Neutrino oscillations :Short,Long and Magic baselines

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Certificate

This is to certify that the thesis entitled "Neutrino oscillations" submitted by Miss Simran Jeet Kaur , Roll No. 301304011 in partial fulfilment of the requirement for award of the degree of MASTER OF PHYSICS from School of Physics and Material Science, Thapar University, Patiala. It is to certify that the matter embodied in this report is of the candidates own record and has not been submitted in part or full to any other university or institute for award of any degree

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Abstract

Neutrino oscillations are signature of small masses of neutrinos which are considered to be mass less in standard model, thus it points going beyond standard model. Thus determination of neutrino oscillations is important for formulating theories beyond standard model. In this work, we study phenomenon of two and three flavour neutrino oscillations giving theoretical frame on solar and atmospheric neutrino oscillation probabilities for three generation case, we also discuss currently going neutrino parameters and some detectors, reactors working on neutrino oscillations. After that discuss about baselines (short, long and magic) studying oscillations using different parameters (L,E),then plots for electron muon and taun are drawn with analysis and further motivation in neutrino oscillations

Contents

Chapter 1 :
Introduction

Chapter 2 :
Neutrinos

Chapter 3 :
Two and Three flavour oscillations

Chapter4:
Baselines with Oscillation plots, analysis and further motivation

Chapter one : Introduction

1.1 Elementary Particle Physics

In Elementary particle physics at fundamental level i.e. on subatomic scale matter is made of tiny chunks, with large empty spaces in between. Even these tiny chunks come out as small number of different neutrinos and many more, which are then replicated in astronomical quantities to make all the 'stuff' around us. And these replicas are completely perfect copies. This feature of great similarity has no comparison in the physical world. It just clearly simplifies the task of particle physics that we don't have to worry about big electrons, small ones, new and old ones.

The motivation towards elementary particle physics experiments is the desire to know about nature of fundamental forces and particles. Our current standard model can describe it, which can generate more effective theory in next generation of experiments. In the electroweak sector, where great success was achieved which unifies both electromagnetic and weak nuclear forces, where analysis taken at CERN large electron positron collider (LEP) and the Fermi lab proton-antiproton collider (Tevatron) require that there will be either a light Higgs particle having mass 200 GeV or something wiggling between interactions.

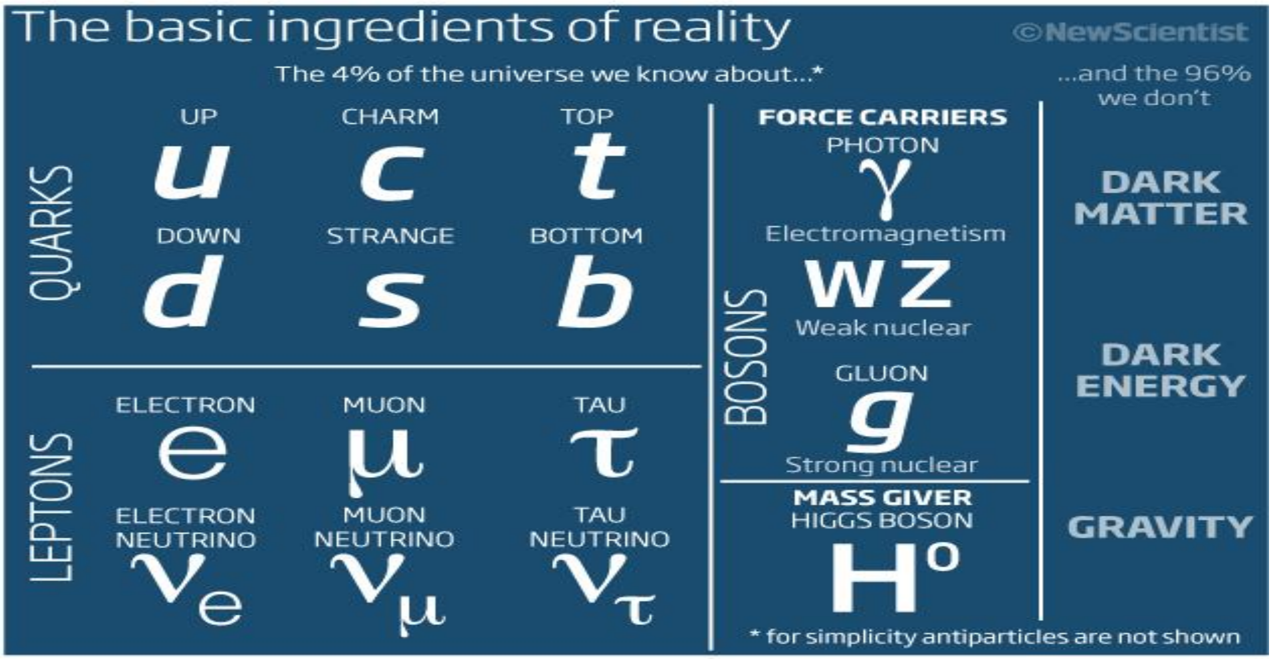
1.2 Standard model

If we look around us question arises that IS EVERYHING in universe is real? The question echoes only one answer: yes it is. But, try to kick a rock science will give different answer.

Keeping aside that question think of what are you actually kicking? When it boils down to it, not a lot. Science investigates differently for rock through lots of different particles, the forces responsible for their interactions and some rules of quantum mechanics.

Taking rock aside, science finds that its fundamental ingredient is atoms – which can be 1000 trillion or more in number, varying according to rock size. Atoms further are divided into smaller subatomic particles, namely protons and neutrons which are built of quarks and electrons with mostly empty space. We would find that distance between electron and nucleus in atom is 2.5 greater than in comparison between earth and sun with large amount of emptiness in between .If real world is made of so much emptiness, then what gives structure and bulk in rocks and other objects? In Physics answer will be electrons. Quantum physics says that no two electrons can occupy the same quantum state that is two atoms cannot be fixed together in same space

It does not mean the nucleus is superfluous. Atoms is majorly composed of protons, neutrons and fundamental forces which binds it together, each having a force carrier called gluons, these all together makes most of the matter around us. But standard model of particle physics says there are basic total 17 constituents which makes matter, thus whole world around us in which photons and neutrinos are common zipping through us but top ,bottom quarks and the heavy, electron-like tau particle are not so common part of everyday life. This makes standard model great significant in world of particles whose basis can be found in symmetry and group theory which is believe to raise puzzles of reality and mathematics



Everything around us comes from basic ingredients called fundamental particles and forces governing them. Standard model of particle physics give us deep insight that how both are related to each other

Matter particles (fermions)

Fundamental structure to matter is given by elementary particles called quarks and leptons. Both group carry 6 particles each forming pairs, also called generations, there are 3 generation, first generation carry up, down quark, second consists of charm and strange and third consists of top and bottom quarks. First generation are lightest and becomes heavy as move to next generations. Stability order also decreases as move from first to third generation, thus third generation particles quickly decays. Quarks also have colours namely red, blue, and green, these coloured quarks combine to form colourless particles which can be identified. Leptons also form 3 generations- first consists of electron, electron neutrino, second have muon ,

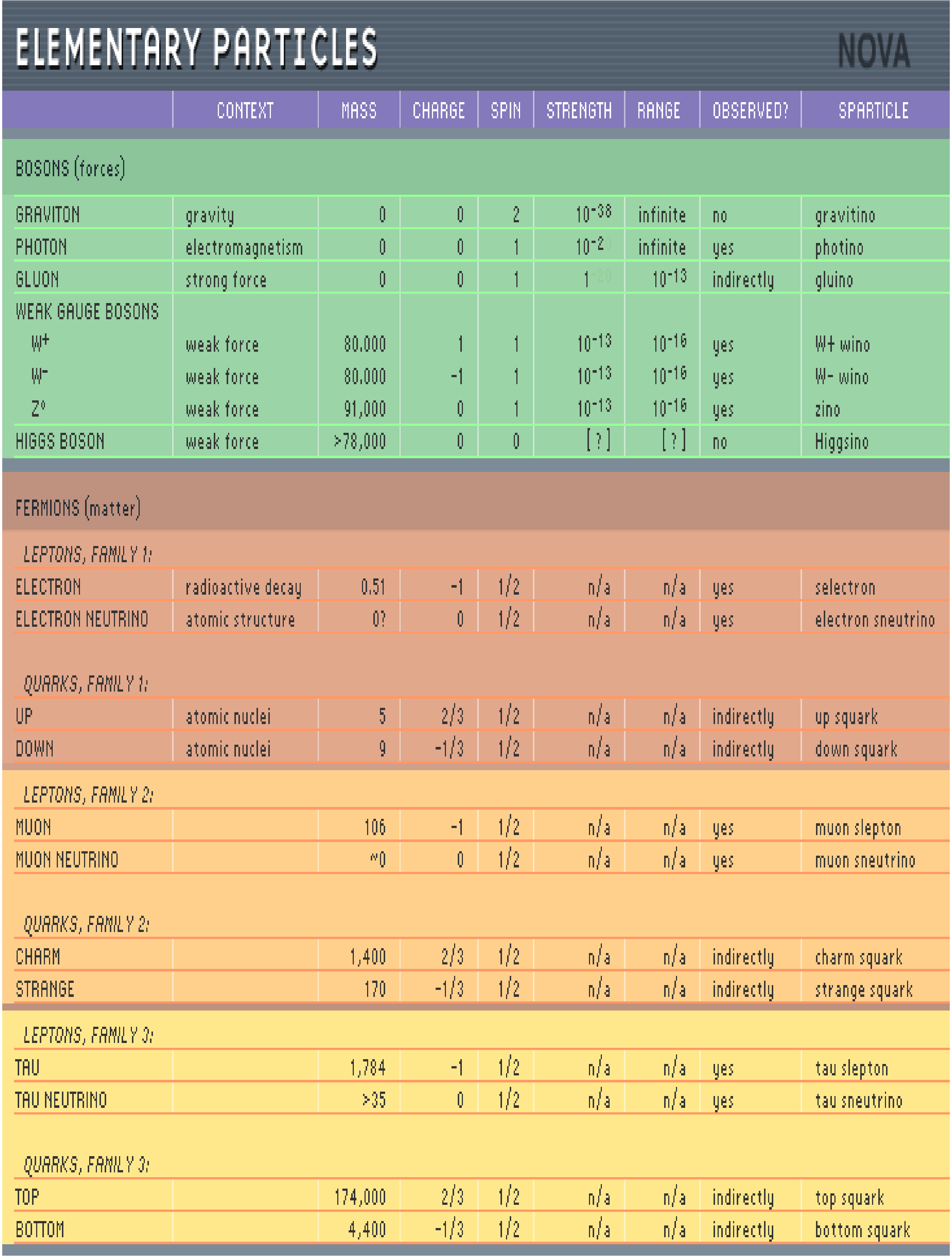
muon neutrino and third carry tau ,tau neutrino. Neutrinos don’t have any charge and very little mass also, but other leptons have charge as well as significant masses

Basic Forces and Interacting Particles

There are four fundamental forces governing everything around us: the strong force, tweak force, electromagnetic force, and gravitational force where each differs in ranges and strengths. Gravity and electromagnetic has infinite range whereas weak and strong forces influence to some very short range. In terms of strength gravity is weakest then strength increases from weak to electromagnetic to strong force, thus strong force is strongest force of all .These fundamental forces exist due to exchange of force-mediatating particles also called bosons i.e. gluon act as exchanging particle in strong force, photon in electromagnetic and W, Z bosons in weak force ,graviton responsible for gravity is not experimentally confirmed yet.

All these forces and exchanging particles are described in standard Model explaining how these forces govern us .However, gravity is still not fitted in standard model. dark matter and dark energy is also not explained in standard model, which is most dominate in universe, about 96% is dark matter and dark energy altogether

Table shows fundamental particles , forces and their characteristics



ELEMENTARY PARTICLES

NOVA

	CONTEXT	MASS	CHARGE	SPIN	STRENGTH	RANGE	OBSERVED?	SPARTICLE
BOSONS (forces)								
GRAVITON	gravity	0	0	2	10^{-38}	infinite	no	gravitino
PHOTON	electromagnetism	0	0	1	10^{-2}	infinite	yes	photino
GLUON	strong force	0	0	1	1	10^{-13}	indirectly	gluino
WEAK GAUGE BOSONS								
W^+	weak force	80,000	1	1	10^{-13}	10^{-16}	yes	W+ wino
W^-	weak force	80,000	-1	1	10^{-13}	10^{-16}	yes	W- wino
Z^0	weak force	91,000	0	1	10^{-13}	10^{-16}	yes	zino
HIGGS BOSON	weak force	>78,000	0	0	[?]	[?]	no	Higgsino
FERMIONS (matter)								
LEPTONS, FAMILY 1:								
ELECTRON	radioactive decay	0.51	-1	1/2	n/a	n/a	yes	selectron
ELECTRON NEUTRINO	atomic structure	0?	0	1/2	n/a	n/a	yes	electron sneutrino
QUARKS, FAMILY 1:								
UP	atomic nuclei	5	2/3	1/2	n/a	n/a	indirectly	up squark
DOWN	atomic nuclei	9	-1/3	1/2	n/a	n/a	indirectly	down squark
LEPTONS, FAMILY 2:								
MUON		106	-1	1/2	n/a	n/a	yes	muon slepton
MUON NEUTRINO		~0	0	1/2	n/a	n/a	yes	muon sneutrino
QUARKS, FAMILY 2:								
CHARM		1,400	2/3	1/2	n/a	n/a	indirectly	charm squark
STRANGE		170	-1/3	1/2	n/a	n/a	indirectly	strange squark
LEPTONS, FAMILY 3:								
TAU		1,784	-1	1/2	n/a	n/a	yes	tau slepton
TAU NEUTRINO		>35	0	1/2	n/a	n/a	yes	tau sneutrino
QUARKS, FAMILY 3:								
TOP		174,000	2/3	1/2	n/a	n/a	indirectly	top squark
BOTTOM		4,400	-1/3	1/2	n/a	n/a	indirectly	bottom squark

Family of fermions are described below

Family 1(Leptons and Quarks)

Electron and Electro neutrino

Electron is a charged particle having charge -1, due to its negative charge it attracts positive charges like protons which is responsible for bounding them together in nucleus. Negative charge of electron also cause repulsion from other atoms and keep them apart. J.J Thomson was first to detect this subatomic particle, around end of 19th century. Neutrino are least massive particles (nearly mass less) and they are almost noninteracting , they are so delicate that even every second trillions of pass through our body giving only one to two reactions in whole lifetime . They interact through matter via gravity and weak forces. Pauli was to discover in 1930 then confirmed by reines and cowan experiment in 1956

Up and down quark

Up and down quark together makes up the matter around us. Both quarks combination makes protons and neutrons responsible to give volume to all matter. Neutron consists of udd quark combination ((-1/3) +(-1/3) + (2/3) = 0) and proton has uud combination ((-1/3) + (2/3) + (2/3) = 1). In 1960’s Murray Gell-Mann and George Zweig hypothesis confirmed existence of quarks.

Family 2

Muon And Muon neutrino

Muon is a negatively charged, massive and unstable subatomic particle which quickly disintegrates into electron, ν_μ, and $\overline{\nu}_e$ .it was identified in 1937 by Jabez C street and Edward C Stevenson

Muon neutrino confirmed in 1961 by Steinberger and two others is different from electron neutrino on the basis of radioactive decays. Electron neutrinos are related with radioactive decay which makes electron along with a neutrino. Muon neutrino is linked with radioactive decay which makes muons also.

Charm quark and strange quark

Charm quark is massive and fourth in number found indirectly in 1974 after up,down and strange. It is more massive then up quark but some properties are similar also with up quark. Strange particle was also discovered indirectly in 1947, found within V particle along with u and d quarks. Its name was also given due to its strange behaviour

Family 3

Tau and Tau neutrino

Martin perl discovered tau lepton in 1975. Tau lepton is alike to electron, but very massive and highly unstable, it quickly disintegrate to other particles in less than trillionth of second. Tau neutrino is heaviest of all other neutrinos whose existence was established in 2000 if it produces tau lepton on hitting the atomic nuclei

Top and bottom quark

Top was sixth and bottom was fifth quarks discovered in 1955 and 1976 respectively .one top quark mass is analogues to gold nuclei consisting of 197 protons and neutrons that are made up of 591 up and down quarks

Exchanging Bosons

Graviton

It is an exchanging particle of gravitational force, which is weakest force than others. It is only attractive force which becomes stronger as larger the mass is concentrated in an area. Other forces like electromagnetic is repulsive as well as attractive also. Graviton is not experimentally confirmed yet

Photon

Electromagnetic forces are due to exchanging particle called photon. In an atom attraction between negative charged electron and positive charged nuclei holds atom together due to presence of photon, electromagnetic force. Atoms and molecules are bounded (that makes our body) due to electromagnetic force, without which we will not be able to do most of our daily life works

Weak gauge bosons

There are two charged (W^+,W^-) and one neutral(Z^0) weak gauge bosons, mediatators of the weak force. The weak force generates radioactive decay by altering down to up quark in neutron, thus changed into proton with release of electron. Weak force can also convert proton to neutron. It was confirmed experimentally by Carlo Rubbia with group of 130 physicists in 1983

Gluon

The gluon, which is boson of the strong force, having range of 10^{-13} cm or less. Protons, neutrons and other particles are formed due to some combination of quarks, these quarks are always bounded together due to strong force. It was discovered indirectly in Deutsches Elektronen Synchrotron in 1979.

Higgs boson

Higgs boson reasoned that why weak gauge bosons has significant mass which was first thought as mass less. Higgs boson creates a higgs field, which slows down weak bosons and gives a significant mass. As the temperature is increased higgs field is suppressed, and weak gauge bosons become mass less travelling at speed of light .Higgs boson was discovered by Peter Higgs

The Quantum Numbers Involved In Particle Physics

Mass

acc to string theory ,all elementary particles are made of small, vibrating strings whose energy can tell us mass of particle as mass and energy are connected by $E=Mc^2$. Figure shows mass units in MeV .

Charge

particle having charge equal 1 resembles a positive electrical charged, one with -1 is negatively charged, and that of 0 tells that it is electrically neutral. Quarks have fractional charges, which merge in such a way resulting charges +1,-1 or 0 forming particles like protons and neutrons

Spin

Experimentally intrinsic angular momentum known as spin is confirmed in every elementary particle. All fermions have spin 1/2, whereas bosons have whole integer spin.

Strength

All four forces, having different ranges, differs in strength, in which strongest is strong force whereas gravity is weakest. Figure shows all other forces are with respect to strong force which is having strength 1

Range

all exchanging particles have certain regions of influence (range) i.e. photon, graviton has infinite range but others have limits on ranges. Figure displayed have range units in cm

1.3 Beyond standard model

Standard Model, is still not able to explain origin of mass, the strong CP problem, neutrino oscillations, matter– antimatter asymmetry, and the nature of dark matter and dark energy, thus it needs more construction to explain these limitations and going beyond it. There are several theories which are beyond standard model like super symmetry, string theory, M-theory and extra dimensions.

Limitation of Standard Model

Standard model is able to tell so much about elementary particles much insufficient in explaining following things

1. Gravity: gravity exclusion is one of major limitation of standard model. Experimentally also graviton existence is not confirmed which is reason that it is not able to fit in standard model.

2. Dark matter and dark energy: Cosmological observations says that universe contains 26% of dark matter, and 69% dark energy, which are both exempted out of standard model

3. Neutrino masses: In standard model neutrino are considered as mass less, but neutrino oscillation experiments reveals mass of neutrino .Even if neutrino mass is added in standard model, it can give rise to new theoretical problems.

4. Matter-antimatter asymmetry: Everything around us is made of matter but standard model tells that matter and antimatter should be in equal proportions, but still now symmetry between them is not confirmed.

Super Symmetry

Super symmetry is the theory that the Standard Model, our best picture of how the universe works, is only half the story. Super symmetry says that every particle in the Standard Model has a corresponding super symmetric particle yet to be discovered. The principle is attractive to scientists because it would tie up serious loose ends in our understanding of how the universe hangs together. If super symmetry bears out, it would explain minor mass of Higgs boson and how the universe's four forces are really different aspects of a single force. It would also get us closer to a good, working description of dark matter. At micro scale, particles posses different symmetries like matter and antimatter differ in charge though they are identical. Some of them differ in mass also like Photons of EM force and Z bosons in weak force differ in mass. Thus particle physics can tell lot about symmetries in nature. Super symmetry can reveal lot about symmetry in matter and forces, higgs boson light mass due to which it is very difficult to observe. Exact super symmetry can also tell us of squarks which are particles of force having same mass as particle of matter but they are not experimentally confirmed yet. Whereas broken symmetry can tell us how this big universe is unified .various searches are still going on supersymmetry like at LHC should be very general and limited applicability

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CHAPTER TWO : NEUTRINOS

2.1 Neutrino sources

Below are following points which tell of types of neutrino sources available

1. **Atmospheric**: muon,electron neutrinos and their antineutrinos are constituents of atmospheric neutrinos .The interactions of cosmic rays with atmospheric nuclei results in muon,pions and kaons whose further decay produce atmospheric neutrinos whose energy ranges between 100 MeV to 100 GeV with total flux proportional to inverse of square of its energy

2. Solar: electron neutrinos are solar neutrinos whose formation occurs in sun through thermonuclear reaction ,and there are mainly two types of these reactions, called pp(proton-proton) chain and CNO(carbon ,Nitrogen, oxygen) cycle, which are responsible for fusion of protons into 4–helium and production of electron neutrinos the solar neutrino fluxes and energy spectra are computed using specific solar models incorporating many observed parameters like the solar surface luminosity, the mass and age of the sun etc. One such model known as the standard solar model

3. Astrophysical: neutrinos of very high energies are produced by astrophysical sources like gamma ray burst and active galactic nuclei jets. Charged pions generated by reactions with ambient photons and protons within the source decay to give muons and moun neutrinos. The muon further decay to give electron and muon neutrinos. In most models, the ratio of electron to muon neutrinos is 1:2 and that of muon anti-neutrinos to muon neutrinos is 1:1 while the electron anti neutrinos to electron neutrino ratio is 0 for $p\Upsilon$ reactions and 1:1 for pp reaction, they have energy up to order of 10^6 TeV and are thus known as ultra high energy neutrinos

4. Reactors: neutrinos may also be produced by nuclear reactors, they produces an isotropic ν_e flux with neutrino energies of order of MeV . Oscillation experiments at reactors all disappearance experiments with low neutrino beam energy due to which smaller value of the neutrino mass squared difference can be accessed ex CHOOZ,D-CHOOZ ,Palo Verde

5.**Neutrino factories** : These factories are able to control 10^{20} muons in a year ,using muon storage rings which accelerates muons(~20 GeV) by circulating in these long ,parallel connected tracks . Muon decays in rings produce high flux neutrinos

Experiment	**Country**	**Type of Detector**	**Major expt. Type**	**Time schedule**
Super-Kamiokande	Japan	Water Cerenkov	Solar, supernova, atmospheric	1996-
K2K	Japan	Water Cerenkov	Long baseline	1999-
Hyper-Kamiokande	Japan	Water Cerenkov	Atmospheric, long baseline	Non Decided
SNO)	Canada	D^2O Cherenkov	Solar, supernova	1999-
GNO)	Italy	Gallium	Solar	1998-
ICARUS	Italy	Liquid Argon	Atmospheric, long baseline	2010-
KamLAND	Japan	Scintillator	Reactor	2001-
Double-chooz (DC)	France	Scintillator	Reactor	2007
MiniBooNE	USA	Scintillator	Short baseline(from fermilab booster)	2003-
MINOS	USA	Mag iron cal	Long baseline(from fermilab main injector),	2005
OPERA	Italy	Lead/emulsion	Long baseline(from CERN),tau appearance	2003-
UNO	USA	Water Cerenkov	Atmospheric, long baseline	Non Decided
ICAL/INO	India	Mag iron cal.	Atmospheric, long baseline	Non Decided

Table gives information of currently working and future detectors

2.2 Neutrino oscillations

To determine age of sun in 1830-1860, when Lord Rayleigh believed gravity to be origin of sun's energy. On the basis of the known rate of solar radiation, he showed that the maximum possible age of the sun was substantially shorter than the age of the earth. In 1896, Becquerel discovered radioactivity this suggested that nuclear fission, not gravity, might be the source of the sun's energy, and this would allow for a much longer lifetime. In 1920 Eddington suggested process nuclear fusion powers the sun .In 1938, Hans Bethe worked out the details. In heavy stars the dominant mechanism is the CNO cycle, But in the sun (and other relatively light stars) the dominant routes the so-called pp chain it all starts out as hydrogen(protons), and it all ends up as a particles (helium-4 nuclei) - plus some electrons, positrons, photons and neutrinos. But how can we tell what is going on inside the sun? Photons take a thousand years to work their way out from the centre to the surface, and what we see from earth doesn't tell us much about the interior. But neutrinos -because they interact so weakly, emerge virtually unscathed by passage through the sun. Neutrinos, therefore, are the perfect probes for studying the interior of the sun. There are certainly plenty of neutrinos coming from the sun. Majority of estimation of solar neutrino abundances was predicted by John Bahcall, who said that every second more than 10 billion neutrinos are zipping through our body, but no worries they do not harm us, leaving only one or two neutrino induced reactions in our body for our whole lifetime,this property tells that they are very delicate or non interacting. In 1968 Ray Davis reported the first experiments to measure solar neutrinos, using a huge lank of chlorine in the Homestake mine in South Dakota (you have to do it deep underground to eliminate background from cosmic rays). The Davis experiment - for which he was finally awarded Nobel Prize in 2002 - collected argon atoms for several months .The total accumulation was less than estimation of bahcall (about a third). This pointed out the famous solar neutrino problem.

At the time, most physicists assumed the experiments were wrong. But gradually the community came to take the solar neutrino problem seriously - especially when other experiments, using quite different detection methods, confirmed the deficit .In 1968, Bruno Pontecorvo suggested a beautifully simple explanation for the solar neutrino problem. He proposed that the electron neutrinos produced by the sun are transformed in flight into a different species (muon neutrinos say, or even antineutrinos) to which Davis' experiment was insensitive. This is the mechanism we now call neutrino oscillation. The theory is basically the quantum mechanics of mixed states which is analogues to the classical theory of coupled harmonic oscillators.

In 2001, the Super-Kamiokande collaboration presented its results on solar neutrinos using process elastic neutrino-electron scattering. They recorded 45% of the predicted number assuming all of these neutrinos were still electron neutrinos but their detector is less efficient in counting μ and τ neutrinos. If some of the ν_e 's had converted to ν_μ or ν_τ then the actual flux would be higher- but how much higher they could not say,

because they had no way of knowing what fraction of the neutrinos had in fact converted.
Meanwhile, at Sudbury Neutrino Observatory (SNO) a very similar experiment was under way, using heavy water and enables one to evaluate separately electron neutrino flux and the total neutrino flux. In April 2002 SNO collaboration solved the solar neutrino problem, and neutrino oscillations confirmed

However, neutrino oscillations was first verified at Kamiokande in early 1990s using atmospheric neutrinos suggesting that the muon neutrinos are converting to a different flavour.

Consider the propagation of neutrinos (generated as ν_e) towards the Earth, where our detector is situated. We shall see that, because of neutrino oscillation, some of our ν_e are "magically" transformed into different neutrinos (but the secret lies in the laws of quantum mechanics).This means neutrino have masses which in standard model assumed to be mass less, initial known neutrinos are called flavour eigenstate and other as mass eigen states whose mixing decides the oscillations of flavour states i.e. we can say that if neutrino oscillations are present ,than as one neutrino flavour is generated from source say ν_μ via weak interaction ,as they travel certain distance L, having some energy E ,the probability $P(\nu_\mu \rightarrow \nu_\tau; E, L)$ to get different neutrino say ν_τ will not be zero ,this probability is known as transition probability and ν_μ not changing to other type is called survival probability $P(\nu_\mu \rightarrow \nu_\mu; E, L)$ which is smaller than one .The experiment in which muon neutrinos oscillations are detected as it traverse distance from its source to detector is called disappearance experiment

The ideal test of neutrino oscillations involves a fixed source (a reactor or an accelerator) and a movable detector. As the separation increases, one would monitor the sinusoidal variation. Probability of neutrino changing type is relayed to distance and energy from point of production to destination neutrinos travelling greater distances exhibit greater depletion from oscillation

2.3 Neutrino parameters

For Three neutrino flavours, the information available for neutrino masses and mixings are the following:

- There are two independent mass squared differences which are $|\Delta m_{31}^2| \sim 10^{-3}$ eV^2 and $\Delta m_{21}^2 \sim 10^{-5}$ eV^2
- There are three mixing angles , in which two (θ_{12} ,θ_{23}) are large , but θ_{13} is small with limited upper bound ($\sin^2\theta_{13} < 0.16$)

Global analyses of the neutrino oscillation data available from solar(HOMESTAKE,SAGE,GNO,SK,SNO),atmospheric(SuperKamiokande),reactor (KamLAND and CHOOZ) and accelerator(K2K,MINOS) experiments, made possible to get values of three neutrino oscillation parameters. Below table shows best fit values and allowed ranges of these parameters, from table we can say that θ_{23} is close to, but possibly to be different from $\pi/4$, $\theta_{12} \cong \pi/5.4$ and $\theta_{13} \cong \pi/20$. Note also that Δm^2_{21}, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ are with 1σ uncertainty (= 1/6 of the 3σ range) of approximately 2.6%, 5.4%,2.6%, 9.6% and 8.5%, respectively. Currently there is no experimental value available for CP violation phases .

Parameter	**best-fit(±1σ)**	**3σ**
$\Delta m^2_{21}[10^{-5}$ eV$^2]$	$7.54^{+.26}_{-.22}$	6.99-8.18
$\Delta m^2[10^{-3}$ eV$^2]$	2.49±.06(2.38±.06)	2.23-2.61(2.19-2.56)
$\sin^2\theta_{12}$, $\Delta m^2>0$	0 .308±.017	0.259-0.359
$\sin^2\theta_{23}$, $\Delta m^2<0$	$0.437^{+.033}_{-.023}$	0.374-0.628
$\sin^2\theta_{23}$, $\Delta m^2>0$	$0.455^{+.039}_{-.031}$	0.380-0.641
$\sin^2\theta_{13}$, $\Delta m^2>0$	$0.0234^{+.0020}_{-.0019}$	0.0176-.0295
$\sin^2\theta_{13}$, $\Delta m^2<0$	$0.0240^{+.0019}_{-.0022}$	0.0178-.0298

Table shows best–fit values and 3σ allowed ranges of neutrino parameters, obtained from global fit of current neutrino oscillation data. The values (in bracket) correspond $m_1<m_2<m_3$ ($m_3<m_1<m_2$). The definition of Δm^2 used is $\Delta m^2=(\Delta m^2_{31}-\Delta m^2_{21})/2 >0$ if $m_1<m_2<m_3$ and $\Delta m^2=(\Delta m^2_{32}+\Delta m^2_{21})/2 <0$ if $m_3<m_1<m_2$

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CHAPTER THREE: TWO AND THREE FLAVOUR OSCILLATIONS

3.1 Two Flavour neutrino oscillation

In 1969 Bruno Pontecorvo told that if neutrinos have significant masses with mixing which is resulting into oscillations, it may be solution for arising Solar Neutrino Problem. For example, if we have two neutrino flavours whose original state is called flavour eigenstates e.g. e and μ but travel as mass eigenstates labelled as 1 and 2 having different masses .If flavour do not remains the same and masses m_1 and m_2 also differ, than these two eigenstates are mixed as written below

$$\nu_1=\nu_e \cos\theta + \nu_\mu \sin\theta \quad \nu_2=\nu_e \sin\theta+ \nu_\mu \cos\theta$$

Cosθ and sinθ makes sure that from starting there is neither more or less neutrinos born due to mixing. It is known as “unitary” transformation

Consider an electron neutrino is generated, then expression of its survival probability at distance L is

$P_{ee}=P_{\mu\mu}=1- \sin^2 2\theta \sin^2((\Delta m^2 L)/E)$

Let ν_e,ν_μ be the flavour eigen states and ν_1 ,ν_2 be the mass eigen states with masses m_1 and m_2, respectively and both are having momentum p. States in the flavour and mass can be combined with mixing matrix U ,which effects an orthogonal transformation in two dimensions:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

Where

$$U = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

Where θ is the mixing angle, and states are orthonormalised as

$<\nu_j|\nu_k>=\delta jk$; j,k=μ,e or 1,2 (δ= delta function)

It should be noted in an interaction, neutrinos are always produced as flavour eigenstates, but evolution in terms of their mass eigenstates

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \qquad (1)$$

$$|\nu_e(t=0)> = \nu_e = \cos\theta|\nu_1> + \sin\theta\,|\nu_2> \qquad (2)$$

$$|\nu_\mu(t=0)> = \nu_\mu = -\sin\theta|\nu_1> + \cos\theta\,|\nu_2> \qquad (3)$$

The weak eigen states(ν_μ ,ν_e) are related with mass eigen states(ν_1 ,ν_2)through inclination θ. After some time t(the time evolution of a flavour state can be simply expressed in terms of time evolution of mass eigen states which enter into flavour state at t=0)

$$|\nu_\mu(t=t) = |\nu_\mu(t)> = -\sin\theta|\nu_1> e^{-\frac{iE_1t}{\hbar}} + \cos\theta|\nu_2> e^{-\frac{iE_2t}{\hbar}}$$

$$= -\sin\theta|\nu_1> e^{-\frac{i\left(p+\frac{m_1{}^2}{2p}\right)t}{\hbar}} + \cos\theta|\nu_2> e^{-\frac{i\left(p+\frac{m_2{}^2}{2p}\right)t}{\hbar}} \qquad (4)$$

Here we have used $E_1 = \sqrt{p^2 + m_1^2}$ and $E_2 = \sqrt{p^2 + m_2^2}$

E_1 and E_2 being the energies corresponding to the two mass states m_1 and m_2 . expanding $E_i=(p^2+m_i^2)^{1/2} \cong p+m_i^2/2E$, where p is neutrino momentum and we have assumed the neutrino masses to be small compared to neutrino energy($p>>m_i$),thus

$$|\nu_\mu(t)> = e^{-\frac{i\left(p+\frac{m_1{}^2}{2p}\right)t}{\hbar}}\left(-\sin\theta|\nu_1> + \cos\theta|\nu_2> e^{-\frac{i\left(p+\frac{m_1{}^2-m_2^2}{2p}\right)t}{\hbar}}\right) \qquad (5)$$

Transition probability of ν_e to ν_μ state is given as square of quantum mechanical amplitude, which is given as

$P(\nu_e \rightarrow \nu_\mu) = |<\nu_\mu|\nu_e(t)>|^2$

Where $<\nu_\mu| = \cos\theta <\nu_1| + \sin\theta <\nu_2|$

$P(\nu_e \rightarrow \nu_\mu) = |<\nu_\mu|\nu_e(t)>|^2$

$$=(e^{-iz}(-\sin\theta\cos\theta + \sin\theta\cos\theta\, e^{\frac{i\Delta m^2 x}{2p}}))^2$$

$$=e^{iz-iz}\sin^2\theta\cos^2\theta\,(1- e^{\frac{i\Delta m^2 x}{2p}})(1-e^{-\frac{i\Delta m^2 x}{2p}}) \qquad (6)$$

As neutrinos are relativistic in nature, we made following exchange in expressions below used: $p=E_\nu$ and $x=L$

Probability of finding other flavour, referred as the appearance probability is

$$P(\nu_e \rightarrow \nu_\mu)=\sin^2\theta \;\; \cos^2\theta \,(1\text{-}e^{\frac{i\Delta m^2 L}{2E_\nu}})(1\text{-}e^{\frac{i\Delta m^2 L}{2E_\nu}}) \qquad (7)$$

$$P(\nu_e \rightarrow \nu_\mu)(L,E)=\sin^2 2\theta \;\; \sin^2(1.27\, \Delta m^2 \frac{L}{E\nu}) \qquad (8)$$

Probability of finding the original flavour, referred as the survival probability is $P(\nu_e \rightarrow \nu_\mu)=1\text{-}\sin^2 2\theta \;\; \sin^2(1.27\, \Delta m^2 L/E_\nu \;\;)$

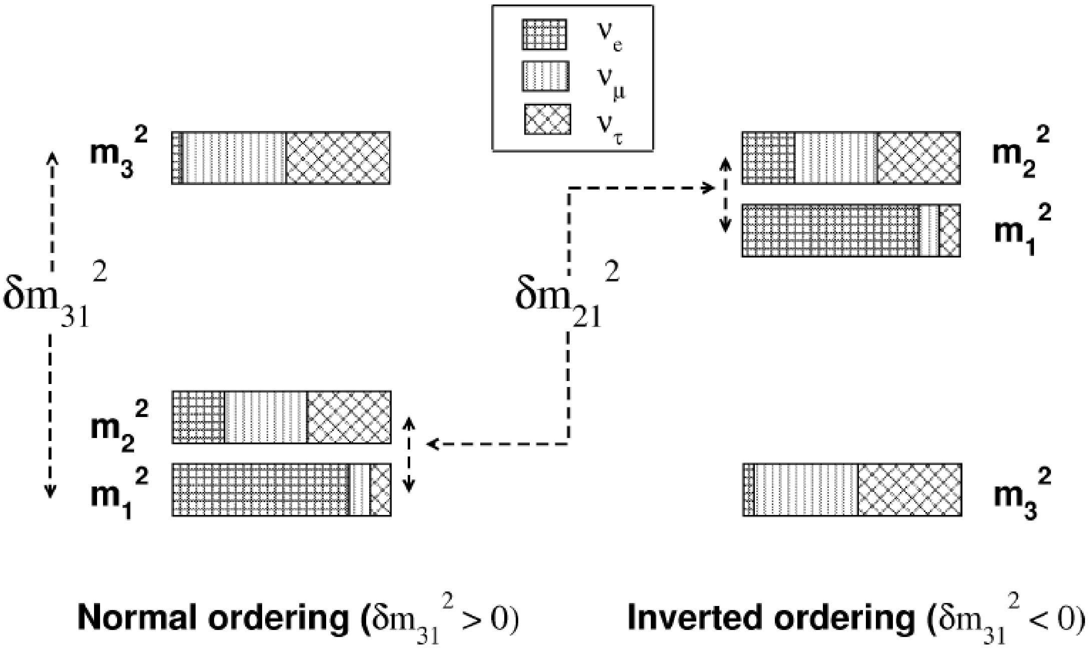
Where θ is mixing angle which signifies amount of coupling between two mass eigen states, L is length of source from the detector (km), E(GeV) is energy of neutrinos produced from its origin. Here solar, atmospheric oscillations are equivalent to 2 flavour oscillations due to weak mixing of 1,3. Scientist investigates on different mass eigen values which makes possible to get mixing at these different mass eigen states. Here Mixing angle related to transition probability is shown as $(\sin 2\theta)^2$. If replacing θ to $\pi/2\text{-}\theta$, the oscillation probability remains same ,thus concluding its degeneracy property which points out two different possibilities of two different mixings for two mass eigen states, the electron neutrino will contain more of ν_1 for $\theta<\pi/4$and if $\theta>\pi/4$ then ν_2 is more in muon neutrino. Moreover, if $(\Delta m^2 L)/2E<<1$ we will get only survival probability not transition probability

3.2 Mass hierarchy

Survival and oscillation probability rely on mixing parameters (θ_{12},θ_{13},θ_{23} and δcp phase) .survival and transition probabilities rely on given mixing parameters and two independent mass squared differences:$\Delta m_{21}^2=m_2^2-m_1^2$ and $\Delta m_{31}^2=m_3^2-m_1^2$.$\Delta m_{21}^2$ known as solar mass squared difference , it governs oscillations of solar neutrinos and Δm_{31}^2 as atmospheric mass squared difference. Δm_{21}^2 is positive in case of solar neutrino, but value of Δm_{31}^2 is limited from atmospheric neutrinos and accelerators neutrino experiments (K2K and MINOS) but does not restricts its sign, determination of sign(Δm_{31}^2) is called mass hierarchy determination

If sign of(Δm_{31}^2) is positive, we will have following mass pattern $m_3>>m_2>>m_1$. This is referred as normal hierarchy (NH)

If sign(Δm_{31}^2) is negative, then mass pattern is $m_2\geq m_1>>m_3$ which is known as inverted hierarchy (IH)



3.3 Three favour oscillation probability in vacuum

Here we derive expressions for probability in case of three flavours .The oscillation probability $P(\nu_l\rightarrow\nu_{l'})$ is given by absolute square of overlap of the observed flavour state $|\nu_{l'}>$, with time evolved initially produced flavour state, $|\nu_l>$, where l and l’ may be e,μ or τ. In vacuum yields the following result for the probability that ν_l becomes $\nu_{l'}$ after propagating a distance L

$$P(\nu_l\rightarrow\nu_{l'}) = |\textstyle\sum^3_{i=1} U_{l'i}\, e\frac{-i\Delta m^2_{ji}L}{2E}\, U_{li}{}^*|^2 \qquad (1)$$

$$P(\nu_l\rightarrow\nu_{l'})=\textstyle\sum_{i,k} U_{l'i}U_{l'k}{}^*U_{li}{}^*U_{lk}\;\; e\frac{-i\Delta m^2_{ji}L}{2E}$$

$$= P_{CP\,-even}\,(\nu_l\rightarrow\nu_{l'}) \quad + P_{CP\,-odd}\,\,(\nu_l\rightarrow\nu_{l'}) \qquad (2)$$

$$P_{CP\,-even}\,\,(\nu_l\rightarrow\nu_l') \,=\delta_{ll'} - 2\,\mathrm{Re}\textstyle\sum_{i>k}(U_{l'i}U_{l'i}{}^*U_{li}{}^*U_{lk})\,\,(1\text{- } \mathrm{Cos}\frac{\Delta m^2_{ki}L}{2E}) \qquad (3)$$

$$P_{CP\,-odd}\,\,(\nu_l\rightarrow\nu_l') \,=\, 2\textstyle\sum_{i>k}\mathrm{Im}\,(U_{l'i}U_{l'k}{}^*U_{li}{}^*U_{lk}) \qquad (4)$$

Total probability is given as :

$$P(\nu_l\rightarrow\nu_{l'})= \delta_{ll'} - 2\,\mathrm{Re}\textstyle\sum_{i>k}(U_{l'i}U_{l'i}{}^*U_{li}{}^*U_{lk})\,\,(1\text{-}\mathrm{Cos}\frac{\Delta m^2_{ki}L}{2E})+ 2\sum_{i>k}\mathrm{Im}(U_{l'i}U_{l'k}{}^*U_{li}{}^*U_{lk})\,\,\mathrm{Sin}\frac{\Delta m^2_{ki}L}{2E}\,) \qquad (5)$$

oscillation probability expression for antineutrinos is given as

$$P(\nu_l\rightarrow\nu_{l'})= \delta_{ll'} - 2\,\mathrm{Re}\textstyle\sum_{i>k}(U_{l'i}U_{l'i}{}^*U_{li}{}^*U_{lk})\,\,(1\text{- } \mathrm{Cos}\frac{\Delta m^2_{ki}L}{2E})\text{-}2\sum_{i>k}\mathrm{Im}\,(U_{l'i}U_{l'k}{}^*U_{li}{}^*U_{lk})\,\,\mathrm{Sin}\frac{\Delta m^2_{ki}L}{2E}) \qquad (6)$$

Below is full 3 flavour expressions for oscillations probability in vacuum using above general formulism

$P(\nu_e\rightarrow\nu_\mu)$	i=2 ,k=1	$-1/4\ \mathrm{Cos}[\theta_{13}]^2\mathrm{Sin}[2\theta_{12}]((\mathrm{Sin}[2\theta_{12}](\mathrm{Cos}[\theta_{23}]^2 - \mathrm{Sin}[\theta_{13}]^2\ \ \mathrm{Sin}[\theta_{23}]^2) + \mathrm{Cos}[2\theta_{12}]\mathrm{Sin}[2\theta_{23}]\mathrm{Sin}[\theta_{13}]\ \ \cos\delta_{13})$
	i=3,k=2	$-(\ \mathrm{Cos}[\theta_{13}]^2\ \mathrm{Sin}[\theta_{13}]\mathrm{Sin}[\theta_{12}]\mathrm{Sin}[\theta_{23}])(\mathrm{Sin}[\theta_{23}]\mathrm{Sin}[\theta_{13}]\mathrm{Sin}[\theta_{12}] - \mathrm{Cos}[\theta_{12}]\mathrm{Cos}[\theta_{23}]\mathrm{Cos}\delta_{13})$
	i=3,k=1	$-(\mathrm{Cos}[\theta_{13}]^2\ \mathrm{Sin}[\theta_{13}]\mathrm{Cos}[\theta_{12}]\mathrm{sin}[\theta_{23}](\mathrm{Cos}[\theta_{23}]\mathrm{e}^{\mathrm{i}\delta}\ \sin[\theta_{12}] + \mathrm{Sin}[\theta_{13}]\mathrm{cos}[\theta_{12}]\mathrm{Sin}[\theta_{23}])$

$P(\nu_e\rightarrow\nu_\tau)$	i=2 ,k=1	$\frac{1}{4}\mathrm{Cos}[\theta_{13}]^2\mathrm{Sin}[2\theta_{12}]((\mathrm{Sin}[2\theta_{12}](\mathrm{Cos}[\theta_{23}]^2\mathrm{Sin}[\theta_{13}]^2 - \mathrm{Sin}[\theta_{23}]^2) + \mathrm{Cos}[2\theta_{12}]\mathrm{Sin}[2\theta_{23}]\mathrm{Sin}[\theta_{13}])$
	i=3,k=2	$-(\mathrm{Cos}[\theta_{13}]^2\ 2\mathrm{Sin}[\theta_{13}]\mathrm{Sin}[\theta_{12}]\mathrm{Cos}[\theta_{23}])(\mathrm{Cos}[\theta_{23}]\mathrm{Sin}[\theta_{13}]\mathrm{Sin}[\theta_{12}] + \mathrm{Cos}[\theta_{12}]\mathrm{Sin}[\theta_{23}])$
	i=3,k=1	$-(\mathrm{Cos}[\theta_{13}]^2\mathrm{Sin}[\theta_{13}]\mathrm{Cos}[\theta_{12}]\mathrm{Cos}[\theta_{23}](\mathrm{Cos}[\theta_{23}]\mathrm{Sin}[\theta_{13}]\mathrm{Cos}[\theta_{12}] - \mathrm{Sin}[\theta_{12}]\mathrm{Sin}[\theta_{23}])$

And survival probability as

$P(\nu_e\rightarrow\nu_e)$	i=2 ,k=1	$\frac{1}{4}\mathrm{Sin}[2\theta_{12}]^2\mathrm{Cos}[\theta_{13}]^4$
	i=3,k=2	$\frac{1}{4}\sin[\theta_{12}]^2\mathrm{Sin}[2\theta_{13}]$
	i=3,k=1	$\frac{1}{4}\cos[\theta_{12}]^2\mathrm{Sin}[2\theta_{13}]^2$

In atmospheric oscillation probability we use

$\Delta m_{32}^2 \cong \Delta m_{31}^2 \cong \Delta m_{atm}^2$

$\Delta m_{21}^2 \cong \Delta m^2_{sol} \cong 0$

$P(\nu_e\rightarrow\nu_\mu)= \sin^2\theta_{23}\sin^2 2\theta_{13}\sin\Delta m^2_{atm}L/4E$ (7)

$P(\nu_e\rightarrow\nu_\tau)= \cos^2\theta_{23}\sin^2 2\theta_{13}\ \sin\Delta m^2_{atm}L/4E$ (8)

electron neutrinos survival probability is

$P(\nu_e \rightarrow\nu_e)=1-\ [P(\nu_e \rightarrow\nu_\mu)+P(\nu_e \rightarrow\nu_\tau)$

$=1-\sin^2 2\theta_{13}\sin\Delta m^2_{atm}L/4E$ (9)

In solar neutrino oscillations, experimental data constraints the value of mass squared difference in which Δm^2_{12} is only used neglecting Δm_{31}^2,Δm_{32}^2 ,thus we have

$P(\nu_e\rightarrow\nu_\mu)=[\sin^2 2\theta_{12}\cos 2\theta_{13}(\cos^2\theta_{23}-\sin^2\theta_{23}\sin^2\theta_{13})+1/4\sin 4\theta_{12}\sin 2\theta_{13}\cos\theta_{13}]$

$\sin\Delta m_{21}^2L/2E$ (10)

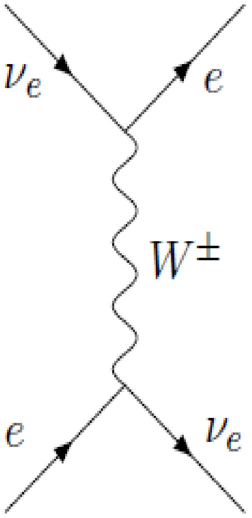
Similarly,

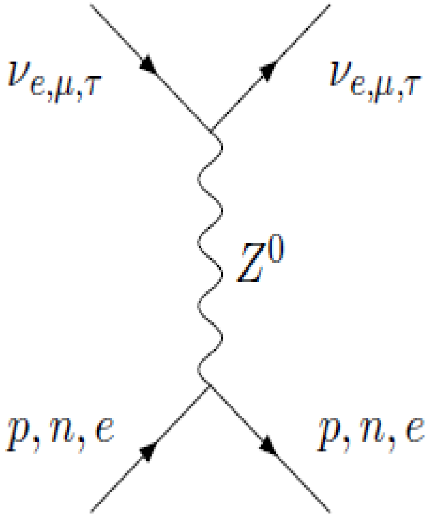
$P(\nu_e\rightarrow\nu_\tau)=[-\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23}\ \sin^2\theta_{13}-\ \sin^2\theta_{23})-1/4\ \sin 4\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}$ $\cos\theta_{13}]\ \ \sin\Delta m_{21}^2L/2E$ (11)

Survival probability
$P(\nu_e\rightarrow\nu_e)=1-[\sin^2 2\theta_{12}\cos^4\theta_{13}]\sin\Delta m_{21}^2L/2E$ (12)

3.4 Neutrino oscillations through matter

Oscillation in matter is totally different than vacuum, as it propagates through any matter which can be sun, supernova or earth neutrino gets integrated with matter through weak interactions ,as they move on few hard scattering and coherent forward elastic scattering also take place. This motion of neutrinos can be compared with the visible light travelling through glass. The crucial point to be noted here is that the coherent forward elastic scattering amplitudes differ for different types of neutrinos. The ordinary matter consists of electrons, protons and neutrons but does not contain any muons or tau-leptons. One can readily see from Fig.





That non coherent interaction takes place between all neutrinos (ν_e, ν_μ and ν_τ) and electrons, protons and neutrons via exchange of Z_0 bosons each and these contributions are same for neutrinos of all three flavours and therefore these interactions don't have any impact on neutrino oscillation probabilities. Interestingly, ν_e gives greater contribution due to their CC interactions (see left panel of Fig) with electrons of the medium by exchanging the W± bosons. This extra matter potential comes in the form

$$A=\pm 2\sqrt{2}\ G_f\ Ne\ E$$

Where G_F is Fermi coupling constant, Ne is the electron number density inside the Earth and E is the neutrino energy. The + indicates neutrinos and – for antineutrinos. Ne related to the matter density (ρ) in following way

$$V_{CC} = \sqrt{2} G_F Ne \simeq 7.6 Ye \frac{\rho}{1014 g/cm3\ eV}$$

Where $Ye = \frac{Ne}{Np+Nn}$ is the relative number density. Np, Nn are proton and neutron densities in Earth matter respectively. In an electrically neutral medium, we have Ne = Np = Nn and Ye comes out to be 0.5.

In the presence of matter, the vacuum oscillation parameters are mapped to the new parameters in the following way

$(\Delta m^2)^m = ((\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2))^{1/2}$

$\sin 2\theta^m = \sin 2\theta \frac{\Delta m^2}{(\Delta m^2)m}$.

The so-called MSW-resonance condition is met at

$\Delta m^2 \cos 2\theta = A$.

At MSW-resonance, $\sin 2\theta^m = 1$ which immediately signifies that it do not rely on vacuum mixing parameter θ, thus in matter mixing is largest at $\theta^m = \pi/4$.

In the presence of three neutrinos the time evolution of flavour eigenstates in matter is written as

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger + \begin{bmatrix} A(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Also diagonalisation is exact if the solar mass squared difference Δm_{21}^2 is neglected, this is known as one mass scale dominant (OMSD) approximation

OMSD expressions

It accounts neglecting Δm_{21}^2 in comparison to Δm_{31}^2 , here to solve full 3 flavour neutrino propagation equation assuming Preliminary Reference Earth Model (PREM) density profile for earth .The condition on neutrino energy and baseline for OMSD approximation to be valid is

$$\Delta m_{21}^2 L/E << 1 ,$$

alternatively it is $L/E<<10^4$ km/GeV. Additionally OMSD approximation is not applicable at very small θ_{13} . thus we can derive following approximation condition on θ_{13} for OMSD approximation to valid : $\sin^2 2\theta_{13} >> 0.004$

Making this approximation and assuming constant matter density computed using PREM density profile of earth , $(\Delta m_{31}^2)^m$ and mixing angle $\sin^2 2\theta_{13}^m$ in matter can be expressed as

$$(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2} \qquad (1)$$

$$\sin 2\theta_{13}^m = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}} \qquad (2)$$

where $\theta_{23}^m = \theta_{23}$ and $\theta_{12} = 0$

using above substitutions OMSD probability in matter is given by

$$P_{\mu e}^m = \sin 2\theta_{23} \sin^2 2\theta_{13} m \sin^2[(1.27 (\Delta m_{31}^2)^m L)/E] \qquad (3)$$

Also $P_{e\mu}^m = P_{\mu e}^m$

$$P_{ee}^m = 1 - \sin^2 2\theta_{13}^m \sin^2[(1.27 (\Delta m_{31}^2)^m \frac{L}{E})] \qquad (4)$$

$$P_{e\tau}{}^{m} = \cos 2\theta_{23} \sin^2 2\theta_{13}{}^{m} \sin^2[(1.27\,(\Delta m_{31}{}^{2})^{m}\,\frac{L}{E})] \qquad (5)$$

$$P_{\mu\mu}{}^{m} = 1 - \cos^2\theta_{13}{}^{m} \sin^2 2\theta_{23} \sin^2[1.27\,\frac{(\Delta m^2_{31} + A + (\Delta m^2_{31})m)L}{2E}] - \sin^2\theta_{13}{}^{m} \sin^2 2\theta_{23}$$

$$\sin^2[1.27\,\frac{(\Delta m^2_{31} + A - (\Delta m^2_{31})m)L}{2E}] - \sin^4\theta_{23} \sin^2 2\theta_{13}{}^{m} \sin^2[1.27\,(\Delta m^2_{31})^{m}\,\frac{L}{E}] \qquad (6)$$

$$P_{\mu\tau}{}^{m} = \cos^2\theta_{13}{}^{m} \sin^2 2\theta_{23} \sin^2[1.27\frac{(\Delta m^2_{31} + A + (\Delta m^2_{31})m)L}{2E}] + \sin 2\theta_{13}{}^{m} \sin^2 2\theta_{23}$$

$$\sin^2[1.27\frac{(\Delta m^2_{31} + A - (\Delta m^2_{31})m)L}{2E}] - \sin^2\theta_{23}\, \cos^2\theta_{23} \sin^2 2\theta_{13}{}^{m} \sin^2[1.27\,(\Delta m^2_{31})^{m}\,\frac{L}{E}] \qquad (7)$$

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CHAPTER FOUR: BASELINES WITH OSCILLATION PLOTS ,ANALYSIS AND FURTHUR MOTIVATION

4.1 Short and Long baseline

The transition ($\alpha\neq\beta$) or survival probability ($\alpha=\beta$) for $\nu_\alpha\to\nu_\beta$ is given by

$$P(\nu_\alpha\to\nu_\beta)=\left|\sum_{j=1}^{n} U_{\beta j}\, U_{\alpha j}^{*} e^{\frac{-iL\,\Delta m_{jk}^{2}}{2E}}\right|^{2} \qquad (1)$$

Where $\Delta m_{jk}^2 = m_j^2 - m_k^2$,L is distance from neutrino source to detector and E is neutrino energy .In this equation an important feature is, phases of form

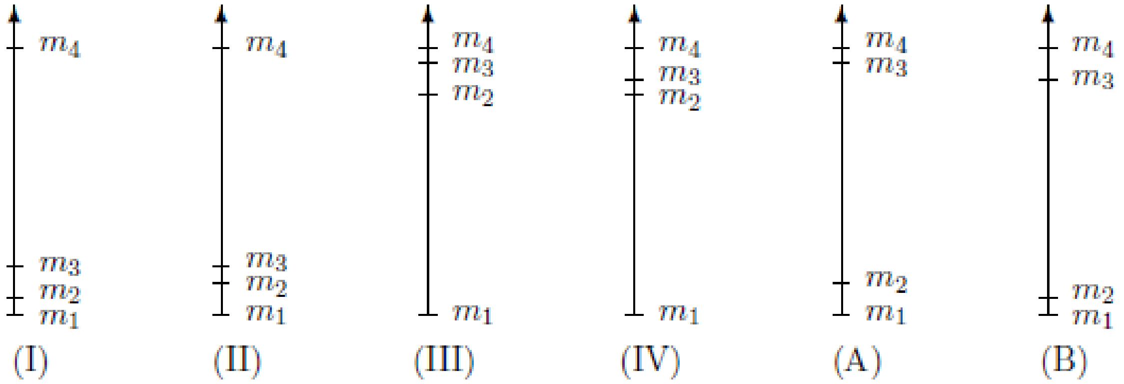
$$\frac{\Delta m^2 L}{2E} \cong 2.53 \text{ x } \left(\frac{\Delta m^2}{1\text{ eV}^2}\right)\left(\frac{E}{1\text{MeV}}\right)^{-1}\left(\frac{L}{1\text{m}}\right)$$

Which determine the order of magnitude of Δm^2 to which neutrino oscillation experiment is sensitive . Clearly experiments can only see phases if they are not too small, say of order 1, it is concerned in SBL experiments

By convention SBL are influenced by mass squared difference, $\Delta m^2 > 0.1$ ev^2 , with E approx 1 MeV , which follows that L>10m is sufficient distance between neutrino source and detector

Today, neutrino mass and oscillations are confirmed through some solar neutrino experiments (Homestake,Super-Kamiokande,Gallium Experiment(GALLEX), SAGE ,MACRO and in LSND experiment. The evidence for ν_u to ν_e oscillations experiments lead to $\Delta m^2_{LSND} \sim 1eV^2$ (the result of LSND experiment is only evidence for neutrino appearance),thus due to different scales of Δm^2 ,which tells that in nature there can be four light neutrinos having mass in order to match with observations of all neutrino oscillations experiments.

With four massive neutrinos there are six possibilities for neutrino mass spectra which accommodate 3 mass squared difference required by experimental data. In four of them three masses from cluster separated from fourth mass by gap which is needed to describe LSND experiment. There are two groups of close masses divided by LSND gap of 1eV. The small mass-squared differences correspond to Δm^2_{sun} (the smallest one, Δm^2_{21} in schemes I and B, Δm^2_{32} in schemes II and IV, Δm^2_{43} in schemes III and A) and Δm^2_{atm}(Δm^2_{31} in schemes I and II, Δm^2_{42} in schemes III and IV, Δm^2_{21} in scheme A, Δm^2_{43} in scheme B) and the largest mass squared difference $\Delta m^2_{41} = \Delta m^2_{LSND}$ is significant for oscillations discovered in LSND experiment. Let us consider short baseline oscillations in case of neutrino mass spectra as shown in Fig below



Considering the following assumptions

$\frac{\Delta m^2_{sun} L}{E}$<<1 and $\frac{\Delta m^2_{atm} L}{E}$ <<1 (2)

Conversion and survival probability of $\nu_\alpha \rightarrow \nu_\beta$ for short baselines are

$P^{(SBL)}(\nu_\alpha \rightarrow \nu_\beta) = \frac{1}{2} A_{\alpha;\beta}(1-\cos\frac{\Delta m^2_{41} L}{2E})$ $(\alpha \neq \beta)$ (3)

$P^{(SBL)}(\nu_\alpha \rightarrow \nu_\beta) = \frac{1}{2} B_{\alpha;\beta}(1-\cos\frac{\Delta m^2_{41} L}{2E})$ (4)

The oscillation amplitudes $A_{\alpha;\beta}$ and $B_{\alpha;\alpha}$ rely on neutrino *mixing* parameters and type of mass spectra

$A_{\alpha;\beta} = 4\,|\sum U^*_{\alpha k} U_{\beta k}|^2$ (5)

$B_{\alpha;\alpha} = 4 \sum U_{\alpha k}{}^2 \,(1- \sum U_{\alpha k}{}^2)$ (6)

Where index k runs over the indices of the first or second group of neutrino masses (see Fig). Equations (3) and (4) are equivalent to two-neutrino expressions. Eq.(5) tells us that $A_{\alpha;\beta}$ is amplitude for $\nu_\alpha \rightarrow \nu_\beta$ oscillations and it holds same for anti neutrinos as well , hence no CP violation in SBL neutrino oscillations

For two flavours α=e and μ, SBL disappearance experiments are also done, using bugey reactor disappearance and other accelerator experiments ,no disappearance has been seen in SBL experiments which sets an upper bounds on B_α and a_α(which is dependent on c_α which is related to B_α as $B_\alpha = 4c_\alpha(1- c_\alpha)$ which are functions of Δm^2_{SBL} ,we have B_α greater than 1 and a_α greater than 0.5

It is found that there is smallness of $\sum_{k=1,2} U^2_{ek}$ in scheme A and of $\sum_{k=3,4} U^2_{ek}$ which implies that the mixing between ν_e and other neutrinos is small whose Δm^2 important parameter for atmospheric neutrino oscillations (e.g. ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B). Thus in atmospheric and long baseline experiments conversion probability for electron neutrinos and antineutrinos is hidden. For scheme A we have following condition

$\frac{\Delta m^2_{43} L}{E} << 1$, $\frac{\Delta m^2_{41} L}{E} >> 1$

The transition probability for long base line for $\nu_\alpha \rightarrow \nu_\beta$ averaged over fast oscillation due to Δm^2_{41}

$$P^{(LBL)}(\nu_\alpha \rightarrow \nu_\beta) = |U_{\beta 1} U^*_{\alpha 1} + U_{\beta 2} U^*_{\alpha 2} \exp(-i(\Delta m_{21})^2 L/2E)|^2 + |\sum_{k=3,4} U_{\beta k} U^*_{\alpha k}|^2 \qquad (7)$$

In case of scheme B probability can be found by replacing index 1 with 3 and 2 with 4. From equation 7, the lower bounds on survival probability comes out to be

$P(LBL) > (1-a^0_e)^2$ $(a^0_e = \sum_{k=1,2} U^2_{ek})$

And an upper bound

$1 - P^{(LBL)} < a^0_e (2 - a^0_e)$

Lower bound implies that survival probability for electron antineutrinos is close to one and upper bound depend on Δm^2_{41} , and very difficult to measure in future LBL experiments. Further calculations shows that conversion probability of ν_e and $\overline{\nu_e}$ in atmospheric and longbaseline experiments is very small, and $\nu_\mu \rightarrow \nu_\tau$ probability oscillations is small for short base line experiments

4.2 Magic baseline

In particle physics neutrino oscillations are now fully confirmed after Super-k and KamLAND experiments. Getting θ_{13} and order of neutrino mass states are two major problems to solve, as both can reveal about δcp and neutrino mass hierarchy. Discovery of θ_{13} value could decide sign(m^2_{31}) and thus neutrino mass hierarchy. In addition, solar neutrino oscillations can only be described by solar LMA(large mixing angle).$\sin^2 2\theta_{13}$ is responsible for describing mixing between the solar and atmospheric neutrino oscillations ,whose value is limited by CHOOZ-experiment to $\sin^2 2\theta_{13} < 0.1$ .The value of $\sin^2 2\theta_{13}$ in addition with the solar m^2_{21} lies within large mixing angle , which is closely connected for determining nature 3-flavour neutrino oscillations , for example lepton CP violation. Three-flavour and other suppressed effects will be examined in future reactor and long-baseline experiments, such as super beam and neutrino factory experiments. Due to systematic errors, super beams and super beam are constrained to $\sin^2 2\theta_{13} > 10^{-3}$, whereas neutrino factories can, be sensitive to three-flavour effects for $\sin^2 2\theta_{13} < 10^{-4.}$ .electron to muon transition probability (also known as golden channel) gives us ideal method to discover mixing angle θ_{13} and the CP phase, δ_{CP} . Highly pure, intense fluxes of ν_e (or $\overline{\nu_e}$) can be produced using “Beta Beams” (β-beam), maybe in future at CERN(European Organization for Nuclear Research) accelerator complex and tevatron at FNAL is also proposed. Majority of searches on neutrino oscillations involving beta-beams uses megaton water Cerenkov detectors through muon neutrinos and antineutrinos. Main cause of that is these experiments take low to medium acceleration values for β unstable ions, leading to production of relatively low energy (anti)neutrino beam. This Water Cerenkov detector has ability of observing the resultant low energy muons also.

Neutrino industries studying oscillations have certain limitations called degeneracy’s occurring in neutrino oscillation expressions written as follows:

- the (θ_{13}, δ_{CP}) intrinsic degeneracy
- the (sign(m^2_{31}), δ_{CP}) degeneracy
- the (θ_{23}, $\pi/2 - \theta_{23}$) degeneracy

i.e. overall “eight-fold” degeneracy with many “clone” solutions also weakens the quality of any experiment. Problem of clone solutions due two first two degeneracy’s can be removed by selecting the baseline of experiment equal to the characteristic refraction length due to the matter inside earth .this baseline is known as magic baselines ,which is independent of order of mass hierarchy, θ_{13} , energy E and δ_{CP}. In future CERN-INO distance of 7152 km is hopeful to reach magic baseline distance. At this baseline matter effects increases, thus need of denser regions of the earth. Thus, neutrinos or antineutrinos having energies of about 3-8 GeV greater matter effects are developed for both NH, IH. Most important consequence of $\nu_e \rightarrow \nu_\mu$ channel is resonance condition in matter effects. At magic baseline Probability is also independent of delta cp for both NH and IH , also quite large for NH and is essentially 0 for IH over a wide range of energy

In fact, to our knowledge, long baseline experiments can use resonant matter effects to study the neutrino mass matrix. Great advantage of near-maximal earth matter effect is clean measurement of mixing angle θ_{13} and mass hierarchy. Sensitivity to θ_{13} and mass hierarchy’s achieved by studying oscillations ($P_{e\mu}$) under matter resonance effects ,under this condition rise in $P_{e\mu}$ is balanced with drop of β-beam flux due to the very long baseline. These experimental set ups are said to be most convenient to study oscillations in comparison to other proposed one. Thus β-beam experiment from CERN to INO could appear as a important tool for measurement of neutrino mass hierarchy and θ_{13}..

The current best-fit value for the atmospheric angle is $\theta_{23} = \pi/4$, the first two of these degeneracy's together with multiparameter associations fully restrict future long-baseline experiments. This problem is overcome by combining neutrino factories with super beam up-grades and detector's so that probability oscillations should increase. Combination of baselines is an easy and effortless choice in comparison to other detectors, these detectors are also cheap relative to accelerator complex. proper combination of two neutrino industries, in which one is magic baseline of length ∼7300 km−7600 km ,has ability to remove degeneracy very accurately ,still for $\sin^2 2\theta_{13} < 10^{-3}$.Since , 3 flavour v_u to v_e oscillation probability including matter effects

$P_{app} \cong \sin^2 2\theta_{13} \ \sin^2\theta_{23} \ \sin^2[(1-A)\Delta]/(1-A)^2 \ \pm \ \alpha \ \sin 2\theta_{13} \ \varepsilon \ \sin\delta cp \ \sin\Delta \ \sin(A\Delta) \ \sin[(1-A)\Delta]/(1-A)A \ + \sin 2\theta_{13} \ \varepsilon \ \cos\delta cp \ \cos\Delta \ \sin(A\Delta) \ \sin[(1-A)\Delta]/(1-A)A \ + \alpha^2 \cos^2\theta_{23} \ \sin^2 2\theta_{12} \sin^2(A\Delta)/A^2$

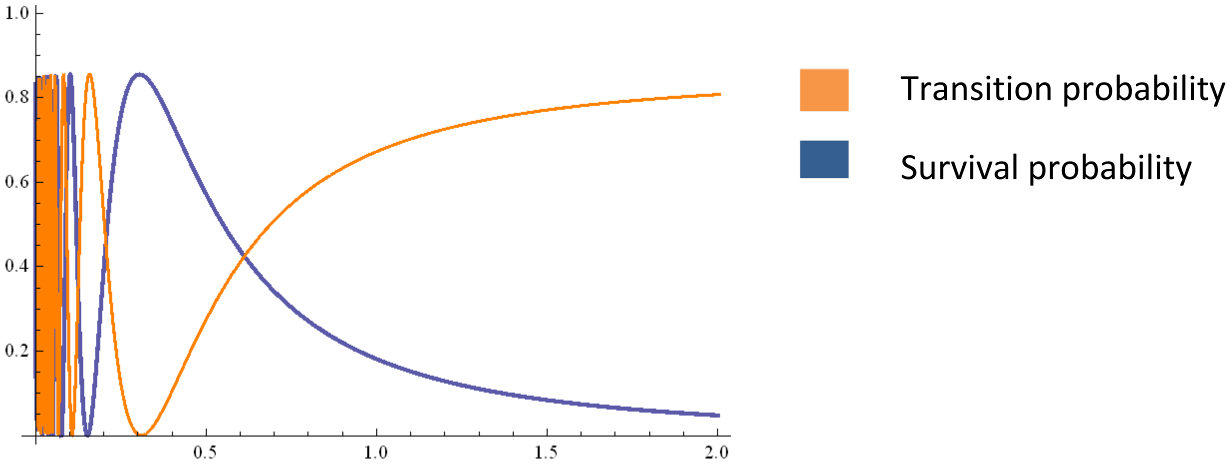
$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \cong \pm.03$, $\Delta = \frac{\Delta m_{31}^2 L}{4E}$, $\varepsilon = \sin 2\theta_{12} \sin 2\theta_{23}$, $A = \frac{2\sqrt{2} GFneE}{\Delta m312}$

At magic baseline $\sin(A\Delta) = 0$, thus except first term all other terms vanishes and give us clear value of $\sin^2 2\theta_{13}$ and order of $m^2{}_{31}$ independent of CP phase . Under condition $\sin(A\Delta) = 0$, $\sqrt{2} GFn_e L = 2\pi$, in terms of constant matter density Lmagic [km] $\simeq 32726(\frac{1}{p\ [g/cm3]})$ .thus length is effected by matter density, but independent of energy E and neutrino parameters. For constant matter density ≃ 4.3 g/cm^3, we get Lmagic ≃ 7630 km. Numerically, it comes out that Lmagic ∼ 7250 km if using realistic PREM (Preliminary Reference Earth Model) profile by limiting the δcp factor in appearance rates. For e.g, distance between Fermilab to Gran Sasso is magic baseline in sense of this definition. Magic baseline has limitations also that the statistics provided at this baseline is pretty low and no δcp calculations can be done due to exclusion of second and third terms in probability expression. These disadvantages can be overcome by joining magic baseline with shorter baseline in which magic baseline allow us to access θ_{13} and shorter baseline measure δcp with better statistics.

4.3 Plot and discussion

For studying probability of neutrinos at fixed distances and varying energy we plot following graphs in vacuum as well as matter affects. Probability is along Y-axis and energy is along X-axis. In this plot all the three probabilities are shown i.e. muon to tauon and muon to muon survival probability and for electron survival and transition probability as well.

Here first solar neutrino probability is drawn for 2 flavours, orange curve is transition probability and blue for survival .Here energy is in MeV and length at long base line

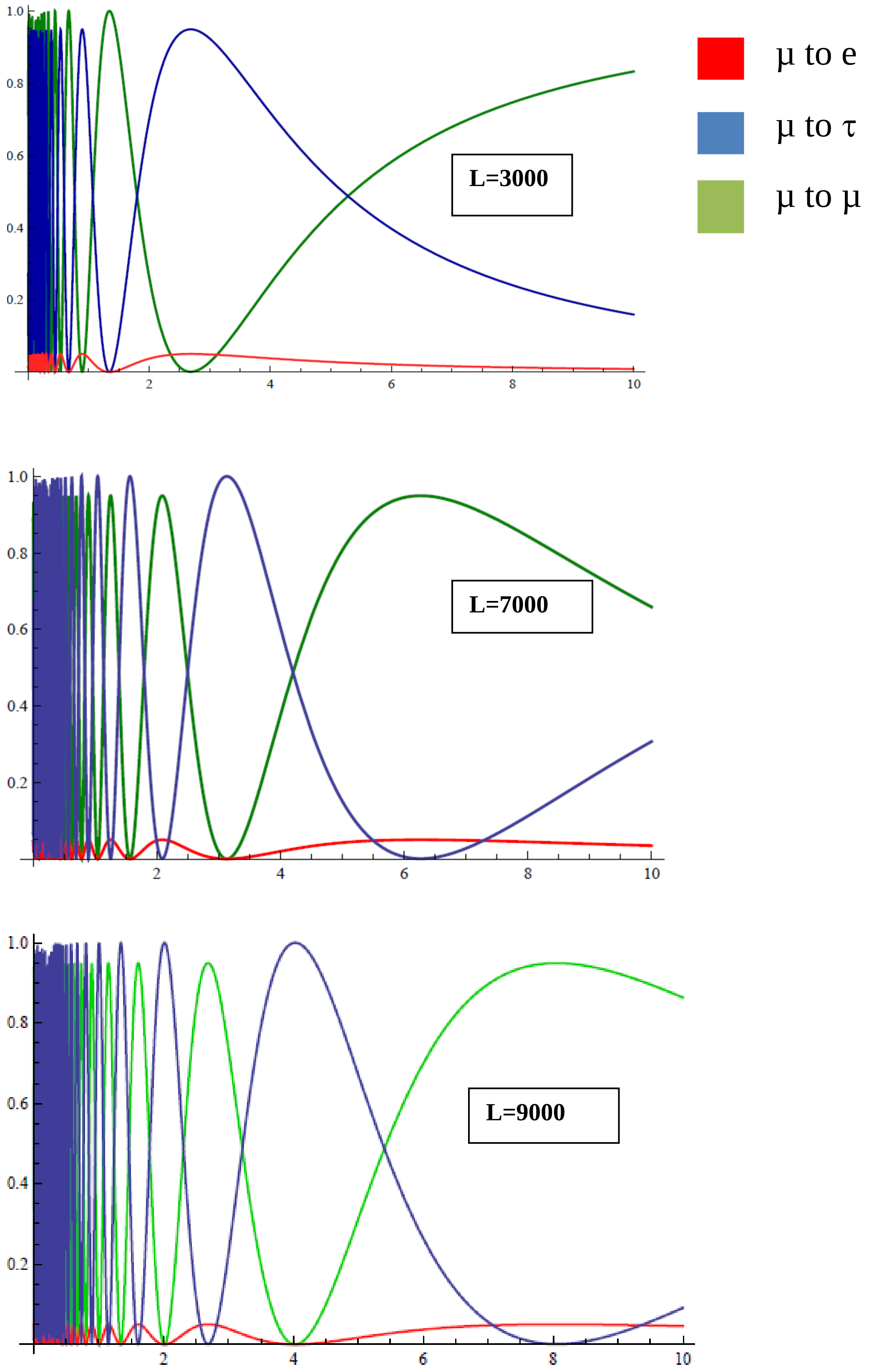


Plot : Probability Vs Energy (MeV) for solar neutrinos

Probability plots for 3 neutrino case

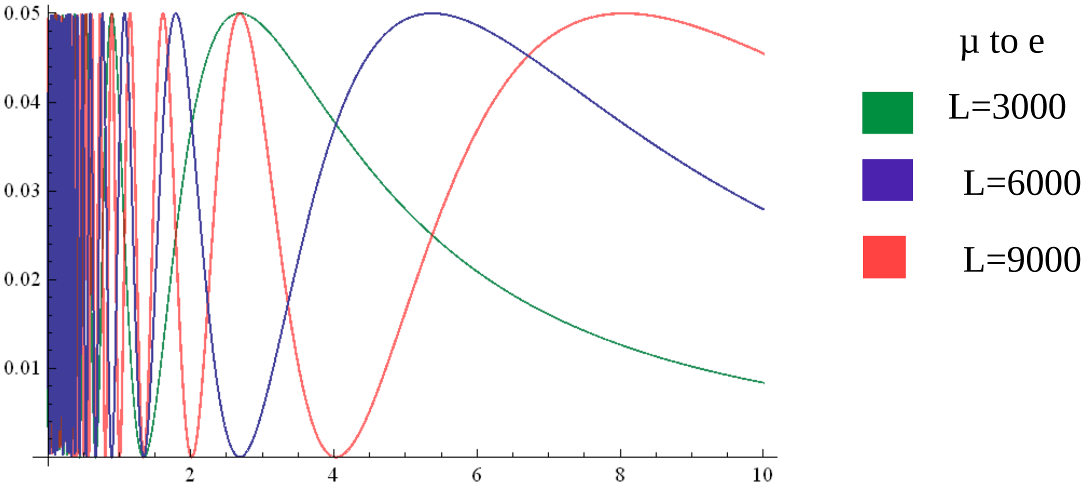
The atmospheric neutrino consists initially of muon neutrinos and as they propagate muon changes into either electron or muon but change from moun to tauon is more frequent rather than electron. The energy range for this probability lies in GeV's. At short baseline the change of flavour transition is not very rapid.

But if we plot the same graph for long baseline i.e. we have to increase the length for long baseline here we are taking a length L=9000 m and other parameters remains the same the oscillations are more rapid and change of flavour is more fast as they propagate in vacuum.



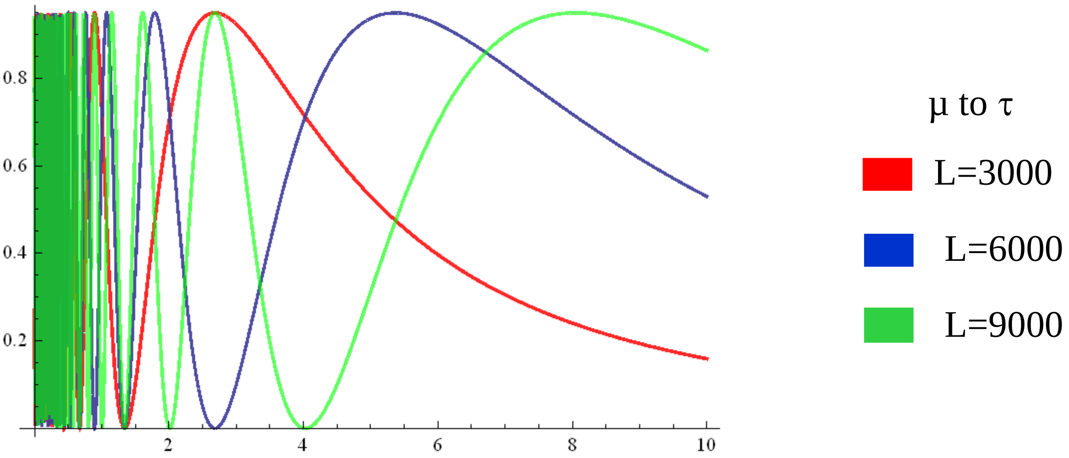
Plot : Probability Vs Energy (GeV) for atmospheric vacuum oscillations at different lengths

All the parameters remains same as in the case of muon neutrinos change into electron neutrinos. We can change only length .In this plot probability is along Y-axis and energy is along X-axis .When length is taken as 3000m it is shown by green line when we further increase the length from 3000 to 6000m it is shown by blue line then further increase in length from 6000 to 9000m it is shown by the red line.



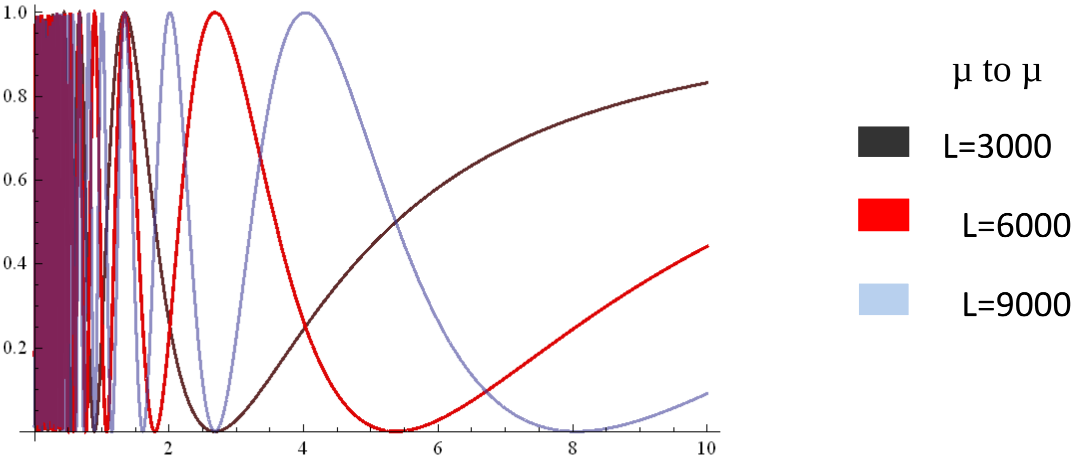
Plot : Probability Vs Energy (GeV) for atmospheric vacuum oscillations at different lengths for muon neutrino changing to electron neutrino

All the parameters remains same as in the case of muon neutrinos change into taun neutrinos. We can change only length .In this plot probability is along Y-axis and energy is along X-axis .When length is taken as 3000m it is shown by red line when we further increase the length from 3000 to 6000m it is shown by blue line then further increase in length from 6000 to 9000m it is shown by the green line.

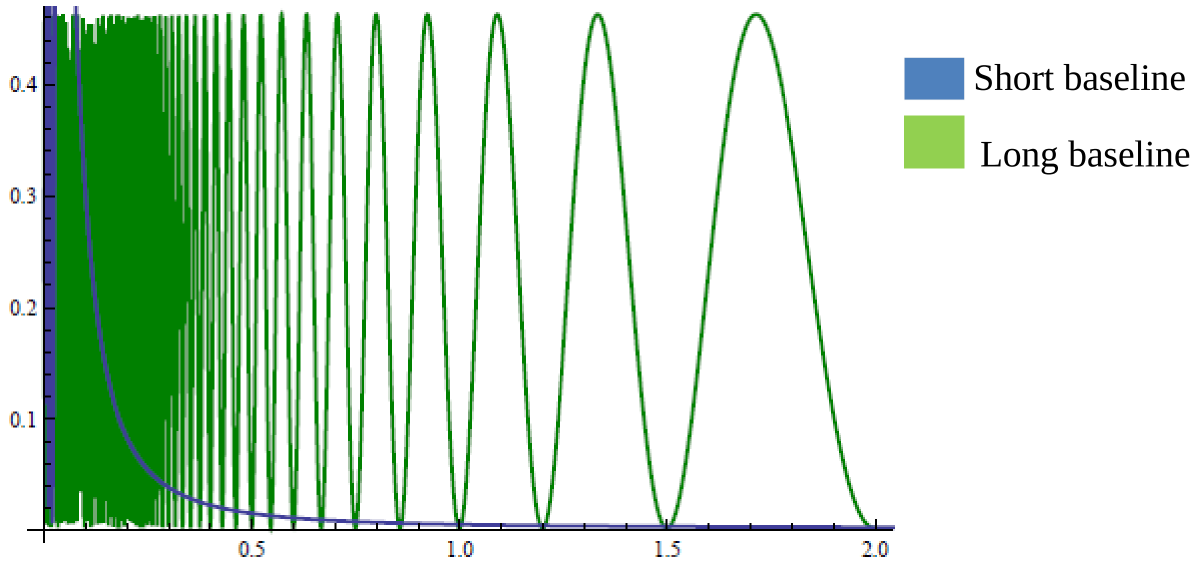


Plot : Probability Vs Energy (GeV) for atmospheric vacuum oscillations at different lengths for muon neutrino changing to taun neutrino

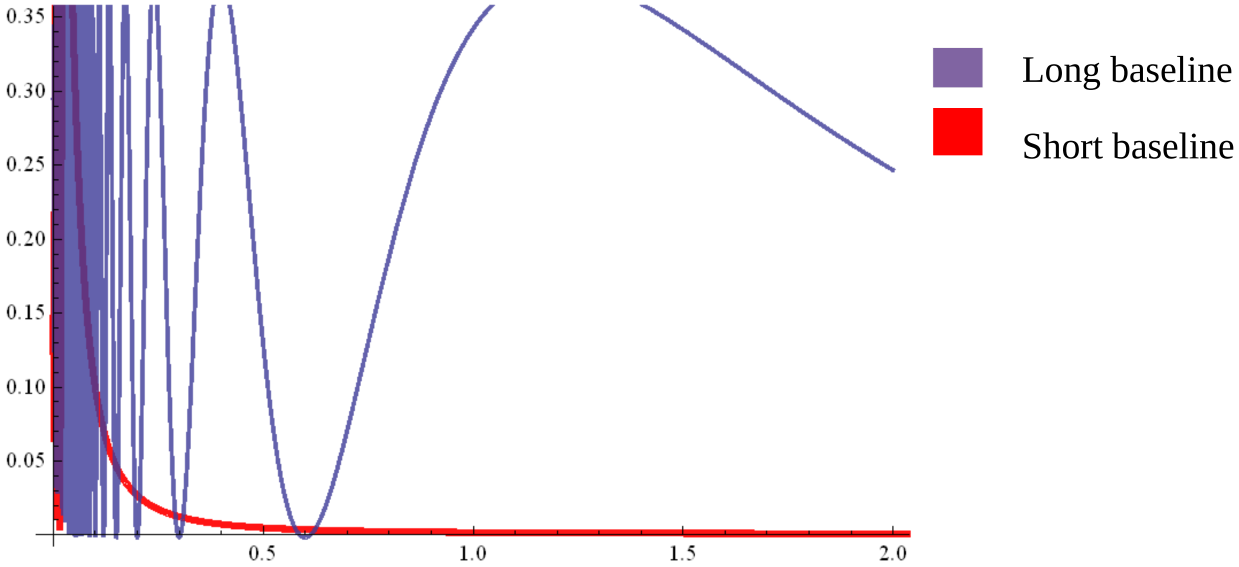
All the parameters remain same as in the case of muon neutrinos change into muon neutrinos. We can change only length .In this plot probability is along Y-axis and energy is along X-axis .When length is taken as 3000m it is shown by black line when we further increase the length from 3000 to 6000m it is shown by red line then further increase in length from 6000 to 9000m it is shown by the blue line



Plot : Probability Vs Energy (GeV) for atmospheric vacuum oscillations at different lengths for muon neutrino changing to muon neutrino



Plot : Probability Vs Energy (MeV) for solar vacuum oscillations at different lengths for electron neutrino changing to muon neutrino



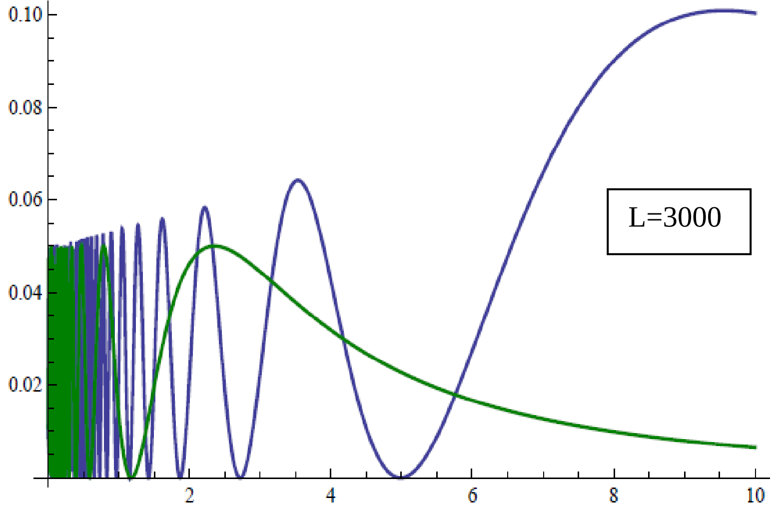
Plot : Probability Vs Energy (MeV) for solar vacuum oscillations at different lengths for electron neutrino changing to taun neutrino

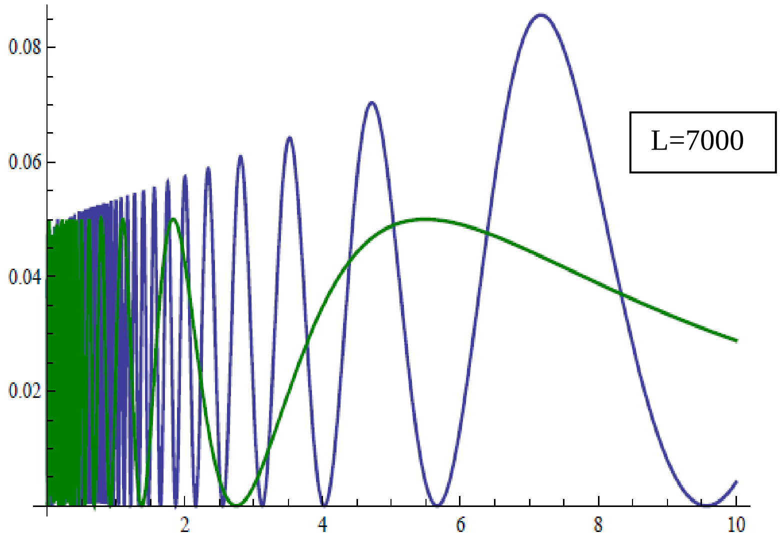
Probability Vs Energy (GeV) for matter plus vacuum oscillations at different lengths for muon neutrino changing to electron neutrino

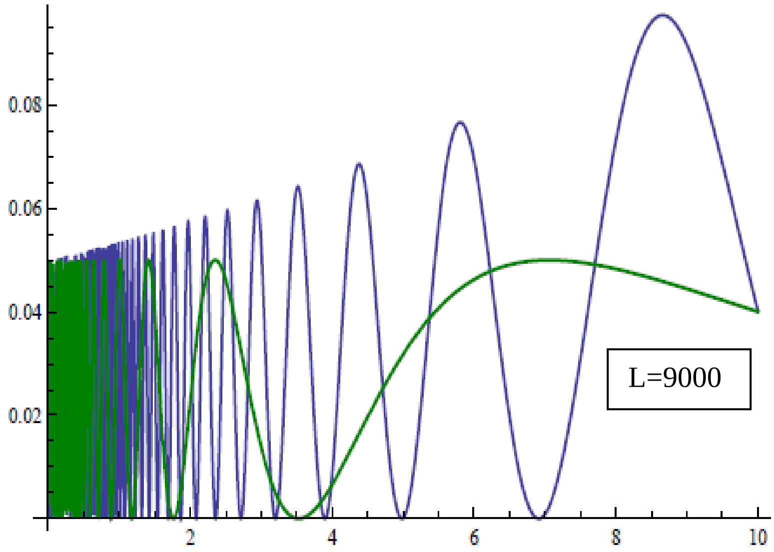
μ to e oscillation

🟦 Matter

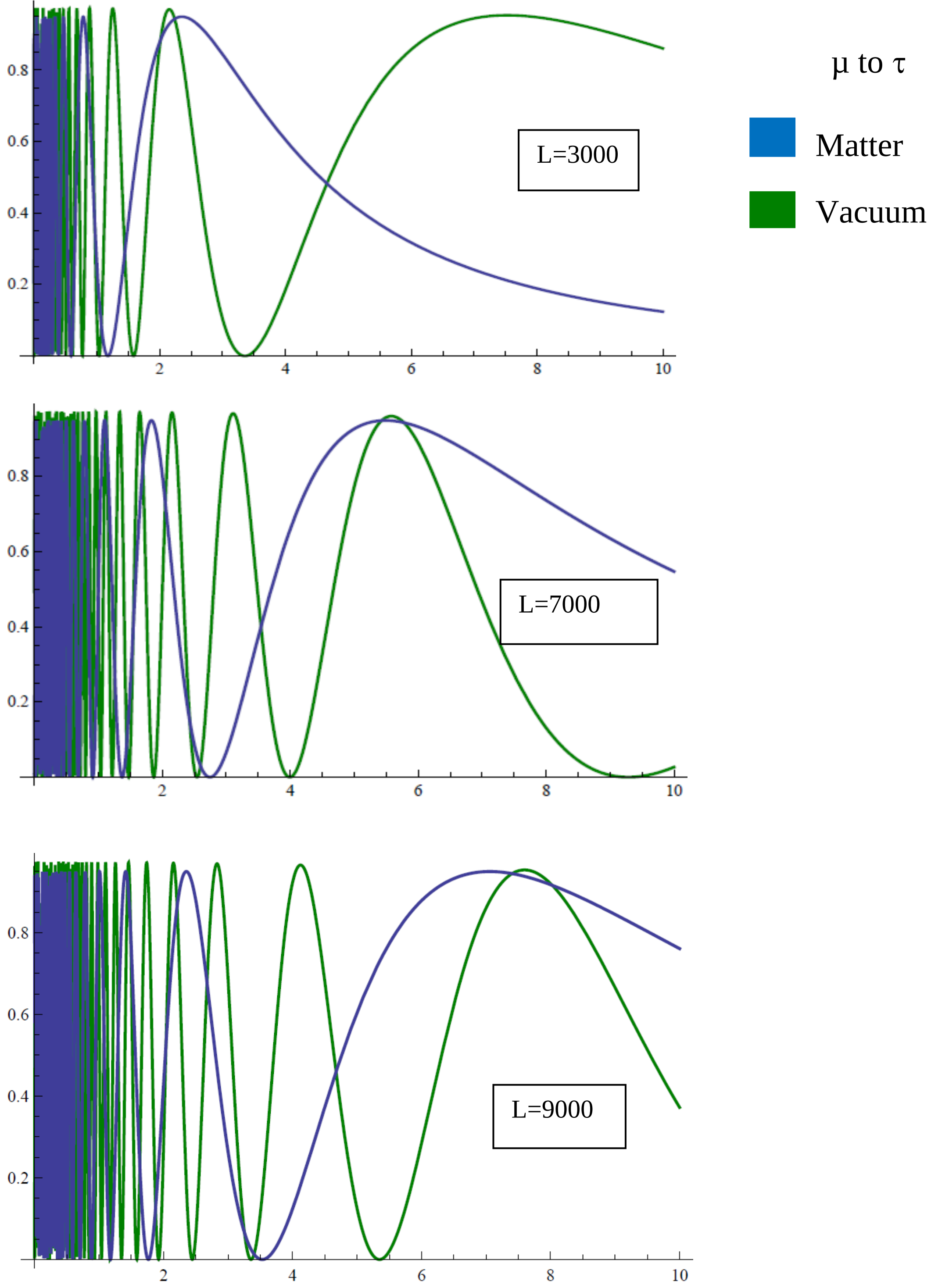
🟩 Vacuum



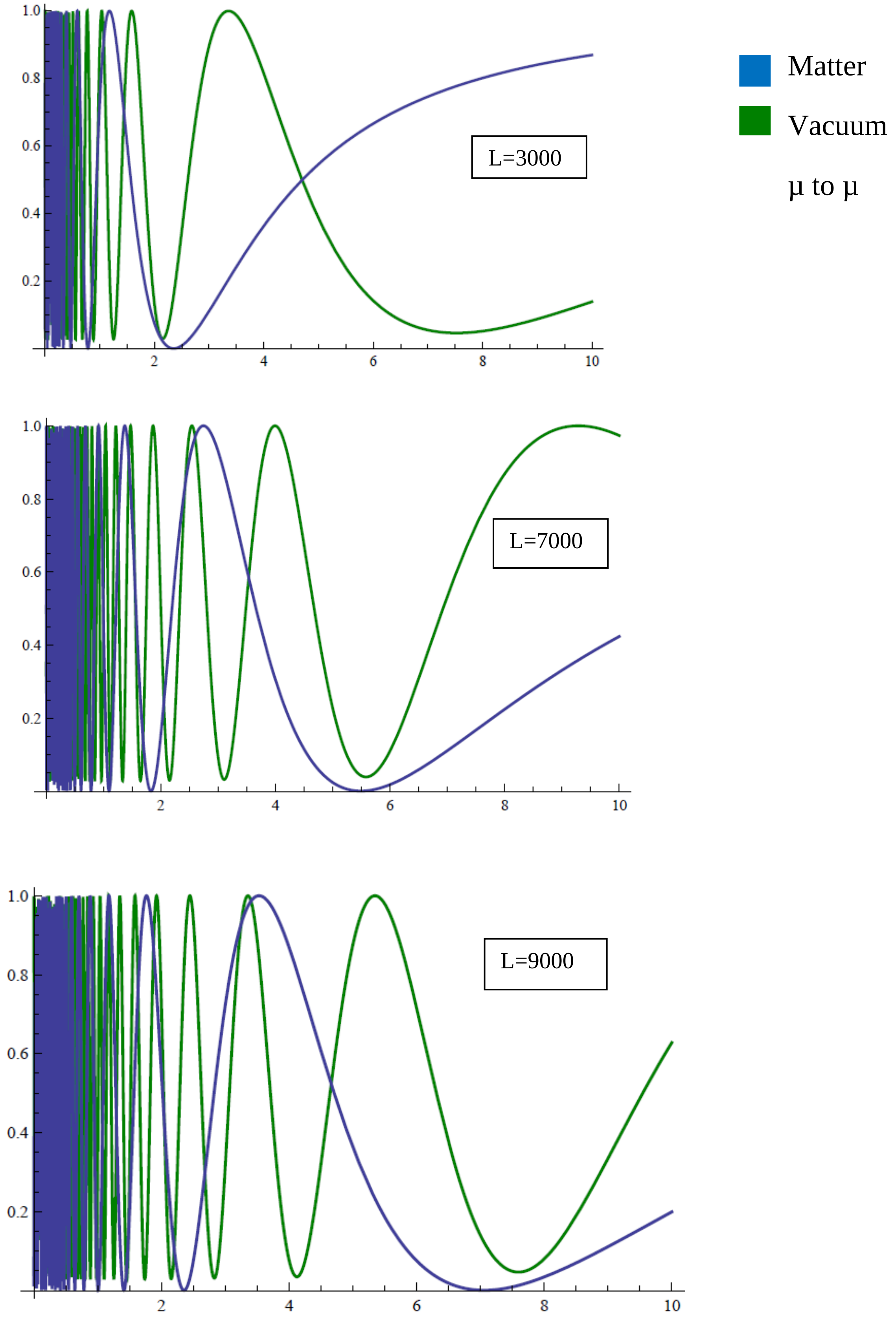




Probability Vs Energy (GeV) for matter plus vacuum oscillations at different lengths for muon neutrino changing to taun neutrino



Probability Vs Energy (GeV) for matter plus vacuum oscillations at different lengths for muon neutrino changing to muon neutrino



Graphs of muon transition and survival probability in vacuum as well as in matter shows that muon to electron probability in matter is maximum. This is due to the fact that CC interaction with electrons of matter effects most for electron neutrino oscillations, thus matter effects rely on type of flavour. From graphs it can be seen that oscillations are rapid at low energy and greater length, as the length is increased energy range also expands but decrease probability oscillations results. μ to τ probability oscillations are more in magnitude relative to electron. Thus in atmospheric neutrinos mixing between mass eigen states is greatest for μ and τ in comparison to electrons

4.4 Recent activities in neutrino oscillations

In late 2013 IceCube revealed that it had captured the first signals from neutrinos with extremely high energies, which suggests that the particles came from outside of our galaxy. While neutrinos generated inside the Sun and by cosmic rays colliding with the Earth's atmosphere have been detected for many years, neutrinos from much farther away had remained elusive.

In 2014 Olga Mena, Sergio Palomares-Ruiz and Aaron Vincent of the University of Valencia in Spain did an independent analysis of IceCube data from 2010 to 2012, and concluded that the best-fit flavour ratio was 1:0:0 – an abundance of electron neutrinos with no muon or tau neutrinos present. If true, this unexpected result would mean that rare neutrino decays were taking place or that the detected particles were mixing with a fourth and very hypothetical "sterile" neutrino. In both cases, the discovery could have pointed to exotic physics beyond our current understanding. Information was gathered in 2010 and 2013 by Tokia to kamioka long-baseline neutrino experiment.In survey of muon neutrino disappearance experiments, following information is revealed for both mass hierarchy at 68% confidence intervals :
In Normal hierarchy: $\sin 2\theta_{23}=0.514+0.055-0.056$ and $\Delta m^2_{32}=(2.51\pm0.10)\times10^{-3}\mathrm{eV}^2/c^4$ and for Inverted Hierarchy $\sin 2\theta_{23}=0.511\pm0.055$ and $\Delta m^2_{13}=(2.48\pm0.10)\times10^{-3}\ \mathrm{eV}^2/c^4$. Experiment use combination of muon neutrino disappearance and electron neutrino appearance measurements in order to get these parameters, $|\Delta m^2|$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, δ_{CP}. Combinations of these parameters are present in Frequentist and Bayesian intervals, with and without including recent reactor measurements. At 90% confidence level and including reactor measurements, exclude the region $\delta_{CP}=[0.15,0.83]\pi$ for normal hierarchy and $\delta_{CP}=[-0.08,1.09]\pi$ for inverted hierarchy. NH is favoured by T2K and reactor with Bayes factor of 2.2. The most probable values of oscillation parameters, when reactor data is included,
are $\sin^2\theta_{23}=0.528+0.055-0.038$ and $|\Delta m_{23}{}^2|=(2.51\pm0.11)\times10^{-3}\ \mathrm{eV}^2/c^4$.

Although fifty years of intense efforts have been made by researchers in various ways to understand this fundamental particle yet the information about this subatomic particle is not complete. Solar neutrino unknown facts are only revealed by neutrino oscillations but search for exact neutrino masses is still going on. Current experiments on neutrinos are focused on the estimation of parameter θ_{13} and there velocity. If we get more data on order of mass hierarchy and δcp, we can get to know nature of neutrino oscillations. Currently we have large data available on neutrino parameters, still the future measurements are more sensitive to θ_{13}. For solar neutrinos, a combined analysis of solar experiments can provide upper limits on θ_{13} up to a higher significance level. It is claimed that if θ_{13} is greater than the limit as expected then faster atmospheric oscillations can be predicted at about 1-2 km.

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