Analysis of MIMO system with MMSE-SIC Receiver for Equalization

A Thesis submitted in partial fulfilment of the requirements for the award of Degree of

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In

Wireless Communication

Submitted by

KARAN KUMAR ARYA

Roll No. 801463013

Under guidance of

Dr. AMANPREET KAUR

Assistant Professor, ECED

T.U, Patiala

ELECTRONICS AND COMMUNICATION ENGINEERING DEPARTMENT

THAPAR UNIVERSITY

(Established under the section 3 of UGC Act, 1956)

PATIALA – 147004 (PUNJAB)

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CERTIFICATE

I Karan Kumar Arya, hereby, declare that the work presented in the thesis entitled "**Analysis of MIMO system with MMSE-SIC Receiver for Equalization**" by me in partial fulfilment of the requirements for the award of degree of Master of Engineering in Wireless Communication from Thapar University, Patiala, is an authentic record of my own work under the supervision of Dr. Amanpreet Kaur, Assistant Professor, Electronics and Communication Engineering Department. The matter presented in this thesis has not been submitted in any other University/Institute for the award of any other degree.

Date: 29/7/2016

Karan Kumar Arya

Karan Kumar Arya

Roll no: 801463013

This is to certify that the above statement made by the student is correct to the best of my knowledge and belief.

Date: 29/7/2016

Amanpreet Kaur

Dr. Amanpreet Kaur

Assistant Professor

ECED

Thapar University, Patiala

Countersigned by:

(Dr. Sanjay Sharma)

Professor and Head, ECED

Thapar university, Patiala

(Dr.S.S. Bhatia)

Dean of Academic Affairs

Thapar university, Patiala

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ABSTRACT

Multiple-input–multiple-output (MIMO) technology is a latter development in wireless communications and has been shown to increase channel capacity in single user systems. The use of multiple antennas at both the transmitter and receiver can considerably increase the channel capacity. These systems are called the multiple input multiple-output (MIMO) systems.

Growth of mobile data applications has increased the requirement for wireless communication systems which offers high throughput, wide coverage, and improved reliability. The main challenges in the design of such systems are the limited resources (like transmission power, scarce frequency bandwidth and limited implementation complexity) and the impairments of the wireless channels which includes noise, interference, and fading effects. MIMO system is one of the favourable wireless technologies that can attain above demands.

In this thesis, the performance of Multiple-input–multiple-output (MIMO) scheme is analyzed under Rayleigh fading channel using Maximum Ratio Combining (MRC) method for Minimum mean square error (MMSE), MMSE-SIC, Zero-forcing (ZF), ZF-Successive interference cancellation (ZF-SIC) equalization techniques. The analysis of this combining method is done on the basis of two major factors, Signal to Noise Ratio (SNR) and Bit Error Rate (BER) performance using BPSK and QPSK modulation techniques. With increase in number of antenna on receiver side, SNR gets maximized and BER gets minimized. It is analyzed that MRC with QPSK modulation has shown better evaluation results than BPSK.

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MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
SIMO	Single Input Multiple Output
MISO	Multiple Input Single Output
MRC	Maximal Ratio Combining
EGC	Equal Gain Combining
SC	Selection Combining
SIC	Successive Interference Cancellation
OSIC	Ordered Successive Interference Cancellation
MMSE	Minimum Mean Square Error
ZF	Zero Forcing
SINR	Signal To Interference Noise
BER	Bit Error Rate

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CHAPTER 1

INTRODUCTION

Communication between humans was first by the sound through voice. With the desire for slightly more distance communication, there came devices like drums, some visual methods like, smoke signals and signal flags were used. The optical communication devices, utilized the light portion of the electromagnetic spectrum. With the advancement of the technology, now the electromagnetic spectrum, outside the visible region, has been employed for communication, through the use of radio. One of the humankind's greatest natural resource is the electromagnetic spectrum and antenna is key factor for utilizing this resource.

1.1. Wireless Communication:

Transmission from one place to another without using wires or cables is termed as Wireless Communication. In general, Wireless communication is considered as a subdivision of telecommunications. In broadcasting systems such as radio and TV is termed as one-way communication and in mobile phones is two-way communication. People do not remain in the same location, they can move one place to another, like office or home, and they need reliable connections. Wireless communication allows these facilities like users communicate to one another while travelling in car or bus or walking in streets. In traditional wire-based systems (such as telephones) wires required installation of cables or wires in one fixed location but now wireless communication provides such technologies those are available without any difficulties. Wired communication includes many types of fixed, cellular telephone, portable two-way radios communication devices and mobiles. Other examples are satellite television, headsets, keyboards and wireless computer mice, broadcast television [1].

Generations of Wireless Technology:

Mobile telephone's history can be classified into four generations. The first (pre-cellular) generation implied mobile telephones that mainly used a frequency band in a specific area. Such telephones had terrible problems with call completion and congestion. When a single user was using specific frequency in a particular geographic area, if another user could try to call in the same frequency the call was denied. In the following, the features of each generation are presented.

First Generation:

Cellular mobile telephones were developed in the first generation about the world using various, in-compatible analog technologies. Analog modulation techniques were used in these systems such as FM (Frequency Modulation) or FDMA (Frequency Division Multiple Access). The bandwidth is divided into some particular frequencies and then assigns these frequencies to the unique calls by using the frequency division multiplexing. The examples of first generation cellular systems are TACS (Total Access Communication System), NMT (Nordic Mobile Telephone) system, C-450, AMPS (Advanced Mobile Phone System) etc [2]. These systems had limitations such as long call set up time, insecure transmission, low service quality, etc.

Second Generation:

CDMA (Code Division Multiple Access) or TDMA (Time Division Multiple Access) digital access techniques are used in the second generation systems. To enhance the number of channels these systems use the combination of FDMA and TDMA access methods. In today's world GSM (Global System for Mobile) has widely popular among communication networks. Other technology in use is CDMA. Second generation systems gives better service functionality and reliability than the cellular mobile telephones of the first generation systems and more efficiently use the provided bandwidth. This generation systems support facsimile, voice and data services. The 2G digital services provided very useful features like; expanded capacity, caller ID, call forwarding, and short messaging [2].

Third Generation:

This generation system provides both data and voice communication at the very fast speed. For today's need the data rate provided by this system with the help of data transmission is quite less. 3G networks use a various forms of wireless network technologies, considering CDMA, EDGE, CDMA2000, UMTS, TDMA, GSM and WCDMA, and this leads a great deal of flexibility. Third generation systems provide better functionality and quality of communication and better call origin delay than second generation systems [2]. These system also provide high data rate and the user can access the wide variety of services through wireless. The biggest feature of this generation is video conferencing.

Fourth Generation:

A 4G system provides an end-to-end IP solution where voice, data and streamed multimedia can be served to users on an “Anytime, Anywhere” basis at higher data rates than previous generations.

This was followed, in 2001 by 3G, in 2011/2012 expected to be followed by real “real” 4G, which refers to all Internet Protocol (IP) packet switched networks giving Ultra Mobile Broadband (gigabit speed) access. This generation systems have collection of various techniques, OFDM (Orthogonal Frequency Division Multiplexing) and it is a fully packet-switched network. Apart from advantages, wireless communication systems are prone to some problems too [2].

1.2. Drawbacks of Wireless Communication:

A wireless Communication system is prone to lot of losses in transmitting the signal from one point to other through air. A major disadvantage is Multipath. It is used to identify the multi paths, a radio wave may follow between source and destination in LOS (Line Of Sight) communication. The signal received at a receiver, that was originated from the transmitter may come from different directions with different, amplitudes, phases, propagation delays and angle of arrival. These components compound vector ally at the receiver side and cause Fade and Distort to the signal. Issues of multipath:

- A signal arrived at the receiver get multiple copies of that signal with different phases.
- Detection may get more difficult if phase add acoustically, the signal relative to noise declines.
- Delay propagates resulting in ISI (Inter-Symbol Interference) one or more retarded copies of a signal may attain the same as the original signal for a resultant bit.

The most difficult and discouraging job in receiving radio signals is fluctuations in signal strength, this is termed as Fading. This leads to rapid variations of the amplitude and phase of the signal if the vehicle moves over a distance in the order or wave length or more. Fading is the attenuation that a carrier-modulated telecommunication signal goes through over certain propagation media in the wireless communication. The fading may changed with time, radio frequency or geographical location and is often simulated as a random process.

A technique used in wireless communication systems to overcome fading is the use of MIMO (Multiple Input Multiple Output) Systems [3].

1.3. Benefits of MIMO systems:

MIMO leads to significant increase in data rates that are possible in wireless communication systems and therefore this is a very crucial technology in 3G, 4G wireless communications since these 3G, 4G systems are based on very high data rates. They enable transmission of very high data rates above the wireless lines.

MIMO systems are used to enhance or improve the throughput (data rate) of wireless access, even under circumstances of multipath, interference and signal fading for far distances.

1.4. MIMO Systems:

MIMO communication systems can be defined intuitively $[GSS^{+}03, PNG03]$ by considering that multiple antennas are used at the transmitting end as well as at the receiving end. The core idea behind MIMO is that signals samples in the spatial domain at both ends are combined in such a way that they either create effective multiple parallel spatial data pipes (therefore increasing the data rate), and/or add diversity to improve the quality (bit-error rate or BER) of the communication[5].

MIMO makes antennas work smarter by enabling them to merge data streams arriving from different paths and at different times to effectively increase receiver signal-capturing power. MIMO Systems have multiple transmit antennas and receive antennas having identical Receiver and Transmitter. These transmit Several Information flow in parallel in available space so termed as Spatial Multiplexing. Spatial diversity technology is used in smart antennas, which sets surplus antennas to better use. MIMO technology takes advantages of a natural radio-wave phenomenon known as multipath. Problems with multipath consider refraction, atmospheric ducting, and ionospheric reflection, terrestrial objects and reflection from water bodies such as hills and buildings. Where there is more than single antenna at transmitter and receiver end of the radio link, this is termed MIMO. MIMO can be used to provide improvement in channel throughputs as well as channel robustness. Figure 1.1 shows a typical MIMO model.

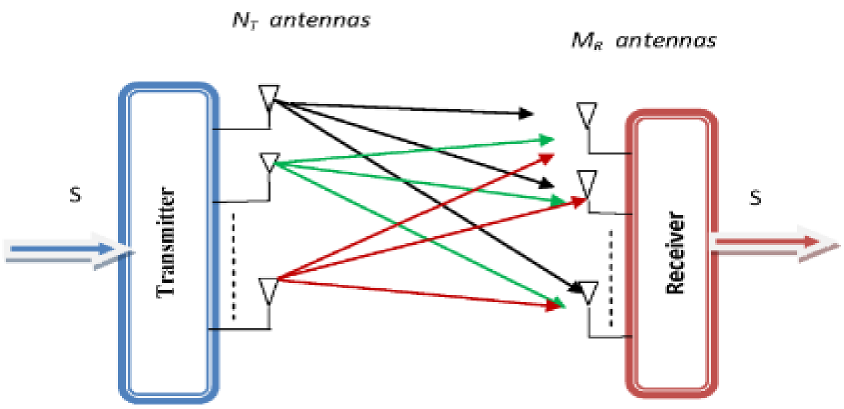

Fig 1.1 MIMO system for N_T transmit antenna and M_R receive antenna [6]

Figure 1.1 shows that MIMO system consists of N_T transmitting antennas and M_R receiving antennas. In order to derive the received signal matrix from this MIMO system.

Consider 2×2 MIMO system with Rayleigh faded channel assumed.

The received signal on the first receive antenna is:

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 \tag{1.1}$$

The received signal on the second receive antenna is:

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \tag{1.2}$$

Where y_1 and y_2 are received symbols on the first and second antenna respectively, h_{11} is the channel from 1st transmit antenna to 1st receive antenna, h_{12} is the channel from 2nd transmit antenna to 2nd receive antenna, h_{22} is the channel from 2nd transmit antenna to 2nd receive antenna, x_1 and x_2 are the transmitted symbols and n_1 and n_2 is the noise on 1st and 2nd receive antenna respectively.

Equation (1.1) and Equation (1.2) can be represented in matrix form as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Therefore, the received vector can be expressed as:

$$y = Hx + n \tag{1.3}$$

In order to and reduce multipath and fading, some simpler versions of MIMO systems are used sometimes. These are as mentioned in the hierarchical order in which these were introduced in the industry and is mentioned in next subsections.

1.4.1. MIMO-SISO:

SISO - Single Input Single Output is the simplest term of the radio link is specified in MIMO. This system works with a single antenna at transmitter as well as receiver side. There is no requirement of addition processing and diversity. Figure 1.2 shows a SISO system block diagram [7].

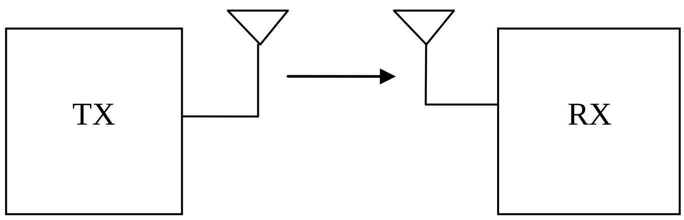

Fig 1.2 Block Diagram of SISO [7]

1.4.2. MIMO-SIMO:

The multiple output version of MIMO is SIMO (single Input Multiple Output), take place where the single antenna at the transmitter side and the multiple antennas at the receiver side. It is frequently used to capable a receiver system which receives signals from a number of commutative sources to combat the effects of fading. It has been used for many years with short wave listening/receiving stations to combat the effects of and interference ionospheric fading. The SIMO systems are satisfactory in some applications except where the receiving system is situated in the mobile device like mobile phones, the execution may be restricted by size battery, cost and cost [7]. Figure 1.3 shows a SIMO system block diagram.

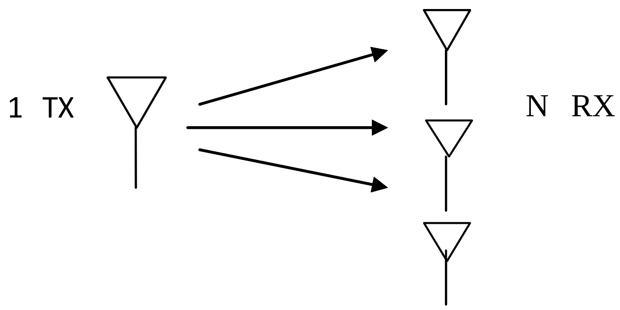

Fig 1.3 Block Diagram of SIMO [7]

1.4.3. MIMO-MISO:

MISO or the multiple input and single output is a scheme in which there are multiple antennas at the transmitters side and single receiving antenna at the receiver side. It is a scheme of Radio Frequency (RF) wireless communication system. MISO is like SIMO except at the receiver side, a single antenna is employed. MISO is known as transmit diversity as well. In this scenario, the identical data is communicated unnecessarily from the two transmitter antennas. Then the receiver is capable to receive the best signal then that can be use to take out the necessary data. Figure 1.3 shows a MISO system block diagram [7].

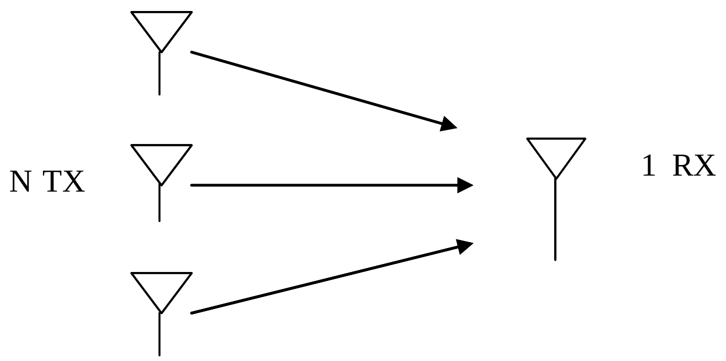

Fig 1.4 Block Diagram of MISO [7]

Using MISO, the advantage is that, multiple antennas and the unneeded processing is displaced from the source to the destination. This strategy has different applications like in W-LANS, Digital television. These systems are beneficial as the unneeded processing has been moved from receiving side for the transmitting side and therefore say in illustration of less power, processing and mobile phones is needed at the receiver end or the user end.

Multipath is the propagation phenomenon in the wireless communication that results in radio signals caused by two or more paths received at the receiving end. Causes of multipath consider refraction, atmospheric ducting, and ionospheric reflection, terrestrial objects and reflection from water bodies such as hills and buildings. Thus there would be multipath interference, problem with multipath fading. These paths include re-radiation by the ionospheric layers, ionospheric refraction, ground waves, etc.

1.5. Fading:

Fading is a popular issue when transmitting a signal in wireless communication channel. Fading causes a lot of degradation in the performance of any modulation technique in wireless communication and therefore we have to device ways and means to overcome the effects of fading. The knowledge of multipath is essential to understand fading. In wireless telecommunications, Multipath is the propagation phenomenon in the wireless communication that results in radio signals caused by two or more paths received at the receiving end. Causes of multipath consider refraction, atmospheric ducting, and ionospheric reflection, terrestrial objects and reflection from water bodies such as hills and buildings. Multipath fading happens in any surrounding where there is multipath extension and within the radio communication system there is some movement of elements. In the ionosphere, fading gives result from absorption of the RF energy.

1.5.1. Types of Fading:

In wireless communication channel estimation is necessary to neutralize the consequences of channel noise on the signal. The wireless channel is basically characterised by the fluctuations of the channel strength over frequency and time. The type of fading being intimated by the signal transmitted in a wireless channel is dependent on the relationship of the signal arguments and the channel arguments. Signal arguments include symbol period, bandwidth, and channel arguments include Doppler and RMS (Root Mean Square) delay spread. According to the time dispersive nature of the channel the types of fading can be characterized as [1].

- **Large-scale fading:** When path loss of the transmitted signal occurs as a purpose of shadowing and distance by large obstacles such as mountains and buildings, it is known as large-scale fading. In general, large scale fading is traceable to diffraction.
- **Small-scale fading:** When the multiple path signals are interfered constructively or destructively in the radio link, fading occurs and is known as small-scale fading. In general, small scale fading is described by a Rician or Rayleigh probability function. It is classified by its rate of fluctuation (slow or fast) and spectral properties (flat or frequency-selective).

Various techniques can be implemented at the receiver side to overcome the effect of fading.

1.5.2. Channel Models:

To counter the effects of multipath signal fading various methods are designated at the receiving end. For this various mathematical frameworks are used to speculate the unspecialized behaviour of the concerned channel. Some important channel frameworks are [8]:

1.5.2.1. Rayleigh Faded channel:

At the receiver side, addition of different signals with different phases causes multipath fading in wireless communication systems. The phase difference is generated in this received signal is because of signals having travelled different distances by travelling on different paths of a propagation surrounding of wireless communication. As the phases of the coming paths are changing quickly phase and amplitude of the received signal experiences quick variations that are frequently modelled as a random variable with a specific distribution. To simulate distributed signals Rayleigh distributions are used which reaches at a receiver by more than one path [9]. Rayleigh distribution is most normally used for fast fading. The probability density function of Rayleigh distribution is given by:

$$f_{ray} = \frac{r}{\sigma^2} e^{-r^2/\sigma^2}, r \geq 0 \tag{1.4}$$

Where, r is random variable and σ^2 is the variance. To distinguish multipath fading/attenuation channels Rayleigh flat fading channel is normally used that impact the execution of wireless communication systems when there are no LOS (Line-Of Sight) exists among receiver and transmitter.

1.5.2.2. Rician Fading:

When the position of the receiver is on a line of sight (LOS) with respect to the transmitter the Rician distribution is particularly used, hence there will be an LOS signal element in the arriving signal owing to the multipath [9]. For some urban scenarios and for satellite communications are acceptable by Rician fading is admirable. It is type of small-scale fading as the possibility of deep fades is fewer than that in the Rayleigh-fading case. The probability density function for the Rician distribution and is mathematically stated as follows:

$$P_R(R) = \frac{R}{\sigma^2} e^{\frac{-(R^2 - A^2)}{\sigma^2}} I_0\left(\frac{RA}{\sigma^2}\right), 0 \leq R \leq \infty \tag{1.5}$$

where, σ^2 is the local mean scattered power.

A is the peak amplitude of the Line-of Sight (LOS) signal.

I_0 is the modified Bessel Function of the first kind.

1.5.2.3. Nakagami Fading:

Another essential distribution in wireless communication is Nakagami-m distribution. This fading was developed from experimental measurements to model the statistical fading of the multipath scenario [9]. Sometimes Nakagami-m distribution is denoted by m-distributions, a broad class of fading channel circumstances can be modelled. In indoor mobile propagation, scintillating ionospheric radio links and land-mobile Nakagami-m distribution gives best fit. Also more recent studies showed that Nakagami-m gives the best fit for satellite-to-outdoor and satellite-to-indoor radio wave propagation.

The probability density function for the Nakagami-m distribution and is mathematically stated as follows:

$$p_R(R) = \frac{2m^m R^{2m-1}}{\Omega^m \Gamma(m)} exp\left(-\frac{mR^2}{\Omega}\right) \tag{1.6}$$

where, $R \geq 0$is the amplitude of channel, $\Omega = \mathrm{E}(R^2)$ is average fading power, $\mathrm{E}(\cdot)$ is the expectation operator, and $\Gamma(.)$ is gamma function. Above, m is the Nakagami fading parameter which determines the severity of the fading. *m* is the inverse of the normalized variance of R^2.

$$m = \frac{(E(R)^2)^2}{Var\,(R^2)} \tag{1.7}$$

where, *m* is the shape factor of the Nakagami or the gamma distribution.

1.6. Additive White Gaussian Noise Channel:

Additive White Gaussian Noise (AWGN) channel is common channel representation for examining all modulation techniques. In this channel, white Gaussian noise to signal passing all the way through it, because it is known that it has uniform power across the frequency band for the overall information system. It is used in information theory to imitate the consequence of several random processes that arise in nature. According to above theory, the received signal can be written as:

$$r(t) = x(t) + n(t) \tag{1.8}$$

Where $n(t)$, is additive white Gaussian noise. $n(t)$ is stationary random process with flat power spectral density given by $N_o/2$.

The effects of Multipath, fading, ISI and the fading in a specific kind of environment can be overcome by a process called equalization.

1.7. Equalization:

The distortion is established in the channel in terms of delay and amplitude when the signal passes throughout the channel those results in Inter Symbol Interference (ISI). ISI deforms the transmitted data that causing bit error at the receiver side. ISI has been known as the main hazard in high speed data transmission over wireless media. Therefore, Equalizers are used to deal with ISI. An equalizer is executed at the baseband or at IF in a receiver. And the essential receiving method of any communication resides with noise signal performance. Therefore to reduce the noise element exists in communication, equalization techniques are used. To present baseband waveforms the baseband complex envelope term can be used, the channel answer demodulated signal and adaptive equalizer algorithms are generally simulated and applied at the baseband.

1.7.1. MIMO EQUALIZERS (RECEIVERS):

Before demodulating the signal received at the receiver end the equalization process should be performed. Reducing the effect of Inter Symbol Interference (ISI) in MIMO system which occurs because of spatial diversity is the important objective of equalization. Equalization can be performed by using both linearly and non-linearly manners. In the Linear category there are two main equalizing algorithms 1: Minimum Mean Square Error (MMSE equalizer) 2: Zero Forcing (ZF-equalizer).

1.7.1.1. ZF-Equalizer:

In wireless communication systems zero forcing comes under linear equalization algorithm, it reverses the frequency response of the channel. The term Zero forcing indicates to reduce the Inter Symbol Interference (ISI) to zero in a noise free scenario [10]. ZF equalizer selects the minimum deterministic squared error vector between all feasible transmit vectors $\bar{x}$ i.e. norm of error $||\bar{y}-\bar{H}\bar{x}||^2$ should be reduced and from the vector differentiation techniques estimated error reducing transmit vector can be given as:

$$\hat{x} = ((\bar{H}^H\bar{H})^{-1}\bar{H}^H)\,\bar{y} \qquad (1.9)$$

where, H is a full rank square matrix. This is nothing except executing multiplication process of a pseudo inverse of channel matrix with received vector. This can successfully diminish the ISI except it results in vast noise enrichment. Since the new noise vector is:

$$\hat{n} = ((\overline{H}^H \overline{H})^{-1} \overline{H}^H)^{-1} \overline{n} \quad (1.10)$$

Noise power will be significantly increased if the channel transfer function is highly reduced at any frequency among bandwidth of interest, as is common in frequency selective channels. For better optimization between ISI mitigation and noise enrichment an equalizer is required. The suitable equalizer for this scenario is MMSE-Equalizer.

1.7.1.2. MMSE-Equalizer:

MMSE equalizer is proposed such that it reduces the average mean square error between transmitted symbol $\overline{x}$ and its estimate $\hat{x}$ at output of equalizer i.e. now the problem is Minimizing $E\{||\hat{x} - \overline{x}||^2\}$ and the resulting estimating vector is:

$$\hat{x}_{MMSE} = P_d (P_d \overline{H}^H \overline{H} + \sigma_n^2 [I]_r) \overline{H}^H \overline{y} \quad (1.11)$$

Where, P_d is the symbol power. In this case as h->0 the estimated transmit symbol value will be surrounded only. Therefore MMSE estimator is ROBUST to noise. In addition to, at high SNR relevance it works as a ZF-receiver only as its noise element will be subjugated by the symbol power. And at low SNR it involves a matched filter with transfer function$\overline{H}^H$.

A MIMO system is proficient of overcoming the problem of ISI and fading by the use multiple antennas at Receiver and Transmitter. With multiple antennas used, the way in which output is combined from these antennas leads to a different kind of output at the Receiver side, this is known as diversity combining and is of various types as mentioned in section below.

1.8. Diversity:

Diversity is primarily used to counter act fading effects and we have seen fading causes a big degradation in the performance of any modulation techniques and channel impairments. There are different types of diversity techniques. The popular ones are antenna diversity, frequency diversity, time, polarization, angle and code diversity. Of this antenna diversity is one of the more admired ones where in we use multiple antennas.

Now these antennas can be used at both transmitter and receiver either at transmitter or receiver.

Frequency diversity requires using frequency signals which are split by the coherence bandwidth of the channel. In Time diversity, we must separate the transmission more than the channel’s coherence time. Polarization diversity, the horizontal and the vertical polarize signals fade differently and independently, angle diversity and used different codes so we can have code diversity. To counter the effects of inter symbol interference or ISI equalization is used, diversity is frequently employed to decrease the depth and period. So the depth of fade and the period of fades experienced by a receiver in a flat fading scenario can be overcome by using diversity. These methods can be employed both at mobile receivers and at base station, any of the diversity techniques can be deployed either at the mobile receivers or at base station.

Spatial diversity is the most widely used diversity technique that employs multiple antennas at the base station because comprising many antennas in our handset is a little inconvenient or you can even have antennas on different base stations. So we really get a good spatial diversity. The key point in spatial diversity is the division of the antenna components depending upon how clattered is the multipath environment, our antenna separations may have to be increased or decreased.

1.8.1. Diversity Techniques:

Different types of diversity techniques available are:

1.8.1.1. MRC (Maximal Ratio Combining):

For independent AWGN channels Maximal-ratio combining is the best combiner. MRC is a linear combining scheme, where different signal inputs are independently added and weighted. In this combining technique, the signals are weighted and co-phased before combining and summing from individual diversity branches. In the MRC combining method, weighting, summing circuits, and co-phasing are needed. For increasing the compounded Carrier to Noise Ratio (CNR), the weights have to be chosen as relative to the individual signals level. The employed weighting to the all diversity subdivision has to be weighted related to the value of SNR. Figure 1.5 shows the block diagram of MRC.

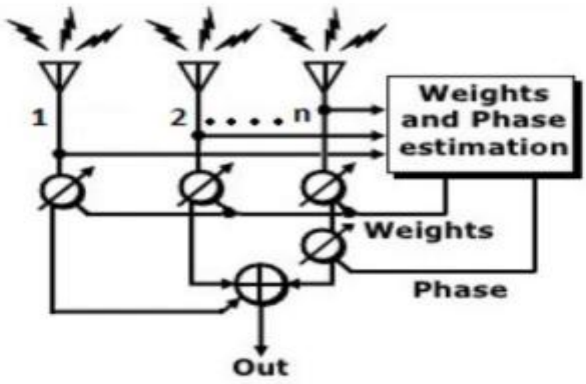

Fig 1.5 Block diagram of Maximal Ratio Combining (MRC) diversity scheme [11]

On the i^{th} receiver antenna, the received signal is:

$$y_i = h_i x + n_i \tag{1.12}$$

where, y_i = received symbol on the i^{th} receive antenna

h_i = channel on i^{th} receive antenna

x = transmitted symbol

n_i = noise on i^{th} receive antenna

The equalized symbol is:

$$\hat{x} = \frac{h^H y}{h^H h} = \frac{h^H}{h} + \frac{h^H n}{h^H h} = x + \frac{h^H n}{h^H h} \tag{1.13}$$

Where $h^H h = \sum_{i=1}^{N} |h_i|^2$,

Sum of all the channel powers across all the receive antennas.

1.8.1.1a. Effective signal to noise ratio(E_b/N_o) with MRC:

The instantaneous bit energy to noise ratio, in the presence of channel h_i at i^{th} receive antenna is

$$\gamma_i = \frac{|h_i|^2 E_b}{N_o} \tag{1.14}$$

where, h_i = channel on i^{th} receive antenna

E_b/N_o is SNR

Equalizing channel withh^H, with the N receive antenna case, the effective bit energy is:

$$\gamma_i = \sum_{i=1}^{N} \frac{|h_i|^2 E_b}{N_o} \tag{1.15}$$

Efficient bit energy to noise ratio in an N receive antenna case is N times the bit energy to noise ratio for single antenna.

1.8.1.1b. Bit Error Rate with MRC:

If h_i is a Rayleigh distributed random variable, then ${h_i}^2$ is a chi-squared random variable with two degrees of freedom. The probability distribution function of γ is:

$$p(\gamma_i) = \frac{1}{(E_b/N_o)} e^{\frac{-\gamma_i}{(E_b/N_o)}} \tag{1.16}$$

As the efficient bit energy to noise ratio γ is the sum of N such random variables, the pdf of γ is a chi-squared random variable with 2N degrees of freedom. The pdf of γ is:

$$p(\gamma) = \frac{1}{(N-1)(E_b/N_o)^N} \gamma^{N-1} e^{\frac{-\gamma_i}{(E_b/N_o)}} \;,\; \gamma \geq 0 \tag{1.17}$$

where, E_b/N_o is SNR (Signal-to-Noise Ratio)

N is diversity branches

BER figuring in AWGN, the efficient bit energy to noise ratio with MRC is γ, the total bit error rate is the integral of the conditional BER integrated over all possible values of γ.

$$P_e = \int_0^\infty \frac{1}{2} erfc(\sqrt{\gamma})\; p(\gamma) \mathrm{d}\gamma$$

$$= \int_0^\infty \frac{1}{2} erfc(\sqrt{\gamma}) \frac{1}{(N-1)!(E_b/N_o)^N} \gamma^{N-1} e^{\frac{-\gamma}{(E_b/N_o)}} d_\gamma \tag{1.18}$$

1.8.1.2. EGC (Equal Gain Combining):

The EGC is similar to MRC except the weighting circuits are omitted in this. The performance betterment of EGC is lower than MRC only up to little extent as there is an opportunity to assemble the accompanying noise and interference, high quality signals, which are free from noise and interference free. In EGC's typical process non-coherently combined but individual signal subdivisions are combined coherently. MRC is considered as the most ideal combining method but expensive in designing to set the gain in all subdivision at receiver side. It requires proper tracking for the tangled fading, which is very ambitious to attain practically. Yet, a easy phase lock summing circuit is used, because of this it is very simple to design an EGC. Figure 1.6 shows the block diagram of EGC.

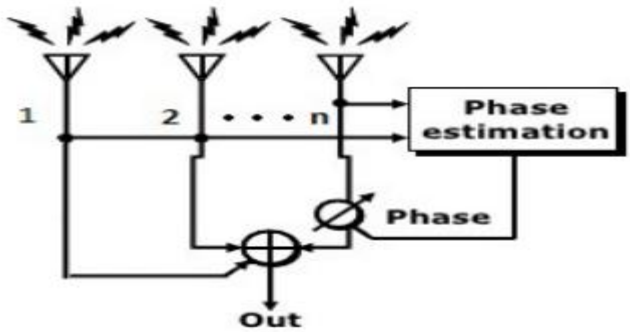

Fig 1.6 Block diagram of Equal Gain Combining (EGC) diversity scheme [11]

EGC can be applied in the reception of diversity with coherent modulation. The diversity subdivisions are compounded here with close weights but envelope gains of diversity channels and conjugate phase are ignored in EGC. The gross system of EGC is as following, because there is no envelope gain estimation of the channel.

On the i^{th} receive antenna, equalization is performed at the receiver by dividing the received symbol y_i by the known phase of h_i. The channel h_i is represented in polar form as $|h_i|e^{j\theta_i}$. The decoded symbol in sum of the phase compensated channel from all the receive antennas.

$$\hat{y} = \sum_i \frac{y_i}{e^{j\theta_i}}$$

$$= \sum_i |h_i| x + \tilde{n}_i \qquad (1.19)$$

Where, $\tilde{n}_i = \frac{n_i}{e^{j\theta_i}}$ is the additive noise scaled by the phase of the channel coefficient. For PSK (Phase Shift Keying) modulation, by the phase of channel coefficient, equalization is adequate. Withal, for QAM (Quadrature Amplitude Modulation) modulation, counterbalance for the amplitude is also essential whenever equalizing.

1.8.1.3. SC (Selection Combining):

In SC, through the number of antennas, the division that receives the signal with the highest SNR is used and linked to the demodulator. Greater the number of accessible branches, higher the possibility of having a large SNR at the output [12]. Figure 1.7 shows the block diagram of SC.

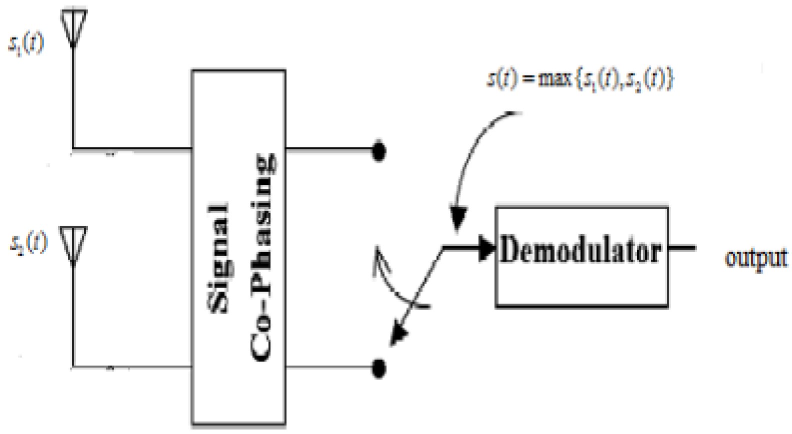

Fig 1.7 Block diagram of Selection Combining (SC) diversity scheme [12]

The combined output is given by:

$$y(t) = Ae^{j\theta_i}s(t) + z(t)\text{n} \qquad (1.20)$$

A= max$\{A_0, A_1, \ldots., A_{M-1}\}$

The received SNR is:

$$\Gamma = \frac{A^2 E_b}{N_o} = \max\{\Gamma_0, \Gamma_1, \ldots., \Gamma_{M-1}\} \qquad (1.21)$$

The CDF of Γ is:

$$P_\Gamma(\gamma) = \prod_{i=0}^{M-1} P_{\Gamma_i}(\gamma) \qquad (1.22)$$

The bit-error rate probability is:

$$P_e = \frac{1}{2}\sum_{k=0}^{N}(-1)^k \binom{N}{k} \left(1 + \frac{k}{E_b/N_o}\right)^{-1/2} \qquad (1.23)$$

1.9. Thesis Objectives:

Based upon the Literature survey has been carried out for MIMO systems. It was observed that fading and ISI (Inter Symbol Interference) are the major hurdles in these systems to support a good SNR (Signal to Noise Ratio) and thus a good data rate in the system. To overcome this, diversity techniques can be used at the receiver, it was studied that MRC (Maximal Ratio Combining) outperforms other diversity techniques.

To combat ISI, MMSE equalizers have also been presented in the literature. The research gaps show that not much work is available in literature to overcome ISI by the use of Minimum Mean Square Error - Successive Interference Cancellation (MMSE-SIC) receivers. So the objectives of this thesis are defined as:

1. Compare the performance of existing Zero Forcing - Successive Interference Cancellation (ZF-SIC) and Minimum Mean Square Error - Successive Interference Cancellation (MMSE-SIC) receivers to propose that MMSE-SIC is better in a fading environment.
2. Many modulation techniques like (Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) have been used with the MMSE-SIC receiver to propose that QPSK perform better in a fading environment with MMSE-SIC receiver.
3. ML (Maximum Likelihood) Detection has also been introduced as a good detection scheme with MIMO system to combat fading and ISI.

1.10. Thesis Organisation:

The thesis is divided into six chapters:

- **Chapter 1:** is dedicated to the Overview of Wireless communication, MIMO communication, Diversity and Equalization.
- **Chapter 2:** includes the Literature Survey on the Minimum Mean Square Error (MMSE) Zero Forcing (ZF) equalizers and ML detection with SIC (Successive Interference Cancellation) using different modulation schemes.
- **Chapter 3:** includes theoretical and mathematical analysis of SIC, ZF, MMSE equalizers and ML detection (i.e. ZF-SIC and MMSE-SIC), BER performance analysis of ZE-SIC and MMSE-SIC using BPSK modulation scheme.
- **Chapter 4:** includes the Results and Discussions.
- **Chapter 5:** is dedicated to Conclusions and Future Scope.

CHAPTER 2
LITERATURE SURVEY

In this chapter, a brief summary of the Literature survey and review for the current research work is provided. The review of the ensuing papers motivate in building this research work.

Abhishek Rawat [10] the author in this paper presented MIMO system for the detection of the symbol with minimum BER. Flat fading Rayleigh multipath is supposed in the channel and the modulation is BPSK. Finally this research observed that with Maximum Likelihood (ML) detection gives low Bit Error Rate (BER) for a given SNR.

Navdeep Kaur [11] the author in this paper presented the execution of an un-coded Single Input Multiple Output (SIMO) scheme is examined under Rayleigh fading channel using Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC) combining methods. On the basis of Bit Error Rate (BER) and Signal to Noise Ratio (SNR) performance using PSK and QAM modulation schemes, MRC and EGC studied examined that with QAM modulation MRC gives better results than EGC.

X. Zhang et al., [13] the authors in this paper presented a model for fading with Path Loss Process based model and analysed the execution of successive interference cancellation (SIC) *d*-dimensional fading scheme with power law density functions.

J. Ketonen et al., [14] the authors in this paper presented comparison of Linear Minimum Mean Square Error (LMMSE) K-best LSD (List Sphere Decoder) with the iterative K-best LSD and iterative successive interference cancellation (SIC) detector evaluated in 3G LTE (Long Term Evolution) system and the research found that K-best LSD performs better than SIC when highly correlated MIMO channels are used.

T. Im et al., [16] the authors in this paper presented a new signal detection technique termed as MMSE-OSIC^2. The MMSE-OSIC permits a number of users, with user choice supported on the Maximum Likelihood (ML) metric values that are again used to estimate the Log Likelihood Ratio (LLR) values. They compared this technique by MMSE-OSIC and establish that the projected MMSE-OSIC^2 signal detection technique attains near ML performance, involving a complication that is analogous to the MMSE-OSIC technique.

Tsung-Hsien Liu [17] the author in this paper presented that with optimal detection order, the real-valued MMSE-SIC detector using fast algorithm. The amount of real-valued multiplications required by this fast algorithm is nearly 1.57 times that needed by the complex-valued LDL^H Decomposition Based Algorithm with Backward Substitution) (LDBABs) algorithm.

J. Shen et al., [20] the authors in this paper presented some blind algorithms (Zero Forcing channel equalization). It can be devoted to single input multiple output systems precisely to remove the ISI. These algorithms employ second order statistics for multichannel equalization and based on the subspace removal of a preselected block column of the channel convolution matrix.

V. Pohl et al., [22] the authors in this paper presented filters that are generally Finite Impulse Response (FIR) filters and are decided by the number of inputs and outputs. In the time domain, equalization is well-known method for combating inter-symbol interference (ISI) in frequency selective channels. They have proposed the common formation of the linear zero forcing (ZF) equalizing filters for MIMO communication systems with more output and less inputs.

S. P. Jadhav et al., [23] the authors in this paper, compared the BER of various modulation schemes 64-QAM, BPSK,16-QAM and QPSK system under MRC and they found that in the presence of co channel interference and noise the performance of Maximal Ratio Combiner (MRC) maximizes the output SNR in the. The effect of fading and interference degrades the wireless communication system.

S. Mahanta et al., [24] the authors in this paper presented developed V-BLAST (Vertical Bell Labs Layered Space-Time) with spatial multiplexing method used in MIMO systems. They found that with V-BLAST that utilize successive interference scheme, offers the execution very near to the optimal turbo-MIMO approach, while providing astonishing enhancements in computational complexity.

Rick S. Blum et al., [25] the authors in this paper contemplate a system involves 2-antenna 16-state space–time codes with Successive Interference Cancellation (SIC) and channel approximation tht they used to decrease the complexity of 4-antenna space-time code systems. Using 2-antenna space –time code they achieved 2-dB improvement.

N. Satish Kumar et al., [26] the authors in this paper presented the execution of Minimum Mean Square Error (MMSE) equalizer receiver for MIMO communication channel. They found that as the $m \times n$ antenna increases with respect to number of transmitting antenna the BER decreases in MMSE equalizer based MIMO communication channel. Therefore this equalizer brings down the total power of the ISI and noise elements in the output.

P. Wadhwa et al., [27] the authors in this paper examined that the equalization problems for MIMO systems and with the use of numerical results simulated execution of the equalization methods for MIMO systems is carried out and it is proposed that as number of receiving antenna increases the BIT Error Rate (BER) decreases.

Allanki S. Rao et al., [28] the authors in this paper presented a model for BER analysis using multiple equalizers and then best equalization technique is proposed. 2dB betterment for BER is achieved with optimal ordering ZF-SIC compared to ZF-SIC, results in around 2dB of improvement for BER.

Kai-Kit Wong et al., [29] the authors in this paper presented a model to the issue of increasing the execution of a multiuser multiple-input–multiple-output (MIMO) system for communicating from one base station to many mobile stations in both frequency-selective and frequency-flat fading channels. They developed the antenna weights a lower bound of product SINR with a closed-form solution. They also found that MT×SINR system that operates closely gradient search-based optimization way for a 2-user MIMO system.

T. Yoo et al., [30] the authors in this paper presented Zero Forcing Beam forming (ZFBF) method and compared it with Dirty Paper Coding (DPC) method in MIMO system and proved that ZFBF method can attain the same asymptotic sum rate as the optimal DPC method as the number of users goes to infinity. This is because with a large number of users the transmitter can select user channels that are nearly orthogonal to each other.

C. Manchon et al., [31] in this paper authors have examined different MIMO receiver arrangements based on MMSE filtering and SIC (Sequential Interference Cancellation) for the downlink of the 3GPP LTE (Long Term Evolution) systems. They examined these arrangements using two parts: codeword-SIC structures, for each independently-coded stream by including the turbo-decoder inside the feedback loop processing are carried out and symbol-SIC receiver, in which the interference cancellation and detection is done independently for each subcarrier. They found that the optimum trade-off among computational complexity and receiver execution is accomplished by only cancelling the interference of a codeword when this has been efficiently decoded.

J.H. Park et al., [32] the authors in this paper presented a new time-domain recursive method for minimizing the complications of the MMSE-SIC (Minimum Mean Square Successive Interference Cancellation) method in Orthogonal Frequency Division Multiplexing (OFDM) systems. The complications of classical minimum mean square successive interference cancellation equalizer for reducing ICI (Inter-Carrier Interference) caused by time-varying multipath channels in OFDM systems are minimized by this method and provide the same execution as that of best MMSE-SIC equalizer.

A. K. Sahu [33] the author in this paper presented BER analysis of the Rayleigh fading channel using BPSK modulation scheme and showed it is slowly changed by changing the number of antenna either in receiver side or in transmitter side. Research showed that the better BER analysis is attained if transmitter diversity is less than receiver diversity in the MIMO systems conditions. He proposed that ML detector gives better result in comparison to ZF and MMSE equalizers.

J. Jalden [34] the author in this paper presented two algorithms for the implementation of ML detection in MIMO systems. The first algorithm is Semi-definite Relaxation and second algorithm is Sphere Decoder. He proposed in this paper that the anticipated complexity is lower bounded by an exponential function for a large number of detection problems.

P. Shang et al., [36] the authors in this paper presented a soft ZF (Zero Forcing) method for turbo-coded MIMO systems. They divided this method in three steps to reduce the computational complexity and these steps includes soft de-mapping, application of proper channel gain, ZF detection of the transmitted modulation symbol and observed that soft zero forcing method can attain just a dB power loss with considerably reduced detection complexity compared to ML (Maximum Likelihood) detection.

D. B. Keogh [37] the author in this paper presented a couple of settled MIMO radio channel’s capacity. Their descriptions are dependent on a single argument, the fluctuation of that offers some insight into capacity multiplication or instead the loss of it. He studied that any channel with an adequately high scattering characteristic is likely to be of full rank and sustain MIMO capacity multiplication.

D. Gesbert et al., [39] the authors in this paper presented a novel model framework for MIMO outdoor wireless fading channels that is more common and realistic than the normal i.i.d. (independent and identically distributed) model and expresses the effect of definite propagation geometry parameters in distributing conditions such as the range & the scattering radius.

B.A. Bjerke et al., [40] the authors in this paper presented the system as a Serially Concatenated Convolutional Code (SCCC) in which multipath channel and the code adopt the roles of component codes. They also presented an iterative ("turbo") MAP (Maximum A Posteriori) supported decoding scheme and equalization and measure its execution when utilized to a system with N transmit antennas and M receive antennas and showed that important interleaving gains can be accomplished by executing algorithmic precoding compared to without precoding system.

S. M. Alamouti [41] the author in this paper presented a simple two-branch transmit diversity scheme. Using one receiving antenna and two transmitting antennas, the novel system offers the same diversity order as Maximal Ratio Receive Combining (MRRC) with two receiving antennas one transmitting antenna and the use of M receiving antennas to offer a diversity order of 2M. He showed that this new system does not need any increase of bandwidth.

A. J. Paulraj et al., [42] the authors in this paper presented the use of multiple antennas at receiver side & transmitter side, commonly termed MIMO system, is an promising cost-effective technology that recommends significant benefits in making 1-Gb/s wireless links a reality.

L. Zheng et al., [43] the authors in this paper presented a novel slant that both types of gain can, in fact, are simultaneously achievable in a given channel, except there is a trade-off among them. For the highly distributed Rayleigh-fading channel they use a simple characterization of the optimal trade-off curve and use it to estimate the execution of present many antenna methods.

L. Szczecinski et al., [44] the authors in this paper presented two new algorithms to analyse the weights of the linear minimum mean square error & successive interference receivers in MIMO systems. These methods are compared to the fast V-BLAST method. They studied that the first proposed method has much easier implementation except the same arithmetic complexity, although the second proposed method lowers by 33% the needed number of arithmetic operations.

E. Biglieri et al., [45] the authors in this paper presented some information-theoretic characteristics of digital communications above fading channels. By emphasizing capacity as the most significant execution measure and examining both single user and multiuser transmission they have focused interest on the information theory of fading channels and finally they depicted Code design and equalization techniques.

G. J. Foschini [46] the author in this paper presented that with the tight capacity bound signals is layered in time and space. He proposed, with 21-dB average SNR at each receiving element, n = 8 at 1%, the data rate of42 b/s/Hz are attained. With the same bandwidth, total radiated transmitter power, the capacity is more than 40 times that of a (1, 1) system .

F. He et al., [49] in this paper the authors presented that to increase the secrecy capacity, cooperative diversity gives benefits to the authorized users. With amendment in diversity order and the transmission power, the users can obtain the target secrecy capacity with a practically low outage probability.

A. Trimeche et al., [50] the authors in this paper presented depth analysis of the ZF (Zero Forcing) and MMSE (Minimum Mean Squared Error) equalizers in MIMO systems. They proposed, in MIMO communication multiple antennas at both transmitter and receiver side with ZF, MMSE, ZF-SIC, MMSE-SIC and optimal ordering.

S. Patel et al., [51] the authors in this paper presented signal detection technology for Multiple Input Multiple Output – Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems in MMSE (Minimum Mean Square Error) equalization.

CHAPTER 3

SUCCESSIVE INTERFERENCE CANCELLATION (SIC)

3.1. Introduction:

As simultaneous wireless systems are flattering increasingly interference-restricted, there is a rising concern in using advanced interference moderation methods to better the system execution in addition to the conventional scheme of treating interference as background noise [13]. One important scheme is successive interference cancellation (SIC).

The optimum signal is perceived first rather than of jointly perceiving signals from all the antennas and its interference is cancelled from the individual received signal in the SIC receiver. Then the second optimum signal is perceived and its interference cancelled from the rest of the signals and so on. This technique is known as successive interference cancellation (SIC) [14].

3.2 SIC:

SIC is also called as OSIC (Ordered Successive Interference Cancellation) method. It is a collection of linear receivers where every receiver perceives one of the parallel data streams, with the perceived signal elements successively cancelled by the received signal at individual stage. Particularly the received signal in is subtracted by the perceived signal so that in the subsequent stage rest of the signals among the less interference can be used. Figure 3.1 shows the block diagram of SIC signal estimation.

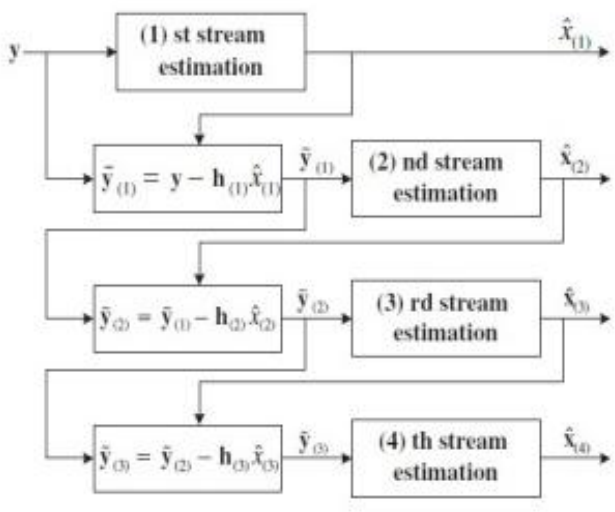

Fig 3.1 SIC signal estimation [14]

The above figure describes the OSIC signal recognition method for four spatial streams. Let $x_{(i)}$ indicate the symbol to be perceived in the *i*th order, that may be varied from the transmitted signal at the *i*th antenna, since $x_{(i)}$ depends on the order of recognition. Let $\hat{x}_{(i)}$ indicate a sliced value of $x_{(i)}$ [15]. Figure 3.2 shows the SIC flow chart.

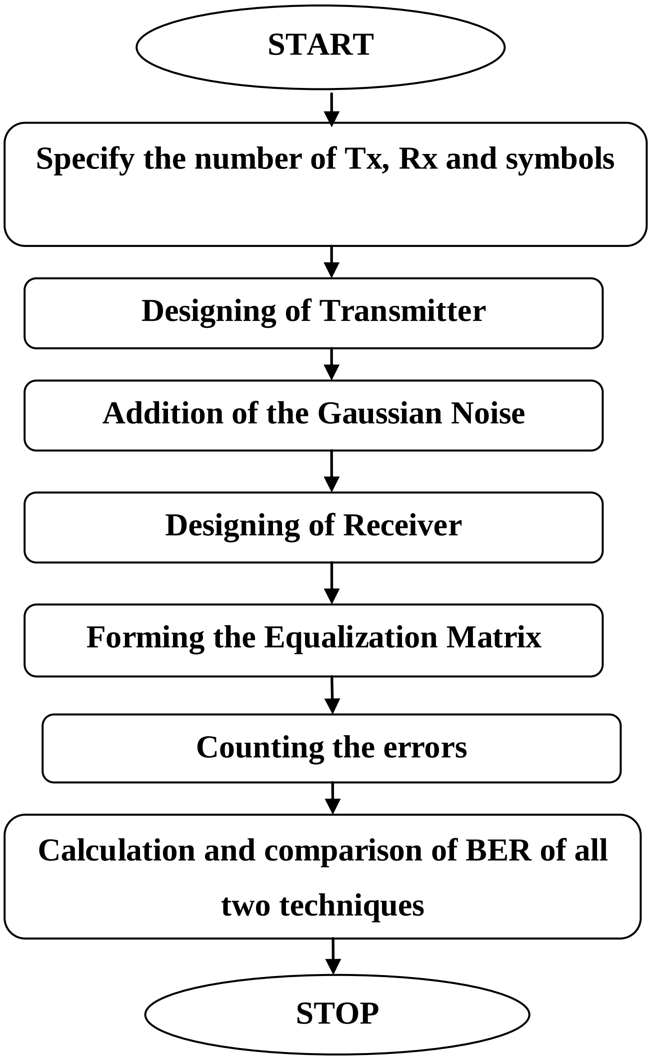

Fig 3.2 SIC Flow Chart

3.2.1. MMSE based SIC detection:

MMSE based SIC detection MMSE method is used for symbol estimation. The MMSE weight matrix is given by:

$$W_{MMSE} = (H^H H + I)^{-1} H^H \quad (3.1)$$

Where σ_n^2 is the noise variance.

The 1st stream is calculated with the 1st row vector of the MMSE weight matrix. After calculation and segment to produce $\hat{x}_{(1)}$ the rest of the signal in the first stage is formed by subtracting it from the received signal, that is:

$$\hat{y}_{(1)} = y - h_{(1)}\hat{x}_{(1)}$$
$$= h_{(1)}\big(x_{(1)} - \hat{x}_{(1)}\big) + h_{(2)}x_2 + \cdots + h_{(M_T)}x_{(M_T)} + n \qquad (3.2)$$

If $x_{(1)} = \hat{x}_{(1)}$, in that case the interference is effectively rejected in the course of calculating $x_{(2)}$; but, if $x_{(1)} \neq \hat{x}_{(1)}$, in this, error propagation is sustained because the minimum mean square error weight, which has been considered under the situation of $x_{(1)} = \hat{x}_{(1)}$ is used for calculating $x_{(2)}$.

In the previous stage, owing to the error propagation caused by incorrect decision, the order of detection has major control on the overall performance of ordered successive interference cancellation detection. So we use the post-detection SINR (Signal-to Interference-Noise-Ratio) for order. Signals having higher post-detection SINR's are perceived first [16]. Consider the linear minimum mean square error detection with the following post-detection SINR:

$$SINR_i = \frac{E_x \left|W_{i,MMSE}\ h_i\right|^2}{E_x \sum_{l\neq i}\left|W_{i,MMSE}\ h_l\right| + \sigma^2 \left\|W_{i,MMSE}\ \right\|^2} i = 1,2,\ldots,M_T \qquad (3.3)$$

Where, E_x is the energy of the transmitted signals, h_i is the ith column vector of the channel matrix H.

3.2.2. ZF based SIC detection:

In this ZF weight matrix is used for symbol estimation that is given by:

$$W_{ZF} = (H^H H)^{-1} H^H \qquad (3.4)$$

The 1st stream vector is calculated with the 1st row vector of the ZF weight matrix. After calculation and segment to produce $\hat{x}_{(1)}$, the rest of the signal in the first stage is formed by subtracting it from the received signal, that is:

$$\hat{y}_{(1)} = y - h_{(1)}\hat{x}_{(1)}$$
$$= h_{(1)}\big(x_{(1)} - \hat{x}_{(1)}\big) + h_{(2)}x_2 + \cdots + h_{(M_T)}x_{(M_T)} + n \qquad (3.5)$$

If $x_{(1)} = \hat{x}_{(1)}$, in that case the interference is effectively cancelled in the course of calculating $x_{(2)}$; but, if $x_{(1)} \neq \hat{x}_{(1)}$, in this case, error propagation is sustained because the zero forcing weight that has been considered under the condition of $x_{(1)} = \hat{x}_{(1)}$ is used for estimating $x_{(2)}$.

In the previous stage, owing to the error propagation caused by incorrect decision, the order of detection has major control on the overall performance of ordered successive interference cancellation detection. So we use the post-detection SINR (Signal-to Interference-Noise-Ratio) for order. Signals having higher post-detection SINR's are perceived first

Signals with a higher post-detection Signal-to Interference-Noise-Ratio are perceived first. Once M_T SINR values are estimated by using the zero forcing weight matrix, we select the resultant layer with the highest SINR. In the course of selecting the second perceived symbol, the interference owing to the first perceived symbol is rejected from the received signals. Assume that $(1) = l$ (i.e., the lth symbol has been rejected first). Then, the channel matrix is customized by removing the channel gain vector corresponding to the lth symbol as follows:

$$H^{(1)} = [h_1 h_2 \dots h_{l-1} h_{l+1} \dots h_N] \tag{3.6}$$

Using the modified channel matrix $H^{(1)}$in the place of H the ZF weight matrix is re-estimated. Now $(M_T - 1)$ SINR values, $[SINR_i]_{i=1, i \neq 1}^{M_T}$, are estimated to select the symbol with the maximum SINR. The same procedure is repeated with the rest of the symbols after rejecting the after that symbol with the maximum SINR [17].

3.3. Minimum Mean Square Error (MMSE):

To reduce the error between the transmitted and the estimated symbols MMSE method is used. MMSE minimizes both the noise as well as the interference elements but the ZF (Zero Forcing) equalizer minimizes only the interference elements. So, in MMSE equalizer the error between the transmitted and the estimated symbols at the receiver is reduced. Therefore, MMSE equalizer executes best in the occurrence of noise.

3.4. Zero Forcing with SIC:

Ordered successive interference cancellation is essentially based on subtraction of interference of already perceived components of **s** from the receiver vector **r** which results in a modified receiver vector with a few interferers. In other terms, successive interference cancellation is based on the subtraction of interference of already perceived components **s** from the received vector **x** which results in a modified receiver vector with a few interferers. When SIC is applied, the order in which the components of **s** are perceived is important to the overall execution of the method. To verify a good detection order, the covariance matrix of the estimation error $s - s_{est}$ is used. We know that the covariance matrix is given by:

$$Q = E[\in.\in^{H}] = \sigma_n^2 (H^H H)^{-1} \tag{3.7}$$

$$Q = E[(s - s_{est})(s - s_{est})^{H}] = \sigma_n^2 (H^H H)^{-1} \equiv \sigma_n^2 P \tag{3.8}$$

Where P= $H^{+}(H^{+})^{H}$. Let $s_{est}P$ be the pth entry of s_{est}, then the "best" is the one for which P_{pp} (i.e., the p-th diagonal element of **P**) is the smallest, because this is estimate with the smallest error variance. From the eq^{n} (3.8) it becomes clear that P_{pp} is equal to the squared length of row p of H^{+}.

Therefore, finding the minimum squared length row of H^{+} is equivalent. Summarizing, the decoding algorithm consist of three parts:

- Ordering
- Interference Nulling
- Interference Cancellation

Figure 3.2 shows the block diagram of SIC ZF detector.

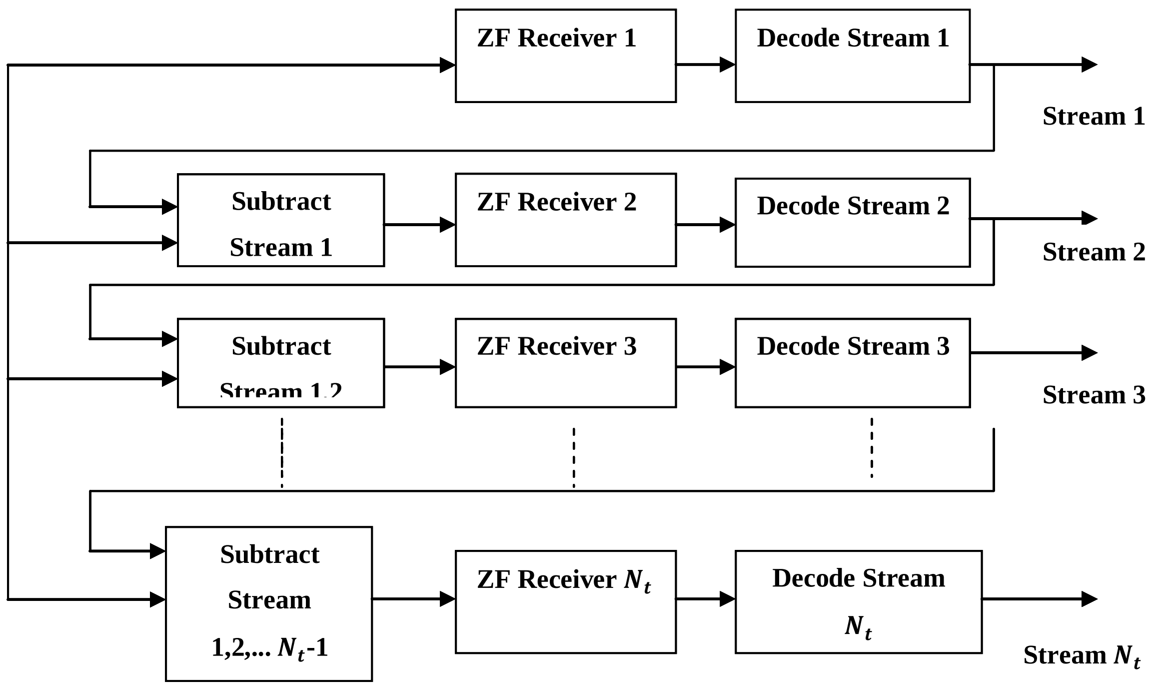

Fig 3.3 SIC Zero Forcing Detector

Figure 3.3 shows a, ZF detector to detect the data streams$s_1(m)$ decodes it and then subtracts this decoded stream from the received vector. Supposing the first stream is effectively decoded, and after that the second ZF detector only needs to deal with $s_3 \dots \dots \dots s_{N_t}$ as interference, sinces_1 has been accurately subtracted off. Thus, the second ZF detector projects onto a subspace which is orthogonal to$h_3 \dots \dots \dots h_{N_t}$ .This procedure is continued until the last ZF detector does not have to deal with any interference from the other data streams. We suppose subtraction is effective in all preceding stages. This SIC (Successive Interference Cancellation) ZF detector structural design is illustrated in Figure 3.2 so we can see here with respect to the ZF with OSIC and the ZF method introduces extra complexity.

The detection procedure was carried out using MATLAB 2007 b version for different Modulation Schemes.

3.5. Proposed Methodology:

For ZF-SIC equalizers, MRC, ML detection and MMSE-SIC equalizers with 2x2 MIMO system with BPSK (Binary Phase Shift Keying) and QPSK (Quadrature Phase Shift Keying) in Rayleigh fading environment.

3.5.1. BPSK Modulation:

In Binary Phase Shift Keying (BPSK) only one sinusoid is taken as basis function modulation. Modulation is achieved by varying the phase of the basis function depending on the message bits.

The BER for BPSK modulation in Rayleigh faded channel is defined as:

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_o}{(E_b/N_o)+1}}\right) \qquad (3.9)$$

3.5.1.1. ZF-SIC for a 2 X 2 MIMO with BPSK modulation:

System Model:

In order to compare the performance of ZF, MRC and ZF-SIC combine in a typical MIMO system with 2x2 receivers was considered. The receivers were using BPSK modulation with BER defined by equation (3.9).

Simulations were carried out using MATLAB 2007 and the results are shown in figure 3.4.

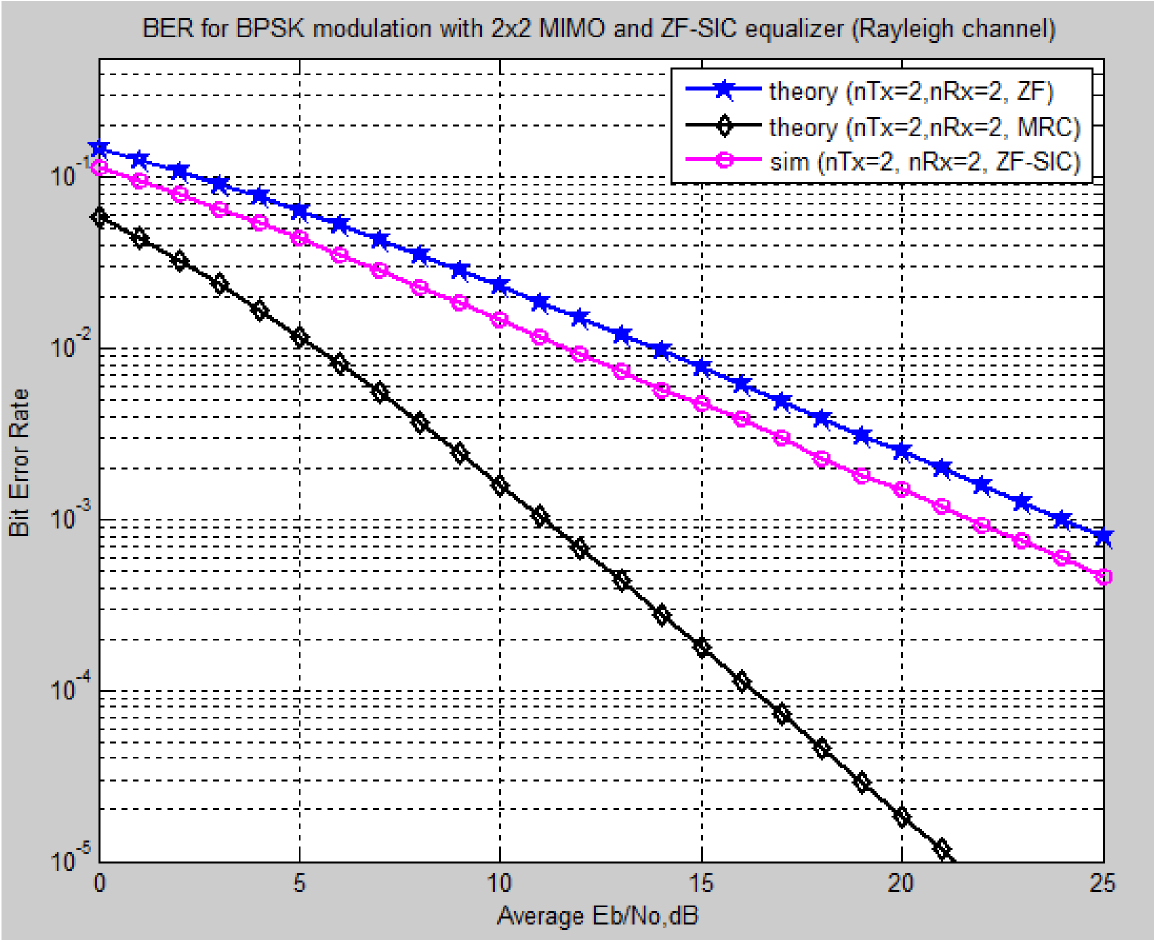

Fig 3.4 BER performance of the ZF based SIC detector for a 2 × 2 MIMO System with BPSK modulation.

It can be observed from the figure 3.4 that:

- MRC has lowest BER (Bit Error Rate) with lowest E_B/N_o (SNR).
- ZF-SIC (Zero Forcing – Successive Interference Cancellation) has a little higher BER and a much higher SNR (Signal to Noise Ratio).
- ZF has highest BER and higher SNR.

The optimised is ZF-SIC for a BPSK modulation environment.

3.5.1.2. MMSE-SIC for a 2 X 2 MIMO with BPSK modulation:

System Model:

To propose that MMSE-SIC is even better than ZF-SIC we again consider a 2x2 MIMO system with MRC, MMSE-SIC, ZF receiver and MMSE-SIC sort receivers. The receivers were using BPSK modulation with BER defined by equation (3.9).

Simulations were carried out using MATLAB 2007 and the results are shown in figure 3.5.

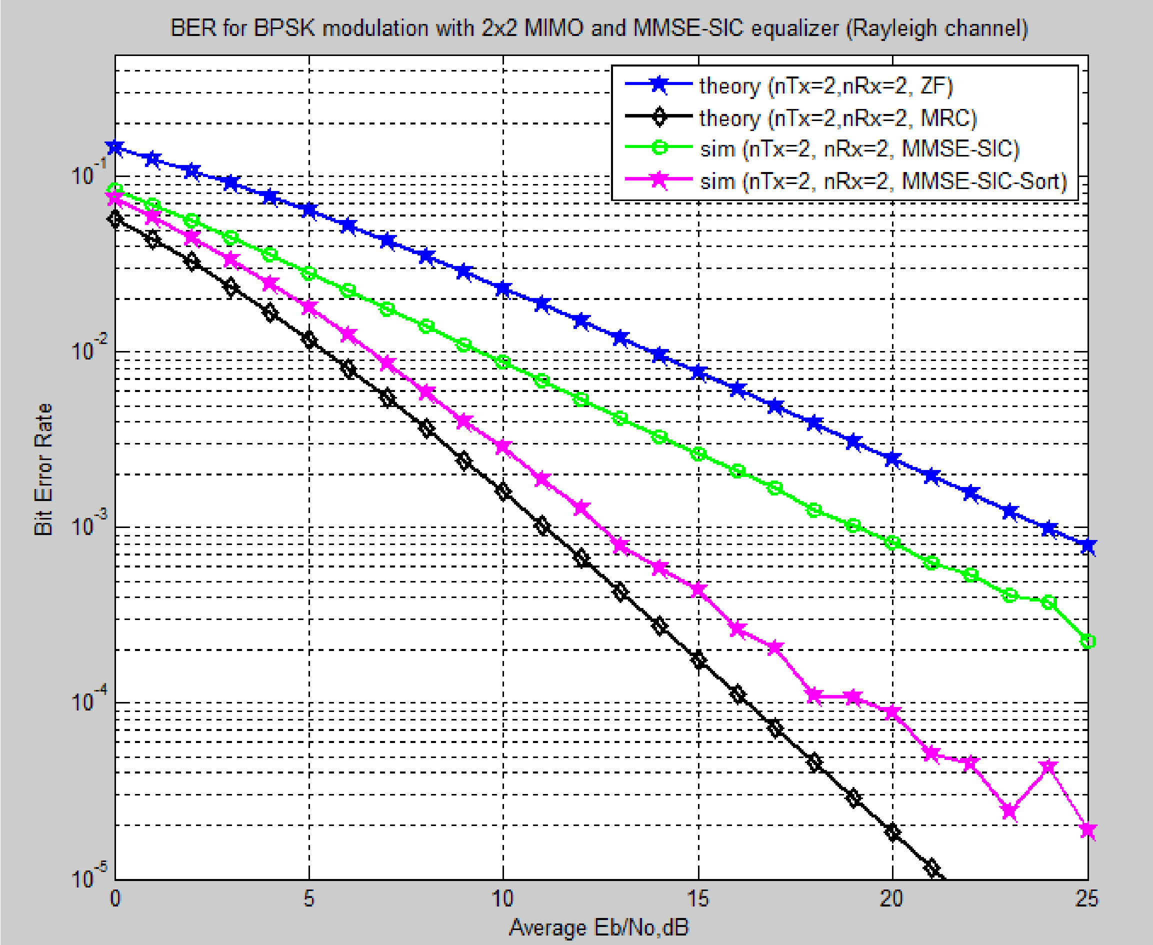

Fig 3.5 BER performance of the MMSE-SIC detector for a 2 × 2 MIMO System with BPSK modulation.

It can be observed from the figure 3.5 that:

- MRC has lowest BER with lowest E_B/N_o (SNR).
- MMSE-SIC sort has a little higher BER and SNR (Signal to Noise Ratio).
- MMSE-SIC has better BER and with better SNR.
- ZF has highest BER and higher SNR.

Since MSE-SIC give an optimized performance in terms of SNR and BER thus it is considered for current 2x2 MIMO systems with BPSK modulation scheme.

Since ZF-SIC and MMSE-SIC have shown good results in a typical 2x2 system with BPSK modulation, to see which performs better, we compare MMSE-SIC and ZF-SIC receivers in terms of E_B/N_o vs BER using equations (3.3), (3.5) and (3.9).

3.5.1.3. BER comparison of MMSE-SIC and ZF-SIC:

Figure 3.6 shows the BER vs SNR comparison of MMSE-SIC and ZF-SIC equalizer to see which performs better in a Rayleigh faded environment with BPSK modulation scheme used.

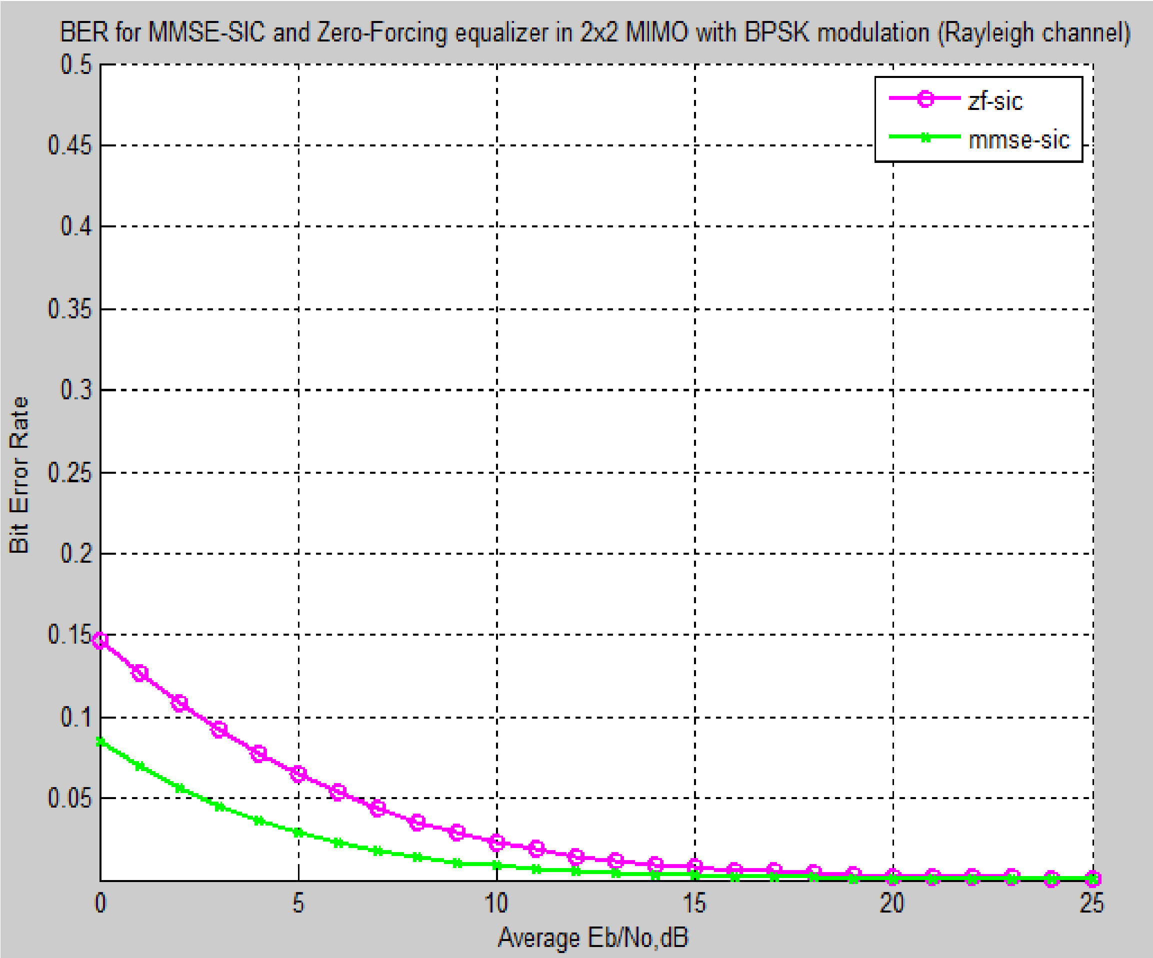

Fig 3.6 BER Comparison of the MMSE based SIC and ZF detector for a MIMO System with BPSK modulation.

Results:

From figure 3.6, it shows that both MMSE-SIC and ZF-SIC have the same data rate but the BER (Bit Error Rate) is less in MMSE-SIC as compare to ZF-SIC so in MMSE-SIC BER is less with same data rate so it is better.

An improved modulation scheme can also be used to improve the system performance, so MMSE-SIC and ZF-SIC receivers are analysed with QPSK (Quadrature Phase Shift Keying) modulation scheme to see which one performs better in a Rayleigh faded environment.

3.5.2. QPSK Modulation:

The equation below is the ratio of Energy consumed in Bit to the signal spectral noise and represent bit error rate in QPSK:

$$P_b = Q\sqrt{\frac{2E_b}{N_o}} \tag{3.10}$$

System Model:

In order to compare the performance of ZF-SIC, MMSE-SIC and ML detection in a typical MIMO system with 2x2 receivers was considered. The receivers were using QPSK modulation with BER defined by equation (3.10).

3.5.2.1. MMSE-SIC and ZF-SIC with QPSK Modulation:

Figure 3.7 shows the BER comparison of ZF-SIC and MMSE-SIC using QPSK modulation scheme.

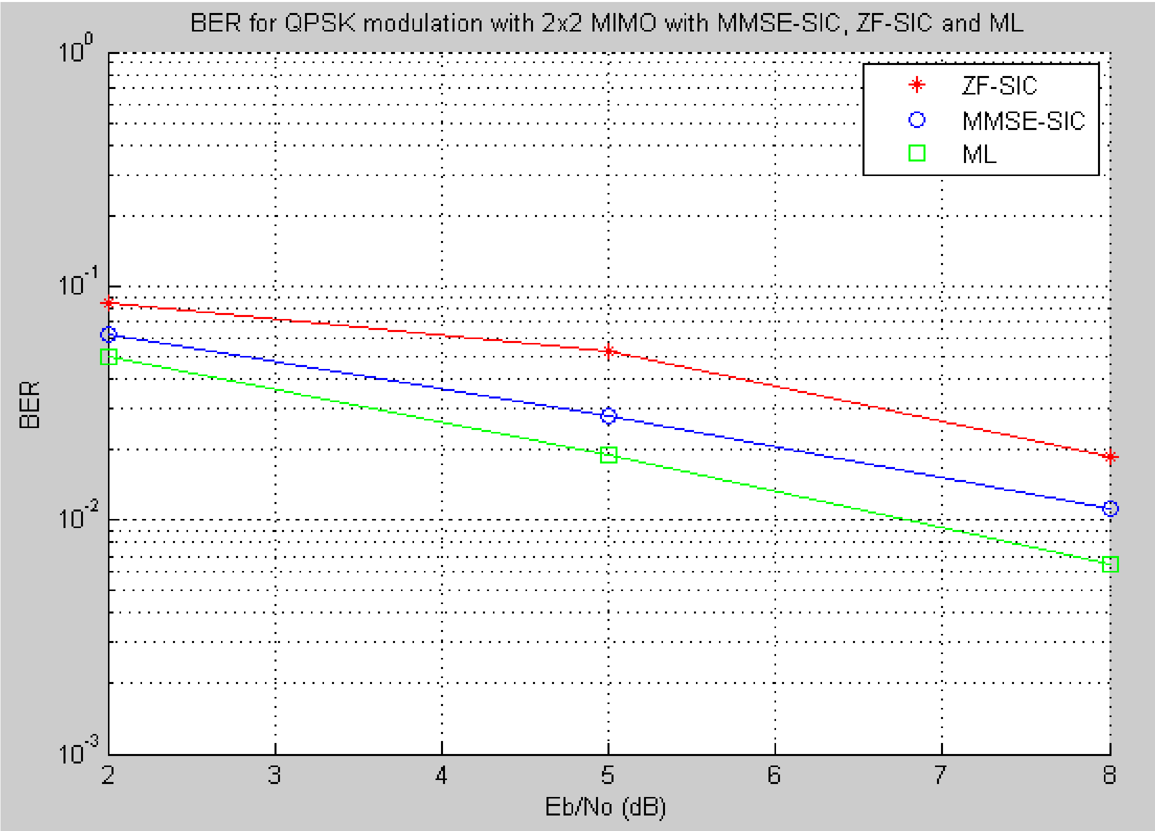

Fig 3.7 BER Comparison of MMSE-SIC, ZF-SIC and ML Equalizers with QPSK modulation

From figure 3.7, we observe that the ML detection receiver is the best in performance followed by the ZF-SIC and MMSE-SIC equalizers, as also seen. In terms of complexity of the receivers, ML raises exponentially with the number of transmitting antennas whereas the MMSE-SIC and ZF-SIC are linear receivers combined with (SIC) Successive Interference Cancellation so we take here optimized one that is MMSE-SIC equalizer.

First the BER versus SNR comparison of ZF-SIC with ZF and MRC is performed in a typical 2x2 MIMO system with BPSK modulation scheme in Rayleigh faded channel, ZF-SIC performs better than ZF and MRC so ZF-SIC is optimized.

Then the BER versus SNR comparison of ZF, MRC, MMSE-SIC and MMSE-SIC Sort is performed in a typical 2x2 MIMO system with BPSK modulation scheme in Rayleigh faded channel, MMSE-SIC gives better results than ZF and MRC so MMSE-SIC equalizer is optimized.

CHAPTER 4

RSULTS AND DISCUSSIONS

Multipath is the propagation phenomenon in the wireless communication that results in radio signals caused by two or more paths received at the receiving end. Causes of multipath consider refraction, atmospheric ducting, and ionospheric reflection, terrestrial objects and reflection from water bodies such as hills and buildings. Thus there would be multipath interference, problem with multipath fading.

Fading is a popular issue when transmitting a signal in wireless communication channel. Fading causes a lot of degradation in the performance of any modulation technique in wireless communication and therefore we have to device ways and means to overcome the effects of fading.

Among the different types of distortions in communication, Inter Symbol Interference (ISI) is an occurrence deforms the transmitted data that causing bit error at the receiver side. ISI has been known as the main hazard in high speed data transmission over wireless media. Typically the digital information that is transmitted will be in the appearance of square waveform representing the 0's and 1's.

Methods to overcome the effects of these all types of hazards such as Multipath, Fading and ISI in wireless communication systems, different diversity techniques are used such as EGC (Equal Gain Combining), (SC) Selection Combining and MRC (Maximal Ratio Combining). The BER comparison between MMSE-SIC and ZF-SIC in a typical 2x2 MIMO system using BPSK and QPSK modulation scheme in a Rayleigh faded channel is performed. The result shows QPSK performs better than BPSK in Rayleigh faded channel.

The Tables 4.1 and 4.2 shows the results of ZF-SIC equalizer, MRC, ML detection and MMSE-SIC equalizer in a typical 2x2 MIMO environment different modulation schemes such as BPSK and QPSK.

Table 4.1 BER and SNR dB values for MMSE-SIC, MRC and ZF Equalizer in (2×2) MIMO system with BPSK modulation

Parameters	**Environment**	**ZF**	**ZF-SIC**	**MMSE-SIC**	**MRC**
BER	2x2 MIMO	1.464	0.1136	0.08481	0.05806
SNR	2x2 MIMO	0.0007	0.0004	0.00026	1.16e-05

Table 4.2 BER and SNR dB values for ML detection, MMSE-SIC and ZF-SIC Equalizers in (2×2) MIMO system with QPSK modulation.

Parameters	**Environment**	**ZF-SIC**	**MMSE-SIC**	**ML**
BER	2x2 MIMO	0.0849	0.0621	0.0497
SNR	2x2 MIMO	0.0185	0.0112	0.0064

CHAPTER 5
CONCLUSION AND FUTURE SCOPE

Equalization techniques are of more importance in the design of high data rate wireless systems. They can tackle for inter symbol interference even in mobile fading Channel with high efficiency. Here we analysed the performance of different equalization techniques to find out suitable equaliser for 2x2 MIMO channel in Rayleigh multipath fading environment.

We analyzed BER of BPSK modulation with 2x2 MIMO with ZF-SIC and MMSE-SIC with MRC diversity technique and it shows that MMSE-SIC with MRC gives better BER performance than ZF-SIC equalizer. Also we compare ZF-SIC, MMSE-SIC and ML equalization with QPSK modulation scheme here ML gives better BER performance than MMSE-SIC and ZF-SIC but the complexity of ML is very difficult to implement so we use MMSE-SIC equalization than ML and ZF-SIC.

Zero Forcing equalizer performs effectively only in theoretical assumptions that are when noise is zero. Its performance reduces in mobile fading environment. Minimum Mean Square Error (MMSE) equalizer uses LMS (Least Mean Square) as standard to compensate ISI. The MMSE equalizer results in improvement when compared to zero forcing equalizer.

From the results we conclude that Minimum Mean Square Equalization with simple successive interference cancellation (MMSE-SIC) case, addition of optimal ordering results in improvement for BER. So, by observing the simulation results we conclude that by using MMSE with SIC optimal ordering, interference can be cancelled at optimum level even in a mobile fading channel.

5.1. Conclusion:

We first compare ZF equalizer, MRC, ZF-SIC equalizer in a typical 2x2 MIMO system and conclude that best one is ZF-SIC equalizer then we compare the ZF-SIC equalizer, MMSE-SIC equalizer and MRC in the same 2x2 MIMO system and conclude that MMSE-SIC equalizer is better than ZF-SIC equalizer and MRC and both these comparisons with BPSK modulation scheme. Further we only compare MMSE-SIC and ZF-SIC equalizers and conclude that MMSE-SIC equalizer is optimized then ZF-SIC in terms of BER (Bit error Rate).

Finally we compare ML (Maximum Likelihood) detection, MMSE-SIC and ZF-SIC equalizers in a typical 2x2 MIMO system with QPSK modulation scheme and conclude that MMSE-SIC equalizer is optimized than ZF-SIC equalizer and ML detection.

5.2. Future Scope:

- In future work, for further improvement of the BER performance of MIMO systems will use more number of antennas both at Transmitter and Receiver side (2x4 and 4x4).
- Also there is a chance to implement the MIMO system by using different Modulation types such as QAM (Quadrature Amplitude Modulation), MSK (Minimum Shift Keying).
- We examined our work with Rayleigh fading environment. For further improvement we can use different fading environments such as Rician fading and Nakagami fading.
- We used in this AGWN noise and further we can use shot noise, thermal noise etc.
- We also can use MMSE-SIC with LTE (Long Term Evolution) systems.
- Here in this thesis we used ideal environment with channel information known and further we can use with this unknown channel information.
- MMSE-SIC equalizers can also br predicted by using Pilot Sequences.

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