A thesis report on

TOOL PATH GENERATION FOR FREE FORM SURFACES USING B-SPLINE SURFACE

Submitted in partial fulfillment of the requirements for the award of degree of

Master of Engineering

In

CAD/CAM & ROBOTICS

Submitted By

SIMRANPREET SINGH RANDHAWA

Roll No. 801081029

Under the Guidance of

Dr. J.S. SAINI

Assistant Professor

Department of Mechanical Engineering

Thapar University, Patiala

Department of Mechanical Engineering

Thapar University, Patiala

July, 2012

CERTIFICATE

I, hereby declare that the matter which is being presented in the thesis report on **"TOOL PATH GENERATION FOR FREE FORM SURFACES USING B-SPLINE SURFACE"**, in partial fulfillment of the requirements for the award of degree of Master of Engineering in *CAD / CAM & ROBOTICS* submitted in Mechanical Engineering Department of Thapar University, Patiala, is a work carried out by me.

The matter embodied in this report has not been submitted in part or full to any other university or institute for the award of any other degree.

Simranpreet Singh

Simranpreet Singh Randhawa
801081029
M.E. (CCR)

This is to certify that above declaration made by the student concerned is correct to the best of my knowledge and belief.

11/7/12

Dr. Jaswinder Singh Saini
Assistant Professor,
Department of Mechanical Engineering,
Thapar University,Patiala.

Dr. Ajay Batish
Professor & Head,
Mechanical Engineering Department,
Thapar University, Patiala.

Dr. S.K. Mohapatra
Dean Academic Affairs,
Thapar University, Patiala

ACKNOWLEDGEMENT

Words are often less to reveal ones deep regards. With an understanding that work like this can never be the outcome of a single person. I take this opportunity to express my profound sense of gratitude and respect to all those who directly or indirectly helped me through the duration of this work.

I take the opportunity to express my heartfelt adulation and gratitude to my supervisor **Dr. Jaswinder Singh Saini** for his unreserved guidance, constructive suggestions, thought provoking discussions and unabashed inspiration in the nurturing work. It has been a benediction for me to spend many opportune moments under the guidance of the perfectionist at the acme of professionalism. The present work is testimony to his activity, inspiration and ardent personal interest, taken by him during the course of this work in its present form.

I take pride of myself being son of ideal parents for their everlasting desire, sacrifice, affectionate blessings, and help, without which it would not have been possible for me to complete my studies.

No words acknowledge the support I received from my friends for their valorous help and co-operation. I would like to thank all the members and employees of Mechanical Engineering Department, Thapar University, Patiala for their everlasting support. Above all, I express my indebtedness to the "ALMIGHTY" for all his blessings and kindness.

Simranpreet Singh Randhawa

ABSTRACT

Tool path generation is an important step for the machining of the free form surfaces. The accuracy of machining free form surfaces greatly depends upon the tool path. A number of algorithms had been proposed by different researchers for the accurate and efficient tool path generation for the machining of the free form surfaces.

The presentstudy is focused on implementation of an algorithm given by Choi and Banerjee [29]that generates tool paths for free-form surfaces based on the accuracy of a desiredmanufactured part. Thisalgorithm includes two components. First is the forward-step function that determines the maximum distance between two cutter contact points with a given tolerance. The second component is the side step function which determines themaximum distancebetween two adjacent tool paths with a given scallop height. These functions are independent of the surface type and are applicable to allcontinuous parametric surfaces that are twice differentiable. This algorithm reduces Cutter Contact points while keeping the given tolerance and scallop height in the tool paths. The algorithm is thereafter modified using the B-spline surface. The modified algorithm is then used to machine a wax component which is compared with the desired surface.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 GENERAL

Machining is a form of subtractive manufacturing, in which a collection of material-working processes utilizing power-driven machine tools, such as saws, lathes, milling machines, and drill presses, are used with a sharp cutting tool to physically remove material to achieve a desired geometry [36]. Machining is the most important of the manufacturing processes. Machining can be also defined as the process of removing material from a workpiece in the form of chips. The term metal cutting is used when the material is metallic.

The three principal machining processes are classified as turning, milling and drilling. Other operations falling into miscellaneous categories include shaping, planning, boring, broaching and sawing. Turning operations are operations that rotate the workpiece as the primary method of moving metal against the cutting tool. Lathes are the principal machine tool used in turning.Milling operations are operations in which the cutting tool rotates to bring cutting edges to bear against the workpiece. Milling machines are the principal machine tool used in milling.Drilling operations are operations in which holes are produced or refined by bringing a rotating cutter with cutting edges at the lower extremity into contact with the workpiece. Drilling operations are done primarily in drill presses but sometimes on lathes or mills.Miscellaneous operations are operations that may not produce chips duringmachining but these operations are performed at a typical machine tool. Burnishing is an example of a miscellaneous operation. Burnishing produces no chips but can be performed at a lathe, mill, or drill press. More recent, advanced machining techniques like Electrical Discharge Machining (EDM), Electro-Chemical Erosion, Laser Cutting, or Water Jet Cutting are used to shape metal workpieces.

The machining of materials using machines is not very accurate, to some extent, it also depends upon the skill of the worker operating the machine. The production rate on a machine also depends on the work piece. Hence there are compromises in production rate as well as in accuracy. These drawbacks led people moving towards NC/CNC machines. The accuracy is greater as compared to a conventional machine, accuracy of NC/CNC machine depends upon the part program prepared by the programmer. The production rate is also high as compared to the conventional machines. The advantages of the NC/CNC machines increased their usage as compared to the conventional machines.

1.2NC/CNC MACHINES

Numerical Control (NC) refers to the automation of machine tools that are operated by abstractly programmed commands encoded on a storage medium, as opposed to manually controlled via handwheels or levers, or mechanically automated via cams alone[34]. The first NC machines were built in the 1940s and 1950s, based on existing tools that were modified with motors that moved the controls to follow points fed into the system on punched tape. These early servomechanisms were rapidly augmented with analog and digital computers, creating the modern Computer Numerical Control (CNC) machine tools that have revolutionized the machining processes.

In modern CNC systems, end-to-end component design is highly automated using Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) programs. The programs produce a computer file that is interpreted to extract the commands needed to operate a particular machine via a postprocessor, and then loaded into the CNC machines for production. Since any particular component might require the use of a number of different tools-drills, saws, etc., modern machines often combine multiple tools into a single "cell". In other cases, a number of different machines are used with an external controller and human or robotic operators that move the component from machine to machine. In either case, the complex series of steps needed to produce any part is highly automated and produces a part that closely matches the original CAD design. In CNC, a "crash" occurs when the machine moves in such a way that is harmful to the machine, tools, or parts being machined, sometimes resulting in bending or breakage of cutting tools, accessory clamps, vises, and fixtures, or causing damage to the machine itself by bending guide rails, breaking drive screws, or causing structural components to crack or deform under strain. A mild crash may not damage the machine or tools, but may damage the part being machined so that it must be scrapped.

In CNC machine the program is put into the machine and the machine is then ready for production. Some machined components will generally require a number of different tooling applications such as drilling, reaming and tapping etc., and most modern machines will combine tools within a single cell. This cell will move or rotate to apply the required tooling application, and this will also be controlled by the CNC system.

The main benefit of CNC machine is that the cutting process is controlled by a central computer. This eliminates a lot of the human error which can exist with standard machine. The precise nature of computer control means that a lot of the tasks impossible with human controlled are possible with CNC machines. The vertical cutting also can move along a z-axis, allowing precise methods of cutting (such as engraving), not possible with manual machines. Although originally expensive, CNC machine has dropped in price, thanks to the low cost of computers and open source software.

CNC machines are traditionally programmed using manual part programming consisting of G-codes and M-codes. It is the most common method of programming, some machine-tool/control manufacturers also have invented their own proprietary "conversational" methods of programming, trying to make it easier to program simple parts and make set-up and modifications at the machine easier .These have met with varying success.It is also known as G-code programming. Used mainly in automation, it is part of Computer-Aided Engineering. G-code is sometimes also called G programming language. In decades past G-code was the more common term, and remains such among many users. G-codes are also called preparatory codes, and are any word in a CNC program that activates or prepare the compiles, begins with the letter "G". This includes:

i. Rapid move (transport the tool through space to the place where it is needed for cutting; do this as quickly as possible).
ii. Controlled feed move in a straight line or arc.
iii. Series of controlled feed moves that would result in a hole being bored, a workpiece cut (routed) to a specific dimension, or a profile (contour) shape added to the edge of a workpiece.
iv. Set tool information such as offset.
v. Switch coordinate systems.

M codes are miscellaneous codes used to perform various functions like spindle start/stop, coolant on/of , automatic tool change , end of program etc.

G and M codes are used for the machining of regular surfaces, but for the machining of irregular surfaces or free form surfaces different approaches are developed for the tool path generation. As there are irregularities in the surface the effort is done to plan the tool path efficiently so that no area of the surface remainunmachined.

1.3FREE FORM SURFACES

The surfaces which are not regular in their shape and are made by joining different patches together by defining some continuity at points of joining are called free form surfaces. Freeform surfaces, also called sculptured surfaces, have been widely used in aerospace, automobile, consumer products andthe die/mold industry. Freeform surfaces are usually designed tomeet or improve an aesthetic and/or functional requirement. Often they are defined as surfaces containing oneor more non-planar, non-quadratic surfaces generally representedby parametric and/or tessellated models.Freeform surface, or freeform surfacing, is used in CAD and other computer graphics software to describe the skin of a 3D geometric element. Freeform surfaces do not have rigid radial dimensions, unlike regular surfaces such as planes, cylinders and conic etc. surfaces. They are used to describe forms such as turbine blades, car bodies,boat hulls. Initially developed for the automotive and aerospace industries, freeform surfacing is now widely used in all engineering design disciplines from consumer goods products to ships.

The smoothness between patches, known as continuity, is often referred to in terms of a C value:

C0: Position Continuity

C1: Tangent continuity

C2: Curvature Continuity

Free form surfaces usually comprise different surfaces joined together by defining any of the above continuity at the joining edges. The different surfaces can be Bezier surface, B-Spline surface, Hermite surface etc.

1.4 LITERATURE REVIEW

Free-form surfaces are widely used in CAD systems to describe the surfaces of parts, such as molds and dies. There are different methods available to machine free form surfaces on CNC machines. Many researchers have worked on different algorithms to machine free form surfaces on CNC machines. The following section gives a brief view of research work performed by different researchers to develop efficient algorithms for generating free form surfaces.

Bobrow [1] presented a method for generating numerically controlled milling machine tool path directly from constructive solid geometry part representations. The improvements in the geometric modeling systems had led to the need of more reliable and highly automated software for tool path generation. The commonly used machine algorithms required that the part information that cannot be specified by modeling software can be specified by user. More geometric information was needed if the model used to represent the part was incomplete. The most common types of geometric modeling systems used for machine tool path generation were boundary representation systems. Most machining algorithms which use boundary representation systems, including APT, do not require the system to possess all the information necessary to represent the true solid model, i.e., to contain all the necessary face, edge, and vertex data relationships needed to represent the solid. The algorithm presented required less user interaction than APT boundary representation methods. A wide variety of parts when machined using the algorithm, the algorithm was computationally more efficient.

Barnhill [2]focused on the representation and design of surfaces in a computer graphics environment. The subject was approached from two points of view: the design of surfaces which included the interactive modification of geometric information and the representation of surfaces for which the geometric information was relatively fixed. Design takes place in 3-D space whereas representation can be higher dimensional. Triangular patches were used to interpolate and approximate the arbitrarily located data and required the preprocessing steps of triangulation and derivative estimation.

LoneyandOzsoy [3]discussed the development of an interactive Computer Aided Design and Manufacturing system used in defining and machining of free form surfaces. The system used a menu driven front end with graphical feedback to guide a user through curve and free form surface definition resulting in a mathematical model which was used to generate NC machine cutter location source file. The free form curves and surfaces do not possess analytic forms. They are defined in piecewise segments and joined together with some specified continuity. With the advent of the computer, computational methods were devised defining the composite surfaces as an assembly of curvilinear quadrilateral patches. By using parametric forms, independence is gained from a particular system of coordinates. A general theory of surface patches showed how four boundary curves can be blended into a smooth patch using general blending functions allowing any order of continuity between patches. The introduction of the control polygon by Bezier helped bring designers the intuition needed in the efficient control of surface geometry. Modifying the polygon modifies the curve in a predictable way. Surfaces are defined in a similar way by the input of open polyhedrons. B-splines applied to curve and surface definitions were found to be useful for making local modification possible on curves and surfaces.

Gregory andMahn [4] presenteda theory of geometric continuity between parametric surface patches. The theory explained that a convex combination of parametrically defined C2 surfaces was not necessarily C2.

Choi *et al.* [5] presented a method for modeling and machining compound surfaces commonly found in die cavities and punches. A Constructive Solid Geometry scheme was employed to model compound surfaces that consists of planar surface elements, general quadratic surface elements and composite parametric surfaces. A prototype modeling system was implemented on an IBM system. It took lessertime to generate cutter location data for a compound surface of realistic complexity. The work presented a surface modeling method for machining of analytic compound surfaces. A portion of a solid object to be machined was sectioned by a series of planes to obtain cutter contact points. The cutter path generation method used was similar to the one used by Bobrow [1]. To generate a cutter contact point, the intersection point between a vertical line (a point on the xy plane) and the compound surface was computed together with a surface normal vector at the point. One implication of the difference (i.e. plane intersection versus line intersection) was that cutter paths can be planned by following the isoparametric curves of the base surface.

Liu andHoschek [6]presented the necessary and sufficient conditions for Geometric C1 Continuity (GC1) that covered the different combinations between rectangular and triangular Bezier surface patches.

Suhand Lee [7] presented a procedure to machine a pocket with a convex or concave free surface bounded by lines, circular arcs and free curves. Although this pocket cutting capability was implemented in many numerical control packages, most of them could handle only convex shaped pockets bounded by curves of limited types and numbers. The cutter location data were computed directly with better computational efficiency than normal, without using an iterative method.

Pham [8] discussed problems that are encountered in the designing of offsets, gave a brief survey of existing techniques, and suggested some directions for future research. The designing of offset curves and surfaces was essential task in many industrial applications such as the generation of tool-path geometry in numerical-control machining and robot-path planning.

Huang and Oliver [9] presented an algorithm for three-axis NC tool path generation on sculptured surfaces. Non-constant parameter tool contact curves were defined on the part by intersecting parallel planes with the part model surface. The results of the technique were compared to those generated from a commercially available Computer-Aided Manufacturing program, and indicated that equivalent accuracy was obtained with many fewer cutter location (CL) points. Automatic NC tool path generation systems typically produce CL data directly from a mathematical model of a product design. The cutter location data was then post-processed into machine executable controlling codes for actual production. This capability integrates CAM activities with CAD systems to shorten product lead time and reduce the cost of production. The approach was generally efficient because the tool contact curves were easy to retrieve from the surface definitions. The drawback of this approach, however, was that the relationship between the parametric coordinate and the corresponding Cartesian coordinate was not uniform. Therefore, the accuracy and efficiency of the constant parameter approach for tool path generation vary depending on the geometry of the part surfaces.

Ma and Peng [10]presented a unified method for smoothing free-form surfaces with Bezier patches. A non-smooth surface consisting of bicubic patches was turned into a G1 surface. Using this method, solid objects bounded with bicubic Bezier patches were G1 smoothed with bicubic patches. The operation was local. It provided designers with the facility to adjust the shape of a smoothed object interactively. The method could also be used for constructing smooth closed surfaces whose modeling is important for CAD.

The presented method allowed one to model the fillet and rounding surfaces using bicubic patches flexibly. Since the transition surfaces were represented with the same type of surface, it had great advantage for both surface and solid modeling. A good number of shape parameters were provided so as to modify and control the shape of resultant surfaces with ease and convenience.

Leon andTrompette [11] described a method to provide deformation methods involving simultaneous movements of control vertices.Deformation of free-form surfaces encounters difficulties because more than one control point must be moved to achieve satisfying results. The method uses an analogy between the control polyhedron of a surface and the mechanical equilibrium of a bar network. Surface deformation was then carried out through changes of mechanical parameters. New equilibrium positions of the network match the solution of linear system of equations allowing real time shape modifications. An analytical approach of vertices movements was developed to demonstrate the method's behavior and the influence of mechanical parameters changes.

Konno andChiyokura [12]proposed a new surface representation that enables the smooth interpolation of an irregular curve mesh with NURBS curves and surfaces. Designers required a means of designing complex free-form surfaces easily and intuitively. One general approach to designing such surfaces was to first define a curve mesh consisting of characteristic lines, such as cross sections and boundary curves, then to interpolate the curve mesh using free-form surfaces. NURBS surfaces were widely used but make the interpolation of an irregular curve mesh difficult. This has been a major limiting constraint on designers.

Wang [13]presented a method to calculate intersection curves of offsets of two parametric surfaces. The method was based on the concept of normal projection. The intersection was represented in the parameter spaces of the base surfaces and no offset surface approximation was needed. Offset surfaces had a number of indispensable applications in CAD/CAM, including, for instance, the generation of tool paths for NC machining and the constant radius blending in surface modeling. An intersection of two offset surfaces was often of great interest in these applications. For example, a blend surface of two surfaces could be constructed by moving the center of a sphere of given radius along the intersection curve of two surfaces that are offset from the base surfaces by the radius of the sphere. In general, numerical methods must be employed because no analytical representation of the intersection exists, especially in the case that the base surfaces are of parametric forms, for example, Non-Uniform Rational B-Spline (NURBS) surfaces. The work presented a method for numerical computation of intersection of offsets of a pair of parametric surfaces.

Korenand Lin [14] presented an analytical method for planning an efficient tool path in machining free form surfaces on 3-axis milling machines. The approach used a non-constant offset of the previous tool path which guaranteed that the cutter moving in an un-machined area of the part surface and without redundant machining. The approach comprised three steps:

i. Calculation of tool path interval.
ii. Conversion of the path interval to parametric interval.
iii. Synthesis of efficient tool path planning.

This approach increased the efficiency of machining.

Ge*et al.* [15]presented a special class of rational Bezier curves that correspond to low-harmonic trajectory patterns. For given high-speed machinery, a significant source of the internally induced vibrational excitation was the presence of high frequency harmonics in the trajectories that the system was forced to follow. Harmonic Bernstein polynomials and

Harmonic De-Casteljau algorithm were also introduced as two major tools for generating the Harmonic Rational Bezier curves. These curves were used to synthesize trajectories within the dynamic response limitations of the actuators while avoiding the excitation of the natural modes of vibration of the system.

Nasri [16] described an algorithm to generate recursive subdivision surfaces that interpolate B-spline curves. The control polygon of each curve is defined by a path of vertices of the polyhedral network describing the surface. The method consists of applying a one-step subdivision of the initial network and modifying the topology in the neighborhood of the vertices generated from the control polygons. Subsequent subdivisions of the modified network generated sequences of polygons each of which converges to a curve interpolated by the limit surface. In the case of regular networks, the method could be reduced to a knot insertion process. Recursive subdivision provides definition of surfaces over irregular networks.

Dragomatzand Mann [17] presented a classified bibliography of literature on NC milling path generation.The focus was work that included some aspect of path generation, in particular, computing and creating roughing and finishing paths for 2 to 5 axis machine tools.

Lo [18] presented and analyzed methods based on linear, curve, and surface interpolators.The feed-rate was one of the most important factors to machining efficiency and quality. Number of methods of tool path generation adopt a constant velocity along the cutter location path and do not satisfy the desired feed-rate along the sculptured surface.

The proposed method adapted the CL velocity one segment by one segment so that a constant CC velocity is maintained. Although this method was the simplest one, it required a large CL file for these CL segments, and usually results in feed-rate fluctuation. The other method required a modified curve interpolator in the CNC machine. In addition to the curve geometric parameters, the parameters related to the CL velocity profile (that corresponds to a constant CC velocity) were fed to the modified interpolator for the path generation. Consequently off-line curve fitting for the CL path and the CL velocity was needed. This method solved the problem of fluctuation of feed rate and condensed the CL file. The third method required a surface interpolator that was new to current CNC machines. The surface interpolator consists of three real-time algorithms for the interpolation, the offsetting and the path interval calculation (for path scheduling of the next CC path), respectively. This method maintained the desired feedrate along the sculptured surface and condensed the CL file significantly. Although a lot of real-time computation was needed for the surface interpolator, it did not cause a problem for current CNC systems.

Maekawa [19] presented a literature survey on offset curves and surfaces after 1992. The review up to 1992 was carried out by Pham[8]. The article focused on five active areas of research on offsets:

i. Representing exact offsets in Bezier/B-spline format
ii. Approximations
iii. Self-intersections
iv. Geodesic offsets
v. General offsets

Offset curves/surfaces, also called parallel curves/ surfaces, are defined as locus of the points which are at constant distant along the normal from the generator curves/surfaces. Offsets are widely used in various applications, such as tool path generation for 2.5D pocket machining, 3D NC machining, definition of tolerance regions, access space representations in robotics, curved plate (shell) representation in solid modeling, rapid prototyping where materials are solidified in successive two-dimensional layers, brush stroke representation and in feature recognition through construction of skeletons or medial axes of geometric models.

Yang and Han [20]presented a systematic tool-path generation methodology which incorporates interference detection and optimal tool selection for machining free-form surfaces on 3-axis CNC machines using ball-end cutters. In the work, the global and local interference was first detected and prevented, and then the optimal tools in terms of machining time were selected and tool paths were generated. A system of five algorithms was developed to determine the interference area. On the basis of these algorithms, the machining time of each available tool was estimated by considering tool size, scallop height, and accessible surface area. Finally, the combination of tools which possesses the minimum overall machining time was selected as the optimal tool sizes**.**

The optimal tool selection method presented in was designed for any type of parametric surfaces to be machined; and it was used as an inline process in producing interference-free high-efficiency tool paths for 3-axis NC machining using ball-end cutters.

Lartiguea*et al.* [21] presented an accurate and efficient method to generate a tool path for a smooth free form surface using planar cubic B-spline. A three axis CNC machine tool with a ball end mill cutter was used. The break points were generated by computing the offset surface driving plane intersection curve reflecting the curvature by a planar cubic B-spline curve. Maximum scallop height was calculated by computing the stationary points of the distance function between the scallop curve and the design surface.

Fengand Li [22] presented a new approach for determination of efficient tool paths in the machining of sculptured surfaces using three axis ball end milling. The objective was to keep the scallop height constant across the machined surface so that the redundant tool paths were reduced. The work determined the tool paths without restoring to the approximated 2D representations of the 3Dcutting geometry. Two offset surfaces of the design surface, the scallop surface and the tool center surface were employed to successively establish scallop curves on the scallop surface and the cutter location tool paths for the design surface. The effectiveness of presented approach was demonstrated through the machining of typical sculptured surface. The result indicated that the constant scallop height machining achieves the specified machining accuracy with fewer and shorter tool paths than existing tool path generation.

The advantage of constant scallop height machining compared with iso-parametric and iso-planar methods was demonstrated for a practical convex surface. The constant scallop height tool paths were 7-21% shorter and more efficient than those generated by the other methods. The improvement was dependent on the design surface geometry.

Narayanaswamiand Pang [23] investigated the applicability of multi-resolution analysis using B-spline wavelets to NC machining of contoured 2D objects. A complex curve was decomposed using wavelet theory into lower resolution curves. The low-resolution (coarse) curves were similar to rough-cuts and high-resolution (fine) curves to finish cuts in NC machining. High-resolution curves were used close to the object boundary similar to conventional offsetting while lower resolution curves are used farther away from the object boundary. Experimental results indicate that wavelet-based tool path planning improves machining efficiency. Tool path length was reduced, sharp corners were smoothed out thereby reducing uncut areas and larger tools can be selected for rough-cuts.

Ding *et al.* [**24**] presented an algorithm that overcomes the disadvantage of iso planar method while keeping its advantage of robustness and simplicity. The isophote concept was used for the partition of different regions of the surface. The tool path side steps were adaptive to the surface features in each region. The machining efficiency was increased by applying the region by region or global local machining strategy. The drawback of the iso planar method used later was that in the region where the direction of the surface normal is close to that of the parallel intersecting planes, the intersecting plane intervals have to be reduced because of influence of surface slopes.

As compared with the conventional iso planar tool path generation algorithm the presented algorithm improved machining efficiency by reducing total tool path length needed to machine a free form surface.

Yin [25] proposedprogressive fitting and multi-resolution tool path generating techniques, by which multi-level (LOD) models fitting for different subsets of sampled points were obtained, and then multi-resolution rough-cut and finish-cut tool paths were generated based on the LOD models. The advantages of the proposed method were:

i. The user need not care for data reduction in CAD modeling
ii. Final result was obtained by interpolating two lower-level reconstructed surfaces, and each lower multi-resolution CAD representation can be used to generate rough-cut tool paths
iii. Different manufacturing requirements utilize different level models to generate tool paths
iv. Selective refinement can be applied by interpolating selected areas at different levels of details

The key advantage of the progressive fitting algorithm was that it used different level surfaces to generate adaptive rough-cut and finish-cut tool path curves directly. Therefore, based on the proposed techniques, tool path length was reduced. Sharp corners were smoothed out and large tools were selected for rough machining. The efficiency of this algorithm was demonstrated, and it results in a 20% reduction in machining time.

Renner [26] presented a method to explicitly compute the curves in three-dimensions, practical algorithmic issues were discussed concerning the efficiency of the implementation. The paper presented two approximate solutions to the problem. The first was derived from the exact representation, while the second extends conventional least-squares approximation by incorporating the geometry of the surface as well. The efficiency and behavior of the algorithms were evaluated by means of examples.

Wei and Lin [27] established a systematic general analytical method for CNC machining of the free-form surfaces, and developed the postprocessor to obtain the NC code. The method comprises five steps:

i. To find the equation of surface
ii. Curvature analysis

iii. The selection of tool
iv. The error calculation of the linear incremental kinematics
v. The calculation of the tool-path interval

Using the discussed analytical method to machine free-form surfaces, one can make the cut-length greatly reduced on the fundamental of the priority with promising an invariant machining precision which enhanced the potential of Computer Numerical Control machine, also greatly enhanced the product capacity to carry out the automatic production.

Choi and Banerjee [28] focused on generating tool path for free form surfaces based on accuracy of desired manufactured part. The mathematical curves and surfaces used to represent a specific manufactured part were used to generate near optimal tool paths and cutter location files. The algorithm included two components; first was forward step which was defined as maximum distance between two cutter contact points for given tolerance; second was side step which determines the maximum distance between two adjacent tool paths for a given scallop height. This algorithm reduced the manufacturing time, computing time as well as cutter contact points. True machining errors were verified by comparing designed surface and machined surface using point cloud method. The method, independent of surface type, can be applied to any continuous parametric surface which is twice differentiable.

Choi and Banerjee [29] presented the tool path generation method for multi axis machining of free form surfaces using Bezier curves and surfaces. This method of tool path generation included two steps; forward step function and side step function. Bezier curves and surfaces were used for generating cutter contact pointsand cutter location data files for free form surfaces. The presented algorithm of tool path generation was very efficient. It reduced the cutter location points by which NC code was generated significantly. The method was independent of surface type and was applicable to all continuous parametric surfaces that were twice differentiable.

Chen and Shi [30]presented a new method for tool path generation which was based on approximating free form surfaces by triangular meshes. The vertices of triangular meshes were offseted along surface normal. All offset vertices were then connected together to form offset triangular meshes sliced by a group of planes to obtain tool paths. The proposed algorithm ensured that all tool paths were planar curves, so that it was easy to control NC machine feed rate.

Lasemi*et al.* [31] provided a state-of-the- art review on research development in CNC machining of freeform surfaces. Various methodologies and computer tools are developed to improve efficiency and quality of freeform surface machining. This review primarily focused on three aspects in freeform surface machining: tool path generation, tool orientation identification, and tool geometry selection. For each aspect, first concepts, requirements and fundamental research methods were briefly introduced.

It was said that the specific constraints have to be applied in path planning for different machining stages to achieve the optimal time and quality. For example in finish machining, the machining time should be minimized while the scallop height must be maintained below the specified level. An ideal tool path should generate uniformly distributed scallops across the whole surface. Smaller scallop size does not necessarily mean a better tool path, since it is achieved at the cost of increased machining time. On the other hand, the minimum machining time could be achieved when the scallop height set to the maximum allowable measure. Tool path planning composed of 2 aspects: path topology and path parameters. The former was defined by the pattern that the cutter moves to produce the surface, and the latter was modeled by the tool side step between successive paths and the tool's forward step in each path. Many researches have been carried out on the optimization of the tool path in these two areas. The tool path generation problem was converted into the following sub-problems with a defined cutting tool:

i. Specify path pattern and the linking strategy (path direction)
ii. Specify points on the surface that the tool should track
iii. Check tool local and global interferences

In tool path generation, iso-scallop tool paths along with the curvature matching method have significantly lead to the improved surface quality and reduced machining time. Many achievements have also been observed in the machining of polyhedral surfaces, point clouds and compound surfaces.

1.5CONCLUSION FROM LITERATURE REVIEW

From literature review it is concluded that the current methods for machining free form surfaces require some important human decisions, such as determination of the precise interval between successive tool-paths. A tool path interval that is too large can result in a rough surface; one that is too small can increase machining time, making the process inefficient. Another critical decision is to find efficient tool paths of the entire part surface. The need for human decisions in the CAD/CNC process causes difficulties in the integration of the design with the manufacturing stages. To automate the design/manufacturing process, algorithms for efficient path planning based on accurate tool-path intervals are needed.

CHAPTER 2

MATHEMATICAL FORMULATIONS

As per the literature review, a large number of algorithms represent the free-form surface in the form of Bezier and B-spline surface. One such algorithm given by Choi and Banerjee [29] uses the Bezier surface for representing the free form surface. The purpose of the study is to implement the algorithm given by Choi and Banerjee [29] and then implement the same algorithm using the B-spline surface representation.

The following section gives the basic mathematical formulation of the algorithm.

2.1 BEZIER CURVE

Bezier curve was designed by French Scientist P.Bezier in 1962. It is an approximate curve. Degree of this curve depends upon the number of control points. Mathematically, Bezier curve is represented as following

$$p(u) = \sum_{i=0}^{n} p_i \, B_i^n(u), \qquad 0 \le u \le 1 \tag{2.1}$$

where $n+1$ number of control points, B_i^n is the Bernstein's function, p_i is the control point

Bernstein's function is defined as

$$B_i^n = C(n,i)u^i(1-u)^{n-i}$$

where,

$$C(n,i) = \frac{n!}{i!(n-i)!}$$

The first order derivative of Bezier curve, represented as $\dot{x}$, and is given by,

$$\dot{x} = n\sum_{j=0}^{n-1}(p_{j+1} - p_j)B_j^{n-1}(u) \tag{2.2}$$

The equation (2.2) can be simplified by the introduction of the forward difference operator, Δ

$$\Delta p_j = p_{j+1} - p_j$$

Hence the derivative of the Bezier curve can be given by,

$$\dot{x}(u) = n\sum_{j=0}^{n-1}\Delta p_j B_j^{n-1}(u) \tag{2.3}$$

In generalized form, the forward difference operator Δ, can be defined as,

$$\Delta^r p_j = \Delta^{r-1}p_{j+1} - \Delta^{r-1}p_j$$

The r^{th} derivative of a Bezier curve is now given by,

$$\frac{d^r}{dt^r}p^n(u) = \frac{n!}{(n-r)!}\sum_{j=0}^{n-r}\Delta^r p_j B_j^{n-r}(u) \tag{2.4}$$

Two important special cases of the above equation are as following,

1. $\frac{d^r}{dt^r} p^n(0) = \frac{n!}{(n-r)} \Delta^r p_0$
2. $\frac{d^r}{dt^r} p^n(1) = \frac{n!}{(n-r)} \Delta^r p_{n-r}$

Thus the r^{th} derivative of a Bezier curve at an end point depends only on $r + 1$ Bezier points near and including that end point. For r=0 we have already the established property of endpoint interpolation. The case r=1 states that p_0 and p_1 define the tangent at u=0, provided they are distinct. Similarly, p_{n-1} and p_n determine the tangent at u=1.

2.2 BEZIER SURFACE

A surface is the locus of a curve that is moving through space and thereby changing its shape [35]. We now formalize this intuitive concept in order to describe a surface patch. First, it is assumed that the moving curve is a Bezier curve determined by a set of control points. Each original control point moves through space on a curve. The next assumption is that this curve is also a Bezier curve, and that the curves on which the control points move are all of the same degree. Thus the Bezier cubic patch can be represented as,

$$p(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} p_{ij} B_i^n(u) B_j^m(v), 0 \le u \le 1, \ \ 0 \le v \le 1 \quad (2.5)$$

where$n$+1 and m+1 are the number of control points in $u$and $v$direction respectively.

2.3 B-SPLINE CURVE

B-Spline is a generalization of Bezier curve. The main advantage of B-Spline over Bezier is that it provides local control, which means that if we change the location of one point it cannot change the shape of whole curve, it will only effect the area of the curve corresponding to that point. The mathematical representation of the B-Spline curve is as following:

$$p(u) = \sum_{i=0}^{n} p_i N_i^k(u), \qquad 0 \le u \le u_{max} \quad (2.6)$$

where$n + 1$ is the number of control points,$N_i^k$is the B-Spline function, $k$is the parameter which defines degree of the curve

The B-spline function is given by,

$$N_i^1 = \begin{cases} 1\ , & t_i \le u \le t_{i+1} \\ 0, & otherwise \end{cases}$$

$$N_i^k = (u - t_i)\frac{N_i^{k-1}}{t_{i+k-1} - t_i} + (t_{i+k} - u)\frac{N_{i+1}^{k-1}}{t_{i+k} - t_{i+1}}$$

In the above equation $t_i$are the knot values given by

$$t_i = \begin{cases} 0 \ , & i < k \\ i - k + 1 \ , & k \le i \le n \\ n - k + 2 \ , & i > n \end{cases}$$

with $0 \le i \le n + k$, and $u_{max} = n - k + 2$

2.4 B-SPLINE SURFACE

The mathematical representation of B-Spline surface is the extension of equation (2.6),

$$p(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} p_{i,j} N_i^k(u) N_j^l(v), \tag{2.7}$$

where n+1 and m+1 are the number of control points in u and v directions respectively, k and l are the parameters to define degree in the u and v directions respectively.

2.5 FIRST AND SECOND FUNDAMENTAL FORM OF A SURFACE

Given an embedded curve $p\big(u(t), v(t)\big)$, the quadratic form

$$\mathrm{I} = p^t . p^t = E \frac{du}{dt}\frac{du}{dt} + 2F \frac{du}{dt}\frac{dv}{dt} + G \frac{dv}{dt}\frac{dv}{dt}$$

is known as the first fundamental form with *t* as the independent variable along the path and,

$$E = p^u . p^u$$

$$F = p^u . p^v$$

$$G = p^v . p^v$$

These are the coefficients of the first fundamental form and $p^u and\, p^v$ are the partial derivatives along the u and v direction at point p on the surface.

The quadratic,

$$\mathrm{II} = L \frac{du}{dt}\frac{du}{dt} + 2M \frac{du}{dt}\frac{dv}{dt} + N \frac{dv}{dt}\frac{dv}{dt}$$

is referred as second fundamental form where,

$$L = p^{uu} .n$$

$$M = p^{uv} .n$$

$$N = p^{vv} .n$$

are the coefficients of the second fundamental form.

2.6 TOOL PATH GENERATION

Tool path generation is an important step in the machining of the free form surfaces.. The accuracy of the machined surface depends greatly on the tool path, so in order to have good accuracy the tool path generation should be accurate and proper. In tool path generation the main job is to calculate the cutter location points by specifying forward step and side step, the basic movements of the tool to reach the next cutter location point.

2.7CUTTER CONTACT AND CUTTER LOCATION POINT

The Cutter Location (CL) point is a reference point on the cutter, with which the machine tool moves linearly along the tool path. The CC point is the point on the cutter that touches the work piece surface. Ideally, it is desired that the CC point lie on the work piece surface and is an addressable point which is a point along the axis at which the tool can be placed so as to minimize the manufacturing error. The CC and CL points are shown in figure 2.1.

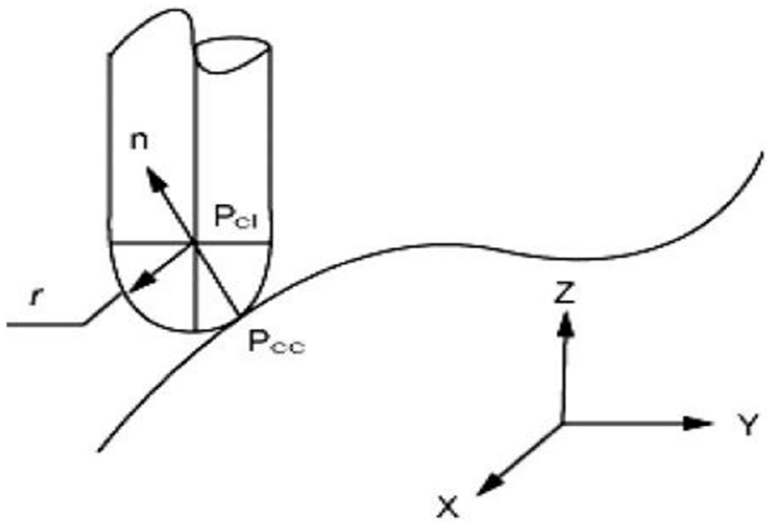

Figure 2.1: CC and CL point

The surface normal *n* at an arbitrary point on the surface is expressed as:

n = ———

where and are the partial derivatives along and direction at a point on the surface.

2.8PROPOSED APPROACH

The proposed approach contains calculation of forward step and side step.These are given in the following sections:

2.8.1 Calculation of Forward Step

Each tool path is approximated by a series of line segments. The accuracy of tool path is controlled by deviation. Each segment amounts is a forward-step and the maximum deviation is called tolerance. To calculate forward step, first and second derivatives are used. Therefore, this function is independent of the surface type and is applicable to all continuous parametric surfaces that are twice differentiable. The forward step is denoted by *s*. The mathematical formula for forward step can be derived from iso-parametric curve on the surface and some thermos shown below.

Theorem 1. The function $f(t,y)$is Lipschitzian with respect to$y$ in a region R if there is a constant K such that $|f(t,y) - f(t,z)| \leq k|y - z|$, $(t,y) \in R$, $(t,z) \in R$.

Theorem 2. Let $f(x)$ be a differentiable function with $|\dot{f(x)}| \leq k_1$ for all $x \in (a,b)$. If $\hat{f}(x)$ is the piecewise linear approximation of $f(x)$ in (a,b) with subdivision interval *s,* then $max_{x\in(a,b)}|f(x) - \hat{f}(x)| \leq k_1\frac{s}{2}$.

Since twice differentiability of a function $f(x)$ implies a Lipschitz condition on, have. Now the case in which both $|\dot{f}|$ and $|\ddot{f}|$ have known bounds $k_1$and k_2, respectively, is handled in the following theorem.

Theorem 3. Let $f(x)$ be a differentiable function with $|\ddot{f}(x)| \leq k_1$ for all $x \in (a,b)$. If $\hat{f}(x)$ is the piecewise linear approximation of $f(x)$ in (a,b) with subdivision interval *s,* then$max_{x\in(a,b)}|f(x) - \hat{f}(x)| \leq k_{1\frac{s}{8}}$.

Theorem 4. Let $f(x)$ be a twice differentiable function passing through the points (x_1, y_1) $(x_2, y_2)$with $|\dot{f}| \leq k_1, |\ddot{f}| \leq k_2$. Let $x_2 \geq x_1$, $(x_2 - x_1)$=*s*, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ $(|m| \leq k_1)$.

Then, distance between cutter contact points can be determined by using maximum deviation, k_1, k_2.It should be noted that Theorems 2-4imply the existence of the derivatives since they are stated in terms of the first, second, or both the derivatives. Thus, this method is independent of the surface type and is applicable to all continuous parametric surfaces that are twice differentiable.

The derivatives are partial derivatives. The partial derivative is tangent vector of iso parametric curve. It can be calculated by derivative of equation (2.5),

$$\frac{\partial}{\partial u} p^{m,n}(u,v) = \sum_{j=0}^{n}\left[\frac{\partial}{\partial u}\sum_{i=0}^{m} p_{i,j}B_i^m(u)\right]B_j^n(v) \qquad (2.8)$$

where, m is the degree of the Bezier curve in u direction and n is the degree in v direction.

The equation (2.8) can be written in terms of the forward difference operator as given by equation (2.9)

$$\frac{\partial}{\partial u} p^{m,n}(u,v) = \mathrm{m}\sum_{j=0}^{n}\sum_{i=0}^{m-1}\Delta^{1,0}p_{i,j}B_i^{m-1}(u)B_j^n(v) \qquad (2.9)$$

In generalized form the formulas for higher order partial can be written as equation (2.10)

$$\frac{\partial^r}{\partial u^r} p^{m,n}(u,v) = \frac{m!}{(m-r)!}\sum_{j=0}^{n}\sum_{i=0}^{m-r}\Delta^{r,0}p_{i,j}B_i^{m-r}(u)B_j^n(v) \qquad (2.10)$$

where, the difference operator,

$\Delta^{r,0}p_{i,j} = \Delta^{r-1,0}p_{i+1,j} - \Delta^{r-1,0}p_{i,j}$

and,

$$— \quad = —$$

where, the difference operator,

The mixed partial of arbitrary order can be written as equation (2.11)

$$— \quad = — \qquad (2.11)$$

Partial and mixed derivatives of a point on the surface can be determined using equation (2.11). From these, we are interested in four boundary curves. We can obtain one of the boundary curves ,— , using the following equation,

$$— \quad = — \qquad (2.12)$$

First and second derivatives and at a specific point on the surface can be calculated using equation (2.12). Thereafter maximum forward step can be determined with the given tolerance.

Let be the maximum second derivative of current curve. The forward-step size can be calculated using equation(2.13)

$$— \qquad (2.13)$$

where, e is the given tolerance.

2.8.2 Calculation of Side Step

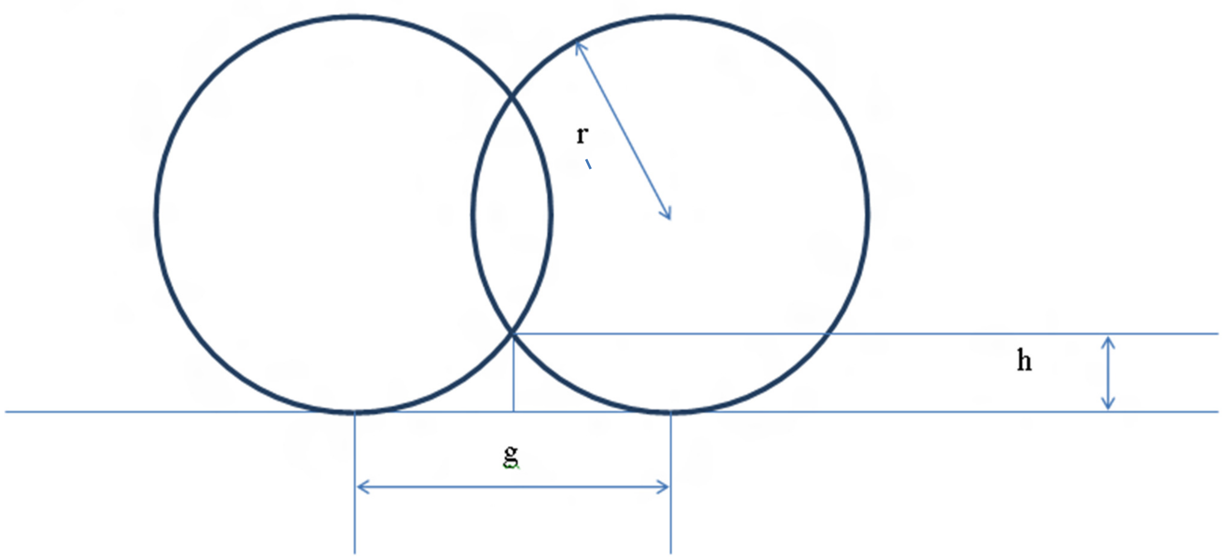

Figure 2.2: Flat Surface

The side step, *g* is a function of the scallop height *h*, tool-radius *r,* and the local radius of the curvature, R. In this section, a new method to calculate side-step size with the given constraint of scallop height is developed. The designed part's surface can be classified into convex, concave, and flat surface. The curvature of convex, concave, and flat surfaces are positive, negative, and zero, respectively. Therefore, we consider three different cases to calculate side-step size,*g*.

First we shall consider a flat surface as shown in figure 2.2,

The scallop height,*h* is given by equation (2.14)

$$= - \overline{\qquad\quad} - \ , \qquad (2.14)$$

where is the radius of the tool and *g* is the side step. The value of the side step is given by the equation (2.15).

$$= 2 \overline{\qquad\qquad} \qquad (2.15)$$

Secondly, a convex curvature is considered as shown in figure 2.3. To find the side step for a convex surface we have to find δ, which is the difference between a designed curve and a linear tool path.

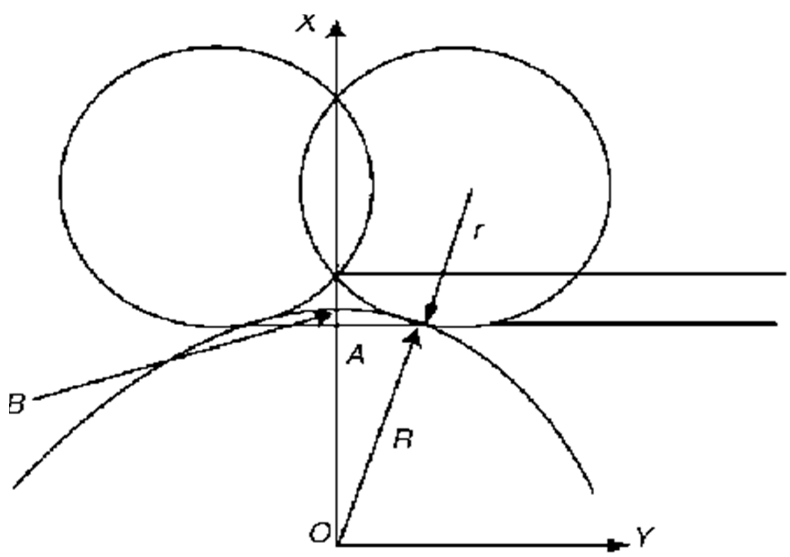

Figure 2.3: Convex Surface

From figure 2.3, δ = OB-OA,

To find OA we first calculate step size *g,* using equation (2.15). As the step size is very small we can use = /2 as the initial value. From the figure 2.3.

OA = ,

OB = R,

So the scallop height,*h* and side step,*g* is given by equation (2.16) and (2.17) respectively,

= - +δ, (2.16)

= 2 (2.17)

In the similar manner we can calculate the side step for a concave surface, as shown in figure 2.4,

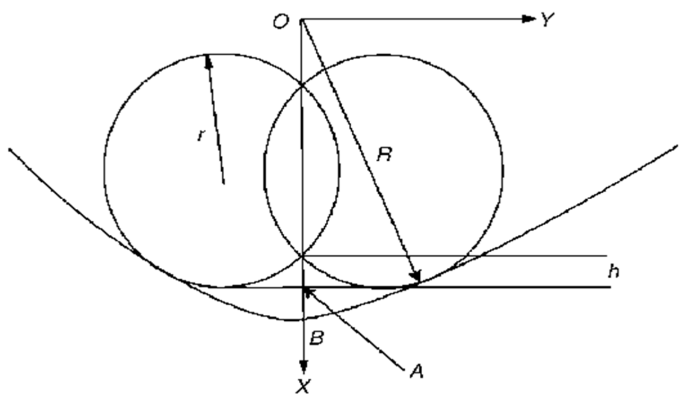

Figure 2.4: Concave Surface

From figure 2.4, δ = OB-OA,

OA = ,

OB = R,

The scallop height,*h* and side step, *g* for a concave curvature are given by equation (2.18) and (2.19) respectively,

= δ, (2.18)

= 2 , (2.19)

where, R is the local radius of the curvature of the surface, Hence the value of side step can be calculated for any type of the surface using the discussed equations.

2.9CONVERSION OF THE PHYSICAL DOMAIN TO PARAMETRIC DOMAIN

The forward-and side-step sizes described in the previous section are determined with physical unit instead of the parametric value (u, v). Since the part being processed is represented by a Bezier surface described by using u, v parameters, we have to convert the forward and side step with the physical unit to the parametric value in order to calculate the next CC point on the surface. First, we consider the direction of the forward step that is parallel to the x-axis and not in the direction of the iso-parametric curve on the surface to obtain u, v parameter values corresponding to the forward-step size in the physical domain as shown in figure 2.5, where is the forward step in the current parametric curve direction and s is the forward step in the direction of the physical domain. y is the angle between the tangent vector of the forward step in the physical domain, T, and the tangent vector of the parametric curve that can be calculated by

cosθ= — ,

θ = — ,

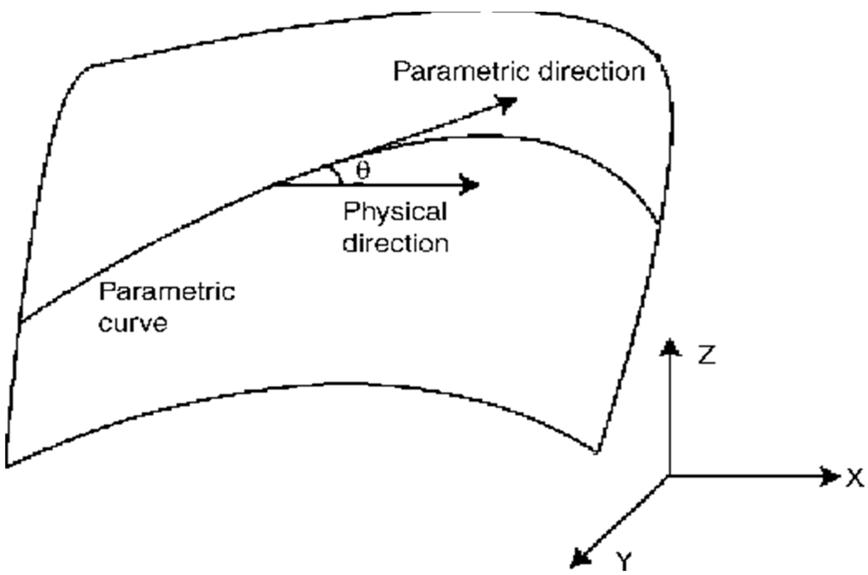

Figure 2.5: Direction of the Tool

CHAPTER3

IMPLEMENTATION

To verify the algorithm the cutter contact points are generated for a surface. The surface is then machined using a 3-axis milling machine. The algorithm is coded in MATLAB on a personal computer. We machined free form shaped part using a block of wax. After machining, a point cloud method is used to scan the machined surface to compare them. The desired surface is generated by modeling software RHINOCEROS 4.0 using the x, y, z-coordinates generated by MATLAB.

3.1 MATLAB GENERATION

The proposed algorithm as discussed in the previous chapter for the generation of tool path is coded in MATLAB. The different steps involved are discussed in the following subsections:

3.1.1 Generation of Bezier and B-spline Surface

The Bezier and B-spline surfaces are generated by coding their respective algorithm in MATLAB. These algorithms, as discussed in the previous chapter contain general approach to generate Bezier and B-spline surfaces. The same control points are used for both Bezier and B-Spline surface. The generated Bezier and B-spline surface are shown in figure 3.1 and 3.2 respectively.

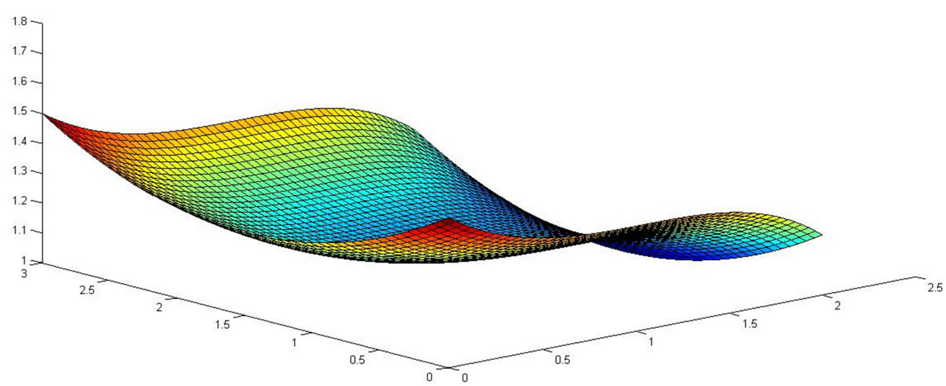

Figure 3.1: Generated Bezier Surface

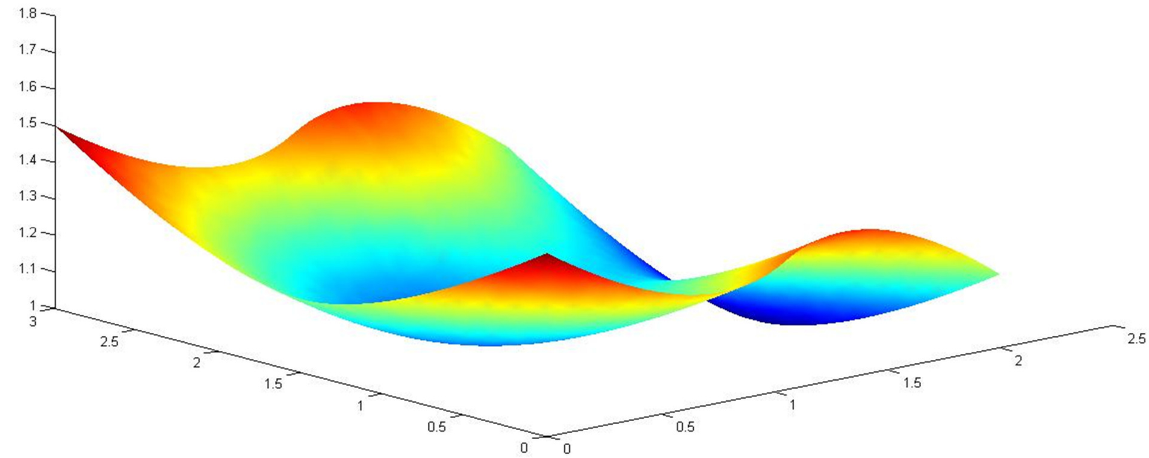

Figure 3.2: Generated B-spline Surface

3.1.2 Generation of Cutter Contact Points

The forward step and side step are calculated using equation (2.13) and (2.15), to generate the cutter contact points in the forward direction at specified forward step and in the side at specified side step. The x, y, z-coordinates of all the cutter contact points are stored in the data base to generate the CNC program [34]. The generated cutter contact points on Bezier and B-spline surface are shown in figure 3.3 and 3.4 respectively.

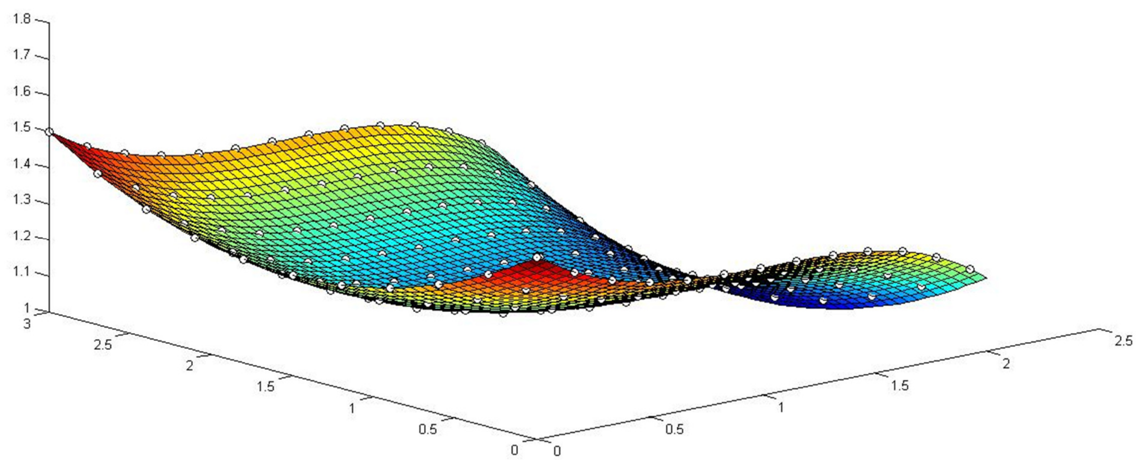

Figure 3.3: Cutter Contact points on Bezier Surface

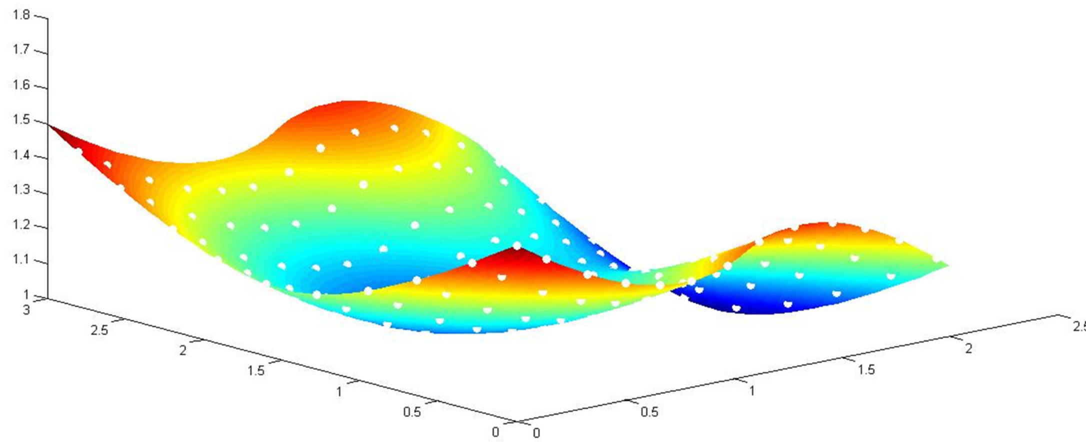

Figure 3.4: Cutter Contact points on B-spline Surface

3.2 MODELING OF DESIRED SURFACE FROM GENERATED POINTS

The cutter contact points generated in MATLAB are used to model the desired surface in RHINOCEROS 4.0, a surface modelling software. Figure 3.5 and 3.6 shows the generated Bezier and B-spline surfaces respectively.

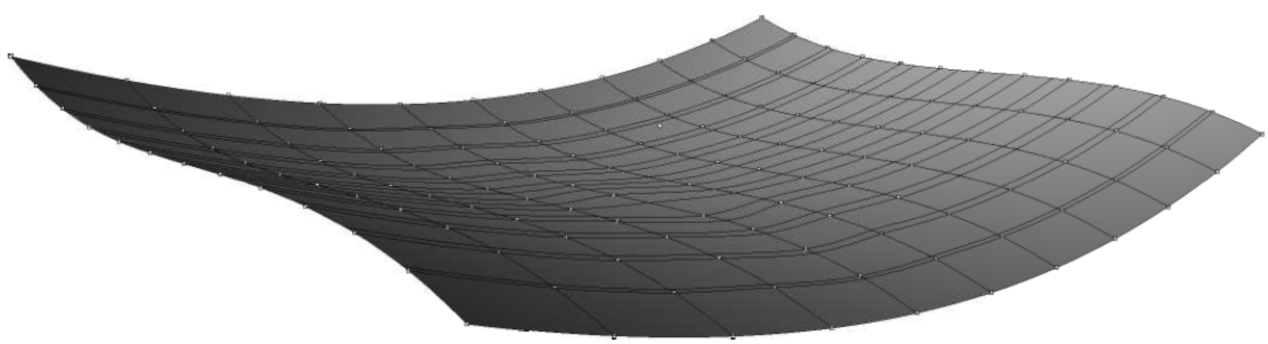

Figure 3.5: Bezier surface generated using RHINOCEROS

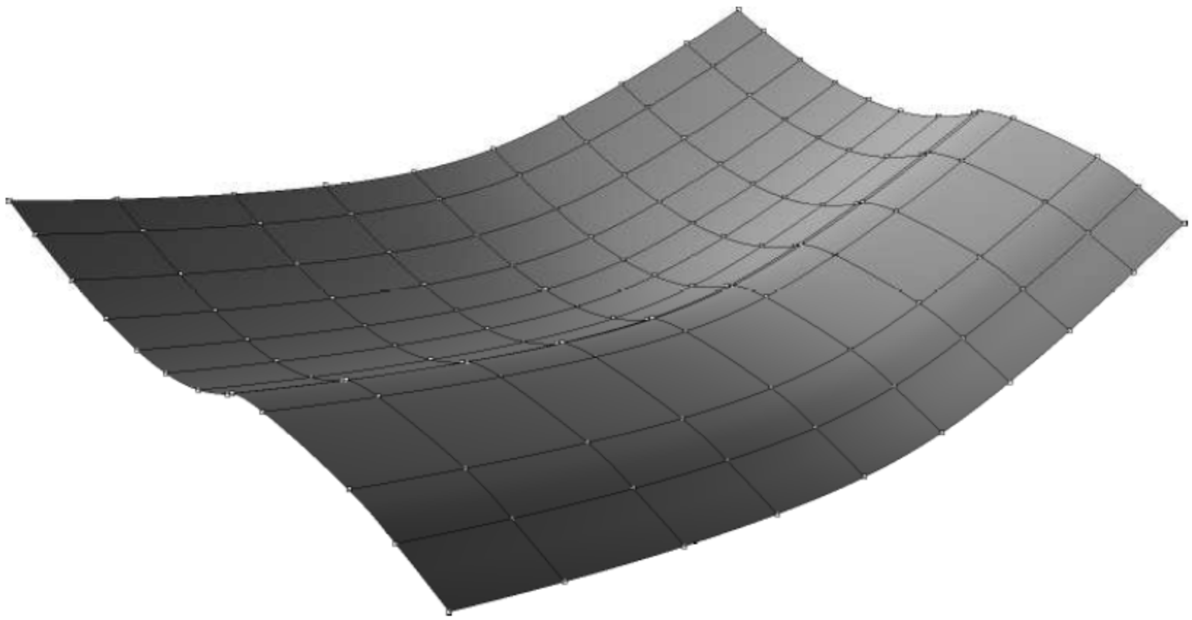

Figure 3.6: B-spline Surface generated using RHINOCEROS

3.3MACHINING

The CNC program is generated using calculated cutter contact points using word address manual part programming [34]. The program is simulated on a vertical milling centre shown in figure 3.7. The written code can be transferred on to the milling centre by using a flash card or can be entered manually.

Figure 3.7: Vertical Milling Centre

The wax piece is used to machine the generated surface on the milling machine. The machining is done using a ball end milling tool of 4 mm diameter. The work piece and the tool setup are shown in figure 3.8.

Figure 3.8: Work piece and tool setup

After machining the wax piece with the entered CNC program, the generated Bezier and B-spline surfaces are shown in figure 3.9 and 3.10 respectively.

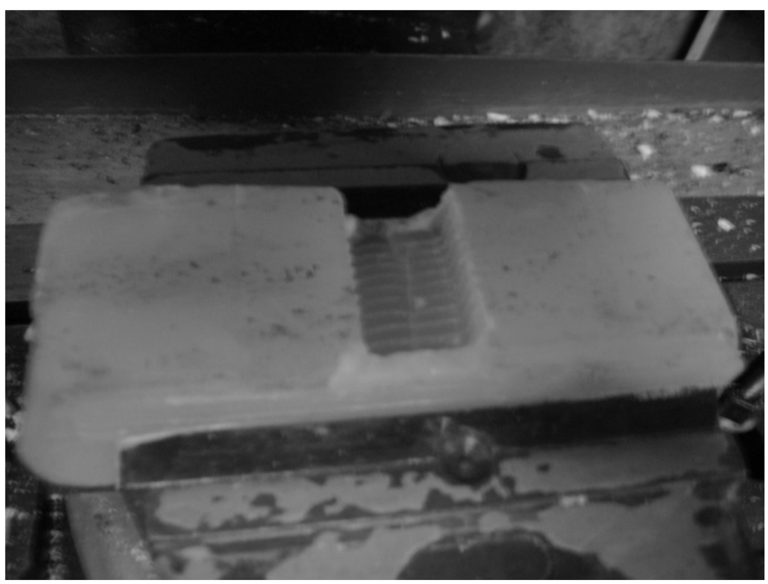

Figure 3.9: Generated Bezier Surface

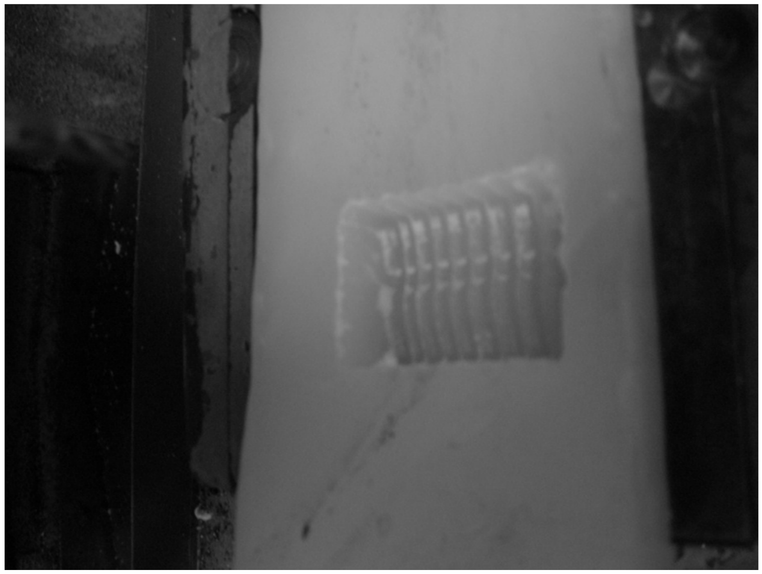

Figure 3.10: Generated B-spline Surface

Thereafter, the Coordinate Measuring Machine (CMM) is used make a point cloud of the machined Bezier and B-spline surface. This point cloud is used to model the surface in RHINOCEROS 4.0. The surfaces generated from the point cloud of machined Bezier and B-spline surface are shown in figure 3.11 and 3.12 respectively.

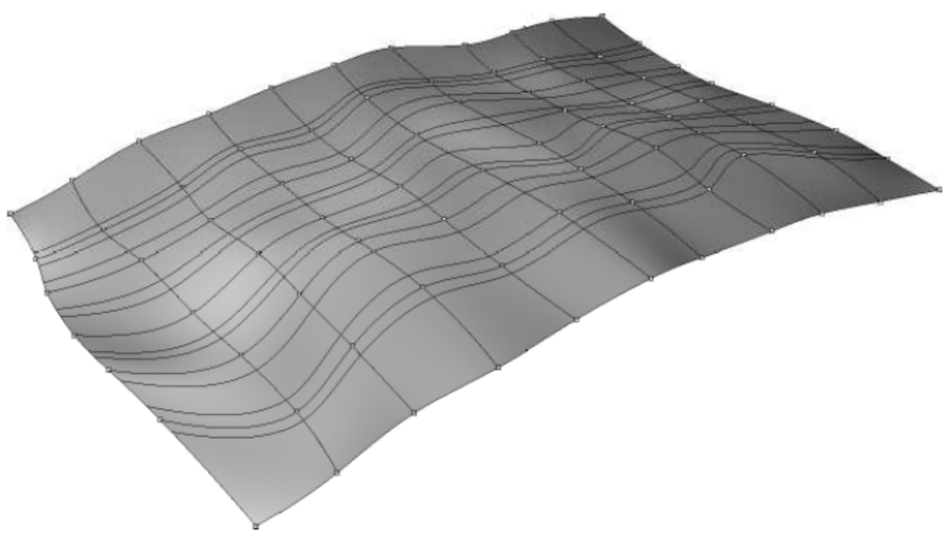

Figure 3.11: Surface from point cloud for Bezier surface

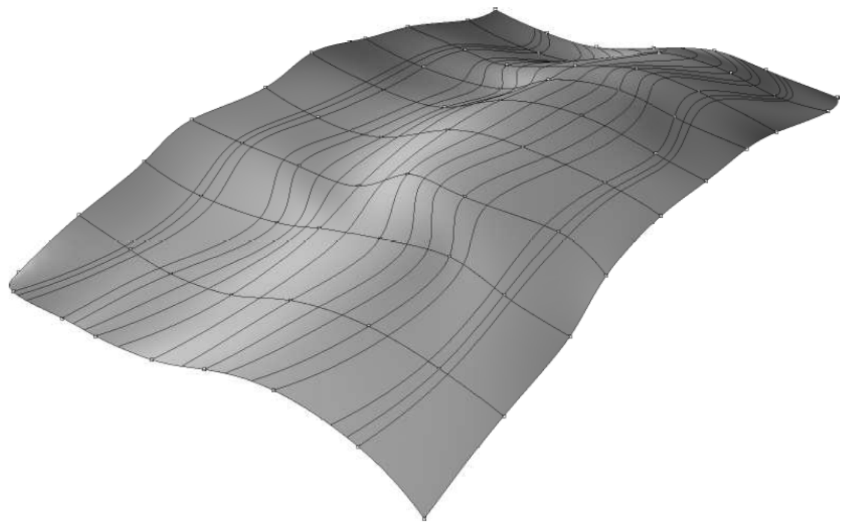

Figure 3.12: Surface from point cloud for B-spline surface

3.4 COMPARISON OF DESIRED AND MACHINED SURFACE

To check the accuracy of the machined surface, the desired and the machined surfaces are overlapped to observe the regions of the surfaces which are merging within one another. Thin overlapping will clearly show the difference between machined surface and desired surface. The overlapping of surfaces is done in ANSYS WORKBENCH 14.0.

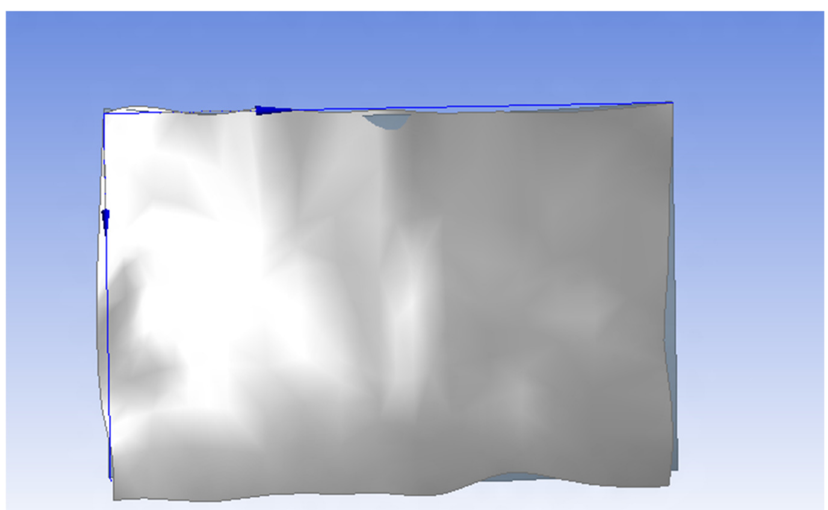

Figure 3.13: Overlapping of Bezier machined and desired surface

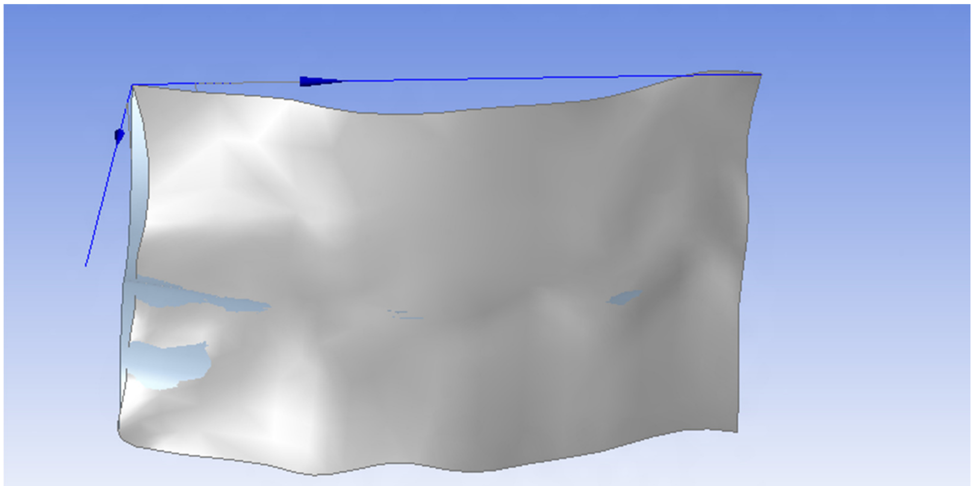

Figure 3.14: Overlapping of B-spline machined and desired surface

The editing of the overlapped surface is not effective in ANSYS. So to have a more effective view, the overlapping is also done in Pro-E software. The surfaces are shown in different colours to make a more effective view. In the figure 3.15 and 3.16, the green and blue colour shows the desired and the machined surface respectively.

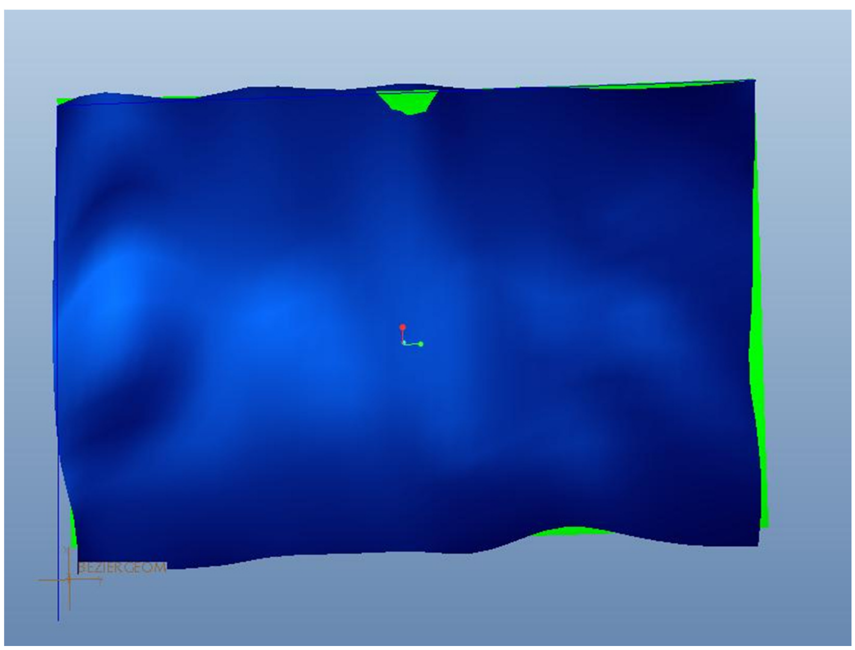

Figure 3.15: Overlapping of Bezier surface in Pro-E

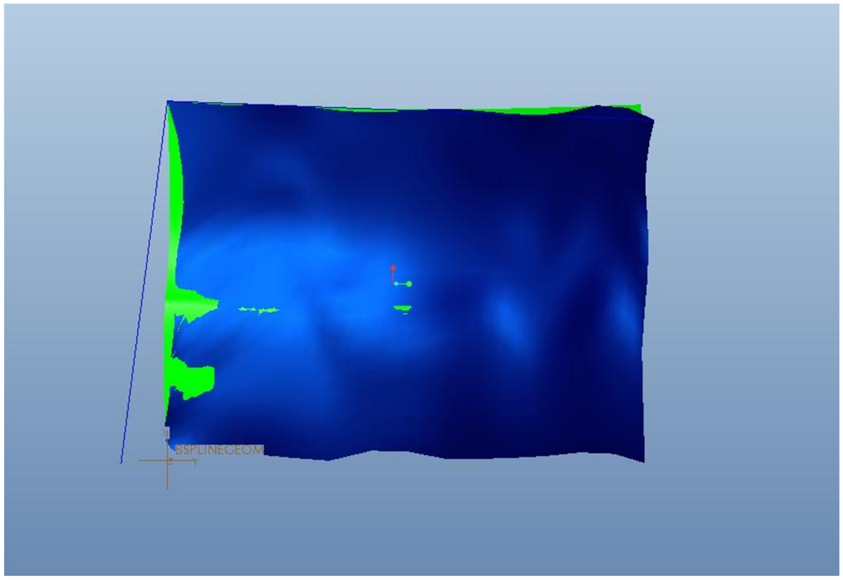

Figure 3. 16: Overlapping of B-spline surface in Pro-E

3.4.1 Comparison on scallop height basis

The comparison between desired surface and machined surface is also done by measuring the scallop height of the machined surface. The scallop height of the machined surface should be within range of the assumed scallop height. Tables 3.1 and 3.2 shows the maximum scallop height of machined Bezier and B-spline surface, number of cutter location points, and average scallop height of the machined Bezier and B-spline surface for assumed scallop height.

Table 3.1: Scallop Height for Bezier Surface

Assumed Scallop Height (mm)	Maximum Scallop Height of Machined Surface (mm)	Average Scallop Height of Machined Surface (mm)	Number of Cutter Location Points (mm)
0.254	1.06	0.91	143

Table 3.2: Scallop Height for B-spline Surface

Assumed Scallop Height (mm)	Maximum Scallop Height of Machined Surface (mm)	Average Scallop Height of Machined Surface (mm)	Number of Cutter Location Points (mm)
0.254	1.19	0.96	130

Thus the table 3.1 and 3.2 show that the maximum scallop height and average scallop height of the machined surface lies within the limits of the assumed scallop height for Bezier and B-spline surface respectively. In case of Bezier surface the number of cutter location points corresponding to scallop height 0.254 mm is 143. If the scallop height is increased to 1.27 mm the number of cutter location points reduces to 53. Whereas, in case of B-spline surface number of cutter location points corresponding to 0.254 mm are 130. If the scallop height is increased to 1.27 mm the number of cutter location points reduces to 40.

CHAPTER 4
CONCLUSION

4.1 CONCLUSION

The following conclusions are drawn from the present work:

1. The algorithm given by Choi and Banerjee [29], is implemented successfully through the integration of mathematical modeling used for calculating the forward step and side step size into the core of the algorithm.
2. The algorithm is modified by using the B-spline surface as against the Bezier surface used by Choi and Banerjee [29].
3. As claimed by Choi and Banerjee [29], the algorithm is applicable to all the continuous parametric surfaces that are twice differentiable, so the implementation is done by using B-spline surface.
4. Verification of machining errors are done by comparing the desired surface and machined surface using the point cloud method. This verification is done for both Bezier and B-spline surface.

4.2 SCOPE OF FUTURE WORK

The following points can be used for the further improvement of the present work:

1. The machining error can be further reduced.
2. The algorithm can be used to develop a canned cycle that can be used directly by the part programmer.

REFERENCES

[1] Bobrow, J. E., NC machine tool path generation from CSG part representations, Computer Aided Geometric Design, 17(2), 69–76, 1985.

[2] Barnhill, R.E., Surfaces in computer aided geometric design: a survey with new results, Computer Aided Geometric Design, 2(1), 1-17, 1985.

[3] Loney, G. C.,Ozsoy, T. M., Machining of free form surface, Computer Aided Design, 19(2), 85–89, 1987.

[4] Gregory, J.A., Hahn, J., Geometric continuity and convex combination patches, Computer Aided Geometric Design, 4, 79-89, 1987.

[5] Choi, B. K., Lee, C. S., Hwang, J. S., Jun, C. S., Compound surface modeling and machining, Computer Aided Design, 20(3), 127–136, 1988.

[6] Liu, D.,Hoschek, J., GCl continuity conditions between adjacent rectangular Bezier surface patches, Computer Aided Design, 21(4), 194-200, 1989.

[7] Suh, Y. S,, and Lee, K., NC Milling Tool-Path Generation for Arbitrary Pockets Defined By Sculptured Surfaces, Computer Aided Design, 22(5), 273-283, 1990.

[8] Pham, B., Offset curves and surfaces: a brief survey, Computer Aided Design, 24(4), 223–9, 1992.

[9] Huang, Y., Oliver, J., Non-constant parameter NC tool path generation on sculptured surfaces, International Journal of Advanced Manufacturing Technology, 9(2), 281–290, 1994.

[10] Lizhuang, Ma, Qunsheng,Peng, Smoothing of free-form surfaces with Bezier patches, Computer Aided Geometric Design, 12, 231-249, 1995.

[11] Leon, J.C., Trompette, P., A new approach towards free-form surfaces control, Computer Aided Geometric Design, 12, 395-416, 1995.

[12] KouichiKonnoa, Hiroaki Chiyokura, B., An approach of designing and controlling free-form surfaces by using NURBS boundary Gregory patches, Computer Aided Geometric Design, 13, 825-849, 1996.

[13] Wang, Yu, Intersection of offsets of parametric surfaces, Computer Aided Geometric Design, 13, 453-465, 1996.

[14] Lin, Rong-Shine and Koren, Y., Efficient tool-path planning for machining free-from surfaces, Transactions of the ASME, 118, 20–28, 1996.

[15] Q.J. Ge,Srinivasan, L.,Rastegar, J., Low-harmonic rational Bezier curves for trajectory generation of high-speed machinery, Computer Aided Geometric Design, 14, 251-271, 1997.

[16] Nasri, A., Curve interpolation in recursively generated B-spline surfaces over arbitrary topology, Computer Aided Geometric Design, 14, 13-30, (1997).

[17] Dragomatz, D, Mann, S, A classified bibliography of literature on NC milling pathGeneration, Computer-Aided Design, 29(3), 239-247, 1997.

[18] Chih-Ching Lo, A new approach to CNC tool path generation, Computer-Aided design, 30(8), 649-655, 1998.

[19] Maekawa, T., An overview of offset curves and surfaces, Computer-Aided Design, 31, 165–173, 1999.

[20] Yang, D.C.H., Han., Z., Interference detection and optimal tool selection in 3-axis NC machining of free-form surfaces, Computer Aided Design, 31, 303–315, 1999.

[21] Lartiguea, C.,Thiebauta, F., Maekawa, T., CNC tool path in terms of B-spline curves, Computer-Aided Design, 33, 307-319, 2001.

[22] Hsi Yung Feng, Huiwen, Li., Constant scallop height tool path generation for three axis sculptured surface machining, Computer Aided Design, 34, 647-654, 2002.

[23] Narayanaswami, R., Pang, J., Multiresolution analysis as an approach for tool path planning in NC machining, Computer Aided Design, 35, 167-178, 2003.

[24] Ding, S.,Mannon, M.A., Poo, A.N., Yang, D.C.H, Han, Z., Adaptive iso planar tool path generation for machining of free form surfaces, Computer aided design, 35, 141-153, 2003.

[25] Yin, Z.,Rough and finish tool-path generation for NC machining of freeform surfaces based on a multiresolution method, Computer aided design, 36, 1231-1219, 2004.

[26] Renner, G.,Weib, V., Exact and approximate computation of B-spline curves on surfaces, Computer aided design, 36, 351-362, 2004.

[27] Wei, Eing-Jer, Lin, Ming Chang, Study on general analytical method for CNC machining the free-form surfaces, Journal of Materials Processing Technology, 168, 408–413, 2005.

[28] Choi, Y.K., and Banerjee, A., Tool path generation and tolerance analysis for free-form surfaces, International Journal of Machine Tools and Manufacturer, 47(3–4), 689–696, 2006.

[29] Choi, Y.K., and Banerjee,A.,Jae-Woo Lee, Tool path generation for free form surfaces using Bezier curves/surfaces, Computers and Industrial Engineering, 52, 486–501, 2007.

[30] Chen, T., Shi, Z., A tool path generation strategy for three-axis ball-end milling of free-form surfaces, Journal of Materials Processing Technology, 208, 259–263, 2008.

[31] Lasemi,Ali, Xue, Deyi, Gu, Peihua, Recent development in CNC machining of freeform surfaces: A state-of-the-art review, Computer Aided Design, 42, 641-654, 2010.

[32] Ibrahim, Z., Introduction to CAD/CAM, Tata McGraw Hill, New Delhi, 2010.

[33] Farin, Gerald, Curves and Surfaces for CAGD, Morgan Kaufmann.

[34] Koren, Yoram, Numerical control of machine tools, Tata McGraw Hill, New Delhi, 2010.

[35] Hearn, Donald and Baker, M.pauline, Computer Graphics, Pearson education, 2004.

[36] http://w.w.w.wikipedia.com/machining.