

“Performance Analysis of Super Gaussian Optical Solitons in Cubic-Quintic Medium with Perturbation”

Dissertation submitted in the partial fulfilment of
requirement for the award of degree of

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In
Electronics & Communication Engineering**

Submitted By

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ENGINEERING**

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DECLARATION

I, Shivani, hereby declare that the work, which is being presented in this dissertation entitled "**Performance Analysis of Super Gaussian Optical Solitons in Cubic-Quintic Medium with Perturbation**" by me in partial fulfilment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering from Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Hardeep Singh**.

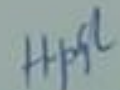
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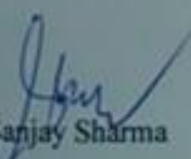

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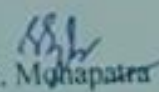
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ABSTRACT

The Significance of fibre optic communication is increasing at very high rate in present world due to its huge capacity and capability to meet high speed data link requirements. Like any communication medium, there are also limiting factors in optical communication medium which disturbs the integrity of information. In an optical communication medium, we have dispersion and nonlinear effects. Dispersion is pulse broadening occurs due to different propagation delay faced by different wavelength signals and nonlinear effects are due to the intensity dependent refractive index of optical fiber. These are two major impairments that affect the quality of optical communication link. The various dispersion compensation techniques are used to control loss of data due to dispersion. Optical soliton is a special pulse which provides a revolutionary solution to these problems by providing an exact balance between nonlinear effects and dispersion broadening. Optical soliton travels through optical fiber with zero dispersion effect and maintaining its constant shape. The commercial optical soliton communication systems are expected to be available in few years. A lot of research has been done in conservative optical systems that a system in which has zero loss in medium. The practical optical systems are of dissipative nature as there is a loss in optical medium and are focus of research in recent times.

Our work is focussed on Stable Super Gaussian Optical Soliton pulse propagation through optical fiber cable since the modal fields of LASER source is of Super Gaussian profile. We investigated Super Gaussian pulses of order $m = 1, 2, 3$. We explored the possibility of having stable soliton pulse in a nonlinear optical medium with perturbation that is a dissipative system. We investigated our system with expanded Nonlinear Schrödinger Equations with introduction of perturbation terms. We analysed the effect of fifth order nonlinearity in addition to Kerr nonlinearity on stability of soliton pulses. We find the parameters for which we get stable super Gaussian soliton propagation in cubic-quintic medium in case of conservative system. We observed after introducing perturbation in medium we don't get stable super Gaussian soliton pulse propagation at same parameters as in case of conservative system. Through our investigation, we found the parameter values of Kerr nonlinearity, Quintic nonlinearity and Group velocity dispersion at which we get stable Super Gaussian solitons of order $m=1,2,3$ in optical medium with perturbation.

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Introduction to fiber optics communication

1.1 Introduction

Fiber optics communication has played major role in telecommunication industry. Telecommunication networks based on optical fiber technology have become major information transmission system due to high capacity optical links. Optical fiber has been used to transmit telephone signals, for Internet communication, and cable television signals. The lightwave technology, together with microelectronics, is believed to be a major factor in the advent of the “information age.” Today, fiber optics is mainstream technology and is growing rapidly. In long haul optical communication systems where bit rate is high, the dispersion phenomenon is a major problem. An easy solution to this problem would be use of optical solitons in communication systems. Optical solitons has drawn considerable attention because of its fundamental nature as well as potential applications for optical fiber communications.

The theoretical concept of propagation of optical solitons through optical fibers was first suggested in 1973 by Hasegawa and Tappert [1], while it was experimentally demonstrated in 1980 by Mollenauer et al. [2]. Since then research on optical solitons had seen rapid development due to its potential applications in long distance optical communication and optical switching [3]. Their stability against perturbation and preservation of their propagation characteristics after collision make them attractive for long- and short-distance information transmission [4-5]. Today, optical solitons are regarded as an important alternative for the next generation of ultrahigh-speed optical telecommunication systems. However, soliton systems are still waiting for the full field deployment.

A fascinating manifestation of the fiber nonlinearity occurs through optical solitons. Solitons are special wave packets that are formed due to delicate balance between optical fiber nonlinearity and dispersion. Solitons have ability to propagate undistorted over long distances.

1.2 Non-Linear and Linear Effects

When material properties are modified by signal itself, then it is called non-linear effects. And the branch of optics that deals with behaviour of light in non-linear medium is named as non-linear fiber optics. The response of any dielectric to light becomes nonlinear for intense electromagnetic fields and same is true for optical fibers. For material like silica, induced polarisation in the material is given by

$$P = \epsilon_0(\chi_1 \cdot E + \chi_2 \cdot E \cdot E + \chi_3 \cdot E \cdot E \cdot E + \dots \dots) \quad (1.2.1)$$

Where, ϵ_0 is free space permittivity, χ_1 is first order susceptibility, χ_2 is second order susceptibility, E is imposed electric field. Generally, we consider first order susceptibility but for larger light intensities, we have to consider higher order term into polarization of the material.

The second-order susceptibility is responsible for second-harmonic generation nonlinear effects [4]. However, it is nonzero only for media that lack inversion symmetry at the molecular level. The second term is negligibly small for SiO_2 (for glass) as this is a symmetric molecule. Therefore, second term is neglected. Equation (1.2.1) now reduces to

$$P = \epsilon_0(\chi_1 \cdot E + \chi_3 \cdot E \cdot E \cdot E) \quad (1.2.2)$$

And refractive index of material is given by

$$n = \sqrt{(1 + \chi)} \quad (1.2.3)$$

Where $\chi = \chi_{LIN} + \chi_{NONLIN}$

$$n = \sqrt{(1 + \chi_{LIN} + \chi_{NONLIN})} = n_0 \left(1 + \frac{1}{2n_0^2} \cdot \chi_{NONLIN}\right) \quad (1.2.4)$$

Therefore refractive index of fiber is given as

$$n = n_0 + n_2 |E|^2 \quad (1.2.5)$$

Where n_0 is $\sqrt{1 + \chi_{LIN}}$

And n_2 is the second-order nonlinearity coefficient. Its value vary for different materials but for glass, its value is of the order of $10^{-20} \frac{m^2}{V^2}$. Thus change in refractive index is

proportional to intensity of light propagating through the medium. This is also known as optical Kerr-effect. This nonlinear effect, along with phenomena of dispersion, is responsible for formation of optical solitons.

1.2.1 Stimulated Inelastic Scattering

Two examples of inelastic scattering are *Raman scattering* and *Brillouin scattering*. Both Raman and Brillouin can be understood as scattering of a photon to a lower energy photon such that the energy difference appears in the form of a phonon. The nonlinear phenomena of stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) become important at high power levels. SRS and SBS has been known since 1970s. The main difference between the two is that in Raman scattering optical phonons participate, whereas acoustic phonons are responsible for Brillouin scattering. Both scattering processes result in a loss of power at the incident frequency. Scattered light's intensity grows exponentially once incident power crosses threshold value.

a. Stimulated Raman Scattering (SRS)

Spontaneous Raman scattering occurs in optical fibers when a pump wave is scattered by the silica molecules. Some pump photons give up their energy to create other photons of reduced energy at a lower frequency; the remaining energy is absorbed by silica molecules, which end up in an excited vibrational state. Scattered light's intensity grows exponentially for the case of Raman scattering once incident power crosses threshold value. SRS can occur in both the forward and backward directions in optical fibers. The threshold power P_{th} is defined as the incident power at which half of the pump power is transferred to the Stokes field at the output end of a fiber of length L . It is estimated from [4]

$$g_R L_{eff} P_{th} / A_{eff} \approx 16 \quad (1.2.1.1)$$

where g_R is the peak value of the Raman gain. As before, L_{eff} can be approximated by $1/\alpha$. If we replace A_{eff} by πw^2 , where w is the spot size, P_{th} for SRS is given by

$$P_{th} \approx 16\alpha\pi w^2 / g_R$$

SRS is especially useful because of its extremely large bandwidth.

b. Stimulated Brillouin Scattering (SBS)

Brillouin scattering is an effect caused by the $\chi^{(3)}$ nonlinearity of a medium, specifically by that part of the nonlinearity which is related to acoustic phonons. An incident photon can be converted into a scattered photon and a phonon. Photon is of slightly lower energy, usually propagating in the backward direction. The coupling of optical fields and acoustic waves is known as electrostriction. The effect can occur spontaneously even at low optical powers, then reflecting the thermally generated phonon field. For higher optical powers, there can be a stimulated effect, where the optical fields substantially contribute to the phonon population. Above a certain threshold power of a light beam in a medium, stimulated Brillouin scattering can reflect most of the power of an incident beam. This process involves a strong nonlinear optical gain for the back-reflected wave: an originally weak counter propagating wave at the suitable optical frequency can be strongly amplified. Here, the two counter-propagating waves generate a traveling refractive index grating; the higher the reflected power, the stronger the index grating and the higher the effective reflectivity.

The SBS gain g_B is frequency dependent because of a finite damping time T_B of acoustic waves (the lifetime of acoustic phonons). For silica fibers, the Brillouin frequency shift is of the order of 10–20 GHz, and the Brillouin gain has an intrinsic bandwidth of typically 50–100 MHz, which is determined by the strong acoustic absorption (short phonon lifetime).

The threshold power $P_{th} = I_p A_{eff}$, where A_{eff} is the effective core area, satisfies the condition [4]

$$g_R L_{eff} P_{th} / A_{eff} \approx 21 \quad (1.2.1.2)$$

where L_{eff} is the effective interaction length defined as

$$L_{eff} = [1 - \exp(-\alpha L)] / \alpha$$

and α represents fiber losses.

1.2.2 Self-Phase Modulation

Self-phase modulation is nonlinear optical effect which arises due to optical Kerr-effect, which was discovered by J. Kerr in 1875. Different intensity portions of light signal, when travelling in a medium, experience different refractive index. The leading edge will

experience a positive refractive index gradient and trailing edge a negative refractive index gradient.

The phase (φ) introduced by a field E over a fiber length L is given as

$$\varphi = \frac{2\pi}{\lambda} nL$$

where λ is wavelength of optical pulse propagating, n is fiber refractive index. This temporally varying index change results in a temporally varying phase change. Since this nonlinear phase change is induced by the signal itself, it is called Self-phase modulation.

Change in frequency spectrum is given as variation in phase with time

$$\omega = \frac{d\varphi}{dt} \quad (1.2.2.1)$$

Now let's consider Gaussian pulse, which modulates an optical carrier frequency ω and the new instantaneous frequency becomes,

$$\omega' = \omega_0 + \frac{d\varphi}{dt} \quad (1.2.2.2)$$

Since $\varphi = -\frac{2\pi}{\lambda} (n_l + n_{nl}I)L$, ω becomes

$$\omega' = \omega_0 - \frac{2\pi}{\lambda} Ln_{nl} \frac{dI}{dt} \quad (1.2.2.3)$$

So when intensity of light increases with time, ω' will be

$$\omega' = \omega_0 - \omega(t) \quad (1.2.2.4)$$

And for falling edge, when intensity decreases with time, ω' will be

$$\omega' = \omega_0 + \omega(t) \quad (1.2.2.5)$$

This shows that the pulse has got chirp, i.e., a temporally varying instantaneous frequency. The time-dependent phase change caused by SPM is associated with a modification of the optical spectrum. There is broadening of spectrum without any change in the pulse in time domain.

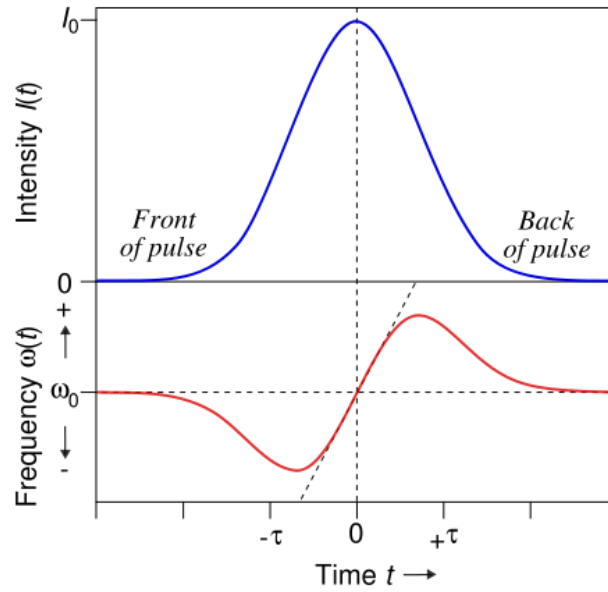


Fig. 1.1 Effect of SPM on frequency [6]

1.2.3 Cross-Phase Modulation

XPM is a similar effect to SPM, but it involves two optical pulses instead of one. In XPM, the intensity modulation of one of the pulses results in a phase modulation of the other. The intensity dependence of the refractive index can also lead to another nonlinear phenomenon known as *cross-phase modulation* (XPM). It occurs in Wavelength division multiplexing technique when two or more optical channels are transmitted simultaneously inside an optical fiber. In such systems, the nonlinear phase shift for a signal depends not only on the power of that signal but also on the power of other signals in other channels.

Phase shift in j th channel is given by:

$$\varphi = \gamma L_{eff}(P_j + 2 \sum_{m \neq j} P_m) \quad (1.2.3.1)$$

1.2.4 Dispersion in Single Mode Fiber

Intermodal dispersion is absent simply in single mode fibers. However, pulse broadening does not disappear altogether. The group velocity associated with the fundamental mode travelling in SMF is frequency dependent because of chromatic dispersion. As a result, different spectral components of the pulse travel at slightly different group velocities, a

phenomenon referred to as *group-velocity dispersion* (GVD), *intramodal dispersion*, or simply *fiber dispersion*. Intramodal dispersion are of two types, material dispersion and waveguide dispersion.

a. Material Dispersion

Material dispersion occurs because the refractive index of silica, the material used for fiber fabrication, is dependent on optical frequency.

The dispersion parameter $D_M = 122(1 - \lambda_{ZD}/\lambda)$

Where λ_{ZD} is zero dispersion wavelength

The dispersion parameter D_M is negative below λ_{ZD} and becomes positive above that.

b. Waveguide Dispersion

Waveguide dispersion is chromatic dispersion which arises from waveguide effects: the dispersive phase shifts for a wave in a waveguide differ from those which the wave would experience in a homogeneous medium. The total dispersion is the combination of material dispersion and waveguide dispersion.

1.2.5 Group Velocity Dispersion

The group velocity is the velocity at which energy or information is conveyed along the wave. Group velocity dispersion is the phenomena that group velocity of light is dependent on optical frequency. Dispersion-induced pulse broadening can be understood from the fact that different frequency components of a pulse travel at slightly different speeds along the fiber because of GVD. More specifically, in the normal-dispersion regime, blue components travel slower than red components while the opposite occurs in the anomalous-dispersion regime. The pulse can maintain its width only if all spectral components arrive together. Any time delay in the arrival of different spectral components leads to pulse broadening. So, it means that different frequencies are going to travel at different velocities, due to which there will be temporal broadening of the pulse.

GVD can be written as,

$$GVD = \frac{\partial}{\partial \omega} \cdot \frac{1}{v_g}$$

Where v_g is group velocity of pulse. For optical fibers, the group velocity dispersion is usually defined as a derivative with respect to wavelength. GVD parameter, in this case, is given as,

$$D = -\frac{2\pi c}{\lambda^2} \cdot GVD$$

D is specified in ps/(nm.km). For 1550nm pulse, its value is typically 20 ps/(nm.km). Depending upon sign of D and GVD, there are two regime of dispersion. If pulse propagates through *anomalously dispersive medium*, high frequency components travel faster than the lower ones, and the pulse becomes negatively chirped, decreasing in frequency with time. And if pulse propagates through *normal dispersive medium*, low frequency components travel faster than the higher ones, and the pulse becomes positively chirped. But overall there will be temporal spreading of the pulse. Broadening of pulses due to dispersion is undesirable as it interferes with the detection process and limits bit rate as it leads to errors if the pulse spreads outside its allocated bit slot.

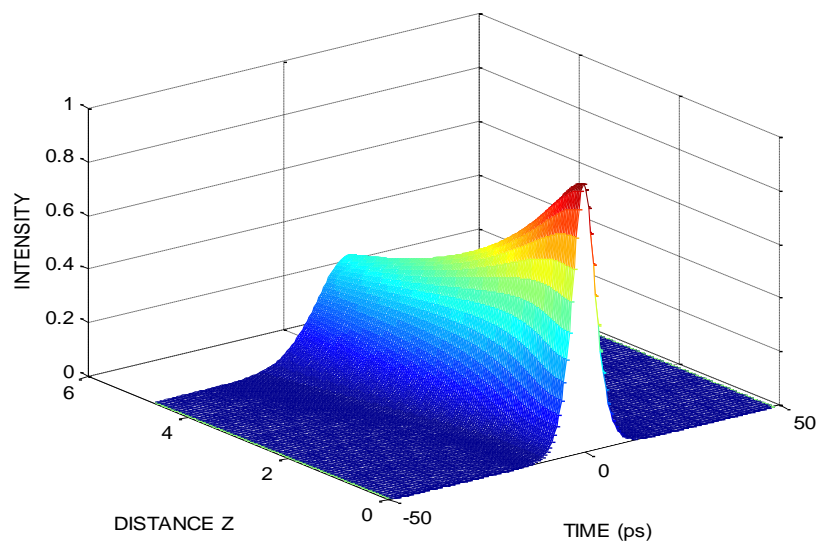


Fig. 1.2 Broadening of sech pulse due to effect of dispersion

1.3 Optical Solitons

1.3.1 A Brief History

Over one hundred and fifty years ago, while conducting experiments to determine the most efficient design for canal boats, a young Scottish engineer named John Scott Russell

(1808-1882) made a remarkable scientific discovery. During his experiments he discovered a phenomenon that he described as the wave of translation. He described his discovery in an article called Report on waves [7] which describes the phenomena of solitons. The phenomenon was based on a water wave that formed in a narrow channel and displayed some strange properties. For one, the wave was stable, it neither dies out or steepened like normal waves. Furthermore, the wave would not merge with other waves, a small wave traveling faster would rather overtake a large slower one. In 1895, Diederik Korteweg and Gustav de Vries derived a nonlinear partial differential equation known as the Korteweg-de Vries equation that describes Russel's solitary waves. Then in 1965 Norman Zabusky and Martin Kruskal published their numerical solutions of the KdV equation. In 1973 Akira Hasegawa was the first to suggest that solitons could exist in optical fibers, due to a balance between self-phase modulation and anomalous dispersion. In 1988 Mollenauer and his team transmitted soliton pulses over 4000 kilometers using a phenomenon called the Raman effect, to provide optical gain in the fiber.

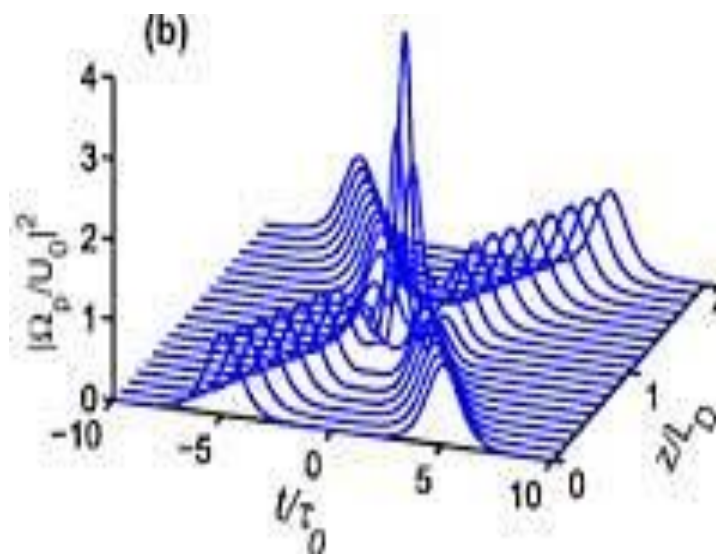


Fig. 1.3 Collision between two optical solitons [8]

1.3.2 Optical Solitons

In optics, the term soliton means any optical field that does not change during propagation because of a balance between nonlinear and dispersion effects in the fiber. Since solitons do not suffer distortion from nonlinearity and dispersion, which are inherent in fibers, they can travel through fibers for long distances. As written above that

dispersion can occur in *normal or anomalous regime*. The combined effect of GVD (in normal regime) and SPM is that there will be rapidly broadening of pulse. For pulse travelling in anomalous dispersion region, there will be negative chirping which balances positive chirp introduced by SPM. So soliton solutions exist only in the region of anomalous dispersion.

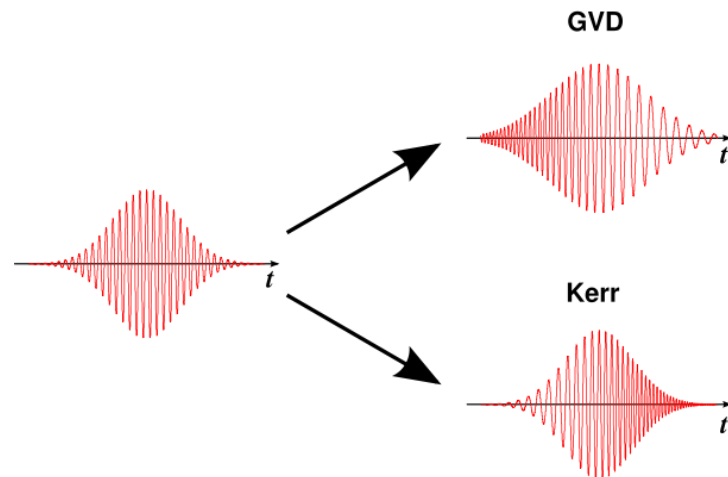


Fig. 1.4 Dispersive (top) and nonlinear effects (bottom) on a Gaussian pulse [10]

Solitons are highly stable and even after interacting with each other, they retain their properties. If initial pulse shape or amplitude is not appropriate for soliton solutions, the pulse adjusts its shape and width and evolves into a soliton with part of pulse energy dispersed away.

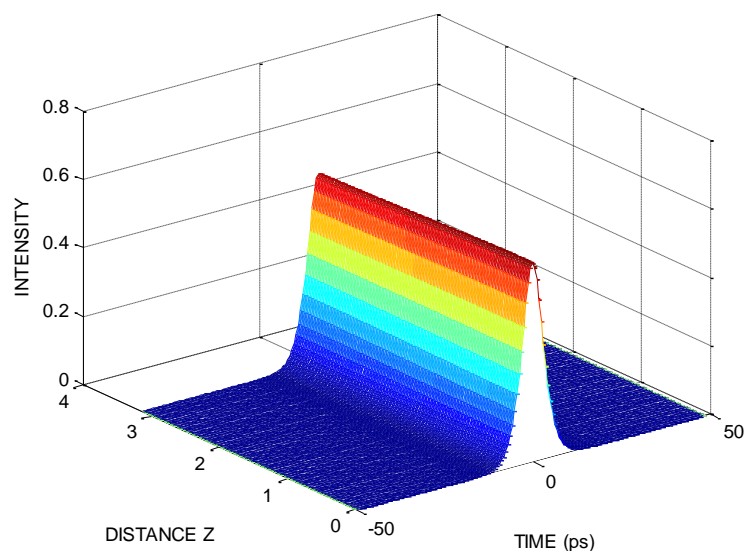


Fig. 1.5 Evolution of optical soliton with normalized propagation distance

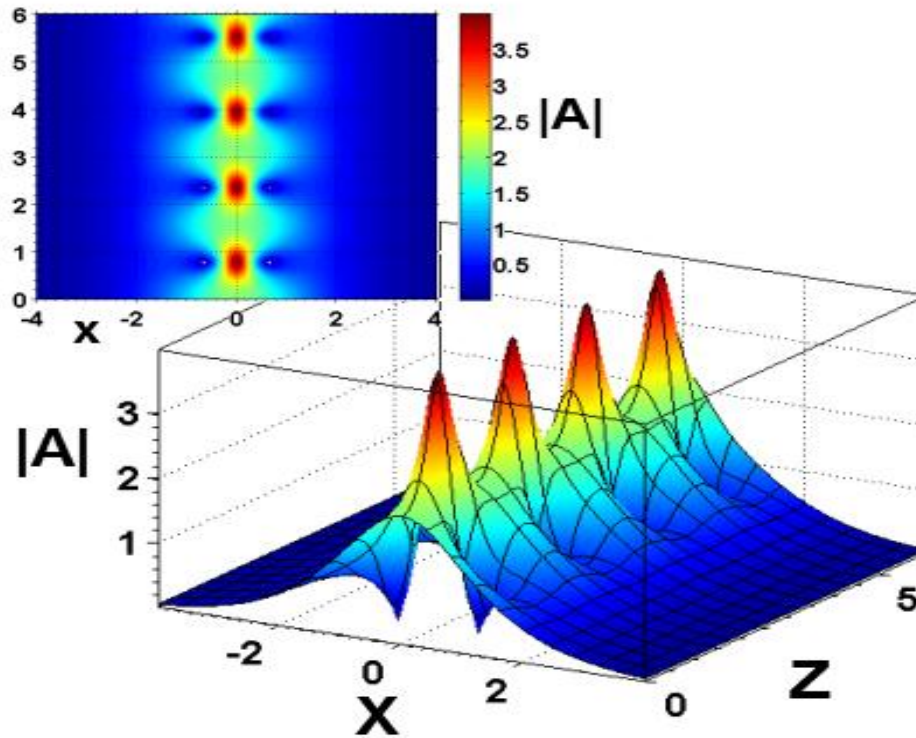
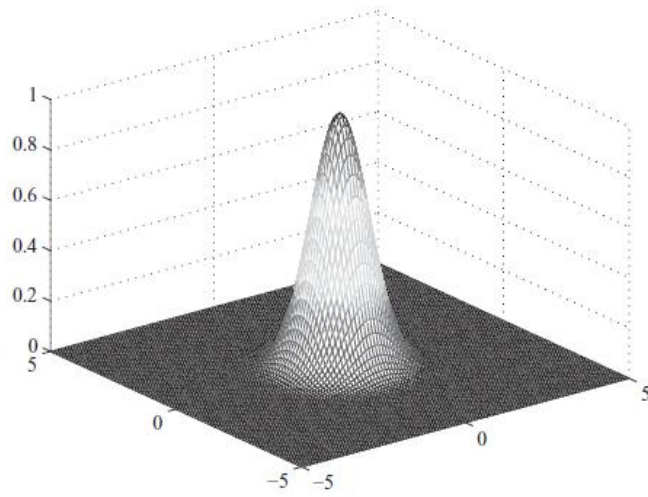
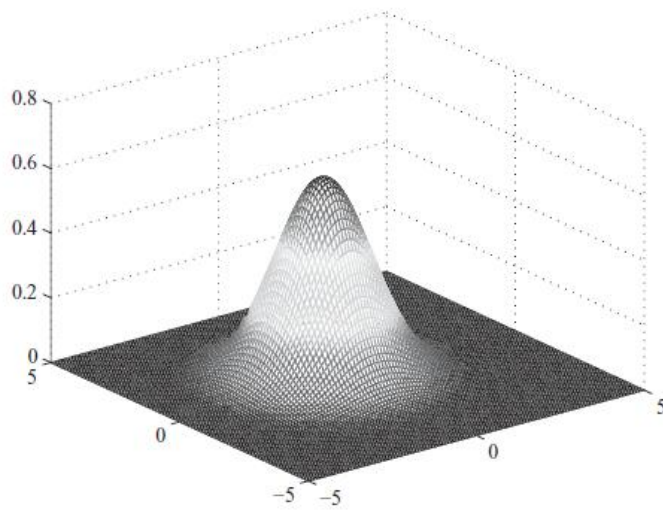


Fig. 1.6 Evolution of breathing soliton with distance of propagation [11]

There are two main kinds of optical solitons: spatial and temporal solitons. Temporal solitons were discovered first and are often simply referred to as solitons in optics. We talk about temporal solitons when the electromagnetic field is already spatially confined (e.g. in optical fibers) and the pulse shape (in time) will not change because the nonlinear effects balance dispersion. Spatial solitons are those when the electromagnetic field is not spatially confined but the pulse shape in space does not change with propagation because of nonlinear effects (self-focusing) balancing out diffraction. Spatial solitons result from the balance between linear diffraction (like there is dispersion in temporal solitons) and non-linear self-focusing. Self-focusing is possible in media with nonlinear effects (such as the Kerr effect). Because of the dependence of the refractive index on the intensity (in other words on the pulse shape), the refractive index thus depends on the position in space. Spatial solitons are formed when effects due to diffraction cancels the effects due to self focusing.



(a)



(b)

Fig. 1.7 Gaussian spatial beam (a) Initial beam (b) Beam after propagation [12]

1.4. Motivation and Goal of This Work

Use of Optical fiber technology in telecommunication and networking is getting more and more prevalent in upcoming days. High data rate capabilities, noise rejection and electrical isolation are just a few of the important characteristics that make fiber optic technology ideal for use in industrial and commercial systems also. In long haul optical communication systems, at high bit rate, the dispersion phenomenon is a major problem that limits transmission bit rate. Dispersion in fibers leads to spreading of pulse in

temporal domain after some distance. But by balancing the effect of GVD with SPM, we can have stable pulse propagation, which is called soliton propagation. Solitons are pulses of light with a certain shape and energy that can propagate unchanged over large distances.

In context of soliton propagation, most of the earlier works are confined to conservative systems having nonlinearity upto third order. In a conservative system, when light is transmitted in the form of solitons, it is possible to achieve transmission at much higher bit rates over much longer distances. But, real fibers are, of course, far from perfect, so the effects of quintic nonlinearity and perturbation in optical medium is of real importance for practical systems.

This work is centred on finding optical soliton solutions for medium with perturbation considering higher order nonlinear effects. Goal of this thesis is to find various parameters in which stable optical solitons can be achieved in real situations in the presence of perturbation.

1.5 Outline of Dissertation

The objectives of our work are given below-

1. The performance analysis of Optical Super Gaussian solitons with order $m = 1, 2, 3$ in cubic – quintic medium (dissipative system).
2. This work focuses on performance analysis of Optical Super Gaussian solitons with order $m = 1, 2, 3$ in cubic – quintic dissipative medium with perturbation.
3. Through simulation analysis, we are finding various nonlinear and linear parameters which result in stable optical soliton propagation.

This work is divided into five chapters. Chapter 2 includes theoretical analysis of Nonlinear Schrödinger equations and various nonlinearities in optical medium.

Chapter 3 gives literature survey related to this work.

Chapter 4 focuses on simulation analysis of our work which results in nonlinear and linear parameters in optical medium that results in stable optical solitons.

Chapter 5 describes conclusion and future scope of our work.

Optical Solitons in Nonlinear Media

2.1 Introduction of Nonlinear Schrödinger Equation

Almost all the complex nonlinear partial differential equations governing the nonlinear systems are in the family of Nonlinear Schrödinger equation. Nonlinear Schrödinger equation is one of the most important nonlinear models and among the most prominent equations in nonlinear physics. In case of propagation of intense optical pulse in a nonlinear medium the dynamics of the nonlinear pulse propagation is also governed by the NLSE. Comprehensive study has been done on NLSE to study influence of combined effects of GVD and SPM.

The nonlinear Schrödinger equation (NLSE) is an appropriate equation for describing the propagation of light in optical fibers:

$$i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0 \quad (2.1.1)$$

where, a is complex envelope

$$\beta_2 \text{ is GVD factor given by } : - \frac{D\lambda^2}{2\pi c}$$

$$\gamma \text{ is non-linear factor given by } : \frac{2\pi n_2}{\lambda A_{eff}}$$

D is dispersion in ps/nm km, λ is wavelength in nm, c is speed of light in vacuum, A_{eff} is effective area of core in mm^2 , n_2 can be found using equation:

$$n = n_0 + n_2 I \quad (2.1.2)$$

Here, time t is to be found using $\tau = \frac{t}{T_0}$, where T_0 is connected with FWHM of fundamental soliton solution and distance of propagation is to be found using $\zeta = \frac{z}{L_D}$, where L_D is characteristic length for effects of dispersion. Similarly, characteristic length for nonlinear effects is L_{NL} . These are length of fiber after which dispersion or nonlinear

effects are prominent. Now if we take N as ratio of ratio of dispersion length ($T_0^2/|\beta_2|$) and non-linearity length ($1/\gamma P_0$), then if,

1. $N \gg 1$, nonlinear effects will be more prominent than dispersive effects and pulse will spectrally broaden due to SPM effects.
2. $N \ll 1$, dispersive effects are dominant and pulse will eventually dies due to broadening.
3. For $N \approx 1$ both SPM and GVD cooperate in such a way that the SPM-induced effects is just right to cancel the GVD-induced broadening of the pulse. The optical pulse would then propagate undistorted in the form of a soliton.

Using above terminology, NLSE can also be written as,

$$\frac{\partial^2 a}{\partial \tau^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0 \quad (2.1.3)$$

By integrating the NLSE, solution of fundamental solitons ($N=1$) can be written as,

$$u(z, t) = \text{sech}(t) \exp\left(\frac{iz}{2}\right) \quad (2.1.4)$$

where $\text{sech}(t)$ is hyperbolic secant function.

2.1.1 Derivation of NLSE

Like all electromagnetic phenomena, the propagation of optical fields in fibers is governed by Maxwell's equations, which can be written as,

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad (2.1.1.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.1.1.2)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (2.1.1.3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.1.1.4)$$

where \mathbf{E} and \mathbf{H} are electric and magnetic field vectors, respectively, and \mathbf{D} and \mathbf{B} are corresponding electric and magnetic flux densities. The current density vector is \mathbf{J} and the charge density is ρ .

In absence of free charges in the medium like optical fiber, \mathbf{J} and $\rho = 0$. So Maxwell's equations are modified in to,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.1.1.5)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (2.1.1.6)$$

$$\nabla \cdot \mathbf{D} = 0 \quad (2.1.1.7)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.1.1.8)$$

As we know that flux density \mathbf{D} is,

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (2.1.1.9)$$

where ϵ_0 is the vacuum permittivity.

The wave equation that describes propagation of optical pulses through fiber can be derived from Maxwell's equations. Taking curl of Equation (2.1.1.5), we get,

$$\nabla \times \nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B})$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

Also, by using mathematical identity,

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

For a homogenous media, we have $\nabla \cdot \mathbf{E} = 0$ and $1/\mu_0 \epsilon_0 = c^2$,

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (2.1.1.10)$$

This is the derivation of wave equation.

Now, we move to derive Nonlinear Schrödinger equation from this wave equation.

\mathbf{P} is induced polarization and is given as,

$$\mathbf{P} = \epsilon_0(\chi^{(1)}.E + \chi^{(2)}.E.E + \chi^{(3)}.E.E.E + \dots) = P_{LIN} + P_{NONLIN}$$

Where, $\chi^{(1)}.E$ is dominant term, $\chi^{(2)}.E.E$ is negligibly small for SiO₂ and $\chi^{(3)}.E.E.E$ term corresponds to nonlinearity. So Equation (2.1.1.10) becomes,

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 (P_{LIN})}{\partial t^2} + \mu_0 \frac{\partial^2 (P_{NONLIN})}{\partial t^2}$$

Let us say that electric field is given by,

$$\mathbf{E} = E_0 e^{j\omega_0 t}$$

Here, ω_0 is signal frequency,

Pulse propagation will be decided by both nonlinear effects and dispersion. To study pulse evolution, it is easier to express \mathbf{E} as function of frequency.

$$\mathbf{E} = E_0 e^{j\omega_0 t}$$

Fourier transform of \mathbf{E} , which is dependent on space r and time t , is taken as $\tilde{\mathbf{E}}$.

$$\tilde{\mathbf{E}}(r, \omega - \omega_0) = \int_{-\infty}^{\infty} \mathbf{E}(r, t) e^{-j(\omega - \omega_0)t} dt$$

Now wave equation will be written as,

$$\nabla^2 \tilde{\mathbf{E}} + \epsilon(\omega) k_0^2 \tilde{\mathbf{E}} = 0$$

Here $\epsilon(\omega) = 1 + \chi^{(1)}(\omega) + \epsilon_{NL}$

We have signal which has equation as follows,

$$\tilde{\mathbf{E}}(r, \omega - \omega_0) = F(\rho, \varphi) \tilde{\mathbf{A}}(z, \omega - \omega_0) e^{-j\beta_0 z}$$

Where $F(\rho, \varphi)$ is transverse function, $\tilde{\mathbf{A}}(z, \omega - \omega_0)$ is envelope function which is evolving as function of z , exponential term is phase factor.

Substitute above function into wave equation and separate out two equations,

$$\nabla^2 F + \{ \epsilon(\omega) k_0^2 - \tilde{\beta}^2 \} F = 0 \quad (2.1.1.11)$$

For envelope, second equation is,

$$-2j\beta_0 \frac{\partial \tilde{\mathbf{A}}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2) \tilde{\mathbf{A}} = 0 \quad (2.1.1.12)$$

Our focus is on above equation, as when light propagates, envelope of pulse changes. Since $\tilde{\beta}$ is very close to β_0 , one can make approximation,

$$\begin{aligned}\tilde{\beta}^2 - \beta_0^2 &= (\tilde{\beta} - \beta_0)(\tilde{\beta} + \beta_0) \\ &\approx 2\beta_0(\tilde{\beta} - \beta_0)\end{aligned}$$

Now main equation has become,

$$\frac{\partial \tilde{A}}{\partial z} + j(\tilde{\beta} - \beta_0)\tilde{A} = 0$$

Using Taylor Series expansion,

$$\beta(\omega) = \beta_0 + (\omega - \omega_0) \frac{\partial \beta}{\partial \omega} + \frac{(\omega - \omega_0)^2}{2} \frac{\partial^2 \beta}{\partial \omega^2} + \dots$$

$$\text{Putting } \beta_n = \frac{\partial^n \beta}{\partial \omega^n}$$

We have

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{(\omega - \omega_0)^2}{2}\beta_2 + \dots$$

Also, Dielectric constant is equal to,

$$\epsilon = (n + \Delta n)^2, \text{ n is refractive index.}$$

$$\approx n^2 + 2n\Delta n$$

$$\Delta n = n_2|E|^2 - j\alpha/2k_0$$

First term is due to nonlinearity and second term is due to loss.

Due to above terms,

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta(\omega)$$

Shifting to time domain, using equation written below,

$$A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(z, \omega - \omega_0) e^{j(\omega - \omega_0)t} d\omega$$

We get final Nonlinear schrödinger equation,

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = -j\gamma|A|^2 A$$

(2.1.1.13)

Where γ is nonlinear parameter given as,

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}}$$

Since β_1 is 1/GVD, as our pulse is moving and we have new frame of reference that is moving with same velocity then,

$$\beta_1 \frac{\partial A}{\partial t} = 0$$

So, NLSE is given by,

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = -j\gamma|A|^2 A$$

(2.1.1.14)

2.2 Optical Nonlinearities

There are many types of medium in which optical soliton propagation is studied like Kerr medium, quintic medium, log-law medium, power law medium, saturable medium and so on. For each case, nonlinear Schrödinger equation would be different. In our study, we are considering optical soliton pulse propagation through cubic-quintic medium in which we are including effect of higher order nonlinearity of fiber.

2.2.1 Cubic Nonlinearity

In homogeneous medium, optical refractive index of medium depends upon intensity of light pulse (or beam) or quadratic in externally applied field. It means that the higher intensities which are present at centre of pulse will feel higher refractive index than those lower intensity portions of pulse which are near end.

$$n = n_0 + n_2 |E|^2$$

Where n is refractive index and n_2 is the second-order nonlinearity coefficient which depends upon intensity. Value of n_2 is dependent on third order susceptibility as

$$n_2 = \frac{3}{8n} \chi^3$$

Dependence of refractive index on intensity leads to temporal self phase modulation, a major nonlinear effect in optical fiber. Nonlinear Schrödinger equation considering cubic nonlinearity is

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A + j\gamma |A|^2 A = 0 \quad (2.2.1.1)$$

Where γ is Kerr coefficient representing cubic nonlinear effects. It is called ‘cubic’ as it comes with third power of amplitude A . β_2 is GVD factor.

2.2.2 Quintic Nonlinearity

At higher peak powers higher order nonlinearity has great influence on propagation of optical pulses in fiber. Therefore, we have to include effect of higher order susceptibility (up to fifth order) in calculations of NLSE. In that case, cubic-quintic nonlinearity becomes,

$$n_{nl}(I) = n_2(I) + n_3(I)^2 \quad (2.2.2.1)$$

Nonlinear Schrödinger equation in cubic-quintic media will be

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A + j\gamma |A|^2 A + js\gamma_2 |A|^4 A = 0 \quad (2.2.2.2)$$

Where s represents sign of quintic nonlinearity. If s has same sign as cubic nonlinearity then it is called focusing nonlinearity. If sign of s is different than cubic nonlinearity, then it is called de-focusing nonlinear effect. Value of quintic nonlinearity γ_2 is just 1-10 percent of γ i.e. cubic nonlinearity.

2.2.3 Power Law Nonlinearity

In materials like semiconductors, power law nonlinearity occurs. It is like generalization of cubic nonlinearity. The well known method of Inverse Scattering Transform which is

used to integrate nonlinear Schrödinger equation can not be used for this type of nonlinearity.

Generalised Nonlinear Schrödinger Equation will take the form,

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + j\gamma F(|A|^2)A = 0$$

For Kerr nonlinearity,

$$F(|A|^2) = (|A|^2)$$

For power law nonlinearity,

$$F(|A|^2) = (|A|^2)^n$$

Where n can be, $0 < n < 2$. So NLSE in this case will be,

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + j\gamma (|A|^{2n})A = 0 \quad (2.2.3.1)$$

2.2.4 Log-Law Nonlinearity

This type of nonlinearity occurs in case of Gaussons (stationary Gaussian beams) and helps in study of periodic and quasi-periodic beam evolution. For log law nonlinearity,

$$F(|A|^2) = \ln(|A|^2)$$

In this case, NLSE will be,

$$\frac{\partial A}{\partial z} - j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + 2j\gamma A \ln(|A|) = 0 \quad (2.2.4.1)$$

2.3 Perturbation Theory

Propagation of temporal or spatial solitons have been extensively studied for conservative medium. It is a medium in which loss is considered negligible. It means that the term $\frac{\alpha}{2}A$ is considered zero. But real fibers, obviously, have attenuation and other loss effects in them. So we can't avoid phenomena of loss while studying propagation of optical pulses through real optical fibers. On the other hand, dissipative systems include the effect of

loss in its study. Here we will study dissipative system with effect of perturbation in medium.

Perturbation in medium can come due to many practical reasons. It mainly comes because of loss which is inherited in optical fibers or due to gain. Soliton propagation in presence of perturbation in input signal with cubic nonlinearity is being studied comprehensively. We will do study on optical pulse propagation through dissipative medium having cubic quintic nonlinearity along with perturbation in medium.

In our study, NLSE which governs dynamics of the system is given by following equation:

$$i \frac{dA}{dz} + \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma_1 |A|^2 A + \gamma |A|^4 A = iR \quad (2.3.1)$$

where γ includes the contribution of fifth order nonlinearity of medium, which is much smaller in comparison to the Kerr nonlinearity γ_1 , β_2 is group velocity dispersion (GVD) coefficient and R is representing perturbation in medium. R can be calculated using following equation:

$$R = \delta A + \beta \frac{\partial^2 A}{\partial t^2} \quad (2.3.2)$$

Where β is small loss in fiber and δ is excess gain which comes after full compensation of loss by optical amplifiers.

A small perturbation in medium can make stable soliton solutions to change. So we will study effect of small perturbation in cubic-quintic medium.

Literature Survey

The word ‘soliton’ or ‘solitary waves’ was first coined in 1965 and since then solitons have been comprehensively studied. Field of optical solitons, specifically, has grown enormously in last few decades. A partial list includes not only the spatial and temporal Kerr solitons described by the already famous nonlinear Schrödinger equation but also many other types of solitons such as spatiotemporal solitons (often called light bullets) [5]. The theoretical concept of propagation of optical solitons through optical fibers was first suggested in 1973 by Hasegawa and Tappert [1], while it was experimentally demonstrated in 1980 by Mollenauer et al. [2]. Since then research on optical solitons had seen rapid development due to its potential applications in long distance optical communication and optical switching [9].

In 1983, Nick J. Doran and Keith J. Blow [12] had calculated the effects of loss on the propagation of soliton solutions of the nonlinear Schrödinger equation. In their study they found that the loss and pulse width are fundamental limiting factors for soliton propagation, and therefore, the highest priority should be minimization of them. By using the nonlinear effects, it should not be necessary to work with dispersion shifted fibers since bit rates in excess of 1 Gbps could easily be obtained for a few hundred kilometers unrepeated, provided high-power pulses can be obtained. Alternatively, work near zero dispersion is required for extremely high bit rate systems (>100 Gbit).

In 1990s, the nonlinear cubic-quintic Schrödinger equation (NLCQSE) had started receiving increasing interest. This was important because this equation is in a regime far away from saturation and also the correct model to describe the propagation of the envelope of a light pulse in actual optical fiber. By 1994, C. De Angelis, [13] determined approximate analytical expressions for the solutions in the cubic-quintic nonlinear Schrödinger equation (NLCQSE) and discussed the stability of the solutions. They also compared their solutions with the available exact ones. Using their approach, all the self-trapped pulses parameters (energy, amplitude, width, propagation constant) are determined and the stability characteristics of the solitary waves were found.

Nakazawa and his group [14] propagated solitons in a fiber loop containing amplitude modulators in addition to the erbium-doped amplifiers over 106 kms. Later, at 10Gbit/s Soliton data was transmitted over one million kilometres by using a new technique. It incorporated synchronous shaping and retiming using a high speed optical modulator. Many research groups worldwide started conducting soliton transmission experiments recognizing the potential of soliton communication and attempting to push the transmission capacity to higher and higher limits.

S. Wabnitz, Y. Kodama and A. B. Aceves [15] proposed method of analysing optical temporal and optical spatial solitons in lossless fiber and their interaction while solitons travel in different wavelength channels or polarization. Theory of interaction of solitons is well explained by them. They gave theory for perturbation in soliton parameters in Nonlinear Schrödinger equation and described inverse scattering theory which gave many quantities that remains conserved during propagation. They have discussed picosecond optical solitons first and then femtosecond pulses considering the effects of higher order dispersion. Few applications of spatial optical solitons are explained by them in their paper. They also presented derivation of nonlinear Schrödinger equation in diffractive slab waveguide which is analogous to spatial soliton case.

George I. A. Stegeman, Demetrios N. Christodoulides, and Mordechai Segev [16] gave details on optical spatial solitons, which propagate undisturbed because of interplay of diffraction and nonlinearity. They possess few properties of real particles like when two optical spatial solitons interact, they retain their properties after interaction. Soliton is found in many fields but, by that time, research had shifted to optical solitons. They have discussed new class of optical solitons like quadratic optical solitons, in which phenomena of beam trapping is obtained due to exchange of energy in fundamental and second harmonics, photorefractive solitons, in which many physical phenomena occurs on a microscopic level every one of them describes change in refractive index, vector solitons and incoherent solitons. For vector solitons to be self trapped, the self-phase and cross-phase modulation coefficients must be equal. They also discussed spatio-temporal solitons.

In 2000, V. S. Grigoryan, R.-M. Mu, G. M. Carter, and C. R. Menyuk, [17] observed in a recirculating fiber loop for the first time stable propagation of dispersion-managed solitons over 28 000 km with zero or average dispersion.

Woo-Pyo-Hong [18] gave analytical bright and dark wave solutions for cubic-quintic nonlinear Schrödinger equation. He demonstrated the effects of femtosecond optical pulse propagation in non-Kerr media. He used the complex envelope function ansatz that was proposed by Li. et. al. He found modal coefficient in all parameters of solitary wave solutions.

Tatiana A. Davydova and Yuri A. Zaliznyak, in 2001 [19] presented study on bright solitons whose phase changes nonlinearly with spatial (or temporal) coordinate. They have used 1D generalized nonlinear Schrödinger equation which includes the cubic-quintic local nonlinearity and dispersive effects upto fourth order. They used variational approach for analytical work. They found out that chirped soliton pulses with perturbation may break up into ordinary solitons and move nonuniformly significantly changing their form.

Roger H. Stolen, gave a paper about major applications and future trends of fiber optic nonlinearity [20]. High capacity lightwave systems were affected by research on soliton. He highlighted theory of nonlinear optical processing. He gave knowledge of Brillouin scattering, Raman lasers, solitons, optical Kerr effect and parametric amplifiers.

Zhiyong Xua, Lu Lic, Zhonghao Lia, Guosheng Zhoua, [21] in their paper incorporated the effects of third order dispersion and self-steepening in soliton propagation. They found out that to increase the bit rate above 100 Gbps in fiber optic communication systems it is required to decrease the pulse width. However, the standard NLS equation is not sufficient as light pulses become shorter. Thus, extra terms describing the effects of third-order dispersion (TOD) and self-steepening must be added to that equation. They investigated higher-order nonlinear Schrödinger (HNLS) equation which was initially proposed by Kodama and Hasegawa.

T. Ohara, H. Takara, A. Hirano, K. Mori, and S. Kawanishi [22], designed all-optical limiter in which spectrally filtered optical solitons are used. They improved optical power received by more than 2dB in all the four channels. For the first time, all optical multichannel limiter was designed. Experimental results proved that AMCL improves efficiency and reduce power consumption. In a nonlinear optical fiber (NLF), WDM signals are fed. When fundamental soliton power is exceeded by peak power of a channel, spectral broadening occurs in the Nonlinear fiber as the optical pulse is compressed, which occurs due to larger chirping by SPM than chromatic dispersion. Difference in the

spectral wings are caused due to difference in the intensity of each pulse, which are removed by WDM multiplexer/de-multiplexer filters and fluctuations in intensity are reduced.

Andrey A. Sukhorukov, Yuri S. Kivshar, Hagai S. Eisenberg, and Yaron Silberberg presented a paper on Spatial Optical Solitons [23] in which a new improved analytical model about discrete NLS is proposed. They have also discussed bright and dark solitons, discrete solitons and gap solitons and demonstrated that the study of discrete gap solitons binary waveguide arrays can be used as a testing ground. They gave theory about waveguide array networks.

In 2004, M. S. Ozyazici, presented a work [24] about the interaction of higher order solitons by solving the nonlinear Schrödinger equation which models the propagation of solitons in the optical fiber for the propagation of two soliton pulse. The Schrödinger equation was modified by including terms related to loss of the optical fiber. In this paper, the Schrödinger equation is solved by using split-step Fourier transform technique, which is well known today. The interaction of the second order solitons was specifically investigated by him. It takes into account the broadening effect of the optical fiber loss and changes it to the one fourth of the initial pulsewidth at the half period. It was found that the loss of the optical fiber increases the interaction of solitons. Next year, G.M. Muslu, H.A. Erbay, presented higher-order split-step Fourier schemes for the generalized nonlinear Schrödinger equation. The numerical experiments reported there that the first-order and second-order schemes were less efficient than fourth-order split-step Fourier scheme.

J. Toulouse, [25] presented a paper about optical nonlinearities in fibers. Controlling nonlinearity and their interplay was one of major issue in those years. Various types of optical nonlinearities that are present in optical fibers are discussed and the effects caused by each type of nonlinearity is presented. The required material and fiber parameters to optimize optical nonlinearity for WDM systems are found out by him. Variations produced by optical nonlinearity for different parameters are also discussed. He proposed few equations according to which the effective area A_{eff} , the effective length L_{eff} , and the nonlinear coefficient γ determine strength of nonlinearities. He also studied Highly nonlinear fibers. By reducing the effective core area A_{eff} and increasing the difference in refractive indices of core and cladding, ($\Delta n \equiv n_{\text{core}} - n_{\text{clad}}$), Highly nonlinear fibers

can be designed. Reduced A_{eff} can be obtained by proper design of index profile. Increasing the difference in refractive indices of core and cladding is needed to confine the mode (travelling in fiber) more tightly in the core.

Same year, Chi-Feng Chena, Sien Chi, studied propagation of the femtosecond second-order solitons in an optical fiber [26]. They showed that the propagation of the second-order soliton was well explained by generalized nonlinear Schrödinger equation. The propagations of a 50 fs and a 10 fs second-order soliton in an optical fiber were numerically simulated. It was found that, for the case of 10 fs second-order soliton, the soliton decay is dominated by the third-order dispersion, in contrast to the case of 50 fs second-order solitons, where the soliton decay is dominated by the delayed Raman response.

T. I. Lakoba [27] presented a paper about Dispersion-Managed Soliton WDM Systems. He showed that in ultra-long haul dispersion-Managed Soliton WDM Systems transmission performance can be improved by proper matching in average path dispersion and pulse width of input optical soliton used. For lower values of average path dispersion, narrower pulse width should be used. It is proved by him that this relation between average path dispersion and pulse width of input optical soliton used is only for dispersion manage regime. It will not work for other regimes. If minimum width of pulse changes then balance between pulse width and path average dispersion will not hold.

Svetlana V. Serak, Nelson V. Tabiryan, Marco Peccianti, and Gaetano Assanto [28] presented a paper in 2006 about all optical logic gates using interaction between spatial solitons. They have demonstrated AND, NOR and XNOR all-optical gates. The signal confined by spatial soliton and spatial soliton itself can be switched by control beams.

Sazzad Muhammad Samaun Imran, [29] in 2006, numerically showed formation and propagation of solitons using the generalized non-linear Schrödinger equation. He found in his study that a phase angle above 0.1 separated the soliton solution into a row of peaks. The results were found to be most useful at a phase angle of 0.07. The sign changes direction of the phase.

A. B. Moubissi, K. Nakkeeran and Abdosllam M. Abobaker, [30] found relation between in the power of a pulse and its chirp, pulse duration, and fiber parameters like group-velocity dispersion and self-phase modulation parameters. They also showed that the

results obtained from the direct numerical simulation of the NLSE are matching the solutions of the variational equations.

Hongjun Zheng, Shanliang Liu, Xin Li, and Zhen Tian [31] presented a paper on temporal characteristics of an optical soliton and how it get affected by distributed Raman amplification (DRA). They have used well known Split-step Fourier method to numerically prove their results which were in agreement with experimental data. Distributed Raman amplification does not change the temporal shape of soliton pulse and can also compensate for fiber loss. Increasing the pumping power of Raman, compensation for optical fiber loss can be increased. They have done their study of distributed Raman amplification on Gaussian optical soliton and hyperbolic secant optical pulses. When the distance of propagation is more than the effective fiber length of the DRA, then loss due to optical fiber can be partially compensated. And when the distance of propagation is less than the effective fiber length, then fiber loss can be completely balanced.

S. Konar, M. Mishra, Soumendu Jana, [32] studied a cosh-Gaussian laser beam both in Kerr and cubic quintic nonlinear media. They studied propagation characteristics both analytically and numerically. They found that cosh-gaussian beams are unstable and convert into sech or Gaussian type beam. In cubic media, they collapse above certain virtual threshold power. In cubic quintic nonlinear media in which quintic nonlinearity is defocusing, with low and moderate power these beams transform into sech or Gaussian beam. They convert to flat top Beam at higher powers, where the length of the flat top and sharpness of flatness increase with the increase in power. The beam first splits into two and again combines and this phenomenon occurs again and again at very large powers.

Jennifer H. Lee, James van Howe, Chris Xu and Xiang Liu, in 2008, [33] presented a paper about fundamentals of Soliton self-frequency shift (SSFS) which is caused by Raman self-pumping that continuously red-shifts a soliton pulse. Redshift of centre frequency of a soliton pulse of sub picosecond pulsewidth with increase in power in standard single-mode fiber was first observed by Mitschke and Mollenauer and they named this concept as Soliton self-frequency shift. Experimental demonstrations are done by them along with discussion about various areas in which this concept could be useful. Many kind of fibers yields Soliton self-frequency shift which is dictated by dispersion

and nonlinearity of fiber. They discussed Soliton self-frequency shift in single mode fiber, microstructured optical fibers and higher order mode fibers.

R. Ganapathy, K. Porsezian, A. Hasegawa and V. N. Serkin, [34] proposed their work on interaction of solitons. Interactions considered by them were of two types, interactions of solitons when their amplitude are equal and when unequal amplitude soliton pulses interact with their velocities equal in magnitude but different in sign. Their study gave various important results. Firstly, parameters of solitons plays major role in suppression of forces of interaction in two solitons. Secondly, when fiber loss is compensated, then soliton pulse amplification is get after soliton-soliton collision. Third, soliton pulse width management is essential for optimal soliton pulse amplification.

Abdosllam M. Abobaker, K. Nakkeeran, A.B. Moubissi and P.Tchofo Dinda, [35] in 2008, gave a design about transmitting chirp-free Gaussian pulses in dispersion-managed fiber systems for very long distances. They also gave an interesting example of bit sequence transmission along with higher order effects was given for all types of DM fiber lines. They also included effect of noise in that example.

Asim Shahzad and M. Zafrullah, in 2009, [36] have done qualitative and mathematical analysis for soliton interaction. They gave the mathematical model based on the Nonlinear Schrödinger (NLSE) equation and they had conducted various simulation experiments using the split-step Fourier transform method to investigate the interaction between solitons. They studied equal amplitude pulses and in-phase solitons that are propagating in same direction in dispersion shifted fibers. The simulation results showed that, solitons evolve as a giant pulse of double amplitude after travelling a certain distance because of attraction between them and get separated if propagated further. Their study also revealed that in order to avoid the soliton interaction a careful choice of the pulse and fiber parameters is required as soliton interactions limits the channel capacity.

Zai-yun Zhang, Zhen-hai Liu, Xiu-jin Miao and Yue-zhong Chen [37] proposed travelling wave solutions for Kerr nonlinearity media. They qualitatively studied Nonlinear schrodinger equation considering perturbation in it. These solutions had Jacobi elliptic function periodic solutions, bell-shaped solitary wave solutions and kink-shaped solitary wave solutions. Relations between the solitary wave solutions and Jacobi elliptic function periodic solutions were found. It is also proved that solitary wave solution can be evolute from the exact Jacobi periodic solutions.

R. Ganapathy, in 2012, presented a paper named Soliton dispersion management in nonlinear optical fibers, [38] in which he considered the concept of quasisoliton propagation in a dispersion management fiber and in cases of soliton dispersion management, soliton energy control and guiding center soliton, he studied the dynamics of soliton and the interaction of soliton.

In 2012, Laila Girgis, Daniela milovic, Tasawar Hayat, Omar M. Aldossary and Anjan Biswas [39] studied dynamics of optical soliton propagation in perturbed log law nonlinear optical fiber, in which solitons are named as Gaussons. Perturbative terms that are studied in this paper are higher order dispersion terms, intermodal dispersion, self-steepening coefficient term, coefficient of nonlinear dissipation induced by Raman scattering, two-photon absorption term, nonlinear amplification or absorption etc.

Pan Wang and Bo Tian [40] studied the non-Kerr media and the effects of the quintic nonlinearity on the ultrashort optical soliton pulse propagation in the same media. They used dependent variable transformation and Hirota method. Solitons propagation and soliton interaction are investigated analytically and graphically in this paper. They found that there are bumps in the interaction regions and the main effect of the two solitons is the repulsion or attraction alternately.

Sergei K. Turitsyn, Brandon G. Bale, Mikhail P. Fedoruk, [41] in 2012, provided mathematical theories of Hamiltonian and dissipative systems and then discussed future trends and upcoming problems in practical implementation of this concept in fibre-optics and lasers. They observed that to achieve high energy and to have minimum peak power (or intensity) DM solitons can be proved to be helpful. This is required to minimize the undesirable nonlinear effects. In general terms, the DM soliton can keep more energy compared to the traditional soliton with a fixed shape and pulse parameters (during evolution). And DM solitons have capacity to avoid possible nonlinear problems with high local intensity.

All optical devices are centre of research in recent times because of elimination of costly electronics in o-e-o conversions. Today, vast research is going on all optical logic gates, all optical adder, all optical subtractor etc. In 2010, B. Knobnob, S.Mitatha, K.Dejhan, S.Chaiyasoonthorn and P.P.Yupapin [42] proposed a system of the dark–bright solitons conversion. The bright optical soliton can be get from the propagating dark soliton in the optical media by using the ring resonator system incorporating the add/drop multiplexer.

The main advantage of the system is that the detection of the bright soliton pulse is normally easier than dark optical soliton pulse. Firstly, a micro-ring resonator is used and a dark soliton pulse is passed into it, then it is propagated into smaller micro- and nano-ring resonators. Then, into the current system the add/ drop filter is connected, where the bright solitons are received from the drop port and the dark solitons are obtained from through ports of the add/drop filter. The results obtained have shown that the input soliton power and the ring resonator coupling coefficient can control power of detected soliton.

Zhengbin Wang, Yijun Feng, Bo Zhu, Junming Zhao, and Tian Jiang [43] in 2010 presented a paper about dark schrodinger solitons and generation of harmonics nonlinear transmission line. One-dimensional nonlinear Schrödinger equation can be derived from the equation used for representing propagation of wave envelope which supports dark solitons propagation. Dark solitons can be formed in a very practical short nonlinear transmission line metamaterial by increasing the dissipation of each unit cell. Both transmission line simulation and analytical investigation are done to verify their theory.

Sappasit Thongmee and Preecha P.Yupapin [44] made new design of Half adder/ Half subtractor using Dark-Bright soliton conversion. The main advantage of this design is that it is all optical. They used microring and nanoring devices in their design. They took input and control light pulse trains which are put into the first add/drop optical filter. For this purpose both Dark or Bright solitons could be used. First, the dark soliton is split dark and bright soliton via the add/drop optical filter. Dark soliton can be seen at through port and bright soliton at drop port. When the optical pulse train X,Y is fed into their designed component from input and add ports, respectively, we get half adder operation at the end.

Recently, Weizhu Bao, Qinglin Tang and Zhiguo Xu [45] showed efficient numerical method for dark and bright soliton computations in Nonlinear Schrödinger equation. They analysed stability of these methods. According to their study, time-splitting finite difference through a transformation (TSFD-T) method is most efficient among existing methods. They gave few very important conclusions. Firstly, even under the small perturbation in the initial data all the bright, type I dark and type II dark solitons are dynamically stable in NLSE. Second, the interactions for type I, type II and bright solitons are elastic and their velocity and shape before and after collision are same. If two type I dark solitons or two type II dark solitons are interacting and their velocities are less

than a critical value, they will be repulsed by each other; otherwise they will be transmitted through each other.

In 2013, P. Phongsanam, C. Teeka, R. Jomtarak, S. Mitatha, P.P. Yupapin, [46] gave a system, by use of Dark-Bright soliton conversion, two input optical logic OR and AND gates simultaneously. Response time of switching is obtained upto nanoseconds in their paper. Optical logic AND gate is found at the drop port and Optical logic OR gate can be formed at the through port. Their scheme can be used as an arbitrary logic switching system as it is a simple and flexible. It can be used to form the advanced complex logic circuits. All optical 2-to-4 line decoder can be created using their system.

Very recently, A.H. Bhrawy, M.A. Abdelkawy and Anjan Biswas [47], gave model of the nonlinear Schrödinger's equation to study the propagation of optical solitons in (1+1) and (2+1) dimensions through nonlinear optical fibers. They introduced special solutions named Jacobi doubly periodic wave solutions. When modulus is approaching to 1, the Jacobi functions reduces to the hyperbolic functions and we can have solutions by the extended tanh method (extended sech method). When m is approaching to 0, the Jacobi functions degenerate to the triangular functions and we can use the solutions by extended sine method (extended cosine method).

Propagation of optical solitons has been studied analytically and numerically taking different pulse shapes and taking different optical media like cubic-quintic, power law, saturable nonlinear media. Anjan Biswas, Huaizhong Ren and Swapan Konar [48] presented their study on optical solitons in non-Kerr nonlinearity media along with perturbation. They considered the power law, parabolic law and the dual-power law nonlinearities in their paper. They have studied both deterministic as well as stochastic type of perturbation. Detailed mathematical analysis is done by them in all type of nonlinearities. They found out that due to nonsaturable nature of Kerr non-linearity, it is inadequate to describe the soliton dynamics in the ultrahigh bit rate transmission.

All-optical wavelength converter that has reshaping, retiming, and regenerating (3R) capability was designed by Lu Gao, Kelvin H. Wagner and Robert R. McLeod. (3+1) dimensional optical bullets are used for its functioning [49]. It has compact size and it is ultra fast having speed in terabits per seconds. It has vast applications in optical time-division multiplexing (OTDM) and wavelength-division multiplexing (WDM) combined communication networks. Numerical simulations and propagation of light bullets are

done using Split-step Fourier transform beam propagation method. Networks with requirement of ultra fast switching speed can benefit from this work.

S. Konar, Manoj Mishra and S. Jana [50], studied optical soliton propagation in cubic-quintic nonlinear medium. They presented their study of optical solitons in dispersion managed optical communication systems. They found that quintic nonlinearity has little effect on optical pulse propagation, when only a single pulse is considered. But in WDM systems, where there are a number of pulses travelling simultaneously, the effect of quintic nonlinearity is that it reduces the collision distance between neighbouring pulses. And increasing the strength of quintic nonlinearity, the collision length decreases.

Samudra Roy and Shyamal K. Bhadra, [51] introduced Rayleigh's Dissipation Function to solve Soliton Perturbation Problems. They have used a variational approach for analytical work. Perturbation terms which include two-photon absorption, two soliton interactions, intrapulse Raman scattering, self-steepening and two-photon absorption are included in the extended nonlinear Schrödinger equation. The related standard governing equations of the pulse propagation can be generated from the generalized Euler-Lagrange equation with the help of Rayleigh's dissipation function and assuming some modifications. They have also used another method named as Kantorovitch approach which deals with such perturbing problems analytically.

Russell Kohl, Anjan Biswas, Daniela Milovic and Essaid Zerrad, [52] studied Gaussian optical solitons in the presence of different perturbation terms. Dispersion managed fiber is considered and they have included the fourth and sixth order dispersion terms. In this paper linear amplification or attenuation terms, two-photon absorption terms, nonlinear amplification-absorption terms, bandpass filtering term, the self-steepening coefficient, and the higher order dispersion coefficient are extensively studied in the perturbation term.

In 2009, solutions for bright and dark spatial solitons were given by Gabriel F. Calvo, Juan Belmonte-Beitia and Víctor M. Pérez-García [53]. They studied the nonlinear Schrödinger equation which describes media having saturable nonlinearity. They have qualitatively studied the stability of bright solitary waves, dark and grey solitary waves. Study of the stationary equation is done for all types of waves.

Same year, Anjan Biswas and Daniela Milovic [54], presented a paper in which they studied optical soliton propagation in power law media along with fourth order dispersive effects as for short pulse widths of optical solitons fourth order dispersion can not be neglected. Anjan Biswas [55] has published a paper about optical soliton propagation in power law media with time dependent dispersion and nonlinearity. He also included the effects of attenuation on optical soliton propagation in that media. Velocity of solitons using coefficient of soliton equation is also found mathematically.

Nonlinear Schrödinger equations with saturable nonlinearities are studied extensively by Alexander Pankov [56] for gap solitons. He provided nontrivial periodic solutions with arbitrarily large periods to periodic stationary discrete NLSE. Gap solitons are also named as breathers. These are spatially localised solitary standing waves.

Zai-yun Zhang, Zhen-hai Liu, Xiu-jin Miao and Yue-zhong Chen, [57] have used the modified mapping method and the extended mapping method to provide solutions for nonlinear Schrödinger equation along with perturbation. They have studied Kerr law nonlinearity in their work. They studied the linear combination of two different Jacobi elliptic functions.

Anjan Biswas, Carl Cleary, James E. Watson Jr. and Daniela Milovic [58] in 2010 studied optical soliton propagation in log-law media along with perturbation having time dependent coefficients. Time dependent dispersion and nonlinearity are considered in this paper. The solitons for log law nonlinearity are named as Gaussons. The perturbation terms that are included in this study are linear attenuation and inter-modal dispersion. They further observed that coefficient of GVD and nonlinearity must be in right ratio.

Stanley Johnson, Stanley Pau, and Franko Küppers [59] demonstrated experimentally optical re-timing of data stream. Their experimental retiming scheme let the data pulses that are out of synchronization as far as 16 ps after and 10 ps before bit center time to be received within considerable range of bit error rate. They have used pulses of 2 picoseconds (full width at half maximum (FWHM)) at 1550 nm generated by a tunable mode locked laser. Modulation is done by Mach- Zehnder modulator (MZM). They experimentally performed their scheme and also numerically simulated it. Results found experimentally are in close agreement with numerically found data. Then they extended their study to advanced version which include clock pulses for every data pulse. Pulses having offset up to 23 ps from bit center could be retimed and this is great performance.

Xianqiong Zhong, XiaoxiaZhang, AnpingXiang and KeCheng [60] studied spectra, frequency chirp etc of initial sech pulse in quintic nonlinearity in fibers. Dispersion considered is in normal regime, not anomalous regime. Role of quintic nonlinearity is same as for initial Gaussian pulse. They also studied wave breaking which will be more intense for more oscillation peaks will appear in the pulse wings. Increasing effect of the quintic nonlinearity and the soliton order, the distance of wave breaking will decrease.

Anjan Biswas, Megan Fessak, Stephen Johnson, Siercke Beatrice, Daniela Milovic, Zlatko Jovanoski, Russell Kohl and Fayequa Majid [61] gave travelling wave solutions for non-Kerr law optical nonlinearity and studied optical soliton propagation through such fibers. They studied, analytically and numerically, five different types of nonlinearities i.e. Kerr-law, power-law, parabolic-law, dual-power law and the log-law nonlinearity. It was found that the nonlinearity parameter and the degree of nonlinearity in the nonlinear term of the NLSE must be same for any type of nonlinearity.

K. M. Aghdami, M. Golshani, and R. Kheradmand [62] investigated Optical bistability along with modulational instability (MI) of a 2-D array of coupled optical cavities. They have included Kerr nonlinearity in it and 2-D discrete optical solitons are simulated. The interaction of three propagating beams in 2-D adjacent waveguides is also studied by them. Then they have proposed designs of various all optical gates using discrete cavity solitons like NOR, X-NOR, NAND. Their work and designs are used for rapid routing and processing of data in form of optical pulses.

Very recently, Qin Zhou [63] provides analytic study on optical soliton propagation in the nonlinear fibers considering Raman effect. He considered time-modulated parabolic law nonlinearity in his work. He used new method named as non-auto-Bäcklund transformation (NATB) method to get analytical solutions.

Numerical Analysis of Super Gaussian Optical Solitons in Cubic-Quintic Medium with Perturbation

4.1 Introduction

Research on optical solitons had seen rapid development due to its potential applications in long distance optical communication and optical switching. Main attractive features of optical solitons are,

1. It can travel for longer distances without change in its shape.
2. It maintains its shape even after collision with other soliton pulse.

In our study, we have considered dissipative systems because they matches practical optical fibers. Dissipative systems are those in which effects of loss and perturbation are included in the study. They are different from ideal conservative systems in which no fiber loss is taken into account. Perturbation in medium can come due to many practical reasons. Soliton propagation in presence of perturbation in input signal with cubic nonlinearity is being studied comprehensively. We will extend study on optical pulse propagation through dissipative medium having cubic quintic nonlinearity along with perturbation in medium.

In our study, nonlinear Schrödinger equation used is different from NLSE for conservative system. In fact, it is extended version of nonlinear Schrödinger equation, often called as perturbed NLSE. The NLSE used in our study is,

$$j \frac{dA}{dz} + \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma_1 |A|^2 A + \gamma |A|^4 A = jR \quad (4.1.1)$$

where γ includes the contribution of fifth order nonlinearity of medium, which is much smaller in comparison to the Kerr nonlinearity γ_1 , β_2 is group velocity dispersion (GVD) coefficient and R is representing perturbation in medium. R can be calculated using following equation:

$$R = \delta A + \beta \frac{\partial^2 A}{\partial t^2} \quad (4.1.2)$$

Where β is small loss in fiber and δ is excess gain which comes after full compensation of loss by optical amplifiers.

4.2 Methodology: Split Step Fourier Method

We employed Split Step Fourier method to analyse evolution of optical pulse propagation in perturbed medium. Split step fourier method is a numerical method used to solve Partial differential equations. This method computes the solution in small steps .So the distance is propagated by applying SSFM at small step size. It solves the nonlinear terms in time domain and linear parts in frequency domain. So fourier transforms are used to do solve in frequency domain. It is very effective to study light pulse propagation in optical fibers where linear and nonlinear mechanisms make it difficult to find general analytic solutions. First linear part that is dispersion effect is applied at distance half of Δz ,then nonlinearity terms effect is included for step size Δz at distance half of Δz . After this, again dispersion terms effect is applied for rest half of Δz . In this way pulse propagation is analyzed by propagating small distances equal to step size to cover whole distance.

In SSFT we take dispersion operator \hat{D} as a linear parameter,

$$\hat{D} = j \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} + \frac{\alpha}{2}$$

And nonlinear operator \hat{N} as

$$\hat{N} = j\gamma_1 |A|^2$$

In perturbed NLSE, we take dispersion operator \hat{D} as a linear parameter,

$$\hat{D} = j \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} + \beta \frac{\partial^2}{\partial t^2} + \delta$$

And nonlinear operator \hat{N} as

$$\hat{N} = j\gamma_1 |A|^2 + j\gamma |A|^4$$

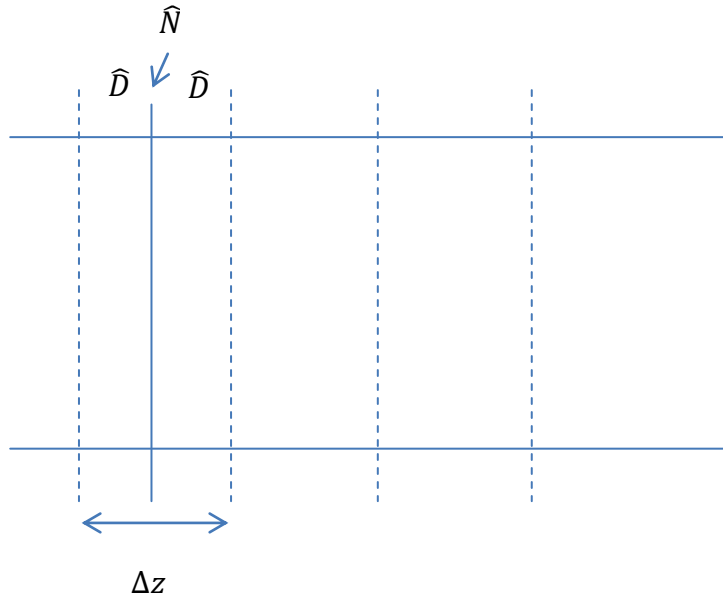


Fig. 4.1 Applying Dispersion and nonlinearity in Split-Step Fourier method

To apply split step fourier method ,we first split our equations into linear and nonlinear terms, the linear terms (Dispersion terms) are written as

$$\frac{dA}{dz} = j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \beta \frac{\partial^2 A}{\partial t^2} + \delta A$$

The nonlinear terms are obtained as

$$\frac{\partial A}{\partial z} = j(j\gamma_1 |A|^2 + \gamma |A|^4)$$

The nonlinear terms are solved in time domain. To solve linear terms we have to take Fourier transform. Then we take inverse Fourier transform to take it back in time domain. The nonlinear terms are applied in time domain. After that again we take Fourier transform and apply linear terms for rest of distance.

4.3 Results and Discussion

In our study, we considered operating wavelength $\lambda=1.55\mu m$, dispersion $d=11ps/nm-km$, Kerr nonlinearity parameter $\gamma_1 = 3.5 W^1/km$, Quintic nonlinearity parameter γ is 1% of cubic nonlinearity . We consider following initial conditions:

$$A(z,t) = \sqrt{P_0} \exp \left\{ -\left(\frac{t}{2T_0} \right) \right\}^{2m} \quad (4.3.1)$$

Where P_0 is initial power and m is order of Gaussian pulse and can be taken as $m = 1, 2, 3$. T_0 is initial pulse width.

Effect of m (order of Gaussian pulse) can be seen from following graph.

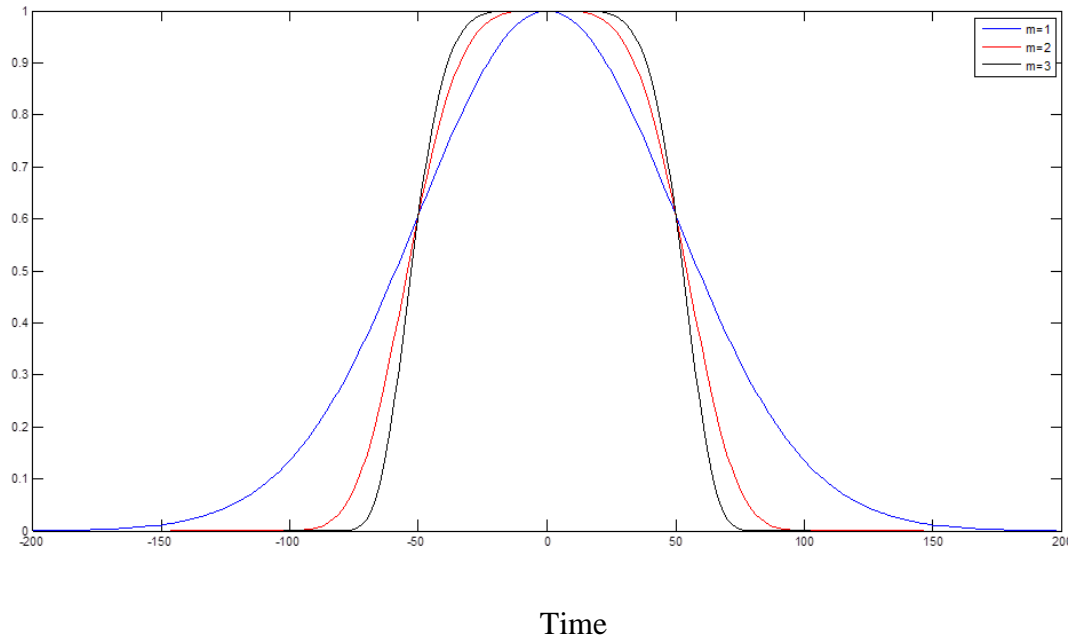


Fig. 4.2 Effect of order of pulse on Initial Gaussian Pulse

It can be seen from above figure that as order of Gaussian Pulse is increased, we are getting more flat-top pulse. Gaussian pulse with $m=3$ is much more flat at the top than Gaussian pulse with order $m=1$. Also base width of pulse is narrower for Gaussian pulse with order $m=3$ than for $m=1$.

Figure 4.3 shows propagation of optical Gaussian soliton pulse in Kerr medium without considering effects of quintic nonlinearity. We have taken Kerr nonlinearity $\gamma_1 = 3.5 \text{ W}^{-1}/\text{km}$, Quintic nonlinearity parameter $\gamma = 0$, and dispersion $d=11\text{ps}/\text{nm}\cdot\text{km}$. As we can observe from the figure that pulse is not spreading with the propagation distance. Also amplitude of optical pulse is constant after travelling some distance. It means that it is stable optical soliton and it will travel maintaining its shape in optical fiber for longer distances.

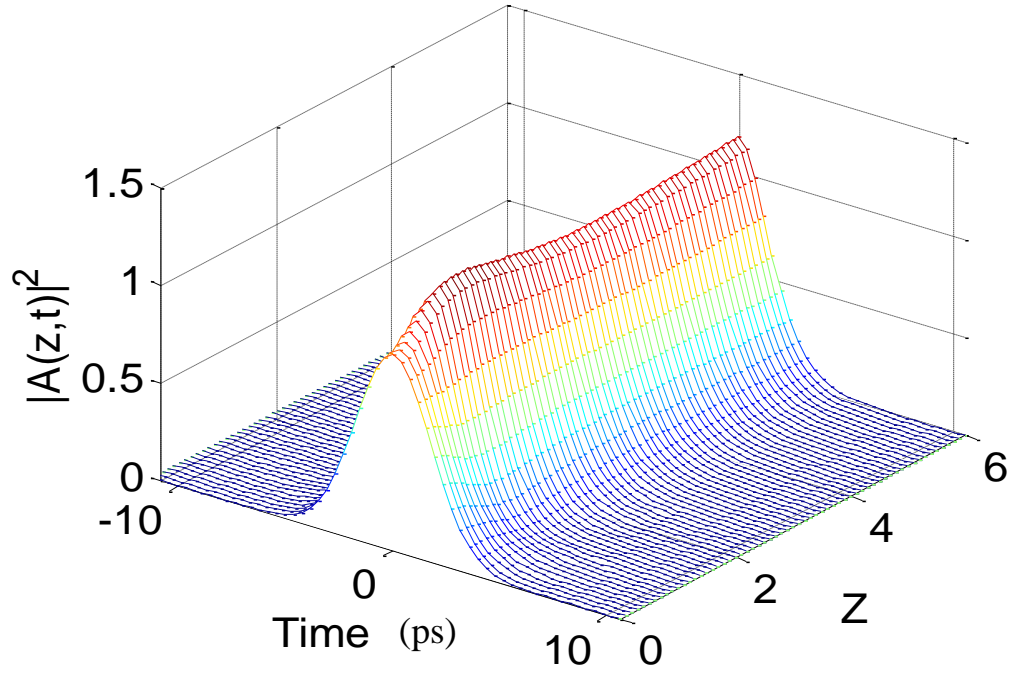


Fig. 4.3 Evolution of Gaussian soliton as function of normalized propagation distance in Kerr medium without quintic nonlinearity

Figure 4.4 shows propagation of optical Gaussian soliton in presence of quintic nonlinear effects. We have taken quintic nonlinearity $\gamma = 50\%$ of Kerr nonlinearity γ_1 . We are using De-focussing type quintic nonlinearity which means that it acts opposite to Kerr nonlinearity. And it is shown that as we increase quintic nonlinearity, behaviour of pulse propagation changes into oscillatory motion. As we can see that these oscillations are periodic in nature, so it is also a stable soliton also called ‘Breathing Solitons’. In this type of solitons, amplitude rises and falls periodically.

In further study, we will use the value of quintic nonlinearity equals to 1 per cent of Kerr nonlinearity. In practical systems, the optical fibers are fabricated by semiconductor materials which has value of fifth order nonlinearity about 1 per cent of third order nonlinearity i.e. Kerr nonlinearity.

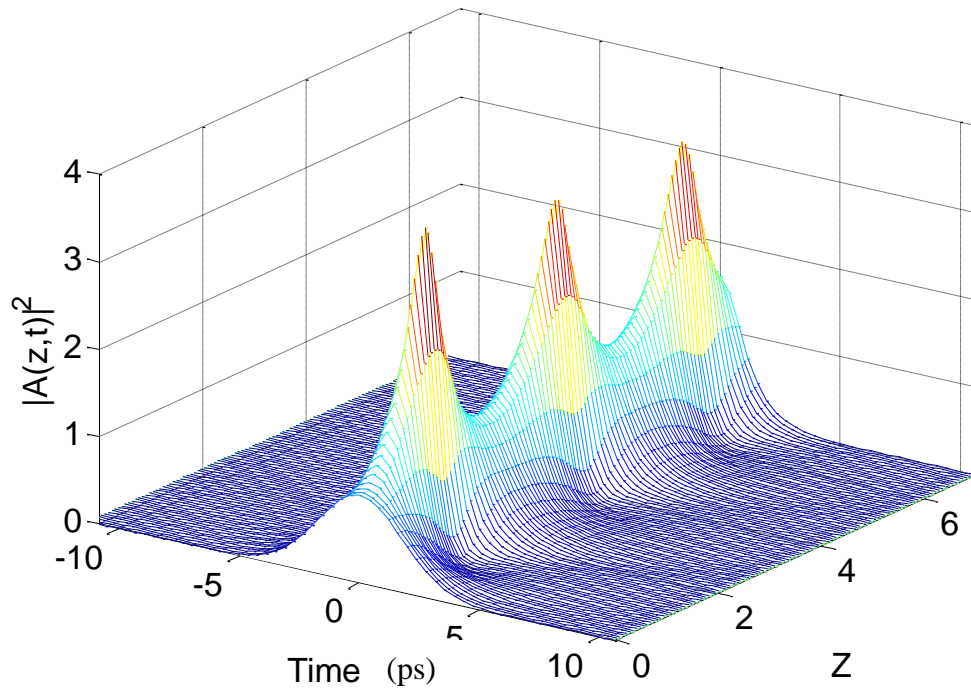


Fig. 4.4 Evolution of optical soliton with normalised distance Z in presence of cubic quintic nonlinearity with $\gamma = 50\%$ of Kerr nonlinearity γ_1

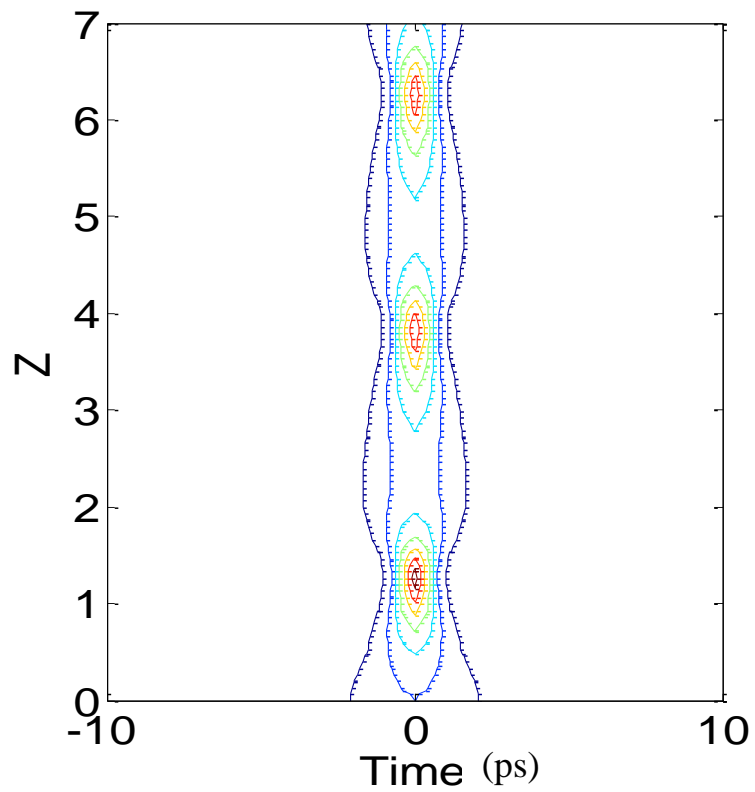


Fig. 4.5 Contour plots of evolution of optical Gaussian soliton along with the propagation distance in presence of high quintic nonlinearity.

A. Cubic-quintic medium (without perturbation)

Firstly, we will consider cubic-quintic medium with quintic nonlinearity $\gamma = 1\%$ of Kerr nonlinearity γ_1 . We are taking defocussing type of quintic nonlinearity which means that its effect is opposite to Kerr nonlinearity. Here we are not introducing perturbation so we put $\delta = 0$ and $\beta = 0$.

The figure 4.6 shows propagation of optical Super-Gaussian soliton with order $m=1$ along with distance of propagation. Here we have taken cubic nonlinearity $\gamma_1 = 3.5 \text{ W}^1/\text{km}$, Quintic nonlinearity parameter $\gamma = 1\%$ of γ_1 , and dispersion $d=11\text{ps/nm-km}$. As we can see from the figure 4.6 that stable optical soliton propagation is achieved.

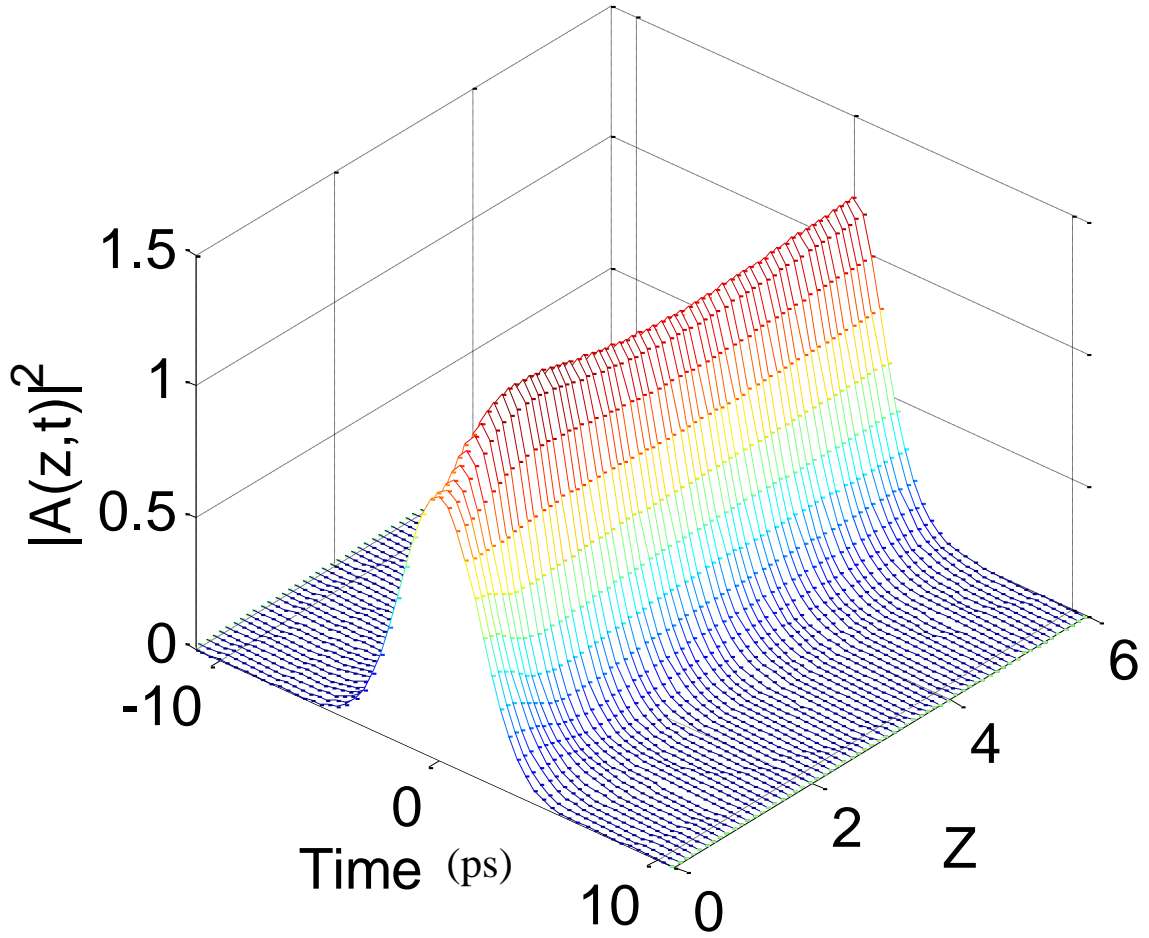


Fig. 4.6 Evolution of optical Super-Gaussian soliton ($m=1$) with normalized propagation distance in cubic quintic medium with $\gamma = 1\%$ of γ_1 .

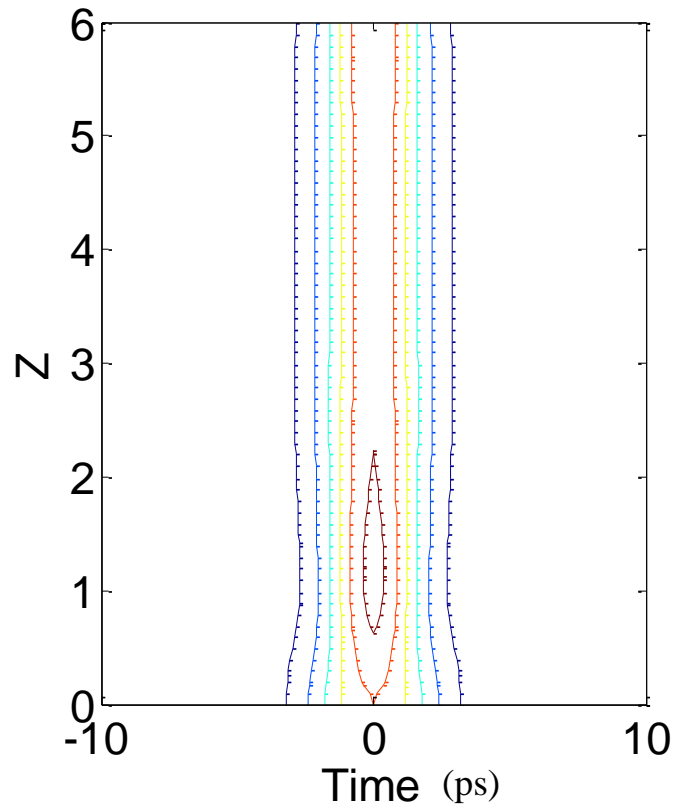


Fig. 4.7 Contour plot of propagation of optical Super-Gaussian soliton with order $m=1$

Contour of propagation of optical Super-Gaussian soliton with order $m=1$ is shown in figure 4.7. This figure shows that as distance of propagation increases, there is no effect on propagation of optical soliton. It is not spreading or attaining height as we can observe from its contour. It can be observed from the figure that initially there is a peak that vanishes with distance. This peak comes because optical soliton takes some time to get stable. Once it has catch a behaviour, it continues to travel in same form for longer distances. Figure 4.8 shows evolution of Super Gaussian optical soliton pulse taking order $m=2$ with normalized distance Z . We have taken quintic nonlinearity as 1% of Kerr nonlinearity. As we can observe from the figure that Super-Gaussian optical soliton with order $m=2$ changes its shape to Gaussian first and then propagate straight. Contour plot of propagation of Super Gaussian optical soliton pulse with normalized propagation distance Z is shown in figure 4.9. Contour plot also reveals that we get stable optical soliton in case of Super Gaussian pulse with order $m=2$.

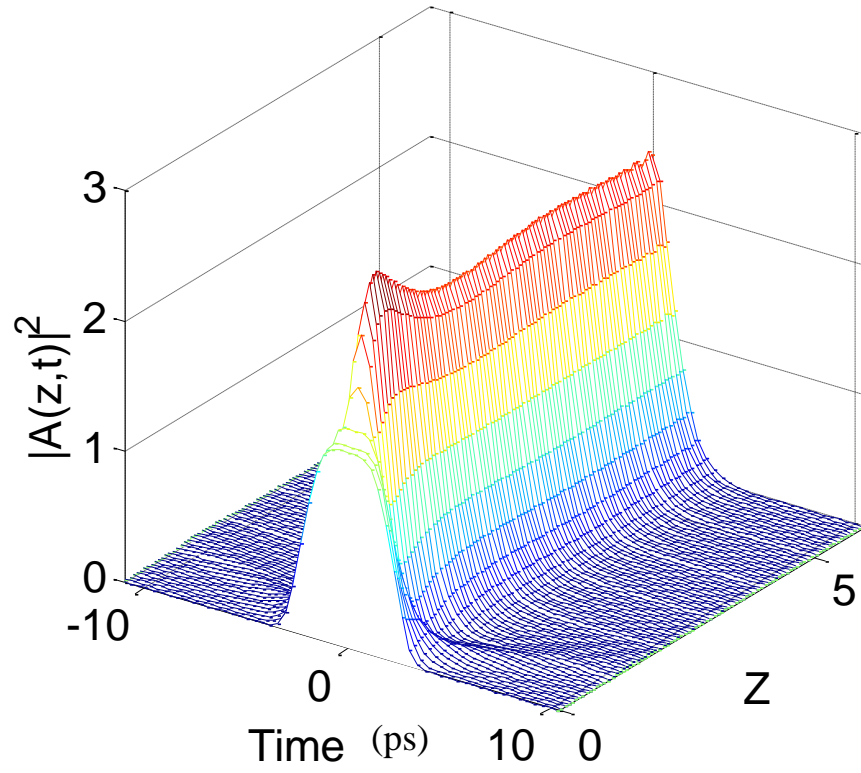


Fig. 4.8 Propagation of optical Super-Gaussian soliton with order $m=2$ with normalized distance

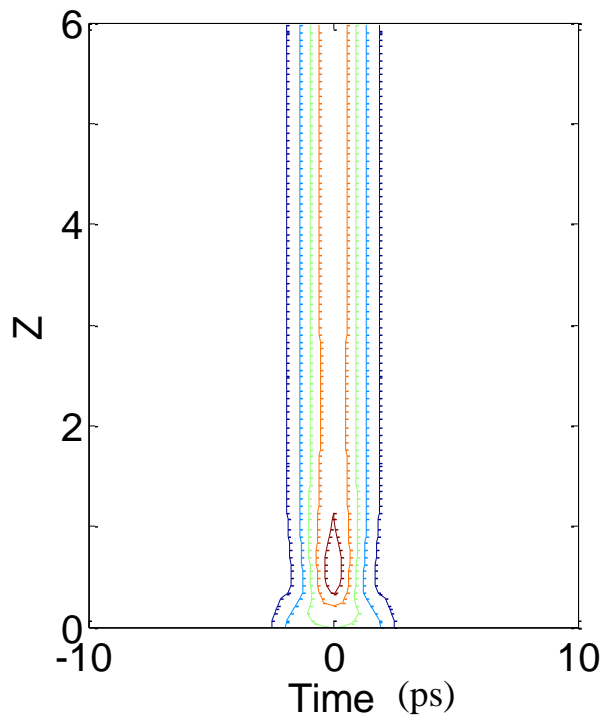


Fig. 4.9 Contour plot of propagation of optical Super-Gaussian soliton with order $m=2$

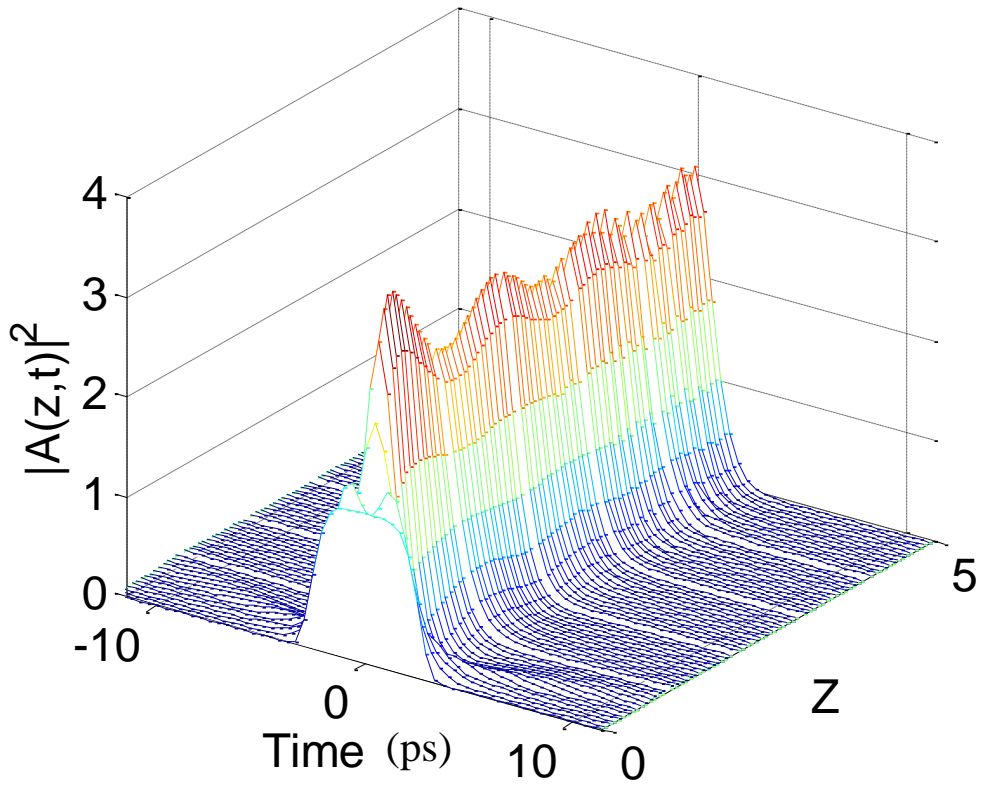


Fig. 4.10 Evolution of Super Gaussian optical soliton pulse taking order $m=3$ with normalized distance

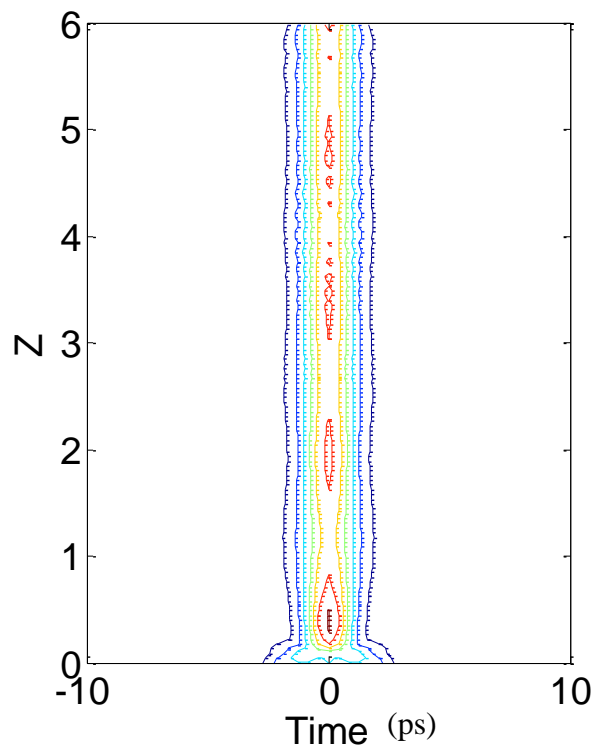


Fig. 4.11 Contour plot of propagation of optical Super-Gaussian soliton with order $m=3$

Figure 4.10 shows Evolution of Super Gaussian optical soliton pulse taking order $m=3$ with normalized distance Z . It is observed that Super-Gaussian pulse with order $m=3$ also changes its profile to Gaussian and then it propagates periodically.

B. Cubic-quintic medium (with perturbation)

Now we will extend our study to propagation of optical Super-Gaussian solitons with order $m=1,2,3$ in Cubic-quintic medium with effects of perturbation in medium. We have taken dispersion $d = 11ps/nm - km$. Kerr nonlinearity is slightly different for different order of Super Gaussian optical soliton pulse. Quintic nonlinearity is taken 1% of Kerr nonlinearity.

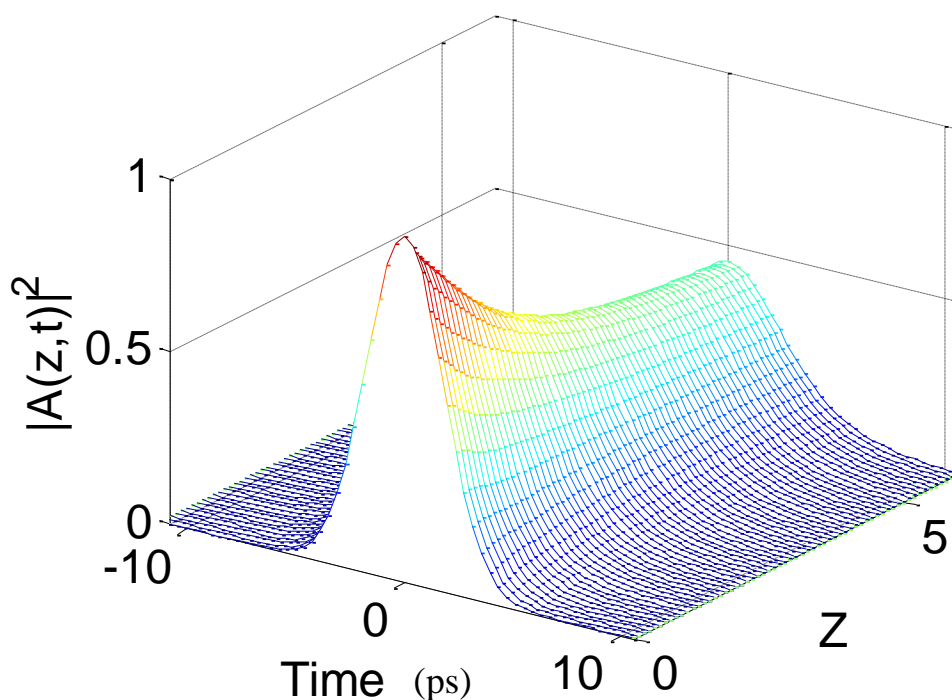


Fig. 4.12 Evolution of optical soliton with propagation distance Z in presence of perturbation in medium with $\delta = 0$ and $\beta = 0.15$

The figure 4.12 shows evolution of optical Gaussian soliton with normalized propagation distance Z in presence of perturbation in medium. We have considered effects of loss in medium so we put $\beta = 0.15$ and $\delta = 0$. It shows that since term indicating gain in fiber

is zero but loss term is non-zero, so we are not achieving stable optical soliton propagation along the fiber. As it propagates along the fiber, pulse starts dispersing.

Figure 4.13 shows evolution of optical Gaussian soliton with normalized propagation distance Z in presence of perturbation in medium with gain. It is observed that as the initial Gaussian pulse propagates in a medium in which there is only gain, pulse attains height and amplitude increases because of gain in medium. Because of this amplitude increase, we are not getting stable optical soliton propagation.

We have observed that by applying only gain or loss in medium, stability can't be achieved. Now to get stable optical soliton solutions, there needs to be balance between gain and loss in medium.

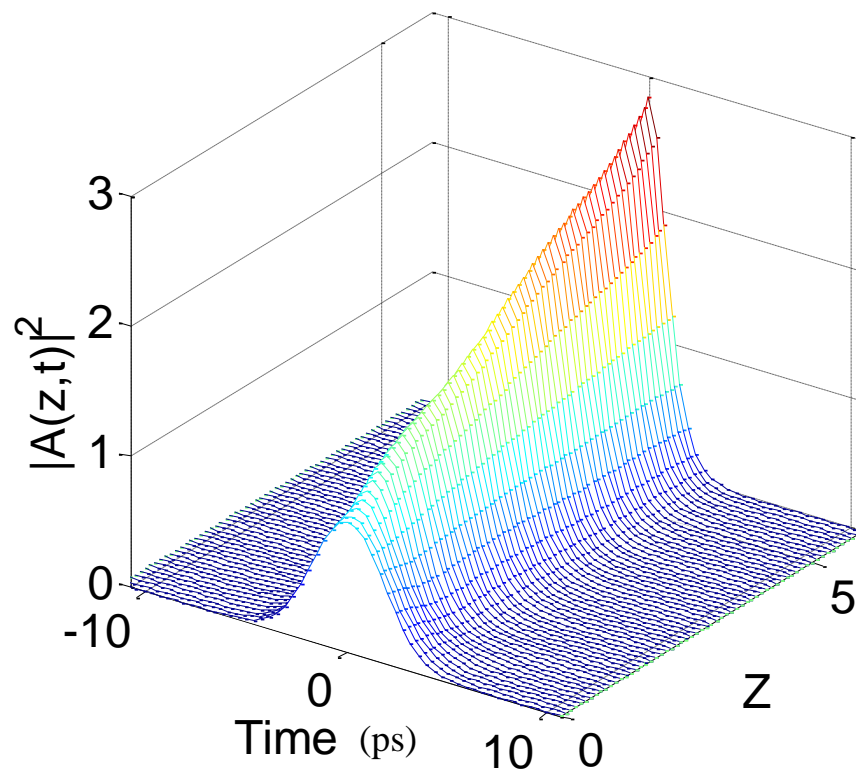


Fig. 4.13 Evolution of optical soliton with propagation distance Z in presence of perturbation in medium with $\delta = 0.04$ and $\beta = 0$

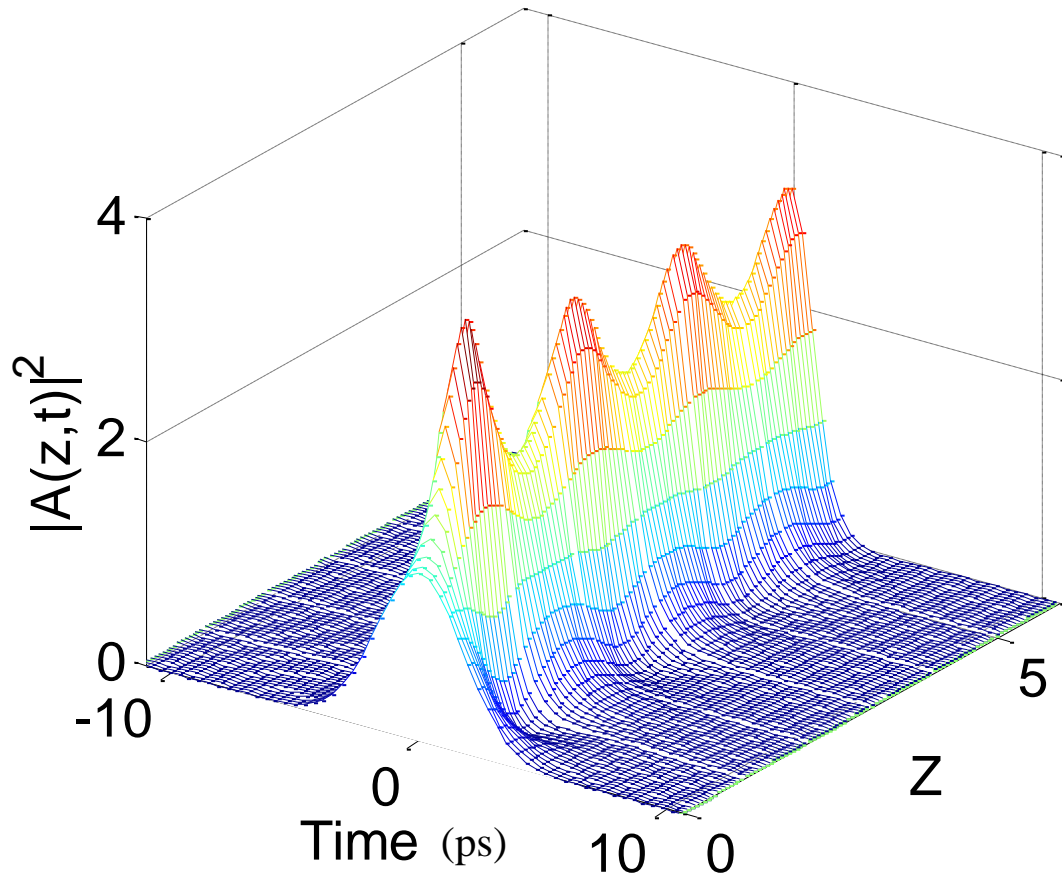


Fig. 4.14 Evolution of Super Gaussian optical soliton with order $m=1$ with normalized propagation distance in cubic quintic medium with perturbation

After balancing gain and loss in medium, we get stable optical soliton solutions. But nature of propagation changes for optical Super-Gaussian pulse with order $m=1,2,3$. Figure 4.14 shows propagation of Super Gaussian optical soliton with order $m=1$ with normalized propagation distance in cubic quintic medium with perturbation, $\delta = 0.04$ and $\beta = 0.15$. Value of Kerr nonlinearity taken is $\gamma_1 = 4 \text{ W}^1/\text{km}$. Quintic nonlinearity is 1% of Kerr nonlinearity and it is defocussing type. Contour plot of this figure is shown below.

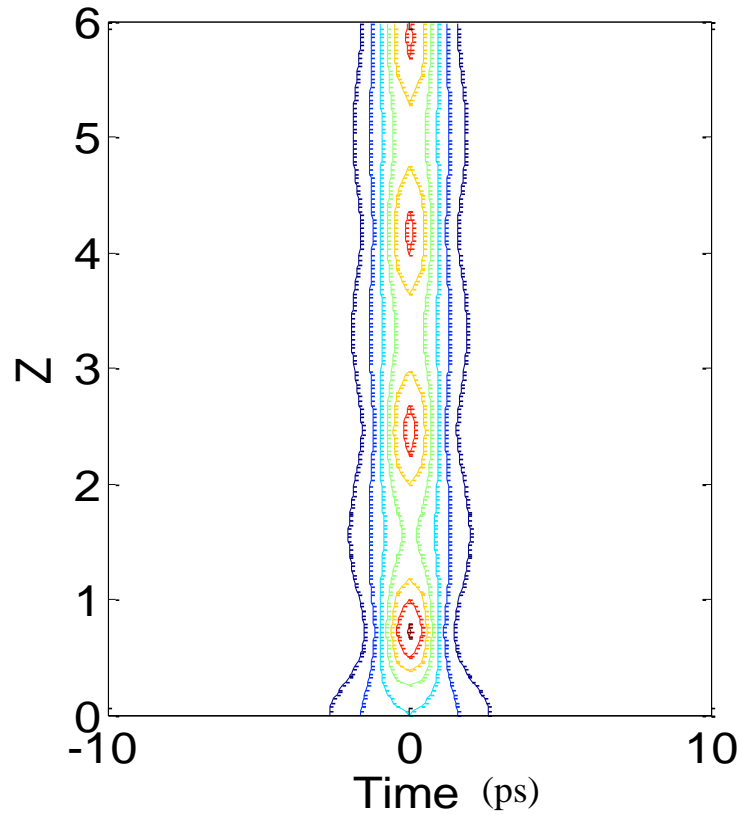


Fig. 4.15 Contour plot of propagation of Super Gaussian optical soliton ($m=1$) with normalized propagation distance in presence of perturbation

It is observed that after adding small perturbation to medium, behaviour of optical soliton propagation changes from straight to periodic. Now, optical soliton is propagating periodically with distance.

Figure 4.16 shows evolution of Super Gaussian optical soliton (order $m=2$) with normalized distance of propagation Z . Value of cubic nonlinearity taken in this case is $\gamma_1 = 3.6 \text{ W}^l/\text{km}$. Small perturbation $\delta = 0.04$ and $\beta = 0.15$ is applied. It is found that, similar to case of optical pulse propagation without perturbation, Super Gaussian optical pulse changes its shape to Gaussian. It is further observed that effect of adding perturbation to medium is that pulse propagation is now periodic in nature with oscillatory behaviour.

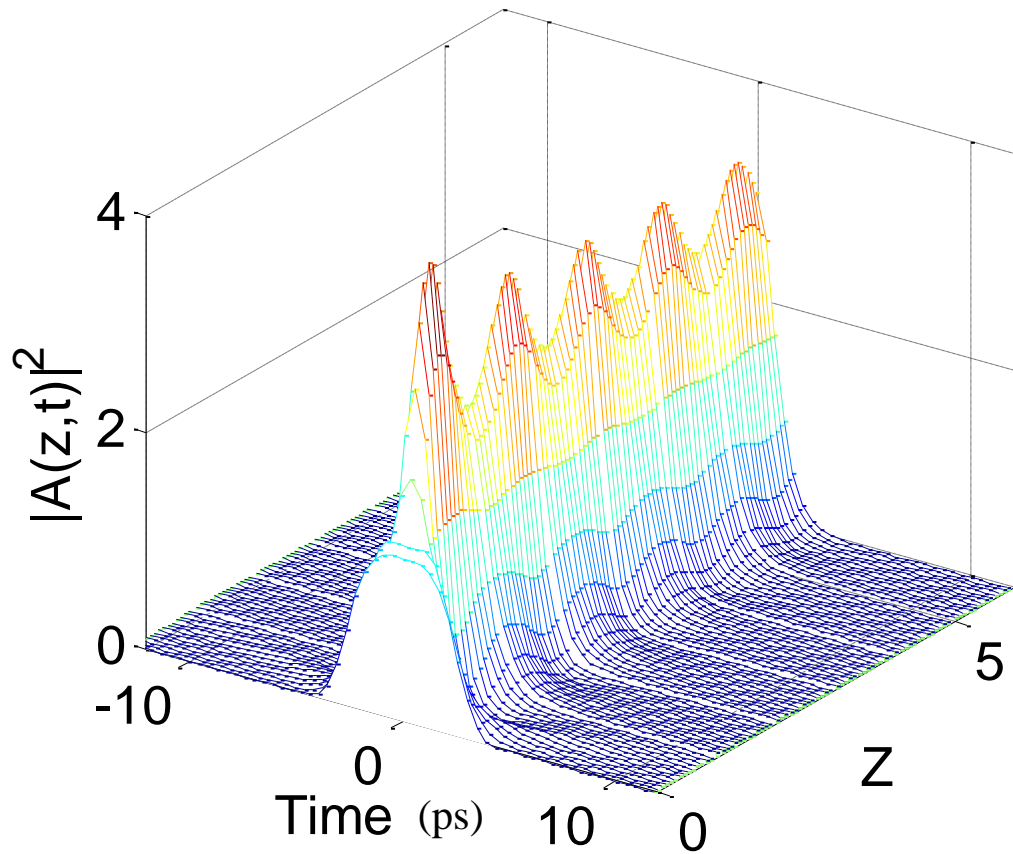


Fig. 4.16 Propagation of Super Gaussian optical soliton (with order $m=2$) with normalized distance of propagation in presence of perturbation

Contour plot of propagation of Super Gaussian optical soliton with order $m=2$ with propagation distance is shown in figure 4.17. It can be observed from the figure that initially there are few peaks that vanishes with distance. This peak comes because optical soliton takes some time to get stable. Once it has catch a behaviour, it continues to travel in same form for longer distances. As we can observe from contour plot that after some distance, self similar pattern is repeating. It proves that we are getting stable optical soliton pulse propagation for Super Gaussian optical solitons of order $m=2$.

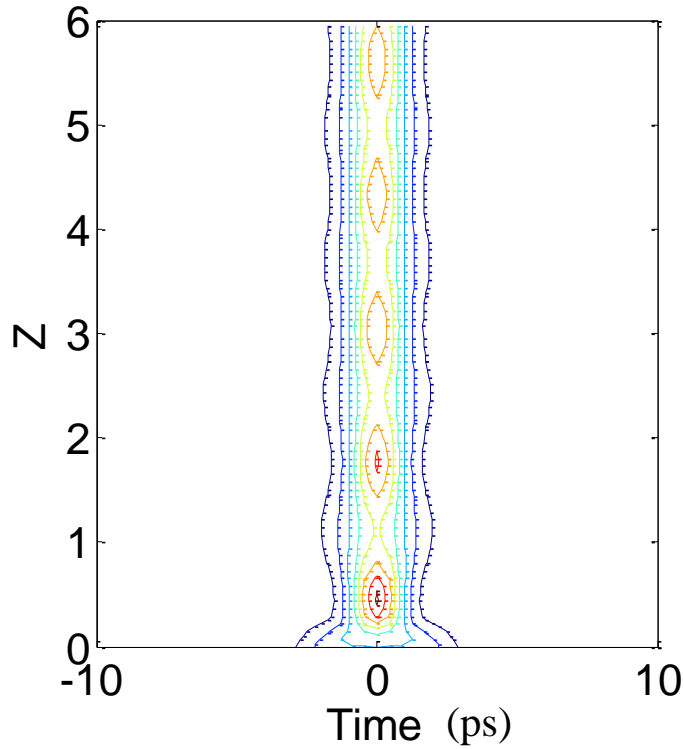


Fig. 4.17 Contour plot of propagation of Super Gaussian optical soliton ($m=2$) with normalized propagation distance in presence of perturbation

We have applied perturbation in Super Gaussian optical soliton with order $m=3$. We observed that stable optical solutions can be achieved for a fiber with Kerr nonlinearity $\gamma_1 = 3.8 \text{ W}^1/\text{km}$ and quintic nonlinearity set to 1% of cubic nonlinearity. It is well known that Super Gaussian optical soliton with $m=3$ is a complicated pulse. It is observed from figure 4.18 that Super Gaussian optical soliton with order $m=3$ changes its shape to Gaussian. It is also observed that after changing its profile to Gaussian, it travels by undergoing periodic oscillations.

We observed that in this case we don't get stable soliton pulse propagation at the parameter values of kerr nonlinearity, quintic nonlinearity and group velocity dispersion as we used in case of conservative optical medium. It is because of interplay between the perturbation terms that we introduced to study dissipative optical medium. We set the values of perturbation terms accordingly that we get stable soliton propagation. In order to get that we change the parameter values of kerr nonlinearity factor to $3.8 \text{ W}^1/\text{km}$. Value of Group velocity dispersion remains same in this case.

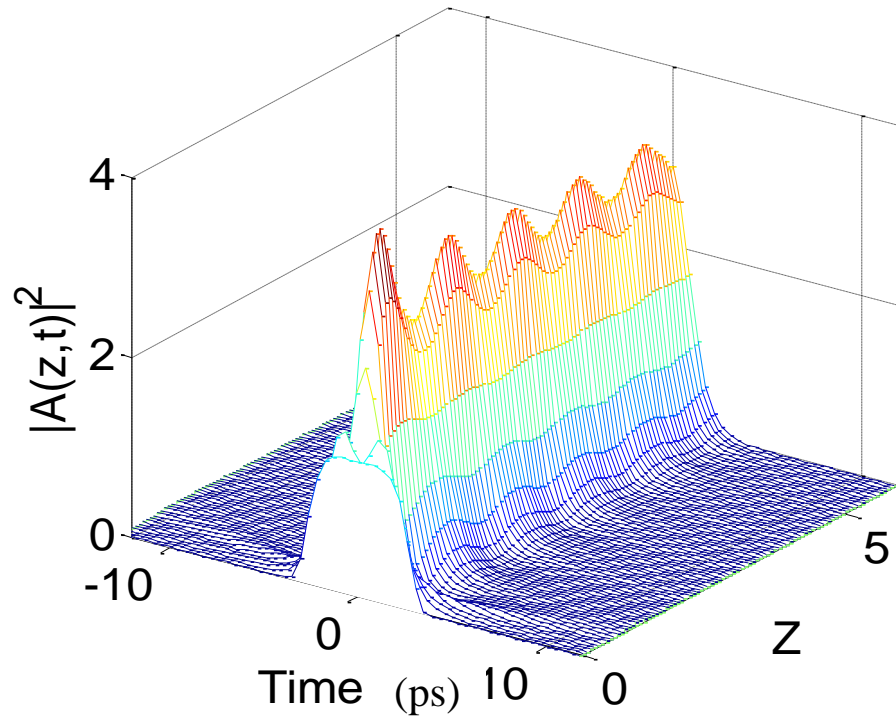


Fig. 4.18 Evolution of Super Gaussian optical soliton pulse taking order $m=3$ with normalized distance in presence of perturbation

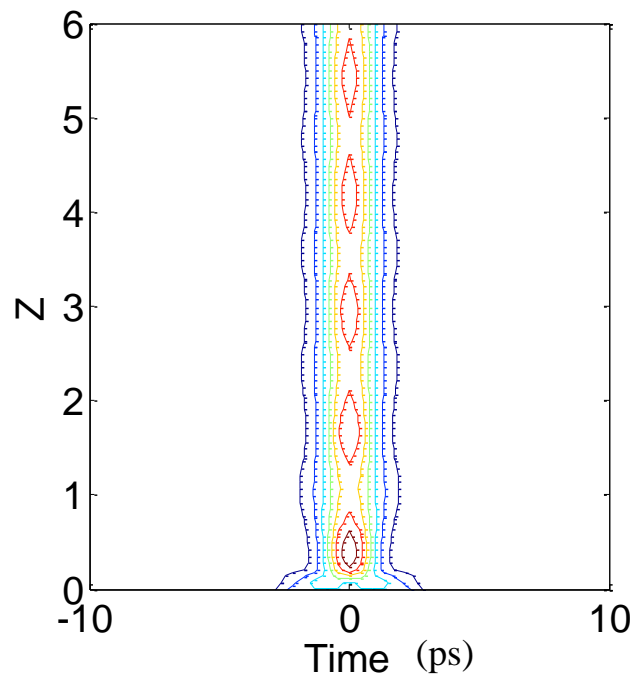


Fig. 4.19 Contour plot of propagation of Super Gaussian optical soliton with order $m=3$ in presence of perturbation

Conclusion and Future Scope

In this work, we studied and investigated evolution of Super Gaussian optical solitons in Cubic-Quintic medium with perturbation. Main motivation of this work was to find out set of various parameters at which we can get stable optical soliton propagation. We observed effect of adding higher order nonlinearity (Quintic nonlinearity) in Kerr medium. We studied effects of perturbation in fiber having Cubic- Quintic nonlinearity. We also observed effect of only gain or only loss in fiber. Following conclusions can be made from our study:

1. As value of Defocusing quintic nonlinearity increases, behaviour of pulse propagation changes into oscillatory motion and these oscillations are periodic in nature, so it is also a stable soliton also called ‘Breathing Solitons’. So value of quintic nonlinearity is taken between 1-10% of Kerr nonliarity.
2. It is observed from numerical simulation experiments that Super Gaussian optical pulses are unstable in Cubic-Quintic nonlinear media. In this media, they convert into Gaussian type pulse.
3. Exact balance of loss and gain in perturbed medium is required for stable optical soliton formation in Cubic-Quintic medium. In medium with only gain or only loss we can’t get Super Gaussian optical soliton propagation.
4. We can’t get stable optical soliton solutions for medium, in which perturbation is added, keeping same values of Kerr nonlinearity as in case of propagation of optical solitons in non-perturbed medium. We have to adjust value of Kerr nonlinearity in perturbed medium.
5. In our study, we found out that our fiber can tolerate gain upto 4% and loss upto 15%. More loss or gain will have detrimental effect on optical soliton propagation.

The future scope of our work is that the results could be used by designers of optical fibers that can be used for propagation of optical solitons up to longer distances. Even in presence of gain or loss in medium, soliton propagation will not

be effected. Further, these fibers can be used in optical communication systems which will increase reliability of overall system.

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