LONGITUDINAL DISPERSION OF CONSERVATIVE POLLUTANTS IN OPEN CHANNELS

A THESIS

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DOCTOR OF PHILOSOPHY
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By

ZULFEQUAR AHMAD

DEPARTMENT OF CIVIL ENGINEERING
THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY
(Deemed University)
PATIALA–147 001 (INDIA)

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CERTIFICATE

Certified that the work presented in the thesis entitled "**LONGITUDINAL DISPERSION OF CONSERVATIVE POLLUTANTS IN OPEN CHANNELS**" which is being submitted by Mr. Zulfequar Ahmad in fulfilment of the requirement for the award of the Degree of **Doctor of Philosophy** in the Department of Civil Engineering, Thapar Institute of Engineering and Technology (Deemed University), Patiala is an authentic record of candidate's own work carried out during a period from March 1994 to July 1995 at University of Roorkee, Roorkee and from August 1995 to July 1997 at Thapar Institute of Engineering and Technology (Deemed University), Patiala under the supervision of **Dr. R.P. Mathur, Dr. K.G. Ranga Raju** and **Dr. U.C. Kothyari.** The matter presented in the thesis has not been submitted for the award of any other degree in any University.

Kothyari

(Dr. U.C. Kothyari)
Asstt. Professor,
Department of Civil Engg.,
University of Roorkee,
ROORKEE-247 667.

RangaRaju

(Dr. K.G. Ranga Raju)
Professor & Head,
Department of Civil Engg.,
University of Roorkee,
ROORKEE-247 667.

(Dr. R.P. Mathur) 8/7.97
Professor Emeritus,
Department of Civil Engg.,
Thapar Institute of Engg. & Tech.,
(Deemed University)
PATIALA-147 001.

ABSTRACT

Whenever rivers act as sinks for intentional or accidental spillage of effluents resulting from the activities of society, it becomes necessary to know the concentration downstream of its point of injection into the river to check whether the required environmental standards are being satisfied or not. As the injected effluent travels downstream, its concentration decreases due to advection and diffusion (molecular and turbulent) processes and variation of velocity along the depth of flow in the channel. After sufficient time of the pollutant injection, the pollutant is completely mixed laterally, so that there is only a slight variation of pollutant concentration in the lateral direction. After lateral mixing has been accomplished the primary variation of concentration is in one direction and the dispersion is known as **longitudinal dispersion**. The time required from instant of injection to complete lateral mixing is called 'Convective period' or the 'Mixing time', and the distance travelled during this time is called 'Mixing length'. After the mixing length the spreading of the pollutant is primarily due to variation of velocity along the depth of flow, the contribution of the molecular and turbulent diffusion being negligible (Holley, 1969; Fischer *et al.*, 1979). However, molecular and turbulent diffusion alongwith secondary flow are associated with lateral mixing of the pollutant. The longitudinal dispersion equation for a conservative pollutant and steady and non-uniform flow after the mixing length is assumed to be of the form (Cunge *et al.* 1980):

$$\frac{\partial(AC)}{\partial t}+\frac{\partial(AUC)}{\partial x}=\frac{\partial}{\partial x}\left(AD_L\frac{\partial C}{\partial x}\right)$$

where x is the distance downstream, t is time, C and U are the cross-sectional average concentration and velocity respectively, A is the flow area and D_L is the longitudinal dispersion coefficient.

Taylor (1954) was probably the first to study the process of dispersion. He asserted that the phenomenon of dispersion in turbulent flow can be described by one-dimensional Fickian diffusion equation. Elder (1959) extended the theory of Taylor and stated that dispersion in open channels beyond the mixing length may also be described by the one-dimensional diffusion equation.

Several investigators (Sayre, 1965, 1968; Fischer, 1967; Yotsukura and Cobb, 1972; Ward, 1973; McQuivey and Keefer, 1976) studied the variation of the mixing length with flow conditions and proposed relationships for the same. These relationships were mainly derived using experimental data on the mixing process in lateral and vertical directions. In the present study it has been found that the relationships for mixing length produce results of variance from the observed values in many cases.

Analytical and numerical solutions of the longitudinal dispersion equation are available in literature. These solutions were obtained under simplifying assumptions which may generally not hold good in practice. Numerical solutions have been attempted for solving the dispersion equation for real situations; the finite difference method of solution with either Split operator approach or Combined operator approach has been used quite commonly. However, in all these studies the parameters A, U and D_L were considered to be invariant with distance.

In the present study a new finite difference numerical scheme for solution of longitudinal dispersion equation has been proposed. The proof-of-the-concept tests for the proposed scheme have been made with Fischer's (1968) analytical model and Jaque and Ball's (1994) numerical method. The proposed scheme is also applicable in non-uniform flow and for varying dispersion coefficient. The Combined operator scheme has been adopted in the numerical solution used, wherein the exact solution of advection process is obtained by developing a variable spatial grid so that the root of the trajectory of the concentration characteristics passes through the computational nodes. Solution of the diffusion process is achieved using Crank-Nicholson scheme by using a temporal weighting coefficient.

Conventionally, non-uniform flow is treated as an equivalent uniform flow and C-t curves are predicted. But in the present study it has been found that large differences occur in the predicted C-t curves, predicted by treating the flow as non-uniform and approximating it as uniform.

The experiments were conducted in the Hydraulics Laboratory at Civil Engineering Department of the University of Roorkee, Roorkee in a recirculating flume of 0.20 m width and 30.0 m length. First the experiments were carried out in clear-water flows and these were followed by experiments in sediment-laden flows. Rhodamine WT was used as the tracer whose concentration was monitored in the flume using a Turner Fluorometer 10-AU-005 of continuous flow type. Two uniform sands of average diameter 0.064 mm and 0.164 mm were used as sediment. The concentration of sediment in suspension was varied from 500 ppm to 11,000 ppm. These data were analysed alongwith the data available from previous studies on dispersion.

Determination of the value of D_L from the observed temporal variation of concentration (C-t curve) along the river is possible by the change of moment method (Taylor, 1954), the routing procedure and diffusive transport method (Fischer, 1966). However, all of these methods suffer from some limitations. In the present study the proposed numerical scheme has been extended by incorporating in it the one-dimensional grid search method for determination of D_L values using the observed C-t curves at two or more stations.

One needs a predictor for D_L in terms of flow and channel variables to be able to make calculations of C-t curves at different downstream locations given the C-t curve at an upstream station. Several investigators have proposed predictors for D_L. However, computations have revealed that the available predictors for D_L do not produce satisfactory results. In the present study a new predictor has been proposed. The proposed relationship gives better results than the existing methods. Nevertheless, the predicted and the observed D_L values depart from each other by a maximum factor of ten. A sensitivity analysis for D_L is made, which indicates that large errors in D_L cause relatively small errors in the prediction of C-t curves at downstream stations.

Nordin and Sabol (1974), Day (1975), Day and Wood (1976) and Singh *et al.* (1992) found D_L to vary with distance in the downstream direction even for constant flow conditions, although several other studies indicated D_L to remain constant with distance. Therefore, there was a need to study this aspect using a wide range of data from laboratory and field studies. Analysis of field and laboratory data revealed that D_L is constant along the flow distance for uniform flow, while in non-uniform flow D_L varies with distance because of its strong dependence on shear velocity. If shear velocity increases, D_L increases and vice-versa.

Arora (1983), Samaga *et al.* (1986) and Umeyama and Gerritsen (1992) have studied the velocity distribution in open channels carrying suspended sediment load. They found these velocity distributions to deviate from the velocity distributions in flows without sediment. In the present experimental program also, it was observed that variation of velocity over the depth of flow from the mean velocity is more in sediment-laden flow than in corresponding clear-water flows. Dispersion is considered to be significantly affected by the convection process (Holley, 1969). Therefore, the dispersion characteristics of flows transporting suspended sediments may be expected to be different from those of clear-water flows. Singh (1987) has studied the effect of suspended sediment on longitudinal dispersion through a limited series of experiments. In the present study it was found that more dispersion occurs in sediment-laden flows than in the corresponding clear-water flows. Dispersion is found to increase with an increase in sediment concentration in the flow, as the deviation of velocity distribution from the mean velocity is more in sediment-laden flows than in clear-water flows. No effect of sediment size is observed on longitudinal dispersion within the range of experimental data used in the present study. An equation is proposed for prediction of D_L values in sediment-laden flows. Predicted values of D_L from the proposed relationship indicate a maximum error of ±50 percent from the observed D_L values.

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(ZULFEQUAR AHMAD)

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CONTENTS

CHAPTER **Page No.**

LIST OF SYMBOLS

Symbol **Description**

A = Cross-sectional area of flow

a = Proportionality constant

A^* = Dimensionless flow area

$A_1,..,A_4$ = HPI coefficients of interpolating function

$a_1,..,a_4$ = HPI concentration interpolating coefficients

A_c = A constant

AINT = FORTRAN function used for ignoring fractions

a_j, b_j, c_j = Author unknown concentration coefficients

A_u = Integration constant used in Umeyama and Gerritsen's velocity distribution equation

B = Bed width of the channel

b = Transverse size of the roughness element

$b_1,..,b_4$ = HPI concentration derivatives interpolating coefficients

B_c = A constant

BS_i = Bed width of the channel at station, i

C = Cross-sectional average concentration

c = Time-averaged concentration at a point

$C(x,t)$ = Concentration at distance x and time t

C^* = Dimensionless cross-sectional average concentration

C-t = Concentration versus time

$c_1,....,c_6$ = Concentration interpolating coefficients (Six-Point scheme)

C_B = Coefficient in Leopold and Madhock's relation for stream bed width

C_b = Chezy's coefficient for bed roughness of channel

c_b = The reference concentration at $y=b'$

C_D = Coefficient in Leopold and Madhock's relation for depth of flow

C_j^{i+1} = Concentration at grid (j, i+1)

C_n = Weighted average concentration at a time level n

c_p = Peak concentration of tracer at a station

C_P = Peak concentration at the first sampling site

C_r = Courant number

C_s = Cross-sectional average concentration of sediment

CS(j,i) = Concentration at station j at time level i

c_t = Instantaneous concentration at a point

C_U = Coefficient in Leopold and Madhock's relation for flow velocity

C_V = Coefficient of variation

C_w = Chezy's coefficient for side roughness of channel

D = Depth of flow

$d_1,\ldots,d_6$ = Concentration interpolating coefficients (SOWMAC scheme)

d_a = Arithmetic-mean size of sediment mixture

dA = Elemental area

D_C = Critical depth of flow

D_L = Longitudinal dispersion coefficient

D_L^* = Dimensionless longitudinal dispersion coefficient

D_{L0} = Dispersion coefficient at $x=0$

D_{LC} = Dispersion coefficient in clear-water flow

D_{LP} = Optimum dispersion coefficient

D_{LS} = Dispersion coefficient in sediment-laden flow

D_m = Molecular diffusion coefficient

D_n = Normal depth of flow

dx = Side of elemental area, dA in x-direction

D_y = Dispersion coefficient in vertical direction

dy = Side of elemental area , dA in y-direction

D_z = Dispersion coefficient in lateral direction

e = Clear transverse gap between roughness elements

ERS = Error between observed and predicted C-t curves

f = Foot of trajectory on the temporal axis

g = Acceleration due to gravity

h = Height of roughness element

H = Hydraulic depth of flow

h_1 = Spur length

HPI = Holly-Preissmann interpolation

i,j = Grid index point

I_A = Lateral velocity distribution of the depth average velocity

K = Multiplying constant

k = A constant

K_0 = Coefficient for the loss of dye in a reach

K_1 = Coefficient for loss of dye due to decay and adsorption

K_d = Regional dispersion factor

K_s = Equivalent sand roughness

l = Distance from the point of maximum surface velocity to the most distant bank in a channel

L_B = Overall bend length measured along the centre line of a curved channel

L_m = Required mixing length for application of Taylor's model

L_{my} = Longitudinal distance required for complete vertical mixing

L_{mz} = Longitudinal distance required for complete lateral mixing

M = Total number of temporal grid

m = Reach-back time level number

m_B = Exponent in Leopold and Maddock's relation for stream bed width

m_D = Exponent in Leopold and Maddock's relation for depth of flow

m_U = Exponent in Leopold and Maddock's relation for velocity of flow

M_V = Volume of the tracer introduced in the flow

N = Total number of spatial grid

n = Manning's coefficient

NS = Total number of stations

O = Order of error

ppm = Parts per million

P_r = Generalised roughness parameter
Q = Discharge
q = Discharge per unit width
Q_i = Initial discharge at the first sampling site
R = Hydraulic radius
r = Pipe radius
R_b = Hydraulic radius with respect to bed of the channel
r_c = Centre line radius of a channel bend
R_R = Recovery ratio
R_w = Hydraulic radius with respect to sides of the channel
S = Slope of the channel bed
s = Centre to centre longitudinal spacing of elements
S.D. = Standard deviation
S_f = Slope of energy line
S_x = Diffusion parameter
T = Top width of stream
t = Time
t^* = Dimensionless time
$t_1,\ldots,t_5$ = Travelled time of solute cloud
T_{1F} = Elapsed time to the arrival of the 25% peak concentration of the tracer cloud on the following limb
T_{1R} = Elapsed time to the arrival of the 25% peak concentration of the tracer cloud on the rising limb
T_{2F} = Elapsed time to the arrival of the 50% peak concentration of the tracer cloud on the following limb
T_{2R} = Elapsed time to the arrival of the 50% peak concentration of the tracer cloud on the rising limb
T_{3F} = Elapsed time to the arrival of the 75% peak concentration of the tracer cloud on the following limb
T_{3R} = Elapsed time to the arrival of the 75% peak concentration of the tracer cloud on the rising limb
T_B = Elapsed time to the arrival of the leading edge of the tracer cloud
T_E = Elapsed time to the trailing edge of the tracer cloud

t_p = Elapsed time to the peak concentration of the tracer cloud
U = Mean cross-sectional velocity
u = Time-averaged velocity at a point in x-direction
U^* = Dimensionless mean cross-sectional velocity
U_* = Shear velocity
U_{max} = Maximum velocity in cross-section
u_t = Instantaneous velocity at a point in x-direction
v = Time-averaged velocity at a point in y-direction
v_t = Instantaneous velocity at a point in y-direction
w = Time-averaged velocity at a point in z-direction
x = Distance in the longitudinal direction
x^* = Dimensionless longitudinal distance
x_1 = Distance of the first station
x_2 = Distance of the second station
xs_i = Distance of station, i from x=0
y = Distance along vertical direction, Elevation above the bed
YHI = Yang-Hsu interpolation
z = Distance along lateral direction
A'_i = Cross-sectional area of ith sub area
C'_i = Concentration in ith sub area
C''_i = Deviation of concentration from mean concentration of ith sub area
D'_m = Maximum depth in cross-section
$\overline{U}$ = Velocity of centroid of dye
U'_* = Shear velocity due to grain resistance
U_{*c} = Shear velocity with respect to centre line shear
$a'_1,\ldots,a'_4$ = YHI concentration interpolating coefficients
$b'_1,..,b'_4$ = YHI concentration derivatives interpolating coefficients
$\bar{t}$ = Time of arrival of centroid of the tracer cloud at the observation station
$\bar{t}_1$ = Time of arrival of centroid of the tracer cloud at first station
$\bar{t}_2$ = Time of arrival of centroid of the tracer cloud at second station

U'_i = Mean velocity of ith sub area

$\frac{\Delta u}{U_{*c}}$ = Velocity defect

$\Delta \bar{x}_j$ = Average spacing of the grid

a'_j, b'_j, c'_j = Author known concentration coefficients

β' = Ratio of computed D_L value and observed D_L value

b' = The reference depth of flow

u' = Fluctuations from time-averaged velocity in x-direction at a point

v' = Fluctuations from time-averaged velocity in y-direction at a point

c' = Fluctuations from time-averaged concentration at a point

u'' = Deviation of velocity from cross-sectional average velocity at a point in x-direction

c'' = Deviation of concentration from cross-sectional average concentration at a point

$c'_1, \ldots, c'_6$ = Concentration interpolating coefficients (Improved six-point scheme)

$\forall$ Control Volume

ψ = Mass of tracer produced or consumed within the flow per unit volume and per unit time

κ = von Karman constant

τ = Time parameter

η = Dimensionless shape coefficient

ς = Coordinate of an observer moving with mean flow velocity

Δ = Prefix for incremental quantity

α = Interpolating parameter = $1/C_r$-m

θ = Temporal weighting coefficient

ξ = Foot of trajectory on spatial axis

β = Constant for magnitude for sediment concentration

δ = Boundary layer thickness

$\Delta\gamma_s$ = Difference of specific weight of sediment and water

τ_0 = Average bed shear stress

ω_0 = Fall velocity in clear water

θ_1 = Non-dimensional lateral mixing coefficient

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ = Coefficients used in Jaque and Ball scheme

ρ_f = Mass density of water

ρ_s = Density of sediments

σ_t^2 = Temporal variance of a C-t curve

ϕ_x = Rate of transport of mass flux of solute per unit area

ε_x = Turbulent diffusion coefficient along x-direction

ε_y = Turbulent diffusion coefficient along y-direction

σ_y^2 = Variance of the lateral concentration distribution

ε_z = Turbulent diffusion coefficient along z-direction

σ_z^2 = Variance of the vertical concentration distribution

LIST OF FIGURES

LIST OF TABLES

1

INTRODUCTION

1.1 GENERAL

Rivers serve as source of water for various organised uses and also as passive conduit of human wastes. These wastes emanate from diverse sources - industrial and municipal, toxic and/or hazardous and are responsible for widespread pollution. Therefore, knowledge of pollutant mixing and transport in rivers is necessary for determination of both the acceptable limits of effluent input and the concentration along the river course. Fischer *et al.* (1979) list the following pollutants;

- Natural inorganic salts and sediments.
- Waste heat carried by the effluent from once-through cooling systems of thermal power plants.
- Domestic sewage waste.
- Industrial waste containing heavy metals such as lead, mercury and cadmium.
- Industrial wastes having synthetic organic chemicals.
- Radioactive wastes.
- Chemical and biological warfare agents.

A common approach for evaluating the mixing characteristics of a stream is to inject a slug of conservative pollutant across the stream width and then to observe its concentration versus time (C-t) curves at various cross-sections downstream. Even when the pollutants are dumped near one of the river banks, they mix with the flow in lateral and vertical directions for some distance beyond which they are transported longitudinally. A decrease in concentration occurs with distance due to longitudinal dispersion which is a combination of diffusion (molecular and turbulent) and convection (caused by velocity variation along the vertical in the cross-section). It is, however, known that in river channels diffusion is generally insignificant compared to convection as a means of mixing (Holley, 1969; Fischer *et al.*, 1979). Even in the case of injection of the effluent at one of the banks of a stream, mixing of the effluent in vertical and transverse directions will lead to a nearly uniform concentration over the cross-section after some distance, and further dilution is only a consequence of longitudinal dispersion. As such, the knowledge regarding phenomenon of longitudinal dispersion is of importance in effective pollution control and water quality management programmes.

1.2 DISPERSION PROCESS

When a pollutant is injected into a stream, the concentration of the pollutant decreases in the downstream direction. This is a result of diffusion and dispersion of the pollutant. Diffusion is the process which is entirely due to the random motion of the fluid particles. The spreading out is accomplished by different paths randomly followed by different fluid particles. Thus diffusion is associated with molecular action and turbulent action. Dispersion also involves the process of spreading out, but with the help of spatial velocity differences. After a time lapse the pollutant is completely mixed laterally, such that there is only slight variation in concentration in lateral direction. After lateral mixing has taken place the primary variation of concentration is in one direction and the dispersion from that section onwards is known as longitudinal dispersion and is independent of the geometric configuration and type of the source. The process of lateral mixing and longitudinal dispersion due to point injection in an open channel is depicted in Fig. 1.1. The time required for the beginning of longitudinal dispersion is known as 'mixing time' or the 'convective period', the corresponding distance from the source being known as the 'mixing length'. The period beyond mixing time is known as 'dispersive period' or

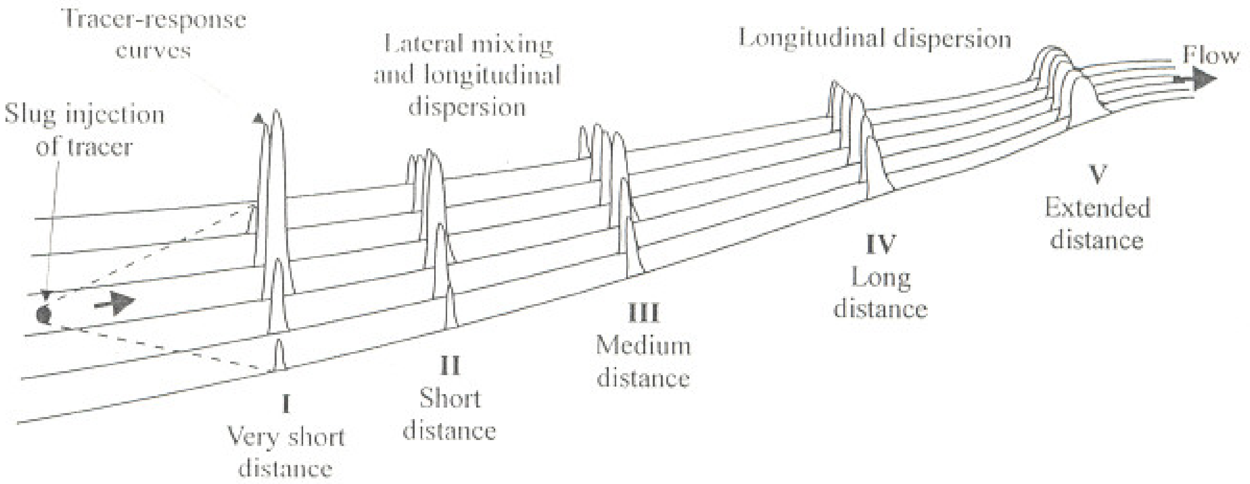


Fig. 1.1 Lateral Mixing and Longitudinal Dispersion Process

'Taylor period' and the process governing spreading of the pollutant in this period is also known as 'one-dimensional longitudinal dispersion'. After the mixing length the spreading of the pollutant is primarily due to spatial velocity distribution, the contributions of the molecular and turbulent diffusion being negligible. However, molecular and turbulent diffusion alongwith the secondary flow are associated with lateral mixing of the pollutant.

For a conservative pollutant and steady and non-uniform flow the process of longitudinal dispersion is described by the following equation (Cunge *et al.*, 1980):

$$\frac{\partial(AC)}{\partial t}+\frac{\partial(AUC)}{\partial x}=\frac{\partial}{\partial x}\left(AD_L\frac{\partial C}{\partial x}\right) \tag{1.1}$$

where C and U are the cross-sectional mean values of the concentration and flow velocity respectively, A is the flow cross-sectional area, x is the distance in flow direction, t is the time and D_L is the longitudinal dispersion coefficient.

Equation (1.1) represents two kinds of transport mechanisms viz. advection and diffusion. These processes are separately explained by the following equations:

$$\text{Advection :}\quad \frac{\partial(AC)}{\partial t}+\frac{\partial(AUC)}{\partial x}=0 \tag{1.2}$$

$$\text{Diffusion} \ : \quad \frac{\partial(AC)}{\partial t} = \frac{\partial}{\partial x}\left(AD_L \frac{\partial C}{\partial x}\right) \tag{1.3}$$

However, the variation of D_L with distance can be neglected particularly in the case of dispersion in uniform flows in prismatic channels. The process could be then described by the one-dimensional Fickian equation (Fischer *et al.*, 1979), which can be written as

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_L\left(\frac{\partial^2 C}{\partial x^2}\right) \tag{1.4}$$

In many field situations the process is found to be not truly Fickian (Beltaos and Day, 1976; Chatwin, 1980; Day and Wood, 1976; Miller and Richardson, 1974). Hence the dispersion process in these cases has been described through the similarity approach (Day and Wood, 1976; Singh, 1987) and storage zone model (Young and Wallis, 1993; Valentine and Wood, 1979). A brief review of these and other studies regarding longitudinal dispersion in open channels is given below.

1.3 BRIEF REVIEW

Taylor (1954) was the first to study the process of dispersion. He asserted that the phenomenon of dispersion in turbulent flows can be described by a one-dimensional Fickian diffusion equation. Elder (1959) extended the theory of Taylor and stated that dispersion in open channels beyond the mixing length may also be described by the one-dimensional diffusion equation.

Several investigators (Fischer, 1967; Sayre, 1965, 1968; McQuivey and Keefer, 1976; Yotsukura and Cobb, 1972; Ward, 1973) studied the variation of the mixing length with flow conditions and proposed relationships for the same. These relationships were mainly derived using experimental data on mixing process as in lateral and vertical directions. It is shown in Chapter-IV that the relationships for the determination of the mixing length produce varying results for a given set of flow parameters.

Analytical solutions of the dispersion equation are available in literature (Sayre, 1968; Fischer, 1968; Bansal, 1971). These solutions were obtained under simplifying assumptions which may generally not hold good in practice. Numerical solutions have been attempted for solving the dispersion equation for real situations; the finite difference method of solution has been used quite commonly. Several investigators solved the dispersion equation for constant values of A, U and D_L using different initial and boundary conditions. Sayre (1968) solved the one-dimensional dispersion equation for the initial condition of an instantaneous plane source distributed uniformly over the cross-section. Fischer (1968) obtained a solution wherein the observed C-t curve on an upstream section of the channel is used as the initial tracer distribution. Bansal (1971) extended the solution given by Sayre (1968) by accounting for the loss of tracer in the downstream direction.

The finite difference numerical solution of Eq. (1.1) is possible through independent solution of advection and diffusion processes. This is termed as the Split operator approach. Investigators such as Holly and Preissmann (1977), Wang *et al.* (1988), Leismann (1989), Li (1990), Li and Yu (1994), Yang *et al.* (1990), Schohl and Holly (1991), Reichert and Wanner (1991), Jalil *et al.* (1994) and Wu and Tsanis (1994) used this approach to solve Eq. (1.1). Also both the advection and the diffusion processes can be handled concurrently and then the solution method is termed as the Combined operator approach. Investigators such as Stone and Brian (1963); Jaque and Ball (1994) and others used this approach to solve Eq. (1.1). However, in all of the above studies the parameters A, U and D_L were considered to be invariant with distance.

Determination of the value of D_L from the observed temporal variation of concentration (C-t curve) along the river is possible by the Change of moment method proposed by Taylor (1954). The Routing Procedure and Diffusive Transport method were proposed as better alternatives by Fischer (1966). However, all these methods suffer from some limitations which are discussed in Chapter-II in detail.

One needs a predictor for D_L in terms of flow and channel variables to be able to make calculations of C-t curves at different downstream locations given the C-t curve at an upstream station. Some predictors are given by Elder (1959), Yotsukura and Fiering (1964), Thackston and Krenkel (1967), Sooky (1969), Sumer (1969), Gumer and West

(1992), McQuivey and Keefer (1974), Jain (1976), Hou and Christensen (1976), Asai *et al.* (1991) and others. However, computations have revealed (details given in Chapter-V) that the available predictors for D_L do not produce satisfactory results.

Singh *et al.* (1992), Nordin and Sabol (1974), Day (1975) and Day and Wood (1976) found D_L to vary with distance in the downstream direction even for constant flow conditions, although several other studies indicated D_L to remain constant with distance. Therefore, there is a need to study this aspect using a wide range of data from laboratory and field studies.

Arora (1983), Samaga *et al.* (1986) and Umeyama and Gerritsen (1992) have studied the velocity distribution in open channels carrying suspended sediment load. They found these velocity distributions to deviate from the velocity distributions in flows without sediment. Dispersion is considered to be significantly affected by the convection process (Holley, 1969). Therefore, the dispersion characteristics of flows transporting suspended sediments may be expected to be different from those of clear-water flows. Singh (1987) has studied the effect of suspended sediment on longitudinal dispersion through a limited series of experiments.

1.4 OBJECTIVES

The present investigation was taken up keeping in view the above gaps in knowledge. The broad objective of the investigation was to conduct a carefully controlled set of experiments to study in detail the dispersion process in clear-water and sediment-laden flows. To cover a wide range of relevant parameters in analysis, laboratory and river data collected elsewhere by previous investigators have also been used. The specific objectives were:

(i) To evolve a numerical model for the solution of the complete dispersion equation i.e. Eq. (1.1).

(ii) To study the variation of D_L with distance in the direction of flow.

(iii) To study the sensitivity of the computed C-t curves at different locations to the variations in D_L.

(iv) To develop a relationship for D_L using a wide range of laboratory and field data.

(v) To study the effect of suspended sediment load on the dispersion process.

1.5 LIMITATIONS

The laboratory experiments for the present investigation are limited to steady, subcritical uniform flows in a rigid rectangular channel. Two uniform sands of average diameter 0.064 mm and 0.164 mm with concentrations ranging from 500 ppm to 11,000 ppm were used as suspended sediment. The tracer used was conservative and neutrally buoyant and was a solution of Rhodamine WT dye prepared in distilled water. The effects of dead zones and river meandering on dispersion characteristics have not been studied due to the lack of availability of these details in the data.

1.6 STRUCTURE OF THE THESIS

For the purpose of lucid presentation, this thesis is divided into six chapters. Chapter-I gives introduction, objectives and limitations of the study. Chapter-II presents the basic theory and literature review of the subject. The literature of review consists of solution of dispersion equation, prediction of D_L, mixing length requirement and Fickian and non-Fickian behavior of the dispersion process. Chapter-III presents the development of a new numerical scheme for solution of longitudinal dispersion equation i.e. Eq. (1.1). The numerical method for determination of D_L is also given in this chapter. Chapter-IV describes the experimental programme and collection of data from previous investigator's work. Chapter-V gives the details of data analysis, new predictors for D_L in both clear-water flow and suspended sediment-laden flow and use of constant and varying values of D_L in computations of C-t curves. Chapter-VI summarises the conclusions of the present study.

2

BASIC THEORY AND REVIEW OF LITERATURE

2.1 PRELIMINARY REMARKS

Mixing process in natural streams is highly complex and the mechanisms involved therein have not been understood completely. However, in a straight channel, the main mechanisms are the shearing action due to velocity gradient and molecular and turbulent diffusions. Because molecular diffusion is negligibly small as compared to the others, its contribution to the total mixing process is ignored. The present chapter deals with the basic theory and a review of the previous studies on longitudinal dispersion in turbulent shear flow in open channels.

2.2 BASIC THEORY

The basic aim of a dispersion study is to predict the transport of a dispersing material called 'tracer' by the flow. This is best comprehended by way of analysis of mass-balance equation of the tracer being transported. The mass-balance is a differential equation, which incorporates all the aspects affecting the transport of the pollutant. The transport of pollutant consists of molecular diffusion, turbulent diffusion and longitudinal dispersion.

To formulate mass-balance equation of a tracer, let us visualise a tracer injected in a two-dimensional open channel flow as shown in Fig. 2.1. The time-average velocity distribution along the depth is shown in the figure. The position of tracer cloud at different time t_1, t_2 and t_3 is shown. As is clear from the figure, as the tracer moves

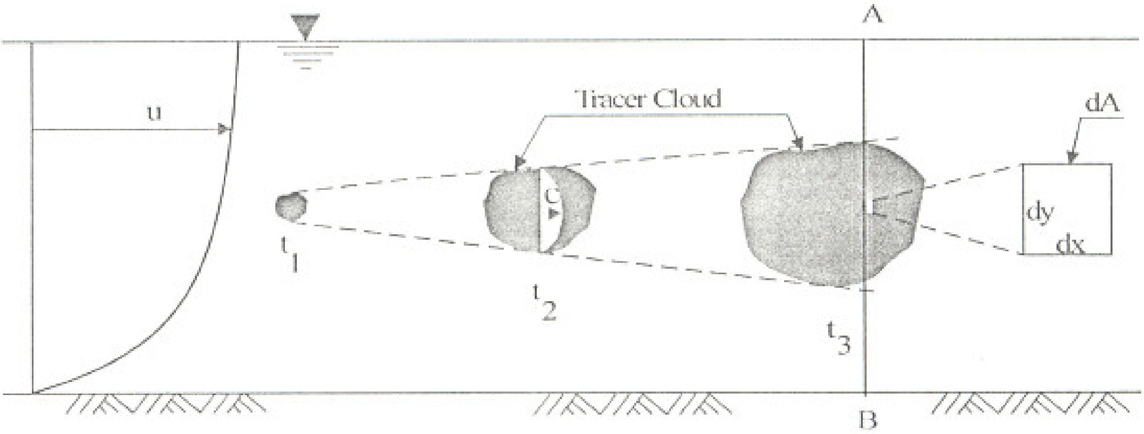


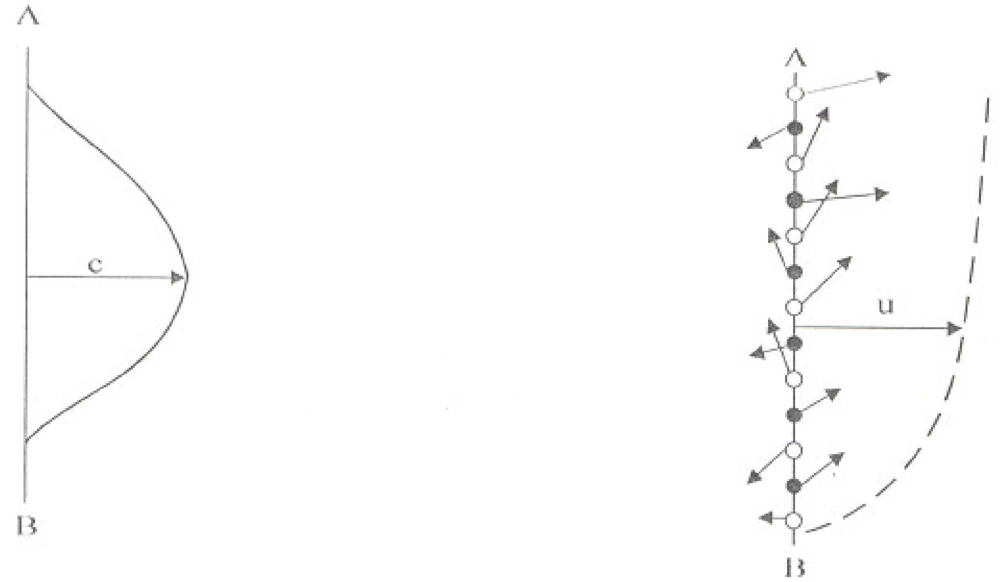
Fig. 2.1 Transport in Two-Dimensional Flow

downstream, it spreads out and the maximum concentration in the cloud thus continuously decreases.

Let us now consider an elemental area, dA with sides dx and dy in x-direction and y-direction respectively. The differential mass-balance equation for mixing process can be obtained by writing the mass-balances of the tracer entering and leaving through the sides of the area. The following consideration centers on what happens to the left side dy of the area dA. Similar phenomena occur on all the four sides of dA.

2.2.1 Molecular Diffusion

Consider the tracer along line AB of Fig. 2.1. The concentration distribution is shown in Fig. 2.2(a). Fig. 2.2(b) shows the velocity distribution along AB, when the fluid is considered as a continuum. Actually at any instant, there are a large number of molecules distributed along the region of the line AB. Either the tracer molecules (shown by filled circle) or the fluid molecules (open circles) move off in all directions with greatly varying velocities, most of which are much greater than u. If we consider the true molecular motion, then to write the mass-balance equation, it is necessary to know the number of molecules passing through dy, the direction and the speed of each molecule. This is not an easy task to find these parameters. Therefore the continuum approach is commonly used.



(a) Concentration (b) Velocity

Fig. 2.2 Transport due to Molecular Diffusion

The rate at which tracer is carried by fluid flow through dy depends on the time-averaged concentration, c and time-averaged fluid velocity, u in x-direction. The difference between the true molecular motion and the manner chosen to represent the motion is accounted for by the terms referred to as molecular diffusion. This representation lead to (Holley, 1969);

$$\frac{\partial c}{\partial t}+\frac{\partial (uc)}{\partial x}=\frac{\partial}{\partial x}\left(D_m\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_m\frac{\partial c}{\partial y}\right) \tag{2.1}$$

as the two-dimensional mass-balance equation. Here D_m is molecular diffusion coefficient. This equation is based on the analogy with Fick's law for molecular diffusion (Fischer *et al.*, 1979), in which it is considered that the turbulent diffusive flux in any direction is proportional to the concentration gradient in that direction (Fischer, 1967). For a one-dimensional process, Fick's law can be expressed as

$$\phi_x=-D_m\frac{\partial c}{\partial x} \tag{2.2}$$

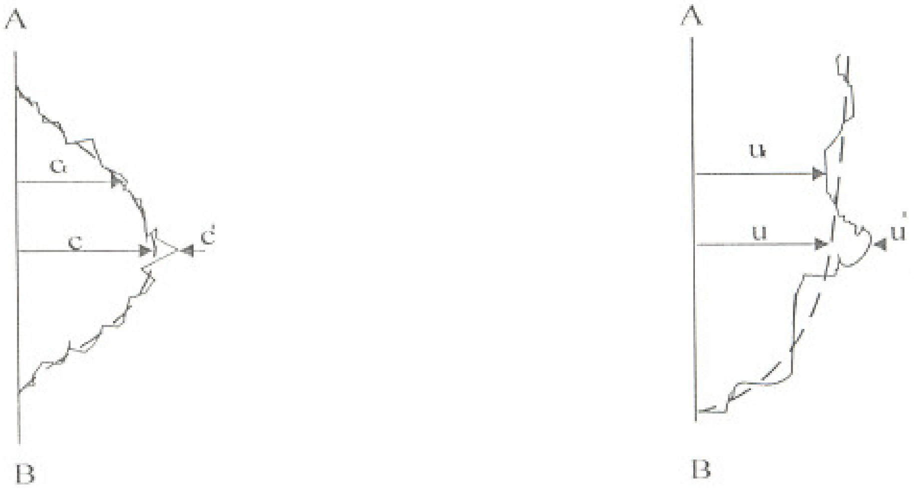
Here ϕ_x is the rate of transport of mass flux of diffusing solute per unit area normal to the direction x. Using principles of conservation of mass, Eq. (2.2) can be written as

$$\frac{\partial c}{\partial t} = -\frac{\partial \phi_x}{\partial x} = \frac{\partial}{\partial x}\left(D_m \frac{\partial c}{\partial x}\right) \tag{2.3}$$

Equation (2.2) is called Fick's first law and Eq. (2.3) is called Fick's second law. The term 'Fickian process' is commonly used for a transport process that is described by Eq. (2.2) or Eq. (2.3).

2.2.2 Turbulent Diffusion

Consider the tracer in a turbulent flow. Figs. 2.3(a) and 2.3(b) show the concentration and velocity distributions along the line AB at time t_3. The u' and c' are fluctuations from the time-average value of velocity and concentration respectively in x-direction.



(a) Concentration

(b) Velocity

Fig. 2.3 Transport due to Turbulent Diffusion

The mass-balance equation for turbulent flow is (Holley, 1969):

$$\frac{\partial (c_1)}{\partial t} + \frac{\partial (u_1 c_1)}{\partial x} + \frac{\partial (v_1 c_1)}{\partial y} = \frac{\partial}{\partial x}\left(D_m \frac{\partial c_1}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_m \frac{\partial c_1}{\partial y}\right) \tag{2.4}$$

Here u_t and v_t are instantaneous velocities at a point in x- and y-direction respectively. c_t is the instantaneous concentration at a point. Even if the primary flow is in x-direction, the turbulent flow exists in y-direction, therefore a term is included in Eq. (2.4) for convection in y-direction. The instantaneous velocities and concentration are given by

$$u_t = u + u'$$

$$v_t = v'$$

$$c_t = c + c' \tag{2.5}$$

Now putting the values of u_t, v_t and c_t from Eq. (2.5) into Eq. (2.4) and integrating Eq. (2.4) with respect to time, gives

$$\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} = \frac{\partial}{\partial x}\left(D_m \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_m \frac{\partial c}{\partial y}\right) + \frac{\partial(-\overline{u'c'})}{\partial x} + \frac{\partial(-\overline{v'c'})}{\partial y} \tag{2.6}$$

Here bars on $u'c'$ and $v'c'$ show time-average values. By analogy to molecular diffusion, turbulent diffusion coefficients, ε_x and ε_y in x- and y-direction respectively may be introduced as

$$\overline{u'c'} = -\varepsilon_x \frac{\partial c}{\partial x} \quad \text{and} \quad \overline{v'c'} = -\varepsilon_y \frac{\partial c}{\partial y} \tag{2.7}$$

Equation (2.6) may now be written as

$$\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} = \frac{\partial}{\partial x}\left((D_m + \varepsilon_x)\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left((D_m + \varepsilon_y)\frac{\partial c}{\partial y}\right) \tag{2.8}$$

Equation (2.8) is the mass-balance equation for turbulent flow in x-direction in terms of the time-averaged quantities and it represents the process called turbulent diffusion. This equation thus includes dispersion due to both molecular and turbulent diffusions.

2.2.3 Longitudinal Dispersion

As the tracer cloud moves downstream, it continues to spread out, as shown in Fig. 2.4, until at time t_4 the cloud completely fills the cross-section. Still later, at time say t_5, the tracer is essentially completely mixed laterally, so that there is only slight variation of

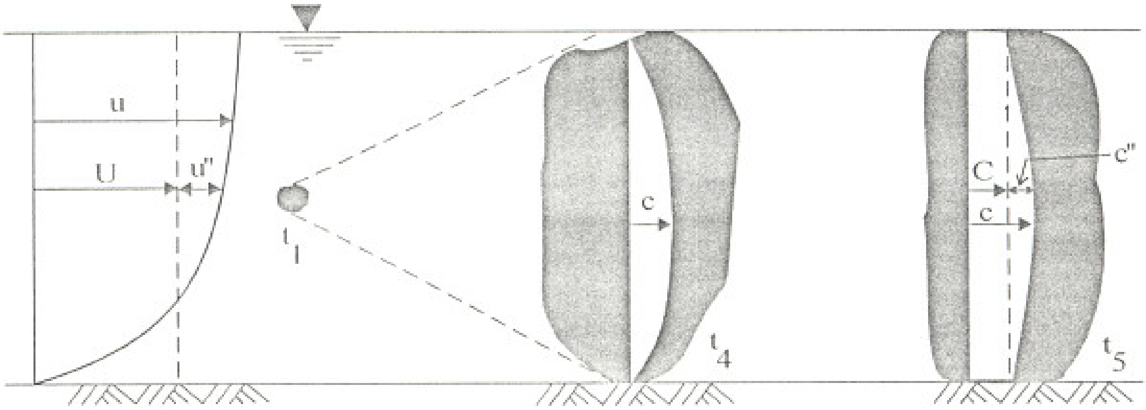


Fig. 2.4 One-Dimensional Transport

concentration in the y-direction. After lateral mixing has taken place, the primary variation of concentration is in just one direction and dispersion beyond this section is known as longitudinal dispersion.

The general mass-balance equation of a tracer in three-dimensions within a control volume is given as below (Sooky, 1969):

$$\frac{\partial c}{\partial t}+\frac{\partial (uc)}{\partial x}+\frac{\partial (vc)}{\partial y}+\frac{\partial (wc)}{\partial z}=\frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial c}{\partial x}+\varepsilon_y\frac{\partial c}{\partial y}+\varepsilon_z\frac{\partial c}{\partial z}\right)+$$

$$\frac{\partial}{\partial x}\left(D_m\frac{\partial c}{\partial x}+D_m\frac{\partial c}{\partial y}+D_m\frac{\partial c}{\partial z}\right)+\Psi \qquad (2.9)$$

Here w is time-average point velocity in z-direction, ε_z are turbulent diffusion coefficient along z-direction and Ψ is the mass of tracer produced or consumed within the flow per unit volume and per unit time. For dispersion of conservative tracer in x-direction Eq. (2.9) gets reduced to Eq. (2.8).

Equation (2.9) cannot be solved analytically or numerically because neither the three-dimensional velocities nor the distribution of mixing coefficients are known. In the case of rivers, however, in which solute transport eventually becomes a one-dimensional process, we do not need to use the three-dimensional form of the equation. The time-average values of velocity and concentration may be expressed as

$$u = U + u''$$

$$c = C + c'' \tag{2.10}$$

where u'' and c'' are deviations of velocity and concentration from cross-sectional average values U and C respectively. Substituting the values of u and c from Eq. (2.10) into Eq. (2.8) and integrating with respect to control volume $\forall = Adx$, gives

$$\frac{\partial(AC)}{\partial t} + \frac{\partial(AUC)}{\partial x} = \frac{\partial}{\partial x}\left(A(D_m + \varepsilon_x)\frac{\partial C}{\partial x} - A\overline{u''c''}\right) \tag{2.11}$$

Taylor (1953,1954) was the first to show that the convection associated with u'' is proportional to the longitudinal gradient of cross-sectional average concentration, so that

$$D_L \frac{\partial C}{\partial x} = (D_m + \varepsilon_x)\frac{\partial C}{\partial x} - \overline{u''c''} \qquad ; c'' << C \tag{2.12}$$

Here both the velocity distribution and the diffusion (molecular and turbulent) contribute to longitudinal dispersion. In Eq. (2.12) D_L is called the dispersion coefficient. Combining Eqs. (2.11) and (2.12) one can get (Orlob, 1983)

$$\frac{\partial(AC)}{\partial t} + \frac{\partial(AUC)}{\partial x} = \frac{\partial}{\partial x}\left(AD_L \frac{\partial C}{\partial x}\right) \tag{1.1}$$

This is a one-dimensional dispersion equatión which describes the time-dependent dispersion of a neutrally buoyant, conservative tracer substance in an open channel subject to steady but non-uniform flow. This equation is also valid for cases where deviation of concentration from the cross-sectional average concentration is much less than the cross-sectional average concentration. The value of D_L in Eq. (1.1) needs to reflect all the processes which affect the longitudinal spreading or dispersion of a solute cloud. It is considered that the dominant mechanism producing longitudinal dispersion is differential advection caused by the cross-sectional variation of the local longitudinal velocity. In uniform flow, the molecular and turbulent diffusion contribute only about 1% or less of the total dispersion in turbulent flow (Holley, 1969).

2.2.4 Taylor's One-Dimensional Model of Longitudinal Dispersion

Taylor (1921, 1953 and 1954) made the following assumptions for the dispersion process (Chatwin, 1980; Chatwin and Allen, 1985):

(i) The tracer (solute) cloud has been evolving for a sufficiently long time.
(ii) The turbulence is statistically stationary in time.
(iii) The flow cross-section is constant.
(iv) The solute is conservative.

Under these conditions, Eq. (1.1) can be simplified to the usual form known as the Advection Dispersion Equation (ADE) or Fickian equation, viz.

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_L\frac{\partial^2 C}{\partial x^2} \qquad (1.4)$$

The solution of Eq. (1.4) for an impulsive input of a finite mass tracer, with suitable choice for the origins of x and t (Chatwin, 1970 and 1971; Holley and Tsai, 1977; Sayre, 1968) is given by:

$$C(x,t) = \frac{M_v}{2A\left(\pi D_L t\right)^{\frac{1}{2}}}\exp\left[\frac{-(x-Ut)^2}{4D_L t}\right] \qquad (2.13)$$

Here M_v is the volume of tracer introduced in the flow. On the basis of theoretical consideration for turbulent flow in pipe, Taylor proposed a relationship for dispersion coefficient as $D_L = 10.1rU_*$. Here r = radius of pipe and U_* = shear velocity. By performing experiments in smooth and very rough pipes, he verified his theoretical predictions.

It is obvious that Eq. (2.13) represents a Gaussian distribution in time. However, the measured concentration profiles are not always Gaussian. This implies deviation of the dispersion process from the Fickian theory. A detailed discussion on this aspect is presented later in this chapter.

2.2.5 Mixing Length Requirement for the Beginning of Fickian Dispersion Process

Fischer (1967) has shown that Taylor's one-dimensional diffusion model viz. Eq. (1.4) is a realistic description of a dispersing cloud of tracer particles only after the variance of the observed concentration versus time curves, σ_t^2, increases linearly with the longitudinal distance x. The time required for this condition to be attained is called the convective period or the mixing time and the corresponding distance from the source is called the mixing length. Taylor's one-dimensional model i.e. Fickian dispersion process is not applicable within this length. Taylor period is the time during which the transport of the solute is described by a one-dimensional dispersion equation. Fischer (1967) performed experiments in the laboratory to observe the convective time. To identify in each experiment the beginning of the Taylor period, three tests were applied: (a) The concentration distribution over any cross-section should approach the steady-state profile; (b) The growth rate of the variance should become linear; and (c) The concentration distribution should decay according to the dispersion equation (i.e. Eq. 1.1). He proposed a relationship for the mixing length. The relationships for mixing length proposed by Fischer and several other investigators are summarised in Table 2.1.

Fischer's mixing length relationship is simple and gives a higher value of the mixing length than the other equations. In terms of width, Fischer's relationship predicts a requirement of about 20 times the top width for a rough stream and 200 times the top width for a smooth stream. In Sayre (1968) relationship for mixing length the lower limit corresponds to the condition of no lateral velocity gradient and is of the order of 100 times the depth of flow while the upper limit corresponds to the condition where there is lateral velocity gradient and can easily exceed 50,000 times the depth of flow. On the basis of dispersion experiments in small mountainous streams, Day (1975), observed that the complete cross-sectional mixing was attained at distance 24 times the width of flow downstream of the point of injection of the tracer.

Table 2.1 : Relationships for Mixing Length

S. No.	Investigator	Mixing length	Remarks
1	Fischer, 1967	$L_m = 1.8 \frac{l^2}{R} \frac{U}{U_*}$	On the basis of analytical and experimental results.
2	Sayre, 1965	$L_m = \max\left(L_{mz}, L_{my}\right)$ $L_{mz} = \frac{U}{2\alpha_z^2} \frac{B^2}{D_z}$ $L_{my} = \frac{2U}{\alpha_y^2} \frac{D^2}{D_y}$	Considering mixing in lateral and vertical direction.
3	Kilpatrick *et al.*, 1967	$L_m = 0.4 \frac{U}{U_*} \frac{B^2}{D}$	On the basis of dispersion theory.
4	Sayre, 1968	$7.5 \frac{U}{U_*} \le \frac{L_m}{D}$ $\frac{L_m}{D} \le 2.5 \frac{U}{U_*} \left(\frac{B}{D}\right)^2$	For flow having lateral velocity gradients
5	McQuivey and Keefer, 1976	$L_m = 0.3 \frac{l^2}{R} \frac{U}{U_*}$	Based on a quasi two-dimensional solution of the convective equation and laboratory and field data.
6	Yotsukura and Cobb*, 1972	$L_m = 0.22 \frac{T^2}{D} \frac{U}{U_*}$	99% mixing of dosage, central injection.
7	Ward, 1973	$L_m = 0.39 \frac{T^2}{D} \frac{U}{U_*}$	99% mixing of $\int_{-\infty}^{\infty} C dx$, central injection.

* for shape factor = 1.55, the shape factor being defined by Yotsukura and Cobb as $\frac{1}{TU^2D^3}\int_0^T u^2 D^3 dz$. Here L_m = mixing length, l = distance from the point of maximum surface velocity to the most distant bank, U_* = shear velocity, R = hydraulic radius, $\alpha_z = \frac{B}{\sigma_z}$ and $\alpha_y = \frac{2D}{\sigma_y}$ where L_{mz} and L_{my} are the longitudinal distances for the complete mixing in lateral and vertical direction respectively, B = width of the channel, D = depth of flow, σ_y and σ_z are the standard deviations of the lateral and vertical concentration distributions respectively, D_z and D_y are dispersion coefficients in lateral and vertical directions respectively and T = Top width of the channel.

2.3 SOLUTION OF LONGITUDINAL DISPERSION EQUATION

Equation (1.1) describes the time-dependent dispersion of a neutrally-buoyant, conservative tracer substance in a natural channel subject to steady but non-uniform flow. To solve longitudinal dispersion problems, it is necessary to solve Eq. (1.1) either analytically or numerically. Analytical solution of the equation is difficult because of variable coefficients (McQuivey and Keefer, 1976). Therefore analytical solutions to the equation exist only for some particular simplified flows and tracer injection configuration, but for most practical situations only numerical solution methods are used.

2.3.1 Analytical Solutions

The analytical solutions of Eq. (1.1) are generally of little practical use, for the reason that A, U and D_L are never constant over any appreciable reach of a natural river. Some solutions of the simplified form of the Eq. (1.1) are, nevertheless, described in the following section.

2.3.1.1 Fischer's method

Fischer (1968) obtained a solution of Eq. (1.1) for constant values of A, U and D_L. The observed C-t curve on an upstream section of the channel is used as initial tracer distribution. The solution given is

$$C(x_2,t) = \int_{-\infty}^{\infty} C(x_1,\tau) \frac{\exp\left[\dfrac{-\{U(\bar{t}_2 - \bar{t}_1 - t + \tau)\}^2}{4D_L(\bar{t}_2 - \bar{t}_1)}\right]}{\sqrt{4\pi D_L(\bar{t}_2 - \bar{t}_1)}} U d\tau \qquad (2.14)$$

Here $C(x_2,t)$ is the predicted value of concentration at station x_2 at time t, $C(x_1,\tau)$ is the value of concentration at station x_1 at time τ, and $\bar{t}_1$ and $\bar{t}_2$ are the respective times of passage of the tracer cloud past the two stations calculated assuming that the cross-sectional mean velocity is equal to the velocity of the tracer cloud. Thus if the C-t curve, flow parameters and D_L value are known at one station, one can find the C-t curve at the second station.

2.3.1.2 Bansal's method

Bansal (1971) observed that the loss in concentration of a non-conservative solute increases downstream as the distance increases. Using the appropriate initial and boundary conditions he gave a modified solution for Eq. (1.1) for constant values of A, U and D_L as

$$C(x,t) = \frac{1}{K_d} \frac{M_v}{A\sqrt{4\pi D_L t}} \exp\left[-\frac{(x-Ut)^2}{4D_L t}\right]\left(1-K_0 \frac{x}{Ut}\right) \tag{2.15}$$

where K_0 = coefficient for the loss of dye in a reach and is given by

$$K_0 = \frac{1}{4}\left[\frac{UT}{4D_L} + \frac{\log\left(2\frac{H}{L_m}\right)}{\log\left(D_L \frac{t_p}{L_m^2}\right)} + K_1\right]$$

Here K_d = regional dispersion factor defined for a stream, K_1 = a coefficient defined for a given reach for the loss of dye due to decay and adsorption - assumed to be 0.5, a constant for all streams studied by him, t_p = time to peak arrival of concentration and H = hydraulic depth. Prakash (1977), Nokes *et al.* (1984) and Alavian (1986) have also proposed analytical solutions of the dispersion equation under simplified assumptions.

2.3.2 Numerical Solutions

The finite difference numerical method has been commonly used in dispersion studies. Eq. (1.1) is a linear parabolic partial differential equation. For numerical solution, it would require use of one upstream and one downstream boundary condition as well as the condition corresponding to the initial state. Two approaches namely the Split operator approach and the Combined operator approach which are used for the solution are briefly discussed below.

2.3.2.1 Split operator approach

In the Split operator approach the advection and the diffusion components of the dispersion process are computed independently for each time step. It has the advantage in allowing use of accurate (but different) methods for each process. Computation of the diffusion components can be performed accurately using a variety of finite difference and finite element methods (Morton, 1981). However, it is rather difficult to attain sufficient accuracy in numerical computation of advection (Li, 1990; Schohl and Holly, 1991). Most finite difference methods for the calculation of the advection portion of Eq. (1.1) are plagued by an artificial or numerical diffusion, which is sometimes stronger than the physical diffusion rendering the calculation useless.

(A) Characteristics-Based Computation of Advection

The backward characteristics method with interpolating approximation of the exact solution is widely used in the solution of the pure advection equation. The choice of the interpolation function affects the accuracy of the solution significantly. The one-dimensional advection equation for constant A and U is written as

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = 0 \tag{2.16}$$

The above equation may be written in terms of total derivatives as

$$\frac{dC}{dt} = 0 \quad \text{along} \quad \frac{dx}{dt} = U \tag{2.17}$$

These ordinary differential equations simply state that the concentration must be constant along the space-time trajectory defined by U(x,t). Implementation of Eq. (2.17) in a mathematical model on the fixed Eulerian space-time grid of Fig. 2.5 implies that if the concentrations are known at all grid points at time level t_i, the unknown concentration at any at time level t_{i+1} is obtained from the solution of Eq. (2.17) as

$$C_j^{i+1} = C_\xi^i \tag{2.18}$$

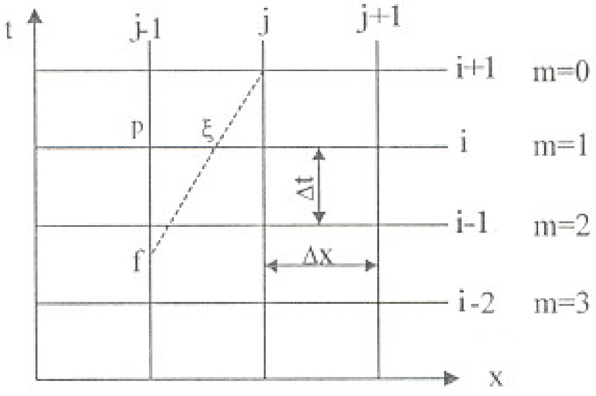


Fig. 2.5 Finite Difference Grid

Here superscript i denotes time level, subscript j denotes a spatial point x_j, and ξ denotes the coordinate of the foot of the trajectory along spatial axis leading to the point $(x_j\ ,\ t_{i+1})$. The problem of finding the concentration at point x_j at time t_{i+1} reduces to the problem of knowing the concentration at ξ at time t_i. For that reason the accuracy of the scheme is dependent on how accurately one estimates C^i_ξ. The following schemes have been used for the determination of C^i_ξ.

(i) Upwind finite difference scheme

Here C^i_ξ is calculated by linear interpolation using two nodes as below:

$$C^i_\xi = C^i_{j-1} + \frac{C^i_j - C^i_{j-1}}{x_j - x_{j-1}}(x_\xi - x_{j-1}) \tag{2.19}$$

Such linear interpolation introduces large dissipation in the solution (Li, 1990).

(ii) Holly-Preissmann two-point fourth order scheme

Holly and Preissmann (1977) suggested two-point fourth order scheme to determine C^i_ξ. This scheme uses a third-degree interpolating polynomial of the form

$$C^i_\xi = A_1 + A_2 x_\xi + A_3 x^2_\xi + A_4 x^3_\xi \tag{2.20}$$

in which the coefficients A_1, A_2, A_3 and A_4 can be determined using two computational grid points bracketing ξ such that the following four conditions are satisfied;

$$\text{At} \quad x_\xi = x_{j-1} \quad C_\xi^i = C_{j-1}^i \,, \qquad \left.\frac{\partial C}{\partial x}\right|_\xi^i = \left.\frac{\partial C}{\partial x}\right|_{j-1}^i$$

$$\text{and} \qquad \text{at} \quad x_\xi = x_j \quad C_\xi^i = C_j^i \,, \qquad \left.\frac{\partial C}{\partial x}\right|_\xi^i = \left.\frac{\partial C}{\partial x}\right|_j^i \tag{2.21}$$

Performing the algebra necessary to evaluate A_1, A_2, A_3 and A_4 and then substituting them into Eq. (2.20) and simplifying one can obtain

$$C_\xi^i = a_1 C_{j-1}^i + a_2 C_j^i + a_3 \left.\frac{\partial C}{\partial x}\right|_{j-1}^i + a_4 \left.\frac{\partial C}{\partial x}\right|_j^i \tag{2.22}$$

in which

$$a_1 = C_r^{\,2}(3 - 2C_r) \quad ; \quad a_2 = 1 - a_1 \quad ; \quad a_3 = C_r^{\,2}(1 - C_r)(x_j - x_{j-1})$$

$$\text{and} \qquad a_4 = -C_r(1 - C_r)^2 (x_j - x_{j-1}) \tag{2.23}$$

where $C_r = \dfrac{(x_j - x_\xi)}{(x_j - x_{j-1})}$ is Courant number.

Equation (2.22) requires initial condition and upstream boundary condition as well as spatial concentration derivatives at t_i to determine concentration at next time step, t_{i+1}. But to continue the computation, the concentration derivatives must be known at time step t_{i+1}. Derivatives for time t_{i+1} are obtained in exactly the same manner as the advection of the concentration itself as, i.e.

$$\left.\frac{\partial C}{\partial x}\right|_\xi^i = b_1 C_{j-1}^i + b_2 C_j^i + b_3 \left.\frac{\partial C}{\partial x}\right|_{j-1}^i + b_4 \left.\frac{\partial C}{\partial x}\right|_j^i \tag{2.24}$$

in which

$$b_1 = 6C_r(C_r - 1)(x_j - x_{j-1})^{-1} \quad ; \quad b_2 = -b_1 \quad ; \quad b_3 = C_r(3C_r - 2)$$

$$\text{and} \qquad b_4 = (C_r - 1)(3C_r - 1) \tag{2.25}$$

By analogy with Eq. (2.18) written for the derivative set

$$\left.\frac{\partial C}{\partial x}\right|_{j}^{i+1} = \left.\frac{\partial C}{\partial x}\right|_{\xi}^{i} \tag{2.26}$$

This scheme achieves excellent accuracy (Schohl and Holly, 1991), but requires solution of auxiliary advection equation, viz. Eq. (2.24), for the first derivative of the concentration field. Even at the initial state the upstream boundary spatial derivative of concentrations is not known which is required to initiate the scheme.

(iii) Yang-Hsu time-line interpolation scheme

In Holly-Preissmann (1977) scheme the trajectory of the characteristics line has been written as a function of a dependent variable and its spatial derivative for two points on the spatial grid axis. Yang and Hsu (1990) used dependent variables for points on the time axis and its temporal derivatives instead of dependent variables for points on the spatial grid axis and its spatial derivatives. This means that the trajectory of the characteristics line should fall on the time axis at f (see Fig. 2.5) instead of on the spatial axis at ξ. Hence

$$C_j^{i+1} = C_{j-1}^{f} \tag{2.27}$$

Using the third-degree interpolating polynomial of Holly-Preissmann (1977), Yang-Hsu expressed C_{j-1}^{f} as follows:

$$C_{j-1}^{f} = a'_1\, C_{j-1}^{i-2} + a'_2 \left.\frac{\partial C}{\partial t}\right|_{j-1}^{i-2} + a'_3\, C_{j-1}^{i-1} + a'_4 \left.\frac{\partial C}{\partial t}\right|_{j-1}^{i-1} \tag{2.28}$$

in which

$$a'_1 = \alpha^2(3-2\alpha)\ ;\ a'_2 = \alpha^2(1-\alpha)\Delta t\ ;\ \ a'_3 = 1 - a_1 \ \text{ and } \ a'_4 = -\alpha(1-\alpha)^2\Delta t \tag{2.29}$$

where α = interpolating parameter $= \dfrac{1}{C_r} - m$, m is the reach-back time level number (see Fig. 2.5).

The temporal derivative of the concentration may also be obtained in the same way the spatial derivative of the concentration has been obtained . One can obtain

$$\left.\frac{\partial C}{\partial t}\right|_{j-1}^{i+1} = \left.\frac{\partial C}{\partial t}\right|_{j-1}^{r} = b'_1\, C_{j-1}^{i-2} + b'_2 \left.\frac{\partial C}{\partial t}\right|_{j-1}^{i-2} + b'_3\, C_{j-1}^{i-1} + b'_4 \left.\frac{\partial C}{\partial t}\right|_{j-1}^{i-1} \qquad (2.30)$$

in which

$$b'_1 = 6\alpha(\alpha - 1) / \Delta t \quad ; \quad b'_2 = \alpha(3\alpha - 2) \quad ; \quad b'_3 = -b'_1 \quad \text{and} \quad b'_4 = (\alpha - 1)(3\alpha - 1) \qquad (2.31)$$

Equations (2.28) and (2.30) complete the calculation of pure advection of the concentration and its derivative for one time step.

(iv) The six-point scheme

Komatsu *et al.* (1985) proposed a six-point scheme to calculate the value of C_ξ^i with high accuracy. Using values of concentration at six known points around ξ at time t_i the following algebraic expression is obtained for C_ξ^i.

$$C_\xi^i = c_1 C_{j-3}^i + c_2 C_{j-2}^i + c_3 C_{j-1}^i + c_4 C_j^i + c_5 C_{j+1}^i + c_6 C_{j+2}^i \qquad (2.32)$$

where

$$c_1 = -0.01806 C_r^{\,3} - 0.03828 C_r^{\,2} + 0.05633 C_r$$

$$c_2 = 0.2570 C_r^{\,3} + 0.05276 C_r^{\,2} - 0.3097 C_r$$

$$c_3 = -0.6806 C_r^{\,3} + 0.6480 C_r^{\,2} + 1.033 C_r$$

$$c_4 = 0.6806 C_r^{\,3} - 1.394 C_r^{\,2} + 0.2869 C_r + 1$$

$$c_5 = -0.2570 C_r^{\,3} + 0.8236 C_r^{\,2} - 0.5667 C_r$$

$$c_6 = 0.01806 C_r^{\,3} - 0.09245 C_r^{\,2} + 0.07439 C_r$$

(v) Improved six-point scheme

When applied to two- or three-dimensional advection-diffusion equation, the six-point scheme has proved to be less accurate than for the one-dimensional case. This is due to increased opportunity of accumulation of truncation error because of repeated use of the interpolating polynomial. To improve the results, an improved six-point scheme

(Komatsu *et al.*, 1989) was developed in which an artificial correction term was added to the results.

The improved six-point scheme is as follows:

$$C_\xi^i = c_1' C_{j-3}^i + c_2' C_{j-2}^i + c_3' C_{j-1}^i + c_4' C_j^i + c_5' C_{j+1}^i + c_6' C_{j+2}^i \tag{2.33}$$

where

$$c_1' = c_1 + \frac{0.275 C_r}{12} D_1$$

$$c_2' = c_2 - \frac{C_r \lambda S_1}{2} D_2 - \frac{(-1 + 3C_r) \times 0.275}{12} D_1$$

$$c_3' = c_3 - \frac{(1 - 3C_r) \lambda S_1}{2} D_2 - \frac{(1 - C_r) \times 0.275}{6} D_1$$

$$c_4' = c_4 - \frac{(-2 + 3C_r) \lambda S_1}{2} D_2 + \frac{C_r \times 0.275}{6} D_1$$

$$c_5' = c_5 - \frac{(1 - C_r) \lambda S_1}{2} D_2 - \frac{(-2 + 3C_r) \times 0.275}{12} D_1$$

$$c_6' = c_6 - \frac{(1 - C_r) \times 0.275}{12} D_1$$

$$\lambda = \begin{cases} \dfrac{\omega}{|\omega|} & (\omega \neq 0) \\ 0 & (\omega = 0) \end{cases}$$

$$\omega = -\frac{\partial C}{\partial x} \frac{\partial^2 C}{\partial x^2} \frac{\partial^4 C}{\partial x^4} \frac{\partial^5 C}{\partial x^5}, \quad \frac{\partial C}{\partial x} = \frac{C_j - C_{j-1}}{\Delta x}, \quad \frac{\partial^2 C}{\partial x^2} = \frac{C_{j+1} - C_j - C_{j-1} + C_{j-2}}{2(\Delta x)^2}$$

$$\frac{\partial^4 C}{\partial x^4} = \frac{C_{j+2} - 3C_{j+1} + 2C_j + 2C_{j-1} - 3C_{j-2} + C_{j-3}}{2(\Delta x)^4}$$

$$\frac{\partial^5 C}{\partial x^5} = \frac{C_{j+2} - 5C_{j+1} + 10C_j - 10C_{j-1} + 5C_{j-2} - C_{j-3}}{(\Delta x)^5}$$

$$D_1 = 4.74 \times 10^{-2} \left(C_r^2 - C_r \right)$$

$$D_2 = -3.16 \times 10^{-2} \left(C_r^2 - C_r \right)$$

$$S_1 = \frac{23.6}{(i+18.7)^{.758}}$$

c_1 .....c_6 are the coefficients which are equivalent to those used in the six-point scheme.

(vi) SOWMAC scheme

Taking into account the highly accurate and stable features of the finite difference of second order derivative, a refined implicit scheme namely SOWMAC for solution of advection process was developed by Komatsu *et al.* (1992), based on the concept of solving second order wave equation instead of first order advection equation. SOWMAC scheme is finally given as follows;

$$d_1C_{j-1}^{i+1} + d_2C_j^{i+1} + d_3C_{j+1}^{i+1} = d_4C_{j-1}^{i} + d_5C_j^{i} + d_6C_{j+1}^{i} \tag{2.34}$$

where

$$d_1 = 0.3776C_{r0+} + 0.3152C_{r0-} - 0.5467C_{r1+} + 0.4843C_{r1-} + 0.1691C_r^{\,2}$$

$$d_2 = 1.3072 + 0.0624\left|C_r\right| - 0.3382C_r^{\,2}$$

$$d_3 = 0.3152C_{r0+} + 0.3776C_{r0-} + 0.4843C_{r1+} - 0.5467C_{r1-} + 0.1691C_r^{\,2}$$

$$d_4 = 0.3776C_{r0+} + 0.3152C_{r0-} + 0.5157C_{r1+} - 0.4533C_{r1-} + 0.1381C_r^{\,2}$$

$$d_5 = 1.3072 + 0.0624\left|C_r\right| - 0.2762C_r^{\,2}$$

$$d_6 = 0.3152C_{r0+} + 0.3776C_{r0-} - 0.4533C_{r1+} + 0.5157C_{r1-} + 0.1381C_r^{\,2}$$

$$C_{r0+} = \mathrm{AINT}\left\{\frac{C_r + 1}{1 + \left|C_r\right|}\right\}$$

$$C_{r0-} = \mathrm{AINT}\left\{\frac{1 - C_r}{1 + \left|C_r\right|}\right\}$$

$$C_{r1+} = \frac{\left|C_r\right| + C_r}{2}$$

$$C_{r1-} = \left|\frac{C_r - \left|C_r\right|}{2}\right|$$

In the above, AINT is an incorporated function used in FORTRAN for ignoring fractions. The above scheme uses only three computational grid points, so there is no need for special treatment of the boundary condition.

The six-point scheme, the improved six-point scheme and SOWMAC scheme retain almost the same accuracies as the Holly and Preissmann scheme. Besides these other schemes which are also available for determination of C_{ξ}^{i} include those proposed by Komatsu *et al.* (1992), Leonard (1979), Jalil *et al.* (1994), Li (1990) and Schohl and Holly (1991).

(B) Computations of Diffusion

Morton (1981) stated that the computation for diffusion component can be accurately executed using a variety of finite difference schemes. Holly and Preissmann (1977) solved the diffusion part using the cubic interpolation formula which is similar to those used for the advection part. Yang and Hsu (1990) used centered finite difference scheme and Toda (1986) used the Crank-Nicholson method which give the best solution.

2.3.2.2 Combined operator approach

In the Combined operator approach the advection and diffusion processes are treated simultaneously. This means that the computed pollutant concentration at the next time step (i+1) accounts for the contribution due to both the advection and diffusion processes. Stone and Brian (1963) solved the longitudinal dispersion equation using the Combined operator approach by assigning weighting coefficients in both the temporal and the spatial derivatives. The use of weighting coefficients in both the temporal and spatial derivatives resulted in a complicated algorithm to be used especially in the determination of the coefficient values. Jaque and Ball (1994) method in which the weighting coefficient was applied only in the temporal derivatives produced a simplified algorithm. Their method is given below in detail.

(i) Jaque and Ball Method

The solution technique of Jaque and Ball (1994) is based on the use of a finite difference approximation as shown by the spatial-temporal grid illustrated in Fig. 2.5. Using such a grid they obtained a finite difference approximation to Eq. (1.1) for constant U, A and D_L as

$$\begin{aligned}&\frac{C_n^{i+1}-C_n^i}{\Delta t}+\frac{U}{2\Delta x}\left[\theta\left(C_{j+1}^{i+1}-C_{j-1}^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-C_{j-1}^{i}\right)\right]= \\ &\frac{D_L}{\Delta x^2}\left[\theta\left(C_{j+1}^{i+1}+C_{j-1}^{i+1}-2C_j^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-C_{j-1}^{i}-2C_j^{i}\right)\right]\end{aligned} \tag{2.35}$$

where θ = temporal weighting coefficient and C_n = the weighted average concentration at a time level which is given by

$$C_n=\varepsilon_1 C_{j-1}+\varepsilon_2 C_j+\varepsilon_3 C_{j+1} \tag{2.36}$$

subjected to the constraint that

$$\varepsilon_1+\varepsilon_2+\varepsilon_3=1 \tag{2.37}$$

$\varepsilon_1, \varepsilon_2$ and ε_3 are arbitrary weighting coefficients. They are obtained by truncation error analysis for the best accuracy as

$$\varepsilon_1=\varepsilon_3=\frac{1}{6}-S_x(1-\theta)+\theta\left(S_x+\frac{C_r^2}{2}\right)-\frac{C_r}{6} \tag{2.38}$$

ε_2 is obtained by Eq. (2.37) and θ can be taken as 0.5 and C_r = the Courant number which is defined by

$$C_r=\frac{U\Delta t}{\Delta x} \tag{2.39}$$

and S_x = diffusion parameter which is defined by

$$S_x=\frac{D_L\Delta t}{\Delta x^2} \tag{2.40}$$

Substituting Eq. (2.36) into Eq. (2.35) results, after rearrangement of the terms as

$$\begin{aligned}&\left(\varepsilon_1-\frac{\theta C_r}{2}-\theta S_x\right)C_{j-1}^{i+1}+\left(\varepsilon_2+2S_x\theta\right)C_j^{i+1}+\left(\varepsilon_3+\frac{\theta C_r}{2}-\theta S_x\right)C_{j+1}^{i+1}=\\&\left(\varepsilon_1+\frac{C_r}{2}-\frac{\theta C_r}{2}+S_x-\theta S_x\right)C_{j-1}^{i}+\left(\varepsilon_2-2S_x+2S_x\theta\right)C_j^{i}+\left(\varepsilon_3-\frac{C_r}{2}+\frac{\theta C_r}{2}+S_x-\theta S_x\right)C_{j+1}^{i}\end{aligned}\tag{2.41}$$

Application of Eq. (2.41) along a spatial domain results in a set of finite difference equations which are banded about the central diagonal. The bandwidth of this set of equations is three and these are solved by using simple tridiagonal matrix manipulation techniques. Eq. (2.41) requires to use for its solution the upstream boundary condition, the downstream boundary condition and the initial condition.

It is seen from the above that to use Eq. (1.1) for prediction of concentration profiles, estimate of the D_L value is needed first. Methods available for determination of D_L are reviewed below.

2.4 DETERMINATION OF DISPERSION COEFFICIENT USING CONCENTRATION CURVES

Dispersion coefficient represents the dispersive characteristics of a stream. The dispersion coefficient for longitudinal dispersion can be obtained by using curves of the temporal variation of concentration (C-t curves) at two or more stations downstream of the injection point. The following methods are available for determination of D_L using C-t curves and other properties of the stream.

2.4.1 Change of Moment Method

This method is based on the solution of Taylor's one-dimensional dispersion equation. The method does not make any assumption regarding the initial concentration distribution of the tracer (Fischer *et al.*, 1979). For the one-dimensional dispersion method, the dispersion coefficient is proportional to the rate of change of variance of the C-t curves, i.e.

$$D_L=\frac{\overline{U}^3}{2}\frac{d\sigma_t^2}{dx}=\frac{\overline{U}^2}{2}\frac{d\sigma_t^2}{dt}\tag{2.42}$$

where $\overline{U}$ is the velocity of centroid of dye. The variance σ_t^2 of C-t curve at any station x is given by

$$\sigma_t^2 = \frac{\int_{-\infty}^{\infty} Ct^2 dt}{\int_{-\infty}^{\infty} Cdt} - (\bar{t})^2 \tag{2.43}$$

where $\bar{t}$, the time to the centroid of the tracer cloud is given by

$$\bar{t} = \frac{\int_{-\infty}^{\infty} Ctdt}{\int_{-\infty}^{\infty} Cdt} \tag{2.44}$$

The value of $\frac{d\sigma_t^2}{dx}$ may be obtained by plotting σ_t^2 against the distance of the sampling station from the injection point and D_L computed by Eq. (2.42). This method requires minimum two C-t curves at two stations to find D_L. The method suffers from the drawback that long tails observed usually in C-t curves contribute significantly to the second moment although they correspond to small mass of flow. The D_L calculated by this method is thus not very accurate.

2.4.2 Routing Procedure

Fischer (1968) proposed the routing procedure given by Eq. (2.14) for solving Taylor's one-dimensional equation. In this method an initial value of D_L is selected and using C-t curve of upstream station, the C-t curve at downstream station is obtained by Eq. (2.14). The predicted and observed C-t curves at the downstream station are compared. The D_L is obtained by iteration for best matching of the predicted and observed C-t curves. The D_L calculated by the moment method is used as the initial value of D_L in this method. The method gives a better estimate of D_L than the moment method. This method also requires a minimum of two C-t curves at two stations for computation of D_L.

2.4.3 The Diffusive Transport Method

Fischer (1968) also proposed 'diffusive transport method' for the determination of D_L. By definition, D_L is the average mass transport through the entire section divided by the mean concentration gradient and is expressed as:

$$D_L = -\frac{1}{A\frac{\partial C}{\partial \varsigma}} \sum_{i=1}^{n} U_i' A_i' C_i'' \qquad (2.45)$$

in which $C = \frac{1}{A}\sum_{i=1}^{n} A_i C_i'$; $\varsigma = x - Ut$ and $C_i'' = C_i' - C$ (2.46)

where A is total area of the section divided into n sub areas, A_1', A_2', ..., A_n', U_i' and C_i' are mean velocity and concentration of tracer of i^{th} sub area. The diffusive transport method gives good results but the accuracy of the method is limited by three factors; (i) the channel should be uniform and its geometry must be accurately defined. (ii) measurement of concentration must be made at enough points in the cross-section to adequately define the concentration variation, and (iii) measurement of C_i' must be very accurate. However, it seems to be a difficult task to have measurements regarding all the above-mentioned parameters.

Out of the above, the change of moment method is easiest to use but it produces unrealistic results. The routing procedure gives better results than moment method but its application is limited to the simplified form of Eq. (1.1); when U, A and D_L are varying, this method also cannot be used.

2.5 EMPIRICAL RELATIONSHIPS FOR DISPERSION COEFFICIENT

Many experimental studies have been carried out by several investigators to develop predictors for D_L in terms of the known hydraulic characteristics of the stream. Empirical equations for computation of D_L recommended by various investigators are presented in Table 2.2.

Table 2.2 : Relationships for D_L

S. No.	Investigator	Equation	Remarks
1	Taylor, 1954	$D_L = 10.1\,U_* r$	Pipe flow, dispersion mainly due to diffusive transport. r = pipe radius
2	Elder, 1959	$D_L = 6.3\,U_* D$	Wide channel.
3	Sumer, 1969	$D_L = 6.23\,U_* D$	Considering velocity profile and vertical turbulent diffusion.
4	Yotsukura and Fiering, 1964	$D_L = 9.0$ to $13.0\,U_* D$	For hydraulically rough and smooth boundary.
5	Thackston and Krenkel, 1967	$D_L = 7.25\,U_* D\left(\frac{U}{U_*}\right)^{\frac{1}{4}}$	Using both the laboratory and the field data.
6	Fischer, 1966	$D_L = 0.011\frac{U^2B^2}{DU_*}$	Using data for smooth and rough laboratory flume and field data.
7	McQuivey and Keefer, 1974	$D_L = 0.058\frac{Q}{SB}$	Based on data from 18 rivers. S = bed slope and Q= Discharge
8	Jain, 1976	$D_L = \frac{U^2T^3}{kAU_*}$	Considering shape of the stream, k = 0.1 to 0.2 and increases with B/D
9	Liu, 1978	$D_L = \frac{Q^2}{2U_*R^3}\left(\frac{U_*}{U}\right)^2$	Considering transverse diffusion coefficient, $\varepsilon_z = 0.23DU_*$
10	Marivoet and Craenenbroeck, 1986	$D_L = 0.0021\frac{U^2B^2}{DU_*}$	Based on data from canals.
11	Sooky, 1969	$D_L = K_1' + K' + K''$ $K_1' = 0.2222\frac{U_*}{\kappa^3}\frac{D_m'}{a}\eta$ $K'' = \frac{1}{9}\kappa U_* D_m'$ $K' = aK''$	Considering shape of the stream and velocity distribution. D_m'=max. depth in cross-section a = proportionality constant η = Dimensionless Shape factor κ = von Karman constant

Table 2.2 Continued ......

12	Fukuoka and Sayre, 1973	$\frac{D_L}{DU_*} = 1.0\left(\frac{Br_c^3}{L_B^2 D^2}\right)^{0.86}$ and $\frac{D_L}{RU_*} = 0.8\left(\frac{r_c^2}{L_B D}\right)^{1.4}$	Consider the meandering effect, r_c is the radius of curvature of bend and L_B is the overall bend length measured along the centerline of the channel.
13	Hou and Christensen, 1976	$\frac{D_L}{RU_*} = 1.5239 - 0.1395\log\frac{R}{K_S}$ $+0.0081\left(\log\frac{R}{K_S}\right)^2$	Considering roughness of bed, K_S is equivalent sand roughness.
14	Magazine, 1983	$\frac{D_L}{R_b U} = \frac{D_L}{R_w U} = 75.86(P_r)^{-1.632}$ $P_r = \frac{C_b}{\sqrt{g}}\left(\frac{s}{h}\right)^{0.3}\left(\frac{s}{b}\right)^{0.3}\left(1.5+\frac{e}{h}\right)$ for rough-bed channels $P_r = 1.5\frac{C_w}{\sqrt{g}}\left(\frac{s}{h_1}\right)^{0.3}\left(\frac{s}{D}\right)^{0.3}$ for rough-side channels	Studied large-scale bed and side roughness. (Notations are given below the Table)
15	Beltaos, 1978	$\frac{D_L}{RU_*} \propto \left(\frac{B}{R}\right)^2$	Used Sooky's finding.
16	Asai *et al.*, 1991	$\frac{D_L}{DU_*} = 2.0\left(\frac{B}{D}\right)^{1.5}$	By analysing field and lab data.

Notations used in Magazine equation;

C_w = Chezy's coefficient for sides roughness
C_b = Chezy's coefficient for bed roughness
s = Centre to centre longitudinal spacing of roughness elements
h = Height of roughness element
h_1 = Spur length
b = Transverse size of the roughness elements
e = Transverse gap between the elements
R_b = Hydraulic radius with respect to bed used for bed roughness.
R_w = Hydraulic radius with respect to sides used for side roughness

Amongst the equations given in Table 2.2, the relationship of Asai *et al.* (1991) is expected to produce realistic results, because this equation has a theoretical basis and is verified for a wide range of experimental and field conditions. Theoretically it was found that the dispersion coefficient in wide channels is closely related with (i) lateral velocity distribution of the depth-averaged velocity, I_A (ii) non-dimensional lateral mixing coefficient, θ_l (iii) the width to depth ratio and (iv) resistance factor U/U_*. The theoretical expression of D_L obtained was as below :

$$\frac{D_L}{DU_*} = \frac{I_A}{\theta_l}\left(\frac{U}{U_*}\right)^2\left(\frac{B}{D}\right)^2 \tag{2.47}$$

However, the equation given in Table 2.2 (S. No. 16) was finally proposed on the basis of analysis of data made by Asai *et al.* (1991).

2.6 DISPERSION IN SEDIMENT - LADEN FLOW

Alluvial streams carry sediments as suspended load and bed load. To make the dispersion study more oriented to field application, it is necessary to know the effects of sediment on the dispersion process. No literature is available except the study of Singh (1987) on the dispersion characteristics of suspended sediment-laden flows. Singh (1987) performed experiments in a flume and observed that the presence of sediments does not have any noticeable effect on the longitudinal dispersion in the range of concentration studied (<5000 ppm). However, it is well known that the presence of sediment affects the friction factor, which can have a significant effect on dispersion.

Longitudinal dispersion occurs mainly due to the non-uniform velocity along the vertical in a section of the stream. It is well known that velocity distribution for sediment-laden flow departs from the velocity distribution for clear-water flow under similar flow conditions (Einstein and Chein, 1955; Vanoni, 1946; Coleman, 1981; Arora, 1983; Lau, 1983; Samaga, 1986; Umeyama and Gerritsen, 1992). Therefore it is likely that, presence of sediment would affect the process of longitudinal dispersion. To analyse the effect of sediment on the dispersion process, the velocity distribution in sediment-laden flow should be known. The following section describes the velocity distribution in sediment-laden flow as studied by different investigators.

2.6.1 Velocity Distribution in Sediment - Laden Flow

It has been indicated from previous research that Prandtl's logarithmic equation which was originally derived for clear-water flow, accurately describes the velocity distribution in sediment-laden flows also except that the value of the von Karman coefficient is different. Studies of Vanoni (1946) and Einstein and Chein (1955) indicated that the value of the von Karman coefficient decreases with increase in sediment concentration. However, this result has not been uniformly supported by others. Imamoto *et al.* (1977) found that the value of von Karman coefficient increases with sediment concentration but Itakura and Kishi (1980) found constant value of κ=0.4 same as for clear water to hold good for sediment-laden flows also.

A plausible hypothesis was made by Coleman (1981) regarding this aspect, who proposed that the inner region in which the logarithmic law is valid remains unaffected by the presence of sediment. Lau (1983) confirmed Coleman's finding on the basis of the flow-resistance concept, showing that the velocity is smaller near the bed and larger near the water surface for sediment-laden flow than for clear-water flow for given discharge and slope. Arora (1983) also observed that the velocity is smaller near the bed and larger near the water surface for sediment-laden flow. Samaga (1986) studied the velocity distribution for sediment-laden flow, using a two-layer model, which consists of a parabolic equation in the wall region and the logarithmic equation in the outer region. Umeyama and Gerritsen (1992) predicted the velocity profile in sediment-laden flow based on new mixing length concept which consists of a logarithmic term and a power series term. The studies of Arora (1983), Samaga (1986) and Umeyama and Gerritsen (1992) on velocity distribution are discussed below in detail.

2.6.1.1 Arora's velocity distribution law

Arora (1983) states that a simple logarithmic law does not describe the velocity distribution in sediment-laden flows. He examined the velocity distribution laws given by other investigators with his own experimental data and found that none of the velocity distribution laws satisfactorily applied to those data particularly at high sediment concentration flows. He proposed the following velocity distribution law for smooth rigid bed channels of various shapes.

$$\frac{u - U_{max}}{U_{*c}} = \frac{2.3}{\kappa}\log\frac{y}{D} - \frac{\Delta u}{U_{*c}} \tag{2.48}$$

where U_{max} = maximum velocity of flow, U_{*c} = shear velocity with respect to centreline shear stress and equal to 1.1 times of the average shear velocity U_* for rectangular channels and $\frac{\Delta u}{U_{*c}}$ is velocity defect.

The velocity defect was found to be a function of the term $C_s\sqrt{\frac{\omega_0}{U_{*c}}}$ which is shown in Figs. 2.6 and 2.7. Here ω_0 is fall velocity of the sediment in clear water and C_s is average concentration of sediment. Arora (1983) also proposed a relation for U_{max}, which is a function of the aspect ratio i.e. top width of channel / depth of flow.

As is clear from Fig. 2.6, the velocity defect decreases as y/D increases and at y/D=0.8 the velocity defect becomes zero. This means that velocity is lower near the bed compared to clear water case and that it is more near the surface. This results in larger variation of velocity over the vertical in sediment-laden flow causing more dispersion.

2.6.1.2 Samaga's velocity distribution law

Samaga (1986) described the velocity distribution in alluvial channels with non-uniform sediment, using a two-layer model. In the lower layer the velocity was assumed to follow a parabolic equation and in the upper layer it was assumed to follow the logarithmic law as below.

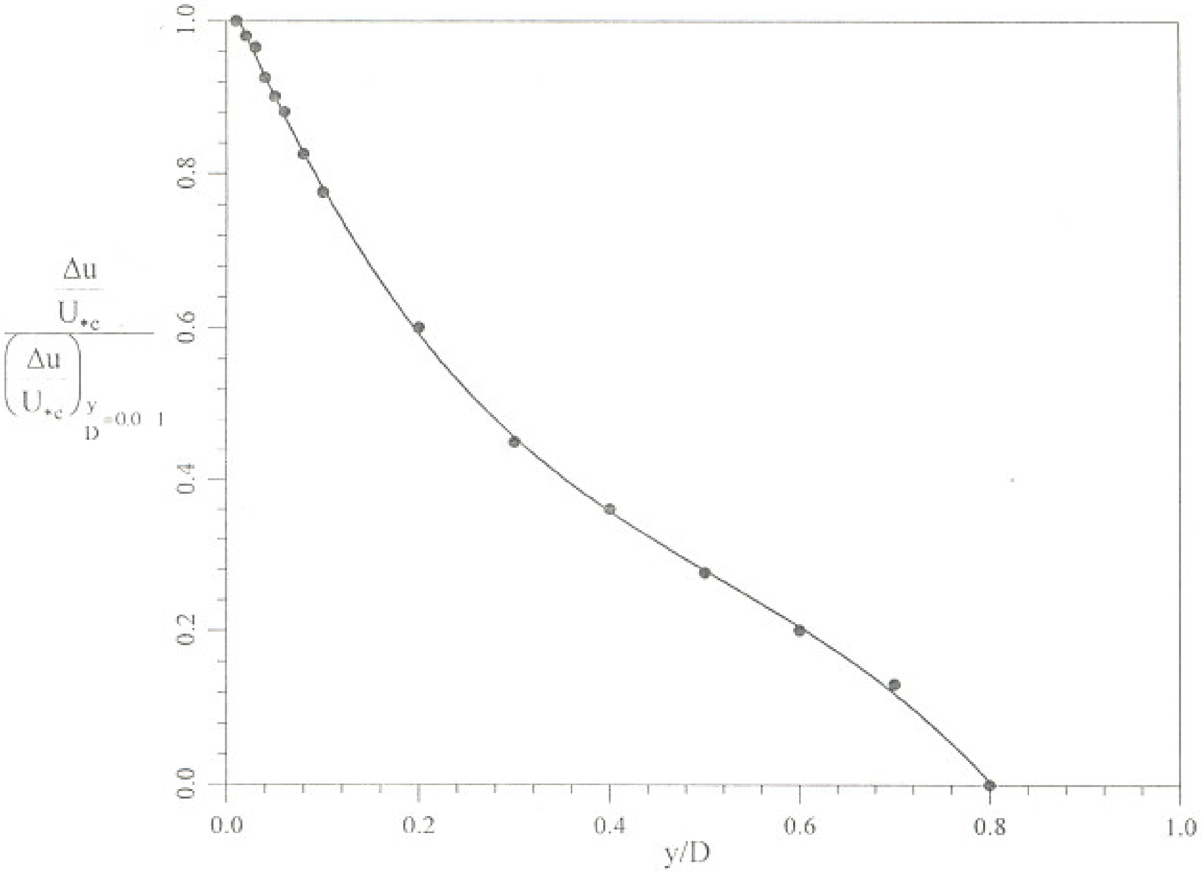


Fig. 2.6 Variation of Velocity Defect with y/D

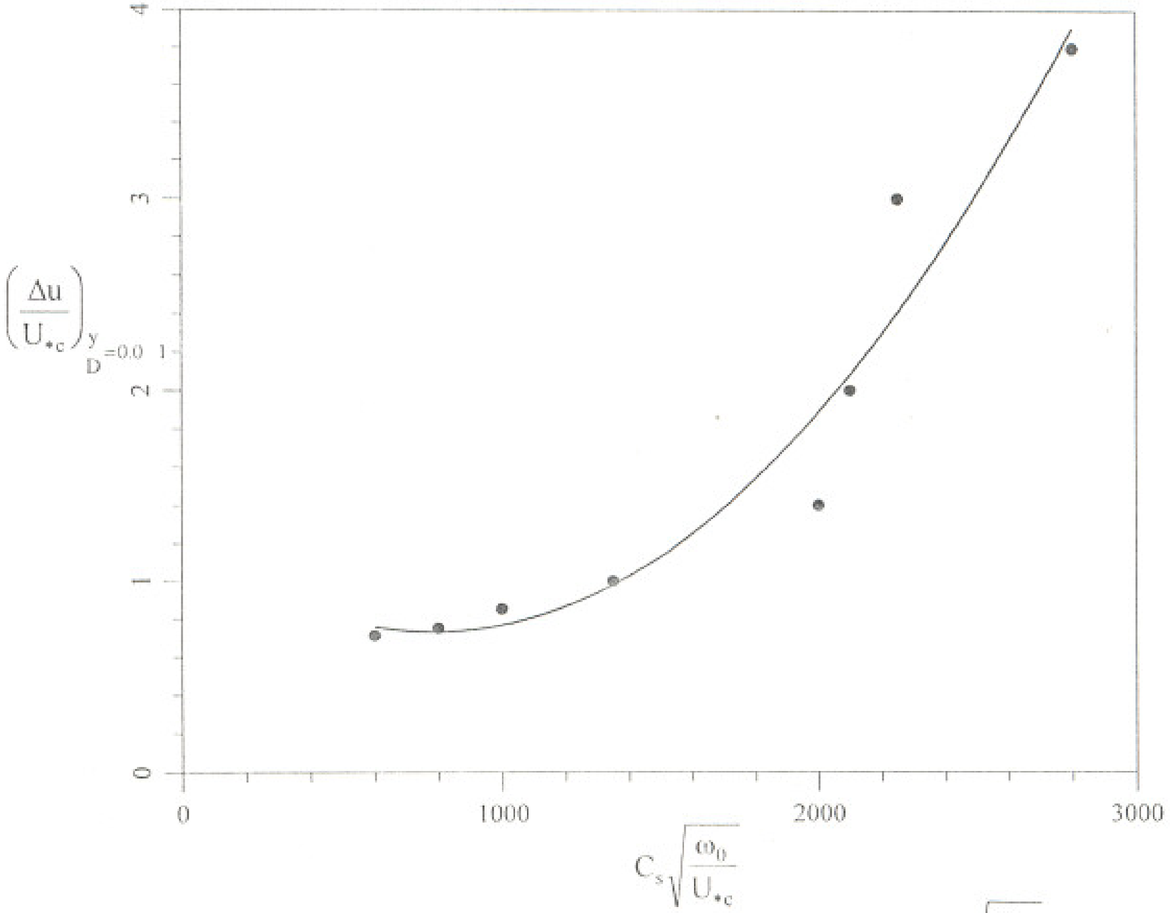


Fig. 2.7 Variation of Velocity Defect with $C_s\sqrt{\dfrac{\omega_0}{U_{*c}}}$

$$\frac{u}{U_*} = 5.75\log\left(\frac{y}{d_a}\right) + A_c \qquad \text{for} \qquad \frac{y}{D} \geq 0.2 \tag{2.49}$$

and

$$\frac{u}{U_*} = 28.75\frac{y}{D}\left(1 - \frac{y}{2D}\right) + B_c \qquad \text{for} \qquad \frac{d_a}{D} < \frac{y}{D} \leq 0.2 \tag{2.50}$$

where d_a = arithmetic-mean size of sediment mixture, A_c and B_c are functions of D/d_a as shown in Fig. 2.8. For the undulated bed, the term U'_* (shear velocity due to grain resistance) is used in place of U_*. Samaga verified his results using the data on velocity distribution in river Rhine as shown in Fig. 2.9.

2.6.1.3 Umeyama and Gerritsen's velocity distribution law

Umeyama and Gerritsen (1992) proposed a theoretical model for velocity distribution in sediment-laden flow by means of a new mixing-length concept. The proposed model is

$$\frac{u}{U_*} = \frac{1}{\kappa}\left\{\ln(y) + \frac{\beta}{2}\frac{c}{c_b}\frac{y}{\delta} + \frac{1}{2!}\frac{\beta}{2}\frac{c}{c_b}\left(1 + \frac{\beta}{2}\frac{c}{c_b}\right)\frac{y^2}{2\delta^2} + .. + \frac{1}{\kappa!}\frac{\beta}{2}\frac{c}{c_b}\left(1 + \frac{\beta}{2}\frac{c}{c_b}\right)..\left((\kappa - 1) + \frac{\beta}{2}\frac{c}{c_b}\right)\frac{y^\kappa}{\kappa\delta^\kappa}..\right\}$$
$$\left.1 + \left(\frac{\rho_s}{\rho_f} - 1\right)c_b\right]^{-1/2} + A_u \tag{2.51}$$

where β= constant for magnitude of sediment concentration, c_b= the reference concentration at $y=b'$, δ= boundary layer thickness, A_u= integration constant ρ_s= density of sediment and ρ_f = density of clear water. This equation consists of a logarithmic term and power series terms. The power series terms explain the departure from the logarithmic law due to sediment-laden flow. The departure from the logarithmic law is larger with increase in the value of β, which depends on the size of sediment and sediment concentration.

From the above, one can conclude that the deviation of velocity from the mean value is more in sediment-laden flow than in clear-water flow, which may result in more dispersion in sediment-laden flow.

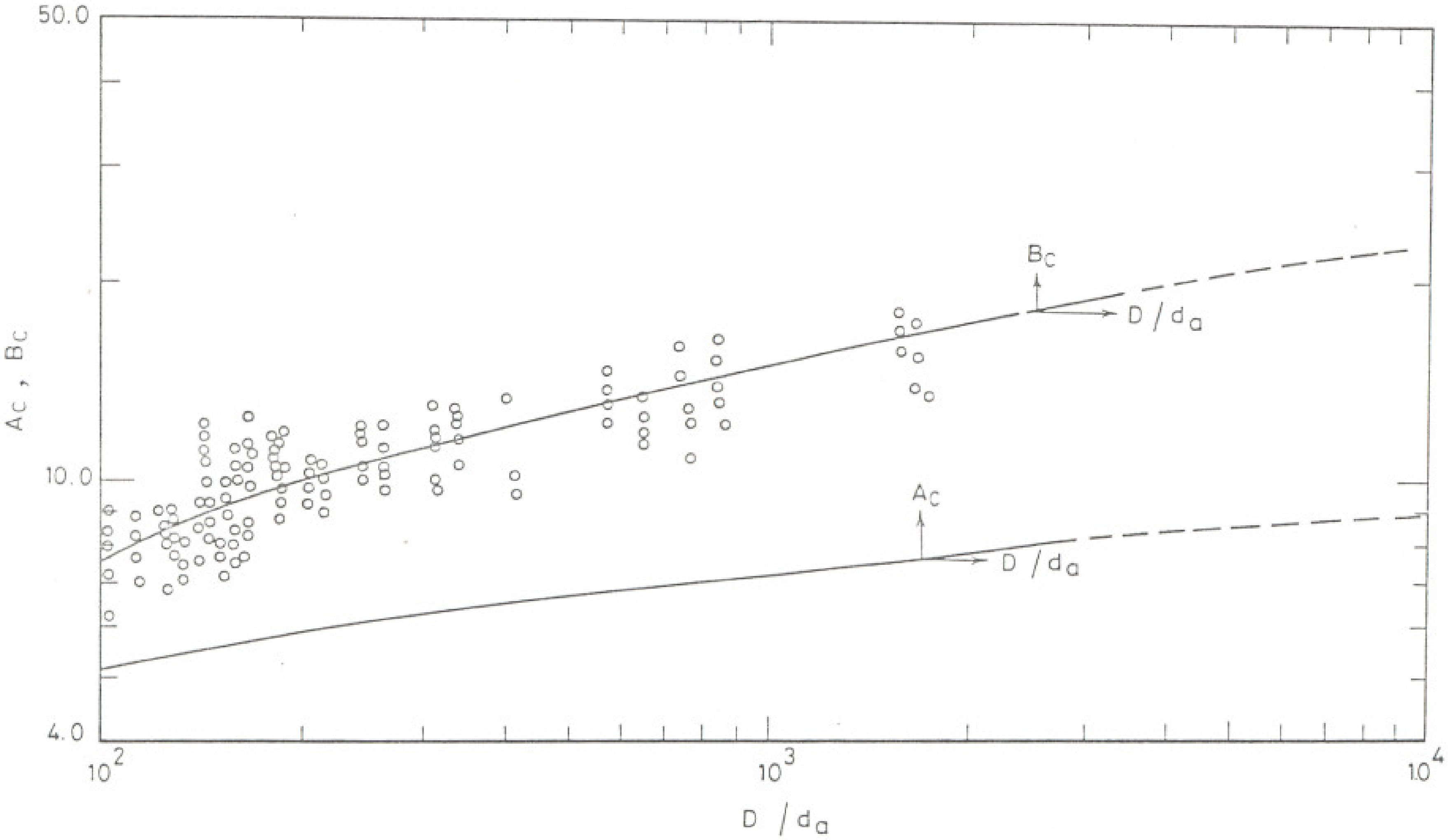


Fig.2. 8 Relation Between A_c , B_c and D/d_a (Samaga,1986)

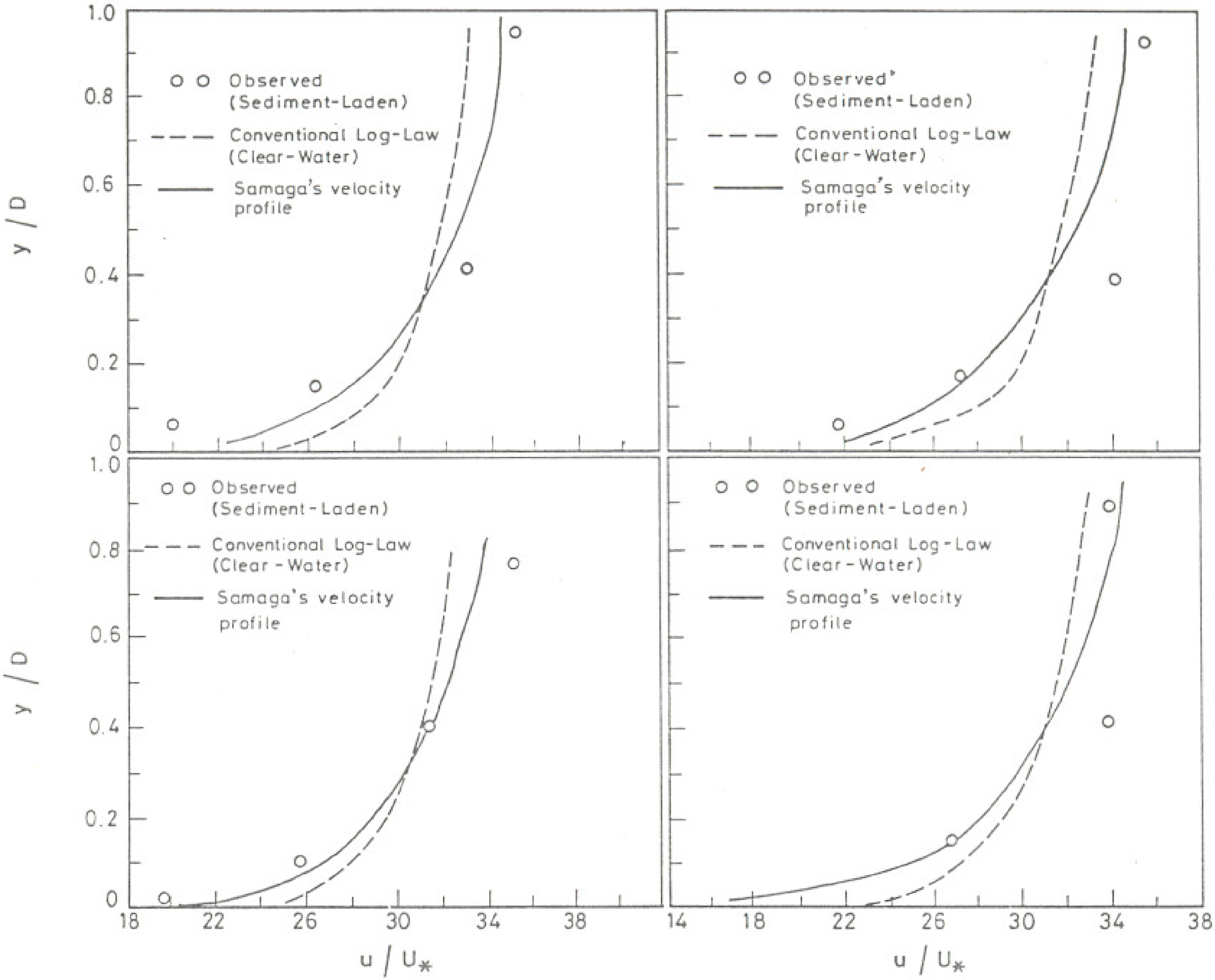


Fig.2.9 Comparison of Samaga's Velocity Profile Model with Rhine River Data

2.7 NON-FICKIAN BEHAVIOUR OF THE DISPERSION PROCESS

It is clear that non-Fickian behaviour of dispersion prevails in the mixing length region of a stream. But it is also found to exist beyond the mixing length in some cases due to presence of eddies, advection flow (short-circuiting), dead zones and dikes or spurs present in the path of pollutant flow. Also if the non-uniformity in the river channels is large, it results in changes in area, mean velocity and dispersion coefficient with longitudinal distance. The Fickian one-dimensional dispersion equation is not applicable in such streams. Several investigators have studied the dispersion process taking into account the non-Fickian behaviour. These include Beltaos (1980), Day (1975), Chatwin (1980), Day and Wood (1976), Sabol and Nordin (1978, 1981), Liu and Cheng (1980, 1982), Beltaos (1978) and Wu and Tsanis (1994).

Beltaos (1980) categorically stated that 'Fickian behaviour is not likely to be encountered in large streams within lengths of practical interest'. Day (1975), after conducting dispersion studies in five mountainous streams in New Zealand, concluded that, it is not proper to apply Taylor's analysis for studying dispersion in natural streams. He also observed that concentration distributions were not Gaussian but maintained a persistent asymmetry resulting in an ever-increasing value of D_L in the downstream direction. Fischer (1966) explained the long tail of C-t curve, as a result of the slow release of tracer/pollutant from the laminar sub-layer. Chatwin (1980) contended that because of the deviation from Gaussian distribution, the observed C-t curves cannot be described by Taylor's model even approximately.

Sabol and Nordin (1978, 1981) and Beer and Young (1983) also expressed the view that the long tails of low concentration that resulted from temporary storage and release mechanics within the flow are responsible for the deviation from the Fickian behaviour. They proposed a Dead Zone model for dispersion process. Li and Cheng (1980, 1982) modified the one-dimensional dispersion equation by replacing the constant dispersion coefficient by a time or distance dependent dispersion coefficient. This model was tested using the field data of Godfrey and Frederick (1970) and Nordin and Sabol (1974) and it was concluded that the model yields more realistic value of peak concentration and D_L in natural streams even without considering explicitly the dead zones and other complicated factors. It was also pointed out that the pure Fickian model was not capable of satisfactorily predicting the C-t curves in natural streams.

Wu and Tsanis (1994) made a systematic study of the circulation, pollutant transport and residence time in a distorted model using a numerical scheme. They also studied the effects of eddies, dike and advection flow. They concluded that a two-dimensional advection-diffusion model is successful in modelling the transport and dispersion of a pollutant emitted into the basin. The presence of dikes or spurs was found to significantly affect the residence time.

2.8 CONCLUDING REMARKS

A review of pertinent literature concerning the dispersion process of neutrally buoyant dispersants in laboratory flumes and natural streams presented in this chapter indicates that the following points regarding dispersion process need careful consideration.

(i) Models which are based on Fickian-diffusion theory are not capable of describing the dispersion process in open channel flows in the light of the fact that these models as such, cannot take care of the dispersion due to varying flow cross-sections, dead zone effects and also because of the fact that the observed concentration distributions are not Gaussian.

(ii) Numerous relationships are available for estimation of the mixing length required before the longitudinal dispersion process begins. These relationships need to be checked for their validity.

(iii) Numerical methods which have been used for solution of the dispersion process face difficulty while back-tracking the trajectory of concentration characteristics line.

(iv) A large number of predictors are available for estimation of D_L. They need to be verified for their accuracy using carefully collected data from laboratory and field.

(v) Relatively little information is available on dispersion in sediment-laden flows.

3

NUMERICAL MODELLING OF DISPERSION

3.1 INTRODUCTION

The literature pertaining to the numerical solution of the longitudinal dispersion equation was reviewed in Chapter-II. It was observed that none of the numerical schemes available in literature is directly applicable when U, A and D_L are functions of the distance. Almost all the numerical schemes for computation of the advection part of dispersion equation suffer from the numerical diffusion to some extent due to the computational step involving evaluation of concentration at the root of its characteristic line through interpolation. In this chapter a new numerical scheme has been developed which is applicable to the cases when U, A and D_L are functions of distance also. Here an exact solution is proposed for the advection process and Crank-Nicholson scheme is used for solution of the diffusion process. The scheme is extended for evaluation of D_L taking as input the observed C-t curves at two or more stations.

3.2 NUMERICAL SCHEME FOR SOLUTION OF LONGITUDINAL DISPERSION EQUATION

Presented herein is a numerical solution technique based on a Combined operator approach, in which the advection and diffusion processes are treated simultaneously. The variables U, A and D_L are considered as functions of the distance. The variation of dispersion coefficient is assumed to be linear with distance because such variation is simpler to study. The exact solution of the advection process has been achieved by making the computations on a spatial grid the size of which varies with the distance. To eliminate the possibility of numerical oscillations, weighting coefficients are introduced to the pollutant concentration in the time stepping. For unconditional stable solution of diffusion process the six-point Crank-Nicholson scheme has been used.

3.2.1 Non-Dimensional Form of Longitudinal Dispersion Equation

The longitudinal dispersion Eq. (1.1) is a linear parabolic partial differential equation requiring for its solution, one upstream and one downstream boundary condition as well as an initial state. The assumptions involved in the development of the equation are as follows.

- pollutant is conservative.
- flow is steady but non-uniform.
- buoyancy effects are neglected.
- complete mixing of the pollutant over the cross-section has taken place.

The non-dimensional variables defined here are as follows:

$$C^* = \frac{C}{C_p} \quad ; \quad x^* = \frac{x}{D} \quad ; \quad t^* = t\sqrt{\frac{g}{D}} \quad ;$$

$$U^* = \frac{U}{\sqrt{gD}} \quad ; \quad A^* = \frac{A}{D^2} \quad \text{and} \quad D_L^* = \frac{D_L}{D\sqrt{gD}} \tag{3.1}$$

where superscript '*' indicates the dimensionless value of that variable and C_p is the peak concentration at the first station. Substituting the values of variables from Eq. (3.1) into Eq. (1.1), and after dropping the superscript '*' from these variables one can get

$$\frac{\partial(AC)}{\partial t} + \frac{\partial(AUC)}{\partial x} = \frac{\partial}{\partial x}\left(AD_L \frac{\partial C}{\partial x}\right) \tag{1.1}$$

Assuming linear variation of D_L with distance in downstream x, one can write

$$D_L = D_{L0} + Kx \tag{3.2}$$

where D_{L0} is the dispersion coefficient at x = 0 and K is a multiplying coefficient. Substituting the value of D_L from Eq. (3.2) into Eq. (1.1) and differentiating one can get:

$$C\frac{\partial A}{\partial t}+A\frac{\partial C}{\partial t}+C\frac{\partial (AU)}{\partial x}+AU\frac{\partial C}{\partial x}=AD_L\frac{\partial^2 C}{\partial x^2}+\left(AK+D_L\frac{\partial A}{\partial x}\right)\frac{\partial C}{\partial x} \tag{3.3}$$

For steady flow, $\frac{\partial A}{\partial t}=0$ and for constant discharge flow , $\frac{\partial (AU)}{\partial x}=0$. Hence for steady and constant discharge flow Eq. (3.3) is reduced to the form

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=D_L\frac{\partial^2 C}{\partial x^2}+\left(K+\frac{D_L}{A}\frac{\partial A}{\partial x}\right)\frac{\partial C}{\partial x} \tag{3.4}$$

The second term on the right hand side of Eq. (3.4) is significant only in non-uniform flows when the dispersion coefficient would vary with the distance. Each term appearing in Eq. (3.4) is non-dimensional in nature. For getting the results to be used in practice the outputs are multiplied by the corresponding scaling parameters.

3.2.2 Finite Difference Method

Equation (3.4) may be solved by finite difference method using variable spatial-temporal grids as shown in Fig. 3.1.

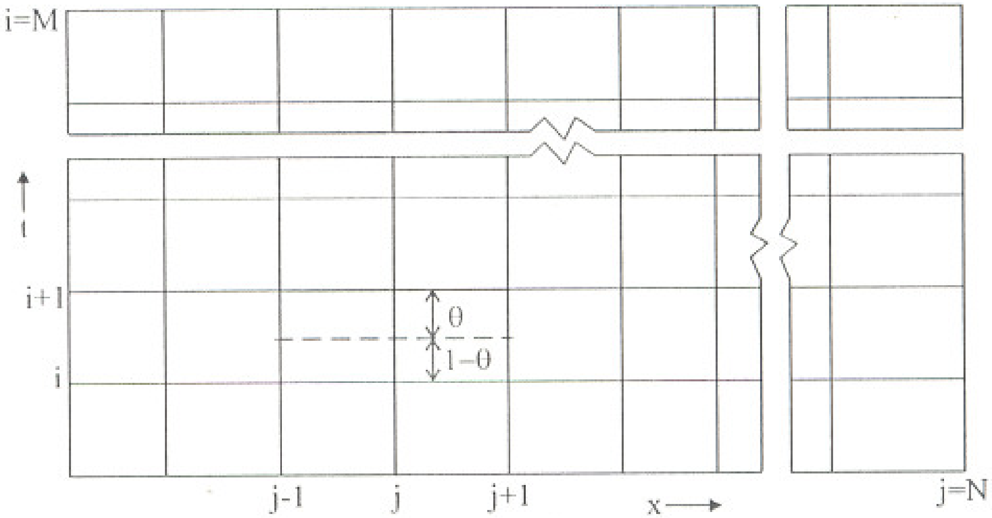


Fig. 3.1 Variable Spatial-Temporal Grid

Advection Process: The advection equation is

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = 0 \tag{3.5}$$

To obtain the exact solution of Eq. (3.5), forward difference for $\frac{\partial C}{\partial t}$ and backward difference for $\frac{\partial C}{\partial x}$ has been taken. Now Eq. (3.5) is written as (Abbot, 1980)

$$\frac{C_j^{i+1} - C_j^i}{\Delta t} + U_j \frac{C_j^i - C_{j-1}^i}{\Delta x_{j-1}} = 0 \tag{3.6}$$

where $\Delta x_{j-1} = x_j - x_{j-1}$ and Uj = velocity at section , j and x_j = distance of section, j. When Courant number is unity

$$\Delta x_{j-1} = U_j \Delta t \tag{3.7}$$

Equation (3.7) is used to generate the spatial - temporal grid in which Δx is dependent on U, hence the spatial grid length varies with the velocity. By substituting Eq. (3.7) into Eq. (3.6) one can get

$$C_j^{i+1} = C_{j-1}^i \tag{3.8}$$

This is the exact solution of the advection process. Here the Δx values are so chosen that root of characteristic line falls at the nodal point p (see Fig. 2.5), where the concentration is known. Hence, interpolation is not required to determine the concentration at root of trajectory of characteristic as in case of the methods by Holley and Preissmann (1977) and Yang and Hsu (1990).

Diffusion Process: The diffusion equation is

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} + \left(K + \frac{D_L}{A}\frac{\partial A}{\partial x}\right)\frac{\partial C}{\partial x} \tag{3.9}$$

A centered finite difference scheme is used for first order partial derivatives and the Crank-Nicholson scheme is used for the second order partial derivatives as follows:

$$\frac{\partial C}{\partial x}=\frac{1}{\left(\Delta x_j+\Delta x_{j-1}\right)}\left\{\theta\left(C_{j+1}^{i+1}-C_{j-1}^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-C_{j-1}^{i}\right)\right\}+O\left(\Delta\bar{x}^2\right)$$

$$\frac{\partial A}{\partial x}=\frac{\left(A_{j+1}-A_{j-1}\right)}{\left(\Delta x_j+\Delta x_{j-1}\right)}+O\left(\Delta\bar{x}^2\right)$$

$$\frac{\partial^2 C}{\partial x^2}=\frac{1}{0.5\left(\Delta x_j^2+\Delta x_{j-1}^2\right)}\left\{\theta\left(C_{j+1}^{i+1}-2C_j^{i+1}+C_{j-1}^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-2C_j^{i}+C_{j-1}^{i}\right)\right\}$$

$$+O\left(\Delta\bar{x}^2\right) \qquad (3.10)$$

where $\Delta\bar{x}=\frac{\Delta x_j+\Delta x_{j-1}}{2}$ and the term $O\left(\Delta x^2\right)$ is the order of local truncation error. Hence under the assumption $\Delta\bar{x}\to 0$, the difference and the differential equations become equivalent. Substituting Eq. (3.10) into Eq. (3.9) to get second order accuracy, we have

$$\frac{C_j^{i+1}-C_j^{i}}{\Delta t}=\frac{\left(D_{L0}+Kx_j\right)}{0.5\left(\Delta x_j^2+\Delta x_{j-1}^2\right)}\left\{\theta\left(C_{j+1}^{i+1}-2C_j^{i+1}+C_{j-1}^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-2C_j^{i}+C_{j-1}^{i}\right)\right\}+$$

$$\frac{1}{\left(\Delta x_j+\Delta x_{j-1}\right)}\left\{K+\frac{\left(D_{L0}+Kx_j\right)}{A_j}\frac{\left(A_{j+1}-A_{j-1}\right)}{\left(\Delta x_j+\Delta x_{j-1}\right)}\right\}\left\{\theta\left(C_{j+1}^{i+1}-C_{j-1}^{i+1}\right)+(1-\theta)\left(C_{j+1}^{i}-C_{j-1}^{i}\right)\right\}$$

$$(3.11)$$

Equations (3.6) and (3.11) may be combined to get the solution of complete advection-diffusion equation. Combining Eqs. (3.6) and (3.11) and by arranging the known quantities on one side and the unknown quantities on the other side of the equation, we get

$$a_j C_{j-1}^{i+1}+b_j C_j^{i+1}+c_j C_{j+1}^{i+1}=E_j^{i} \qquad (3.12)$$

in which $$E_j^i = a'_j C_{j+1}^i + b'_j C_j^i + c'_j C_{j-1}^i \qquad (3.13)$$

where $$a_j = -\theta(R_j - P_j) \quad ; \quad b_j = 1 + 2\theta R_j \quad ; \quad c_j = -\theta(R_j + P_j)$$

$$a'_j = (1-\theta)(R_j + P_j) \quad ; \quad b'_j = -2R_j(1-\theta) \quad ; \quad c'_j = 1 + (1-\theta)(R_j - P_j) \qquad (3.14)$$

and

$$R_j = \frac{\left(D_{L0} + Kx_j\right)\Delta t}{0.5\left(\Delta x_j^2 + \Delta x_{j-1}^2\right)} \quad ;$$

$$(3.15)$$

$$P_j = \frac{\Delta t}{\left(\Delta x_j + \Delta x_{j-1}\right)}\left\{K + \frac{\left(D_{L0} + Kx_j\right)}{A_j}\frac{\left(A_{j+1} - A_{j-1}\right)}{\left(\Delta x_j + \Delta x_{j-1}\right)}\right\}$$

Also θ = temporal weighting coefficient, which must be more than 0.5 for stable solution (Smith, 1978). In the present study θ is taken equal to 0.55 to minimize the numerical diffusion.

Equation (3.12) cannot be solved by itself because it contains three unknown values at time level (i+1). However, this equation can be written for all the computational nodes, thus producing a system of (N-2) implicit linear equations involving N number of unknowns. Here N is the total number of nodes in the spatial grid. To solve this system of equations two boundary conditions are to be used one each at the upstream and the downstream of the flow domain. The procedure of solving these equations is described in the following section.

3.2.3 Solution of the System of Equations

The price to be paid for unconditional stability of the Crank-Nicholson method is that the system of algebraic equations (Eq. 3.12) is to be solved at every time step. This is, however, a system with a very special form (tridiagonal form) that makes it easy to solve.

Making use of the boundary conditions, the system of equations can be written for time step (i+1) as

$$\begin{bmatrix} b_2 & c_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_3 & b_3 & c_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_j & b_j & c_j & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{N-2} & b_{N-2} & c_{N-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{N-1} & b_{N-1} \end{bmatrix} \times \begin{bmatrix} C_2^{i+1} \\ C_3^{i+1} \\ \cdot \\ \cdot \\ C_j^{i+1} \\ \cdot \\ \cdot \\ \cdot \\ C_{N-2}^{i+1} \\ C_{N-1}^{i+1} \end{bmatrix} = \begin{bmatrix} E_2^i \\ E_3^i \\ \cdot \\ \cdot \\ E_j^i \\ \cdot \\ \cdot \\ \cdot \\ E_{N-2}^i \\ E_{N-1}^i \end{bmatrix} \qquad (3.16)$$

The value of C_1^{i+1} and C_N^{i+1} are known as upstream and downstream boundary conditions respectively. Hence the (old) value of E_2^i and E_N^i, calculated by Eq. (3.13) can be modified as

$$\begin{aligned} E_2^i(\text{new}) &= E_2^i(\text{old}) - a_2 C_1^{i+1} \\ E_{N-1}^i(\text{new}) &= E_{N-1}^i(\text{old}) - c_{N-1} C_N^{i+1} \end{aligned} \qquad (3.17)$$

and the new values of E_2^i and E_N^i are taken in Eq. (3.16). The solution of such a system of equations can be done using the Thomas or Double Sweep algorithm. A computational procedure due to Thomas is described below (Sastry, 1995).

(i) Set $\alpha_2 = b_2$ and compute

$$\alpha_j = b_j - \frac{a_j C_{j-1}}{\alpha_{j-1}} \qquad j = 3,4,\ldots\ldots\ldots,N\ - \qquad (3.18)$$

(ii) Set $\beta_2 = \dfrac{E_2^i}{b_2}$ and compute

$$\beta_j = \frac{E_j^i - a_j \beta_{j-1}}{\alpha_j} \qquad j = 3,4,\ldots\ldots\ldots,N-1 \qquad (3.19)$$

(iii) Finally, compute C_j^{i+1} from

$$C_j^{i+1} = \beta_j - \frac{c_j C_{j+1}^{i+1}}{\alpha_j} \qquad j = N-1, N-2, \ldots\ldots, 2 \tag{3.20}$$

where $C_{N-1}^{i+1} = \beta_{N-1}$.

This procedure has been found to be very efficient for use on a digital computer (Sastry, 1995). The computer code for the above algorithm is named as subroutine CONCNT. The flow chart for the same is shown in Fig. 3.2.

3.2.4 Generation of Variable Size Spatial Grid

The exact solution of advection process is obtained by generating a variable size spatial and constant size temporal grid as per Eq. (3.7). The flow between any two consecutive stations is considered as gradually varied flow (see Fig. 3.3). Since U is considered to vary with distance, one has to integrate velocity with respect of time to get the grid size Δx_j . The flow and channel properties are measured only at the stations. To obtain the velocity or depth of flow at any point between the two measuring station, the equation for gradually varied flow equation needs to be solved. The channel geometry at such points are obtained through interpolation, e.g. the width of the channel at any grid position j+1 is obtained as:

$$B_{j+1} = B_j + \frac{BS_{i+1} - BS_i}{xs_{i+1} - xs_i} \Delta x_j \tag{3.21}$$

where BS_i is the width of channel at station i and xs_i is the distance of the station i from the first station. The other channel geometric parameters are also determined in the same manner.

Now let it be assumed that j^{th} grid size has been determined and $(j+1)^{th}$ grid is to be determined. Initially for approximation the velocity between the grid j and j+1 is assumed to be constant and equal to velocity at the grid j. The size of the grid is then given by

$$\Delta x_j = U_j \Delta t \tag{3.7}$$

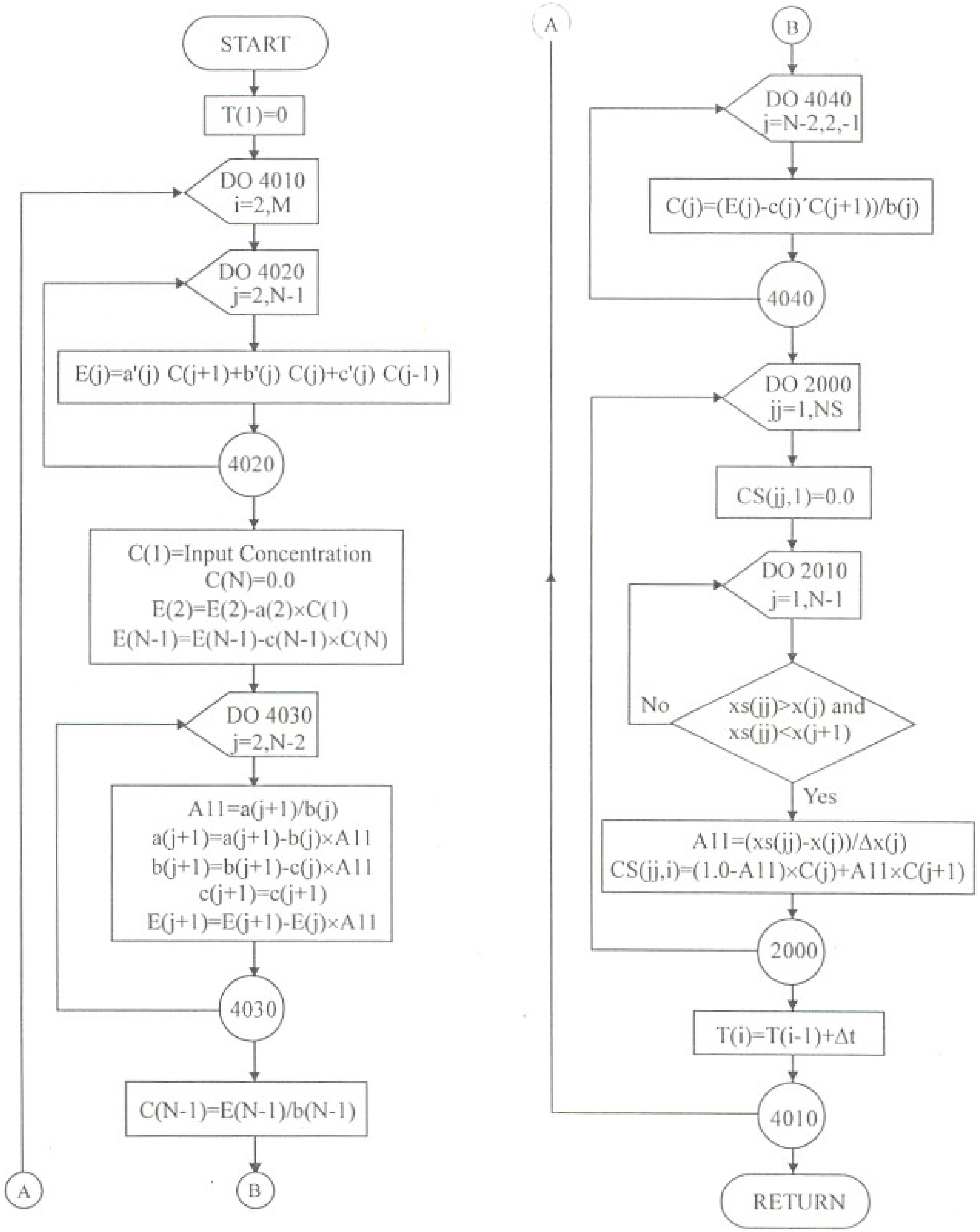


Fig. 3.2 Flow Chart for Subroutine, CONCNT

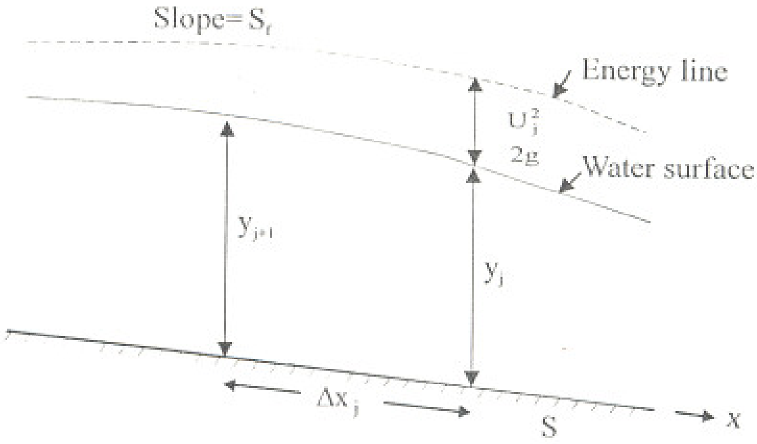


Fig. 3.3 Sketch of Gradually Varied Flow

The basic differential equation of gradually varied flow is (Henderson, 1967)

$$\frac{dy}{dx} = F(y) \tag{3.22}$$

in which $F(y) = \dfrac{S_0 - S_f}{1 - \dfrac{Q^2 T}{gA^3}}$ where $S_f = \dfrac{n^2 Q^2}{A^2 R^{4/3}}$

$\frac{dy}{dx}$ represents the water-surface slope relative to the bottom of the channel, n is the Manning's coefficient, S is the bed slope and S_f is the slope of the total energy line. Equation (3.22) is a non-linear differential equation and Runge-Kutta method is used for its numerical solution. The depth of flow at (j+1) can be obtained by the Standard Fourth Order Range-Kutta method as below

$$y_{j+1} = y_j + \frac{1}{6}\left(K'_1 + 2K'_2 + 2K'_3 + K'_4\right) \tag{3.23}$$

where

$$K'_1 = \Delta x_j \times F(y)$$

$$K'_2 = \Delta x_j \times F(y + \frac{K'_1}{2})$$

$$K'_3 = \Delta x_j \times F(y + \frac{K'_2}{2})$$

$$K'_4 = \Delta x_j \times F(y + K'_3) \tag{3.24}$$

The velocity at grid (j+1) for a rectangular channel is obtained as

$$U_{j+1} = \frac{Q}{B_{j+1} \times Y_{j+1}} \tag{3.25}$$

The Δx_j as obtained from Eq. (3.7) is then revised as below:

$$\Delta x_j = \frac{1}{2}\left(U_j + U_{j+1}\right)\Delta t \tag{3.26}$$

The computational steps involved between Eqs. (3.23) to (3.26) are repeated until U_{j+1} values computed in successive iterations are not different by more than the chosen tolerance limit. The location of the grid (j+1) is then given by

$$x_{j+1} = x_j - \frac{1}{2}\left(U_j + U_{j+1}\right)\Delta t \tag{3.27}$$

The computer code for generation of variable size spatial grid is named as subroutine GRID. The flow chart for the same is shown in Fig. 3.4.

3.2.5 Initial and Boundary Conditions

The initial condition and two boundary conditions are required to obtain the solution of the longitudinal dispersion model discussed as above. The initial condition is to be specified over the considered reach length of the stream for the initial time t_0 in the form of spatial profile of the concentration $C(x,t_0)$. Generally before the injection of pollutants, $C(x,t_0)=0$. However, if in some cases $C(x,t_0)\neq 0$, its value should be known at various locations of the stream.

The boundary conditions required to solve the equations specify the temporal profile of the concentration at the upstream end, x_1 and at the downstream end, x_N. However, at a very large distance from the injection point of the pollutant, the concentration of the pollutant may be considered to be zero. At x=0 the temporal variation of concentration is either measured or it can be determined using any mathematical model, which is valid within the mixing length.

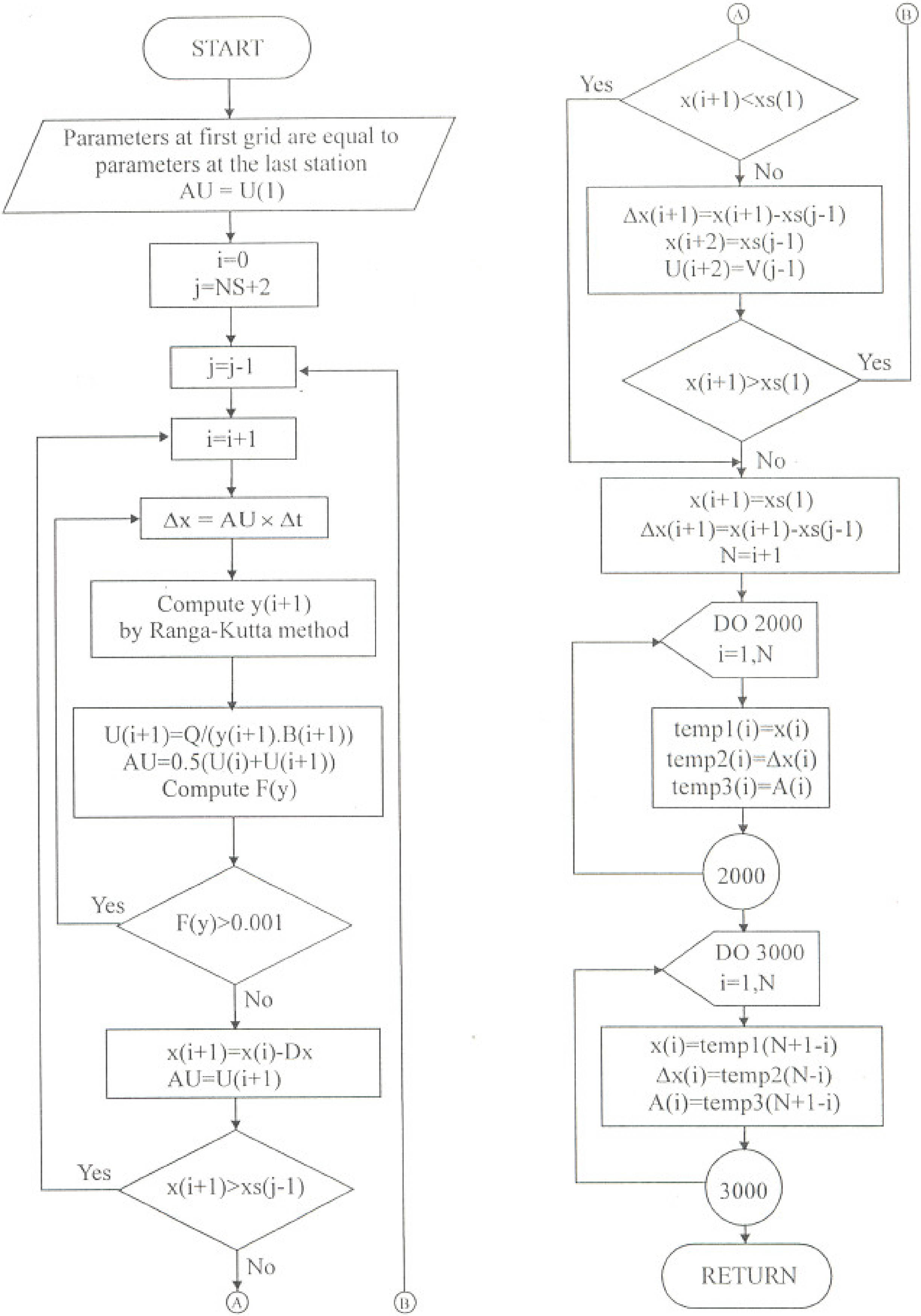


Fig. 3.4 Flow Chart for Subroutine, GRID

The initial and boundary conditions may be summarized as follow

Initial Condition:

(a) $C(x, t_0) \quad 0$ For $x \geq 0$ (3.28)

Boundary Conditions:

(a) $C(0,t) = \text{Known}$ For all t (3.29)

(b) $C(\infty,t) = 0$ For $t \geq 0$ (3.30)

3.3 DETERMINATION OF D_L USING THE NUMERICAL MODEL

As pointed out by Fischer (1966), the change of moment method produces unrealistic values for the dispersion coefficient particularly in natural streams. Routing procedure is also not accurate particularly for non-Gaussian concentration distribution. The diffusive transport method needs to use a number of parameters, which are not so easy to observe in natural streams. The change of moment method and routing procedure have been used, so far, in uniform flow. But in natural streams the flow is generally non-uniform. Therefore, it is proposed to extend the proposed numerical method for the purpose of determination of D_L value in non-uniform flow. The method of one-dimensional grid search used for this purpose is described below.

3.3.1 One-Dimensional Grid Search Method

This method is based on the Bisection procedure, which is used to find roots of one-dimensional functions. Also one can find the optimum value of D_L , which is such that it produces maximum agreement between the predicted and the observed C-t curves at the stations. The agreement between predicted and observed C-t curves has been judged in the present study in terms of the error (ERS), which is defined as (see Fig. 3.5)

$$\text{ERS} = \left(1 - \frac{\text{Overlapping (shaded) area}}{\text{Area of observed C - t curve}}\right) \times 100\% \tag{3.31}$$

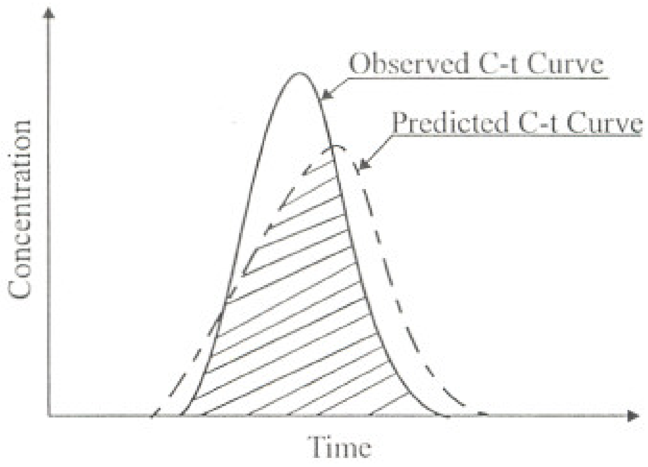


Fig. 3.5 Definition of Error

The measured C-t curve at the first station is used as input and C-t curves at the other downstream stations are predicted by the proposed numerical scheme. Using some trial value of D_L the error at each station is calculated by Eq. (3.31) and the average value of error (ERS) is obtained. The value of D_L producing minimum value of ERS is considered as the optimum one.

A global optimum value of dispersion coefficient exists due to parabolic nature of Eq. (1.1) (see Fig. 3.6). The initial approximate value of D_L is calculated by change of moment method. Let the initial D_L correspond to point ① (see Fig. 3.6). An incremental value, ΔD_L of dispersion coefficient equal to one tenth of the initial D_L is taken and error is calculated at the points ② and ③. The point corresponding to minimum error is searched. Let in this case, the point be ②. Now error is calculated at the next point ④ and again minimum error is searched; let that point be again ②. This means that the optimum value of D_L lies somewhere in the bracketing triplet (①,② and ④). Now the grid is bisected and ERS(5) and ERS(6) are calculated. If ERS(2) < (ERS(5) and ERS(6)) then the grid is again bisected and same process is followed, until one reaches upto the desired accuracy. But if ERS(6) or ERS(5) is minimum then the same process is continued in the required direction.

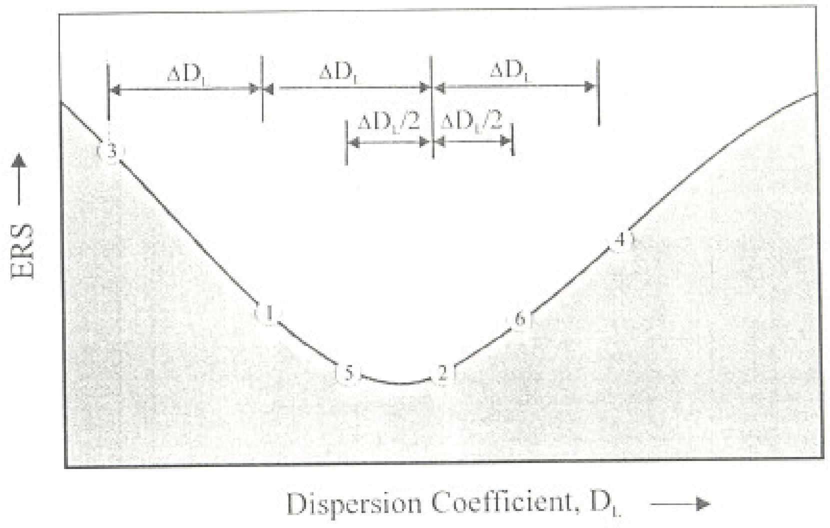


Fig. 3.6 One-Dimensional Grid Search Method

The computer code for the above algorithm for determination of D_L using the proposed numerical model is described by a flow chart given in Fig. 3.7.

3.4 PROCEDURE FOR COMPUTATION OF C-t CURVES

The stepwise procedure for the determination of C-t curves in clear-water and sediment-laden flows using the channel and flow characteristics, D_L value and the concentration curve of tracer which is input to the channel is as below:

i) Determine the concentration at the input station at equal interval of time Δt through interpolation.

ii) Generate the computational spatial grids from known values of velocity along the channel. Estimate the values of velocity at intermediate computational sections using the gradually varied flow computation as explained in section 3.2.4.

iii) Using the proposed numerical scheme compute the C-t curves at the desired stations in downstream.

The computer code for the above procedure is described in a flow chart given in Fig. 3.8.

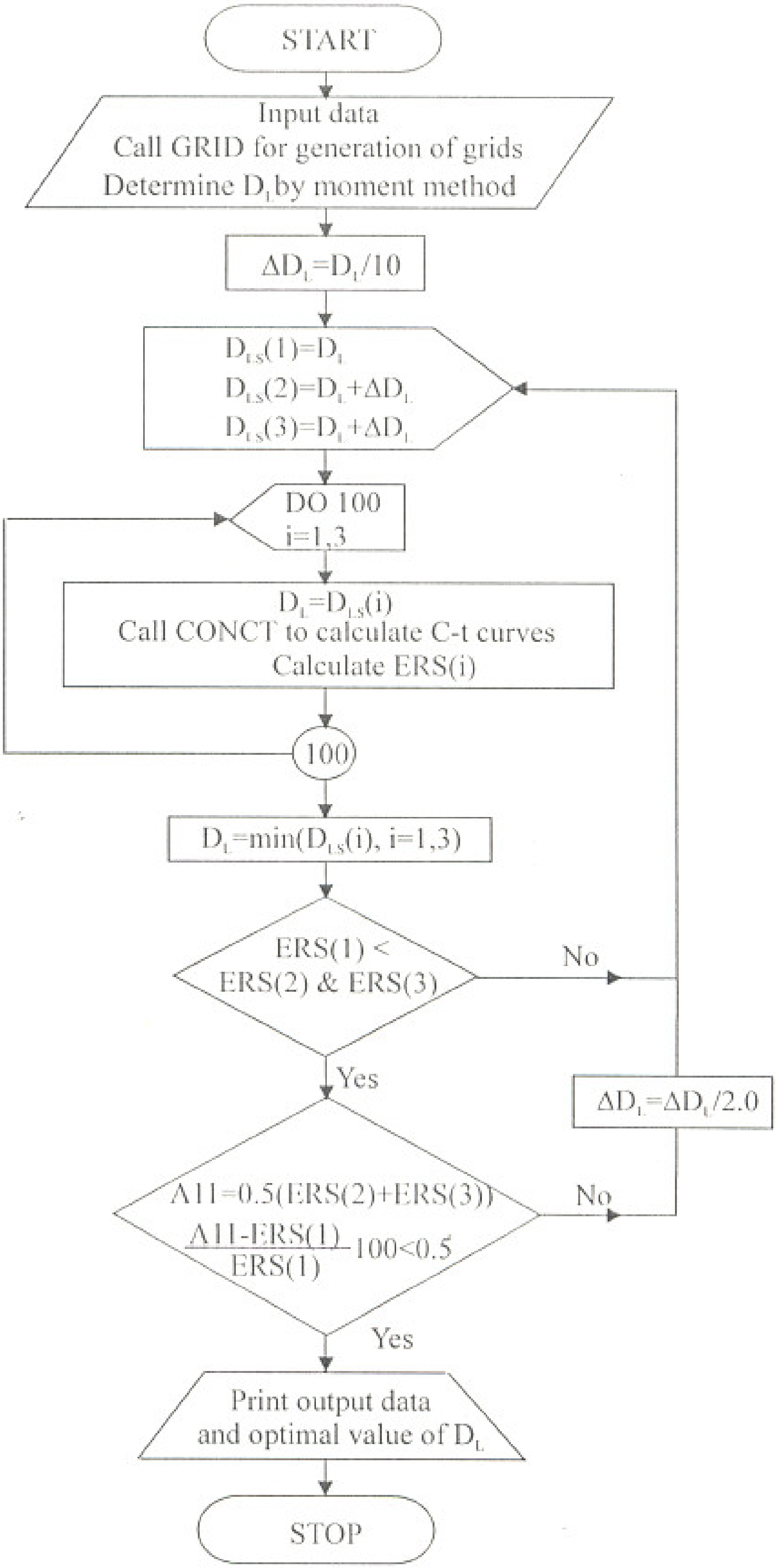


Fig. 3.7 Flow Chart for Determination of D_L

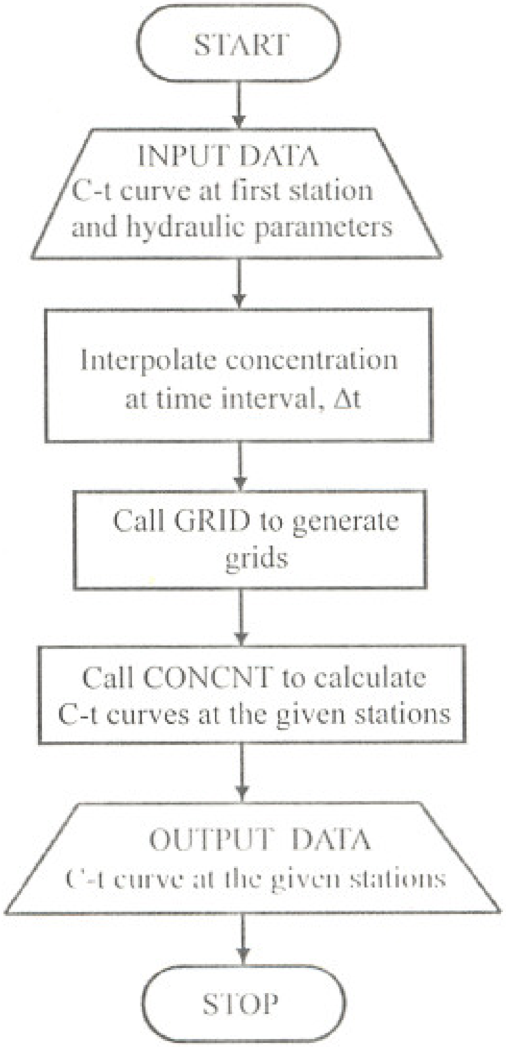


Fig. 3.8 Flow Chart for Determination of C-t Curves

3.5 THE COMPUTER CODE

The formulations described in this chapter are developed into a computer program to study the various aspects pertaining to the longitudinal dispersion of conservative pollutants in the open channels. The computer code has been developed in FORTRAN-77 and has a modular structure which makes it easy to incorporate new features or to test the program without invoking some existing features.

The flow charts for various modules of the programs have been shown in Figs. 3.2, 3.4, 3.7 and 3.8. The computer program has been tested on IBM compatible PC-Pentium and

TATA ELXSI POWER SERIES 3220 main frame computer at University of Roorkee, Roorkee using IRIX FORTRAN-77 compiler version 4.0.5.

3.6 CONCLUDING REMARKS

A numerical scheme based on the Combined operator approach is proposed for the solution of the longitudinal dispersion equation when U, A and D_L vary with distance. Exact solution of advection process has been obtained by developing a variable spatial grid so that the root of the trajectory of the concentration characteristic passes through the nodal points. For diffusion process, Crank-Nicholson scheme has been used. The proposed scheme possesses a computational accuracy of the second order. By introducing the weighting coefficient to the pollutant concentration in the time stepping the scheme could become highly accurate in the prediction of pollutant concentration. The proposed scheme has been extended by incorporating in it the one-dimensional grid search method for determination of D_L values.

4

EXPERIMENTAL PROGRAMME

4.1 INTRODUCTION

This chapter describes the experimental work carried out for the present study. The experiments were conducted in the Hydraulics Laboratory of the University of Roorkee, Roorkee. In addition, a large quantity of data from earlier field and laboratory studies was compiled. Details regarding these are given below.

The review of literature as presented in Chapter-II and the theoretical work presented in Chapter-III revealed that more detailed data are required on the following aspects of longitudinal dispersion.

(i) Determination of the mixing length.
(ii) Dispersion in clear-water flows.
(iii) Dispersion in sediment-laden flows.

The present experimental programme was aimed at acquiring data on all these aspects.

4.2 EQUIPMENT

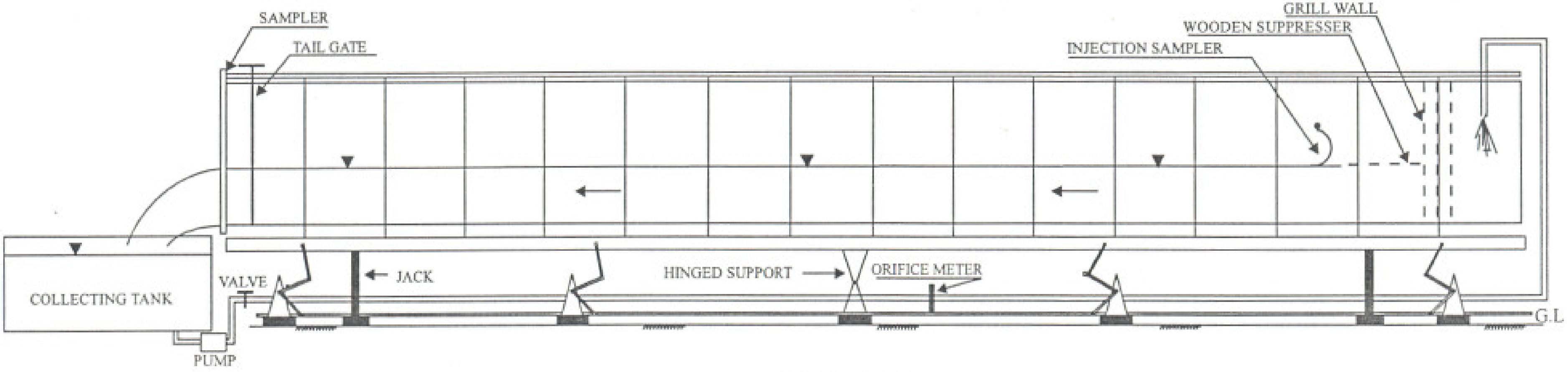
The experiments were conducted in a rectangular flume. An Orificemeter was used to measure the discharge through the flume. Point velocities were measured by a Prandtl Pitot-tube. A Fluorometer was used to measure the tracer concentration downstream of the injection point. Tracer injection sampler and collecting sampler were used to inject the tracer and collect the tracer respectively. The fluorescent dye used as the tracer was Rhodamine WT. This Rhodamine dye is very popular as a hydrologic tracer because of

(i) its conservative nature.
(ii) neutral behaviour with water and sediment.
(iii) its high sensitivity.
(iv) satisfactory recovery ratio and
(v) its detectability and simplicity of fluorometric testing procedures.

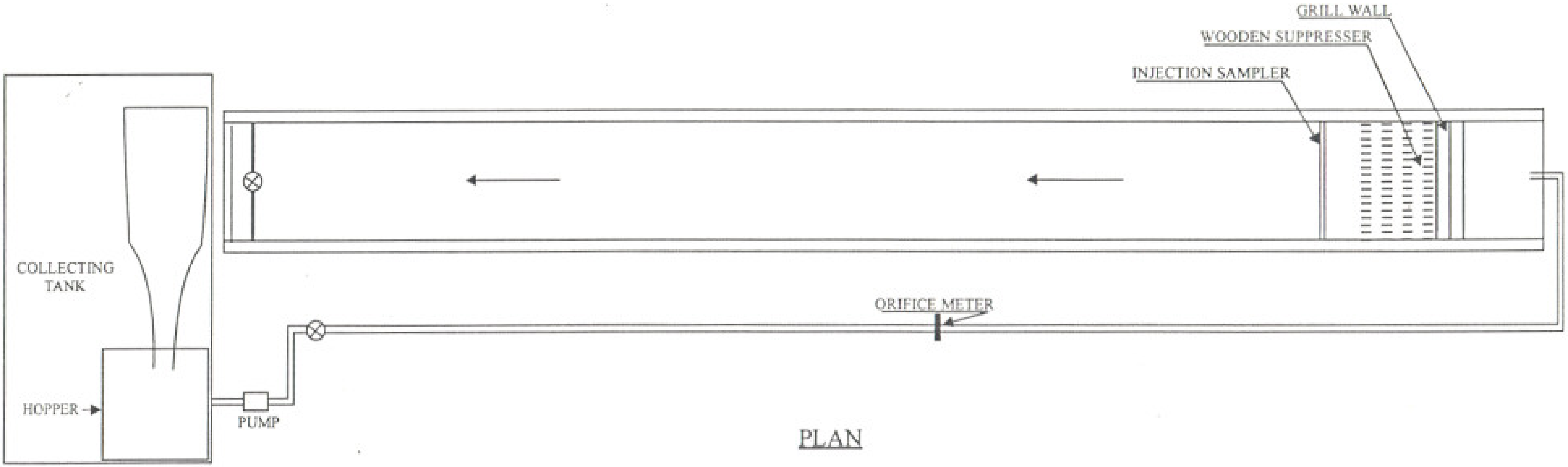
4.2.1 Flume

The flume used in the present study is a 20 cm wide, 50 cm deep and 30 m long recirculatory tilting flume (see Fig. 4.1). A photograph of the flume is shown in Fig. 4.2. The flume is provided with glass side wall on one side and a painted steel sheet wall on the other side. The presence of different materials on two sides necessitates a modification to the side wall correction procedure valid for a flume with the same material on both sides. Such a modified method was given by Soni (1975) and the same was used for computations. The bed of the flume was made smooth using cement plaster. The flume is also provided with circular pipe rails (adjusted parallel to the bed) for positioning the pointer gauge, tracer injection sampler and continuous tracer sampling system.

The recirculating system of the flume consisted of a rectangular tank with a hopper for collecting the sediment at the downstream end of the flume (see Fig. 4.1). A pump is connected with the tank and a supply pipe for maintaining the recirculation. The supply line is connected to the upstream end of the flume. An orificemeter is installed in the supply pipe for the measurement of discharge and this was calibrated. The discharge was controlled by a valve. Three brick grill walls and a floating wooden wave suppresser were provided at the entrance of the flume for damping the disturbances at the free surface (see Fig. 4.3).



ELEVATION



PLAN

Fig. 4.1 Layout of the Flume

Fig. 4.2 A Photograph of the Flume

Fig. 4.3 A Photograph of Grill Wall, Wooden Suppresser and Tracer Injection Sampler

4.2.2 Accessories of the Flume

(i) Orificemeter

An orificemeter (designated as A in Fig. 4.6) with a 7.5 cm diameter orifice installed in the 10 cm diameter supply pipe was used to measure the discharge through the flume. The orificemeter was calibrated using a sharp-crested weir installed at the downstream end of the flume. The calibration curve for the orificemeter is shown in Fig. 4.4.

(ii) Prandtl Pitot-tube

Prandtl Pitot-tube of diameter 1 mm was used to measure the velocity of flow in clear-water flow. In case of the sediment-laden flow the Pitot-tube having 3 mm aperture size was used to avoid the risk of clogging. Regular cleaning of the Pitot-tube was done for its proper functioning. The Pitot-tube was connected to an inclined manometer for measuring the pressure difference.

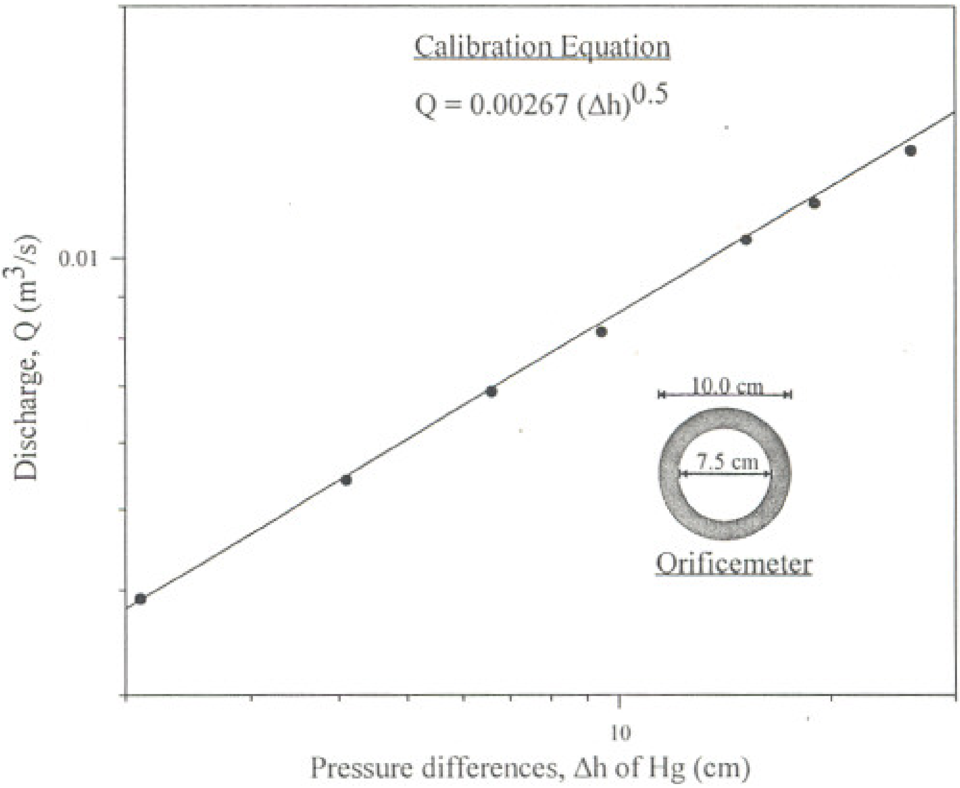


Fig. 4.4 Calibration Chart of Orificemeter-A

(iii) Tracer injection sampler

The tracer was introduced in the flume as a slug by an injection device which consisted of a metallic tray of semi-circular cross-section of 10 cm diameter. The tray was hinged to the sides of the flume keeping it in a horizontal position spanning the full width of the flume. Tracer solution was filled up in it and it was positioned slightly above the water surface. The tray was tipped instantaneously to inject the tracer (see Figs. 4.3 and 4.5)

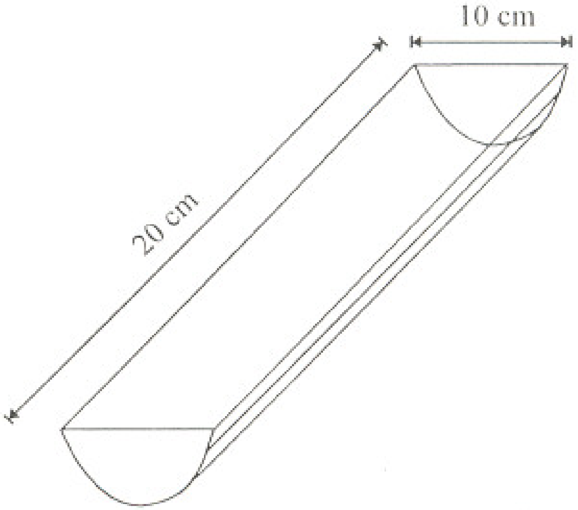


Fig. 4.5 Tracer Injection Sampler

(iv) Tracer concentration monitoring

The Rhodamine dye WT was injected using the injection sampler at one section in the upstream part of the flume. Concentration samples were collected continuously by using another sampler at four different stations in the flume located downstream of the injection section. These concentration samples were passed through the Fluorometer for monitoring. The samples were returned to the flume with the help of a Tullu-pump. An orificemeter (Designated as B in Fig. 4.6) was used to measure the discharge of this pump while a valve (Designated as B in Fig. 4.6) was used to regulate the discharge. The line diagram of water flow for the whole system is depicted in Fig. 4.6. The calibration chart for orificemeter-B is shown in Fig . 4.7.

The valve-B was so regulated as to maintain a velocity through the monitoring system which is equal to the velocity of the flow in the channel. A photograph of the monitoring system is shown in Fig. 4.8.

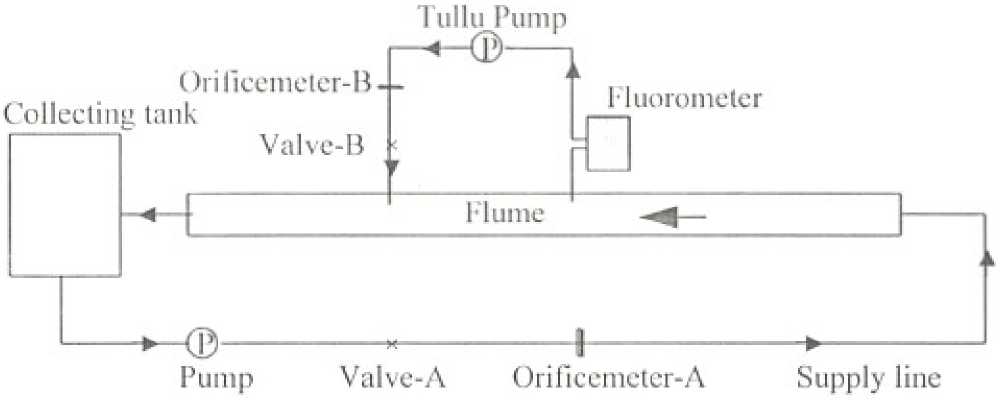


Fig. 4.6 Line Diagram of Water Flow

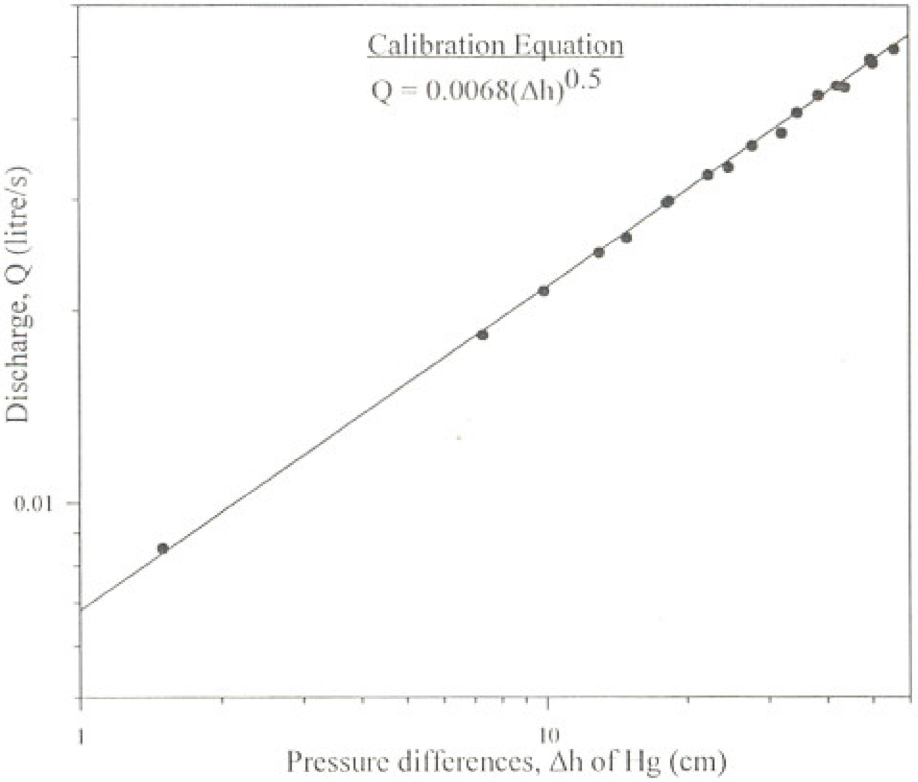


Fig. 4.7 Calibration Chart of Orificemeter-B

Fig. 4.8 A Photograph of Tracer Monitoring System

(v) Fluorometer

Fluorometer is an instrument which is used to measure the concentration of various analytes in samples of interest via fluorescence. The Fluorometer used in this study was model 10 AU - 005 from the Turner Designs Sunnyvale, USA, 1991.

The operating principle of the Fluorometer 10 - AU is based on the properties of the fluorescent material from which it absorbs light at one wavelength and almost instantly emits light at a new and longer wavelength. Light from a light source (the lamp) is passed through an excitation filter that transmits light of the chosen wavelength range. This light passes through the sample which emits light proportional to the concentration of the fluorescent material present and proportional to the intensity of the exciting light. The emitted light is passed through another optical filter (emission filter) to prevent any scattered exciting light from reaching the detector (in this case a photomultiplier tube) and to pass the emitted color that is specific to the analyte of the interest. The photomultiplier tube looks something like a vacuum tube. It generates electrons (electric current) in response to photons (light). The photomultiplier contains nine stages to multiply the electrons coming from the previous stage. Since the reference light path is taken from the same source of light, it makes the instrument stable. The optical system of the Fluorometer 10-AU is given in Fig. 4.9.

Operation of Fluorometer consists of three basic steps: (i) Activation and initial verification of operational parameters; activation involves supplying power and operational parameters involve review of default values which are assigned according to application. (ii) Calibration, which consists of setting the sensitivity of the instrument to a level appropriate to the samples and blank. A sample of known concentration is used as standard. (iii) Running the samples. Once the instrument is calibrated, simply inserting a cuvette containing sample into the sample compartment or by letting the sample flow through the flow cell, the concentration is read on the home screen. The data are stored in the Fluorometer. The Fluorometer can log upto 64,800 data points, including index, time and date, sample read out and temperature. The log up data is sent to an external Computer (IBM-compatible) through the data logging software. The data logging interval

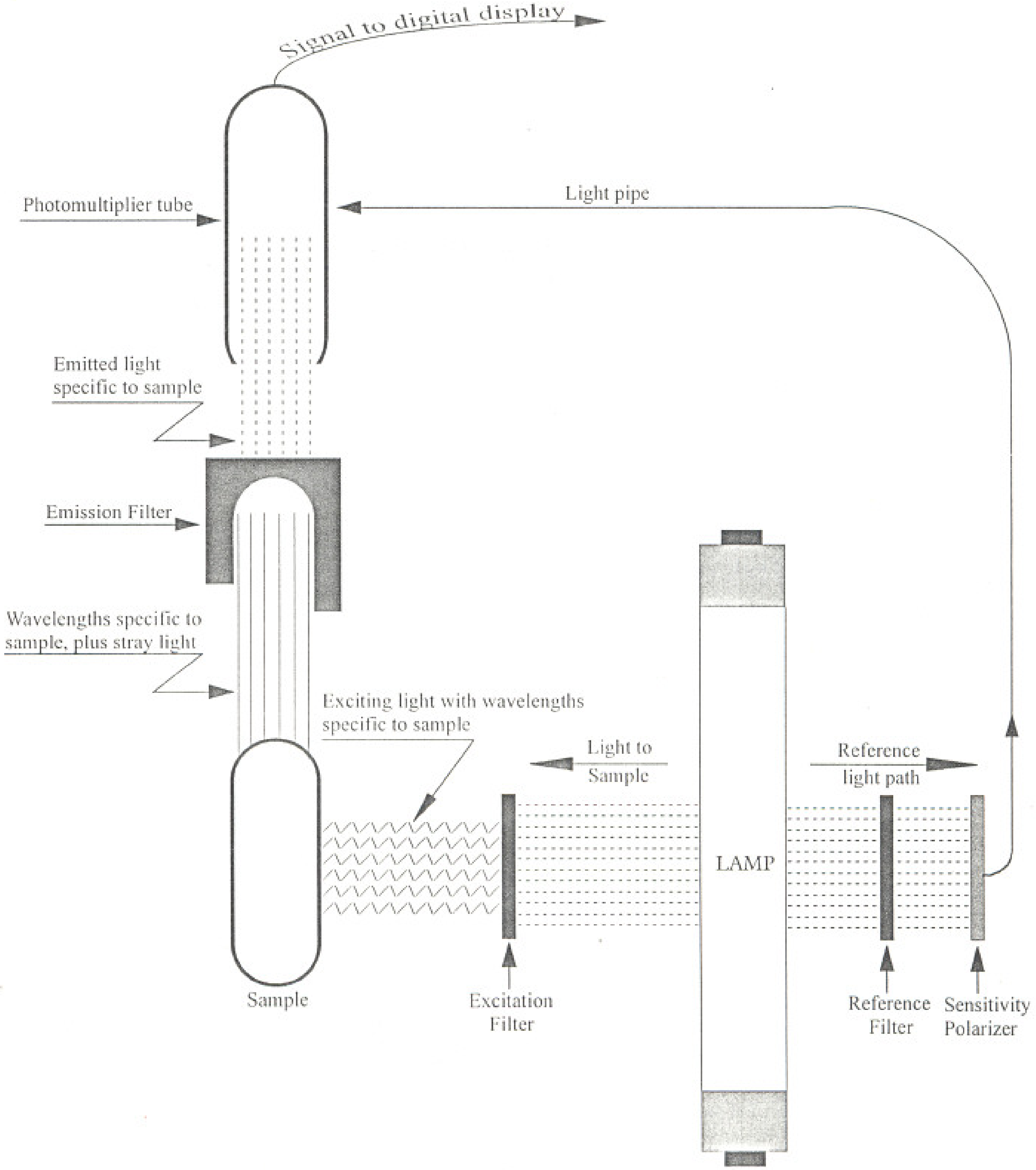


Fig. 4.9 Optical System of the Fluorometer Model 10 - AU-005

can be chosen as 1, 2, 3, 5, 10, 20 or 30 seconds. The limit of detectability of the instrument is 5 parts per trillion of Rhodamine WT in potable water.

(vi) Tracer collecting sampler

The tracer collecting sampler, designed and fabricated in the laboratory, consisted of a sampling probe having three vertical copper tubes of 6 mm diameter (see Fig. 4.10) fixed at 8 cm centre to centre spacing, and pointing against the flow direction. The purpose of providing three tubes is to collect continuously the width-integrated sample. The sample from these tubes entered into a manifold of 35 mm diameter and 15 mm height, whose outlet is connected to the inlet point of the Fluorometer through a 12.5 mm diameter polyethylene pipe. The small diameters of the tubes ensured that dispersion in the connecting tubes is negligible. The use of manifold ensured that a width-integrated sample having a concentration equal to the cross-sectional average concentration of tracer in the flume is flowing through the Fluorometer.

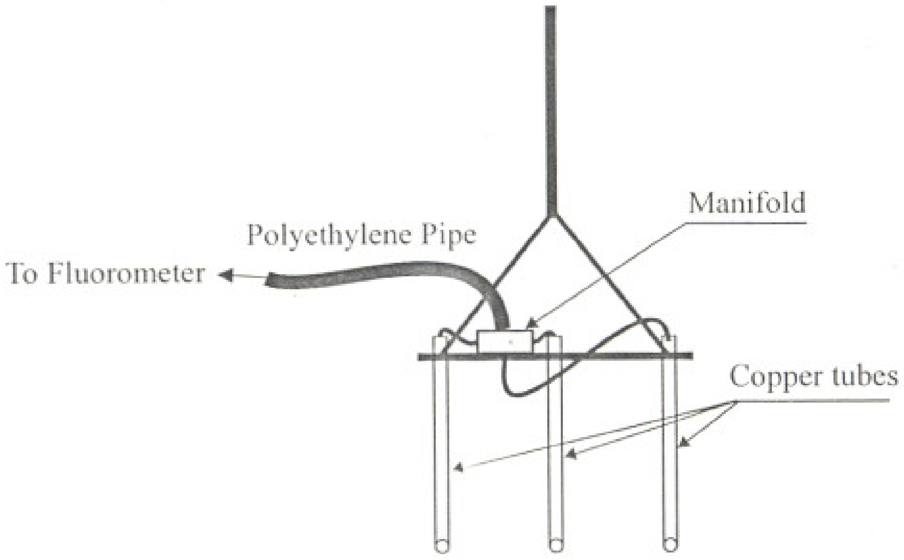


Fig. 4.10 Tracer Collecting Sampler

(vii) Sediment sampling and sediments used

To know the effect of suspended sediment on the dispersion, experiments have been conducted in sediment-laden flows. For each set of experiments in sediment-laden flow, a sample for the sediment load was taken.

Fine sand was used as sediment in the present experimental work, so that only suspended load occurs even during the low velocities. Two sizes of sand were used. The first size of sand passed through 90 μ sieve and was retained on 45 μ sieve giving an average diameter equal to 0.064 mm. The second size of sediment passed through 180 μ sieve and was retained on 150 μ sieve, giving average diameter equal to 0.164 mm. The relative density of these sands was 2.65. The average concentration of sediments in suspension was varied from 500 ppm to 11,000 ppm by volume.

The average concentration of the suspended load in the flow was measured by a width-integrating sampler (see Fig. 4.11) capable of sampling the entire depth. This sampler was installed at the downstream end of the flume. The sample of sediment-laden flow was collected from the sampler through a rubber tube in a collector. The collected samples from the collector were filtered through a thick filter paper. The filtered material was dried in an oven and were weighed on a chemical balance and concentration in parts per million (ppm) for each sample was determined.

4.3 EXPERIMENTAL PROCEDURE

In the beginning the bed and railing of the flume were made parallel to each other by adjusting the railing screws. The bed slope was measured using a surveyor's level and pointer gauge and also by a second method using glass boxes. Two glass boxes connected to each other through a rubber tube were placed at two locations and filled with water. The water levels in the boxes were measured. The difference in water level divided by the distance between the two locations gives the slope of the bed.

To cover a wide range of data, experiments were conducted for three values of bed slopes equal to 4.545×10^{-3}, 2.470×10^{-3} and 1.488×10^{-3} designated as SET1, SET2 and SET3

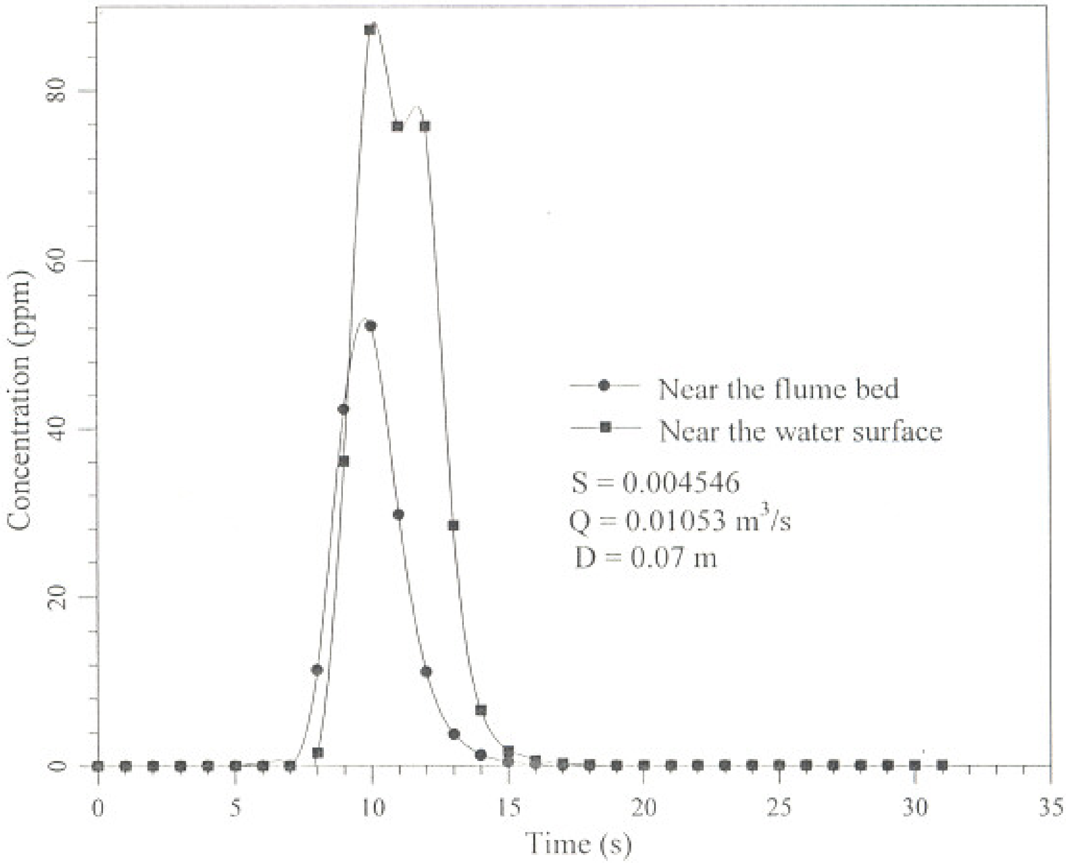
Fig. 4.11 A Photograph of Sediment Collecting Sampler

respectively. For each set, the experiments were carried out for three or more than three discharges. For each discharge, the uniform flow was first maintained by adjusting the tailgate and measuring the depth of flow along the flume. Once uniform flow was established the mixing length was determined. After the mixing length determination, clear- water runs were taken. Sediment-laden flow runs followed the clear-water runs. A minimum of three runs were taken for the sediment-laden flow for each discharge by increasing the sediment concentration.

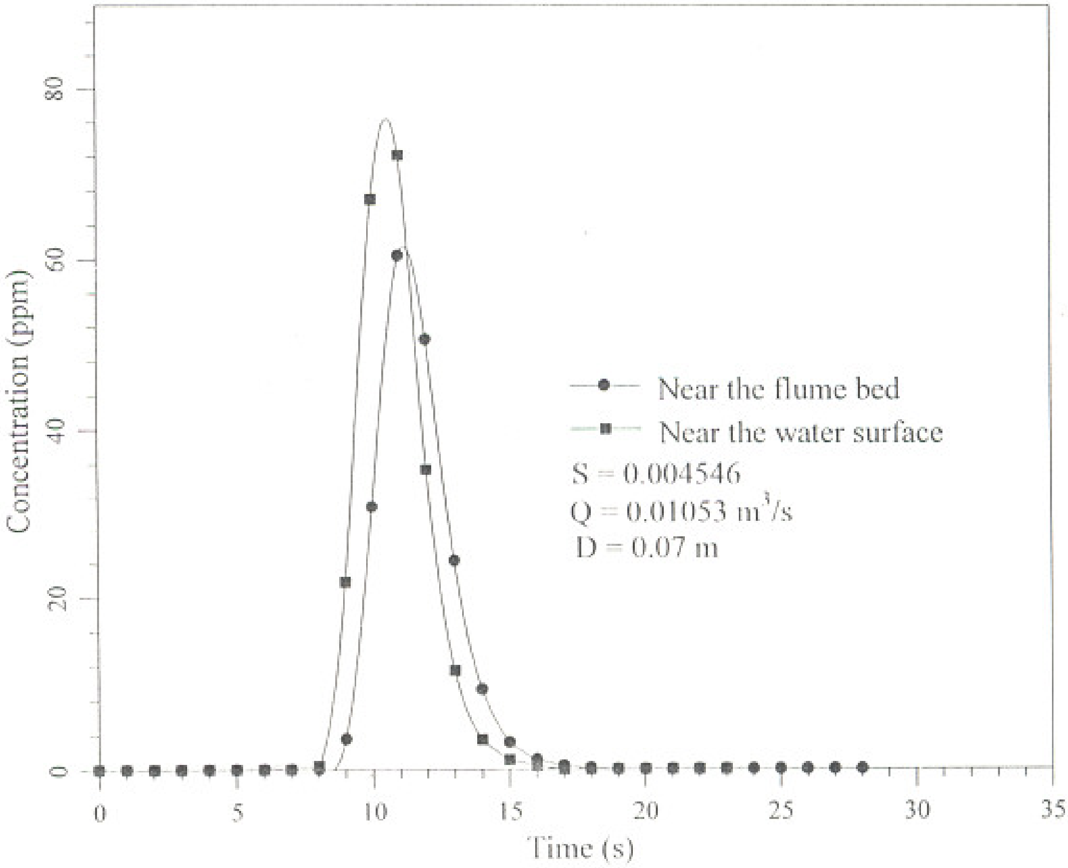
4.3.1 Mixing Length

The tracer injection sampler was fitted near the upstream end of the flume. The collecting sampler was located at a station which was near to the injection section. At that station, the sampler was placed just below the water surface. The monitoring system was activated. Valve-B was regulated to equate the velocity through monitoring system to the velocity in the flume. The required quantity of the dye solution was poured into the injection tray and was tipped instantaneously and in synchronisation with the switching on of the data logging of the Fluorometer. After the passage of dye cloud the data logging was stopped. Now the collecting sampler was lowered and placed near the bed at the same station. An equal quantity of dye solution was injected and data was again logged up in the Fluorometer. Between the two tracer injections a sufficient time was allowed to thoroughly mix up the dye in the recirculating water.

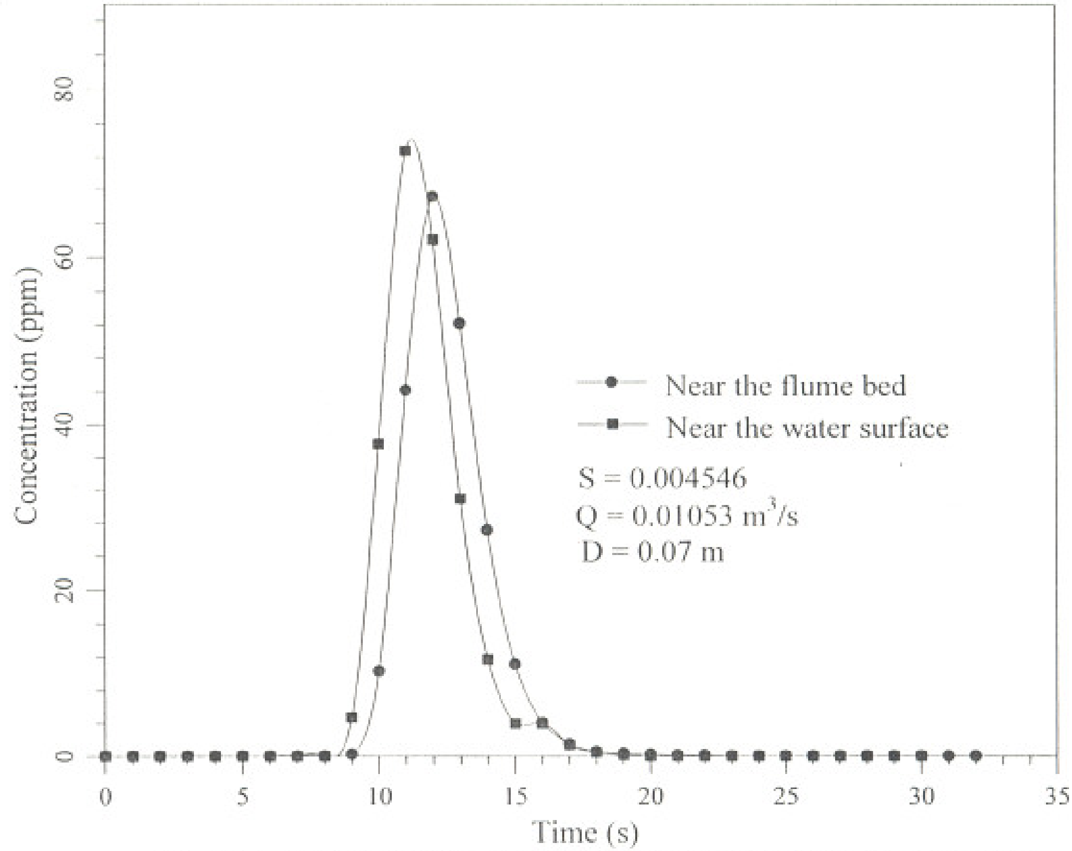
The movable trolley housing the monitoring system was shifted to another station which was one metre downstream from the first station. In the same way as at the first station, the C-t curves at just below the water surface and near the bed were observed using the same quantity of dye. This procedure was repeated for a number of stations. The C-t curves near the bed and at the water surface of a station were compared. The station, where both the C-t curves are practically the same is the one where mixing has taken place over the cross-section and the distance of the first such station from the injection station is the mixing length. For example, Fig. 4.12(a) to Fig. 4.12(d) illustrate the above procedure for determination of the mixing length for a particular run. In this particular



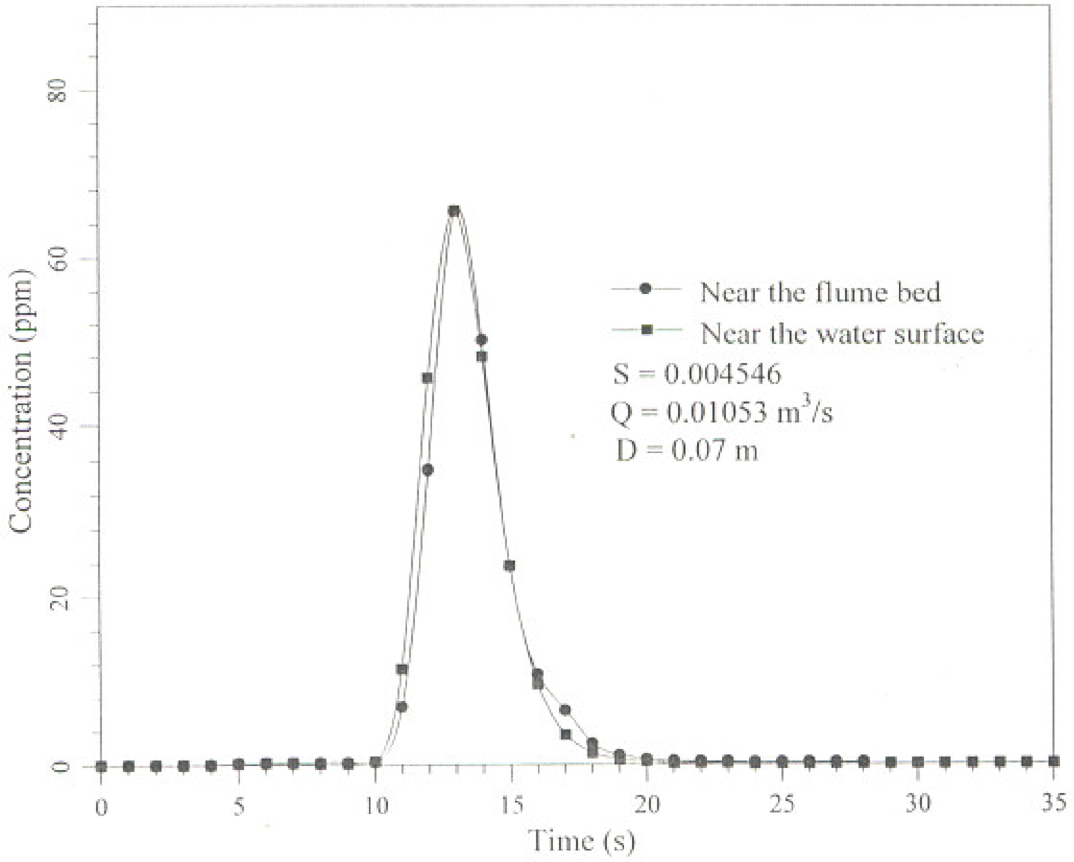
(a) Observed C-t Curves at 2.0 m Distance from the Injection Station



(b) Observed C-t Curves at 3.0 m Distance from the Injection Station



(c) Observed C-t Curves at 4.0 m Distance from the Injection Station



(d) Observed C-t Curves at 5.0 m Distance from the Injection Station

Fig. 4.12 Determination of Mixing Length case the mixing length lies between 4 m to 5 m from the injection station. Also, Table 4.1 gives the estimates of mixing length as obtained through the use of existing predictors for the run described above. A wide variation in these values may be noted.

Table 4.1 : Estimated Mixing Length

Investigator(s)	Mixing length, L_m (m)
Fischer, 1967	7.67
Kilpatrick *et al.*, 1967	4.01
McQuivey and Keefer, 1976	1.28
Yotsukura and Cobb, 1972	2.21
Ward, 1973	3.91

4.3.2 Clear-Water Flow

Once the mixing length for a particular discharge and bed slope was known, the next step was to locate the stations where the C-t curves had to be measured. Four stations were fixed for each discharge at equal spacing. The range of the station-spacing was from 4m to 6m depending upon the discharge and bed slope of the flume. To measure the C-t curves at the first station, the collecting sampler was placed at mid depth of flow. A known amount of dye was injected through the injector and data logging was started at the same time. The monitoring system was then moved to the second, third and fourth stations and C-t curves were observed for the same quantity of dye injection. Fig. 4.13 illustrate the observed C-t curves for one typical clear-water run.

4.3.3 Sediment-Laden Flow

After the clear-water runs were completed a predetermined amount of sediment was added to the recirculating system and sufficient time was given for mixing the sediment uniformly in the recirculating water. Then C-t curves at all the four stations were observed in the same manner as described earlier for clear-water flow.

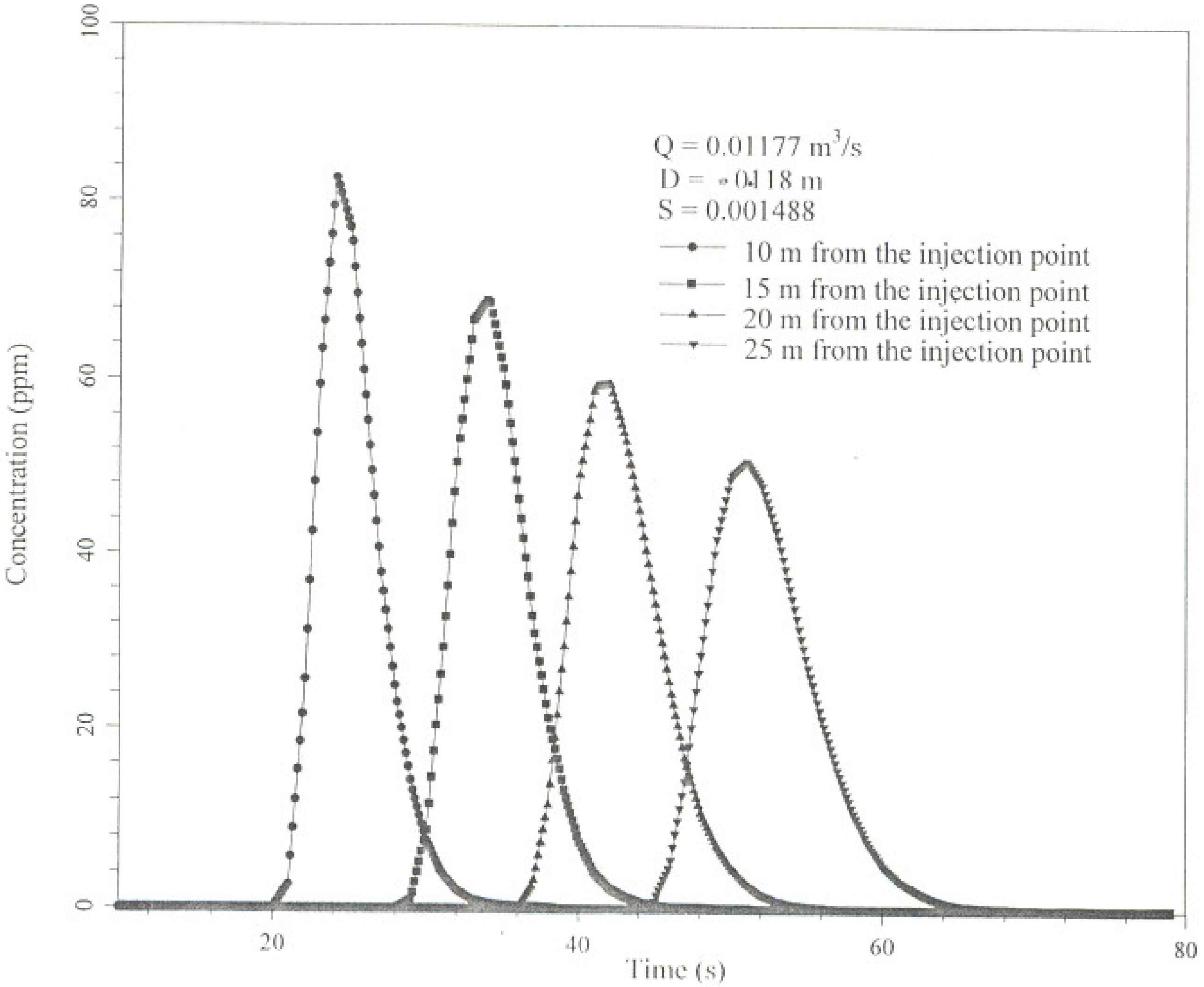


Fig. 4.13 A Typical Series of Observed C-t Curves

The sediment load was determined by collecting the sample in the collector described earlier. The collected sample was passed through a filter paper and dried in an oven. The weight of sediment was measured by a chemical balance. The average concentration of sediment was then expressed in parts per million (ppm) as volume of solids per unit volume of sediment water mixture.

More sediment was then added to the recirculating water and at least two more runs for two more sediment concentrations were taken. After taking the runs at different sediment concentrations for a given discharge, the water-sediment mixture was removed and the tank filled up with the fresh water. The same discharge of clear water was maintained.

The second sediment was then added to the recirculating water and at least three runs were taken at different sediment concentrations.

The log-up data in the Fluorometer were transferred to a 386 personal computer by internal data logging software (IDL). Further analysis of data was made through the computer.

For each run either in clear-water or sediment-laden flow, the velocity distribution at a section was measured by Prandtl Pitot-tube. As already stated, the withdrawal velocity (during measurement of tracer concentration) was kept equal to the velocity in the flume to obtain the true concentration.

All the experimental data collected during the present study are listed in Appendix-I and Appendix-II.

4.4 DESCRIPTION OF DATA OF PREVIOUS INVESTIGATORS USED IN THE PRESENT STUDY

To cover a wide range of relevant parameters in the analysis, laboratory and field data collected by previous investigators during various dispersion and time of travel studies have been compiled and used. For all the data used herein, it was reported that the observed C-t curves were obtained beyond the convective period. Some of these data were compiled by Singh (1987). These and many more data sets compiled during this study may be grouped into the following four categories.

(i) The data for which the observed C-t curves for clear-water flow and sediment-laden flow under the corresponding hydraulic conditions were available (see Table 4.2(A)).

(ii) The data wherein observed C-t curves are available, flow properties and channel properties are known. Sediment load is not reported. (see Table 4.2(B)).

(iii) The data wherein observed C-t curves are not available but salient points of C-t curves are known. The flow properties and channel properties are also known. Sediment load is not reported. (see Table 4.2(C)).

Table 4.2 : Sources and other Relevant Information of Previous Investigators Data

(A) First Group

Sl. No.	Investigators & Data Source	Type of Study	No. of Flumes/Streams	No. of Experiments	No. of C-t Curves used	Size of Sediments	Method used for computation of D_L
1	Singh (1987)	Laboratory	1	43	169	0.058 mm and 0.0058 mm	Present study*

(B) Second Group

2	Fischer, Report No. KH-R-12, CALTECH, (1966)	Laboratory	3	10	26	---	Present study
3	Beltaos and Day Report No. REH/76/1 T.S.W.E.D. Alberta Research Council, (1976)	Field	1	1	7	--	Present study
4	Nordin and Sabol USGS, Report No. 20-74 Water Resour. Investigations, (1974)	Field	13	13	52	--	Present study
5	Berkas, Written communication with Ranga Raju, (1986)	Field	1	3	8	---	Present study
6	James and Helinsky, Report No. 84-4203 Water Resour. Investigation, (1984)	Field	1	3	9	---	Present study
7	McQuivey and Keefer, (1976)	Field	1	1	5	---	Present study

Table 4.2 Continued .....

(C) Third Group

8	Graf, USGS, Open File Report 84-868, (1984)	Field	10	23	47	---	Present study
9	Calandro, USGS, Report No. 17, (1978)	Field	11	23	64	---	Present study
10	Taylor, Information Circular 9 Maryland Geological Survey, Baltimore, Maryland, (1970)	Field	1	3	25	---	Present study

(D) Fourth Group

11	Hou and Christensen,(1976)	Field & Laboratory	20	45	---	---	Routing Procedure
12	Dorban, (1982)	Field	1	1	---	---	Routing Procedure
13	McQuivey and Keefer, (1974)*	Field	17	36	---	---	by their own equation
14	Miller and Richardson, (1974)	Laboratory	1	9	---	---	Change of Moments
15	Liu, (1977)	Field	---	13	---	---	---

* Described in section 3.3.1.

(iv) The data wherein the C-t curves are not known but values for D_L, flow properties and channel properties are known. Sediment load is not reported. (see Table 4.2(D)).

The source and other relevant information pertaining to other data used in the present study is summarised in Table 4.2. All the data used during the present study are listed in Appendix-I. Table of range for these data is given later.

4.5 DATA REDUCTION

For field data, the observed concentrations were adjusted for zero tracer loss by dividing the observed concentrations by the recovery ratio, R_R. For this the approach as used by Singh (1987) was followed. If the stream discharge was constant in the study reach no further adjustments of concentrations were done but for those cases where discharge changed along the study reach, the concentrations adjusted to zero dye loss were further corrected by multiplying each concentration value by the ratio $\frac{Q}{Q_i}$, where Q_i is the initial discharge at the first sampling site in the study reach. Such adjustment to the concentrations of the observed C-t curves was also made by Nordin and Sabol (1974).

If the discharge changed in the study reach, in addition to the above adjustment in concentration, the depth D, the width B, and the mean velocity U, at each of the downstream sampling sites were also modified corresponding to the initial discharge using Leopold and Maddock's (1953) relations. These relations describe the variation of mean depth, width and velocity at a particular cross-section with variations in discharge based on the data collected on several American streams. Leopold and Maddock showed that the relations can be expressed as follows:

$$D = C_D Q^{m_D} \tag{4.1}$$

$$U = C_U Q^{m_U} \tag{4.2}$$

and

$$B = C_B Q^{m_B} \tag{4.3}$$

where C_D, C_U, C_B, m_D, m_U and m_B are constants and may be determined using two sets of observed D, U, B and discharge Q values. The sum of m_D, m_U and m_B is unity. After modifying the depth, width and mean velocity as per the above relations, the average value of these characteristics were obtained for the study reach.

4.6 RANGE OF DATA

The range of data collected in the present study is given in Table 4.3. The range of data compiled from the earlier studies is given in Table 4.4.

Table 4.3 : Range of Data Collected in the Present Study

Sl. No.	Parameters	Range
1	Slope	1.488×10^{-3} to 4.546×10^{-3}
2	Discharge	21.88×10^{-3} m^3/s to 0.5889×10^{-3} m^3/s
3	Flow velocity	0.4352 m/s to 0.9117 m/s
4	Depth of flow	0.058 m to 0.1388 m
5	Dispersion Coefficient	0.02273 m^2/s to 0.11451 m^2/s
6	Width of Flume	0.20 m
7	Spacing of stations	4.0m to 6.0 m
8	Hydraulic radius, R	0.039m to 0.0547 m
9	Size of sediment	0.064 mm to 0.164 mm
10	Concentration of sediment	462.26 ppm to 10995.5 ppm

Table 4.4 : Range of Data Compiled from Earlier Studies

Sl. No.	Investigators	No. of data	D_L (m^2/s) Min. Max.	B (m) Min. Max.	D (m) Min. Max.	Q (m3/s) Min. Max.	S ---- Min. Max
1	Singh (1987)	28	0.00147 0.0805	0.4 0.4	0.034 0.155	0.008 0.041	0.0013 0.00603
2	Fischer (1966)	10	0.01 0.5306	0.317 1.100	0.021 0.184	0.00603 0.05101	0.000267 0.00104
3	Beltaos and Day (1976)	6	11.0 20.0	9.96 54.6	2.49 3.74	70.51 80.94	0.0001 0.0001
4	Nordin and Sabol (1974)	13	11.53 1417.81	12.34 739.14	0.25 17.51	1.26 7126.0	0.000007 0.0006
5	Calandro (1978)	23	0.52 6800.0	7.92 268.22	0.24 3.66	1.11 251.45	0.000007 0.000365
6	James and Helinsky (1984)	3	10.35 12.75	9.36 12.95	0.30 0.61	0.62 2.22	0.00341 0.00341
7	McQuivey and Keefer (1976)	4	139.68 295.07	867.00 867.00	16.76 16.76	18454.27 21636.54	0.000002 0.000002
8	Taylor (1970)	9	4.29 350.0	34.9 48.46	0.29 0.88	2.10 21.24	0.000556 0.000956
9	Dorban (1982)	4	1.24 3.36	8.6 12.0	0.285 0.30	0.95 1.18	0.00087 0.001583
10	McQuivey and Keefer (1974)	36	4.64 891.84	12.5 201.17	0.25 3.84	0.99 933.0	0.00015 0.00355
11	Miller and Richardson (1974)	8	0.0522 6.12	0.5974 0.5974	0.1247 0.132	0.0231 0.0606	0.001 0.0296

Table 4.4 Continued ...

12	Berkas (1986)	5	14.14	29.87	1.25	35.2335	0.000273
			81.465	80.60	2.393	39.62	0.00352
13	Graf (1984)	22	8.85	8.83	0.17	0.4896	0.00235
			2880.0	118.77	42.25	45.1385	0.000289
14	Liu (1977)	13	6.5	12.4	0.38	0.99	0.000121
			1490.0	183.95	3.05	957.0	0.00292
15	Hou and Christensen (1976)	45	0.01914	0.90	0.15	0.2269	.0000002
			46.445	67.96	7.46	118.657	.001334

5

ANALYSIS OF DATA AND RESULTS

5.1 INTRODUCTION

Data collected during earlier studies as well as during the present study are analysed in this chapter. The proposed numerical scheme is first verified by comparing its results with those obtained from the analytical model of Fischer (1968) and the numerical model of Jaque and Ball (1994). The effect of suspended sediment load is also studied using the experimental data. Predictors for D_L are proposed for clear-water flow and sediment-laden flow. The sensitivity of the C-t curves to errors in D_L is also studied.

5.2 PROOF-OF-THE-CONCEPT TESTS FOR THE PROPOSED NUMERICAL SCHEME

Fischer (1968) proposed the analytical solution of Eq. (1.1) under the assumptions of uniform flow, constant D_L and Gaussian input C-t curve. Fischer's solution was first obtained by using a Gaussian input C-t curve shown in Fig. 5.1(a) with $U^* = 1.034$ and $D_L^*=0.88$. In Fig. 5.1(b) the C-t curves as computed at the downstream locations using Fischer's analytical method and the present numerical scheme are shown for comparison. Close agreement between these may be noted.

Jaque and Ball (1994) proposed a numerical method for prediction of C-t curve under the assumption of uniform flow and constant D_L value and they reported this method to be superior compared to that of Holly and Preissmann (1977) and Stone and Brian (1963). An example from the field data given by Nordin and Sabol (1974) is taken for

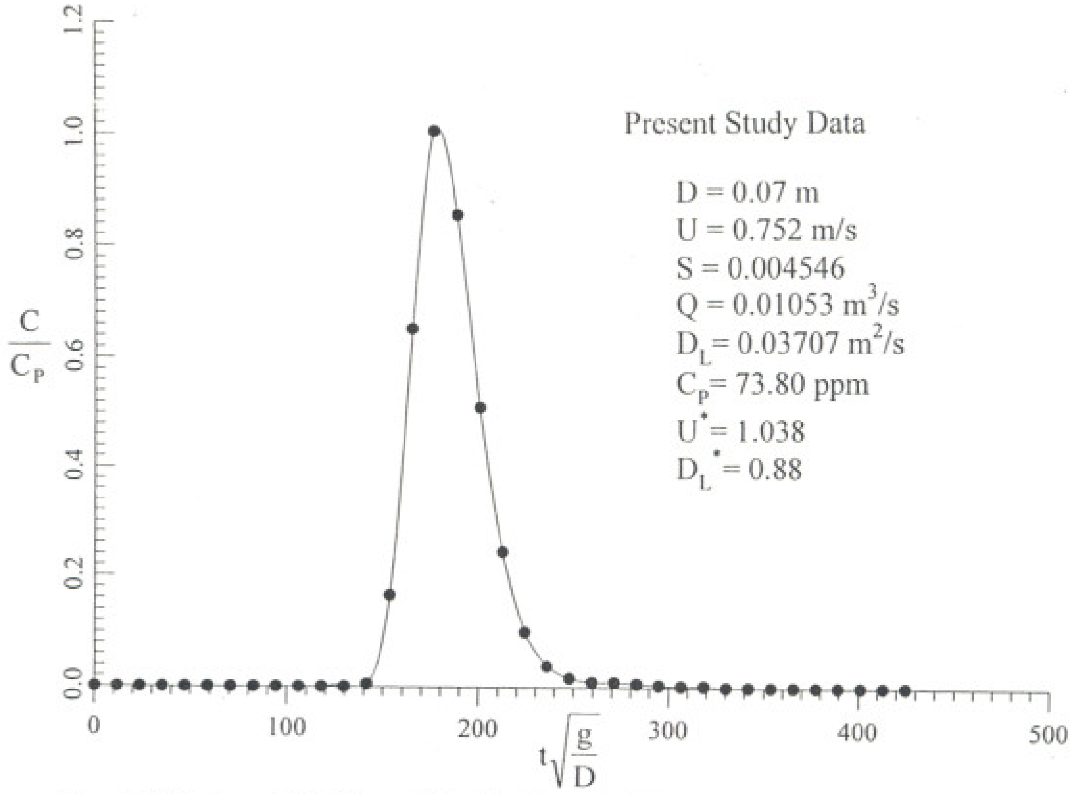


Fig. 5.1(a) Input C-t Curve Used in Proof-of-Concept Test for the Proposed Numerical Scheme with the Fischer's Analytical Solution

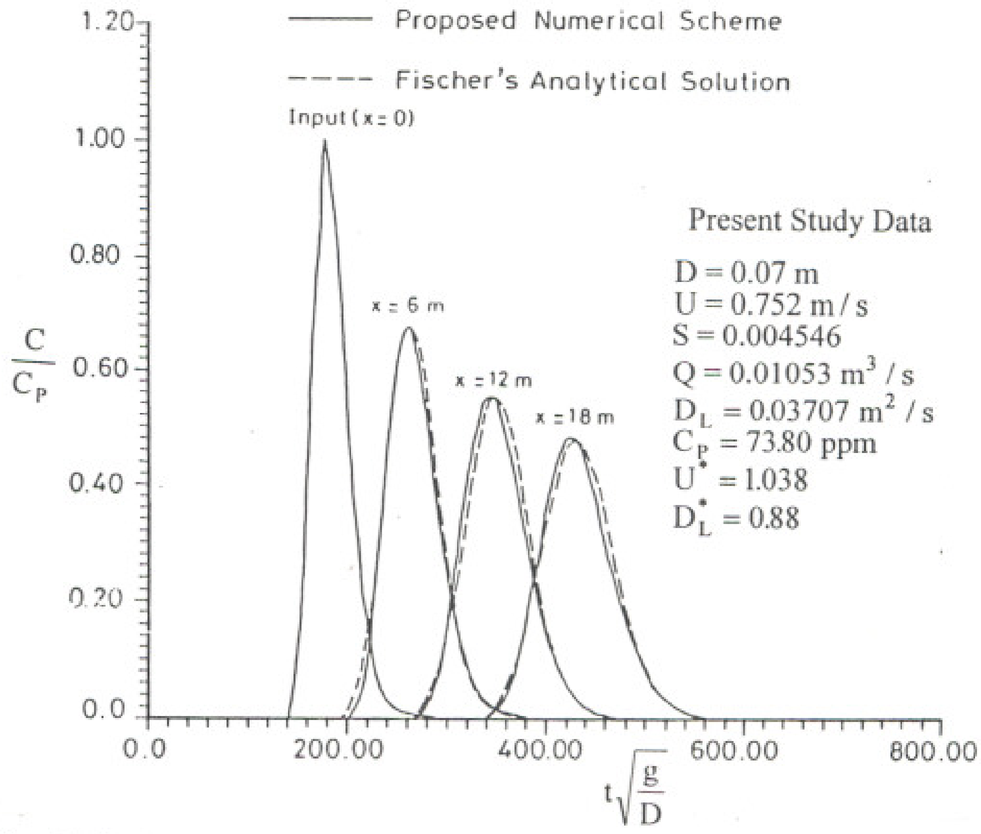


Fig. 5.1(b) Comparison of Results Obtained Using Fischer's Analytical Solution and the Proposed Numerical Scheme

computations with $U^* = 0.144$ and $D_L^* = 36.0$ and a non-Gaussian input C-t curve (see Fig. 5.2(a)). Computation are made for C-t curves at downstream locations using Jaque and Ball method and the proposed numerical scheme at a Courant number of unity. Fig. 5.2(b) depicts this comparison, wherein a close agreement is indeed obtained. However, Jaque and Ball solution is not repeatable for other values of the Courant number, as illustrated in this figure by the computed C-t curves with $C_r = 0.5$.

The results presented as above are considered to be the proof-of-concepts used and software coding made in the present study. The methods of Fischer, Jaque and Ball for predictions of C-t curves are intended for uniform flows. However, these methods can also be used in non-uniform flows by considering mean flow properties between successive computational sections. The approximations made through such an averaging of flow parameters are evaluated by making the computations using the proposed numerical scheme. For this M_1 type flow profile with laboratory and field data range are considered as shown in Tables 5.1 and 5.2.

Table 5.1 Uniform Flow Data

Nature of Data	Q (m^3/s)	B (m)	S	n	D_n (m)	D_c (m)	D_L (m^2/s)
Laboratory Data	0.0105	0.20	0.004546	0.012	0.076	0.0656	0.0371
Field Data	2625.6	501.30	0.000118	0.020	3.95	1.41	922.00

Table 5.2 Gradually Varied Flow Data

Nature of Data	Flow Profile	Station	Distance (m)	D (m)	U (m/s)	D_L (m^2/s)
Laboratory Data	M1	1	0.0	0.08	0.658	0.0435
		2	5.6	0.10	0.527	0.0577
		3	13.6	0.12	0.439	0.0723
Field Data	M1	1	0.0	4.00	1.31	922.08
		2	12613.20	5.00	1.05	1286.12
		3	40487.5	6.00	0.873	1687.35

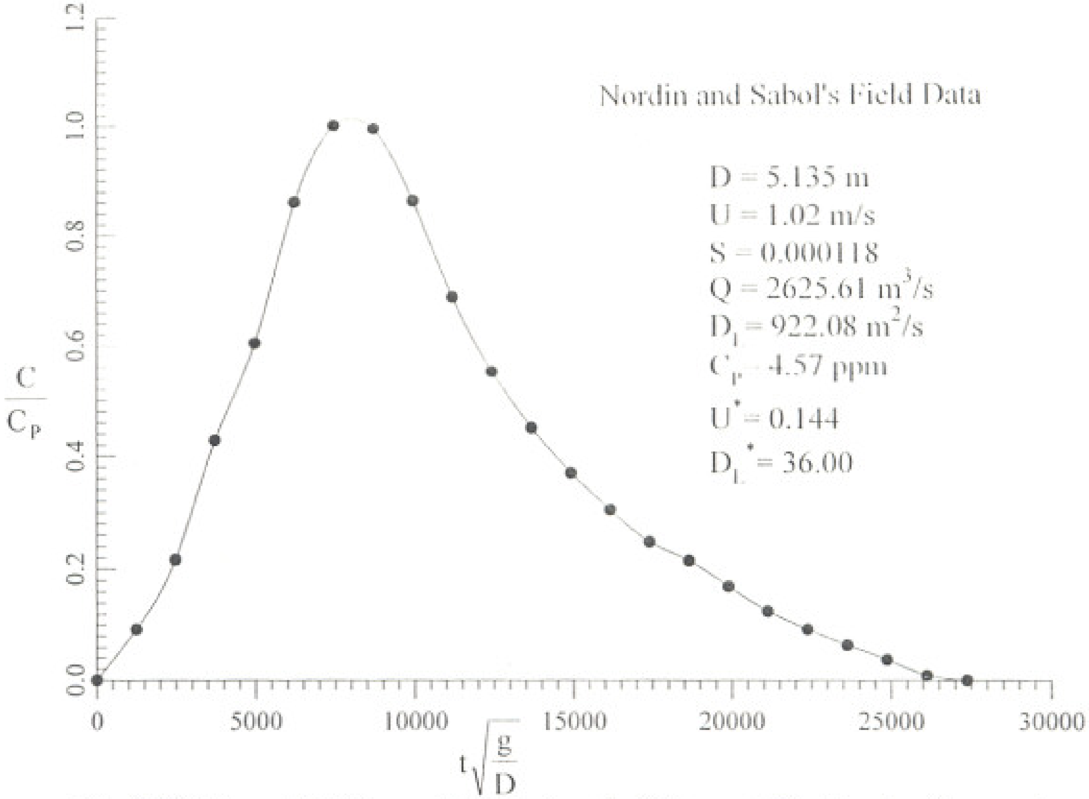


Fig. 5.2(a) Input C-t Curve Used in Proof-of-Concept Test for the Proposed Numerical Scheme with the Jaque and Ball Scheme

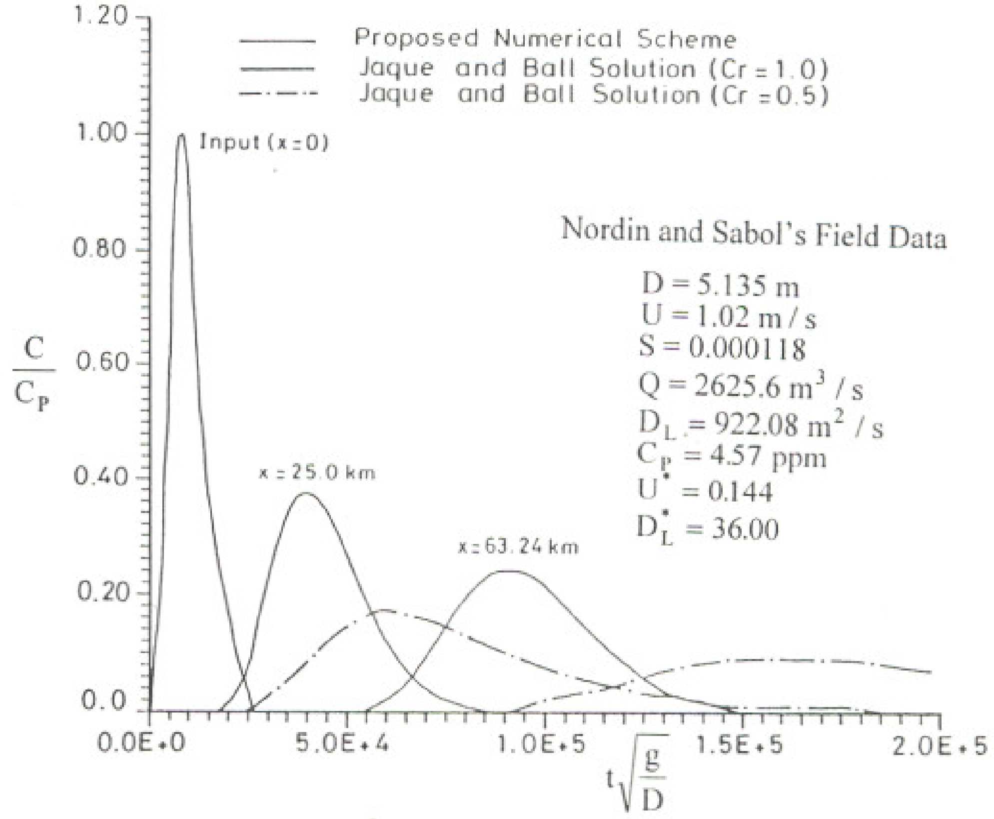


Fig. 5.2(b) Comparison of Results Obtained Using Jaque and Ball Scheme and the Proposed Numerical Scheme

In Table 5.1 D_n and D_c are normal depth and critical depth of flow respectively and n is the Manning's coefficient. For the gradually varied flow the D_L values are considered as constant multiples of DU_*. The C-t curves computed as per these are presented in Figs. 5.3 and 5.4. It can be seen from these figures that both the peak concentration and the time to peak are affected because of averaging the flow parameters.

5.3 ANALYSIS FOR THE COEFFICIENT OF DISPERSION

The variation of coefficient of dispersion D_L with the distance and flow parameters is studied using the numerical scheme proposed herein. As mentioned in Chapter-II the analysis of Nordin and Sabol (1974), Day and Wood (1976) and Singh (1987) indicated the D_L value to increase with distance. Therefore, in the present study, D_L is considered to vary with distance as per Eq. (3.2). Data of Nordin and Sabol (1974), Beltaos and Day (1976) and Fischer (1966) are re-analysed for this purpose as below.

Optimum value of the dispersion coefficient D_{L0} and the parameter K in Eq. (3.2) are determined using the two-dimensional grid search method. Initial estimates for D_{L0} and K were chosen using moment method for the C-t curves at the first two sections. Those values of D_{L0} and K are considered to be the optimum ones for which the error is minimum between observed and computed C-t curves at all the downstream sections. To make the search for the optimum values a two-dimensional grid is constructed as per Fig. 5.5. The incremental value of D_{L0} and K are taken equal to 1/10 and 1/2 of initial value of D_{L0} and K respectively. The feasible region is defined on this truth that the values of D_L will never be less than zero. The feasible region is shown by the shaded portion in Fig. 5.5.

Let the computation start from a point P. The error ERS as defined in section 3.3 of Chapter-III is calculated at the neighbourhood of P, at R and Q lying vertically above and below P. If the error at point P is minimum the grid was redefined by taking $\Delta K=\Delta K/2.0$ and computation is repeated till one got the desirable accuracy. But if the error at either point R or Q is minimum, the computation is shifted in the required direction, unless one got some other point where the error is minimum in its neighbourhood, say at point M_C

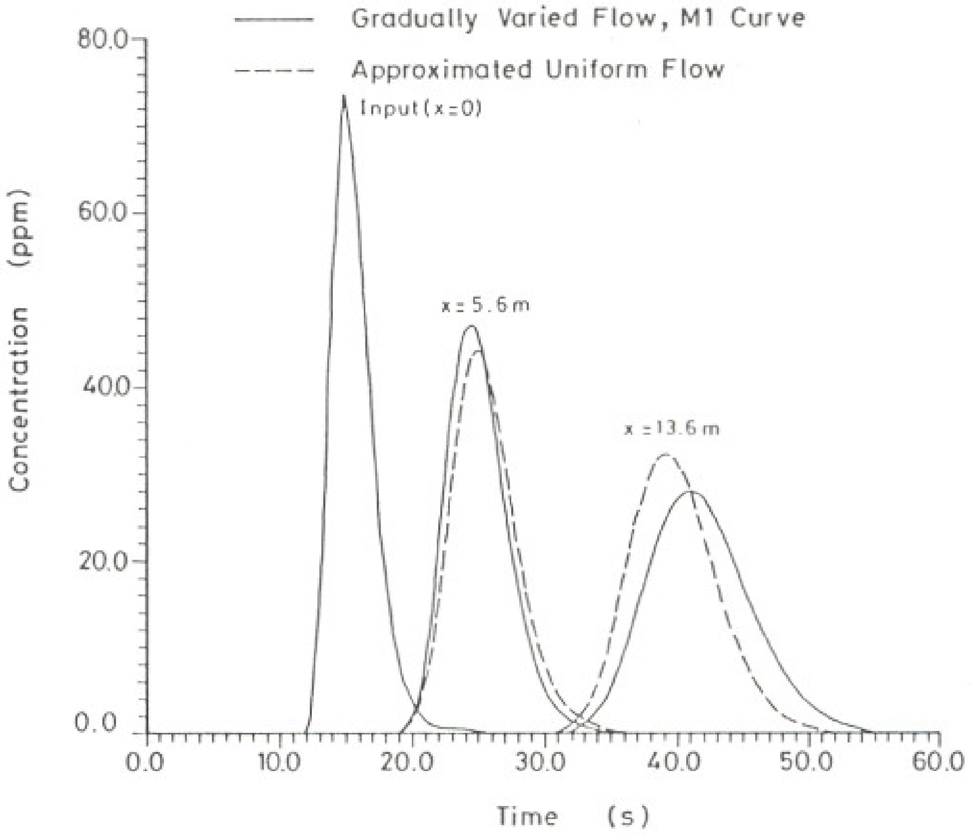


Fig. 5.3 Temporal Variation of Concentration Using Gradually Varied Flow and Approximated Uniform Flow. (Present Study Data)

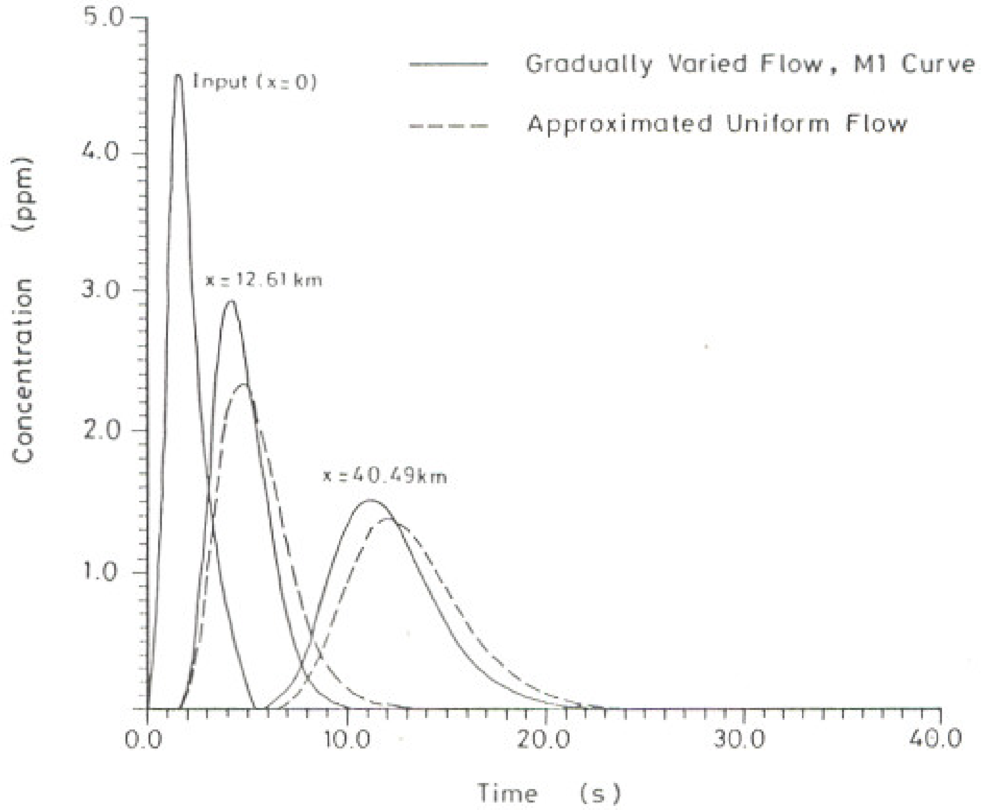


Fig. 5.4 Temporal Variation of Concentration Using Gradually Varied Flow and Approximated Uniform Flow. (Nordin and Sabol's Field Data)

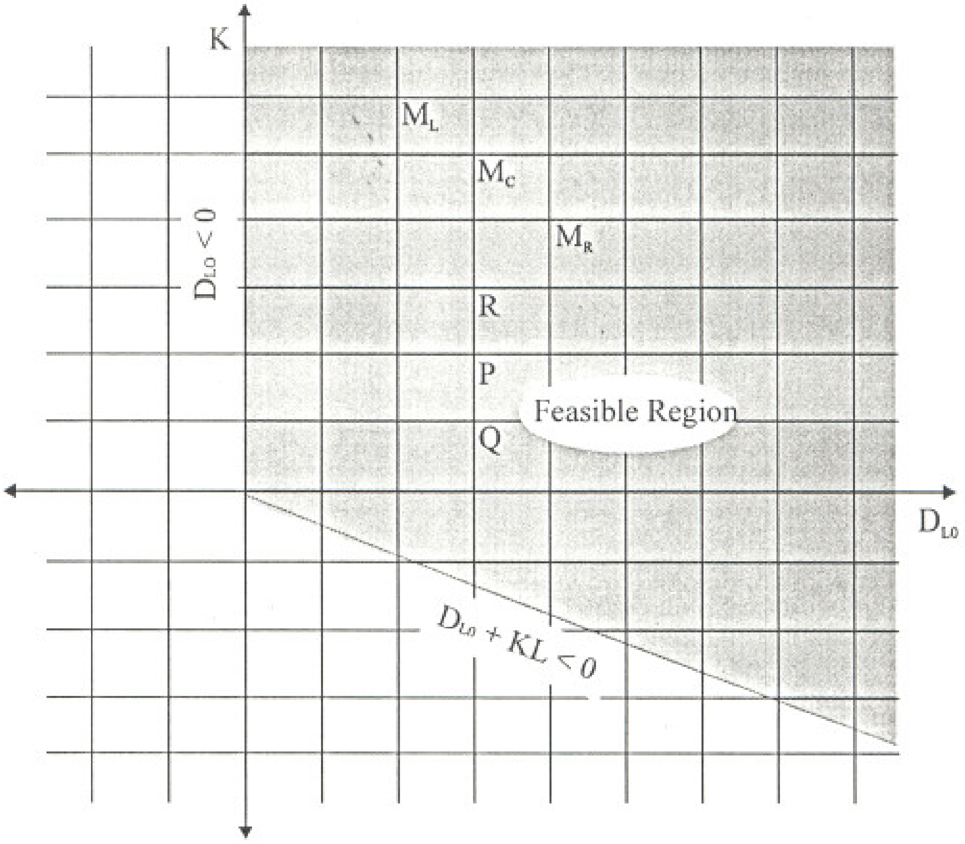


Fig. 5.5 Determination of Multiplying and Additive Constant of Dispersion Coefficient

(see Fig. 5.5). The value of D_{L0} and K at point M_C are the local optimum values. To get global optimum value of D_{L0} and K, the computation is shifted horizontally in both the directions for D_L equal to $D_{L0}+\Delta D_L$ and $D_{L0}-\Delta D_L$. The local minimum is determined in the same way for both values of D_L. Let the local optimum points be M_R and M_L for values of D_L equal to $D_{L0}+\Delta D_L$ and $D_{L0}-\Delta D_L$ respectively. Out of the three local optimum points, one point is selected which has minimum error. If this point is M_C, the point is considered as global optimum value. But if the point is M_R or M_L, the computation is shifted in the required direction, till one got a global optimum point.

The variation of the dispersion coefficient obtained as above for Antietam Creek, Chattahoochee river and Conocheague Creek with distance is shown in Fig. 5.6. Also the variation of shear velocity with distance for the corresponding flow conditions is presented in Fig. 5.7. It may be noted that positive values of K were obtained through the grid search for the flow cases marked as 3 and 4 for river Chattahoochee in Fig. 5.6 and negative values were obtained for the flow cases marked as 1 and 2 for Antietam Creek.

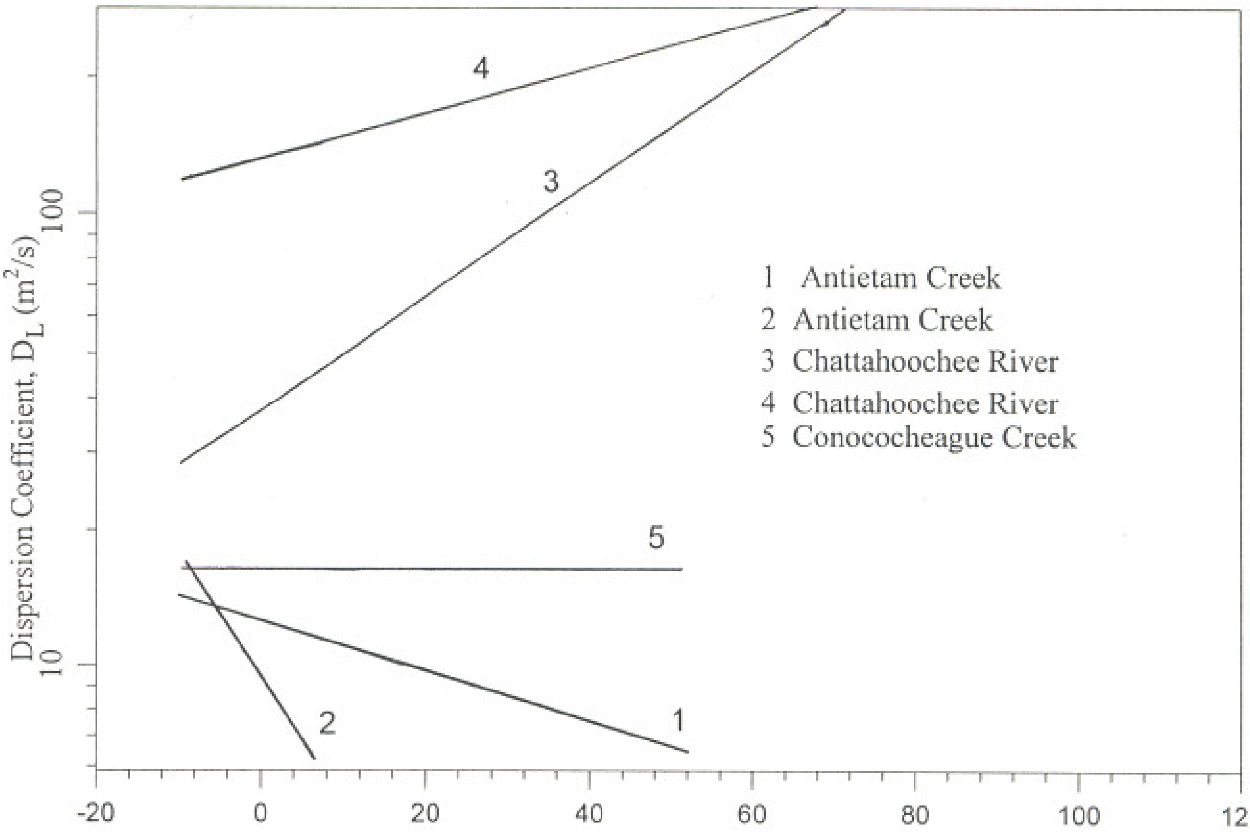


Fig. 5.6 Spatial Variation of Dispersion Coefficient

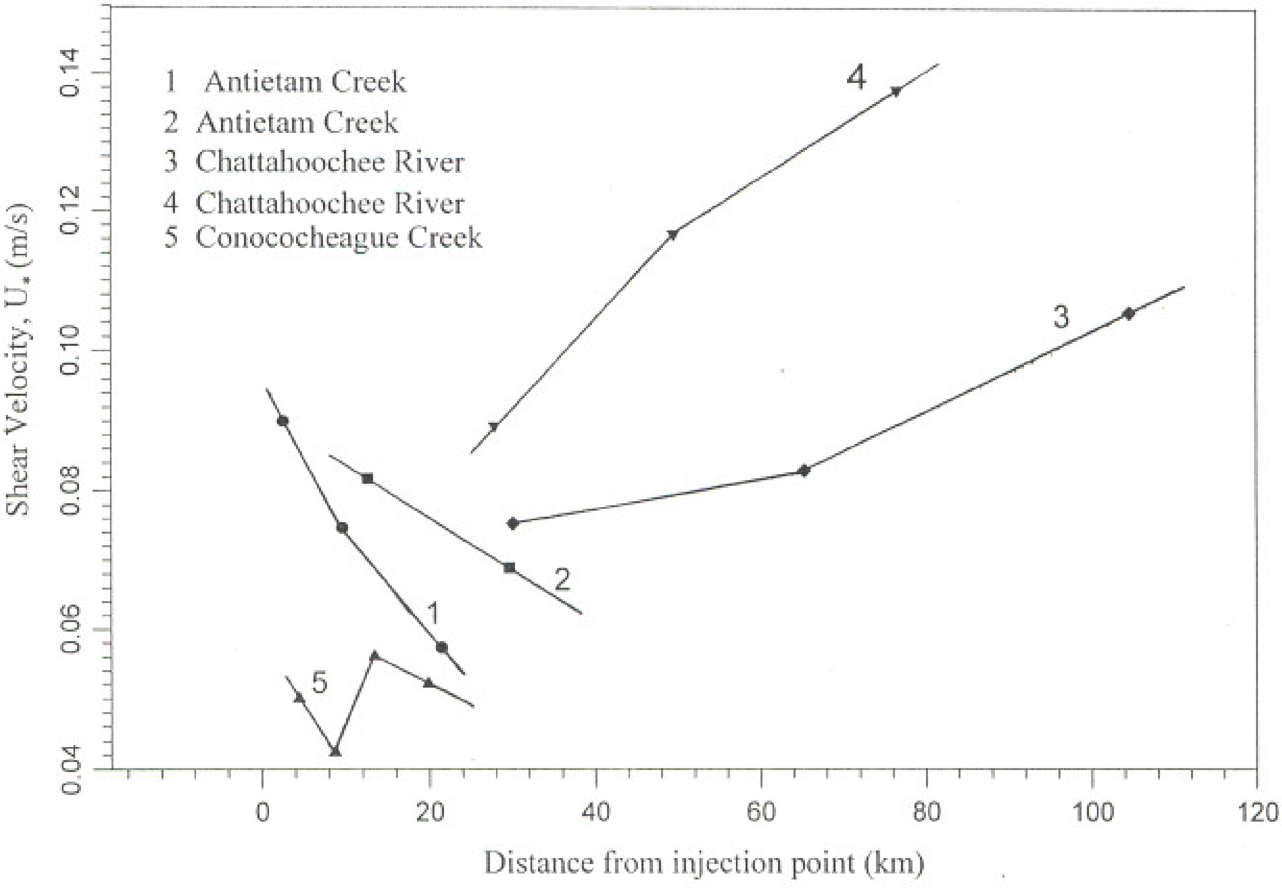


Fig. 5.7 Spatial Variation of Shear Velocity

For the flow marked as 5 for Conocheague Creek in Fig. 5.6 the K value was practically zero. These results do indicate direct proportionality of D_L and U_*.

Besides the above, reanalysis is also made of the field data of Beltaos and Day (1976) and laboratory data of Fischer (1966). The field data of Beltaos and Day (1976) consisted of observed C-t curves at seven stations at distances of 1.6, 4.2, 8.2, 11.1, 13.9 and 18.5 km from the injection station. The standard deviation of shear velocity in the study reach was 0.026 indicating that flow was almost uniform. The optimum values of D_{L0} and K as determined here are 14.4 m^2/s and 0.0000221 m/s. Using these values the predicted C-t curves are plotted in Fig. 5.8 along with the observed C-t curves. With K=0, the optimum value of dispersion coefficient calculated by the method described in section 3.3 is 16.0 m^2/s. The predicted and observed C-t curves with constant D_L are shown in Fig. 5.9. A comparison of Figs. 5.8 and 5.9 shows that the predicted curves in both the cases are practically the same. Similar analysis is presented for one set of Fischer's (1966) laboratory data. The flow was uniform with shear velocity equal to 0.0136 m/s. The stations at which C-t curves were observed are located at 7.0, 14.0, 21.0 and 28.0 m from the injection station. Fig. 5.10 shows the predicted C-t curves for optimum value of D_{L0} =0.0092 m^2/s and K=0.00093 while Fig. 5.11 shows the predicted C-t curves when K=0 and corresponding optimum value of D_L=0.01 m^2/s. In this case also the predicted curves as given in Figs. 5.10 and 5.11 are the same.

In the light of the above analysis, one can say that for uniform flow, the dispersion coefficient remains constant along the distance, for example, in case of data for Conocheague Creek; in non-uniform flow D_L varies with distance because of its strong dependence on U_*, for example, in case of data for Chattahoochee river and Antietam Creek. The D_L values are recomputed for all the data in the light of the above conclusion as below: Initial estimate of D_L was obtained using the method of moments. The proposed numerical scheme was then used alongwith the grid search method for computation of the optimum value of D_L. The D_L values thus computed are associated with minimum error between the computed and observed C-t curves as illustrated by Figs. 5.12 and 5.13. The values of D_L computed as above are listed in Appendix-I.

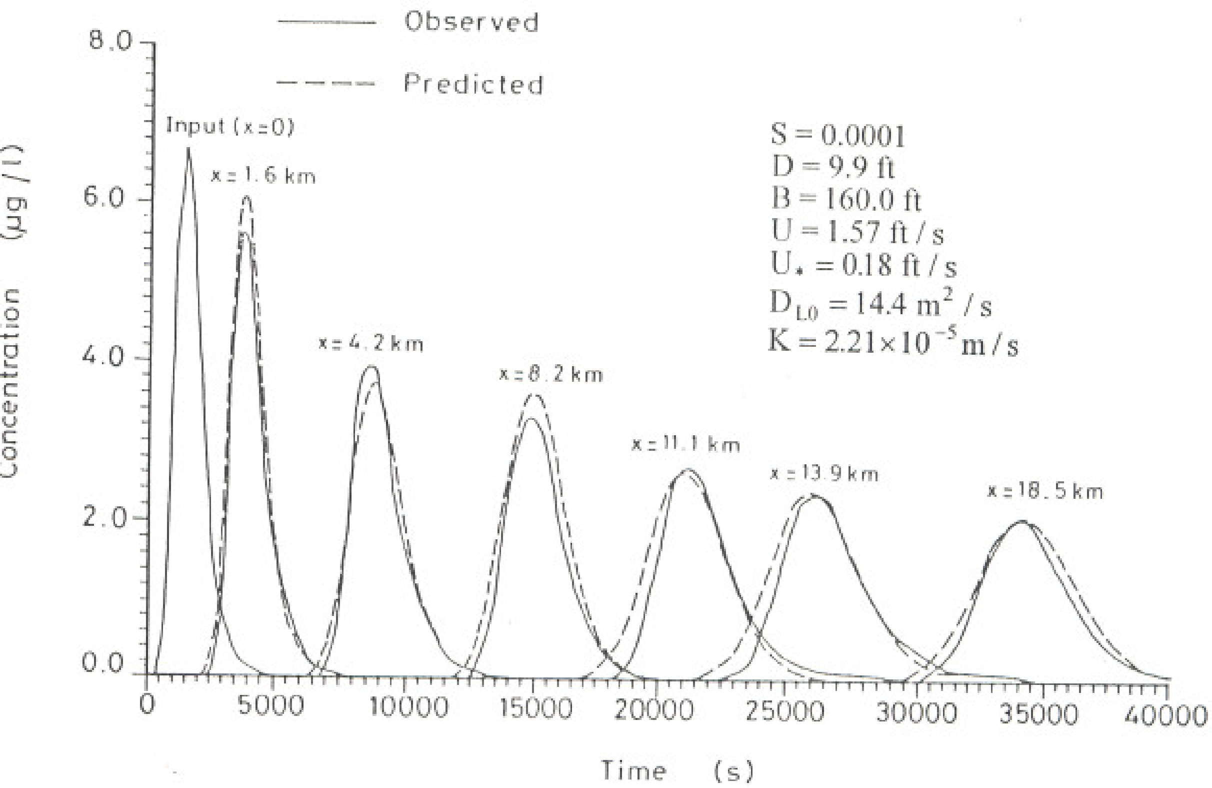


Fig. 5.8 Comparison of Observed and Predicted C-t Curves for K≠0. (Beltaos and Day's Field Data)

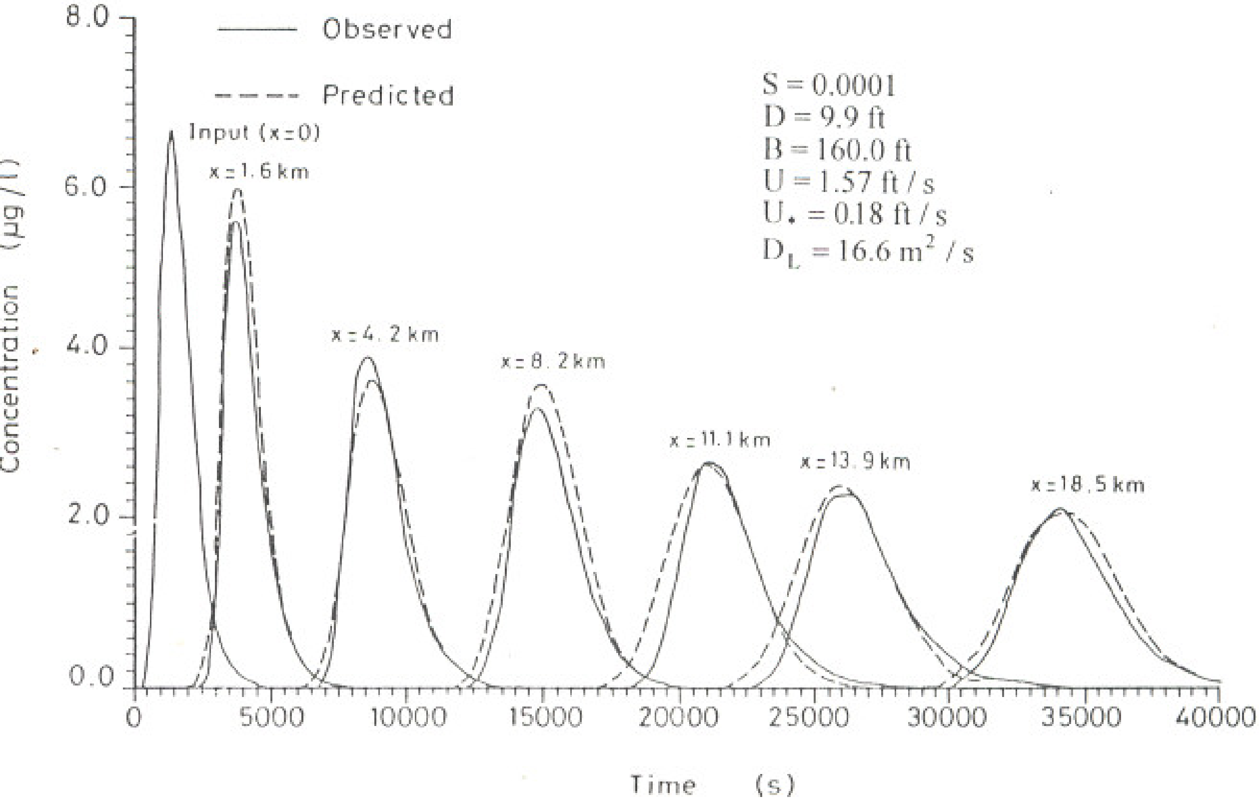


Fig. 5.9 Comparison of Observed and Predicted C-t Curves for K=0. (Beltaos and Day's Field Data)

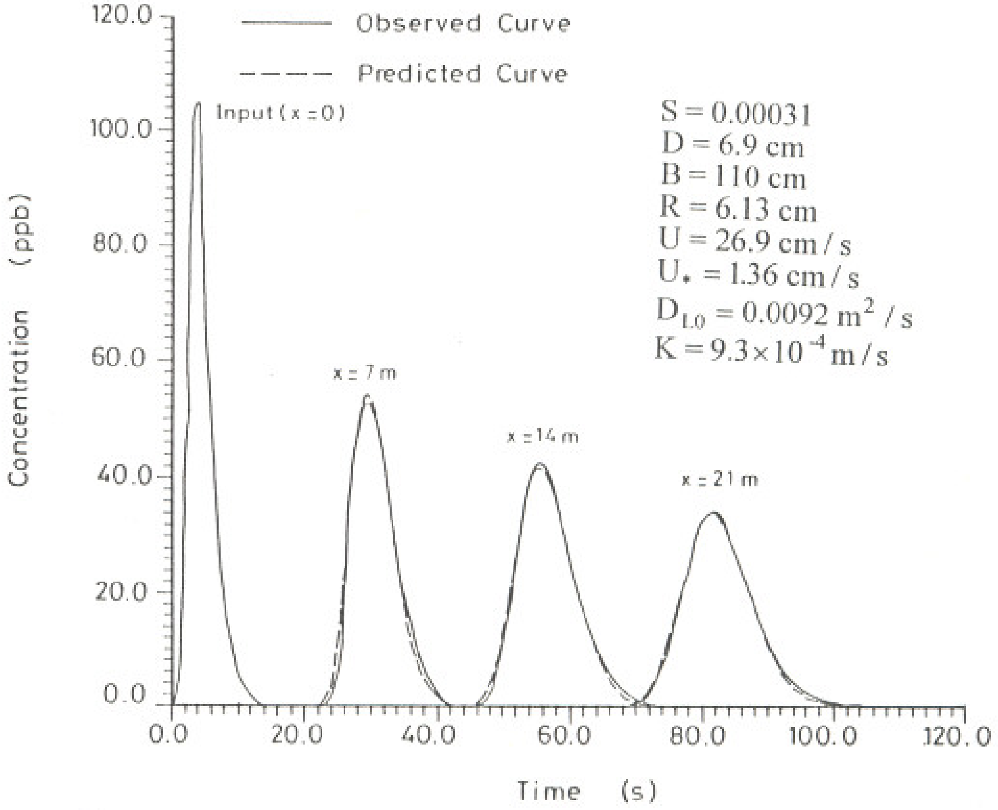


Fig. 5.10 Comparison of Observed and Predicted C-t Curves for K≠0.
(Fischer's Laboratory Data)

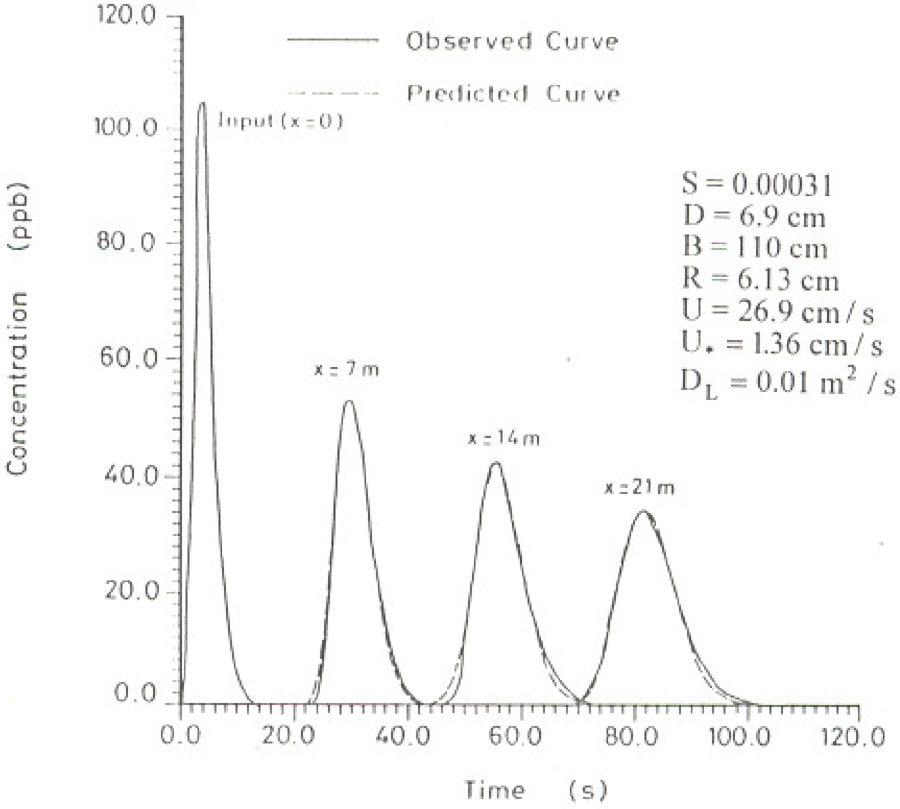


Fig. 5.11 Comparison of Observed and Predicted C-t Curves for K≠0.
(Fischer's Laboratory Data)

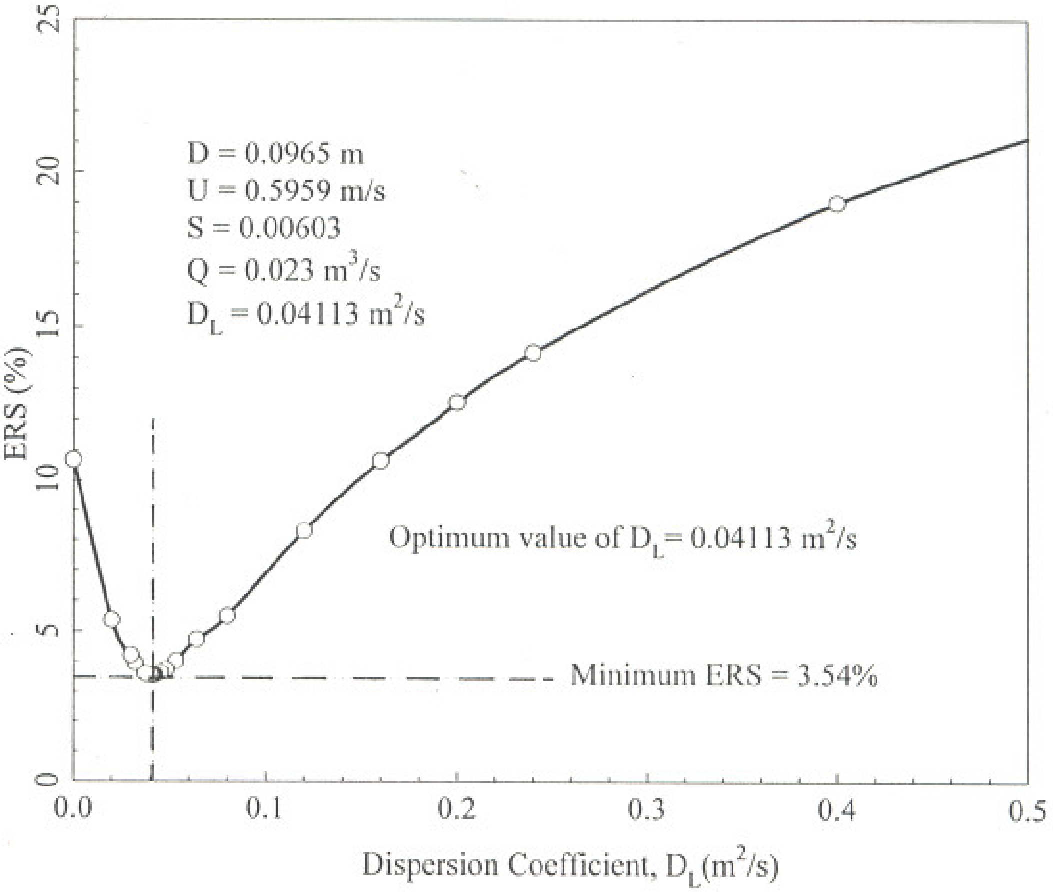


Fig. 5.12 Determination of Dispersion Coefficient (Singh's Laboratory Data)

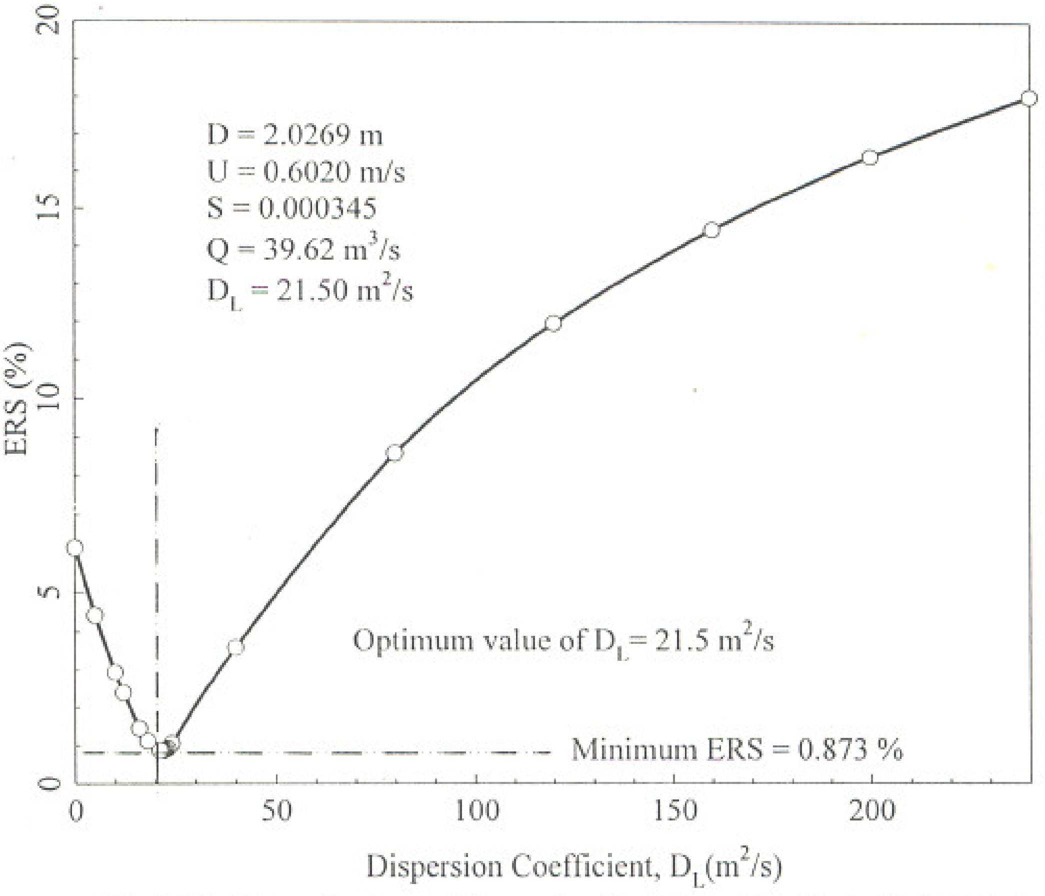


Fig. 5.13 Determination of Dispersion Coefficient (Berkas's Field Data)

5.4 VERIFICATION OF RELATIONSHIPS FOR D_L

All the available data from laboratory and field have been first used to verify the existing predictors for D_L. For this purpose the predictors selected are those proposed by Fischer (1966), Asai *et al.* (1991), Liu (1978), Marivoet and Craenenbroeck (1986), McQuivey and Keefer (1974) and Thackston and Krenkel (1967). The comparison between the observed values of D_L and those computed using the above predictors is depicted in Figs. 5.14 to 5.19. Notations used for data of different investigator(s) are given in Table 5.3.

Table 5.3 : Notations Used for Data of Different Investigator(s)

S.No.	Name of Investigator(s)	Symbol Used
1	Author's data	○
2	Singh (1987)	□
3	Fischer (1966)	△
4	Beltaos and Day (1976)	▽
5	Nordin and Sabol (1974)	◇
6	Calandro (1978)	⊙
7	James and Helinsky (1984)	⊡
8	McQuivey and Keefer (1976)	◬
9	Taylor (1970)	⧨
10	Dorban (1982)	⟐
11	McQuivey and Keefer (1974)	⊕
12	Miller and Richardson (1974)	⊞
13	Berkas (1986)	⨹
14	Graf (1984)	⧩
15	Liu (1977)	◈
16	Hou and Christensen (1976)	⊕

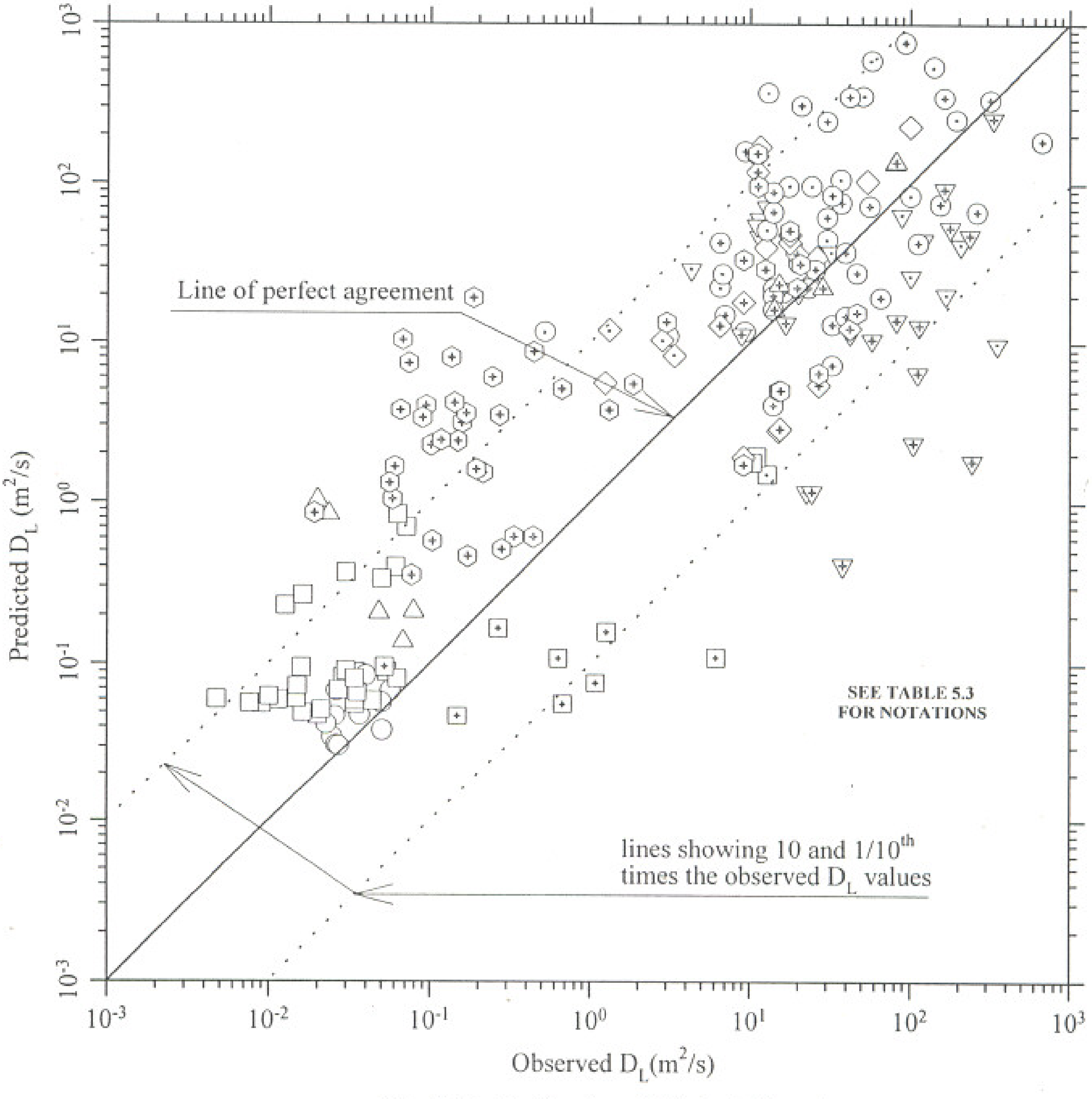


Fig. 5.14 Verification of Fischer's Equation

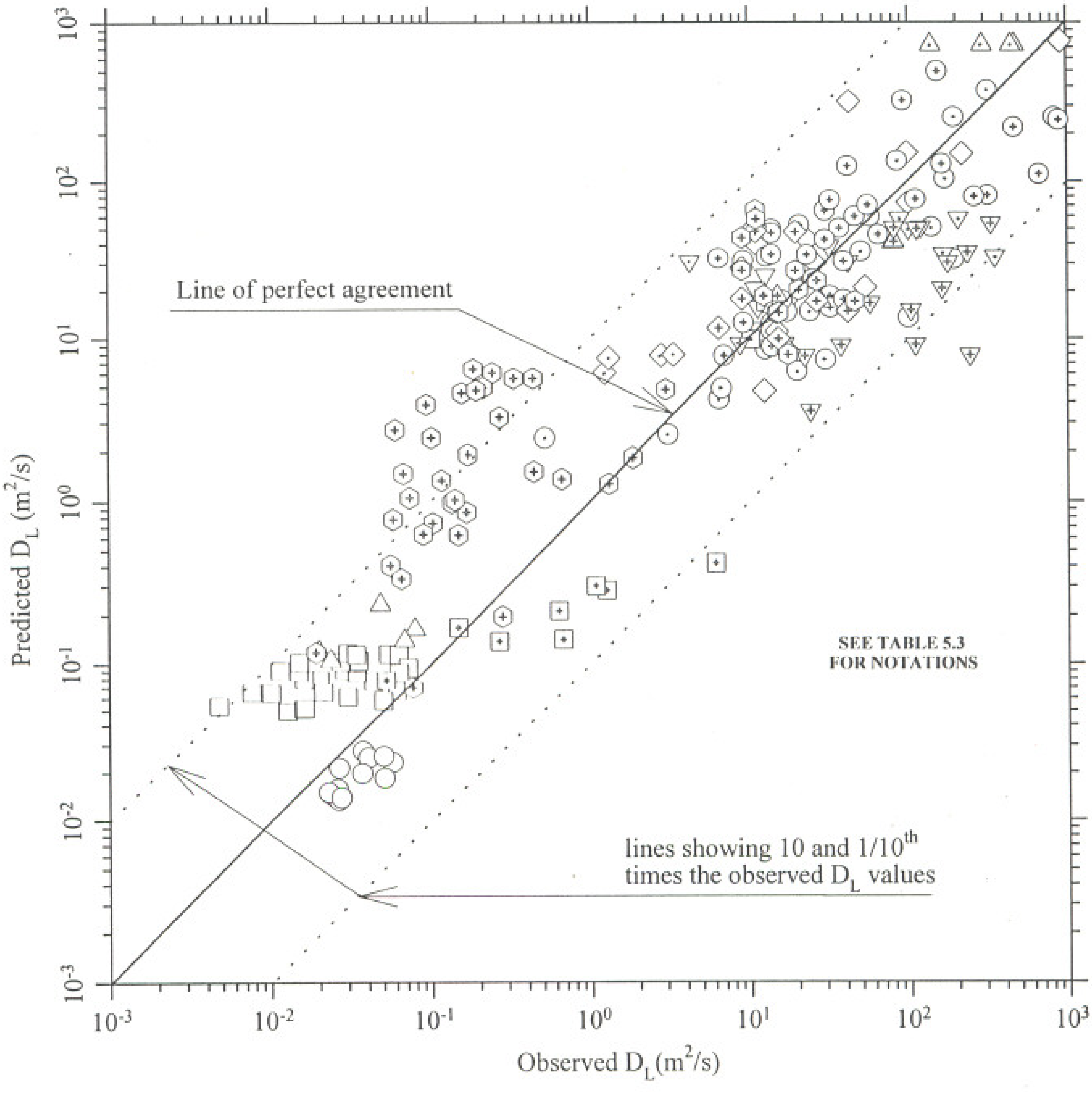


Fig. 5.15 Verification of Asai's *et al.* Equation

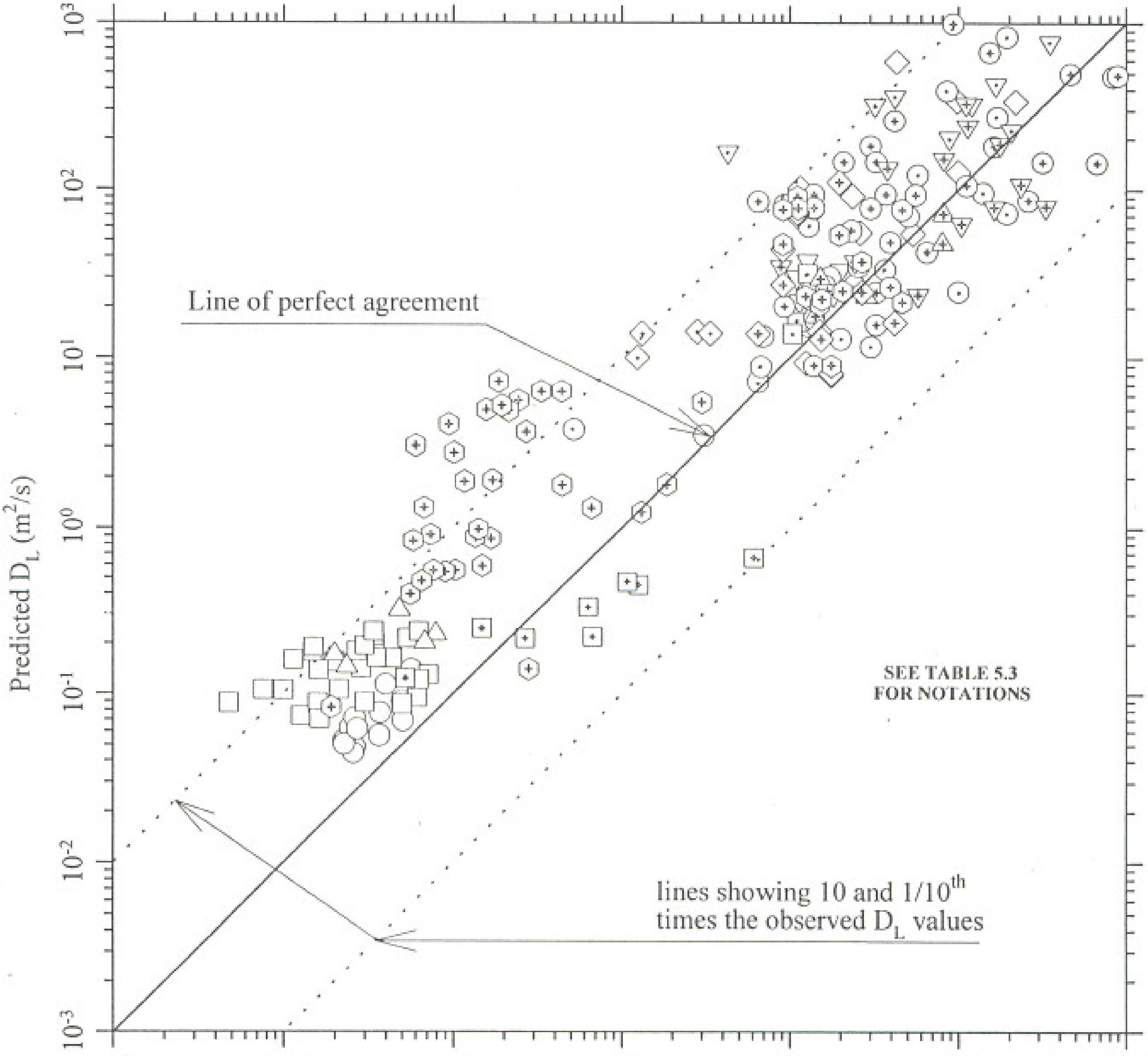


Fig. 5.16 Verification of Liu's Equation

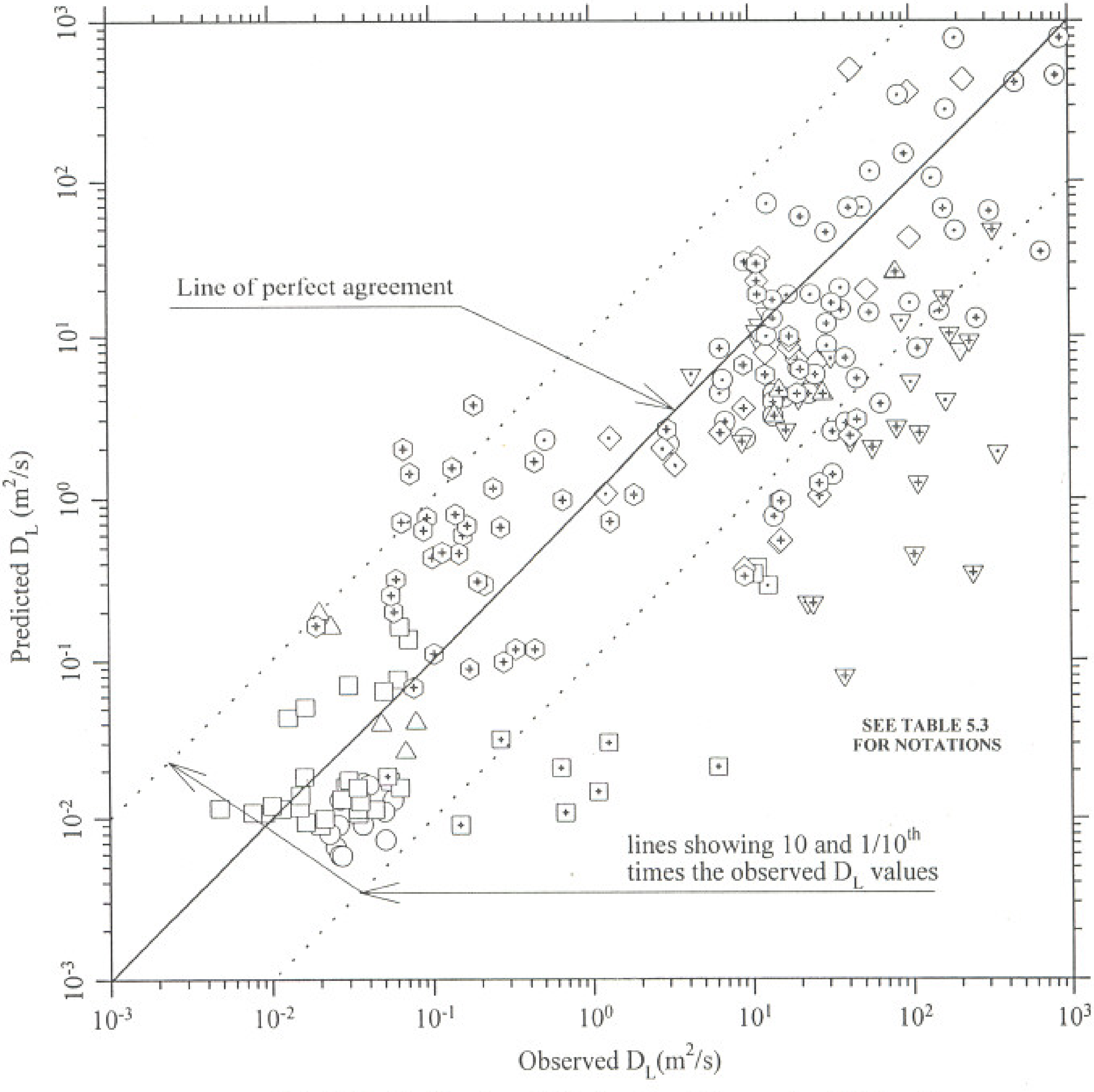


Fig. 5.17 Verification of Marivoet and Craenenbrock's Equation

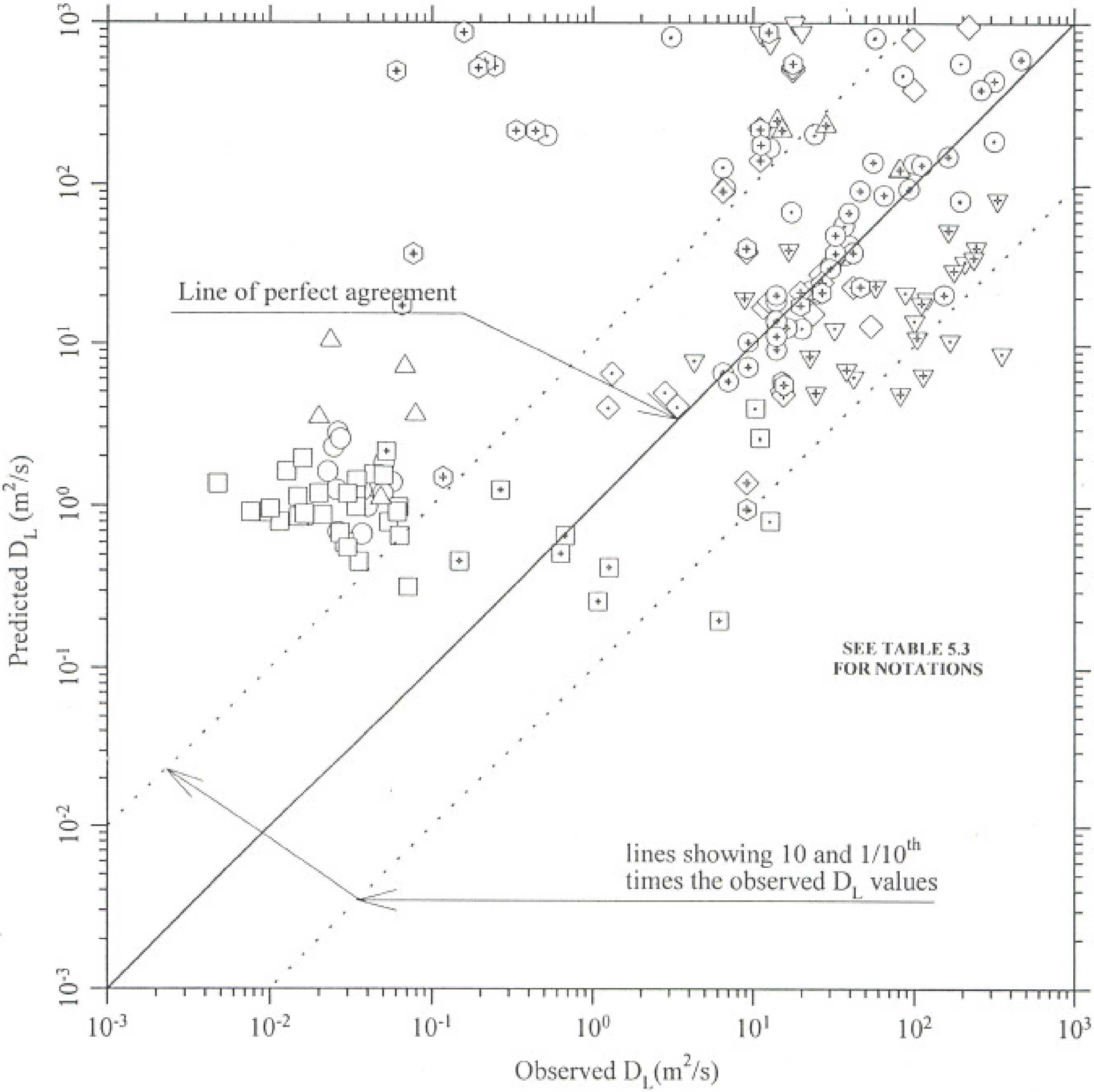


Fig. 5.18 Verification of McQuivey and Keefer's Equation

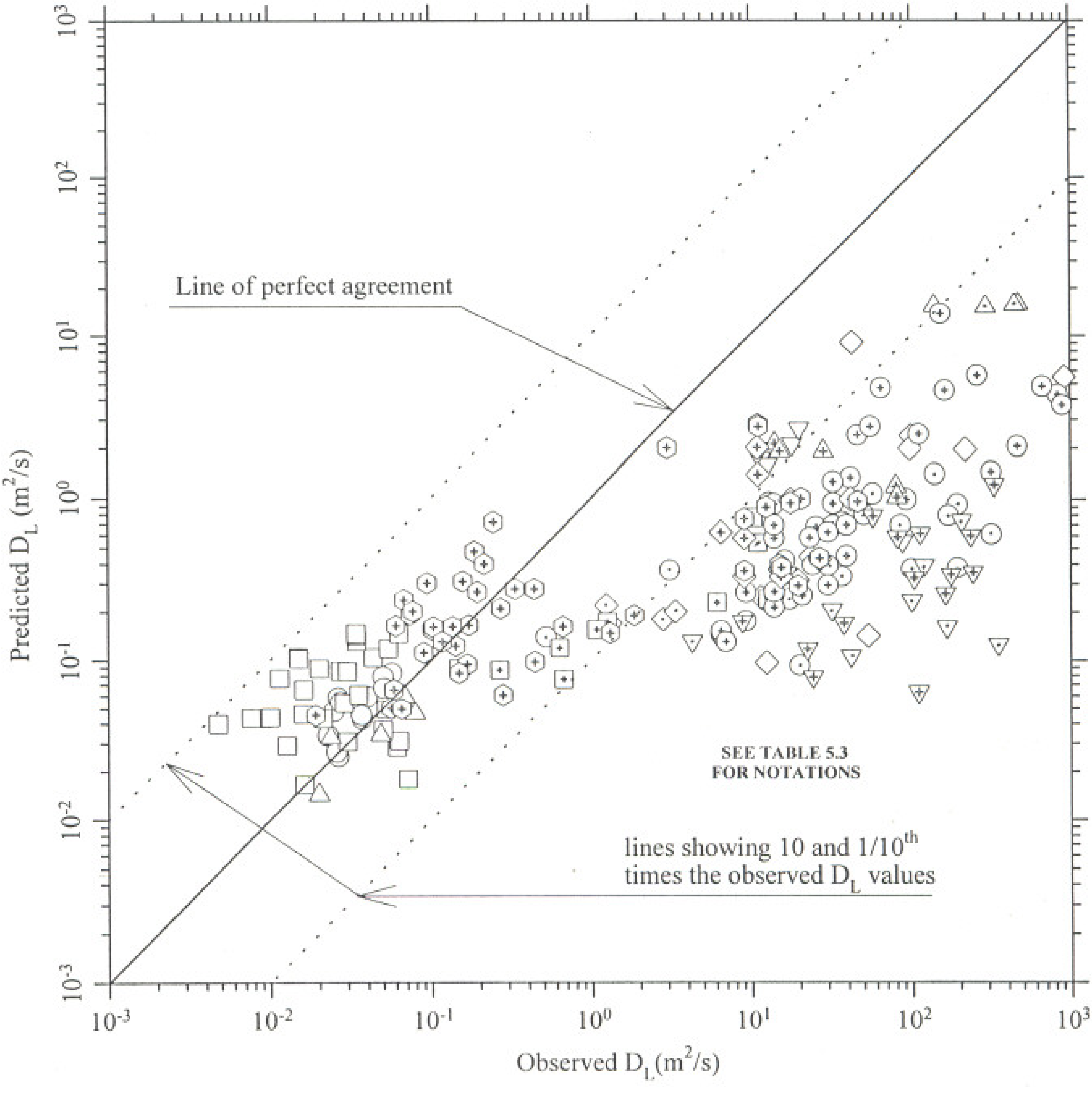


Fig. 5.19 Verification of Thackston and Krenkel's Equation

A close study of these figures reveals that none of the existing relationships has been able to predict the value of D_L satisfactorily for all the data. Nevertheless, the relationship of Asai *et al.* (1991) gives a better prediction than the other relationships. Nevertheless, even in this case, the predicted and the observed D_L values depart from each other by a factor of ten or more; see Fig. 5.15.

5.5 PROPOSED RELATIONSHIP FOR D_L

As discussed in Chapter-II, Asai *et al.* (1991) made a theoretical analysis for identification of the flow and the channel characteristics which significantly influence the dispersion coefficient. Through such analysis it was found that the non-dimensional parameter $\frac{D_L}{DU_*}$ should be affected by $\frac{B}{D}$, $\frac{U}{U_*}$ and the lateral velocity distribution for wide channel. However, Asai *et al.* (1991) related the parameter $\frac{D_L}{DU_*}$ with $\frac{B}{D}$ only. In order to further improve the accuracy of relationship for D_L, the following functional relationship is considered to hold good.

$$\frac{D_L}{qS} = f_1\left(\frac{B}{R}, \frac{U}{U_*}, S\right) \tag{5.1}$$

Here f_1 represents the functional relationship and q is discharge per unit width. The variable qS has been included in the above relationship to account for the effect of stream power ($\tau_0 U$) on the longitudinal dispersion. Here τ_0 is average bed shear stress. Likewise the parameters $\frac{U}{U_*}$ and S have been included to account for the effect of velocity distribution on D_L. Multiple regression analysis produced the following relationship for D_L:

$$\frac{D_L}{qS} = 0.4 \times \left(\frac{B}{R}\right)^{2.12} \left(\frac{U}{U_*}\right)^{-0.72} (S)^{-0.81} \tag{5.2}$$

The multiple correlation coefficient for above equation is 0.94. Data having a wide range (see Tables 4.3 and 4.4) are used for obtaining Eq. (5.2). The data used for this purpose included the laboratory data of present study, Singh (1987), Fischer (1966), Miller and Richardson (1974) and Hou and Christensen (1976) and the field data of Beltaos and Day (1976), Nordin and Sabol (1974), Calandro (1978), James and Helinsky (1984), McQuivey and Keefer (1974 and 1976), Taylor (1970), Dorban (1982), Berkas (1986), Graf (1984), Liu (1977) and Hou and Christensen (1976). In order to check the accuracy, the values of D_L computed through Eq. (5.2) are graphically compared with corresponding observed values as shown in Fig. 5.20. A comparison of Fig. 5.20 with the similar figures for existing relationships, viz. Figs. 5.14 to 5.19, revealed that Eq. (5.2) produces the estimate of D_L with better accuracy. Nevertheless, some of the computed D_L values differ from the observed ones by a factor of ten.

In addition to the graphical comparison of the existing equations for D_L with the equation developed in the present study (i.e. Eq. 5.2), the D_L values computed by all the equations were compared with the observed D_L values by computing a factor, β' representing the bias between the computed and observed D_L values. The bias is defined as

$$\beta' = \frac{D_L \text{ (computed)}}{D_L \text{ (observed)}} \tag{5.3}$$

The bias computed for each relationship is compared for different ranges of B/R. By examining the bias for different ranges of B/R, it is possible to determine which relationships are best suited under various ranges of B/R. Table 5.4 provides the bias for each of eleven relationships in selected ranges of B/R. For each range of B/R, average, standard deviation (S.D.) and coefficient of variation C_V of β' are listed in the table. It can be observed from the table that the average bias is comparatively large for the relationships of Elder (1959), Yotsukura and Fiering (1964), Fischer (1966), Thackston and Krenkel (1967), Sumer (1969), McQuivey and Keefer (1974), Jain (1976) and Liu (1978) and small for the relationships of Marivoet and Craenenbroeck (1986), Asai *et al.* (1991) and the relationship of the present study, when the whole range of data is considered. However, the relationship of Marivoet and Craenenbroeck (1986) produces higher coefficient of variation. According to Table 5.4, performance of the equation proposed in the present study is equally good in almost the entire range of B/R values

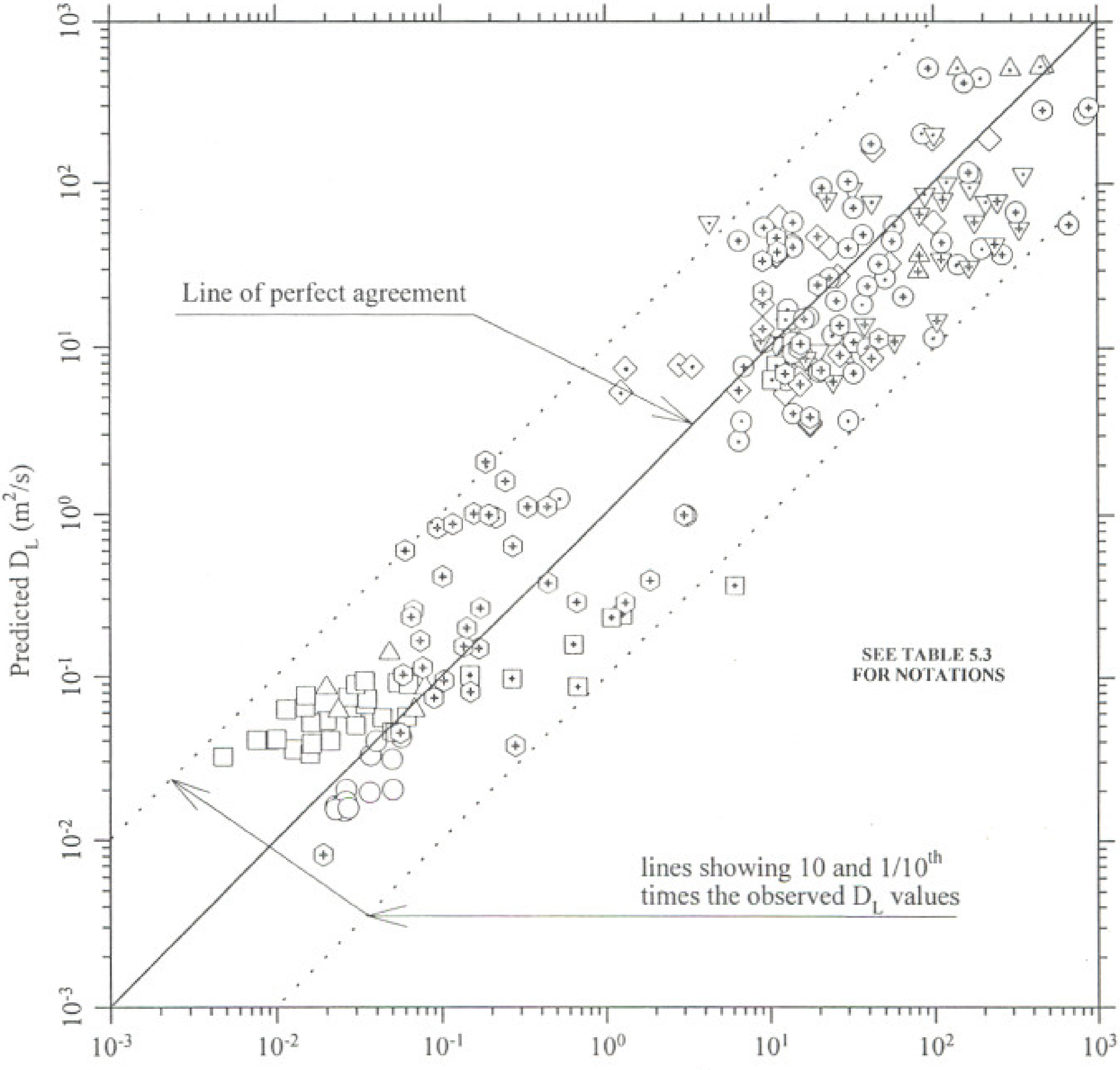


Fig. 5.20 Verification of Author's Equation

Table 5.4 : Bias for Eleven D_L Prediction Equations in Selected Ranges of B/R

Source	β'	$B/R < 10$	$10 \leq B/R < 20$	$20 \leq B/R < 40$	$40 \leq B/R < 60$	$60 \leq B/R < 100$	$B/R \geq 100$
Elder, 1959	Average	1.1094	0.4154	0.0758	0.0162	0.0044	0.0057
	S.D.	1.0160	0.4694	0.2115	0.0190	0.0045	0.0055
	C_V	0.9158	1.1300	2.7913	1.1721	1.0156	0.9692
Yotsukura and Fiering, 1964	Average	2.2893	0.8573	0.1563	0.0334	0.0091	0.0118
	S.D.	2.0964	0.9687	0.4363	0.0392	0.0093	0.0114
	C_V	0.9158	1.1300	2.7913	1.1722	1.0156	0.9692
Fischer, 1966	Average	4.0439	18.5454	5.3173	9.9775	3.9920	10.0909
	S.D.	6.6335	28.1890	15.5583	20.9852	4.6843	8.2841
	C_V	1.6404	1.5200	2.9260	2.1033	1.1734	0.8209
Thackston and Krenkel, 1967	Average	2.2862	0.8228	0.1433	0.0320	0.0084	0.0105
	S.D.	1.9756	0.9141	0.4134	0.0402	0.0082	0.0096
	C_V	0.8641	1.1109	2.8853	1.2551	0.9772	0.9070
Sumer, 1969	Average	1.0971	0.4108	0.0749	0.0160	0.0044	0.0057
	S.D.	1.0047	0.4642	0.2091	0.0188	0.0044	0.0055
	C_V	0.9158	1.1300	2.7913	1.1722	1.0156	0.9692
McQuivey and Keefer, 1974	Average	73324.78	30924.87	337.3075	88.7402	1.8469	1.2831
	S.D.	530660.10	71070.91	1305.25	311.89	2.6977	1.4610
	C_V	7.2371	2.2982	3.8696	3.5146	1.4607	1.1387
Jain, 1976	Average	2452.26	11253.08	3222.05	6043.92	2418.10	6111.48
	S.D.	4026.04	17103.16	9431.63	12718.76	2839.30	5024.84
	C_V	1.6418	1.5199	2.9272	2.1044	1.1742	0.8222
Liu, 1978	Average	4.6031	8.8858	3.8743	5.1070	2.5929	9.4238
	S.D.	4.0912	11.2459	6.7201	13.6759	2.4303	9.0654
	C_V	0.8888	1.2656	1.7345	2.6779	0.9373	0.9620
Marivoet and Craenenbroeck, 1986	Average	0.7720	3.5405	1.0151	1.9048	0.7621	1.9264
	S.D.	1.2664	5.3815	2.9702	4.0063	0.8943	1.5815
	C_V	1.6404	1.5200	2.9260	2.1033	1.1734	0.8209
Asai et al., 1991	Average	2.6105	8.2448	2.7435	1.5846	0.8818	2.3296
	S.D.	2.6107	10.5834	5.9949	1.6679	0.8580	2.0040
	C_V	1.0001	1.2836	2.1852	1.0525	0.9730	0.8602
Present Study,1997	Average	1.7005	2.0553	1.3567	1.4106	1.1465	4.1305
	S.D.	1.5979	2.3741	1.9161	1.3203	1.1034	3.3834
	C_V	0.9397	1.1551	1.4123	0.9360	0.9624	0.8191

except for $B/R > 100$. The performance of McQuivey and Keefer (1974) relation is better when $B/R > 100$. For $10 \leq B/R < 20$, the performance of relationships of Yotsukura and Fiering (1964) and Thackston and Krenkel (1967) is good, and for $B/R < 10$, the performance of the equations of Elder (1959) and Sumer (1969) is better.

It may be mentioned that for non-Gaussian input concentrations value of D_L for given data changes greatly depending upon the method used for its determination. The author determined D_L values using the method developed during this study for the laboratory data of Singh (1987), Fischer (1966) and those collected in the present study and field data of Beltaos and Day (1976), Nordin and Sabol (1974), Calandro (1978), James and Helinsky (1984), McQuivey and Keefer (1976), Taylor (1970), Berkas (1986) and Graf (1984). For the remaining data D_L values as determined by the respective investigators were taken since observed C-t curves were not available for these data to enable computation of D_L using author's method. One may, therefore, suppose that part of the scatter of the data seen in Fig. 5.20 is due to the possible errors in the values of D_L themselves. An analysis was thus, made only for those data for which D_L values were determined as per the method developed in the present study. The following equation for D_L was obtained for these data by multiple regression analysis:

$$\frac{D_L}{qS} = 0.253 \times \left(\frac{B}{R}\right)^{2.11} \left(\frac{U}{U_*}\right)^{-1.06} (S)^{-0.98} \tag{5.4}$$

The multiple correlation coefficient for the above equation is 0.98. It is to be observed that values of the exponents of the dependent variables in Eqs. (5.2) and (5.4) are comparable. The D_L values computed as per Eq. (5.4) were plotted in Fig. 5.21 against corresponding observed D_L values. The figure reveals that the scatter is much less than in Fig. 5.20. It is , however, recommended that Eq. (5.2) be used for predictive purposes, because of the much larger range of data used in developing this equation.

Figures 5.20 and 5.21 give an idea of the possible errors in the values of D_L computed from Eqs. (5.2) and (5.4). It would be worthwhile investigating what effect these errors would have on the C-t curves computed using these values of D_L.

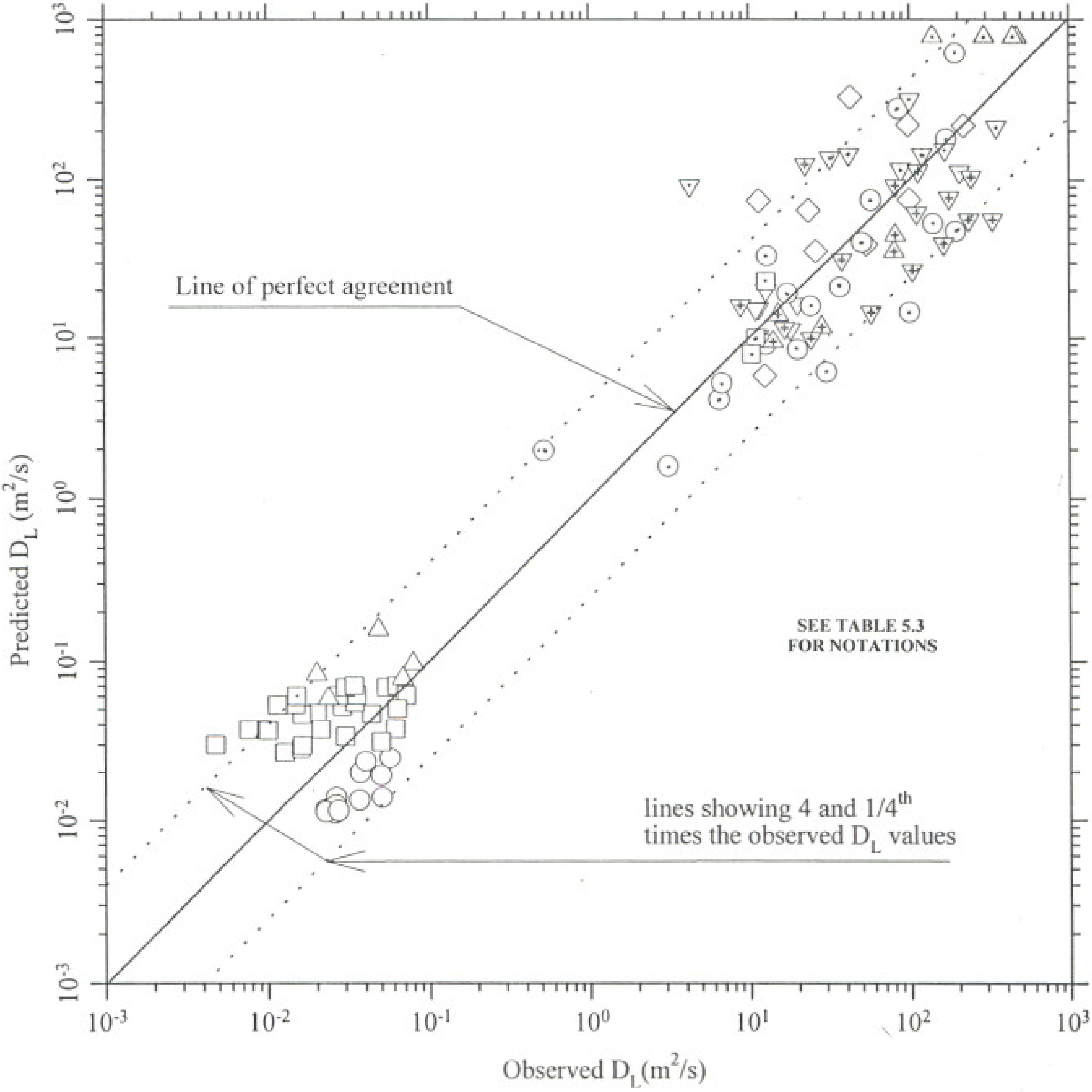


Fig. 5.21 Verification of Author's Equation (derived using only those data where D_L is determined by the method proposed in this study)

A sensitivity analysis of D_L values is, therefore, carried out for determining the effect of such inaccuracy in D_L on computation of the pollutant concentration.

5.6 SENSITIVITY ANALYSIS FOR DISPERSION COEFFICIENT

The scheme proposed in the present study for numerical solution of Eq. (1.1) is used to compute the C-t curves. Four types of input C-t curves are used as shown in Fig. 5.22. These are designated as DLSNT1, DLSNT2, DLSNT3 and DLSNT4. Out of these the C-t curve namely DLSNT1 is symmetrically distributed whereas the others are having skewed distribution. Other data regarding these are presented in Tables 5.5 and 5.6. A wide variation in the range of hydraulic variables in these may be noted. Using the present scheme the C-t curves are predicted at the remaining sections in the downstream using the value of dispersion coefficient called here as D_{LP} as obtained through grid search method. The C-t curves are recomputed using the present scheme with altered values of D_{LP}. For this the values of D_{LP} are altered by multiplying factors of .01, 0.02, 0.05, 0.1, 10, 20, 50 and 100. For comparison the ERS values (see Eq. 3.31) are computed with the C-t curves computed with D_{LP} values taken as the reference. The accuracy of computations is then defined as (100-ERS).

Table 5.5 : Details of Data Used for Sensitivity Analysis

Sl. No.	Data Set Name	D (m)	U (m/s)	S	Q (m3/s)	D_L (m2/s)	C_P (ppm)	D_{LP}*	U*
1	DLSNT1	0.07	0.752	0.004546	0.01053	0.03707	73.80	0.88	1.034
2	DLSNT2	5.135	1.02	0.000118	2625.61	922.08	4.57	36.0	0.144
3	DLSNT3	0.25	0.405	0.000314	12.34	124.78	49.87	360.0	0.2586
4	DLSNT4	0.12	0.6896	0.00603	0.0331	0.0539	42.0	0.37	0.6356

In above table C_P is the peak concentration at the first station.

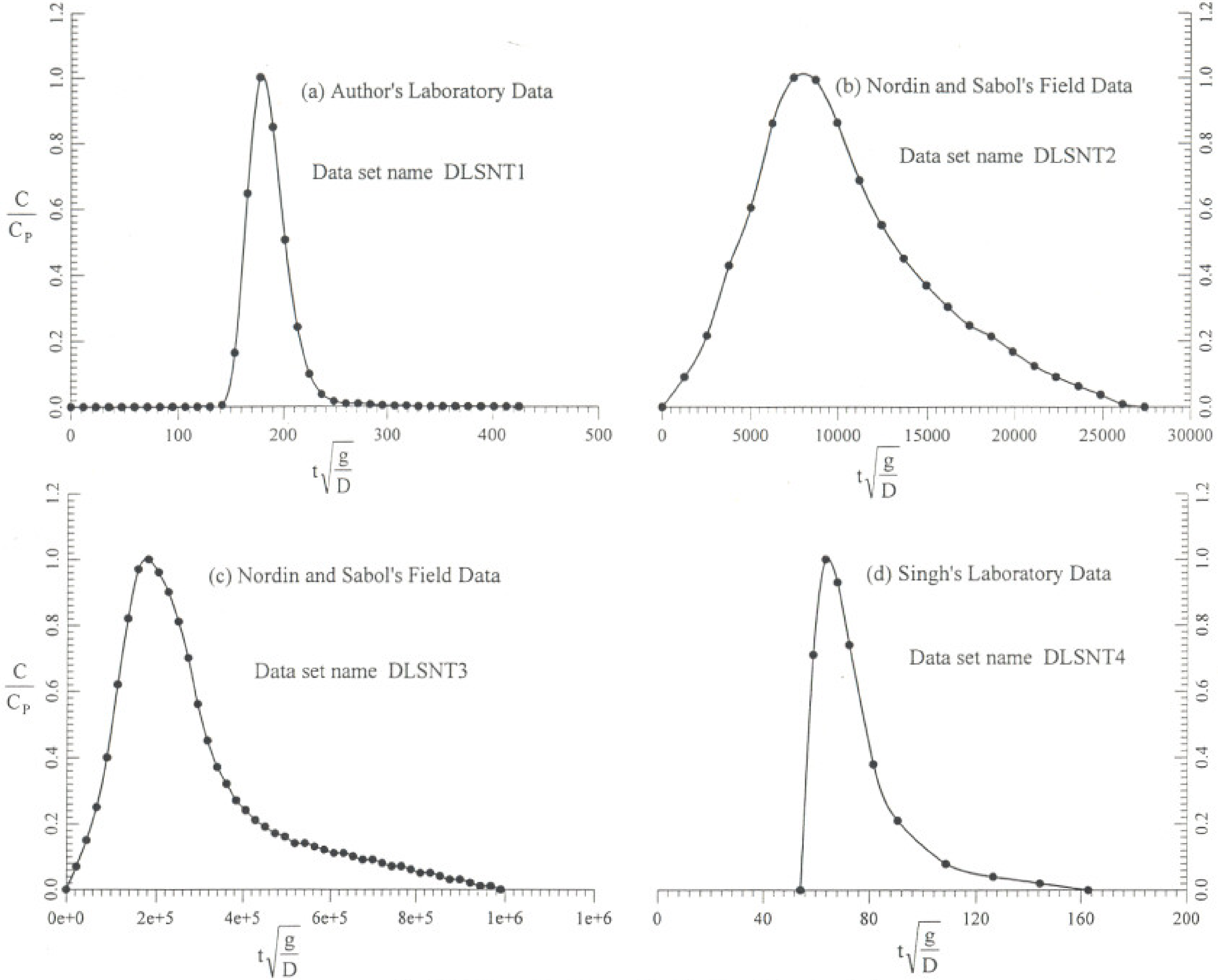


Fig. 5.22 Input C-t Curves Used in Sensitivity Analysis

Table 5.6 : Distance of Stations Selected for Sensitivity Analysis

Sl. No.	Data Set Name	Distance of Stations From First Station (m)			
		First Station	Second Station	Third Station	Fourth Station
1	DLSNT1	0.00	6.00	12.00	18.00
2	DLSNT2	0.00	25000.00	63233.7	----
3	DLSNT3	0.00	10000.00	18503.5	----
4	DLSNT4	0.00	2.00	4.00	6.00

The values of the accuracy so computed are plotted against the corresponding D_L/D_{LP} values for all the four cases in Fig. 5.23. A close study of Fig. 5.23 reveals that accuracy for computation of C-t curves changed by a factor of only about 0.75 when the dispersion coefficient is altered with a multiplying factor varying from 0.1 to 10, although accuracy of the computations of C-t curves reduces drastically for still larger variation in D_L values. For instance even with a value of D_L which is ten times larger than the correct value the ERS is only of the order of 30% for the data set DLSNT1 and of the order of 28% for the data set DLSNT4. Likewise the variation of peak of the C-t curves with variation in the value of D_L is presented in Fig. 5.24 for all the four cases. This figures also reveal that the peak of C-t curve does not change very much when the value of D_L is altered within the range of 0.1 to 10.0 times the actual value.

In the light of these results it is concluded that the likely error in D_L in using Eq. (5.2) does not result in errors of the same order of magnitude in the C-t curves obtained using these D_L values.

5.7 DISPERSION IN SEDIMENT-LADEN FLOW

To study the effect of suspended sediment on longitudinal dispersion, C-t curves are observed for clear-water flow and for sediment-laden flow under nearly identical flow conditions, using sands of average diameter 0.064 mm and 0.164 mm with their concentration in suspension ranging from 500 ppm to 11,000 ppm. Such data collected by

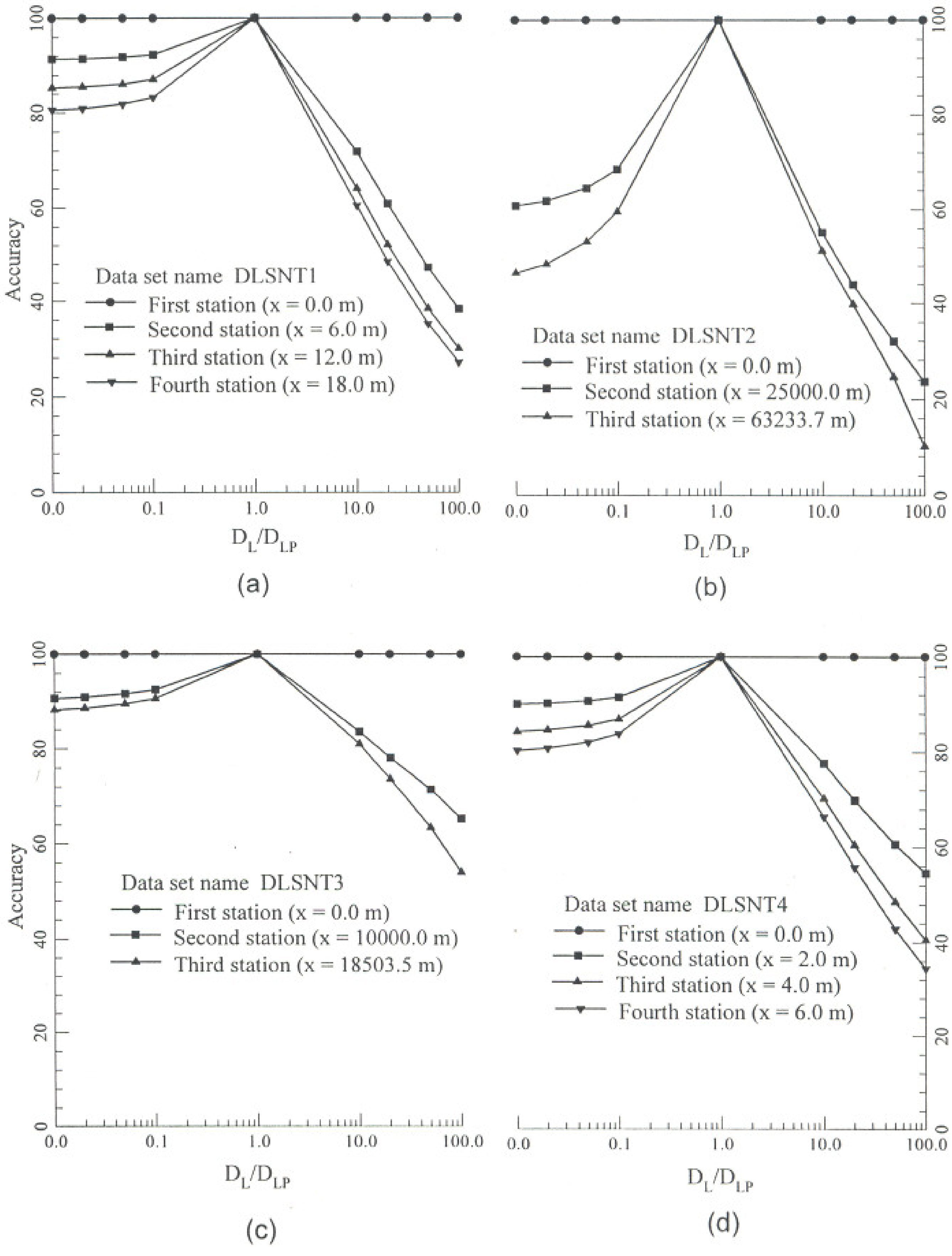
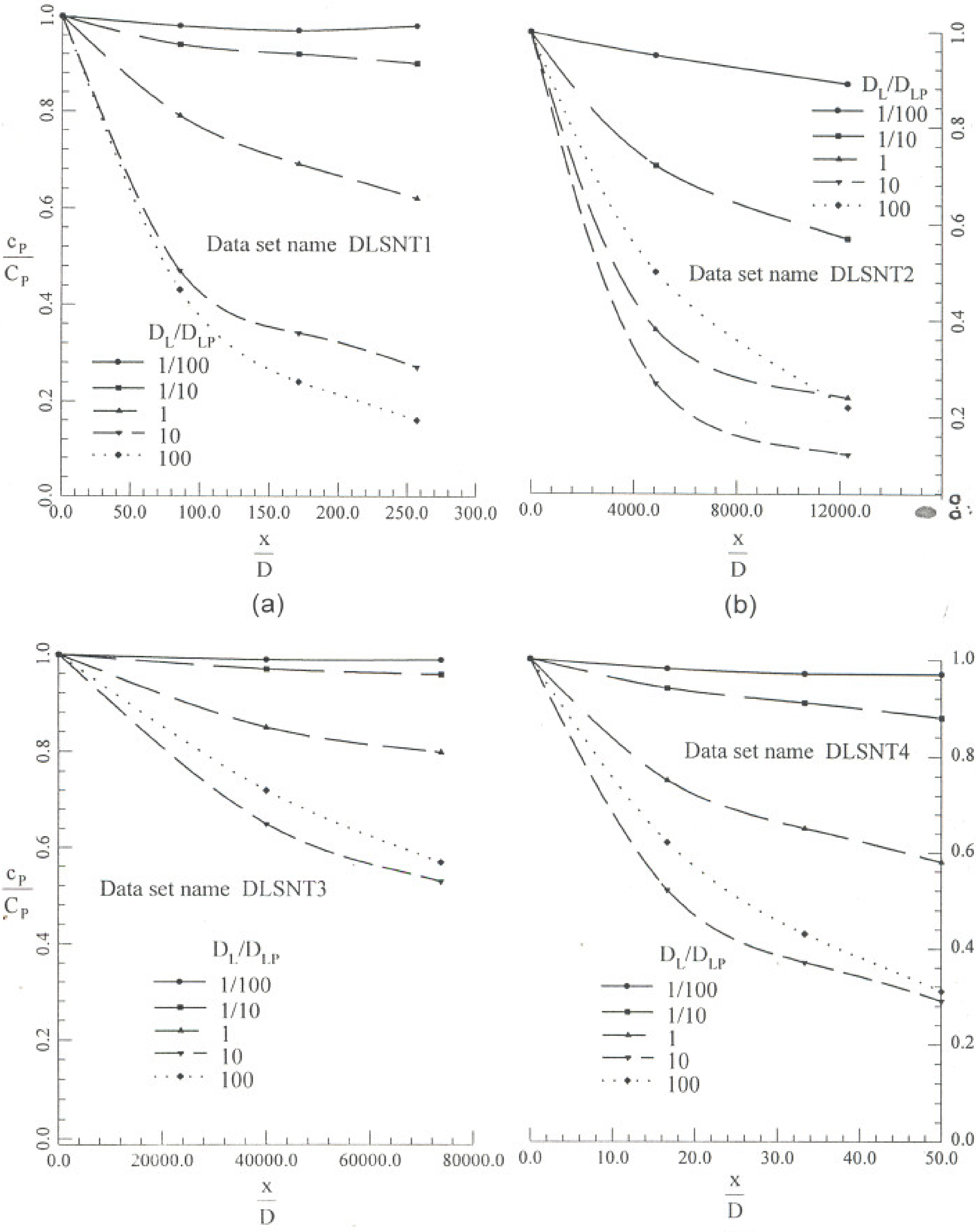


Fig. 5.23 Effect of Variation of D_L on Accuracy of Predicted C-t Curves.



(a) (b)

(c) (d)

Fig. 5.24 Effect of Variation of D_L on Accuracy of Peak Concentration

Singh (1987) are also utilised in the present study. The velocity distributions observed in clear-water flow (CWF) and corresponding sediment-laden flows (SLF) for two runs are shown in Fig. 5.25 for illustration. Study of these and other such figures revealed that the velocity distribution becomes more non-uniform as the concentration of sediment load increases in the flow. However, the gross flow characteristics such as the depth-averaged velocity does not change appreciably because of the presence of suspended load. These results are in conformity with the findings of earlier studies (Samaga, 1986; Arora, 1983; and Umeyama and Gerritsen, 1992).

As mentioned earlier, the longitudinal dispersion is significantly affected by the non-uniformity of velocity distribution because the former occurs mainly due to convection. Since sediment-laden flows are having greater non-uniformity in velocity distribution, it is expected that such flows will have different dispersion characteristics as compared to clear-water flows.

The C-t curves observed for clear-water flows and corresponding sediment-laden flows are superimposed on each other for studying the effect of sediment concentration on C-t curves. In Fig. 5.26 such C-t curves are depicted for illustration for two runs. More attenuation of the peak of C-t curve as expected during the sediment transporting flows is evident from this and many other such figures (not shown here). However, the magnitude of difference between C-t curves of clear-water and sediment-laden flows depends upon sediment concentration, flow and the channel parameters.

The values of dispersion coefficient, D_{LS} for sediment-laden flows was, therefore, analysed to account for these effects. Following functional relationship is assumed for D_{LS}:

$$D_{LS} = \phi_1\left(D_{LC}, C_S, S, d_a, U_*, \rho_f, \Delta\gamma_S\right) \tag{5.5}$$

where D_{LC} = dispersion coefficient in clear-water flow, C_s = cross-sectional average concentration of sediment, d_a = arithmetic-mean size of sediment, ρ_f = mass density of water and $\Delta\gamma_s$ = difference of specific weights of sediment and water.

Equation (5.5) can be written as below in non-dimensional form

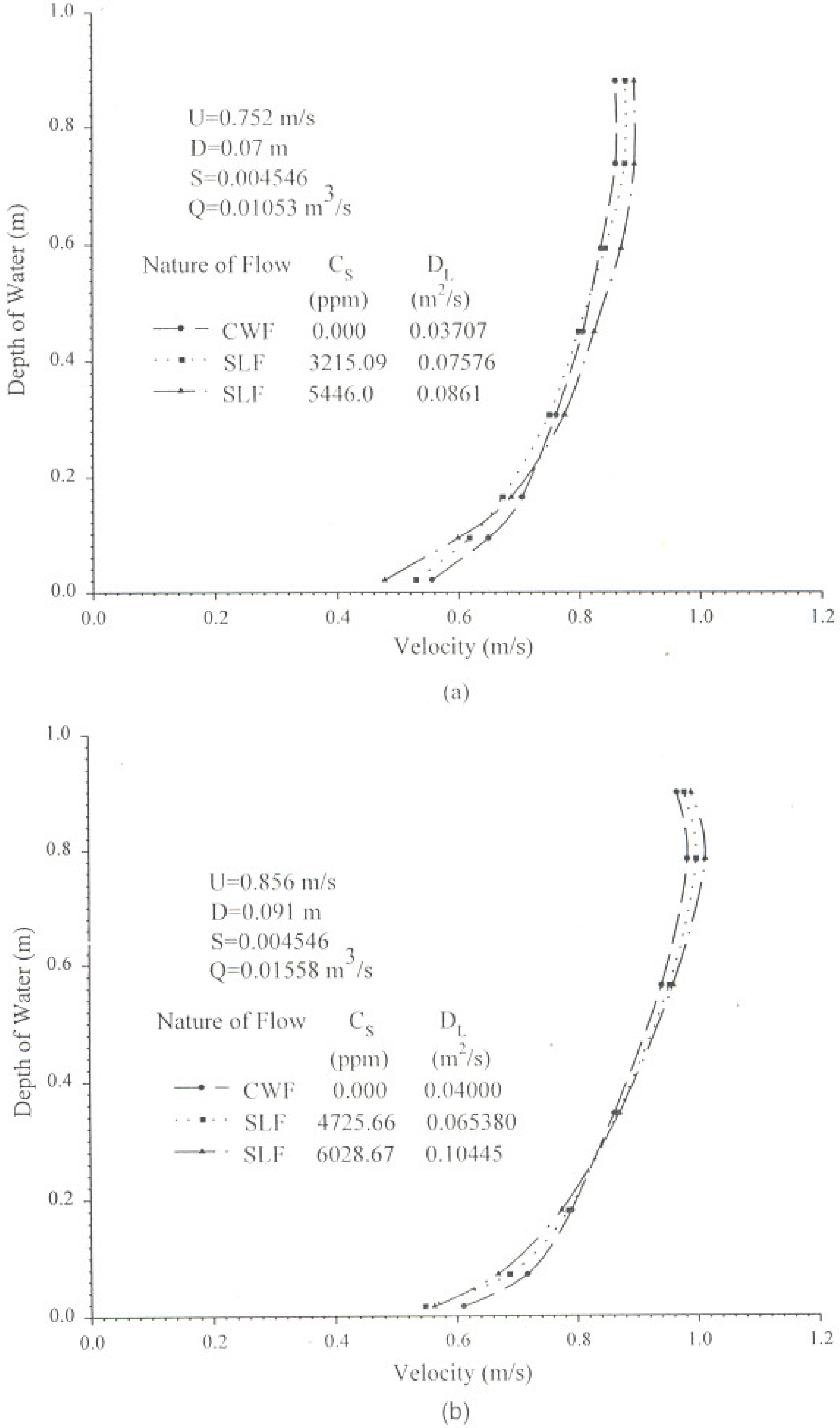


Fig. 5.25 Velocity Distribution in Clear-Water and Sediment-Laden Flow

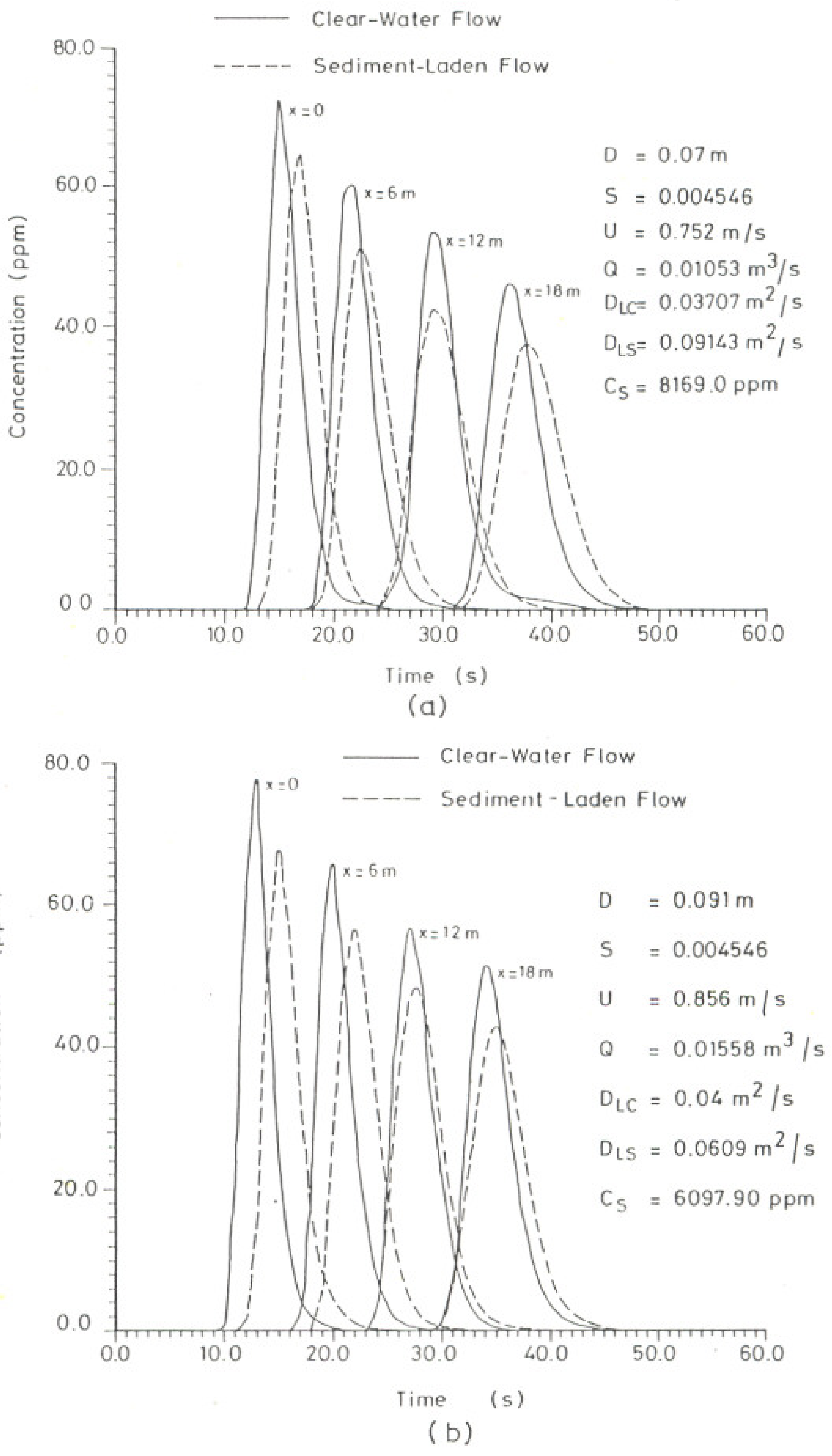


Fig. 5.26 Observed C-t Curves for Clear-Water Flow and Sediment-Laden Flow

$$\frac{D_{LS} - D_{LC}}{D_{LC}} = \phi_2\left(C_S, S, \frac{\sqrt{\frac{\Delta\gamma_S}{\rho_f} d_a}}{U_*} \right) \tag{5.6}$$

Through multiple regression based on least square method, the following equation is obtained:

$$\frac{D_{LS} - D_{LC}}{D_{LC}} = 2.47 \times 10^{-9} \frac{C_S^{1.3}}{S^{1.6}} \left(\frac{\sqrt{\frac{\Delta\gamma_S}{\rho_f} d_a}}{U_*} \right)^{1.278} \tag{5.7}$$

The multiple correlation coefficient for Eq. (5.7) is 0.60. The data used for this purpose included the laboratory sediment-laden flow data of present study and Singh's (1987) study. The observed values of D_{LS} are graphically compared with the values computed using Eq. (5.7) in Fig. 5.27.

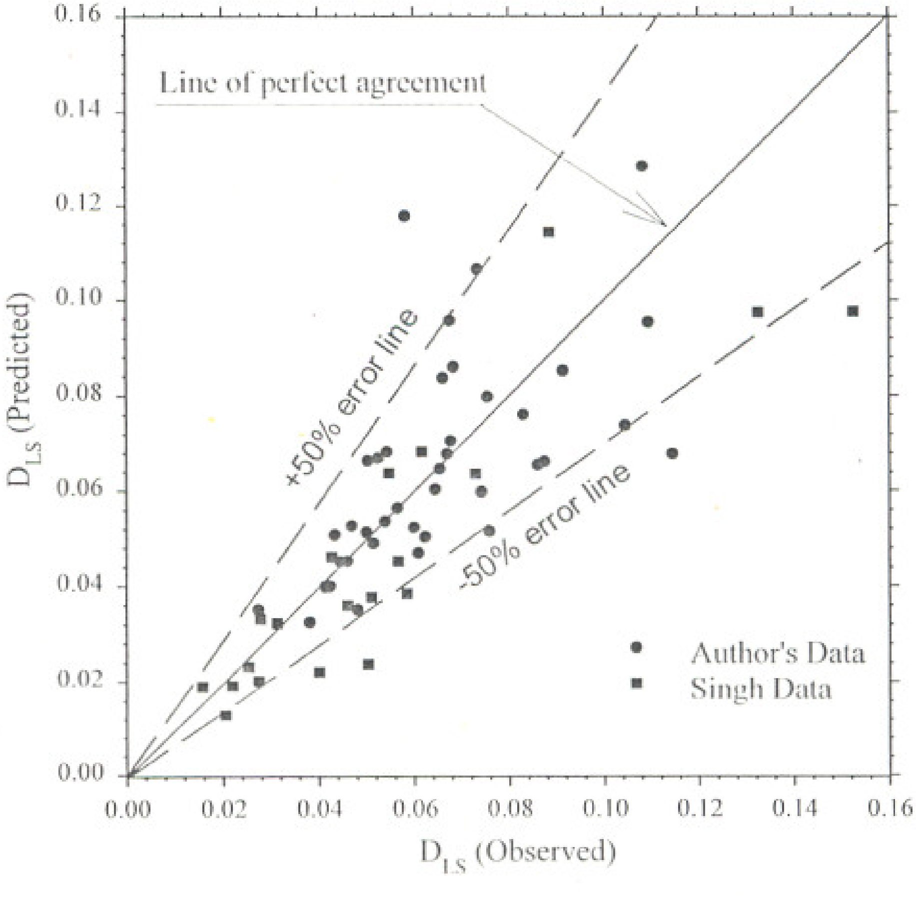


Fig. 5.27 Predicted and Observed Dispersion Coefficient for Sediment-Laden Flow

This figure reveals that Eq. (5.7) predicts the D_{LS} values with an accuracy of larger than ±50%. Considering that C-t curves are not very sensitive to D_L values, the accuracy obtained herein is satisfactory.

5.8 COMPUTATION OF C-t CURVES

For computation of C-t curves the stepwise procedure was described in section 3.4. For this the D_L value is first estimated by using Eq. (5.2) for clear-water flows and by using Eq. (5.7) for sediment-laden flows. Next, the C-t curves are computed using the algorithm described in Fig. 3.8 and the D_L value computed as above.

The computed C-t curves were plotted alongwith the corresponding observed C-t curves for graphical comparison using all the data for clear-water and sediment-laden flows. In Figs. 5.28 to 5.36 such a comparison is illustrated for some of the data. As revealed by these figures, generally the prediction of C-t curves is satisfactory as shown by Figs. 5.28, 5.29, 5.31, 5.34, 5.35 and 5.36 and many other such figures which are not shown here. Excellent prediction of C-t curves was also obtained for some of the data, for example in Fig. 5.32. However, poor prediction (Figs. 5.30 and 5.33, for instance) was obtained only for few data. Thus, it can be stated that the C-t curves can be satisfactorily predicted using the method proposed in the present study.

5.9 CONCLUDING REMARKS

Analysis regarding the dispersion characteristics in open channel flows has been presented using the data obtained in the present study and the data available from studies of previous investigators. A new numerical scheme for the solution of Eq. (1.1) for longitudinal dispersion has been proposed. The proof-of-the-concept tests for the proposed scheme have been made with Fischer's (1968) analytical model and Jaque and Ball's (1994) numerical method. The proposed scheme is also applicable in non-uniform flow and for varying dispersion coefficient. The numerical scheme has been extended to determine the dispersion coefficient using one-dimensional grid search. Various predictors for estimating D_L have also been checked and it has been shown that none of the predictions is in good agreement with the actual values of D_L. Further, they give widely varying results. A new predictor for D_L has been proposed and it is shown that the new predictor gives comparatively good results. Even the new predictor for D_L gives D_L values in many cases which are $\frac{1}{10^{th}}$ or 10 times the actual D_L values. Therefore a sensitivity analysis has been made and it has been found that even if the value of D_L varies by a factor of ten, there is not a very large change in the predicted C-t curve at the downstream stations. It has been also shown through analysis of field and laboratory data that the value of D_L for uniform flow is constant but for non-uniform flow D_L varies with hydraulic parameters. The D_L value was found to decrease with distance when shear velocity decreases in the flow direction also D_L value was found to increase with distance when shear velocity was increasing in the flow direction. In the case of sediment-laden flow the dispersion is more than in the corresponding clear-water flow. This is also highlighted by observing the velocity distribution along the depth of flow and it has been observed that the deviation of velocity from the mean velocity is more in sediment-laden flow than in the clear-water flow. A predictor for D_L in sediment-laden flow has been also proposed.

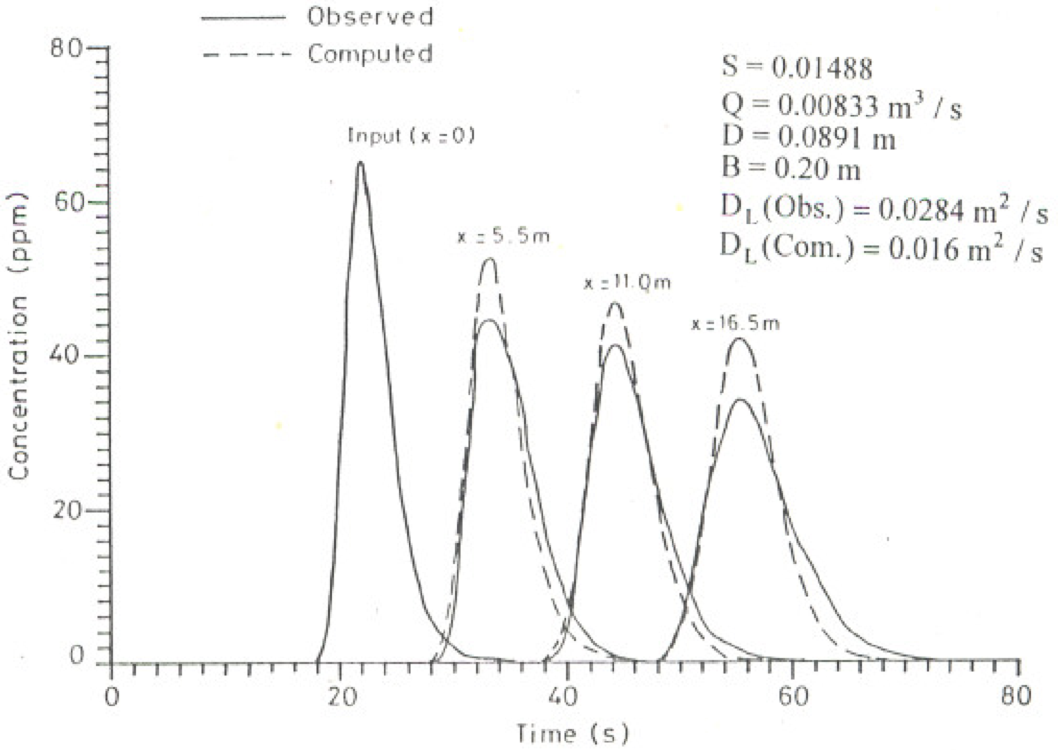


Fig. 5.28 Computed and Observed C-t Curves for Clear-Water Flow. (Present Study Data)

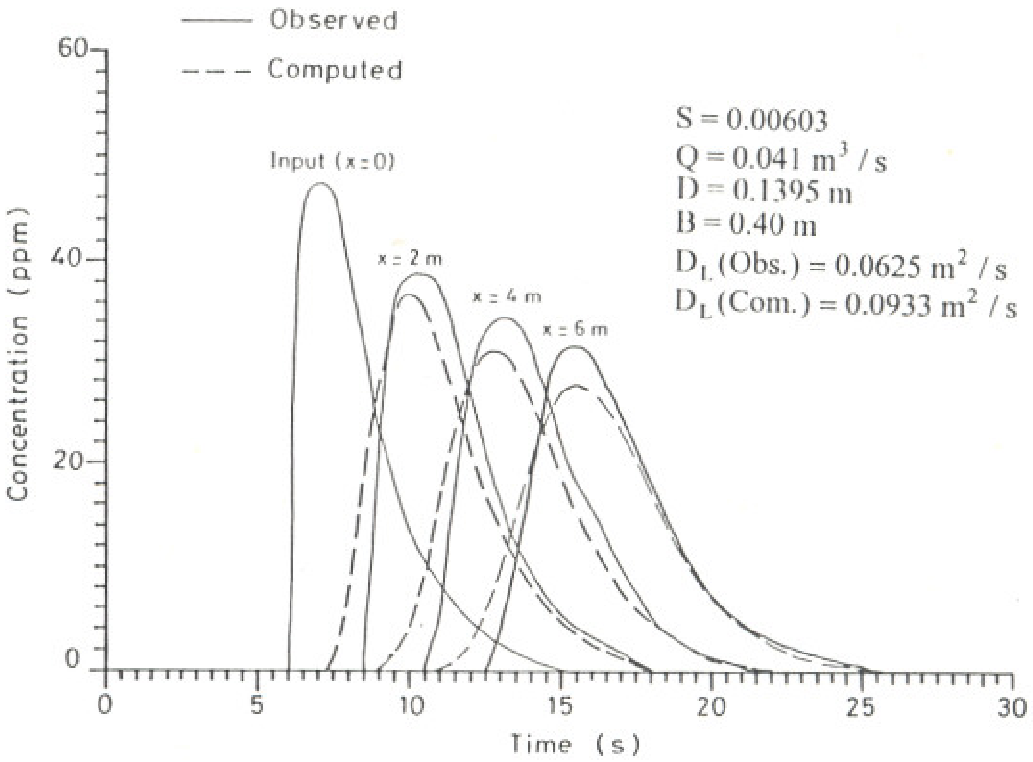


Fig. 5.29 Computed and Observed C-t Curves for Clear-Water Flow.
(Singh's Laboratory Data)

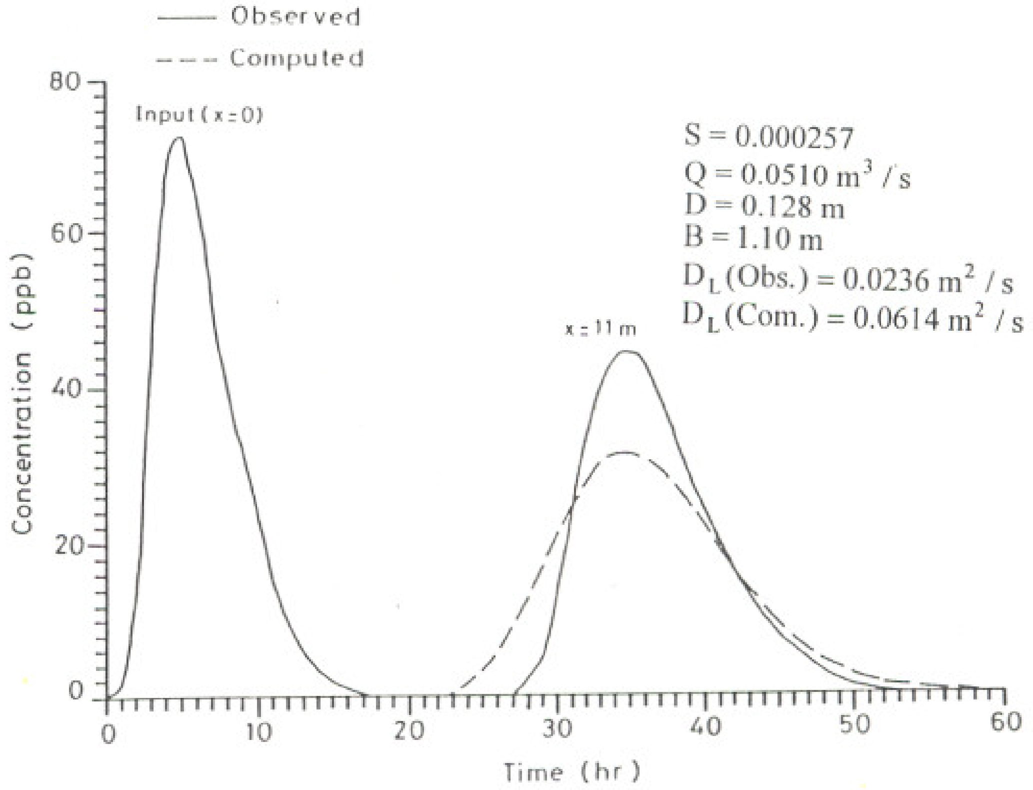


Fig. 5.30 Computed and Observed C-t Curves for Clear-Water Flow.
(Fischer's Laboratory Data)

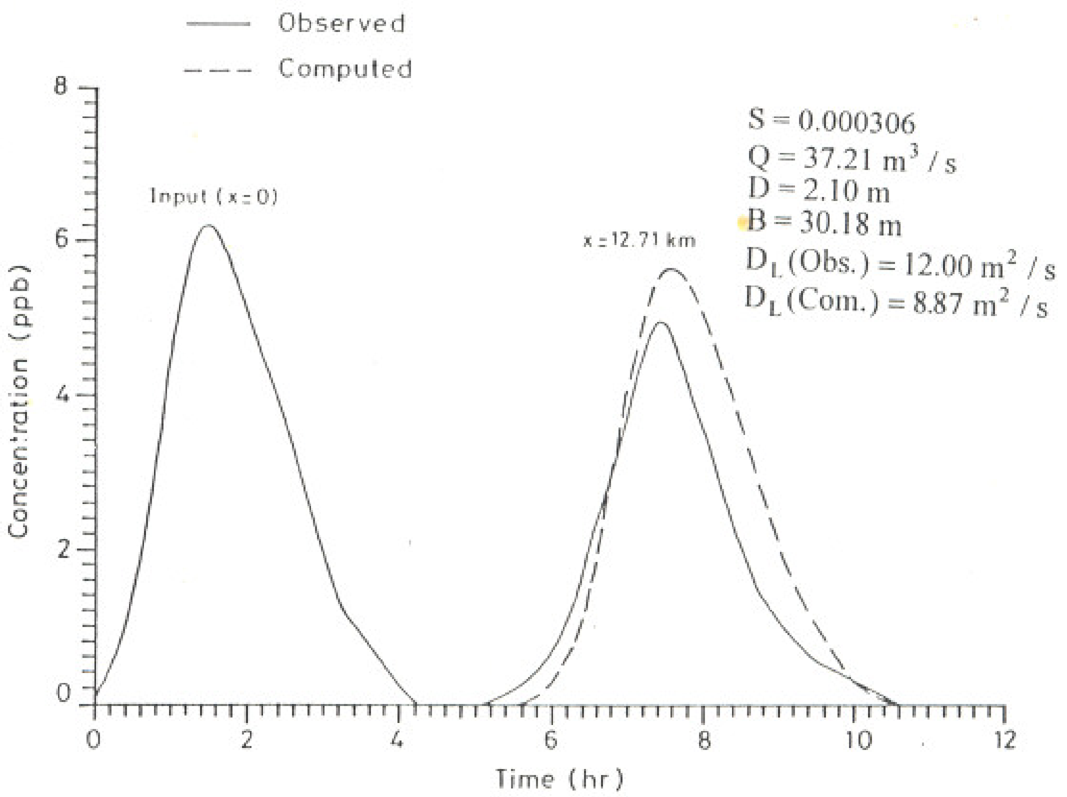


Fig. 5.31 Computed and Observed C-t Curves for Clear-Water Flow.
(Berkas's Field Data)

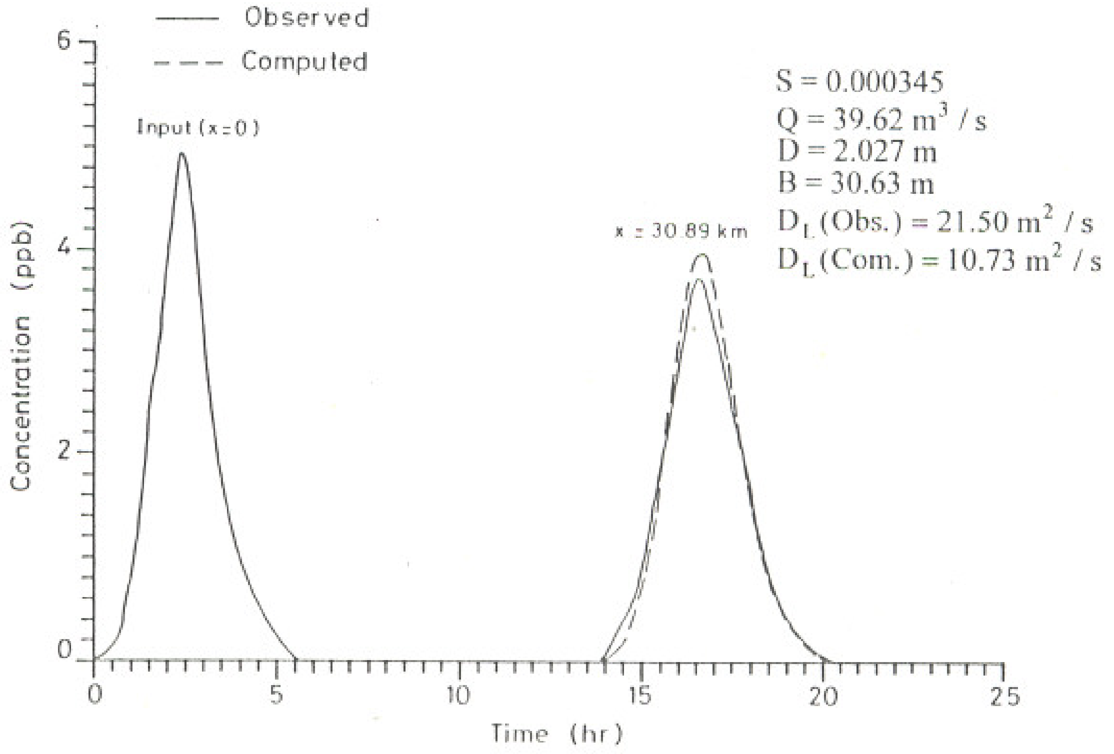


Fig. 5.32 Computed and Observed C-t Curves for Clear-Water Flow.
(Berkas's Field Data)

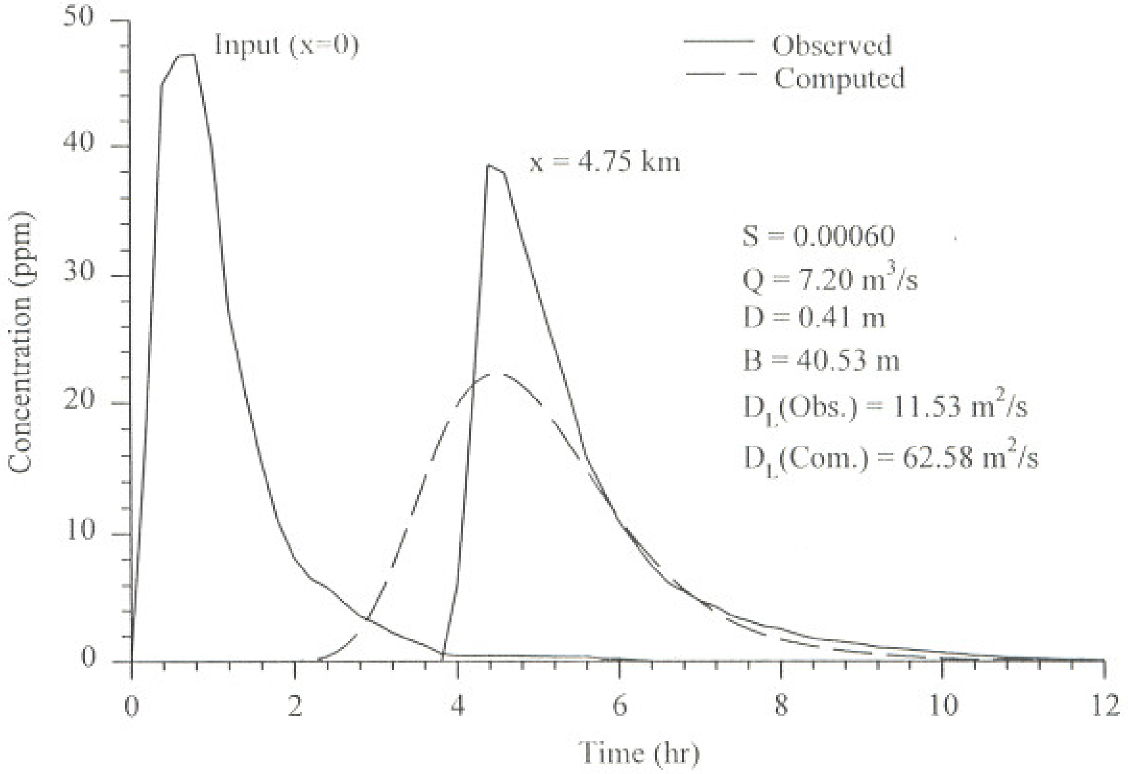


Fig. 5.33 Computed and Observed C-t Curves for Clear-Water Flow (Nordin and Sabol's Field Data)

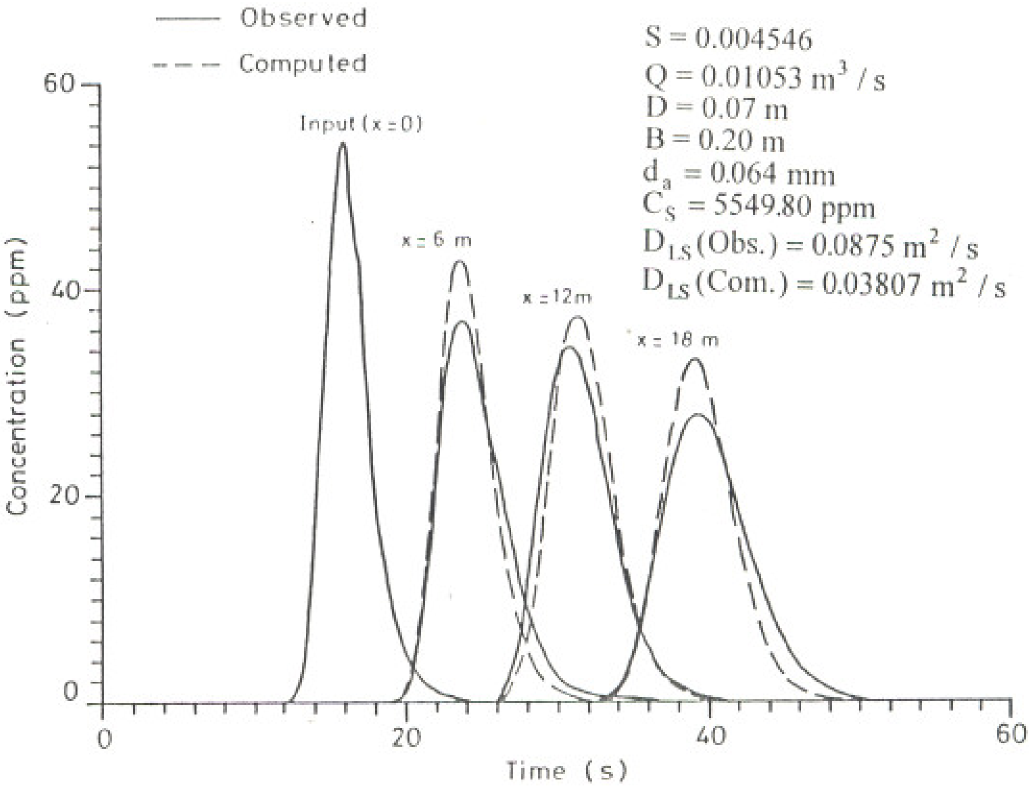


Fig. 5.34 Computed and Observed C-t Curves for Sediment-Laden Flow. (Present Study Data)

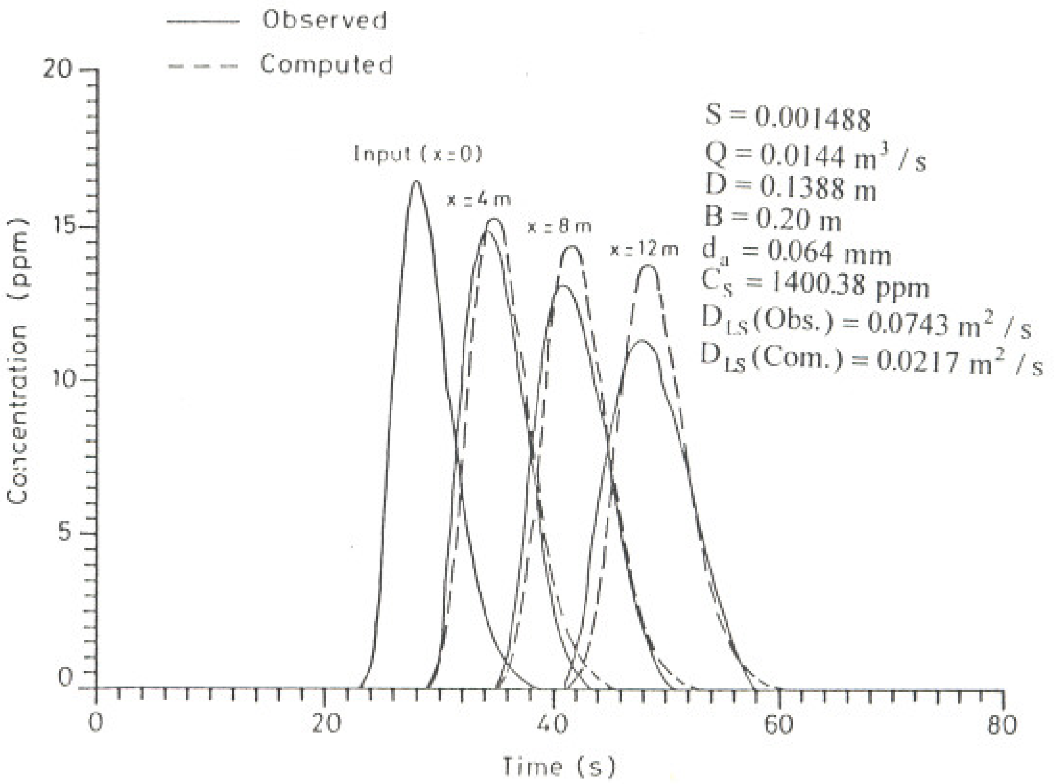


Fig. 5.35 Computed and Observed C-t Curves for Sediment-Laden Flow. (Present Study Data)

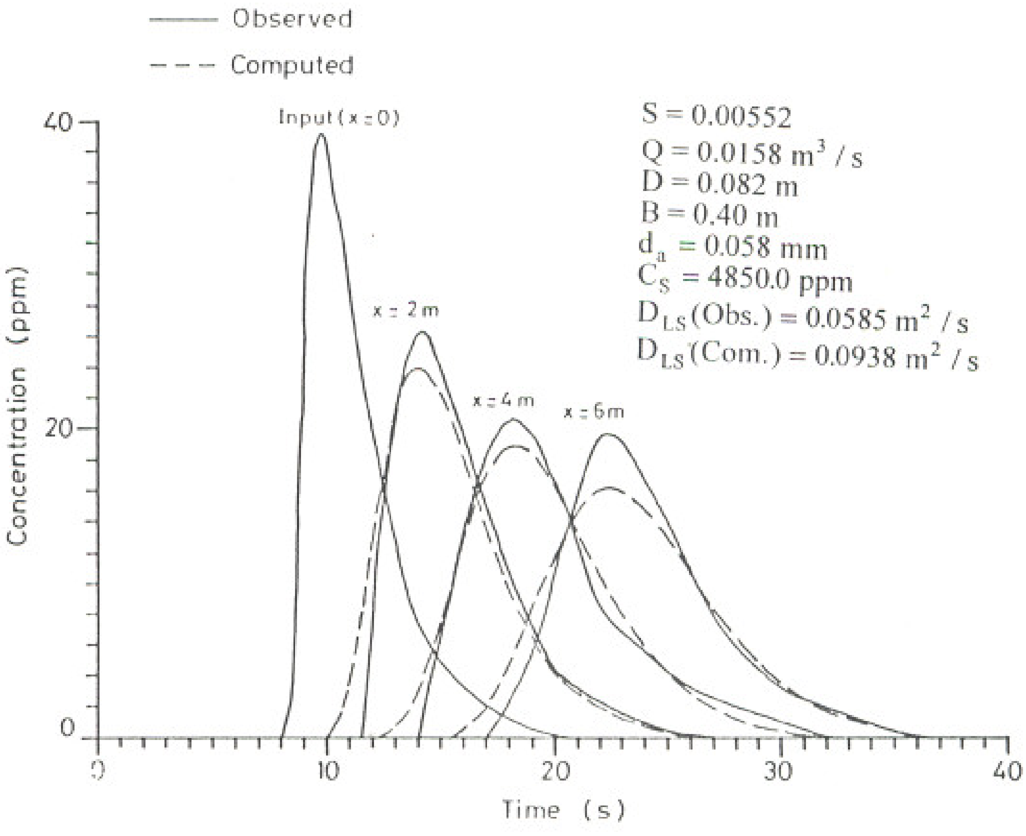


Fig. 5.36 Computed and Observed C-t Curves for Sediment-Laden Flow. (Singh's Laboratory Data)

6

CONCLUSIONS

The present study was carried out to evolve a numerical solution of the longitudinal dispersion process represented by Eq. (1.1). The other aim of the study was to investigate the effect of suspended sediment load on the longitudinal dispersion of a conservative pollutant.

From the analysis of the experimental data collected in the present investigation and the laboratory and river data covering a wide range of pertinent variables available from earlier studies, the following conclusions have been drawn.

1. A new finite difference numerical scheme for solution of Eq. (1.1) for longitudinal dispersion has been proposed. The proof-of-the-concept tests for the proposed scheme have been made with Fischer's (1968) analytical model and Jaque and Ball's (1994) numerical method. The proposed scheme is also applicable in non-uniform flow and for varying dispersion coefficient. The Combined operator scheme has been adopted in the numerical solution used, wherein exact solution of advection process is obtained by developing a variable spatial grid so that the root of the trajectory of the concentration characteristics passes through the computational nodes. Solution of the diffusion process is achieved using Crank-Nicholson scheme by using a temporal weighting coefficient.

2. The proposed numerical scheme is particularly useful for prediction of pollution concentration in non-uniform flows. Conventionally such predictions are made by treating the non-uniform flow as an equivalent uniform flow. However, large differences are found to occur in the predicted C-t curves by treating the flow as non-uniform and approximating it as uniform.

   The proposed scheme has been extended by incorporating in it the one-dimensional grid search method for determination of D_L values using the observed C-t curves at two or more stations.

3. Analysis of field and laboratory data revealed that D_L is constant along the flow distance for uniform flow, while in non-uniform flow D_L varies with distance because of its strong dependence on shear velocity. In non-uniform flows variation of D_L can be explained using Eq. (3.2). The multiplying coefficient K in Eq. (3.2) becomes zero when flow is uniform i.e. under constant shear velocity (for example in the case of Conocheague creek). Its value is negative when shear velocity decreases in the flow direction (for example in the case of Antietam creek), while value of K is positive when the shear velocity increases along the flow direction (for example in the case of Chattahoochee river).

4. Predictors for D_L. proposed by Fischer (1966), Asai *et al.* (1991), Liu (1978), Marivoet and Craenenbroeck (1986), McQuivey and Keefer (1974) and Thackston and Krenkel (1967) were checked for their accuracy using a wide range of data collected in the present study. Predictions of D_L thus obtained are compared graphically with observed D_L values in Figs. 5.14 to 5.19. It was noticed that none of these predictors produce reasonable estimate of D_L values. Predictions from Asai *et al.* (1991) have relatively less error and even here they differ from the actual by a factor of about ten or more. Predictions from Elder (1959) and Sumer (1969) perform well for $B/R < 10$. For $B/R > 100$, McQuivey and Keefer (1974) predictor is good.

5. A new relationship, viz. Eq. (5.2), is obtained for prediction of D_L. Comparison was made between the values predicted from this equation with the observed D_L values. The proposed relationship gives better results than the existing methods, Nevertheless, the predicted and the observed D_L values depart from each other by a maximum factor of ten. Overall performance of new relationship is well except for B/R>100.

6. The errors in the predicted C-t curve due to errors in D_L are considerably smaller than the errors in D_L. Even when the D_L value is varied by a factor varying from 0.1 to 10, the accuracy for computation of C-t curve varied by a factor of only about 0.75. The accuracy decreases with the increase of distance of downstream stations. Likewise the peak of C-t curves did not change very much, when the value of D_L was altered within the range of 0.1 to 10.0 times the actual value.

7. More dispersion occurs in sediment-laden flows than in the corresponding clear-water flows. Dispersion is found to increase with an increase in sediment concentration in the flow, as the deviation of velocity distribution from the mean velocity is more in sediment-laden flows than in clear-water flows. No effect of sediment size is observed on longitudinal dispersion within the range of experimental data used in present study.

8. A relationship, viz. Eq. (5.7), is proposed for prediction of D_L values in sediment-laden flows. Predicted values of D_L from the proposed relationship indicate a maximum error of ±50 percent from the observed D_L values. The C-t curves computed in sediment-laden flows using D_L calculated by Eq. (5.7) and the proposed numerical scheme show a good agreement with the observed C-t curves.

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APPENDIX-I

(A) DISPERSION COEFFICIENT AND FLOW PARAMETERS IN CLEAR-WATER RUNS

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U_* (m/s)	R (m)	U (m/s)	A (m^2)
	Present study - Laboratory Data								
1	.004546	.01053	.0700	.2000	.03707	.0408	.0374	.7521	.0140
2	.004546	.01558	.0910	.2000	.04000	.0420	.0395	.8560	.0182
3	.004546	.02188	.1200	.2000	.05678	.0450	.0454	.9117	.0240
4	.004546	.01863	.1118	.2000	.05000	.0477	.0511	.8332	.0224
5	.002470	.00589	.0580	.2000	.02631	.0288	.0343	.5078	.0116
6	.002470	.01053	.0915	.2000	.03661	.0335	.0464	.5754	.0183
7	.002470	.01558	.1243	.2000	.05060	.0364	.0547	.6267	.0249
8	.001488	.00833	.0891	.2000	.02273	.0248	.0421	.4675	.0178
9	.001488	.01177	.1180	.2000	.02475	.0269	.0494	.4987	.0236
10	.001488	.01442	.1388	.2000	.02623	.0278	.0531	.5195	.0278
11	.001488	.00645	.0741	.2000	.02579	.0239	.0390	.4352	.0148
12	.001488	.00833	.0900	.2000	.02273	.0251	.0433	.4628	.0180
13	.001488	.01317	.1309	.2000	.02719	.0280	.0538	.5031	.0262
	Singh (1987) - Laboratory Data								
14	.002790	.02300	.1300	.4000	.02000	.0559	.0788	.4423	.0520
15	.002790	.01650	.1060	.4000	.01613	.0511	.0693	.3892	.0424
16	.002790	.03000	.1430	.4000	.04350	.0575	.0834	.5245	.0572
17	.003630	.02000	.1080	.4000	.01143	.0586	.0701	.4630	.0432
18	.003630	.02850	.1320	.4000	.01487	.0638	.0795	.5398	.0528
19	.003630	.01500	.0835	.4000	.02867	.0517	.0589	.4491	.0334
20	.003630	.03600	.1550	.4000	.03422	.0685	.0873	.5806	.0620
21	.004712	.03750	.1500	.4000	.03400	.0773	.0857	.6250	.0600
22	.004712	.02220	.1061	.4000	.02700	.0662	.0693	.5231	.0424
23	.004710	.01480	.0847	.4000	.03525	.0600	.0595	.4368	.0339
24	.004710	.02780	.1190	.4000	.01500	.0694	.0746	.5840	.0476
25	.006030	.02300	.0965	.4000	.04113	.0714	.0651	.5959	.0386
26	.006030	.03310	.1200	.4000	.05390	.0785	.0750	.6896	.0480
27	.006030	.04100	.1395	.4000	.06244	.0840	.0822	.7348	.0558
28	.001300	.01460	.0730	.4000	.01270	.0263	.0535	.5000	.0292
29	.001300	.01750	.1030	.4000	.01594	.0324	.0680	.4248	.0412
30	.001300	.00800	.0490	.4000	.01626	.0225	.0394	.4082	.0196
31	.002370	.01500	.0580	.4000	.06100	.0320	.0450	.6466	.0232
32	.004240	.00930	.0340	.4000	.07144	.0343	.0291	.6838	.0136
33	.004240	.01920	.0500	.4000	.06281	.0386	.0400	.9600	.0200

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U_* (m/s)	R (m)	U (m/s)	A (m^2)
34	.001910	.01202	.0900	.4000	.00900	.0388	.0621	.3339	.0360
35	.001910	.01206	.0900	.4000	.00759	.0388	.0621	.3350	.0360
36	.001910	.01267	.0900	.4000	.01000	.0386	.0621	.3519	.0360
37	.001910	.01156	.0900	.4000	.02117	.0390	.0621	.3211	.0360
38	.001310	.01235	.0950	.4000	.00477	.0325	.0644	.3250	.0380
39	.006030	.04100	.1395	.4000	.03400	.0840	.0822	.7348	.0558
40	.001880	.02000	.0750	.4000	.05000	.0311	.0545	.6667	.0300
41	.002040	.01670	.0648	.4000	.03004	.0308	.0489	.6443	.0259
	Fischer (1966) - Laboratory Data								
42	.000540	.03703	.1390	1.1000	.07910	.0265	.1110	.2420	.1530
43	.001040	.02194	.0940	1.1000	.04810	.0308	.0803	.2120	.1035
44	.000330	.04497	.1840	1.1000	.06810	.0259	.1379	.2220	.2026
45	.000310	.02043	.0690	1.1000	.01000	.0136	.0613	.2690	.0760
46	.000257	.05101	.1280	1.1000	.02364	.0162	.1038	.3620	.1409
	Belta and Day (1976) - Field Data								
47	.000100	71.03	2.82	39.96	11.48	.052	2.480	.628	113.11
48	.000100	70.51	2.61	48.34	11.50	.050	2.368	.556	126.81
49	.000100	70.73	2.49	54.60	12.76	.049	2.296	.517	136.80
50	.000100	71.79	2.71	47.43	11.00	.052	2.408	.564	127.28
51	.000100	71.07	2.93	41.33	18.13	.055	2.579	.584	121.70
52	.000100	80.94	3.74	53.40	20.00	.061	3.392	.392	206.48
	Nordin and Sabol (1974) - Field Data								
53	.000600	7.20	.41	40.53	11.53	.048	.405	.430	16.74
54	.000600	6.81	.69	42.21	23.54	.064	.670	.233	29.21
55	.000272	145.01	2.34	79.40	99.89	.079	2.210	.780	185.80
56	.000468	3.07	.30	28.96	53.73	.037	.294	.354	8.69
57	.000314	1.26	.25	12.34	12.478	.027	.241	.405	3.10
58	.000690	10.80	.79	32.31	26.17	.073	.755	.422	25.59
59	.000164	348.72	2.27	153.62	98.22	.060	2.205	1.000	348.72
60	.000146	378.78	2.30	158.20	217.91	.057	2.235	1.041	363.86
61	.000007	6812.74	17.51	701.04	43.14	.036	16.677	.555	12275.2
62	.000118	2625.61	5.14	500.80	922.08	.077	5.032	1.021	2571.61
63	.000118	7126.02	9.07	535.68	1417.81	.102	8.771	1.467	4857.55
	Calandro (1978) - Field Data								
64	.000261	16.87	.67	46.63	194.00	.041	.652	.539	31.27
65	.000062	2.69	.58	19.81	6.50	.018	.547	.235	11.47
66	.000109	14.41	.91	37.19	24.30	.031	.872	.424	34.00
67	.000027	7.29	1.43	19.20	3.10	.018	1.247	.265	27.51
68	.000076	6.52	.73	29.26	12.73	.023	.697	.305	21.40
69	.000318	8.90	.58	28.65	36.89	.042	.557	.536	16.59
70	.000077	2.58	.52	20.42	6.765	.019	.493	.244	10.58
71	.000365	1.11	.24	14.02	20.125	.029	.236	.324	3.42
72	.000035	90.32	2.13	100.28	50.781	.026	2.047	.422	213.96
73	.000101	249.73	1.62	253.59	194.06	.040	1.595	.610	409.67
74	.000007	183.40	3.96	161.54	13.00	.016	3.777	.287	640.10
75	.000089	126.04	1.40	172.97	85.39	.035	1.380	.520	242.52
76	.000019	251.45	3.66	152.40	140.00	.026	3.490	.451	557.42

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U. (m/s)	R (m)	U (m/s)	A (m^2)
77	.000030	188.19	2.19	211.53	169.53	.025	2.150	.405	464.22
78	.000190	165.29	1.01	268.22	315.63	.043	.998	.613	269.79
79	.000084	120.66	1.90	103.39	57.65	.039	1.837	.613	196.95
80	.000154	11.41	.79	31.49	100.16	.034	.754	.457	24.95
81	.000023	16.26	1.52	39.78	30.31	.018	1.416	.268	60.62
82	.000191	6.90	.55	30.66	17.41	.032	.535	.405	17.01
83	.000039	2.20	.67	16.31	.52	.015	.620	.201	10.93
	James and Helinsky (1984) - Field Data								
84	.003410	.62	.30	12.95	12.75	.099	.291	.156	3.95
85	.003410	1.54	.48	9.94	11.00	.121	.439	.322	4.79
86	.003410	2.22	.61	9.36	10.35	.134	.539	.389	5.71
	McQuivey and Keefer (1976) - Field Data								
87	.000020	21636.54	16.76	867.00	471.68	.058	16.760	1.489	14530.9
88	.000020	20662.97	16.76	867.00	447.61	.058	16.760	1.422	14530.9
89	.000020	19471.43	16.76	867.00	139.68	.058	16.760	1.340	14530.9
90	.000020	18454.27	16.76	867.00	295.07	.058	16.760	1.270	14530.9
	Taylor (1970) - Field Data								
91	.000649	4.32	.35	36.12	166.00	.047	.351	.154	12.66
92	.000718	5.81	.38	37.34	32.00	.052	.381	.222	14.23
93	.000956	9.42	.40	40.54	100.31	.061	.396	.191	16.06
94	.000550	2.63	.31	35.20	4.29	.041	.314	.166	11.05
95	.000813	12.04	.56	43.28	120.20	.067	.564	.288	24.41
96	.000883	15.10	.67	46.18	88.13	.076	.671	.375	30.96
97	.000752	21.24	.88	48.46	205.79	.081	.884	.342	42.84
98	.000553	2.10	.29	34.90	42.00	.040	.290	.100	10.11
99	.000622	3.39	.32	35.66	350.00	.044	.320	.100	11.41
	Dorban (1982) - Field Data								
100	.001137	1.10	.285	11.28	2.82	.055	.271	.342	3.21
101	.001583	.95	.300	8.60	1.24	.066	.280	.368	2.58
102	.001426	1.07	.290	10.53	3.36	.062	.275	.350	3.05
103	.000870	1.18	.295	12.00	1.32	.049	.281	.332	3.54
	McQuivey and Keefer (1974) - Field Data								
104	.000600	2.41	.32	35.05	6.50	.044	.322	.213	11.27
105	.000600	5.21	.44	36.58	13.94	.051	.445	.320	16.27
106	.000600	18.40	.88	47.55	37.16	.072	.875	.442	41.62
107	.001000	1.98	.39	15.85	9.29	.062	.387	.323	6.13
108	.001000	4.36	.52	19.81	16.26	.071	.516	.427	10.21
109	.001000	8.91	.71	24.38	25.55	.083	.706	.518	17.20
110	.000200	379.22	2.22	182.88	464.50	.066	2.223	.933	406.59
111	.000200	911.26	3.56	201.17	836.10	.084	3.555	1.274	715.24
112	.000200	933.90	3.10	196.60	891.84	.078	3.105	1.530	610.35
113	.000600	9.20	.84	47.24	13.94	.070	.840	.232	39.70
114	.000600	50.94	2.09	53.34	46.45	.111	2.089	.457	111.42
115	.000600	84.90	2.13	59.44	55.74	.112	2.130	.671	126.61
116	.000500	2.41	.41	19.81	13.94	.045	.415	.293	8.22
117	.000500	8.21	.94	25.91	32.51	.068	.936	.338	24.26

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U_* (m/s)	R (m)	U (m/s)	A (m^2)
118	.000500	13.44	.92	36.58	39.48	.067	.920	.399	33.67
119	.009800	302.81	2.93	85.95	153.29	.531	2.934	1.201	252.15
120	.001300	57.87	.98	67.06	41.81	.112	.976	.884	65.47
121	.001300	230.65	2.16	68.58	162.57	.166	2.164	1.554	148.37
122	.000730	4.24	.30	32.61	9.29	.046	.301	.433	9.81
123	.000530	9.90	.42	50.90	20.90	.047	.420	.463	21.38
124	.003550	14.15	.56	24.99	13.94	.140	.563	1.006	14.07
125	.001350	69.05	2.46	34.14	65.03	.180	2.458	.823	83.91
126	.000780	.99	.25	12.50	6.97	.044	.255	.311	3.19
127	.000780	2.41	.41	15.85	13.94	.056	.412	.369	6.52
128	.000610	8.63	.81	36.58	23.23	.069	.806	.293	29.50
129	.000610	14.15	.80	42.37	30.19	.069	.800	.418	33.89
130	.000150	118.86	2.03	103.63	315.86	.055	2.034	.564	210.79
131	.000150	389.13	4.75	127.41	668.88	.084	4.749	.643	605.05
132	.000440	70.75	2.33	70.10	111.48	.100	2.332	.433	163.46
133	.000440	212.25	3.84	71.63	260.12	.129	3.843	.771	275.24
134	.000830	3.96	.81	13.41	13.94	.081	.808	.366	10.83
135	.000830	10.61	1.20	19.51	32.51	.099	1.198	.454	23.37
136	.000180	7.36	.98	35.05	39.48	.042	.984	.213	34.49
137	.000360	12.74	.55	67.06	30.19	.044	.547	.347	36.65
138	.000520	29.01	1.13	65.53	32.51	.076	1.126	.393	73.77
139	.000320	105.84	1.35	202.69	92.90	.065	1.349	.387	273.43
	Miller and Richardson (1974) - Laboratory Data								
140	.001040	.0231	.1265	.5974	.0522	.0301	.0889	.3050	.0756
141	.003100	.0402	.1265	.5974	.2666	.0530	.0889	.5330	.0756
142	.003400	.0231	.1247	.5974	.6726	.0541	.0880	.3110	.0745
143	.007630	.0402	.1262	.5974	.6327	.0814	.0887	.5330	.0754
144	.014000	.0606	.1320	.5974	1.2635	.1109	.0915	.7680	.0789
145	.004890	.0231	.1271	.5974	.1477	.0646	.0892	.3140	.0759
146	.015000	.0402	.1262	.5974	1.0805	.1158	.0887	.5330	.0754
147	.029600	.0606	.1311	.5974	6.1200	.1631	.0911	.7740	.0783
	Berkas (1986) - Field Data								
148	.000273	35.2335	2.3927	29.8704	14.1420	.0743	2.0623	.5456	71.4703
149	.000306	37.2145	2.1031	30.1752	12.0000	.0744	1.8458	.5944	63.4621
150	.000345	39.6200	2.0269	30.6324	21.5000	.0778	1.7900	.6020	62.0894
151	.000352	37.3560	1.3716	50.2920	80.3400	.0671	1.3007	.6660	68.9805
152	.000352	38.2050	1.2497	50.5968	81.4650	.0642	1.1909	.6233	63.2298
	Graf (1984) - Field Data								
153	.002234	14.5462	.41	9.14	243.00	.0908	.3765	.2719	3.75
154	.002148	2.8654	.21	9.14	22.46	.0643	.1963	.1317	1.88
155	.000744	.4896	.21	7.62	24.38	.0384	.2020	.1219	1.63
156	.000289	9.0277	.53	34.08	162.88	.0381	.5118	.3844	17.98
157	.000289	1.7922	.24	19.53	110.78	.0259	.2364	.0981	4.73
158	.000625	10.8814	.53	33.87	176.25	.0564	.5183	.3584	18.10
159	.000639	1.1688	.44	15.13	38.00	.0509	.4135	.0607	6.62
160	.000840	7.1684	.89	20.92	57.56	.0823	.8215	.4017	18.66
161	.002350	8.0372	.59	30.48	113.38	.1140	.5636	.2926	17.84
162	.002339	6.1694	.57	30.78	81.79	.1119	.5454	.2935	17.41

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U* (m/s)	R (m)	U (m/s)	A (m^2)
163	.002328	.7358	.36	24.25	1040.00	.0896	.3516	.0436	8.78
164	.000383	4.8393	.63	18.29	16.66	.0469	.5867	.3289	11.47
165	.000371	2.3149	.41	18.29	8.85	.0378	.3924	.2192	7.50
166	.000650	45.1385	1.11	49.20	331.08	.0823	1.0619	.9385	54.60
167	.000645	14.7585	.77	37.06	234.69	.0686	.7437	.4118	28.72
168	.000654	2.6404	.61	21.06	104.14	.0607	.5732	.1338	12.77
	Liu (1977) - Field Data								
169	.001300	1.54	.49	13.34	15.00	.079	.491	.235	6.55
170	.000360	9.14	.95	37.99	9.10	.058	.953	.252	36.20
171	.002920	.99	.38	14.02	9.10	.104	.378	.187	5.30
172	.000324	3.96	.85	28.44	26.80	.052	.851	.164	24.20
173	.000324	6.80	.67	56.30	19.70	.046	.666	.181	37.50
174	.001300	8.49	.85	16.27	41.80	.104	.848	.616	13.80
175	.000400	84.90	1.84	54.89	11.00	.085	1.840	.838	101.00
176	.000121	25.50	1.56	23.91	17.70	.043	1.560	.683	37.30
177	.000121	26.90	1.56	24.18	17.70	.043	1.560	.713	37.70
178	.000400	51.36	1.47	52.66	11.10	.076	1.470	.658	77.40
179	.001300	1.36	.45	12.40	15.40	.076	.454	.241	5.63
180	.000200	957.00	3.05	183.95	1490.00	.077	3.050	1.710	561.00
181	.000224	8.78	1.10	24.90	6.50	.049	1.100	.320	27.40
	Hou and Christensen (1976) - Laboratory and Field Data								
182	.00000728	6.62	2.16	38.30	.09420	.012	2.164	.080	82.89
183	.00000880	2.08	1.37	27.43	.06020	.011	1.372	.055	37.63
184	.00000140	10.07	2.62	34.08	.06754	.006	2.621	.113	89.32
185	.00004637	11.82	2.04	27.57	.24452	.030	2.042	.210	56.30
186	.00001926	15.00	1.89	37.80	.18581	.019	1.890	.210	71.42
187	.00001512	5.14	2.19	34.45	.21331	.018	2.195	.068	75.61
188	.00001005	5.67	2.01	37.62	.15728	.014	2.012	.075	75.68
189	.00000106	7.37	2.53	30.36	.07386	.005	2.530	.096	76.80
190	.00000005	1.37	2.87	21.49	.01914	.001	2.865	.022	61.57
191	.00001079	3.68	1.89	37.80	.19389	.014	1.890	.052	71.42
192	.00000158	3.81	2.26	45.11	.10136	.006	2.256	.037	101.75
193	.00000319	1.80	2.10	31.13	.17113	.008	2.103	.027	65.46
194	.00000112	2.48	1.68	26.82	.16760	.004	1.676	.055	44.97
195	.00001815	2.36	1.77	35.00	.33296	.018	1.768	.038	61.88
196	.00000420	1.44	1.71	15.36	.10312	.008	1.707	.055	26.22
197	.00000045	1.31	1.80	31.11	.05806	.003	1.798	.023	55.95
198	.00000045	1.31	1.80	31.11	.05834	.003	1.798	.023	55.95
199	.00000072	7.21	2.62	34.08	.13582	.004	2.621	.081	89.32
200	.00000028	3.82	2.53	30.36	.08956	.003	2.530	.050	76.80
201	.00001815	2.36	1.77	35.00	.44036	.018	1.768	.038	61.88
202	.00000320	5.43	2.26	45.11	.27063	.008	2.256	.053	101.75
203	.00000259	2.66	1.46	21.95	.14177	.006	1.463	.083	32.11
204	.00000016	1.23	1.83	26.52	.05593	.002	1.829	.025	48.50
205	.00000037	2.26	1.98	27.74	.14883	.003	1.981	.041	54.95
206	.00000224	3.19	1.37	31.55	.44315	.005	1.372	.074	43.27
207	.00000022	1.00	2.19	17.34	.27927	.002	2.195	.026	38.05
208	.00000421	3.41	1.46	21.95	.66379	.008	1.463	.106	32.11
209	.00000662	2.37	1.22	18.29	1.31458	.009	1.219	.106	22.30

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	D_L (m^2/s)	U_* (m/s)	R (m)	U (m/s)	A (m^2)
210	.00000465	4.21	1.58	25.36	1.85806	.009	1.585	.105	40.19
211	.00003737	34.84	2.47	61.72	12.44901	.030	2.469	.229	152.38
212	.00003313	43.09	2.77	67.96	20.62448	.030	2.774	.229	188.49
213	.00361948	.2269	.15	2.42	.11700	.0690	.1341	.6251	.363
214	.00048427	.2815	.45	.90	.07660	.0275	.1591	.7010	.402
215	.00023351	.1708	.17	2.42	.06530	.0183	.1460	.4249	.402
216	.00000605	98.9848	7.46	48.80	3.00000	.0191	6.1499	.2719	364.073
217	.00052498	118.6568	2.13	59.40	11.10006	.1040	2.1001	.9379	126.517
218	.00056944	92.5593	2.09	53.30	11.20039	.1070	2.0501	.8309	111.399
219	.00046592	15.4559	1.04	47.20	9.09985	.0686	1.0299	.3149	49.088
220	.00374940	1.0640	.39	17.10	9.09985	.1190	.3850	.1579	6.739
221	.00011415	26.9846	1.56	24.40	17.69989	.0408	1.4874	.7090	38.062
222	.00133083	2.1324	.50	15.80	15.10046	.0803	.4941	.2710	7.869
223	.00133405	2.0665	.49	16.00	15.49994	.0792	.4791	.2630	7.856
224	.00126523	9.2794	.84	18.20	46.44501	.1010	.8220	.6041	15.360
225	.00035502	4.4432	.85	33.80	26.79974	.0543	.8461	.1539	28.866
226	.00040290	4.3935	.58	35.80	19.70009	.0478	.5779	.2109	20.830

(B) DISPERSION COEFFICIENT AND FLOW PARAMETERS IN SEDIMENT-LADEN RUNS

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	U_* (m/s)	d_s (mm)	C_s (ppm)	D_{LS} (m^2/s)	D_{LS}/D_{LC} -------
	Present study - Laboratory Data								
1	0.004546	0.01053	0.0700	0.200	0.0408	0.064	8169.06	0.09144	2.47
2	0.004546	0.01053	0.0700	0.200	0.0408	0.064	3215.10	0.07576	2.04
3	0.004546	0.01053	0.0700	0.200	0.0408	0.064	5549.80	0.08750	2.36
4	0.004546	0.01053	0.0700	0.200	0.0408	0.064	5446.00	0.08607	2.32
5	0.004546	0.01053	0.0700	0.200	0.0408	0.164	4382.64	0.08297	2.24
6	0.004546	0.01053	0.0700	0.200	0.0408	0.164	2017.73	0.05000	1.349
7	0.004546	0.01053	0.0700	0.200	0.0408	0.164	3564.91	0.05242	1.414
8	0.004546	0.01558	0.0910	0.200	0.0420	0.064	6097.93	0.06090	1.5225
9	0.004546	0.01558	0.0910	0.200	0.0420	0.064	4725.66	0.06538	1.634
10	0.004546	0.01558	0.0910	0.200	0.0420	0.064	6028.67	0.10445	2.6113
11	0.004546	0.01558	0.0910	0.200	0.0420	0.064	2767.92	0.06000	1.5
12	0.004546	0.01558	0.0910	0.200	0.0420	0.164	4813.77	0.06826	1.706
13	0.004546	0.01558	0.0910	0.200	0.0420	0.164	2178.86	0.05649	1.412
14	0.004546	0.01558	0.0910	0.200	0.0420	0.164	3520.75	0.06771	1.693
15	0.004546	0.01863	0.1118	0.200	0.0477	0.064	10995.47	0.10824	2.165
16	0.004546	0.01863	0.1118	0.200	0.0477	0.164	6199.43	0.05819	1.164
17	0.002470	0.00589	0.0580	0.200	0.0288	0.064	1336.98	0.04141	1.57
18	0.002470	0.00589	0.0580	0.200	0.0288	0.064	1740.75	0.04600	1.748
19	0.002470	0.00589	0.0580	0.200	0.0288	0.064	1355.09	0.04244	1.613
20	0.002470	0.00589	0.0580	0.200	0.0288	0.164	462.26	0.03806	1.447
21	0.002470	0.00589	0.0580	0.200	0.0288	0.164	1092.08	0.04463	1.696
22	0.002470	0.01053	0.0915	0.200	0.0335	0.064	1250.56	0.04338	1.185
23	0.002470	0.01053	0.0915	0.200	0.0335	0.064	2220.37	0.05031	1.374

Run No.	S -----	Q (m^3/s)	D (m)	B (m)	U_* (m/s)	d_s (mm)	C_s (ppm)	D_{LS} (m^2/s)	D_{LS}/D_{LC} ------
24	0.002470	0.01053	0.0915	0.200	0.0335	0.064	1381.88	0.04692	1.282
25	0.002470	0.01053	0.0915	0.200	0.0335	0.164	1172.45	0.06436	1.758
26	0.002470	0.01053	0.0915	0.200	0.0335	0.164	0772.08	0.06227	1.701
27	0.002470	0.01053	0.0915	0.200	0.0335	0.164	1451.32	0.06700	1.83
28	0.002470	0.01558	0.1243	0.200	0.0364	0.064	2046.04	0.06613	1.31
29	0.002470	0.01558	0.1243	0.200	0.0364	0.064	2578.50	0.06750	1.334
30	0.002470	0.01558	0.1243	0.200	0.0364	0.064	3020.75	0.07331	1.44
31	0.002470	0.01558	0.1243	0.200	0.0364	0.064	1244.52	0.05430	1.07
32	0.002470	0.01558	0.1243	0.200	0.0364	0.164	1277.73	0.07538	1.56
33	0.002470	0.01558	0.1243	0.200	0.0364	0.164	1614.70	0.10936	2.16
34	0.002470	0.01558	0.1243	0.200	0.0364	0.164	2012.08	0.05313	1.05
35	0.001488	0.00833	0.0891	0.200	0.0248	0.064	650.19	0.02726	1.2
36	0.001488	0.00833	0.0891	0.200	0.0248	0.064	1087.92	0.06094	2.68
37	0.001488	0.01177	0.1180	0.200	0.0269	0.064	1107.55	0.05138	2.08
38	0.001488	0.01177	0.1180	0.200	0.0269	0.064	1258.11	0.05386	2.176
39	0.001488	0.01177	0.1180	0.200	0.0269	0.064	1715.10	0.11452	4.63
40	0.001488	0.01442	0.1388	0.200	0.0278	0.064	1400.38	0.07425	2.83
41	0.001488	0.01442	0.1388	0.200	0.0278	0.064	1921.13	0.02414	0.928
42	0.001488	0.01442	0.1388	0.200	0.0278	0.064	2643.39	0.02713	1.034
43	0.001488	0.01442	0.1388	0.200	0.0278	0.064	0504.52	0.04806	1.833
	Singh (1987) - Laboratory Data								
44	0.005520	0.01580	0.0820	0.400	0.06383	0.0580	1750.00	0.03131	1.044
45	0.005520	0.01580	0.0820	0.400	0.06383	0.0580	4850.00	0.05850	1.95
46	0.004700	0.02300	0.1040	0.400	0.06516	0.0580	3350.00	0.02775	1.028
47	0.004700	0.02300	0.1040	0.400	0.06516	0.0580	5100.00	0.05100	1.889
48	0.003920	0.01680	0.0910	0.400	0.06512	0.0580	2500.00	0.04600	1.6
49	0.003920	0.02500	0.1150	0.400	0.06210	0.0580	4450.00	0.04000	2.69
50	0.003920	0.02500	0.1150	0.400	0.06210	0.0580	2950.00	0.02193	1.474
51	0.003920	0.02500	0.1150	0.400	0.06210	0.0580	5200.00	0.05030	3.38
52	0.003920	0.02500	0.1150	0.400	0.06210	0.0580	5000.00	0.02525	1.698
53	0.006030	0.02380	0.1000	0.400	0.07272	0.0580	4950.00	0.04266	1.1286
54	0.005520	0.02810	0.1140	0.400	0.07373	0.0580	3850.00	0.05468	1.015
55	0.005520	0.02810	0.1140	0.400	0.07373	0.0580	5150.00	0.06156	1.1422
56	0.004700	0.02700	0.1180	0.400	0.06917	0.0580	3850.00	0.01568	1.0453
57	0.004700	0.02700	0.1180	0.400	0.06917	0.0580	4800.00	0.02740	1.827
58	0.004700	0.03100	0.1270	0.400	0.07126	0.0580	4600.00	0.02051	2.1
59	0.006030	0.02380	0.1000	0.400	0.07272	0.0580	4550.00	0.05656	1.496
60	0.006030	0.02950	0.1150	0.400	0.07743	0.0580	4450.00	0.07289	1.352
61	0.004240	0.01920	0.0500	0.400	0.03861	0.0820	2100.00	0.15250	2.428
62	0.002568	0.01870	0.0690	0.400	0.03624	0.0580	1500.00	0.13250	2.172
63	0.002568	0.01870	0.0698	0.400	0.03655	0.0580	2050.00	0.08850	1.45

APPENDIX-II

Notations used to report the data of the present study are depicted in the following C-t curve.

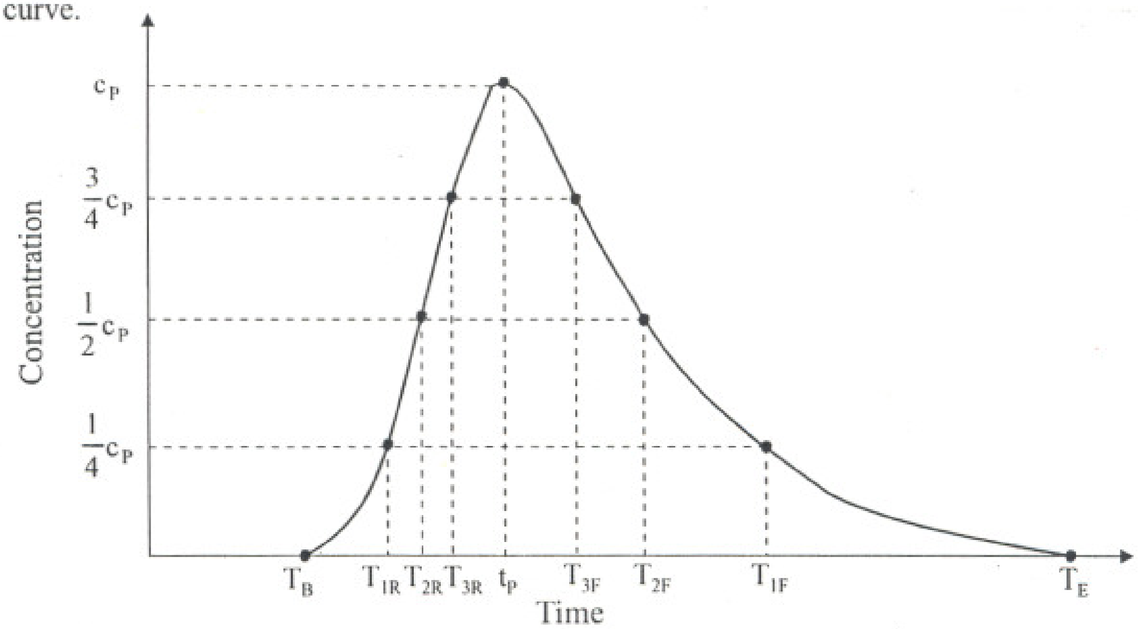

Fig. A1 Definition Sketch of a C-t Curve

c_P	Peak concentration of the tracer cloud
T_B	Elapsed time to the arrival of the leading edge of the tracer cloud
T_{1R}	Elapsed time to the arrival of the 25% peak concentration of the tracer cloud on the rising limb
T_{2R}	Elapsed time to the arrival of the 50% peak concentration of the tracer cloud on the rising limb
T_{3R}	Elapsed time to the arrival of the 75% peak concentration of the tracer cloud on the rising limb
t_P	Elapsed time to the peak concentration of the tracer cloud
T_{3F}	Elapsed time to the arrival of the 75% peak concentration of the tracer cloud on the following limb
T_{2F}	Elapsed time to the arrival of the 50% peak concentration of the tracer cloud on the following limb
T_{1F}	Elapsed time to the arrival of the 25% peak concentration of the tracer cloud on the following limb
T_E	Elapsed time to the trailing edge of the tracer cloud

TRAVELTIME AND LONGITUDINAL DISPERSION DATA COLLECTED IN THE PRESENT STUDY

(A) CLEAR-WATER RUNS

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
1	.00	73.80	11.00	13.18	13.70	14.29	15.00	16.65	17.02	17.75	28.00
	6.00	60.45	17.00	19.20	19.80	20.42	22.00	22.98	23.95	24.59	36.00
	12.00	52.35	24.00	26.73	27.44	28.07	29.00	30.74	31.72	32.63	48.00
	18.00	46.40	30.00	33.19	34.02	34.78	36.00	38.09	39.18	40.48	53.00
2	.00	76.93	9.00	11.01	11.48	11.96	13.00	13.83	14.66	15.25	27.00
	6.00	66.17	16.00	17.74	18.37	18.88	20.00	21.16	22.33	22.75	39.00
	12.00	57.54	22.00	24.65	25.34	25.92	27.00	29.36	29.48	30.51	46.00
	18.00	52.65	25.00	31.37	32.17	32.84	34.00	35.58	36.74	37.67	52.00
3	.00	67.70	13.00	15.19	15.71	16.30	17.00	18.32	19.64	19.78	27.00
	5.00	56.81	17.00	19.41	20.07	20.62	22.00	22.80	23.80	24.46	31.00
	10.00	50.35	22.00	24.13	24.74	25.39	27.00	28.00	28.95	29.64	41.00
	15.00	45.90	26.00	28.80	29.51	30.18	32.00	33.03	34.07	34.73	44.00
4	.00	63.90	14.00	15.96	16.49	17.00	18.00	19.21	20.42	20.62	31.00
	5.00	57.27	19.00	20.58	21.28	21.85	23.00	24.40	25.02	26.04	38.00
	10.00	48.37	24.00	26.12	26.79	27.50	29.00	30.46	31.24	32.36	40.00
	15.00	44.20	29.00	31.02	31.73	32.50	34.00	36.26	36.72	37.71	45.00
5	.00	75.94	13.00	18.16	18.75	19.39	21.00	22.08	23.17	24.18	44.00
	6.00	55.48	26.00	28.28	29.09	29.81	31.00	33.55	34.76	36.33	45.00
	12.00	43.79	36.00	38.81	39.78	40.71	43.00	45.19	46.11	47.77	55.00
	18.00	39.60	46.00	50.12	51.34	52.45	55.00	57.53	59.34	61.67	74.00
6	.00	73.29	17.00	19.21	19.79	20.42	22.00	23.17	24.09	25.31	42.00
	5.00	58.93	24.00	26.34	27.12	27.84	29.00	31.37	32.48	33.77	47.00
	10.00	51.27	31.00	34.16	35.01	35.88	38.00	40.37	41.19	42.67	56.00
	15.00	47.40	39.00	42.09	43.08	44.05	46.00	48.35	49.71	51.61	61.00
7	.00	71.21	21.00	23.16	23.81	24.50	26.00	27.28	28.11	29.22	36.00
	4.00	57.61	24.00	27.56	28.32	28.96	30.00	32.46	33.48	34.71	46.00
	8.00	52.69	30.00	32.96	33.76	34.59	36.00	38.12	39.41	40.77	50.00
	12.00	49.58	31.00	39.05	39.89	40.70	42.00	44.51	45.74	47.35	60.00
8	.00	62.91	18.00	19.70	20.35	20.90	22.00	23.92	24.50	26.01	40.00
	5.50	45.50	28.00	30.37	31.14	31.62	34.00	36.11	37.46	39.23	53.00
	11.00	42.20	38.00	40.60	41.54	42.44	44.00	47.42	48.28	50.23	64.00
	16.50	34.10	48.00	50.69	51.86	53.00	55.00	58.26	60.15	62.63	78.00
9	.00	82.57	20.00	21.86	22.47	23.03	24.00	25.75	26.83	28.19	39.00
	5.00	69.15	28.00	30.45	31.28	32.01	34.00	36.08	36.79	38.31	52.00
	10.00	59.47	35.00	38.17	39.02	39.80	42.00	44.44	45.44	47.17	57.00
	15.00	50.59	41.00	46.80	47.77	48.73	51.00	53.83	55.28	57.50	72.00
10	.00	92.06	22.00	24.59	25.34	25.97	27.00	28.98	30.01	31.39	43.00
	4.00	74.44	28.00	30.76	31.56	32.40	34.00	36.69	37.87	39.29	50.00
	8.00	71.07	34.00	38.63	39.60	40.51	42.00	44.58	46.01	47.61	59.00
	12.00	67.33	41.00	44.61	45.69	46.68	49.00	51.15	52.63	54.39	70.00

Note: Run number of Appendix-I and Appendix-II of the data of the present study are same for both clear-water flows and sediment-laden flows.

Continued ...

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
11	.00	120.18	16.00	17.71	18.34	18.86	20.00	21.43	22.22	23.34	36.00
	6.00	83.51	23.00	28.90	29.73	30.57	32.00	34.18	35.46	37.16	52.00
	12.00	67.27	36.00	41.10	42.15	43.18	45.00	48.38	49.46	51.51	68.00
	18.00	58.09	50.00	53.31	54.52	55.66	58.00	61.09	63.05	65.36	78.00
12	.00	71.38	20.00	21.84	22.44	22.98	24.00	26.97	27.03	28.15	40.00
	5.00	53.67	28.00	30.38	31.18	31.90	34.00	35.81	36.78	38.48	52.00
	15.00	46.46	38.00	39.45	40.45	42.43	44.00	46.51	47.96	49.65	60.00
	20.00	41.77	45.00	48.55	49.63	50.70	53.00	55.66	57.09	59.29	75.00
13	.00	67.25	24.00	26.23	26.88	27.58	29.00	30.80	32.15	33.45	44.00
	4.00	59.41	27.00	31.73	32.55	33.39	36.00	38.24	39.72	41.89	56.00
	8.00	54.90	38.00	40.24	41.24	42.21	44.00	46.44	47.66	49.25	58.00
	12.00	52.77	42.00	46.41	47.44	48.40	50.00	53.18	54.29	56.12	70.00

(B) SEDIMENT-LADEN RUNS

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
1	.00	27.75	13.00	14.53	15.19	15.71	17.00	18.07	19.13	19.68	25.00
	6.00	22.20	18.00	19.87	20.53	21.18	22.50	24.18	25.21	26.44	34.00
	12.00	19.54	24.00	26.00	26.84	27.70	29.00	31.45	32.53	34.04	41.00
	18.00	14.99	31.00	34.06	35.07	35.99	38.00	40.29	41.57	43.18	49.00
2	.00	77.01	12.00	14.16	14.70	15.31	16.00	16.80	18.38	19.31	31.00
	6.00	60.28	18.00	20.26	20.93	21.59	23.00	24.64	25.42	26.56	33.00
	12.00	49.95	25.00	27.24	27.64	28.10	30.00	32.89	33.81	35.24	45.00
	18.00	43.78	33.00	35.27	36.26	37.15	39.00	41.74	42.22	43.69	49.00
3	.00	49.62	11.00	14.03	14.51	15.00	16.00	17.00	18.00	18.46	25.00
	6.00	39.06	19.00	21.10	21.74	22.44	24.00	25.57	26.45	27.63	37.00
	12.00	33.36	25.00	27.78	28.59	29.41	31.00	33.32	33.82	35.04	42.00
	18.00	30.18	29.00	35.45	36.47	37.44	39.00	41.61	42.83	44.39	51.00
4	.00	59.16	13.00	14.52	15.18	15.72	17.00	18.11	19.22	19.70	28.00
	6.00	44.25	19.00	20.43	21.17	21.81	23.00	24.70	25.83	26.74	31.00
	12.00	40.65	24.00	28.72	29.47	30.16	32.00	34.01	36.09	37.38	43.00
	18.00	31.17	33.00	35.39	36.38	37.38	39.00	41.49	42.66	44.15	48.00
5	.00	63.07	12.00	13.72	14.32	14.82	16.00	17.13	18.27	19.12	24.00
	6.00	44.82	18.00	19.63	20.36	20.99	23.00	24.60	25.48	26.57	31.00
	12.00	39.53	23.00	25.87	26.70	27.55	29.00	31.07	32.28	33.66	43.00
	18.00	35.02	31.00	34.05	35.07	36.02	38.00	40.08	41.49	44.13	49.00
6	.00	77.35	12.00	13.30	13.85	14.42	15.00	16.75	17.64	18.51	23.00
	6.00	66.45	18.00	19.65	20.38	21.00	22.00	24.16	25.13	26.31	31.00
	12.00	63.20	24.00	26.34	27.16	27.87	29.00	31.34	32.37	33.64	42.00
	18.00	58.46	32.00	34.35	35.29	36.17	38.00	40.01	41.30	43.13	53.00
7	.00	73.52	8.00	13.83	14.39	14.87	16.00	17.06	18.12	18.60	29.00
	6.00	58.19	19.00	20.68	21.38	21.96	23.00	24.90	26.09	27.26	32.00
	12.00	46.31	25.00	26.95	27.70	28.52	30.00	32.01	33.18	34.57	39.00
	18.00	44.83	31.00	33.81	34.79	35.73	38.00	40.24	41.17	42.78	53.00

Continued...

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
8	.00	56.11	11.00	13.04	13.54	14.07	15.00	16.28	17.55	17.73	27.00
	6.00	52.13	17.00	19.37	20.06	20.68	22.00	23.55	24.27	25.34	34.00
	12.00	46.51	19.00	24.64	25.40	26.08	28.00	29.53	30.41	31.53	40.00
	18.00	40.34	27.00	31.47	32.41	33.29	35.00	37.80	38.02	39.37	48.00
9	.00	36.07	13.00	14.61	15.25	15.77	17.00	18.07	19.15	19.65	23.00
	6.00	33.06	18.00	20.58	21.27	21.64	22.00	23.76	24.71	26.03	34.00
	12.00	26.50	24.00	26.31	27.11	27.83	29.00	33.73	32.12	33.10	38.00
	18.00	23.57	29.00	32.81	33.76	34.66	36.00	38.67	39.57	40.71	53.00
10	.00	28.78	16.00	17.26	17.82	18.41	19.00	20.90	21.70	22.51	30.00
	6.00	23.02	22.00	23.84	24.49	25.16	27.00	28.56	29.15	30.11	33.00
	12.00	23.89	24.00	29.87	30.78	31.65	33.00	34.66	36.98	38.03	45.00
	18.00	19.84	32.00	33.73	34.67	35.64	38.00	40.36	41.03	42.54	51.00
11	.00	72.57	12.00	13.65	14.25	14.73	16.00	16.80	17.56	18.29	26.00
	6.00	57.47	11.00	19.01	19.58	20.21	21.00	22.75	23.72	24.60	35.00
	12.00	49.12	24.00	25.77	26.49	27.17	29.00	30.31	31.20	32.35	36.00
	18.00	43.46	30.00	31.91	32.68	33.50	35.00	37.65	37.96	39.24	44.00
12	.00	38.91	7.00	13.29	13.85	14.42	15.00	16.60	17.45	18.34	28.00
	6.00	37.56	17.00	18.80	19.43	19.97	21.00	22.76	23.67	24.62	29.00
	12.00	31.80	19.00	24.66	25.47	26.24	28.00	29.94	30.76	32.04	38.00
	18.00	27.41	29.00	31.22	32.11	32.99	35.00	37.64	38.13	39.45	44.00
13	.00	71.17	11.00	12.83	14.23	14.61	16.00	16.80	17.68	18.52	29.00
	6.00	59.74	17.00	18.41	19.06	19.64	21.00	22.70	23.03	24.11	30.00
	12.00	52.34	23.00	25.22	25.92	26.63	28.00	30.15	30.57	31.55	35.00
	18.00	44.85	30.00	32.26	33.03	33.77	35.00	37.00	38.06	39.35	43.00
14	.00	56.80	11.00	13.16	13.66	14.23	15.00	16.19	17.39	17.55	28.00
	6.00	45.51	14.00	18.30	18.97	19.59	21.00	22.35	23.20	24.33	36.00
	12.00	38.37	22.00	24.35	25.15	25.85	27.00	29.25	30.36	31.70	42.00
	18.00	32.20	28.00	30.75	31.65	32.54	34.00	36.23	37.37	38.67	43.00
15	.00	30.84	14.00	15.54	16.18	16.68	18.00	15.34	19.66	20.36	27.00
	5.00	27.39	19.00	21.46	22.21	22.85	24.00	27.42	26.80	27.72	34.00
	10.00	26.11	25.00	27.47	28.27	28.98	31.00	32.67	33.48	34.55	45.00
	15.00	24.61	26.00	30.82	31.76	32.66	34.00	36.65	37.64	38.78	44.00
16	.00	21.50	14.00	15.25	15.82	16.40	17.00	18.60	20.73	22.24	27.00
	5.00	19.84	15.00	19.54	20.27	20.89	22.00	24.02	25.07	26.36	35.00
	10.00	18.07	20.00	24.70	25.52	26.31	28.00	29.96	30.68	32.02	37.00
	15.00	15.23	28.00	29.82	30.66	31.52	34.00	36.15	37.32	38.66	47.00
17	.00	95.84	13.00	17.17	17.77	18.44	20.00	21.48	22.33	23.43	32.00
	6.00	71.30	24.00	26.01	26.81	27.66	29.00	31.63	32.80	34.30	45.00
	12.00	59.10	32.00	35.82	36.85	37.83	40.00	41.66	43.37	45.47	52.00
	18.00	50.80	45.00	47.67	48.82	49.95	53.00	55.71	57.52	59.80	71.00
18	.00	87.90	15.00	16.20	16.75	17.35	19.00	20.09	21.19	22.09	32.00
	6.00	62.50	24.00	25.79	26.69	27.59	29.00	32.06	33.28	35.08	46.00
	12.00	54.10	33.00	35.81	36.90	37.97	40.00	42.68	44.34	46.35	60.00
	18.00	46.90	43.00	47.46	48.78	49.97	53.00	55.93	57.39	59.59	71.00
19	.00	62.25	14.00	16.04	16.59	17.21	18.00	19.86	20.91	21.78	25.00
	6.00	42.71	26.00	27.76	28.61	29.48	32.00	35.56	35.08	36.48	42.00
	12.00	36.54	38.00	39.94	40.93	41.90	44.00	46.39	50.08	50.39	55.00
	18.00	33.13	41.00	47.47	49.70	51.66	55.00	58.76	60.45	63.09	73.00

Continued ...

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
20	.00	112.79	14.00	16.20	16.67	17.26	19.00	20.28	21.26	23.15	37.00
	6.00	89.10	24.00	26.47	27.35	28.17	30.00	32.40	34.21	35.53	50.00
	12.00	72.00	34.00	37.35	38.42	39.43	41.00	44.11	45.50	47.41	63.00
	18.00	61.00	44.00	47.48	48.73	49.88	52.00	55.18	56.82	59.13	73.00
21	.00	91.90	14.00	16.19	16.76	17.39	19.00	20.28	21.21	22.40	28.00
	6.00	67.60	23.00	26.36	27.21	27.99	30.00	32.22	33.02	34.65	41.00
	12.00	52.30	34.00	37.22	38.16	39.09	41.00	44.38	45.88	47.69	57.00
	18.00	47.70	44.00	48.25	49.51	50.71	53.00	56.05	57.73	59.84	66.00
22	.00	65.90	17.00	18.94	19.53	20.14	21.00	23.21	24.15	25.37	35.00
	5.00	53.00	24.00	26.17	26.95	27.78	30.00	31.87	32.87	34.26	41.00
	10.00	45.60	32.00	34.45	35.39	36.33	38.00	40.68	41.95	43.61	50.00
	15.00	40.80	38.00	41.29	42.44	43.58	46.00	48.41	49.88	51.68	64.00
23	.00	45.30	19.00	20.61	21.29	21.85	23.00	25.22	25.70	27.11	32.00
	5.00	35.20	25.00	26.42	26.90	27.70	30.00	31.70	33.08	35.42	42.00
	10.00	28.60	33.00	36.07	37.03	38.09	40.00	43.05	44.02	45.58	52.00
	15.00	27.60	38.00	40.89	42.10	43.33	46.00	48.42	49.95	51.72	56.00
24	.00	60.14	17.00	19.04	19.59	20.20	21.00	23.30	24.20	25.37	35.00
	5.00	49.81	24.00	26.47	27.29	28.01	30.00	32.22	32.97	34.26	40.00
	10.00	41.57	31.00	33.95	34.94	35.92	38.00	40.28	41.68	43.53	54.00
	15.00	34.61	39.00	41.76	42.83	43.88	46.00	48.86	50.22	52.09	58.00
25	.00	110.10	17.00	18.46	19.12	19.71	21.00	22.97	23.79	25.19	38.00
	5.00	80.46	25.00	26.58	27.37	28.14	30.00	32.18	33.45	34.79	39.00
	10.00	68.61	32.00	34.27	35.17	36.06	38.00	40.84	42.28	44.04	51.00
	15.00	61.17	39.00	41.36	42.57	43.71	46.00	49.45	50.78	52.69	63.00
26	.00	60.60	18.00	19.07	19.61	20.22	21.00	22.42	23.40	26.51	34.00
	5.00	46.90	25.00	26.52	27.28	27.95	30.00	32.17	33.10	34.61	42.00
	10.00	39.80	32.00	34.73	35.61	36.49	38.00	40.71	41.93	43.66	54.00
	15.00	35.20	39.00	41.43	42.55	43.67	46.00	48.83	50.18	52:15	64.00
27	.00	46.10	13.00	19.36	20.01	20.62	22.00	23.72	24.50	25.63	30.00
	5.00	38.50	24.00	26.32	27.11	27.82	29.00	31.88	32.95	34.37	49.00
	10.00	30.00	33.00	34.77	35.73	36.79	39.00	41.36	42.76	44.05	54.00
	15.00	27.60	39.00	41.10	42.22	43.27	45.00	48.03	49.63	51.43	58.00
28	.00	61.23	23.00	25.01	25.66	26.39	28.00	30.01	30.80	32.17	43.00
	4.00	51.87	27.00	30.61	31.50	32.36	34.00	36.15	37.41	39.20	48.00
	8.00	42.45	34.00	35.84	36.77	37.76	40.00	42.29	43.62	45.23	50.00
	12.00	41.29	39.00	40.77	41.85	42.93	45.00	47.94	49.13	51.03	67.00
29	.00	68.05	20.00	21.54	22.22	22.82	24.00	27.40	27.13	28.32	33.00
	4.00	54.50	26.00	27.40	28.16	28.89	31.00	33.09	34.08	35.55	40.00
	8.00	50.61	29.00	32.13	32.99	33.97	36.00	38.63	40.02	41.67	49.00
	12.00	47.88	35.00	38.23	39.28	40.33	42.00	45.34	46.70	48.62	55.00
30	.00	49.30	20.00	22.02	22.61	23.28	25.00	26.79	27.81	29.37	36.00
	4.00	43.83	26.00	27.70	28.45	29.19	31.00	33.06	34.33	35.70	41.00
	8.00	35.74	30.00	32.80	33.77	35.26	37.00	39.15	43.07	43.34	49.00
	12.00	32.90	34.00	37.52	38.57	39.74	43.00	45.39	47.12	49.10	55.00
31	.00	81.73	18.00	21.02	21.67	22.39	24.00	25.63	26.46	27.63	40.00
	4.00	71.00	21.00	27.50	28.32	29.01	31.00	33.68	34:66	36.22	44.00
	8.00	62.01	30.00	32.38	33.28	34.15	36.00	38.25	39.46	40.70	46.00
	12.00	58.45	30.00	38.35	38.98	40.32	43.00	45.50	47.44	48.74	58.00

Continued...

Run No.	Distance (m)	C_P (ppm)	T_B (s)	T_{1R} (s)	T_{2R} (s)	T_{3R} (s)	t_P (s)	T_{3F} (s)	T_{2F} (s)	T_{1F} (s)	T_E (s)
32	.00	51.43	20.00	22.27	22.95	23.63	25.00	27.26	27.70	28.69	39.00
	4.00	44.04	26.00	27.61	28.33	28.94	30.00	32.22	33.26	34.50	42.00
	8.00	38.86	28.00	33.43	34.33	35.23	37.00	39.47	40.62	42.28	54.00
	12.00	32.03	36.00	37.54	38.51	39.47	41.00	44.20	45.10	46.51	50.00
33	.00	57.57	20.00	22.00	22.55	23.15	24.00	26.24	27.20	28.36	32.00
	4.00	46.71	25.00	27.28	28.00	28.72	30.00	32.76	33.85	35.42	46.00
	8.00	40.14	30.00	31.76	32.67	33.57	35.00	38.19	39.23	40.77	45.00
	12.00	36.15	33.00	35.49	36.52	37.59	40.00	42.22	43.67	45.50	51.00
34	.00	41.24	17.00	24.35	25.11	25.80	27.00	29.45	30.66	32.34	38.00
	4.00	36.42	24.00	26.24	26.98	28.47	30.00	32.29	33.54	35.19	41.00
	8.00	31.51	30.00	32.35	33.21	34.07	36.00	39.08	40.17	41.77	48.00
	12.00	28.21	34.00	37.29	39.12	40.10	42.00	44.11	47.55	47.35	55.00
35	.00	61.06	18.00	19.72	20.36	20.89	22.00	24.73	24.96	26.14	38.00
	5.50	42.91	29.00	30.38	31.11	31.79	33.00	36.36	37.26	39.05	52.00
	11.00	36.80	36.00	39.09	41.05	41.96	44.00	46.95	48.43	50.42	57.00
	16.50	32.10	49.00	52.45	53.80	55.18	58.00	61.21	62.72	65.20	79.00
36	.00	98.60	18.00	19.37	19.99	20.57	22.00	23.38	24.40	25.74	31.00
	5.50	63.20	31.00	33.89	34.82	35.77	38.00	40.36	41.88	43.49	50.00
	11.00	54.90	36.00	39.44	40.63	41.79	44.00	47.11	48.85	50.61	63.00
	16.50	49.00	56.00	58.75	59.91	61.06	64.00	67.33	68.88	70.77	79.00
37	.00	28.84	19.00	21.13	21.68	22.30	24.00	25.50	26.46	27.79	44.00
	5.00	21.36	27.00	28.86	29.58	30.36	32.00	34.72	35.92	37.30	41.00
	10.00	18.50	35.00	37.95	38.91	39.91	42.00	44.60	46.05	47.46	51.00
38	.00	28.50	20.00	21.16	21.68	22.28	23.00	24.80	26.03	27.65	32.00
	5.00	19.40	28.00	30.13	30.84	31.65	34.00	36.27	37.70	39.24	43.00
	10.00	15.10	40.00	41.68	42.72	43.85	47.00	49.54	51.15	53.25	64.00
	15.00	14.70	43.00	45.67	47.15	48.38	51.00	54.48	56.29	58.62	64.00
39	.00	28.30	27.00	28.46	28.97	30.85	33.00	35.90	37.21	39.09	47.00
	5.00	22.90	35.00	36.80	37.71	38.76	41.00	44.25	46.05	47.59	55.00
	10.00	18.70	44.00	45.69	46.80	47.98	51.00	54.08	55.94	58.33	66.00
40	.00	18.25	23.00	24.92	25.60	26.34	28.00	30.05	31.31	32.72	39.00
	4.00	15.23	29.00	30.66	31.47	32.28	34.00	36.78	38.13	39.59	43.00
	8.00	13.08	35.00	36.92	37.83	38.75	41.00	44.05	45.26	47.06	51.00
	12.00	10.48	41.00	42.72	43.94	45.22	48.00	51.02	52.69	54.44	58.00
41	.00	28.40	26.00	28.28	29.07	29.80	31.00	33.84	34.94	36.48	41.00
	4.00	25.70	29.00	30.77	31.54	32.35	34.00	36.97	38.23	40.37	45.00
	8.00	21.70	34.00	36.00	36.79	37.72	40.00	43.01	44.76	46.67	54.00
	12.00	18.50	42.00	43.84	45.44	45.96	48.00	51.37	54.20	56.35	61.00
42	.00	51.70	23.00	24.37	25.04	25.70	27.00	29.24	30.56	32.19	42.00
	4.00	44.20	29.00	31.04	31.71	32.48	34.00	37.08	38.92	41.03	50.00
	8.00	39.70	35.00	37.36	38.34	39.33	41.00	44.48	45.52	47.62	54.00
	12.00	30.80	42.00	43.41	44.51	45.92	51.00	56.32	58.08	60.41	66.00
43	.00	90.50	24.00	25.64	26.33	26.90	28.00	30.43	31.53	33.03	44.00
	4.00	73.60	31.00	32.53	33.34	34.11	36.00	38.30	39.72	41.38	52.00
	8.00	62.90	37.00	39.19	40.07	40.98	43.00	46.41	47.65	49.47	54.00
	12.00	58.90	42.00	44.91	45.95	47.01	50.00	53.06	54.41	56.35	65.00

End.