NONLINEAR PROPAGATION OF ELLIPTICAL SUPER GAUSSIAN BEAM

A thesis submitted in partial fulfillment of the requirements for the award of
degree of

Master of Science
In
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Submitted by:

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I affectionately dedicate this thesis to my niece ‘Parneet’.

Certificate

I hereby certify that the work which has been presented in this thesis entitled, "**Nonlinear Propagation of Elliptical Super Gaussian Beam",** submitted in partial fulfillment of the requirements for the award of degree of **Master of Science** in **Physics** at **Thapar University**, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Soumendu Jana, Assistant Professor, SPMS** and refers other researcher's work which are duly listed in reference section.

The matter embodied in this thesis has not been submitted for the award of any other degree of this or any other university.

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This to certify that the above statement made by the candidate is correct and true to best of my knowledge.

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Chapter 1

1.1 Introduction

Spatial optical soliton is a fascinating fundamental and applied research topic in nonlinear optics. These are optical beams confined in direction/(s) transverse to the propagation axis. They are extremely versatile in nature over their temporal counterpart owing to their additional degrees of freedom along transverse dimension. Other than optics, spatial solitons are a familiar mane in mechanical engineering, fluid dynamics, thermodynamics, pattern formation, biology etc. Even we encounter spatial solitons in nature. For example, the tsunami waves, sand dunes, some strange cloud formation ('morning glory' cloud / rolling cloud), oasis in a desert. However, the mechanism of such soliton formation is very complicated and topic of fundamental multidisciplinary research. Our topic of interest, i.e., optical spatial soliton, is one of the most important areas of nonlinear optics, which arises mainly due to the high intensity of the radiation field. Optical phenomena get modified due to nonlinear refractive index and thus the name 'nonlinear optics' has been coined [1-10]. This branch includes all phenomena in which the optical parameters of materials are changed with irradiation by light. Usually this requires high optical intensities sources like lasers. Note in case of linear optics the light of low intensity is used but here we use very intense light. Nonlinear optical phenomena are "nonlinear" in the sense that they occur when the response of a material system to an applied optical field depends in a nonlinear manner on the strength of the optical field. The first nonlinear-optical experiment using the laser was performed by Franken in 1961 [1]. A ruby laser radiation with a wavelength of 694.2nm was used to generate the second harmonic in a quartz crystal at the wavelength of 347.1 nm [2]. This was the beginning of non linear optics. This seminal work was followed by the discovery of a rich diversity of nonlinear optical effects, including sum-frequency generation, stimulated Raman scattering, self-focusing, optical rectification, four-wave mixing, and many others [1-5].

The NLO has large effect on science and technology, it increases our understanding about interaction of light with matter and its further study provide as great applications in science and technology. Because the nonlinear optical phenomenon play very important role in the optical fibers communication, hence the major factor behind the large growth of the non linear optics was the advancement of optical fibers. Up to 1979 the loss in the silica fiber was reduced up to 0.2 db/km. This low loss silica fibers do not gave rise only to optical communication but also increases this new field of non linear optics. Stimulated Raman- and Brillion-scattering processes in optical fibers were studied in 1972 [3-4]. This work stimulated the study of other nonlinear phenomena such as optically induced birefringence, parametric four-wave mixing, and self-phase modulation [5-9]. In 1973, it was suggested that optical fibers can support soliton-like pulses which is formed due to counterbalance between the dispersive and nonlinear effects. Thus with the advancement of fiber communication system large number of non linear optical phenomena are came into picture. NLO appears to have a strong future in areas of photonics devices and scientific investigations [10].

1.2 Origin of Nonlinearity

Before the invention of laser it was believed that interaction between light and matter is linear i.e. as the matter properties changes linearly with linear change in light. Such that electric polarization ($\vec{P}$) of a material is directly proportional to electric field ($\vec{E}$) of incident light.i.e.

$$\vec{P} = \chi\vec{E}$$

where, χ is susceptibility of the optical medium. We see that the optical phenomenon which is observed with the weak light or less intense light are liner and the theory related with it is called linear optics or optics of weak light. But when we have light whose intensity is very high, like, laser light then polarization is not linearly dependent upon the apply electric field, nonlinear dependence of polarization on electric field is observed such as [1]:

$$\chi = \chi + \chi^2\vec{E} + \chi^3\vec{E}^{\,2} + \ldots\ldots.$$

Where χ^n= nth order susceptibility.

Here we see the non liner effect of radiation when the intense light interacts with matter. Thus, there is the origin of non-linear polarization. This non linear effect is related to the anharmonic motion of the bound electron of the medium under the influence of electromagnetic radiations. This non linear response of the matter when interacts with light give rise to many non linear optical phenomena. These non-linear effects are called optics of intense light these non-linear effects are studied under a separate branch of physics called non linear optics [3].

1.3 Light Matter Interaction

Because in the non liner optics we deal with interaction of matter with light, so depends upon the nature of material and coherence property of light these interactions are categorized as following [11]:

- Nonresonant interaction with incoherent light.
- Nonresonant interaction with coherent light.
- Resonant interaction with incoherent light.
- Resonant interaction with coherent light

Topic of our investigation is ‘Nonresonant interaction with coherent light’. So, we will deal with a no loss conservative system dynamics.

There are many approaches through which interaction of light and matter is studied. These different approaches are given as following:

- Density Matrix Formalism
- Rate Equation Approach
- Nonlinear Electric Polarization Approach

Nonlinear polarization can be achieved in different systems in different ways. Among these, electron cloud distortion model is the most familiar.

1.4 Electron Cloud Distortion Model

In an optical material there is collection of negatively and positively charged particles. The negatively charged particles are electrons and positively charged are cores. In the absence of electromagnetic radiation center of negatively charged particles coincide with center of positively charged particles, as a result, they counter balance each other. When the incident light is made to fall on material, due to the presence of electric field of electromagnetic radiations positively charged core aligns in direction of applied field and the negatively charged aligns in opposite direction of electric field .Hence there is separation (elongation) between the two charges .Hence there is resultant dipole moment [12].

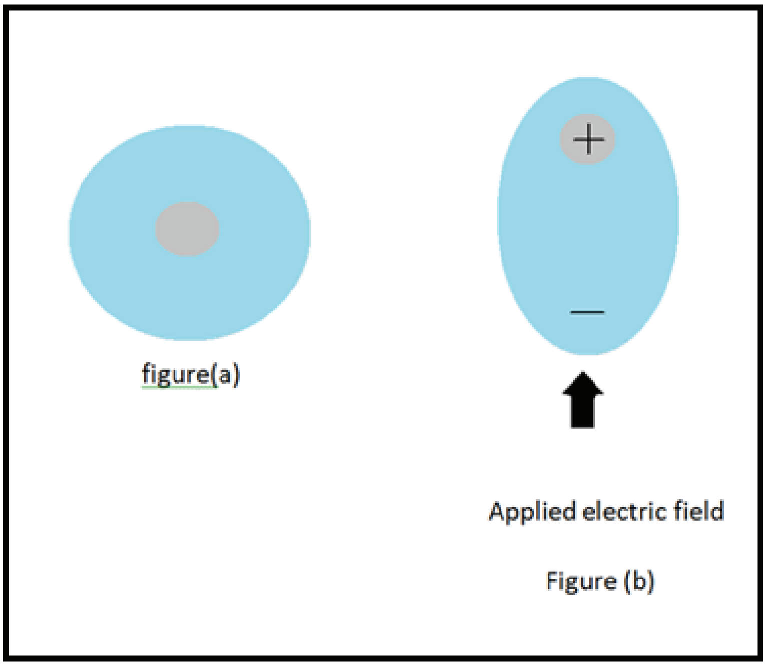


Figure 1 (a) the atom in absence of electric field E

Figure 1(b) the elongated atom in the influence of the electric field.

1.5 Soliton Formation

Solitary waves commonly called as solitons are subject of deep theoretical as well as experimental studies in different field like hydrodynamic, plasma physics, biology and nonlinear optics and so on [13]. The history of soliton formation is very old. The first reported observation of solitons was made in 1834 by Sir John Scott Russell, a Scottish engineer and later famous Victorian engineer and shipbuilder, while studying water waves in the Glasgow-Edinburgh channel .He was observing a boat which is pulled by the pair of horses. Suddenly the boat was stopped and due to the sudden stop a strong wave is generated in water. The heap of water move forward with the constant velocity inside narrow channel. He observed the wave up to few kilometers. He saw the wave kept its shape and velocity preserved for long distance. He named such wave as 'wave of translation'. These waves latterly called as solitary waves. However, the properties of soliton were not understood completely until appropriate mathematical models like inverse scattering method were introduced. This phenomenon attracted significant research attention. The work of Stokes, Boussinesq and Rayleigh was acknowledged in early period. In 1898 two Dutch physicists, Korteweg and de Vries, presented their famous KdV equation [14]. The wave of translation was thus recognized as 'soliton' in 1965 by Martin Kruskal and Norman Zabusky, who solved KdV equation.

Soliton

A wave propagates keeping its shape and width unaltered even after collision with another similar wave, it will be considered as soliton or we can say Soliton is a non linear wave which has the following properties [15]

- A localized wave that maintains its shape while propagates
- These localized waves are stable against mutual collisions.

1.6 Types of Solitons

Optical solitons are mainly classified into various categories

- Spatial soliton

- Temporal soliton
- Spatiotemporal soliton

1.6.1 Spatial Soliton

These are self guided or self trapped optical beams that propagate in slab waveguides or bulk nonlinear media. To get knowledge about the spatial soliton first we must know about the two phenomenon self diffraction and self focusing. Optical beams have an innate tendency to spread or diffract during their propagation in any homogenous medium. It behaves as concave lens and tries to spread the beam. Diffraction creates curved wave front similar as produced by a concave lens and spreads the beam to a wider region [13]. Self-focusing of light is the process in which an intense beam of light modifies the optical properties of a material medium in such a manner that the beam is caused to come to a focus within the material. As a result, the laser beam induces a refractive index variation within the material with a larger refractive index at the center of the beam than at its edges. Thus the material acts as if it were a convex lens, causing the beam to focus within the material [1].

As we know that light is an optical beam with finite transverse dimension, broadens due to self-diffraction. If this beam is propagating in a nonlinear medium, then the nonlinearity induced lensing may self-focus or defocus the beam. When both of these effects counteracts and thus balances each other and produces an unaffected beam which is not self focused and not self diffracted, this type of beam is called spatial soliton because it is invariant in its spatial dimension. When a beam progratеs in a Kerr nonlinear medium with positive Kerr coefficient. The medium posses properties of self focusing, because the intensity of light is maximum in centre of beam then at its edges, so refractive index is also maximum in centre then its edges, so phase velocity is minimum in center as compared to its edges. This medium acts as self focusing lens and tries to focus the light, so in this medium both effects [16] i.e. self focusing and diffraction counter balance each other. Due to these effects a spatial beam is generated.

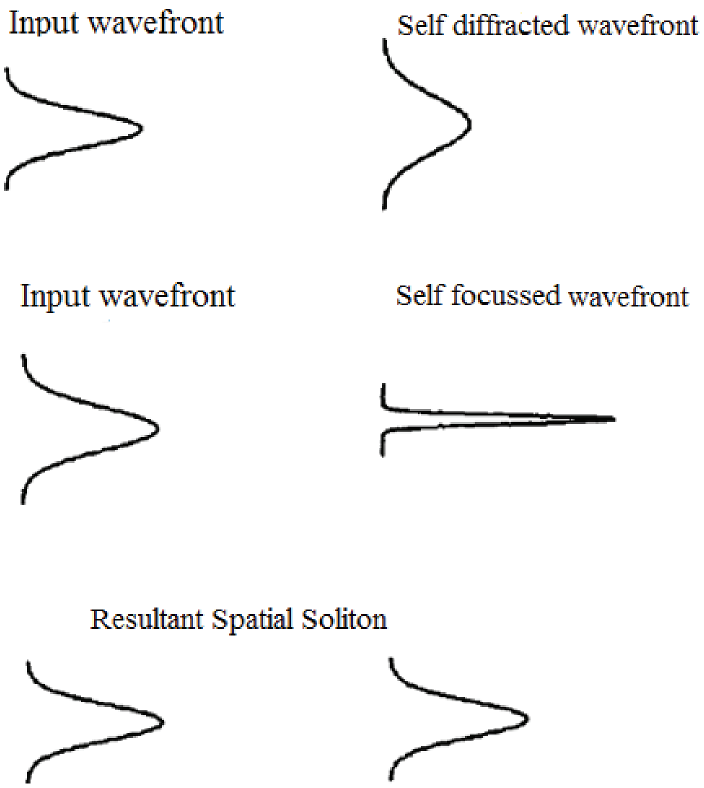


Fig.2 Formation of spatial Soliton [17]

These self trapped beams are fundamental in nature and have promising practical applications in the modern communication technology, for example, a new range of all optical signal routing devices can be built based on this phenomenon [18]. Spatial solitons, possessing one dimension, which exist in a slab waveguide are stable under propagation and collides in a Kerr nonlinear medium [19]. The one dimensional spatial solitons have been experimentally observed in different materials both at high and very recently at low powers for instance they have been observed in CS_2 waveguides [20], single mode glass waveguides. Optical beams which are self trapped in two transverse directions are unstable [16]. Below certain threshold power they diffract away and became self defocused. This situation may be changed by introducing quintic nonlinearity [21] or saturating type nonlinearity.

The mechanism of stability of soliton (with two transverse directions) in higher order cubic quintic medium is that in these mediums critical power of self-focusing is inversely proportional to beam radius. As the beam with power more than critical power propagates in a medium it self-focuses, the beam radius decreases. Due to decrease in the beam radius, its critical power increases and at a point the power becomes less than critical power. As a result again critical power decreases and beam again moves towards focusing .This process goes on and results to an oscillating beam width ,i.e., self trapped beam. Hence spatial soliton become stable in higher medium. [12]

Spatial solitons have been observed in different types of systems, like bulk medium [22], slab waveguides, thin surface waveguides and optical cavities**.** Some spatial solitons are 1D and 2D spatial soliton, photo refractive soliton , incoherent or partially coherent soliton quadratic soliton , Bragg soliton , discrete soliton , vortex soliton ,solitons of complex Ginzburg-Landau equation , cavity soliton , parametric soliton , nonlinear magneto optic soliton , walking soliton and gap soliton [12].

1.6.2 Temporal Soliton

These are optical pulses with a certain shape and energy that can propagate unchanged over large distances. Their existence was predicted in 1973 in the context of optical fibers [23] and by 1980 such solitons had been observed experimentally. These are light pulses that neither broaden nor squeeze in both temporal and spectral domain [12]. When a pulse propagates through a dispersive medium pulse broadening occurs and a chirp is produced. Similarly due to nonlinearity self-phase modulation (SPM) occurs. SPM induces chirp that may broaden or contract the pulse. When both the opposite types of chirp combine they counter balance each other. Also the GVD induced pulse broadening nullified by the SPM. The resultant pulse produced is called temporal soliton. When pulse propagates the dispersion of pulse also occurs, so due to only dispersion when pulse propagates the leading edge of pulse get blue shifted and trailing edge gets red shifted. The combining effect of both dispersion and self focusing is given below i.e. formation of a soliton, shown in below Fig.

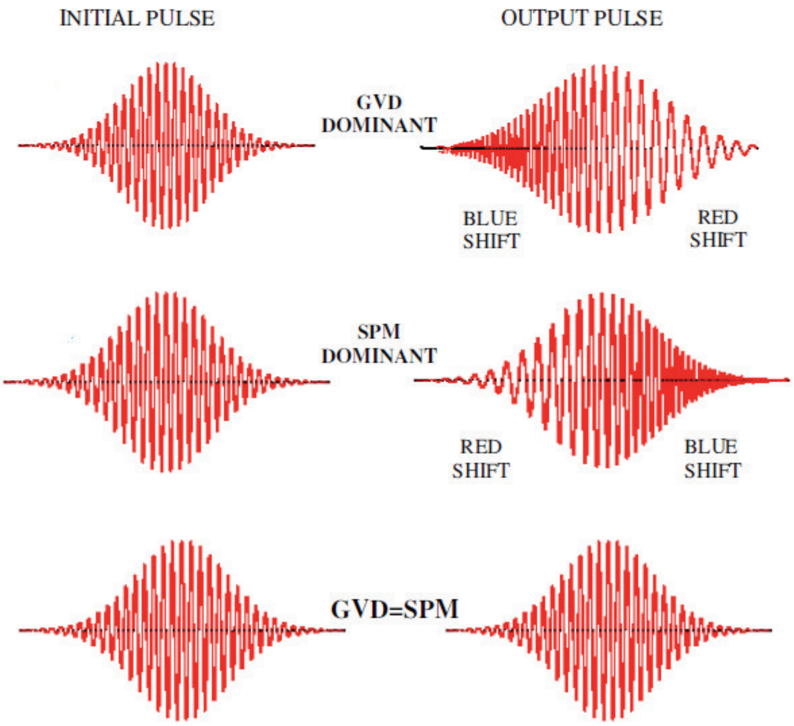


Fig 3.Formation of temporal Soliton [12]

1.6.3 Spatiotemporal Soliton (STS)

These are the hybrid type of soliton which is localized in all three dimensions of space as well as time [24]. These result from the balance of diffraction by self-focusing and GVD by nonlinear phase-modulation. The possibility of these beams is predicated by Silberberg, who also gave name light bullet to these pulsed beams [25].

1.7 Recent Research and Motivation:

Our topic of investigation is on spatial soliton propagation. These are more attractive than their temporal counterpart as the former have greater dimension. Spatial soliton has been widely studied both theoretically and experimentally. Simple Gaussian beam has been used for the early stage research. The propagation of an elliptical Gaussian beam passing through misaligned optical systems was studied [26]. The sinusoidal Gaussian beams were also studied in 1997[27].The propagation properties of cosh-Gaussian beams, Hermite cosh Gaussian beams and Hermite cosine-Gaussian beams have been widely studied [28-31]. The role of higher order nonlinearities in beam propagation and induced focusing has been examined in detail [32]. Also different beam profile have been used for such investigation. For example Gaussian beam, super Gaussian beams [33], self-trapping of degenerate modes of a laser beam [34], off axis mode [35], spiral self-trapping [36], elliptical Gaussian beams [36-39], self-focusing of necklace beams [40], self-trapped vector waves [41] and self-trapping of Bessel beams [42]. Elliptical Gaussian beams have been studied in saturating nonlinear medium [43]. However, the nonlinear propagation of elliptical super Gaussian beam in higher order nonlinear medium was not paid proper attention. So we propose the propagation of elliptical super Gaussian beam in a cubic quintic nonlinear media.

1.8 Objective:

1. To study the propagation of elliptical super Gaussian beam in a cubic quintic nonlinear media.
2. To investigate the effect of beam ellipticity on the propagation dynamics.
3. To investigate the effect of quintic nonlinearity on the propagation dynamics

Chapter 2

2.1 MATHEMATICAL FORMULATION

We consider the propagation of elliptical super Gaussian beam in a cubic quintic nonlinear media. The beam propagation equation is a nonlinear Schrodinger equation (NLSE) of the following form

$$-i\frac{\partial A}{\partial Z}+\frac{\partial^2 A}{\partial X^2}+\frac{\partial^2 A}{\partial Y^2}+n_1|A|^2A+sn_2|A|^4A=0................(1)$$

Where, A is the slowly varying of the field of the beam. The first term describes the evolution of the optical field along propagation length. Second and third term represents the diffraction along two orthogonal spatial dimensions, i.e., along x axis and y axis. The fourth term corresponding to the cubic nonlinearity and the fifth represents the quintic nonlinearity. s represents the relative strength of quintic nonlinearity.

We took our elliptical super Gaussian beam profile of the following form

$$A=A(Z)e^{-\left(\frac{x}{ra}\right)^{2m}-\left(\frac{y}{rb}\right)^{2m}}e^{i(\alpha x^2+\beta y^2)}e^{i(Vx+Uy)}.............(2)$$

Where ra, rb are the width parameters of beam respectively along the x and y directions. The free parameters $A(Z)$ is amplitude, α,β are the chirps respectively along the x and y directions. U and V are the tilt angles respectively along the x and y directions. m is super Gaussian parameters that are responsible for beam flatness and consider three values1,2,3 of them.

Fig. 4(a), 5(a) and 6(a) depict the beam profile for m=1, 2 and 3 respectively. These show that with increasing m parameter the beam becomes flater. Fig. 4(b), 5(b) and 6(b) are the cross-sectional view of them, which clearly show the elliptical nature.

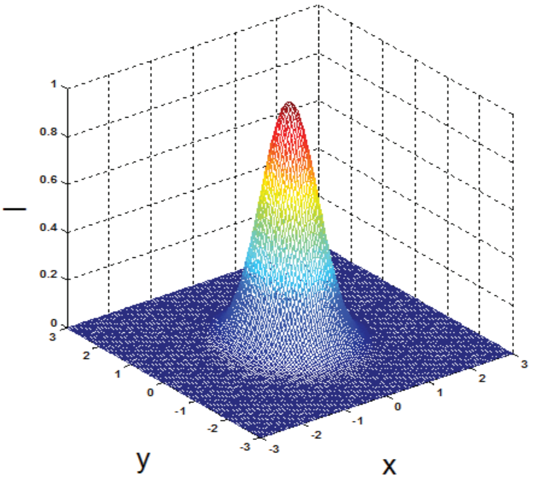


Fig.4a: Beam profile for $m=1$

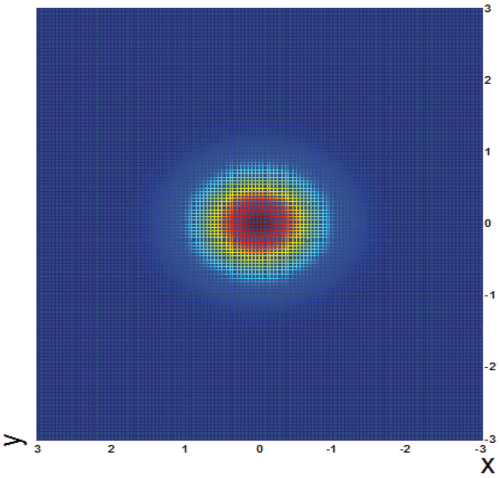


Fig.4b: cross-sectional view of beam for $m=1$

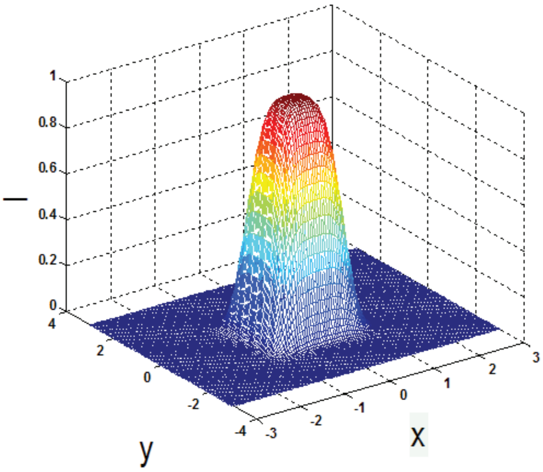


Fig.5a: Beam profile for $m=2$

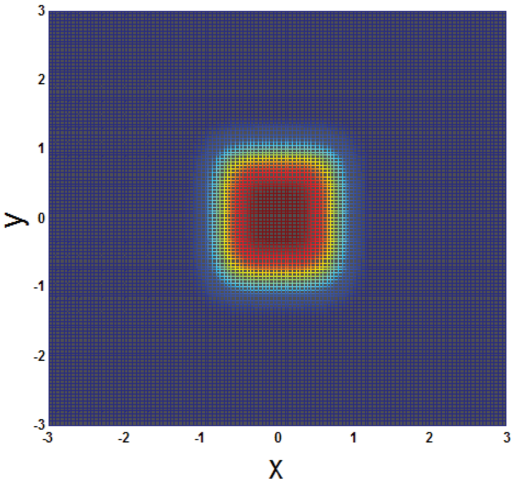


Fig.5b: cross-sectional view of beam for $m=2$

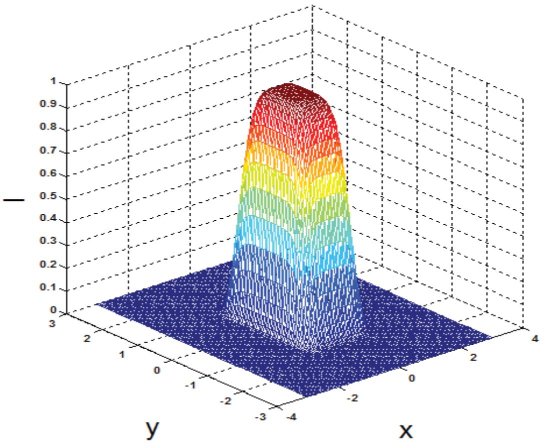


Fig.6a: Beam profile for $m=3$

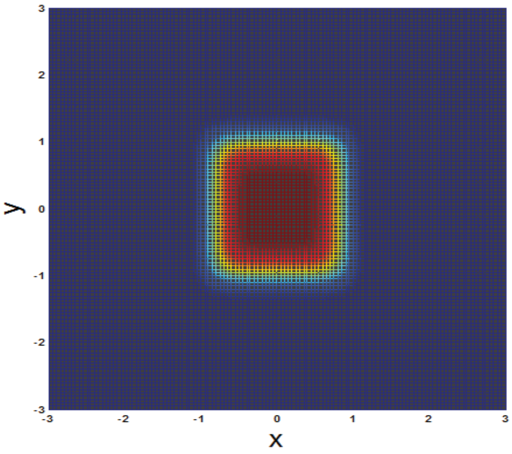


Fig.6b: cross-sectional view of beam for $m=3$

Now our primary task is to solve the NLSE for the above mentioned beam profile. Several analytical method are there for the solution. For example, inverse scattering method, Lux pair method, AKNS method, Fourier series, transform techniques, Green function techniques etc. With the advancement of computers numerical methods are developed to solve these equations. However, analytical methods have their own importance as they give the evolution of beam parameters in form of formula which can be further investigated to get the insight about the beam propagation. Also there are certain disadvantage of the aforesaid direct analytical methods. For many nonintegrable problem they stumble. Then there are some approximate analytical methods, which may solve the NLSE without changing the physics. Such a very useful method which is used in nonlinear optics is variational method based on trial functions and Rayleigh-Ritz optimization. The main advantage of the method is that it gives simple and explicit evolution equations for physically relevant quantities (here, the beam parameters) in situations where exact analytical solutions are either unavailable or may be too complicated to provide the proper physical insight and also in situations where only numerical solutions can be found. The drawback of the method is that it is based on the proper choice of trial functions. More importantly if the beam or pulse rest its shape this method cann’t capture this effect and thus fails catastrophically. However, in our case we can depend on this method as earlier research work suggest self-similar structure can be formed in such higher order nonlinear media.

The essential stage of the method is to find the Lagrangian density and hence reduced Lagrangian for the systems. For our model the field Lagrangian density L is given by

$$L=\frac{i}{2}\left(A^{*}\frac{\partial A}{\partial Z}-A\frac{\partial A^{*}}{\partial Z}\right)+\left|\frac{\partial A}{\partial X}\right|^{2}+\left|\frac{\partial A}{\partial Y}\right|^{2}-\frac{n_1|A|^{4}}{2}-\frac{Sn_2|A|^{6}}{3}\text{.................}(3)$$

We put trail function in given Lagrangian to get the following:

$$L=\Bigg[\bigg[-\left(\frac{\partial\theta}{\partial Z}+x^{2}\frac{\partial\alpha}{\partial Z}+y^{2}\frac{\partial\beta}{\partial Z}\right)-\left(x\frac{\partial V}{\partial Z}+y\frac{\partial U}{\partial Z}\right)+4\alpha^{2}x^{2}+4\beta^{2}y^{2}+U^{2}+V^{2}+4\alpha xV+4\beta yU$$

$$\text{........}+\frac{4m^{2}x^{4m-1}}{(ra)^{4m}}+\frac{4m^{2}y^{4m-1}}{(rb)^{4m}}\bigg]\varphi_0{}^{2}e^{-2\left(\frac{x}{ra}\right)^{2m}-2\left(\frac{y}{rb}\right)^{2m}}-\frac{n_1\varphi_0{}^{4}e^{-4\left(\frac{x}{ra}\right)^{2m}-4\left(\frac{y}{rb}\right)^{2m}}}{2}-\frac{Sn_2\varphi_0{}^{6}e^{-6\left(\frac{x}{ra}\right)^{2m}-6\left(\frac{y}{rb}\right)^{2m}}}{3}\Bigg]\text{.......}(4a)$$

We obtain total Lagrangian which is given as $< L >= \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} L dx dy$

$$< L >= \left[-A_0^{\,2}\frac{\partial \alpha}{\partial Z} + 4\alpha^2 A_0^{\,2} \right] A_{m_i} + \left[-A_0^{\,2}\frac{\partial \beta}{\partial Z} + 4\beta^2 A_0^{\,2} \right] B_{m_i} + \frac{4A_0^{\,2}m^2}{(ra)^{4m}} C_{m_i} + \left[A_0^{\,2}V^2 + A_0^{\,2}U^2 \right] D_{m_i} + \ldots$$

$$\ldots\ldots\ldots + \left[\frac{4A_0^{\,2}m^2}{(rb)^{4m}} \right] E_{m_i} - \frac{n_1 A_0^{\,4}}{2} F_{m_i} - \frac{Sn_2 A_0^{\,6}}{3} G_{m_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

Using Euler Lagrangian equation following equations are obtained .

$$\varphi^2 ab = \text{Constant} \ldots\ldots\ldots\ldots\ldots (5a)$$

$$\frac{\partial U}{\partial Z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

$$\frac{\partial V}{\partial Z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5c)$$

$$\frac{\partial^2 a}{\partial Z^2} = \frac{16m^2}{r^{4m}a^3}\frac{C_i}{A_i} - \frac{n_1\varphi_0^{\,2}}{a}\frac{F_i}{A_i} - \frac{4Sn_2\varphi_0^{\,4}}{3a}\frac{C_i}{A_i} \ldots\ldots\ldots (6)$$

$$\frac{\partial^2 b}{\partial Z^2} = \frac{16m^2}{r^{4m}b^3}\frac{E_i}{B_i} - \frac{n_1\varphi_0^{\,2}}{b}\frac{F_i}{B_i} - \frac{4Sn_2\varphi^4}{b}\frac{G_i}{B_i} \ldots\ldots\ldots (7)$$

We normalize equation (6) and (7) using following equations.

$P = \frac{n_1\varphi_0^{\,2}r^2}{4}$ => the critical power of self-focusing in cubic or Kerr media , $N = \frac{Sn_2\varphi_0^{\,4}}{n_1\varphi_0^2}$ => the relative strength of quintic nonlinearity. ....(8)

The normalized equation looks as follows:

$$\frac{\partial^2 a}{\partial Z^2}=\frac{16m^2}{r^{4m-4}}\frac{C_i}{A_i}\left[\frac{1}{a^3}-\frac{P}{a^2b}-\frac{4SPNr^2}{3a^3b^2}\frac{G_i}{F_i}\right]\text{...............(9)}$$

$$\frac{\partial^2 b}{\partial Z^2}=\frac{16m^2}{r^{4m-2}}\frac{C_i}{B_i}\left[\frac{{}^{E_i}\!/\!_{C_i}}{b^3}-\frac{P}{ab^2}-\frac{4SPNr^2}{3a^2b^3}\frac{G_i}{F_i}\right]\text{............(10)}$$

The eq. (9) and (10) are the evolution equations of beams widths along x and y directions. In other way they represent the oscillating particles of unit mass. These are utilized to study the beam dynamics.

Chapter 3

3.1 RESULTS AND DISCUSSION:

We first study the beam propagation in cubic medium, i.e., s=0. As expected, at $P=1$ we get self trapped beam, whereas for a higher P the beam get self-focused and at lower P it is self-diffracted. This has been depicted in Fig. 7. However, for all values of m we got the similar effect.

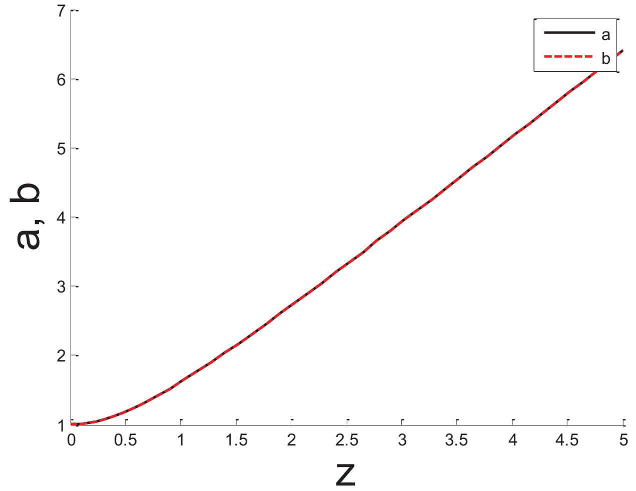


Fig. 7a: Self defocusing of symmetrical beam in cubic medium at $P=0.9$,a=b, $m=1$

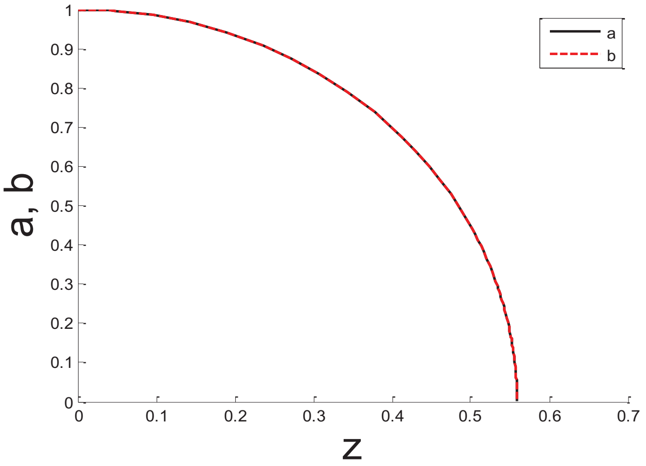


Fig. 7b: Self focusing of symmetrical beam in cubic medium at $P=1.2$,a=b, $m=1$

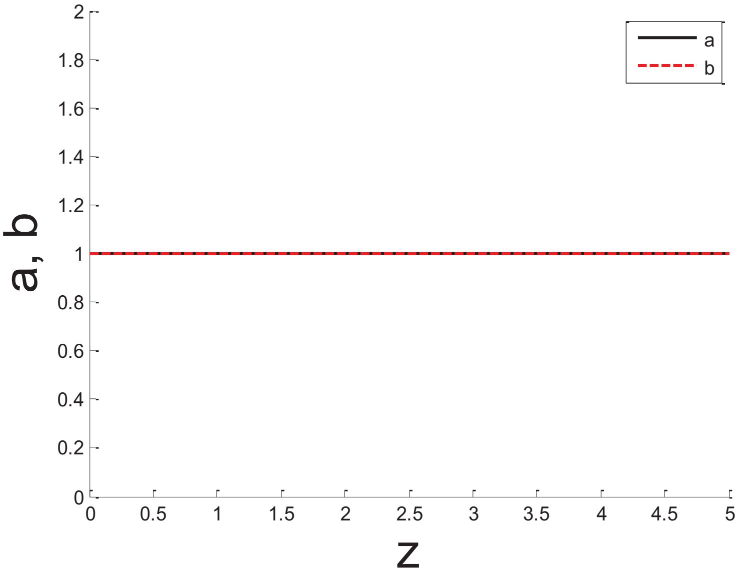


Fig. 7c: Self -trapping of symmetrical beam in cubic medium at. $P=1$,a=b, $m=1$

We investigate the effect of initial beam width on critical power of self focusing. Fig. 8 depicts that critical power of self focusing increases with beam width.

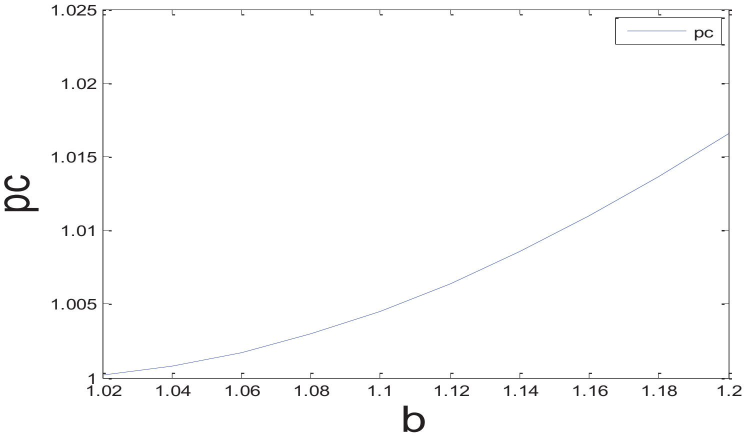


Fig. 8: Variation of critical power of self focusing with *b*

Above in Fig.7 we show the beam dynamics for different powers when the initial beam widths were same, i.e., for symmetrical beam. We took asymmetrical beam which also show (Fig.9(a)) self trapping but at higher slightly critical power. The beam width oscillates periodically during propagation in contrary to the cubic case where beam widths were unchanged. Also notable thing is that the beam widths never cross their initial values during the propagation. In this case too we get self-focusing and defocusing at higher and lower beam power. This has been depicted through Fig.9(b) and (c) respectively.

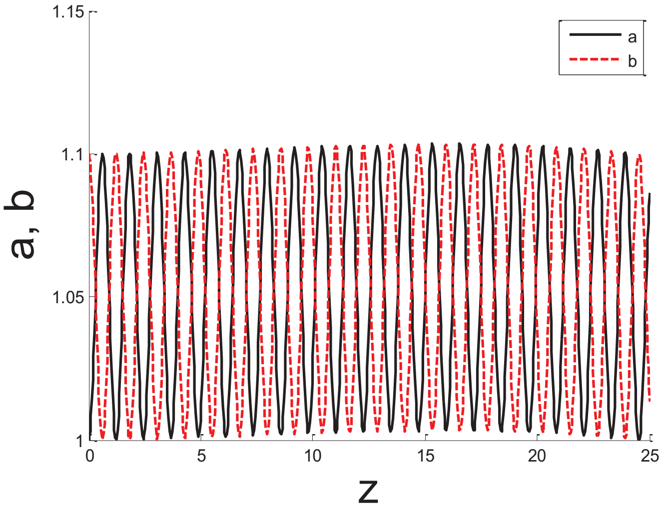


Fig. 9a: Self -trapping of asymmetrical beam in cubic medium at $P = 1.004527$,a $\neq$ b, $m = 1$

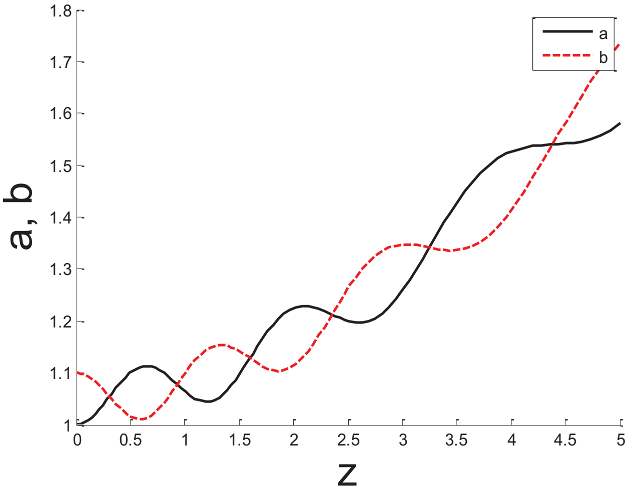


Fig. 9b: Self -diffracting of asymmetrical beam in cubic medium at $P = 1$ a $\neq$ b, $m = 1$

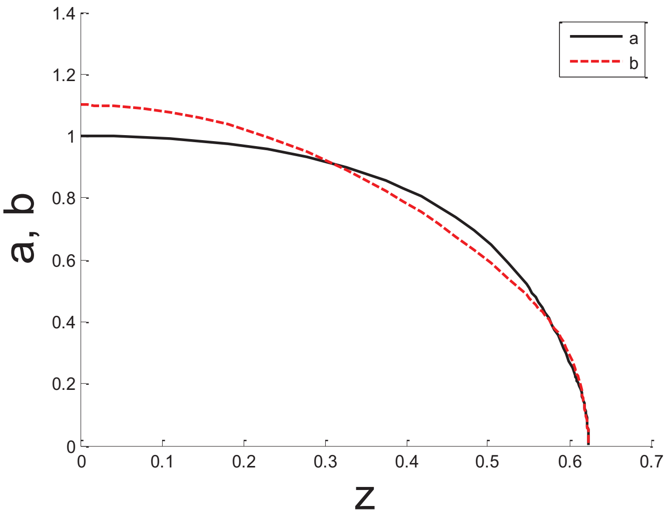


Fig. 9c :Self -focusing of asymmetrical beam in cubic medium at $P = 1.2$ a $\neq$ b, $m = 1$

At higher value of m we get similar behavior of the beam. However marginal change in self trapping power has been observed.

Now we switch over quintic medium. We consider *s=-1*, which implies opposite sign of nonlinearity with respect to the cubic nonlinearity. This kind of combination of positive cubic and negative quintic nonlinearity is referred as competing nonlinearity. In quintic medium beam collapse has been avoided, rather oscillation of symmetrically beam widths has been observed. Fig. 10(a), 10(b), and 10(c), show the aforesaid behavior for m=1,2 and 3 respectively.

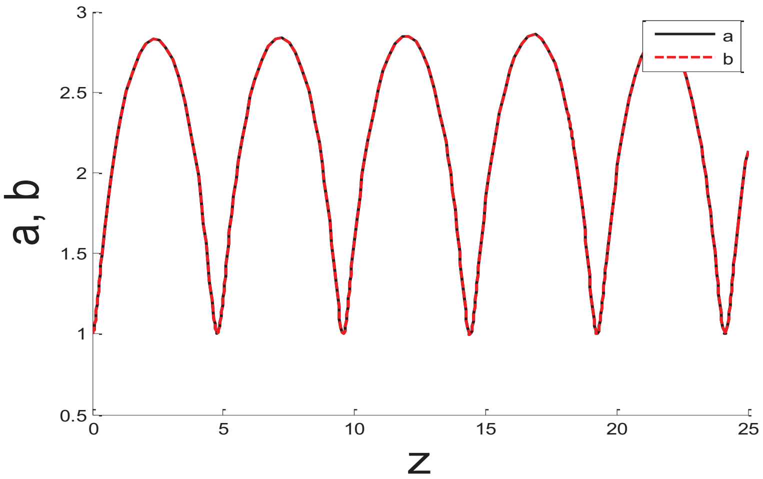


Fig. 10a: Self -trapping of symmetrical beam in cubic-quintic medium at P=2. *S*=-1*, a=b, m*=1

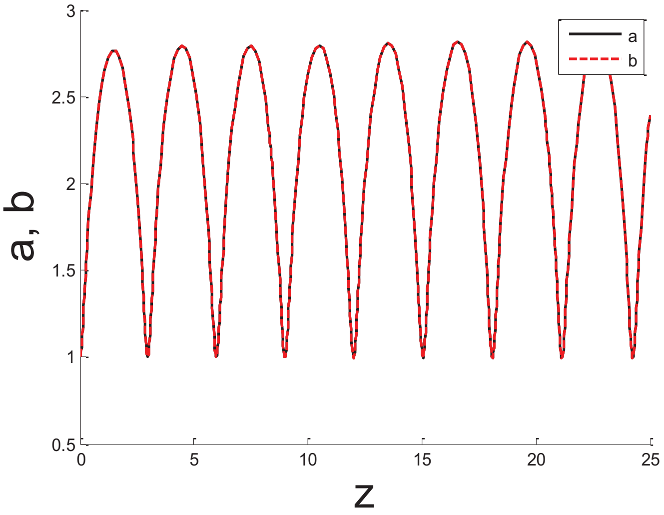


Fig. 10b: Self -trapping of symmetrical beam in cubic-quintic medium at P=2.6. *S*=-1*, a= b, m*=2

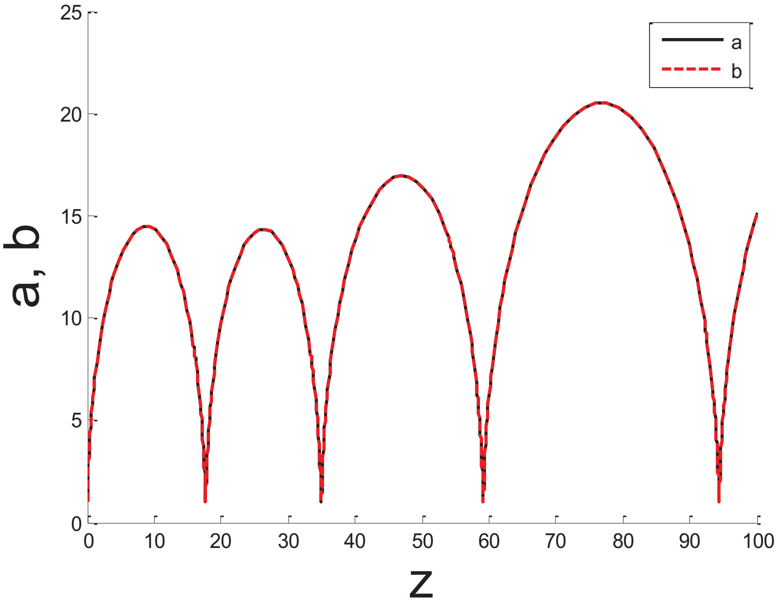


Fig.10c: Self -trapping of symmetrical beam in cubic-quintic medium at $P = 20$,*S*=-1,*a*=*b*,*m*=3

It is clear that higher power is required for self trapping of beam with higher superGaussian parameter. We also study the dynamics of asymmetrical. In this case beam more complicated beam dynamics have been observed, particularly at higher order superGaussian beam. Fig. 11(a), 11(b), and 11(c), show the self-trapping for m=1,2 and 3 respectively.

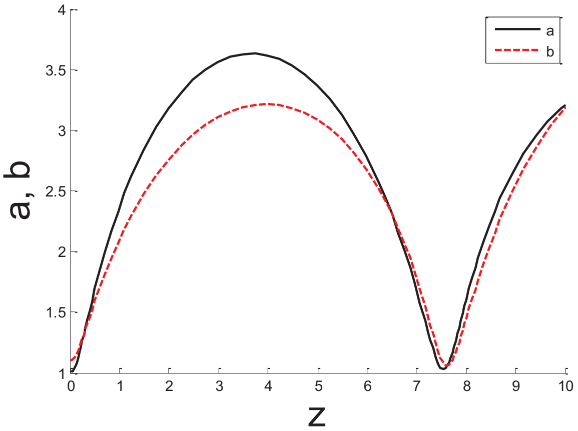


Fig.11a:Self -trapping of asymmetrical beam in cubic-quintic medium at $P = 1.8$, S=-1, $a \neq b$, m=1.

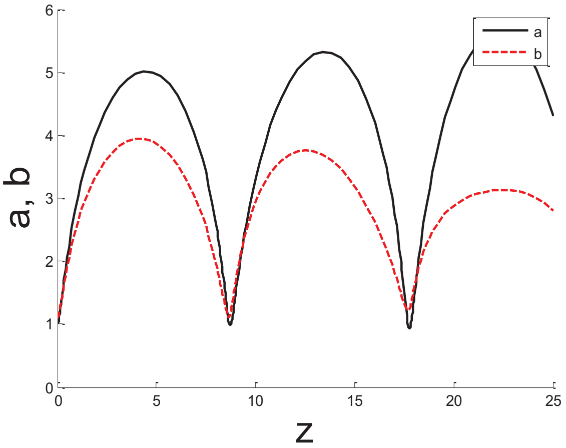


Fig.11b : Self -trapping of asymmetrical beam in cubic-quintic medium at. $P = 1.2$ S=-1, $a \neq b$, m=2.

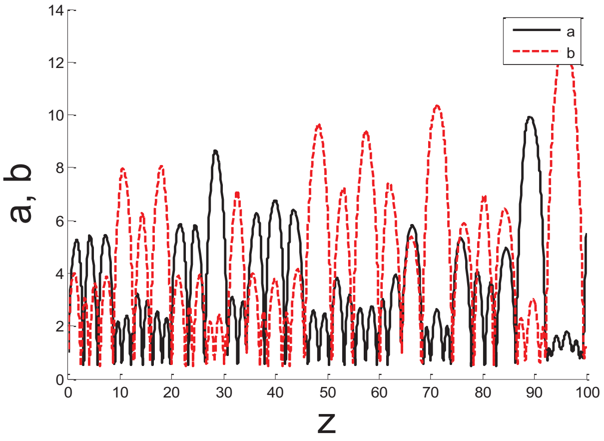
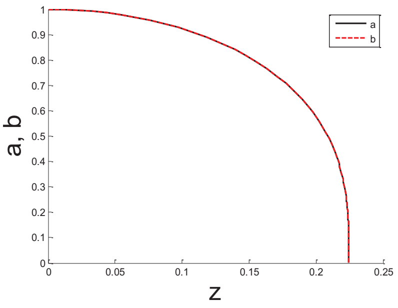
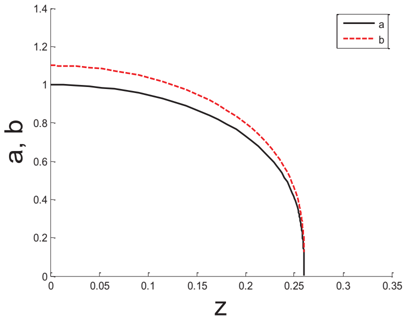
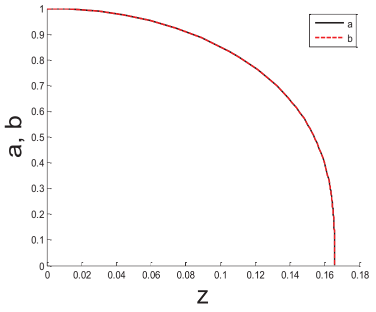
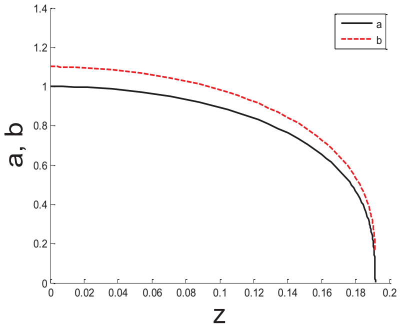


Fig. 11c: Self -trapping of asymmetrical beam in cubic-quintic medium at $P = 10.5$,S=-1, $a \neq b$, m=3.

The other type of quintic nonlinearity, i.e., s=1, however, shows self-focusing at the power of self-trapping in cubic medium. The reason is obvious as s=+1 strengthen the cubic nonlinearity. The figures in the following table show self-focusing for all *m*. A simple observation reveals that self –focusing is easier at higher *m*. This matches with cubic case. This simply means that s=+1 merely enhance the cubic effect. Whereas s=-1 shows more intriguing beam dynamics.

m	Symmetrical beam	Asymmetrical beam
m=1		
m=2		

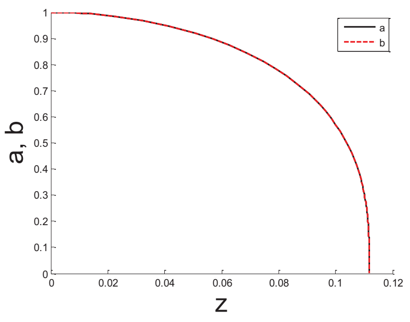
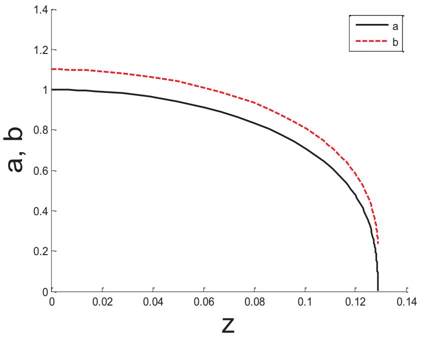
m=3		

Fig. 12 : self-focusing for s=1 at P=1.

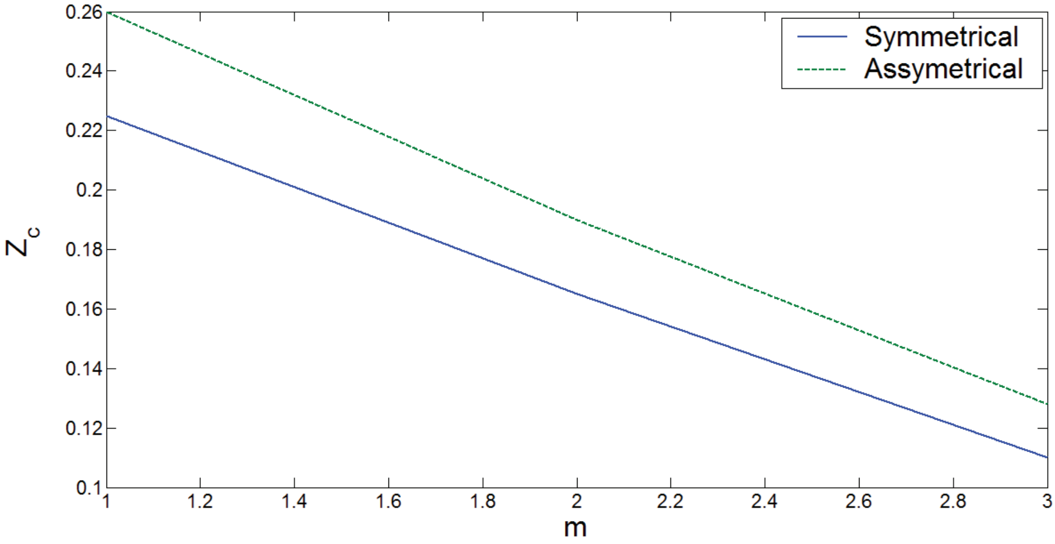


Fig.13 Variation of collapse length with super Gaussian parameter in quintic medium s=1.

At this point its worthy to discuss the confinement or self trapping with the help of phase plot. The following plot, i.e., Fig. 14 and 15 will depict self-trapped beam dynamics and corresponding phase plot for s=0 and -1 respectively. The bound oscillation of the phase plot indicates self-trapped beam propagation.

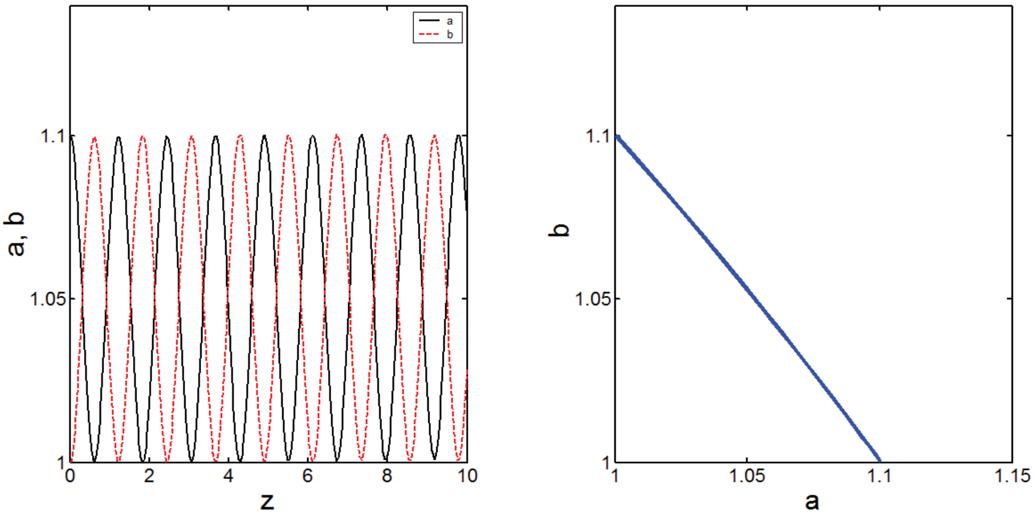


Fig.14 Typical self-trapped beam dynamics and corresponding phase plot for s=0.

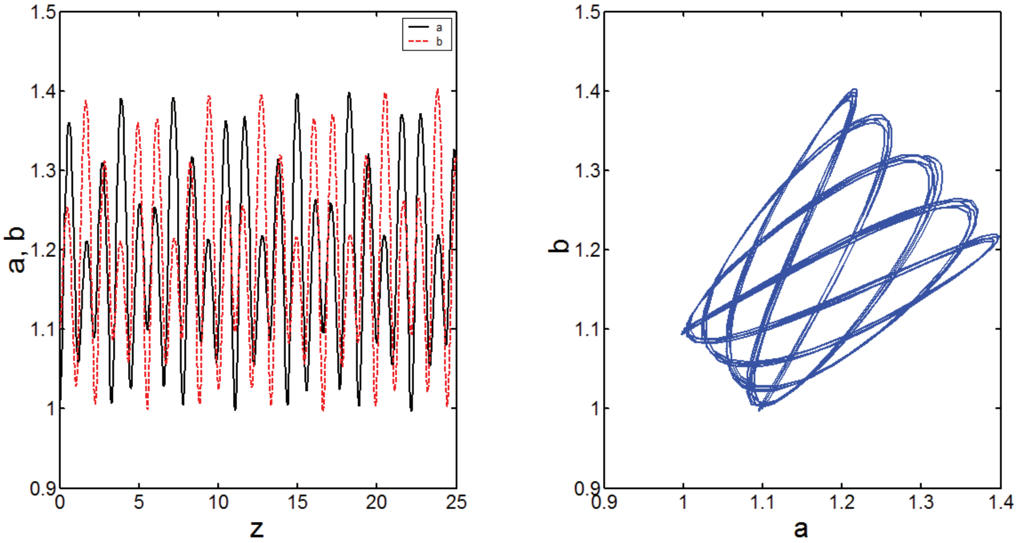


Fig.15 Typical self-trapped beam dynamics and corresponding the phase plot for s=-1

For further analysis we construct a scalar potential V from which can derive the beam evolution equations following the condition: $\frac{\partial^2 a}{\partial Z^2} = -\frac{\partial V}{\partial a}$ and $\frac{\partial^2 b}{\partial Z^2} = -\frac{\partial V}{\partial b}$.

$$V = \frac{16m^2}{r^{4m-4}} \frac{C_i}{A_i} \left[\frac{1}{2a^2} + \frac{1}{2b^2} - \frac{P}{ab} - \frac{2SPNr^2}{3a^2b^2} \frac{G_i}{F_i} \right]$$

Also we find the corresponding Hamiltonian of the system as follows:

$$H = \left[\frac{\dot{a}^2 + \dot{b}^2}{2} \right] + J \left[\frac{1}{2a^2} + \frac{1}{2b^2} - \frac{P}{ab} - \frac{2SPNr^2}{3a^2b^2} \frac{G_i}{F_i} \right]$$

Where $J = \frac{16m^2}{r^{4m-1}} \frac{C_i}{A_i}$.

We find effective beam radius $\rho = \sqrt{a^2 + b^2}$ as follows:

It can be easily shown that

$$\frac{\partial^2 \left(a^2 + b^2 \right)}{\partial Z^2} = 4H - \frac{8spnG_i r^2 J}{3F_i a^2 b^2} .$$

Integrating twice, we get

$$\rho^2 = a^2 + b^2 = a_0{}^2 + b_0{}^2 + 2HZ^2 + 2(a_0\dot{a}_0 + b_0\dot{b}_0)Z + J_q \ln R - \frac{J_q Z}{R_0} - J_q \ln R_0$$

With $J_q = \frac{8SPNG_i r^2 J}{3F_i}$, $R = ab$ $a(0) = a_0, b(0) = b_0$.

We consider our beam is not initially diverging, i.e., $\dot{a}_0 = 0 = \dot{b}_0$. This leads to

$$\rho = \left[\rho_0{}^2 + 2\rho\dot{\rho}_0 Z + 2HZ^2 + J_q \ln R - \frac{J_q Z_c}{R_o} - J_q \ln R_0 \right]^{1/2} ,$$

Where, $\rho_0 = \sqrt{a_0{}^2 + b_0{}^2}$, We can find the critical length Z_C for self-focusing of the beam putting $\rho = 0$, which is as follows:

$$Z_C = \frac{J_q \pm \sqrt{J_q^{\ 2} - 8HR_0^{\ 2}(J_q \ln R - J_q \ln R_0 + \rho_0^{\ 2})}}{4HR_0}$$

Now since in self-trapping case no beam collapse occurs, in such case Z_C is infinity. This condition yields the threshold power P_{th} of self trapping.

$$P_{th} = \frac{3F_i a_0 b_0}{2\left(3F_i a_0 b_0 + 2SNG_i r^2\right)}\left(\frac{b_0}{a_0} + \frac{a_0}{b_0}\right)$$

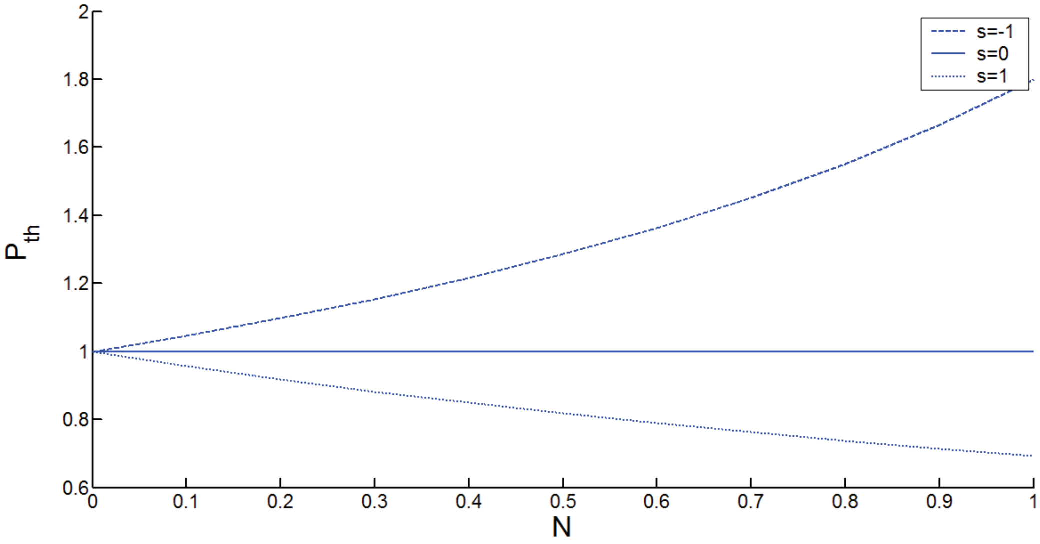


Fig. 16: Variation of the P_{th} w.r.t. relative strength of quintic nonlinearity.

3.1 Conclusion

We investigated propagation of elliptical super Gaussian beam in a cubic quintic nonlinear media. We identified the condition for self trapped propagation, i.e., spatial soliton formation. Also the effect of beam ellipticity on the propagation dynamics has been shown. The effect of quintic nonlinearity on the propagation dynamics has also been discussed. It has been found that defocusing quintic nonlinearity significantly change the beam dynamics and yields intriguing situations. In other hand the asymmetrical beam has more interesting dynamics in comparison to its symmetrical counterpart.

The outcome of this investigation can be utilized for further experimental verification. Also it will be helpful for all optical switching and data processing, terrestrial communication etc.

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