STRUCTURAL EVALUATION OF FLEXIBLE PAVEMENT FROM FWD DEFLECTIONS, DEFLECTION BOWL PARAMETERS & SURFACE MODULI USING MACHINE LEARNING

A Dissertation Submitted in Fulfillment of the Requirement for the Award of the Degree of

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IN

INFRASTRUCTURE ENGINEERING

Submitted by

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DECLARATION

I, Vikrant Payal hereby declare that the work presented in the Dissertation entitled **“STRUCTURAL EVALUATION OF FLEXIBLE PAVEMENT FROM FWD DEFLECTION, DEFLECTION BOWL PARAMETERS & SURFACE MODULI USING MACHINE LEARNING”** in fulfillment of the requirement for the award of degree of Master of Engineering (Infrastructure Engineering) submitted at Department of Civil Engineering, **Thapar Institute of Engineering and Technology (Deemed to be University), Patiala** is an authentic record of work carried out under the supervision of **Dr. Tanuj Chopra, Assistant Professor, CED, TIET, Patiala** and **Dr. Manpreet Singh, Assistant Professor, CED, TIET, Patiala.** The matter presented in this has not been submitted either in part or full to any other university or institute for the award of any other degree.

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This is to certify that the above declaration made by the student concerned is correct according to the best of my knowledge and opinion.

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ABSTRACT

A pavement's evaluation is necessary so that the required maintenance and rehabilitation schedule can be made in order to maintain a certain level of serviceability. Numerous structural and non-structural methods can be employed for determination of structural and functional evaluation of a pavement. Measuring both the structural and functional adequacy of pavement is time and cost intensive. In this Dissertation, a cost-effective approach for Structural evaluation of pavement by the strains developed in the pavement, Horizontal tensile strains at bottom of bituminous layer and Vertical compressive strains at top of subgrade, is presented by modelling them from the FWD (Falling Weight Deflector) Deflections, Deflection Bowl Parameters (DBPs) and Surface Modulus (SM) using various Machine Learning Techniques. Parameter FWD deflections were determined from FWD testing, from them DBPs and Surface Moduli were calculated. Data was collected from a 50km stretch of flexible pavement of a 4-lane National Highway in state of Haryana in India. For modelling the strains, Regression models and Machine Learning approaches, i.e., Artificial Neural Network, Gaussian Process Regression, Support Vector Machine, Regression Trees, etc., were used. The developed models presented good results for both horizontal and vertical strains and can be employed by stakeholders for analysis of pavement life, distress analysis in pavements and further guide the future detailed investigation at the project level and is less resource and time intensive with relatively simpler approach. Applicability of the developed models for preliminary determination of maintenance and rehabilitation schedule of pavement at both Project and Network levels is possible using this.

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1.1 PAVEMENT ANALYSIS

1.1.1 Flexible Pavement

These are pavements constructed of bituminous and granular materials. It is also known as **asphalt pavements.**

Following are the types of flexible pavements:

- **Conventional flexible pavement**: these are layered systems having better materials on top as there is a higher intensity of stresses and relatively inferior materials at the bottom due to the intensity of stresses being low.

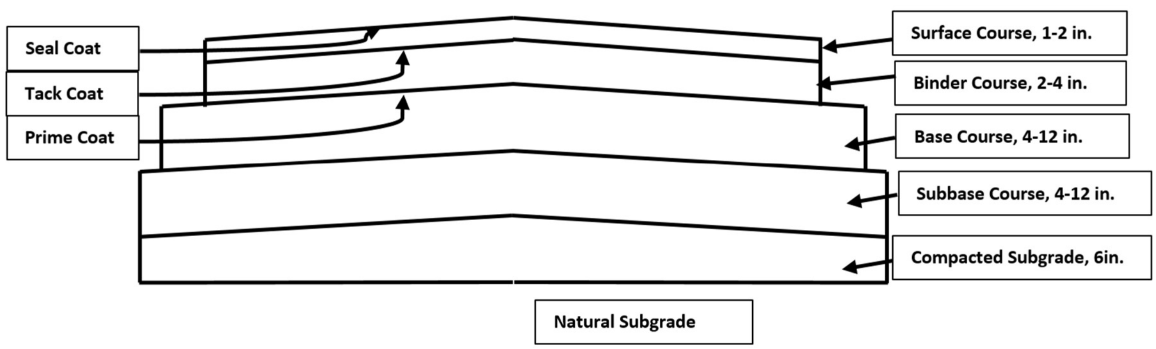

Figure 1. 1 Typical cross-section of Conventional flexible pavement (Huang, 2004)

- **Full-depth asphalt pavement**: One or more layers of HMA are directly placed over subgrade or improved subgrade.

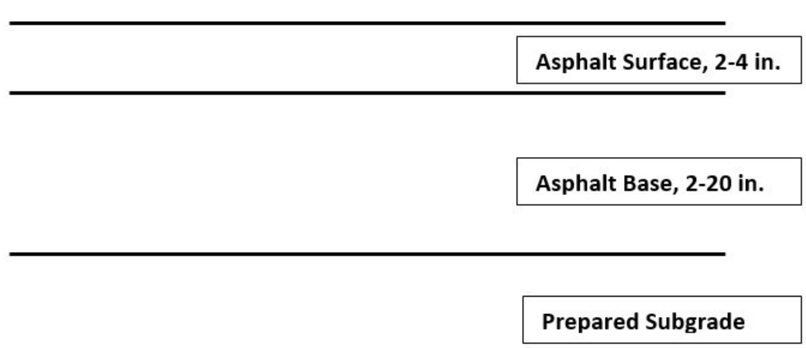

Figure 1. 2 Cross-section of Full-depth asphalt pavement (Huang, 2004)

Distresses in asphalt pavements:

Table-1. 1 Distresses in Asphalt Pavement (Huang, 2004)

TYPES OF DISTRESSES	STRUCTURAL	FUNCTIONAL	LOAD ASSOCIATED	NON-LOAD ASSOCIATED
ALLIGATOR OR FATIGUE CRACKING	✘		✘	
BLEEDING		✘		✘
BLOCK CRACKING	✘			✘
CORRUGATION		✘		✘
DEPRESSION		✘		✘
JOINT REFLECTION CRACKING	✘			✘
LANE/SHOULDER DROP-OFF OR HEAVE		✘		✘
LANE/SHOULDER SEPARATION		✘		✘
LONGITUDINAL OR TRANSVERSE CRACKING	✘			✘
PATCH DETERIORATION	✘	✘	✘	
POLISHED AGGREGATE		✘	✘	
POTHOLES	✘	✘	✘	
PUMPING AND WATER BLEEDING	✘	✘	✘	
RAVELING AND WEATHERING		✘		
RUTTING		✘	✘	
SLIPPAGE CRACKING	✘		✘	
SWELL	✘	✘		✘

1.1.2 Serviceability of flexible pavement

It is the ability of any section of pavement under consideration to serve traffic in its present condition. There are two ways to determine serviceability.

1.1.2.1 Present serviceability index (PSI)

Using the **present serviceability index (PSI)** developed at the AASHO Road Test, it is based both on pavement roughness and on distress conditions, like rutting, cracking, and patching.

1.1.2.2 Steps for framing PSI:

- Establishment of definitions
- Establishment of rating group or panel
- Orientation and training of rating panel
- Selection of pavement for rating
- Field rating
- Replication
- Validity of rating panel.
- Physical measurements
- Summaries of measurements
- Derivation of PSI

1.1.2.3 Limitations of PSI equations

- It is evaluated on the basis of a rating panel. As with time, vehicles, highway characteristics and travel time changes. With this, so does the public's perception about it.
- It consists of rideability along with surface defects, which is needed to be separated for the management of the pavement inventory. Thus, need for a new rating scale to evaluate them separately and reasonably.
- Many equipments used in Road Test are not in use today. Thus, it will generate errors when using the original PSI equation.

1.1.3 Non-destructive deflection testing

Deflection measurement is an important tool for the evaluation of the structural capacity of in-situ pavement. It is used to determine elastic moduli of the pavement components by means of back-calculation. It is utilized in the concrete pavement to evaluate load transfer efficiency across joints and cracks, along with the location and extent of voids.

1.1.3.1 Type of NDT Equipment for deflection measurements:

- **Static/Slowly moving loads:** Benkelman Beam, California Travelling Deflectometer, LaCroix Deflectometer
- **Steady-State Vibration:** Dynaflect, Road Rater, Rolling Dynamic Deflectometer
- **Impulse Load:** Various Falling Weight Deflectometer

1.1.3.2 Factors Influencing Deflections:

- Loading
- Climate
- Pavement Conditions

1.1.3.3 Back-calculation of Elastic moduli:

The deflection data obtained from the NDT deflection tests are used to back-calculate the moduli of various components (Bituminous layer, Base course, Subgrade, etc.). In this, deflection basin is measured and, the different sets of moduli are varied in order to get the best match of computed deflections with measured deflections.

1.1.3.4 Methods:

- Manual method
- KGPBACK Program
- MODULUS Program
- WESDEF Program
- ILL-BACK
- Multi-depth Deflectometer System

1.2 PAVEMENT MANAGEMENT SYSTEMS (PMS)

1.2.1 Introduction

Pavement management refers to the various strategies that can be employed so as to define a pavement restoration and rehabilitation policy. These are the plans that establish minimum standards for the acceptable pavement condition and seek to define the type of treatment required and the time frame for the project completion. Strategies for rehabilitation management requires items like pavement condition, initial cost, annual maintenance, user costs, and safety, physical, environmental and economic limitations. LCA is the basis for pavement management and includes total capital for the project, which includes initial construction, routine maintenance, and major rehabilitation activities over the pavement's lifetime.

Pavement management includes a systematic process for maintaining, upgrading, and operating physical pavement assets in an economical fashion. It is a combination of the application of established engineering principles with sound business practices and economic theory, thereby making a decision in an organized and scientific approach. This process involves:

- assess present pavement condition,
- predict the future conditions,
- conduct an alternatives analysis,
- select an appropriate rehabilitation strategy

A computer-driven protocol employed to achieve this is called A Pavement management system having following elements:

- an inventory and condition database,
- mathematical models to forecast future pavement condition,
- procedures for conducting alternative analyses,
- Reporting and visualization tools to facilitate the interpretation and display of results.

1.2.2 PMS Activity Levels

The complete framework for Pavement Management consists of two distinct levels:

- Network Level
- Project Level

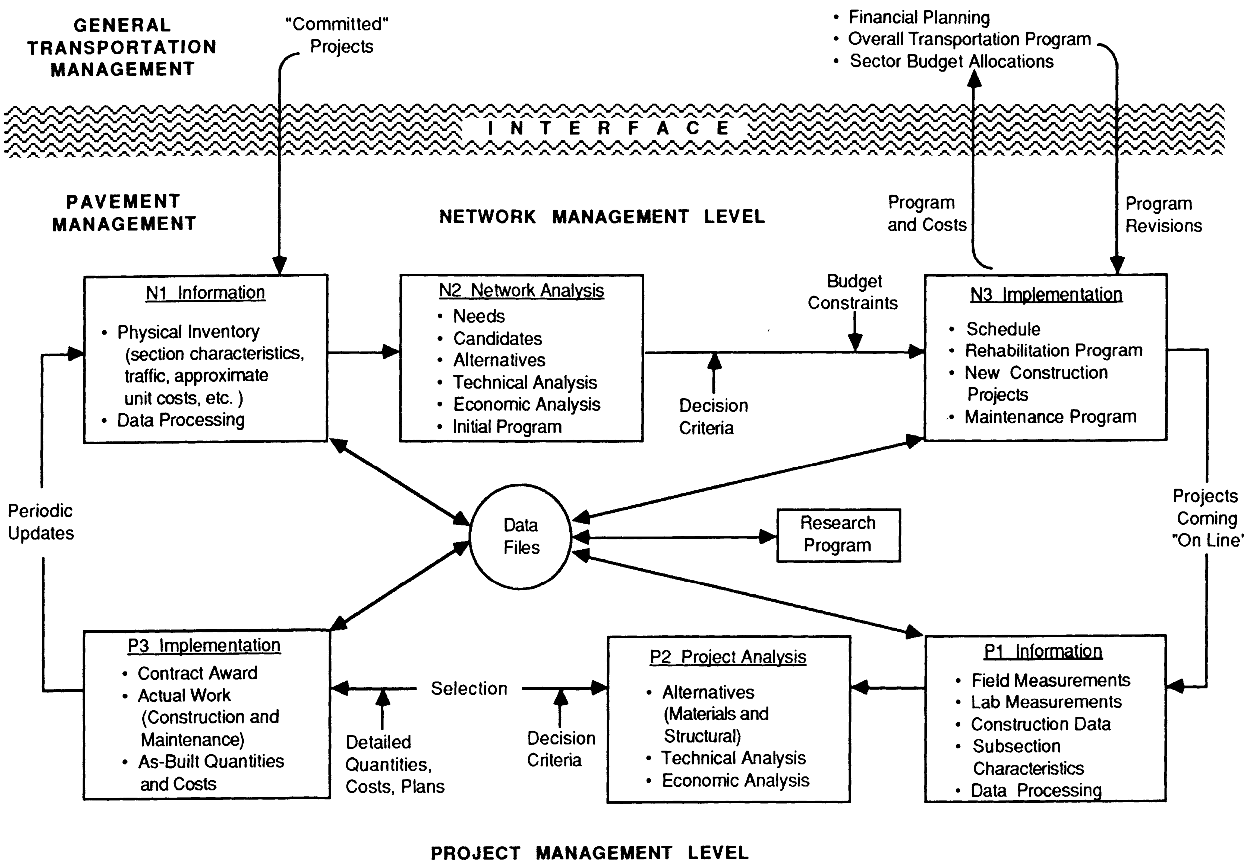

Figure 1. 3 Framework for Total Pavement Management System (SOURCE: R. Haas, W.R. Hudson, and J. Zaniewski, Modern Pavement Management, Krieger Publishing, Malabar, FL, 1994, p. 38.(Haas et al., 1994))

1.2.2.1 Network Level

It is the global view of pavement infrastructure and deals with overall budget and planning issues. Activities included in this are:

- Identification of various pavement maintenance, reconstruction, and rehabilitation needs
- Outlining funds needed to address the above needs
- Selecting viable funding options and approaches to be tested
- Defining the impact these funding options on the pavement performance as well as the overall safety of the driving public may have
- Developing ideal pavement budget recommendations

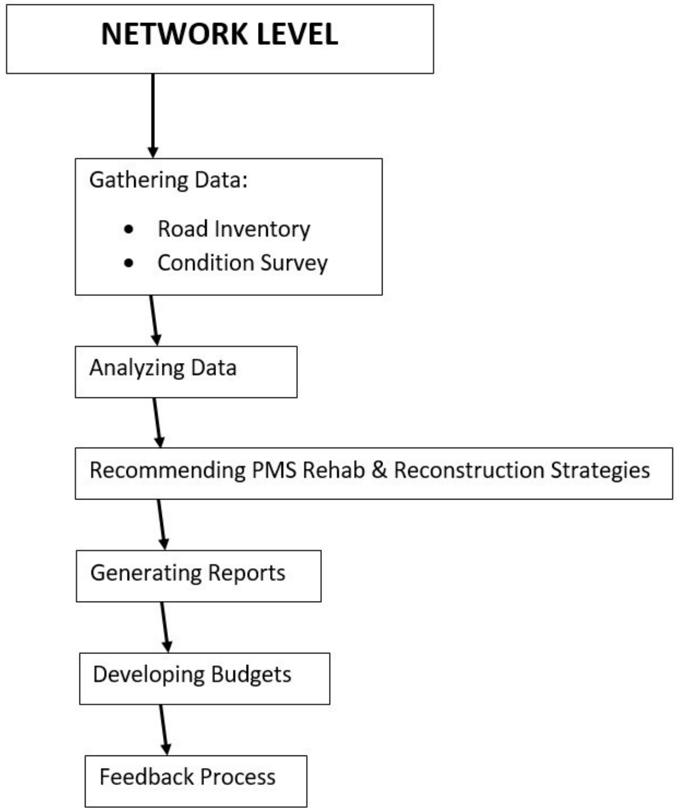

Figure 1. 4 Network-level data elements in PMS

1.2.2.2 Project Level

It is focused locally on a limited element of the larger network. Overall the network level PMS comprises multiple Project level PMS activities. Here specific decisions on maintenance strategies along with funding allocations are taken. This normally includes:

- Assessing the need for construction or cause of deterioration
- Recognizing feasible design, maintenance, rehabilitation, and reconstruction approaches
- Investigation of the cost-effectiveness of various alternatives
- Defining the imposed constraints
- Selecting the most cost-effective strategy within the imposed constraints

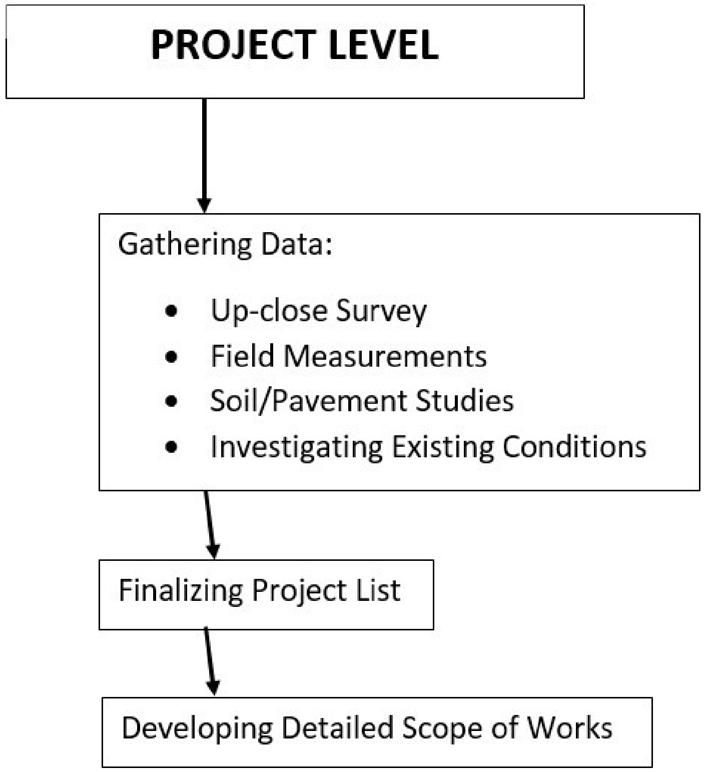

Figure 1. 5 Project-level data elements in PMS

1.3 REGRESSION ANALYSIS

1.3.1 Introduction

Regression analysis is a set of statistical processes in statistical modeling, which are used to define or estimate the relationship between a dependent variable/response variable/outcome and one or multiple independent variables/predictors/explanatory variables. These are used for two main purposes, firstly being as a tool for prediction and forecasting. And to surmise the relationship between the independent and dependent variables, i.e. to estimate casual relationships based on observational data.

1.3.2 Structure/Components

For analysis, firstly, a model is selected and, then the chosen method is used in estimating parameters of that model. Components involved in regression models:

- Unknown parameters: These are denoted by scalar or vector β
- Independent variables: These are observed in data. Denoted by vector X_i
- Dependent variables: These are observed in data. Denoted by scalar Y_i

- Error terms: Not observed in the data directly. (e_i)

Regression models propose that Y_i is a function of X_i and β, where e_i is an additive error term which stands in for un-modeled determinants of Y_i i.e.

$$Y_i = f(X_i, \beta) + e_i$$

Here, function $f(X_i, \beta)$ is estimated to determine the best fit for the data.

1.3.2.1 Types

- **Simple Linear Regression**

For modeling $n$data points, there is one independent variable x_i and two parameters β_0 and β_1

Straight line $y_i = \beta_0 + \beta_1 x_i + e_i;\ i = 1, \dots, n$

- **Multiple Linear Regression**

In this, there are several independent variables/functions of independent variables.

In the case of a parabola, $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + e_i;\ i = 1, \dots, n$

For p independent variables, general model is:

$$yi = \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + e_i$$

Other models:

- Binomial Regression
- Binary Regression
- Logistic Regression
- Multinomial Logistic Regression
- Probit Regression
- Nonlinear Regression

1.4 ARTIFICIAL NEURAL NETWORK (ANN)

1.4.1 Introduction

Artificial Neural Networks (ANNs), also known as neural networks (NNs), are computer systems that are inspired by biological neural networks akin to that in animal brains. It is basically a machine that is designed so as to model the way in which the brain executes a particular task or function of interest. In this, the network usually either gets executed using electronic components or is simulated in software on computers.

It is established on a collection of connected units or nodes known as artificial neurons. These neurons roughly model the neurons in an organic brain where each connection, similar to the synapses, can transmit signals to other neurons. This artificial neuron then receives a signal which processes it and then signals neurons connected to it. This signal is a real number, and the output of each neuron is computed by some non-linear function of sum its output. The connections so formed are called edges. These neurons and the edges have a weight assigned to them. It gets adjusted as the learning process proceeds. The weight alters the strength of the signal at a connection.

1.4.2 Components/ Structure

ANN consists of a collection of simulated neurons, each of which is a node that is connected to other nodes via links. These links correspond to the organic axon-synapse-dendrite connections. Each link has a weight, which determines the strength of influence of one node on another.

- **Neurons:** These artificial neurons have the I/Ps and produce a single O/P which is sent to multiple other neurons. I/Ps can be feature-value of some external data or O/Ps other neurons.
  In order to obtain O/P from a neuron, weighted sum of all I/Ps, which is done by weights of connections from I/Ps to neuron. Then a bias is added to the sum. It is sometimes called the activation. These are passed through activation functions to obtain O/P
- **Connections:** Network consists of connections, each of which provides the O/P of one neuron as I/P to another neuron. Neurons may have Multiple I/P and O/P connections.
- **Weights:** All of the connections are assigned weight that represents the relative importance.
- **Propagation function:** It computes the I/P to a neuron from O/Ps of its predecessor neurons along with their connections as a weighted sum. A bias term may be added to the result of the propagation.

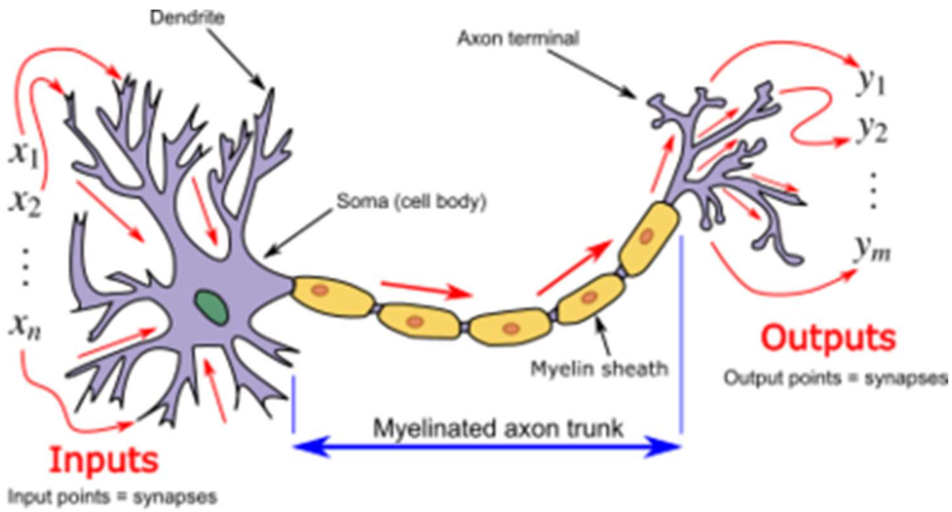

Figure 1. 6 Neuron and myelinated axon, with signal flow from inputs at dendrites to outputs at axon terminals

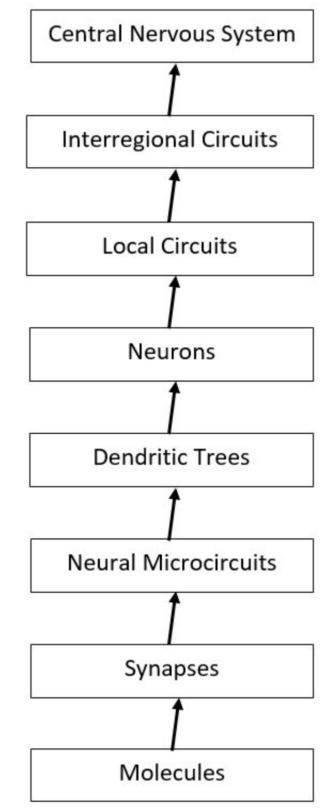

Figure 1. 7 Structural organization of levels in the brain

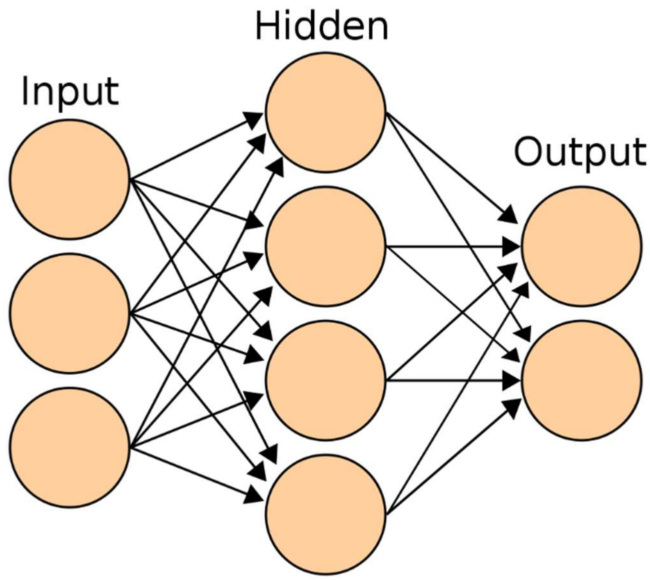

Figure 1. 8 An artificial neural network.

1.5 DECISION TREE

1.5.1 Introduction

The Decision tree is a decision support tool having a tree-like model of choices and their likely consequences. It is used to display an algorithm having only conditional control statements.

It is used in supervised learning approach in domain of statistics, data mining and machine learning. It is one way to display an algorithm which have only abovementioned conditional control statements. A decision tree is a flowchart-like structure in which every internal node is used to represent a test on an attribute. Branch is used to represent the outcome of that test (combinations of features that lead to the class labels), and the leaf node is used to represent a class label, i.e., decision taken after computation of all the attributes. The classification rules are denoted by the paths from the root to the leaf.

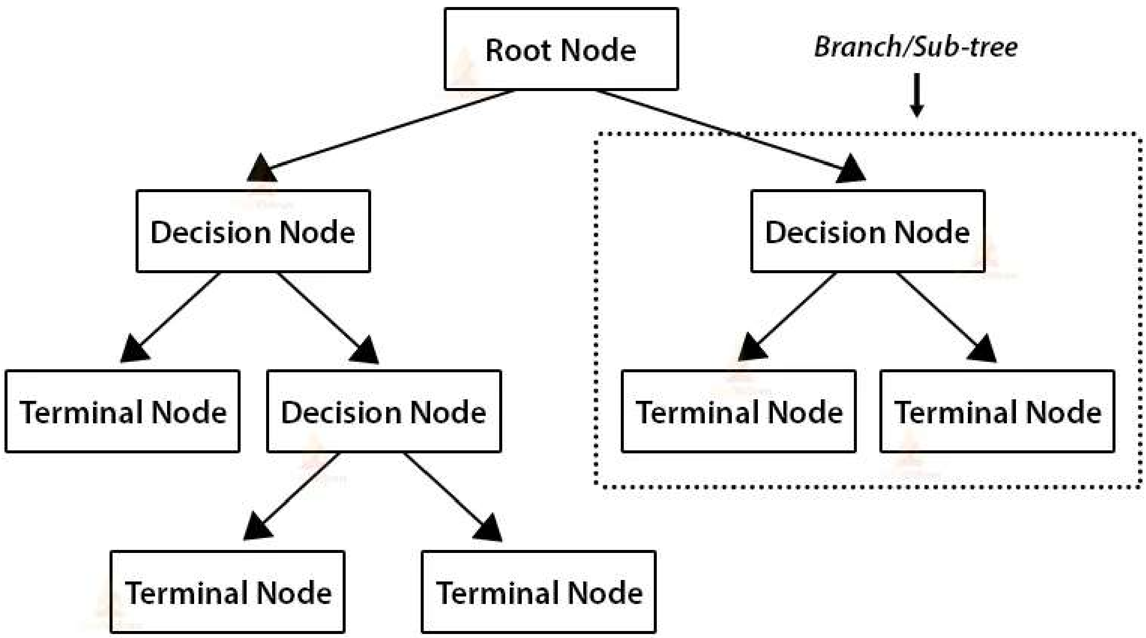

Figure 1. 9 A typical Decision tree

1.5.1.1 Types of decision trees:

- **Classification Trees**: When the decision tree has definite target variable, i.e., discrete set of values.
- **Regression Trees:** When the decision tree has a continuous target variable or an outcome that can be considered as a real number, it is termed as the regression trees.

1.5.1.2 Terminology of Regression Trees

- **Root:** This is the starting point of the decision tree, which also represents the population sample.
- **Leaf:** The terminal node is called the leaf node.
- **Decision Node:** Here the other nodes are divided into the further categories. (Child Node and Parent Nodes)
- **Child Node:** When a node is divided into other subparts, these subparts are called child nodes.
- **Parent Node**: The node which is divided into subparts is called the parent node.

1.5.2 Advantages of Regression Trees

- Each step can be visualized, thus assisting in making rational decisions.

- Priority can be given to a decision criterion.
- Taking a decision based on regression is easier as compared too other methods. As much of the undesired data gets filtered after each step, there is less data to work on as on proceeds further in the tree.
- It is easy to prepare a regression tree. A user can present it to the higher authorities in a much easier way as it can be represented on a simple chart or diagram.

1.6 ENSEMBLES OF TREES

1.6.1 Introduction

Ensemble methods are those which combines numerous decision trees to yield better predictive performance than employing a single decision tree. The major principle behind the ensemble model is that a collection of weak learners come together to form a much strong learner.

Following are the techniques to perform ensemble decision trees:

1. Bagging or Bootstrap Aggregation.

2. Boosting

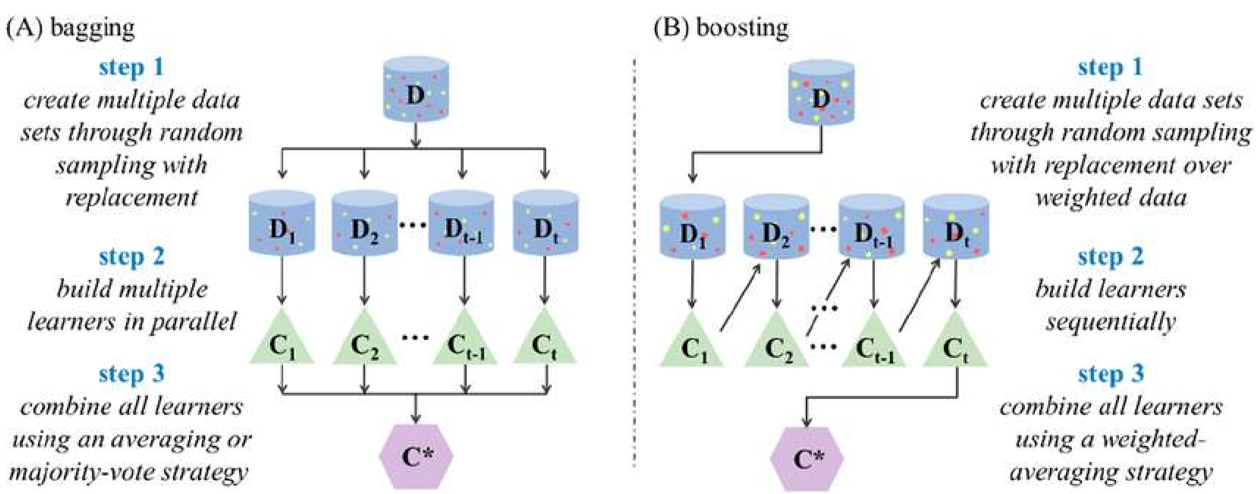

Figure 1. 10 Illustrations of (A) bagging and (B) boosting ensemble algorithms (Yang et al., 2019)

1.6.2 Bagging or Bootstrap Aggregation.

It is employed to reduce the variance of the decision tree. It is done by creating several subsets of data from training sample chosen randomly with replacement. Subsequently, each collection of subset data is used to train their respective decision trees. Thus, ending up with an ensemble of different models. The average of all the predictions from various decision tree are employed which in turn is more robust than a single decision tree.

Random Forest is a further extension of over bagging. It is taken further by taking random selection of features, in addition to taking random subset of data, rather than using all features to grow trees. Thus, creating random forest.

1.6.2.1 Implementation of Random Forest:

(Training Dataset: N observations, M features)

1. First, a sample from training data set is taken randomly with replacement.

2. A subset of M features are selected randomly, and the feature that gives the best split is used to split the node iteratively.

3. The tree is grown to the largest.

4. Above steps are repeated, and prediction is given based on the aggregation of predictions from n number of trees.

1.6.2.2 Advantages of using Random Forest technique:

- Handles higher dimensionality data very well.
- Handles missing values and maintains accuracy for missing data.

1.6.2.3 Disadvantages of using Random Forest technique:

- Since final prediction is based on the mean predictions from subset trees, it won't give precise values for the regression model.

1.6.3. Boosting

Boosting is another ensemble technique used to generate a group of predictors. Here, the learners are learned sequentially with early learners fitting simple models to the data and then analyzing data for errors, i.e., trees are fitted consecutively (random sample) and at each step, the objective is to solve for net error from the preceding tree.

If any input is misclassified by a hypothesis, its weight is increased in order to classify it correctly in the next hypothesis. By combining the whole set at the end, the weak learners are converted into better performing model.

Gradient Boosting is an extension of over boosting method.

$$Gradient\ Boosting = Gradient\ Descent + Boosting.$$

In this, the gradient descent algorithm is employed as it can optimize any differentiable loss function. An ensemble of trees is built one at a time and individual trees are added in sequence. Then the next tree tries to recover the loss (difference between actual and predicted values).

1.6.3.1Advantages of using Gradient Boosting technique:

- Supports different loss function.
- Works well with interactions.

1.6.3.2 Disadvantages of using Gradient Boosting technique:

- Prone to over-fitting.
- Requires careful tuning of different hyper-parameters

1.7 SUPPORT VECTOR MACHINES (SVM)

1.7.1 Introduction

Support Vector Machine or SVM is a type of a Supervised Learning Algorithm which is used for regression and classification problems in Machine Learning problems. The objective in SVM is to create a best line or a decision boundary, also known as hyperplane, which can segregate n dimensional space into classes so as to place new data point in the correct category.

In this, the extreme points (or vectors) are chosen which in turn assist in creation of the hyperplane. These extreme cases are known as the support vector, hence the name of the algorithm- Support Vector Machine.

The Figure 1.11 shows the classification of two different categories using the decision boundary or the hyperplane.

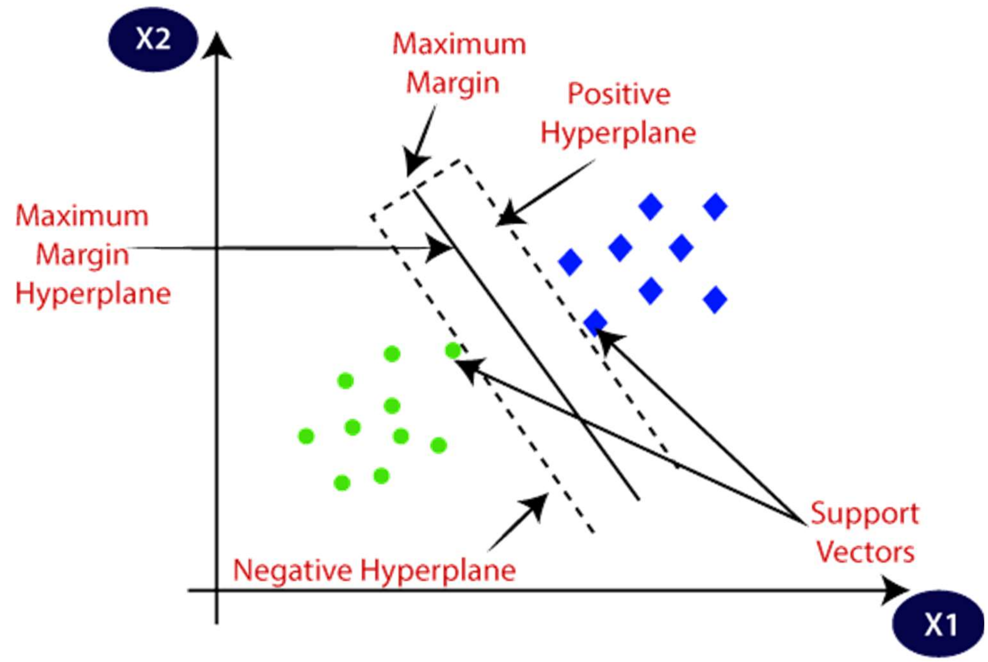

Figure 1. 11 Classification using hyperplane

1.7.2 Types of SVM

SVM can be of two types:

Linear SVM: These are used for linearly separable data, i.e., the dataset that can be classified into two classes by using a single straight line. The classifier that is employed is known as Linear SVM classifier.

Non-linear SVM: Non- these are used for non-linearly separated data, i.e., dataset that cannot be classified by using a straight line. The classifier that is in such case is called Non-linear SVM classifier. Nonlinear classification is performed efficiently using kernel trick. In this, the inputs are mapped implicitly into high-dimensional feature spaces.

Following SVMs were used in MATLAB:

- **Linear SVM:** SVM that follows simple linear structure in the data, using linear kernel. It is the easiest SVM to interpret.
- **Quadratic SVM:** A SVM that uses quadratic kernel.
- **Cubic SVM:** A support vector machine that uses Cubic Kernel.

- **Fine Gaussian SVM:** A SVM that follows finely detailed structure in data. It uses a gaussian kernel with kernel scale: sqrt(P)/4, where P is the no. of predictors.
- **Medium Gaussian SVM** A SVM that finds less fine structure in data. It uses a gaussian kernel with kernel scale: sqrt(P), where P is the no. of predictors.
- **Coarse Gaussian SVM:** In this the SVM follows a much coarse structure in data. kernel scale: sqrt(P)*4, where P is the no. of predictors.
- **Optimizable SVM:** A support vector machine that optimizes hyperparameters.

1.8 GAUSSIAN PROCESS REGRESSION (GPR)

1.8.1 Introduction

Gaussian process regression was developed, for vector-valued function, to solve multi-output prediction problem. In this method, the correlations between all the input and output variables (taken in N points in the desired domain) is described by constructing a covariance.

Gaussian process regression, or GPR, is nonparametric in nature (i.e., not limited by a functional form) kernel-based probabilistic models Instead of calculating the probability distribution of parameters of a specific function, probability distribution over all admissible functions that fit the data is calculated.(Williams and Rasmussen, 2006)

It includes:

- Specifying a prior (on the function space)
- Calculating the posterior (by means of the training data)
- Computation of the predictive posterior distribution on points of interest.

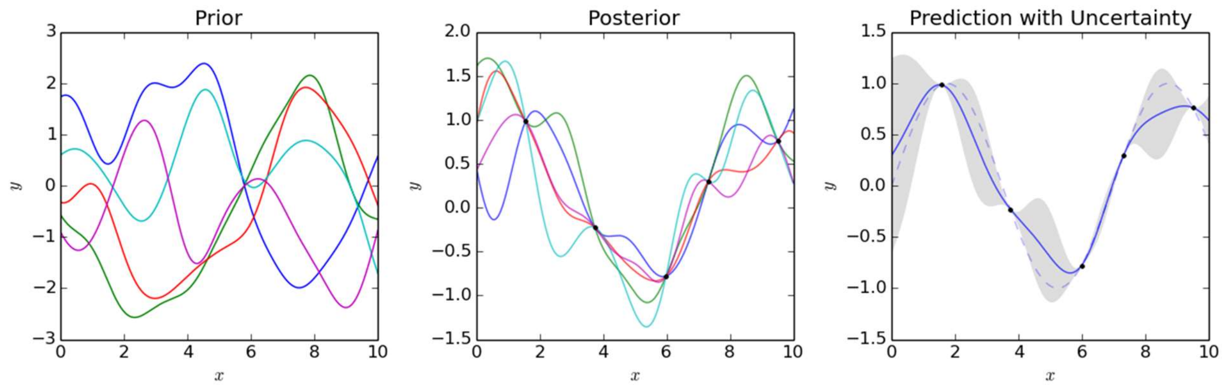

Figure 1. 12 Gaussian Process Regression (prediction) with a squared exponential kernel. Left: draws from the prior function distribution. Middle: draws from the posterior. Right: mean prediction with one standard deviation shaded. (By Cdipaolo96, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=47589433)

1.8.2 GPR has several advantages:

- It works well on small datasets
- Can provide uncertainty measurements on the predictions.
- The prediction interpolates the observations
- It is versatile as various kernels can be specified.

1.8.3 The disadvantages of Gaussian processes:

- They are not sparse, i.e., they utilize the whole features information to perform the prediction.
- They become less efficient in high dimensional spaces.

CHAPTER-2 LITERATURE REVIEW

2.1 PAVEMENT ANALYSIS AND MANAGEMENT

2.1.1. Illustration of pavement management: From Data Inventory to Priority Analysis (Karan et al., 1981)

In this paper, the authors presented a summary of a case study for the implementation of the Pavement management system/process. Traffic information, alternative rehabilitation strategies, information about cost and info about budgeting served as input for the pavement management program. The network management system that was utilized composed of 3 phases as shown in figure1.12

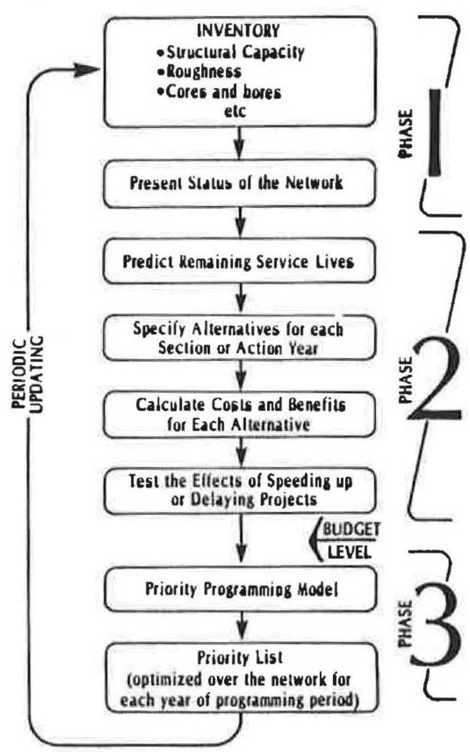

Figure 2. 1 General Structure of System (Karan et al., 1981)

As per the authors, the result obtained in this paper presents the guideline for implementation of the future management of the network. Hence by using the inventory data, the following goals were achieved:

- Establishment of the improvement needs within a 10-year programming period

- The optimum list of pavement-improvement priorities determined by an economic analysis
- Assessment of the efficacy of the anticipated budget with respect to the average network serviceability
- Effect of decrease of funds for rehabilitation on the average serviceability levels. (as shown in Figure 1.12)

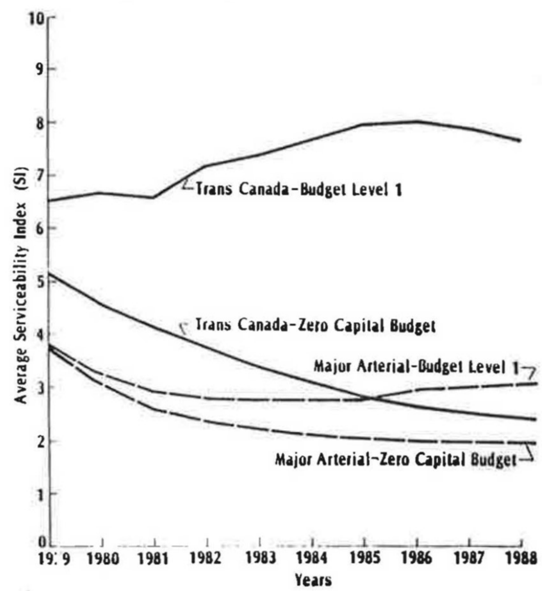

Figure 2. 2 Budget level analysis for Trans Canada and major-arterial sections (Karan et al., 1981)

2.1.2. Integrated model for project-*level management of flexible pavements (Rada et al., 1986)*

In this paper, the authors presented an integrated model that can be utilized for project-level pavement management. As the decisions in pavement management are made of multiple and often conflicting objectives (for users, operators and, highway agencies). Thus, the selection of optimal pavement strategy requires analysis by taking into account the multiple types of costs and benefits and their comparative scale and importance. The model that they proposed consisted of

- Life cycle cost model (LCCM) that generates the feasible pavement management strategies for a highway segment under consideration along with the design, performance, characteristics and, costs for each of them
- Cost-effective model for evaluation, leading to the identification of the optimal strategies.

The analysis results of the authors indicated recognition of multiple types of costs along with measures of pavement performance upon evaluation of pavement strategies. Thus, presenting an effective method for making the optimal choice which may not always have minimum life cycle cost.

2.1.3. PMSC: Pavement Management System for Small Communities (Tavakoli et al., 1992)

Here authors reiterated the importance of an appropriately designed and developed Pavement Management System as it enables highway agencies in doing a more effective allocation of the funds available to them. Moreover, it empowers them to estimate future needs more successfully. They presented a PMS for small communities, called PMSC, for decision backing system in order to effectively recommend and make decisions about scheduling and accounting at several levels of M&R works within the local transportation system. The PMSC created consisted of seven modules, i.e., Pavement Inventory, Distress Survey, Maintenance and Rehabilitation (M&R) strategies, Unit Costs, Deterioration Rates, Priority Rating and Goals, and Backup Module.

The resultant management system was comprehensive as well as user-friendly. Its database could be modified with ease to include local unit costs and to reflect policies and procedures particular to that site by incorporating complete scope of field conditions and probable maintenance measures. It enables short as well as long-range planning in terms of fiscal as well as physical conditions for financially realistic optimum management practices.

2.2 REGRESSION ANALYSIS

2.2.1. Regression analysis of soil compressibility (Azzouz et al., 1976)

In this paper, the authors tried to use statistical techniques in order to analyze and evaluate the experimental data from the consolidation tests on numerous undisturbed soil samples. For this, they developed regression equations which were then used to estimate the compression index and compression ratio using the classification/index data. From the simple linear regression analysis, they reported that the compression index and the compression ratio could be expressed in the best form in terms of the initial void ratio. They came to this conclusion as upon introduction of other independent variables, the accuracy of the resulting regression models did not improve significantly. The correlations were examined within the context of the large amounts of similar relationships as reported by other investigators and, the authors found significant differences.

2.2.2. Estimating key characteristics of the concrete delivery and placement process using linear regression analysis (Dunlop and Smith, 2003)

In this paper, the authors used multiple linear regression analysis for estimating the productivity of the concrete operations by developing a model so that the efficiency and effectiveness of these operations can be improved.

Table-2. 1 Correlation Coefficient b/w all ten explanatory variables (Dunlop and Smith, 2003)

	Type of pour	Av. volume	No. of trucks on job	Av. cycle time	Concrete mix	Weather	Start time	No. of loads	Total volume
Type of pour	1.00								
Av. volume	0.09	1.00							
No. of trucks on job	0.36	0.09	1.00						
Av. cycle time	−0.15	0.19	0.34	1.00					
Concrete mix	−0.16	0.11	−0.38	−0.26	1.00				
Weather	0.11	0.19	0.15	0.07	0.00	1.00			
Start time	−0.38	−0.07	−0.23	−0.10	0.07	−0.17	1.00		
No. of loads	0.48	0.16	0.79	0.26	−0.34	0.20	−0.41	1.00	
Total volume	0.49	0.22	0.78	0.26	−0.32	0.21	−0.42	**0.99**	1.00

The authors reported that the developed regression model for the actual productivity describes over 80% of the variability in the data set of the observed concrete operations. They were able to designate the significant factors in order of its importance, i.e., number of loads, average volume, type of pour, weather, number of trucks on the job and, the average cycle time.

The validation exercise for the model by the authors showed that it produces good results for production productivities greater than about $6m^3hour^{-1}$.

2.2.3. Linking roadway crashes and tire-pavement friction: a case study (Najafi et al., 2017)

In this paper, the authors presented a study in which they utilized regression analysis to verify the effect of friction on the rate of wet and dry condition vehicle crashes for various types of urban roads. They investigated the effect friction has on the rate of fatal and the injury causing vehicle crashes on urban roads.

Thus, authors came to the conclusion that friction is a significant factor affecting the ratio of wet as well as dry condition vehicle crashes. As opposed to other studies where the emphasis was laid only on wet conditions, authors asserted that friction impacts dry condition crashes as well. As per the authors, using normal probability plots of residual and residual plots for the predicted values and regresses suggested that the relationship is not linear, and the transformation was needed that also improved its coefficient of determination. They also suggested the relationship between friction and crash rates to be utilized by highway agencies to define the acceptable levels of friction for various types of roadways present in their networks.

2.2.4. Effect of frost heave on long-term roughness deterioration of flexible pavement structures (Sylvestre et al., 2019)

In this paper, the authors were able to introduce pavement roughness deterioration models having reasonable predictive capacity of pavement IRI deterioration using multiple linear regression analysis. The first proposed model was used to assess the degradation of in-service pavements using the cracking performance index, which was able to help in determining the cause of possible accelerated degradation of pavement for recuperation projects. From the second model, authors were able to link long-term roughness performance of flexible pavement to frost heave along with other various related degradation mechanisms. As per the authors, it will enable highway administrators and contractors to adapt the design of pavements subjected to the frost action as per their needs and design goals. And also developing a better understanding of the effect frost heave have on pavement deterioration.

2.3 ARTIFICIAL NEURAL NETWORK (ANN)

2.3.1. Artificial Neural networks and regression analysis for predicting Faulting in jointed concrete pavements considering base condition (Saghafi et al., 2009)

Authors used Artificial Neural Networks (ANNs) and Multivariate Linear Regression (MLR) for predicting joint faulting in this paper by modeling the effect of base layer condition and pavement age on joint faulting distress. Since significant joint faulting has a major impact on the life cycle cost of the pavement in terms of vehicle operation as well as early rehabilitation costs, the prediction for future pavement conditions becomes more crucial.

ANN was successful in predicting measure joint faulting with the coefficient of R^2 values of 0.96 for testing data and MAE, used to inspect the amount of errors between measured and predicted faulting values, values for training and testing being very low, i.e., 0.09 and 0.23, respectively.

A simplified model was developed for predicting joint faulting using MLR and stepwise algorithm of MLR method. Thus, they demonstrated in this paper that ANN could be used for modeling the complex relationship between Pavement age and the parameters related to base layer conditions and joint faulting in jointed concrete pavements. Here with the ANN, they were able to come up with a model with higher R^2 values, and it also had the ability to predict joint faulting more accurately when compared with MLR models developed using the same data.

2.3.2. Artificial Neural Network Model of Bridge Deterioration (Huang, 2010)

In this paper, the author presented a study where they used statistical analysis to identify significant factors that influence the deterioration of the bridge and then use it to develop an application model such that it can estimate its future conditions. They also utilized the historical maintenance and inspection data to identify 11 significant factors and, then an artificial neural network (ANN) model was established to predict the associated deterioration.

To evaluate the possibility that which of the maintenance history will have the statistically significant effect on the mean age of transition, ANOVA was conducted. And two null hypotheses were tested. After that, the dependency of condition in various inventory attributes are studied and, the results are summarized in Table 1.3, where it is observed that 5 out of 11 parameters influenced A_{12} and A_{23} significantly.

Table-2. 2 Significance of Parameters studied (Huang, 2010)

	p-value	
Parameters	A_{12}	A_{23}
County	0.9148	Not tested
District	0.0014[a]	0.8643
Design load	0.0001[a]	0.0211[a]
Deck length[b]	0.2045	0.0158[a]
Deck area[b]	0.1599	0.0019[a]
Number of lanes	0.4157	0.8897
Function class	0.1423	0.8758
ADT[b]	0.0158[a]	0.6125
Environment	0.0005[a]	0.0053[a]
Degree of skew[b]	0.0500[a]	0.6100
Number of spans	0.4699	0.0149[a]

[a]Significant at 95% level of confidence.

[b]The confidence level may be different when different categorizations for data samples are used. The categorization with highest confidence level was used in the analysis and the smallest *p* value was reported in this table.

Those parameters were considered to be the features of the ANN model, which were found to be significant, and then the author scaled the characteristics on a uniform scale as shown in table5 by feature transformation such that the influence of different features had equalized. Neurons in the hidden layer in ANN used hyperbolic tangent activation function, and the output layers used sigmoid activation function. And different ANN architectures were employed.

Here ANN prediction model reached classification rates of 84.66 and 75.39% for the training sets and testing sets. Also, eleven statistically significant factors for deck deterioration were determined. The author came to the conclusion that the ANN performs well when modeling deck deterioration when presented as a pattern classification problem. Also, the developed model can predict the condition of the bridge fairly accurately and thus can provide pertinent information for maintenance planning and the decision-making process at both the project as well as network levels.

2.3.3. Artificial Neural Networks in Construction Engineering and Management (Waziri et al., 2017)

In this paper, the authors presented a review of the trend and new applications of different ANN algorithms. They presented successful applications of ANNs in risk assessment, cost prediction, optimization and scheduling, claim and dispute resolution outcomes, and decision making. Here ANN was applied to problems that are quite difficult to solve using the traditional mathematical and statistical models. ANN was integrated with other soft computing methods like Genetic Algorithm, Fuzzy Logic, Ant Colony Optimization, Artificial Bee Colony, and Particle Swarm Optimization so as to get better results when compared to the conventional ANNs.

Authors reported ANN to lack general procedure while selecting its initial weights and parameters for its effective application. It was also observed that it is unsuitable for long-term forecasting when trends are changing. To counter these shortcomings, it can be integrated successfully with the GA, ACO, PSO, fuzzy logic, and so on.

2.3.4. Application of ANN in Pavement Engineering: State-of-Art (Abambres and Ferreira, 2017)

In this paper, the authors provided the extensive and detailed state of the art of ANN models utilized in pavement engineering from fields such as pavement management, materials and, design problems. In this work, they highlighted numerically as well as graphically the effectiveness and possible shortcoming of each ANN application. The applications included (a) Pavement Management: Pavement Distresses, Pavement Condition Indices, Maintenance; (b)

Materials and Pavement Design: Material Physical Properties; Layer Moduli/Thickness, Poisson's Ratio; Stress, Strain, Deflection and Creep; Equivalent Single Axle Load (ESAL)

Based on the detailed analysis and study, the authors reported the lack of utilization of advanced techniques, like ANN, for design. In most of the cases, conventional/traditional techniques like Multi-Layer Perceptron (MLP) and learning algorithms like Back-Propagation (BP) and Levenberg-Marquard (LM) were utilized. Also, there was not a lot of references made to special trimming and data preprocessing techniques so that improvement in the generalization ability of the network.

2.4 BENCHMARKING THE STRUCTURAL CONDITION

2.4.1 Falling Weight Deflectometer bowl parameters as an analysis tool for pavement structural evaluations (Horak and Emery, 2006)

Here the authors showed an approach for the use of the parameters, like DBPs and mechanistically determined structural evaluation, to analyze payments in a semi-empirical-mechanist fashion. These parameters were used in conjunction with the other assessment methodologies like visual surveys so as to describe the structural capacity of the pavements by regarding the state of the pavements as sound, warning and, severe. This technique can be used to filter the state of the condition of the section to identify sections near or at failure and pinpoint it to specific layers that need further detailed investigation. Thus, making the identification of distress and explaining the probable cause of the structural distress along with the failure mechanism possible. After that, one can carry out further detailed investigations of the distressed area by the field and the laboratory testing.

2.4.2 Surface Moduli determined with the Falling Weight Deflectometer used as Benchmarking Tool (Horak, 2007)

In this paper, the author used a benchmarking process that employs a semi-empirical-mechanistic approach, created by the DBPs along with described behavior states of the flexible pavement, along with visual survey data to designate, and based on the structural capacity and behavior state of pavement, pavement sections as sound, warning and severe. Here surface modulus (SM) is also utilized (determined using Boussinesq equation) to define the behavior of the subgrade, i.e., linear elastic, stress stiffening or, stress softening. As the subgrade characteristics, largely being the defining factor for the structural analysis of remaining of the pavement structure. By using this modified methodology, the author was able to identify distress locations along with the possible relative cause of defects by means of the visual distress surveys. This approach is useful for guiding the detailed structural analysis required more effectively.

2.4.3 Benchmarking the structural condition of flexible pavements with deflection bowl parameters (Horak, 2008)

In this paper author presented a relative benchmarking methodology using deflection bowl parameters (DBPs) measured with the Falling weight deflectometer (FWD), which can be used in conjunction with the standardized visual survey methodology to give guidance on individual layer conditions and come to a decision for necessary rehabilitation requirements as BLI, MLI, LLI have been found to correlate well with relevant pavement structural condition. Also, the correlation of calculated surface moduli with DBPs for granular base pavements is presented to augment this benchmarking methodology.

In benchmarking using DBPs, visual condition rating is linked with the behavior states of pavement and used for distress diagnosis on a preliminary investigation level to establish its direct cause.

Upon correlating DBPs and calculated surface moduli for each zone and linked DBPs separately (BLI, MLI and, LLI with zones 1, 2, and 3, respectively), the author concluded that it is possible to use SM_0, SM_{300} and SM_{600} instead of BLI, MLI and, LLI in benchmarking. It was concluded that the SM values could help in the identification and then pinpoint the probable cause of distress in the structure of the pavement at base and surface layers and was confirmed by the above-mentioned three-tiered visual survey condition rating.

It was concluded by author that the DBPs could be used for the semi-mechanistic empirical design of pavement and with complementary visual surveys to describe pavement structural layers. The Benchmarking methodology with the associated visual condition rating enables to identify a uniform section and pinpoint the cause of structural distress. SM correlates well with related zones and linked DBPs, and thus, identification of cause and effect of structural failure can be done.

2.4.4 Review of Falling Weight Deflectometer Deflection Benchmark Analysis on Roads and Airfields (Horak et al., 2015a)

For comparative evaluation of the structural condition of flexible pavements, the authors presented a benchmark analysis method that can be performed using the FWD data. Finally, they presented a 3-tired condition assessment method. This included gauging-potential of existing deflection bowl parameters for inclusion in benchmarking analysis at the network level by assessing the condition of the layers. Regression analysis is used for correlating DBPs to various layers of pavement. They utilized it for the calculation of structural number (SNP) and the pavement number (PN). Recommendation of parameters for evaluating each layer is presented, i.e., for subbase, subgrade and surface and base course. The FWD-derived parameters and structural indicators can help in the identification of potential distress areas along with their probable origin and thus directing the need for further detailed survey or analysis.

2.4.5 Flexible Road Pavement Structural Condition Benchmarking Methodology incorporating Structural Indices derived from Falling Weight Deflectometer deflection bowls (Horak et al., 2015b)

In this study, a benchmarking tool for comparative evaluation of flexible pavements using the Falling Weight Deflectometer is presented. The parameters and indices are analyzed and associated with different zones of the pavement structure. the Radius of Curvature (RoC) is a good indicator of asphalt and base layer while, Base Layer Index (BLI), Middle Layer Index (MLI) and, Lower Layer Index (LLI) is indicative of the condition of the base layer, subbase layer and the structural response of the subgrade respectively. Ultimately these parameters are evaluated and were found to strengthen existing benchmarking tools for analysis at the network level. The SNPeff, SCI, PNeff were found to perform quite well when DBPs were used in addition to them. The relevance of its application in pavement management systems is evident at the network as well as in project-level investigations.

2.5 EVALUATION USING FWD DEFLECTIONS AND DEFLECTION BOWL PARAMETERS

2.5.1 Pavement structural evaluation based on roughness and surface distress survey using neural network model (Fakhri and Dezfoulian, 2019)

Here a method for pavement structural evaluation is presented for evaluating pavement layers condition and identification of the rehabilitation needs. For this author devised a relation between Deflection Bowl Parameters (DBPs) which were calculated from Falling Weight Deflector (FWD) in terms of structural indices and with International Roughness Index (IRI) & Pavement Surface Evaluation and Rating Index (PASER), i.e., pavement performance indices using ANN and Regression models.

Regression was used for analysis between DBPs (RoC, BLI, MLI & LLI) with lane roughness IRI_{AVE} and IRI_R and then, using simple linear and polynomial equations for structural indices with IRI and PASER index. R^2 And MSE values were presented along with plots and equations.

It is observed that ANN produces better results than Regression as it had R^2 values 3 times higher than the linear model R^2 values. Hence it is concluded that the evaluation of pavement structural condition (especially for upper and middle layers) could be done by estimating structural indices based on speedy and cost-effective methods using both IRI and PASER indices concurrently.

Further, more variables are also needed to be considered like AADT, percentage of trucks, the thickness of asphalt, base and sub-base, time since last maintenance activity, environmental and climate data (average annual temperature, annual precipitation, temperature range) and asphalt temp, etc.

2.5.2 Implementation of Deflection Bowl measurements for Structural evaluation at Network Level of Airport Pavement Management System (Pigozzi et al., 2014)

Here authors provided the structural condition-based threshold values for utilization in the airport pavement management system. The deflections from heavy weight deflectometer, standard equipment for NDT testing at the airport, were done to define the relation between selected deflection parameters and the back-calculated layer moduli for the benchmarking values. The developed model so produced provided good prediction results for the determination of remaining service life and the evaluation of airport pavement's health.

2.5.3 Simulating pavement structural condition using artificial neural networks (Plati et al., 2016)

The authors investigated the feasibility of ANN being incorporated for modeling pavement structural conditions in the pavement management systems. They defined the strain-assessment criteria to be able to characterize the structural condition of the flexible asphalt pavement in respect of fatigue failure. Strain assessment was done by ANN using asphalt tensile strain and FWD deflections along with the thickness of the asphalt layer.

The results from both conventional regression analysis and the ANN technique were compared (which is only indicative, not definitive), and ANN models were found to outperform the linear regression but cannot capture the cause-effect relationship between variables which helps in understanding the relationship like in regression analysis.

They demonstrated the application of the Developed ANN models for characterizing pavement structural conditions used in strain assessment for making provisions for the maintenance and rehabilitation activities for pavement management. For this, pavement sections in a network were characterized, and their needs were thus defined by establishing trigger values for strains. There is a need for further research by taking physical conditions and traffic volumes into consideration.

2.6. DEFINING STRAIN CRITERIA IN FLEXIBLE PAVEMENTS

2.6.1 Fatigue endurance limit for highway and airport pavements (Carpenter et al., 2003)

Authors analyzed fatigue data using the conventional manner, and based on this, found out there is variation in data at normal strain levels as per the recommendation for the fatigue testing and at low strain levels. The dissipated energy thus shows a clear change in the behavior of the material at low flexural strains. This is because, at low levels of strains, the damage amassed from each loading cycle is excessively lower than that were predicted from extrapolating fatigue testing at usual strain levels. This is accredited to the healing process. Thus, laboratory testing can be said to verify the presence of fatigue endurance limit around 90 to 70 micro strains. Lower than that of the fatigue life of the prepared mixture is dramatically extended as compared to normal design considerations.

2.6.2 Perpetual pavement design in China (Yang et al., 2005)

Here strain criteria for the different perpetual pavements under consideration are presented, and their design is also discussed. These had varying thickness of the HMA layers and also the difference in the surface treatment of the underlying layer. The strain criteria presented are of 70με and 200με for fatigue and rutting thresholds, respectively based on the previous laboratory and field research. Also, for the less conservative design, fatigue criteria are taken as 125με, and 200με was set as the criteria for rutting.

2.6.3 First findings from the Kansas perpetual pavements experiment (Romanoschi et al., 2008)

The appropriateness of the perpetual pavements was investigated from the said construction done by KDOT. Their structural design was based on the 1993 AASHTO Design Guide. Analysis was performed by measuring the pavement response under the known load of the vehicle. It was done prior to the opening of the section for traffic and once more after it. Based on the analysis of the perpetual pavements, authors proposed that the limitation of strain from 60με to 100με solely based upon the laboratorial testing.

2.6.4 Development of stochastic perpetual pavement design criteria (Willis and Timm, 2010)

The authors conducted research with the purpose of determining field-based strain thresholds that can be utilized for the perpetual pavement design. It was reported that after the 55th percentile strain corresponded to 193με, all the sections that were experiencing fatigue cracking had greater strain magnitude than those with no cracking at all.

2.6.5 IRC: 37-2018 Guidelines for the design of flexible pavements (Congress, 2018)

As per IRC 37 design code, the limiting strain is taken as 80με for the tensile strain caused by traffic in the bituminous layer and 200με for the vertical subgrade strain. Its values are less than these, the bituminous layer will never crack, and very little rutting will be there in the subgrade for climatic conditions existing in the plains of India as the Average Annual Pavement Temperature is about 35°C.

2.7. INTERNATIONAL ROUGHNESS INDEX

2.7.1 International Roughness Index: Relationship to Other Measures of roughness and Riding Quality (Paterson, 1986)

The road roughness is measured using various means and methods worldwide. Thus, in this paper author presented a comprehensive study of various scales for roughness measurements and their characteristics, along with sources of variation and the range of applications of the conversions are presented. The International Roughness Index (IRI), which was developed from International Road Roughness Experiment (IRRE) as an appropriate calibration standard for all profilometric instruments, is a transferable reference scale. The author presented a two-way conversion relationship along with their confidence intervals. Road roughness measures included in here are Quarter-car index (QI), Bump integrated trailer (BI), Analyseur de Profil en Long (APL) and serviceability index (SI) for other sources.

A chart was presented for the approximate conversion between IRI and major roughness scales with a high degree of confidence. In case of the Bump Integrator, and other systems using hardware as a reference, applicability is stated to be dependent on the similarity of hardware.

2.7.2 Machine learning approach for pavement performance prediction (Marcelino et al., 2021)

In this paper, the authors presented a general machine learning approach for the development of the pavement performance prediction model in Pavement Management Systems (PMS). The Machine learning algorithm is used instead of ANN models as neural network models are trained for high accuracy and neglect the models' generalization capability. The authors here want to propose an approach supporting different machine learning algorithms and put an emphasis on generalization performance. For this, they predicted IRI for 5 and 10 years.

As it was proposed to be used at network level PMS on a daily basis, the variable selection was done such that the variables representing these factors are easily available. Engineering decree was applied in the selection of these variables with the effort being made to include performance, structural, climatic, and traffic data.

Different datasets were created for the 5 and 10-year predictions having complete and comprehensive (lagged) values. For the modeling, a random forest algorithm is used as it helps in reducing the variance of the machine learning model by combining various models.

Based on the error analysis, it was reported that the 5-year model (MSE=0.064) performs better than the 10-year model (MSE=0.104) with both having acceptable accuracy for long-term IRI predictions when concerned with network level. The low values of standard deviation the MSE, it is concluded to have better generalization capabilities. Based on sensitivity analysis, and models are sensitive to previous IRI values. The applicability of this model to other networks is evident because of the approach presented by the authors being quite generic.

2.7.3 Prediction of Pavement Remaining Service Life Using Roughness Data (Al-Suleiman and Shiyab, 2003)

In this paper, the authors presented a methodology by which they were able to predict Pavement Remaining Service Life, known as RSL, by using data pertaining to serviceability, roughness and age of the sections consisting of asphalt surfaced pavements in terms of the International Roughness Index (IRI). They estimated the Present Serviceability Index (PSI) using models based on the direct roughness measurements and the comparison to other studies were made. Separate regression models for heavy trafficked and fast trafficked lanes were presented and the roughness- serviceability and the roughness- age models were found to be statistically significant and thus can be used for prediction. Thus making use of roughness data for estimation of RSL as a decision-making indicator of maintenance and rehabilitation (M&R) activities at the project level possible. A criteria for decision-making of selecting treatment type based on RSL is also presented. The author recommended the setting up of maintenance and rehabilitation activities on the lane-by-lane basis due to variation in performance of each lane in the pavement when considering models developed based on the roughness measurements.

2.7.4 An ANN model to correlate roughness and structural performance in asphalt pavements (Sollazzo et al., 2017)

In this paper, the authors presented a preliminary approach for developed Artificial Neural Network to be able to estimate the structural performance of asphalt pavements from the roughness data, i.e., estimation of the structural state of pavement using its functional conditions. Its significance is recognized because of the advantages of present high-performance survey devices for the procurement of functional parameters of road pavement. As it allows road agencies to reduce the frequency of deflection tests which are costly, time-consuming as well as present great disruption to the traffic flow as compared to functional surveys. They included various input parameters for traffic (average of annual ESALs, Average of Annual Average Daily number of trucks), weather (average temperature, standardized temperature range) and structural aspects (pavement total thickness, asphalt layer thickness) in the analysis to be able to differentiate numerous road sections.

Based on the analysis, the effectiveness of adoption of ANN for modeling (IRI and SNeff) the given relation using the large database is established by the correlation. Here ANN is observed to perform better than the Linear Regression by the MSE and R2. It is evident from the high accuracy in both the testing and validation phase.

They trained three different ANNs for the analysis of modifying datasets and using different variables. From the results, as presented by the authors, it can be established that the roughness and structural performance can be correlated with the good accuracy by the approach presented where structural-survey are either not conducted regularly or not readily available.

2.7.5 Modeling for pavement roughness using the ANFIS approach (Terzi, 2013)

In this paper author modeled IRI for flexible pavements using a hybrid machine learning approach known as ANFIS. For this, they used cumulative equivalent single axle load (ESAL), Age of the pavement (AGE) and Structural number (SN, whose most values are from back-calculation of FWD readings) as the inputs to simulate IRI.

Inputs were entered one at a time and combined together by seven different models. The model showed remarkable ability to successfully estimate IRI based on SUM ESAL, SN and AGE as is evident from the high regression coefficient (0.97) for the ANFIS model

As highway agencies have the necessary equipment for measurement of roughness, the current methodology for developing the IRI model makes it promising to be applied in different regions where agencies may lack personnel and time for taking measurements. To make estimate for current as well as to forecast of future pavement condition possible and thus making it possible to be incorporated in pavement management decision making for maintenance and rehabilitation activities at the network level.

2.7.6 Development of Relationship between Roughness (IRI) and Visible Surface Distresses: A Study on PMGSY Roads (Prasad et al., 2013)

In this paper, the authors presented a study on PMGSY roads which focuses on the development of a relationship between roughness and other functional surface distresses (Cracking, potholes, raveling, patching, rutting, edge failures), based on their extent and severity. It is done as the roughness, determined here by IRI, is a measure for the texture of the pavement surface and is dependent on the other functional distresses present on the surface of the road. Data collection for distress was done for 50m sections. And normalized by converting them into percentage w.r.t. total area considered.

A relation was developed between IRI value and visible distress data using regression analysis. In this, 70% data was used for model development and the remaining was used for its validation. The Multiple-linear regression model was developed by SPSS using IRI as a dependent variable and 12 variables out of 27 variables being the independent one. Based on the student-t and p-values validity of the model was checked. Based on it, dependent variables follow a normal distribution across the observation with a constant variation. Also the variables included in the model were acceptable along with value of Multiple R being 0.815. Special attention was given while conduction tests to be able to account for the effect of edge failures in this model.

According to the authors if roughness data due to distresses is known, under similar conditions, the given model can be used for predicting total roughness of pavements. With knowledge of large data, vehicle operating costs (VOC) from roughness estimated because of pavement distresses.

2.7.7 Development of Pavement Roughness Models using Artificial Neural Network (ANN) (Alatoom and Al-Suleiman, 2021)

Realizing that the pavement roughness plays an instrumental role in determining the riding quality of the road networks and being a performance measure for the pavement along with being a determinant in establishing optimal time for pavement Maintenance and Rehabilitation works, authors presented the research in this paper to develop pavement roughness models using ANN based on the smartphone measurements. For this, pavement age, traffic loading and traffic volumes were taken as input variables for examination on the IRI values were done. The 44 arterial streets considered were divided into 204 road sections and a comparative analysis between ANN and regression models were performed.

Using MSE, RMSE, MAE and R2 as the performance measure, the authors concluded that the model with Age, ∑ESAL, ∑HV, ∑TV (MODEL 7) was most accurate and complex as well with three hidden layers. And MODEL 4, 5, 6 were able to predict roughness reasonably with less complex architecture. ∑TV representing the accumulated total volume of vehicles was of the greatest importance based on the sensitivity analysis. Also developed ANN model performed better than comparative regression models when predicting IRI in addition to being more complex. The Maintenance and rehabilitation alternatives were suggested by authors based on the present and predicted IRI values. When ANN was used, the average error was 14.1% as compared to 48.4% as predicted by the regression approach. Thus, ANN have high prediction accuracy and low average error when compared to its counterpart.

Authors recommended consideration of initial IRI, pavement distresses, material properties, structural capacity, and layer thickness on pavement roughness. The Maintenance and Rehabilitation alternatives can be improved based on the relationship between IRI, PCI, and pavement structural capacity.

2.7.8 A simplified Pavement Condition Index Regression model for Pavement Evaluation (Elhadidy et al., 2021)

The authors presented a simplified approach for linking the Pavement Condition Index (PCI) with the Roughness (IRI) using the regression model. A sigmoidal function was used for this to get the best possible relationship between PCI and IRI having R2 value of 0.995 and the accuracy is indicated by the validation process. Thus, the bias the being pretty low in the prediction of IRI values. In the end, they proposed a pavement condition rating system based solely on the roughness in form of IRI, yielding conditions equivalent to those by PCI-based rating system. This might enable engineers to better identify the proper maintenance strategies on a network level for budget allocation.

2.7.9 International Roughness Index prediction model for flexible pavements (Abdelaziz et al., 2020)

In this, the authors developed an accurate model to predict IRI for the flexible pavements based on the LTPP database. For modeling, they used multiple regression as well as the Artificial Neural Network method. In the models, pavement age, initial IRI (IRI just after pavement construction), transverse cracks, alligator cracks and standard deviation of the rut depth served as the inputs or the independent variables. Based on the analysis of correlation coefficients, it is determined that the pavement age, initial IRI, the standard deviation of total pavement rut depth, and transverse and alligator fatigue cracking were the most influencing parameters. From the regression model, the R2 value obtained was 0.57. Whereas using ANN for modeling yielded a much higher R2 value of 0.75.

2.7.10 A frame work for Development and Comprehensive Comparison of Empirical Pavement Performance Models. (Kargah-Ostadi and Stoffels, 2015)

Here authors provided an outline to develop and compare roughness prediction models for flexible pavements using LTPP data. Diverse architectures are used for defining the guidelines for pavement selection, including the Artificial Neural Network (ANN), Radial Basis Function (RBF) networks and Support Vector Machines (SVM). The qualitative and quantitative performance assessment of these models was done. Based on quantitative analysis, having higher accuracy may not provide a model that can be generalized over the database without being applied while developing a model. Scatter plots and sensitivity analysis were used for Qualitative analysis of the models. Based on them, Generalized RBF (GRBF) is the model that can be used to develop models for roughness prediction. The ML models seems to provide greater accuracy and generalization when compared to nonlinear regression models, with the downside being higher complexity.

2.7.11 Network-Level Pavement Roughness Prediction Model for Rehabilitation Recommendations (Kargah-Ostadi et al., 2010)

Here authors developed, using ANN, a model for variation in IRI over time. It can thus be utilized in the prediction and comparative analysis of trends in pavement roughness upon application of various rehabilitation activities. Based on the result of the model, ANN makes IRI prediction possible with minimum errors. As the model was made for a specific pavement and environmental considerations, it is only applicable to similar conditions. They presented the application of the roughness model in conjunction with the Life Cycle Cost Analysis (LCCA) to provide recommendations for future pavement rehabilitation strategies. Thus, being a useful tool in PMS decision-making at the Network Level.

2.7.12 Prediction of pavement roughness using a hybrid gene expression programming-neural network technique (Mazari and Rodriguez, 2016)

The authors presented an application of a hybrid technique for IRI modeling. In this, Artificial Neural Network was combined with Gene Expression Programming (GEP). The development of the model was done using two different LTPP databases, initially with Gene Expression Programming techniques and the other with a hybrid one after combining it with ANN. In the first, the predictors were AGE, ESAL, and SN, while in other IRI_0, AGE_0, $ESAL_0$, SN, ΔAGE and ΔESAL were the predictors to model IRI and IRI_P respectively. The hybrid method was found to predict the IRI quite effectively. While employing GEP for the prediction of present IRI using an independent set of historical LTPP roughness data,the prediction was reasonable.

2.7.13 Prediction of Pavement Performance: Application of Support Vector Regression with Different Kerenels. (Ziari et al., 2016a)

The applicability of Support Vector Machine (SVM) for predicting pavement condition is shown in the paper. For this, 9-input variables were considered, i.e., surface thickness, pavement thickness, Equivalent single-axle load, annual average daily traffic, annual average daily truck traffic, annual average precipitation, annual average temperature, annual average freeze index and the Pavement age. In the analysis, five kernel types of the SVM algorithm were formed. Here IRI serves as the pavement performance index. The Pearson VII Universal kernel of the SVM algorithm is shown to be capable of short and long-term pavement performance prediction, while others only showed reasonable accuracy in the short term.

2.7.14 Prediction of IRI in short and long terms for flexible pavements: ANN and GMDH methods (Ziari et al., 2016b)

The authors presented an approach for pavement condition prediction for flexible pavements using Artificial Neural Networks (ANN) and Group Method of Data Handling (GMDH). Analysis was performed for short-terms (1-year and 2-year) as well as long term (Pavement life cycle). The variables considered for roughness (IRI) analysis were Surface thickness (ST), pavement thickness (PT), Equivalent single axle load (ESAL), average annual daily traffic (AADT), annual average precipitation (AAP), annual average temperature (AAT) and annual average freeze index (AAFI). Based on the analysis and sensitivity check, ANN has a utility for long and short-term predictions of pavement condition due to its high accuracy. Whereas GMDH showed unacceptable accuracy levels for condition prediction. The ANN model can achieve better results when it is optimized by the means of homogeneous data.

2.8. REVISION OF EXISTING METHODS OR NEW APPROACH

2.8.1 Modeling the pavement serviceability ratio (PSR) of flexible highway pavements by artificial neural networks (Terzi, 2007)

In this paper, the author used Artificial Neural Network (ANN) technique to model the Pavement Serviceability Index (PSI) of Flexible pavement of highway and compared with the AASHO model, where PSI is used as a common evaluator in order to describe a functional condition of the pavement with respect to its ride quality. Here a methodology is developed using ANN such that the estimate of Pavement Serviceability Index is made such that the need for any restrictive assumption is eliminated by taking into account factors such as the slope variance, rut depth, patches, cracking and, longitudinal cracking as the input variables and the AASHO panel data as output variable. ANN used an inductive-learning principle to learn from a set of examples called training set (~80%) and residual data was selected as a test set (~20%) where the input variables included slope variance, rut depth, patches, cracking and longitudinal cracking with the output being panel data.

ANN model was developed for pavement serviceability ratio (PSR) determined on the surface of flexible pavements with repeated application of Levenberg-Marquardt back-propagation training. It also estimated better results than PSI to the panel of AASHO panel data of PSR (based on R^2 Values) and thus is appropriate for evaluating the functional capacity of flexible pavement from PSR for pavement management systems.

2.8.2 A modified pavement condition rating index for flexible pavement evaluation in Egypt (Ibrahim et al., 2020)

In this paper, the authors presented a revision and modification for the existing Pavement Condition Rating (PCR) method for pavement evaluation in order to incorporate it in an effective PMS for roads and transport directorates in Egypt. It is done such that the modified PCR method yields results comparable to the existing method, which uses Pavement Condition Index (PCI). The modified PCR thus presented had revised weights for distress.

The authors observed that the PCR values are usually much higher than the PCI values. Trends observed were similar though values were different that was due to different weights of each distress in each method. As the PCI method is more accurate, reliable and, common, the PCR method needs to be modified while maintaining its simplicity

The least-square method was used to improve the current distress weights in the PCR method to produce values comparable to the PCI method. New distresses with their weights were also incorporated in the MPCR method. The author observed, MPCR method yields reasonable values comparable to PCI values with R^2 value of 0.741 and comparatively minimal bias. Also, upon verification author concluded that MPCR values are very close to PCI values and the suggested weights were more reliable and rational as even slightly better accuracy was observed when compared to the data used for development with R^2 of 0.82. Thus the pavement condition using the MPCR method are found to better express the pavement condition when compared to the PCR method. The author also validated it further by employing this method on an existing pavement.

2.8.3 Comprehensive performance indicators for road pavement condition assessment (Marcelino et al., 2018)

In this, the authors presented a unique approach for the development of pavement condition indicators (PCI) using a machine learning algorithm (regularized regression with lasso) in order to find a sense of balance between data collected and practical requirements for the process.

Following data was collected averaged over 100m segments. And for 3% missing data cases (mean substitution method used)

- long evenness (IRI)
- trans evenness (rut depth, RD)
- friction (side force coefficient, SFC)
- texture (mean profile depth, MPD)
- cracking
- structural condition (FWD deflections)

the author described three distinct approaches for pavement condition assessment in this paper. First being one where information regarding various technical road aspects were considered (as a reference). Then another one in which information about visual distresses and ride comfort is then combined. And lastly, the one which is based on a regression model learned from data by a machine learning algorithm called regularized regression with lasso. In this, it does both feature selection and regularization with an enhanced prediction accuracy along with an interpretability of the model.

Here General Performance Indicator (GPI, as in COST Action 354 GPI) was evaluated using Quality Index (QI). Another parameter evaluated is GPI by the ML approach. In the ML approach, the author used an algorithm known as regularized regression with Lasso. It was applied for a supervised learning problem

Following conclusions were made by the authors:

- Application of ML can improve the accuracy of Pavement Condition Indicators (PCI) when less data is available
- ML model has lower MSE and higher R^2 as compared to QI
- Adjusted QI performs better than QI but worse than Regularized regression with lasso

2.8.4 Back-propagation Neural Network to Estimate Pavement Performance: Dealing with Measurement Errors (Amin and Amador-Jiménez, 2017)

In this paper, the authors applied Back-propagation Neural Network (BNN) with the main purpose of reducing the measurement errors while modeling pavement performance, thus making a model more robust. For this, a Generalized Delta Rule (GDR) algorithm is applied in the Multi-Layer Perceptron (MLP) network along with a sigmoidal activation function for the Pavement Condition Index (PCI). For this Average Annual Daily Traffic (AADT), Equivalent Single Axle Loads (ESALs), Structural Number (SN), pavement's age, slab thickness and difference of PCI between current and preceding year (ΔPCI) acted as the input for PCI. The results obtained were that the PCI has an inverse-relationships with AADT, ESAL and Pavement's age for both rigid as well as flexible pavements that have received treatments recently. But has a positive relationship with SN and slab thickness. ΔPCI influences PCI values significantly and AADT and ESAL have considerable importance in its estimate. However, there is an insignificant influence of the age of the pavement on the PCI values but not in the case of Flexible pavements. Also, as per the authors' conclusion, the complete historical record of pavement condition will enable in accurate performance models.

2.9 CONCLUSION OF LITERATURE

2.9.1 Present Scenario

Here we can see that the distresses in the pavement are the cause of the failure in flexible pavement. The measure of these distresses then helps in determining the structural as well as functional performance and capacity from the parameters observed. The measure of the distresses is used to determine the need for the maintenance and rehabilitation activities based on the decisions matrix PMS for the pre-defined cost-effectiveness and performance criteria for the pavement.

As the data collection process is both time and cost-intensive process, not every distress and performance data collection are possible as it being uneconomical. Thus, the development for the performance indices that are important for the pavement performance prediction becomes hampered and not every highway and road agencies are able to create or define proper and effective Maintenance and Rehabilitation (M&R) activities or schedule for their road networks for budget allocation for these activities and structural and performance evaluation at network and project level, which can be done using a pre-defined PMS or defining/developing PMS for the road network.

Also, by correlating the various performance indicators and further model development, one can dramatically reduce the need for measurement and data collection of all these parameters and the process becoming more cost as well as time efficient. It can be seen from the literature.

The pre-existing models (like in 1993 AASHTO) can be used for the analysis after calibration as per the site conditions, which may not be suitable for some of the sites due to dramatic differences in the site conditions. This problem is countered by the road and highway agencies by developing models for pavement performance evaluation based on the site-specific conditions. The measurement of both the functional and structural parameters is not necessarily needed. One can evaluate/correlate the structural performance of the pavement by measurement of some of the functional characteristics like the roughness or vice-a-versa because of some degree of correlation, as the roughness being one of the best indicators for the functional performance of the flexible pavement.

The correlation or model development is also possible by the use of tools like regression analysis, which provides the correlation between the dependent variable and various independent variables. With advent of computational prowess, there was development of techniques that are too complex to be employed by an individual. Techniques like ANN are better and, their superiority over the regression and other analysis is evident. Thus, serving as a better tool for the model development or correlation analysis between various parameters.

2.9.2 Gaps in existing research

- Strains developed in pavement are not determined by FWD Deflection directly. (FWD deflections are used to backcalculate modulus of elasticity pavement layers, which are then used to calculate stresses or strains based on different elastic models)

- Effectiveness of various Deflection Bowl Parameters (DBPs) in determining structural capacity of pavement. (DBPs provides improvement when evaluating the deflection bowl from the structural response of the pavement as they describe various aspects of the measured deflection bowl. They can further determine the effect and nature of response of various layers application of loading)
- Use of Machine Learning methods to calculate strains in pavement directly from FWD Deflections, DBPs and Surface Moduli.
- Performance of various Machine Learning approaches in calculating, or modelling, strains in pavement and their effectiveness.

2.9.3 Aims and Objectives:

The specified aims and objectives of the research are to be:

- Structural evaluation of bituminous layer by correlating Horizontal Tensile Strains with FWD Deflections, DBPs.
- Structural evaluation of base course and subgrade by developing correlation between Vertical Compressive Strains and FWD Deflections, DBPs.
- Comparison of the effectiveness of various Machine Learning techniques used for modeling: Multiple Linear Regression, Artificial Neural Network (ANN), and Gaussian Process Regression (GPR), Regression Trees, etc.
- Define effectiveness of developed models into Project and Network level analysis in the pavement management decision-making process.

CHAPTER-3 DESIGN & METHODOLOGY

3.1 DATA COLLECTION

Data was collected for 50km stretch of NH-9 (Hisar to Dabwali) in state of Haryana in India. Pavement under consideration is flexible having 4 lanes and a divided carriageway. The testing was performed in the summer season. The data collected included deflection measurements using Falling Weight Deflectometer. This data was then further processed to calculate Deflections Bowl Parameters and Surface Moduli, which were used to model strains in the pavement using Machine Learning.

FWD testing was done as per specifications in the IRC: 115-2014. As the pavement condition was classified as Good, measurements were taken at 250m intervals along the outer wheel paths of the outer lanes at 0.75m from the outer edge of outer lane, and at 500m intervals along the outer wheel path of the inner lane at 4.2m from outer edge of the outer lane. The Peak Normalized Deflections (in mm) is observed at 7 points at a radial distance of 0mm, 200mm, 300mm, 450mm, 600mm, 900mm and 1200mm. Pavement temperature is noted with the record of the Season of testing along with the thickness of Bituminous and Granular surface is also noted.

3.2 DATA ANALYSIS

In the analysis, the FWD Deflections were used to determine Deflection Bowl Parameters (DBPs), Surface Moduli and Horizontal and Vertical strains in the pavement. Strains were calculated from elastic moduli of each layer which were backcalculated from FWD Deflections and other inputs. These DBPs, Surface Moduli and Deflections were used individually as predictors to model Horizontal and Vertical strains as response in order to evaluate the pavement structurally as shown in Figure 3.1.

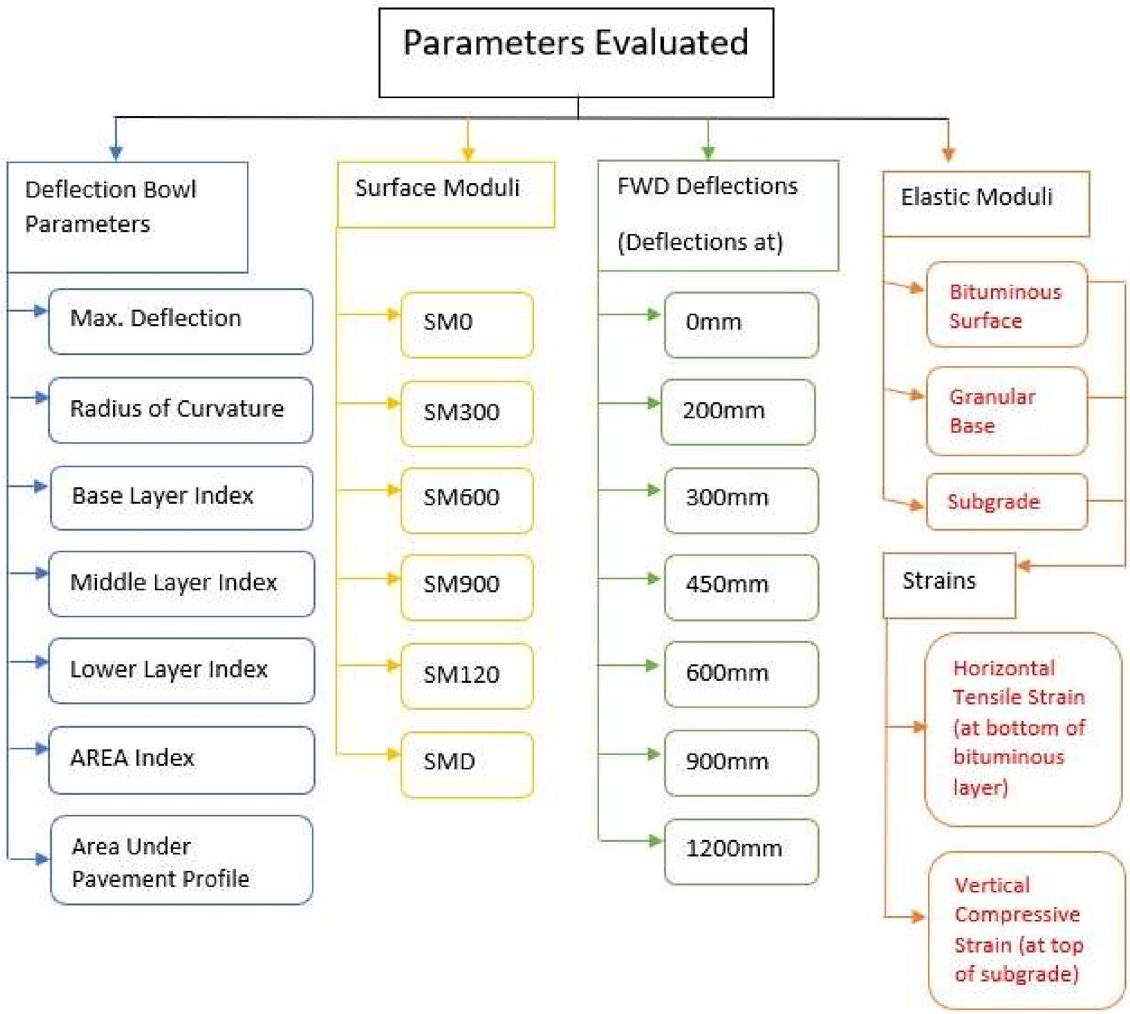

Figure 3. 1 Parameters evaluated for Strain Modelling

3.2.1 Deflections Bowl Parameters

The standardized position of the geophones to measure deflections is at 0mm, 200mm, 300mm, 450mm, 600mm, 900mm and 1200mm from the center of the load, giving a fair description of the entire deflection bowl, are utilized in order define various deflection bowl parameters. These DBPs can thus be used to describe various aspects of the deflection bowl that has been measured.

Using FWD deflections various deflection bowl parameters were calculated in order to enable one to assess the structural condition of the various layers of the flexible pavements. Each of these DBP is reported to correlate to specific zones of the pavement layers. DBPs selected and how they are calculated, along with the zone they correlate to, is presented in Table-3.1. They may either help or direct cause of distress determination or analysis on a preliminary level.

Table-3. 1 DBPs and zones they correlate to. (Horak, 2008)

DBPs		Formula	Zone Correlated
D_{MAX}	Maximum Deflection	$D_0\ measured\ at\ point\ of\ loading$	Gives an indication of all structural layers with about 70% contribution by the subgrade
RoC	Radius of Curvature	$RoC = \left(\frac{L^2}{2D_0\left(1 - {D_{200}}/{D_0}\right)}\right);$ $L = 200mm\ for\ FWD$	Gives an indication of the structural condition of the surfacing and base condition
BLI	Base Layer Index	$BLI = D_0 - D_{300}$	Gives an indication of primarily the base layer structural condition
MLI	Middle Layer Index	$MLI = D_{300} - D_{600}$	Gives an indication of the subbase and probably selected layer structural condition
LLI	Lower Layer Index	$LLI = D_{600} - D_{900}$	Gives an indication of the lower structural layers like the selected and the subgrade layers
AREA	Area	$A = \frac{6(D_0 + 2D_{300} + 2D_{600} + D_{900})}{D_0}$	Supposed to reflect the structural response of the whole pavement structure, but with weak correlations
AUPP	Area Under Pavement Profile	$AUPP = \frac{(5D_0 - 2D_{300} - 2D_{600} - D_{900})}{2}$	Characterizing condition of the pavement upper layers

3.2.2 Surface Moduli

Contribution of the subgrade to the central deflection is around 60% to 80%. (Ullidtz, 1987). Analysis and evaluation of the pavement response can be formed by correct classification as well as the by determination of the subgrade strength in pavement structural evaluation. Surface Moduli, which is weighted mean modulus of equivalent half space calculated from surface deflection by using adapted Boussinesq's equations, can be used for further investigation of nature of the response of the subgrade.

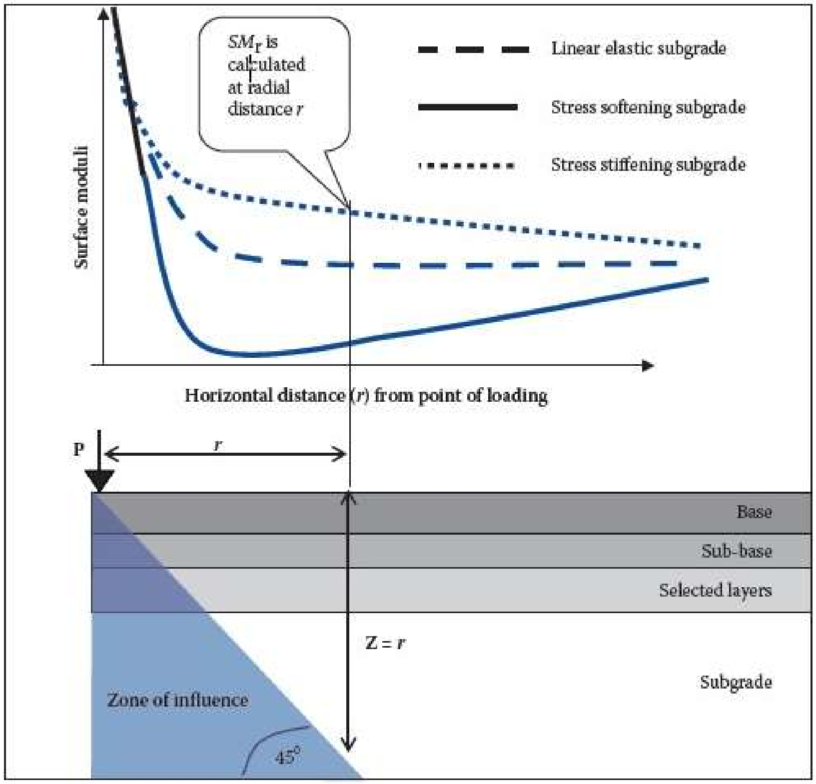

Figure 3. 2 Surface Moduli Plots for pavement structures(Ullidtz, 1987)

Using FWD Deflections, following Surface moduli can be calculated to define structural condition of the different layers of the flexible pavement.

SM under the point of loading having maximum deflection as well as the at any point away from point of deflection can be calculated as shown in Table-3.2

Table-3. 2 Surface Moduli calculation and corresponding zone.

Surface Moduli	Formula	Indicator of
SM0	$SM_0 = 2\sigma_0(1-\mu^2)\left({}^{a}/_{d_0}\right); r = 0$	Representative of compressed materials in the zone of influence below depth z
SM300	$SM_r = \sigma_0(1-\mu^2)\left({}^{a^2}/_{r} \times d_0\right)$ $r = radial\ distance\ from\ centre\ of\ loading$	
SM600		
SM1200		
SMD (Surface Modulus Differential))	$SMD = SM_{600} - SM_{1200}$	SM gradient over zones 2 and 3

3.2.3 Strain Calculation

Strains were calculated using the measured deflections as shown in Figure 3.3. The measured deflections and other inputs, like configuration of geophones, peak loading, thickness of each layer and poison's ratio, were used to backcalculate Elastic Moduli using KGPBACK. These elastic moduli were then used along with wheel load and configuration, Poison's ratio and thickness of layer and the depth and radial distance of the analysis points to calculate strains, i.e., horizontal strain at bottom of the bituminous layer and vertical strains at top of the subgrade, using IITPAVE.

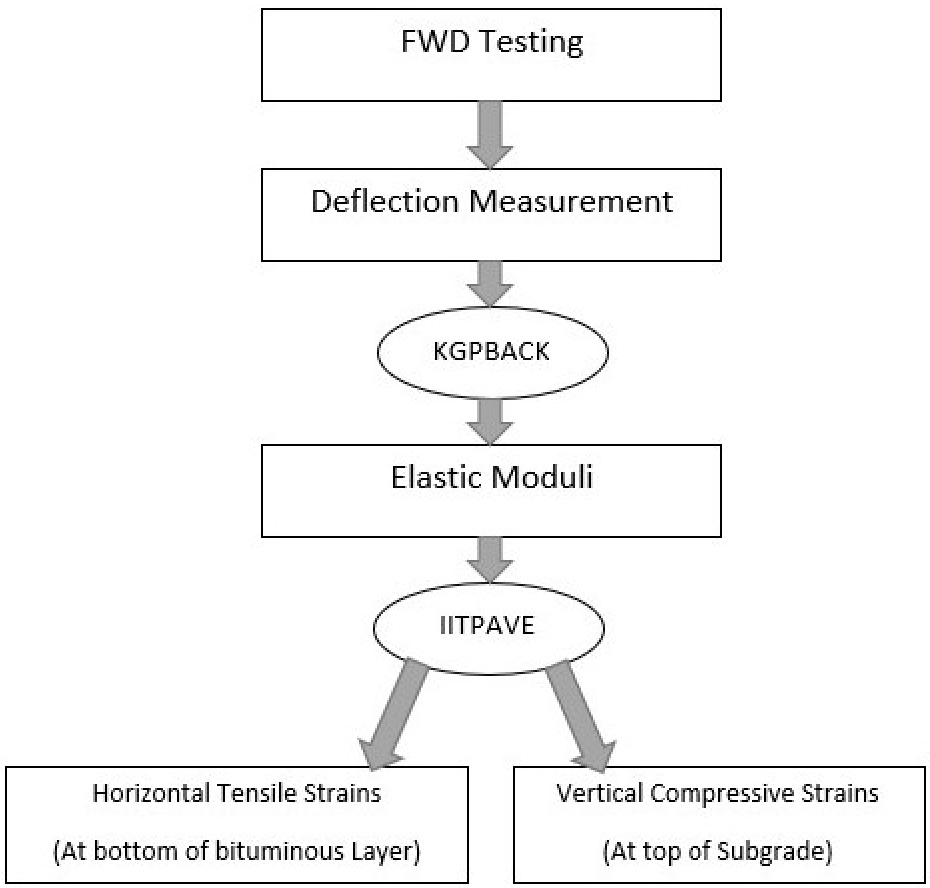

Figure 3. 3 Strain Calculation using FWD Deflections

3.2.3.1 Moduli of Layers (using KGPBACK)

Elastic Moduli of the pavement layers, i.e., Bituminous layer, Granular layer, and Subgrade, was determined by the process of Backcalculation. For this, KGPBACK software was used. It determines the elastic moduli of layers using Genetic Algorithm. These Backcalculated moduli thus calculated are used for the analysis of the in-service pavements as well as for the assessment of the structural condition pf the pavement. Elastic moduli are calculated using KGPBACK using inputs as shown in the Figure 3.4.

From the KGPBACK Software we get Backcalculated elastic moduli for the layer system. As these values are influenced by the pavement temperature of the bituminous layer as well as the moisture content of the granular layer and subgrade, appropriate corrections are applied to them for further utilization

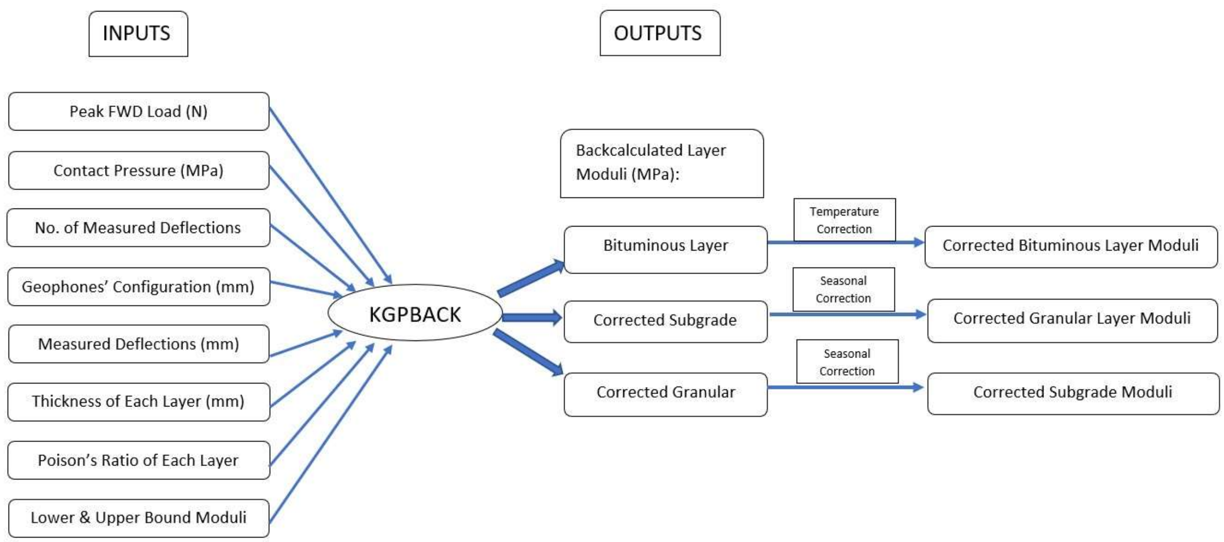

Figure 3. 4 Layer moduli of pavement calculated from FWD Deflections and other inputs using KGPBACK Software.

Correction applied to Backcalculated Moduli

<u>Bituminous Layer (Temperature Correction)</u>

The backcalculated moduli values at the temperature other than at the identified standard temperature needs to be corrected as per IRC:115-2014. Backcalculated Moduli obtained by FWD survey at temperature "T_2"°C are corrected to estimate Elastic modulus corresponding to the temperature "T_1"°C using the following equation:

$$E_{T1} = \lambda\, E_{T2}\ where, \lambda = Temperature\ Correction\ Factor$$

$$\lambda = \frac{1 - 0.238 \ln T_1}{1 - 0.238 \ln T_2}$$

$$E_{T1} = Backcalculatedd\ Modulus\ (MPa) at\ temperature\ T_1 ℃$$

$$E_{T2} = Backcalculatedd\ Modulus\ (MPa) at\ temperature\ T_2 ℃$$

Seasonal Variation Correction

As per IRC:114-2014, pavement layer moduli values should relate to the period when the subgrade is at its weakest state. In India, this period usually occurs during the end of the monsoon. Thus, it is desirable to conduct FWD Testing during this period. However, when it is not possible to do so, a correction procedure should be adopted.

Granular Layer:

Following equations are applicable for the monsoon granular layer modulus of 60 MPa or more. Where Minimum Winter modulus of 80 MPa and minimum Summer Modulus of 100 MPa is there.

$$E_{gran\_mon} = -0.0003 \times \left(E_{gran\_sum}\right)^2 + 0.9584 \times \left(E_{gran\_sum}\right) - 32.989$$

$$E_{gran\_mon} = 1.05523 \times \left(E_{gra\_win}\right)^{0.624} - 113.857$$

$where,$

$$E_{gran\_mon} = granular\ layer\ modulus\ in\ Monsoon\ (MPa)$$

$$E_{gran\_sum} = granular\ layer\ modulus\ in\ Summer\ (MPa)$$

$$E_{gran\_win} = granular\ layer\ modulus\ in\ Winter\ (MPa)$$

Subgrade:

Following equations are used for estimating the modulus value of subgrade layer from the modulus value backcalculated from deflections collected during winter and summer seasons. These equations are applicable for monsoon subgrade modulus of 20 MPa or more. Where Winter/Summer Modulus of 30 MPa is present.

$$E_{sub\_mon} = 3.351 \times \left(E_{sub\_win}\right)^{0.7688} - 28.9$$

$$E_{sub\_mon} = 0.8554 \times \left(E_{sub\_sum}\right) - 8.461$$

$where,$

$$E_{sub\_mon} = subgrade\ modulus\ in\ Monsoon\ (MPa)$$

$$E_{sub\_win} = subgrade\ modulus\ in\ Winter\ (MPa)$$

$$E_{sub\_sum} = subgrade\ modulus\ in\ Summer\ (MPa)$$

3.2.3.1 Strain Calculation (using IITPAVE)

IITPAVE is developed for the analysis of elastic layered pavement system. Using this software, the stresses, strains, deflections at different locations in a pavement system can be computed.

Here a uniformly distributed single load is applied over a circular contact area. In IITPAVE, wheel load and the contact pressure are the load inputs that are used to describe the single vertical load applied at the surface. (Congress, 2018)

For the analysis, Horizontal tensile strains at bottom of the bituminous layer and Vertical compressive strains at top of subgrade are required. These are calculated using IITPAVE using inputs as shown in the Figure 3.5

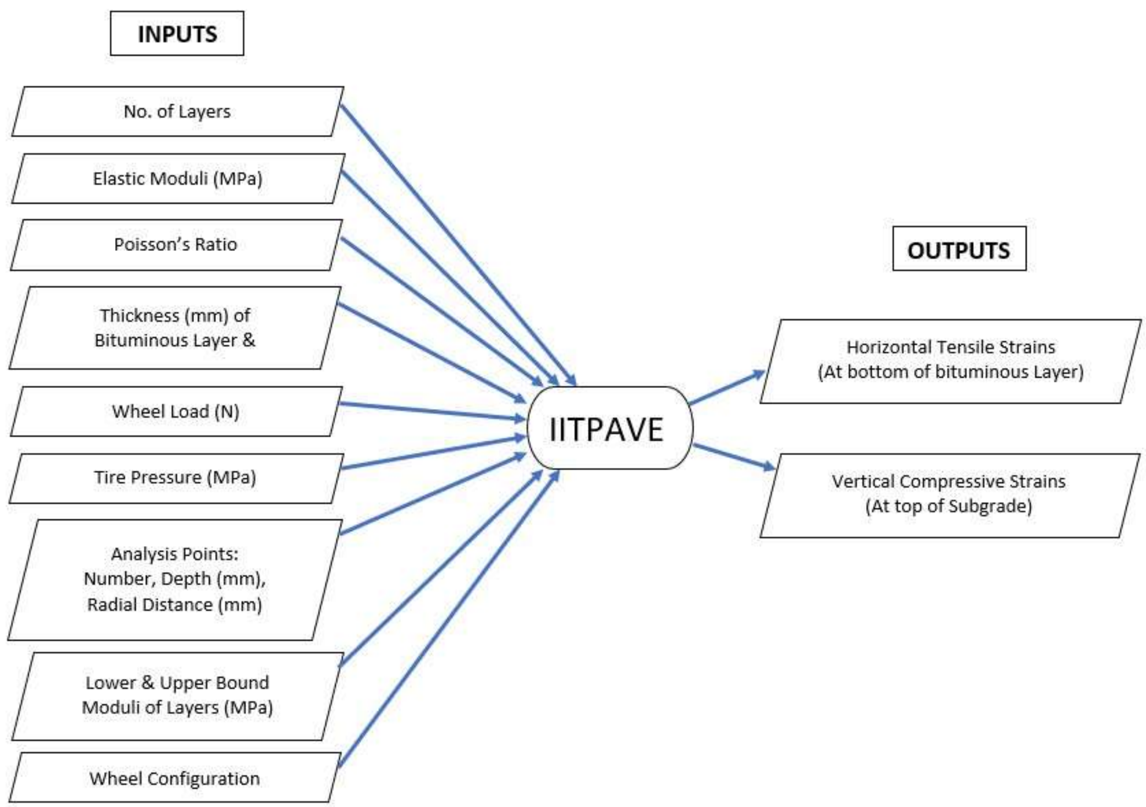

Figure 3. 5 Strain calculation from Layer Moduli and other inputs using IITPAVE

3.3 MODEL DEVELOPMENT

The measured deflections during FWD testing were used to model the Horizontal as well as the Vertical strains. Similarly, various deflection bowl parameters (DBPs) like Maximum measured deflection, Radius of Curvature, Base Layer Index, Middle Layer Index, Lower Layer Index, Area and Area under pavement profile, were also used for modelling abovementioned strains. The effect of pavement thickness on the model development is also considered. The extent to which various surface moduli can be used in the model development is also studied.

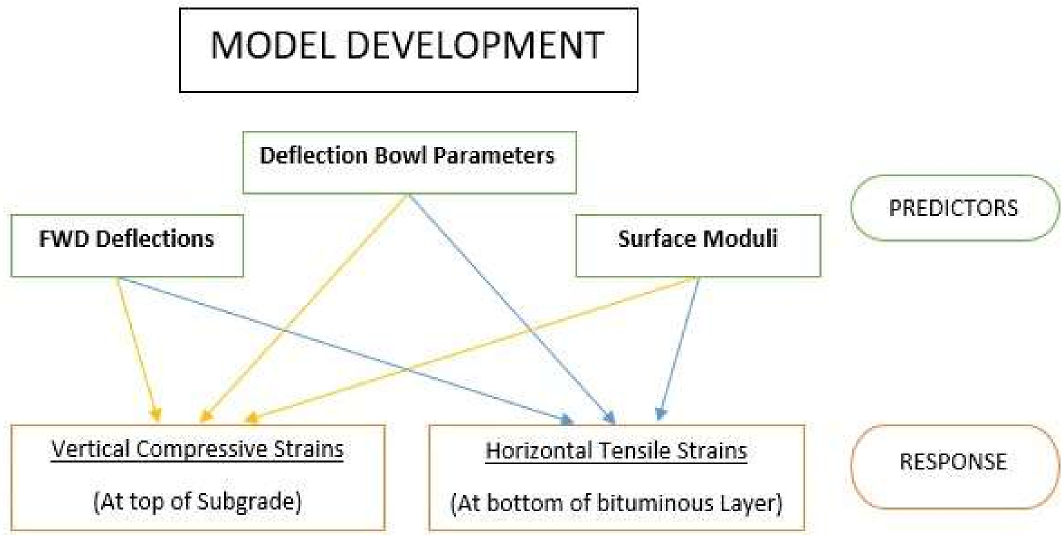

Figure 3. 6 Predictors and Response for model development

3.3.1 For modelling following software/programs were used:

3.3.1.1 IBM SPSS Statistics 26

This software suite was used for regression analysis. Here individual parameters were used for modelling the strains. Following models were employed:

- Linear
- Logarithmic
- Inverse
- Quadratic
- Cubic
- Compound
- Power
- S
- Growth
- Exponential
- Logistic

3.3.1.2 MATLAB 2022a

This is a platform for programming and numerical computation. An interactive environment is provided in this for algorithm development, data visualization, data analysis and numerical computation.

A. Neural Net Fitting App:

Included in *Deep Learning Toolbox 14.4*. In this two-layer feed-forward network was used for ANN Modelling of Individual parameters with the strains.

B. Regression Learner App:

Included in *Statistics and Machine Learning Toolbox 12.3* was used to train regression models to make predictions using supervised machine learning. In this, models (with multiple parameters) were developed after utilizing the feature ranking algorithms, i.e., MRMR, F Test and RReliefF, based on the importance scores.

Models that were developed includes the following:

- Linear Regression Models
  - Preset: Linear (Terms: Linear, Interactions, Quadratic, Pure Quadratic)
  - Preset: Stepwise Linear
- Regression Trees
  - Fine Tree
  - Medium Tree
  - Coarse Tree
  - Optimizable Trees
- Support Vector Machines (SVM)
  - Fine Gaussian SVM
  - Medium Gaussian SVM
  - Coarse Gaussian SVM
  - Quadratic
  - Optimizable
- Gaussian Process Regression Models (GPR)
  - Linear GPR

    - Cubic GPR
    - Rational Quadratic GPR
    - Exponential GPR
    - Squared Exponential GPR
    - Matern 5/2 GPR
    - Optimizable GPR
- Ensembles of Trees
    - Boosted Trees
    - Bagged Trees
    - Optimizable Ensembles
- Artificial Neural Networks
    - Narrow Neural Networks
    - Medium Neural Networks
    - Wide Neural Networks
    - Bi-Layered Neural Network
    - Tri-Layered Neural Networks
    - Optimizable Neural Networks

3.3.2 Ranking Algorithms

Multiple parameters were selected based on the feature ranking algorithms (MRMR, F Test, RReliefF)

3.4.2.1 MRMR Algorithm (Minimum Redundancy Maximum Relevance):

In this algorithm, at each iteration, feature is selected that has the maximum importance w.r.t. the target variable and minimum idleness w.r.t. the features that have been selected at the prior iterations.(Ding and Peng, 2005)

That is, it attempts to find a small set of features that are relevant w.r.t. the target variable and are hardly redundant with each other.

3.4.2.2 F-tests:

It is a statistical test having F-Distribution under null hypothesis for test statistic. It assesses the amount of variability between the group means in the context of the variation within groups to determine whether the mean differences are statistically significant. Here the standing of each predictor is examined independently using an F-test.

3.4.2.3 RReliefF:

RReliefF or ReliefF is used to find weightage of predictors in the case where the response is continuous or a multiclass categorical variable. In this the algorithm predictors are penalized when they give values different than the values of the neighbors of the same class and are awarded for giving different values to the neighbors of different class. However, assigning intermediate weights to compute final weights of the predictors in case of RreliefF (Robnik-Šikonja and Kononenko, 2003)

3.4 DEVELOPED MODELS

Both single as well as multiple parameters were used as predictors to model Horizontal and Vertical Strains.

3.4.1 Using Single Parameters

Regression analysis (IBM SPSS) and ANN (MATLAB) was used to model both Horizontal Strains and Vertical Strains using:

- FWD Deflections
    - 0mm
    - 200mm
    - 300mm
    - 450mm
    - 600mm
    - 900mm
    - 1200mm
- Deflections Bowl Parameters (DBPs)
    - Maximum Deflection (D.Max)
    - Radius of Curvature (RoC)
    - Base Layer Index (BLI)
    - Middle Layer Index (MLI)
    - Lower Layer Index (LLI)
    - AREA Index (A)

- Area Under Pavement Profile (AUPP)

- Surface Moduli
  - SM0
  - SM300
  - SM600
  - SM1200
  - SMD

3.4.1.1 Regression Models

Regression models to model strains were developed using the IBM SPSS Statistics. Linear, Logarithmic, Inverse, Quadratic, Cubic, Logistic and many more models were used. Here the DBPs, Surface Moduli and FWD Deflections were used individually, which served as independent variable, and to predict Strains in the pavement, i.e., dependent variable. Here the performance of the models was determined based on the Coefficient of Determination (R2) and Mean Squared Error (MSE).

3.4.1.2 ANN Models

Neural Net Fitting App included in Deep Learning Toolbox of MATLAB was used for developing ANN Models. In this two-layer feed-forward network was used for ANN Modelling of parameters with the strains.

In ANN models, dataset was randomly divided into 3 parts, i.e., for Training, Validation and Testing, consisting of 70%, 15%, 15% of data respectively. Model was prepared using the data in training set by calculating gradients and the network weights. and then its performance was evaluated from the testing dataset. Then the dataset in the validation phase was used for optimal neural network training. Data in the validation phase is utilized to for regularization to prevent overfitting. Then at last, the test dataset was used to evaluate the performance of the network.

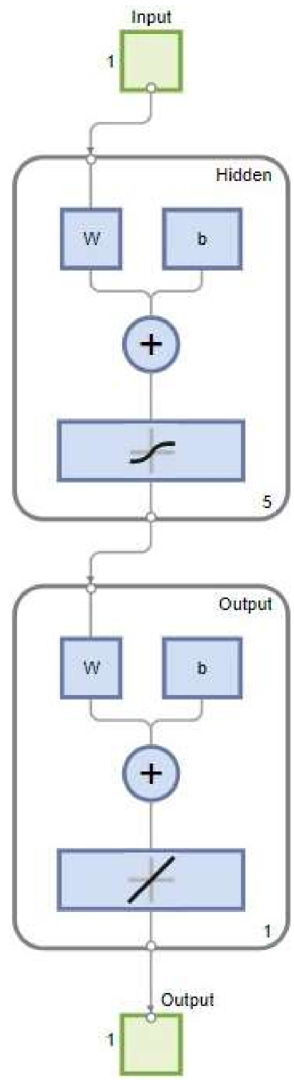

Figure 3. 7 Network diagram of ANN (Hidden Layer size:5; I/P=1; O/P=1)

Three different algorithms were used for modelling, i.e., Levenberg-Marquardt, Bayesian Regularization and Scaled Conjugate Gradient. Also, ANN models with 5 and 10 hidden layers were created for each algorithm in order to determine the best way to model strains using different parameters.

3.4.2 Using Multiple Parameters

Multiple parameters were selected based on the feature ranking algorithms (MRMR, F Test, RReliefF) to serve as the predictors or independent variable to model two different dependent variables, i.e., horizontal tensile strain bottom of bituminous layer and Vertical compressive strain at top of subgrade. The data was then divided into two sets randomly, i.e., training and testing, consisting of 70% and 30% data respectively. Model was prepared using the data in training set and then its performance was evaluated from the testing dataset.

A. Horizontal Strain using FWD Deflections

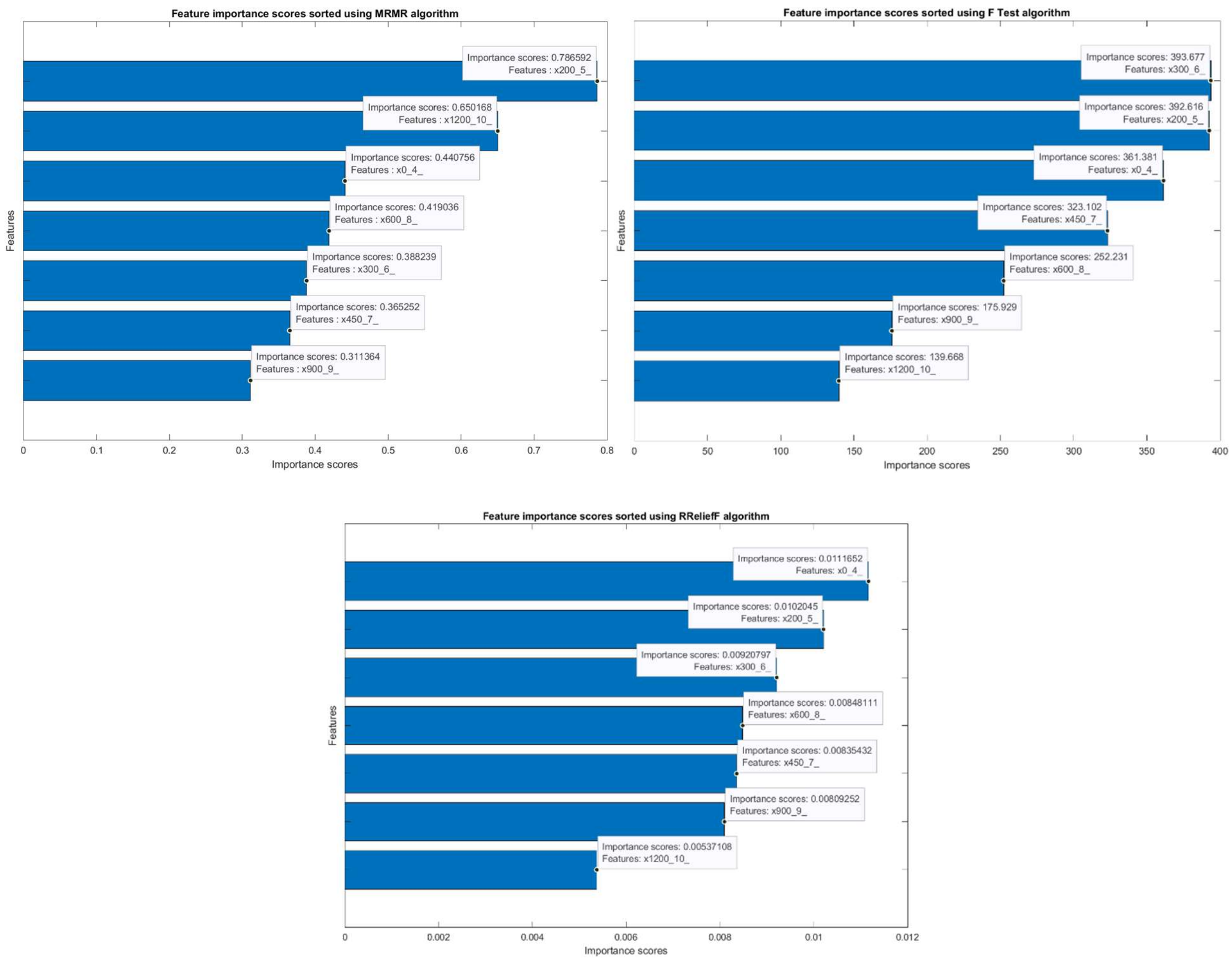

Figure 3. 8 Feature importance scores of FWD Deflections sorted by MRMR, F Test and RReliefF algorithms for modelling Horizontal Strains

Based on the relatively low feature importance scores in the ranking algorithms, deflection at 900mm and 1200mm were excluded. Following 5 features with good scores were selected to create a model.

- FWD Deflection (A): (0mm, 200mm, 300mm, 450mm, 600mm)

B. Horizontal Strain using DBPs

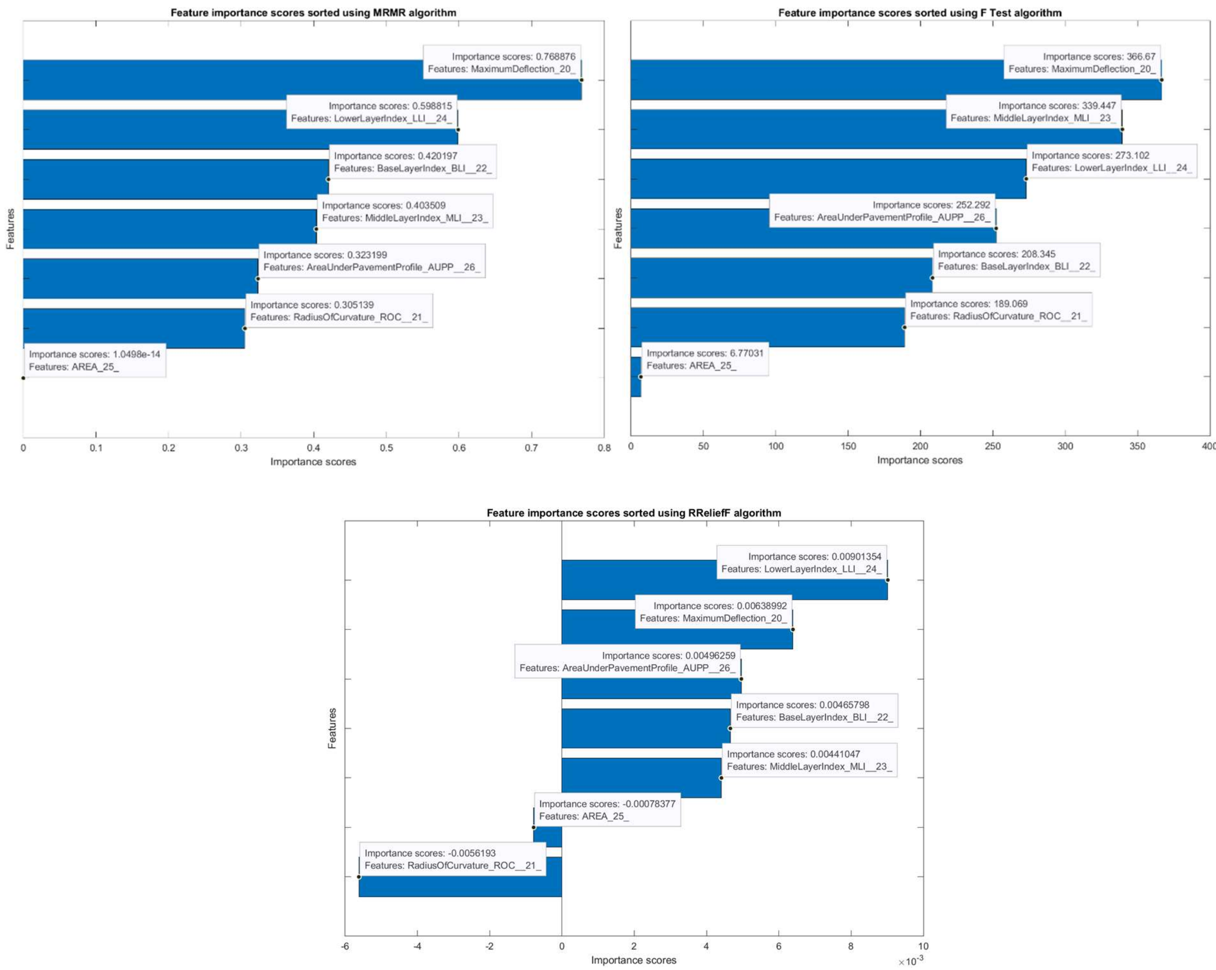

Figure 3. 9 Feature importance scores of DBPs sorted by MRMR, F Test and RReliefF algorithms for modelling Horizontal Strains

Maximum Deflection, Radius of Curvature and Area Under Pavement Profile correlate to the zone having bituminous layers and are thus selected. AREA index, BLI and MLI although having weak correlation to top zone are selected based on high importance scores in the ranking algorithm. Therefore, following models are developed based on the suitability:

- Deflection Bowl Parameters (A) (Max Def., RoC, BLI, MLI, AREA, AUPP)
- Deflection Bowl Parameters (B) (RoC, BLI, AREA, AUPP)

C. Horizontal Strain using Surface Moduli

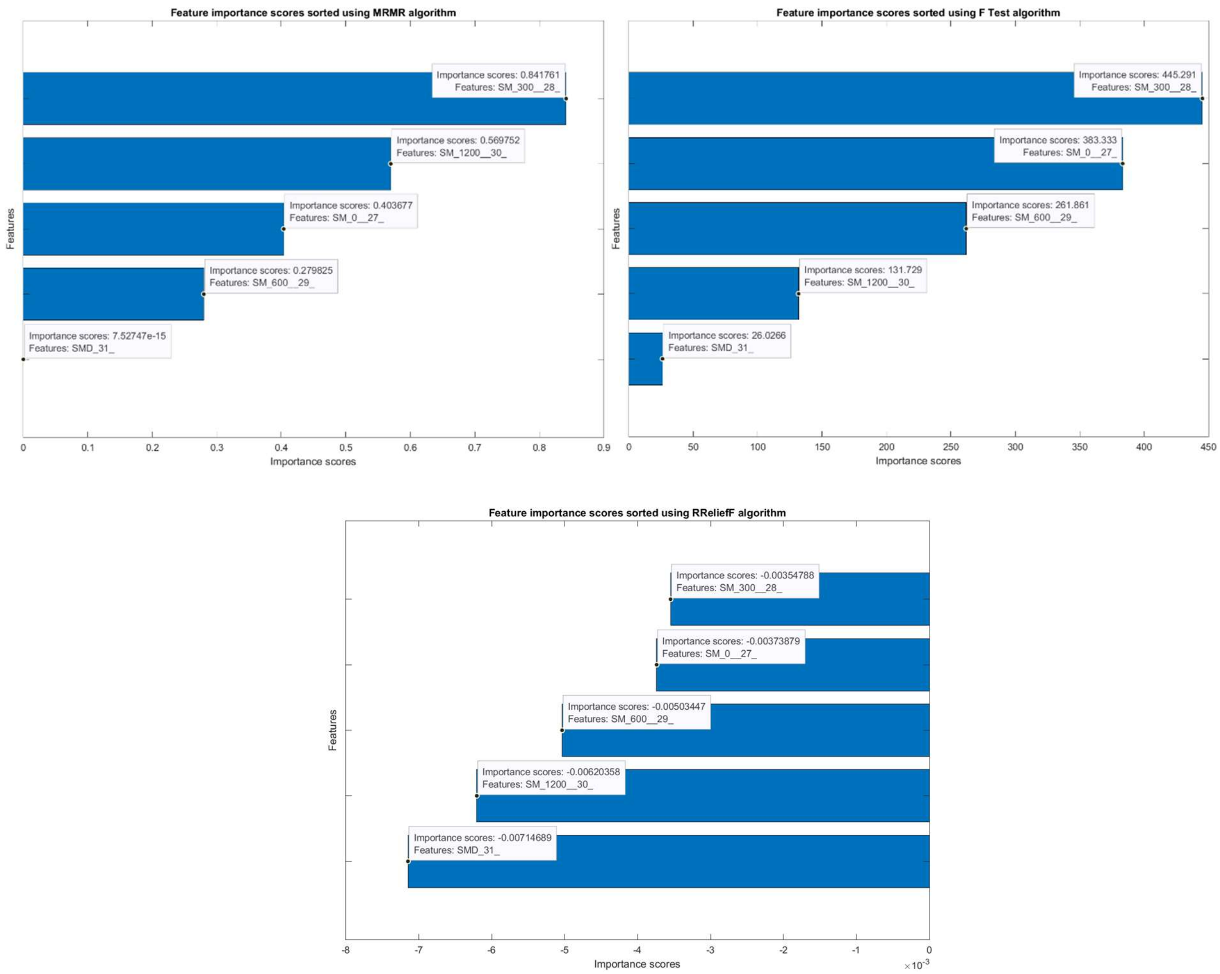

Figure 3. 10 Feature importance scores of Surface Moduli sorted by MRMR, F Test and RReliefF algorithms for modelling Horizontal Strains

Based on the relative high importance scores of the ranking algorithms as well as them representing elastic response of the top layers of pavement, SM0 and SM300 were considered for modelling. As SM600 and SM1200 also showed good scores, they were also selected to see their effect in modelling. Thus, following models were developed:

- Surface Moduli (A) (SM0, SM300, SM600, SM1200)
- Surface Moduli (B) (SM0, SM300, SM600)
- Surface Moduli (C) (SM0, SM300)

D. Vertical Strain using FWD Deflections

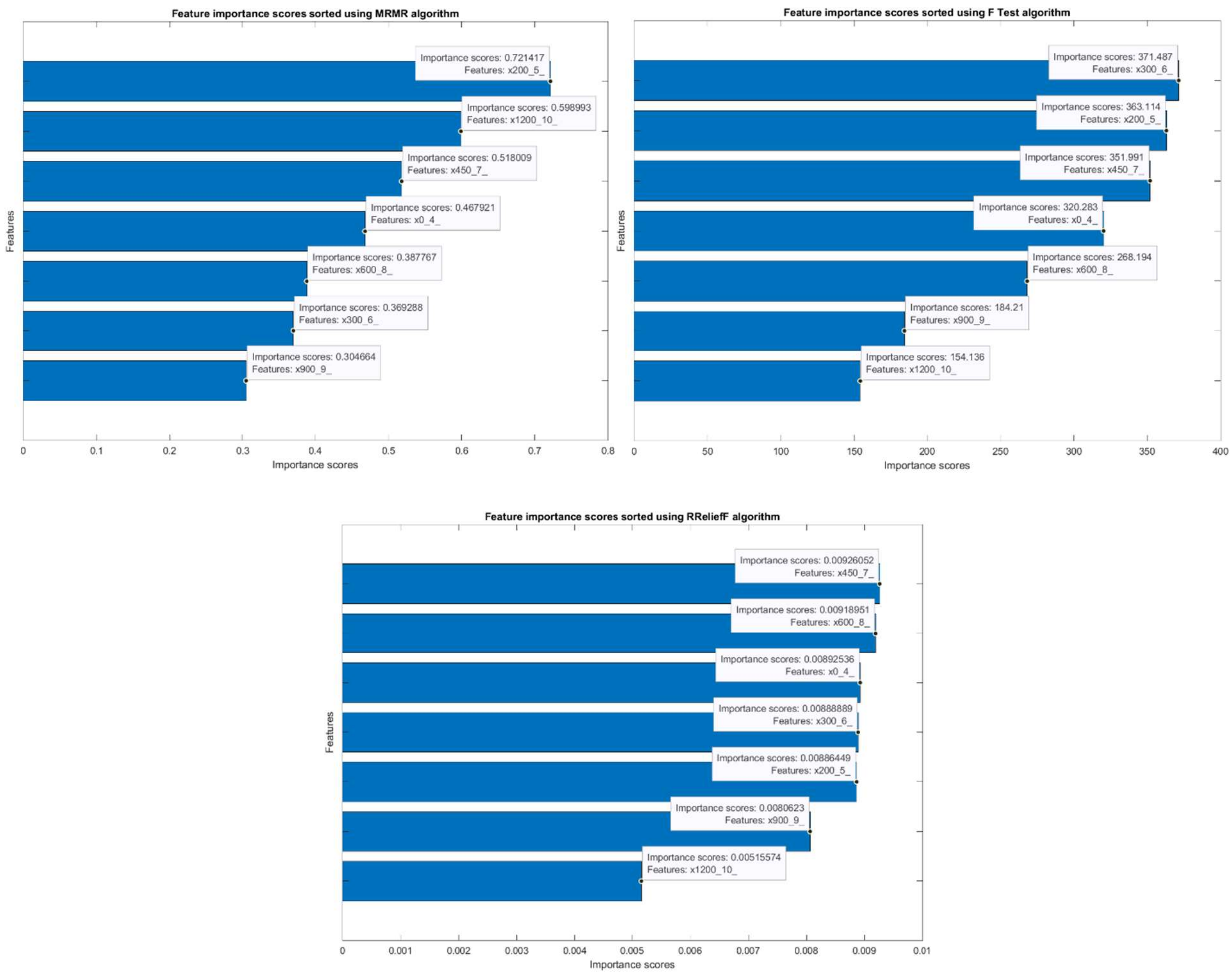

Figure 3. 11 Feature importance scores of FWD Deflections sorted by MRMR, F Test and RReliefF algorithms for modelling Vertical Strains

As deflections at 0mm and 200mm upon loading reflect response of nearly whole of pavement, in addition to high importance scores, they are selected for modelling along with deflection at 450mm. Based on the scores of the ranking algorithms, deflection at 300mm, 600mm and 1200mm are also selected for modelling. Following models were developed:

- FWD Deflection (A) (0mm, 200mm, 450mm, 1200mm)
- FWD Deflection (B) (0mm, 200mm, 450mm, 600mm)
- FWD Deflection (C) (0mm, 200mm, 300mm, 450mm, 600mm)

E. Vertical Strain using DBPs

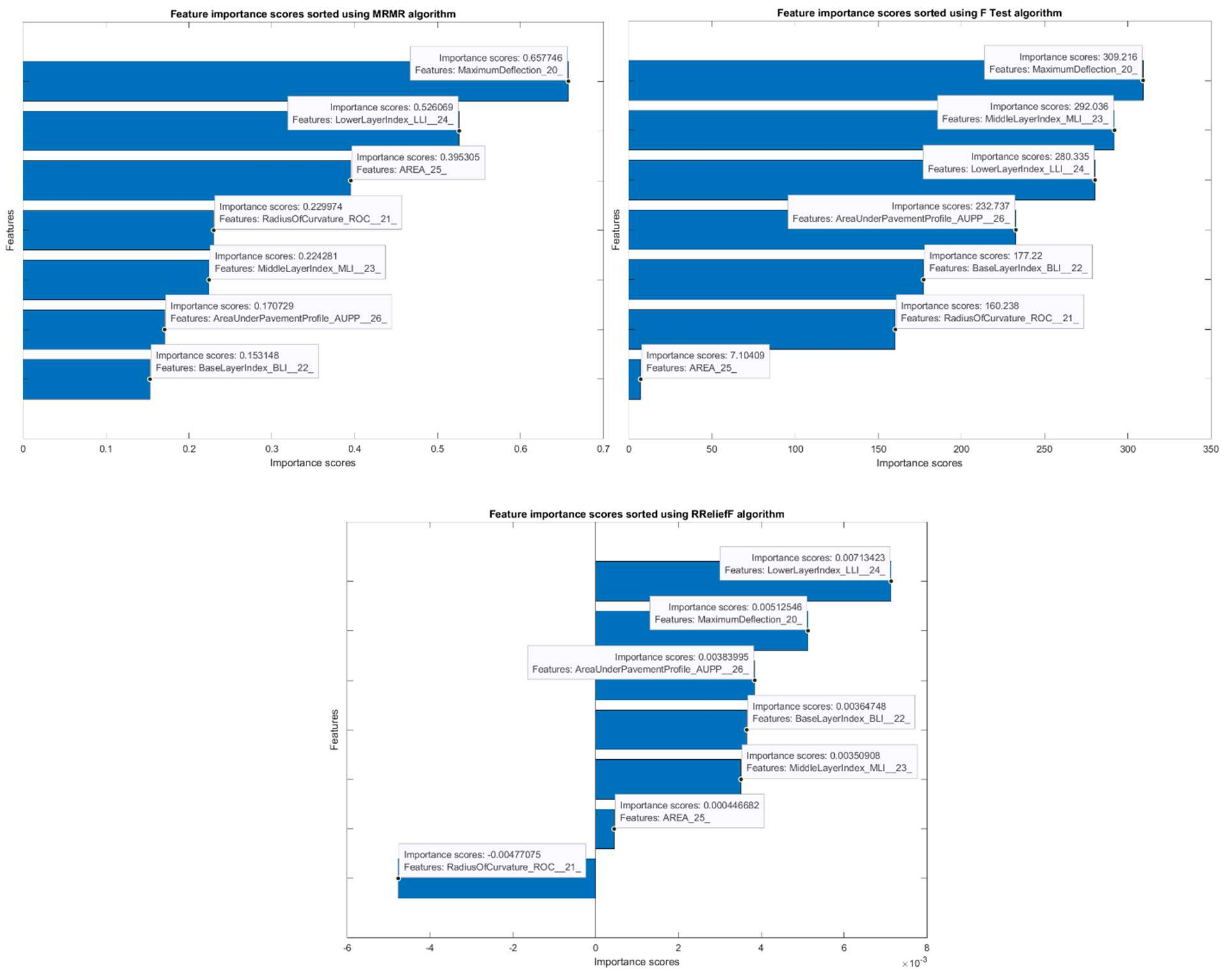

Figure 3. 12 Feature importance scores of DBPs sorted by MRMR, F Test and RReliefF algorithms for modelling Vertical Strains

Maximum Deflection and Lower Layer Index correlate to the zone having subgrade and subbase and are thus selected. Although AREA index and MLI shows weak correlational to lower layers, they are selected (along with ROC) based on high importance scores in the ranking algorithm. Therefore, following models are developed based on the suitability:

- Deflection Bowl Parameters (A) (Max.Def, LLI, AREA)
- Deflection Bowl Parameters (B) (Max.Def, MLI, LLI)
- Deflection Bowl Parameters (C) (Max.Def, ROC, MLI, LLI, AREA)

F. Vertical Strain using Surface Moduli

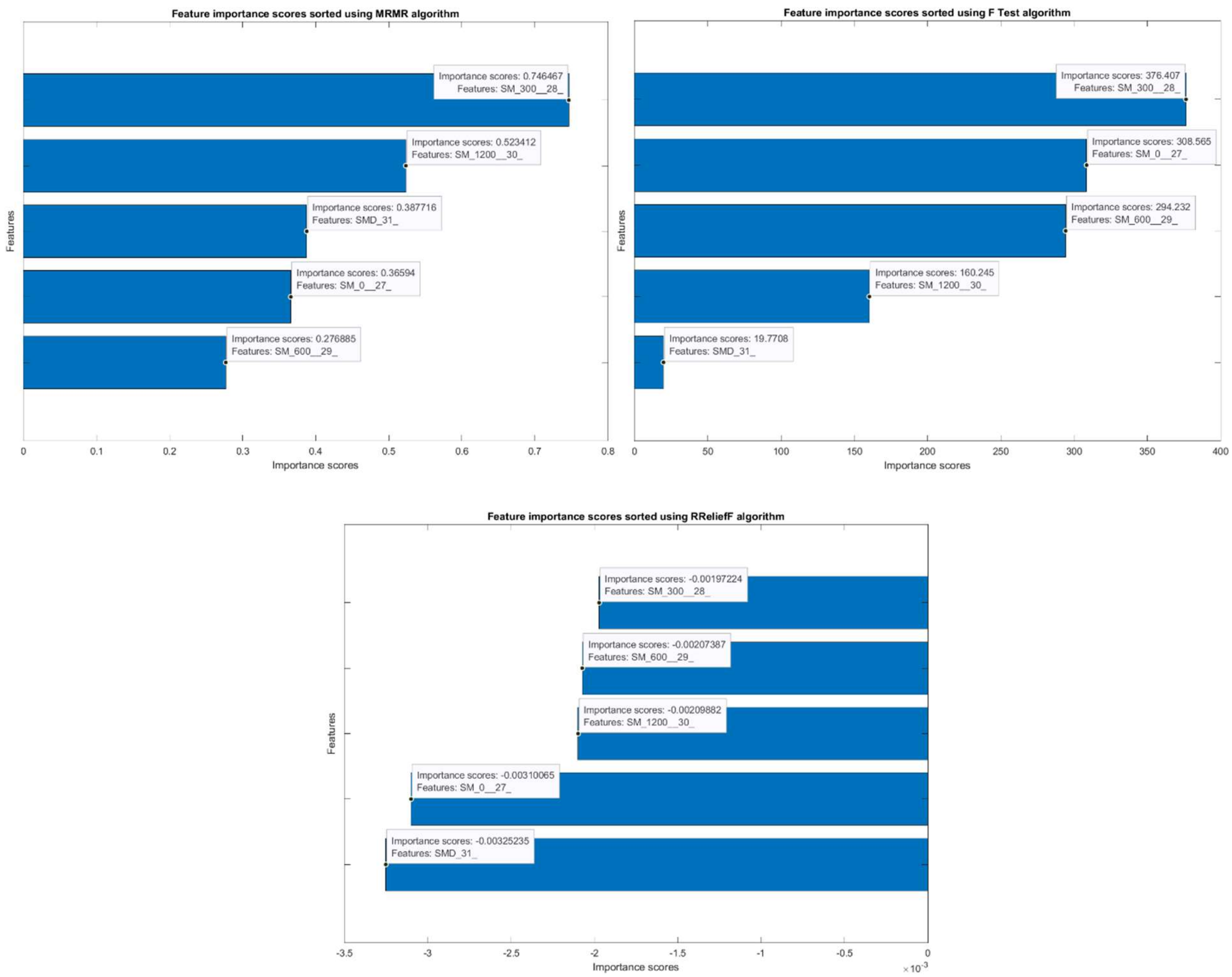

Figure 3. 13 Feature importance scores of Surface Moduli sorted by MRMR, F Test and RReliefF algorithms for modelling Vertical Strains

Based on the relative high importance scores of the ranking algorithms as well as them representing elastic response of the bottom layers of pavement, SM600 and SM1200 were considered for modelling. As SM0 and SM300 are representative of elastic response of top layer of the pavement which may affect condition of the lower layers and they had high importance scores, they were also selected for modelling. Thus, following models were developed:

- Surface Moduli (A): (SM0, SM300, SM600, SM1200)

Therefor Table-3.3. contains the various models (and their IDs) that were developed using various Machine Learning Techniques:

Table-3. 3 Models Developed using Machine Learning techniques with their IDs

Response/ Dependent Variable	Predictors/ Independent Variables	Model ID
Horizontal Strain	FWD Deflection: 0mm, 200mm, 300mm, 450mm, 600mm	H.FWD.A
	DBP: Max Def, RoC, BLI, MLI, LLI, AREA, AUPP	H.DBP.A
	DBP: RoC, BLI, AREA, AUPP	H.DBP.B
	Surface Moduli: SM0, SM300, SM600, SM1200	H.SM.A
	Surface Moduli: SM0, SM300, SM600	H.SM.B
	Surface Moduli: SM0, SM300	H.SM.C
Vertical Strain	FWD Deflection: 0mm, 200mm, 450mm, 1200mm	V.FWD.A
	FWD Deflection: 0mm, 200mm, 450mm, 600mm	V.FWD.B
	FWD Deflection: 0mm, 200mm, 300mm, 450mm, 600mm	V.FWD.C
	DBP: Max Def, LLI, AREA	V.DBP.A
	DBP: Max Def, MLI, LLI	V.DBP.B
	DBP: Max Def, RoC, MLI, LLI, AREA	V.DBP.C
	Surface Moduli: SM0, SM300, SM600, SM1200	V.SM.A

CHAPTER-4 RESULT

4.1 MODELS WITH SINGLE PARAMETERS

4.1.1 Regression Results (IBM SPSS)

For regression analysis, IBM SPSS Statistics was used. In this regression analysis was done using Linear, Logarithmic, Inverse, quadratic, Cubic, Compound, Power, S, Growth, Exponential, and logistic models. Of these only Quadratic and Cubic models showed promising results when FWD Deflections and DBPs were used to model the Strains, horizontal as well as vertical. But in case of Surface Moduli, regression showed results with low R2 values which were significant in case of some parameters but not good enough to be used to model Horizontal and Vertical strains with good confidence.

4.1.1.1 Horizontal Strains using FWD Deflections

Table-4. 1 R^2-values of Regression Models used to develop Horizontal Strains using FWD Deflections

Model	Coefficient of Correlation (R^2)						
	0mm	**200mm**	**300mm**	**450mm**	**600mm**	**900mm**	**1200mm**
Linear	0.695	0.7	0.688	0.654	0.614	0.525	0.46
Logarithmic	0.489	0.481	0.471	0.459	0.44	0.391	0.343
Inverse	0.252	0.22	0.204	0.212	0.211	0.198	0.152
Quadratic	0.792	0.783	0.767	0.716	0.671	0.582	0.526
Cubic	0.868	0.883	0.873	0.82	0.749	0.616	0.538
Compound	0.694	0.704	0.697	0.67	0.635	0.552	0.493
Power	0.516	0.512	0.504	0.495	0.477	0.428	0.38
S	0.277	0.244	0.227	0.238	0.237	0.223	0.173
Growth	0.694	0.704	0.697	0.67	0.635	0.552	0.493
Exponential	0.694	0.704	0.697	0.67	0.635	0.552	0.493
Logistic	0.694	0.704	0.697	0.67	0.635	0.552	0.493

As can be seen from Table-4.1, deflections at distances 0mm, 200mm, 300mm, 450mm and 600mm correlate well with quadratic and cubic models, with latter having better results, based on regression analysis. The R^2 values of 0mm, 200mm and 300mm were 0.868, 0.883, 0.873 respectively showing a good correlation. Meanwhile deflections at 450mm and 600mm had R^2 values of 0.820 and 0.749, requiring further evaluation to be incorporated for modelling the Horizontal strains. The model summary of these models is provided in Table-4.2

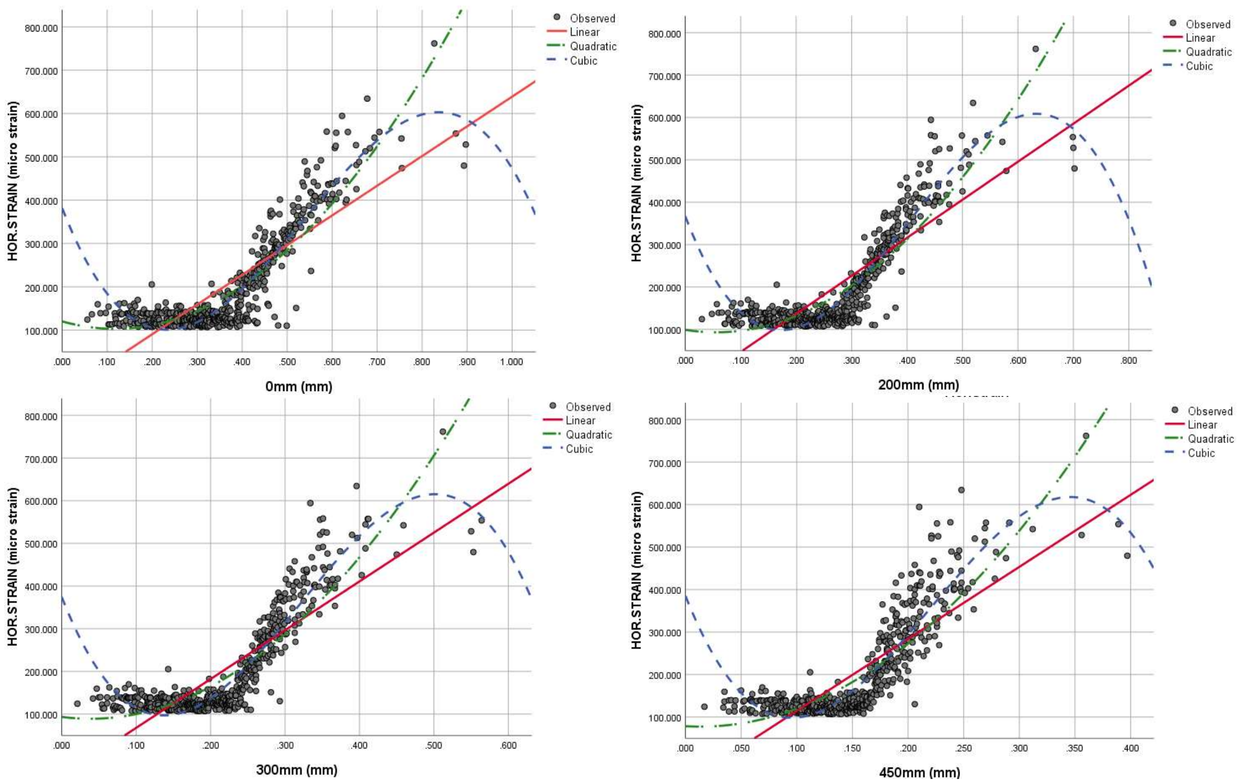

Figure 4. 1 Regression plot of Horizontal Strains Modelled using FWD Deflections at 0mm, 200mm, 300mm, 450mm

The comparison of the best performing models with the linear model is shown in the regression plots in Figure 4.1. Here we can see the models developed from the FWD deflections measured at 0mm, 200mm, 300mm and 450mm.

Table-4. 2 Model Summary and Parameter Estimation of best Regression Models used to develop Horizontal Strains using FWD Deflections

Dependent Variable: HORIZONTAL STRAIN									
A. The independent variable is *Deflection at 0mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.695	1277.389	1	561	0	-46.624	685.47		
Quadratic	0.792	1068.755	2	560	0	120.169	-293.361	1243.629	
Cubic	0.868	1224.547	3	559	0	381.527	-2658.629	7303.618	-4553.566
B. The independent variable is *Deflection at 200mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.7	1306.829	1	561	0	-42.887	897.395		
Quadratic	0.783	1010.037	2	560	0	98.892	-206.743	1852.822	
Cubic	0.883	1405.823	3	559	0	367.07	-3426.055	12632.275	-10453.709
C. The independent variable is *Deflection at 300mm*									
Equation	**Model Summary**					**Parameter Estimates**			

	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear	0.688	1237.769	1	561	0	-46.579	1143.647		
Quadratic	0.767	923.177	2	560	0	93.412	-224.219	2897.912	
Cubic	0.873	1281.224	3	559	0	374.579	-4447.734	20619.23	-21523.031
D. The independent variable is *Deflection at 450mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.654	1061.677	1	561	0	-54.989	1694.596		
Quadratic	0.716	706.747	2	560	0	78.491	-148.897	5608.377	
Cubic	0.82	849.749	3	559	0	384.963	-6596.799	44007.526	-66464.384

4.1.1.2 Horizontal Strains using DBPs

Table-4. 3 R^2-values of Regression Models used to develop Horizontal Strains using DBPs

Model	**Coefficient of Correlation (R^2)**						
	Max Def	**RoC**	**BLI**	**MLI**	**LLI**	**AREA**	**AUPP**
Linear	0.695	0.245	0.624	0.69	0.637	0.047	0.666
Logarithmic	0.489	0.431	0.447	0.445	0.433	0.044	0.465
Inverse	0.252	0.591	0.241	0.162	0.191	0.04	0.237
Quadratic	0.792	0.449	0.719	0.775	0.68	0.055	0.766
Cubic	0.868	0.607	0.735	0.862	0.779	0.054	0.805
Compound	0.694	0.261	0.611	0.688	0.651	0.04	0.656
Power	0.516	0.442	0.461	0.473	0.467	0.036	0.485
S	0.277	0.578	0.259	0.181	0.214	0.033	0.258
Growth	0.694	0.261	0.611	0.688	0.651	0.04	0.656
Exponential	0.694	0.261	0.611	0.688	0.651	0.04	0.656
Logistic	0.694	0.261	0.611	0.688	0.651	0.04	0.656

As it can be seen from the Table-4.3, results from cubic and Quadratic models used in regression analysis outperformed others in case of DBPs like in case of Deflections. Maximum deflection and MLI had good performance with R^2 values being 0.868 and 0.862. Care should be taken when using MLI as it often gives mixed signals regarding structural condition of the bituminous layer. Furthermore, in the analysis BLI, LLI and AUPP also had promising results with R^2 values being 0.735, 0.779 and 0.805. And the parameter like RoC needs further evaluation to be used effectively. Although AREA parameter is known to show weak correlation with the pavement layers, they showed very poor correlation when modelling was done using conventional regression models.

The comparison of various regression models used to develop horizontal strain from D.Max, RoC, BLI and AUPP are shown in the Figure 4.2

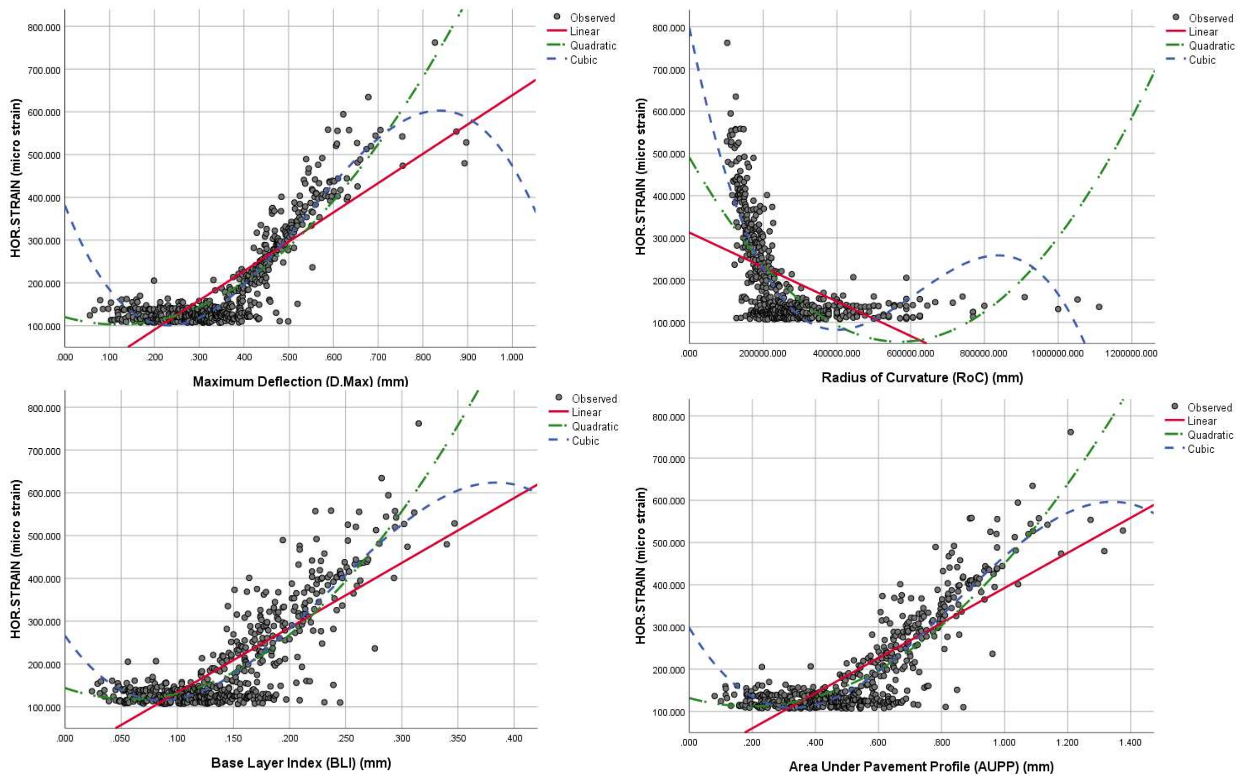

Figure 4. 2 Regression plot of Horizontal Strains Modelled using DBPs: D.Max, RoC, BLI, AUPP

The further summary of the developed models and the estimates of the parameters are given in the Table-4.4.

Table-4. 4 Model Summary and Parameter Estimation of best Regression Models used to develop Horizontal Strains using DBPs

Dependent Variable: HORIZONTAL STRAIN									
A. The independent variable is *Maximum Deflection (D.Max)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.695	1277.389	1	561	0	-46.624	685.47		
Quadratic	0.792	1068.755	2	560	0	120.169	-293.361	1243.629	
Cubic	0.868	1224.547	3	559	0	381.527	-2658.629	7303.618	-4553.566
B. The independent variable is *Radius of Curvature (RoC)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**

Linear	.245	182.085	1	561	.000	312.460	.000		
Quadratic	.449	228.058	2	560	.000	490.508	-.002	1.341E-9	
Cubic	.607	287.902	3	559	.000	797.345	-.004	7.850E-9	-4.230E-15
C. The independent variable is *Base Layer Index (BLI)*									
Equation	**Model Summary**					**Parameter Estimation**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.624	930.071	1	561	0	-17.959	1514.045		
Quadratic	0.719	714.782	2	560	0	143.978	-884.182	7534.835	
Cubic	0.735	517.94	3	559	0	266.085	-3789.292	27013.187	-38331.898
D. The independent variable is *Area Under Pavement Profile (AUPP)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.666	1117.176	1	561	0	-22.991	415.536		
Quadratic	0.766	914.241	2	560	0	131.716	-201.307	519.98	
Cubic	0.805	770.766	3	559	0	298.348	-1251.736	2363.776	-943.031

4.1.1.3 Horizontal Strains using Surface Moduli

Table-4. 5 R^2-values of Regression Models used to develop Horizontal Strains using Surface Moduli

Model	**Coefficient of Correlation (R^2)**				
	SM0	**SM300**	**SM600**	**SM1200**	**SMD**
Linear	0.252	0.204	0.211	0.152	0.016
Logarithmic	0.489	0.471	0.44	0.343	.
Inverse	0.695	0.688	0.614	0.46	.
Quadratic	0.451	0.348	0.365	0.288	0.037
Cubic	0.667	0.585	0.565	0.452	0.04
Compound	0.277	0.227	0.237	0.173	0.016
Power	0.516	0.504	0.477	0.38	.
S	0.694	0.697	0.635	0.493	.
Growth	0.277	0.227	0.237	0.173	0.016
Exponential	0.277	0.227	0.237	0.173	0.016
Logistic	0.277	0.227	0.237	0.173	0.016

As it can be seen from the Table-4.5, regression analysis showed poor results with only Inverse, Cubic and S Models having correlation worth considering. Of these SM0, SM300 and SM600 outperformed others but had low R^2 values that cannot be used to model Horizontal Strains successfully. The comparison of regression models developed using surface moduli, i.e., SM0 and SM300, in modelling strains is shown in Figure 4.3.

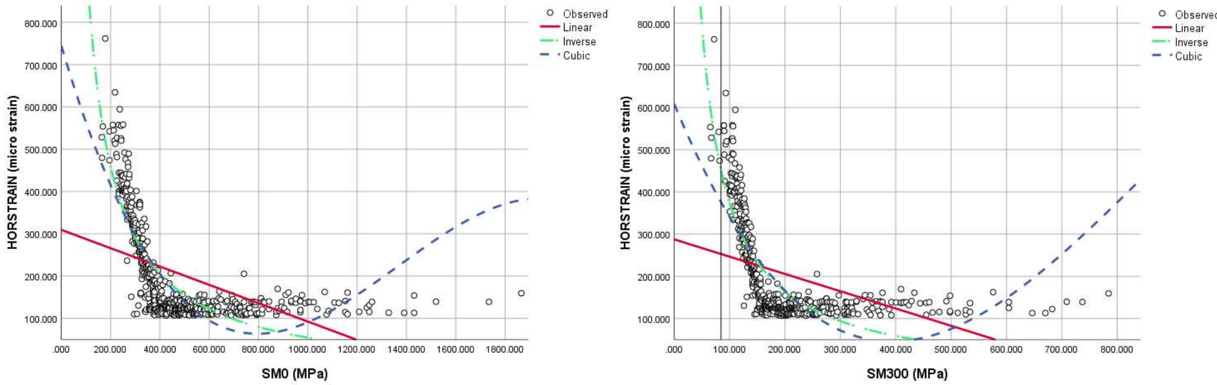

Figure 4. 3 Regression plot of Horizontal Strains Modelled using Surface Moduli: SM0, SM300

Also, the parameters SM1200 showed weak correlation but the SMD showed no correlation whatsoever. Furthermore, the summary of the best performing models is shown in the Table-4.6.

Table-4. 6 Model Summary and Parameter Estimation of best Regression Models used to develop Horizontal Strains using Surface Moduli

Dependent Variable: HORIZONTAL STRAIN									
A. The independent variable is *SM0*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.252	189.341	1	561	0	309.284	-0.217		
Inverse	0.695	1277.389	1	561	0	-46.624	101052.005		
Cubic	0.667	373.286	3	559	0	744.974	-1.985	0.002	-0.0000004297
B The independent variable is *SM300*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.204	143.533	1	561	0	287.51	-0.409		
Inverse	0.688	1237.769	1	561	0	-46.579	42149.118		
Cubic	0.585	262.181	3	559	0	608.65	-3.184	0.005	-0.000002068

4.1.1.4 Vertical Strains using FWD Deflections

Table-4. 7 R^2-values of Regression Models used to develop Vertical Strains using FWD Deflections

Model	**Coefficient of Correlation (R^2)**						
	0mm	**200mm**	**300mm**	**450mm**	**600mm**	**900mm**	**1200mm**
Linear	0.683	0.693	0.689	0.667	0.632	0.55	0.49
Logarithmic	0.509	0.505	0.498	0.491	0.475	0.427	0.379
Inverse	0.277	0.244	0.227	0.238	0.239	0.226	0.176
Quadratic	0.729	0.732	0.728	0.699	0.661	0.581	0.53
Cubic	0.803	0.823	0.825	0.79	0.736	0.621	0.552

Compound	0.67	0.681	0.679	0.659	0.628	0.551	0.495
Power	0.51	0.507	0.502	0.496	0.481	0.435	0.389
S	0.282	0.249	0.233	0.245	0.246	0.233	0.183
Growth	0.67	0.681	0.679	0.659	0.628	0.551	0.495
Exponential	0.67	0.681	0.679	0.659	0.628	0.551	0.495
Logistic	0.67	0.681	0.679	0.659	0.628	0.551	0.495

When FWD Deflections were used for regression analysis to determine Vertical Strains, just like Horizontal strains, Quadratic and Cubic models performed best as shown in Table-4.7. this is due too similar distribution of strains in both, as seen in the regression plot shown in Figure 4.4. FWD Deflections at 0mm, 200mm, 300mm and 450mm had good correlation with R^2 values being around 0.80. are lower when in this case as compared to when modelling Horizontal strains. Also, the correlation of deflections at 600mm and 900mm were lower requiring further evaluation to model vertical strains effectively.

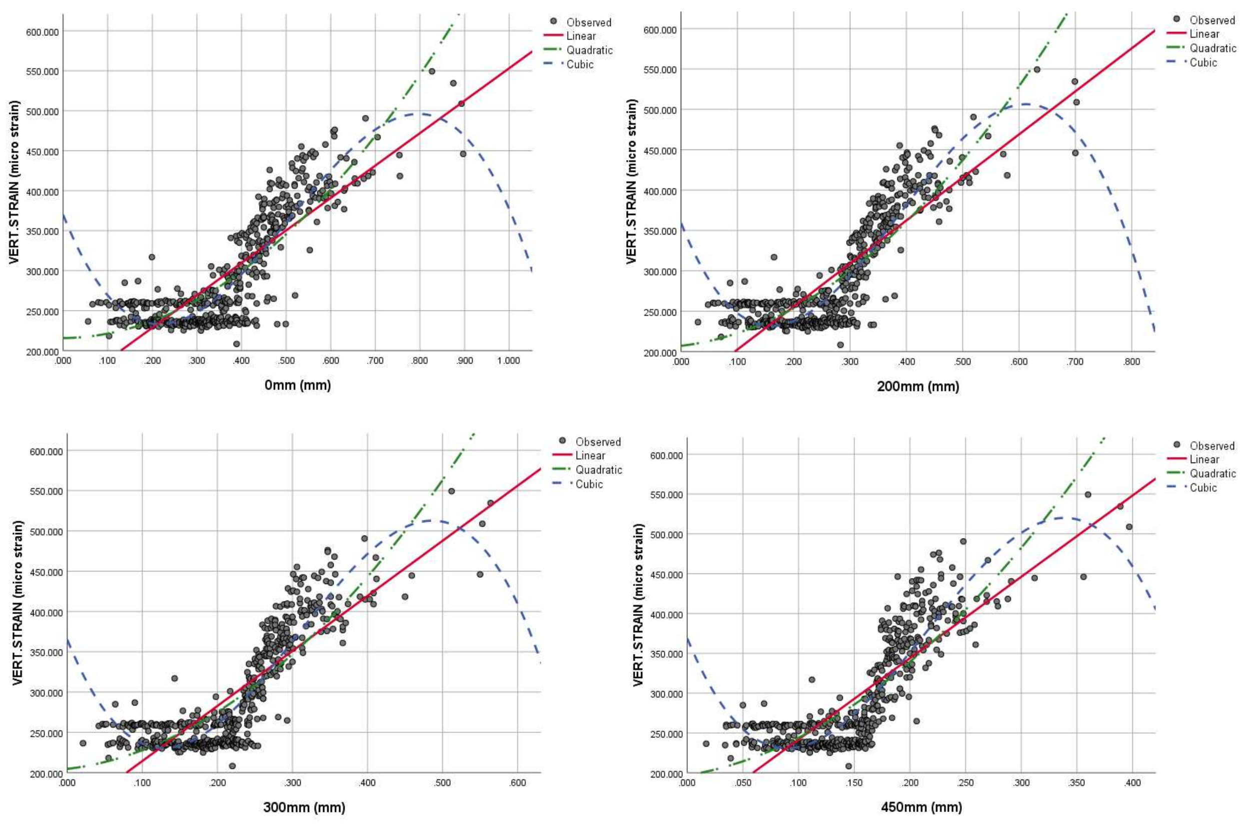

Figure 4. 4 Regression plot of Vertical Strains Modelled using FWD Deflections at 0mm, 200mm, 300mm, 450mm

Based on the R square values and F score, as can be seen in the Table-4.8, quadratic models in case of normal linear models outperformed by a large margin when FWD Deflections at 0mm, 200mm, 300mm and 400mm were used too model vertical strains.

Table-4. 8 Model Summary and Parameter Estimation of best Regression Models used to develop Vertical Strains using FWD Deflections

Dependent Variable: VERTICAL STRAIN									
A. The independent variable is *Deflection at 0mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.683	1209.791	1	561	0	147.513	405.623		
Quadratic	0.729	752.925	2	560	0	215.662	5.685	508.131	
Cubic	0.803	758.081	3	559	0	369.809	-1389.331	4082.264	-2685.657
B. The independent variable is *Deflection at 200mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.693	1267.982	1	561	0	149.16	533.09		
Quadratic	0.732	766.257	2	560	0	207.132	81.614	757.609	
Cubic	0.823	865.106	3	559	0	359.287	-1744.912	6873.495	-5931.07
C. The independent variable is *Deflection at 300mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.689	1245.322	1	561	0	146.145	683.139		
Quadratic	0.728	750.418	2	560	0	204.663	111.349	1211.373	
Cubic	0.825	878.846	3	559	0	365.201	-2300.147	11329.698	-12288.986
D. The independent variable is *Deflection at 450mm*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.667	1122.887	1	561	0	139.796	1020.947		
Quadratic	0.699	649.003	2	560	0	196.824	233.333	2396.122	
Cubic	0.79	702.563	3	559	0	368.7	-3382.798	23931.247	-37274.754

4.1.1.5 Vertical Strains using DBPs

Table-4. 9 R^2-values of Regression Models used to develop Vertical Strains using DBPs

Model	**Coefficient of Correlation (R^2)**						
	Max Def	**RoC**	**BLI**	**MLI**	**LLI**	**Area**	**AUPP**
Linear	0.683	0.264	0.596	0.676	0.649	0.038	0.643
Logarithmic	0.509	0.438	0.455	0.466	0.463	0.035	0.478

Inverse	0.277	0.566	0.26	0.18	0.214	0.031	0.259
Quadratic	0.729	0.459	0.636	0.715	0.668	0.056	0.687
Cubic	0.803	0.586	0.662	0.792	0.752	0.054	0.731
Compound	0.67	0.265	0.581	0.661	0.64	0.035	0.627
Power	0.51	0.434	0.452	0.467	0.468	0.031	0.477
S	0.282	0.551	0.262	0.184	0.219	0.027	0.262
Growth	0.67	0.265	0.581	0.661	0.64	0.035	0.627
Exponential	0.67	0.265	0.581	0.661	0.64	0.035	0.627
Logistic	0.67	0.265	0.581	0.661	0.64	0.035	0.627

In this case, when modelling vertical strains from the deflection bowl parameters like Max Deflection, MLI, LLI, AUPP showed good results with R^2 being 0.803, 0.792, 0.752, 0.731 as shown in Table-4.9. Whereas parameters like RoC, BLI require further evaluation to be used effectively. As MLI gives mixed signals regarding structural condition of subbase and thus give random correlation with it, care should be taken when used. Based on the R squared values and F score of the parameter MLI, from Table-4.10, it may be used in modelling in this particular case.

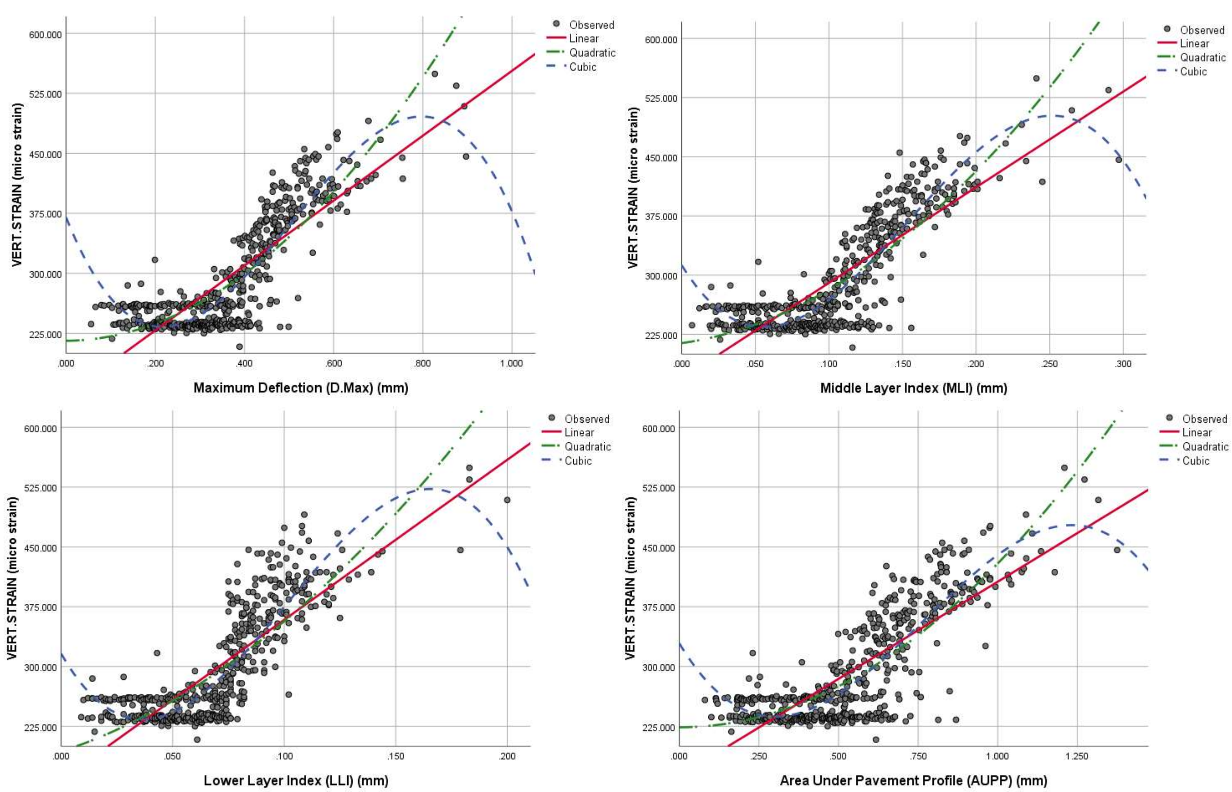

Figure 4. 5 Regression plot of Vertical Strains Modelled using DBPs: D.Max, MLI, LLI, AUPP

Table-4. 10 Model Summary and Parameter Estimation of best Regression Models used to develop Vertical Strains using DBPs

Dependent Variable: VERTICAL STRAIN									
A. The independent variable is *Maximum Deflection (D.Max)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.683	1209.791	1	561	0	147.513	405.623		
Quadratic	0.729	752.925	2	560	0	215.662	5.685	508.131	
Cubic	0.803	758.081	3	559	0	369.809	-1389.331	4082.264	-2685.657
B. The independent variable is *Middle Layer Index (MLI)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.676	1171.697	1	561	0	169.001	1211.908		
Quadratic	0.715	700.919	2	560	0	213.877	262.968	4133.713	
Cubic	0.792	710.681	3	559	0	312.517	-3016.683	32957.918	-71440.736
C. The independent variable is *Lower Layer Index (LLI)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.649	1035.273	1	561	0	157.915	2006.935		
Quadratic	0.668	562.502	2	560	0	193.295	922.739	7115.799	
Cubic	0.752	564.429	3	559	0	316.198	-4870.87	81468.665	-268883.711
D. The independent variable is *Area Under Pavement Profile (AUPP)*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.643	1010.891	1	561	0	162.671	243.732		
Quadratic	0.687	614.213	2	560	0	223.804	-0.012	205.469	
Cubic	0.731	507.403	3	559	0	329.023	-663.307	1369.737	-595.478

4.1.1.6 Vertical Strains using Surface Moduli

Table-4. 11 R^2-values of Regression Models used to develop Vertical Strains using Surface Moduli

Model	**Coefficient of Correlation (R^2)**				
	SM0	**SM300**	**SM600**	**SM1200**	**SMD**
Linear	0.277	0.227	0.239	0.176	0.015
Logarithmic	0.509	0.498	0.475	0.379	.
Inverse	0.683	0.689	0.632	0.49	.
Quadratic	0.478	0.379	0.4	0.324	0.039
Cubic	0.677	0.615	0.601	0.487	0.042
Compound	0.282	0.233	0.246	0.183	0.014
Power	0.51	0.502	0.481	0.389	.

S	0.67	0.679	0.628	0.495	.
Growth	0.282	0.233	0.246	0.183	0.014
Exponential	0.282	0.233	0.246	0.183	0.014
Logistic	0.282	0.233	0.246	0.183	0.014

The best performing models, unlike when FWD Deflections and DBPs were used, inverse cubic and S models performed the best. As it can be seen from Table-4.11, only SM0, SM300 and SM600 can be used in evaluation of Vertical strains but require further evaluation as their values are not high enough (0.683, 0.689, 0.632 respectively) to instill confidence in development of vertical strains from these models. Their regression plots and model summary is presented in Figure 4.6 and Table-4.12. Also, the Parameter SM1200 and SMD performed very poorly and cannot be used in any form or shape to be used in modelling vertical strains

The

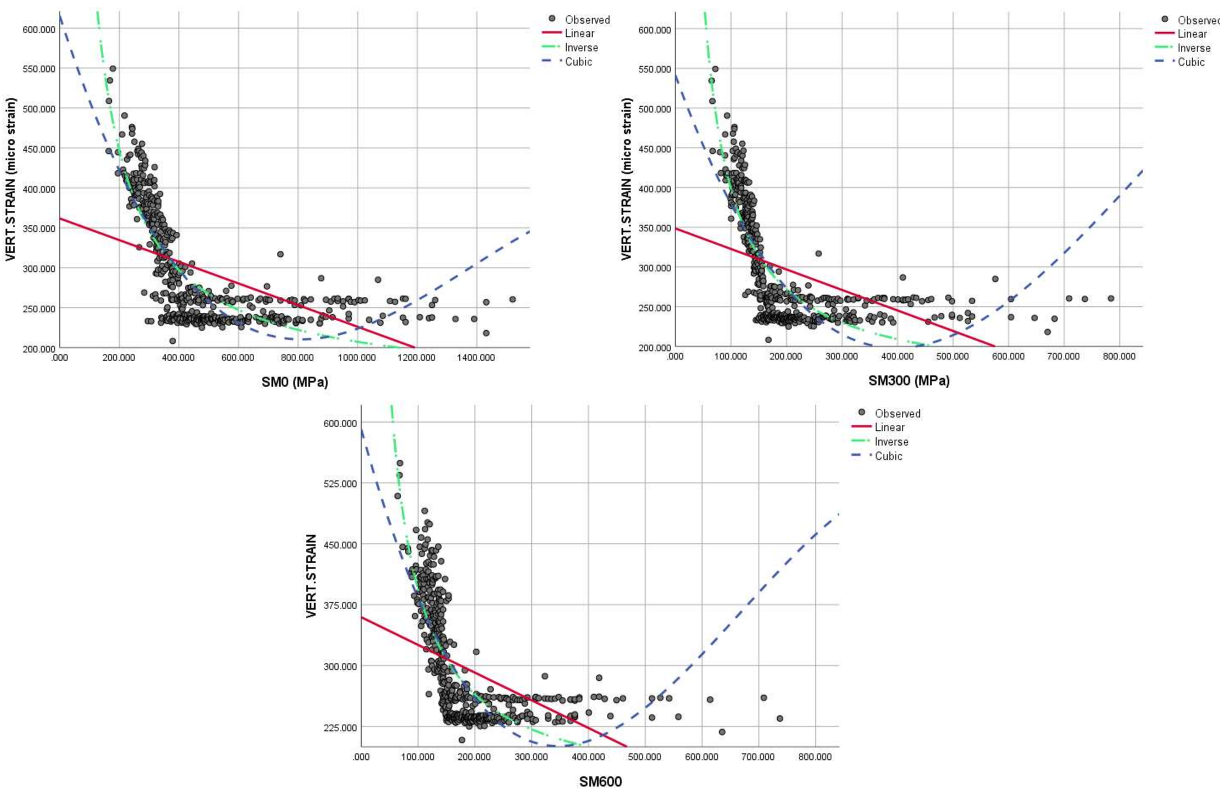

Figure 4. 6 Regression plot of Vertical Strains Modelled using Surface Modul: SM0, SM300, SM600

Table-4. 12 Model Summary and Parameter Estimation of best Regression Models used to develop Vertical Strains using Surface Moduli

Dependent Variable: VERTICAL STRAIN									
A. The independent variable is *SM0*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.277	215.128	1	561	0	361.704	-0.136		
Inverse	0.683	1209.791	1	561	0	147.513	59796.97		
Cubic	0.677	389.929	3	559	0	615.107	-1.161	0.001	-0.0000002462
B. The independent variable is *SM300*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.227	165.09	1	561	0	348.508	-0.258		
Inverse	0.689	1245.322	1	561	0	146.145	25177.075		
Cubic	0.615	297.62	3	559	0	541.359	-1.922	0.003	-0.000001234
C. The independent variable is *SM600*									
Equation	**Model Summary**					**Parameter Estimates**			
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**	**b3**
Linear	0.239	176.177	1	561	0	359.496	-0.341		
Inverse	0.632	965.103	1	561	0	135.34	25826.916		
Cubic	0.601	280.21	3	559	0	590.723	-2.546	0.005	-0.000002469

4.1.2 ANN Results (MATLAB: ANN, Levenberg-Marquardt 5 Neurons)

In case of modelling with ANN, result of all three algorithms used LVM was to some degree better than the others based on MSE and R^2 values. Also, the no. of hidden layers, 5 or 10 hidden layers, made no observable difference in modelling. So, ANN models developed using LVM with 5 hidden layer was used for comparison. The model developed using ANN performed better in all stages, i.e., Training, Validation and Testing stages, based on their R^2 and MSE values.

The results of the modelling done using the ANN is presented below.

4.1.2.1 Horizontal Strains using FWD Deflections

In this case, deflections at 0mm, 200mm, 300mm performed well having R values of around 0.95 in case of training testing and validations stages as shown in Table-4.13. Also, the R values for 450mm and 600mm are in rage of 0.93 and 0.88 respectively. The R values of 900mm and 1200mm are in range of 0.76 suggesting good correlation but requiring further evaluation when used to develop model to find out Horizontal Strains.

Table-4. 13 Performance of ANN Models used to develop Horizontal Strains from FWD Deflections

Parameters		**FWD Deflections**						
		0mm	**200mm**	**300mm**	**450mm**	**600mm**	**900mm**	**1200mm**
Training	**R**	0.9457	0.9574	0.9629	0.9258	0.8808	0.7623	0.7305
	MSE	1425.6	1234.2	986.2354	1956.5	2919.2	5358.7	6081.3
Validation	**R**	0.9567	0.9599	0.958	0.9479	0.8867	0.8183	0.8271
	MSE	1163.2	789.1274	898.5478	1428.7	3672.4	5915.7	5107.3
Test	**R^2**	0.9367	0.9711	0.9375	0.9165	0.8854	0.8123	0.7548
	MSE	1627.7	693.739	2061	2419.3	2720.9	5590.4	5472.3

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Horizontal strains from FWD Deflection (0mm, 200mm, 300mm, 450mm) are shown in Figures 4.7, 4.8, 4.9, 4.10. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.11, 4.12, 4.13, 4.14

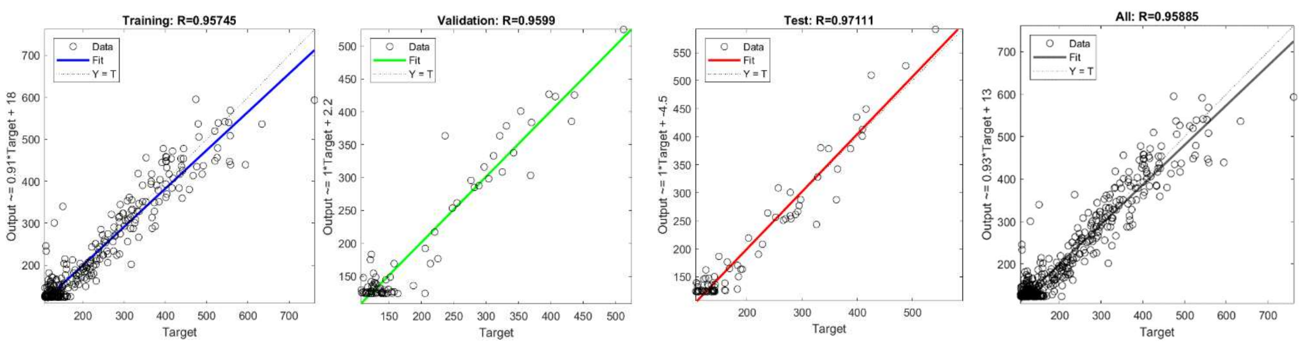

Figure 4. 7 R-values of ANN Model for modelling Hor. Strain using FWD Deflection (0mm)

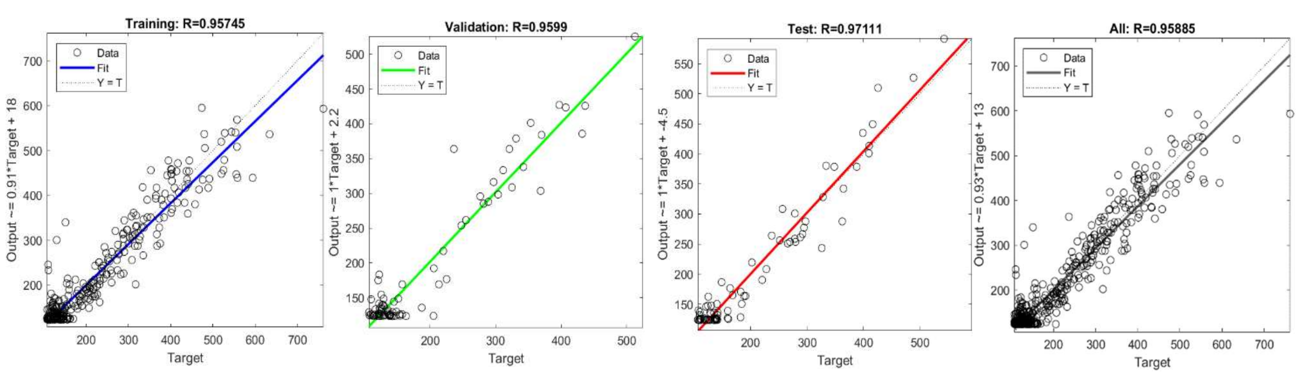

Figure 4. 8 R-values of ANN Model for modelling Hor. Strain using FWD Deflection (200mm)

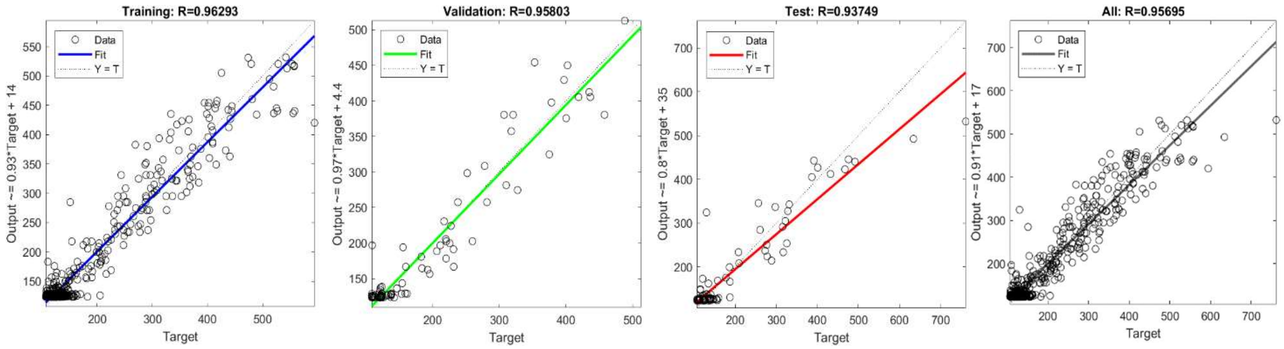

Figure 4. 9 R-values of ANN Model for modelling Hor. Strain using FWD Deflection (300mm)

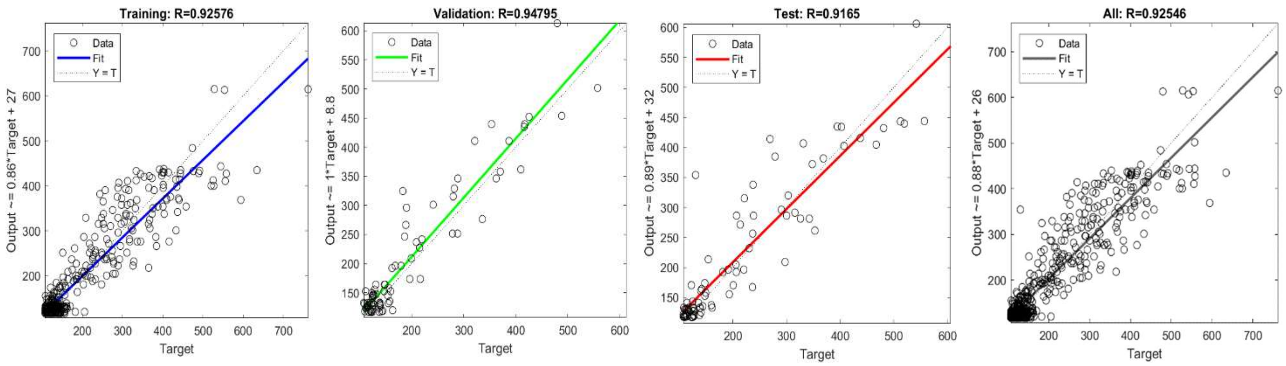

Figure 4. 10 R-values of ANN Model for modelling Hor. Strain using FWD Deflection (450mm)

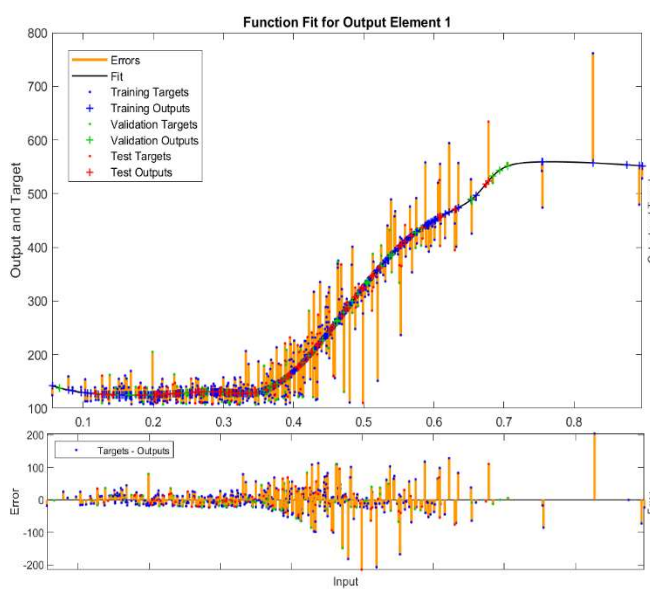

Figure 4. 11 Line Fit Plot of ANN Model for modelling Hor. Strain using FWD Deflection (0mm)

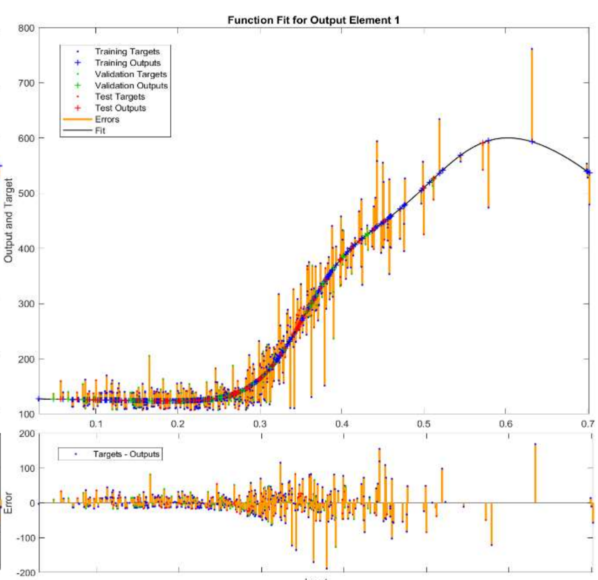

Figure 4. 12 Line Fit Plot of ANN Model for modelling Hor. Strain using FWD Deflection (200mm)

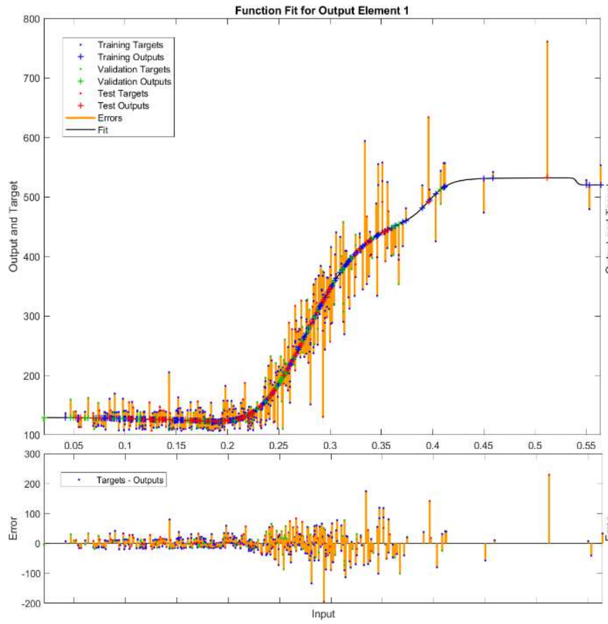

Figure 4. 13 Line Fit Plot of ANN Model for modelling Hor. Strain using FWD Deflection (300mm)

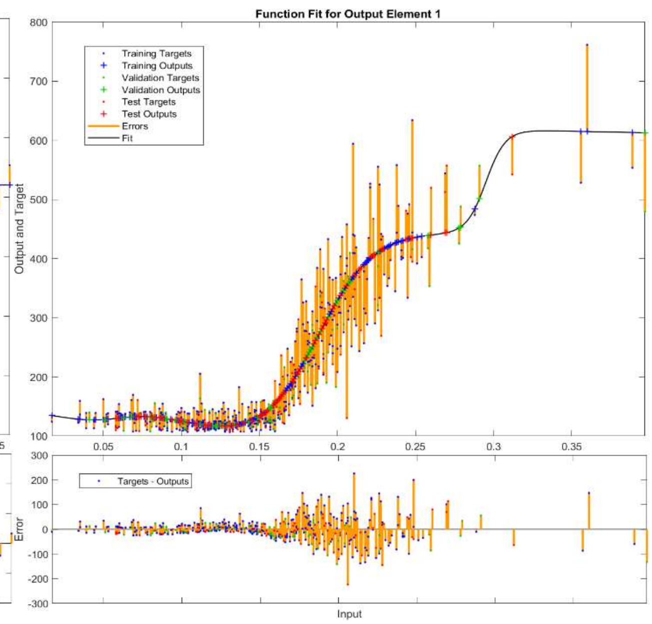

Figure 4. 14 Line Fit Plot of ANN Model for modelling Hor. Strain using FWD Deflection (450mm)

4.1.2.2 Horizontal Strains using DBPs

Here the Max Deflection and MLI had R values around 0.95 showing very good correlation. In case of RoC, BLI, LLI, AUPP, the R values are still high enough, as shown in Table-4.14, and gives a very good correlation and can also be used to model horizontal strains with a good significance.

Table-4. 14 Performance of ANN Models used to develop Horizontal Strains from DBPs

Parameters		DBP						
		Max. Def.	RoC	BLI	MLI	LLI	Area	AUPP
Training	R	0.9468	0.8315	0.854	0.9464	0.8831	0.2457	0.9086
	MSE	1408.4	4403.8	3333.3	1471.9	2710.4	12490	2356.3
Validation	R	0.9446	0.9049	0.8501	0.9433	0.9288	0.3562	0.8534
	MSE	1143.5	1930.8	3939	1511.9	2248.1	13689	2266.9
Test	R	0.9495	0.8236	0.9064	0.8895	0.9232	0.299	0.9091
	MSE	1681.6	4098.4	3731.2	2053.3	2496.1	11364	3171.7

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Horizontal strains from DBPs (Max Def, RoC, BLI, MLI, AUPP) are shown in Figures 4.15, 4.16, 4.17, 4.18, 4.19. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.20, 4.21, 4.22, 4.23, 4.24

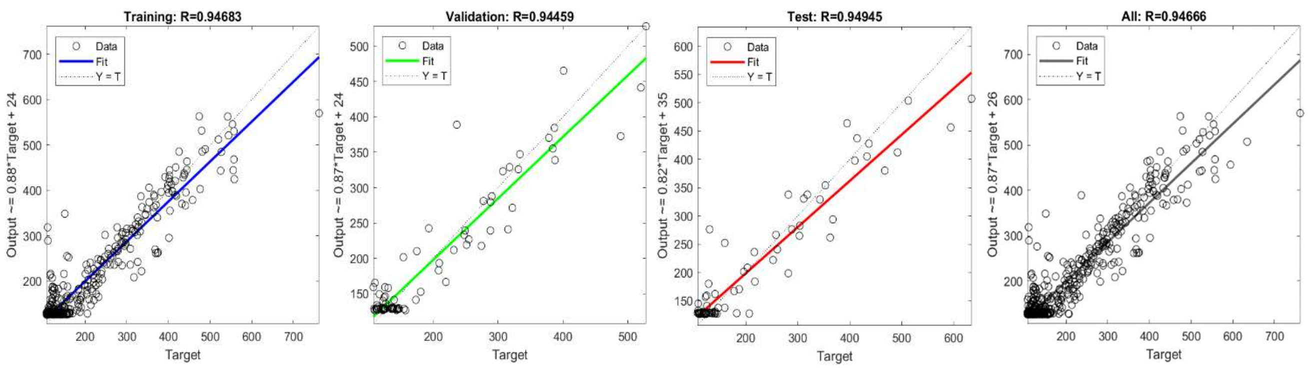

Figure 4. 15 R-values of ANN Model for modelling Hor. Strain using DBP (Max Def)

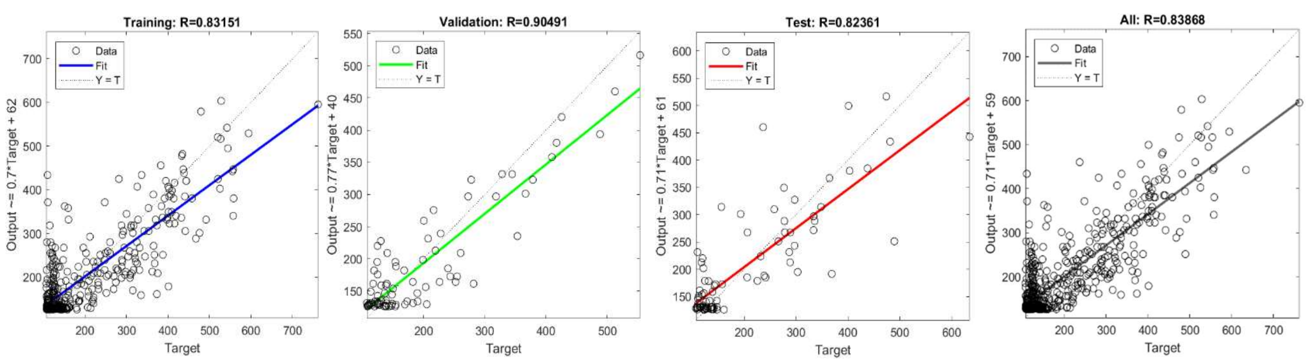

Figure 4. 16 R-values of ANN Model for modelling Hor. Strain using DBP (RoC)

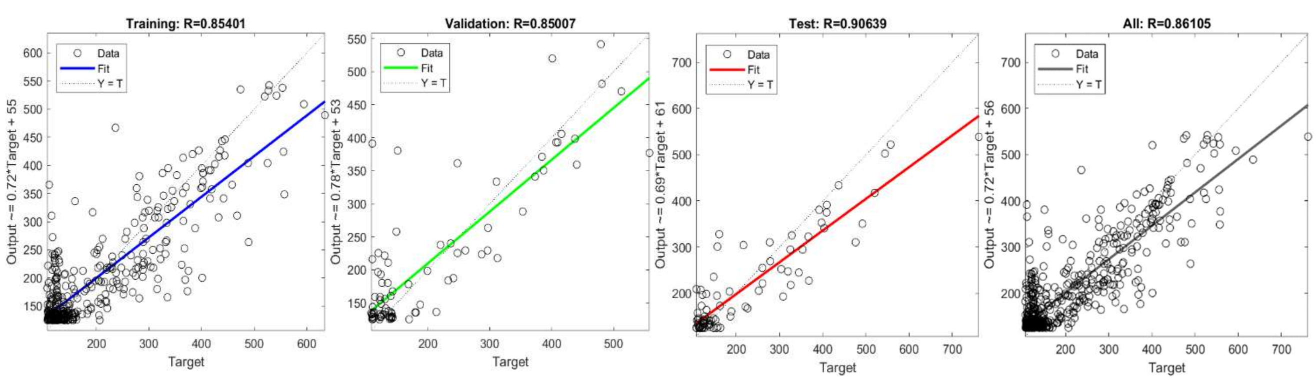

Figure 4. 17 R-values of ANN Model for modelling Hor. Strain using DBP (BLI)

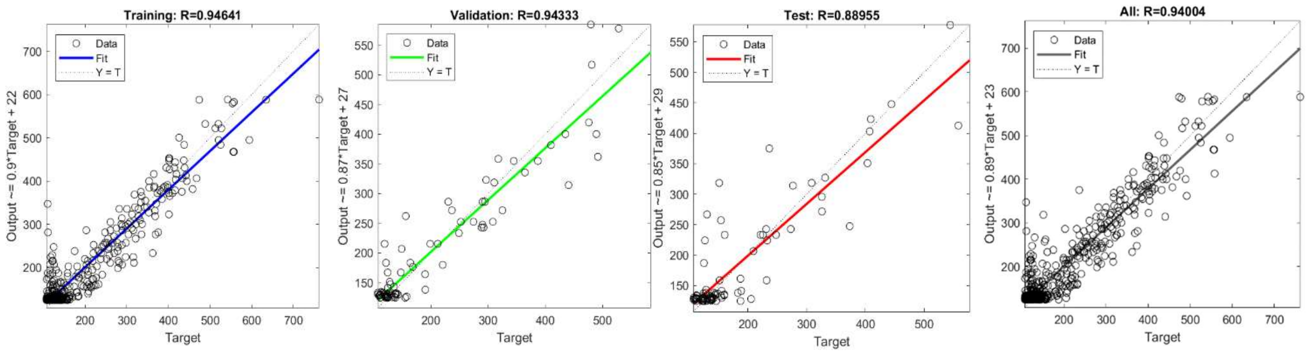

Figure 4. 18 R-values of ANN Model for modelling Hor. Strain using DBP (MLI)

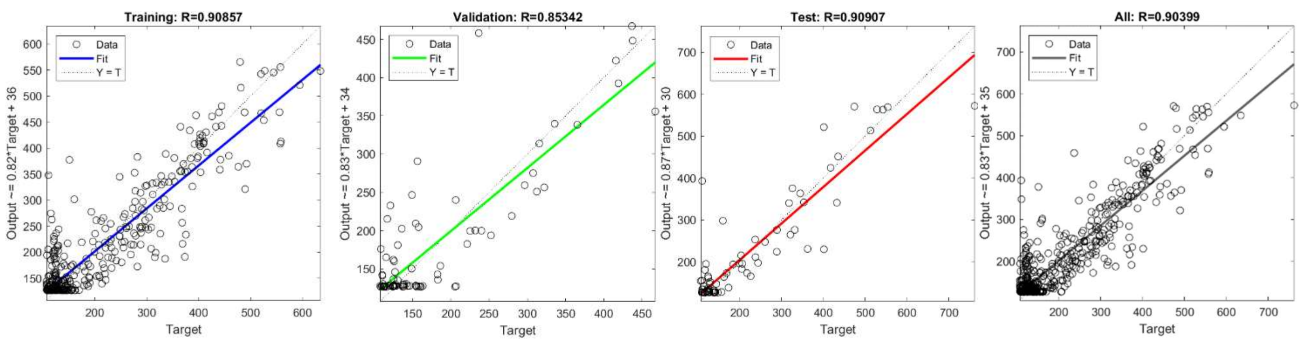

Figure 4. 19 R-values of ANN Model for modelling Hor. Strain using DBP (AUPP)

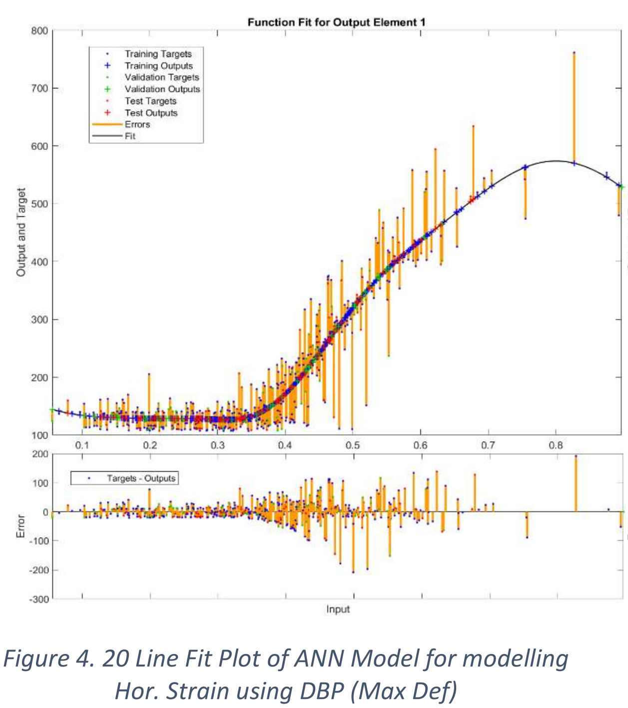

Figure 4. 20 Line Fit Plot of ANN Model for modelling Hor. Strain using DBP (Max Def)

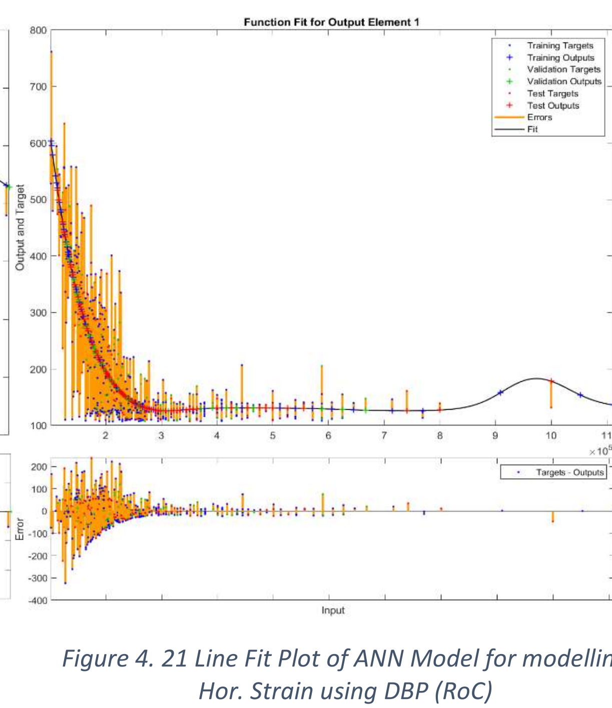

Figure 4. 21 Line Fit Plot of ANN Model for modelling Hor. Strain using DBP (RoC)

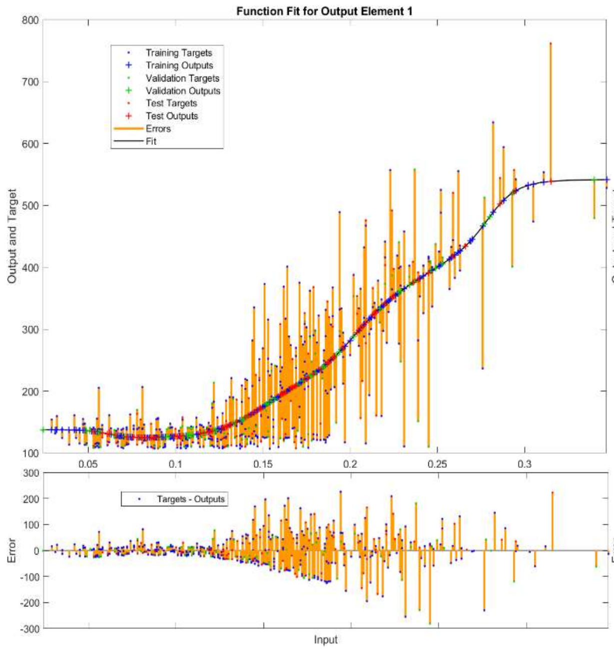

Figure 4. 22 Line Fit Plot of ANN Model for modelling Hor. Strain using DBP (BLI)

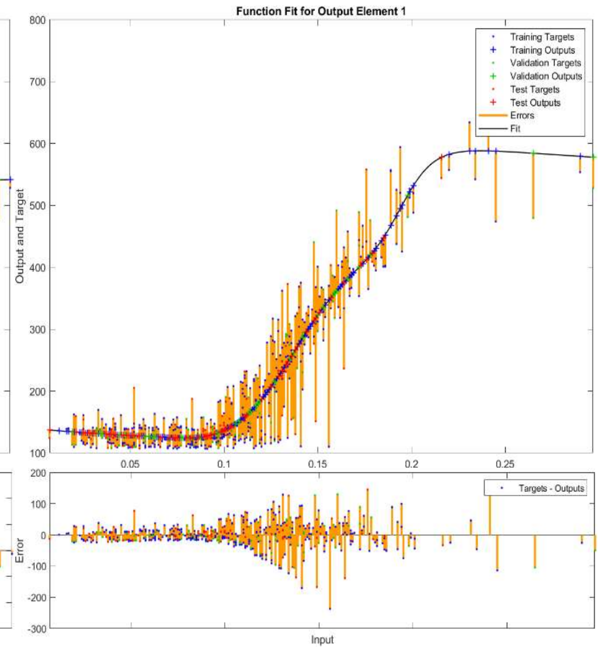

Figure 4. 23 Line Fit Plot of ANN Model for modelling Hor. Strain using DBP (MLI)

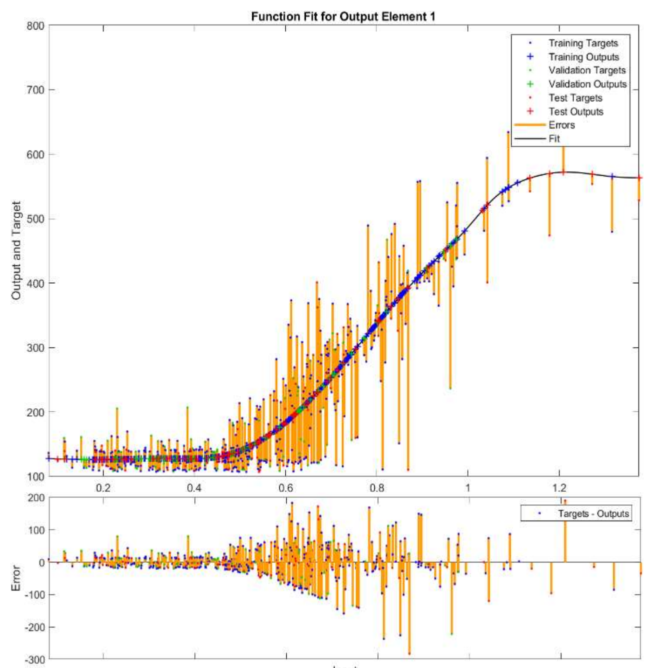

Figure 4. 24 Line Fit Plot of ANN Model for modelling Hor. Strain using DBP (AUPP)

4.1.2.3 Horizontal Strains using Surface Moduli

In case modelling Horizontal Strains using Surface Moduli, SM0 and SM300 have a good correlation with R values as shown in Table-4.15. Moreover, the SM600 also shown a good R value of around 0.85 that may be used in modeling after further assessment. But SM1200 should be avoided due to moderate correlation. Also, the SMD showed better correlation than it did in normal regression models, but it is still poor and cannot be used for modelling.

Table-4. 15 Performance of ANN Models used to develop Horizontal Strains from Surface Moduli

Parameters		Surface Moduli				
		SM0	**SM300**	**SM600**	**SM1200**	**SMD**
Training	**R**	0.9499	0.9496	0.8831	0.731	0.3896
	MSE	1287.6	1351.1	2899.1	6272.8	12035
Validation	**R**	0.9273	0.9513	0.8919	0.7625	0.4577
	MSE	1922.8	1497.9	3512	5698.8	8205.6
Test	**R**	0.9402	0.948	0.8511	0.7815	0.2822
	MSE	1697.5	1156.8	3124.6	5621.7	12246

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Horizontal strains from Surface Moduli (SM0, SM300) are shown in Figures 4.25, 4.26. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.27, 4.28.

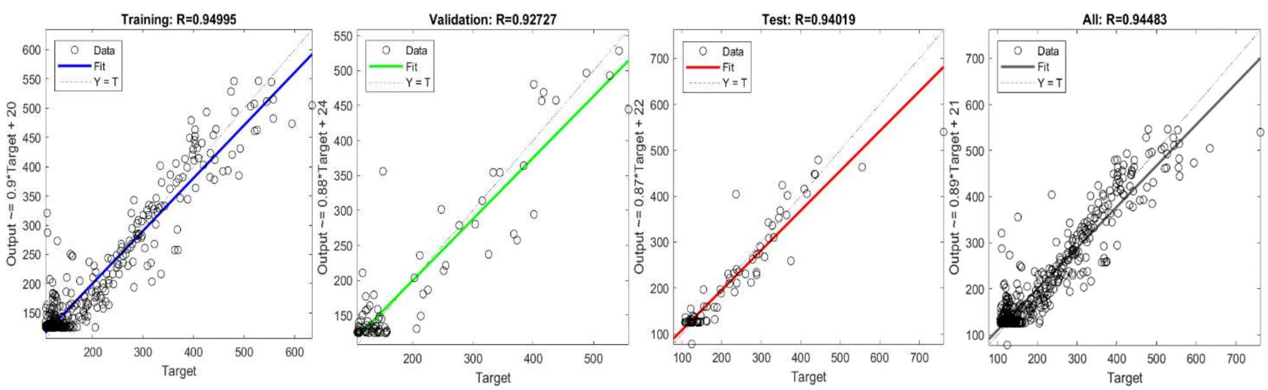

Figure 4. 25 R-values of ANN Model for modelling Hor. Strain using Surface Moduli (SM0)

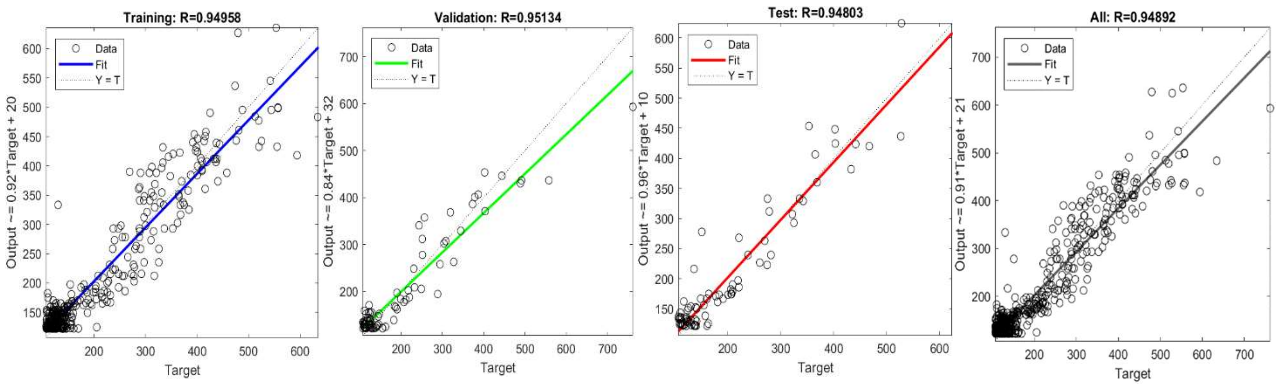

Figure 4. 26 R-values of ANN Model for modelling Hor. Strain using Surface Moduli (SM300)

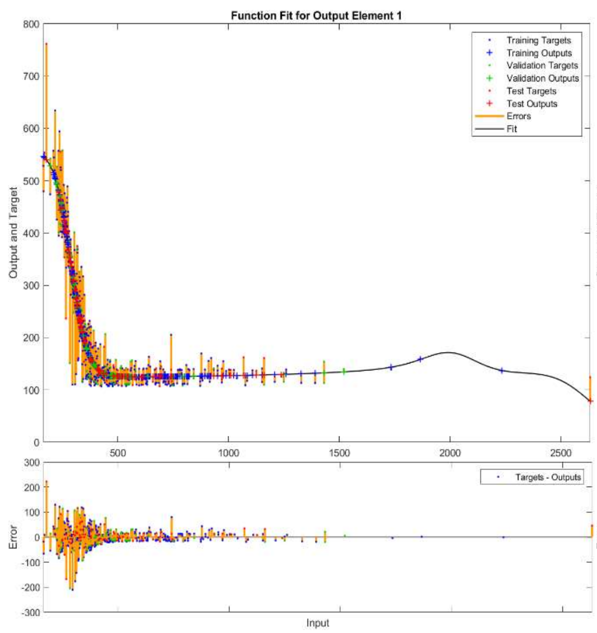

Figure 4. 27 Line Fit Plot of ANN Model for modelling Hor. Strain using Surface Moduli (SM0)

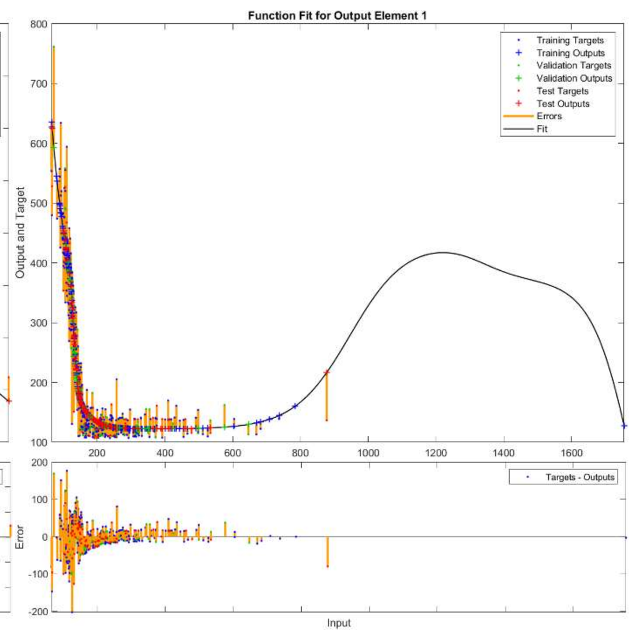

Figure 4. 28 Line Fit Plot of ANN Model for modelling Hor. Strain using Surface Moduli (SM300)

4.1.2.4 Vertical Strains using FWD Deflections

For modelling Vertical Strains, FWD Deflections at 300mm and 450mm shows higher R values (around 0.94), thus greater significance, Table-4.16. Deflections at 0mm, 200mm and 600mm have very good R values (above 0.9) making them appropriate for modelling vertical strains. Whereas 900mm and 1200mm have relatively low R values but are sufficiently high enough to be used after further evaluation.

Table-4. 16 Performance of ANN Models used to develop Vertical Strains from FWD Deflections

Parameters	**FWD Deflections**						
	0mm	**200mm**	**300mm**	**450mm**	**600mm**	**900mm**	**1200mm**

Training	**R**	0.9116	0.9339	0.9447	0.9378	0.9	0.8144	0.7814
	MSE	792.891	582.226	486.984	617.05	946.968	1569.4	1894.6
Validation	**R**	0.9307	0.9059	0.9418	0.9295	0.8954	0.887	0.7788
	MSE	848.04	854.532	583.452	727.063	741.223	1152.4	1583.5
Test	**R**	0.9243	0.9527	0.9454	0.9414	0.9041	0.8151	0.7643
	MSE	547.748	546.983	601.021	518.935	920.781	1696	2201.2

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Vertical strains from FWD Deflection (0mm, 200mm, 300mm, 450mm, 600mm) are shown in Figures 4.29, 4.30, 4.31, 4.32, 4.33. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.34, 4.35, 4.36, 4.37, 4.38.

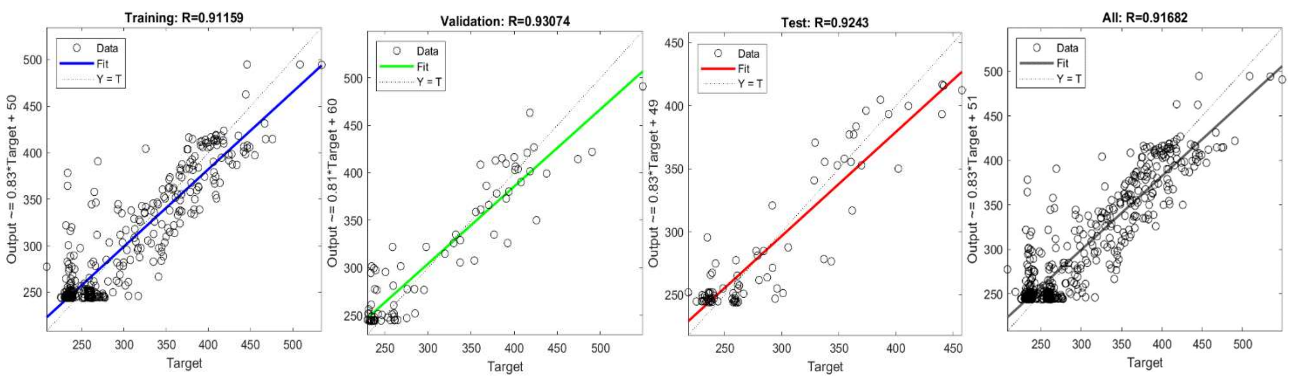

Figure 4. 29 R-values of ANN Model for modelling Vert. Strain using FWD Deflection (0mm)

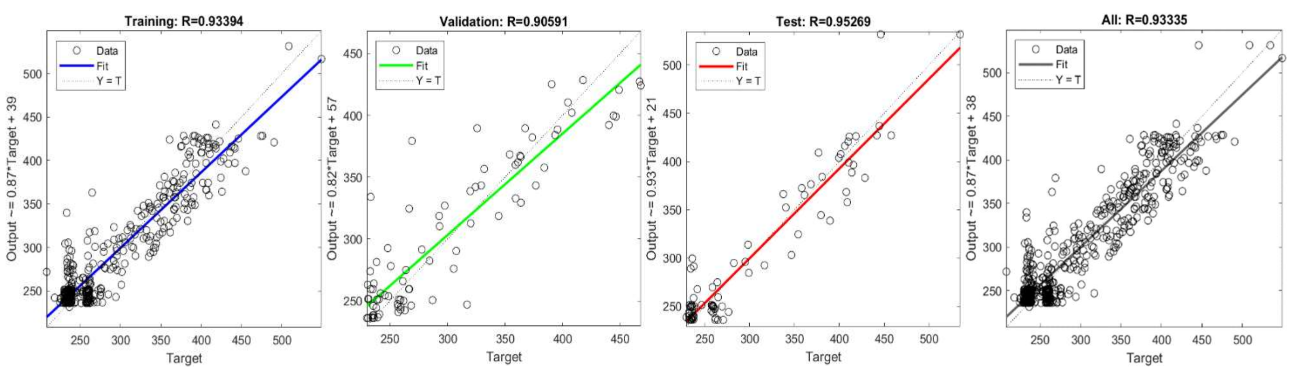

Figure 4. 30 R-values of ANN Model for modelling Vert. Strain using FWD Deflection (200mm)

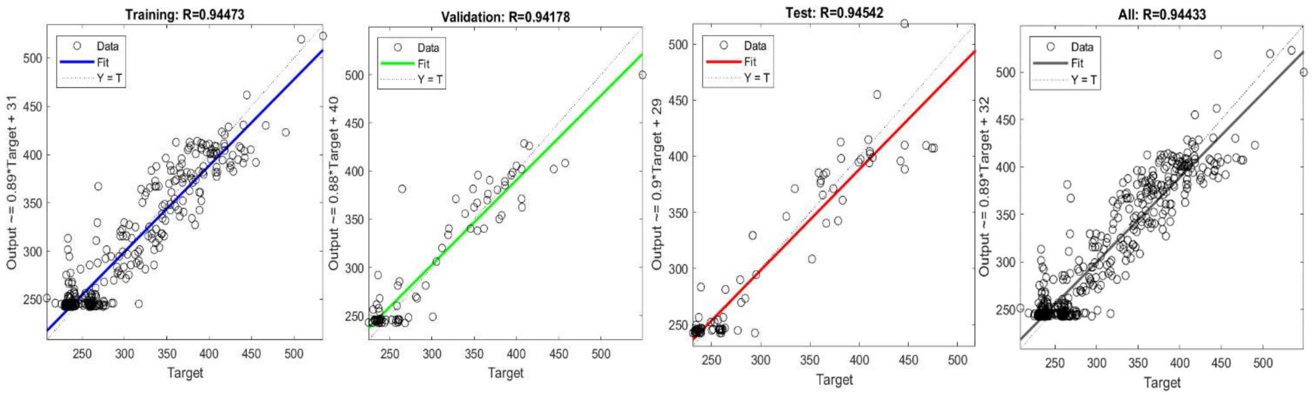

Figure 4. 31 R-values of ANN Model for modelling Vert. Strain using FWD Deflection (300mm)

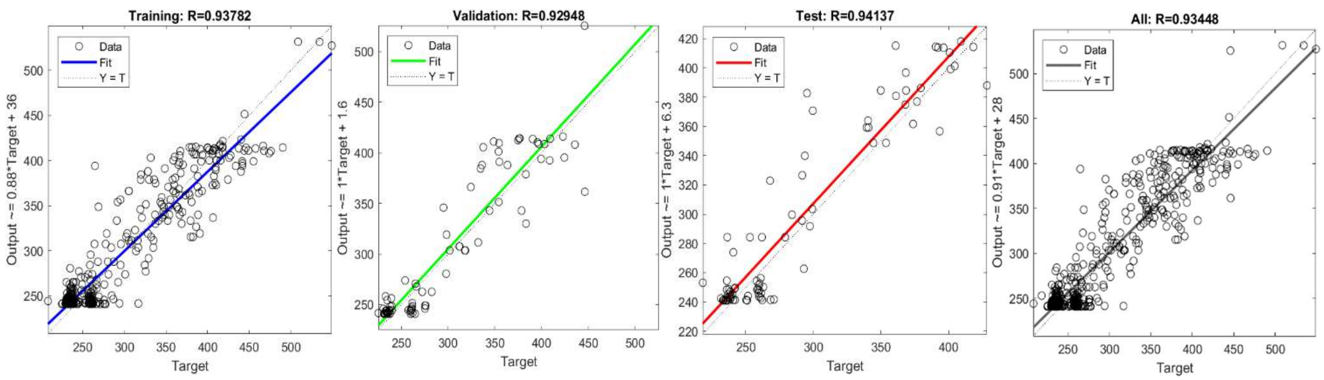

Figure 4. 32 R-values of ANN Model for modelling Vert. Strain using FWD Deflection (450mm)

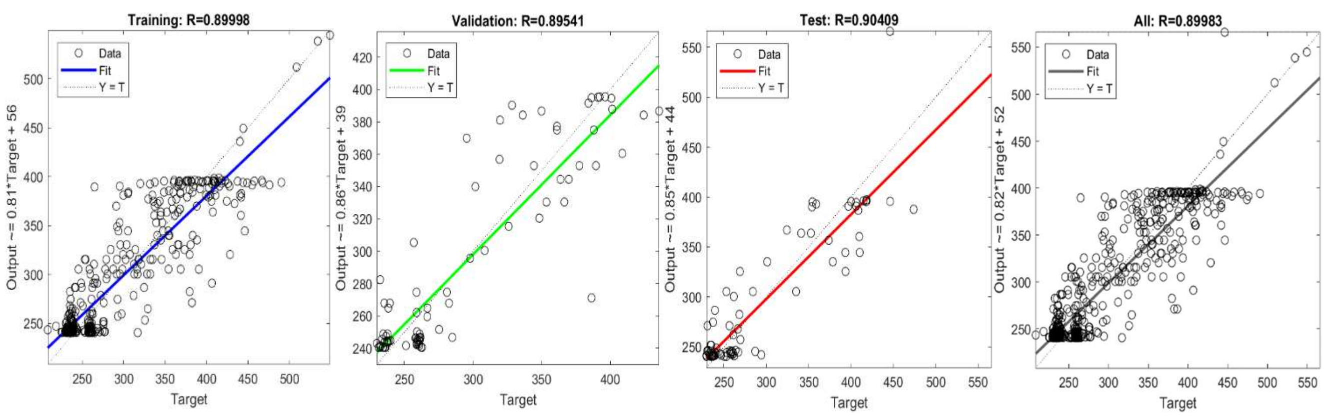

Figure 4. 33 R-values of ANN Model for modelling Vert. Strain using FWD Deflection (600mm)

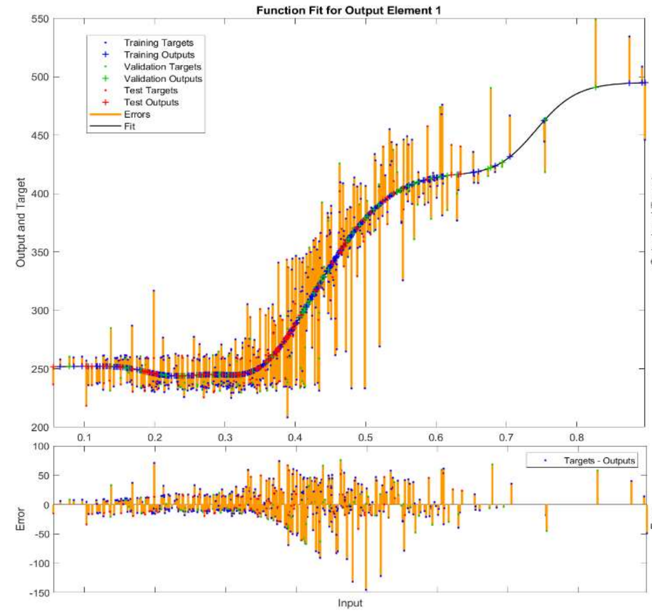

Figure 4. 34 Line Fit Plot of ANN Model for modelling Vert. Strain using FWD Deflection (0mm)

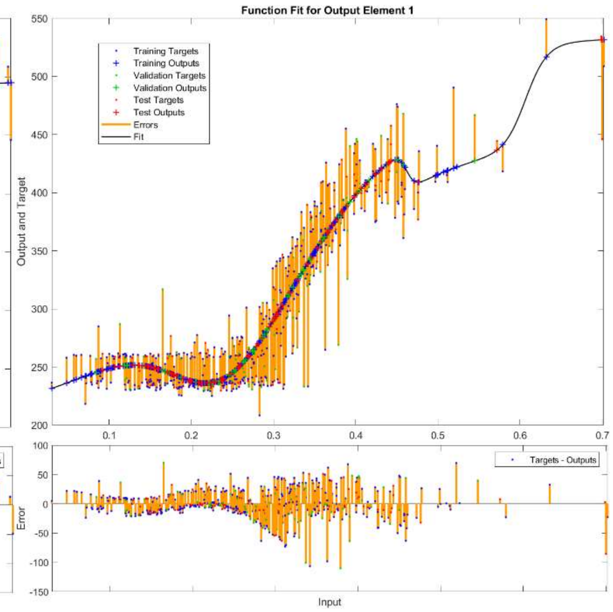

Figure 4. 35 Line Fit Plot of ANN Model for modelling Vert. Strain using FWD Deflection (200mm)

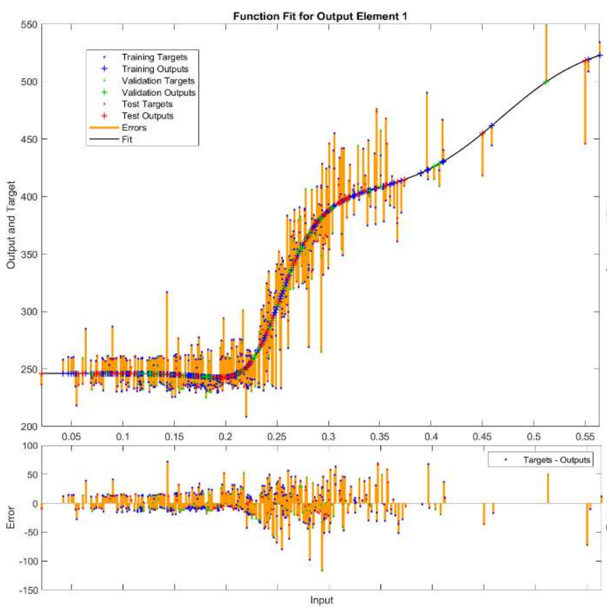

Figure 4. 36 Line Fit Plot of ANN Model for modelling Vert. Strain using FWD Deflection (300mm)

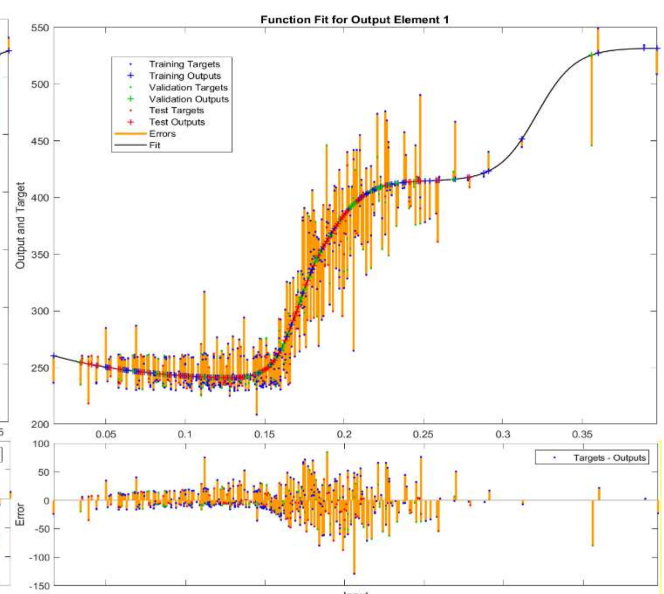

Figure 4. 37 Line Fit Plot of ANN Model for modelling Vert. Strain using FWD Deflection (450mm)

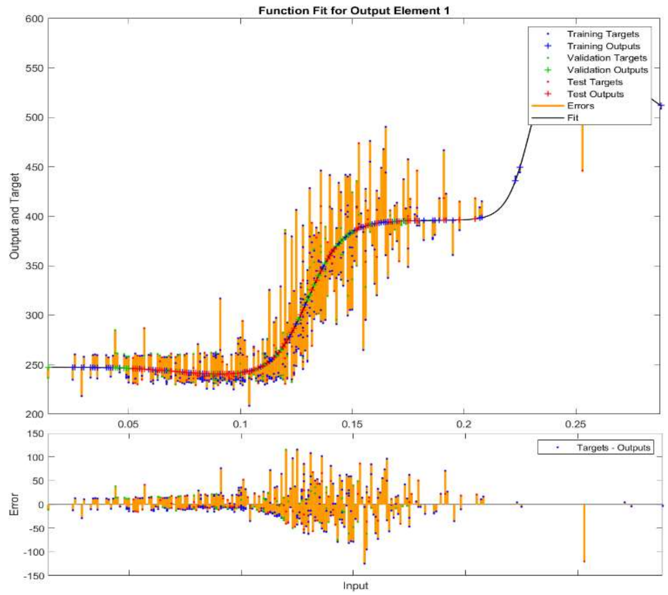

Figure 4. 38 Line Fit Plot of ANN Model for modelling Vert. Strain using FWD Deflection (600mm)

4.1.2.5 Vertical Strains using DBPs

In case of DBPs, as shown in Table-4.17, Max Deflection, MLI, LLI and AUPP can be used for modelling as they have high R value (Around 0.9). As R values of RoC, BLI are around 0.8 having good significance, they can also be employed in evaluation of vertical strains.

Table-4. 17 Performance of ANN Models used to develop Vertical Strains from DBPs

Parameters		**DBP**						
		Max. Def.	**RoC**	**BLI**	**MLI**	**LLI**	**Area**	**AUPP**
Training	**R**	0.9151	0.7804	0.8126	0.9032	0.9	0.2758	0.8606
	MSE	832.73	1962	1527.2	912.457	946.562	4582.8	1299.7
Validation	**R**	0.9015	0.8063	0.8435	0.9034	0.8814	0.2543	0.8733
	MSE	720.372	1330.8	1410.8	992.787	899.593	4015.6	1177.7
Test	**R**	0.9353	0.8521	0.8017	0.9004	0.9136	0.2588	0.8538
	MSE	513.591	1293.3	2147.5	940.276	730.344	4252.2	1175

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Vertical strains from DBPs (Max Def, MLI, LLI, AUPP) are shown in Figures 4.39, 4.40, 4.41, 4.42. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.43, 4.44, 4.45, 4.46

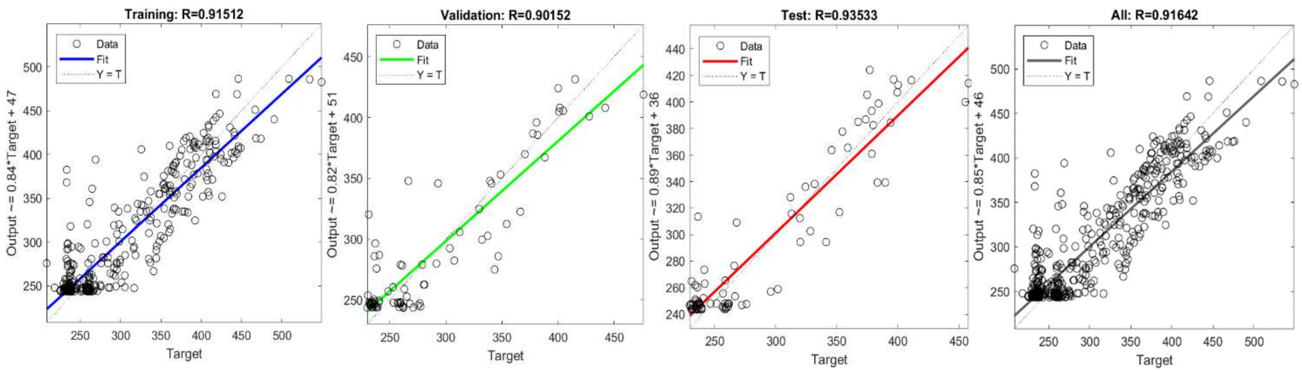

Figure 4. 39 R-values of ANN Model for modelling Vert. Strain using DBPs (Max Def)

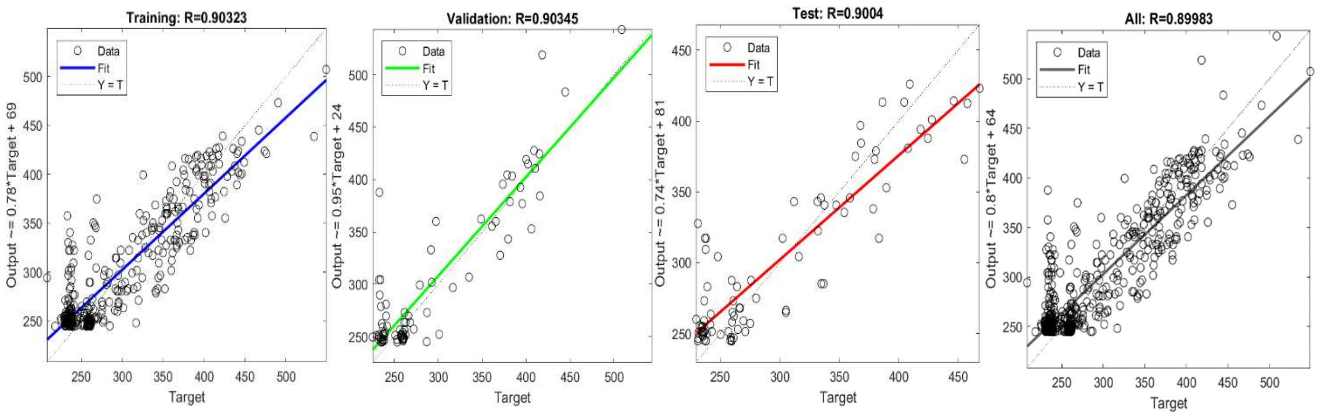

Figure 4. 40 R-values of ANN Model for modelling Vert. Strain using DBPs (MLI)

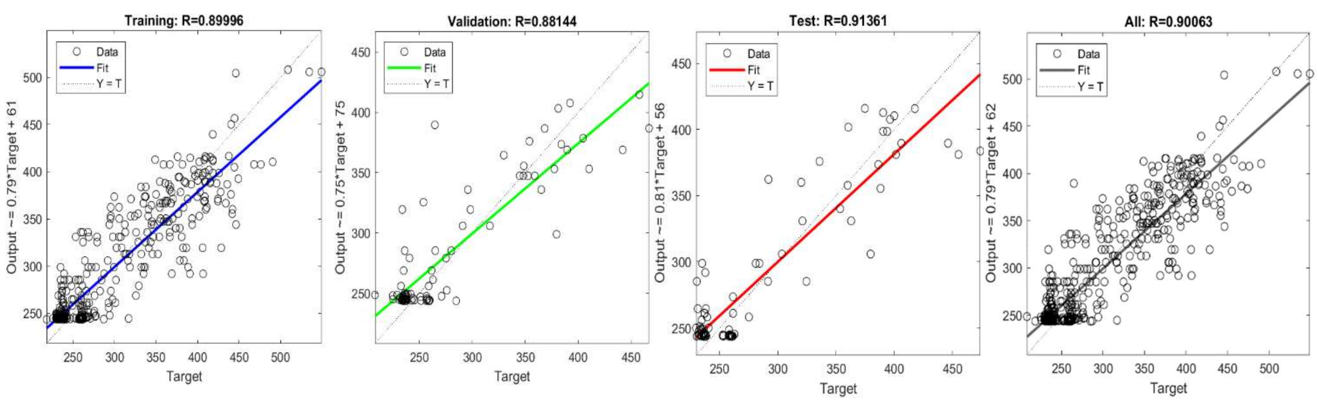

Figure 4. 41 R-values of ANN Model for modelling Vert. Strain using DBPs (LLI)

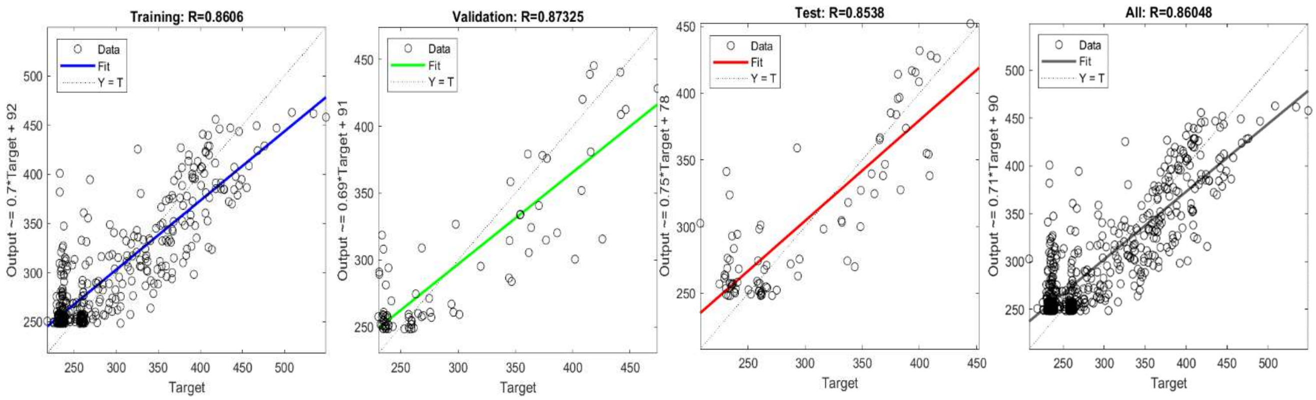

Figure 4. 42 R-values of ANN Model for modelling Vert. Strain using DBPs (AUPP)

Line fit Plot

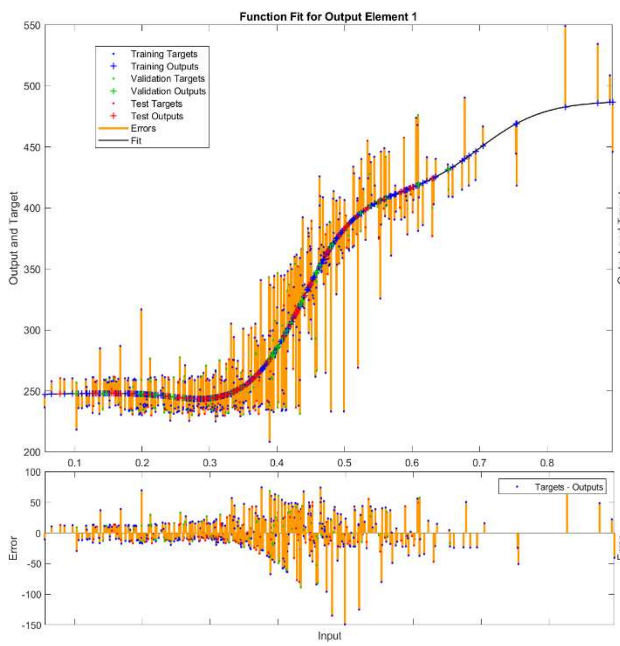

Figure 4. 43 Line Fit Plot of ANN Model for modelling Vert. Strain using DBP (Max Def)

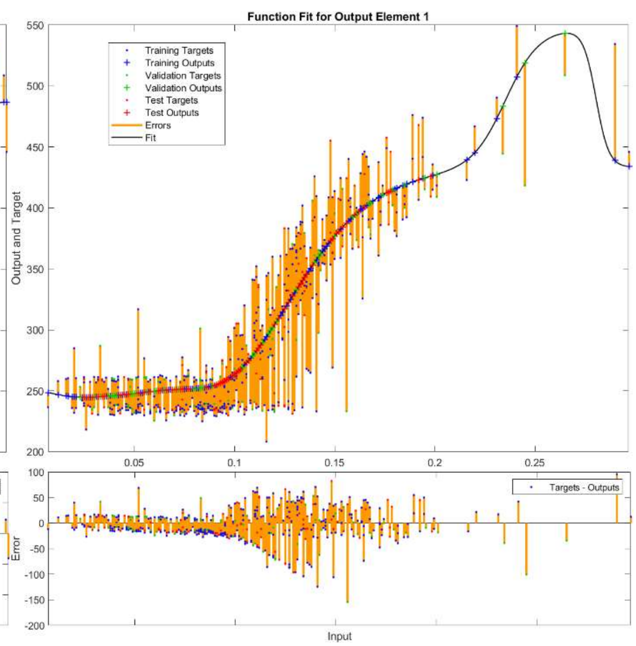

Figure 4. 44 Line Fit Plot of ANN Model for modelling Vert. Strain using DBP (MLI)

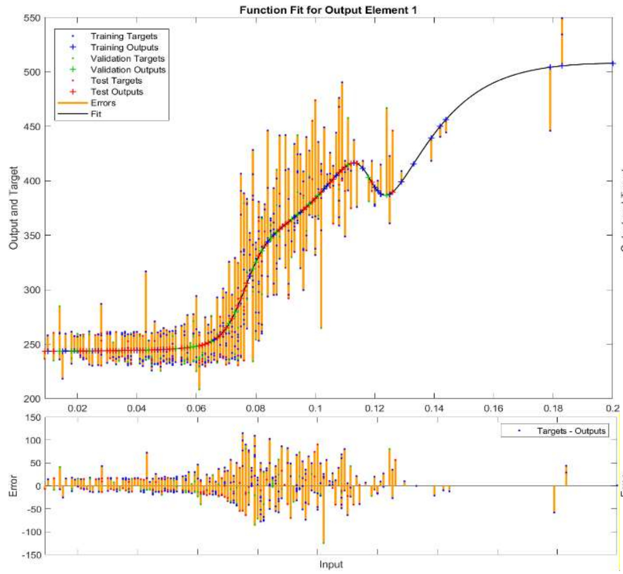

Figure 4. 45 Line Fit Plot of ANN Model for modelling Vert. Strain using DBP (LLI)

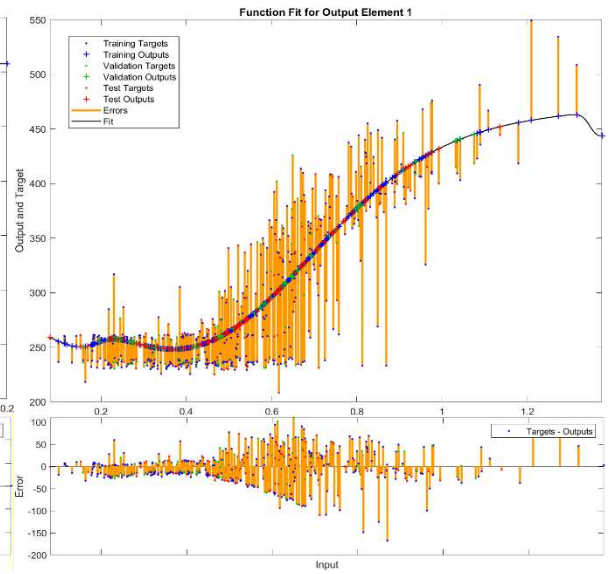

Figure 4. 46 Line Fit Plot of ANN Model for modelling Vert. Strain using DBP (AUPP)

4.1.2.6 Vertical Strains using Surface Moduli

Like that in horizontal strains, Surface Moduli, like SM0, SM300, SM600, can be used in determination of vertical strains effectively as they have R values around 0.9, Table-4.18. SM1200 having R values around 0.8 can also be used as it is good enough to be employed in determining Vertical Strains.

Table-4. 18 Performance of ANN Models used to develop Vertical Strains from Surface Moduli

Parameters		**Surface Moduli**				
		SM0	**SM300**	**SM600**	**SM1200**	**SMD**
Training	**R**	0.9205	0.9211	0.8921	0.7336	0.3855
	MSE	734.68	736.621	1055.8	2195.4	4216.4
Validation	**R**	0.8882	0.9361	0.8736	0.8064	0.4501
	MSE	1028.7	521.668	1022.5	1805.5	3926.9
Test	**R**	0.9094	0.9126	0.8899	0.8394	0.3787
	MSE	846.205	886.727	1016.8	1380.9	3391

Plot of the R values of the ANN models in Training, Validation and Testing phases along with Net R values of the models used to develop Vertical strains from Surface Moduli (SM0, SM300, SM600) are shown in Figures 4.47, 4.48, 4.49. Also, the Line Fit Plots with input and output targets along with the fitted lines are shown in Figures 4.50, 4.51, 4.52.

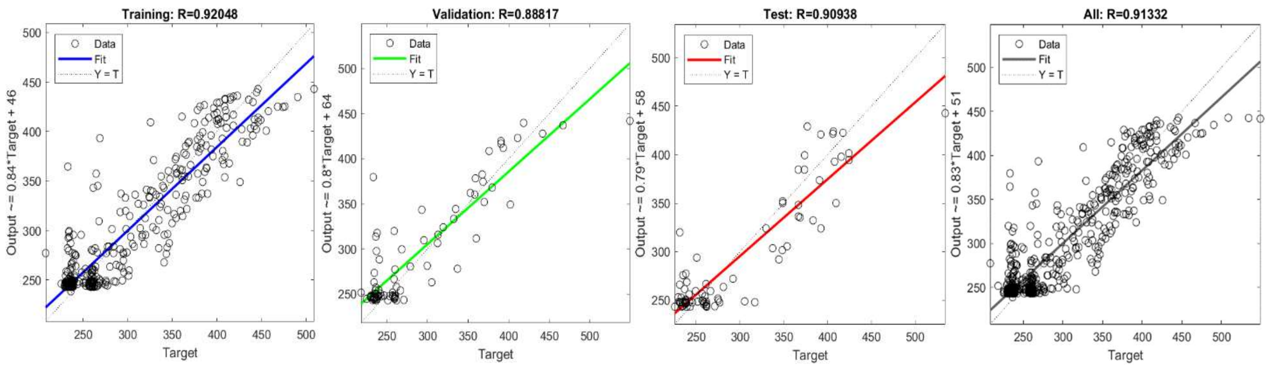

Figure 4. 47 R-values of ANN Model for modelling Vert. Strain using Surface Moduli (SM0)

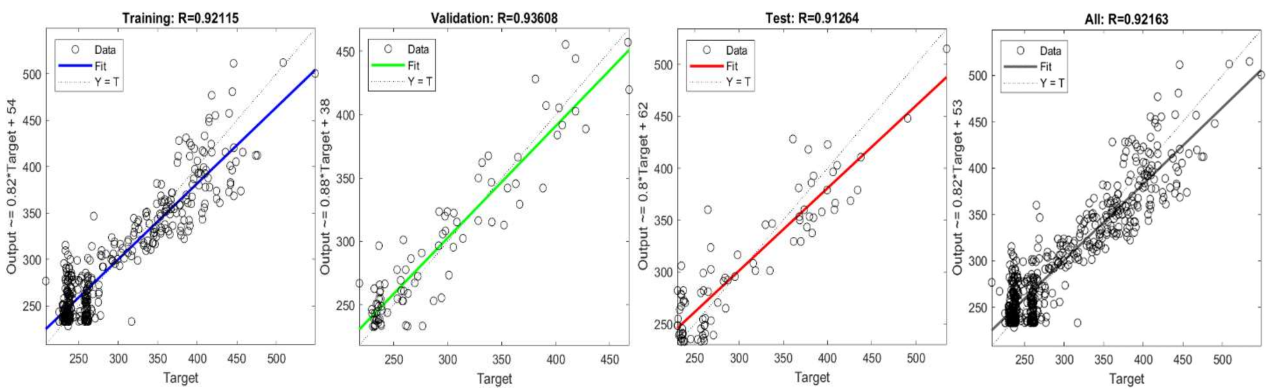

Figure 4. 48 R-values of ANN Model for modelling Vert. Strain using Surface Moduli (SM300)

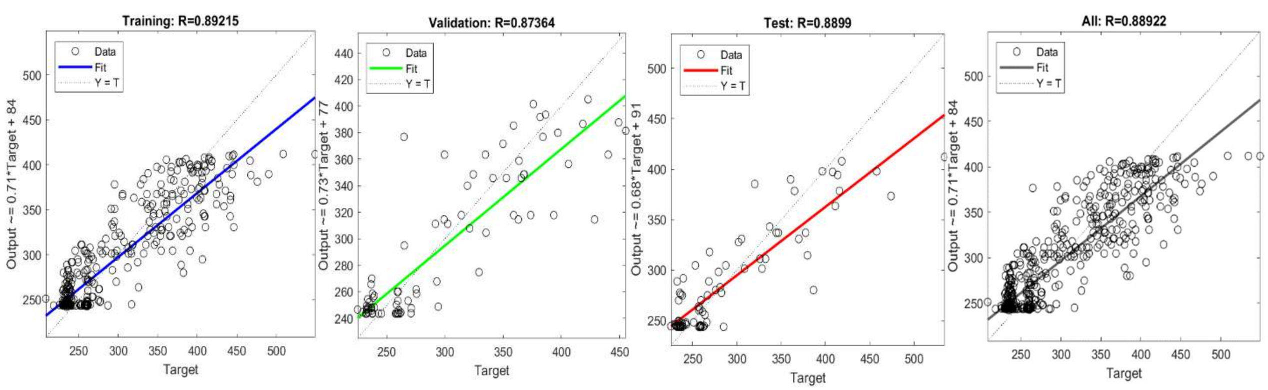

Figure 4. 49 R-values of ANN Model for modelling Vert. Strain using Surface Moduli (SM600)

Line fit

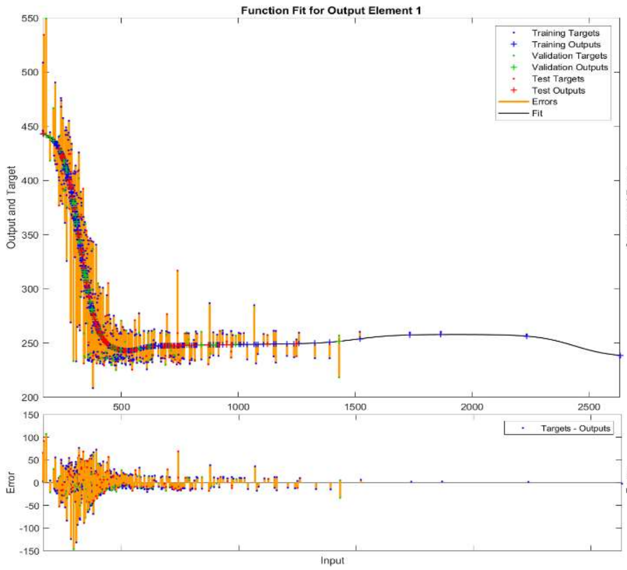

Figure 4. 50 Line Fit Plot of ANN Model for modelling Vert. Strain using Surface Moduli (SM0)

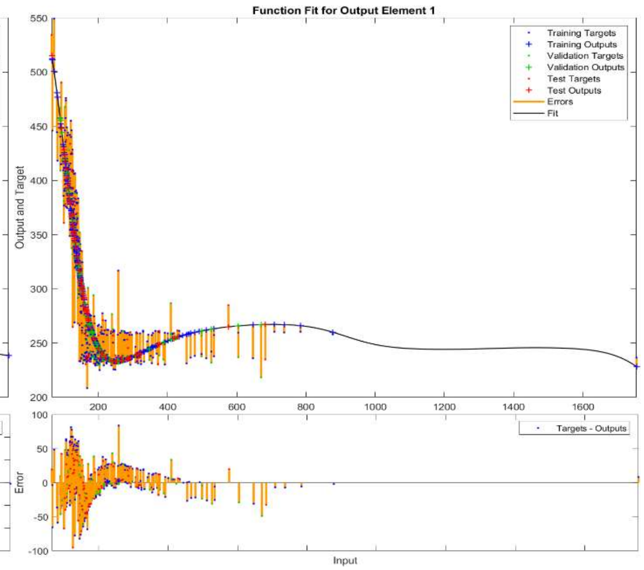

Figure 4. 51 Line Fit Plot of ANN Model for modelling Vert. Strain using Surface Moduli (SM300)

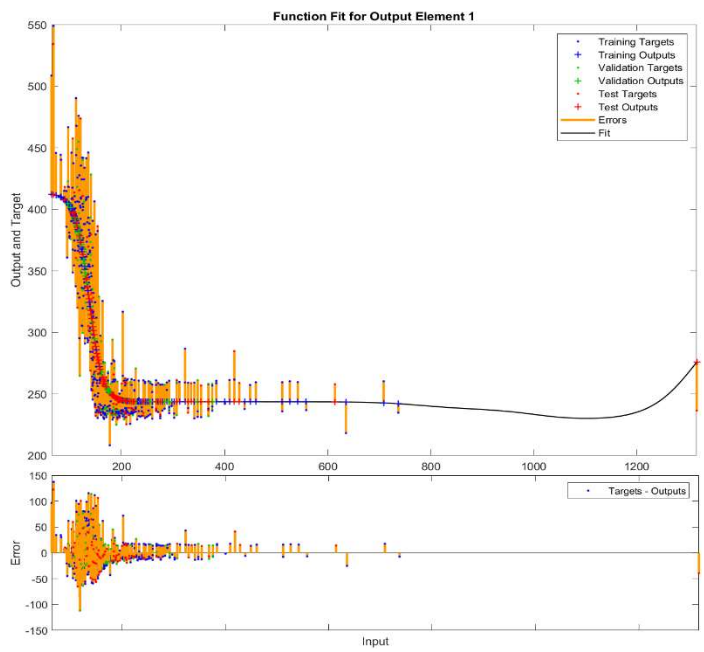

Figure 4. 52 Line Fit Plot of ANN Model for modelling Vert. Strain using Surface Moduli (SM600)

4.2 MODELS WITH MULTIPLE PARAMETERS

4.2.1 Linear Regression

4.2.1.1 Horizontal Strains using FWD Deflections

In this model, Pure Quadratic algorithm had the best performance among those with linear preset, as shown in Table-4.19, with highest R squared value and minimum MSE and MAE values. Also, the algorithms with the stepwise linear preset had almost similar performance but outperformed the Pure Quadratic algorithm with linear preset.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 19 Model: H.FWD.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				**Preset: Stepwise Linear Initial Term: Linear**		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.68	0.72	0.74	0.69	0.75	0.75	0.75
	RMSE	64.595	60.736	58.8	63.646	57.132	57.829	57.308
	MSE	4172.5	3588.9	3457.4	4050.9	3264.1	3344.2	3284.2
	MAE	51.155	37.988	37.715	38.545	37.915	37.516	37.559
Test	R^2	0.71	0.73	0.75	0.74	0.73	0.78	0.78
	RMSE	63.882	62.093	59.787	60.832	62.037	55.896	56.003
	MSE	4080.9	3855.5	3574.5	3700.6	3848.6	3124.3	3136.4
	MAE	51.73	39.13	38.699	38.704	38.288	37.938	37.824

The Response plots of the model is shown in the Figure 4.53. Also, the predicted vs actual plot of the validation and test data is shown in Figure 4.54. it shows that the performance of the model is better at the lower range of the strain. But as the strains increased, the magnitude of deviation of the predicted data from that of the actual one also increased.

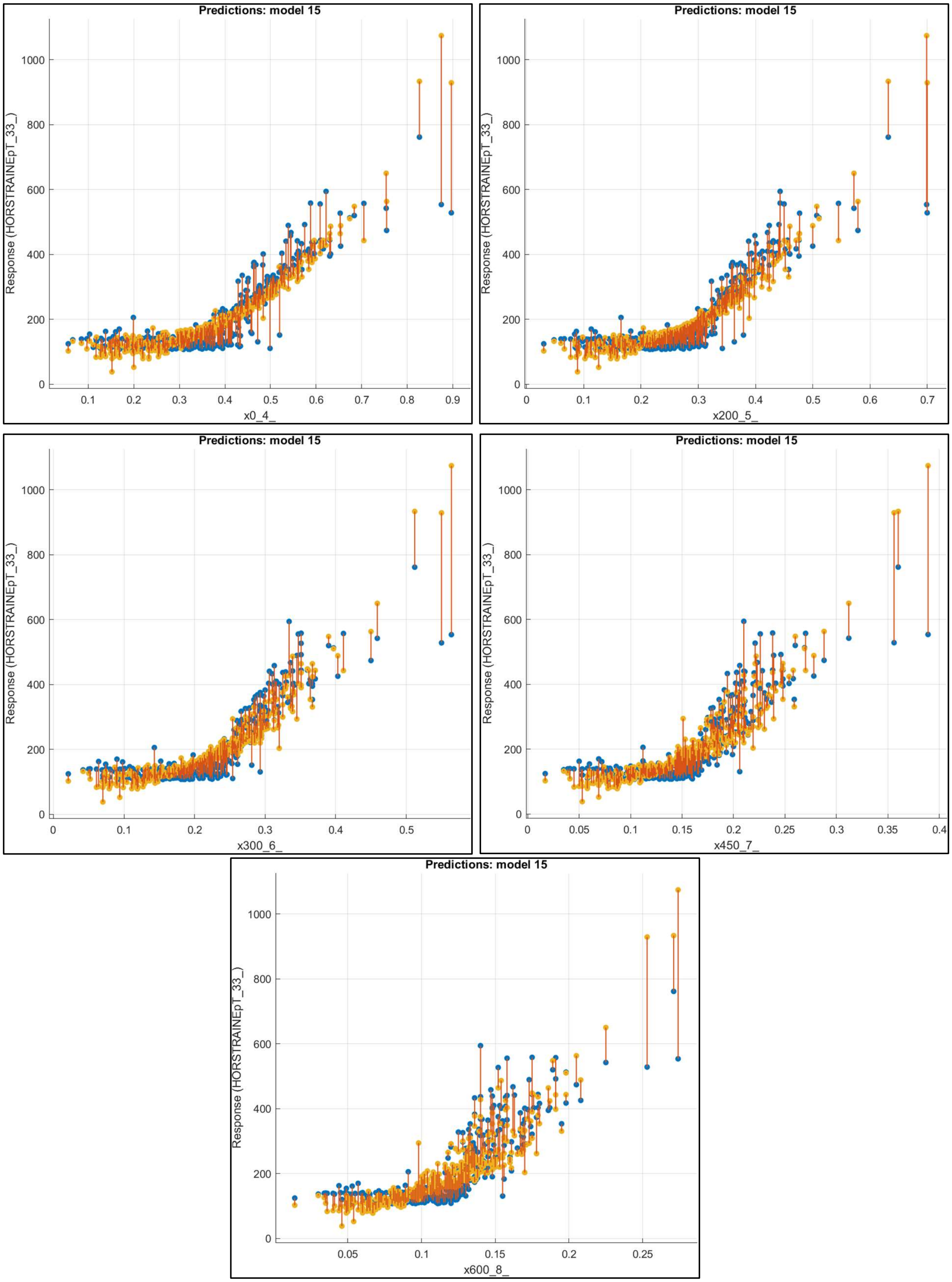

Figure 4. 53 Response Plots of Model: H.FWD.A using Linear regression (0mm, 200mm, 300mm, 450mm, 600mm)

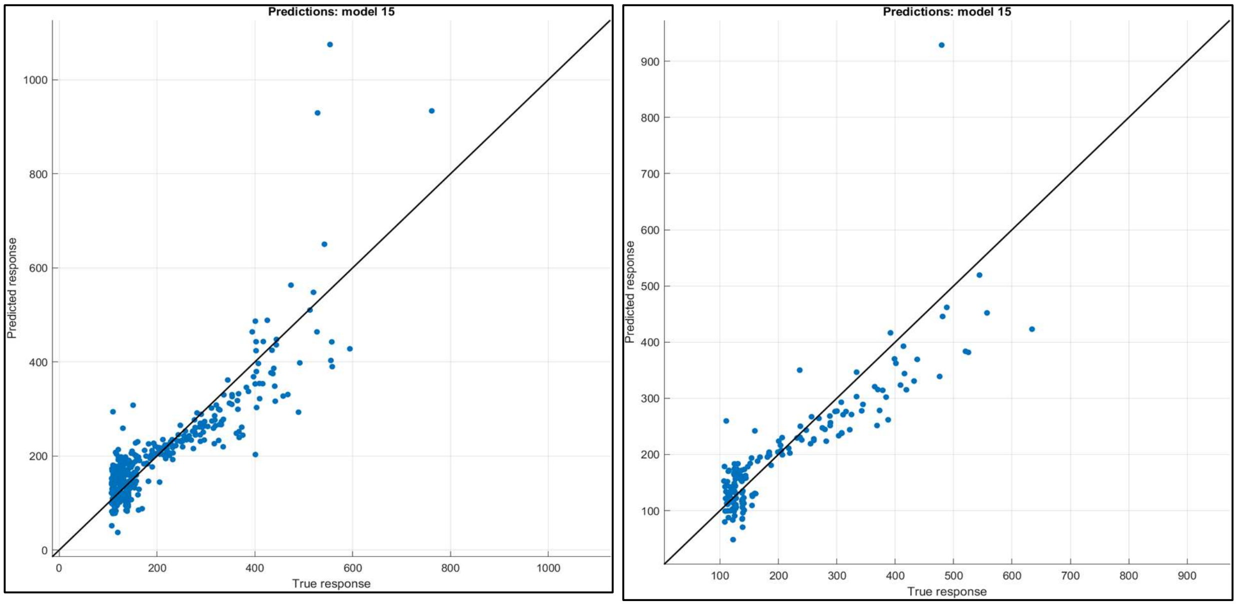

Figure 4. 54 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A using Linear Regression)

4.2.1.2 Horizontal Strains using DBPs

In both models, i.e., H.DBP.A and H.DBP.B, the quadratic algorithm with linear preset was the best among every algorithm. This is based on their high R squared values and low MSE and MAE values in both training and test phase of the dataset. The algorithms with the step linear preset had almost identical performance. (Table-4.20 and Table-4.21)

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, LLI, AREA, AUPP

Table-4. 20 Model: H.DBP.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.78	0.74	0.79	0.78	0.78	0.77	0.75
	RMSE	53.871	59.142	53.137	53.791	53.464	55.167	57.619
	MSE	2902.1	3497.7	2823.6	2893.5	2858.4	3043.4	3319.9
	MAE	40.978	40.972	36.501	35.607	36.209	36.737	36.195
Test	R^2	0.78	0.76	0.8	0.83	0.75	0.74	0.74
	RMSE	55.02	58.316	52.664	49.337	59.664	60.813	60.616
	MSE	3027.2	3400.7	2773.5	2434.1	3559.8	3698.2	3674.3
	MAE	41.273	39.986	37.791	33.127	38.166	38.592	38.729

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 21 Model: H.DBP.B using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.76	0.79	0.77	0.8	0.79	0.77	0.79
	RMSE	56.705	53.191	55.561	51.799	52.851	54.975	52.959
	MSE	3215.4	2829.3	3087.1	2682.7	2793.2	3022.2	2804.6
	MAE	42.849	36.446	39.124	35.695	36.319	38.27	36.597
Test	R^2	0.8	0.82	0.8	0.85	0.82	0.8	0.78
	RMSE	53.166	50.824	52.664	46.352	50.897	53.028	55.458
	MSE	2826.6	2583.1	2773.5	2184.5	2590.5	2812	3075.6
	MAE	40.488	35.731	37.791	32.327	35.41	36.459	37.237

4.2.1.3 Horizontal Strains using Surface Moduli

Every model (H.SM.A, H.SM.B and H.SM.C) had very poor performance in modelling the Horizontal strains and cannot be used in any way. They were not able to create a proper model that can be utilized.

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 22 Model: H.SM.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.28	0.32	-0.17	0.08	0.29	-0.22	0.34
	RMSE	98.715	95.921	126.25	112.19	98.543	129.05	94.853
	MSE	9744.6	9200.9	15939	12586	9710.7	16653	8997.1
	MAE	77.992	64.251	67.079	65.033	65.874	67.666	65.387
Test	R^2	0.28	-14.02	-19.1	-139.26	-17.01	-18.67	-17.01
	RMSE	98.163	447.16	517.38	1366.66	489.68	511.77	489.68
	MSE	9635.9	199950	267680	1867700	239790	261910	239790
	MAE	73.166	99.483	98.438	174.57	100.01	97.935	100.01

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 23 Model: H.SM.B using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	Interactions	Pure Quadratic	Quadratic	Upper Bound on Terms: Interactions	Upper Bound on Terms: Pure Quadratic	Upper Bound on Terms: Quadratic
Training (Validation)	R^2	0.29	0.4	-0.26	-0.05	0.34	-0.28	0.14
	RMSE	98.609	90.316	131.24	119.81	94.771	131.86	108.08
	MSE	9723.7	8157	17223	14355	8991.6	17384	11682
	MAE	78.136	64.196	68.851	65.642	65.046	70.07	65.856
Test	R^2	0.24	-0.14	0.52	-27.79	-0.14	-7.75	-0.59
	RMSE	100.71	123.23	80.359	619.11	123.23	341.41	145.72
	MSE	10142	15186	6457.6	383300	15186	116570	21234
	MAE	74.312	70.869	61.362	116.67	70.869	85.713	74.015

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 24 Model: H.SM.C using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	Interactions	Pure Quadratic	Quadratic	Upper Bound on Terms: Interactions	Upper Bound on Terms: Pure Quadratic	Upper Bound on Terms: Quadratic
Training (Validation)	R^2	0.24	0.34	0.3	0.2	0.34	0.29	0.34
	RMSE	101.63	95.042	97.562	104.25	95.042	98.292	95.037
	MSE	10329	9032.9	9518.4	10869	9032.9	9661.2	9031.9
	MAE	80.574	67.403	68.122	69.205	67.403	67.903	67.757
Test	R^2	0.19	-1.53	-6.03	-5.68	-1.53	-7.24	-7.24
	RMSE	104.09	183.46	305.86	298.17	183.46	331.34	331.34
	MSE	10834	33658	93548	88904	33658	109780	109780
	MAE	76.329	75.422	84.887	84.779	75.422	86.704	86.704

4.2.1.4 Vertical Strains using FWD Deflections

In case of modelling vertical strains from FWD Deflections, the models (V.FWD.A, V.FWD.B, V.FWD.C) have almost identical performance (R squared in range of 0.65 and 0.72) when compared with each other in similar algorithm but the algorithms with stepwise linear preset had the best performance among them based on the R squared values, MSE and MAE (Table-4.25, Table-4.26, Table-4.27). Performance of the models is low and cannot be used satisfactorily at any level of the PMS

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 25 Model: V.FWD.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				**Preset: Stepwise Linear Initial Term: Linear**		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.7	0.74	0.74	0.72	0.74	0.73	0.73
	RMSE	39.407	36.809	36.613	37.967	36.361	37.172	37.381
	MSE	1552.9	1354.9	1340.5	1441.5	1322.1	1381.8	1397.3
	MAE	32.505	28.863	28.761	29.173	28.682	28.933	29.074
Test	R^2	0.65	0.66	0.65	0.64	0.65	0.65	0.66
	RMSE	38.054	37.011	37.981	38.25	37.566	37.575	37.307
	MSE	1448.1	1369.8	1442.6	1463.1	1411.2	1411.9	1391.8
	MAE	30.763	27.446	27.726	27.629	27.766	27.46	27.476

MODEL V.FWD.B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 26 Model: V.FWD.B using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				**Preset: Stepwise Linear Initial Term: Linear**		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.7	0.73	0.74	0.72	0.74	0.74	0.74
	RMSE	39.394	37.125	36.7	38.099	36.817	36.744	36.758

	MSE	1551.9	1378.3	1346.9	1451.5	1355.5	1350.1	1351.2
	MAE	32.447	29	28.828	29.381	29.012	28.94	28.947
Test	R^2	0.64	0.65	0.64	0.63	0.68	0.65	0.65
	RMSE	38.28	37.856	38.296	38.807	36.39	38.04	38.04
	MSE	1465.4	1433.1	1466.6	1506	1324.3	1447	1447
	MAE	30.877	27.963	28.269	28.46	27.346	28.119	28.119

MODEL V.FWD.C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 27 Model: V.FWD.C using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				**Preset: Stepwise Linear Initial Term: Linear**		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.72	0.7	0.73	0.61	0.71	0.71	0.7
	RMSE	39.521	39.488	37.546	44.936	38.323	38.658	39.153
	MSE	1561.9	1559.3	1409.7	2019.2	1468.7	1494.4	1533
	MAE	32.606	29.119	28.288	30.146	28.65	28.847	29.142
Test	R^2	0.64	0.62	0.6	0.62	0.6	0.61	0.62
	RMSE	38.361	39.571	40.424	39.436	40.211	39.999	39.412
	MSE	1471.6	1565.8	1634.1	1555.2	1616.9	1599.9	1553.3
	MAE	30.898	27.524	27.883	27.682	27.801	27.592	27.395

4.2.1.5 Vertical Strains using DBPs

In this case, the performance of models were although decent, the model V.DBP.A and V.DBP.C had better performance as compared to V.DBP.B. (Table-4.28, Table-4.29, Table-4.30). The R squared values of the best performing models in this case were in range of 0.75, they still cannot be used to model vertical strains effectively.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 28 Model: V.DBP.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.69	0.72	0.71	0.71	0.71	0.7	0.7
	RMSE	38.191	36.766	37.214	36.921	37.104	37.817	37.839
	MSE	1458.6	1351.8	1348.9	1363.2	1376.7	1430.1	1431.8
	MAE	31.247	28.242	28.596	28.022	28.482	28.876	28.788
Test	R^2	0.69	0.72	0.73	0.71	0.73	0.73	0.74
	RMSE	39.122	36.871	36.029	37.57	36.044	36.029	35.804
	MSE	3027.2	3400.7	2773.5	2434.1	3559.8	3698.2	3674.3
	MAE	41.273	39.986	37.791	33.127	38.166	38.592	38.729

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 29 Model: V.DBP.B using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	**Interactions**	**Pure Quadratic**	**Quadratic**	**Upper Bound on Terms: Interactions**	**Upper Bound on Terms: Pure Quadratic**	**Upper Bound on Terms: Quadratic**
Training (Validation)	R^2	0.69	0.66	0.66	0.66	0.66	0.67	0.66
	RMSE	38.687	40.386	39.967	40.422	40.172	39.463	40.072
	MSE	1496.7	1631	1597.4	1633.9	1613.8	1557.3	1605.8
	MAE	31.619	28.836	28.8	28.637	28.774	28.562	28.598
Test	R^2	0.68	0.74	0.75	0.74	0.74	0.75	0.74
	RMSE	39.421	35.382	35.193	35.427	35.452	35.183	35.639
	MSE	1554.1	1251.9	1238.6	1255.1	1256.9	1237.9	1270.2
	MAE	32.411	28.964	28.913	28.976	28.981	28.876	29.138

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 30 Model: V.DBP.C using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	Interactions	Pure Quadratic	Quadratic	Upper Bound on Terms: Interactions	Upper Bound on Terms: Pure Quadratic	Upper Bound on Terms: Quadratic
Training (Validation)	R^2	0.73	0.74	0.73	0.77	0.76	0.7	0.75
	RMSE	35.738	35.102	35.686	32.776	33.964	37.981	34.566
	MSE	1277.2	1232.1	1273.5	1074.2	1153.6	1442.5	1194.8
	MAE	28.199	25.391	27.528	24.322	24.859	27.363	25.377
Test	R^2	0.77	0.68	0.77	0.69	0.64	0.73	0.64
	RMSE	33.667	39.575	33.255	38.779	42.119	36.031	42.119
	MSE	1133.4	1566.1	1105.9	1503.8	1774	1298.2	1774
	MAE	27.414	28.194	26.614	28.677	29.691	28.724	29.691

4.2.1.6 Vertical Strains using Surface Moduli

Similar to the models H.SMA, H.SM.B AND H.SM.C, the model V.SM.A cannot be used to model vertical strains because of their drastically poor performance.

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 31 Model: V.SM.A using Linear Regression (R^2, MSE values for Training and Test sets)

		Preset: Linear				Preset: Stepwise Linear Initial Term: Linear		
		Linear	Interactions	Pure Quadratic	Quadratic	Upper Bound on Terms: Interactions	Upper Bound on Terms: Pure Quadratic	Upper Bound on Terms: Quadratic
Training (Validation)	R^2	0.32	-7.76	-4.6	-	-3.84	-	-3.94
	RMSE	57.275	206.27	164.9	-	153.35	-	154.89
	MSE	3280.4	42547	27194	-	23515	-	23991
	MAE	47.341	499.069	46.902	-	46.146	-	47.006
Test	R^2	0.3	0.49	0.39	-	0.38	-	0.39
	RMSE	57.995	49.383	54.283	-	54.754	-	54.119
	MSE	3363.4	2438.7	2946.6	-	2998	-	2928.8
	MAE	46.96	38.758	39.517	-	40.741	-	39.475

4.2.2 Regression Trees

4.2.2.1 Horizontal Strains using FWD Deflections

In this model, performance of all the algorithms except Coarse Tree are somewhat similar (R squared value 0.88 and MSE and MAE about 1500 and 24) in the training phase. But during their Testing phase, their R square value increased and the MSE and MAE values decreased. Best performance was observed in the Medium Tree algorithm. Its R squared value increased to 0.94 and MSE value became almost half during testing phase. This kind of performance makes it suitable to be employed at the network level of the PMS and even Project level activities as a preliminary exercise.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 32 Model: H.FWD.A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	**Medium Tree**	**Coarse Tree**	**Optimizable Tree**
Training	R^2	0.89	0.88	0.85	0.89
	RMSE	38.381	39.848	44.728	38.336
	MSE	1473.1	1587.9	2000.6	1469.6
	MAE	23.563	24.378	26.677	23.97
Test	R^2	0.92	0.94	0.88	0.92
	RMSE	34.236	29.884	40.21	34.484
	MSE	1172.1	893.05	1616.9	1189.1
	MAE	23.168	20.422	26.086	23.465

The Response plots of the model is shown in the Figure 4.55. Also, the predicted vs actual plot of the validation and test data is shown in Figure 4.56. it shows that the performance of the model is similar throughout the range of strains. Making it suitable to model strains.

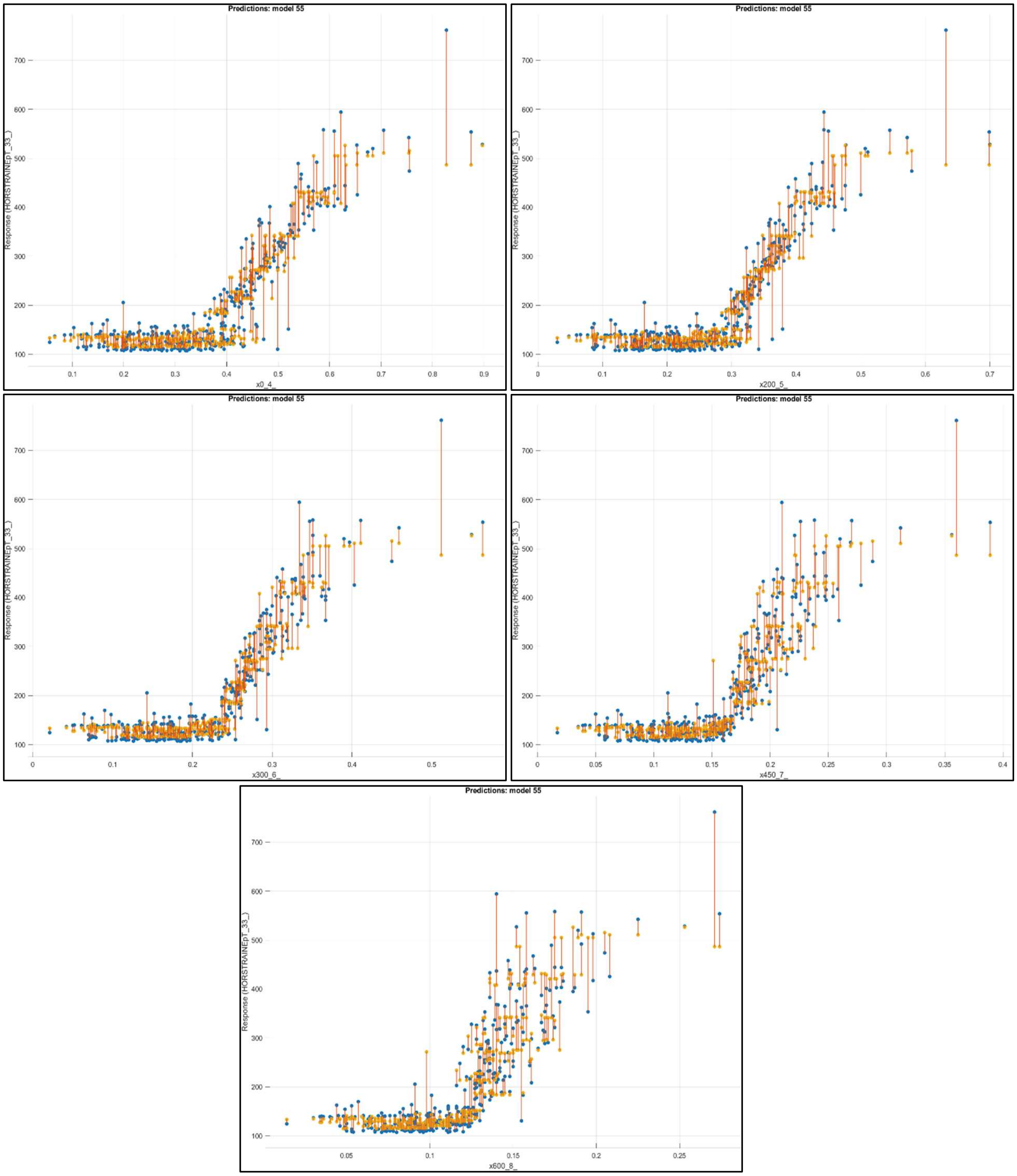

Figure 4. 55 Response Plots of Model: H.FWD.A using Regression Trees (0mm, 200mm, 300mm, 450mm, 600mm)

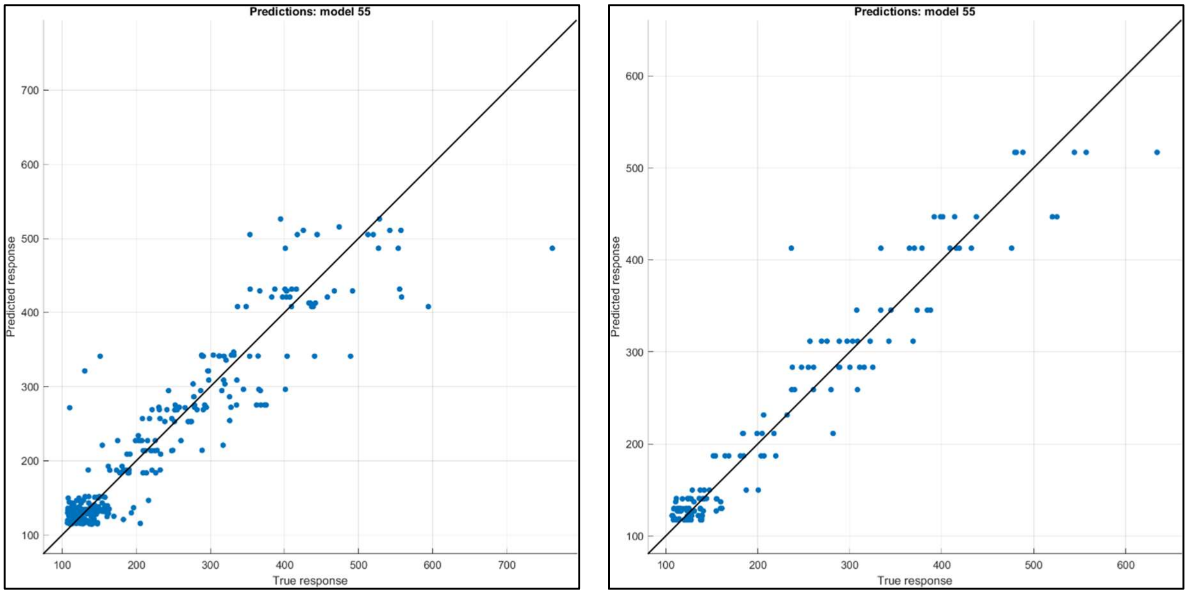

Figure 4. 56 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A using Regression Trees)

4.2.2.2 Horizontal Strains using DBPs

While modelling horizontal strains using Deflection Bowl Parameters, the Model H.DBP.A performed better. Also, the values of R square were higher in case of Medium Tree Algorithm and the Optimizable Trees. The R squared value was in the range of 0.9 and the MAE and MSE values were in rage of 22 and 1150. Such performance makes them suitable to model horizontal strains.

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, AREA, AUPP

Table-4. 33 Model: H.DBP.A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.88	0.89	0.78	0.88
	RMSE	40.316	38.856	53.794	39.199
	MSE	1625.4	1509.8	2893.8	1536.5
	MAE	25.305	24.876	34.499	24.798
Test	R^2	0.91	0.91	0.87	0.92
	RMSE	35.581	34.618	42.367	33.53
	MSE	1266	1198.4	1794.9	1124.3
	MAE	23.546	22.52	26.802	21.993

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 34 Model: H.DBP.B using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.83	0.82	0.71	0.84
	RMSE	46.922	48.362	61.758	45.443
	MSE	2201.7	2338.9	3814	2065.1
	MAE	29.35	31.034	38.959	29.134
Test	R^2	0.9	0.89	0.83	0.89
	RMSE	37.424	40.139	49.391	38.529
	MSE	1400.6	1611.2	2439.5	1484.5
	MAE	23.724	25.756	31.937	24.893

4.2.2.3 Horizontal Strains using Surface Moduli

When modelling Horizontal strains from surface moduli, the optimizable tree had the best performance in every Model, i.e., performance in case of models H.SM.A, H.SM.B, H.SM.C were somewhat similar. Also, their R squared values were in range of 0.92 and MSE of about 1150 (Optimizable Trees).

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 35 Model: H.SM.A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.91	0.91	0.89	0.92
	RMSE	34.994	35.492	38.563	33.826
	MSE	1224.6	1259.7	1487.1	1144.2
	MAE	21.848	22.821	24.88	21.426
Test	R^2	0.88	0.89	0.82	0.89
	RMSE	39.248	38.274	48.658	38.281
	MSE	1540.4	1464.9	2367.6	1465.5
	MAE	24.22	24.383	31.017	23.009

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 36 Model: H.SM.B using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.91	0.91	0.89	0.92
	RMSE	35.765	35.535	38.653	33.792

	MSE	1279.2	1262.8	1494.1	1141.9
	MAE	23.039	22.917	25.077	22.215
Test	R^2	0.89	0.89	0.82	0.9
	RMSE	39.121	38.291	48.676	36.265
	MSE	1530.4	1466.2	2369.3	1315.2
	MAE	24.538	24.64	31.016	23.481

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 37 Model: H.SM.C using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.91	0.91	0.89	0.92
	RMSE	35.935	35.412	38.479	33.503
	MSE	1291.3	1254	1480.6	1122.5
	MAE	23.054	22.42	24.968	22.03
Test	R^2	0.88	0.89	0.83	0.89
	RMSE	39.982	38.222	47.904	37.848
	MSE	1598.6	1460.9	2294.8	1432.5
	MAE	25.696	24.363	30.527	25.659

4.2.2.4 Vertical Strains using FWD Deflections

On modelling Vertical strains from the FWD Deflections, the Model V.FWD.A had the best performance. Also, the performance of the Medium and Optimizable trees algorithms was best in case of their respective model. R squared values of about 0.89 and very low MSE and MAE values (about 430 and 15 respectively) makes V.FWD.A most suitable.

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 38 Model: V.FWD.A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.86	0.87	0.84	0.87
	RMSE	26.373	25.412	28.506	25.412
	MSE	695.52	645.76	812.6	645.76
	MAE	19.63	19.207	21.038	19.207
Test	R^2	0.87	0.89	0.87	0.89
	RMSE	22.997	20.807	23.127	20.807

	MSE	528.88	432.95	534.85	432.95
	MAE	16.707	15.884	17.021	15.884

MODEL V.FWD.B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 39 Model: V.FWD.B using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.87	0.88	0.84	0.88
	RMSE	25.619	25.262	28.406	24.789
	MSE	656.33	638.15	806.88	614.49
	MAE	19.426	19.079	20.823	18.868
Test	R^2	0.87	0.89	0.87	0.87
	RMSE	23.464	20.888	23.041	23.109
	MSE	550.58	436.32	530.89	534.04
	MAE	17.815	15.96	16.906	17.53

MODEL V.FWD.C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 40 Model: V.FWD.C using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.88	0.88	0.86	0.89
	RMSE	25.126	24.297	26.357	23.742
	MSE	631.32	590.36	694.72	563.7
	MAE	18.993	18.015	19.351	18.059
Test	R^2	0.84	0.88	0.87	0.87
	RMSE	25.352	22.058	22.684	23.418
	MSE	642.73	486.57	514.59	548.41
	MAE	18.972	16.211	16.466	17.602

4.2.2.5 Vertical Strains using DBPs

While modelling Vertical strains from the DBPs, although the Models had similar performance, their performance varied greatly when the different algorithms were used, with Optimizable trees being the best performing one.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 41 Model: V.DBP..A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.85	0.83	0.79	0.85
	RMSE	26.5	28.515	31.574	26.5
	MSE	702.25	813.08	996.89	702.23
	MAE	19.274	19.921	22.023	19.274
Test	R^2	0.86	0.87	0.87	0.86
	RMSE	26.211	25.599	25.047	26.211
	MSE	687.03	655.311	627.34	687.03
	MAE	18.95	18.873	18.187	18.95

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 42 Model: V.DBP.B using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.84	0.84	0.78	0.85
	RMSE	27.2	27.201	32.435	26.385
	MSE	739.85	739.89	1052	696.16
	MAE	19.741	19.547	22.389	18.864
Test	R^2	0.87	0.86	0.86	0.88
	RMSE	25.262	25.846	26.587	24.38
	MSE	638.19	668.01	706.87	594.38
	MAE	18.461	19.278	18.959	18.296

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 43 Model: V.DBP.C using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.84	0.84	0.78	0.85
	RMSE	27.264	27.871	32.446	26.573
	MSE	743.34	776.78	1052.7	706.12
	MAE	19.211	19.685	22.284	18.537
Test	R^2	0.84	0.85	0.86	0.87
	RMSE	28.151	26.896	26.499	25.419
	MSE	792.45	723.41	702.18	646.12

	MAE	20.744	19.753	18.869	19.318

4.2.2.6 Vertical Strains using Surface Moduli

In this case, the performance of different algorithms was almost similar in case of the training dataset (R squared value around 0.87). But during testing phase, the optimizable tree performed best with R squared value of 0.92 and MSE and MAE of about 400 and 15. Making it suitable for vertical strain modelling.

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 44 Model: V.SM.A using Regression Trees (R^2, MSE values for Training and Test sets)

		Fine Tree	Medium Tree	Coarse Tree	Optimizable Tree
Training	R^2	0.86	0.87	0.85	0.87
	RMSE	25.947	25.567	26.606	25
	MSE	673.24	653.66	707.9	625
	MAE	19.197	18.406	18.727	18.233
Test	R^2	0.9	0.9	0.88	0.92
	RMSE	22.402	21.621	23.79	19.932
	MSE	502.15	467.48	565.96	397.29
	MAE	16.651	16.578	17.972	15.302

4.2.3 Support Vector Machines

4.2.3.1 Horizontal Strains using FWD Deflections

In case of support vector machine, various algorithms having different kernels were employed (each having different way of determining solution). Therefore, it was expected for them to have drastically different performance as is evident from the Table-4.45. here the best performance was seen in optimizable and Medium Gaussian SVM and poor performance in case of Linear, Quadratic and Cubic SVM. The high R squared value and low MSE and Mae values makes Medium Gaussian SVM and Optimizable SVM from modelling horizontal strains.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 45 Model: H.FWD.A using SVM algorithms (R^2, MSE values for Training and Test sets)

	Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM

Training	R^2	0.68	0.68	-0.43	0.81	0.9	0.84	0.92
	RMSE	65.115	65.475	137.6	50.692	36.224	46.536	32.95
	MSE	4240	4286.9	18932	2569.6	1312.2	2165.6	1085.7
	MAE	50.947	34.471	71.945	24.976	21.329	32.462	19.866
Test	R^2	0.69	0.73	0.58	0.89	0.95	0.86	0.95
	RMSE	66.462	61.163	76.717	39.535	27.625	44.411	25.917
	MSE	4417.2	3740.9	5885.5	1563	763.16	1972.3	671.7
	MAE	52.718	30.831	55.988	19.707	17.502	29.689	16.759

The Response plots of the model using optimizable SVM algorithm is shown in the Figure 4.57. Also, its predicted vs actual plot of the validation and test data is presented in Figure 4.58. it shows that the performance of the model is similar throughout the range of strains. Thus, it being better suited for modelling.

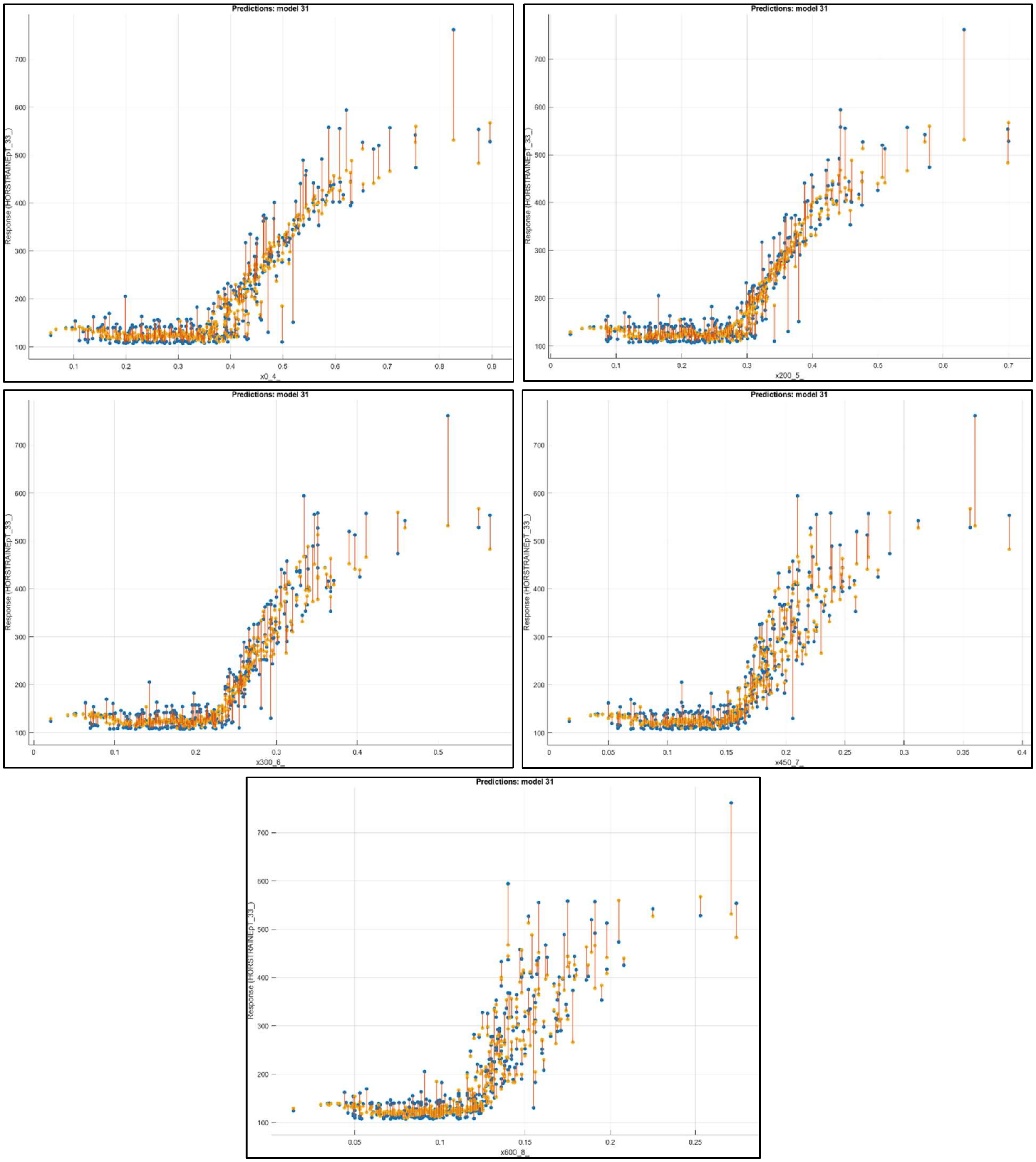

Figure 4. 57 Response Plots of Model: H.FWD.A using SVM (0mm, 200mm, 300mm, 450mm, 600mm)

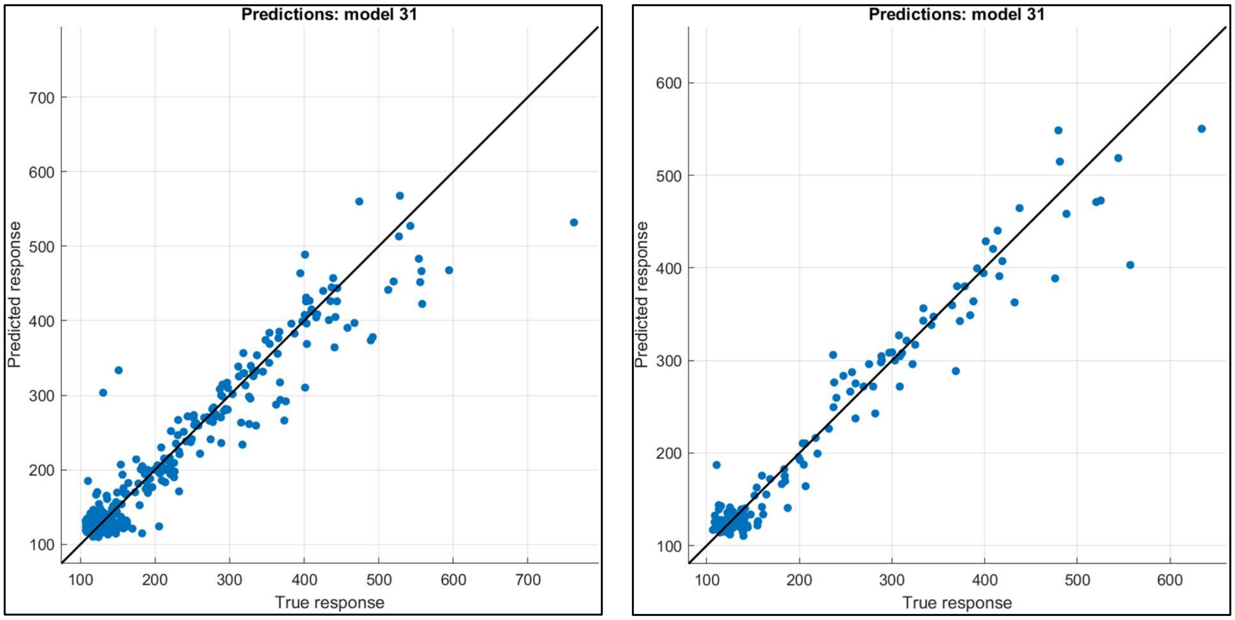

Figure 4. 58 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A using SVM)

4.2.3.2 Horizontal Strains using DBPs

In this case, the models performed best in case of Medium Gaussian SVM followed by optimizable SVM and Coarse Gaussian SVM with Model H.FWD.A performing better by a small amount. Thus, both being ideal for modelling horizontal strain.

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, AREA, AUPP

Table-4. 46 Model: H.DBP.A using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	**Quadratic SVM**	**Cubic SVM**	**Fine Gaussian SVM**	**Medium Gaussian SVM**	**Coarse Gaussian SVM**	**Optimizable SVM**
Training	R^2	0.77	0.78	0.63	0.73	0.9	0.83	0.86
	RMSE	55.274	53.939	69.656	59.312	36.716	47.328	42.521
	MSE	3055.2	2909.4	4851.9	3518	1348.1	2239.9	1808
	MAE	40.93	33.821	31.84	30.105	22.81	33.291	28.991
Test	R^2	0.8	0.83	0.87	0.83	0.93	0.87	0.9
	RMSE	52.661	49.52	41.948	49.16	30.276	42.42	34.64
	MSE	2773.1	2452.2	1759.6	2416.7	916.61	1799.5	1199.9
	MAE	39.148	31.456	24.03	24.837	19.197	29.238	24.43

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 47 Model: H.DBP.B using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.75	0.76	0.81	0.78	0.89	0.8	0.86
	RMSE	57.329	56.742	50.19	54.072	38.328	51.632	42.581
	MSE	3286.6	3219.7	2519	2923.8	1469	2665.9	1813.2
	MAE	42.587	33.795	30.171	28.38	24.267	37.209	29.35
Test	R^2	0.8	0.8	0.92	0.87	0.93	0.83	0.87
	RMSE	53.089	53.062	33.244	42.096	31.629	49.022	42.308
	MSE	2818.4	2815.5	1105.2	1772.1	1000.4	2403.2	1790
	MAE	39.046	30.739	23.565	22.515	21.16	35.158	28.596

4.2.3.3 Horizontal Strains using Surface Moduli

In this case, the fine gaussian and SVM and Medium Gaussian SVM were the best performing algorithms. All three models tested has somewhat similar performance with H.SM.B being the best one by only a small margin.

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 48 Model: H.SM.A using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.13	-0.36	0.17	0.92	0.91	0.54	0.35
	RMSE	108.97	136.29	106.29	33.636	35.343	79.348	93.893
	MSE	11875	18575	11297	1131.4	1249.1	6269.1	8815.8
	MAE	75.483	63.559	74.756	20.735	23.578	53.921	61.589
Test	R^2	0.12	-0.55	-69.18	0.9	0.88	0.56	0.37
	RMSE	108.3	43.69	966.68	37.263	39.851	76.688	91.669
	MSE	11729	20647	934480	1388.5	1588.1	5881.1	8403.3
	MAE	74.024	63.967	151.73	22.627	25.297	51.003	59.843

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 49 Model: H.SM.B using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.14	-0.78	-2.65	0.93	0.92	0.57	0.06
	RMSE	108.26	155.96	223.03	31.934	33.078	76.921	113.38
	MSE	11721	24324	49744	1019.8	1094.2	5916.8	12854
	MAE	74.438	66.276	145.32	19.454	22.321	53.007	76.69
Test	R^2	0.1	-0.74	-2661.97	0.9	0.89	0.58	0.1
	RMSE	109.29	152.23	5954.8	36.624	37.655	74.975	109.65
	MSE	11944	23173	35460000	1269.1	1417.9	5621.3	12023
	MAE	75.249	63.56	557.52	20.696	24.427	50.24	74.914

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 50 Model: H.SM.C using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.04	-0.62	-8.16	0.92	0.92	0.57	0.2
	RMSE	114.19	148.52	353.38	32.995	32.947	76.776	104.24
	MSE	13040	22058	124880	1088.7	1085.5	5894.6	10865
	MAE	78.076	66.208	192.09	21.3	21.97	53.456	77.054
Test	R^2	0.04	-3.39	-1731.48	0.89	0.89	0.57	0.115
	RMSE	112.86	241.82	4803.1	37.434	37.804	75.417	106.3
	MSE	12736	58475	23069000	1401.3	1429.2	5687.7	11299
	MAE	76.965	78.305	608.42	22.798	24.717	51.566	75.062

4.2.3.4 Vertical Strains using FWD Deflections

While modelling Vertical strain too, the Medium Gaussian SVM and Optimizable SVM were the best performers. With all three models having not much difference when their performance is determined using R squared value and the MSE and MAE values.

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 51 Model: V.FWD.A using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.7	0.61	0.77	0.81	0.9	0.82	0.89
	RMSE	39.41	44.553	34.681	31.351	22.543	30.573	23.647
	MSE	1553.2	1984.9	1202.8	982.89	508.2	934.73	559.2
	MAE	32.471	28.837	24.945	21.549	17.465	23.998	17.603
Test	R^2	0.63	0.53	0.81	0.84	0.91	0.8	0.87
	RMSE	38.893	43.8	28.197	25.7	19.693	28.558	23.05
	MSE	1512.7	1918.4	795.08	660.5	387.81	815.58	531.3
	MAE	31.214	26.5	20.873	17.736	15.034	21.234	15.929

MODEL V;FWD;B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 52 Model: V.FWD.B using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.69	0.57	0.46	0.83	0.91	0.82	0.89
	RMSE	39.562	46.804	52.736	29.71	21.822	30.069	23.337
	MSE	1565.1	2190.6	2781	882.69	476.18	904.17	544.62
	MAE	32.475	28.928	37.948	19.83	17.032	23.5	18.428
Test	R^2	0.63	0.54	0.47	0.87	0.9	0.8	0.87
	RMSE	39.047	43.601	46.573	22.997	20.216	28.345	22.784
	MSE	1524.6	1901.1	2169	528.84	408.69	803.46	519.13
	MAE	31.287	26.668	27.525	16.368	15.327	20.986	16.423

MODEL V;FWD;C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 53 Model: V.FWD.C using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.69	0.65	0.58	0.83	0.91	0.82	0.9
	RMSE	39.68	42.31	46.279	29.254	21.912	29.958	22.8817
	MSE	1574.5	1790.2	2141.8	855.78	480.14	897.47	520.62
	MAE	32.662	27.749	30.352	19.603	17.142	23.384	16.837
Test	R^2	0.63	0.43	0.68	0.88	0.9	0.8	0.89

	RMSE	38.967	48.229	36.067	22.318	20.155	28.329	21.268
	MSE	1518.4	2326	1300.8	498.12	404.61	802.55	452.32
	MAE	31.24	26.541	28.181	16.053	15.222	20.88	14.928

4.2.3.5 Vertical Strains using DBPs

Here too the Medium Gaussian SVM and Optimizable SVM had the best performance. With all three models having not much difference when their performance is determined using R squared value and the MSE and AE values.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 54 Model: V.DBP.A using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.69	0.55	0.84	0.76	0.89	0.8	0.68
	RMSE	38.453	46.151	27.925	33.974	23.323	31.065	38.734
	MSE	1478.6	2129.9	779.81	1154.3	543.97	965.02	1500.3
	MAE	31.11	27.389	21.182	22.292	17.327	23.891	31.175
Test	R^2	0.68	0.63	0.44	0.82	0.89	0.81	0.68
	RMSE	39.478	42.456	52.141	30.032	22.668	30.758	39.861
	MSE	1558.5	1802.5	2718.7	901.94	513.82	946.03	1588.9
	MAE	32.18	28.782	28.363	20.078	17.973	24.382	32.4

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 55 Model: V.DBP.B using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.68	0.52	0.81	0.8	0.88	0.82	0.82
	RMSE	38.881	47.67	30.157	30.586	23.903	28.918	29.406
	MSE	1511.8	2272.5	909.45	935.53	571.627	436.26	864.74
	MAE	31.544	27.528	21.945	20.248	17.627	21.885	21.662
Test	R^2	0.67	0.57	0.8	0.85	0.9	0.82	0.81
	RMSE	39.921	45.621	31.193	27.465	22.345	29.829	30.792
	MSE	1593.7	2081.3	973	754.33	499.32	889.76	948.18
	MAE	32.89	28.873	25.275	18.428	17.181	23.176	23.78

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 56 Model: V.DBP.C using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.72	0.74	0.81	0.76	0.89	0.8	0.85
	RMSE	36.198	35.057	30.22	34.046	23.28	30.878	27.113
	MSE	1310.3	1229	913.26	1159.1	541.95	953.46	735.1
	MAE	28.085	24.187	20.79	22.156	17.287	23.799	19.829
Test	R^2	0.75	0.74	0.33	0.79	0.89	0.8	0.35
	RMSE	35.26	35.433	57.255	32.082	23.586	30.916	56.333
	MSE	1243.3	1255.5	3278.1	1029.2	556.32	955.79	3173.4
	MAE	28.013	26.31	29.019	20.775	18.541	24.345	28.887

4.2.3.6 Vertical Strains using Surface Moduli

Like in the case of modeling in the horizontal strains, Fine and Medium Gaussian algorithms have the best performance

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 57 Model: V.SM.A using SVM algorithms (R^2, MSE values for Training and Test sets)

		Linear SVM	Quadratic SVM	Cubic SVM	Fine Gaussian SVM	Medium Gaussian SVM	Coarse Gaussian SVM	Optimizable SVM
Training	R^2	0.3	-8.5	-1116.8	0.88	0.87	0.62	0.32
	RMSE	58.38	214.78	-	24.627	25.505	43.21	57.632
	MSE	3480.3	46132	-	606.49	650.52	1867.1	3321.4
	MAE	47.265	48.058	-	17.575	19.15	34.962	47.174
Test	R^2	0.23	-0.07	-	0.91	0.87	0.63	0.25
	RMSE	60.025	71.796	-	20.55	25.269	42.128	60.092
	MSE	3603	5154.7	-	422.68	638.55	1774.8	3611
	MAE	46.444	36.372	-	15.629	19.405	34.379	46.534

4.2.4 Gaussian Process Regression

4.2.4.1 Horizontal Strains using FWD Deflections

In this case, all of the algorithms tested for the model presented a very R squared value (upto 0.96) with Low MSE and MAE values (600 and 17 respectively) making it suitable for modelling strains.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 58 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	**Squared Exponential GPR**	**Matern 5/2 GPR**	**Exponential GPR**	**Optimizable GPR**
Training	R^2	0.92	0.92	0.92	0.91	0.92
	RMSE	32.786	32.704	32.609	34.506	33.011
	MSE	1074.9	1069.6	1063.3	1190.7	1089.7
	MAE	20.315	21.072	20.187	20.519	20.172
Test	R^2	0.96	0.96	0.96	0.95	0.96
	RMSE	24.298	24.402	24.396	26.821	24.776
	MSE	590.41	595.45	595.14	719.38	613.87
	MAE	16.664	17.173	16.714	17.991	16.908

The Response plots of the model using Rational Quadratic GPR algorithm is shown in the Figure 4.59. Also, its predicted vs actual plot of the validation and test data is presented in Figure 4.60. it shows that the performance of the model is similar throughout the range of strains. Thus, it being best for modelling horizontal strains.

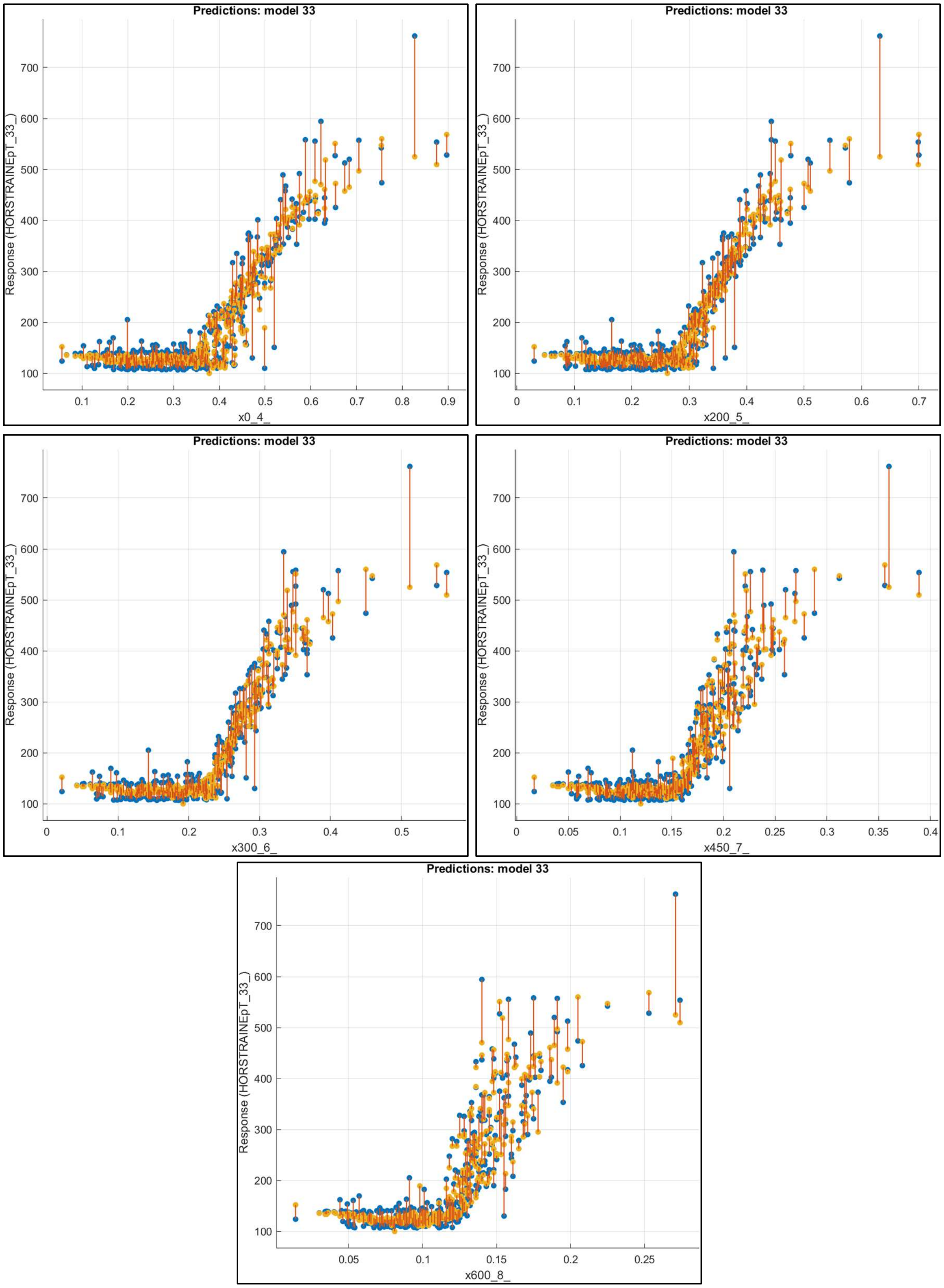

Figure 4. 59 Response Plots of Model: H.FWD.A using GPR (0mm, 200mm, 300mm, 450mm, 600mm)

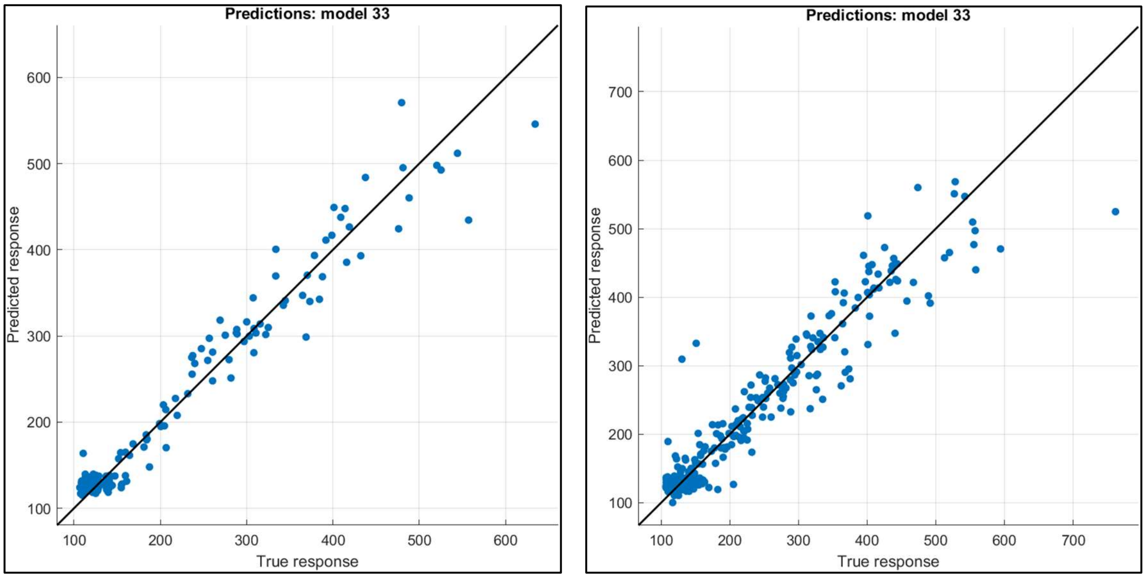

Figure 4. 60 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A GPR)

4.2.4.2 Horizontal Strains using DBPs

Here too the high R squared and low MSE and MAE values both models suitable to model strains. Also, the performance of the algorithms except Squared Exponential GPR is nearly identical. Thus, making them ideal for modeling

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, AREA, AUPP

Table-4. 59 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.91	0.9	0.91	0.91	0.91
	RMSE	34.822	35.48	35.208	35.332	34.24
	MSE	1212.6	1258.8	1239.6	1248.3	1172.4
	MAE	21.815	22.576	22.023	21.801	21.108
Test	R^2	0.94	0.92	0.94	0.95	0.93
	RMSE	29.39	32.677	29.535	27.314	30.333
	MSE	863.77	1067.8	872.34	746.03	920.11
	MAE	18.834	19.724	18.867	18.073	18.878

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 60 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.9	0.89	0.9	0.9	0.9
	RMSE	36.044	37.395	36.436	36.222	36.398
	MSE	1299.1	1398.4	1327.6	1312	1324.8
	MAE	22.394	23.371	22.59	22.147	22.218
Test	R^2	0.94	0.93	0.94	0.94	0.94
	RMSE	28.998	31.512	28.91	28.807	29.399
	MSE	840.88	993.02	835.77	829.84	864.29
	MAE	19.309	20.128	19.246	18.928	19.468

4.2.4.3 Horizontal Strains using Surface Moduli

In case of modelling Horizontal strains from the Surface Moduli, all three models under consideration performed very well and have nearly identical performance (measured in terms of R squared values, MSE and MAE). Thus, in this case too, all models with any GPR algorithms can be utilized for modeling.

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 61 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.94	0.94	0.94	0.94	0.94
	RMSE	28.062	28.733	28.378	28.546	28.168
	MSE	787.46	825.59	805.33	814.87	793.42
	MAE	18.535	19.331	18.672	18.228	18.402
Test	R^2	0.91	0.9	0.91	0.9	0.9
	RMSE	35.548	36.174	35.566	36.375	36.641
	MSE	1263.6	1308.6	1265	1323.2	1342.5
	MAE	21.663	22.5	21.718	22	21.903

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 62 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.94	0.94	0.94	0.94	0.94

	RMSE	28.139	28.597	28.209	28.787	27.893
	MSE	791.78	817.78	795.77	828.71	778
	MAE	18.731	18.442	18.773	18.51	18.439
Test	R^2	0.91	0.9	0.91	0.9	0.91
	RMSE	35.053	35.77	35.161	35.607	34.792
	MSE	1228.7	1279.5	1236.3	1267.8	1210.5
	MAE	21.173	22.098	21.296	21.296	20.935

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 63 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.93	0.93	0.93	0.93	0.93
	RMSE	30.461	30.405	30.635	31.034	31.034
	MSE	927.9	924.46	938.5	963.11	963.11
	MAE	20.626	20.614	20.571	20.229	20.229
Test	R^2	0.89	0.89	0.9	0.89	0.89
	RMSE	37.445	37.446	37.356	37.773	37.773
	MSE	1402.1	1402.1	1395.5	1426.8	1426.8
	MAE	24.404	24.405	24.185	23.78	23.78

4.2.4.4 Vertical Strains using FWD Deflections

Similarly, all three models, V.FWD.A, V.FWD.B and V.FWD.C, can be used for modelling vertical strains as they too have high R squared and low MSE and MAE values.in both training and testing phases.

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 64 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.91	0.91	0.91	0.91	0.9
	RMSE	21.76	21.825	21.578	21.657	22.72
	MSE	473.48	476.32	465.61	469.02	516.21
	MAE	17.385	17.405	17.329	17.197	17.614
Test	R^2	0.91	0.91	0.91	0.9	0.9
	RMSE	19.618	16.695	19.514	20.021	20.46
	MSE	384.86	387.9	380.78	400.85	418.6

	MAE	14.938	15.078	14.897	14.909	15.368

MODEL V.FWD.B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 65 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.91	0.9	0.91	0.91	0.91
	RMSE	21.349	22.534	21.403	21.491	21.154
	MSE	455.78	507.77	458.07	461.88	447.48
	MAE	17.11	17.855	17.194	16.925	16.933
Test	R^2	0.9	0.9	0.9	0.9	0.9
	RMSE	20.011	19.841	19.892	20.105	20.685
	MSE	400.45	393.68	395.7	404.2	427.87
	MAE	14.935	15.004	14.947	15.136	15.419

MODEL V.FWD.C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 66 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.91	0.9	0.91	0.91	0.91
	RMSE	21.33	22.493	21.423	21.412	21.6618
	MSE	454.96	505.93	458.94	458.47	467.35
	MAE	17.067	17.808	17.16	16.87	17.222
Test	R^2	0.9	0.9	0.9	0.9	0.9
	RMSE	19.963	19.86	19.815	20.149	20.287
	MSE	398.5	394.43	392.61	405.98	411.58
	MAE	14.911	15.041	14.907	15.146	15.141

4.2.4.5 Vertical Strains using DBPs

Here R squared values are somewhat lower in modelling Vertical strains as compared to that in the Horizontal strains. Nonetheless, it can be employed in modelling as the values of the performance parameters is still considerably higher.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 67 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.9	0.9	0.9	0.89	0.9

	RMSE	22.214	22.319	22.09	22.667	22.159
	MSE	493.47	498.14	487.97	513.79	491
	MAE	16.708	16.854	16.631	16.976	16.694
Test	R^2	0.9	0.9	0.9	0.91	0.91
	RMSE	21.973	22.43	21.885	21.122	21.351
	MSE	482.8	503.12	478.97	446.16	455.86
	MAE	18.037	18.387	17.965	17.039	17.441

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 68 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.89	0.89	0.89	0.89	0.89
	RMSE	22.678	22.846	22.528	22.587	22.741
	MSE	514.31	521.92	507.53	510.17	517.17
	MAE	17.117	17.14	17.016	16.89	17.158
Test	R^2	0.9	0.89	0.9	0.9	0.9
	RMSE	22.492	22.781	22.441	21.742	22.554
	MSE	505.87	518.99	503.58	472.72	508.7
	MAE	17.529	17.926	17.501	16.734	17.592

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 69 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.9	0.9	0.9	0.9	0.9
	RMSE	21.909	22.069	21.826	22.114	21.941
	MSE	480	487.02	476.35	489.01	481.42
	MAE	16.578	16.665	16.541	16.608	16.671
Test	R^2	0.9	0.9	0.9	0.91	0.89
	RMSE	22.073	22.635	22.145	21.542	23.243
	MSE	487.21	512.35	490.42	464.04	540.25
	MAE	17.869	18.355	17.919	17.21	18.612

4.2.4.6 Vertical Strains using Surface Moduli

Here too the values of performance parameters are lower in modelling vertical strains than its counterpart in modeling horizontal ones. But still, this model can be utilized to model vertical strains with good confidence.

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 70 Model: H.FWD.A using GPR algorithms (R^2, MSE values for Training and Test sets)

		Rational Quadratic GPR	Squared Exponential GPR	Matern 5/2 GPR	Exponential GPR	Optimizable GPR
Training	R^2	0.9	0.9	0.9	0.9	0.91
	RMSE	21.986	22.518	22.247	21.817	21.315
	MSE	483.37	507.06	494.92	475.98	454.35
	MAE	16.728	17.074	16.846	16.512	16.283
Test	R^2	0.91	0.91	0.91	0.92	0.92
	RMSE	20.393	20.498	20.603	20.035	20.263
	MSE	415.89	420.18	424.49	401.4	402.52
	MAE	16.112	16.087	16.257	15.84	15.875

4.2.5 Ensembles of Trees

4.2.5.1 Horizontal Strains using FWD Deflections

For modelling horizontal strains from the FWD deflection model, both boosted as well as the bagged trees gives good performance parameters with the Bagged Tree algorithm being little better.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 71 Model: H.FWD.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.89	0.9	0.91
	RMSE	38.605	35.6	34.368
	MSE	1490.4	1267.4	1181.2
	MAE	22.587	21.44	21.065
Test	R^2	0.94	0.95	0.94
	RMSE	29.818	27.101	27.992
	MSE	889.12	734.46	783.55
	MAE	19.749	18.177	18.857

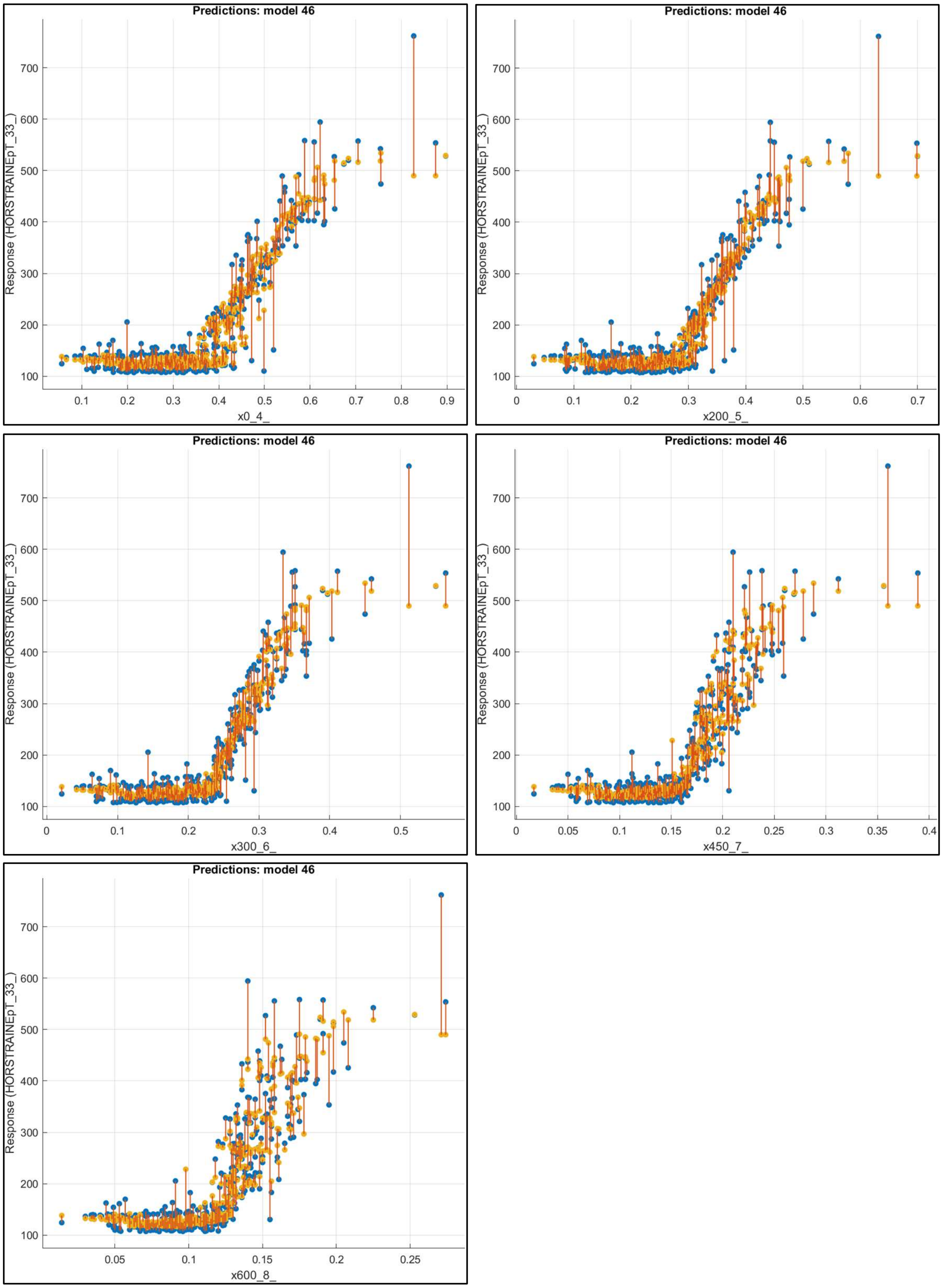

Figure 4. 61 Response Plots of Model: H.FWD.A using Ensemble of Trees (0mm, 200mm, 300mm, 450mm, 600mm)

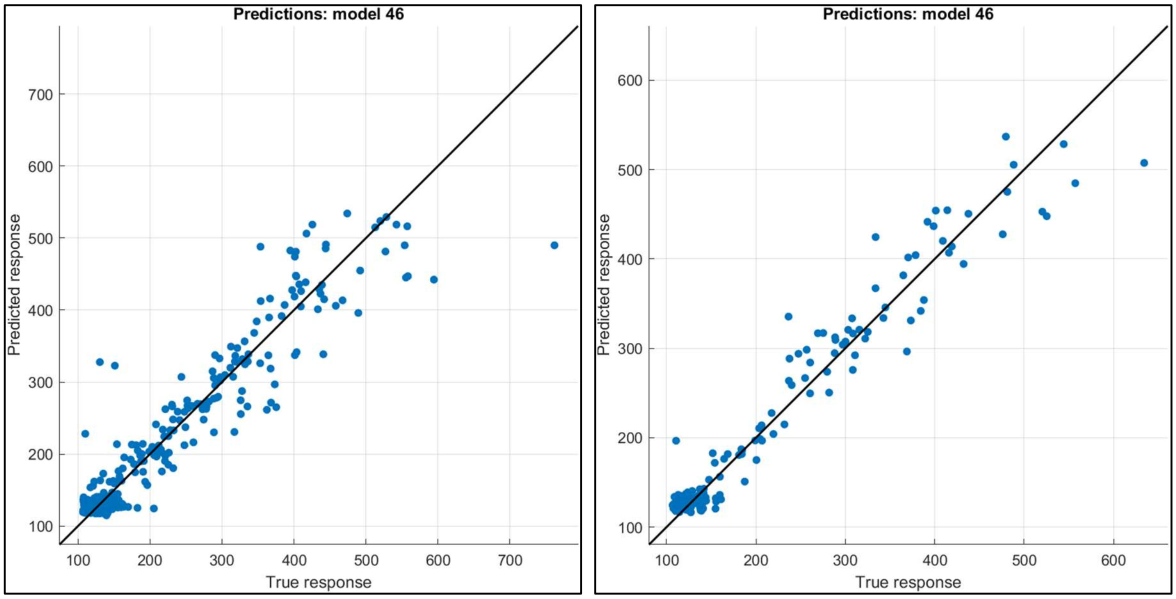

Figure 4. 62 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A Ensemble of Trees)

Here we can see that the Response plots of the model using Bagged Trees is shown in the Figure 4.61. Also, its predicted vs actual plot of the validation and test data is presented in Figure 4.62. it shows that the performance of the model is similar throughout the range of strains. Thus, it being best for modelling horizontal strains.

4.2.5.2 Horizontal Strains using DBPs

In this case, the performance of Model H.DBP.A is slightly better in terms of R squared, MSE and MAE parameters for the same algorithm. Also, the performance of boosted tree algorithm as compared to the bagged on. Therefore, boosted Trees is preferred for modeling strains using given model

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, AREA, AUPP

Table-4. 72 Model: H.DBP.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	**Bagged Trees**	**Optimizable Ensembles**
Training	R^2	0.9	0.89	0.9
	RMSE	36.069	38.812	35.965
	MSE	1301	1506.3	1293.5
	MAE	22.016	24.101	22.574
Test	R^2	0.94	0.92	0.93
	RMSE	30.217	33.287	31.782

	MSE	913.04	1108	1010.1
	MAE	19.861	21.223	21.446

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 73 Model: H.DBP.B using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.88	0.86	0.89
	RMSE	39.278	42.856	38.512
	MSE	1542.8	1836.6	1483.2
	MAE	23.241	25.842	23.311
Test	R^2	0.91	0.9	0.92
	RMSE	36.379	37.034	34.29
	MSE	1323.4	1371.5	1175.8
	MAE	22.709	21.837	21.552

4.2.5.3 Horizontal Strains using Surface Moduli

In case of Modelling Strains from surface moduli, Bagged trees algorithm yields better performance. Also, as performance parameters of all three models from same algorithm is identical, each model is suitable.

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 74 Model: H.SM.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.93	0.93	0.93
	RMSE	31.901	30.154	31.464
	MSE	1017.7	909.29	989.97
	MAE	19.725	19.27	19.974
Test	R^2	0.9	0.9	0.9
	RMSE	37.023	35.618	37.29
	MSE	1370.7	1268.6	1390.5
	MAE	22.272	22.042	22.895

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 75 Model: H.SM.B using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.92	0.93	0.92

	RMSE	32.43	30.829	32.502
	MSE	1051.7	950.46	1056.4
	MAE	20.054	20.115	20.653
Test	R^2	0.9	0.9	0.9
	RMSE	35.916	36.213	35.913
	MSE	1290	1311.3	1289.7
	MAE	21.444	21.504	22.098

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 76 Model: H.SM.C using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.92	0.93	0.93
	RMSE	33.721	31.917	31.676
	MSE	1137.1	1018.7	1003.4
	MAE	21.243	20.88	20.485
Test	R^2	0.89	0.89	0.89
	RMSE	37.618	37.441	37.455
	MSE	1415.1	1401.8	1402.9
	MAE	23.326	23.353	23.712

4.2.5.4 Vertical Strains using FWD Deflections

Like the case of modelling the Horizontal strains using FWD models, bagged trees algorithm also has higher performance in all three FWD models for modelling vertical strains. With them having minimal difference in terms of performance parameters, each of them is suitable for modeling.

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 77 Model: V.FWD.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.86	0.9	0.88
	RMSE	26.579	23.09	24.345
	MSE	706.45	533.16	592.67
	MAE	18.756	19.82	18.391
Test	R^2	0.86	0.89	0.89
	RMSE	23.694	21.191	21.071
	MSE	561.42	449.08	443.99

	MAE	16.835	15.983	15.776

MODEL V.FWD.B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 78 Model: V.FWD.B using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.86	0.9	0.9
	RMSE	26.522	22.861	22.31
	MSE	703.4	522.64	497.72
	MAE	18.625	17.691	17.408
Test	R^2	0.86	0.89	0.89
	RMSE	23.589	20.938	20.821
	MSE	556.44	438.41	433.53
	MAE	16.652	16.11	15.208

MODEL V.FWD.C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 79 Model: V.FWD.C using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.86	0.9	0.85
	RMSE	26.5	22.914	27.391
	MSE	702.26	525.07	750.24
	MAE	18.483	17.75	20.057
Test	R^2	0.86	0.89	0.85
	RMSE	23.733	20.758	24.481
	MSE	563.24	430.88	599.31
	MAE	16.876	15.881	17.966

4.2.5.5 Vertical Strains using DBPs

In case of modelling vertical strains using the DBPs, the bagged trees algorithm has better performance parameters (with R squared value being 0.89 and the MSE and MAE values being around 600 and 17 respectively). Thus, being ideal for modelling using all three DBP models.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 80 Model: V.DBP.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.84	0.87	0.87
	RMSE	27.305	24.763	24.952

	MSE	745.57	613.2	622.59
	MAE	19.176	18.214	18.019
Test	R^2	0.86	0.89	0.89
	RMSE	25.851	23.406	23.063
	MSE	668.3	547.85	531.89
	MAE	18.719	17.095	17.135

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 81 Model: V.DBP.B using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.84	0.87	0.88
	RMSE	27.467	24.493	24.024
	MSE	754.42	599.89	577.16
	MAE	19.277	18.148	17.745
Test	R^2	0.86	0.89	0.9
	RMSE	26.524	23.123	22.032
	MSE	703.53	534.66	485.41
	MAE	18.809	17.524	16.9778

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 82 Model: V.DBP.C using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.84	0.87	0.87
	RMSE	27.196	24.596	24.383
	MSE	739.6	604.98	594.54
	MAE	18.889	18.019	17.208
Test	R^2	0.85	0.89	0.89
	RMSE	27.052	23.182	22.891
	MSE	731.79	537.39	524.01
	MAE	19.467	17.723	17.533

4.2.5.6 Vertical Strains using Surface Moduli

In this case, the optimizable ensembles and bagged trees were the ones with having better performance criteria and thus being utilized in modelling.

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 83 Model: V.SM.A using Ensemble of Trees (R^2, MSE values for Training and Test sets)

		Boosted Trees	Bagged Trees	Optimizable Ensembles
Training	R^2	0.85	0.89	0.9
	RMSE	27.122	22.929	21.508
	MSE	735.61	525.74	462.59
	MAE	18.408	16.868	16.331
Test	R^2	0.86	0.91	0.92
	RMSE	25.541	20.551	19.307
	MSE	652.32	422.33	372.78
	MAE	19.666	15.909	15.217

4.2.6 Artificial Neural Networks

4.2.6.1 Horizontal Strains using FWD Deflections

In this case, the Narrow and Bilayer Neural Network has better values in terms of the performance parameters. Thus, they will be utilized in modelling horizontal strains using FWD models.

MODEL H.FWD.A: Independent Variables-0mm, 200mm, 300mm, 450mm, 600mm

Table-4. 84 Model: H.FWD.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.91	0.9	0.88	0.9	0.91	0.87
	RMSE	35.083	35.923	39.123	36.89	34.843	41.12
	MSE	1230.8	1290.5	1530.6	1360.9	1214.1	1690.9
	MAE	20.965	23.102	23.047	21.121	21.178	25.378
Test	R^2	0.94	0.91	0.91	0.94	0.9	0.95
	RMSE	29.085	35.353	34.728	29.44	36.535	26.091
	MSE	845.94	1249.8	1206	866.74	1334.8	680.72
	MAE	18.366	18.833	20.323	18.364	21.066	16.904

Here we can see that the Response plots of the model using Bilayered Neural Network is shown in the Figure 4.63. Also, its predicted vs actual plot of the validation and test data is presented in Figure 4.64. it shows that the performance of the model is similar throughout the range of strains, i.e., less difference or deviation in actual value of strain and its modelled response. Thus, it being best for modelling horizontal strains.

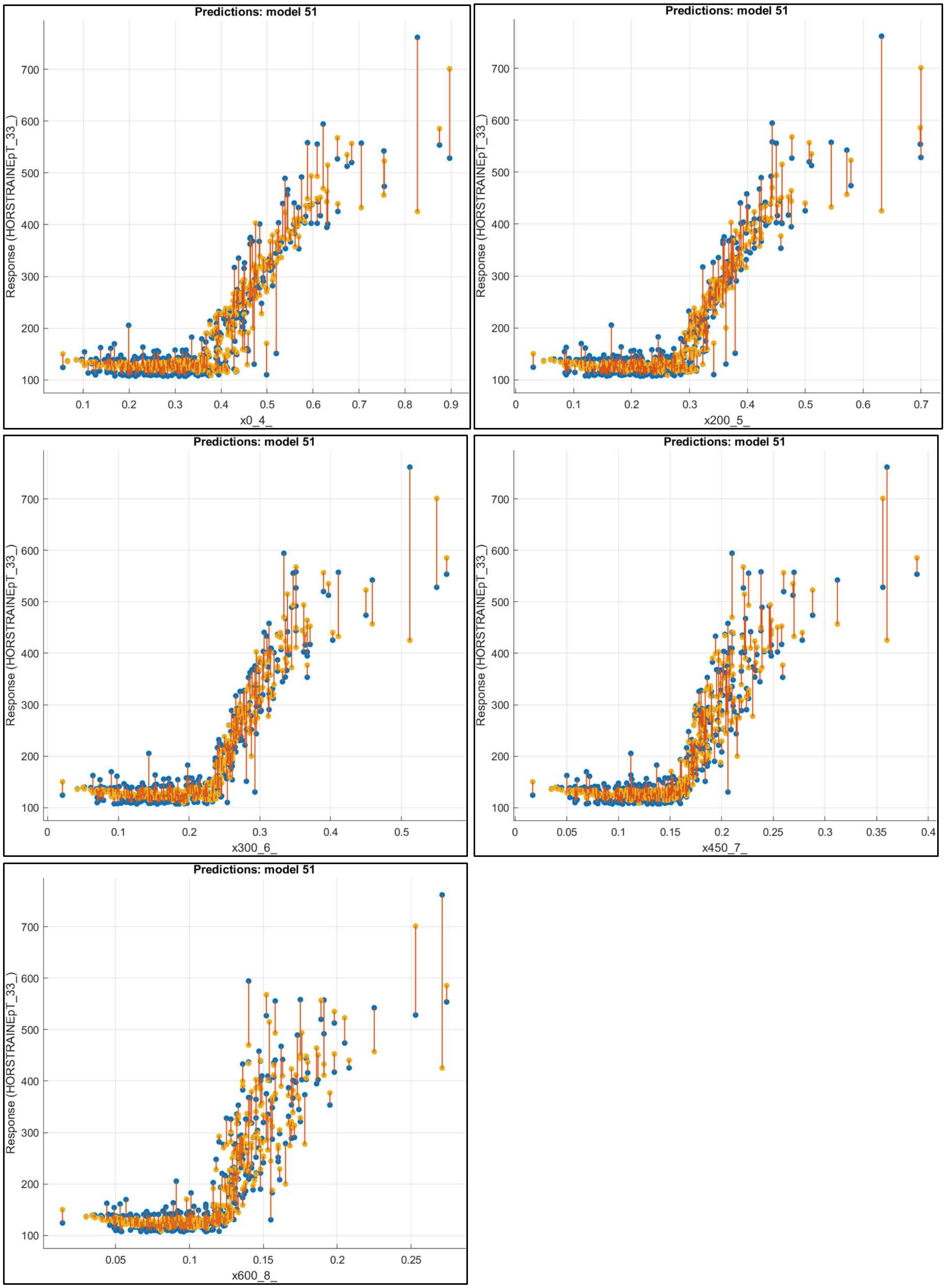

Figure 4. 63 Response Plots of Model: H.FWD.A using ANN (0mm, 200mm, 300mm, 450mm, 600mm)

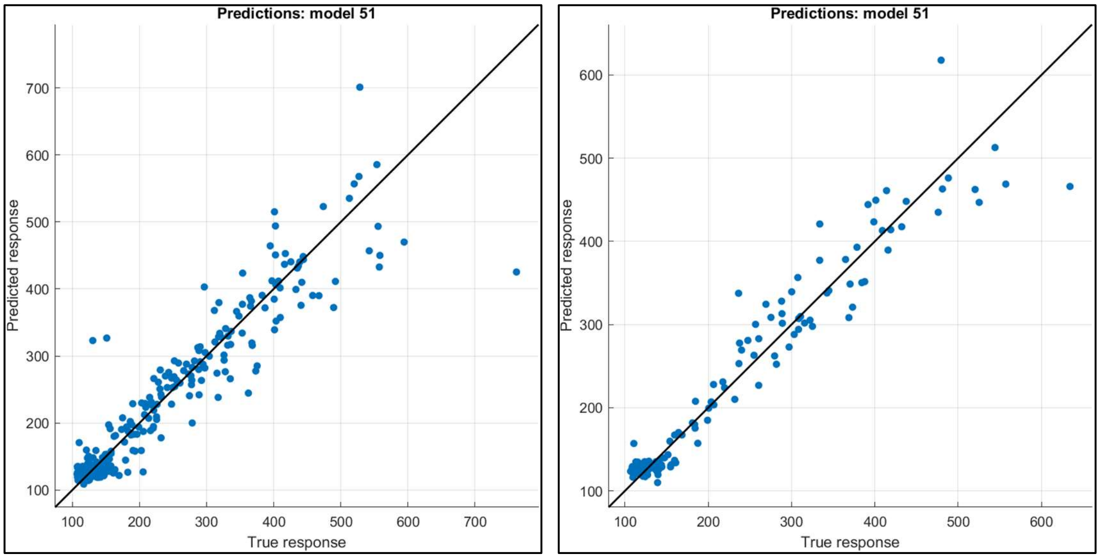

Figure 4. 64 Predicted vs Actual Plot of Validation Data (Left) and Test Data (Right): (Model H.FWD.A ANN)

4.2.6.2 Horizontal Strains using DBPs

In the case of modelling horizontal strains by DBPs, trilayerd, optimizable and narrow neural network have better R squared values, and lower MSE and MAE in both Training and Testing phase. Thus, being suitable for this purpose. With model H.FWD.A yielding better results.

MODEL H.DBP.A: Independent Variables- Max Def., RoC, BLI, MLI, AREA, AUPP

Table-4. 85 Model: H.DBP.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	**Medium Neural Network**	**Wide Neural Network**	**Bilayered Neural Network**	**Trilayered Neural Network**	**Optimizable Neural Network**
Training	R^2	0.89	0.9	0.87	0.9	0.91	0.91
	RMSE	38.22	36.938	40.721	36.517	34.136	33.52
	MSE	1406.7	1364.4	1658.2	1333.5	1165.2	1123.6
	MAE	22.097	23.711	26.502	22.272	21.408	21.457
Test	R^2	0.94	0.92	0.85	0.92	0.92	0.94
	RMSE	28.52	34.349	46.326	33.459	32.726	29.583
	MSE	813.42	1179.8	2146.1	1119.5	1071	875.13
	MAE	18.343	20.654	24.683	19.041	18.916	17.318

MODEL H.DBP.B: Independent Variables- RoC, BLI, AREA, AUPP

Table-4. 86 Model: H.DBP.B using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.89	0.86	0.83	0.89	0.9	0.89
	RMSE	37.556	42.393	46.806	38.331	35.51	37.841
	MSE	1410.5	1797.2	2190.8	1469.3	1261	1432
	MAE	22.47	24.581	25.578	22.798	21.909	24.306
Test	R^2	0.92	0.93	0.87	0.94	0.9	0.94
	RMSE	33.812	30.886	42.421	29.589	36.85	29.858
	MSE	1143.2	953.97	1799.5	875.48	1357.9	891.52
	MAE	13.236	20.038	23.686	18.921	19.403	20.918

4.2.6.3 Horizontal Strains using Surface Moduli

In this case, although following models have almost identical values of the performance parameter in training phase, Bilayered and trilayered Neural Networks are well suited for the task of modelling horizontal strains. Here H.SM.B yields better results.

MODEL H.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 87 Model: H.SM.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.93	0.93	0.92	0.93	0.94	0.93
	RMSE	31.461	31.304	32.395	30.38	28.907	29.778
	MSE	989.79	979.96	1049.5	922.96	835.63	886.74
	MAE	20.049	19.469	21.104	19.35	18.572	20.037
Test	R^2	0.91	0.83	0.89	0.9	0.9	0.89
	RMSE	35.117	47.8	38.708	37.29	37.298	38.191
	MSE	1233.2	2284.9	1498.3	1390.6	1391.1	1458.6
	MAE	21.48	23.716	23.279	22.408	22.057	25

MODEL H.SM.B: Independent Variables- SM0, SM300, SM600

Table-4. 88 Model: H.SM.B using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.92	0.92	0.93	0.93	0.76	0.93
	RMSE	32.508	32.673	31.276	31.129	57.51	31.293
	MSE	1056.8	1067.5	978.21	969.04	3307.4	979.23
	MAE	20.488	20.697	20.03	20.176	33.156	19.757
Test	R^2	0.9	0.92	0.89	0.91	0.9	0.89
	RMSE	36.221	33.634	38.45	34.098	37.11	38.417
	MSE	1312	1131.3	1478.4	1163.7	1377.1	1475.9
	MAE	21.614	20.478	22.352	22.322	21.991	24.824

MODEL H.SM.C: Independent Variables- SM0, SM300

Table-4. 89 Model: H.SM.C using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.92	0.92	0.91	0.93	0.92	0.92
	RMSE	33.663	32.488	34.305	30.95	32.125	32.55
	MSE	1133.2	1055.5	1176.8	957.88	1032	1059.5
	MAE	21.089	20.757	21.771	19.964	21.091	21.465
Test	R^2	0.91	0.91	0.89	0.9	0.89	0.89
	RMSE	34.965	34.349	37.676	36.715	37.822	38.645
	MSE	1222.5	1179.9	1419.5	1348	1430.5	1493.4
	MAE	22.859	22.669	24.43	23.38	24.201	25.019

4.2.6.4 Vertical Strains using FWD Deflections

Here as bilayered Neural Network has higher R squared values and lower MSE and Mae values, it is better suited for modelling vertical strains using FWD deflection models.

MODEL V.FWD.A: Independent Variables- 0mm, 200mm, 450mm, 1200mm

Table-4. 90 Model: V.FWD.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.87	0.9	0.87	0.9	0.89	0.9
	RMSE	25.339	23.149	25.489	22.407	23.571	22.734
	MSE	642.05	535.86	649.7	502.05	555.59	516.83
	MAE	18.968	18.081	19.102	17.42	19.64	17.764
Test	R^2	0.9	0.9	0.9	0.9	0.9	0.9
	RMSE	20.54	20.052	20.275	19.798	20.023	20.664
	MSE	421.9	402.09	411.06	391.96	400.93	426.99
	MAE	14.957	14.73	14.851	14.863	15.079	15.707

MODEL V.FWD.B: Independent Variables- 0mm, 200mm, 450mm, 600mm

Table-4. 91 Model: V.FWD.B using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.9	0.9	0.88	0.89	0.9	0.91
	RMSE	22.388	22.339	24.425	23.386	22.866	22.006
	MSE	501.23	499.02	596.59	546.92	522.88	484.27
	MAE	17.467	17.713	18.364	18.333	17.949	17.031
Test	R^2	0.89	0.91	0.9	0.9	0.91	0.9
	RMSE	21.17	19.301	20.483	19.899	19.643	20.24
	MSE	448.15	372.53	419.56	395.99	385.86	409.64
	MAE	14.878	14.27	15.39	15.187	14.618	15.176

MODEL V.FWD.C: Independent Variables- 0mm, 200mm, 300m, 450mm, 600mm

Table-4. 92 Model: V.FWD.C using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.9	0.9	0.9	0.9	0.9	0.88
	RMSE	23.17	22.989	22.74	22.536	22.482	24.416
	MSE	536.83	528.51	517.1	507.44	505.45	596.15

	MAE	18.028	17.833	17.717	19.651	17.8	18.529
Test	R^2	0.9	0.89	0.88	0.91	0.91	0.88
	RMSE	19.754	20.855	21.848	19.31	19.064	22.491
	MSE	390.21	434.92	477.31	372.9	363.43	505.83
	MAE	14.58	15.703	16.236	14.59	14.368	16.795

4.2.6.5 Vertical Strains using DBPs

Here all models have nearly identical values in terms of performance parameters. Thus, making all the methods ideal for strain determination with Trilayered Neural Network being slightly better.

MODEL V.DBP.A: Independent Variables- Max Def., LLI, AREA

Table-4. 93 Model: V.DBP.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	**Medium Neural Network**	**Wide Neural Network**	**Bilayered Neural Network**	**Trilayered Neural Network**	**Optimizable Neural Network**
Training	R^2	0.87	0.88	0.86	0.88	0.88	0.89
	RMSE	24.791	23.509	25.932	24.23	23.866	23.322
	MSE	614.57	552.68	672.45	587.11	569.6	543.93
	MAE	17.741	17.885	19.223	18.185	17.791	17.552
Test	R^2	0.9	0.9	0.89	0.91	0.9	0.9
	RMSE	21.83	22.145	23.326	21.277	22.337	21.734
	MSE	476.56	490.38	544.1	452.69	498.94	472.37
	MAE	17.195	17.767	18.216	16.79	17.516	17.196

MODEL V.DBP.B: Independent Variables- Max Def., MLI, LLI

Table-4. 94 Model: V.DBP.B using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	**Medium Neural Network**	**Wide Neural Network**	**Bilayered Neural Network**	**Trilayered Neural Network**	**Optimizable Neural Network**
Training	R^2	0.89	0.87	0.87	0.87	0.89	0.89
	RMSE	22.997	24.83	25.172	24.556	23.197	22.853
	MSE	528.84	616.53	633.65	603.02	538.09	522.25
	MAE	17.063	18.607	18.233	17.898	17.528	17.361
Test	R^2	0.9	0.89	0.88	0.9	0.9	0.9
	RMSE	21.841	22.997	24.025	21.701	22.166	22.488

	MSE	477.04	528.87	577.2	470.92	491.33	505.7
	MAE	17.182	17.657	18.247	16.525	16.988	17.664

MODEL V.DBP.C: Independent Variables- Max Def., RoC, MLI, LLI, AREA

Table-4. 95 Model: V.DBP.C using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.89	0.88	0.83	0.86	0.89	0.87
	RMSE	22.568	24.093	28.634	26.024	22.811	25.276
	MSE	509.31	580.48	819.92	677.26	520.33	638.87
	MAE	16.889	17.865	20.577	19.245	17.467	18.862
Test	R^2	0.87	0.89	0.85	0.88	0.89	0.86
	RMSE	24.824	22.673	26.805	23.806	23.149	26.456
	MSE	616.23	514.07	718.49	566.74	535.89	699.93
	MAE	18.628	17.693	19.988	18.321	17.961	20.769

4.2.6.6 Vertical Strains using Surface Moduli

In this case to as Bilayered Neural Network has slightly better values of performance parameters as compared to its counterparts, it should be considered for the analysis of Vertical strains using the Surface Moduli Models.

MODEL V.SM.A: Independent Variables- SM0, SM300, SM600, SM1200

Table-4. 96 Model: V.SM.A using Neural Network (R^2, MSE values for Training and Test sets)

		Narrow Neural Network	Medium Neural Network	Wide Neural Network	Bilayered Neural Network	Trilayered Neural Network	Optimizable Neural Network
Training	R^2	0.89	0.88	0.89	0.9	0.88	0.89
	RMSE	23.187	23.865	22.848	22.319	23.844	22.658
	MSE	537.63	569.52	522.05	498.13	568.53	513.37
	MAE	17.838	17.953	17.455	17.172	17.613	16.998
Test	R^2	0.87	0.9	0.88	0.92	0.91	0.89
	RMSE	25.289	21.631	23.681	20.117	20.647	22.549
	MSE	639.53	467.9	560.81	404.69	426.32	508.46
	MAE	19.045	16.405	18.067	16.55	16.459	18.118

CHAPTER-5 CONCLUSION

5.1 CONCLUSIONS MADE IN THIS WORK:

Following conclusion are made in this work

5.1.1 Modelling with Single Parameter:

5.1.1.1 Regression analysis

- Best model for modelling with FWD Deflections: Cubic, Quadratic
- Best model for modelling with DBPs: Cubic, Quadratic
- Best model for modelling with Surface Moduli: Cubic, Inverse
- Poor performance in terms of R squared and MSE values as compared to ANN

5.1.1.2 Artificial Neural Network:

- Outperforms Regression models in training, validation as well as in testing stages.
- Significantly lower MSE values and higher R squared values as compared to regression models

Table-5. 1 Best Individual Parameters for Modelling strains

Modelled Parameter (Response)	**Modelling Parameters (Predictors)**
Horizontal Tensile Strains (Bottom of Bit. Layer)	FWD Deflections: 0mm, 200mm, 300mm, 450mm
	Deflection Bowl Parameters: Max Def, RoC, BLI, AUPP
	Surface Moduli: SM0, SM300
Vertical Compressive Strains (Top of Subgrade}	FWD Deflections: 0mm, 200mm, 300mm, 450mm, 600mm
	Deflection Bowl Parameters: Max Def, MLI, LLI, AUPP
	Surface Moduli: SM0, SM300, SM600

5.1.2 Modelling with Multiple Parameters

- The best approach is to consider every parameter that affects or define the structural condition of the layer in which the strain is being modelled
- Individual parameter having low or weak correlation might not make much of a difference individually but when presented in a suitable and carefully thought-out dataset might assist in modelling and reduce the variance or distribution

- Linear regression is worst for modelling. It may give unreliable models with poor performance.
- Gaussian Process Regression (GPR) is the best approach for modelling. It gives highest R squared and low MSE and MAE values.
- Followed by performance of Ensemble of Trees and Neural Networks.

Table-5. 2 Best Performing Models made of multiple parameters

Modelled Parameter (Response)	**Modelling Parameters (Predictors)**	**Best Performing Model with parameters (Model ID, Parameters)**	
Horizontal Tensile Strains (Bottom of Bit. Layer)	FWD Deflections	H.FWD.A	0mm, 200mm, 300mm, 450mm, 600mm
	DBPs	H.DBP.A	Max Def, RoC, BLI, MLI, LLI, AREA, AUPP
	Surface Moduli	H.SM.B	SM0, SM300, SM600
Vertical Compressive Strains (Top of Subgrade}	FWD Deflections	V.FWD.A	0mm, 200mm, 450mm, 1200mm
	DBPs	V.DBP.C	RoC, MLI, LLI, AREA
	Surface Moduli	V.SM.A	SM0, SM300, SM600, SM1200

5.1.3 Performance of Parameters with the Strains

5.1.3.1 Horizontal tensile strains at bottom of the bituminous layer can be modelled from:

- Deflections at 0mm, 200mm and 300mm correlates greatest.
- Maximum Deflection (D. Max): Showed very good correlation
- Radius of Curvature (RoC): Shows good correlation (R^2 up to 0.81)
- Base Layer Index (BLI) Shows decent correlation (R^2 up to 0.82)
- Middle Layer Index (MLI): Correlates well with the horizontal strains (R^2 about 0.9)
- Lower Layer Index (LLI): Shows good R^2 values (About 0.86)
- AUPP (Area Under Pavement Profile): Showed good correlation (R^2 value of 0.82)
- Area Parameter (AREA): Weak correlation with all layers of flexible pavement.

5.1.3.1 Vertical compressive strains at bottom of subgrade can be modelled from:

- Response at the deflections 0mm through 600mm in FWD testing show very good correlation
- Response at 900mm and 1200mm: Good but requires further evaluation to be used for modelling.

- Maximum Deflection (D_0): Showed very good correlation
- Radius of Curvature (RoC): Does not show a good correlation with the vertical strain
- Base Layer Index (BLI): Shows poor correlation (R^2 about 0.7)
- Middle Layer Index (MLI): Correlates soundly (R^2 about 0.8)
- Lower Layer Index (LLI): Shows good R^2 values (About 0.84).
- AUPP (Area Under Pavement Profile): Showed good correlation with the Horizontal strains as compared to Vertical strains (R^2 value of 0.82 against 0.75)
- Area Parameter (AREA): Have weak correlation with all layers of flexible pavement.

5.2 SCOPE OF APPLICATION

- Horizontal tensile strains at bottom of bituminous layer and Vertical compressive strains at top of subgrade can be determined using machine learning techniques with relative accuracy.
- The Methodology using Machine Learning Techniques can be applied successfully in Pavement Management Systems (PMS).
- The methodology presented here has a prominent application in PMS at network level as well as Project level analysis
- Although the MSE and MAE values are not that small to be entirely relied upon, but they are still insignificant enough to permit its application even at the project level of PMS.
- Above methodology can serve as the preliminary approach at project or network level of PMS for:
    - Analysis of pavement life (Based on strains)
    - Distress analysis in pavements
    - Guide the future detailed investigation at the project level
- Advantage of methodology by eliminating use of Backcalculation softwares to calculate elasticity and then calculating strains:
    - A Quick and simpler approach
    - Less resource intensive as compared to conventional methodology.

5.3 FUTURE SCOPE OF WORK

- Further research is needed by including factors like

- Structural Parameters: Pavement total thickness, Asphalt layer thickness, Base layer thickness, etc.
- Traffic Parameters: Average of annual ESALs, Average daily number of trucks annual
- Climatic Parameters: Average Temperature, Standardized Temperature range, Av. No. of days with av. Annual Temp. >32°C, Av. No. of days with annual av. Temp. <0°C
- Performance Parameters: Initial IRI, IRI at specified time, Av. Pavement surface temp, Years since first profilometer survey

• Development of possible methodology for strain modeling in case of flexible and rigid pavements with different structural composition like:
  - unbounded and bounded layers
  - cement treated, lime treated, chemically treated layers.

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