

Analysis and Design of Speed Controller for Permanent Magnet Synchronous Motor Using Reduced Modelling

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award of degree of*

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in
Electronic Instrumentation and Control**



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DECLARATION


I hereby certify that the work is being presented in this thesis work entitled “**Analysis and Design of Speed Controller for Permanent Magnet Synchronous Motor Using Reduced Modelling**” in partial fulfillment of award of degree of Master of Engineering in Electronic Instrumentation & Control submitted in Electrical & Instrumentation Engineering Department, Thapar University, Patiala is an authentic record of my own work carried under the supervision of **Dr. Gagandeep Kaur**, Assistant Professor, Department of Electrical & Instrumentation Engineering, Thapar University, Patiala, Punjab.

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ABSTRACT

Stabilization is a process which controls the linear time invariant system under certain performance indices either in time domain or in frequency domain. In classical control system, the stabilization of linear time invariant system is achieved by state variable feedback technique or selection of proportional-plus-integral (PID) controller or compensator. The design of controllers and compensators for higher order system involves computationally difficult and cumbersome tasks. Hence there is a need for the design of a higher order system through suitable reduced order models. The controller designed on the basis of reduced order models should effectively control the original higher order system.

Permanent magnets to replace electromagnets, which have windings and require an external electric energy source, resulted in compact dc machines. The synchronous machine, with its conventional field excitation in the rotor, is replaced by the PM excitation; the slip rings and brush assembly are dispensed with. In the presence of a d axis stator current, the d and q current channels are cross-coupled and the model is non-linear, as a result of the torque term. Under the assumption that the d axis current being zero, then the system becomes linear and resembles that of a separately-excited dc motor with constant excitation. From then on, the block-diagram derivation, current loop approximation, speed-loop approximation and derivation of the speed controller by using symmetric optimum are identical to those for a dc motor drive speed controller design.

The model order reduction method proposed in this work gives better approximated reduced order model for the given PMSM drive system. Because of this we get the reduced order system performance as close as possible to the higher order system response. This will result in reduction in design cost and system complexity. This study focuses on the reduction of models it minimizes the complexity in direct design of controller. A PID controller is sufficient for many industrial applications; hence, it is considered in this work. The approximate values for PID controller are calculated from the reduced order model and suitably tuned to meet the required performance specifications involved in direct design of controller. The approximate values for PID Controller parameters are calculated from the pole zero cancellation method and suitably

tuned to meet the required performance specifications. The tuned values of these controller parameters are attached with the original system and its closed loop response for a unit step input is found to be in good accord with the response of reduced order model. The step response of the original plant with reduced order controller is almost similar to original plant with original controller.

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List of Symbols and Abbreviations

$G(s)$	Original transfer function
$G_r(s)$	Reduced transfer function
$H(s)$	Feedback transfer function
G_{red}	Controller reference transfer function
C_f	Full order controller transfer function
C_r	Reduced order controller transfer function
K_p	Proportional gain
K_i	Integral gain
K_d	Derivative gain
$e(t)$	Error of control system

Chapter 1

Introduction

1.1 Overview

In this work permanent magnet synchronous motor is taken as a case study. Objective is to design a speed controller for the permanent magnet synchronous motor using reduced order modeling. Permanent magnet manufacturing and technology are primarily responsible for lowering the cost and increasing the applications of dc motors. Ferrite or ceramic magnets are the most popular choices for low-cost motors. The availability of modern Permanent Magnets (PM) with considerable energy density led to the development of dc machines. Introduction of PM to replace electromagnets, which have windings and require an external electric energy source, resulted in compact dc machines [1]

The controlling part is the most important part in the processes industries, as here the process is to design of speed controller for the permanent magnet synchronous dc motor using PID i.e. proportional, integral, derivative. In this thesis conventional controller and the Pole zero cancellation method are used to tune the PID parameters [2]. The PID controller involves three separate parameters P i.e. proportional, I i.e. integral and D i.e. derivative. The proportional value evaluates the effect to the current error, the integral value evaluates the effect based on the sum of recent errors, and the derivative value determines the effect based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process through control element. These values can be interpreted in terms of time i.e. P depends on the present error, I on the accumulation of past errors, and D is a prediction of future errors, based on current rate of change. By tuning the three constants in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point and the degree of system oscillation. The use of the PID algorithm for control does not guarantee

optimal control of the system or the stability of system. To reduce the higher order system to lower order system the mixed method is applied. There are other mixed method are also available pade approximation, clustering method, factor division method, routh criteria, continuous fraction expansion method, and the eigen spectrum analysis. In this thesis factor division and routh criteria algorithm are used to reduce the numerator and denominator respectively.

1.2 Organization of thesis

- Chapter 1** It includes the introduction of the thesis.
- Chapter 2** It includes the literature review that contains previous work in this field which has been carried out.
- Chapter 3** It includes the control theory of control system and designing of controller for a given specification.
- Chapter 4** It includes introduction of the permanent magnet synchronous motor.
- Chapter 5** It includes the model order reduction methods.
- Chapter 6** It includes the result and discussion.
- Chapter 7** It includes the conclusion and future scope.
- Chapter 8** Check for originality

K. Ayyar *et.al.* proposed a model order reduction (MOR) method is proposed for reducing higher order system into lower order system. Pade approximation is used to reduce the higher order system to lower order system, PID controller parameter set by conventional method and genetic algorithm. Application implemented in Permanent magnet synchronous DC motor [1].

M. A. Rahman *et.al.* proposed designs review of the permanent magnet Synchronous Motors. A procedure has been developed to predict the steady state and dynamic performances of a brushless permanent magnet synchronous motor. Finite element analysis has been combined with a lumped parameter circuit model in order to provide satisfactory engineering information. One is to develop a unified lumped parameter circuit model for both steady state and dynamic analysis. The second step is to extract the individual lumped parameters from finite element solutions based on corresponding equivalent circuits [3].

B. S. Peter *et.al.* proposed modeling of Permanent magnet synchronous motors. This paper describes the development of a two-axis circuit model for permanent magnet synchronous motor by taking machine magnetic parameter variations and core loss. The circuit model can be applied to both surface mounted magnet and interior permanent magnet rotor configurations. A method for on-line parameter identification scheme based on no-load parameters and saturation level, to improve the model, is discussed in detail. Test schemes to measure the equivalent circuit parameters, and to calculate saturation constants which govern the parameter variations are also presented. [4].

T. Sebastian *et.al.* proposed equivalent circuit models for such motors, the model parameters are related to the motor structure and dimensions. Computed values of Parameters are compared with those measured from two d-axis q-axis experimental motors with different rotor construction [5].

P. Pillay *et.al.* proposed permanent magnet synchronous motor (PMSM) and the brushless dc motors. The permanent magnet synchronous motor has a sinusoidal back emf and requires sinusoidal stator currents to produce constant torque while the brushless dc motor has a trapezoidal back emf and requires rectangular stator currents to produce constant torque. The PMSM is very similar to the wound rotor synchronous machine [6].

B. Singh proposed the latest developments in Permanent Magnet Brushless dc motor drives. A comprehensive account of the state-of-the-art on types of construction of the motor, closed loop controllers in position, speed and current/torque control and recent trends in inverters, sensors are given. Techniques for mechanical sensors elimination are discussed in detail. Special efforts made to reduce torque ripples, noise and vibrations are described. The impact of microelectronics through integrated chips used in the control of permanent magnet brushless dc motor drives is given. The increasing applications of this drive due to improved performance and its cost reduction are also enlisted [7].

B. Philip *et.al.* proposed a new model order reduction algorithm taking the advantages of reciprocal transformation and principal pseudo break frequency estimation is presented. The denominator polynomial is constructed using the approximate dominant poles obtained. Ultimately the denominator polynomial formation is based on simple calculations involving high order system characteristic polynomial. Numerator polynomial is then determined using a recently proposed evolutionary computation -Big Bang Big Crunch algorithm. The method is simple and yields stable reduced order models. Difficulty may arise in finding complex poles in the reduced order model. However a modification in the algorithm by introducing search method to find the imaginary parts of such poles helps in overcoming this. [8].

D. Kumar *et.al.* proposed development of an algorithmic approach for model order reduction based on balanced truncation technique. Reduction of stable systems based on balanced truncation is extended which may be applicable for any Linear time invariant stable or unstable, minimal or non-minimal, continuous or discrete system. For reduction of unstable system, a system decomposition algorithm is given to decompose the unstable system into stable and unstable parts and then reduction of stable

part is carried out and finally the subsystem addition of reduced stable part and already separated unstable part is done to get the final reduced model [9].

R. Prasad *et.al.* proposed A mixed method is proposed which combines the factor division algorithm with the eigen spectrum analysis for deriving reduced order models of high-order linear time invariant systems. Pole centroid and system stiffness of both original and reduced order systems remain same in this method. The proposed method guarantees stability of the reduced model if the original high-order system is stable and is comparable in quality with the other well known existing methods of order reduction. [10].

R. Komarasan *et.al.* proposed improved clustering algorithm is used to obtain the reduced order denominator polynomial, this method also used for multivariable system described by matrix transfer function [11].

J. S. Rana *et.al.* proposed a mixed technique for reducing the order of the high order dynamic systems. In this technique, the denominator polynomial of the reduced order model is determined by using the modified pole clustering while the coefficients of the numerator are obtained by factor division method. This technique is simple and gives stable reduced models for the stable high-order system. C. B. Vishwakarma, modified pole clustering technique is suggested, which generates the more effective cluster .If a cluster contains r number of poles, then criterion is repeated r times with the most dominant pole available in that cluster. The factor division algorithm has been successfully used to find reduced order approximants of high order systems [12].

S. Skogestad *et.al.* proposed analytic tuning rules which are as simple as possible and still result in a good closed-loop behavior. The starting point has been the PID tuning rules of Rivera, Morari and Skogestad which have achieved widespread industrial acceptance. The integral term has been modified to improve disturbance rejection for integrating processes. Furthermore, rather than deriving separate rules for each transfer function model, started by approximating the process by a first-order plus delay processes and then use a single tuning rule. This is much simpler and appears to give controller tunings with comparable performance [13].

M. Jain *et.al.* Proposed the PID controller, this paper presents the and effective and fast tuning method using multi objective genetic algorithm to find the optimized parameter of the PID Controller and compared the result with conventional tuning method [14].

M. S. Saad *et.al.* proposed the implementation of PID controller tuning using two sets of evolutionary techniques which are differential evolution (DE) and genetic algorithm (GA). The optimal PID control parameters are applied for a high order system, system with time delay and non-minimum phase system. The performance of the two techniques is evaluated by setting its objective function with mean square error (MSE) and integral absolute error (IAE). Both techniques will compete to achieve the globally minimum value of its objective functions. Meanwhile, reliability between DE and GA in consistently maintaining minimum MSE is also been studied. This paper also compares the performance of the tuned PID controller using GA and DE method with Ziegler-Nichols method [15].

A. Mirzal *et.al.* proposed time delay component that makes time lag in system, responses in physical, chemical, biological and economic system as well as in process of measurement and computation genetic algorithm is used to determine PID controller parameter and result compared with iterative method and Ziegler-Nichols method [16]

K. Ahmed *et.al.* proposed a genetic algorithm (GA) based electronic PID controller tuner, the tuning is based upon the maximization of comprehension fitness function constructed as the inverse peak overshoot, it give the simpler representation of the problem simplifies chromosome construction and totally avoids using binary encoding and decoding [17]

J. P. Tiwari *et.al.* proposed for finding stable reduced order models of single-input-single-output large-scale systems using factor division algorithm and the clustering technique. The denominator polynomial of the reduced order model with respect to original model is determined by forming the clusters of the poles of the original system, and the coefficients of numerator polynomial with respect to original model are obtained by using the Factor division algorithm. The mixed methods are simple and guarantee the stability of the reduced model if the original system is stable. [18]

B. Tendon *et.al.* proposed PID controller is tuned using Genetic Algorithm & Ziegler-Nichols Tuning Criteria. Tuning methods for PID controllers are very important for the process industries. Traditional methods such as Ziegler-Nichols method often do not provide adequate tuning. Genetic Algorithm (GA) as an intelligent approach has also been widely used to tune the parameters of PID. Genetic algorithms are used to create an objective function that can evaluate the optimum PID gains based on the controlled systems overall error. [19]

3.1 Introduction

A control system is interconnection of components forming a system configuration that will provide a desired system response. The basis for analysis of a system is the foundation provided by linear system theory, which assume a cause effect relationship for components of a system. Therefore a component process shown in figure (3.1). The input output relationship represents the cause-and-effect relationship of the process, which in turns, represents a processing input signal to provide an output signal variable, often with power amplification. An open loop control system utilizes a controller or control actuator to obtain the desired response [20].

An open-loop controller, also called a non-feedback controller, is a type of controller that computes its input into a system using only the current state and its model of the system. A characteristic of the open-loop controller is that it does not use feedback to determine if its output has achieved the desired goal of the input. This means that the system does not observe the output of the processes that it is controlling.

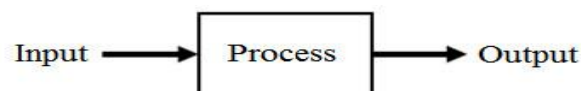


Figure 3.1: Process to be controlled

An open-loop control system utilizes an actuating device to control the process directly without using feedback.

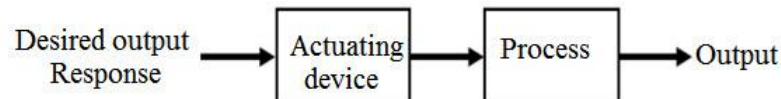


Figure 3.2: Open-loop control system.

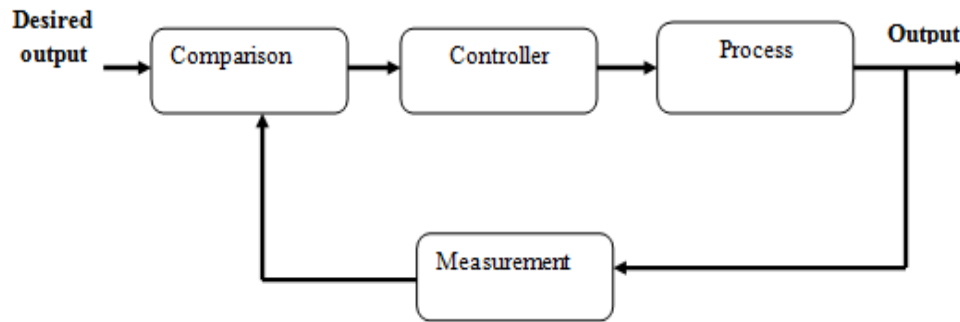


Figure 3.3: Closed-loop control system.

In contrast to an open-loop control system, a closed-loop control system utilizes an additional measure of the actual output to compare the actual output with the desired output response. The measure of the output is called the feedback signal. A simple closed-loop feedback control system is shown in Figure 3.3. A feedback control system is a control system that tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control. A feedback control system often uses a function of a prescribed relationship between the output and reference input to control the process. Often the difference between the output of the process under control and the reference input is amplified and used to control the process so that the difference is continually reduced. The feedback concept has been the foundation for control system analysis and design. A closed-loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output [20].

Due to the increasing complexity of the system under control and the interest in achieving optimum performance, the importance of control system engineering has grown in the past decade. Furthermore, as the systems become more complex, the interrelationship of many controlled variables must be considered in the control scheme. A block diagram depicting a multivariable control system is shown in Figure 3.4. A common example of an open-loop control system is an electric toaster in the kitchen. An example of a closed-loop control system is a person steering an automobile by looking at the auto's location on the road and making the appropriate adjustments. The introduction

of feedback enables us to control a desired output and can improve accuracy, but it requires attention to the issue of stability of response.

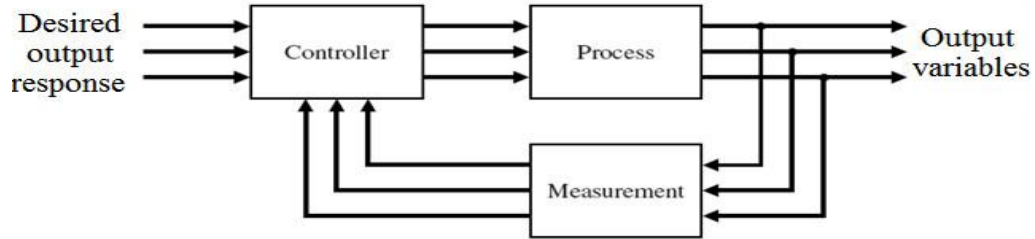


Figure 3.4: multivariable closed loop system

3.3 Control Loop Basics

A familiar example of a control loop is the action taken when adjusting hot and cold faucet valves to maintain the faucet water at the desired temperature. This typically involves the mixing of two process streams, the hot and cold water. The person touches the water to sense or measure its temperature. Based on this feedback Analysis PID Controller for Three Tank Liquid Level Control they perform a control action to adjust the hot and cold water valves until the process temperature stabilizes at the desired value. Sensing water temperature is analogous to taking a measurement of the process value or process variable (PV). The desired temperature is called the set point (SP). The input to the process is called the manipulated variable (MV). The difference between the temperature measurement and the set point is the error (e) that quantifies whether the water is too hot or too cold and by how much.

After measuring the temperature (PV), and then calculating the error, the controller decides when to change the tap position (MV) and by how much. When the controller first turns the valve on, they may turn the hot valve only slightly if warm water is desired, or they may open the valve all the way if very hot water is desired. This is an example of a simple proportional control. In the event that hot water does not arrive quickly, the controller may try to speed-up the process by opening up the hot water valve more-and-more as time goes by. This is an example of an integral control. By using only the proportional and integral control methods, it is possible that in some systems the

water temperature may oscillate between hot and cold, because the controller is adjusting the valves too quickly and over-compensating or overshooting the set point. In the interest of achieving a gradual convergence at the desired temperature (SP), the controller may wish to damp the anticipated future oscillations. So in order to compensate for this effect, the controller may elect to temper their adjustments. This can be thought of as a derivative control method.

Making a change that is too large when the error is small is equivalent to a high gain controller and will lead to overshoot. If the controller were to repeatedly make changes that were too large and repeatedly overshoot the target, the output would oscillate around the set point in a constant, growing, or decaying sinusoid. If the oscillations increase with time then the system is unstable, whereas if they decrease the system is stable. If the oscillations remain at a constant magnitude the system is marginally stable. A human would not do this because we are adaptive controllers, learning from the process history; however, simple PID controllers do not have the ability to learn and must be set up correctly. Selecting the correct gains for effective control is known as tuning the controller [21].

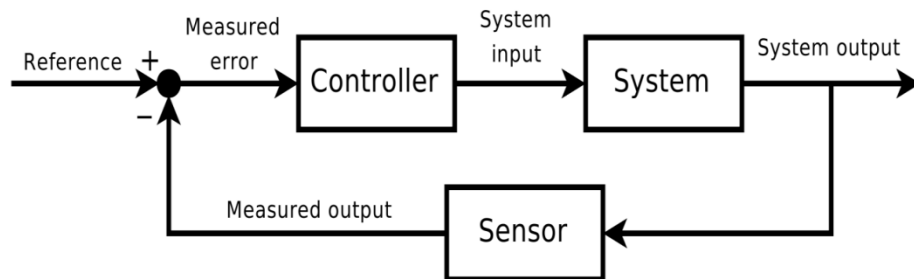


Figure 3.5: Closed loop system

3.4 Closed-Loop Transfer Function

The output of the system $y(t)$ is fed back through a sensor measurement to the reference value $r(t)$. The controller then takes the error difference between the reference and the output to change the inputs u to the system under control P . This is shown in the figure (3.5). This kind of controller is a closed-loop controller or feedback controller. This is called a single-input-single-output (SISO) control system; MIMO (i.e. Multi-Input-

Multi-Output) systems, with more than one input/output, are common. In such cases variables are represented through vectors instead of simple scalar values. For some distributed parameter systems the vectors may be infinite-dimensional. If we assume the controller C, the plant P, and the sensor F are linear and time-invariant (i.e.: elements of their transfer function C(s), P(s), and F(s) do not depend on time), the systems above can be analyzed using the Laplace transform on the variables.

This gives the following relations:

$$Y(s) = P(s)U(s) \quad (3.1)$$

$$U(s) = C(s)E(s) \quad (3.2)$$

$$E(s) = R(s) - F(s)Y(s) \quad (3.3)$$

Solving for Y(s) in terms of R(s) gives

$$Y(s) = \frac{P(s)C(s)}{1+F(s)P(s)C(s)} * R(s) = H(s) * R(s) \quad (3.4)$$

$$H(s) = \frac{P(s)C(s)}{1+F(s)P(s)C(s)} \quad (3.5)$$

The above expression is referred to as the closed-loop transfer function of the system. The numerator is the forward (open-loop) gain from r to y, and the denominator is one plus the gain in going around the feedback loop, the so-called loop gain. If $P(s)C(s) \gg 1$, i.e. it has a large norm with each value of s, and if $F(s) = 1$, then Y(s) is approximately equal to R(s). This simply means setting the reference to control the output. [21]

3.5 PID Controller and Designing

In the following sections we will describe the introduction of the PID controller and designing is explained.

3.5.1 PID Controller

A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism widely used in industrial control systems – a PID is the most commonly used feedback controller. A PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs. In the absence of knowledge of the underlying process, PID controllers are the best controllers. However, for best performance, the PID parameters used in the calculation must be tuned according to the nature of the system – while the design is generic, the parameters depend on the specific system.

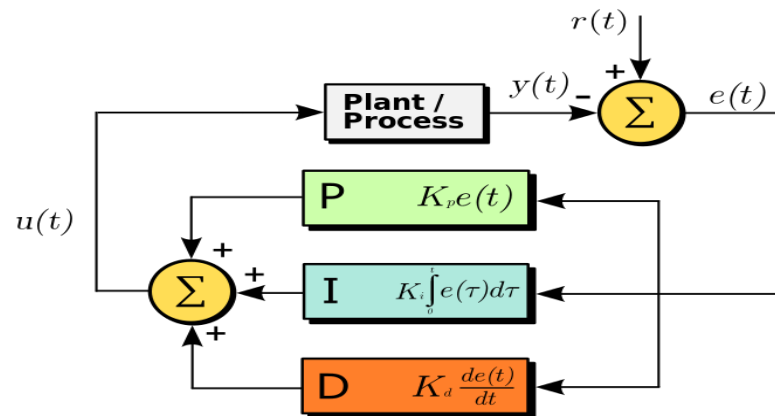


Figure 3.6: Block diagram of PID controller [22]

The PID controller calculation involves three separate parameters, and is accordingly sometimes called three-term control: the proportional, the integral and derivative values, denoted P, I, and D. The proportional value determines the reaction to the current error, the integral value determines the reaction based on the sum of recent errors, and the derivative value determines the reaction based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element. Heuristically, these values can be interpreted in terms of time: P depends on the present error, I on the accumulation of past errors, and D is a prediction of future errors, based on current rate of change. By tuning the three constants in the PID

controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point and the degree of system oscillation. Note that the use of the PID algorithm for control does not guarantee optimal control of the system or system stability.

3.5.2 PID Control Theory

The PID controller is probably the most-used feedback control design. PID is an acronym for Proportional-Integral-Derivative, referring to the three terms operating on the error signal to produce a control signal. If $u(t)$ is the control signal sent to the system, $y(t)$ is the measured output and $r(t)$ is the desired output, and tracking error $e(t) = r(t) - y(t)$, a PID controller has the general form.

$$U(t) = K_p e(t) + K_I \int e(t) dt + K_D \frac{d}{dt} e(t) \quad (3.6)$$

The desired closed loop dynamics is obtained by adjusting the three parameters K_P , K_I and K_D , often iteratively by "tuning" and without specific knowledge of a plant model. Stability can often be ensured using only the proportional term. The integral term permits the rejection of a step disturbance. The derivative term is used to provide damping or shaping of the response. PID controllers are the most well established class of control systems: however, they cannot be used in several more complicated cases, especially if MIMO systems are considered. Applying Laplace transformation results in the transformed PID controller equation

$$U(s) = K_p e(s) + K_I \frac{1}{s} e(s) + K_D s e(s) \quad (3.7)$$

PID controller transfer function

$$C(s) = K_p + K_I \frac{1}{s} + K_D s \quad (3.8)$$

3.5.3 Proportional Term

The proportional term (sometimes called gain) makes a change to the output that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant K_p , called the proportional gain.

The proportional term is given Eq. (3.9)

$$P_{out} = K_p e(t) \quad (3.9)$$

Where:

P_{out} : Proportional term of output

K_p : Proportional gain

e : Error = $SP - PV$

t : Time or instantaneous time

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error, and a less responsive or sensitive controller. If the proportional gain is too low, the control action may be too small when responding to system disturbances. In the absence of disturbances, pure proportional control will not settle at its target value, but will retain a steady state error that is a function of the proportional gain and the process gain.

3.5.4 Integral Term

The contribution from the integral term sometimes called reset is proportional to both the magnitude of the error and the duration of the error. Summing the instantaneous error over time gives the accumulated offset that should have been corrected previously. The accumulated error is then multiplied by the integral gain and added to the controller output. The magnitude of the contribution of the integral term to the overall control action is determined by the integral gain, K_I

The integral term is given by:

$$I_{out} = K_I \int e(t)dt \quad (3.10)$$

Where,

I_{out} : Integral term of output

K_I : Integral gain

e : Error = $SP - PV$

t : Time or instantaneous time

The integral term when added to the proportional term accelerates the movement of the process towards set point and eliminates the residual steady-state error that occurs with a proportional only controller. However, since the integral term is responding to accumulated errors from the past, it can cause the present value to overshoot the set point value.

3.5.5 Derivative Term

The rate of change of the process error is calculated by determining the slope of the error over time i.e., its first derivative with respect to time and multiplying this rate of change by the derivative gain K_d . The magnitude of the contribution of the derivative term to the overall control action is termed the derivative gain K_d .

The derivative term is given by:

$$D_{out} = K_D \frac{d}{d(t)} e(t) \quad (3.11)$$

Where,

D_{out} : Derivative term of output

K_D : Derivative gain

e : Error = $SP - PV$

t : Time or instantaneous time

The derivative term slows the rate of change of the controller output and this effect is most noticeable close to the controller set point. Hence, derivative control is used to reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability. However, differentiation of a signal amplifies noise and thus this term in the controller is highly sensitive to noise in the error

term, and can cause a process to become unstable if the noise and the derivative gain are sufficiently large. Hence an approximation to a differentiator with a limited bandwidth is more commonly used. Such a circuit is known as a Phase-Lead compensator.

The proportional, integral, and derivative terms are summed to calculate the output of the PID controller. Defining $U(t)$ as the controller output, the final form of the PID algorithm is:

$$U(t) = K_P e(t) + K_I \int e(t) dt + K_D \frac{d}{dt} e(t) \quad (3.12)$$

Proportional gain K_P

Larger values typically mean faster response since the larger the error, the larger the proportional term compensation. An excessively large proportional gain will lead to process instability and oscillation.

Integral gain K_I

Larger values imply steady state errors are eliminated more quickly. The trade-off is larger overshoot: any negative error integrated during transient response must be integrated away by positive error before reaching steady state.

Derivative gain K_D

Larger values decrease overshoot, but slow down transient response and may lead to instability due to signal noise amplification in the differentiation of the error [22, 23].

3.6 Loop Tuning

Tuning a control loop is the adjustment of its control parameters i.e gain / proportional band, integral gain/reset, derivative gain/rate to the optimum values for the desired control response. Stability is a basic requirement, but beyond that, different systems have different behavior, different applications have different requirements, and some desiderata conflict. Further, some processes have a degree of non-linearity and so parameters that work well at full-load conditions don't work when the process is starting up from no load; this can be corrected by gain scheduling. PID controllers often provide acceptable control even in the absence of tuning, but performance can generally be improved by careful tuning, and performance may be unacceptable with poor tuning. PID tuning is a difficult problem, even though there are only three parameters and in principle

is simple to describe, because it must satisfy complex criteria within the limitations of PID control. There are accordingly various methods for loop tuning, and more sophisticated techniques are the subject of patents; this section describes some traditional manual methods for loop tuning [24].

3.6.1 Stability

If the PID controller parameters are chosen incorrectly, the controlled process input can be unstable, i.e. its output diverges, with or without oscillation, and is limited only by saturation or mechanical breakage. Instability is caused by excess gain, particularly in the presence of significant lag. Generally, stability of response is required and the process must not oscillate for any combination of process conditions and set points, though sometimes marginal stability is acceptable or desired [24].

3.6.2 Optimum Behavior

The optimum behavior on a process change or set point change varies depending on the application. Two basic desiderata are regulation and command tracking these refers to how well the controlled variable tracks the desired value. Specific criteria for command tracking include rise time and settling time. Some processes must not allow an overshoot of the process variable beyond the set point if, for example, this would be unsafe. Other processes must minimize the energy expended in reaching a new set point [24].

3.6.3 Tuning Methods

There are several methods for tuning a PID loop. The most effective methods generally involve the development of some form of process model, and then choosing P, I, and D based on the dynamic model parameters. Manual tuning methods can be relatively inefficient, particularly if the loops have response times on the order of minutes or longer. The choice of method will depend largely on whether or not the loop can be taken "offline" for tuning, and the response time of the system. If the system can be taken offline, the best tuning method often involves subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters [25, 26].

3.6.4 Manual Tuning

If the system must remain online, one tuning method is to first set K_i and K_d values to zero. Increase then K_p until the output of the loop oscillates, then the K_p should be set to approximately half of that value for a "quarter amplitude decay" type response. Then increase K_i until any offset is correct in sufficient time for the process. However, too much K_i will cause instability. Finally, increase K_d , if required, until the loop is acceptably quick to reach its reference after a load disturbance. However, too much K_d will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the set point more quickly; however, some systems cannot accept overshoot, in which case an over-damped closed-loop system is required, which will require a K_p setting significantly less than half that of the K_p setting causing oscillation [26].

Table 3.1 Effect of increasing a parameter independently

Parameter	Rise Time	Overshoot	Settling Time	Steady State Error	Stability
k_p	Decrease	Increase	Decrease	Decrease	Degrade
k_i	Decrease	Increase	Decrease	Decreases Significantly	Degrade
k_d	Minor Decrease	Minor Decrease	Minor Decrease	No effect	Improve

3.6.5 Ziegler–Nichols Method

Another heuristic tuning method is formally known as the Ziegler–Nichols method, introduced by John G. Ziegler and Nathaniel B. Nichols. As in the method above, the K_i and K_d gains are first set to zero. The P gain is increased until it reaches the ultimate gain, K_U , at which the output of the loop starts to oscillate. K_U and the oscillation period P_U are used to set the gains are shown in table 3.2[24].

Table 3.2 Ziegler–Nichols Method

Control type	k_p	k_i	k_d
P	$0.05K_U$	----	-----
PI	$0.45K_U$	$1.2 \frac{K_P}{P_U}$	-----
PID	$0.60K_U$	$2 * \frac{K_P}{P_U}$	$K_P P_U * \frac{1}{8}$

The closed – loop Ziegler-Nichols method consist of following steps.

1. With P-only closed loop control, increase the magnitude of the proportional gain until the closed loop is in a continuous oscillation. For slightly larger values of controller gain, the closed loop system is unstable, while the slightly lower values the system is stable.
2. The value of controller proportional gain that causes the continuous oscillation is called the critical gain, K_u . The peak- to - peak period is called critical period P_U .
3. Depending upon controller chosen, P, PI, or PID, use the value in table 3.2 for tuning parameters, based on the critical gain and period.

3.7 PID Controller Design

Series controllers are preferred over feedback controllers because for higher order systems, a large number of state variables would require large number transducers to sense during feedback. This makes the use series controllers very common. The controller is attached with the reduced order model and closed loop response is observed. The parameters of the controller are tuned to get a response, meeting the desired specifications. The tuned parameters are introduced into the higher order system for stabilization processes [2]. The transfer function of PID controller is written for a continuous system given in Eq. (3.13)

$$G_c(s) = K_p + \frac{K_i}{s} + K_d * s \quad (3.13)$$

The design involves the determination of the values of the constants K_p , K_d and K_i meeting the required performance specifications.

3.7.1 Conventional method

In the conventional method we design a controller from the given specification. If the plant mathematical model is known and feedback transfer is known then full controller value can be found by using the given formula.

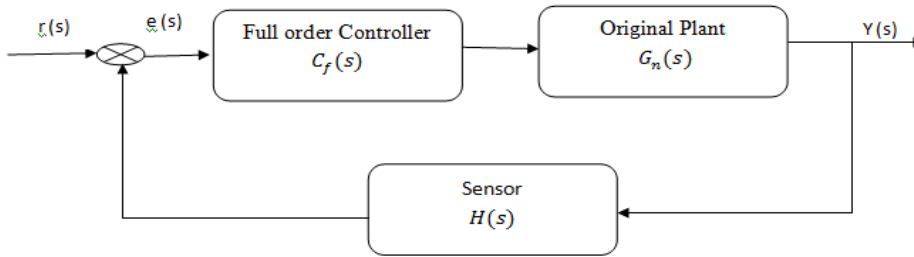


Figure 3.7 full controller with the original system

$$C_f(s) = \frac{G_{ref}(s)}{G_n(s)[1-G_{ref}(s)*H(s)]} \quad (3.14)$$

Where

$G_{ref}(s)$ Is the reference model, the specification are given rise time should be less than 3 sec. maximum overshoot should be less than 3% and settling time should be less than 2 sec.

$H(s)$ is the feedback signal of the closed loop system.

After designing the controller next step is to find out the overall closed loop transfer function and its unit step input. Overall closed loop transfer function can be found by using given equation below.

$$G_{org} = \frac{C_f(s)*G_n(s)}{1+C_f(s)*G_n(s)*H(s)} \quad (3.15)$$

Where

$C_f(s)$, $G_n(s)$ and $H(s)$ are the full controller, original system and feedback signal respectively

G_{org} is overall close loop response of the system

Same procedure is applicable for the reduced order controller designing firstly find out the reduced order of the higher order. After that calculate the $C_r(s)$ by the help of given formula

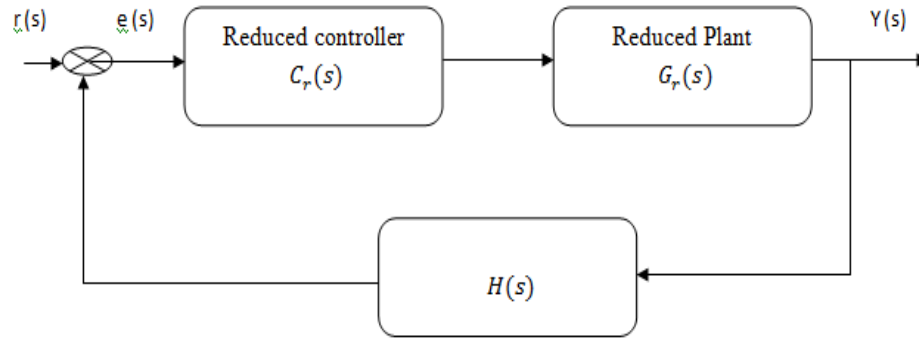


Figure 3.8: Reduced controller with the reduced system

$$C_r(s) = \frac{G_{ref}(s)}{G_r(s)[1-G_{ref}(s)*H(s)]} \quad (3.16)$$

Where,

$G_{ref}(s)$ And $H(s)$ are known parameter $C_r(s)$ can be calculated very easily. Next step to find out the overall closed loop transfer with help of given formula.

$$G_{red} = \frac{C_r(s)*G_r(s)}{1+C_r(s)*G_r(s)*H(s)} \quad (3.17)$$

Where

$C_r(s)$, $G_r(s)$ and $H(s)$ are reduced controller, reduced system, and feedback signal respectively

Now next step is to find out the step response with reduced controller.

3.7.2. Pole zero cancellation method

When an open-loop system has right-half-plane poles in which case the system is unstable. The idea is to improve the problem is to add zeros at the same place as

the unstable poles. It do not effect cancel the unstable poles. Unfortunately, this method is unreliable. The problem is that when an added zero does not exactly cancel the corresponding unstable pole. A part of the root locus will be attentive in the right-half plane. This causes the closed-loop response to be unstable.

The PID equation can be written as

$$G(s) = \frac{k_d s^2 + \frac{k_p}{k_d} * s + \frac{k_i}{k_d}}{s} \quad (3.18)$$

This form is easy to determine the close loop transfer function

$$H(s) = \frac{1}{s^2 + 2 * \xi * \omega * s + \omega^2} \quad (3.19)$$

$$\text{If } \frac{k_i}{k_d} = \omega^2, \frac{k_p}{k_d} = 2 * \xi * \omega \text{ then } G(s)H(s) = \frac{k_d}{s}$$

3.7.3 General Algorithm for Design of Controller Using Reduced Order Model

Step 1: Read the open loop transfer function of the given higher order system.

Step 2: Form the closed loop transfer function.

Step 3: Obtain the step response of closed system.

Step 4: Check the response for the required specifications.

Step5: If the specifications are not met, obtain a reduced order model using the Proposed scheme and design a controller for the reduced order model.

Step6: Obtain the initial values of the parameters K_p , K_i and K_d by pole-zero cancellation method

Step7: Attach the controller with the reduced order model and get the closed loop response with the initial values of the controller parameters.

Step 8: Find the optimum values for the controller parameters which satisfy the required specifications.

Step 9: With the optimum values, attach this controller with the original system.

Step 10: Obtain the closed loop response of the system with the controller.

Step 11: If the specifications are met, exit; else tune the parameters of the controller till it meets the required specifications.

3.7.4. Controller Specifications

Performance specifications are considered with respect to the closed loop response of the compensated system to unit step input. The specifications are chosen as. Maximum Overshoot should be less than 3%, settling time less than 3 seconds, Steady state error less than 2%.

4.1 Introduction

Modeling of physical systems results in complex high order dynamic models. It is of desirable to replace these models by simpler model so that different control can be carried out on this reduced order. There is a great need for model reduction algorithms also. The reduced models of control systems, biomedical systems, chemical system electrical systems and mechanical systems, aeronautical and astronautically systems. Show better modeling while implementation of mathematical model of regulator or the controller is limited. The processes may be complex and nonlinear. Even the computational cost can be reduced by finding a simpler model of the process.

Model order reduction was developed for the systems and control theory. This studies properties of dynamic systems for reducing their complexity. At the same preserving their input-output behavior. Model order reduction is a flourishing field of research both in systems and control theory and in numerical analysis as well as model order reduction tries to quickly capture the essential features of the control and scheme. Basic properties of the original model must already be present in the smaller approximation model only then it can replicate original higher order process. There is limit for reduction all necessary properties of the original model are captured with sufficient precision within the system.

Model-based process control has become popular in the chemical process industries. One reason being highly accurate models can be solved with modern dynamic powerful optimization algorithms and simulator. The increasing accuracy of the models for the complex of these models system requires computational and soft methodology tools which all available now. Many dynamic models derived from the basic mathematical logics and computation for these real time model-based controllers. This presents a need for model reduction techniques. The objective of model reduction for the purpose of controller design is to reduce a high- order model to a lower order system which retains most of the input-output behavior of the system considered.

4.2 Model Simplification and Model Order Reduction

The term reduced order model is frequently used for time domain i.e. state space models. And the model simplification is related to the frequency domain. Reducing the order of state space model is equivalent to search for a coarser representation of the process; It gives lower dimensional state space model for the similar inputs. This reduced model is checked for controllability and an observability hence replicate the original system to reduced system.

Model simplification, of any control system process does not ensure that the resulting transfer function is realizable. The term simplification is used for an approximate transfer function whose denominators are of lower order than those of original system subjected to functional criterion.

4.3 Need for Model Order Reduction

All the physical systems are transformed into mathematical models. These mathematical models give a complete description of a system in the form of higher order differential equations. It is necessary and useful to find a reduced model that adequately reflects the effective characteristics of the system. There are various reasons which describe the need of reduction as listed below.

1. Quick and easy understanding of the system

A higher order model of the system possesses difficulty in its synthesis and analysis identification. An obvious method of dealing with such type of system is to approximate it to a lower order model that reflects the characteristics of the original system i.e. the time constant, the natural frequency and damping ratio.

2. Reduced computational burden

When the order of the system model is higher, the numerical techniques need to compute the system response at the cost of computation time and memory required by digital computer.

3. Reduced hardware complexity

Most controllers are designed on the basis of linear low order model, which are more reliable, less costly and easy to implement and maintain due to less hardware complexity.

4. Making feasible design

Reduced order models may be effectively used in control applications like

- i. Model reference adaptive control schemes
- ii. Hierarchical control scheme
- iii. Suboptimal control
- iv. Decentralized controllers

5. To improve the methodology of computer aided control system design [31].

4.4 Applications of Reduced Order Model

Reduced order models and reduction techniques have been widely used for the analysis and the synthesis of high order systems. In Predicting dynamic error of high order systems using low order equivalents, control system design, reduced order estimator, Suboptimal control derived by simplified models and in Providing guidelines for online interactive modeling.

4.5 Methods of Reduced Order Model in Frequency Domain

The main objective of model order reduction is that the reduced order approximation should reproduce the significant characteristics of the original system as closely as possible. The model order reduction techniques in frequency domain can broadly be classified as, Frequency domain simplification techniques given below.

4.5.1 Continued fraction expansion (CFE) and truncation

Continued-fraction expansion method is one of the most attractive methods for the order reduction of transfer functions. It has many useful properties such as computational simplicity, the fitting of the preservation and time moments, of the steady-state responses for polynomial inputs of the form $\sum \alpha_i t^i$. T.N. Lucas did model reduction by continued fraction method [32].

In this method, the continued fraction expansion of the transfer function is truncated .Get a low order model, with a step response matching closely that of the original system. There are three basic forms for continued fraction representation of a

transfer function. Each form concentrates on a different frequency range for the approximation of the original transfer function.

The reduced order model obtained from the Caue first form cannot preserve the steady state value of the original model. The restriction to the original transfer function is that it requires the order of the numerator to be one less than that of the denominator. The Caue first form does not exist. The reduced order model from the Caue first form is the matching of the original transfer function by the Markov parameters, which is the power series expansion over $1/s$. In Caue second form, continued-fraction is equivalent to the Maclaurin expansion about $s = 0$. This form maths the steady state reponses, but does not provide a good match for the initial transient response

The Caue third form can only be applied when the order of the numerator of the original transfer function is one less than that of the denominator. The mixed form is equivalent to the Maclaurin series about $s = 0$ and $s = \infty$. The reduced order model matches the first m time moments and m Markov parameters of the original model, where m is the order of reduced order model. This method has one serious disadvantage. It may produce unstable reduced models even though when the original high-order system is stable.

$$r = a_0 + \frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 \dots}}}$$
(4.1)

Where

a_i and b_i are either rational numbers ,real number or complex numbers, if $b_i = 1$ is called the simple fraction. If it contains finite number of terms is called finite continued fraction. If the expression contains an infinite number of terms it is called an infinite continued fraction.

4.5.2 Pade approximation technique

This method has a number of advantages, such as computational simplicity, fitting of the initial time moment and the steady state value of the outputs of the system and the model for polynomial inputs belonging to the class of form $\sum \alpha_i t^i$. Serious drawback of pade-

based approximants is that unstable reduced-order models can be generated from original asymptotically stable high order system. O. Ismail and B. Bandyopadhyay modeled reduction of linear interval systems using pade approximation [33].

pade approximation is the method of model order reduction of the higher order system. This gives the simplification of a model after converting it into a reduced order model. This approach stems from the theory of pade. Before understanding the pade approximation method consider the following definitions given below [1].

Consider a function:

$$f(s) = c_0 + c_1s + c_2s + \dots \quad (4.2)$$

A rational function $u_m(s)/v_n(s)$ where $u_m(s)$ and $v_n(s)$ are m^{th} and n^{th} order polynomials in s respectively, and $m \leq n$. The rational function $u_m(s)/v_n(s)$ is said to be Pade approximant of $f(s)$ if and only if the first $(m+n)$ terms of the power series expansions of $f(s)$ and $u_m(s)/v_n(s)$ are identical. For the function $f(s)$ in Eq. 4.2 to be approximated, let the following pade approximant be defined.

$$\frac{u_m(s)}{v_n(s)} = \frac{a_0 + a_1s + \dots + a_{m-1}s^{m-1}}{b_0 + b_1s + \dots + b_{n-1}s^{n-1} + s^n} \quad (4.3)$$

For the first $(m + n)$ terms of Eq. 4.1 and Eq. 4.2 to be equivalent, it becomes apparent that the following set of relations must be configured

$$\begin{aligned} a_0 &= b_0c_0 \\ a_0 &= b_0c_1 + b_1c_0 \\ a_{n-1} &= b_0c_{n-1} + b_1c_{n-2} + \dots + b_{n-1}c_0 \\ 0 &= b_0c_n + b_1c_{n-1} + \dots + b_n c_0 \\ 0 &= b_0c_{2n-1} + b_1c_{2n-2} + \dots + b_{n-2}c_n + c_{n-1} \end{aligned} \quad (4.4)$$

Coefficients c_i , $i = 0, 1, 2, \dots$ Can be found using Eq. 4.4 and

$c_i = (-1)^j a_{j+2,1}$ For the full model

$$G(s) = \frac{N(s)}{D(s)} = \frac{d_0 + d_1 s + \dots + d_{m-1} s^{m-1}}{e_0 + e_1 s + \dots + e_{m-1} s^{m-1} + e_m s^m} \quad (4.5)$$

Eq. 4.5 can be written in the matrix form give below

$$\begin{bmatrix} C_n & C_{n-1} & \dots & C_1 \\ C_{n+1} & C_n & \dots & C_2 \\ C_{n+2} & C_{n+1} & \dots & C_3 \\ \dots & \dots & \dots & \dots \\ C_{2n-1} & C_{2n-2} & \dots & C_n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \dots \\ b_{n-2} \\ b_{n-1} \end{bmatrix} = \begin{bmatrix} -C_0 \\ -C_1 \\ \dots \\ -C_{n-2} \\ -C_{n-1} \end{bmatrix}$$

$$\begin{bmatrix} C_0 & 0 & 0 & 0 \\ C_1 & C_0 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ C_{n-2} & C_{n-1} & \dots & C_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \dots \\ b_{n-2} \\ b_{n-1} \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ \dots \\ a_{n-2} \\ a_{n-1} \end{bmatrix} \quad (4.6)$$

4.5.3 Routh criteria.

The Routh stability technique consists of obtaining the numerator and the denominator polynomials of the reduced order model, respectively, from the numerator and the denominator polynomials of the original system. It is computational methods that develop a sequence of computed numbers from two generating rows of numbers. This method is used to check the stability of the system. The Routh criterion is an efficient test which gives how many roots of the polynomial lie in the right half of the s-plane, that gives information about the roots symmetrically located about the origin. The number of sign changes in the first column of the Routh array determines the number of roots lying in the right of the s-plane. S. Panda et al. used Routh approximation for reduction of liner time invariant systems [34].

$$Dr(s) = \sum_{i=0}^r b_i s^i \quad (4.7)$$

4.5.4 Eigen Spectrum analysis

This method is applicable only for the real poles, in this reduction method both original and reduced order pole centroid and system stiffness are kept exactly same to obtain the reduced order system poles. G. Parmar et al. carried out the system reduction eigen spectrum analysis [36]. Description of method:

Let the transfer function of the high-order system (HOS) of order 'n' is

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s^1 + b_2 s^2 \dots + b_{n-1} s^{n-1}}{(s + \lambda_1)(s + \lambda_2) \dots (s + \lambda_n)} \quad (4.8)$$

Where $-\lambda_1 < -\lambda_2 < \dots < -\lambda_n$ are poles of higher order system (HOS)

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s^1 + \dots + \alpha_{r-1} s^{r-1}}{(s + \tilde{\lambda}_1)(s + \tilde{\lambda}_2) \dots (s + \tilde{\lambda}_r)} \quad (4.9)$$

Where $-\tilde{\lambda}_1 < -\tilde{\lambda}_2 < -\tilde{\lambda}_3 \dots < -\tilde{\lambda}_r$ are poles of lower order system (LOS)

$$\text{Re} \tilde{\lambda}_p' (1 - \lambda s) + M(1 - p') = 0 \quad (4.10)$$

$$\text{Re} \tilde{\lambda}_p' [\lambda s(1 - p') + 1] + MQ = N \quad (4.11)$$

Where,

$\tilde{\lambda}_m$ and $\tilde{\lambda}_s$ are the pole centroid and system stiffness of LOS such that $\tilde{\lambda}_m = \lambda m$ and $\lambda s = \tilde{\lambda}_s$ then following equation can be written as Form the above Eq. 4.10 and 4.11 calculate the value of $\text{Re} \tilde{\lambda}_p'$ and M.

This reduces the denominator of higher order system to lower order system.

4.5.5 Eigen Permutation Algorithm

Eigen Spectrum Analysis both the pole centroid and system stiffness of the original and the reduced order systems are kept exactly same to obtain the reduced order system. In some cases, the difficulty with these methods is tendency to become non-minimum phase

due to equalization of system stiffness. Each method has their advantages and disadvantages. Major number of methods available in the literature but no approach constantly gives the best results for all systems. The reduction procedure is very easy and computer oriented. This method is applicable for the real pole only or complex pole only when real and complex poles comes together it does not reduces the system. Jay Singh et al. used the Eigen Algorithm for system reduction [37]. To obtain the reduced order use the following Eq. 4.12

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^k d_i s^i} \quad (4.12)$$

Here $N_k(s)$ is the numerator c_i is coefficient of the numerator and $D_k(s)$ is denominator and d_i is the coefficient of the denominator.

4.5.6 Stability equation method

This method is based on forming stability equations of the numerator and denominator polynomials of high-order system. This method discards the non-dominant poles and zeros and obtaining the reduced order model. This approach prevents the stability of reduced-order model. The disadvantages are related to the fact that this method is founded on heuristic considerations and besides the structural stability that is preserved in the reduced-order model. The correlations between the original and the reduced model are lost. S.R. Desai and R. Prasad used stability equation for order reduction [38].

$G(s) = \frac{N(s)}{D(s)}$ is given a stable nth order equation. To reduce the denominator truncate it into two parts one is even power and another is in odd power.

$$D(s) = D_e(s) + D_o(s) \quad (4.13)$$

Where

$$D_e(s) = a_{11} \prod_{i=0}^{k_1} \left(1 + \frac{s^2}{z_i^2}\right) \quad (4.14)$$

$$D_o(s) = a_{12} s \prod_{i=0}^{k_1} \left(1 + \frac{s^2}{p_i^2}\right) \quad (4.15)$$

Where

k_1 And k_2 is integer part, $D_e(s)$ and $D_o(s)$ is the even and odd part of the denominator. Remove the large magnitude of zi and pi .it gives the reduced order equation.

$$D_{re}(s) = a_{11} \prod_{i=0}^{r_1} \left(1 + \frac{s^2}{z_i^2}\right) \quad (4.16)$$

$$D_{ro}(s) = a_{12}s \prod_{i=0}^{r_1} \left(1 + \frac{s^2}{p_i^2}\right) \quad (4.17)$$

Where

r_1 And r_2 are the integer part of even and odd respectively. The reduced denominator is given as

$$D_r(s) = D_{re}(s) + D_{ro}(s) \quad (4.18)$$

Eq. 4.18 gives the reduced model of the given higher order system.

4.5.7 Polynomial differentiation method

The differentiation method for model order reduction was introduced by Gautam. This method is based on differentiation of polynomials of the higher order transfer functions are differentiated successively many time to yield the coefficients of the reduced order transfer function. The reduced polynomials are reciprocated back and normalized. The straightforward differentiation is discarded because it has a drawback that zeros with large module tend to be better approximated than those with small module. J. S. Yadav et al. did model reduction of SISO discrete systems using polynomial differentiation [39].

$$\frac{d}{dx} x^n = n * x^{n-1}, n \neq 0 \quad (4.19)$$

4.5.8 Clustering technique

Model reduction has attracted attention in system modeling and design for the last four decades. This continued interest and the huge number of methods available in the literature reflect the importance of producing a reliable reduced order model for the system analysis and design. Some of the papers were proposed, based on matching of Markov parameters and initial time moments of the original and reduced order systems such as Mittal *et al.* (2002), Prasad *et al.* (2003). The concept of retaining the dominant

dynamical characteristics of the original system in the reduced model is intuitive and has two appealing advantages. The reduced-order model retains the basic physical features such as time constants of the original system and the stability of the simplified model is guaranteed. The clustering method differs from the existing pole clustering technique by considering the distance of system poles from the first pole in the group clustering process. This process yields a better approximation in the reduction process. Jasvir Singh Rana et al. made use of modified pole clustering and factor division method for order reduction [10].

Let r real poles in one clusters (p_1, p_2, \dots, p_n) then inverse distance measure criterion identifies the cluster centre as

$$P_c = \left\{ \sum_{i=1}^n \left(\frac{1}{p_i} \right) / r \right\}^{-1} \quad (4.20)$$

Where

P_c Is the cluster center and r is the real poles of the original system.

5.1 Introduction

The principle of operation of a synchronous motor, armature winding of a 3-phase synchronous machine is to a suitable balanced 3-phase source and the field winding to a D.C source of appropriate voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e. at any location in the stator there will be a North Pole at some instant of time and it will become a South Pole after a time period corresponding to half a cycle. (After time $=1/2f$) where f = frequency of the supply). Let us assume that the stationary South Pole in the rotor is aligned with the North Pole in the stator moving in clockwise direction at a particular instant of time, as shown in Fig 5.1. These two poles get attracted and try to maintain this alignment (as per Lenz’s law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. K. Ayyar et al. proposed a Model Order Reduction (MOR) for reduction of higher order to lower order systems [1].

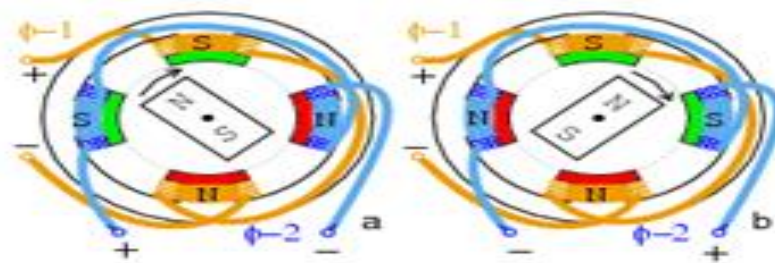


Figure 5.1: Force of attraction between stator pole and rotor poles - resulting in production of torque in clockwise direction

However the rotor cannot move instantaneously due to its mechanical inertia, and so it needs some time to move. In the mean time, the stator pole would quickly a time duration

corresponding to half a cycle change its polarity and becomes a South Pole. So the force of attraction will no longer be present and instead the like poles experience a force of repulsion as shown in Fig.5.2.

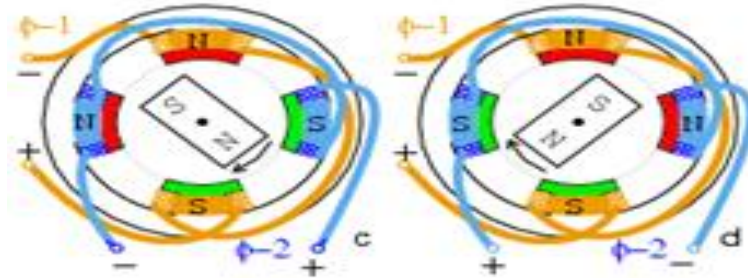


Figure 5.2: Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction

Even this condition will not last longer as the stator pole would again change to North Pole after a time of $1/2f$. Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to $1/2f$ seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only. On the contrary if the rotor is brought to near synchronous speed by some external means say a small motor (known as pony motor-which could be a D.C or AC induction rotor) mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the pony motor is disconnected.

Thus the synchronous rotor cannot start rotating on its own or usually we say that the synchronous rotor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to operate at its synchronous speed. These machines are also called brushless DC machine. Ajith H. Wijenayake and Peter B.

Schmidt described the establishment of a two axis circuit model of permanent magnet synchronous motor [4].

5.2 Machine Construction

Permanent magnet synchronous motors are predominantly surface-magnet machines with wide magnet pole-arcs and concentrated stator windings. The design is based on a square waveform distribution of the air-gap flux density waveform as well as the winding density of the stator phases in order to match the operational characteristics of the self-controlled inverter. Enrique L. Carrillo Arroyo dealt with modeling of permanent magnet synchronous motor drive using simulink [40].

5.2.1 Permanent Magnets

Motors obtain life-long field excitation from permanent magnets mounted on the rotor surface. Advances in permanent magnet manufacturing and technology are primarily responsible for lowering the cost and increasing the applications of DC motors. Ferrite or ceramic magnets are the most popular choices for low-cost motors. These magnets are now available with a remanence of 0.38 T and an almost straight demagnetization characteristic throughout the second quadrant. For special applications, magnetic materials with high-energy products such as neodymium-iron-boron (Nd-Fe-B) are used.

The magnets are constructed in the form of arcs, radially magnetized, and glued onto the surface of the rotor with adjacent rotor poles of opposite magnetic polarity as shown in Fig 4.3. The number of rotor poles is inversely proportional to the maximum speed of rotation, and is frequently chosen to meet manufacturing constraints. Most DC motors have four, six, or eight poles, with four the most popular choice.

5.2.2 Stator Windings

Permanent magnet synchronous motors are often assumed to have three phases, but this is not always the case. Small motors for applications such as light-duty cooling fans have minimal performance requirements, and it is cost effective to build them with just one or two phases. On the other hand, it is preferable to use a high phase number for large drives with megawatt ratings. This reduces the power-handling capacity of a single phase, and

also incorporates some degree of fault tolerance. Machines with as many as 15 phases have been built for ship propulsion. Although these are special-purpose designs, motors with four and five phases are quite common. The number of stator slots is chosen depending on the rotor poles, phase number, and the winding configuration.

5.3 Current Control

The power converter in a high-performance motor drive used in motion control essentially functions as a power amplifier, reproducing the low power level control signals generated in the field orientation controller at power levels appropriate for the driven machine. High-performance drives utilize control strategies which develop command signals for the AC machine currents. The basic reason for the selection of current as the controlled variable is the same as for the DC machine. The stator dynamics effects of stator resistance, stator inductance, and induced electromagnetic field are eliminated. Thus, to the extent that the current regulatory functions as an ideal current supply, the order of the system under control is reduced and the complexity of the controller can be significantly simplified. T. Sebastian et al. also modeled the permanent magnet synchronous motor [5].

Current regulators for AC drives are complex because an AC current regulator must control both the amplitude and phase of the stator current. The AC drive current regulator forms the inner loop of the overall motion controller. As such, it must have the widest bandwidth in the system and must, by necessity, have zero or nearly zero steady-state error. Both current source inverters and voltage source inverters can be operated in controlled current modes. The current source inverter is a "natural" current supply and can readily be adapted to controlled current operation. The voltage source inverter requires more complexity in the current regulator but offers much higher bandwidth and elimination of current harmonics as compared to the CSI and is almost exclusively used for motion control applications. Current controllers can be classified into two groups, hysteresis and PWM current controllers. Both types are discussed below.

5.4 Mathematical model of permanent magnet synchronous motor

Proportional-plus-integral controller is sufficient for many industrial applications; hence, it is considered in this work. Selection of the gain and time constants of such a controller by using the symmetric optimum principle is straightforward if the d axis stator current is assumed to be zero. In the presence of a d axis stator current, the d and q current channels are cross-coupled and the model is non-linear, as a result of the torque term. Under the assumption that the d axis current being zero then the system becomes linear and resembles that of a separately-excited dc motor with constant excitation. From then on, the block-diagram derivation, current loop approximation, speed-loop approximation and derivation of the speed controller by using symmetric optimum are identical to those for a dc motor drive speed controller design [1]. P. Pillay and R. Krishnan, too, has described that permanent magnet motor drives can be compared to induction motor for servo applications [6]. B. Singh showed the latest developments Permanent Magnet Brushless DC motor [7].

5.4.1 Block diagram derivation

The motor q axis voltage equation with the d axis current being zero becomes (Sharma et al., 2008):

$$V_{qs}^r = (R_s + L_q p) i_{qs}^r + \omega \lambda_{af} \quad (5.1)$$

And the electromechanical Eq. (5.2) is

$$\frac{p}{2} (T_e - T_1) = J p \omega_r + B_1 \omega_r \quad (5.2)$$

Where, the electromagnetic torque is given by Eq. (5.3)

$$T_e = \frac{3}{2} * \frac{p}{2} * \lambda_{af} i_{qs}^r \quad (5.3)$$

And if the load is assumed to be frictional, then Eq. (5.4)

$$T_1 = B_1 \omega_m \quad (5.4)$$

Which upon substitution gives the electromechanical Eq. (5.5)

$$(Jp + B_r)\omega_r = \left\{ \frac{3}{2} * \left(\frac{p}{2}\right) * \left(\frac{p}{2}\right)\lambda_{af} \right\} i_{qs}^r = K_t * i_{qs}^r \quad (5.5)$$

The frictional torque coefficient is Eq. (5.6)

$$B_t = \left(\frac{p}{2}\right) B_1 + B_1 \quad (5.6)$$

And torque constant is Eq. (5.7)

$$K_t = \frac{3}{2} \left(\frac{p}{2}\right) * \left(\frac{p}{2}\right) * \lambda_{af} \quad (5.7)$$

The Eq. 5.1 and 5.5, when combined into a block diagram with the current-and speed-feedback loops added (Sharma et al., 2008). The inverter is modeled as a gain with a time lag (Talebi et al., 2007) by Eq.(5.8-5.10)

$$G(s) = \frac{K_{in}}{1+sT_{in}} \quad (5.8)$$

Where:

$$K_{in} = 0.65 * \frac{V_{dc}}{V_{cm}} \quad (5.9)$$

$$T_{in} = \frac{1}{2} * f_c \quad (5.10)$$

Where, V_{dc} is the dc-link voltage input to the inverter (Islam et al., 2011), V_{cm} is the maximum control voltage and f_c is the switching (carrier) frequency of the inverter.

The induced emf due to rotor flux linkages is Eq. 5.11

$$e_a = \lambda_{af} * \omega_r (V) \quad (5.11)$$

Current loop: This induced-emf loop crosses the q axis current loop and it could be simplified by moving the pick-off point for the induced-emf loop from speed to current output point. This gives the current-loop transfer function.

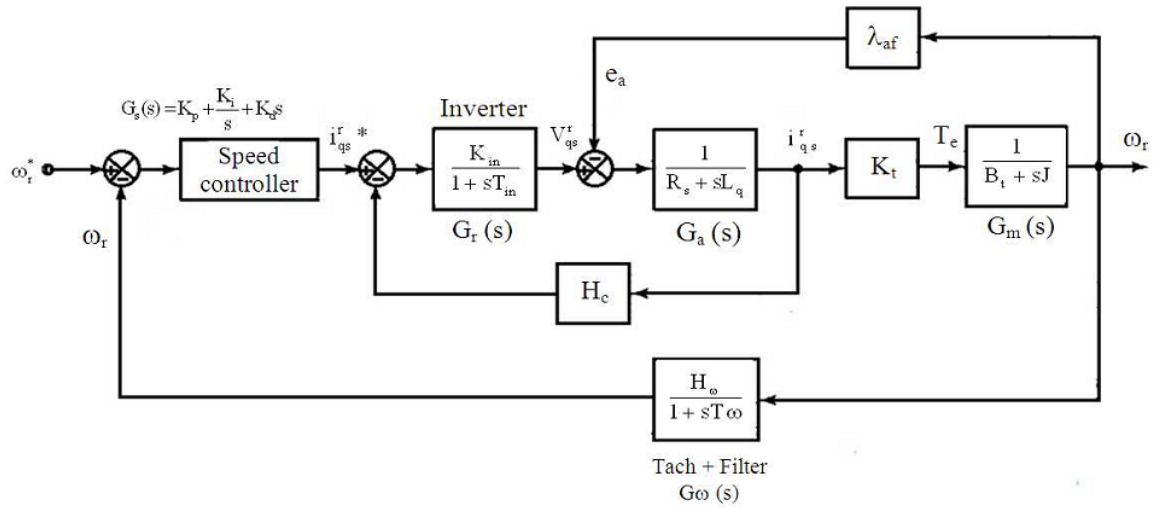


Figure 5.3 Block diagram of the speed controlled PSMS drive

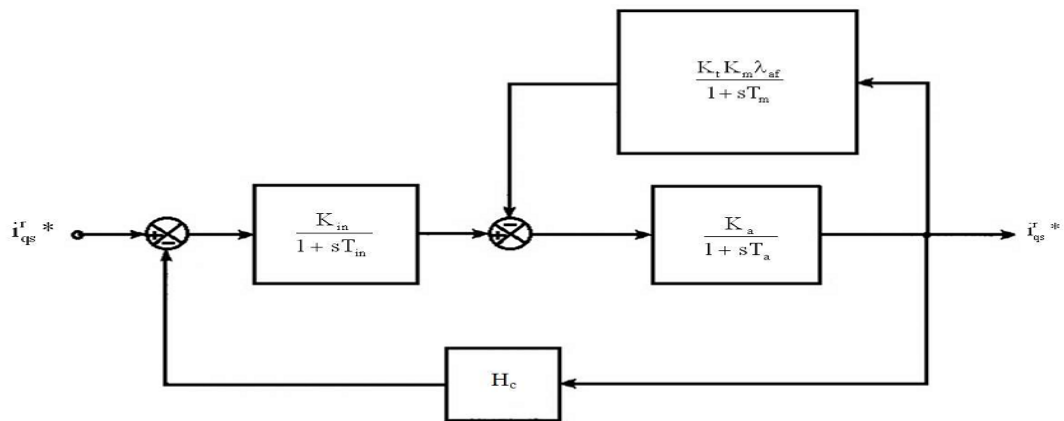


Figure 5.4 Current controller

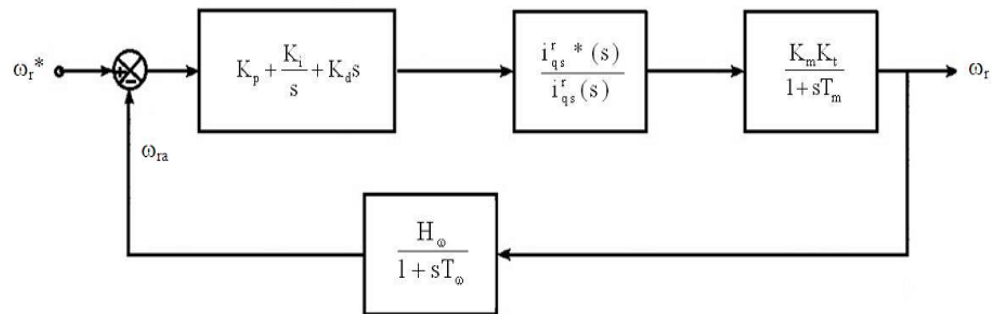


Figure 5.5 Speed control loop

This induced emf loop crosses the q axis current loop and it could be simplified by moving the pickoff point for the induced-emf loop from speed to current output point. This gives the current-loop transfer function from Fig. 5.4

$$\frac{i_{qs}^r(s)}{i_{qs}^{*r}(s)} = \frac{K_{in} K_a (1+sT_m)}{H_c K_a K_{in} (1+sT_m) + (1+sT_{in}) \{K_a K_b + (1+sT_a)(1+sT_m)\}} \quad (5.12)$$

$$K_a = \frac{1}{R_s}, T_s = \frac{L_q}{R_s}, K_b = K_t K_m \lambda_{af}, T_{in} = \frac{J}{B_t}, K_{in} = \frac{1}{B_t} \quad (5.13)$$

Feedback equation can be given as

These values substituted in the Eq. (5.12)

$$H(s) = \frac{H_\omega}{1+T_\omega s} \quad (5.14)$$

6.1 Permanent magnet synchronous motor transfer function

Transfer function of the permanent magnet synchronous motor current loop transfer function is given by Eq. 5.12

$$\frac{i_{qs}^r(s)}{i_{qs}^{r*}(s)} = \frac{K_{in}K_a(1 + sT_m)}{H_cK_aK_{in}(1 + sT_m) + (1 + sT_{in})\{K_aK_b + (1 + sT_a)(1 + sT_m)\}}$$

Sensor feedback can be written from Eq. 5.14

$$H(s) = \frac{H_\omega}{1 + T_\omega s}$$

K_a, K_b, T_{in} and K_{in} values are given in Eq. 5.13. Drive parameter values are given below in the table 6.1.

Table 6.1 Drive parameters

Parameter	Values	Units
R_s	1.4	Ω
L_d	0.0056	H
L_q	0.009	H
λ_{af}	0.1596	$Wb - Turn$
B_t	0.01	$N - \frac{m}{rad}/sec$
J	0.006	$Kg - m^2$
p	6	–
f_c	2	KHz
V_{cm}	10	V
H_ω	0.05	V/V
T_ω	0.002	sec
H_c	0.8	V/A
V_{dc}	285	V

From these parameters the values of K_a , T_a , K_m , T_m and K_b can be calculated
Close loop transfer function of the permanent magnet synchronous motor let the $G(s)$ is the transfer function of the system.

$$G(s) = \frac{N(s)}{D(s)} = \frac{1657.078*s+2763.2}{.000000576*s^4+0.0024*s^3+4.2*s^2+27.78*s+34.63} \quad (6.1)$$

$$H(s) = \frac{.05}{0.002*s+1} \quad (6.2)$$

This is the closed loop transfer function of the permanent magnet synchronous motor drive system. This is the fourth order system.

Where

$G(s)$ Is the original transfer function and $H(s)$ is the sensor transfer function.

Here the original plant transfer function is permanent magnet synchronous motor whose transfer function is taken by Eq. (6.1)

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{1657.078*s+2763.2}{.000000576*s^4+0.0024*s^3+4.2*s^2+27.78*s+34.63}$$

The step response of original system is given in figure (6.1)

Reduction method

The transfer function of the control system is expressed as

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+---+a_1s+a}{b_n s^n + b_{n-1} s^{n-1} + --- + b_1 s + b_0} = \frac{\sum_{i=0}^{n-1} a_i s^i}{\sum_{i=0}^n b_i s^i} \quad (6.3)$$

Where, $N(s)$ and $D(s)$ are numerator and denominator polynomials of original higher order model $G(s)$ respectively. Let the order of $D(s)$ be even. Following construct the routh array for the denominator polynomial of the given transfer functions starting with the first entry as the constant term. To obtain a reduced model of order 'r' a new routh array is formed, where the first (r-1) terms of the above array forms the first column. The remaining entries of the array are now easily filled. M. A. Choghadi and H.

A. Talebi studied the case of multiple poles on the imaginary axis using routh criterion [35].

$$D_r(s) = b_0 + b_1s + b_2s^2 + \dots + s^r \quad (6.4)$$

Denominator is the r^{th} order reduced normalized denominator is given as

$$D_r(s) = \sum_{i=0}^r b_i s^i \quad (6.5)$$

In this work higher order system is reduced to lower order system by using the mix method routh criteria and factor division algorithm the numerator of the higher order system is reduced by the factor division algorithm and the denominator is reduced by the routh criteria method.

$$G_r(s) = \frac{\bar{N}(s)}{\bar{D}(s)} = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0}{bs^n + b_{n-1}s^{n-1} + \dots + b_1s + b_0} = \frac{\sum_{i=0}^{n-1} a_i s^i}{\sum_{i=0}^n b_i s^i} \quad (6.6)$$

Step 1: Construct the routh array for the denominator polynomial of the given transfer function starting with the first entry as the constant term. To obtain a reduced model of order r a new routh array is formed, where the first $(r + 1)$ terms of the above array forms the first column. The remaining entries of the array are now easily filled. Once the array is complete, it will be noted that the last three element in the first column take as it is this gives the reduced model of given higher order system.

$$D_r(s) = s^r + \dots + b_2s^2 + b_1s^1 + b_0 \quad (6.7)$$

Which is the r^{th} order reduced normalized denominator and can be expressed as

$$D_r(s) = \sum_{i=0}^r b_i s^i \quad (6.8)$$

s^4	0.000000576	4.2	34.63
s^3	0.0024	27.78	0
s^2	4.193	34.63	0
s^1	27.760	0	0
s^0	34.63	0	0

Take first Colum last three entries given below

s^2	4.193
s^1	27.760
s^0	34.63

These values can be written as

$$D_r(s) = 4.193 * s^2 + 27.760 * s + 34.63$$

Or this can be written in simple way given below

$$D_r(s) = s^2 + 6.616 * s + 8.25$$

Step 2: Determination of numerator polynomial of $r - 1^{th}$ order using factor division algorithm.

$$N_r(s) = \frac{N(s)}{D(s)} * D_r(s) \quad (6.9)$$

The $D_r(s)$ is found in previous equation and $(N(s))/ (D(s))$ are known now $N_r(s)$ can be found. The $N_r(s)$ of the reduced order model $G_r(s)$ in equation (4.14) will be the series expansion of

$$G_r(s) = \frac{N(s)}{D(s)/D_r(s)} = \frac{\sum_{i=0}^{n-1} a_i s^i}{\sum_{i=0}^{n-1} e_i s^i}, \text{ about } s=0 \text{ up to the term of order } s^{r-1} \quad (6.10)$$

This is calculated by modified form of a moment generating algorithm, which uses the Routh recurrence formula to give the third, fifth, seventh, etc., rows as

$$C_0 = \frac{b_0}{e_0} \begin{cases} 2763.2 & 1657.078 \\ 4.19 & 1.15 * 10^{-3} \end{cases}$$

$$C_1 = \frac{a_0}{e_0} \begin{cases} 1657.078 \\ 4.19 \end{cases}$$

Therefore the numerator of reduced order model is obtained as

Put the values of C_0 and C_1 in Eq. 6.11

$$N_{r-1}(s) = C_{r-1}s^{r-1} + C_{r-2}s^{r-2} + \dots + C_1s^1 + C_0 = \sum_{i=0}^{r-1} C_i s^i \quad (6.11)$$

The final reduced order model is obtained by multiplying the gain factor k with the numerator of the reduced order model.

After applying step 2 the numerator values given below

$$N_r(s) = 395.48 * s + 659.47 \quad (6.13)$$

Thus the reduced model is given as $G_r(s)$

$$G_r(s) = \frac{395.24*s+659.47}{s^2+6.616*s+8.25} \quad (6.14)$$

$N(s)$ and $D(s)$ are the numerator and denominator of the system. This is the fourth order system to reduce this system to second order system, routh criteria and factor division is used. Step 1 show routh criteria method is used to reduce the denominator of

the system. Step 2 show factor division methods are used to reduce to the numerator of the system.

By using the factor division method and the routh criteria the fourth order system reduced in the second order and it retains the input output properties same as the higher order system. The step response $G(s)$ and $G_r(s)$ of the both system is same.

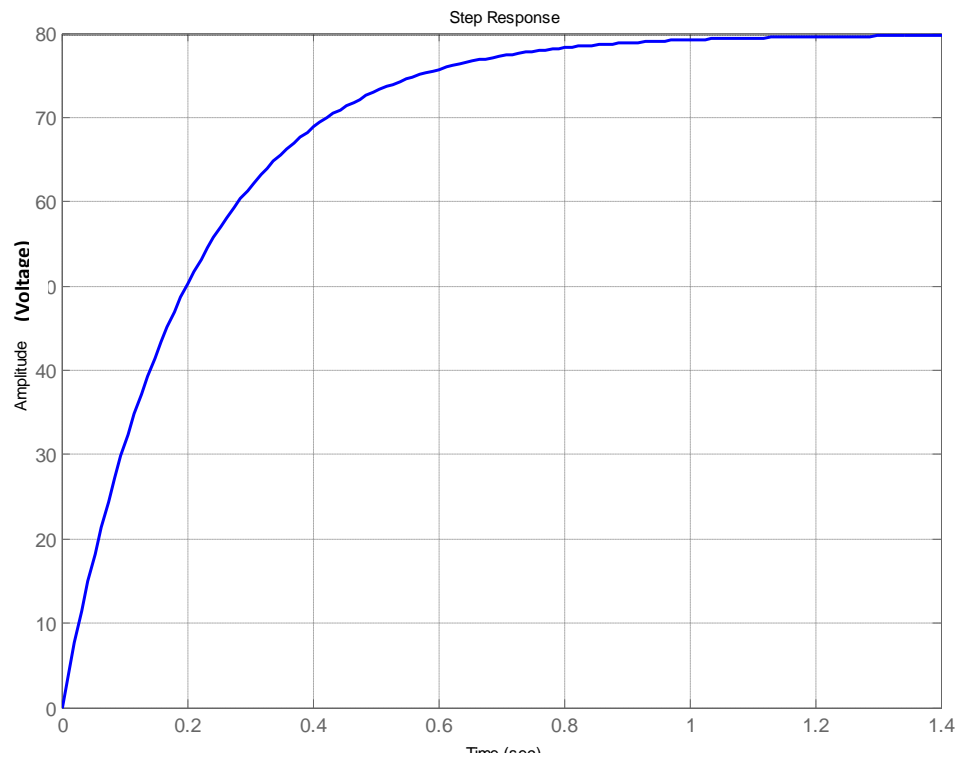


Figure 6.1: original system step response

Figure 6.1 demonstrates the step response of the original transfer function, i.e., transfer function of Permanent magnet synchronous motor.

6.2 Reduced plant transfer function $G_r(s)$

Here reduced plant transfer function is given, there are two term denominator is reduced by the routh approximation method and the numerator is reduced by factor division method. The reduced plant transfer function is given by Eq. (4.20)

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{395.24*s+659.47}{s^2+6.616*s+8.25} \quad (6.15)$$

Here $G_r(s)$ is the plant transfer function

The step response of reduced system is given in fig (6.2)

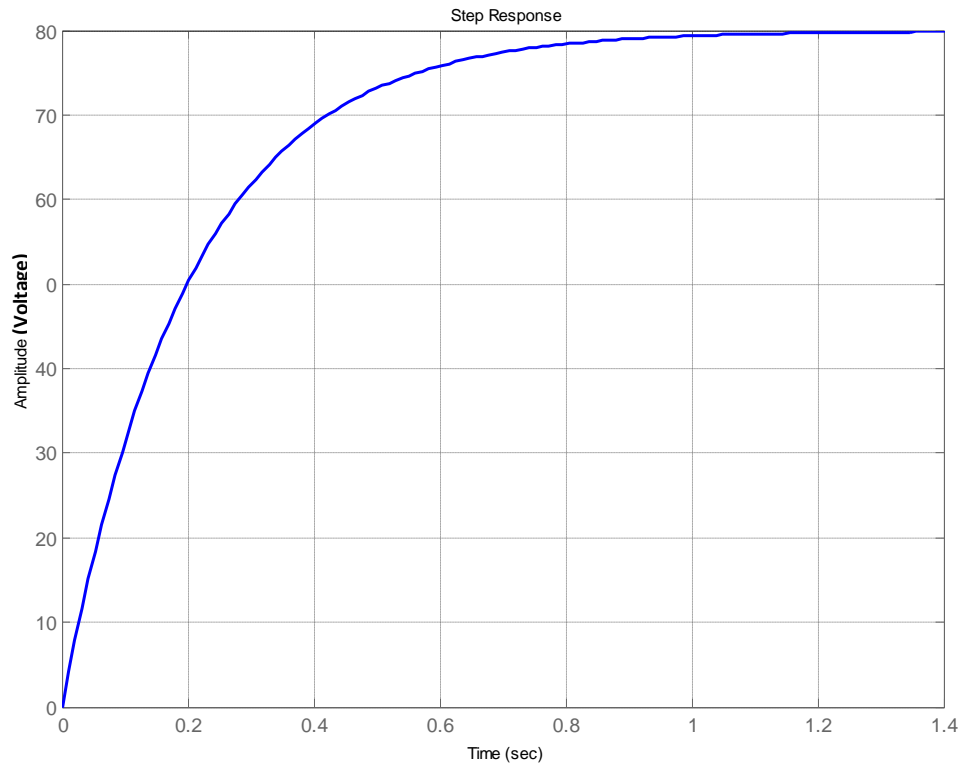


Figure 6.2: Step response of second order system

Figure 6.2 represents the Step response of second order system

It can be inferred from figures 6.1 and 6.2 that both have same originalities, i.e., they have same parameters like rise time, settling time and maximum overshoot.

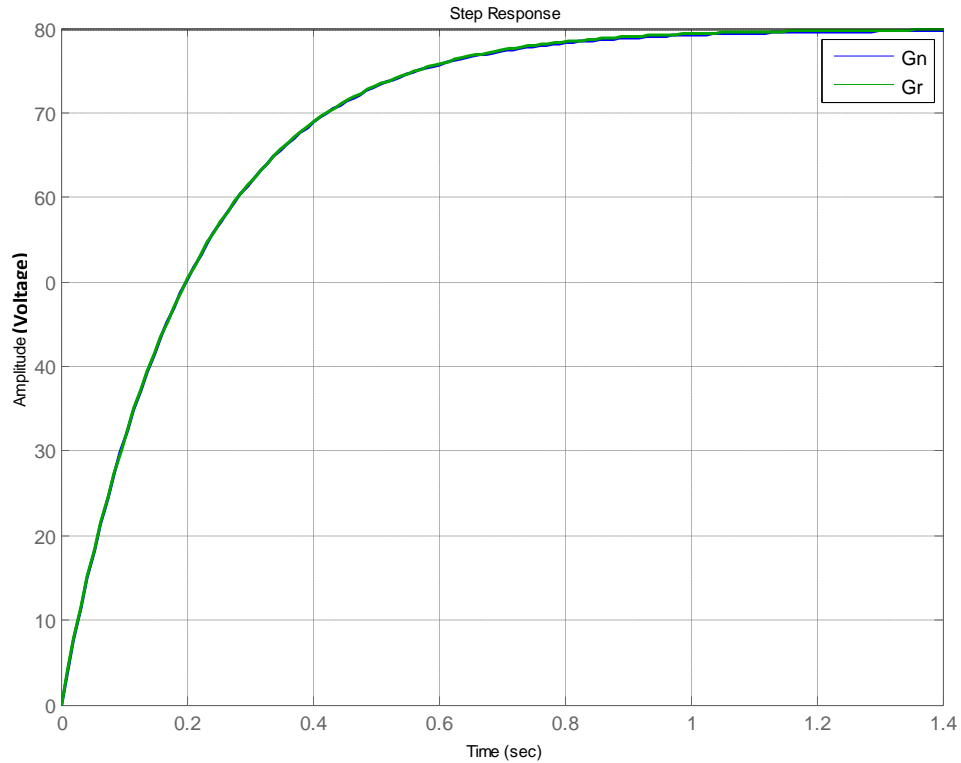


Figure 6.3 step response of original and reduced system

Above figure shows the original system and reduced system step responses. It is clearly shown that both are following the same characteristics. Where G_n is the original system transfer function and the G_r is the reduced order transfer function.

6.3 Comparison table of original system and reduced system

Table 6.2 Comparison table of original system and reduced system

	Rise time (sec)	Setting time (sec)	Max overshoot (%)	ISE
Original system	0.443	0.792	0	
Reduced system (routh criteria)	0.442	0.794	0	0.0045
Reduced system (pade approximation)	0.445	0.788	0	0.013

Table 6.1 shows the step response of the higher order system and the reduced system here it shows the approximately same value of rise time, settling time and maximum overshoot. Here two methods are shown first method is routh criteria and second method is pade approximation. This table it is clearly shown that all parameter are rise time, settling time and maximum overshoot value are same, and the ISE integral square error is small in the routh criteria method.

6.4 Closed loop transfer function of classical full order controller with the original system.

Feedback sensor transfer function is taken from Eq. (5.15)

$$H(s) = \frac{.05}{0.002*s+1} \quad (6.16)$$

Where $G_n(s)$ and $H(s)$ are the plant transfer function and feedback transfer function respectively controller specification is given maximum overshoot should be less than 3%, settling time should be less than 2 sec and the rise time should be less than 3 sec. From the above specification the transfer function given below $G_{ref}(s)$.

$$G_{ref}(s) = \frac{4096}{s^2+32*s+4096} \quad (6.17)$$

The Classical controller $C_f(s)$ is calculated by Eq. 6.5

$$C_f(s) = \frac{G_{ref}(s)}{G_n(s)[1-G_{ref}(s)*H(s)]} \quad (6.18)$$

Where

$C_f(s)$ Full order controller transfer function.

$C_f(s)$ Formula is given in Eq. (6.5) $G_n(s)$ and $H(s)$ values are known so G_{org} values can be calculated by given formula Eq. (6.6)

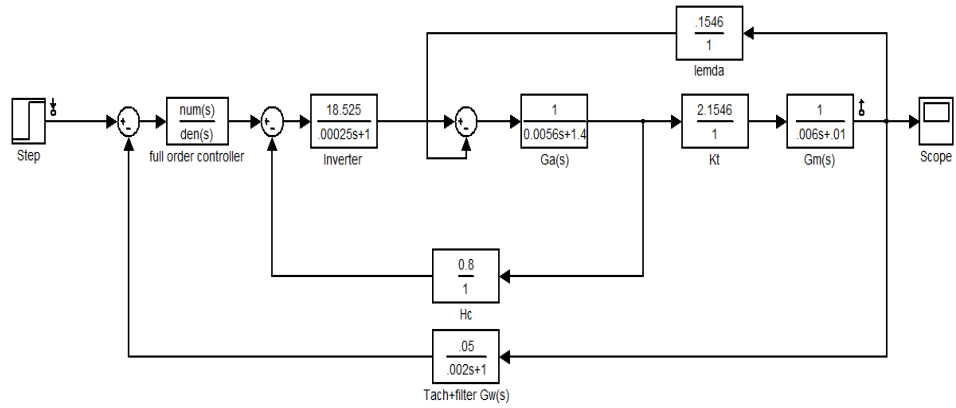


Figure 6.4 Close loop control system with original system

$$G_{org} = \frac{C_f(s) * G_n(s)}{1 + C_f(s) * G_n(s) * H(s)} \quad (6.19)$$

The step response of the original system $G_n(s)$ with full controller $C_f(s)$ is given below:

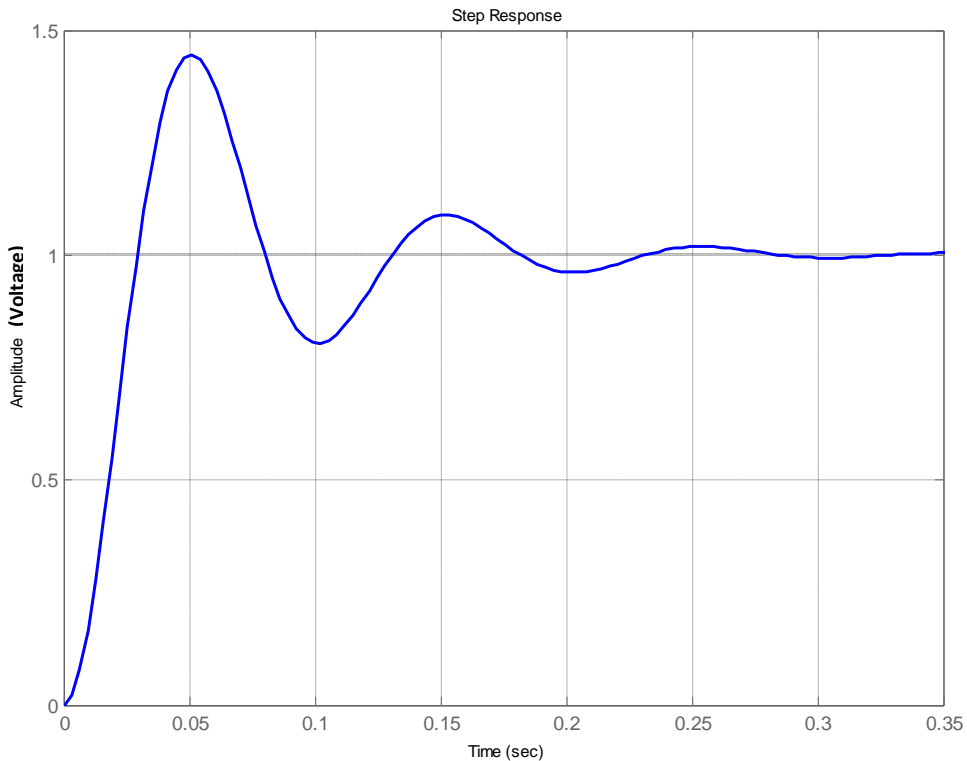


Figure 6.5: Original system with full order controller Step response

Above Figure 6.5 showing the step response of original system with the classical full order controller, the overall close loop response rise time, maximum overshoot, settling time and steady state error are shown in the table 6.3.

6.5 Closed loop transfer function of reduced order controller and reduced system

Here reduced plant transfer function is given, there are two term denominator is reduced by the routh approximation method and the nominator is reduced by factor division method. The reduced plant transfer function is given by Eq. (6.2)

$$G_r(s) = \frac{\check{N}(s)}{\check{D}(s)} = \frac{395.4 * s + 659.47}{s^2 + 6.616 * s + 8.25}$$

The feedback transfer function of the system is given by Eq. (6.3)

$$H(s) = \frac{.005}{0.002 * s + 1}$$

Where $G_r(s)$ and $H(s)$ are the plant transfer function and feedback transfer function respectively

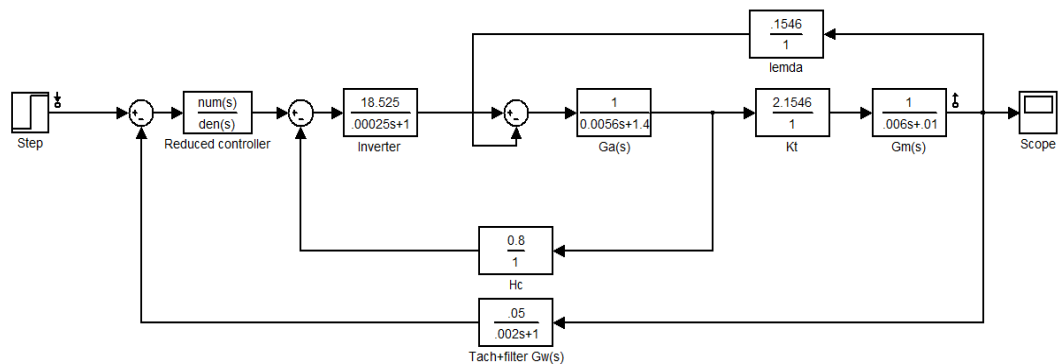


Figure 6.6: Close loop transfer function of reduced system and reduced controller

$G_{ref}(s)$ Is the reference transfer function is derived from the given specification. The reduced order controller $C_r(s)$ is taken from Eq. (3.16)

$$C_r(s) = \frac{G_{ref}(s)}{G_r(s)[1-G_{ref}(s)*H(s)]}$$

Once $C_r(s)$ found then calculate the overall close loop response. The reduced transfer function can be calculated. G_{red} is taken from the Eq. (3.17)

$$G_{red} = \frac{C_r(s)*G_r(s)}{1+C_r(s)*G_r(s)*H(s)}$$

Where G_{red} the overall is closed loop transfer function reduced order controller and reduced order plant transfer function. The step response of the original system $G_r(s)$ with full controller $C_r(s)$ is given in figure 6.7.

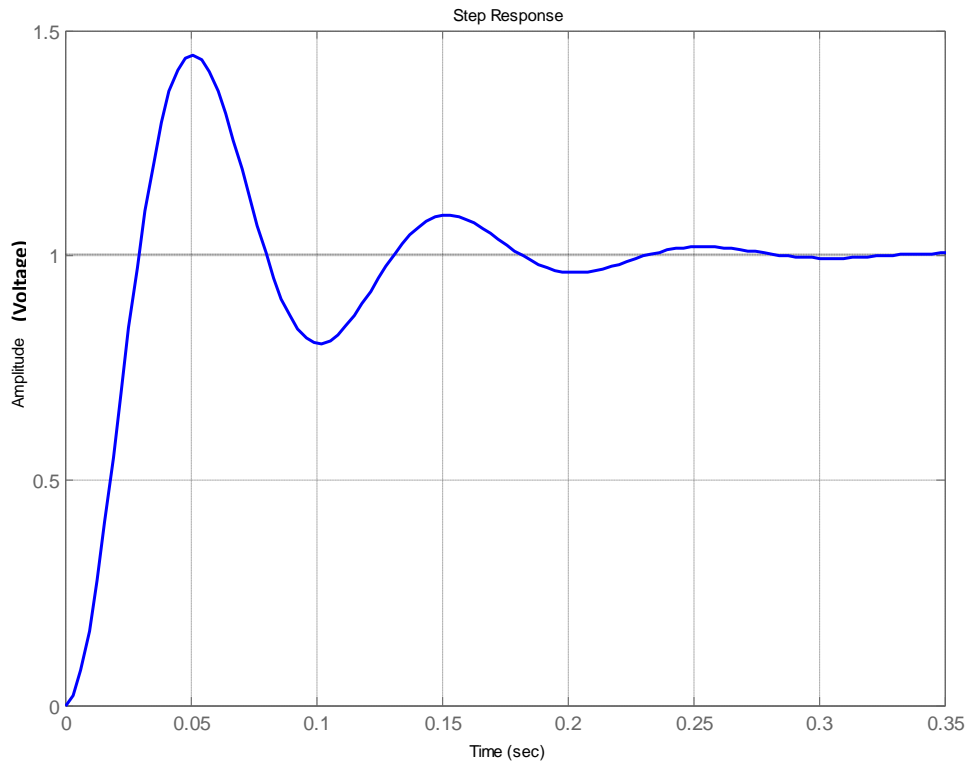


Figure 6.7: Graph of reduced system with reduced controller Step response

Above Figure 6.7 showing the step response of original system with the full order controller, the system response parameter are shown in the graph rise time, maximum overshoot , settling time and steady state error.

6.6 overall close loop response of the reduced controller and original plant

Here closed loop response of the reduced controller and original system and its step response is shown in figure 6.8 it show the same characteristics as the original system. It means that reduced system characteristics are same as the higher order system.

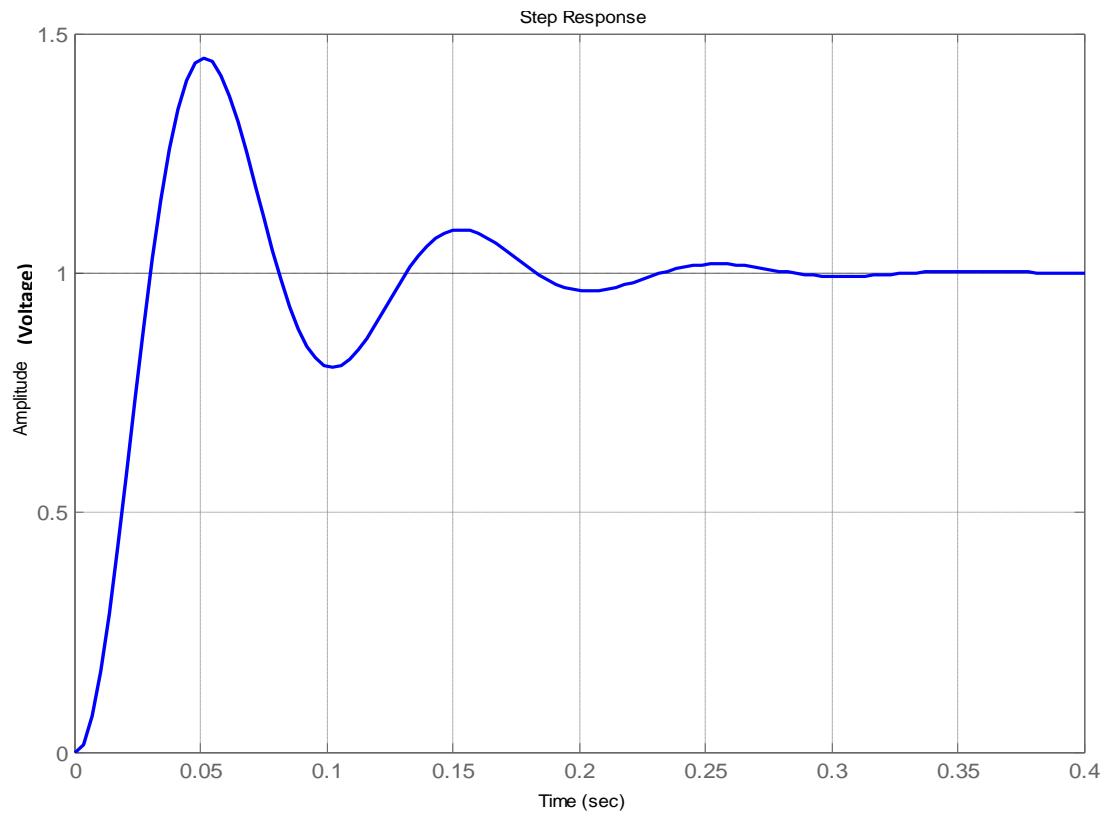


Figure 6.8 Graph of reduced controller and original system step response

Figure 6.9 shows the step response of the reduced controller with the reduced system. It shows the same result as the original system gives.

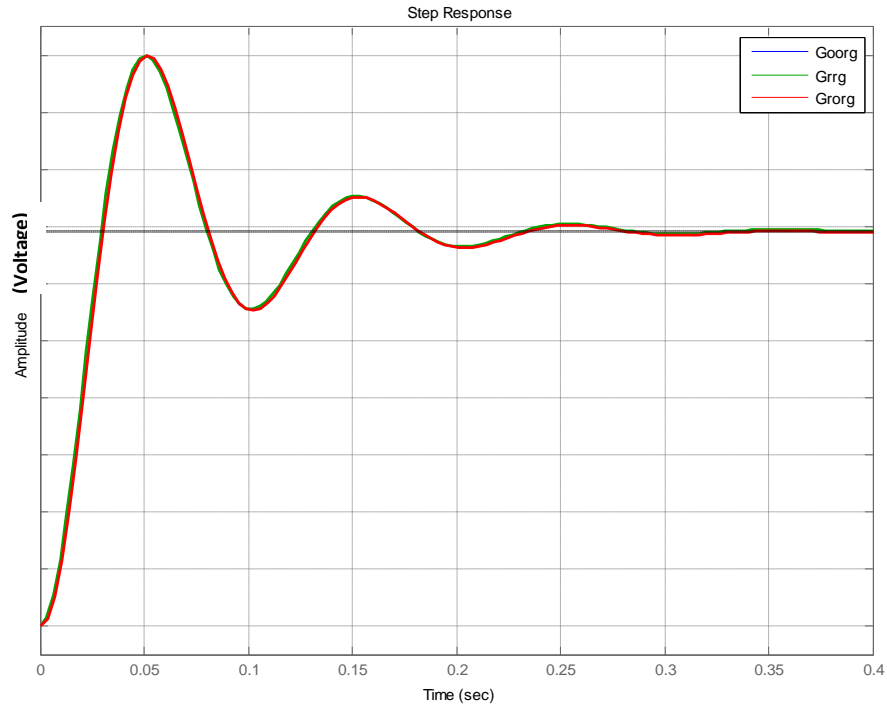


Figure 6.9 Step responses

Figure 6.9 shows all three combination conventional methods step responses in one graph. Here it is clearly shown that all responses are same and giving the same characteristic rise time, maximum overshoot, settling time and steady state error, G_{oorg} shows the full controller with the original system, G_{rrg} shows that reduced system with the reduced controller and G_{rog} shows that reduced controller with original system.

Table 6.3 Step response comparison with original system and reduced system controller

	Rise Time (sec)	Settling Time (sec)	Max. Overshoot (%)
Full control with original system	0.0198	0.2208	44.3968
Reduced controller with reduced system	0.0198	0.2210	44.9288
Original system with reduced controller	0.0198	0.2209	44.3188

6.7 Controller design

In this work aim is to design a speed controller for a permanent magnet synchronous dc motor, the controller specification is given maximum overshoot less than 3%, rise time less than 3 sec, and the settling time is less than 2%. After applying pole zero cancellation method the values of K_p, K_I and K_d values comes. $K_p=29.95, K_i=16.5$ and $K_d=0.003357$

6.8 Closed loop response of higher order system with PID controller.

The closed loop response of the higher order system is given in figure 6.9.

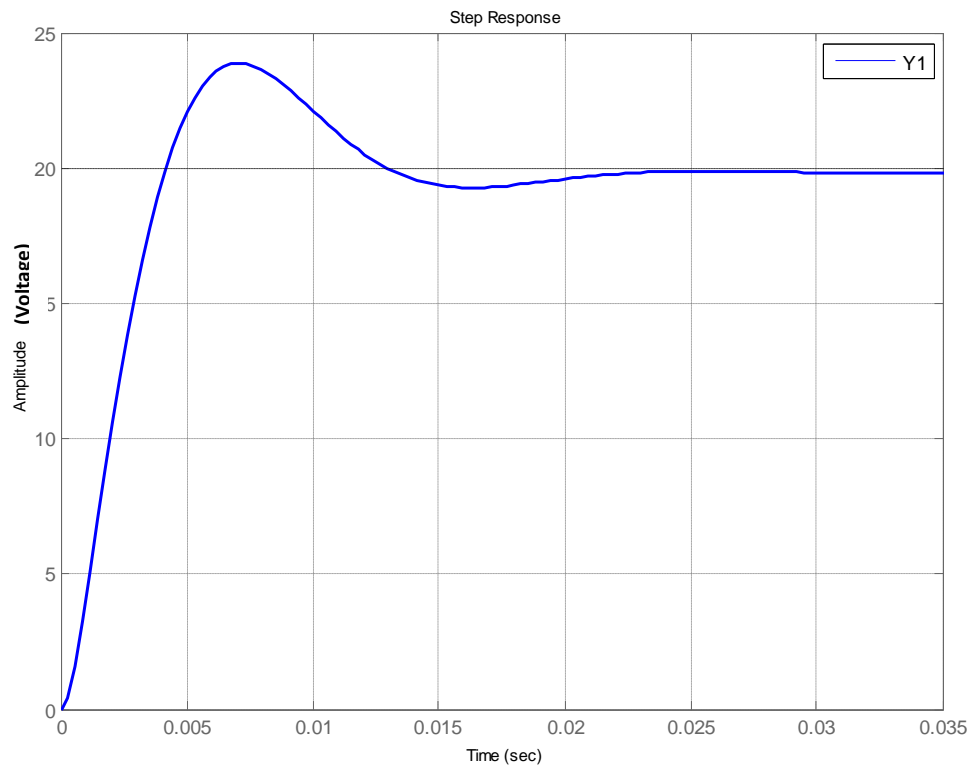


Figure 6.10 closed loop response of the higher order system with controller

Figure 6.10 show the closed loop response of the higher order system with the PID controller in the above figure clearly shown all the parameters values rise time 0.00292 sec settling time .02 sec, maximum overshoot 19.4% and final value 20.

6.9 Closed loop response of lower order system with PID controller

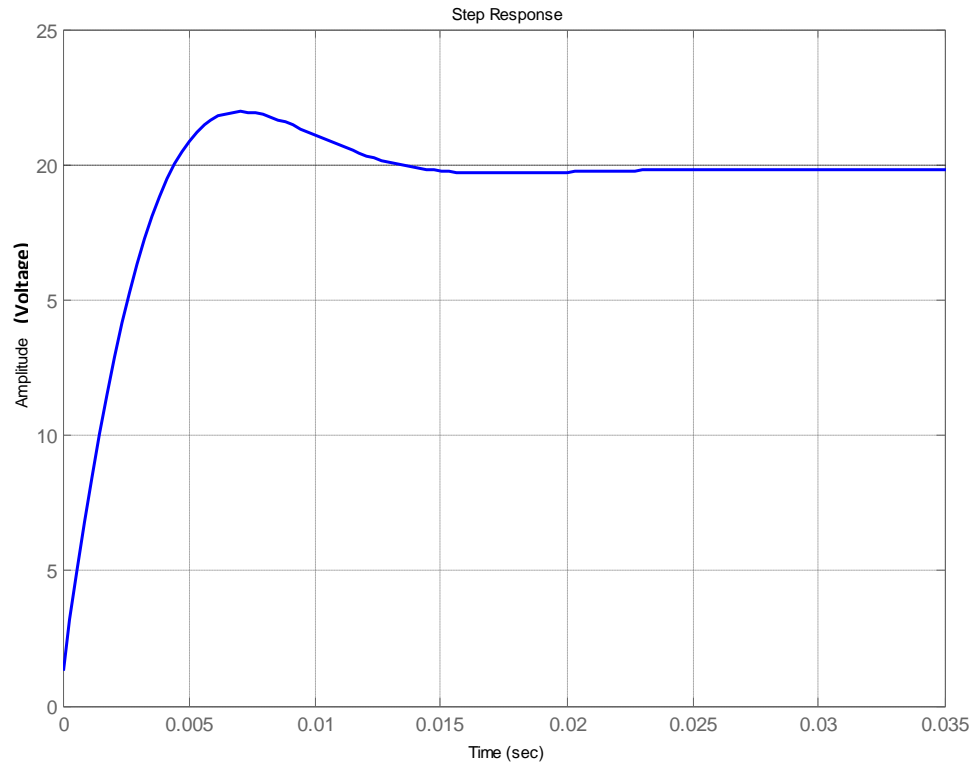


Figure 6.11 Closed loop response of the lower order system with controller

Figure 6.11 shows the closed response of the second order system with the PID controller and its show the same values of the rise time, maximum overshoot and settling time as the original system show the comparison shown in table 6.10. It mean that reduced system preserve the whole characteristics of the original system as the higher order system produces.

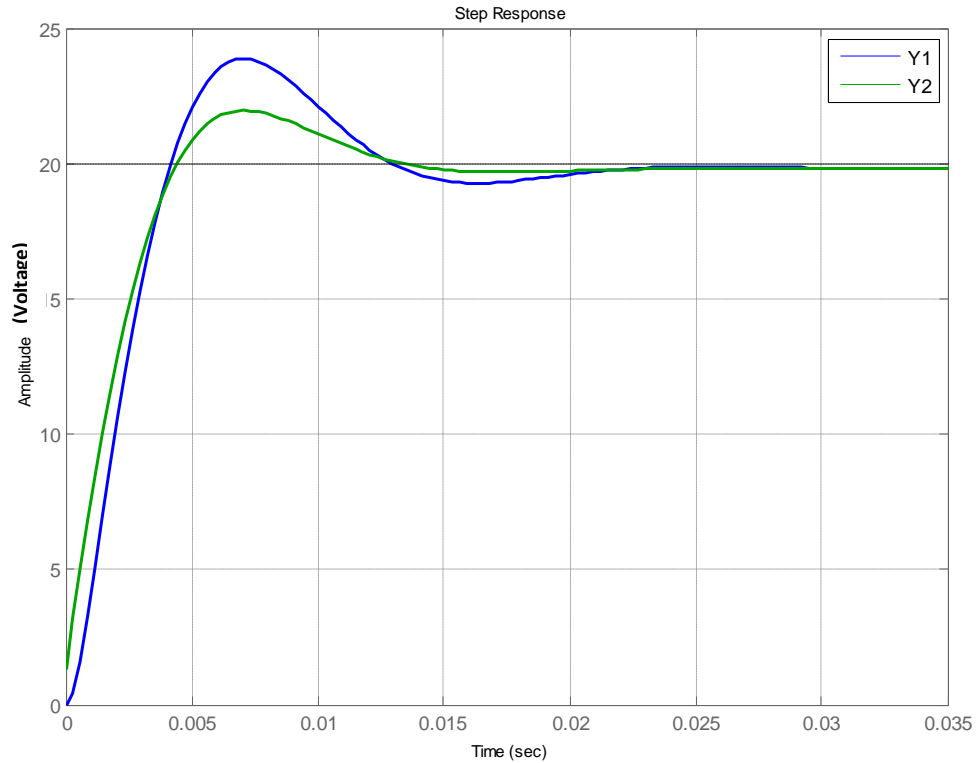


Figure 6.12 step response of original system and reduced system

Figure 6.12 show original systems and reduced system step response with the PID controller and it show the same characteristics rise time, settling time and the steady state error, expect maximum overshoot in the reduced system the overshoot is less than the original system. In the above graph Y1 shows the original system and Y2 shows the lower order system step responses with the PID controller.

6.10 Comparison table between the closed loop system of the higher System and the Lower order system with PID controller

Table 6.4 Comparison table between the closed loop system of the higher system and the lower order system with controller

	Rise time (sec)	Settling time (sec)	Maximum overshoot (%)
Original system with the PID controller	0.00192	0.002	19.4
Reduced system (routh criteria) with PID controller	0.00225	0.0112	9.8
Original system with the PID controller	0.0193	.002	21.8
Reduced system (pade approximation) with PID controller	0.00297	0.0115	11.3

Above table 6.4 shows that the overall close loop response of the system with the PID controller. From this table 6.4 it is clear that the original system and reduced system are showing same rise time, settling time. But the overshoot of the reduced system is less than the original system. There are two method available, one method is pade approximation method to reduced the system and the controller is tuned by genetic algorithm, second method is routh criteria method which is applied in this thesis, and PID controller is tuned by pole zero cancellation method. From the table 6.4 show that rise time, settling time and maximum overshoot values the routh approximation method are less than pade approximation method.

7.1 Conclusion

The permanent magnet synchronous motor has to be reduced from fourth order to second order. As per the previous research, pade approximation had been used which a good advantage of computational simplicity but this study has revolved around the use of factor division to find the numerator of the transfer function, and routh Criteria for the denominator. routh criterion has a number of benefits. This method offers to check the stability without actually solving the characteristic equation. Along with the stability, relative stability can also be computed easily. If the system comes out to be unstable then this criterion gives number of roots of characteristic equation which have positive real part. The study kept these points under consideration for carrying out the research. Afterwards, a conventional method was employed to design the controller. Moreover, a pole-zero cancellation method was used to tune the PID parameters- P, I and D. The outcomes thus obtained were compared with that of the previous studies. Thus, the study concludes to the fact that the rise time, settling time and maximum overshoot is less as compared with the pade approximation methods used.

7.2. Future Scope

Although there are a lot of methods to reduce the higher order complicated system this can be tested on different system or a new method can be developed.

CHAPTER 8

CHECK FOR ORIGINALITY

The dissertation report presented here has been checked for its originality using online plagiarism checker “Paper Rater”, available at http://www.paperrater.com/plagiarism_checker. Various theoretical concepts are explained as per the references from different technical books which I studied during my engineering graduation and post graduation studies. Thanks to all those who are already present in my references text.

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