PERFORMANCE ANALYSIS OF DIVERSITY COMBINING WITH CHANNEL CODING

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Submitted by

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CERTIFICATE

I, Rekha Rani hereby certify that the work which being is presented in this thesis entitled **"Performance Analysis of Diversity Combining with Channel Coding"** by me in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronics and Communication from Thapar University, Patiala is an authentic record of my own work carried under supervision of Ms. Surbhi Sharma and referred other researcher's work which are duly listed in the reference section.

The matter in this thesis has not been submitted in any University/Institution for the award of Master of Engineering.

Rekha Rani

(Rekha Rani)

Date: 20-June-2011 Signature of student

This is certifying that the above statement made by the student is correct to the best of my knowledge and belief.

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ABSTRACT

Wireless communications is the fastest growing segment of the communications industry. There is always a greater demand for capacity, reliability and also need to integrate voice, data and other type of traffic over radio channels. There are many ways to solve all these problems, such as Space-time coding, Spread spectrum techniques and Antenna arrays.

In Wireless communications, the effect of channel fading and co-channel interference is very high. So we need diversity techniques which are used to mitigate all these effects. Indeed, diversity techniques at the receiver, in which two or more copies of the same information-bearing signal are combined skillfully to increase the overall signal-to-noise ratio (SNR), still offer one of the greatest potential for radio link performance improvement to many of the current and future wireless technologies. The channel coding is a rudimentary technique, which used to transmit digital data reliably over a noisy channel and to improve the efficiency of information transmission.

This thesis develops a mathematical framework for analyzing the average bit error rate performance of selection diversity combining, maximal ratio combining and optimum combining schemes with binary phase-shift keying (BPSK) modulation over independent and identically distributed Rayleigh fading channel with channel coding. With channel coding the performance of diversity combining techniques has been improved than the uncoded diversity combining techniques.

It has been shown that the coding gain for coded SC over uncoded SC is 4 dB for N=5 and 4.2 dB for N=6. It has been seen that coding gain obtained by using channel coding is excellent. Also, the coding gain for coded and uncoded MRC and OC has been computed. The coding gain for coded MRC over uncoded MRC is 1.6 dB for N=5, 6 and coding gain for coded OC over uncoded OC is 4.7 dB for N=5 and 5 dB for N=6 when the number of interferers taken as L=1 and coding gain for coded OC over uncoded OC is 7 dB for N=5 and 6.2 dB for N=6 when the number of interferers are equal and greater than the number of receive array element which is taken as L=18. So it has been shown that for a given BER coded signals required less SNR as compared to uncoded signals.

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LIST OF ABBREVIATIONS

DSP	Digital Signal Processing
VLSI	Very Large Scale Integration
BPSK	Binary Phase Shift Keying
MPSK	Multilevel Phase Shift Key
MQAM	Multilevel Quadrature Amplitude Modulation
SC	Selection Combining
SDC	Selection Diversity Combining
SD	Selection Diversity
MRC	Maximal Ratio Combining
SWC	Switched Combining Diversity
GSC	Generalized Selection Combining
ODC	Optimum Diversity Combining
BEC	Binary Erasure Channels

EGC	Equal Gain Combining
dB	Decibel
AWGN	Additive White Gaussian Noise
i.i.d.	Independent Identical Distribution
i.n.i.d	Independent Non Identical Distribution
LDPC	Low Density Parity Check
PDF	Probability Density Function
CDF	Cumulative Density Function
LLR	Log Likelihood Ratio
GA	Gaussian Approximation
DE	Density Evolution
CEP	Conditional Error Probability
MGF	Moment Generating Function
STBC	Space Time Block Code
LOS	Line Of Signal
NLOS	Non Line Of Sight
SNR	Signal To Noise Ratio
SIR	Signal-To-Interference Ratio
SINR	Signal-To-Interference Noise Ratio
BER	Bit Error Rate
SER	Symbol Error Rate
SOI	Signal Of Interest
CCI	Cochannel Interference

CHAPTER 1

Introduction

The first chapter introduces the block diagram of wireless communication system and gives the brief description of the components used in it. The background of channel coding, modulation, fading and different types of diversity has also been discussed in this chapter.

1.1 INTRODUCTION OF WIRELESS COMMUNICATION

Wireless communications is one of the most active areas of technology development of our time. This development is being driven primarily by the transformation of what has been largely a medium for supporting voice telephony into a medium for supporting other services, such as the transmission of video, images, text, and data. Thus, similar to the developments in wireline capacity in the 1990s, the demand for new wireless capacity is growing at a very rapid pace. Although there are, of course, still a great many technical problems to be solved in wireline communications, demands for additional wireline capacity can be fulfilled largely with the addition of new private infrastructure, such as additional optical fibre, routers, switches, and so on. On the other hand, the traditional resources that have been used to add capacity to wireless systems are radio bandwidth and transmitter power. Unfortunately, these two resources are among the most severely limited in the deployment of modern wireless networks: Radio Bandwidth because of the very tight situation with regard to useful radio spectrum, and Transmitter Power because mobile and other portable services require the use of battery power, which is limited. These two resources are simply not growing or improving at rates that can support anticipated demands for wireless capacity. On the other hand, one resource that is growing at a very rapid rate is that of processing power. Moore's Law, which asserts a doubling of processor capabilities every 18 months, has been quite accurate over the past 20 years, and its accuracy promises to continue for years to come. Given these circumstances, there has been considerable research effort in recent years aimed at developing new wireless capacity through the deployment of greater intelligence in wireless networks. A key aspect of this movement has been the development of novel signal transmission techniques and advanced receiver signal processing methods that allow for significant increases in wireless capacity without attendant increases in bandwidth or power requirements.

But mobile radio channel places fundamental limitations on the performance of wireless communication systems. The transmission path between transmitter and receiver can vary from simple line of sight to one that is severely obstructed by buildings, mountains etc. unlike wired channels that are stationary and predictable, radio channel is extremely random and do not offer easy analysis. Even the speed of motion impacts how rapidly the signal level fades as a mobiles terminal moves in space. Modeling the radio channel has historically been one of the most difficult parts of mobile radio system design, and is typically done in a statistical fashion.

1.1.1 Block diagram of a wireless communication system

In Figure (1.1), the data source generates an information signal that is to be transmitted at the receiver. After the message is produced by the data source which is processed at the transmitter end before being transmitted on a noisy channel. The processing is done to compensate the effect of various signal impairments caused by the channel and to enable the waveforms to be detected easily at the receiver end.

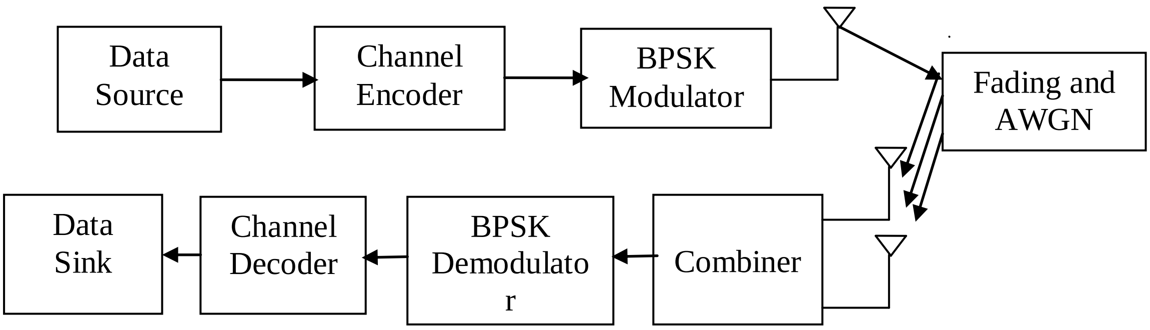

Figure 1.1: Block Diagram of General Communication System

The two main blocks at the transmitter end are encoder and modulator. The message from data source is fed into the channel encoder. Typically, a channel encoder adds redundant bits to the incoming data to make the signal more immune to various channel distortion. These redundant bits are removed at the receiver to recover the original message. The encoded signal is then modulated using the BPSK modulator to transfer the signal waveforms to better withstand the channel impairments. Modulation is the method by which the message symbols are converted to waveforms that are compatible with the requirements imposed by the channel.

The channel is the transmission medium, through which this waveform is transmitted. It is responsible for distortion the message signal and degrading the performance of the wireless communication system. The distortions in wireless channel are often due to noise (AWGN) and fading. After the degraded signal is received at the receiver, it is processed by the demodulator and decoder which try to correct some of the errors in the transmission. The process at the receiver is the reverse of that at the transmitter. The received waveform is demodulated by the BPSK demodulator. The iterative channel decoder then decodes the demodulated waveform and removes some of the errors. If good quality decoders are used, then the effect of this degradation can be mitigated significantly.

The term data sink is used to describe a device or part of computer which is capable of accepting data signals from a data transmission device and storing data for future use. All this trouble is undertaken just to ensure that the correct information which is intelligible is imparted to the data sink.

1.2 CHANNEL CODING

The encoder can be subdivided into two parts, source encoder and channel encoder. In this thesis, we are not concerned with source coding, so the term encoder simply means channel encoder. The channel encoder makes the message signal more immune to noise and distortion by adding redundant information in the discrete domain. This technique can thought of as a medium for accommodating desirable system trade-offs such as error performance vs. bandwidth. It is possible to get as much as 10dB performance improvement by the use of channel coding. At the receiver end the decoder uses this redundancy to efficiently extract the originally transmitted message with as less errors as possible. Basically channel coding is used to maximize the information rate. Channel codes are of three types

1. Block codes
2. Convolutional codes
3 Turbo codes

As number of channel codes have been used over time. A few of them are hamming codes, BCH codes, Reed Solomon codes, Turbo codes, LDPC codes and Trellis codes. A few important properties of these codes are discussed below:

1.2.1 Hamming codes

Named after their inventor, Richard Hamming, Hamming codes are one of the earliest channel codes that were actually effective. They are especially suitable for correcting all single errors and detecting two or fewer errors in a block. These codes have a minimum distance of 3 and are very effective when used with syndrome decoding.

- Block length: $n=2^m-1$
- Information bits: $k=2^m-1-m$
- Parity Check Bits: $n-k=m$
- Correctable Errors: $t=1$

These codes became very popular when they were invented but are not used much now days because of the development of more efficient coding schemes. Due to the simplicity of Hamming codes they are used in computer memory RAM.

1.2.2. BCH Codes

The BCH codes are a generalization of the Hamming codes and allow multiple error correction. The abbreviation BCH comprises of the names of the inventors of these codes, Bose, Chaudhuri and Hocquenghem. These codes belongs to a special class of codes called cyclic codes which enable them to be decoded very easily by the use of elegant algebraic method called syndrome decoding.

BCH codes for $m \geq 3$ and $t< 2^{m-1}$.

- Block Length: $n=2^m-1$
- Parity Check Bits: $n-k \leq mt$
- Minimum Distance: $d \geq 2t + 1$

The very famous R-S codes are a subclass of BCH codes.

1.2.3 Reed-Solomon codes

In 1960, Irving Reed and Gus Solomon introduced a new class of error-correcting codes that are now called Reed-Solomon (R-S) codes. Reed-Solomon is an error-correcting code that works by oversampling a polynomial constructed from the input data. These codes are amongst one of the most popular channel codes and are used in a number of commercial applications and data transmission technologies. Most prominent application of RS codes includes CDs, DVDs, Blu-Ray Discs, DSL and WiMAX. The main ides behind RS codes is that the data which is to be encoded first seen as a polynomial. The code is based on a theorem of algebra that states that any k distinct points uniquely determine a polynomial of degree, at most k-1.

- Block Length: $n=2^m-1$
- Minimum Hamming Distance: $n-k+1$

1.2.4 Low Density Parity Check (LDPC) Codes:

Low-density parity check (LDPC) codes [1], discovered by Robert Gallager in his Ph.D thesis at M.I.T in 1962.Unlike many other classes of codes, LDPC codes are equipped with very fats encoding and decoding algorithm which makes them very useful for practical applications. At the end of time when these codes were invented, the hardware was not available for their implementation. So they were not given much attention. Now with the advancement of VLSI and DSP it is finally possible to implement these codes practically. The main attraction of LDPC codes is that they approach the Shannon limit of channel. There have been simulations that perform within 0.04dB of the Shannon limit. The codes that approached the Shannon limit are not the ones originally proposed by Gallager, but rather a modified version called irregular LDPC codes proposed by MacKay [2]. In 1981 R. M. Tanner [3] introduced a recursive approach to the construction of LDPC codes described in the language of graphical theory. The LDPC codes are characterized by a sparse parity check matrix in which the number of 1's is very less as compared to the number of 0's [4]. LDPC codes have been proved to achieve capacity on binary erasure channels (BEC). Unlike turbo codes, they can be optimized for good bit-error rate performance over a given channel and for a given code-rate [5].

1.3 MODULATION SCHEMES

After the signal has been encoded, it is modulated using a carrier to transform the signal into waveform that can better withstand the channel impairments. By modulating, we vary some parameter of the carrier according to the message signal and sent it over the channel. The basic band-pass modulation/demodulation techniques are divided into two parts:

1. Coherent Modulation
2. Non-coherent Modulation

In coherent modulation schemes, the information about the phase and frequency of the carrier is needed at the receiver for detection. It is more efficient in terms of the performance but the receiver design becomes more complex. In non-coherent modulation schemes, no information about the phase and frequency of the carrier is required for detection. In this scheme the receiver design is less complex but at the cost of degraded performance. A number of modulation schemes have been proposed in the past.

1.4 FADING IN WIRELESS COMMUNICATIONS

Radio waves propagate from a transmitting antenna and travel through free space undergoing absorption, reflection, refraction, diffraction and scattering. They are greatly affected by the ground, buildings, tress, and other objects present in their path. All the things are responsible for the characteristics of the received signal. The two main factors that affect the reliable communication of a message over the channel are noise and fading. Noise in communication system is generally modelled a as AWGN because it is convenient to deal with noise of additive rather than multiplicative nature.

Noise affects transmission over the channel by degrading the signal quality. Over the large distance, signal quality is shown to degrade even without the presence of large quantities of AWGN. This degradation is known as fading [6] and the errors introduced by fading are much difficult to deal with as compared to the errors introduced by noise. Fading is caused by multi-path effect. Multi-path effect means that a signal transmitted from a transmitter may have multiple copies traversing different paths to reach a receiver. Thus, at the receiver, the received signal should be the sum of all these multi-path signals. Because the paths traversed by these signals are different; some are longer and some are shorter. The one at the direction of light of signal (LOS) should be the shortest. These signals interact with each other. If signals are in phase, they would intensify the resultant signal; otherwise, the resultant signal is weakened due to out of phase. This phenomenon is called channel fading. In general, there are two criteria to measure channel fading, including (1) Doppler spread, and (2) delay spread.

Fading are of two types:

Large scale fading**:** This type of fading is seen over the large distance and caused by shadowing introduced by obstacles where a large obstruction such as a hill or large buildings obscures the main signal path between transmitter and the receiver. The amplitude change caused by shadowing is often modelled using log-normal distribution. Large scale fading is also called shadow fading.

Small scale fading**:** Small scale fading is observed over smaller distances as compared to large scale fading. in small scale fading, different replicas of the same signal are created by reflection, diffraction and scattering, Small scale fading can be further categorized as:

- Fast or slow fading
- Frequency selective or flat fading

Small scale fading models that have been widely used for modeling the fading environment are:

- Rayleigh fading
- Rician fading
- Nakagami fading

1.4.1 Rayleigh fading

The Rayleigh distribution is most widely used distribution to describe the received envelope value. The Rayleigh flat fading channel model assumes that all the components that make up the resultant received signal are reflected or scattered and there is no direct path from transmitter to the receiver. The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. In the Rayleigh fading channel model, it is assumed that the channel induce amplitude which varies in time according to the Rayleigh distribution.

When the channel impulse response is modelled as a zero-mean complex valued Gaussian process, the envelope at any instant is Rayleigh-distributed. The Rayleigh distribution of received complex envelope of a signal $z(t) = |x(t)|$ at any time‘t’ is given as

$$p_z(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} \qquad x \geq 0 \tag{1.1}$$

where $E\{x^2\} = 2\sigma^2$ and σ the root mean square value of is received voltage signal before envelope detection and σ^2 is the time average power of received signal before envelope detection [6,7]. It is well known that the envelope of the sum of two quadrature Gaussian noise signal obeys a Rayleigh distribution.

1.4.2 Rician fading

The Nakagami-*n* distribution also known as Rice distribution [7]. When there is a dominant stationary (non fading) signal component present, such as line-of-sight path, the small scale fading envelope distribution is Rician. That is it is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. For a multipath fading channel containing a specular or LOS component, the complex envelope of the received signal can be given by a Rician distribution

$$p_z(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{(x^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ax}{\sigma^2}\right) & A \geq 0, x \geq 0 \\ 0 & x < 0 \end{cases} \tag{1.2}$$

where A denotes the peak amplitude of the dominant or LOS signal and $I_0(.)$ is the zeroth order modified Besselfunction of first kind. The Rician distribution is often described in terms of a parameter K called Rician factor,which is defined as ratio between the deterministic signal power and the variance of the multipath or can also be defined as $K = A^2/2\sigma^2$ is the relation between the power of the LOS componenet and the power of the Rayleigh component. For K=0 we have Rayleigh fading anf for K=∞ we have no fading (that is channel with no multipath and only a LOS component). The fading parameter K is therefore a measure of the severity of the fading: a small K implies severe fading, a large K implies relatively mild fading.

1.4.3 Nakagami fading

The Rayleigh and Rician fading modela described above fall short of describing long distance fading effects with sufficient accuracy. M.Nakagami observed this fact and then formulated a parametric gamma functuon which was inspired by his experimenta in high frequency long distance propagation. The model proposed by Nakagami uses an adaptive m parameter to descirbe the fading conditions. It is shown that fading conditions less and more severe than Rayleigh and Rician fading can also be accurately modeled by Nakagami fading. Nakagami fading model assumes that the signal that has passed through the channel will fade according to the Nakagami distribution. This means that the envelope of the channel response of Nakagami channel will be Nakagami distributed. The PDF for this can be given by:

$$p_Z(x) = \frac{m^m x^{2m-1}}{\Omega^{\mathrm{m}}\Gamma(\mathrm{m})} e^{-\frac{mx^2}{\Omega}} \tag{1.3}$$

$\Gamma(\mathrm{m})$ is the gamma function and m is the shape factor with the constraint($m \geq 1/2$). Experimental and theoretical [7, 8] works have shown that the Nakagami distribution is the best-fit distribution for data obtained from many urban multipath radio channel.

1.5 DIVERSITY

Diversity combining used to mitigating the effect of fading which consists of receiving redundantly the same information-bearing signal over two or more fading channels, then combine these multiple replicas at the receiver to increase the overall received SNR. The intuition behind this concept is to exploit the low probability of concurrence of deep fades in all the diversity channels to lower the probability of error and of outage. So to improve the received signal quality and link performance over the small scale times and distances, Equalization, Diversity and Channel Coding these three techniques are used.

Small scale fading, or simply fading which is used to describe the rapid fluctuation of amplitude, phases or multipath delays of a radio signal over a short period of time or travel distance. For narrowband signal, small scale fading typically results in a Rayleigh fading distribution of signal strength over small distances. In order to prevent deep fades from occurring, microscopic diversity techniques can be exploit the rapidly changing the signal.

Large scale fading is caused by shadowing due to variations in both the terrain profile and the nature of the surrounding. In deeply shadowing conditions, the received signal strength at a mobile can drop well below that of free space. So by selecting the base station which is not shadowed when others are, the mobile can improve substantially the average signal to noise on the forward link. This is called macroscopic diversity.

There are many ways of achieving independent fading paths in a wireless system.

1.5.1 Polarization Diversity

A second method of achieving diversity is by using either two transmit antennas or two receive antennas with different polarization (e.g., vertically and horizontally polarized waves). The two transmitted waves follow the same path. There are two disadvantages of polarization diversity. First, you can have at most two diversity branches, corresponding to the two types of polarization. The second disadvantage is that polarization diversity loses effectively half the power (3 dB) because transmit or receive power is divided between the two differently polarized antennas.

1.5.2 Frequency diversity

Frequency diversity is achieved by transmitting the same narrowband signal at different carrier frequencies, where the carriers are separated by the coherence bandwidth of the channel. This technique requires additional transmit power to send the signal over multiple frequency bands.

1.5.3 Time diversity

Time diversity is achieved by transmitting the same signal at different times, where the time difference is greater than the channel coherence time (the inverse of the channel Doppler spread). Time diversity does not require increased transmit power but it does lower the data rate, since data is repeated in the diversity time slots rather than sending new data in those time slots. Time diversity can also be achieved through coding and interleaving [9]. As mentioned above, diversity has long been recognized as a powerful communication receiver technique for mitigating the detrimental effects of channel fading and co-channel interference. The underlying premise is that if several uncorrelated replicas of a signal are received over multiple diversity paths with comparable signal strengths, then it is improbable that these signals will experience simultaneous deep fades. Diversity methods can be employed either at the base station (macroscopic diversity) or at the mobile station (microscopic diversity), although the antenna separation required differs for each case.

1.5.4 Space Diversity

One method is to use multiple transmit or receive antennas, also called an antenna array, where the elements of the array are separated in wide enough with respect to the carrier wavelength so as to obtain sufficient decorrelation. This type of diversity is referred to as space diversity. Note that with receiver space diversity, independent fading paths are realized without an increase in transmit signal power or bandwidth. Moreover, coherent combining of the diversity signals increases the signal-to-noise power ratio at the receiver over the SNR that would be obtained with just a single receive antenna. This SNR increase, called array gain, can also be obtained with transmitter space diversity by appropriately weighting the antenna transmit powers relative to the channel gains. In addition to array gain, space diversity also provides diversity gain, defined as the change in slope of the error probability resulting from the diversity combining. This technique can be easily implemented at the base stations and does not require extra radio spectrum occupancy [10].

Diversity methods can be implemented by following ways.

- At the transmitter
- At the receiver
- At transmitter and receiver (or both)

Space diversity reception methods can be classified into various categories.

- Selection diversity combining (SDC)
- Maximal ratio combining (MRC)
- Equal gain combining (EGC)
- Switched combining diversity (SWC)
- Generalized selection combining (GSC)
- Optimum diversity combining (ODC)

The brief overview of diversity reception methods are given below

- ***Selection combining diversity (SC)***

SC-type system only process one of the diversity branches. Specifically, in its conventional form, the Sc combiner chooses the branch with the highest SNR. In addition, since the output of SC combiner is equal to the signal on only one of the branches, the coherent sum of the individual branch signal is not required. Therefore, the SC scheme can be used in conjunction with differentially coherent and non coherent modulation techniques since it does not require knowledge of the signal phases on each branch [11].

- ***Threshold Combining***

Selection combining for systems that transmit continuously may require a dedicated receiver on each branch to continuously monitor branch SNR. A simpler type of combining, called threshold combining, avoids the need for a dedicated receiver on each branch by scanning each of the branches in sequential order and outputting the first signal whose SNR is above a given threshold γ_t. Once a branch is chosen, the combiner outputs that signal as long as the SNR on that branch remains above the desired threshold. If the SNR on the selected branch falls below the threshold, the combiner switches to another branch.

- ***Maximal-Ratio Combining***

The signal at the output of the receivers is linearly combined in MRC so as to maximize the instantaneous signal to noise ratio. This is achieved by combining the co - phased signal. The SNR of the combined signal is equal to the sum of the SNRs of all the branch signals. That is, branches with strong signal are further amplified, while weak signals are attenuated. Thus it has a advantage of producing an output with an acceptable SNR even when none of the individual signals are themselves acceptable. This technique gives the best statistical reduction of fading of a known linear diversity combiner. Modern DSP techniques and digital receivers are now making this optimal form of diversity practical [12].

$$\gamma_{\Sigma} = \sum_{i=1}^{M} \gamma_{\mathrm{i}} \tag{1.4}$$

Thus, the SNR of the combined output is the sum of SNRs on each branch. Hence, the average combiner SNR and corresponding array gain increase linearly with the number of diversity branches M, in contrast to the diminishing returns associated with the average combiner SNR in SC.

- ***Equal-Gain Combining***

Maximal-ratio combining requires knowledge of the time-varying SNR on each branch, which can be difficult to measure. A simpler technique is equal-gain combining (EGC), which co-phases the signals on each branch and then combines them with equal weighting$\alpha_i = e^{-\theta_i}$. Performance of EGC is quite close to that of MRC, typically exhibiting less than 1 dB of power penalty. This is the price paid for the reduced complexity of using equal gains [12].

- ***Optimum combining***

In addition to combating multipath fading, space diversity can also be used in cellular radio systems to reduce the relative power of co-channel interferers (CCI's) that are present at each element of the array. When operating in this scenario, the appropriate diversity scheme to employ is one that combines the branch outputs in such a way as to maximize the signal-to-interference plus noise (SINR) ratio at the combiner output. Under such conditions, this scheme, which is referred to as optimum combining (OC), will achieve a larger output SINR than MRC and is thus highly desirable even when the number of interferers exceeds the number of antenna array elements. This improved SINR efficiency can manifest itself in the cellular mobile radio application as a reduction in the number of base stations and/or an increased channel capacity through greater frequency reuse.

The difference between the RAKE receiver for MRC and that for OC lies in selection of the weight vector **w.** Specifically, for MRC the weights are selected for maximum instantaneous SNR at the combiner output, and thus $w = \frac{\alpha_d}{\sigma_n^2}$. For OC the weights are selected for maximum instantaneous SINR at the same location, and thus $w = R_{ni}^{-1}\alpha_d$ where R_{ni} is the noise plus interference covariance matrix. For this receiver, the maximum instantaneous SINR at the combiner output is given by [13]

$$\gamma_{oc} = P_d \alpha_d^H R_{ni}^{-1} \alpha_d \tag{1.5}$$

where the superscript H stands for the Hermitian (transpose complex conjugate) operation.

- ***Transmitter Diversity***

In transmit diversity there are multiple transmit antennas, and the transmit power is divided among these antennas. Transmit diversity is desirable in systems where more space, power, and processing capability is available on the transmit side than on the receive side. Transmit diversity design depends on whether or not the complex channel gain is known to the transmitter [14]. When this gain is known, the system is quite similar to receiver diversity. However, without this channel knowledge, transmit diversity gain requires a combination of space and time diversity via a novel technique called the Alamouti scheme.

The Alamouti scheme is a simple transmit diversity technique that may be applied in systems with M_T= 2 and any number of receive antennas. The transmission strategy for the Alamouti scheme is shown in the schematic in Figure 1.2. Assuming two data symbols s_1 and s_2 to be transmitted, the transmitter launches s_1 and s_2 each with energy $E_s/2$ simultaneously from antennas 1 and 2, respectively. Over the next symbol period symbol $-s_2{}^*$ is transmitted from antenna 1 and symbol $s_1{}^*$ is transmitted from antenna 2, each with symbol energy $E_s/2$.Hence, effectively, only one data symbol is transmitted per symbol period [15].

Mathematically

$$y = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = Hs + n \tag{1.6}$$

$$H_A = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \tag{1.7}$$

s_i is transmitted using half the total symbol energy E_s. The received SNR is thus equal to the sum of SNRs on each branch, identical to the case of transmit diversity with MRC.

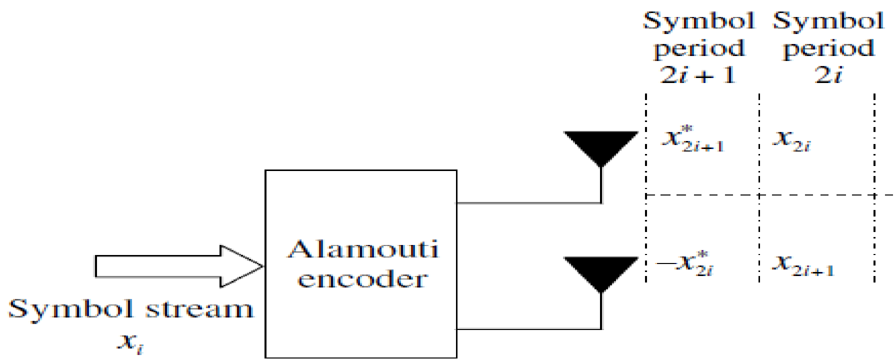

Figure1.2: Schematic Diagram of Alamouti scheme [15]

The Alamouti scheme achieves a diversity order of 2, the maximum possible for a two-antenna transmit system.

1.6 THESIS OBJECTIVE

As discussed above, the main motive of any wireless communication system is to reliably transmit the information to the receiver and reduce the number of errors induced by the channel. In this thesis, we have worked on reducing the transmission errors and improving the BER. We try to combine the best coding that is LDPC and modulation BPSK schemes with SC and MRC at the receiver over the Rayleigh faded channel. Performance analysis of coded SC, MRC was analyzed with different SNR values by varying the number of receive antennas.

- Analytically BER of SC (uncoded system) is calculated over i.i.d Rayleigh fading channel.
- Analytically BER of coded SC is calculated over i.i.d. Rayleigh fading channel.
- Compare the uncoded and coded SC system.
- Analytically BER of MRC (uncoded system) is calculated over i.i.d. Rayleigh fading channel.
- Analytically BER of coded MRC is calculated over i.i.d. Rayleigh fading channel
- Compare the uncoded and coded MRC system.
- Analytically BER of optimum combining is calculated over i.i.d. Rayleigh fading channel when the number of interferers is one.
- Analytically BER of optimum combining is calculated over i.i.d. Rayleigh fading channel when the number of interferers are equal and greater than the number of receive array elements.
- Comparison between MRC and OC in the presence of interference.

1.7 THESIS ORGANIZATION

This thesis includes five chapters. An outline of each chapter is given below:

The *1^st^ chapter* gives an introduction of wireless communication system and diversity techniques. Some of the problems of wireless communications such as modulation, channel coding, fading are also addressed in this chapter.

Chapter 2 is dedicated to the literature survey. The research papers which are relevant to this thesis are discussed here.

Chapter 3 presents a study of selection combining (SC), maximal ratio combining (MRC) and OC. In this chapter simulated and analytically results of outage probability, mean output SNR of SC, MRC and probability of error for SC, MRC and OC combining are calculated and comparison between SC, MRC and OC has been done.

Chapter 4 includes meaningful results which are calculated analytically. In this chapter we have presented the analysis and result of probability of error for coded-SC and coded-MRC over i.i.d Rayleigh fading environment and comparison between coded and uncoded system for these has been done.

Chapter 5 concludes this thesis, summarizing the major results and offering suggestions for further work on this topic.

CHAPTER 2
LITRATURE SURVEY

This chapter surveys some important studies in the field of our research and outlines the main ideas contained in them

LDPC codes were first introduced and analyzed by **Robert Gallager[1]** in his PhD at M.I.T in 1962 .These LDPC codes are characterized by a sparse parity in which the number of 1's is very less as compared to the number of 0's. He discovered an iterative decoding algorithm which he applied to a new class of codes and for these codes to perform well he needed the parity check matrices to be sparse. The codes which he developed give a good performance and are called regular LDPC codes because they use parity check matrices in which the number of 1's and 0's in each row and column are equal. Still, the LDPC codes were ignored for a long time because of the requirement of high complexity computation and lack of hardware support at that time.

In the year 1995, LDPC codes were rediscovered by **Mackay at al. [2]** whose performance is closest to the Shannon limit based on irregular graphs. Mackay at al. investigated constructions of regular and irregular Gallager codes that allow more rapid encoding and have smaller memory requirements in the encoder and find that these fast encoding Gallager codes have equally good performance.

Mackey et al. [4] defined the most powerful error correcting codes based on very sparse matrices that compared to the other good codes such as turbo codes which enabling reliable transmissions at rates close to the Shannon capacity for not only binary-symmetric channel but also for any channel with symmetric stationary ergodic noise.

Selection diversity combining employed in conjunction with LDCP by **Beng Soon Tan et al. [11]** which is able to mitigate the effects of fading. The probability of error of this scheme over i.i.d and i.n.i.d flat Rayleigh fading channel is derived using the extrinsic information transfer chart and Gaussian approximation (GA). The derived expression is compared with the SNR threshold using density evolution (DE) and has the advantage of reduced computational complexity over DE.

Symbol error rate (SER) are derived for maximal ratio combining (MRC) and equal gain combining (EGC) diversity by **A. Annamalai et al. [12]** of multilevel quadrature amplitude modulation (MQAM) on arbitrary Nakagami fading channel. In this paper, MRC in independent and correlated fading and ECG in independent fading have been considered.

Optimum signal combining for space diversity reception in cellular mobile radio systems studied by **Winter [13].** OC maximize the output SINR which is used not only to combat Rayleigh fading of the desired signal (as with MRC) but also to reduce the power of interfering signals at the receiver. Analytical and computer simulation techniques are used to determine the performance of optimum combining. Results show that optimum combining is significantly better than maximal ratio combining even when the number of interferers is greater than the number of antennas and showed that optimum combining increases the output signal to- interference ratio at the receiver by several decibels. Thus, systems can require fewer base station antennas and/or achieve increased channel capacity through greater frequency reuse.

Selection combining cascaded with maximum-ratio combining (SC/MRC), which is particularly useful when site diversity is available, combined with low-density parity-check (LDPC) codes is able to combat the effects of fading. The bit error rate (BER) expressions of uncoded SC/MRC and SC/MRC with LDPC codes over an independent and identically distributed Rayleigh-fading channel are derived by **Beng Soon Tan et al. [16]** based on the Gaussian approximation approach. These expressions are also applicable to the selection combining and maximum-ratio combining as special cases.
These expressions are able to achieve a significant reduction in computational time with a reasonable accuracy for analyzing the BER performance as compared to simulations and DE.

Satoshi Gounai et al. [17] derived the SNR thresholds of a regular LDPC codes and the irregular LDPC codes and optimum degree distributions of the irregular LDPC codes for SIMO systems with several diversity orders. Satoshi Gounai et al. showed that with low diversity order, the optimum degree distributions of the irregular LDPC code depends on the diversity order largely. While with high diversity order, the optimum degree distribution depends both on the diversity order and the combining scheme largely and also showed that comparing the optimum degree distributions for MRC and STBC, when the diversity orders and optimum degree distributions are the same.

Eric Villier [18] analyzed the performance of optimum combining in the presence of multiple equal power interferers and noise when the number of interferers is less than the number of antenna elements. Desired signal and interferers are subject to flat Rayleigh fading and the propagation channels are independent. An approximate expression of the probability density function (PDF) of the output signal-to-interference-plus-noise ratio (SINR), cumulative distribution function (CDF) of the SINR and the bit-error rate (BER) of some binary modulations has been derived.

Kwok Hung Li et al.[19] proposed Low-density parity-check (LDPC) codes combined with cascade combining (SC/MRC and MRC/SC) which are able to achieve excellent bit-error rate (BER) performance over multipath fading channels. The average output signal-to-noise ratio and the uncoded BER expressions of cascade combining and over i.i.d Nakagami-m fading channels are derived. The BER expressions of cascade combining with LDPC codes over i.i.d. Nakagami-m fading channels are derived based on the GA approach. These expressions include SC and MRC as special cases. The expressions obtained from the GA analysis provide better theoretical understanding of the system performance and allow computing the BER and SNR threshold more efficiently as compared to DE and simulations.

Performance evaluation of optimum combining in wireless communication with Rayleigh fading and cochannel interference has been done by **Amit Shah [20].** This paper considered binary phase shift keying (BPSK) modulation in a flat Rayleigh fading environment when the number of interferences is not less than the number of antenna elements. Closed form expression using hypergeometric functions are derived for outage probability and the average probability of bit error.

Wu, Yongpeng et al. [21] studies the optimum combining (OC) system with multiple arbitrary-power interferers and thermal noise in a flat Rayleigh fading environment. The main contribution of the paper is a concise performance analysis for the overload OC system where the number of interferers exceeds or is equal to the number of antennas elements. Simple closed-form formulas are derived for the moment generating function of the output signal-to-interference-plus-noise ratio (SINR) and the symbol error rate (SER) with M-ary phase shift keying (M-PSK). Based on the derived MGF, the closed-form explicit expressions for the moments of the output SINR are determined.

Valentine A. Aalo [22] studied the effect of cochannel interference on the performance of digital mobile radio systems in a Rayleigh fading environment. The average bit error rate (BER) of an antenna array system with an optimum combining scheme that maximizes the output signal-to-interference-plus-noise ratio is analyzed. BER expressions which are easy to evaluate numerically are derived for coherent binary phase-shift keying schemes in an environment with cochannel interference and noise. In this only one and two interferers are considered.

The performance of maximal ratio combining for space diversity reception in digital cellular mobile radio systems is studied by **Amit Shah [23]** for communications in the presence of multiple cochannel interference (CCI) sources and is compared to optimum combining. Using a multivariate statistical analysis approach and assuming equal-power interference sources, analytical expressions are derived for the density function of the array output signal-to-interference ratio (SIR), the outage probability, and the average probability of bit error with maximal ratio combining. In this paper, Rayleigh fading are extended to the case when the SOI is subject to Rice fading.

S. Sharma et al. [24] studied that LDPC codes have outstanding performance in some cases than that of turbo codes, with iterative decoding algorithm which is easy to implement and parallelizable in hardware. LDPC codes when implemented along with SIMO system with MRC under the combination of AWGN and multipath fading is optimum from the standpoint of maximising SNR at combiner output. In addition when operating in interference scenario optimum combining can be employed that maximizes the output SINR. In this paper it is shown that LDPC codes with optimum combining (LDPC-OC) system improves SINR by 1.98 dB and 2.62 dB with 3 and 4 receive antennas respectively in multipath fading channel in the presence of a single interferer.

Vishwanatha R. Gowda et al. [25] evaluated Error performance of selection diversity (SD) receiver over non-identical fading channels, where the fading statistics of individual branches are different. Bit-error rate (BER) results are directly obtained from the cumulative distribution function (CDF), which drastically reduces the complexity of analysis and simplifies BER expressions. In particular, using this approach, we are able to generate unified simple, closed-form BER expressions for different modulation schemes under non-identical fading conditions in Rayleigh and Nakagami fading channels.

Juan M. Romero-Jerez et al. [26] presented the average bit error rate (BER) of uncoded MIMO systems in Rayleigh fading channels with cochannel interference (CCI) and noise. Receiver schemes such as maximal ratio combining (MRC) and interference cancellation (IC) via null steering of the receive array radiation pattern has been considered. In these paper analytical expressions of the BER for different modulation techniques and comparison between the two proposed receiver schemes with fading and interference has been analyzed.

The ergodic capacity of multiple-input multiple-output (MIMO) systems with a single co-channel interferer in the low signal-to-noise-ratio (SNR) studied by **Caijun Zhong et al. [27]**. Exact analytical expressions for the minimum energy per information bit and wideband slope, are derived for Rayleigh and Rician fading channels. Results showed that the minimum energy per information bit is the same for both channels while their wideband slopes differ significantly and indicate that interference degrades the capacity by increasing the required minimum energy per information bit and reducing the wideband slope.

Optimum combining system of a Rician fading signal with unequal power Rician interferers and thermal noise has been investigated **Yongpeng Wu et al. [28]**. Based on the statistical characteristics analysis of the output SINR, Yongpeng Wu derived a closed form upper bound for the symbol error probability of M-ary phase-shift keying.

The statistical properties of the output signal to interference plus noise ratio (SINR) of a spatial combiner has been studied by **Bhaskar D. Rao et al.[29]** where the spatial weights used are either the Maximal Ratio Combiner (MRC) weights or the Optimum Combining weights. The channels are modeled as slow flat Rayleigh fading channels and multiple interferers are assumed present. In particular, the modified F-distribution is introduced to provide an exact characterization of a MRC receiver and to bound the performance of the OC receiver.

Louie, R.H.Y. et al. [30] derived the asymptotic SER and outage probability of MIMO-OC systems with correlated Rayleigh-faded user channels and SIMO-OC systems with correlated Rician-faded user channels. Results are based on new asymptotic expansions which derived for the CDF and PDF of the SINR at the OC output and obtained new closed-form performance results for unequal power, correlated Rayleigh and Rician co-channel interferer channel. For the MIMO-OC system, result showed that receive correlation decreases the array gain, hence increasing the SER at high SNR. For the SIMO-OC system, result showed that the Rician K factor increases the array gain, hence decreasing the SER at high SNR.

Optimum combining (OC) and maximal ratio combining (MRC) are analyzed using a gain ratio method by **Burke, J.P. et al. [31].** Using the receive carrier-to-interference plus noise ratio (CINR), the gain ratio $CINR_{OC}/INR_{MRC}$ is evaluated in a flat Rayleigh fading communications system with multiple interferers. Exact analytical solutions derived for the probability density function (PDF) and the average gain ratio with one interferer. When more than one interferer is present, the PDF of the gain ratio is illustrated using Monte Carlo simulations and its mean value is shown in basic integral form. An upper bound to the gain ratio is derived providing a simple means to determine when OC will exhibit significant gains over MRC.

Closed-form analytical expressions for the outage probability of different diversity schemes, such as maximal ratio combining (MRC) and optimum combining (OC), and also for interference cancellation (IC) analyzed by **J.M. Goldsmith et al. [32]** which is based on antenna beamsteering, which steers nulls in the array radiation pattern in the direction of the strongest interferers. At the receive antenna array, signal from the desired user has been assumed which is affected by Rice, Nakagami or Rayleigh fading, while CCI signals are assumed to experience Rayleigh fading. In this paper, Results showed that IC yields significantly better performance than MRC if the system is interference-limited and the number of dominant interferers is lower than the number of receive antennas, or when the output SINR is low.

CHAPTER 3

PERFORMANCE ANALYSIS OF SC, MRC AND OC

In the 3rd chapter, performance of SC and MRC and OC has been explained analytically and simulated result of the outage probability, mean output SNR for SC and MRC and probability of error are also calculated for SC, MRC and OC. Comparisons between these are also shown in the last section.

3.1 PERFORMANCE CRITERION

Probably the most common and best understood performance metric of a digital communication system is the average SNR (signal to noise ratio). Most often, this is measured at the output of the receiver and is thus related directly to the data detection itself. Of the several possible performance measures that exist, it is typically the easiest to evaluate and most often serves as an excellent indicators of the overall fidelity of the system. Although, the term noise in signal to noise ratio refers to the ever-present thermal noise at the input to the receiver, in the context of the communication system subject to fading impairment, the more appropriate performance metric is average SNR. The word average refers to a statistical averaging over the probability distribution of the fading. In simple mathematical terms, if $\gamma_i$denotes the instantaneous SNR (a random variable) of each branch then the average SNR at the diversity combiner output is

$$\bar{\gamma} = \int_0^\infty \gamma f_\gamma(\gamma) d\gamma \tag{3.1}$$

where $f_\gamma(.)$denotes the PDF of γ.We can rewrite the equation (3.1) in terms of the MGF(moment generating function) associated with γ,namely

$$\psi_\gamma(s) = \int_0^\infty f_\gamma(\gamma) e^{-s\gamma}\, d\gamma \tag{3.2}$$

Now taking the derivative of equation (3.2) with respect to s, then again we the average SNR at the combiner output that is

$$\bar{\gamma} = \frac{d\psi_\gamma(s)}{ds}\Big|_{s=0} \tag{3.3}$$

In other words, the ability to evaluate the MGF of the instantaneous SNR allows immediate evaluation of the average SNR via a simple mathematical operation differentiation.

The second performance criterion is the average bit error rate. This metric can be evaluated by averaging the conditional error probability (CEP) over the PDF of combiner output SNR. Suppose the CEP is of the form

$$P_b(\gamma) = aerfc(\sqrt{b\gamma}) \quad (3.4)$$

Such as would be case for coherent detection of PSK signal or coherent detection of orthogonal FSK signal. Then the average probability of error can be written as

$$P_b = \int_0^{\infty} P_b(\gamma) f_\gamma(\gamma) d\gamma \quad (3.5)$$

3.2. SELECTION COMBINING DIVERSITY (SC)

Selection combining (SC) is a combining mechanism used in conjunction with space diversity. SC type systems can process only one of the diversity branches. The combiner chooses output with the highest SNR that is the output of the SC combiner equal the signal on only one of the branches. The coherent sum (like MRC, EGC) of the individual branch signal is not required. This is equivalent to choosing the ith branch with the highest $r_i^2 + N_i$ if the noise power $N_i = N$ is the same on all branches. Because only one branch is used at a time, SC is easy to implement because all that is needed a side monitoring station and an antenna switch at the receiver. However, it is not an optimal diversity technique because it does not use all of the possible branches simultaneously. The advantage of SC is it is simple because it controls switch. We don’t require perfect channel state information that is amplitude, phase, and delay. We don’t require many RF changes state like RF amplifiers, noise amplifiers, and power amplifiers.

With ideal SC, the path output from the combiner has an SNR equal to the maximum SNR of all the branches.

For N-branch diversity, the instantaneous symbol energy to noise ratio at the output of the SC is γ_{SC}. γ_{SC} can be written as

$$\gamma_{SC} = \max[\gamma_1, \gamma_2 \ldots \ldots . \gamma_N] \quad (3.6)$$

3.2.1 Outage probability in selection diversity

To analyze the bit error rate, let us first find the outage probability on the ith receive antenna. Outage probability is the probability that the bit energy to noise ratio falls below a threshold. If the branches are independently faded, then order statistics gives the cumulative distribution function (CDF) of γ_{SC} is given by [11]

$$P_{\gamma_{SC}}(\gamma) = p(\gamma_1 < \gamma, \gamma_2 < \gamma, \ldots \ldots \gamma_N < \gamma) = [P_\gamma(\gamma)]^L \quad (3.7)$$

For i.i.d channel, the PDF of selection combining

$$p_{\gamma sc}(\gamma) = \frac{d}{d\gamma}\left(P_{\gamma sc}(\gamma)\right) = N\left[P_{\gamma}(\gamma)\right]^{N-1} p_{\gamma}(\gamma) \tag{3.8}$$

Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. The Rayleigh fading distribution has a PDF in terms of received signal r_i that is

$$p(r_i) = \frac{r_i}{P_o} e^{\frac{-r_i^2}{2P_o}} \qquad (0 \leq \mathrm{r} \leq \infty) \tag{3.9}$$

where r_i^2 is the instantaneous power in ith branch, $2P_o$ is mean square signal power per branch.

Then the instantaneous input signal to noise ratio (SNR) of ith branch.

$$\gamma_i = \frac{r_i^2}{\sigma_n^2} \tag{3.10}$$

Average input signal to noise ratio of each branch is the

$$\gamma_0 = \frac{2P_o}{\sigma_n^2} \tag{3.11}$$

$$\frac{\gamma_i}{\gamma_0} = \frac{r_i^2}{2P_o} \tag{3.12}$$

Now to write the Rayleigh PDF in terms of SNR γ_i from equation (3.9) then we do transformation of one variable PDF to other variable PDF which is given by [33]

$$p(y) = \frac{p(x)}{\frac{dy}{dx}} \tag{3.13}$$

Here we know the PDF of x. By using the equation (3.10), we can write the equation (3.9) in terms of SNR , γ_i PDF

Then we get

$$p(\gamma_i) = \frac{1}{\gamma_0} e^{-\frac{\gamma_i}{\gamma_0}} \tag{3.14}$$

Now to get the outage probability of ith receive antenna from equation (3.14) that is

$$\mathrm{P}(\gamma_i \leq \gamma_{th}) = \int_{-\infty}^{\gamma_{th}} p(\gamma_i) d\,\gamma_i \tag{3.15}$$

where γ_{th} is the threshold for signal to noise ratio, SNR. Now, Put the value of equation (3.14) into equation (3.15), Then outage probability of ith receive antenna becomes

$$P(\gamma_i \le \gamma_{th}) = 1 - e^{-\frac{\gamma_{th}}{\gamma_0}} \tag{3.16}$$

For 'N' receive antennas which is assumed as independent. Then the outage probability for 'N' branches is

$$P_{\gamma sc}(\gamma_i \le \gamma_{th}) = \left(1 - e^{-\frac{\gamma_{th}}{\gamma_0}}\right)^N \tag{3.17}$$

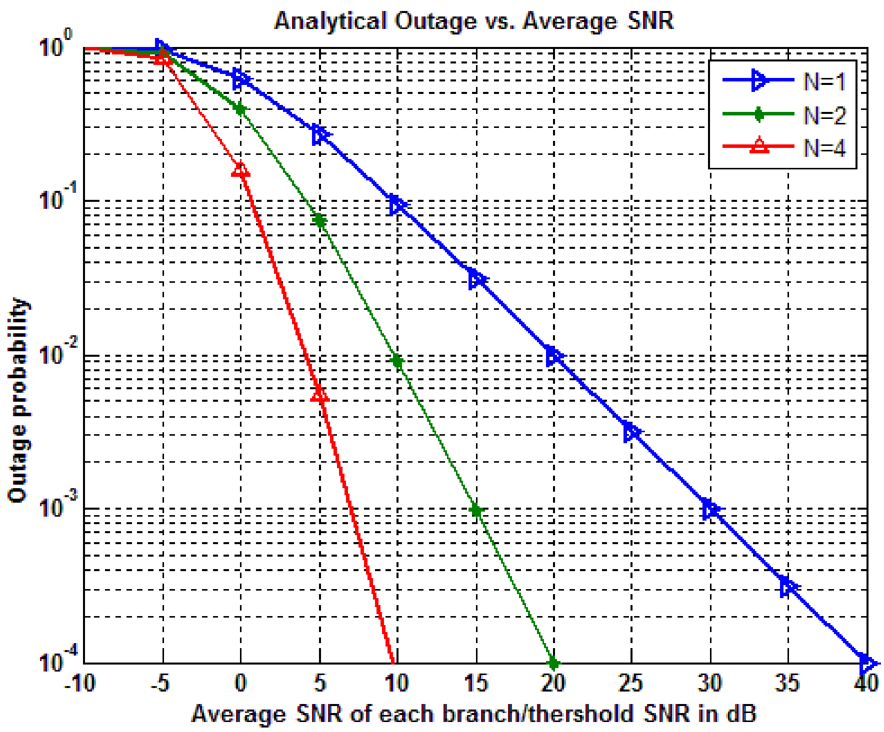

Figure 3.1: Outage probability vs. Average SNR by varying number of receive antennas from N=1, 2, 4.

3.2.2 Calculate the average SNR, $\bar{\gamma}_N$ at the combiner having N branches

To calculate the PDF for 'N' branches which is equal to the derivative of equation (3.17) with respect to γ_{th} becomes

$$p_{\gamma sc}(\gamma_{th}) = \frac{N}{\gamma_0}\left(1 - e^{-\frac{\gamma_{th}}{\gamma_0}}\right)^{N-1} e^{-\frac{\gamma_{th}}{\gamma_0}} \tag{3.18}$$

To calculate the average output bit energy to noise ratio is,

$$\bar{\gamma}_{SC} = \int_0^\infty \gamma_{th} p_{\gamma sc}(\gamma_{th})\, d\gamma_{th} \tag{3.19}$$

Put the value of equation (3.18) into equation (3.19) then we get

$$\bar{\gamma}_{SC} = \int_0^\infty \gamma_{th} \frac{N}{\gamma_0}\left(1 - e^{-\frac{\gamma_{th}}{\gamma_0}}\right)^{N-1} e^{-\frac{\gamma_{th}}{\gamma_0}} d\gamma_{th} \tag{3.20}$$

$$\bar{\gamma}_{SC} = \gamma_0 * \sum_{k=1}^{N} \frac{1}{k} \tag{3.21}$$

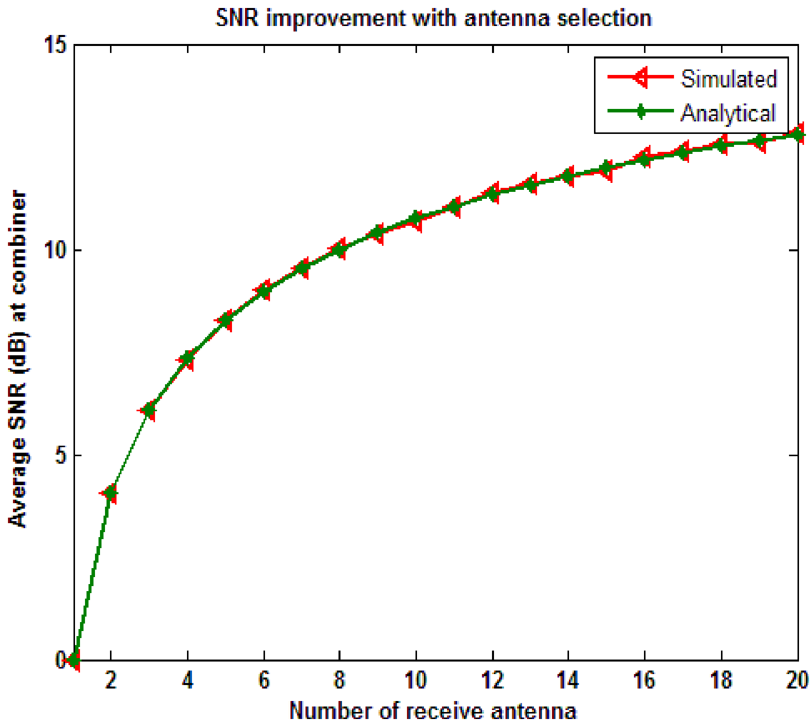

Figure 3.2: Average SNR at combiner vs. Number of receive antenna

With selection diversity we are seeing that the average SNR at the combiner is not increased linearly with increasing the number of receives antennas.

This means that, with '2' receive antennas, the average SNR, $\bar{\gamma}_N$ at combiner is 1.5 times average SNR, $\gamma_0$of each branch. With '3' receive antennas, the average SNR at combiner is 1.833 times average SNR of each branch. With "4" receive antennas, the average SNR at combiner is 2 times average SNR of each branch. That why we do not used this type of combining.

3.2.3 Probability of error, P_e for selection diversity

To calculate the average probability of error at the combiner is computed by integrating the probability of error in AWGN channel over the Rayleigh distribution at the combiner which is given by [9]

$$P_e = \int_0^{\infty} Q(\sqrt{2\gamma_0}) p_{\gamma_{SC}}(\gamma_{th}) d\gamma_{th} \tag{3.22}$$

Putting the value of equation (3.18) into equation (3.22) ,then P_e becomes

$$P_e = \int_0^{\infty} \frac{erfc\left(\sqrt{\gamma_0}\right)}{2} \frac{N}{\gamma_0} \left(1 - e^{-\frac{\gamma_{th}}{\gamma_0}}\right)^{N-1} e^{-\frac{\gamma_{th}}{\gamma_0}} d\gamma_{th} \tag{3.23}$$

Solve the above equation by using Mathematica software and then we get the P_e

$$P_e = \frac{1}{2} \sum_{k=0}^{N} (-1)^k \binom{N}{k} \left(1 + \frac{k}{\gamma_0}\right)^{-\frac{1}{2}} \tag{3.24}$$

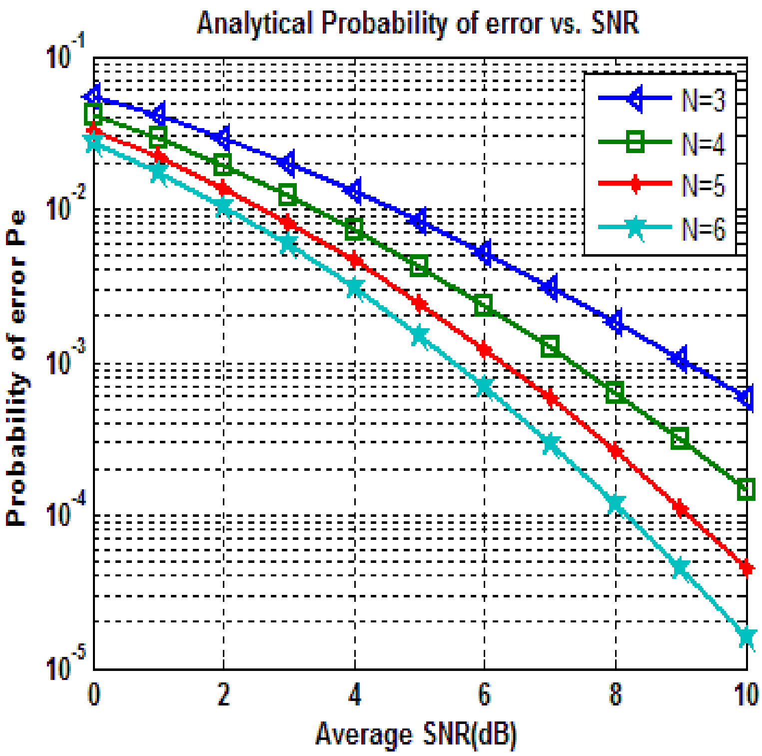

Figure 3.3: Analytical Probability of error vs Average SNR by varying number of antennas from 3,4,5,6.

Table 3.1 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 3.5 dB at BER of 10^{-3}.

Table 3.1: Probability of error of SC at BER of 10^{-3}

Probability of error at BER of 10^{-3}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	9 dB
N=4	7.2 dB
N=5	6.2 dB
N=6	5.5dB

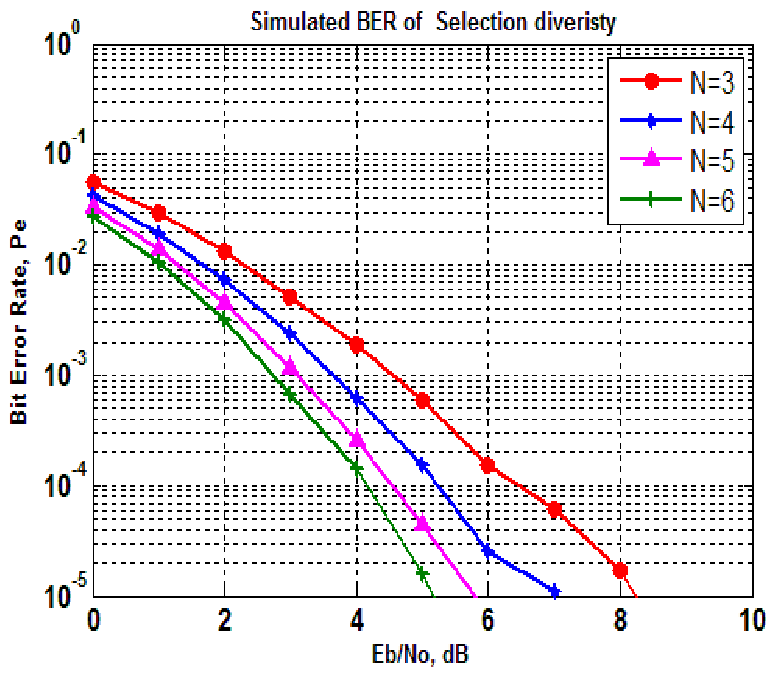

Figure 3.4: Simulated probability of error vs Average SNR by varying number of receive antennas from N=3, 4, 5, 6.

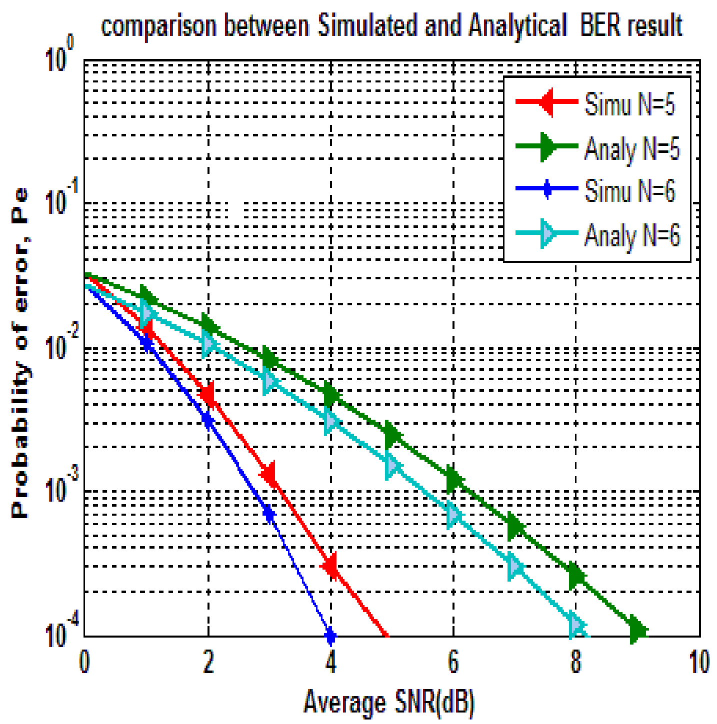

Figure 3.5: Analytical and Simulated probability of error vs. Average SNR by varying N=5, 6.

3.3 MAXIMUM RATIO COMBINING

In SC, the output of combiner equals the signal on one of the branches. In maximal-ratio combining (MRC) the output is a weighted sum of all branches. The signals are co-phased and so $\alpha_i = a_i e^{-j\theta_i}$ where θ_i is the phase of the incoming signal on the i^{th} branch

3.3.1 Calculate the SNR at the combiner having N branches

The envelope of the combiner output will be $r = \sum_{i=1}^{N} a_i r_i$.Assuming the same noise power spectral density (PSD) $N_0/2$ in each branch yields a total noise PSD $N_{tot}/2$ at the combiner output of $\frac{N_{tot}}{2} = \sum_{i=1}^{N} {a_i}^2 N_0/2$. Thus the output SNR of the combiner is

$$\gamma_{MRC} = \frac{r^2}{N_{tot}} = \frac{1}{N_0} \frac{\left(\sum_{i=1}^{N} a_i r_i\right)^2}{\sum_{i=1}^{N} {a_i}^2} \tag{3.25}$$

The goal is to choose the a_i to maximize γ_{MRC} by taking partial derivatives of equation (3.25) or by using the Cauchy-Schwartz inequality **.**Then optimal weights yields ${a_i}^2 = \frac{{r_i}^2}{N_0}$ and the resulting combiner SNR becomes

$$\gamma_{MRC} = \sum_{i=1}^{N} \frac{{r_i}^2}{N_0} = \sum_{i=1}^{N} \gamma_i \tag{3.26}$$

$$\gamma_{MRC} = N\gamma_i \tag{3.27}$$

where $\gamma_i = \frac{{r_i}^2}{N_0}$ is the instantaneous SNR of each branch. Thus the SNR of the combiner output is sum of the SNRs on each branch.

Hence, the average combiner SNR and corresponding array gain increase linearly with the number of diversity branches N, in contrast to the diminishing returns associated with the average combiner SNR in SC given by equation (3.21). As with SC, the distribution of the combined output SNR does not remain exponential even when there is Rayleigh fading on all branches. Effective bit energy to noise ratio in a N receive antenna case is N times the bit energy to noise ratio for single antenna case

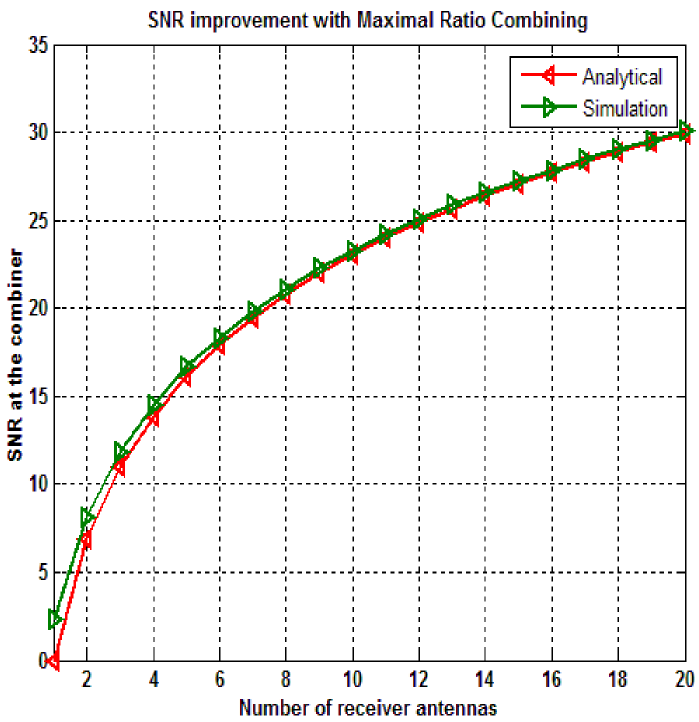

Figure 3.6: SNR at combiner vs. number of receive antennas at γ_i = 0 dB.

3.3.2 Outage probability in maximal ratio combining

We know that, the PDF of γ_i for independent identically distributed (i.i.d.) Rayleigh fading on each branch which is given by equation **(**3.14)

$$p(\gamma_i) = \frac{1}{\gamma_0} e^{-\frac{\gamma_i}{\gamma_0}}$$

To calculate the moment generating function (MGF) of γ_i becomes

$$M_{\gamma_i}(s) = \int_0^{\infty} \frac{1}{\gamma_0} e^{-\frac{\gamma_i}{\gamma_0}} e^{s\gamma_i} d\gamma_i \tag{3.28}$$

$$M_{\gamma_i}(s) = \frac{1}{1 - s\gamma_0} \tag{3.29}$$

For ‘N’ independent branches, The MGF function of equation (3.29) becomes

$$M_{\gamma_{MRC}}(s) = \frac{1}{(1 - s\gamma_0)^N} \tag{3.30}$$

To calculate the PDF of γ_{MRC} from moment generating function(MGF) for ‘N’ branches, Then PDF of γ_{MRC} becomes

$$p_{\gamma_{MRC}}(\gamma) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{(1 - s\gamma_0)^N} e^{-s\gamma} ds \tag{3.31}$$

By solving the equation (3.31), we get

$$p_{\gamma_{MRC}}(\gamma) = \frac{1}{(N-1)!\,{\gamma_0}^N}\gamma^{N-1}\exp\left(-\frac{\gamma}{\gamma_0}\right), \qquad \gamma \geq 0 \tag{3.32}$$

The corresponding outage probability for a given threshold γ_{Th} from PDF at combiner is obtained by integrating the equation (3.32) with respect to γ. Then we get

$$P_{out} = p(\gamma_{MRC} < \gamma_{Th}) = \int_0^{\gamma_{Th}} p_{\gamma_{MRC}}(\gamma)\,d\gamma \tag{3.33}$$

By solving the equation (3.33), then outage probability becomes

$$P_{out} = 1 - e^{-\gamma_{Th}/\gamma_0}\sum_{k=1}^{N}\frac{\left(\gamma_{Th}/\gamma_0\right)^{k-1}}{(k-1)!} \tag{3.34}$$

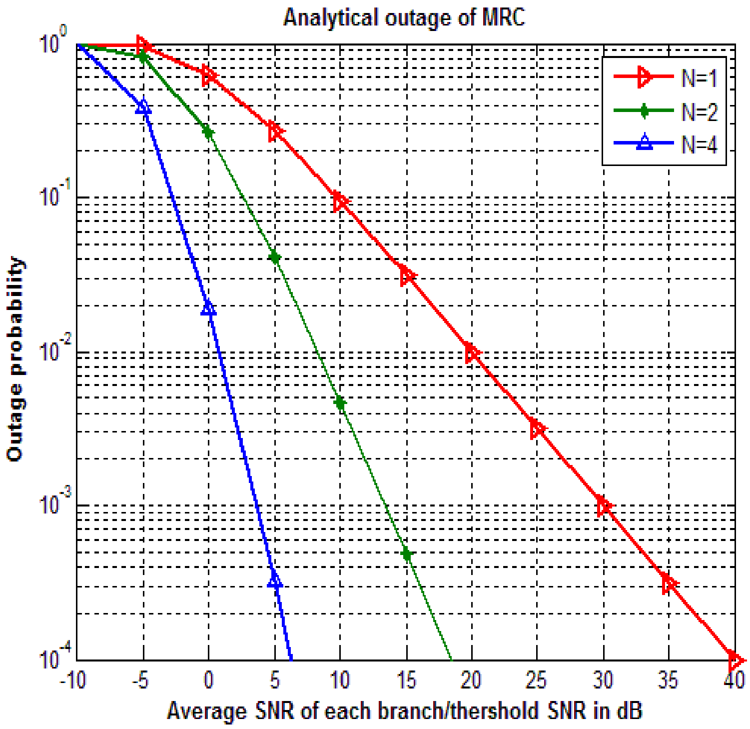

Figure 3.7: Outage probability vs. Average SNR when receive antennas N=1, 2, 4.

3.3.3 Probability of Error with Maximal Ratio Combining

The average probability of bit error is obtained when we put the value of equation (3.32) into equation (3.22). Then the average probability of error P_e becomes

$$P_e = \int_0^{\infty}\frac{1}{2}erfc(\sqrt{\gamma})\frac{1}{(N-1)!\,{\gamma_0}^N}\gamma^{N-1}\exp\left(-\frac{\gamma}{\gamma_0}\right)d\gamma \tag{3.35}$$

By solving the above equation, we get

$$P_e = z^N \sum_{k=0}^{N-1} \binom{N-1+k}{k} (1-z)^k \tag{3.36}$$

where

$$z = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{\gamma_0}\right)^{-1/2}$$

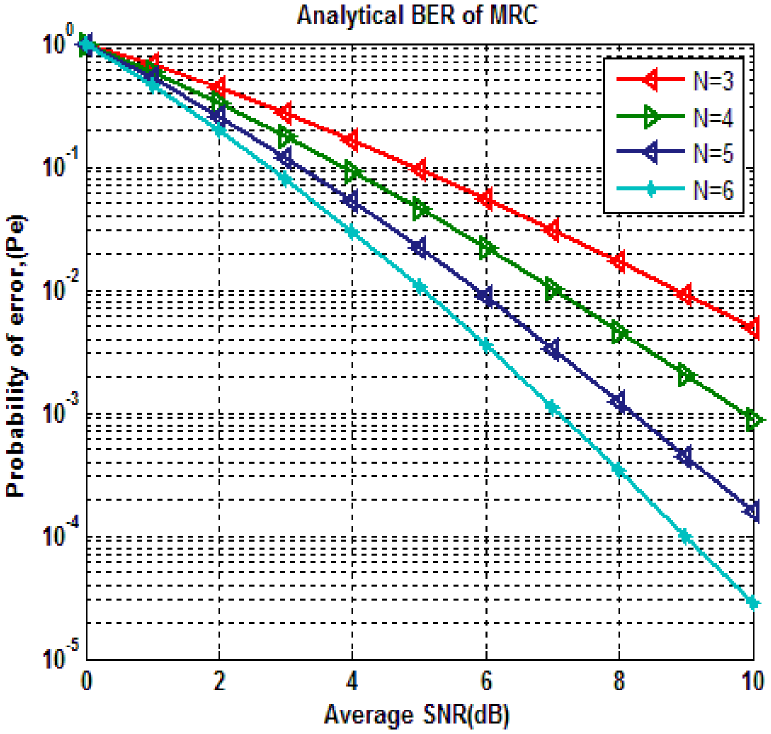

Figure 3.8: Analytical Probability of error vs. Average SNR when receive antennas varies from 3,4,5,6.

Table 3.2 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 4 dB at BER of 10^{-2}.

Table 3.2 : Probability of error of MRC at BER of 10^{-2}

Probability of error at BER of 10^{-2}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	9 dB
N=4	7 dB
N=5	6 dB
N=6	5dB

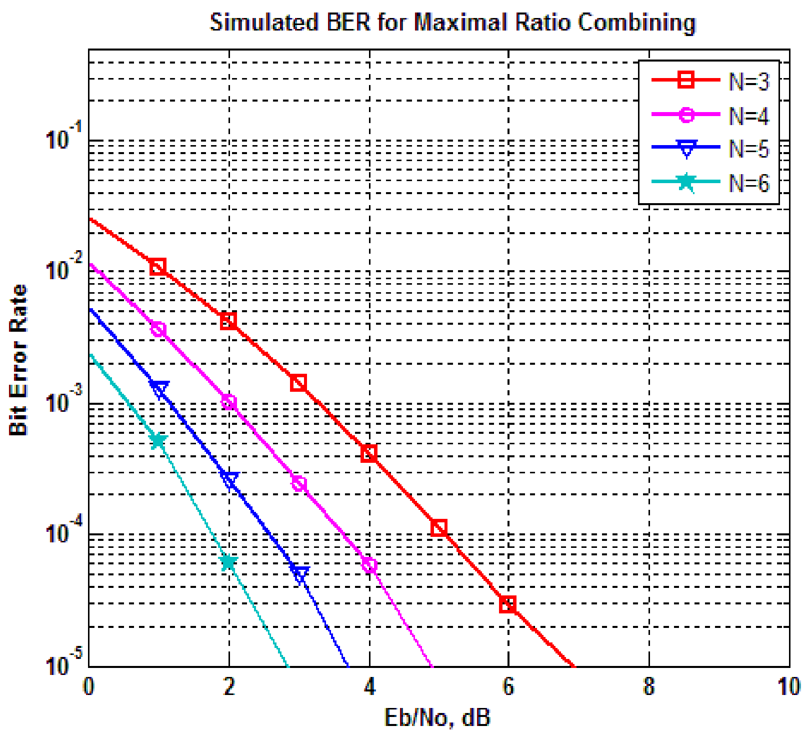

Figure 3.9:Simulated Probability of error vs. Average SNR when receive antennas varies from 3,4,5,6.

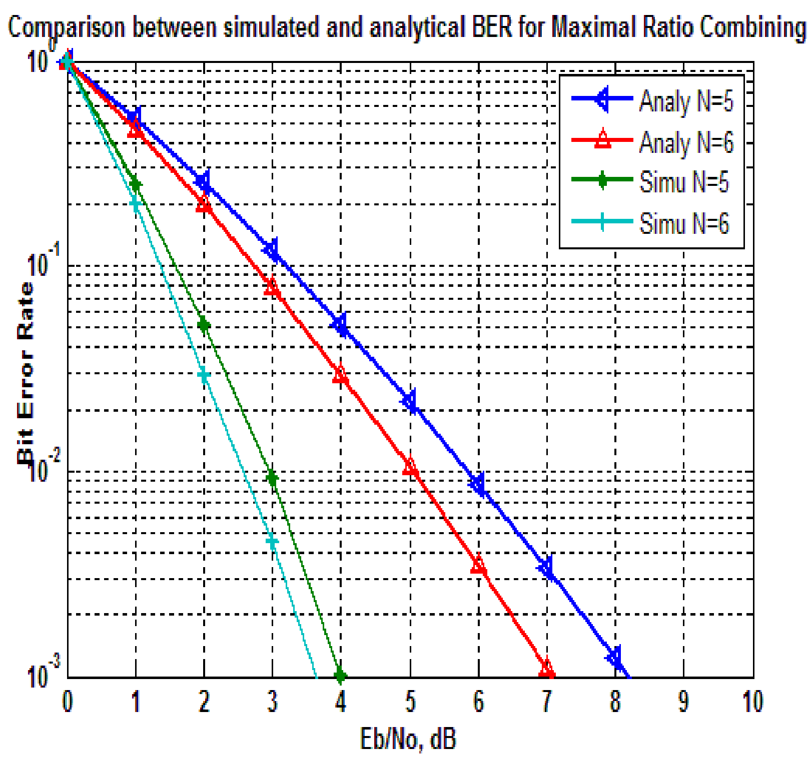

Figure 3.10: Comparison between Simulated and Analytical BER vs. Average SNR when receive antennas varies from 3,4,5,6.

3.4 COMPARISON BETWEEN MRC AND SC

Outage probability and probability of error is much better than in the case of MRC than SC.

3.4.1 Comparison of Outage Probability

Now compare the equation (3.17) of SC and equation (3.34) of MRC

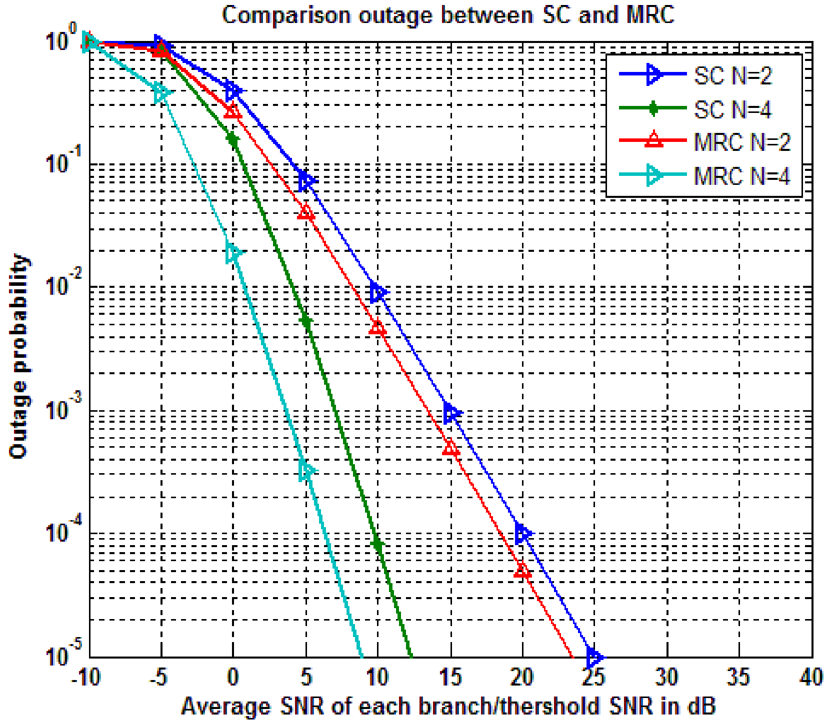

Figure 3.11: Analytical Outage Probability vs. Average SNR of SC and MRC for N=2, N=4.

Table 3.3 shows that for a BER of 10^{-3} for N=2, MRC combining technique required less SNR around 1.5 dB than the SC combining technique. So the performance of MRC is much better than the perfornance of SC.

Table 3.3 : Outage probability of SC and MRC for N=2 at BER of 10^{-3}

Outage probability at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=2	15 dB
MRC,N=2	13.5 dB

Table 3.4 : Outage probability of SC and MRC for N=4 at BER of 10^{-3}

Outage probability at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=4	7 dB
MRC,N=4	4 dB

Table 3.4 shows that for a BER of 10^{-3} for N=4, MRC combining technique required less SNR around 3dB than the SC combining technique. So the performance of MRC is much better than the perfornance of SC

3.4.2 Comparison of Probability of error

Now compare the equation (3.24) of SC and equation (3.36) of MRC.

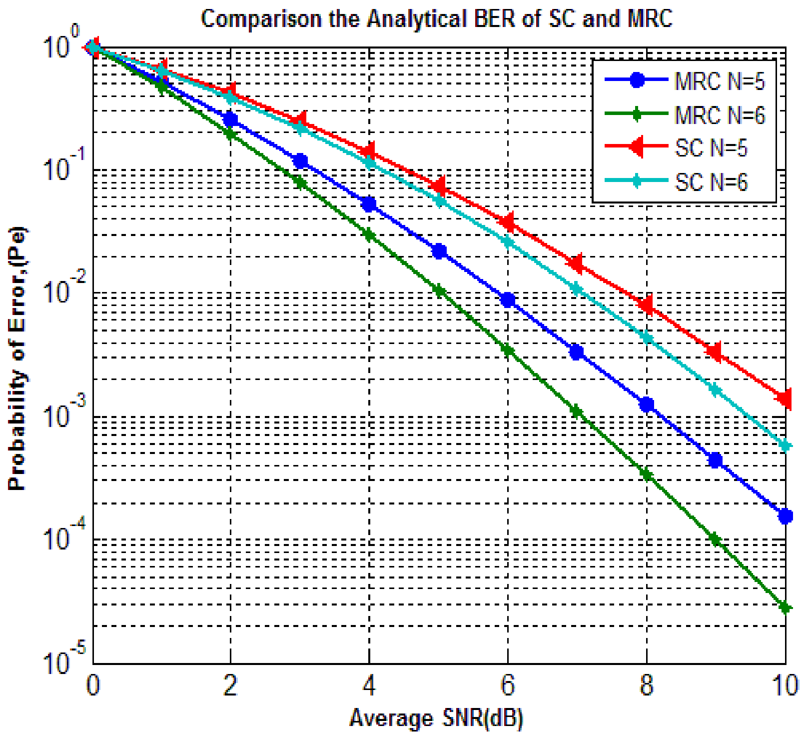

Figure 3.12: Analytical Probability of error vs. Average SNR of SC and MRC for N=5, N=6.

Table 3.5 : Probability of error of SC and MRC for N=5 at BER of 10^{-2}

Probability of Error at BER of 10^{-2}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=5	7.8 dB
MRC,N=5	6 dB

Table 3.5 shows that for a BER of 10^{-2} for N=5, MRC combining technique required less SNR around 1.8 dB than the SC combining technique. So the performance of MRC is much better than the perfornance of SC.

Table 3.6 : Probability of error of SC and MRC for N=6 at BER of 10^{-2}

Probability of Error at BER of 10^{-2}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=6	7 dB
MRC,N=6	5 dB

Table 3.6 shows that for a BER of 10^{-2} for N=6, MRC combining technique required less SNR around 2 dB than the SC combining technique. So the performance of MRC is much better than the perfornance of SC.

3.5 OPTIMUM COMBINING

Space diversity provides an attractive means for improving the performance of mobile radio systems. With space diversity, the signals from the receiving antennas can be combined to combat multipath fading of the desired signal and reduce the relative power of interfering signals. In this chapter, mobile radio systems have considered space diversity only for combating multipath fading of the desired signal. Interference at each receiving antenna is assumed to be independent. Under this condition, MRC produces the highest output signal-to-interference-plus noise ratio (SINR) at the receiver. However, in most systems same interfering signals are present at each of the receiving antennas. Thus, the received signals can be combined to suppress these interfering signals in addition to combating desired signal fading which is referred as a optimum combining (OC). Thereby it achieved higher output SINR than maximal ratio combining and is thus highly desirable even when the number of interferers exceeds the number of antenna array elements. This improved SINR efficiency can manifest itself in the cellular mobile radio application as a reduction in the number of base stations and/or an increased channel capacity through greater frequency reuse [34].

3.5.1 System Model:

Consider a system model which is shown in Figure (3.13) that provides space diversity via an 'N'-elements antenna array. Assume that the antenna elements of the array are placed sufficiently far apart so as to provide independent fading paths and all the interferers have equal power. Furthermore, the system employs binary phase shift keying modulation and channel is characterized by flat Rayleigh fading. At the receiver side, N-elements antenna array operates in the presence of 'L' co-channel interferers. Then the received signal vector **r(t)** at the outputs of the array elements which is expressed as

$$\boldsymbol{r}(t) = \boldsymbol{c_d}\, s(t) + \sum_{k=1}^{L} \boldsymbol{c_k}\, s_k(t) + \boldsymbol{n}(t) \qquad (3.37)$$

where $s(t)$ and $s_k(t)$ are the desired and kth interfering signals respectively.$\boldsymbol{c_d}$ and $\boldsymbol{c_k}(k = 1,2, \dots . L)$ are assumed to be a mutually independent N-dimensional complex Gaussian vector with each component of vectors $\boldsymbol{c_d}$ and $\boldsymbol{c_k}$ having power P_s and P_I respectively and **n**(t) AWGN vector each element of which has zero mean and variance σ_n^2. The desired signal s(t) and the interfering signals $s_k(t)$ are such that $E[s^2(t)] = E[{s_k}^2(t)] = 1$. Here we also assume that the coherence time of the channel is much larger than the duration of signals so that the propagation vectors may be considered as constant.

It is known that the optimum weighting vector in the optimum combining for minimum mean square error between s(t) and weighted and summed array output which is given by [20]

$$w_{OC} = \boldsymbol{R_{ni}^{-1}} \boldsymbol{c_d} \qquad (3.38)$$

For this receiver, the maximum instantaneous SINR at the combiner output is given by [21]

$$\gamma_{OC} = \boldsymbol{c_d}^{\boldsymbol{H}} \boldsymbol{R_{ni}^{-1}} \boldsymbol{c_d} \qquad (3.39)$$

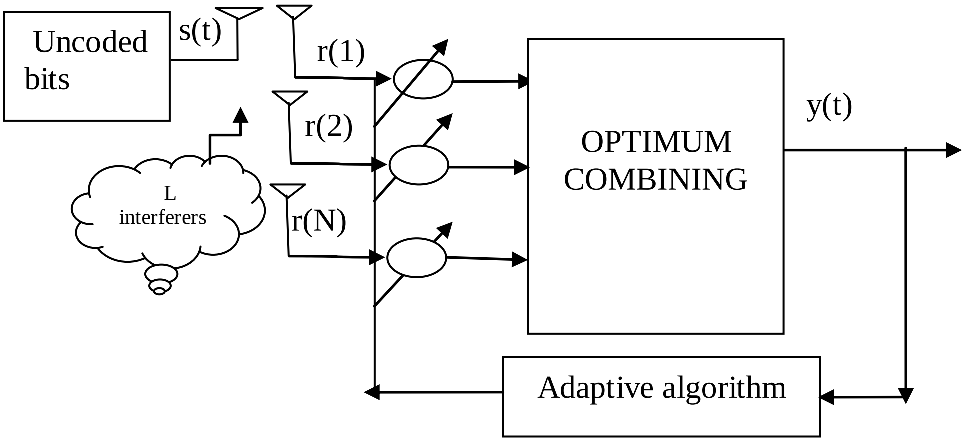

Figure 3.13: System Model of optimum combining

where the superscript 'H' stands for the Hermitian (transpose complex conjugate) operation and $\boldsymbol{R_{ni}}$ is a noise plus interference covariance matrix which is given by

$$\boldsymbol{R_{ni}} = E\left\{\left[\sum_{k=1}^{L} \boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)\right]\left[\sum_{k=1}^{L} \boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)\right]^H\right\} \tag{3.40}$$

$$\boldsymbol{R_{ni}} = \sum_{k=1}^{L} \boldsymbol{c}_k + \sigma_n^2 \boldsymbol{I} \tag{3.41}$$

where $\boldsymbol{I}$ is the identity matrix of dimension LxL. Since $\boldsymbol{R_{ni}}$ can be diagonalized by a unitary transformation [35]

$$\boldsymbol{R_{ni}} = \boldsymbol{V^H} diag\{\lambda_1, \lambda_2 \dots \dots . \lambda_N\}\boldsymbol{V} \tag{3.42}$$

Where $\lambda_1, \lambda_2 \dots \dots . \lambda_N$ are the eigen-values of $\boldsymbol{R_{ni}}$.

3.5.2 Performance of Optimum Combining Receivers

Optimum combining (OC), will achieve a larger output SINR than MRC and is thus highly desirable even when the number of interferers exceeds the number of antenna array elements. This improved SINR efficiency can manifest itself in the cellular mobile radio application as a reduction in the number of base stations and/or an increased channel capacity through greater frequency reuse.

- ***Single Interferer, Independent Identically Distributed Fading***

In this case, only single interfere is considered. Then the received signal at the output of antenna array becomes

$$\boldsymbol{r}(t) = \boldsymbol{c_d}\, s(t) + \boldsymbol{c_I}\, s_I(t) + \boldsymbol{n}(t) \tag{3.43}$$

And from equation (3.40) it becomes

$$\boldsymbol{R_{ni}} = E\{[\boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)][\boldsymbol{c}_k\, s_k(t) + \boldsymbol{n}(t)]^H\} \tag{3.44}$$

$$\boldsymbol{R_{ni}} = \boldsymbol{c}_k\, \boldsymbol{c}_k{}^H + \sigma_n^2 \boldsymbol{I} \tag{3.45}$$

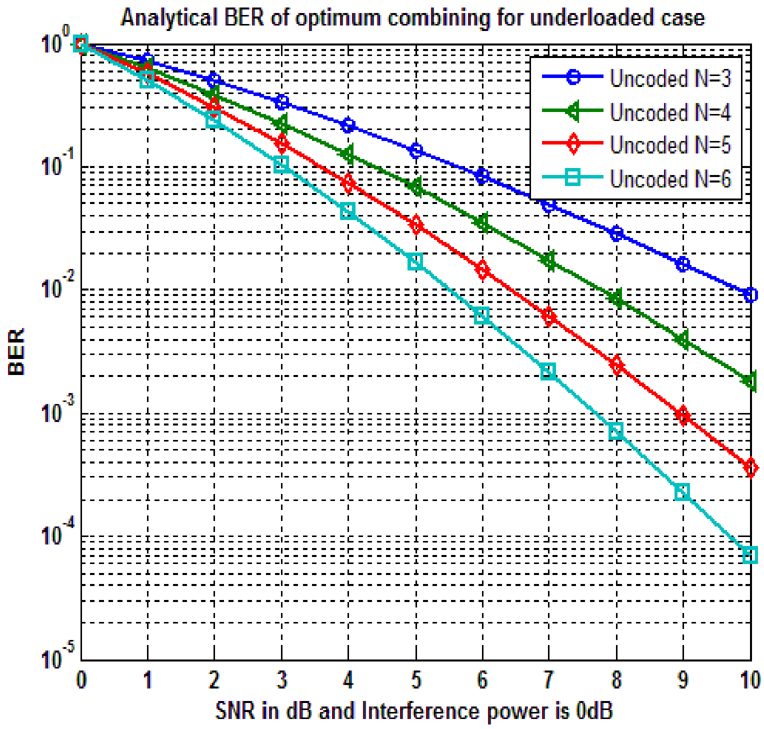

Figure 3.14: BER for uncoded OC system when receive antennas varies from 3 to 6.

Table 3.7 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 4.5 dB at BER of 10^{-2}.

Table 3.7 : Probability of error of OC at BER of 10^{-2}

Probability of error at BER of 10^{-2}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	10 dB
N=4	7.8 dB
N=5	6.5 dB
N=6	5.5dB

➢ ***Number of Interferers Equal to or Greater Than the Number of Array Elements***

Considering optimum diversity combining, the density function of maximum SIR of uncoded system is given by [20]

$$p_{\gamma_{oc}}(\gamma_{oc}) = \frac{\Gamma(L+1)}{\Gamma(N)\Gamma(L+1-N)}\left(\frac{P_s}{P}\right)^{L+1-N} \frac{{\gamma_{oc}}^{N-1}}{\left(\frac{P_s}{P}+\gamma_{oc}\right)^{L+1}} \qquad (3.46)$$

for $\gamma_{oc} \geq 0, 1 \leq N \leq L$

BER for uncoded system over the Rayleigh faded channel

$$P_e = \int_0^{\infty} \frac{1}{2}\mathrm{erfc}(\sqrt{\gamma_{oc}})\, p(\gamma_{oc})d\gamma_{oc} \qquad (3.47)$$

Table 3.8 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 4.5 dB at BER of 10^{-1}.

Table 3.8 : Probability of error of OC at BER of 10^{-1}

Probability of error at BER of 10^{-1}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	7.5dB
N=4	5.5 dB
N=5	4 dB
N=6	3 dB

3.6 COMPARISON BETWEEN MRC AND OPTIMUM COMBINING IN THE PRESENCE OF INTERFERENCE

Using MRC in the presence of interference results in suboptimum performance in that it produces a smaller SINR at the combiner output than OC. The analytical results describing the performance of MRC in the presence of interference can be obtained directly from the results for the performance of MRC in the absence of interference equation (3.36) by replacing the average SNR with the average SINR that is

$$\gamma_0 = \frac{\gamma_d}{1 + L\gamma_I} \qquad (3.49)$$

The PDF in case of MRC in the presence of interference which is given by [23]

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \frac{\Gamma(L+N)}{\Gamma(L)\Gamma(N)}\left(\frac{P_s}{P}\right)^L \frac{\gamma_{MRC}{}^{N-1}}{\left(\frac{P_s}{P}+\gamma_{MRC}\right)^{L+N}} \quad (3.50)$$

BER for MRC in the presence of CCI over the Rayleigh faded channel

$$P_e = \int_0^\infty \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma_{MRC}}\right) p(\gamma_{MRC}) d\gamma_{MRC} \quad (3.51)$$

Putting the value of equation (3.62) into equation (3.63), then BER becomes

$$P_e = \frac{1}{2\sqrt{\pi}\Gamma(N)\Gamma(L)}\left[\frac{\left(\frac{P_s}{P}\right)^N \Gamma\left(\frac{1}{2}-N\right)\Gamma(N+L)}{-N}\ {}_2F_2\left(N+L, N; N+\frac{1}{2}, N+1; \frac{P_s}{P}\right)\right.$$
$$+\frac{\left(\frac{P_s}{P}\right)^{\frac{1}{2}}\Gamma\left(N-\frac{1}{2}\right)\Gamma(-1/2)\Gamma(L+1/2)}{\Gamma\left(\frac{1}{2}\right)}\ {}_2F_2\left(L+\frac{1}{2}, \frac{1}{2}; \frac{3}{2}-N, \frac{3}{2}; \frac{P_s}{P}\right)$$
$$\left. + \Gamma(N)\Gamma(L)\Gamma\left(\frac{1}{2}\right)\right] \quad (3.52)$$

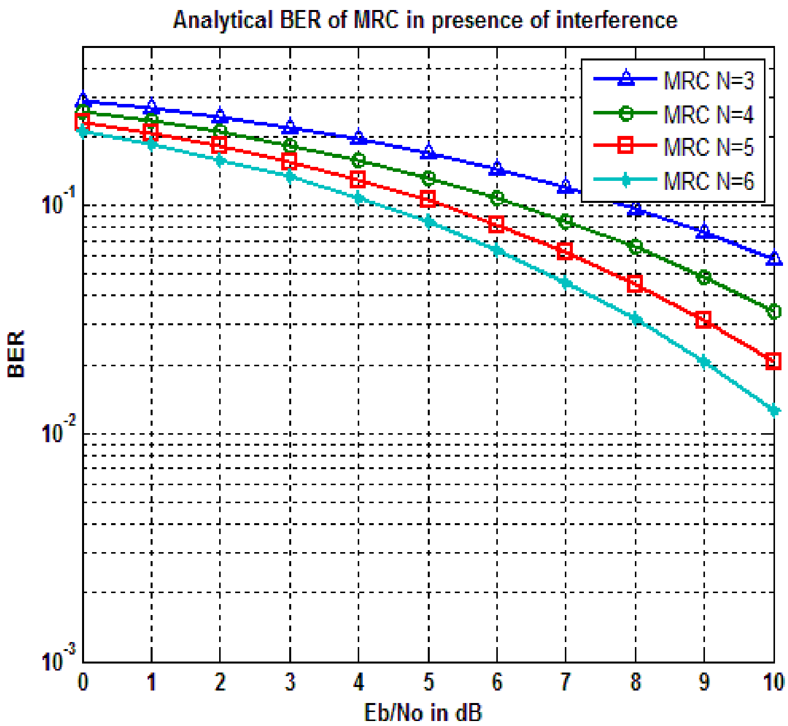

Figure 3.15: BER for MRC system in the presence of interferers taken as L=18 when receive antenna N varies from 3 to 6.

Table 3.9 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 3.7 dB at BER of 10^{-1}.

Table 3.9 : Probability of error of MRC at BER of 10^{-1}

Probability of error at BER of 10^{-1}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	7.9dB
N=4	6.2 dB
N=5	5.2 dB
N=6	4.2dB

Table 3.10 shows that for a BER of 10^{-1} for N=5, OC combining technique required less SNR around 0.7dB than the MRC combining technique in the presence of interferers where number of interferers N=18. So the performance of OC is much better than the perfornance of MRC in the presence of interfereres.

Table 3.10 : Probability of error of MRC and OC for N=5 at BER of 10^{-1}

Probability of Error at 10^{-1}	
Combining Techniques	Signal to Noise ratio(dB)
MRC,N=5	9.7 dB
OC,N=5	9dB

Table 3.11 shows that for a BER of 10^{-1} for N=6, OC combining technique required less SNR around 1dB than the MRC combining technique in the presence of interferers where number of interferers N=18. So the performance of OC is much better than the perfornance of MRC in the presence of interfereres.

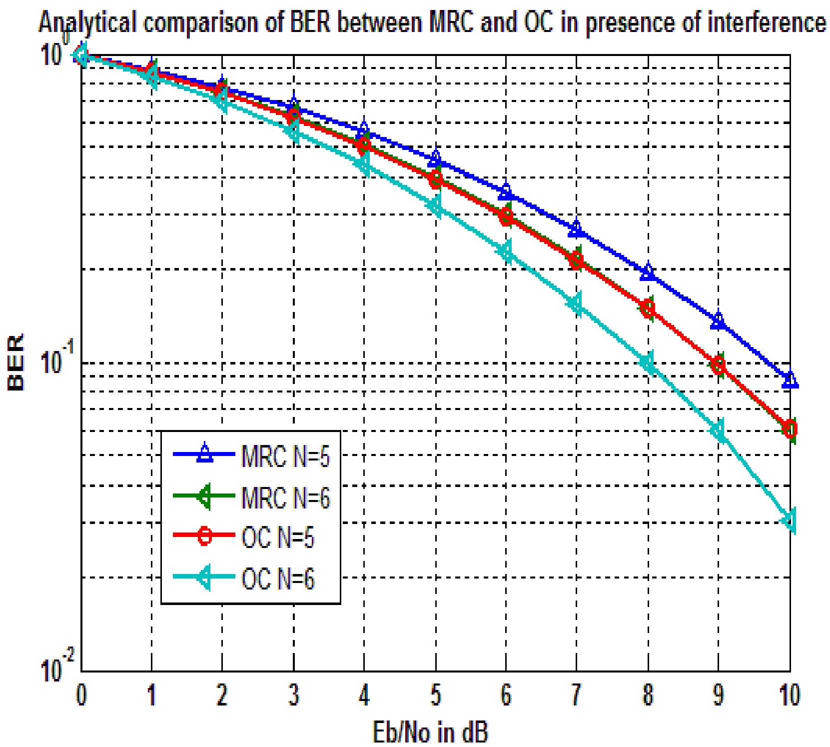

Figure 3.16: Analytical comparison of BER between MRC and OC system in the presence of interferers taken as L=18 when receive antennas N=5, 6.

Table 3.11 : Probability of error of MRC and OC for N=6 at BER of 10^{-1}

Probability of Error at 10^{-1}	
Combining Techniques	Signal to Noise ratio(dB)
MRC,N=6	9 dB
OC,N=6	8 dB

CHAPTER 4

DIVERSITY COMBINING WITH CHANNEL CODING

In the 4rd chapter, brief introduction of channel coding, performance of SC, MRC with channel coding has been evaluated.

4 .1 CHANNEL CODING

Channel coding refers to the class of signal transformations designed to improve communication performance by enabling the transmitted signal to better withstand the effects of various channel impairments, such as noise, interference and fading. Channel coding is one of the better method to improve the error performance of a system without much increase in the hardware and cost. By using channel coding, we can get a performance improvement of about 10dB. Powerful codes have been found that can approach capacity limits closely. Shannon's channel coding theorem state that error free communication is possible for long codes provided that code rate does not exceed the capacity of channel. Different methods are being used nowadays for improving the error performance of wireless communications. If we use these methods together we can get an even better performance from the system. Space-time codes are now concatenated with the channel coder that provides the improvement in bit error rate with good diversity and coding gain. Channel codes concatenated with MIMO systems will help future wireless communication systems to operate at considerably higher data rates with higher reliability than what we have today.

4.1.1 Parity check codes

Parity check codes use linear sums of the information bits, called parity symbols or parity bits, for error detection or correction. A simple parity code is constructed by adding a single parity bit to block of data bits that takes on the value of one or zero as needed to ensure that the summation of all bits in the code word yields an even (or odd) results. This scheme is known as even (or odd) parity scheme. At the receiving terminal, the decoding process consists of testing that modulo-2 sum of the codeword bit yields a zero result (even parity). If the results found to be one instead of zero, the codeword is known to contain errors.

4.1.2 Linear block codes

Linear block codes have the property of linearity that is the sum of any two code words is also a code word, and they are applied to the source bits in blocks, hence the name linear block codes. There are many types of linear block codes, such as

- Cyclic codes
- Repetition codes
- Parity codes
- Polynomial codes (BCH codes are a subset of the polynomial codes)
- Reed Solomon codes
- Algebraic geometric codes
- Reed–Muller codes
- Perfect codes.

First error correcting codes, introduced by R.W. Hamming popularly known as Hamming codes in 1950 to improve the reliability of electro-magnetical computing machine. Mathematically cyclic codes can be Generalized as Generator matrix [**G**] is given by [**G**] = [$\mathbf{I_k}$**: P**] where I_K is the identity matrix and P is the parity check matrix. The code vector is obtained as [C] = [m] $\otimes$[**G**] where [m] is message vector and $\otimes$ used for XOR operation. If the error vector of channel is [E] then the received vector is [W**] = [C]** $\oplus$ **[E]**. The parity check matrix [**H**] should be such that in the decoder, the syndrome **S** obtained as [**S] = [W]*[H^T**] = 0 if no error is presented. Finally Error is given by [E] =[S]. [H^T]-1. At the receiver side, codeword is given by [W + E] =[C]. All the cyclic block codes follow the same procedure as given above only the difference is in generating the polynomials. A generalization of the Hamming codes leads to the BCH codes, which represent a class of cyclic codes that offer a large range of block length, code rates, and error correcting strength. Its Generator matrix is presented as g(X) = LCM [M_1(X), M_2(X) ……… M_{2t}(X)] where M(X) is the message bits.

4.1.3 Reed-Solomon (RS) codes

Reed-Solomon (RS) codes are an extension of BCH codes and were proposed by I.S. Reed and G. Solomon with the corresponding algebraic decoding algorithm by E.R. Berlekamp and J. L. Massey. The RS codes are non-binary cyclic codes in which the code symbols are binary m-tuples. Codes that achieve this "optimal" error correction capability are called maximum distance separable. There exist *t*-error correcting R-S codes for the (n, k, t) set described as (n, k, t) = (2m − 1, 2m − 1 − 2t , t) where 2*t* is the degree of the generating polynomial as well as the number of parity symbols in the code. Encoding of R-S codes is done by using linear feedback shift registers. Decoding algorithms are given by Berlekamp.

4.1.4 Low density parity check code (LDPC)

Low density parity check codes (LDPC) are linear block codes with sparse parity check matrices. The original Gallager's LDPC codes are called regular LDPC codes in which the number of 1's is the same in every row and every column [1]. The following matrix is the parity check matrix of a rate-1/2, length-6 regular LDPC code:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{4.1}$$

In this matrix, the number of 1's in each row is 4 and number of 1's in each column is 2. This parity check matrix is not very sparse because the code is still short.

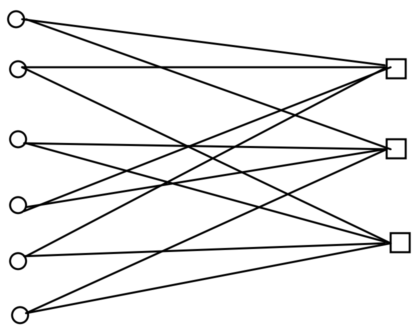

Figure 4.1: Bipartite graph for a regular (2,4) LDPC code.

These codes can be represented in terms of a bipartite graph [37] as shown in Figure (4.1)**.** In this graph, each left node, called variable node. The variable node corresponds to the coded bits in the codeword. Each right node, called a check nodes correspond to the parity check constraints satisfied by the variable nodes. The number of variable nodes corresponds to the number of columns in the parity check matrix **H**, while the number of check nodes corresponds to the number of rows in **H**. Edges connect the variable nodes to the check nodes according to the parity check matrix **H.**

If the number of edges emanating from variable nodes is called a variable node degree d_v and the number of edges emanating from a check node is called check node degree d_c. Then the rate of (d_v , d_c) regular LDPC code is R=1- $d_{v/}$ d_c.

Irregular LDPC codes are LDPC codes that have nodes with different degrees. The degrees of the variable nodes and the check nodes are chosen according to some distribution. For compact description, the degree distribution is often presented in polynomial form. The irregularity is typically specified by a polynomial called the variable (check) node degree profile and given by [24]

$$\left(\lambda(x), \rho(x)\right) = \left(\sum_{j=2}^{d_v} \lambda_j x^{j-1}, \sum_{i=2}^{d_c} \rho_i x^{i-1}\right) \tag{4.2}$$

where d_v and d_c are the maximum degree of variable node 'v' of degree 'j' and check node 'c' of degree 'i' for irregular LDPC code respectively. The coefficient λ_j is the fraction of edges emanating from variable nodes of degree 'j' and ρ_i is the fraction of edges emanating from check nodes of degree 'i'. The polynomial $\rho(x)$ is referred to as the check node degree distribution and $\lambda(x)$ is referred to as variable node degree distribution.

➢ ***Message Passing Decoding Algorithm of LDPC Codes***

Assume that a binary phase shift keying (BPSK) modulation is used and an additive white Gaussian noise (AWGN) vector **n** and Rayleigh faded factor α is present in the codeword. If the k^{th} code symbol that is x_k is mapped into the signal w_k=1-2x_k, then the k^{th} received sampled y_k is

$$y_k = \alpha\, w_k + n_k \tag{4.3}$$

where n_k is the k^{th} noise sample which has a zero mean, Gaussian random variable with variance σ_n^2 and α is the normalized Rayleigh fading factor.

For the uncorrelated Rayleigh fading channel, the conditional PDF of channel output is given by [38]

$$p(y_k|w_k, \alpha) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(y_k - w_k.\alpha)^2}{2\sigma_n^2}\right) \tag{4.4}$$

where $\sigma_n^2 = 1/2R\left(\frac{E_b}{N_0}\right)$ is the variance of noise and R is the code rate.

The steps of the belief propagation algorithm for decoding LDPC codes can be summarized as follows

- **Initiation**

Assuming $P_r(x = 0) = P_r(x = 1) = 1/2$ that is codeword are equally likely then log-likelihood-ratios (LLRs) is represented in terms of which is defined as

$$\psi_{\text{ch}}(x_{\text{k}})=\log\frac{P(x_k=0|y_k,\alpha)}{P(x_k=1|y_k,\alpha)}=\frac{2}{\sigma_n^2}y_k.\alpha \tag{4.5}$$

The message passing decoding algorithm is an iterative algorithm at 'q' iteration which passes messages between variable and check nodes along each edge. x_k is the value of k[th] bit in the codeword corresponding to the variable node 'v'. At the beginning of the first iteration, message from check nodes to variable nodes are set to zero.

$$\psi^{(0)}_{c\rightarrow v,j}(x_{\text{k}})=0 \tag{4.6}$$

where j=1,2,3..........$d_{\text{v}}-1$

- **Update at the Variable Nodes**

Consider the k[th] variable node which is shown in Figure (4.2). It gets extrinsic information $\psi_{\text{ch}}(x_{\text{k}})=\frac{2}{\sigma_n^2}y_k.\alpha$ from the external observation(from the channel) and the incoming edge-LLRs from check node to variable node is $\psi^{(q-1)}_{c\rightarrow v,j}(x_{\text{k}})$. The outgoing LLR from variable node to check node at qth iteration is

$$\psi^{(q)}_{v\rightarrow c,i}(x_{\text{k}})=\psi_{\text{ch}}(x_{\text{k}})+\sum_{j=1,i\neq j}^{\text{d}_{\text{v}}-1}\psi^{(q-1)}_{c\rightarrow v,j}(x_{\text{k}}) \tag{4.7}$$

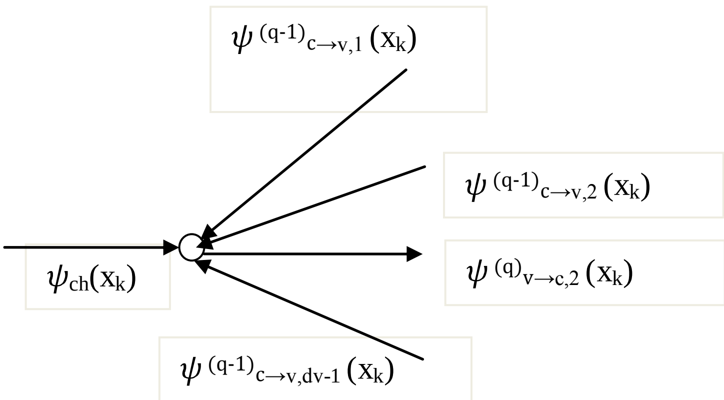

Figure 4.2: sum product decoding at variable node

- **Update at the Check Nodes**

Consider the k[th] check node with degree d_{c} at the qth iteration which is shown in Figure (4.3). If $\psi^{(q)}_{v\rightarrow c,i}(y_{\text{k}})$ represented the LLRs incident on this check node, then the output LLRs on the j[th] edge is governed by:

$$\psi^{(q)}_{c\rightarrow v,j}(y_{\text{k}})=2\tanh^{-1}\prod_{\text{i}=1,}^{\text{d}_{\text{c}}-1}\tanh\left(\frac{|\psi^{(q)}_{v\rightarrow c,i}(y_{\text{k}})|}{2}\right) \tag{4.8}$$

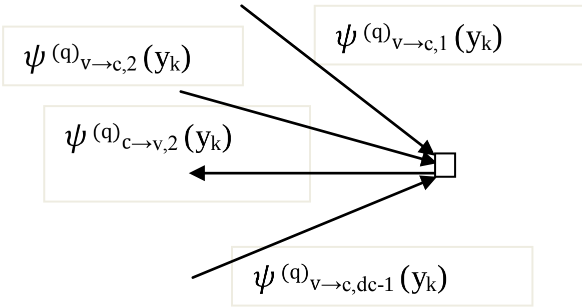

Figure 4.3: sum product decoding at check node

Here the iterations are completed.

- **Decision**

Soft output on the k^th at the end of Q iteration is given as:

$$\psi(x_k) = \psi_{ch}(x_k) + \sum_{j=1, i\neq j}^{d_v - 1} \psi_{c\to v,j}^{Q}(x_k) \qquad (4.9)$$

Hard decision on the bit is obtained as follows:

$$\hat{x}_k = \begin{cases} 1 & if \;\; \psi(x_k) < 0 \\ 0 & if \;\; \psi(x_k) \geq 0 \end{cases} \qquad (4.10)$$

➢ ***Gaussian approximation on Density Evolution***

Density Evolution [39] is the technique which keeps track of the probability density functions (PDF) of the messages (usually LLRs) passed from variable to check nodes or check to variable nodes in iteration. It is a very useful tool which aids in predicting the bit-error rate performance of a given code-ensemble represented by the degree profiles under message passing algorithm decoding. When the channel is symmetric, then Bit-error rate (BER) is independent of the transmitted codeword. In such a case, it sufficient to transmit the all-zero codeword for analysis. For an AWGN channel with BPSK assuming that the all-zero codeword is transmitted, the LLR of the received signal has a Gaussian distribution. In addition, assume that the distribution of the messages (LLRs) passed on any of the edges of the graph, in any given iteration, is Gaussian or a mixture of Gaussians. Then, density evolution simplifies to tracking the means (m) and variances (σ^2) of the Gaussian mixtures. This is called Gaussian approximation [40]. Consistency condition for Gaussians simplifies the variance to $\sigma^2 = 2m$. Hence, in GA, we just have to keep track of the means over iterations to see how the densities evolve. With GA, optimization for thresholds is a linear problem. Sum-product decoding at the variable node is merely sum of the LLRs, which translates to convolution of the densities of LLRs. from comparison of the optimized code profiles and noise thresholds of LDPC codes obtained through GA and actual density evolution, it can be deduced that GA is a good approximation.

4.2 PERFORMANCE EVALUATION OF SELECTION COMBINING WITH CHANNEL CODING

SC-type system only process one of the diversity branches. Specifically, in its conventional form, the SC combiner chooses the branch with the highest SNR. In addition, since the output of SC combiner is equal to the signal on only one of the branches.

4.2.1 System model

In Figure (4.4), Selection diversity combining employed in conjunction with channel decoder which is able to mitigate the effect of fading. This scheme is done over the independent and identically distributed (i.i.d) flat Rayleigh fading channels using the Gaussian distribution. At the combiner, the signal y (t) at the output is fed to the channel decoder given as under

y (t) =a*s (t) + n (t) (4.11)

where 'a' is the optimum combining channel gain over i.i.d Rayleigh fading. s(t) is transmitted BPSK modulated signal and n(t) is additive white Gaussian noise with mean zero and variance σ_n^2.The received signal y(t) is decoded by message passing algorithm.

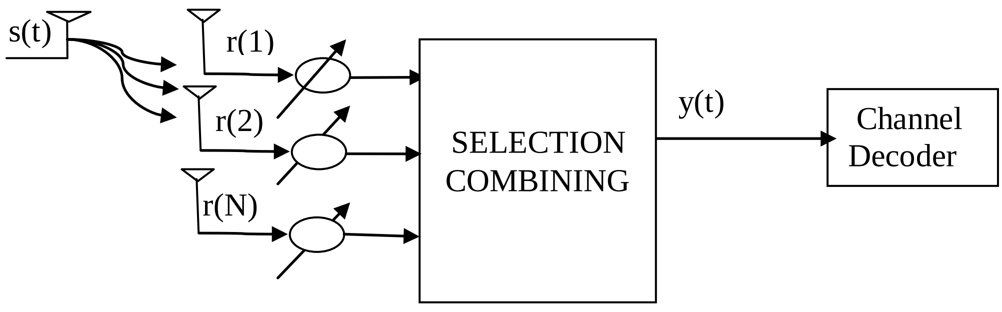

Figure 4.4: System model of Coded SC

4.2.2 Probability of Error of coded Selection Diversity System

To calculate the BER of coded SC system, the message passing algorithm can be analyzed with the assumption that the block length of code is infinity and without loss of generality, all zero codeword is sent. So that the BER at iteration '*l*' is simply the average probability that the variable node messages is negative [41]. The PDF of Log Likelihood Ratio (LLR) of variable nodes, that passes the message to check node on edge 'e', is convolution of the PDF of summation of LLR of check node message received on all incoming edges except 'e' and PDF of channel LLR. But it is very difficult to update these PDFs at '*l*' *iterations.* So we will assume that all the message involved in the decoding process have a symmetric Gaussian distribution in which only mean value needs to be updated iteratively[42**].**

The conditional PDF of channel LLR over the channel gain is given by [38, equation 12]

$$p(q|a, s(t) = +1) = \frac{\sigma_n}{2a\sqrt{2\pi}} \exp\left(-\frac{(q - 2a^2/\sigma_n^2)}{8\,a^2/\sigma_n^2}\right) \tag{4.12}$$

with mean $2a^2/\sigma_n^2$, variance $4a^2/\sigma_n^2$ $\left(\text{where } \sigma_n^2 = \frac{1}{2R*E_b/N_0}, \text{R is code rate}\right)$ and 'q' is the initial message passed from variable node to check node. The conditional PDF of channel LLR is calculated by averaging the equation (4.12) over the channel gain

$$p_0(q) = \int_0^\infty p(q|a, s(t) = +1)\, p(a)da \tag{4.13}$$

after solving , we get

$$p_0(q) = \sum_{i=1}^{N} \sum_{j=1}^{\binom{N}{i}} (-1)^{i-1} \frac{\sigma_n^2 \beta_i^j}{2b} \left[\exp\left(\frac{-b|q|}{2} + \frac{q}{2}\right)\right] \tag{4.14}$$

where $b = \sqrt{1 + 2\sigma_n^2 \beta_i^j}$ Then the PDF of check node message is given by[43]

$$p_c(q) = \frac{1}{\sqrt{4\pi m_c}} \exp\left(-\frac{(q - m_c)^2}{4m_c}\right) \tag{4.15}$$

Now to calculate the PDF of variable node is to convolving the PDF of channel LLR with the PDF of check node that is

$$p_v(q) = p_0(q) \otimes p_c(q) \tag{4.16}$$

$$p_v(q) = \int_{-\infty}^{\infty} p_c(q - \tau)\, p_0(q) d\tau \tag{4.17}$$

Solve the equation (4.17) , we get,

$$p_0(q) = \sum_{i=1}^{N} \sum_{j=1}^{\binom{N}{i}} (-1)^{i-1} \frac{\sigma_n^2 \beta_i^j}{4b} \left[\exp\left(\frac{\sigma_n^2 \beta_i^j m_v}{4b}\right)\right] *$$

$$\left[\left(\exp\left(\frac{1-b}{2}\right)q\right)erfc\left(\frac{-q+m_v b}{2\sqrt{m_v}}\right)+\left(\exp\left(\frac{1+b}{2}\right)q\right)erfc\left(\frac{q+m_v b}{2\sqrt{m_v}}\right)\right] \qquad (4.18)$$

To obtain the probability of bit error P_e, integrate the PDF of variable node given by equation (4.20) from -∞ to 0,

$$P_e=\sum_{i=1}^{N}\sum_{j=1}^{\binom{N}{i}}(-1)^{i-1}\left[\frac{1}{2}erfc\left(\frac{\sqrt{m_v}}{2}\right)-\frac{1}{2b}\exp\left(\frac{\sigma_n^2\beta_i^j m_v}{4b}\right)erfc\left(\frac{b\sqrt{m_v}}{2}\right)\right] \qquad (4.19)$$

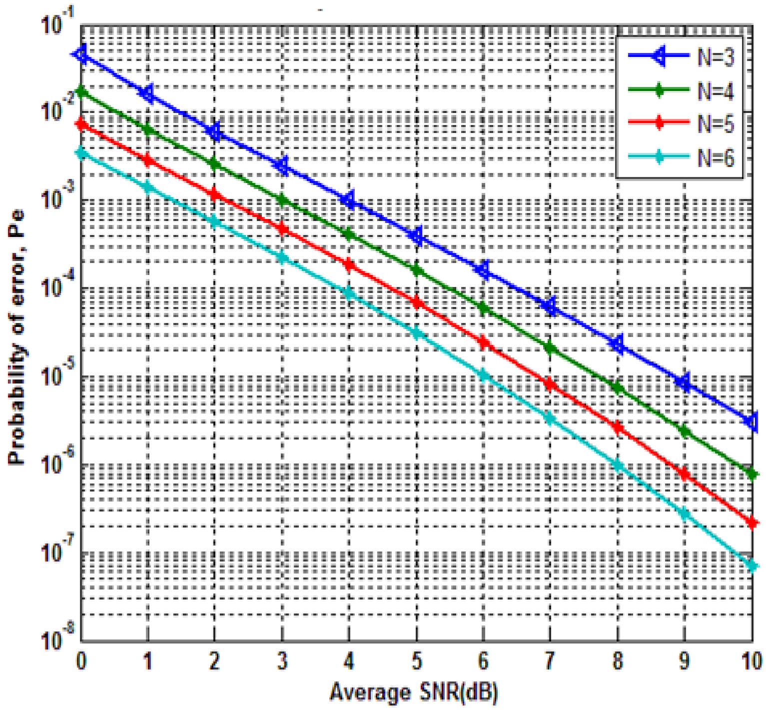

Figure 4.5: Analytical BER of coded SC having number of receive antennas N=3, 4, 5, 6.

Table 4.1 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 2.5 dB at BER of 10^{-4}.

Table 4.1 : Probability of error of coded SC at BER of 10^{-4}

Probability of error at BER of 10^{-4}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	6.5 dB
N=4	5.5 dB

N=5	4.6 dB
N=6	4 dB

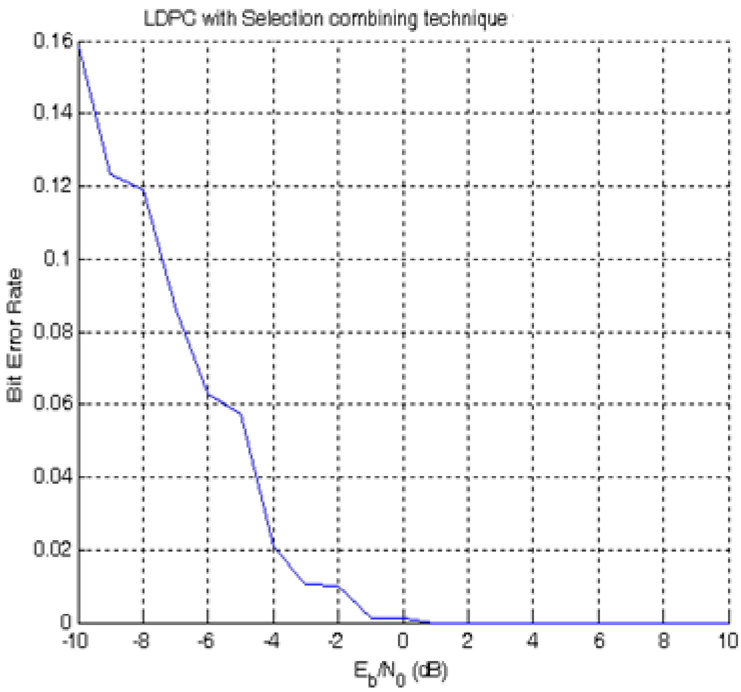

Figure 4.6: Simulated BER of coded SC having number of receive antennas N=4.

4.3 COMPARISON BETWEEN UNCODED SC AND CODED SC

Now compare the equation (3.24) of uncoded SC and equation (4.22) of coded SC

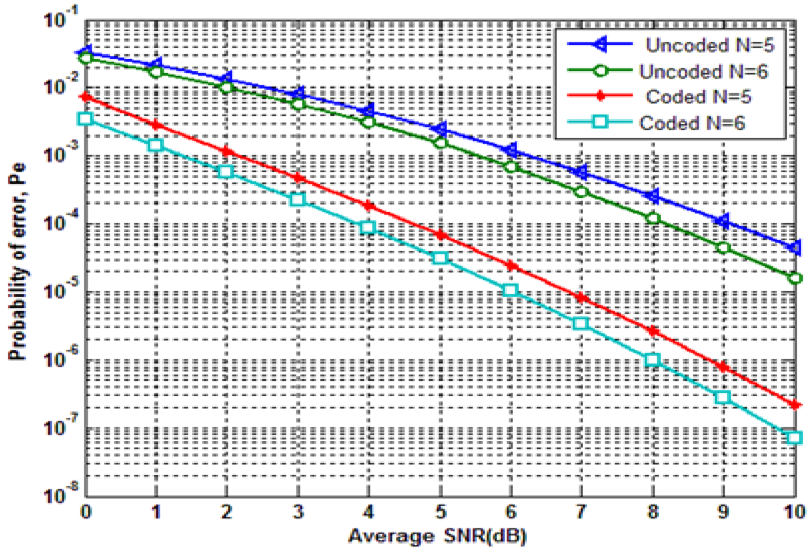

Figure 4.7: Analytical BER of SC and coded SC having number of receive antennas N=5, 6.

Table 4.2 shows that for a BER of 10^{-3} for N=5 coded SC combining technique required less SNR around 4 dB than the SC combining technique. So the performance of coded SC is much better than the perfornance of SC.

Table 4.2: Probability of error of SC and coded SC for N=5 at BER of 10^{-3}

Probability of Error at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=5	6 dB
Coded SC,N=5	2 dB

Table 4.3: Probability of error of SC and coded SC for N=6 at BER of 10^{-3}

Probability of Error at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
SC,N=6	5.5 dB
Coded SC,N=6	1.3 dB

Table 4.3 shows that for a BER of 10^{-3} for N=6, coded SC combining technique required less SNR around 4.2 dB than the SC combining technique. So the performance of coded SC is much better than the perfornance of SC.

4.4 PERFORMANCE EVALUATION OF MRC WITH CHANNEL CODE

The signal at the output of the receivers is linearly combined in MRC so as to maximize the instantaneous signal to noise ratio. This is achieved by combining the co-phased signal. The SNR of the combined signal is equal to the sum of the SNRs of all the branch signals. That is, branches with strong signal are further amplified, while weak signals are attenuated.

4.4.1 System model

In Figure (4.8), Maximal Ratio combining employed in conjunction with channel decoder which is able to mitigate the effect of fading. This scheme is done over the independent and identically distributed (i.i.d) flat Rayleigh fading channels using the Gaussian distribution. At the combiner, the signal y (t) at the output is fed to the channel decoder given as under

y (t) =a*s (t) + n (t) (4.20)

where ‘a’ is the optimum combining channel gain over i.i.d Rayleigh fading. s(t) is transmitted BPSK modulated signal and n(t) is additive white Gaussian noise with mean zero and variance σ_n^2.The received signal y(t) is decoded by message passing algorithm.

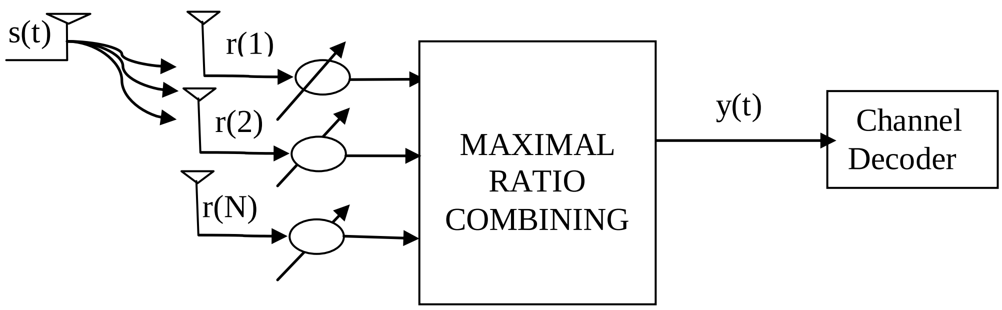

Figure 4.8: System model of coded MRC system

4.4.2 Probability of Error of coded Maximal Ratio Combining System

To calculate the BER of coded MRC system, the message passing algorithm can be analyzed with the assumption that the block length of code is infinity and without loss of generality, all zero codeword is sent. So that the BER at iteration ‘l’ is simply the average probability that the variable node messages is negative [41]. The PDF of Log Likelihood Ratio (LLR) of variable nodes, that passes the message to check node on edge ‘e’, is convolution of the PDF of summation of LLR of check node message received on all incoming edges except ‘e’ and PDF of channel LLR. But it is very difficult to update these PDFs at '*l' iterations.* So we will assume that all the message involved in the decoding process have a symmetric Gaussian distribution in which only mean value needs to be updated iteratively**[**42**].**

The conditional PDF of channel LLR over the channel gain is given by [38, equation 12]

$$p(q|a, s(t) = +1) = \frac{\sigma_n}{2a\sqrt{2\pi}} \exp\left(-\frac{(q - 2a^2/\sigma_n^2)}{8\,a^2/\sigma_n^2}\right) \tag{4.21}$$

with mean $2a^2/\sigma_n^2$, variance $4a^2/\sigma_n^2$ $\left(\text{where } \sigma_n^2 = \frac{1}{2R*E_b/N_0}, \text{R is code rate}\right)$ and 'q' is the initial message passed from variable node to check node. The conditional PDF of channel LLR is calculated by averaging the equation (4.12) over the channel gain

$$p_0(q) = \int_0^\infty p(q|a, s(t) = +1)\, p(a) da \tag{4.22}$$

Now to transformation the PDF of γ_{MRC} which is in equation (3.25) into channel gain *a* is given by

$$p(a) = \frac{2N^N a^{2N-1} \exp(-Na^2)}{(N-1)!} \tag{4.23}$$

Unconditional PDF of channel LLR is calculated by averaging the equation (4.13) over the channel gain PDF. So put the value of equation (4.23) into equation (4.13), Then we get

$$p_0(q) = \frac{N^N \sigma_n}{2\sqrt{2}(N-1)!} \{(-1)^{N-1} \frac{\partial^{N-1}}{\partial b^{N-1}} \left[\frac{1}{\sqrt{b}} \exp\left(-\sigma_n |q| \sqrt{\frac{b}{2}} + \frac{q}{2}\right)\right] \tag{4.24}$$

where $b = N + \frac{1}{2\sigma_n^2}$,

Now to calculate the PDF of variable node is to convolving the PDF of channel LLR with the PDF of check node that is

$$p_v(q) = p_0(q) \otimes p_c(q) \tag{4.25}$$

Now put the value of channel LLR of equation (4.24) and PDF of check node message of equation (4.16) into equation (4.25), then we get PDF of variable node by using the integral in [36,equation 3.322(2)],

$$p_0(q) = \frac{N^N \sigma_n}{4\sqrt{2}(N-1)!} (-1)^{N-1} \frac{\partial^{N-1}}{\partial b^{N-1}} \left[\sqrt{\frac{1}{b}}\, exp\left(\left(\frac{2\sigma_n^2 b - 1}{4}\right) m_c\right)\right] *$$

$$\left\{\left(\exp(\frac{\sigma_n\sqrt{2b}+1}{2})q\right)\left[1-\Phi\left(\frac{q+m_c\sigma_n\sqrt{2b}}{2\sqrt{m_c}}\right)\right]\right.$$

$$\left.*+\exp\left((\frac{-\sigma_n\sqrt{2b}+1}{2})q\right)\left[1-\Phi\left(\frac{-q+m_c\sigma_n\sqrt{2b}}{2\sqrt{m_c}}\right)\right]\right\} \quad (4.26)$$

To obtain the probability of bit error P_e, integrate the PDF of variable node given by equation (4.26) from -∞ to 0,

$$P_e = \frac{N^N}{(N-1)!}(-1)^{N-1}\frac{\partial^{N-1}}{\partial b^{N-1}}\left[\frac{\sigma_n}{\sqrt{2b}-2\sqrt{2}\sigma_n^2 b^{\frac{3}{2}}} erfc\left(\frac{\sigma_n}{2}\sqrt{2bm_c}\right) exp\left(\left(\frac{2\sigma_n^2 b-1}{4}\right)m_c\right)\right.$$

$$\left.-\frac{\sigma_n^2}{1-2b\sigma_n^2} erfc(\frac{\sqrt{m_c}}{2})\right] \quad (4.27)$$

Now at (l +1) th iteration, averaging above equation over all the bit node degrees j is

$$P_e = \frac{N^N}{(N-1)!}(-1)^{N-1}\frac{\partial^{N-1}}{\partial b^{N-1}} *$$

$$\left\{\sum_{j=2}^{d_v}\lambda_k\left[\frac{\sigma_n}{\sqrt{2b}-2\sqrt{2}\sigma_n^2 b^{\frac{3}{2}}} erfc\left(\frac{\sigma_n}{2}\sqrt{2bm_c^{l+1}(j-1)}\right) exp\left(\left(\frac{2\sigma_n^2 b-1}{4}\right)m_c^{l+1}(j\right.\right.\right.$$

$$\left.\left.\left.-1)\right)-\frac{\sigma_n^2}{1-2b\sigma_n^2} erfc\left(\frac{\sqrt{m_c^{l+1}(j-1)}}{2}\right)\right]\right\} \quad (4.28)$$

Because at (l+1) th iteration mean of variable node v of degree j is $m_v^{l+1} = (j-1)m_c^{l+1}$.

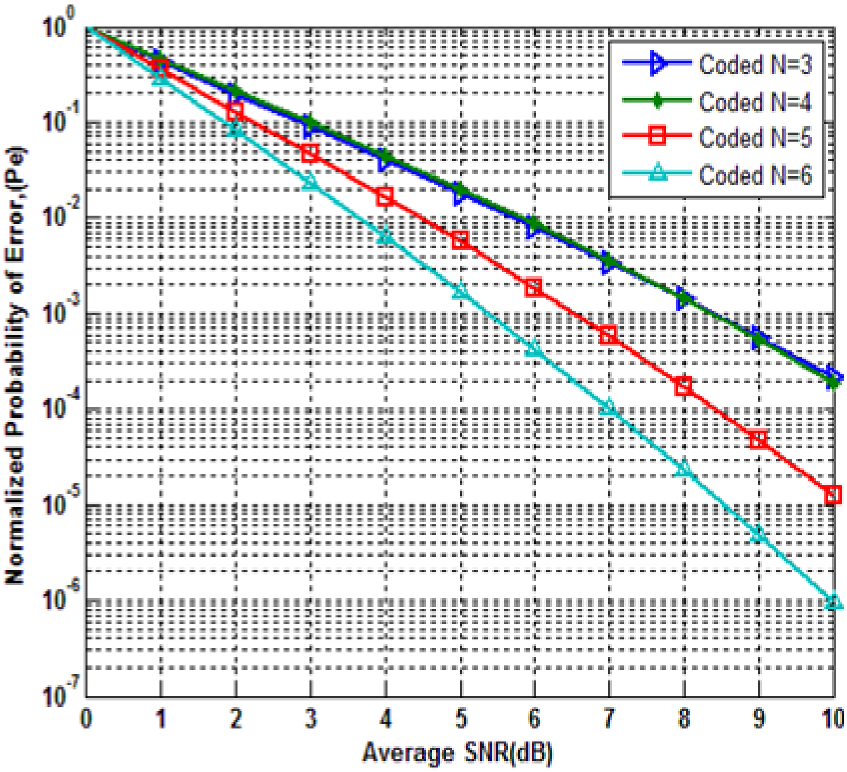

Figure 4.9: Analytical BER of coded MRC having number of receive antennas N=3, 4, 5, 6.

Table 4.4 shows as a number of antennas are increased from 3 to 6 then diversity gain is achieved. From 3 to 6, diversity gain is improved 3.1dB at BER of 10^{-3}.

Table 4.4 : Probability of error of coded MRC at BER of 10^{-3}

Probability of error at BER of 10^{-3}	
Number of receive antenna	Signal to Noise ratio(dB)
N=3	8.5 dB
N=4	8.5 dB
N=5	6.5 dB
N=6	5.4 dB

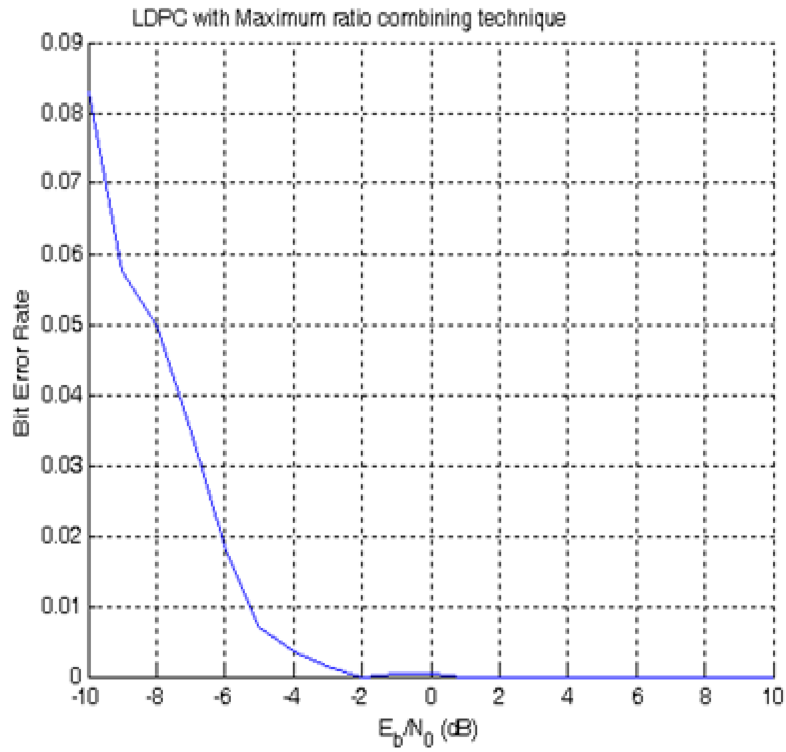

Figure 4.10: Simulated BER of coded MRC having number of receive antennas N=4.

4.5 COMPARISON BETWEEN UNCODED MRC AND CODED MRC

Now compare the equation (3.36) of uncoded MRC and equation (4.28) of coded MRC.

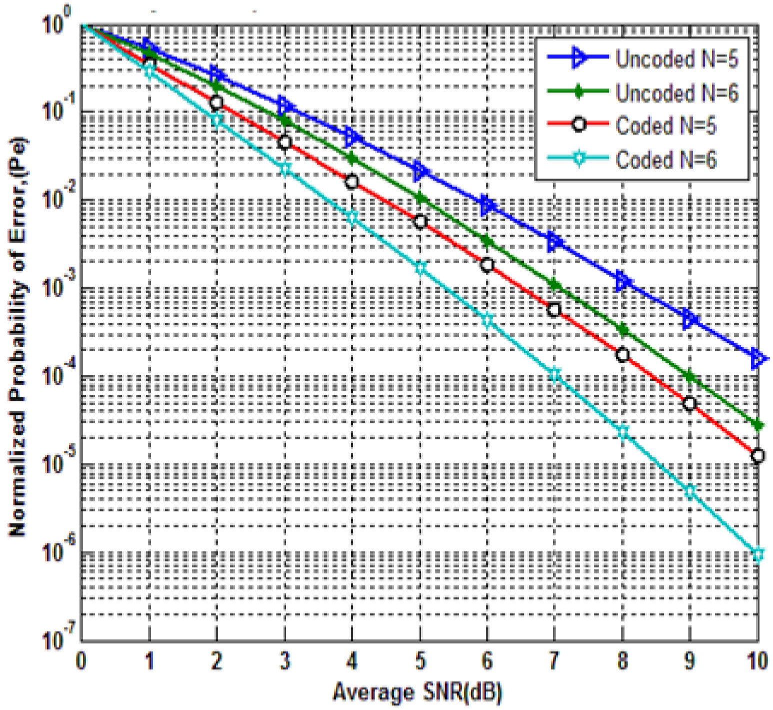

Figure 4.11: Analytical BER of MRC and coded MRC having number of receive antennas N=5, 6.

Table 4.5 shows that for a BER of 10^{-3} for N=5, coded MRC combining technique required less SNR around 1.6 dB than the MRC combining technique. So the performance of coded MRC is much better than the perfornance of uncoded MRC.

Table 4.5 : Probability of error of MRC and coded MRC for N=5 at BER of 10^{-3}

Probability of Error at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
MRC,N=5	8.1 dB
Coded MRC,N=5	6.5 dB

Table 4.6 : Probability of error of MRC and coded MRC for N=6 at BER of 10^{-3}

Probability of Error at BER of 10^{-3}	
Combining Techniques	Signal to Noise ratio(dB)
MRC,N=6	7 dB
Coded MRC,N=6	5.4 dB

Table 4.6 shows that for a BER of 10^{-3} for N=6, coded MRC combining technique required less SNR around 1.6 dB than the MRC combining technique. So the performance of coded MRC is much better than the perfornance of MRC.

CHAPTER 5

CONCLUSIONS AND FUTURE SCOPE OF RESEARCH

CONCLUSIONS

Space Diversity such as selection combining, maximal ratio combining and optimum combining is a very important part of the wireless communication system. It is used to mitigate the effect of fading. So that it helps to improve the performance of a system. Channel coding are amongst the most powerful forward error correcting codes as they have proved their error performance to be the closest to the Shannon limit within 0.0045 dB.

In this thesis, a comparison study of coded and uncoded SC, MRC and OC has been done over the independent and identically distributed Rayleigh fading environment. Coded diversity combining were compared to uncoded diversity combining for the same modulation schemes and coding gain was determined. It has been shown that the coding gain for coded SC over uncoded SC is 4 dB for N=5 and 4.2 dB for N=6. It is seen that coding gain obtained by using channel coding is excellent. Also, the coding gain for coded and uncoded MRC and OC has been computed. The coding gain for coded MRC over uncoded MRC is 1.6 dB for N=5, 6. We can see from the results that by using channel coding diversity combining techniques, a very good coding gain is obtained.

In conclusion, it can be said that the BER performance of channel coding combined with diversity combining techniques is excellent than the uncoded diversity combining techniques.

FUTURE SCOPE

By using the channel coding with diversity combining technique, the BER performance of wireless communication system is improved. The system which we have implemented is over the independent and identically distributed Rayleigh channel fading. It can be further implement on the correlated Rayleigh fading channel.

In this thesis, we have not considered any interference in information. Only noise is considered. So we can extend this work over the interferers which may be have equal power or not.

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