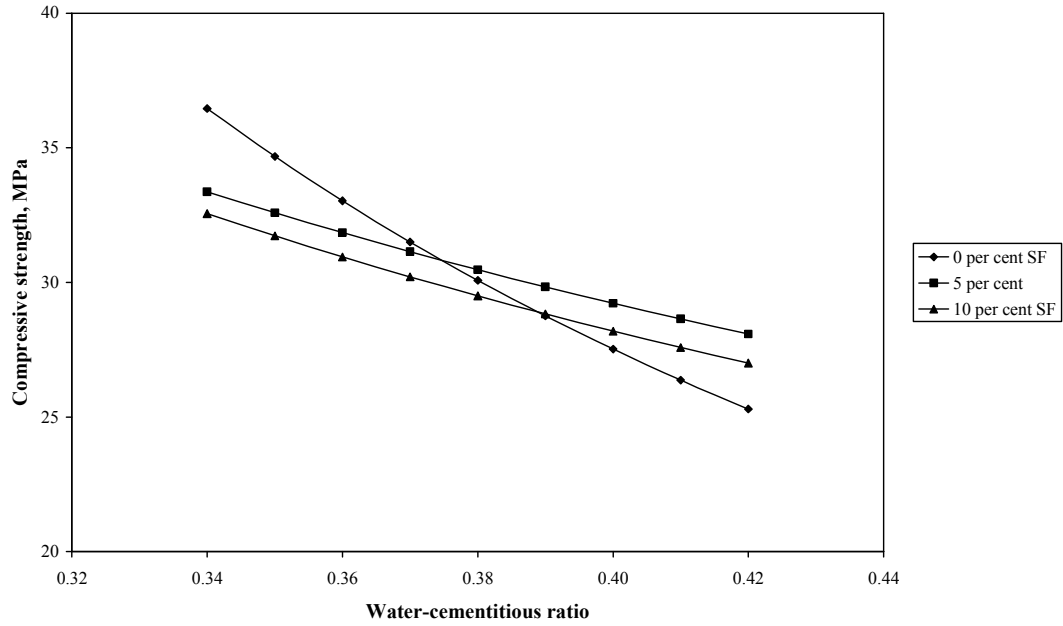


Fig. 6.13 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.0$)

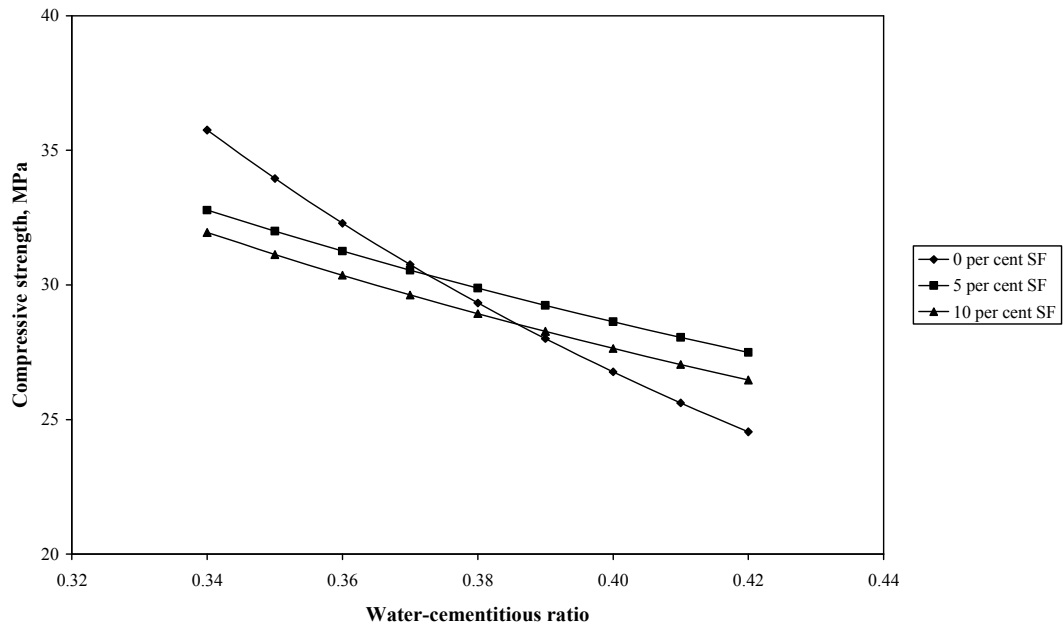


Fig. 6.14 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.3$)

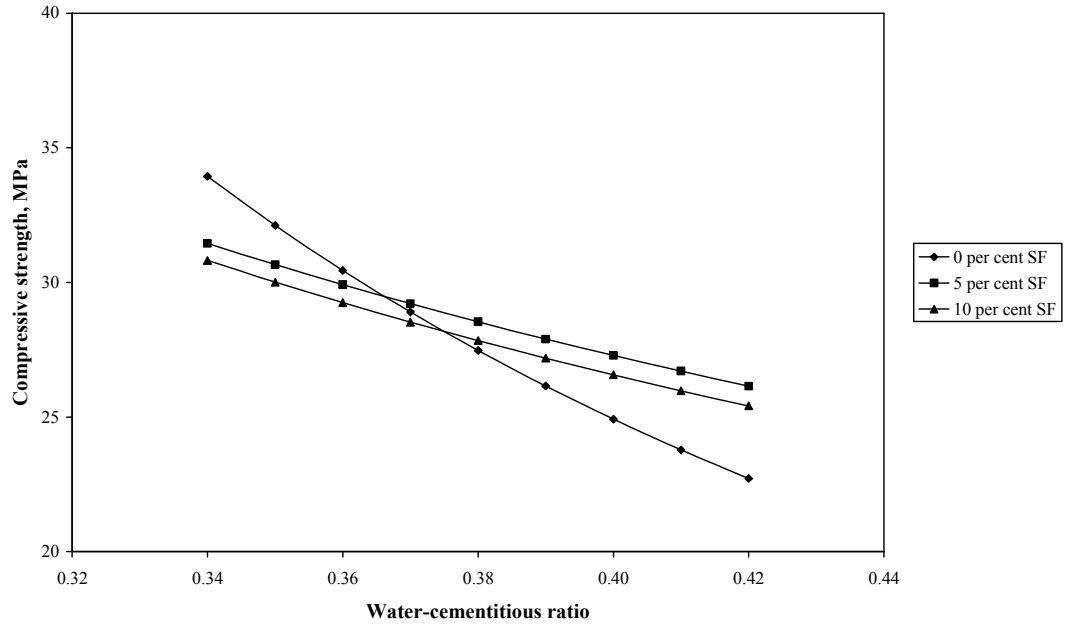


Fig. 6.15 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 2.0$)

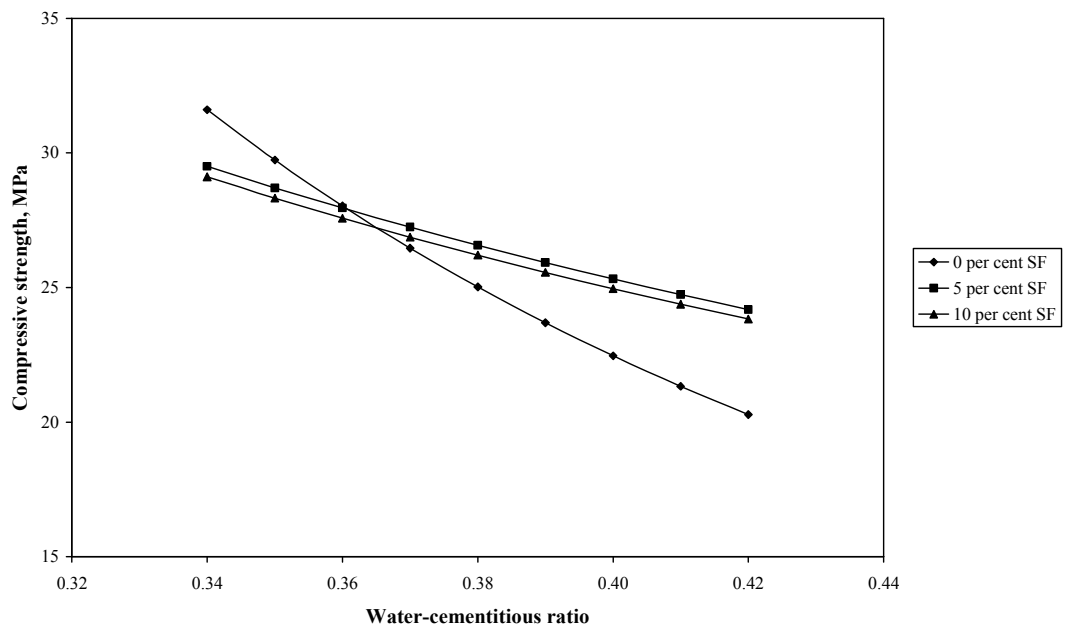


Fig. 6.16 Variation of 7 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 3.0$)

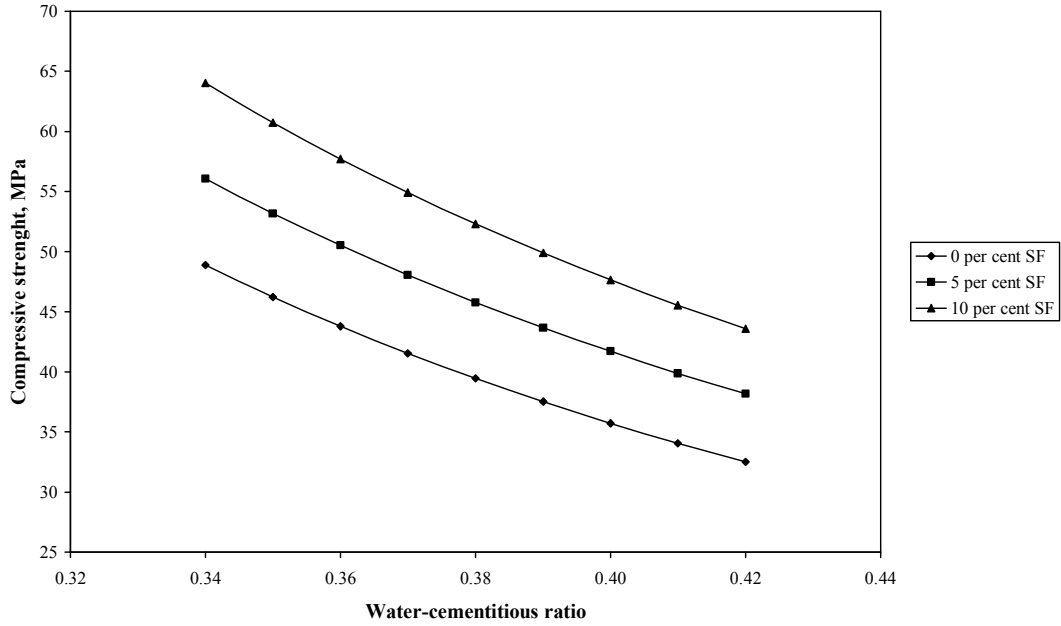


Fig. 6.17 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.0$)

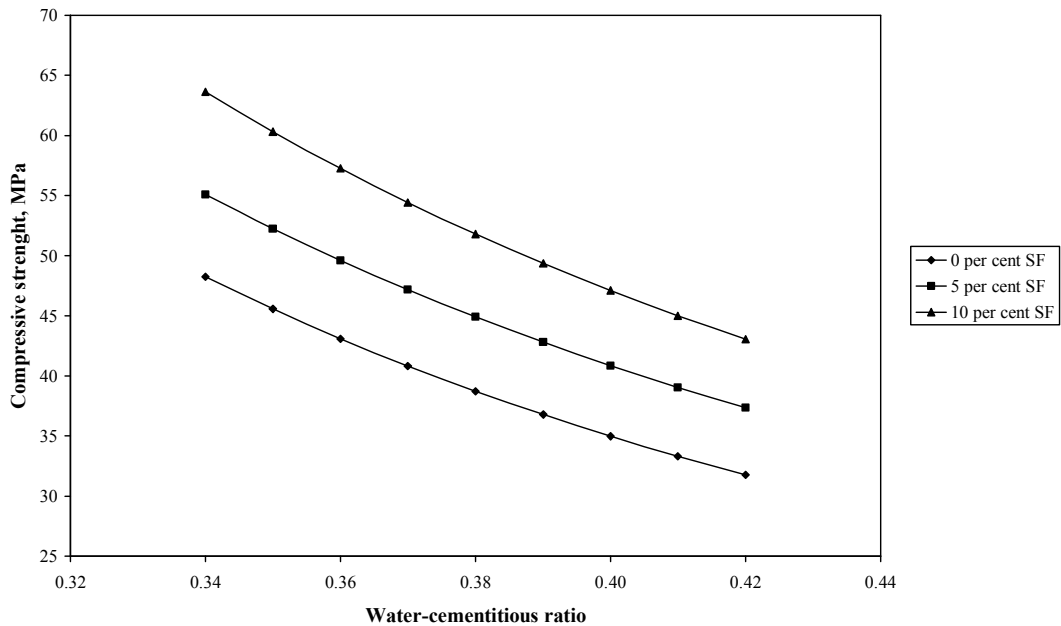


Fig. 6.18 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.3$)

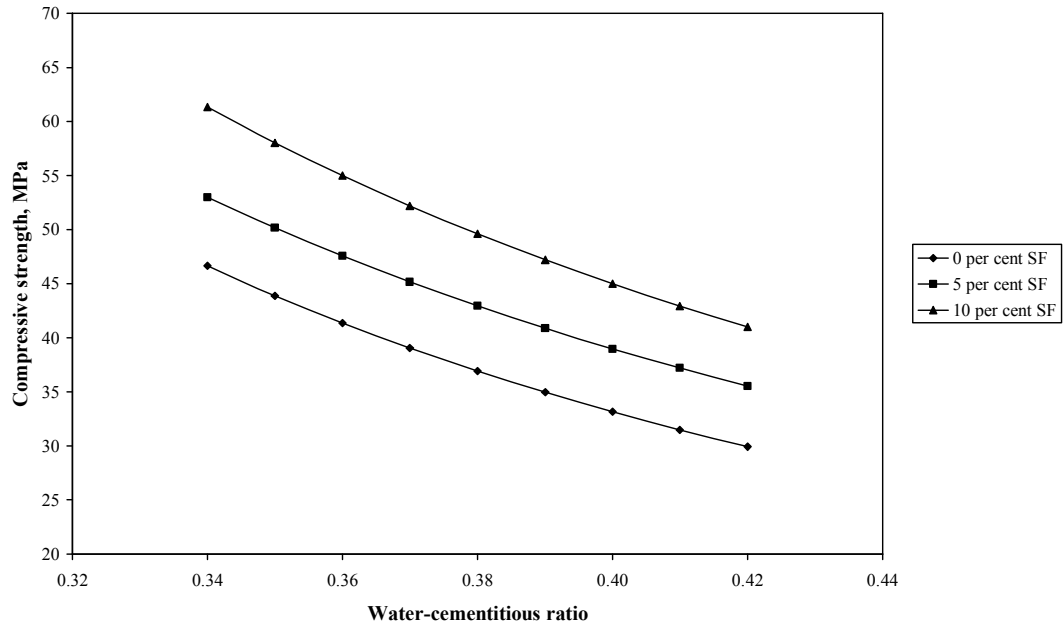


Fig. 6.19 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 2.0$)

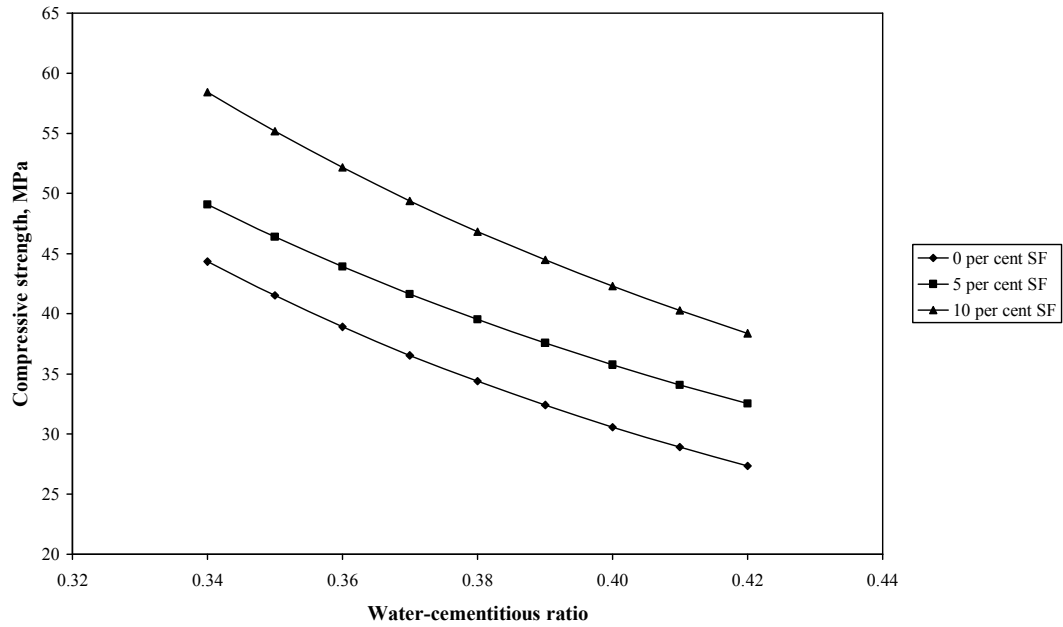


Fig. 6.20 Variation of 28 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 3.0$)

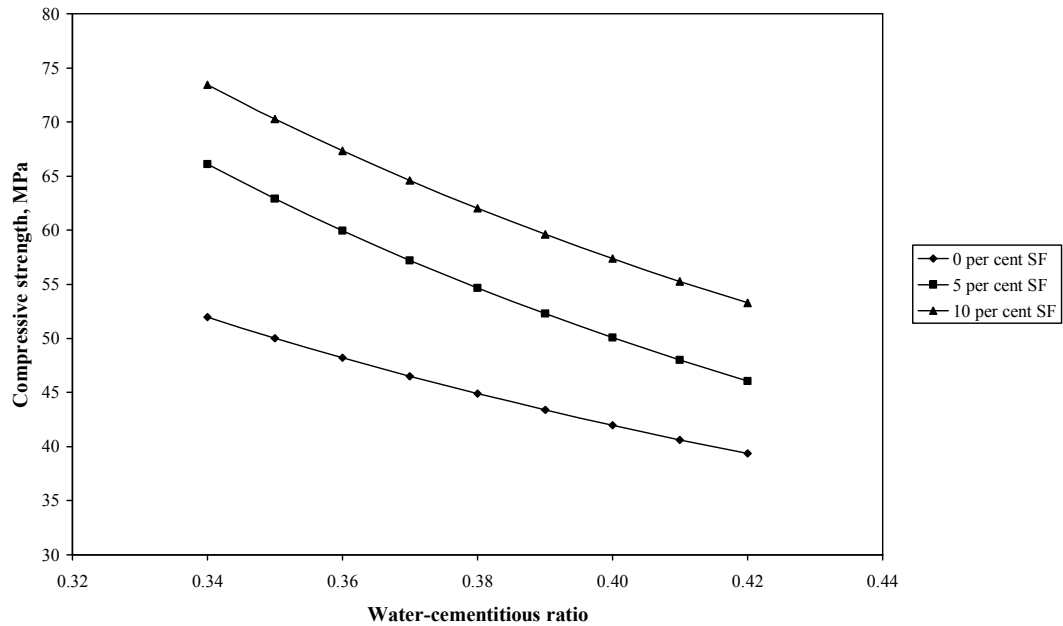


Fig. 6.21 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.0$)

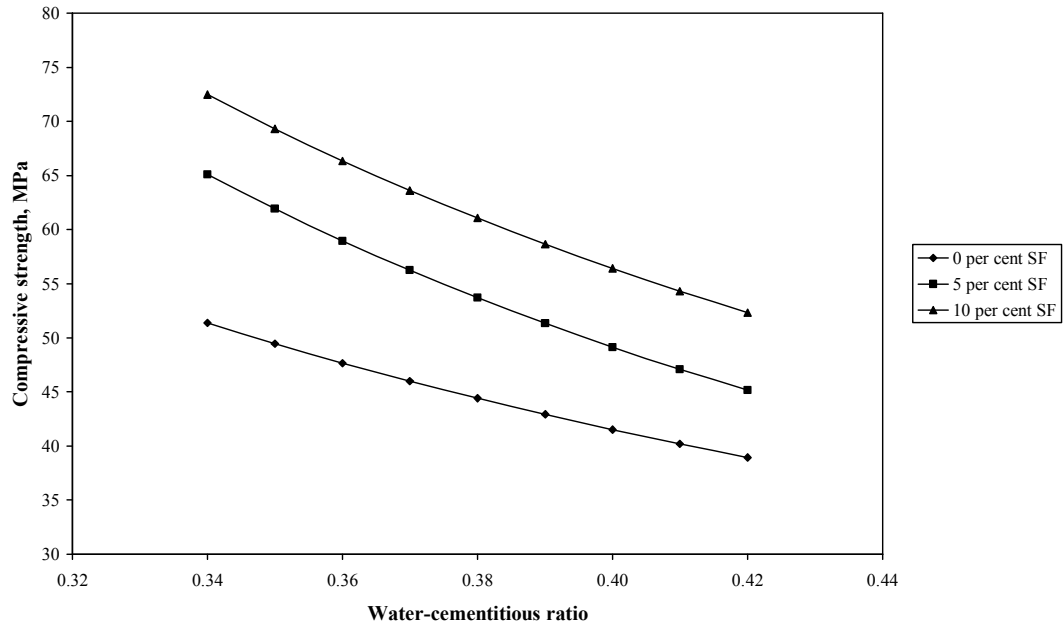


Fig. 6.22 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 1.3$)

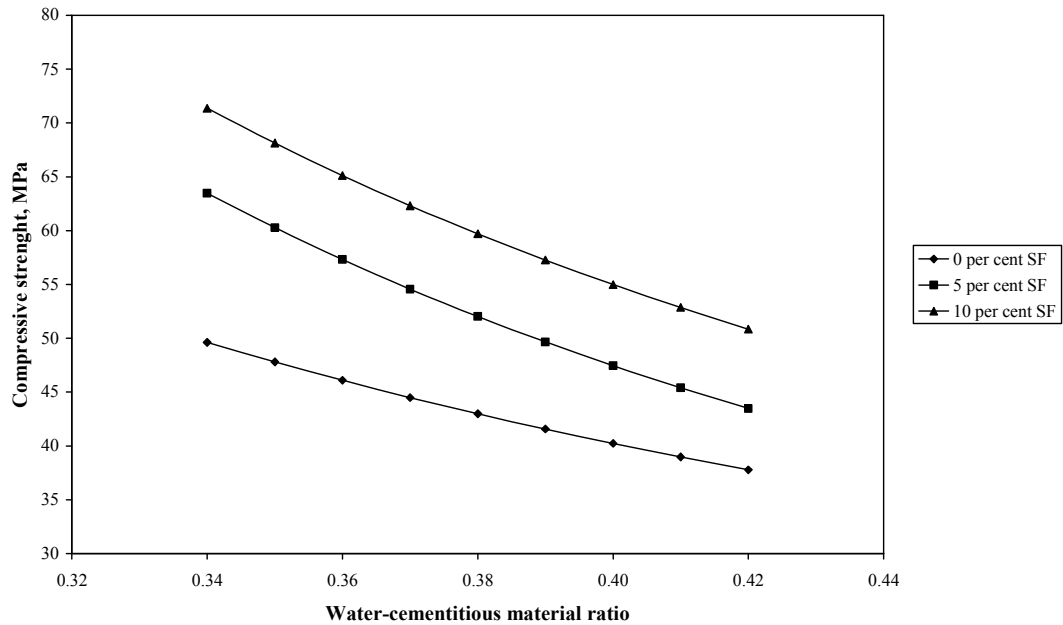


Fig. 6.23 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 2.0$)

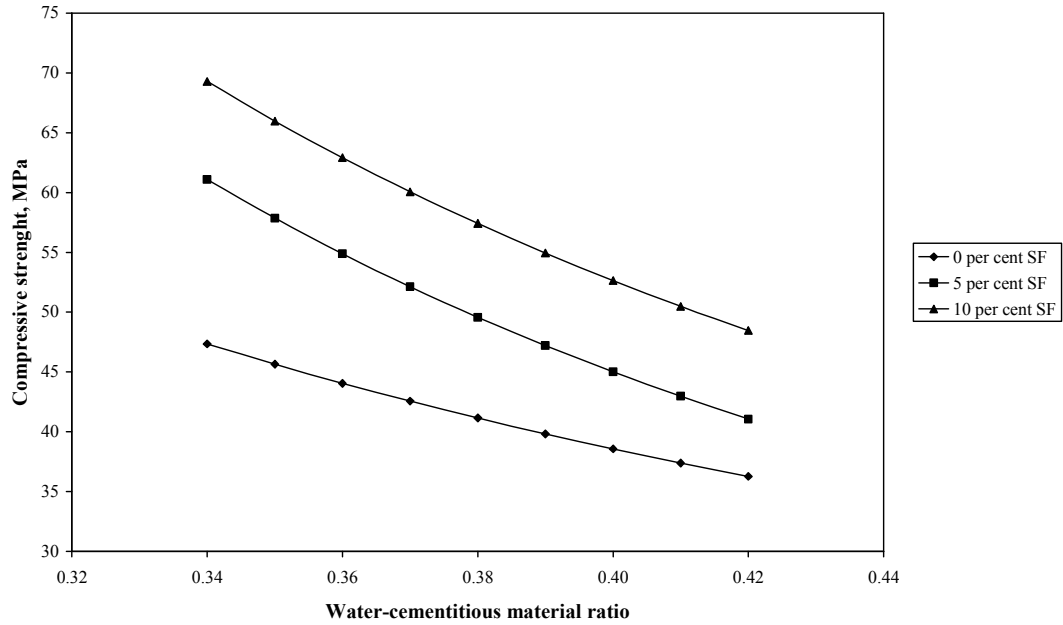


Fig. 6.24 Variation of 56 days compressive strength with water-cementitious material ratio for concrete mixes at 0, 5 and 10 per cent silica fume ($\beta = 3.0$)

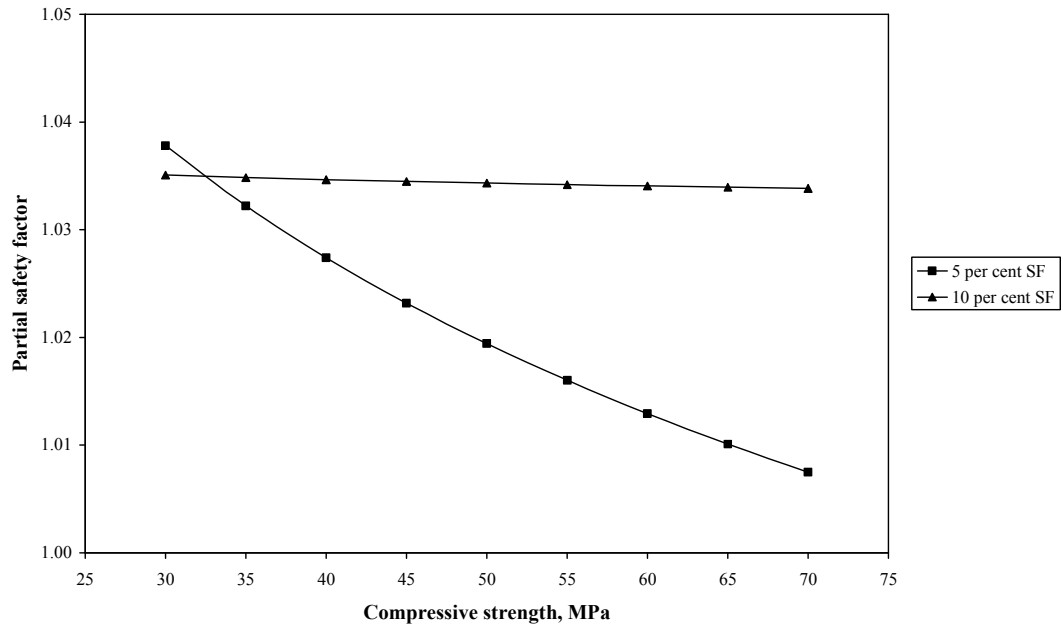


Fig. 6.25 Variation of partial safety factors with 7-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.0$)

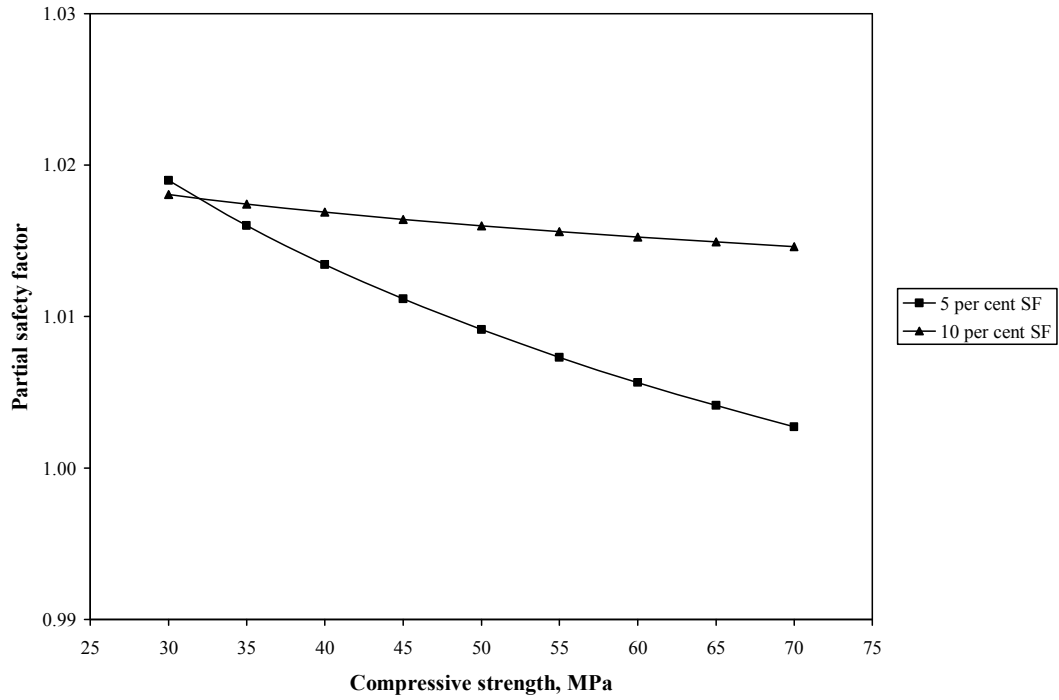


Fig. 6.26 Variation of partial safety factors with 7-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.3$)

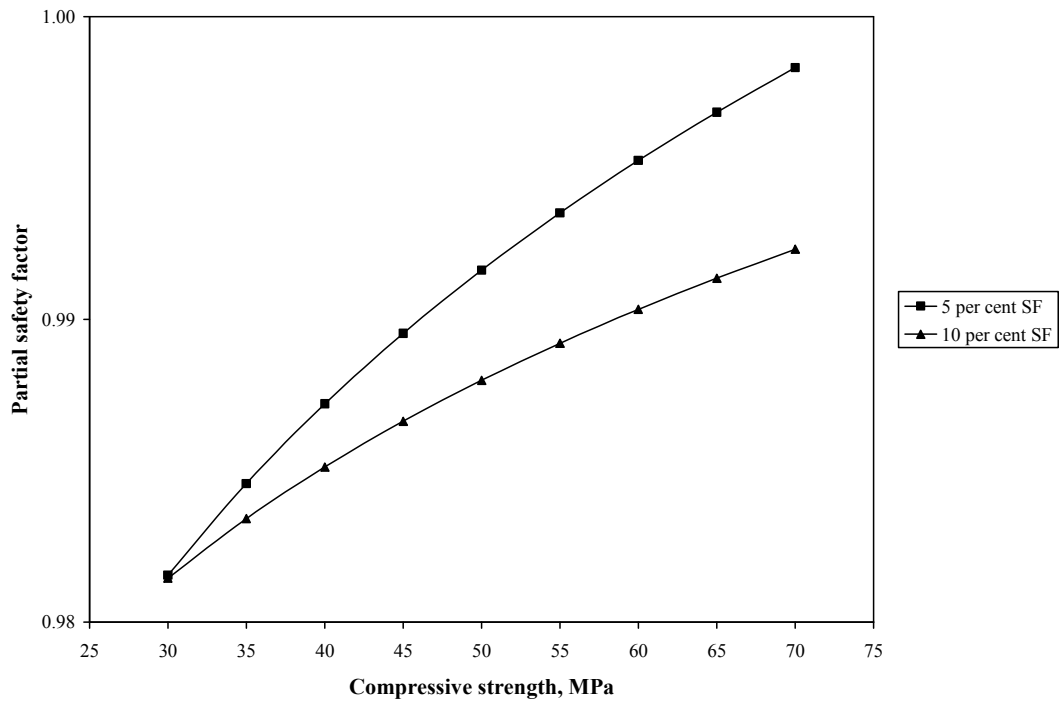


Fig. 6.27 Variation of partial safety factors with 7-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 2.0$)

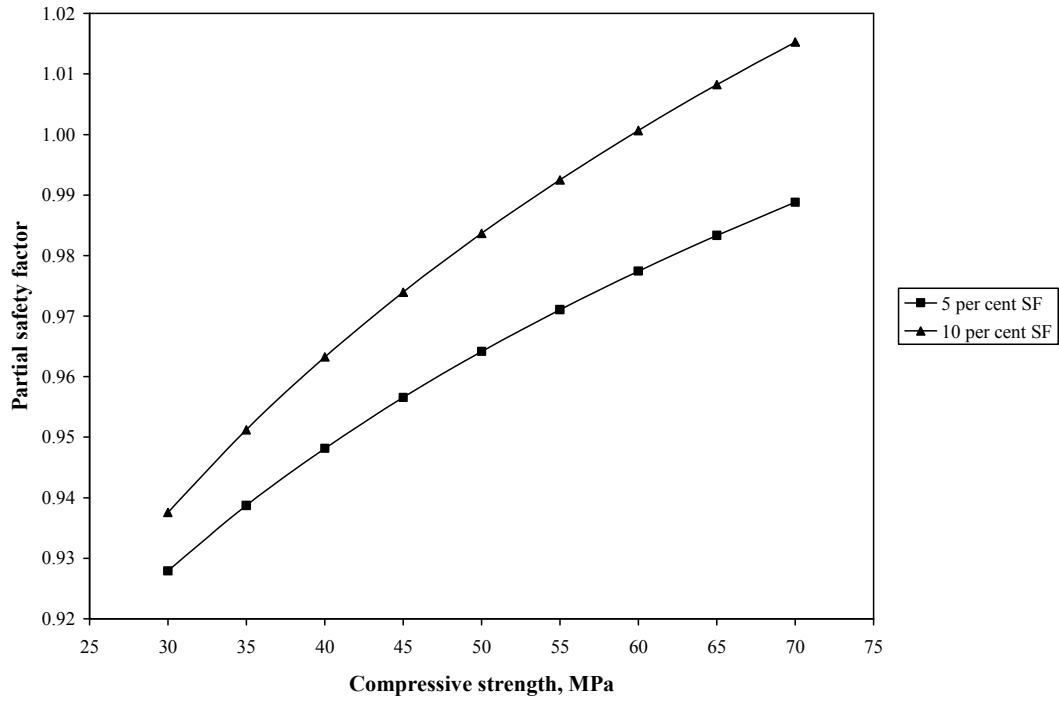


Fig. 6.28 Variation of partial safety factors with 7-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 3.0$)

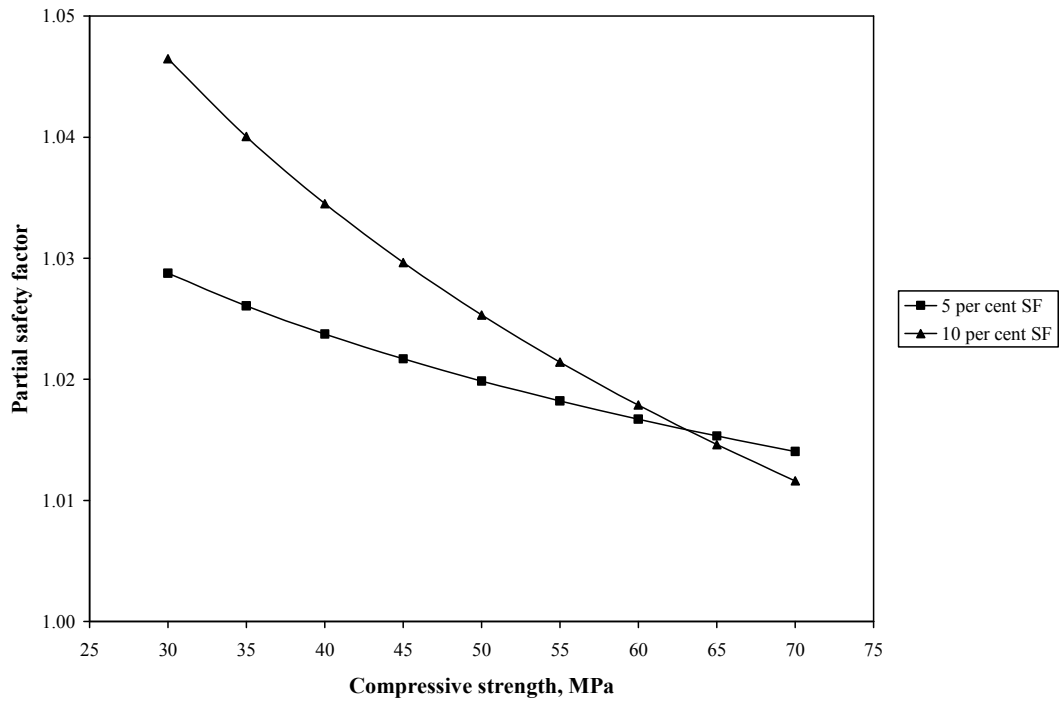


Fig. 6.29 Variation of partial safety factors with 28-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.0$)

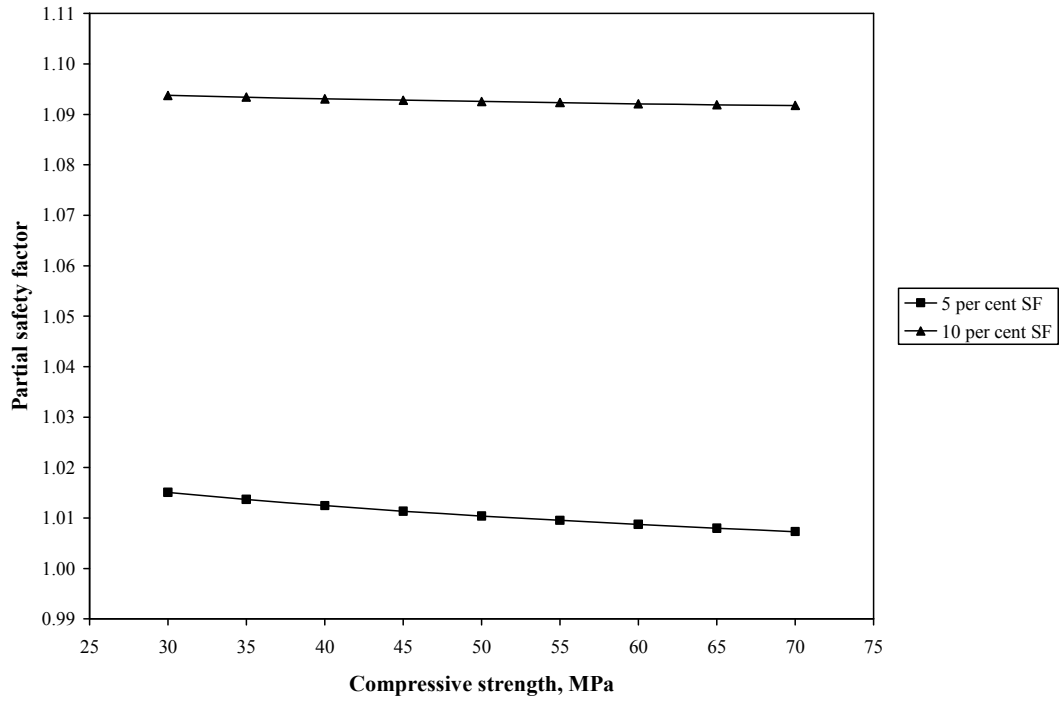


Fig. 6.30 Variation of partial safety factors with 28-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.3$)

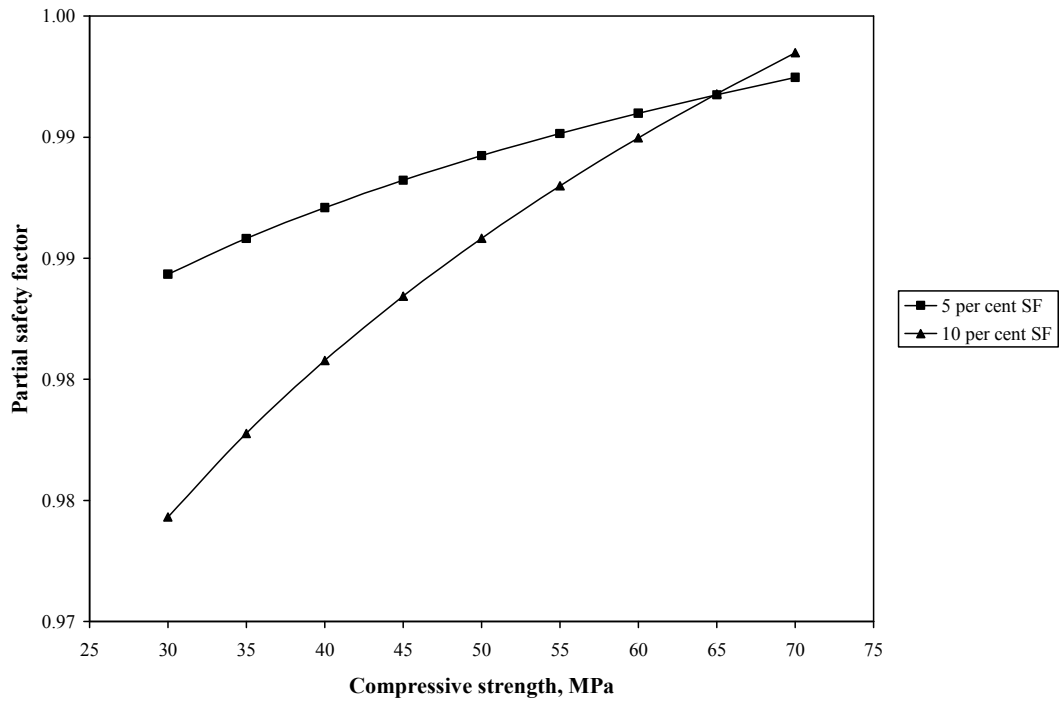


Fig. 6.31 Variation of partial safety factors with 28-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 2.0$)

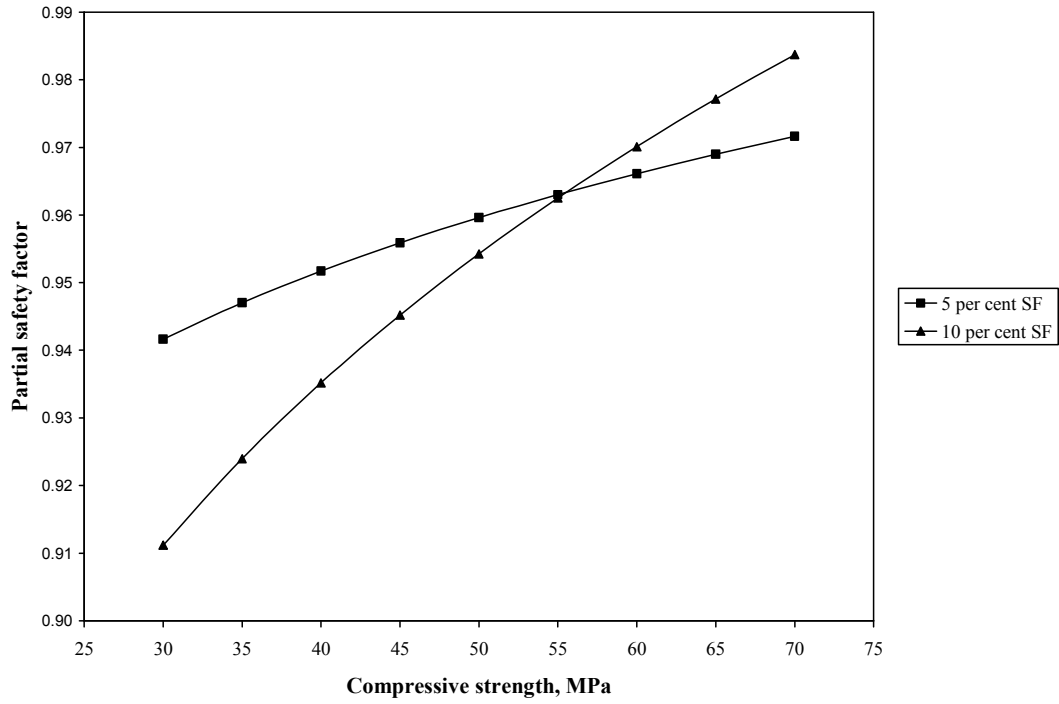


Fig. 6.32 Variation of partial safety factors with 28-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 3.0$)

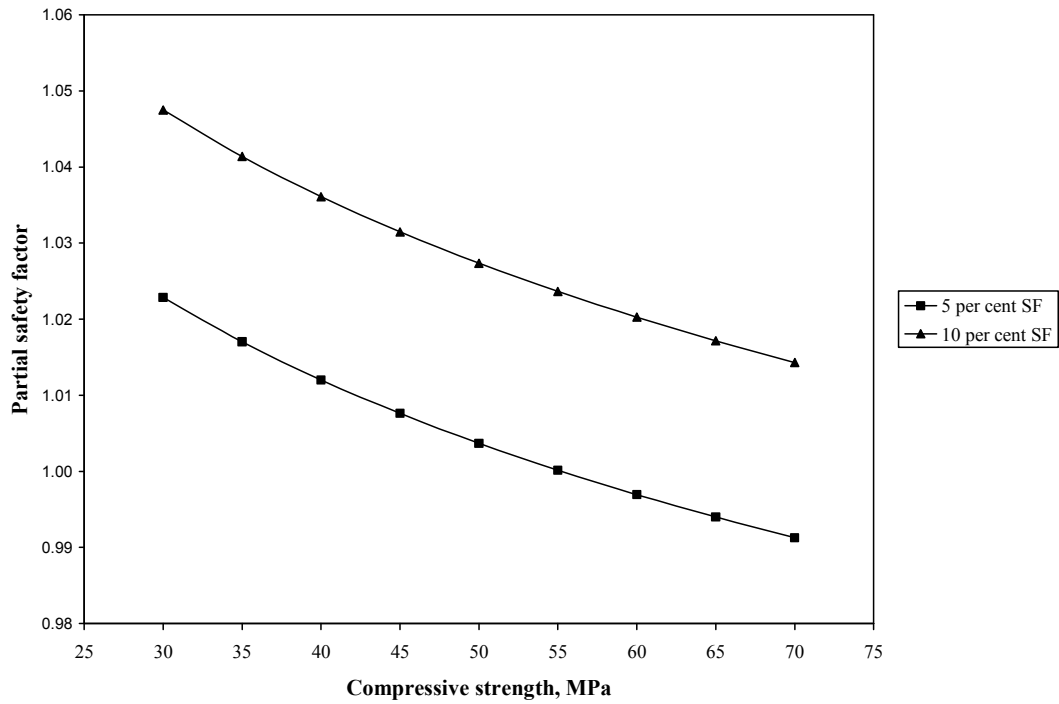


Fig. 6.33 Variation of partial safety factors with 56-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.0$)

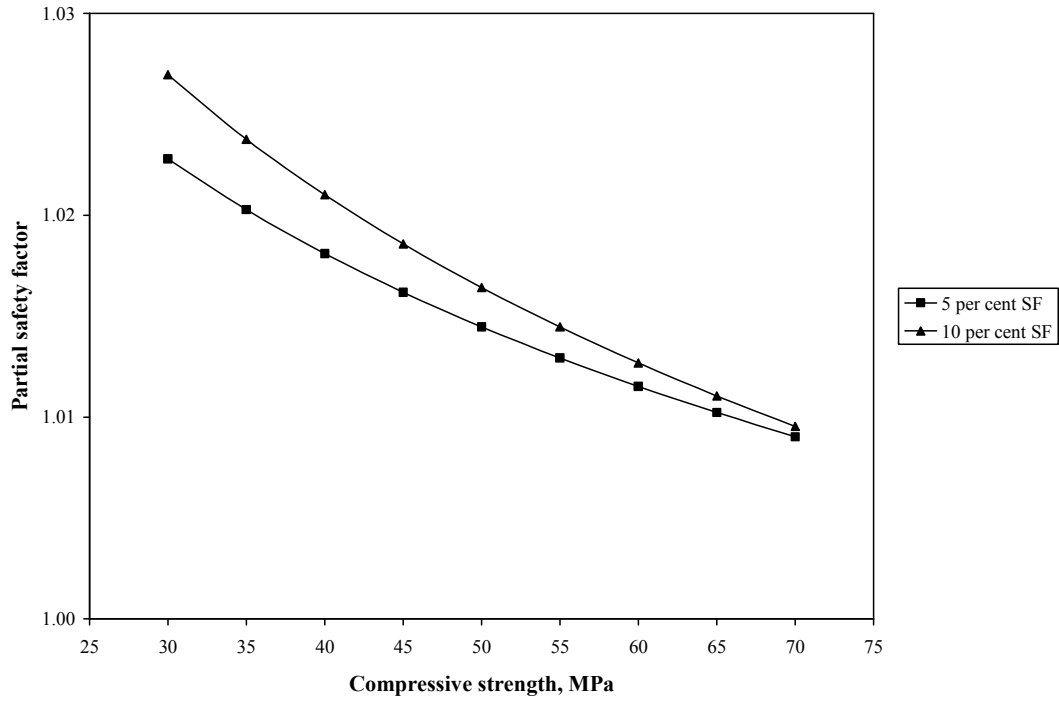


Fig. 6.34 Variation of partial safety factors with 56-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 1.3$)

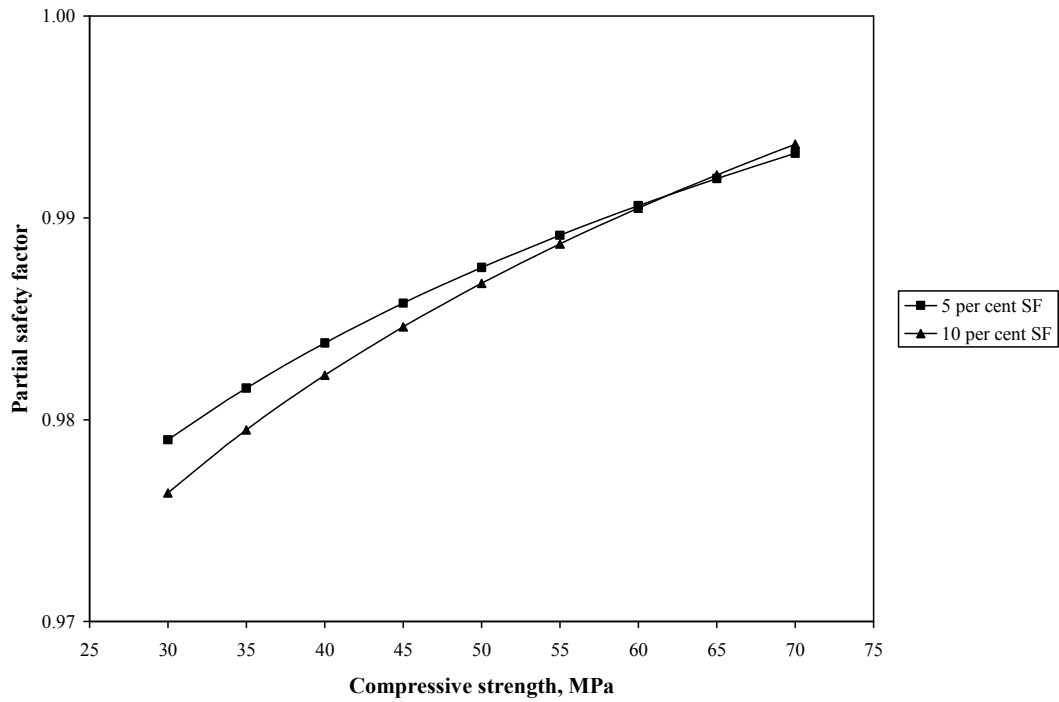


Fig. 6.35 Variation of partial safety factors with 56-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 2.0$)

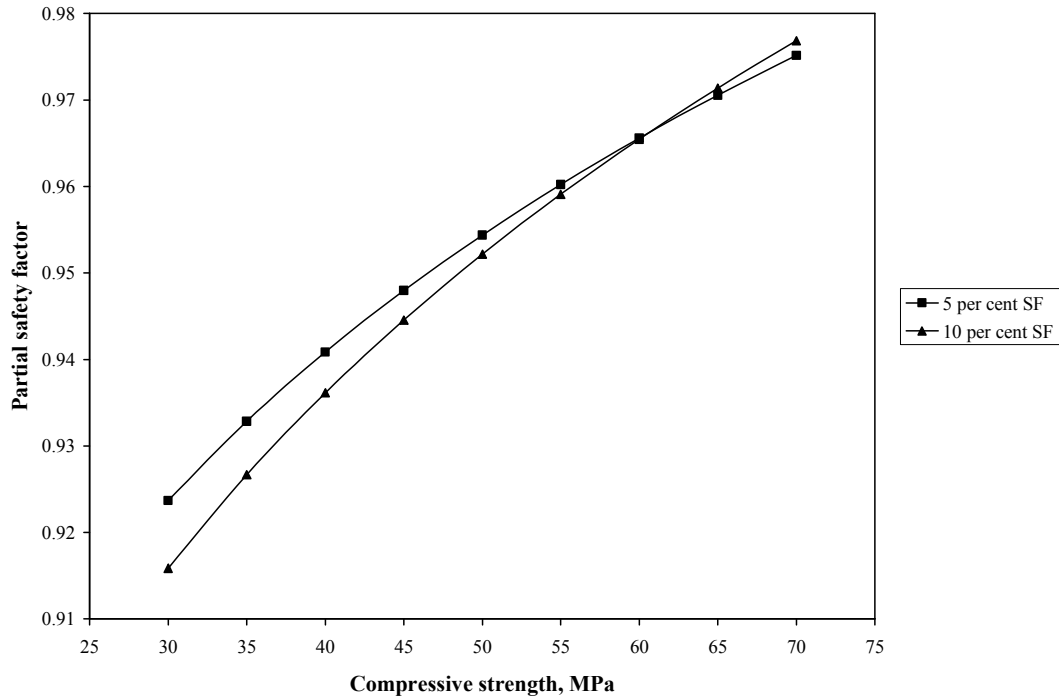


Fig. 6.36 Variation of partial safety factors with 56-days compressive strength for concrete mixes at 5 and 10 per cent silica fume ($\beta = 3.0$)

CONCLUSIONS

7.1 GENERAL

Partial safety factors for variation in the material strengths have been computed, in the present work, from the analysis of the experimental data generated. These partial safety factors have been used for development of the reliability-based design of concrete mixes, without and with 5 and 10 per cent replacement of cement by silica fume proportioned as per the procedure developed in the present work. On the basis of the present study, following conclusions are drawn.

7.2 PROPERTIES OF CONCRETE

7.2.1 Workability

- 1) A concrete mix with silica fume requires less water up to 10 per cent replacement than the mix without silica fume for the same workability. However, the workability gets decreased as the cement replaced by silica fume increases to 15 per cent.
- 2) A concrete mix without silica fume requires more dosage of superplasticizer (SP-430) than concrete mix with 5 and 10 per cent silica fume, respectively, for same workability. However, concrete mix with 15 per cent silica fume requires more dosage of superplasticizer than concrete mix without silica fume.

7.2.2 Statical properties of compressive strength

- 1) With-in-test coefficient of variation being less than the acceptance value of 3.0 per cent [ACI (1998)] indicates that the testing conditions were uniform. It implies that the testing conditions do not affect the results of compressive strength significantly.
- 2) The 7 and 28 days compressive strength of most of the concrete mixes with or without silica fume have coefficient of variation lying between 3.0 to 5.0 per cent, whereas coefficient of variation for concrete mixes cured for 56 days is less than 3 per cent, also the coefficient of variation is less for mixes with silica fume, indicating that the quality of concrete improves with increase in silica fume percentage as well as age of curing.

7.3 COMPRESSIVE STRENGTH

- 1) Concrete mixes proportioned for higher reliability index require lower $w/(c+p)$ ratio to ensure reduced probability of failure.
- 2) Concrete mixes with 5 per cent replacement of cement by silica fume designed for 7 days have more strength than concrete mixes without silica fume.
- 3) Concrete mixes with 5 and 10 per cent replacement of cement by silica fume designed at curing of 28 and 56 days, respectively, have more strength as compared to concrete without silica fume.
- 4) As the $w/(c+p)$ ratio decreases, the compressive strength of concrete mixes with 5 and 10 per cent silica fume increases.

7.4 PARTIAL SAFETY FACTORS

- 1) The partial safety factor decreased with the increase in percentage of silica fume in concrete, also signifying that partial safety factor decreased with increase in compressive strength.
- 2) The concrete mixes with respect to 7 days compressive strength give higher partial safety factors than the concrete mixes proportioned with respect to 28 and 56 days.
- 3) Concrete mixes with silica fume provide lower partial safety factors.
- 4) Partial safety factors vary exponentially with increase in reliability index.

SCOPE FOR FURTHER WORK

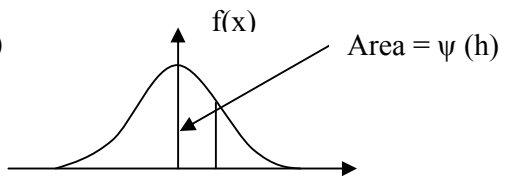
- 1) In the present study only up to 10 per cent replacement of cement by silica fume has been considered. The other percentages i.e. 15 and 20 per cent need investigation.
- 2) In the present study the normal-distribution is considered for the strength of concrete mixes. The other probability distributions like lognormal-distribution etc. for strength of concrete also need investigations.
- 3) In this study only zone-A aggregates have been used. The other proportion of aggregates i.e. CA-I 50 per cent and CA-II 50 per cent, CA-II 50 per cent and CA-III (passing 4.75mm sieve and retained on 2.36mm sieve) 50 per cent also need investigations.

APPENDIX – 1

Table A1.1 Standardized normal variate N (0, 1)

$$\psi(h) = \frac{1}{\sqrt{2\pi}} \int_0^h \exp\left[-\frac{x^2}{2}\right] dx,$$

$$h = \frac{|\bar{x} - x|}{\sigma[x]}$$



h	0	1	2	3	4	5	6	7	8	9
0	0	.00397	.00798	.01197	.01595	.01994	.02392	.02790	.03188	.03586
.1	.0398	.04379	.04775	.05171	.05567	.05962	.06355	.06749	.07142	.07534
.2	.0793	.08317	.08706	.09095	.09483	.09870	.10256	.10642	.11025	.11409
.3	.11791	.12172	.12551	.12930	.13307	.13683	.14057	.14430	.14802	.15173
.4	.15542	.15910	.16275	.16640	.17003	.17364	.17724	.18082	.18438	.18793
.5	.19146	.19497	.19846	.20194	.20540	.20884	.21226	.21566	.21904	.22240
.6	.22575	.22907	.23237	.23565	.23491	.24215	.24537	.24857	.25174	.25490
.7	.25804	.26115	.26423	.26730	.27035	.27337	.27637	.27935	.28230	.28523
.8	.28814	.29103	.29389	.29673	.29954	.30233	.30510	.30785	.31057	.31326
.9	.31594	.31859	.32121	.32381	.32639	.32894	.33147	.33397	.33645	.33891
1.0	.34134	.34375	.34613	.34849	.35083	.35314	.35542	.35769	.35992	.36214
1.1	.36433	.36650	.36864	.37076	.37285	.37492	.37697	.37900	.38100	.38297
1.2	.38493	.38686	.38876	.39065	.39251	.39435	.39616	.39795	.39970	.40147
1.3	.40320	.40490	.40658	.40821	.40987	.41149	.41308	.41465	.41602	.41773
1.4	.41924	.42073	.42219	.42364	.42506	.42647	.42785	.42921	.43056	.43188
1.5	.43319	.43448	.43574	.43799	.43822	.43942	.44062	.44179	.44294	.44408
1.6	.44520	.44630	.44738	.44844	.44949	.45052	.45154	.45254	.45352	.45448
1.7	.45544	.45637	.45728	.45818	.45907	.45994	.46079	.46163	.46246	.46327
1.8	.46407	.46485	.46562	.46635	.46711	.46784	.46855	.46925	.46994	.47062

1.9	.47128	.47193	.47257	.47319	.47361	.47441	.47500	.47558	.47614	.47670
2.0	.47725	.47778	.47830	.47882	.47932	.47981	.48030	.48077	.48123	.4819*1
2.1	.48214	.48257	.48299	.48344	.48382	.48422	.48461	.48499	.48537	.48573
2.2	.48610	.48645	.48679	.48712	.48745	.48776	.48808	.48839	.48869	.48898
2.3	.48928	.48956	.48983	.49009	.49035	.49061	.49086	.49110	.49134	.49157
2.4	.49180	.49202	.49222	.49245	.49265	.49285	.49305	.49324	.49343	.49361
2.5	.49379	.49396	.49413	.49429	.49445	.49461	.49476	.49491	.49506	.49502
2.6	.49534	.49547	.49560	.49573	.49585	.49597	.49609	.49620	.49631	.49642
2.7	.49653	.49664	.49673	.49683	.49692	.49702	.49711	.49719	.49728	.49736
2.8	.49744	.49752	.49759	.49767	.49774	.49781	.49788	.49794	.49801	.49807
2.9	.49813	.49819	.49825	.49830	.49835	.49841	.49846	.49851	.49855	.49860
3.0	.49865	.49869	.49873	.49877	.49881	.49885	.49889	.49893	.49896	.49899
3.1	.49930	.49906	.49909	.49912	.49915	.49918	.49921	.49923	.49926	.49928
3.2	.49931	.49934	.49935	.49938	.49940	.49942	.49994	.49946	.49948	.49949
3.3	.49952	.49953	.49955	.49956	.49958	.49959	.49961	.49962	.49963	.49965
3.4	.49966	.49968	.49968	.49969	.49970	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49980	.49981	.49982	.49982	.49983
3.6	.49984	.49985	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49991	.49992	.49992	.49993
3.8	.49993	.49993	.49993	.49993	.49994	.49994	.49994	.49995	.49995	.49995
3.9	.49995	.49995	.49996	.49996	.49996	.49996	.49996	.49996	.49997	.49998

APPENDIX – II

Table A2.1 Approximate entrapped air content (*SP: 23 – 1982*)

Nominal maximum size of aggregates, mm	Entrapped air, as per cent of volume of concrete
10	3.0
20	2.0
40	1.0

Table A2.2 Approximate sand water content per cubic meter of concrete (*SP: 23 – 1982*)

w/c = 0.60, Workability = 0.80 CF		
Maximum size of aggregates, mm	Water content* per cubic meter of concrete, kg	Sand as per cent of total aggregates by absolute volume
10	208	40
20	186	35

40	165	30
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* Water content corresponding to saturated surface dry aggregates

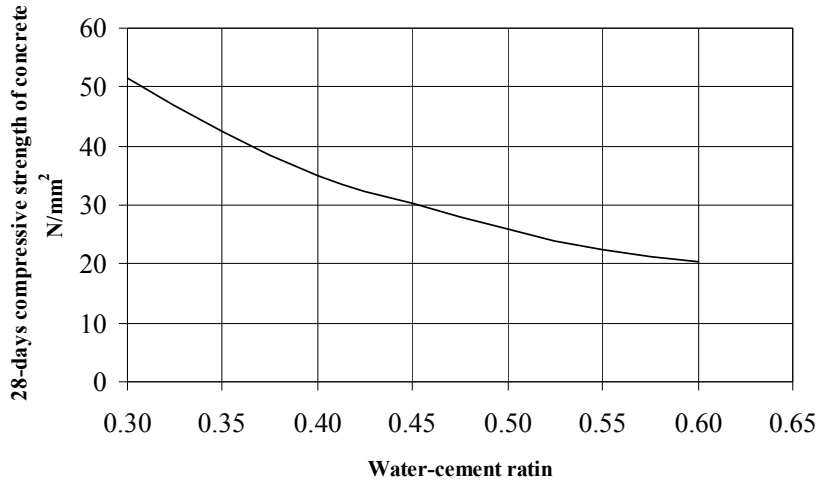


Fig. A2.1 Generalized relationship between free water-cement and compressive strength of concrete (SP: 23 – 1982)

Mix Design as per Indian Standard Recommended Guidelines (SP: 23-1982)

For M40 concrete, with very good quality control, the standard deviation is 5.0

Target strength = $40 + 1.65 \times 5.0$ = 48.25MPa

Water-cementitious material ratio (Fig. A2.1) = 0.32

Water content for medium workability, kg/m³ = 185 kg/m³

Entrapped air (Table A2.1), per cent = 2.0

Fine aggregates in total aggregates by absolute volume (Table A2.2), per cent = 35

Adjustment due to any difference from reference condition (Table A2.2)

	In Water	In Sand
	(per cent)	(per cent)
(i) For decrease in water cement ratio by 0.28	0	-5.6
(ii) For sand conforming to zone – III (IS: 383-1970)	<u>0</u>	<u>-1.5</u>
Total	<u>0</u>	<u>-7.1</u>

Water content = 185 kg/m³

Sand content = 27.9 per cent

Air content = 2.0 per cent

Concrete volume = $1.00 - 0.02$ = 0.98 m³

$$\text{Cement content} = 185/0.32 = 578.12 \text{ kg/m}^3$$

(Although the cement content exceeds the maximum stipulated value, the mix proportions have been computed)

$$\text{Total volume of aggregates} = 0.98 - \left(\frac{578.15}{3.10} + 185 \right) \frac{1}{100} = 0.6085 \text{ m}^3$$

Volume of materials

$$\text{Sand} = 0.1700 \text{ m}^3, \text{ Coarse aggregates-I} = 0.2940 \text{ m}^3, \text{ Coarse aggregates-II} = 0.1445 \text{ m}^3$$

Proportion by mass

$$\text{Sand} = 0.1700 \times (2.66 \times 1000) = 452.20 \text{ kg/m}^3$$

$$\text{Coarse agg.-I} = 0.2940 \times (2.61 \times 1000) = 767.34 \text{ kg/m}^3$$

$$\text{Coarse agg.-I} = 0.1445 \times (2.68 \times 1000) = 387.26 \text{ kg/m}^3$$

Mix proportion

The trial mix proportions (by mass) are:

Water	Cement	Fine aggregates	CA-I	CA-II
0.320	1.00	0.782	1.327	0.670

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