

STUDY OF INDUSTRIAL SYSTEMS USING SUPPLEMENTARY VARIABLE TECHNIQUE

Dissertation submitted in partial fulfilment of the requirement for

the award of the degree of

Master of Science

in

Mathematics and Computing

Submitted by

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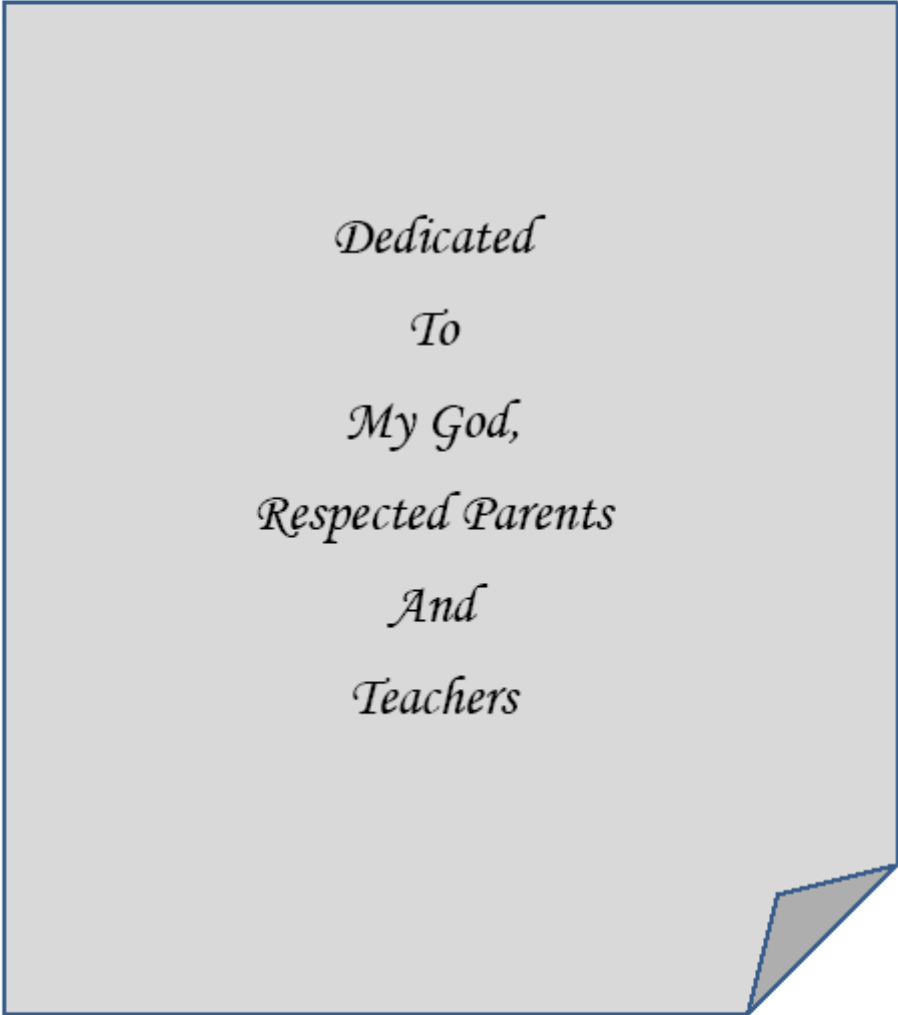


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July, 2012.



*Dedicated
To
My God,
Respected Parents
And
Teachers*

CERTIFICATE

I hereby certify that the work which is being presented in this dissertation entitled "STUDY OF INDUSTRIAL SYSTEMS USING SUPPLYMENTRY VARIABLE TECHNIQUE" which is being submitted for the award of degree of Master of Sciences, School of Mathematics and Computer Applications (SMCA),Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr.Jitender Kumar.

The matter presented in this dissertation has not been submitted for the award of any other degree of this or any other university.


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ACKNOWLEDGEMENT

First of all, I would like to thank the almighty for granting perseverance. I would like to express my gratitude to Dr. Jitender Kumar, School of Mathematics and Computer Applications, Thapar University, Patiala, for their patient guidance and support throughout this work. I was truly very fortunate to have the opportunity to work under him as a student. I take this opportunity to express my sincere thanks to Prof. S.S. Bhatia, Head SMCA, Thapar University Patiala.

My special thanks to my friends who devoted their valuable time and helped me in the completion of this work. I thank all those who have contributed directly or indirectly to this work.

Lastly, and more importantly, I would like to thank my parents for their years of unyielding love and encouragement. They have always wanted the best for me and I admire my parent's determination.

July 13, 2012


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ABSTRACT

The whole range of our work reported in this dissertation is covered into four chapters which are described as:

Chapter-1: This Chapter is introductory in nature and contains the basic concepts including reliability engineering, reliability theory and modeling, system configurations, Lagrange's Method and Runge-kutta 4th order Method.

Chapter-2:

This Chapter discusses the comparison between the availability of a pipe manufacturing industry when the Sub-Systems are subjected to simultaneously and independent failure. The failure rates of the subsystem are constant and repair rates are variable. Availability analysis of a system can benefit the industry in terms of higher productivity and low maintenance cost. It is possible to improve the availability of the of the plant with proper maintenance planning and monitoring. Reliability analysis helps us to obtain the necessary information about the control of various parameters.

Chapter-3:

The main intension of this Chapter is to develop and analyze a reliability model for polytube industry using supplementary variable technique. A mathematical model for the system is given in the form of differential difference equations and solved for obtaining steady state availability. The effect of various subsystems on the availability is also studied taking numerical data.

Chapter-4:

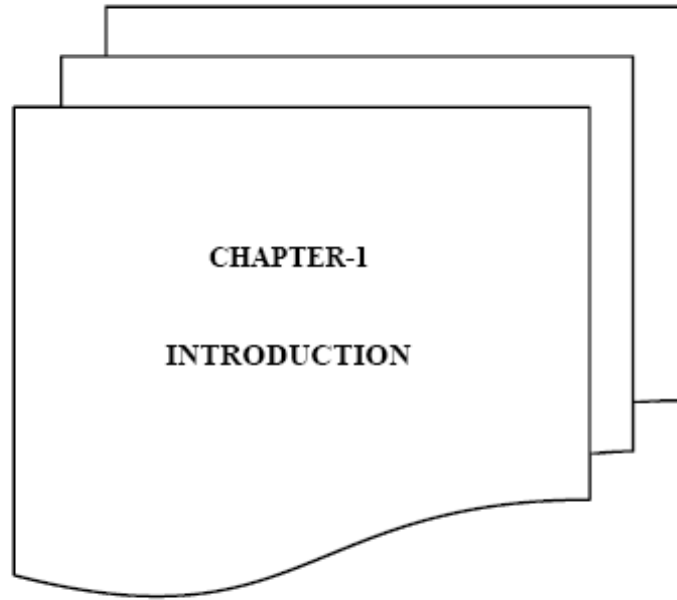
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NOTATIONS

A, B, C, D	indicate that the sub-system is working in full capacity
$\bar{C}, \bar{D}$	indicate the reduced state of the sub-system C and D
a, b, c, d	indicate the failed state of the sub-system
α_i (y) ($i = 1, \dots, 4$)	failure rate of the sub-system A, B, C and D, respectively
$\phi(x), \psi(x), \mu(x)$, and $\sigma(x)$	general repair rates of A, B, C and D, respectively
P_0 (t)	the system is working in full capacity



CHAPTER-1

INTRODUCTION

CHAPTER-1

INTRODUCTION AND LITERATURE SURVEY

The fast pace of industrialization has changed the nature of consumers. Now they are more rational and demanding. Every consumer wants maximum satisfaction in his limited income. Industries have to maintain growth momentum and produce the goods which suit the customers. However, suitability counts on good performance of system under given operating conditions for a given period of time. Failures of the systems cannot be predicted every time. It is inevitable due to various causes such as change in operating conditions, changing in temperature, fluctuation of voltage, vibration, corrosive atmosphere, etc. All these causes don't deny the fact that system can't be made reliable. A system can be made desirable and reliable by providing proper repair facilities, proper selection of components and parts with minimum failure rates, replacement of unit within time, introduction of redundancy, etc. These precautions maximize the reliability of every system. Thus, through our efforts we can produce maximum reliability and minimize the cost of production. First, let us discuss about reliability and reliability concepts.

“Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating condition” This definition brings into focus four important factors, namely,

- First, reliability is a probability. This means that there is always some chance of failure. Reliability engineering is concerned with meeting the specified probability of success, at a specified statistical confidence level.
- Second, reliability is predicated on “intended function:” Generally, this is taken to mean operation without failure.
- However, even if no individual part of the system fails, but the system as a whole does not do what was intended, then it is still charged against the system reliability. The system requirement is the criterion against which reliability is measured.
- Third reliability applies to a specified period of time. In practical terms this means that a system has a specified chance that it will operate without failure before time t . Reliability engineering ensures that components and materials will meet the requirements during the specified time. Units other than time may sometimes be used. The automotive industry might specify reliability in terms of miles; the military might specify reliability of a gun for a certain number of rounds fired. A piece of mechanical equipment may have a reliability rating value in terms of cycles of use.
- Fourth, reliability is restricted to operation under stated conditions. This constraint is necessary because it is impossible to design a system for unlimited conditions. A Mars Rover will have

different specified conditions than the family car. The operating environment must be addressed during design and testing.

Reliability design begins with the development of a model. Reliability models use block diagrams and fault trees to provide a graphical means of evaluating the relationships between different parts of the system. These models incorporate predictions based on parts-counts failure rates taken from historical data. While the predictions are often not accurate in an absolute sense, they are valuable to assess relative differences in design alternatives.

One of the most important design techniques is redundancy. This means that if one part of the system fails, there is an alternate success path, such as a backup system. An automobile brake light might use two light bulbs. If one bulb fails, the brake still operates using the other bulb. Redundancy significantly increases system reliability, and is often the only viable means of doing so. However, redundancy is difficult and expensive, and is therefore limited to critical parts of the system. Another design technique, physics of failure, relies on understanding the physical processes of stress, strength and failure at a very detailed level. Then the material or component can be re-designed to reduce the probability of failure. Another common design technique is component derating: selecting components whose tolerance significantly exceeds the expected stress, as using a heavier gauge wire that exceeds the normal specification for the expected electrical current.

Redundancy is the provision of alternative means or parallel paths in a system for accomplishing a given task such that all means must fail before causing a system failure. It has been noted earlier that any such additional means will increase the system reliability and means life. In a system of complex nature, redundancy can be applied at various levels. The various approaches for introducing redundancy in the system are:

UNIT REDUNDANCY

The simplest and most straight forward approach is to provide a duplicate path for the entire system itself. This is known as system or unit redundancy.

COMPONENT REDUNDANCY

Another approach is to provide redundant paths for each component individually. This is called component redundancy.

MIXED REDUNDANCY

The last approach is to appropriately mix the above technique depending upon the system configuration and reliability requirements. This approach is known as mixed redundancy.

WEAKEST REDUDANCY

The last approach is that the weak components should be identified and strengthened for reliability. This approach is useful when we consider reliability and cost-optimization problems.

We classify redundancy as active and inactive if the redundant system are continuously energized and are sharing a portion of the load, there is active redundancy. If the redundant system do not perform any function unless the primary system fails, it is an inactive redundancy.

We further classify the standby redundancy according to failure characteristics as follows:

- **Hot standby**

Standby components have the same failure rates as the primary component. Since the failure rate of one component is not affected by the other components, the hot standby redundancy consists of statistically independent components.

- **Cold standby**

Standby components do not fail when they are in standby. The failure of the primary component result in the standby component being a primary component and in its failure rate becoming non-zero

- **Warm standby**

A standby component has a smaller failure rate than the primary component but is greater than zero.

Components versus Unit Redundancy

It is easy to duplicate at the unit level rather than at the component level. Redundancy at the component level is better than redundancy at the unit level as far as reliability is concerned.

Now we explain some system configurations which are generally faced when we come across to complex systems.

MARKOV METHOD

This method is to estimate the time-dependent reliability of the system. The method is valid when both the failure and repair rates are constant. When these rates are time-dependent, the Markov process breaks down, except in some special cases. If the probability law of its future state of existence depends only upon the state it is in and not on how the system arrived in that state, the system state behavior is described by a process called a Markov process, i.e. in a Markov process given the 'present' of the process, the future is independent of its past.

Consider a repairable system consisting of two units each with constant failure rate λ and constant repair rate μ . Assume that initially both units are operable and when one unit operates the other unit is on cold standby. As soon as the working unit fails, the standby unit is immediately switched on by a perfect switch and sensing device. If failure occurs when there is no standby unit, then both units are in failed state. At any time, the system is in any of three states: operable state 0 when both units are good with one operating and the other in standby, operable state 1 when one unit is under repair and the other is operating, state 2 when both the units are failed. Let $P_i(t)$, $i=0,1,2$ be the probability that the system is in i^{th} state at time t , then the probability $P_i(t+\Delta t)$, $i=0,1,2$ that the system is in i^{th} state at time $t+\Delta t$ is as follows:

$$P_0(t+\Delta t) = P_0(t) (1-\lambda\Delta t) + P_1(t)\mu\Delta t + O(\Delta t)$$

$$P_1(t+\Delta t) = P_1(t) (1-\lambda\Delta t) (1-\mu\Delta t) + P_0(t) \lambda\Delta t + P_2(t) \mu\Delta t$$

$$P_2(t+\Delta t) = P_2(t) (1-\mu\Delta t) + P_1(t) \lambda\Delta t$$

Taking the limit $\Delta t \rightarrow 0$, the equations become

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$$

$$P_1'(t) = -\lambda P_1(t) + \lambda P_0(t) + \mu P_2(t)$$

$$P_2'(t) = -\mu P_2(t) + \lambda P_1(t)$$

These equations can be solved either using Lagrange's method or by any other method.

SUPPLYMENTARY VARIABLE TECHNIQUE

When the repair rate or failure rate or both are time-dependent, the system loses its Markov character. In this situation, the future event will not depend on the present only (like Markov events) but will depend on the past also. These events are known as non-Markovian events. By introducing a new variable, called supplementary variable, the non Markovian nature of the events is changed to Markovian.

As an example, consider the system consisting of two units discussed in Markov method. Now the system is of non Markovian character. Let $\mu(x)\Delta t$ = the probability that a failed unit is repaired in the interval $(x, x+\Delta x)$ given the elapsed repair time is x . Define $P_i(x, t) dx$ = the probability that the system is in state i at time t and the system stays there between x and $x+\Delta x$ time units $i=1,2$. Introducing this supplementary variable x , the character of the system becomes Markovian, Arguing as in example quoted in previous section, We have the following state equations:

$$P_0(t+\Delta t) = P_0(t) (1-\lambda\Delta t) + \int_0^\infty P_1(x, t) \mu(x) dx \Delta t$$

$$P_1(x+\Delta x, t+\Delta t) = P_1(x, t) (1-\mu(x)\Delta x) (1-\lambda\Delta t) + P_0(t) \lambda\Delta t$$

$$P_2(x+\Delta x, t+\Delta t) = P_2(x, t) (1-\mu(x)\Delta x) + P_1(x, t) \lambda\Delta t$$

So, in view of common problems pertaining to operation of a system, the issue of reliability has invited a good number of scholars of various disciplines. In the late 1950's the subject of reliability is debated vehemently by eminent researchers belonging to the faculty of science and technology.. However, some of them have concentrated on examining the real functioning of the industrial systems with focus on element of reliability. The study of reliability theory has its origin in the discussion of replacement policies of many simple automobile engineering systems.

Reliability / Availability/ Maintainability (RAM) Engineering can benefit the industry in terms of higher productivity and lower maintenance costs. RAM analysis is a powerful tool which utilizes failure information from system components in order to develop probability distributions that the system will be able to perform its intended function.

Reliability theory encompasses issues such as:

- Reliability engineering
- Reliability modeling
- Reliability management
- Reliability prediction, analysis and optimization

Reliability engineering deals with the interdisciplinary use of probability, statistics and stochastic modeling, combined with engineering insights into the design and the scientific understanding of the failure mechanism, to study the various aspects of reliability. Frequently, it is desirable to understand and be able to predict the overall system failure characteristics for any given configuration.

Reliability modeling deals with model building to obtain solutions to problems in predicting, estimating and optimizing the survival or performance of an unreliable system, the impact of the unreliability, and actions to mitigate this impact.

Reliability management deals with the various management issues in the context of managing the design, manufacture and/or operation of reliable products and systems. The emphasis is on the business viewpoint, as unreliability has consequences in cost; time wasted, and in certain cases the welfare or an individual or the security of a nation.

1.1 THE DEVELOPMENT OF RELIABILITY ENGINEERING

The events such as the loss of space shuttle Columbia and Challenger, the Three-Mile Island Nuclear accident, the Chernobyl nuclear accident, and the Chemical spills at Bhopal, India are prime examples of complex system failures. As a result, reliability has emerged as one of the vital ingredient in system planning, design, development and operational phase. 1940-50 was the period of quality control and independent quality control departments were set up in many industries throughout the world. In Korean War of 1950s equipment produced under strict quality control, and deployed in Korea did not function to expectations. This experience led to the birth of reliability engineering in U.S. Defence Departments during 1950s. The increasing complexity of military electronic systems generating higher failure rates, which resulted in greatly reduced availability and increased operational costs. U.S. Department of Defense and Electronics Industry, jointly set up the Advisory Group of Reliability of Electronic Equipment (AGREE) IN 1952. This group brought out a report which became the standard testing procedure for new equipment to be tested to know the weak areas in design before production commencement. The Department of Defence (DOD) re-issued the AGREE Report as U.S. Military Standard (MIL-STD) 781, entitled “Reliability Qualification and Production Approval Tests”. AGREE concepts were also adopted by NASA. In 1965 DOD issued MIL-STD-883 entitled “Reliability Programs

for Systems and Equipment”. In 1981 in U.K., Defence Standard 00-40, “Management of Reliability and Maintainability” was issued. The British Standards Institution issued BS-5760- Guide on reliability of systems and components.

1.2RELIABILITY ENGINEERING AND MANAGEMENT TASKS

Reliability engineers perform a wide variety of special management and engineering tasks to ensure that sufficient attention is given to details that will affect the reliability of a given system. The tasks for production of particular system include the following specific reliability tasks:

- Participate in the planning stages to ensure that resources for a good reliability engineering program are being allocated.
- Prepare a reliability program plan, that is, list the needed reliability tasks and for each give the depth to which it should be performed, when it should be performed, who will use the results, and resources (time, people, money) that the task will use.
- Budget the allowable system failures down to the component level.
- Analyze a proposed design for potential of alternative designs.
- Evaluate the reliability potential of alternative designs.
- Ensure that all components in a design will actually behave as the designer anticipates that they will behave, and will have suitably long lives.
- Formulate and run tests on components, subsystems, and the system itself.
- Provide information to designers on how to improve both the life of a system and its ease of maintenance
- Investigate user complaints and field failures. Set up programs to ensure that information about field failures is timely, accurate, well organized, and used by those who can do something with it.

The industries:

- The chemical and petro-chemical refining industries.
- Power: generation, transmission and distribution systems.
- Communication networks and equipment
- Aviation and defence
- Manufacturing
- Mining

The Applications:

- Evaluate capacity of new facilities, expansions of existing plants
- Plan capital replacements
- Set budgets
- Assess warranty periods
- Develop start up maintenance plans
- Optimizing the setting of preventive maintenance intervals
- Optimizing intervals for condition monitoring programs
- Optimizing intervals for lubrication routines
- Provide audit trail for regulators
- Evaluate alternative technologies

The intended achievable aims of reliability improvement are:

- Better performance
- Lesser maintenance cost
- Convenient safety hazards
- Minimum risk factors
- Minimum operator errors
- Maximum availability of the system /subsystems/components
- Critical ranking of equipment.

1.3 AVAILABILITY

Availability is a performance criterion for repairable systems that accounts for both the reliability and maintainability aspects of a system. It is defined as the probability that the system is operating properly when it is required for use. That is, availability is the probability that a system is not failed or undergoing a repair action when it needs to be used. The numerical value of availability is expressed as a probability from 0 to 1. Availability calculations take into accounts both the failures and repairs of the system. For example, if a lamp has 99.9% availability, there will be one time out of a thousand that someone needs to use the lamp but it is non-operational because the switch is broken, or it is waiting for the replacement of a light bulb, etc.

AVAILABILITY CLASSIFICATION

The definition of availability is somewhat flexible, depending on what types of downtimes are considered in the analysis. As a result, there are a number of different classifications of availability.

- **Point (Instantaneous) Availability**
- **Average Up-Time Availability (Mean Availability)**
- **Steady State Availability**
- **Operational Availability**
- **Point Availability**

Point, or instantaneous, availability is the probability that a system (or component) will be operational at any random time, t . This is very similar to the reliability function in that it gives a probability that a system will function at the given time, t . Unlike reliability, the instantaneous availability measure incorporates maintainability information. At any given time t , the system will be operational if the following conditions are met:

1. It functioned properly during time t with probability $R(t)$, or,
2. It functioned properly since the last repair at time u , $0 < u < t$, with probability:

$$\int_0^t R(t-u)m(u)du$$

Average Up-Time (Mean) Availability

The mean availability is the proportion of time during a mission or time-period that the system is available for use. It represents the mean value of the instantaneous availability function over the period $(0, T)$:

Steady State Availability

The steady state availability of the system is the limit of the instantaneous availability function as time approaches infinity. The instantaneous availability function approaches the steady state value very closely at time approximate to four times the MTBF:

Operational Availability

Operational availability is a measure of availability, which includes all experienced sources of downtime, such as administrative downtime, logistic downtime, etc. The equation for operational availability is :

$$A_o = \frac{\text{Uptime}}{\text{Operating Cycle}}$$

Where the operating cycle is the overall time period of operation being investigated, and uptime is the total time the system was functioning during the operating cycle.

The operational availability is the availability that the customer actually experiences. It is essentially the a posteriori availability based on actual events that happened to the system. The previous availability definitions are a priori estimates based on models of the system failure and downtime distributions. In many cases, operational availability cannot be controlled by the manufacturer due to variation in location, resources, and other factors that are the sole province of the end user of the product.

MAINTAINABILITY

Maintainability is a characteristic of a design, installation, and operation, usually expressed as the probability that a machine can be retained in, or restored to, specified operable condition within a specified interval of time when maintenance is required. In other words maintainability measures the ease and speed with which a system can be restored to operational status after a failure occurs. This refers to the aspects of a product that increase its serviceability and repairability, increase the cost effectiveness of maintenance, and ensure that the product meets the requirements for its intended use.

REPAIRABILITY

This is the probability that a failed product will be repaired to its operational state within a given active repair time.

MAINTENANCE FOR REPAIRABLE SYSTEMS

Maintenance is defined as any action that restores failed units to an operational condition or retains non-failed units in an operational condition. For repairable systems, maintenance plays a vital role in the life of a system. It affects its reliability, its availability, its downtime, its cost of operation, etc. Maintenance actions can be divided into two types : corrective maintenance and preventive maintenance.

CORRECTIVE MAINTENANCE

Corrective maintenance is the action(s) taken so that a failed system is restored to operational status. This usually involves replacing or repairing a component or subsystem that is responsible for the failure of the overall system. Such maintenance is performed at unpredictable intervals, since a component failure time is not known a priori. The objective of corrective maintenance is to restore the system to satisfactory operative state within the shortest possible time. Corrective maintenance is typically carried out in three steps:

Diagnosis of the problem: The maintenance technician must take time to locate the failed parts or otherwise satisfactorily assess the cause of the system failure.

Repair and/or replacement of faulty components: Once the cause of system failure has been determined, actions must be taken into address the cause. Usually by replacing or repairing the components that caused the system to fail.

Verification of the repair action: Once the component in question have been repair or replaced, the maintenance technician must verify that the system is again successfully operating.

Lagrange's Method

Lagrange's equations offer a systematic way to formulate the equations of motion of a mechanical system or a (flexible) structural system with multiple degrees of freedom. A scalar approach is obtained by expressing the scalar quantities of kinetic and potential energy in terms of generalized coordinates. The treatment of Lagrange's method is only brief in this document, but provides enough background information for the vibration analysis of the flexible structure used in the MAE laboratory course.

Runge-Kutta Method

The Runge-Kutta^{4th} order method treats every step in a sequence of steps in identical manner. Prior behavior of a solution is not used in its propagation. This is mathematically proper, since any point along the trajectory of an ordinary differential equation can serve as an initial point. The fact that all steps are treated identically also makes it easy to incorporate Runge-Kutta into relatively simple "driver" schemes. We consider adaptive step size control, discussed in the next section, an essential for serious computing. Occasionally, however, you just want to tabulate a function at equally spaced intervals, and without particularly high accuracy. In the most common case, you want to produce a graph of the function. Then all you need may be a simple driver program that goes from an initial x 's to a final x f_{in} a specified number of steps. To check accuracy, double the number of steps, repeats the integration, and compares results. This approach surely does not minimize computer time, and it can fail for problems whose nature requires a variable step size, but it may well minimize user effort. On small problems, this may be the paramount consideration.

1.4 LITERATURE SURVEY

In the recent past, quite a good number of studies have been carried out by researchers [1]-[25] in the field of reliability, and several methods have been proposed for reliability analysis of productive systems. The goal of maximum production and long run availability under the given operative condition can be achieved by making the system failure free as far as possible by proper maintenance, planning and control. Due to increasing complexity of modern engineering systems, which involve high risks, the concept of reliability has become a very important factor in the overall system design. While dealing with reliability-based design of machines and structures, we can study the relative importance of mechanical

and structural failures from the point of view of loss of human lives. Reliability analysis of such a system helps us to obtain the necessary information about the control of various parameters. Most of the authors discussed the system possessing Markovian properties. The system having non-Markovian property can be converted into a system having Markovian nature by introducing a new variable called supplementary variable.

Initially, Cox [1] used supplementary variable in analyzing non-markovian system and presented a systematic solutions of reliability and availability of that system using supplementary variable technique. Gaver [2] studied a parallel redundant system with constant state dependent hazard rates and arbitrary repair rates. Kumar et al. [3] discussed the availability of the feeding system in the sugar industry. Since then several authors have studied the reliability of the various systems using supplementary variable technique. Singh and Dayal [4] used supplementary variable technique for problem formulation. Subsequently, Kumar et al. [5] used it to study the maintenance planning of pulping system, screening system in paper industry, and availability of crystallization system in paper industry. Alfa and Srinivada Rao [6] discussed supplementary variable technique in stochastic models. Gupta [7] studied availability analysis assuming constant failure rates but repair rates were variable.

The mechanical systems have attracted the attention of researchers in the area of reliability theory. Several authors, such as Dhillon and Natesan [8], Mahajan and Singh [9,10], Singh and Mahajan [11], Gupta et al. [12], Kiureghian and Ditlevson [13], etc. have discussed the problems of reliability and availability of the various manufacturing systems using different approaches. However, the behavior analysis of reliability and availability of serial processes of plastic pipe manufacturing plant was carried out by Gupta et al. [14] assuming constant failure and repair rates. Wang et al. [15] discussed the comparison of reliability and the availability between four systems with warm standby components and standby switching failures. Garg[16] analyzed the reliability and availability of the complex systems. Pac and Mahmet [17] discussed the role of repair strategy in warranty cost minimization. Sharma et al. [18] studied the stochastic behavior of sugar plant with 2-out-of-3: boilers and pre-emptive resume repair. He provided the Laplace transforms solution of the reliability, availability and M.T.T.F of the system. Garg et al. [19] discussed the maintenance scheduling of a repairable block board manufacturing system. Haung and Xu [20] presented a method to study the life time reliability or load sharing redundant systems with arbitrary failure distribution. Gottumullala et al. [21] derived some interesting reliability characteristics for high performance computing application. Yaun and Xu[22] discussed a deteriorating system with its repair man having multiple vacations using the geometric process and the supplementary variable technique and some important reliability indexes are derived. Wang and Chiu [23] discussed the cost benefit analysis of availability systems with warm standby units and imperfect coverage. Gupta [24] studied markov model and availability analysis of a chemical production system by taking failure rates

of the different sub-components of the system are constant while their repair rates are arbitrary distributed. Ke and Chu [25] studied the comparative analysis of availability for redundant repairable system.

The Poly tube industry has not been discussed so far which also plays an important role in our daily life. The assumption of constant failure and repair rates may not be practical in any industry. Keeping this in view, in the present study, we have considered four-unit Poly tube industry subject to variable failure and repair rates of the subsystems and discussed the reliability modeling of Poly tube industry using supplementary variable technique. An attempt has also been made to discuss the availability of this plant by taking constant failure and repair rates.

There has been a phenomenal growth in reliability literature during the past decades. Maintain systems have been studied with a view to obtain performance parameters of interest, such as reliability, availability, mean time to system failure etc. Availability analysis of a system can benefit the industry in terms of higher productivity and low maintenance cost. It is possible to improve the availability of the plant with proper maintenance planning and monitoring. Reliability analysis helps us to obtain the necessary information about the control of various parameters. The poly tube industry involves many processes i.e. Mixture, Extruder, Die and Cutter. The Die and Cutter machine can also work in reduced state. The process starts from the Mixture section in which pipe mixture is prepared with the help of PVC rising, CaCo_3 , citric acid and wax which is heated up to 130°C . The heated material is then cooled up to 100°C and transported to the Extruder by conveyors. With the help of Die and Extruder, pipe is prepared. After smoothing the pipe, sorting process take place. In this process, the pipe carried to Cut teris cut into different sizes as per the need and requirement.

CHAPTER-2

**COMPARATIVE STUDY OF
THE SUBSYSTEMS
SUBJECTED TO
INDEPENDENT AND
SIMULTANEOUS FAILURE**

CHAPTER-2

COMPARITIVE STUDY OF THE SUBSYSTEMS SUBJECTED TO INDEPENDENT AND SIMULTANEOUS FAILURE

In this chapter we discussed the comparison between availability of a pipe manufacturing industry when the subsystems are subjected to simultaneously and independent failure. The failure rates of the sub-systems are constant and the repairs are variable. The governing differential equations of the system are solved using Lagrange's Method. The performance evaluation of system is done by means of long run availability. The tables for various parameters are given which can be useful to the plant management for improving and planning the maintenance schedule.

SYSTEM DESCRIPTION

The Polytube industry consists of four subsystems, namely:

Sub-system A (Mixture): It mixes raw material such as PVC rising, calcium carbonate, wax and other chemicals in appropriate proportion for manufacturing pipe. It consists of a heater by which the raw material is heated up to 130°C and transported to the extruder by conveyors. It consists of blades and a motor whose failure cause complete failure of the system.

Sub-system B (Extruder): Raw material obtained from mixer is heated in this section. It consists of a heater to heat the raw material at different temperatures. The quality of the product depends upon heating process. Its failure causes the complete failure of the system.

Sub-system C (Die): It is used to make different sizes of pipe. Minor failure of the sub-system reduced the capacity of the system and hence loss in production. Major failure results in complete failure of the system.

Sub-system D (Cutter): This sub-system has two units arranged in series. First unit is blade which cut the pipe and the second unit is motor which cut the pipe in different size. Failure of blade reduces the capacity of the system while the failure of motor causes the complete failure of the system.

2.1 Sub-systems subjected to independent failures

2.1.1 ASSUMPTIONS AND NOTATIONS

The assumptions used in developing the performance model are as follows:

1. Failure rates are constant over time and statistically independent.
2. A repaired unit as good as new, performance wise, for a specified duration.
3. Sufficient repair facilities are provided.
4. Service includes repair and/or replacement.

NOTATIONS

A, B, C, D	: Indicates that the sub-system is working in full capacity.
$\bar{C}, \bar{D}$	: Indicate the reduced state of the sub-system C and D.
a, b, c, d	: Indicate the failed state of the sub-system.
λ_1, λ_2	: Transition rate of subsystem C and D.
α_i	: Failure rate of the sub-system A, B, C, D.
$\phi(x)\psi(y)$	: General repair rates of A, B, C, D respectively.
$\mu(z)\sigma(s)$	
$P_o(t)$	: The system is working in full capacity.
$P_i(x, t)$	: Probability that there is failure in subsystem A at time 't' and has an elapsed repair time x. $i = 5, 7, 10, 13, 17$
$P_j(y, t)$	: Probability that there is failure in subsystem B at time 't' and has an elapsed repair time y. $j = 6, 8, 11, 14, 18$.
$P_2(s, t)$	: Probability that there is failure in subsystem D at time 't' and system remains in reduced state and has an elapsed repair time s.
$P_1(z, t)$	: Probability that there is failure in subsystem C at time 't' and system remains in reduced state and has an elapsed repair time z.
$P_4(z, t)$	: Probability when the sub system D is in reduced state and has an elapsed repair time s then the sub system C comes in reduced state and has an elapsed repair time z.
$P_{3(1)}(s, t)$	: Probability when the sub system C comes firstly in reduced state and has an elapsed repair time z then the sub system D comes in failed state and has an elapsed repair time s.

2.4 Mathematical formulations of the system

The differential-difference equations obtained from the state transition is as follows (Fig. 1):

$$\left[\frac{d}{dt} + S_0(t) \right] P_0 = M_0(t) \quad (1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + S_2(z) \right] P_1(z, t) = M_1(z, t) \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + S_2(s) \right] P_2(s, t) = M_2(s, t) \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + S_{3(1)}(s) \right] P_{3(1)}(s, t) = M_3(s, t) \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + S_4(z) \right] P_{4(2)}(z, t) = M_4(z, t) \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_5(x, t) = 0 \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_6(y, t) = 0 \quad (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_7(x, t) = 0 \quad (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_8(y, t) = 0 \quad (9)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z) \right] P_9(z, t) = 0 \quad (10)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_{10}(x, t) = 0 \quad (11)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_{11}(y, t) = 0 \quad (12)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s) \right] P_{12}(s, t) = 0 \quad (13)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_{13}(x, t) = 0 \quad (14)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_{14}(y, t) = 0 \quad (15)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z) \right] P_{15}(z, t) = 0 \quad (16)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s) \right] P_{16}(s, t) = 0 \quad (17)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_{17}(x, t) = 0 \quad (18)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_{18}(y, t) = 0 \quad (19)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z) \right] P_{19}(z, t) = 0 \quad (20)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s) \right] P_{20}(s, t) = 0 \quad (21)$$

Where,

$$S_0(t) = \sum_{i=1}^2 \alpha_i + \lambda_i$$

$$M_0(t) = \int P_5(x, t) \phi(x) dx + \int P_6(y, t) \psi(y) dy + \\ + \int P_2(s, t) \sigma(s) ds + \int P_1(z, t) \mu(z) dz$$

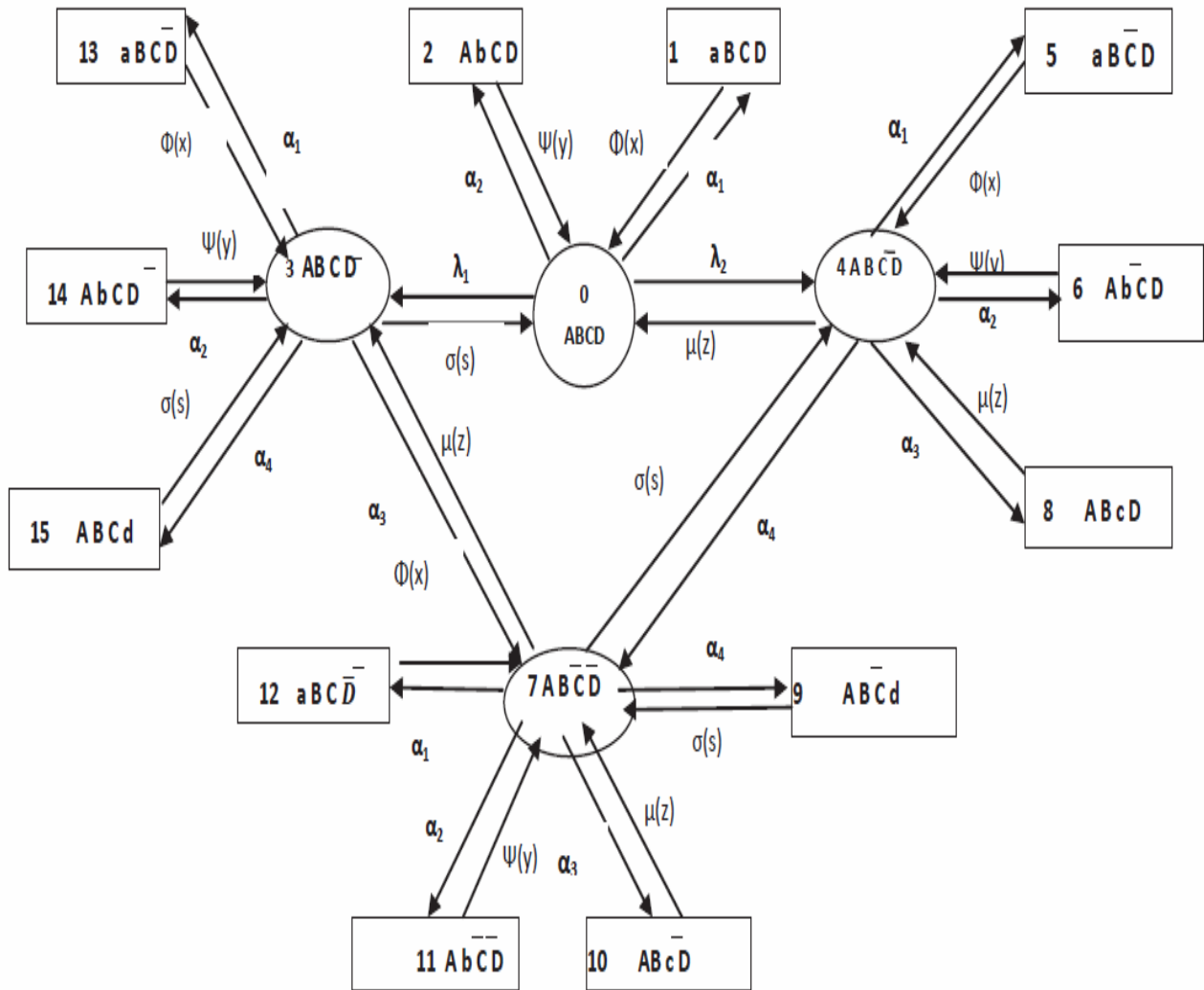


Fig. 1. Transition diagram of Poly Tube Industry when Sub-System simultaneously

$$S_2(s) = \sum_{i=1}^4 \alpha_i + \sigma(s) \quad M_2(s, t) = \lambda_2 P_0(t) + \int P_{10}(x, t) \phi(x) dx + \\ + \int P_{11}(y, t) \psi(y) dy + \int P_{12}(s, t) \sigma(s) ds + \int P_{4(2)}(z, t) \mu(z) dz$$

$$S_{3(1)}(s) = \sum_{i=1}^4 \alpha_i + \sigma(s)$$

$$M_3(s, t) = \int \alpha_4 P_1(z, t) dz + \int P_{20}(s, t) \sigma(s) ds + \\ + \int P_{19}(z, t) \mu(z) dz + \int P_{18}(y, t) \psi(y) dy + \int P_{17}(x, t) \phi(x) dx$$

$$S_{4(2)}(z) = \sum_{i=1}^4 \alpha_i + \mu(z)$$

$$M_4(z, t) = \int \alpha_3 P_2(s, t) ds + \int P_{16}(s, t) \sigma(s) ds + \\ + \int P_{15}(z, t) \mu(z) dz + \int P_{14}(y, t) \psi(y) dy + \int P_{13}(x, t) \phi(x) dx$$

Initial conditions

$$P_0(0) = 1 \text{ otherwise } 0$$

$$P_i(x, 0) = 0 \text{ For } i = 5, 7, 10, 13, 17$$

$$P_j(y, 0) = 0 \text{ For } j = 6, 8, 11, 14, 18$$

$$P_l(z, 0) = 0 \text{ For } l = 1, 9, 15, 17$$

$$P_r(s, 0) = 0 \text{ For } r = 2, 12, 16, 20.$$

$$P_{3(1)}(s, 0) = 0$$

$$P_{4(2)}(z, 0) = 0$$

Boundary conditions

$$P_1(0, t) = \lambda_1 P_0(t)$$

$$P_2(0, t) = \lambda_2 P_0(t)$$

$$P_5(0, t) = \alpha_1 P_0(t)$$

$$P_6(0, t) = \alpha_2 P_0(t)$$

$$P_{3(1)}(0, t) = \int \alpha_4 P_1(z, t) dz$$

$$P_{4(2)}(0, t) = \int \alpha_2 P_2(s, t) ds$$

$$P_7(0, t) = \int \alpha_1 P_1(z, t) dz$$

$$P_8(0, t) = \int \alpha_2 P_1(z, t) dz$$

$$P_9(0, t) = \int \alpha_3 P_1(z, t) dz$$

$$P_{10}(0, t) = \int \alpha_1 P_2(s, t) ds$$

$$P_{11}(0, t) = \int \alpha_2 P_2(s, t) ds$$

$$P_{12}(0, t) = \int \alpha_1 P_2(s, t) ds$$

$$P_{13}(0, t) = \int \alpha_1 P_{4(2)}(z, t) dz$$

$$P_{14}(0, t) = \int \alpha_2 P_{4(2)}(z, t) dz$$

$$P_{15}(0, t) = \int \alpha_3 P_{4(2)}(z, t) dz$$

$$P_{16}(0, t) = \int \alpha_4 P_{4(2)}(z, t) dz$$

$$P_{17}(0, t) = \int \alpha_1 P_{3(1)}(s, t) ds$$

$$P_{18}(0, t) = \int \alpha_2 P_{3(1)}(s, t) ds$$

$$P_{19}(0, t) = \int \alpha_3 P_{3(1)}(s, t) ds$$

$$P_{20}(0, t) = \int \alpha_4 P_{3(1)}(s, t) ds$$

Equation (1) is linear differential equation of first order and other equations (2-21) are partial differential equations of first order. Using the boundary and initial conditions, the equations (1-21) are solved to give the following solution:

$$\begin{aligned}
 P_0(t) &= e^{-\int S_0(t)dt} \left[1 + \int M_0(t) e^{\int S_0(t)dt} dt \right] \\
 P_1(z, t) &= e^{-\int S_1(z)dz} \left[\lambda_1 P_0(t - z) + \int M_1(z, t) e^{\int S_1(z)dz} dz \right] \\
 P_2(s, t) &= e^{-\int S_2(s)ds} \left[\lambda_2 P_0(t - s) + \int M_2(s, t) e^{\int S_2(s)ds} ds \right] \\
 P_{3(1)}(s, t) &= e^{-\int S_{3(1)}(s)ds} \left[\int \alpha_4 P_1(z, t - s) dz + \right. \\
 &\quad \left. + \int M_3(s, t) e^{\int S_{3(1)}(s)ds} ds \right] \\
 P_{4(2)}(z, t) &= e^{-\int S_{4(2)}(z)dz} \left[\int \alpha_3 P_2(s, t - z) ds + \right. \\
 &\quad \left. + \int M_4(z, t) e^{\int S_{4(2)}(z)dz} dz \right]
 \end{aligned}$$

$$\begin{aligned}
 P_5(x, t) &= \alpha_1 P_0(t - x) e^{-\int \phi(x)dx} \\
 P_6(y, t) &= \alpha_2 P_0(t - y) e^{-\int \psi(y)dy} \\
 P_7(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_1(z, t - x) dz \\
 P_8(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_1(z, t - y) dz \\
 P_9(z, t) &= e^{-\int \mu(z)dz} \int \alpha_3 P_1(z, t - z) dz \\
 P_{10}(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_2(s, t - x) ds \\
 P_{11}(y, t) &= e^{-\int \phi(x)dx} \int \alpha_2 P_2(s, t - y) ds \\
 P_{12}(s, t) &= e^{-\int \sigma(s)ds} \int \alpha_4 P_2(s, t - s) ds \\
 P_{13}(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_{4(2)}(z, t - x) dz \\
 P_{14}(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_{4(2)}(z, t - y) dz \\
 P_{15}(z, t) &= e^{-\int \mu(z)dz} \int \alpha_3 P_{4(2)}(z, t - z) dz \\
 P_{16}(s, t) &= e^{-\int \sigma(s)ds} \int \alpha_4 P_{4(2)}(z, t - s) dz \\
 P_{17}(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_{3(1)}(s, t - x) ds \\
 P_{18}(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_{3(1)}(s, t - y) ds \\
 P_{19}(z, t) &= e^{-\int \mu(z)dz} \int \alpha_3 P_{3(1)}(s, t - s) ds \\
 P_{20}(s, t) &= e^{-\int \sigma(s)ds} \int \alpha_4 P_{3(1)}(s, t - s) ds
 \end{aligned}$$

It is evident that all probabilities are obtained in terms of $P_0(t)$ which is given by (1). The time dependent availability $A(t)$ is:

$$A(t) = P_0(t) + \int P_2(s, t) ds + \int P_1(z, t) dz + \int P_{4(2)}(z, t) dz + \int P_{3(1)}(s, t) ds \quad (22)$$

2.5 STEADY STATE AVAILABILITY

In the process industry, we require long run availability of the system, which is obtained by

putting $\frac{d}{dt}=0, \frac{\partial}{\partial t}=0$ and taking probabilities independent of "t" then limiting probabilities from (1-21) are:

$$\begin{aligned} [(\alpha_1 + \alpha_2 + \lambda_1 + \lambda_2)P_0] &= \phi P_5 + \psi P_6 + \sigma P_2 + \mu P_1 \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \mu)P_1] &= \lambda_1 P_0 + \sigma P_{3(1)} + \mu P_9 + \psi P_8 + \phi P_7 \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma)P_2] &= \lambda_2 P_0 + \phi P_{10} + \psi P_{11} + \sigma P_{12} + \mu P_{4(2)} \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma)P_{3(1)}] &= \alpha_4 P_1 + \sigma P_{20} + \mu P_{19} + \psi P_{18} + \phi P_{17} \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \mu)P_{4(2)}] &= \alpha_3 P_2 + \phi P_{13} + \psi P_{14} + \sigma P_{16} + \mu P_{15} \\ \phi P_5 &= \alpha_1 P_0 & \psi P_6 &= \alpha_2 P_0 \\ \phi P_7 &= \alpha_1 P_1 & \psi P_8 &= \alpha_2 P_1 \\ \mu P_9 &= \alpha_3 P_1 & \phi P_{10} &= \alpha_1 P_1 \\ \psi P_{11} &= \alpha_2 P_2 & \sigma P_{12} &= \alpha_4 P_2 \\ \phi P_{13} &= \alpha_1 P_{4(2)} & \psi P_{14} &= \alpha_2 P_{4(2)} \\ \mu P_{15} &= \alpha_3 P_{4(2)} & \sigma P_{16} &= \alpha_4 P_{4(2)} \\ \phi P_{17} &= \alpha_1 P_{3(1)} & \psi P_{18} &= \alpha_2 P_{3(1)} \\ \mu P_{19} &= \alpha_3 P_{3(1)} & \sigma P_{20} &= \alpha_4 P_{3(1)} \end{aligned}$$

$$\text{With initial conditions } P_i(0) = \begin{cases} 1, i = 0 \\ 0, i \neq 0 \end{cases}$$

On solving these equations recursively, we have the steady state probabilities:

$$\begin{aligned} P_2 &= T_2 P_0 & P_1 &= T_1 P_0 \\ P_{4(2)} &= M_2 P_0 & P_{3(1)} &= M_1 P_0 \\ A_v &= [1 + M_1 + M_2 + T_1 + T_2] P_0 \end{aligned}$$

Now using normalizing conditions

$$\sum_{i=0}^{20} P_i = 1$$

$$A_v = [1 + M_1 + M_2 + T_1 + T_2] P_0$$

Where

$$\begin{aligned} P_0 &= \left[1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_3}{\mu} + \frac{\alpha_4}{\sigma} \right) M_1 + \right. \\ &+ \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_4}{\sigma} + \frac{\alpha_3}{\mu} \right) M_2 + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_4}{\sigma} + \frac{\alpha_3}{\mu} \right) T_1 + \\ &\left. + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_4}{\sigma} \right) T_2 \right]^{-1} \end{aligned}$$

$$M_1 = \frac{\lambda_1 \alpha_4}{\sigma \mu} \quad M_2 = \frac{\lambda_2 \alpha_3}{\sigma \mu} \quad T_1 = \frac{\lambda_1}{\mu} \quad T_2 = \frac{\lambda_2}{\sigma}$$

2.6 Sub- system subjected to simultaneously failure:

2.6.1 ASSUMPTIONS AND NOTATIONS

1. Failure rates are constant over time and statistically independent.
2. A repaired unit as good as new, performance wise, for a specified duration.
3. Sufficient repair facilities are provided.
4. Service includes repair and/or replacement.
5. System may work at reduced capacity.
6. There are simultaneous failures.

NOTATIONS

A, B, C, D : Indicates that the sub-system is working in full capacity.

$\overline{C}, \overline{D}$: Indicate the reduced state of the sub-system C and D .

a, b, c, d : Indicate the failed state of the sub-system.

α_i : Failure rate of the sub-system A, B, C, D .

$\phi(x)\psi(y)$: General repair rates of A, B, C, D respectively.

$\mu(z)\sigma(s)$

$P_o(t)$: The system is working in full capacity.

$P_i(x, t)$: Probability that there is failure in subsystem A at time 't' and it is repaired within time interval $(x, x+\Delta)$. For $i = 1, 5, 12, 13$.

$P_j(y, t)$: Probability that there is failure in subsystem B at time 't' and it is repaired within time interval $(y, y+\Delta)$. For $j = 2, 6, 11, 14$.
 ${}_3(s, t)$: Probability that there is failure in subsystem D at time 't' and system remains in reduced state till it is repaired within time interval $(s, s+\Delta)$.

$P_4(z, t)$: Probability that there is failure in subsystem C at time 't' and system remains in reduced state till it is repaired within time interval $(z, z+\Delta)$.

$P_7(s, z, t)$: Probability that there is failure in subsystem C and D at time 't' and system remains in reduced state till it is repaired within time interval $(z, z+\Delta)$ and $(s, s+\Delta)$ respectively.

$P_k(z, t)$: Probability that there is failure in subsystem C at time 't' and it is repaired within time interval $(z, z+\Delta)$. For $k = 8, 10$.

$P_l(z, t)$: Probability that there is failure in subsystem D at time 't' and it is repaired within time interval $(s, s+\Delta)$. For $l = 9, 15$.

2.7 Mathematical formulations of the system

The differential-difference equations obtained from the state transition is as follows (figure 2):

$$\left[\frac{d}{dt} + N_0(t) \right] P_0(t) = C_0(t) \quad (23)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + N_1(s) \right] P_3(s, t) = C_1(s, t) \quad (24)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + N_2(z) \right] P_4(z, t) = C_2(z, t) \quad (25)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \frac{\partial}{\partial z} + N_3(s, z) \right] P_7(s, z, t) = C_3(s, z, t) \quad (26)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_1(x, t) = 0 \quad (27)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_2(y, t) = 0 \quad (28)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_5(x, t) = 0 \quad (29)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_6(y, t) = 0 \quad (30)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z) \right] P_8(z, t) = 0 \quad (31)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s) \right] P_9(s, t) = 0 \quad (32)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z) \right] P_{10}(z, t) = 0 \quad (33)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_{11}(y, t) = 0 \quad (34)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_{12}(x, t) = 0 \quad (35)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_{13}(x, t) = 0 \quad (36)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_{14}(y, t) = 0 \quad (37)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s) \right] P_{15}(s, t) = 0 \quad (38)$$

Where,

$$N_0(t) = \sum_{i=1}^2 \alpha_i + \sum_{i=1}^2 \lambda_i$$

$$C_0(t) = \int P_1(x, t) \phi(x) dx + \int P_2(y, t) \psi(y) dy + \\ + \int P_3(s, t) \sigma(s) ds + \int P_4(z, t) \mu(z) dz$$

$$N_1(s) = \sum_{i=1}^4 \alpha_i + \sigma(s) \quad C_1(s, t) = \lambda_1 P_0(t) + \\ + \int P_{13}(x, t) \phi(x) dx + \int P_{14}(y, t) \psi(y) dy + \\ + \int P_{15}(s, t) \sigma(s) ds + \int P_7(s, z, t) \mu(z) dz ds$$

$$N_2(z) = \sum_{i=1}^4 \alpha_i + \mu(z) \quad C_2(z, t) = \lambda_2 P_0(t) + \\ + \int P_5(x, t) \phi(x) dx + \int P_6(y, t) \psi(y) dy + \\ + \int P_7(s, z, t) \sigma(s) ds dz + \int P_8(z, t) \mu(z) dz$$

$$N_3(s, z) = \sum_{i=1}^4 \alpha_i + \sigma(s) + \mu(z) \quad C_3(s, z, t) = \int \alpha_3 P_3(s, t) ds + \\ + \int \alpha_4 P_4(z, t) dz + \int P_9(s, t) \sigma(s) ds + \int P_{10}(z, t) \mu(z) dz + \\ + \int P_{11}(y, t) \psi(y) dy + \int P_{12}(x, t) \phi(x) dx$$

Initial conditions

$$P_0(0) = 1 \quad \text{otherwise } 0$$

$$P_i(x, 0) = 0 \quad \text{For } i=1,5,12,13$$

$$P_j(y, 0) = 0 \quad \text{For } j=2,6,11,14$$

$$P_k(z, 0) = 0 \quad \text{For } k=4,7,8,10$$

$$P_l(s, 0) = 0 \quad \text{For } l=3,7,9,15$$

$$P_0(0) = 1 \quad \text{otherwise } 0$$

$$P_i(x, 0) = 0 \quad \text{For } i=1,5,12,13$$

$$P_j(y, 0) = 0 \quad \text{For } j=2,6,11,14$$

$$P_k(z, 0) = 0 \quad \text{For } k=4,7,8,10$$

$$P_l(s, 0) = 0 \quad \text{For } l=3,7,9,15$$

Boundary Conditions

$$P_1(0, t) = \alpha_1 P_0(t)$$

$$P_2(0, t) = \alpha_2 P_0(t)$$

$$P_3(0, t) = \alpha_4 P_0(t)$$

$$P_4(0, t) = \alpha_3 P_0(t)$$

$$P_5(0, t) = \int \alpha_1 P_4(z, t) dz$$

$$P_6(0, t) = \int \alpha_2 P_4(z, t) dz$$

$$P_7(0,0,t) = \int \alpha_3 P_3(s,t) ds + \int \alpha_4 P_4(z,t) dz$$

$$P_8(0,t) = \int \alpha_3 P_4(z,t) dz dz$$

$$P_9(0,t) = \int \alpha_4 P_7(s,z,t) ds dz$$

$$P_{10}(0,t) = \int \alpha_3 P_7(s,z,t) ds dz$$

$$P_{11}(0,t) = \int \alpha_2 P_7(s,z,t) ds dz$$

$$P_{12}(0,t) = \int \alpha_1 P_7(s,z,t) ds dz$$

$$P_{13}(0,t) = \int \alpha_1 P_3(s,t)$$

$$P_{14}(0,t) = \int \alpha_2 P_3(s,t) ds$$

$$P_{15}(0,t) = \int \alpha_4 P_3(s,t) ds$$

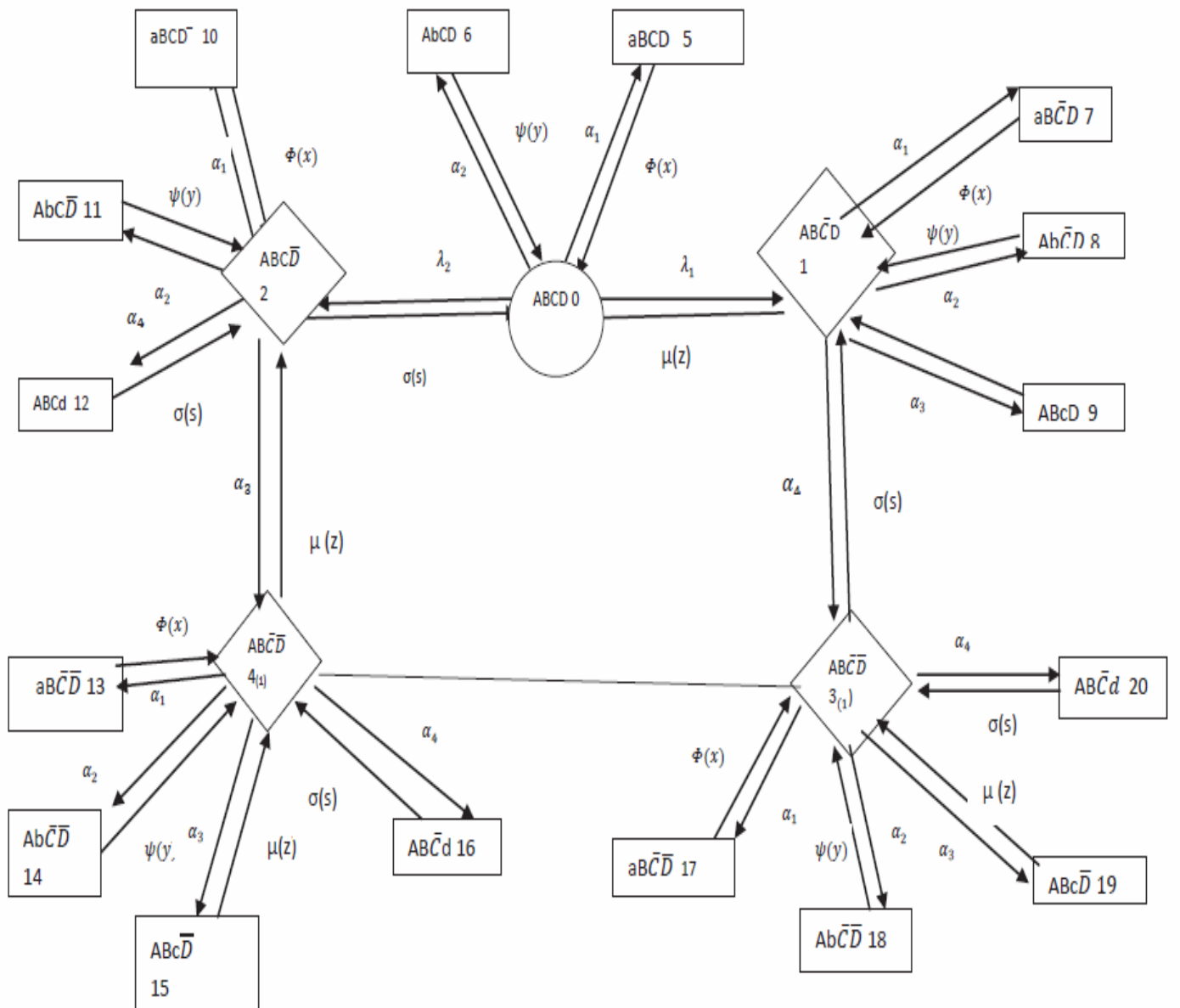


Fig. 2. Transition diagram of Poly Tube Industry when Sub-System failed independently

Equation (23) is linear differential equation of first order and other equations (24-38) are partial differential equations of first order. Using the boundary and initial conditions, the equations (23-38) are solved to give the following solution:

$$\begin{aligned}
P_0(t) &= e^{-\int N_0(t)dt} \left[1 + \int C_0(t) e^{\int N_0(t)dt} dt \right] \\
P_1(x, t) &= \alpha_1 P_0(t - x) e^{-\int \phi(x)dx} \\
P_2(y, t) &= \alpha_2 P_0(t - y) e^{-\int \psi(y)dy} \\
P_3(s, t) &= e^{-\int N_1(s)ds} \left[\alpha_4 P_0(t - s) + \int C_1(s, t) e^{\int N_1(s)ds} ds \right] \\
P_4(z, t) &= e^{-\int N_2(z)dz} \left[\alpha_3 P_0(t - z) + \int C_2(z, t) e^{\int N_2(z)dz} dz \right] \\
P_5(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_4(z, t - x) dz \\
P_6(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_4(z, t - y) dz \\
P_7(s, z, t) &= e^{-\int N_2(s,z)ds} \left[\int \alpha_3 P_3(s, t - z) ds + \right. \\
&\quad \left. + \int \alpha_4 P_4(z, t - s) dz + \int C_3(s, z, t) e^{\int N_2(s,z)ds} ds \right] \\
P_8(z, t) &= e^{-\int \mu(z)dz} \int \alpha_3 P_4(z, t - z) dz \\
P_9(s, t) &= e^{-\int \sigma(s)ds} \int \alpha_4 P_7(s, z, t - s) ds dz \\
P_{10}(z, t) &= e^{-\int \mu(z)dz} \int \alpha_3 P_7(s, z, t - z) ds dz \\
P_{11}(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_{7(3)}(s, z, t - y) ds dz \\
P_{12}(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_{7(3)}(s, z, t - x) ds dz \\
P_{13}(x, t) &= e^{-\int \phi(x)dx} \int \alpha_1 P_3(s, t - x) ds \\
P_{14}(y, t) &= e^{-\int \psi(y)dy} \int \alpha_2 P_3(s, t - y) ds \\
P_{15}(s, t) &= e^{-\int \sigma(s)ds} \int \alpha_4 P_3(s, t - s) ds
\end{aligned}$$

It is evident that all probabilities are obtained in terms of $P_0(t)$ which is given by (23).

The time dependent availability $A(t)$ is:

$$\begin{aligned}
A(t) &= P_0(t) + \int P_3(s, t) ds + \int P_4(z, t) dz + \\
&\quad + \int P_7(s, z, t) dz ds \quad (39)
\end{aligned}$$

2.8 STEADY STATE AVAILABILITY

In the process industry, we require long run availability of the system, which is obtained by putting $\frac{d}{dt}=0, \frac{\partial}{\partial t}=0$ and taking probabilities independent of “t” then limiting probabilities from (23-38) are :

$$\begin{aligned} [(\alpha_1 + \alpha_2 + \lambda_1 + \lambda_2)]P_0 &= \phi P_1 + \psi P_2 + \sigma P_3 + \mu P_4 \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma)]P_3 &= \lambda_1 P_0 + \phi P_{13} + \psi P_{14} + \sigma P_{15} + \mu P_7 \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \mu)]P_4 &= \lambda_2 P_0 + \phi P_5 + \psi P_6 + \sigma P_7 + \mu P_8 \\ [(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma + \mu)]P_7 &= \alpha_3 P_3 + \alpha_4 P_4 + \sigma P_9 + \mu P_{10} + \psi P_{11} + \phi P_{12} \\ \phi P_1 &= \alpha_1 P_0 & \psi P_2 &= \alpha_2 P_0 \\ \phi P_5 &= \alpha_1 P_4 & \psi P_6 &= \alpha_2 P_4 \\ \mu P_8 &= \alpha_3 P_4 & \sigma P_9 &= \alpha_4 P_7 \\ \mu P_{10} &= \alpha_3 P_7 & \psi P_{11} &= \alpha_2 P_7 \\ \phi P_{12} &= \alpha_1 P_7 & \phi P_{13} &= \alpha_1 P_3 \\ \psi P_{14} &= \alpha_2 P_3 & \sigma P_{15} &= \alpha_4 P_3 \end{aligned}$$

On solving these equations recursively, we get:

$$\begin{aligned} P_7 &= \frac{\alpha_3}{s_1} P_3 + \frac{\alpha_4}{s_1} P_4 \\ P_4 &= \frac{\lambda_2}{s_2} P_0 + \frac{\sigma \alpha_3}{s_1 s_2} P_3 \\ P_3 &= \left(\frac{\lambda_1}{s_3} + \frac{\alpha_3 \mu \alpha_4}{s_1 s_2 s_3} \right) P_0 \end{aligned}$$

Now using normalizing conditions:

$$\sum_{i=0}^{15} P_i = 1$$

$$A_v = [1 + M_1 + M_2 + M_3]P_0 \quad (40)$$

Where:

$$\begin{aligned} P_0 &= \left[1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_4}{\sigma} \right) M_1 + \right. \\ &\quad \left. + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_3}{\mu} \right) M_2 + \left(1 + \frac{\alpha_1}{\phi} + \frac{\alpha_2}{\psi} + \frac{\alpha_4}{\sigma} + \frac{\alpha_3}{\mu} \right) M_3 \right]^{-1} \\ s_1 &= \sigma + \mu \\ s_2 &= \alpha_4 + \mu - \frac{\sigma \alpha_4}{s_1} \\ s_3 &= \alpha_3 + \sigma - \frac{\mu \alpha_3}{s_1} - \frac{\sigma \mu \alpha_4 \alpha_3}{s_1 s_2 s_1} \\ M_1 &= \left(\frac{\lambda_1}{s_3} + \frac{\alpha_3 \mu \alpha_4}{s_1 s_2 s_3} \right) \end{aligned}$$

$$M_2 = \frac{\lambda_2}{s_2} + \frac{\sigma \alpha_3}{s_1 s_2} M_1$$

$$M_3 = \frac{\alpha_3}{s_1} M_1 + \frac{\alpha_4}{s_1} M_2$$

Under the available facilities the concern management minimize the failure time of each sub-system adopting the following Measures:

1. Getting the information of failure of each equipment at the earliest moment.
2. Starting the repair work at the earliest moment.
3. Providing trained workers.
4. Providing the special tools required.
5. Making available the spare parts and special parts.

2.9 ANALYSIS AND DISCUSSION

The tables for various parameters can be useful to the plant management for improving and planning the maintenance schedule.

(a) Effect of failure and repair rates of the Sub-System Mixture (A) on Availability when both Sub-Systems failed independently.

$$\begin{array}{ll} \alpha_2=0.007 & \psi=2, \\ \alpha_3=0.01 & \mu=0.33 \\ \alpha_4=0.015 & \sigma=0.02 \\ \lambda_1=0.001 & \\ \lambda_2=0.002 & \end{array}$$

TABLE-

2.1.1

$\phi \backslash a_1$	0.0057	0.0059	0.0061	0.0063	Constant values
0.5	0.9769	0.9732	0.9694	0.9656	$\alpha_2=0.007, \psi=2,$ $\lambda_1=0.001, \lambda_2=0.002$ $\alpha_3=0.01, \mu=0.33$ $\alpha_4=0.015, \sigma=0.02$
0.7	0.9802	0.9775	0.9748	0.9721	
0.9	0.9821	0.9796	0.9770	0.9757	
1.1	0.9832	0.9815	0.9797	0.9780	

(b) Effect of failure and repair rate of the Sub-System Extruder (B) on Availability when both sub-systems failed independently.

$$\alpha_1=0.0057$$

$$\alpha_3=0.01$$

$$\alpha_4=0.015$$

$$\lambda_1=0.001$$

$$\lambda_2=0.002$$

$$\phi = 0.5$$

$$\mu = 0.33$$

$$\sigma = 2$$

TABLE-2.1.2

$\psi \backslash a_2$	0.007	0.009	0.011	0.013	Constant values
2	0.9769	0.9760	0.9751	0.9741	$\alpha_1=0.0057, \phi=0.5$ $\lambda_1=0.001, \lambda_2=0.002$ $\alpha_3=0.01, \mu=0.33$ $\alpha_4=0.015, \sigma=2$
4	0.9786	0.9781	0.9777	0.9772	
6	0.9792	0.9789	0.9785	0.9782	
8	0.9795	0.9792	0.9790	0.9786	

(c) Effect of failure and repair rate of the Sub-System Die (C) on Availability when both sub-systems failed independently.

$$\alpha_1=0.0057$$

$$\phi = 0.5$$

$$\alpha_2=0.007$$

$$\psi = 2$$

$$\alpha_4=0.015$$

$$\sigma = 2$$

$$\lambda_1=0.002,$$

$$\lambda_2=0.002$$

TABLE-2.1.3

$\mu \backslash a_3$	0.01	0.02	0.03	0.04	Constant values
0.33	0.9769	0.9621	0.9382	0.9067	$\alpha_1=0.0057, \phi=0.5$ $\alpha_2=0.007, \psi=2$ $\alpha_4=0.015, \sigma=2$ $\lambda_1=0.002, \lambda_2=0.002$
0.53	0.9788	0.9673	0.99486	0.9236	
0.73	0.99799	0.9705	0.9552	0.9344	
0.93	0.9807	0.9728	0.9597	0.9420	

(d) Effect of failure and repair rate of the sub-system Cutter (D) on Availability when both sub-systems failed independently.

$\alpha_1=0.0057$ $\phi=0.5$
 $\alpha_2=0.007$ $\psi=0.04$
 $\alpha_3=0.33$ $\mu=0.02$
 $\lambda_1=0.001$
 $\lambda_2=0.002$

TABLE-2.1.4

$\sigma \backslash a_4$	0.015	0.030	0.045	0.060	Constant values
2	0.9769	0.9763	0.9756	0.9749	$\alpha_1=0.0057, \phi=0.5$ $\alpha_2=0.007, \psi=0.04$ $\alpha_3=0.33, \mu=0.02$ $\lambda_1=0.001, \lambda_2=0.002$
4	0.9812	0.9809	0.9809	0.9802	
6	0.9825	0.9823	0.9821	0.9818	
8	0.9831	0.9829	0.9827	0.9826	

(e) Effect of failure and repair rate of the Sub-System Mixture (A) on Availability when both Sub-Systems failed simultaneously.

$\alpha_2=0.007$ $\psi=2$
 $\alpha_3=0.01$ $\mu=0.33$
 $\alpha_4=0.015$ $\sigma=0.02$
 $\lambda_1=0.001$
 $\lambda_2=0.002$

TABLE-2.1.5

$\phi \backslash a_1$	0.0057	0.0059	0.0061	0.0063	Constant values
0.5	0.9853	0.9849	0.9845	0.9841	$\alpha_2=0.007, \psi=2$ $\alpha_3=0.01, \mu=0.33$ $\alpha_4=0.015, \sigma=0.02$ $\lambda_1=0.001, \lambda_2=0.002$
0.7	0.9884	0.9882	0.9879	0.9876	
0.9	0.9902	0.9900	0.9898	0.9896	
1.1	0.9913	0.9912	0.9910	0.9908	

(f) Effect of failure and repair rate of the sub-system Extruder (B) on Availability when both sub-systems failed simultaneously.

$$\alpha_1=0.0057$$

$$\phi=0.5$$

$$\alpha_3=0.01$$

$$\mu=0.33$$

$$\alpha_4=0.015$$

$$\sigma=2$$

$$\lambda_1=0.001,$$

$$\lambda_2=0.002$$

TABLE-2.1.6

$\psi \backslash \alpha_2$	0.007	0.009	0.011	0.013	Constant values
2	0.9853	0.9843	0.9832	0.9823	$\alpha_1=0.0057, \phi=0.5$ $\alpha_3=0.01, \mu=0.33$ $\alpha_4=0.015, \sigma=2$ $\lambda_1=0.001, \lambda_2=0.002$
4	0.9869	0.9865	0.9860	0.9855	
6	0.9875	0.9872	0.9869	0.9865	
8	0.9878	0.9876	0.9873	0.9871	

(g) Effect of failure and repair rate of the Sub-System Die (C) on Availability when both Sub-Systems failed simultaneously.

$$\alpha_1=0.0057$$

$$\phi=0.5$$

$$\alpha_2=0.007$$

$$\psi=2$$

$$\alpha_4=0.015$$

$$\sigma=2$$

$$\lambda_1=0.001$$

$$\lambda_2=0.002$$

TABLE-2.1.7

$\mu \backslash \alpha_3$	0.01	0.02	0.03	0.04	Constant values
0.33	0.9853	0.9853	0.9853	0.9853	$\alpha_1=0.0057, \phi=0.5$ $\alpha_2=0.007, \psi=2$ $\alpha_4=0.015, \sigma=2$ $\lambda_1=0.001, \lambda_2=0.002$
0.53	0.9853	0.9853	0.9853	0.9853	
0.73	0.9854	0.9854	0.9854	0.9855	
0.93	0.9854	0.9854	0.9854	0.9855	

(h) **Effect of failure and repair rate of the Sub-System Cutter (D) on Availability when both Sub-Systems failed simultaneously.**

$$\alpha_1=0.0057$$

$$\phi=0.5$$

$$\alpha_2=0.007$$

$$\psi=0.04$$

$$\alpha_3=0.33$$

$$\mu=0.02$$

$$\lambda_1=0.001$$

$$\lambda_2=0.002$$

TABLE-2.1.8

$\sigma \backslash \alpha_4$	0.015	0.030	0.045	0.060	Constant values
2	0.9853	0.9853	0.9852	0.9852	$\alpha_1=0.0057, \phi=0.5$ $\alpha_2=0.007, \psi=0.04$ $\alpha_3=0.33, \mu=0.02$ $\lambda_1=0.001, \lambda_2=0.002$
4	0.9853	0.9853	0.9853	0.9852s	
6	0.9853	0.9852	0.9852	0.9852	
8	0.9854	0.9853	0.9853	0.9852	

CHAPTER-3

**STUDY OF POLYTUBE
INDUSTRY USING
SUPPLEMENTARY VARIABLE
TECHNIQUE**

Chapter-3

A STUDY OF POLYTUBE INDUSTRY USING SUPPLEMENTARY VARIABLE TECHNIQUE

Several researchers including Kumar and Singh[1989], Goyal and Singh[1995-96], Mahajan and Singh[1996-1999], Dhillon and Natesson[1983], have discussed and analysed reliability as well as long run availability of different process industries such as the paper industry, standby complex system having imperfect switch, heating system with warm standby and imperfect switch in sugar industry, straw board mill, utensils manufacturing plant, outdoor power system in fluctuating environment, cement manufacturing plant etc.. Also Gupta et al. [2005] numerically analysed the reliability and availability of butter-oil processing plant. In this Chapter we study a reliability model for polytube industry using supplementary variable technique.

3.2 THE SYSTEM DESCRIPTION

The complete description of the systems and their notations, required in the mathematical formulation, are as follows:

Sub-system A (Mixture)

It mixes raw material such as PVC rising, calcium carbonate, wax and other chemicals in appropriate proportion for manufacturing pipe. It consists of a heater by which the raw material is heated up to 130⁰C and transported to the extruder by conveyors. It consists of blades and a motor whose failure cause complete failure of the system.

Sub-system B (Extruder)

Raw material obtained from mixer is heated in this section. It consists of a heater to heat the raw material at different temperatures. The quality of the product depends on heating process. Its failure causes the complete failure of the system.

Sub-system C (Die)

It is used to make different sizes of pipe. Minor failure of the sub-system reduced the capacity of the system, hence, loss in production. Major failure results in complete failure of the system.

Sub-system D (Cutter)

This sub-system has two units arranged in series. First unit is blade, which cuts the pipe and the second unit is motor which cuts the pipe in different size. Failure of blade reduces the capacity of the system while the failure of motor causes the completes failure of the system.

3.3 ASSUMPTIONS AND NOTATIONS

The assumptions, on which the present analysis is based on, are as follows:

- (i) Repair and failure rates are independent of each other and their unit is taken as per day.
- (ii) Failure and repair rates of the subsystems are taken as variable.
- (iii) Performance wise, a repaired unit is as good as new one for a specified duration.
- (iv) Sufficient repair facilities are provided.
- (v) Service of the subsystem includes repair and/or replacement.
- (vi) System may work at reduced capacity also.
- (vii) There are simultaneous failures.

NOTATIONS

A, B, C, D	: Indicates that the sub-system is working in full capacity.
$\bar{C}, \bar{D}$	: Indicate the reduced state of the sub-system C and D.
a, b, c, d	: Indicate the failed state of the sub-system.
λ_1, λ_2	: Transition rate of subsystem C and D.
α_i	: Failure rate of the sub-system A, B, C, D.
$\phi(x)\psi(y)$	: General repair rates of A, B, C, D respectively.
$\mu(z)\sigma(s)$	
$P_o(t)$	: The system is working in full capacity.
$P_i(x, t)$	: Probability that there is failure in subsystem A at time 't' and has an elapsed repair time x. $i = 5, 7, 10, 13, 17$
$P_j(y, t)$	: Probability that there is failure in subsystem B at time 't' and has an elapsed repair time y. $j = 6, 8, 11, 14, 18$.
$P_2(s, t)$	: Probability that there is failure in subsystem D at time 't' and system remains in reduced state and has an elapsed repair time s.
$P_1(z, t)$	: Probability that there is failure in subsystem C at time 't' and system remains in reduced state and has an elapsed repair time z.
$P_4(z, t)$	: Probability when the sub system D is in reduced state and has an elapsed repair time s then the sub system C comes in reduced state and has an elapsed repair time z.
$P_{3(1)}(s, t)$	: Probability when the sub system C comes firstly in reduced state and has an elapsed repair time z then the sub system D comes in failed state and has an elapsed repair time s.

3.4 Mathematical modeling of the system transient state

When both failure and repair rates are variable

In this section we develop the Chapman–Kolmogorov differential equation assuming variable failure and repair rates of the subsystems by applying supplementary variable technique. In the transient state, Probability considerations give the following system of differential difference equation associated with the state transition diagram (Fig. 3) of the system at time $(t + \Delta t)$. Using mnemonic rule, we have

$$P_0(t + \Delta t) = [1 - \alpha_1(y)\Delta t - \alpha_2(y)\Delta t - \alpha_3(y)\Delta t - \alpha_4(y)\Delta t]P_0(t) + \int \mu(x)P_1(x, y, t)dx\Delta t + \int \sigma(x)P_2(x, y, t)dx\Delta t \\ + \int \phi(x)P_4(x, y, t)dx\Delta t + \int \psi(x)P_5(x, y, t)dx\Delta t,$$

$$P_0(t + \Delta t) - P_0(t) = -[\alpha_1(y)\Delta t - \alpha_2(y)\Delta t - \alpha_3(y)\Delta t - \alpha_4(y)\Delta t]P_0(t) + \int \mu(x)P_1(x, y, t)dx\Delta t + \int \sigma(x)P_2(x, y, t)dx\Delta t \\ + \int \phi(x)P_4(x, y, t)dx\Delta t + \int \psi(x)P_5(x, y, t)dx\Delta t.$$

Dividing both sides by Δt , we get

$$\frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = -[\alpha_1(y) - \alpha_2(y) - \alpha_3(y) - \alpha_4(y)]P_0(t) + \int \mu(x)P_1(x, y, t)dx + \int \sigma(x)P_2(x, y, t)dx \\ + \int \phi(x)P_4(x, y, t)dx + \int \psi(x)P_5(x, y, t)dx,$$

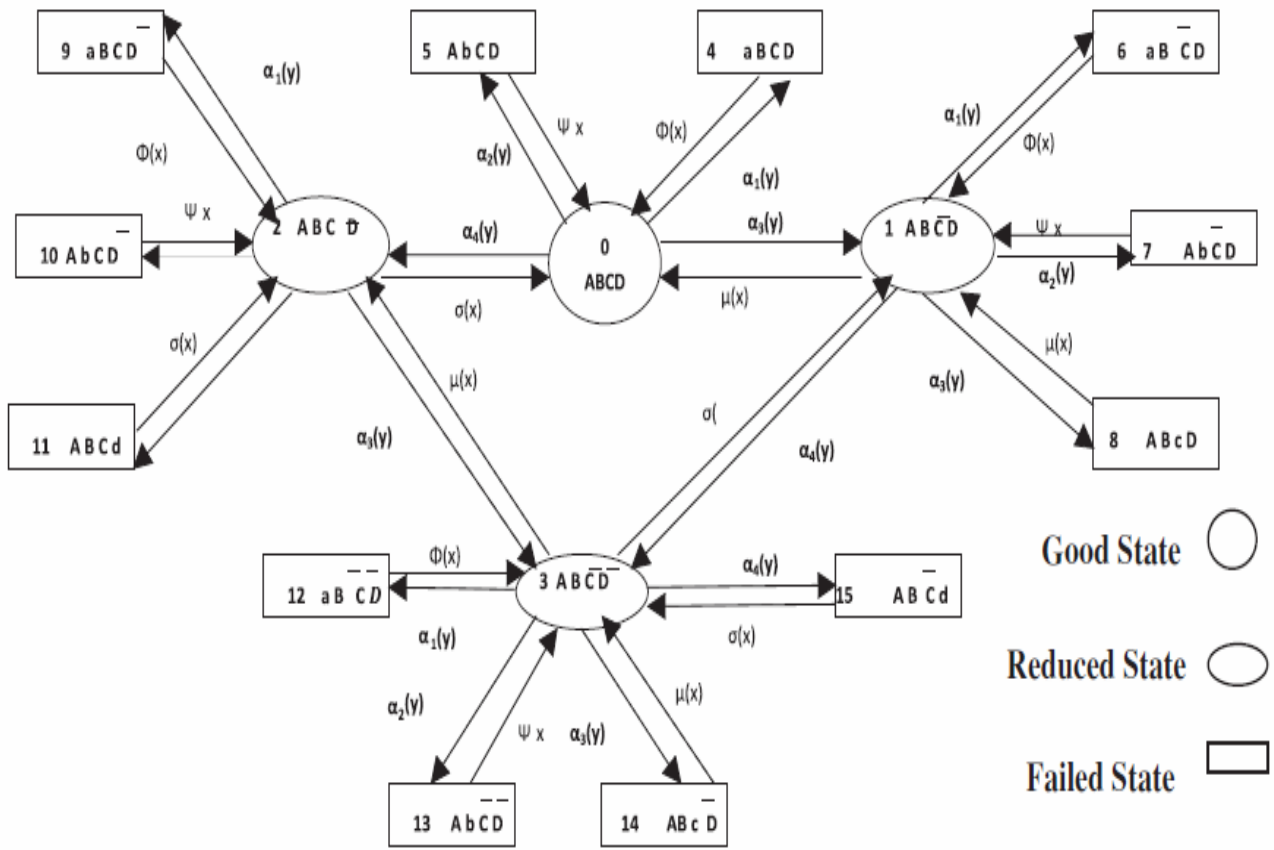


Fig. 3. Transition diagram of Polytube industry.

$$\left[\frac{d}{dt} + T_0 \right] P_0(t) = C_0.$$

(1)

Similarly, for the other states, we can write the differential equation as:

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + T_1(x, y) \right] P_1(x, y, t) = C_1(x, y, t), \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + T_2(x, y) \right] P_2(x, y, t) = C_2(x, y, t), \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + T_3(x, y) \right] P_3(x, y, t) = C_3(x, y, t), \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \phi(x) \right] P_4(x, y, t) = \alpha_1(y) P_0(t), \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \psi(x) \right] P_5(x, y, t) = \alpha_2(y) P_0(t), \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \phi(x) \right] P_6(x, y, t) = \alpha_1(y) P_1(x, y, t), \quad (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \psi(x) \right] P_7(x, y, t) = \alpha_2(y) P_1(x, y, t), \quad (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \mu(x) \right] P_8(x, y, t) = \alpha_3(y) P_1(x, y, t), \quad (9)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \phi(x) \right] P_9(x, y, t) = \alpha_1(y) P_2(x, y, t), \quad (10)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \psi(x) \right] P_{10}(x, y, t) = \alpha_2(y) P_2(x, y, t), \quad (11)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \sigma(x) \right] P_{11}(x, y, t) = \alpha_4(y) P_2(x, y, t), \quad (12)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \phi(x) \right] P_{12}(x, y, t) = \alpha_1(y) P_3(x, y, t), \quad (13)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \psi(x) \right] P_{13}(x, y, t) = \alpha_2(y) P_3(x, y, t), \quad (14)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \mu(x) \right] P_{14}(x, y, t) = \alpha_3(y) P_3(x, y, t), \quad (15)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \sigma(x) \right] P_{15}(x, y, t) = \alpha_4(y) P_3(x, y, t), \quad (16)$$

where

$$T_0 = \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y),$$

$$C_0 = \int \mu(x) P_1(x, y, t) dx + \int \sigma(x) P_2(x, y, t) dx + \int \phi(x) P_4(x, y, t) dx + \int \psi(x) P_5(x, y, t) dx,$$

$$T_1(x, y) = \mu(x) + \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y),$$

$$C_1(x, y, t) = \alpha_3(y) P_0(t) + \phi(x) P_6(x, y, t) + \psi(x) P_7(x, y, t) + \mu(x) P_8(x, y, t) + \sigma(x) P_3(x, y, t),$$

$$T_2(x, y) = \sigma(x) \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y),$$

$$C_2(x, y, t) = \alpha_4(y) P_0(t) + \phi(x) P_9(x, y, t) + \psi(x) P_{10}(x, y, t) + \mu(x) P_3(x, y, t) + \sigma(x) P_{11}(x, y, t),$$

$$T_3(x, y) = \sigma(x) + \mu(x) + \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y),$$

$$C_3(x, y, t) = \alpha_4(y) P_1(x, y, t) + \alpha_3(y) P_2(x, y, t) + \phi(x) P_{12}(x, y, t) + \psi(x) P_{13}(x, y, t) + \mu(x) P_{14}(x, y, t) + \sigma(x) P_{15}(x, y, t),$$

Boundary conditions

$$P_1(0, y, t) = \alpha_3(y)P_0(t), \quad (17)$$

$$P_2(0, y, t) = \alpha_4(y)P_0(t), \quad (18)$$

$$P_3(0, y, t) = \int \alpha_4(y)P_1(x, y, t)dx + \int \alpha_3(y)P_2(x, y, t)dx, \quad (19)$$

$$P_4(0, y, t) = \alpha_1(y)P_0(t), \quad (20)$$

$$P_5(0, y, t) = \alpha_2(y)P_0(t), \quad (21)$$

$$P_6(0, y, t) = \int \alpha_1(y)P_1(x, y, t)dx, \quad (22)$$

$$P_7(0, y, t) = \int \alpha_2(y)P_1(x, y, t)dx, \quad (23)$$

$$P_8(0, y, t) = \int \alpha_3(y)P_1(x, y, t)dx, \quad (24)$$

$$P_9(0, y, t) = \int \alpha_1(y)P_2(x, y, t)dx, \quad (25)$$

$$P_{10}(0, y, t) = \int \alpha_2(y)P_2(x, y, t)dx, \quad (26)$$

$$P_{11}(0, y, t) = \int \alpha_4(y)P_2(x, y, t)dx, \quad (27)$$

$$P_{12}(0, y, t) = \int \alpha_1(y)P_3(x, y, t)dx, \quad (28)$$

$$P_{13}(0, y, t) = \int \alpha_2(y)P_3(x, y, t)dx, \quad (29)$$

$$P_{14}(0, y, t) = \int \alpha_3(y)P_3(x, y, t)dx, \quad (30)$$

$$P_{15}(0, y, t) = \int \alpha_4(y)P_3(x, y, t)dx. \quad (31)$$

Initial condition

$$P_i(x, y, 0) = 0; \quad (i = 1, \dots, 15), \quad (32)$$

$$P_0(0) = 1. \quad (33)$$

The system of differential equations (1)–(16) together with the boundary conditions (17)–(31) and initial conditions (32),(33) is called Chapman–Kolmogorov differential difference equation. Eq. (1) is a linear differential equation of first order and other Eqs. (2)–(16) are linear partial differential equations. In order to find the reliability of the system, the governing Eqs.(1)–(16) will be solved. However, such type of mathematical problems could not be solved analytically so far. An attempt has been made by Gupta [7] to solve Chapman–Kolmogorov differential equation formulated under the assumption of constant failure rates and variable repair rates by Lagrange’s Method. Following the approach of Gupta [7], Eqs.(2)–(16) along with the boundary conditions (17)–(31) have been solved to get probabilities $P_i(t)$ ($i = 1, \dots, 15$) for each state:

$$P_{15}(x, y, t) = e^{-\int \sigma(x) dx} \left[\int \alpha_4(y-x) P_3(x, y-x, t-x) dx + \int \alpha_4(y) P_3(x, y, t) e^{\int \sigma(x) dx} dx \right], \quad (34)$$

$$P_{14}(x, y, t) = e^{-\int \mu(x) dx} \left[\int \alpha_3(y-x) P_3(x, y-x, t-x) dx + \int \alpha_3(y) P_3(x, y, t) e^{\int \mu(x) dx} dx \right], \quad (35)$$

$$P_{13}(x, y, t) = e^{-\int \psi(x) dx} \left[\int \alpha_2(y-x) P_3(x, y-x, t-x) dx + \int \alpha_2(y) P_3(x, y, t) e^{\int \psi(x) dx} dx \right], \quad (36)$$

$$P_{12}(x, y, t) = e^{-\int \phi(x) dx} \left[\int \alpha_1(y-x) P_3(x, y-x, t-x) dx + \int \alpha_1(y) P_3(x, y, t) e^{\int \phi(x) dx} dx \right], \quad (37)$$

$$P_{11}(x, y, t) = e^{-\int \sigma(x) dx} \left[\int \alpha_4(y-x) P_2(x, y-x, t-x) dx + \int \alpha_4(y) P_2(x, y, t) e^{\int \sigma(x) dx} dx \right], \quad (38)$$

$$P_{10}(x, y, t) = e^{-\int \psi(x) dx} \left[\int \alpha_2(y-x) P_2(x, y-x, t-x) dx + \int \alpha_2(y) P_2(x, y, t) e^{\int \psi(x) dx} dx \right], \quad (39)$$

$$P_9(x, y, t) = e^{-\int \phi(x) dx} \left[\int \alpha_1(y-x) P_2(x, y-x, t-x) dx + \int \alpha_1(y) P_2(x, y, t) e^{\int \phi(x) dx} dx \right], \quad (40)$$

$$P_8(x, y, t) = e^{-\int \mu(x) dx} \left[\int \alpha_3(y-x) P_1(x, y-x, t-x) dx + \int \alpha_3(y) P_1(x, y, t) e^{\int \mu(x) dx} dx \right], \quad (41)$$

$$P_7(x, y, t) = e^{-\int \psi(x) dx} \left[\int \alpha_2(y-x) P_1(x, y-x, t-x) dx + \int \alpha_2(y) P_1(x, y, t) e^{\int \psi(x) dx} dx \right], \quad (42)$$

$$P_6(x, y, t) = e^{-\int \phi(x) dx} \left[\int \alpha_1(y-x) P_1(x, y-x, t-x) dx + \int \alpha_1(y) P_1(x, y, t) e^{\int \phi(x) dx} dx \right], \quad (43)$$

$$P_5(x, y, t) = e^{-\int \psi(x) dx} \left[\alpha_2(y-x) P_0(t-x) + \int \alpha_2(y) P_0(t) e^{\int \psi(x) dx} dx \right], \quad (44)$$

$$P_4(x, y, t) = e^{-\int \phi(x) dx} \left[\alpha_1(y-x) P_0(t-x) + \int \alpha_1(y) P_0(t) e^{\int \phi(x) dx} dx \right], \quad (45)$$

$$P_3(x, y, t) = e^{-\int T_3(x, y) dx} \left[\int C_3(x, y, t) e^{\int T_3(x, y) dx} dx + \int \alpha_4(y-x) P_1(x, y-x, t-x) dx + \int \alpha_3(y-x) P_2(x, y-x, t-x) dx \right], \quad (46)$$

$$P_2(x, y, t) = e^{-\int T_{23}(x, y) dx} \left[\int C_2(x, y, t) e^{\int T_{23}(x, y) dx} dx + \alpha_4(y-x) P_0(t-x) \right], \quad (47)$$

$$P_1(x, y, t) = e^{-\int T_{13}(x, y) dx} \left[\int C_1(x, y, t) e^{\int T_{13}(x, y) dx} dx + \alpha_3(y-x) P_0(t-x) \right], \quad (48)$$

$$P_0(t) = e^{-T_0 t} \left[1 + \int C_0(t) e^{T_0 t} dt \right]. \quad (49)$$

If the industry provides the failure and repair rates, one can calculate the reliability $R(t)$ in terms of the probability $P_0(t)$ and using Eq. (1). Thus, the time dependent Reliability $R(t)$ of the system is given by

$$R(t) = P_0(t) + \int \sum_{i=1}^2 P_i(x, y, t) dx dy. \quad (50)$$

When both failure and repair rates are constant

As we have seen in previous case that how difficult is to solve the problem analytically if both failure and repair rates are taken as variables. In order to find the reliability of the system when both failure and repair rates are constant, system of Eqs.(1)–(16) reduces to more simplified form which are given below:

$$\left[\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \right] P_0(t) = \mu P_1(t) + \sigma P_2(t) + \phi P_4(t) + \psi P_5(t), \quad (51)$$

$$\left[\frac{d}{dt} + \mu + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \right] P_1(t) = \alpha_3 P_0(t) + \phi P_6(t) + \psi P_7(t) + \mu P_8(t) + \sigma P_3(t), \quad (52)$$

$$\left[\frac{d}{dt} + \sigma + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \right] P_2(t) = \alpha_4 P_0(t) + \phi P_9(t) + \psi P_{10}(t) + \mu P_3(t) + \sigma P_{11}, \quad (53)$$

$$\left[\frac{d}{dt} + \sigma + \mu + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \right] P_3(t) = \alpha_4 P_1(t) + \alpha_3 P_2(t) + \phi P_{12}(t) + \psi P_{13} + \mu P_{14}(t) + \sigma P_{15}(t), \quad (54)$$

$$\left[\frac{d}{dt} + \phi \right] P_4(t) = \alpha_1 P_0(t), \quad (55)$$

$$\left[\frac{d}{dt} + \psi \right] P_5(t) = \alpha_2 P_0, \quad (56)$$

$$\left[\frac{d}{dt} + \phi \right] P_6(t) = \alpha_1 P_1(t), \quad (57)$$

$$\left[\frac{d}{dt} + \psi \right] P_7(t) = \alpha_2 P_1(t), \quad (58)$$

$$\left[\frac{d}{dt} + \mu \right] P_8(t) = \alpha_3 P_1(t), \quad (59)$$

$$\left[\frac{d}{dt} + \phi \right] P_9(t) = \alpha_1 P_2(t), \quad (60)$$

$$\left[\frac{d}{dt} + \psi \right] P_{10}(t) = \alpha_2 P_2(t), \quad (61)$$

$$\left[\frac{d}{dt} + \sigma \right] P_{11}(t) = \alpha_4 P_2(t), \quad (62)$$

$$\left[\frac{d}{dt} + \phi \right] P_{12}(t) = \alpha_1 P_3(t), \quad (63)$$

$$\left[\frac{d}{dt} + \psi \right] P_{13}(t) = \alpha_2 P_3(t), \quad (64)$$

$$\left[\frac{d}{dt} + \mu \right] P_{14}(t) = \alpha_3 P_3(t), \quad (65)$$

$$\left[\frac{d}{dt} + \sigma \right] P_{15}(t) = \alpha_4 P_3(t). \quad (67)$$

Initial conditions

$$P_i(0) = \begin{cases} 1, & i = 0 \\ 0, & \text{otherwise} \end{cases}. \quad (68)$$

Most of the authors have used Laplace transformation and matrix method to solve the reliability function. But it is difficult to find Laplace inverse since expression for probability transforms are in very complicated form and the complexity increases with the increase in number of equations. To overcome such type of problems, the system of differential Eqs.(51–67), with initial conditions (68), has been solved numerically following the approach earlier used by Gupta et al. [14]. The numerically computation have been carried out starting from $t = 0$ to $t = 360$ days assuming $t = 0.005$ assuming as one day. The availability of Polytube has been obtained by taking different combination of the constant failure and repair rates of the subsystems collected from the concerned industry, the availability $A(t)$ of the system can be computed as,

$$A(t) = \sum_{i=0}^3 P_i(t). \quad (69)$$

3.5 ANALYSIS AND DISCUSSION

(a) Effect of failure rate of mixture (α_1) on availability of the system

The availability of the system has been calculated by varying the failure rate of mixture (α_1) from 0.005 to 0.013 and fixing other parameters as

$\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\alpha_4 = 0.56$, $\varphi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

α_1 Time in days	0.005	0.007	0.009	0.011	0.013
30	0.8394	0.8391	0.8387	0.8384	0.8380
60	0.8148	0.8140	0.8142	0.8139	0.8135
90	0.8087	0.8084	0.8081	0.8078	0.8075
120	0.8072	0.8069	0.8066	0.8063	0.8060
150	0.8068	0.8065	0.8062	0.8059	0.8056
180	0.8067	0.8064	0.8061	0.8058	0.8055
210	0.8067	0.8065	0.8061	0.8058	0.8055
240	0.8067	0.8064	0.8061	0.8058	0.8055
270	0.8067	0.8064	0.8061	0.8058	0.8055
300	0.8067	0.8064	0.8061	0.8058	0.8055
330	0.8067	0.8064	0.8061	0.8058	0.8055
360	0.8067	0.8064	0.8061	0.8058	0.8055

Table-3.1.1

(b) Effect of failure rate of extruder (α_2) on availability of the system

The result shown in Table 2 presents the availability of system computed by varying failure rate of extruder (α_2) from 0.007 to 0.023 and keeping other parameters as

$\alpha_1 = 0.007$, $\alpha_3 = 0.23$, $\alpha_4 = 0.56$, $\varphi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

α_2 Time in days	0.007	0.011	0.015	0.019	0.023
30	0.8394	0.7889	0.7427	0.7004	0.6616
60	0.8148	0.7559	0.7041	0.6585	0.61808
90	0.8087	0.7486	0.6965	0.6511	0.6112
120	0.8072	0.7469	0.6950	0.6498	0.6101
150	0.8068	0.7466	0.6947	0.6496	0.6099
180	0.8067	0.7465	0.6946	0.6495	0.6099
210	0.8067	0.7465	0.6946	0.6495	0.6099
240	0.8067	0.7465	0.6946	0.6495	0.6099
270	0.8067	0.7465	0.6946	0.6495	0.6099
300	0.8067	0.7465	0.6946	0.6495	0.60993
330	0.8067	0.7465	0.6946	0.6495	0.6099
360	0.8067	0.7465	0.6946	0.6495	0.6099

Table 3.1.2

(c) Effect of failure rate of die (α_3) on availability of the system

Now, we have studied the effect of failure rate of sub system, namely die, on availability of the system by varying its failure rate (α_3) from 0.23 to 0.31. Fixing the other failure and repair rates of the subsystem as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_4 = 0.56$, $\varphi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$, the availability of the system is calculated using these data and are shown in Table 3.

α_3 Time in days	0.23	0.25	0.27	0.29	0.31
30	0.8394	0.8380	0.8364	0.8348	0.8331
60	0.8148	0.8135	0.8120	0.81054	0.8089
90	0.8087	0.8074	0.8060	0.8045	0.8029
120	0.8072	0.8059	0.8045	0.8030	0.8014
150	0.8068	0.8055	0.8041	0.8026	0.8010
180	0.8067	0.8054	0.8040	0.8025	0.8009
210	0.8067	0.8054	0.8040	0.8025	0.8009
240	0.8067	0.8054	0.8040	0.8025	0.8009
270	0.8067	0.8054	0.8040	0.8253	0.800
300	0.8067	0.8054	0.8040	0.8025	0.8009
330	0.8067	0.8054	0.8040	0.8253	0.8009
360	0.8067	0.8054	0.8040	0.8025	0.8006

Table 3.1.3

(d) Effect of failure rate of cutter (α_4) on availability of the system

Taking failure and repair rates as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\varphi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$ and changing the failure and repair rates of cutter from 0.56 to 0.64, the availability of the system has been calculated by using this data and results of this study are tabulated in Table 4.

α_4 Time in days	0.56	0.58	0.60	0.62	0.64
30	0.8394	0.8325	0.8301	0.8277	0.8253
60	0.8148	0.8083	0.8061	0.8038	0.8015
90	0.8087	0.8023	0.8001	0.7979	0.7956
120	0.8072	0.8008	0.7986	0.7964	0.7942
150	0.8068	0.8005	0.7983	0.7960	0.7938
180	0.8067	0.8004	0.7982	0.7960	0.7937
210	0.8067	0.8003	0.7982	0.7959	0.7937
240	0.8067	0.8003	0.7982	0.7959	0.7937
270	0.8067	0.8003	0.7982	0.7959	0.7937
300	0.8067	0.8003	0.7982	0.7959	0.7937
330	0.8067	0.8003	0.7982	0.7959	0.7937
360	0.80676	0.8003	0.7982	0.7959	0.7937

Table 3.1.4

(e) Effect of repair rate of mixture (ϕ) on availability of the system

The result shown in Table 5 presents the reliability of system computed by varying repair rate of mixture (ϕ) from 4.2 to 5.0 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

ϕ Time in days	4.2	4.4	4.6	4.8	5.0
30	0.8394	0.8394	0.8395	0.8395	0.8395
60	0.8148	0.8148	0.8149	0.8149	0.8149
90	0.8087	0.8088	0.8088	0.8088	0.8088
120	0.8072	0.8072	0.8073	0.8073	0.8073
150	0.8068	0.8069	0.8069	0.8069	0.8070
180	0.8067	0.8068	0.8068	0.8068	0.8069
210	0.8067	0.8068	0.8068	0.8068	0.8068
240	0.8067	0.8068	0.8068	0.806	0.8068
270	0.8067	0.8068	0.8068	0.8068	0.8068
300	0.8067	0.8068	0.8068	0.8068	0.8068
330	0.8067	0.8068	0.8068	0.8068	0.8068
360	0.8067	0.8068	0.8068	0.8068	0.8068

Table 3.1.5

(f) Effect of repair rate of extruder (ψ) on availability of the system

The results shown in the Table 6 present the availability of system computed by varying repair rate of extruder (ψ) from 0.04 to 0.20 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

ψ Time in days	0.04	0.08	0.12	0.16	0.20
30	0.83943	0.8733	0.8916	0.90255	0.9095
60	0.8148	0.8684	0.8906	0.9023	0.9094
90	0.8087	0.8680	0.89059	0.90231	0.90949
120	0.8072	0.8680	0.89059	0.9023	0.9094
150	0.8068	0.8680	0.89059	0.9023	0.9094
180	0.8067	0.8680	0.8905	0.9023	0.9094
210	0.8067	0.8680	0.8905	0.9023	0.9094
240	0.8067	0.8680	0.89059	0.9023	0.9094
270	0.8067	0.8680	0.8905	0.90231	0.9094
300	0.80676	0.8680	0.89059	0.90231	0.9094
330	0.8067	0.8680	0.8905	0.9023	0.9094
360	0.8067	0.8680	0.8905	0.9023	0.9094

Table 3.1.6

(g) Effect of repair rate of die (μ) on availability of the system

Now, we have studied the effect of repair rate of subsystem, namely die, on availability of the system by varying its repair rate of die (μ) from 2 to 6. Fixing the other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

μ Time in days	2	3	4	5	6
30	0.8394	0.8440	0.8458	0.84671	0.8472
60	0.8148	0.8192	0.8208	0.8216	0.8221
90	0.8087	0.8130	0.8146	0.8155	0.8160
120	0.8072	0.8115	0.8131	0.8139	0.8144
150	0.8068	0.8111	0.8128	0.81361	0.8141
180	0.8067	0.8110	0.8127	0.8135	0.8140
210	0.8067	0.8110	0.8126	0.8135	0.8139
240	0.8067	0.8110	0.8126	0.8134	0.8139
270	0.8067	0.8110	0.8126	0.8134	0.8139
300	0.8067	0.8110	0.8126	0.8134	0.8139
330	0.8067	0.8110	0.8126	0.8134	0.8139
360	0.80676	0.8110	0.8126	0.8134	0.8139

Table 3.1.7

(h) Effect of repair rate of cutter (σ) on availability of the system

The results shown in the Table 8 present the availability of system computed by varying repair rate of cutter (σ) from 2.2 to 6.2 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$.

σ Time in days	2.2	3.2	4.2	5.2	6.2
30	0.8394	0.8577	0.8654	0.8693	0.8715
60	0.8148	0.8320	0.8398	0.8429	0.8450
90	0.8087	0.8257	0.8328	0.8364	0.8385
120	0.8072	0.8241	0.8312	0.8348	0.8369
150	0.8068	0.8238	0.8308	0.8344	0.8365
180	0.8067	0.8237	0.8307	0.8343	0.8364
210	0.8067	0.82369	0.8307	0.8343	0.8364
240	0.8067	0.8236	0.8307	0.8343	0.8364
270	0.8067	0.8236	0.8307	0.8343	0.8364
300	0.8067	0.8236	0.8307	0.8343	0.8364
330	0.8067	0.8236	0.8307	0.8343	0.8364
360	0.8067	0.82369	0.8307	0.8343	0.8364

Table 3.1.8

APPENDIX

```
#include<iostream.h>
```

```
#include<conio.h>
```

```
#include<math.h>
```

```
double f1(double p1,double p2, double p3, double p5, double p6, double p7, double p8, double p17,  
double p18, double lem1, double b1, double b2,  
double b3, double b4, double b5, double b6, double b7, double b8 )
```

```
{  
    return -lem1*p1+b1*p5+b2*p6+b3*p7+b4*p8+b5*p2+b6*p3+b8*p18+b7*p17;  
}
```

```
double f2(double p1,double p2, double p9, double p10, double p11, double p12, double p20, double lem2,  
double b1, double b2,  
double b3, double b4, double b8, double a5 )
```

```
{  
    return -lem2*p2+b1*p9+b2*p10+b3*p11+b4*p12+b8*p20+a5*p1;  
}
```

```
double f3(double p1,double p3, double p13, double p14, double p15, double p16, double p19, double  
lem3, double b1, double b2,  
double b3, double b4, double b7, double a6 )
```

```
{  
    return -lem3*p3+b1*p13+b2*p14+b3*p15+b4*p16+b7*p19+a6*p1;  
}
```

```
double f4(double p2,double p3, double p4, double p21, double p22, double p23, double p24, double lem4,  
double b1, double b2,  
double b3, double b4, double a5, double a6 )
```

```
{  
    return -lem4*p4+b1*p21+b2*p22+b3*p23+b4*p24+a5*p3+a6*p2;  
}
```

```
double f5(double p1,double p5,double b1, double a1 )
```

```
{  
    return -b1*p5+a1*p1;
```

```

}
double f6(double p1,double p6,double b2, double a2 )
{
    return -b2*p6+a2*p1;
}
double f7(double p1,double p7,double b3, double a3 )
{
    return -b3*p7+a3*p1;
}
double f8(double p1,double p8,double b4, double a4 )
{
    return -b4*p8+a4*p1;
}
double f9(double p2,double p9,double b1, double a1 )
{
    return -b1*p9+a1*p2;
}
double f10(double p2,double p10,double b2, double a2 )
{
    return -b2*p10+a2*p2;
}
double f11(double p2,double p11,double b3, double a3 )
{
    return -b3*p11+a3*p2;
}
double f12(double p2,double p12,double b4, double a4 )
{
    return -b4*p12+a4*p2;
}
double f13(double p3,double p13,double b1, double a1 )
{
    return -b1*p13+a1*p3;
}
double f14(double p3,double p14,double b2, double a2 )
{

```

```

    return -b2*p14+a2*p3;
}
double f15(double p3,double p15,double b3, double a3 )
{
    return -b3*p15+a3*p3;
}
double f16(double p3,double p16,double b4, double a4 )
{
    return -b4*p16+a4*p3;
}
double f17(double p2,double p17,double b7, double a7 )
{
    return -b7*p17+a7*p2;
}
double f18(double p3,double p18,double b8, double a8 )
{
    return -b8*p18+a8*p3;
}
double f19(double p4,double p19,double b7, double a7 )
{
    return -b7*p19+a7*p4;
}
double f20(double p4,double p20,double b8, double a8 )
{
    return -b8*p20+a8*p4;
}
double f21(double p4,double p21,double b1, double a1 )
{
    return -b1*p21+a1*p4;
}
double f22(double p4,double p22,double b2, double a2 )
{
    return -b2*p22+a2*p4;
}
double f23(double p4,double p23,double b3, double a3 )

```

```

{
    return -b3*p23+a3*p4;
}

double f24(double p4,double p24,double b4, double a4 )
{
    return -b4*p24+a4*p4;
}

int main()
{
    double p1,p2,p3,p4,p5,p6,p7,p8,p9,p10,p11,p12,p13,p14,p15,p16,p17,p18,p19,p20,p21,p22,p23,p24;

    double a1,a2,a3,a4,a5,a6,a7,a8;
    double b1,b2,b3,b4,b5,b6,b7,b8;
        double lem1,lem2,lem3,lem4,h,rt;
        double t=0.;
    cout<<"enter the step lenth\n";
        cin>>h;
        cout<<" enter the values of alphas\n";
        cin>>a1>>a2>>a3>>a4>>a5>>a6>>a7>>a8;
        cout<<" enter the values of betas\n";
        cin>>b1>>b2>>b3>>b4>>b5>>b6>>b7>>b8;

        lem1=a1+a2+a3+a4+a5+a6;
        lem2=a1+a2+a3+a4+a7+a6+b5;
        lem3=a1+a2+a3+a4+a5+a8+b6;
        lem4=a1+a2+a3+a4+a7+a8;

//      Inital Conditions
    p1=1;

    p2=p3=p4=p5=p6=p7=p8=p9=p10=p11=p12=p13=p14=p15=p16=p17=p18=p19=p20=p21=p22=p23=p24=0;
    do
    {
        double k[4][24];
        k[0][1]=h*f1(p1,p2,p3,p5,p6,p7,p8,p17,p18,lem1,b1,b2,b3,b4,b5,b6,b7,b8 );
        k[0][2]=h*f2(p1,p2,p9,p10,p11,p12,p20,lem2,b1,b2,b3,b4,b8,a5);
    }

```

$k[0][3]=h*f3(p1,p3,p13, p14,p15,p16,p19,lem3,b1,b2,b3,b4,b7,a6);$
 $k[0][4]=h*f4(p2,p3,p4,p21,p22,p23,p24,lem4,b1,b2,b3,b4,a5,a6);$
 $k[0][5]=h*f5(p1,p5,b1,a1);$
 $k[0][6]=h*f6(p1,p6,b2,a2);$
 $k[0][7]=h*f7(p1,p7,b3,a3);$
 $k[0][8]=h*f8(p1,p8,b4,a4);$
 $k[0][9]=h*f9(p2,p9,b1,a1);$
 $k[0][10]=h*f10(p2,p10,b2,a2);$
 $k[0][11]=h*f11(p2,p11,b3,a3);$
 $k[0][12]=h*f12(p2,p12,b4,a4);$
 $k[0][13]=h*f13(p3,p13,b1,a1);$
 $k[0][14]=h*f14(p3,p14,b2,a2);$
 $k[0][15]=h*f15(p3,p15,b3,a3);$
 $k[0][16]=h*f16(p3,p16,b4,a4);$
 $k[0][17]=h*f17(p2,p17,b7,a7);$
 $k[0][18]=h*f18(p3,p18,b8,a8);$
 $k[0][19]=h*f19(p4,p19,b7,a7);$
 $k[0][20]=h*f20(p4,p20,b8,a8);$
 $k[0][21]=h*f21(p4,p21,b1,a1);$
 $k[0][22]=h*f22(p4,p22,b2,a2);$
 $k[0][23]=h*f23(p4,p23,b3,a3);$
 $k[0][24]=h*f24(p4,p24,b4,a4);$

//2d stage

$k[1][1]=h*f1(p1+k[0][1]/2.,p2+k[0][2]/2.,p3+k[0][3]/2.,p5+k[0][5]/2.,p6+k[0][6]/2.,p7+k[0][7]/2.,p8+k[0][8]/2.,p17+k[0][17]/2.,p18+k[0][18]/2.,lem1,b1,b2,b3,b4,b5,b6,b7,b8);$

$k[1][2]=h*f2(p1+k[0][1]/2.,p2+k[0][2]/2.,p9+k[0][9]/2.,p10+k[0][10]/2.,p11+k[0][11]/2.,p12+k[0][12]/2.,p20+k[0][20]/2.,lem2,b1,b2,b3,b4,b8,a5);$

$k[1][3]=h*f3(p1+k[0][1]/2.,p3+k[0][3]/2.,p13+k[0][13]/2.,p14+k[0][14]/2.,p15+k[0][15]/2.,p16+k[0][16]/2.,p19+k[0][19]/2.,lem3,b1,b2,b3,b4,b7,a6);$

$k[1][4]=h*f4(p2+k[0][2]/2.,p3+k[0][3]/2.,p4+k[0][4]/2.,p21+k[0][21]/2.,p22+k[0][22]/2.,p23+k[0][23]/2.,p24+k[0][24]/2.,lem4,b1,b2,b3,b4,a5,a6);$

$k[1][5]=h*f5(p1+k[0][1]/2.,p5+k[0][5]/2.,b1,a1);$
 $k[1][6]=h*f6(p1+k[0][1]/2.,p6+k[0][6]/2.,b2,a2);$
 $k[1][7]=h*f7(p1+k[0][1]/2.,p7+k[0][7]/2.,b3,a3);$
 $k[1][8]=h*f8(p1+k[0][1]/2.,p8+k[0][8]/2.,b4,a4);$
 $k[1][9]=h*f9(p2+k[0][2]/2.,p9+k[0][9]/2.,b1,a1);$
 $k[1][10]=h*f10(p2+k[0][2]/2.,p10+k[0][10]/2.,b2,a2);$
 $k[1][11]=h*f11(p2+k[0][2]/2.,p11+k[0][11]/2.,b3,a3);$
 $k[1][12]=h*f12(p2+k[0][2]/2.,p12+k[0][12]/2.,b4,a4);$
 $k[1][13]=h*f13(p3+k[0][3]/2.,p13+k[0][13]/2.,b1,a1);$
 $k[1][14]=h*f14(p3+k[0][3]/2.,p14+k[0][14]/2.,b2,a2);$
 $k[1][5]=h*f15(p3+k[0][3]/2.,p15+k[0][15]/2.,b3,a3);$
 $k[1][16]=h*f16(p3+k[0][3]/2.,p16+k[0][16]/2.,b4,a4);$
 $k[1][17]=h*f17(p2+k[0][2]/2.,p17+k[0][17]/2.,b7,a7);$
 $k[1][18]=h*f18(p3+k[0][3]/2.,p18+k[0][18]/2.,b8,a8);$
 $k[1][19]=h*f19(p4+k[0][4]/2.,p19+k[0][19]/2.,b7,a7);$
 $k[1][20]=h*f20(p4+k[0][4]/2.,p20+k[0][20]/2.,b8,a8);$
 $k[1][21]=h*f21(p4+k[0][4]/2.,p21+k[0][21]/2.,b1,a1);$
 $k[1][22]=h*f22(p4+k[0][4]/2.,p22+k[0][22]/2.,b2,a2);$
 $k[1][23]=h*f23(p4+k[0][4]/2.,p23+k[0][23]/2.,b3,a3);$
 $k[1][24]=h*f24(p4+k[0][4]/2.,p24+k[0][24]/2.,b4,a4);$

//3rd Stage

$k[2][1]=h*f1(p1+k[1][1]/2.,p2+k[1][2]/2.,p3+k[1][3]/2.,p5+k[1][5]/2.,p6+k[1][6]/2.,p7+k[1][7]/2.,p8+k[1][8]/2.,p17+k[1][17]/2.,p18+k[0][18]/2.,lem1,b1,b2,b3,b4,b5,b6,b7,b8);$

$k[2][2]=h*f2(p1+k[1][1]/2.,p2+k[1][2]/2.,p9+k[1][9]/2.,p10+k[1][10]/2.,p11+k[1][11]/2.,p12+k[1][12]/2.,p20+k[1][20]/2.,lem2,b1,b2,b3,b4,b8,a5);$

$k[2][3]=h*f3(p1+k[1][1]/2.,p3+k[1][3]/2.,p13+k[1][13]/2.,p14+k[1][14]/2.,p15+k[1][15]/2.,p16+k[1][16]/2.,p19+k[1][19]/2.,lem3,b1,b2,b3,b4,b7,a6);$

$k[2][4]=h*f4(p2+k[1][2]/2.,p3+k[1][3]/2.,p4+k[1][4]/2.,p21+k[1][21]/2.,p22+k[1][22]/2.,p23+k[1][23]/2.,p24+k[1][24]/2.,lem4,b1,b2,b3,b4,a5,a6);$

$k[2][5]=h*f5(p1+k[1][1]/2.,p5+k[1][5]/2.,b1,a1);$

$k[2][6]=h*f6(p1+k[1][1]/2.,p6+k[1][6]/2.,b2,a2);$

$k[2][7]=h*f7(p1+k[1][1]/2.,p7+k[1][7]/2.,b3,a3);$
 $k[2][8]=h*f8(p1+k[1][1]/2.,p8+k[1][8]/2.,b4,a4);$
 $k[2][9]=h*f9(p2+k[1][2]/2.,p9+k[1][9]/2.,b1,a1);$
 $k[2][10]=h*f10(p2+k[1][2]/2.,p10+k[1][10]/2.,b2,a2);$
 $k[2][11]=h*f11(p2+k[1][2]/2.,p11+k[1][11]/2.,b3,a3);$
 $k[2][12]=h*f12(p2+k[1][2]/2.,p12+k[1][12]/2.,b4,a4);$
 $k[2][13]=h*f13(p3+k[1][3]/2.,p13+k[1][13]/2.,b1,a1);$
 $k[2][14]=h*f14(p3+k[1][3]/2.,p14+k[1][14]/2.,b2,a2);$
 $k[2][5]=h*f15(p3+k[1][3]/2.,p15+k[1][15]/2.,b3,a3);$
 $k[2][16]=h*f16(p3+k[1][3]/2.,p16+k[1][16]/2.,b4,a4);$
 $k[2][17]=h*f17(p2+k[1][2]/2.,p17+k[1][17]/2.,b7,a7);$
 $k[2][18]=h*f18(p3+k[1][3]/2.,p18+k[1][18]/2.,b8,a8);$
 $k[2][19]=h*f19(p4+k[1][4]/2.,p19+k[1][19]/2.,b7,a7);$
 $k[2][20]=h*f20(p4+k[1][4]/2.,p20+k[1][20]/2.,b8,a8);$
 $k[2][21]=h*f21(p4+k[1][4]/2.,p21+k[1][21]/2.,b1,a1);$
 $k[2][22]=h*f22(p4+k[1][4]/2.,p22+k[1][22]/2.,b2,a2);$
 $k[2][23]=h*f23(p4+k[1][4]/2.,p23+k[1][23]/2.,b3,a3);$
 $k[2][24]=h*f24(p4+k[1][4]/2.,p24+k[1][24]/2.,b4,a4);$
// 4rth stage

$k[3][1]=h*f1(p1+k[2][1]/2.,p2+k[2][2]/2.,p3+k[2][3]/2.,p5+k[2][5]/2.,p6+k[2][6]/2.,p7+k[2][7]/2.,p8+k[2][8]/2.,p17+k[2][17]/2.,p18+k[0][18]/2.,lem1,b1,b2,b3,b4,b5,b6,b7,b8);$

$k[3][2]=h*f2(p1+k[2][1]/2.,p2+k[2][2]/2.,p9+k[2][9]/2.,p10+k[2][10]/2.,p11+k[2][11]/2.,p12+k[2][12]/2.,p20+k[2][20]/2.,lem2,b1,b2,b3,b4,b8,a5);$

$k[3][3]=h*f3(p1+k[2][1]/2.,p3+k[2][3]/2.,p13+k[2][13]/2.,p14+k[2][14]/2.,p15+k[2][15]/2.,p16+k[2][16]/2.,p19+k[2][19]/2.,lem3,b1,b2,b3,b4,b7,a6);$

$k[3][4]=h*f4(p2+k[2][2]/2.,p3+k[2][3]/2.,p4+k[2][4]/2.,p21+k[2][21]/2.,p22+k[2][22]/2.,p23+k[2][23]/2.,p24+k[2][24]/2.,lem4,b1,b2,b3,b4,a5,a6);$

$k[3][5]=h*f5(p1+k[2][1]/2.,p5+k[2][5]/2.,b1,a1);$

$k[3][6]=h*f6(p1+k[2][1]/2.,p6+k[2][6]/2.,b2,a2);$

$k[3][7]=h*f7(p1+k[2][1]/2.,p7+k[2][7]/2.,b3,a3);$

$k[3][8]=h*f8(p1+k[2][1]/2.,p8+k[2][8]/2.,b4,a4);$

$k[3][9]=h*f_9(p_2+k[2][2]/2.,p_9+k[2][9]/2.,b_1,a_1);$
 $k[3][10]=h*f_{10}(p_2+k[2][2]/2.,p_{10}+k[2][10]/2.,b_2,a_2);$
 $k[3][11]=h*f_{11}(p_2+k[2][2]/2.,p_{11}+k[2][11]/2.,b_3,a_3);$
 $k[3][12]=h*f_{12}(p_2+k[2][2]/2.,p_{12}+k[2][12]/2.,b_4,a_4);$
 $k[3][13]=h*f_{13}(p_3+k[2][3]/2.,p_{13}+k[2][13]/2.,b_1,a_1);$
 $k[3][14]=h*f_{14}(p_3+k[2][3]/2.,p_{14}+k[2][14]/2.,b_2,a_2);$
 $k[3][15]=h*f_{15}(p_3+k[2][3]/2.,p_{15}+k[2][15]/2.,b_3,a_3);$
 $k[3][16]=h*f_{16}(p_3+k[2][3]/2.,p_{16}+k[2][16]/2.,b_4,a_4);$
 $k[3][17]=h*f_{17}(p_2+k[2][2]/2.,p_{17}+k[2][17]/2.,b_7,a_7);$
 $k[3][18]=h*f_{18}(p_3+k[2][3]/2.,p_{18}+k[2][18]/2.,b_8,a_8);$
 $k[3][19]=h*f_{19}(p_4+k[2][4]/2.,p_{19}+k[2][19]/2.,b_7,a_7);$
 $k[3][20]=h*f_{20}(p_4+k[2][4]/2.,p_{20}+k[2][20]/2.,b_8,a_8);$
 $k[3][21]=h*f_{21}(p_4+k[2][4]/2.,p_{21}+k[2][21]/2.,b_1,a_1);$
 $k[3][22]=h*f_{22}(p_4+k[2][4]/2.,p_{22}+k[2][22]/2.,b_2,a_2);$
 $k[3][23]=h*f_{23}(p_4+k[2][4]/2.,p_{23}+k[2][23]/2.,b_3,a_3);$
 $k[3][24]=h*f_{24}(p_4+k[2][4]/2.,p_{24}+k[2][24]/2.,b_4,a_4);$

$t+=h;$

$p_1+=(k[0][1]+k[1][1]+k[2][1]+k[3][1])/6.;$
 $p_2+=(k[0][2]+k[1][2]+k[2][2]+k[3][2])/6.;$
 $p_3+=(k[0][3]+k[1][3]+k[2][3]+k[3][3])/6.;$
 $p_4+=(k[0][4]+k[1][4]+k[2][4]+k[3][4])/6.;$
 $p_5+=(k[0][5]+k[1][5]+k[2][5]+k[3][5])/6.;$
 $p_6+=(k[0][6]+k[1][6]+k[2][6]+k[3][6])/6.;$
 $p_7+=(k[0][7]+k[1][7]+k[2][7]+k[3][7])/6.;$
 $p_8+=(k[0][8]+k[1][8]+k[2][8]+k[3][8])/6.;$
 $p_9+=(k[0][9]+k[1][9]+k[2][9]+k[3][9])/6.;$
 $p_{10}+=(k[0][10]+k[1][10]+k[2][10]+k[3][10])/6.;$
 $p_{11}+=(k[0][11]+k[1][11]+k[2][11]+k[3][11])/6.;$
 $p_{12}+=(k[0][12]+k[1][12]+k[2][12]+k[3][12])/6.;$
 $p_{13}+=(k[0][13]+k[1][13]+k[2][13]+k[3][13])/6.;$
 $p_{14}+=(k[0][14]+k[1][14]+k[2][14]+k[3][14])/6.;$
 $p_{15}+=(k[0][15]+k[1][15]+k[2][15]+k[3][15])/6.;$
 $p_{16}+=(k[0][16]+k[1][16]+k[2][16]+k[3][16])/6.;$
 $p_{17}+=(k[0][17]+k[1][17]+k[2][17]+k[3][17])/6.;$

```
p18+=(k[0][18]+k[1][18]+k[2][18]+k[3][18])/6.;  
p19+=(k[0][19]+k[1][19]+k[2][19]+k[3][19])/6.;  
p20+=(k[0][20]+k[1][20]+k[2][20]+k[3][20])/6.;  
p21+=(k[0][21]+k[1][21]+k[2][21]+k[3][21])/6.;  
p22+=(k[0][22]+k[1][22]+k[2][22]+k[3][22])/6.;  
p23+=(k[0][23]+k[1][23]+k[2][23]+k[3][23])/6.;  
p24+=(k[0][24]+k[1][24]+k[2][24]+k[3][24])/6.;
```

```
}while(t<=360);
```

```
rt=p1+p2+p3+p4;
```

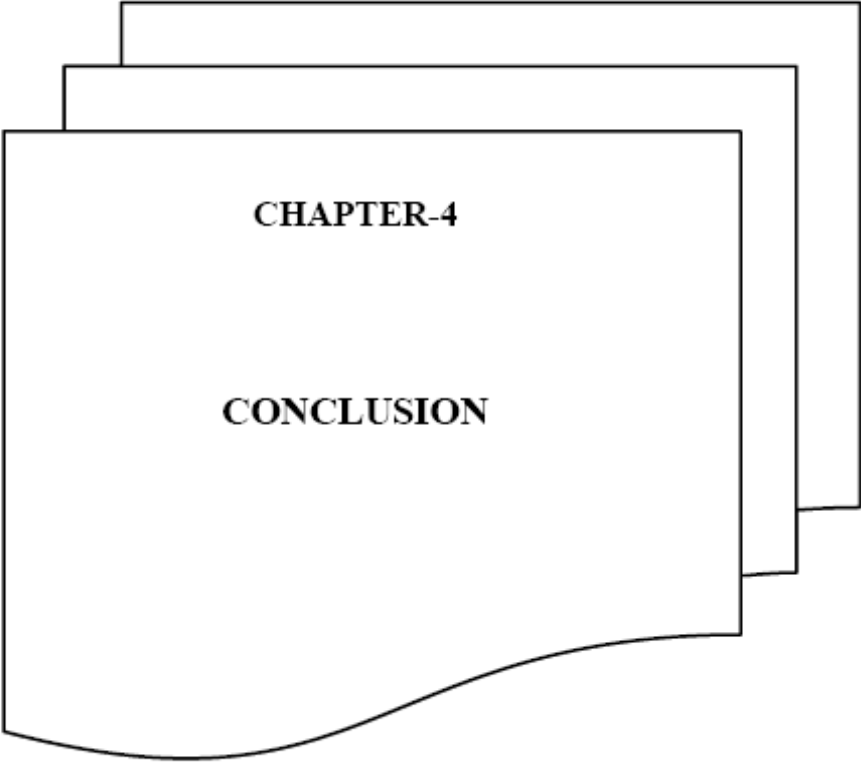
```
cout<<" RT is given by\n";
```

```
cout<<rt;
```

```
getch();
```

```
return 0;
```

```
}
```



CHAPTER-4

CONCLUSION

CHAPTER-4

CONCLUSION

Table 2.1.1 shows that with the increase in failure rate of mixture α_1 from 0.0057 to 0.0063, by keeping other parameters constant ($\lambda_1=0.001$, $\lambda_2=0.002$, $\alpha_2=0.007$, $\alpha_3=0.0133$, $\alpha_4=0.015$, $\psi=2$, $\mu=0.33$, $\sigma=0.02$), the availability of system decreases approximately 1.13%. Whereas, the availability increases approximately 0.63% with the increase repair rate of mixture from 0.5 to 1.1. Similarly, table 2.1.5 shows that availability decreases approximately by 1.12% with the increase of failure rate of mixture α_1 from 0.0057 to 0.0063 keeping other parameters constant ($\lambda_1=0.001$, $\lambda_2=0.002$, $\alpha_2=0.007$, $\alpha_3=0.01$, $\alpha_4=0.015$, $\psi=2$, $\mu=0.33$, $\sigma=0.02$) whereas, the availability increases approximately 0.63% with the increase repair rate of mixture $\square$ from 0.5 to 1.1. Tables 2.1.2, 2.1.3, 2.1.4, 2.1.6, 2.1.7 and 2.1.8 shows that there is almost negligible change in the availability of the Polytube extrusion system with the increase the repair rate of Extruder, Die and Cutter Subsystems.

The comparative study of the table 2.1.1 and table 2.1.5 shows that when the Sub-Systems Die and Cutter failed simultaneously or independently, then in both cases, the Sub- System A i.e. Mixture affects the Availability appreciably. We, thus make an inference that management should take paramount care of Sub-System A in order to improve overall reliability.

The availability of the system has been calculated by varying the failure rate of mixture (α_1) from 0.005 to 0.013 and fixing other parameters as $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\alpha_4 = 0.56$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. The results are shown in Table 3.1.1. It seems that with increase in failure rate of mixture from 0.005 to 0.013 from the Table 3.1.1, the availability of the system decreases by approximately 0.14–0.12% whereas it decreases approximately 3.3–3.2% with increase in the time from 30 days to 360 days.

The result shown in Table 3.1.2 presents the availability of system computed by varying failure rate of extruder (α_2) from 0.007 to 0.023 and keeping other parameters as $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\alpha_4 = 0.56$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. It can be seen that with increase in failure rate of extruder (α_2) from 0.007 to 0.023, the availability of the system decreases approximately from 17.7–19.6% whereas the decrease is approximately 3.3–5.2%, with increase in the time from 30 days to 360 days.

Now, we have studied the effect of failure rate of sub system, namely die, on availability of the system by varying its failure rate (α_3) from 0.23 to 0.31. Fixing the other failure and repair rates of the subsystem as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_4 = 0.56$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$, the availability of the system is calculated using these data and are shown in Table 3.1.3.

It seems that with increase in failure rate of die from 0.23 to 0.31, the availability of the system decreases approximately 0.63–0.58% whereas it decreases approximately 3.3–3.2% with increase in the time from 30 days to 360 days.

Taking failure and repair rates as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$ and changing the failure and repair rates of cutter from 0.56 to 0.64, the availability of the system has been calculated by using this data and results of this study are tabulated in Table 3.1.4. It indicates that with increase in failure rate of cutter from 0.56 to 0.64, the availability of the system decreases approximately 1.4–1.3% whereas it decreases approximately 3.3–3.1% with increase in the time from 30 days to 360 days.

The result shown in Table 3.1.5 presents the reliability of system computed by varying repair rate of mixture (ϕ) from 4.2 to 5.0 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. It indicates that with increase in repair rate of mixture (ϕ) from 4.2 to 5.0, the availability of the system increases approximately 0.01% whereas the availability decreases approximately 3.3–3.2% with increase in the time from 30 days to 360 days.

The results shown in the Table 3.1.6 present the availability of system computed by varying repair rate of extruder (ψ) from 0.04 to 0.20 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. It seems that increase in repair rate of extruder from 0.04 to 0.20 increases the availability of the system by approximately 7.0–10.2% whereas the availability decreases approximately 3.3–0.01% with increase in the time from 30 days to 360 days.

Now, we have studied the effect of repair rate of subsystem, namely die, on availability of the system by varying its repair rate of die (μ) from 3.1.2 to 3.1.6. Fixing the other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. The availability reliability of the system is calculated using these data and results are shown in Table 3.1.7.

It seems that increase in repair rate of die (μ) from 3.1.2 to 3.1.6 increases the availability of the system approximately 0.79–0.72% whereas the availability decreases approximately 3.3% with increase in the time from 30 days to 360 days.

The results shown in the Table 3.1.8 present the availability of system computed by varying repair rate of cutter (σ) from 2.2 to 6.2 and fixing other parameters as $\alpha_1 = 0.005$, $\alpha_2 = 0.007$, $\alpha_3 = 0.23$, $\phi = 4.2$, $\psi = 0.04$, $\mu = 2$ and $\sigma = 2.2$. It indicates that with increase in repair rate of cutter (σ) from 2.2 to 6.2, the availability of the system increases approximately 3.2–2.9% whereas the availability decreases approximately 3.3–3.5% with increase in the time from 30 days to 360 days.

The comparative study of the [Tables 3.1.1–3.1.8](#) shows that the sub system B, i.e. extruder, affects the availability of the whole system more than any of other subsystem. The other subsystems are almost equally affected, which is clearly observable from the tables. We, thus, make an inference that management should take paramount care of sub system B in order to improve overall availability.

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