

# **Reliability Analysis of Regular Multistage Interconnection Networks**

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degree of

**Master of Engineering**  
in  
**Computer Science & Engineering**

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## Certificate

I hereby certify that the work which is being presented in the thesis entitled, "**Computer Science Engineering**", in partial fulfilment of the requirements for the award of degree of Master of Engineering in Computer Science Engineering submitted in Computer Science and Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of Ms. *Rinkle Rani Aggarwal* and refers other researcher's works which are duly listed in the reference section.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

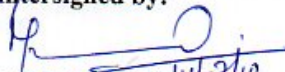
  
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*Sushila*  
(Sushila Baloda)

## Abstract

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The performance of a computer system depends directly on the time required to perform a basic operation and the number of these basic operations that can be performed concurrently. High performance computing systems can be designed using parallel processing. Parallel processing is achieved by using more than one processors or computers together they communicate with each other to solve a given problem. MINs provide better way for the communication between different processors or memory modules with less complexity, fast communication, good fault tolerance, high reliability and low cost. Reliability of a system is the probability that it will successfully perform its intended operations for a given time under stated operating conditions. In this thesis an important issue in the design of MINs is the provision of more than one path between each input-output pair. There are mainly three types of reliability measures, which are important to MINs, namely terminal reliability, broadcast reliability and network reliability. The purpose of this thesis is to develop tighter bounds for network reliability for the extra-stage MIN. In this thesis Reliability analysis of three regular multistage interconnection networks namely SEN, Omega and Cube has been done. With the addition of one and two extra stages more regular MINs namely SEN+, Omega+, Cube+ and SEN+2, Omega+2, Cube+2 are derived from SEN, Omega and Cube respectively. As measures of network performance, the terminal, broadcast and network reliability of all three networks have been evaluated. From the reliability analysis following conclusions have been made. Addition of one stage to any of SEN, Omega or Cube networks provide higher reliability in terms of terminal, broadcast and network, than the addition of two stages in the corresponding network. Further comparative analysis among SEN+, Cube+ and Omega+ has shown that Cube+ is most reliable than SEN+ and Omega+.

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### 1.1 Parallel Systems

There are many different ways to organize computational structures to achieve parallelism. Parallelism is a strategy where multiple systems are working simultaneously for performing large, complex tasks faster. A large task can either be performed serially, one step following another, or can be decomposed into smaller tasks to be performed simultaneously, i.e., in parallel. Parallelism is achieved by:

- Breaking up the task into smaller tasks
- Assigning the smaller tasks to multiple processing elements
- Processing elements solve the problem by coordinating with each other

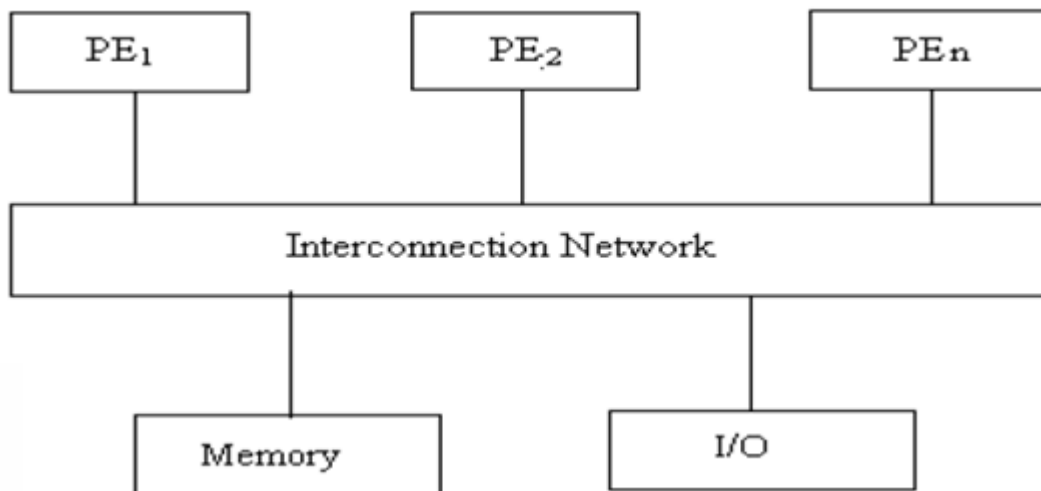
A parallel system is a collection of processing elements that communicate and cooperate to solve large problems faster. Many research efforts around the world are being conducted with the purpose of determining that hardware and software organizations that are best suited for general purpose parallel processing [8]. The processors in a parallel system communicate with and work in cooperation with other processors. The primary reasons for using parallel systems:

- Save time
- Solve larger problems
- Provide concurrency (do multiple things at the same time)

The structure of interconnection network is one of the key objectives to improve system performance. The communication subsystems linking processors, memory modules and I/O controllers in a parallel processing system is one of the most important architecture features and has a profound impact on system capabilities, performance, size and cost [3]. An Interconnection network of the processors provides the desired connectivity and performance at minimum cost is required for communication in parallel processing systems with a large number of components. A multistage interconnection network (MIN) is usually segmented into several stages and links to connect any sources to any destinations [1].

## 1.2 Interconnection Networks

An interconnection network is a complex connection of switches and links permitting processors in a multiprocessor system to communicate among themselves or with memory modules or I/O devices. It is the path, in which the data must travel in order to access memory in a shared memory computer or to communicate with other processes in a distributed memory environment or to use any I/O devices [8].



**Figure 1.1: An Interconnection Network**

## 1.3 Design Dimensions of An Interconnection Network

The key design dimensions for interconnection networks are:

- Switching Methodology
- Operational Mode
- Control Strategy
- Network Topology

### 1.3.1 Switching Methodology

Two major switching methodologies are:

**Circuit switching:** In circuit switching an end to end physical path is actually established between a source and a destination. This path exists as long as the data transmission is not complete. Circuit switching is suitable for bulk transmission of data.

**Packet switching:** In packet switching data is divided into packets and routed through the interconnection network without establishing a physical end to end connection path. Packet switching is more efficient for short messages.

### 1.3.2 Operational Modes

Operational modes can either be synchronous or asynchronous or a combination of the two.

**Synchronous:** This mode is useful for either a data manipulating function or for a data instruction broadcast. Synchronous control techniques are characterized by a global clock, which broadcasts clock signals to all devices in a system so that the entire system operates in lock-step fashion.

**Asynchronous:** Asynchronous communication is needed for multi processing in which connection requests are issued dynamically. Asynchronous techniques do not utilize a single global clock, but rather distribute the control function throughout the system, often utilizing many individual clocks for timing.

### 1.3.3 Control Strategy

A typical interconnection network consists of a number of switching elements and interconnection links. Interconnection functions are realized by properly setting control of switching elements. The control strategy can be of two types:

**Centralized control strategy:** In this all control settings are managed by a centralized controller. A main control system manages all of the switching elements.

**Distributed control strategy:** In this all control settings are managed by individual switching elements.

### 1.3.4 Network Topology

A network can be represented by a graph in which nodes indicate switches and edges represent communication links. Topology is the pattern in which the individual switches are interconnected to other elements such as processors, memories and other switching elements. The topologies can be categorized into two groups:

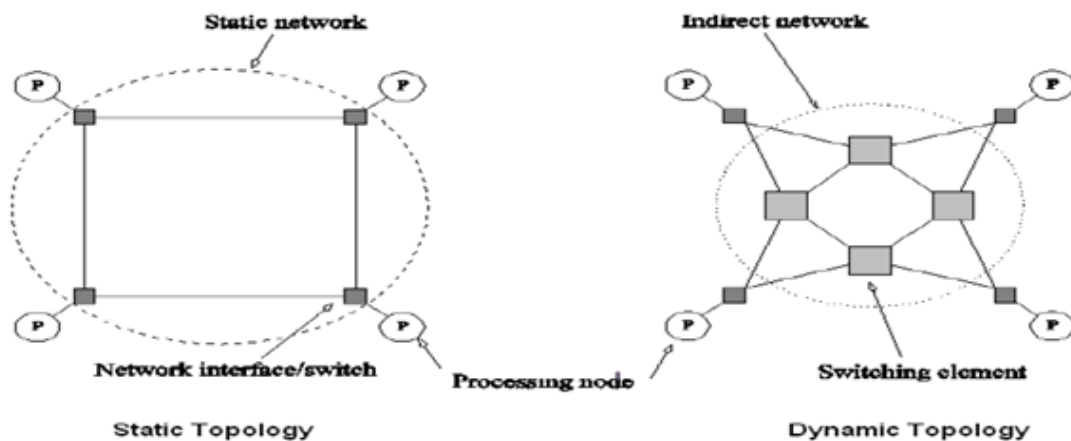
**Static:** In static topology, links between two processors are with passive and dedicated buses which cannot be reconfigured for direct connection with other

processors. Static networks that are generally used in message passing architectures [14]. The following network topologies are commonly used:

- Ring Network
- Star Connected Network
- Completely Connected Network
- Tree Network
- Mesh Network
- Hypercube Network

**Dynamic:** On the other hand, links in dynamic topology can be reconfigured by setting network's active switching elements [14]. Dynamic interconnection networks implement one of the following interconnection techniques:

- Crossbar Networks
- Bus based Network
- Multistage Interconnection Networks



**Figure 1.2: Types of Network Topology**

#### 1.3.4.1 Static Topology

**Ring Network:** In ring network, every device has two adjacent neighbours for communication. In a ring network, all the communication messages travel in the same direction whether clockwise or anticlockwise. Damage of cable or device can result in the breakdown of the whole network [14].

**Star Connected Network:** In the computer networking the most commonly used topology is the star topology. All the computers in the star topologies are connected to

central device like hub, switch or router. The functionality of all these devices is different. The main disadvantage of this kind of topology is that if central device stops working then there will be no transmission between any nodes.

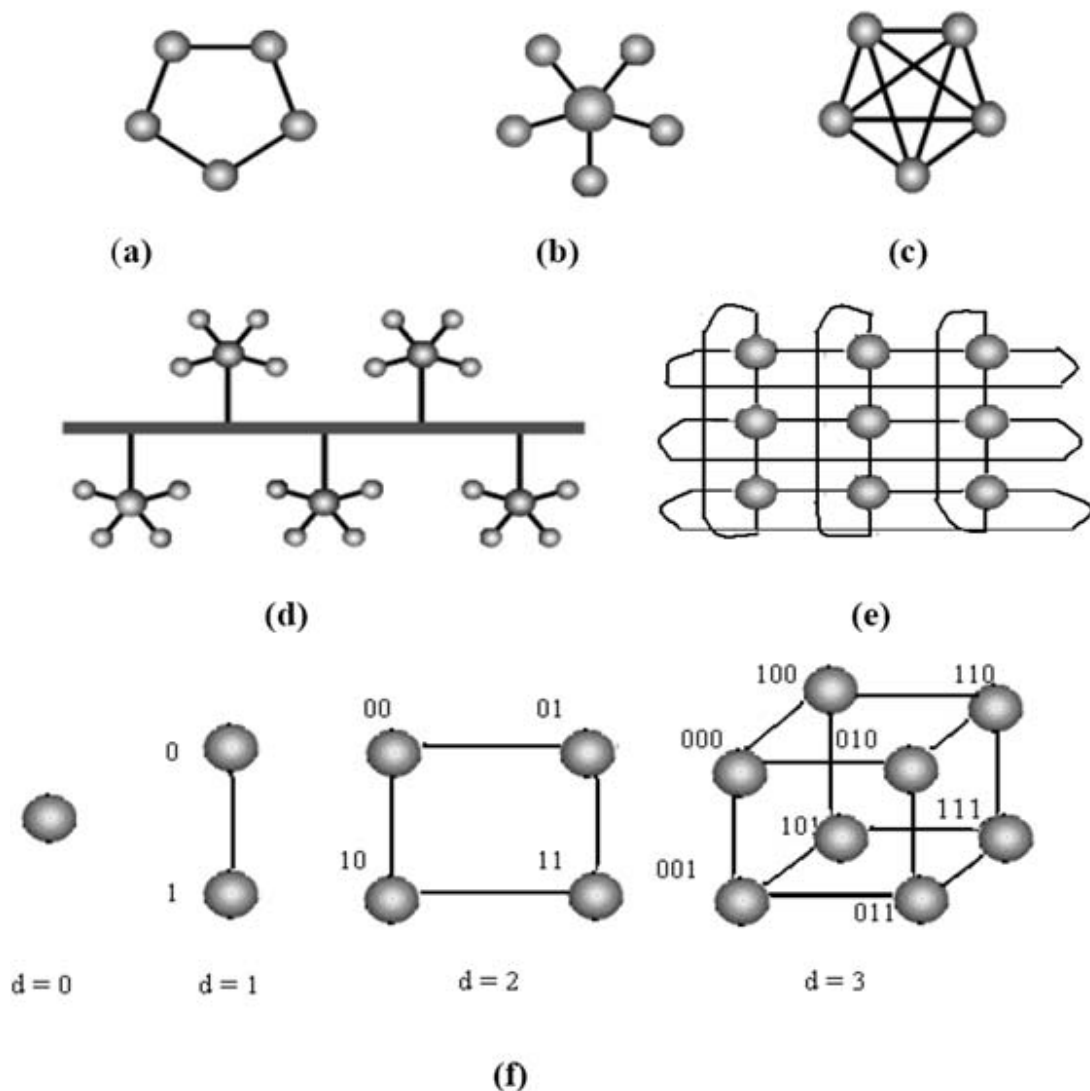
**Completely Connected Network:** In this topology, all nodes are directly connected to every other node with a point to point link. In this message sent to the destination can take any possible shortest, easiest route to reach its destination. In the previous topologies star and ring, messages are usually broadcasted to every computer [14]. In the ring topology message can travel in only one direction i.e. clockwise or anticlockwise.

**Tree Network:** Tree topologies are comprised of the multiple star topologies on a bus. Tree topologies integrate multiple star topologies together onto a bus. Only the hub devices can connect directly with the tree bus and each hub functions as a root of a tree of the network devices, as shown in Figure 1.3 (d). This bus/star/hybrid combination supports future expandability of the computer networks, much better than a bus or star.

**Mesh Network:** A mesh simply connects one processor to four others, as shown in Figure 1.3 (e). Processors along the top and bottom edges would be connected to the processor in the same column along the opposite edge. The processors on the last column would be connected to the first processor on the next row. The processor in the bottom right corner would have a connection to the processor in the top left corner. With this layout, there are  $2N$  connections, but it takes at most  $N-1$  shifts to get data from one processor to any other.

**Hypercube network:** A zero dimensional hypercube is a single processor and a one-dimensional hypercube connects two zero dimensional hypercube i.e. a line connecting two nodes defines a one-dimensional cube. A square with four nodes is a two-dimensional cube. Generally a hypercube of dimension  $d+1$  is constructed by connecting corresponding processors in two hypercube of dimension  $d$  [14]. In hypercube two processors are connected if and only if the binary representation of their labels differs in a single position. It may be considered as a mesh with additional, long distance connections. A disadvantage of the hypercube interconnect is that it is more complex than the mesh. In the Figure 1.3 (f), processors in the cubes of dimension 1, 2, and 3 are labelled with integers, represented as binary numbers. Two

processors are neighbours in dimension  $d$  if and only if their binary labels differ only in the  $d^{\text{th}}$  place.

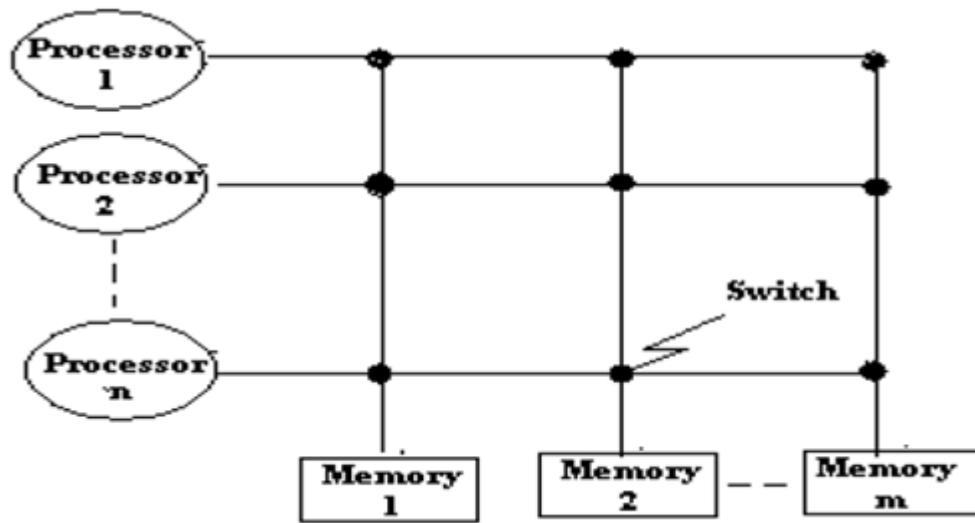


**Figure 1.3: Various Types of Static Network Topologies are (a) Ring (b) Star (c) Completely Connected (d) Tree (e) Mesh (f) Hypercube of Dimensional zero to three**

### 1.3.4.2 Dynamic Topology

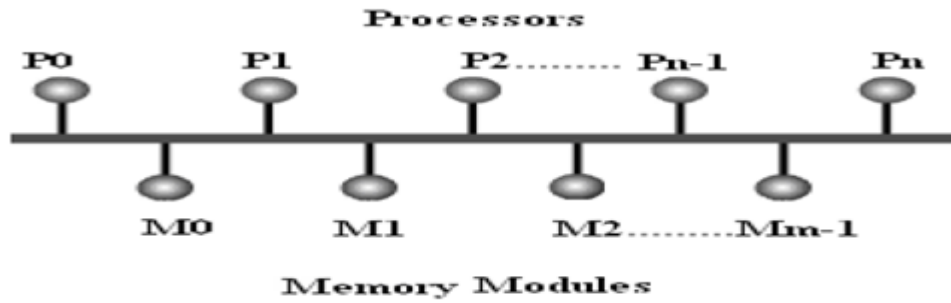
**Crossbar Networks:** The crossbar makes a connection from a given vertical bus to the appropriate horizontal bus and allows traffic to flow along this path. In crossbar network, the other horizontal or vertical buses can be supporting a flow of traffic at the same time. For example, if each horizontal bus needs to talk to a separate vertical bus, then they all can be moving data at the same time. This completely eliminates the

single-shared-resource limitation of the system bus. The crossbar is a preferable approach for high performance multiprocessors [24].



**Figure 1.4: A Crossbar Network**

**Bus network:** Bus topology uses a common backbone to connect all the network devices in a network in a linear shape shown in Figure 1.5. Some of the simplest and earliest parallel machines used bus. All processors access a common bus for exchanging data. The distance between any two nodes is  $O(1)$  in a bus. The bus also provides a convenient broadcast media. However, the bandwidth of the shared bus is a major bottleneck. A bus is highly non scalable architecture, because only one processor can communicate on the bus at a time. A bus network design offers minimum bandwidth. It is highly inefficient and unreliable because of a single bus, the failure of which will make it unusable [8]. Buses are commonly used in shared memory parallel computers to communicate read and write requests to a shared global memory. A single bus organization is simple and inexpensive. By adding more number of processors or memory increases the bus contention, which decrease a bus throughput.

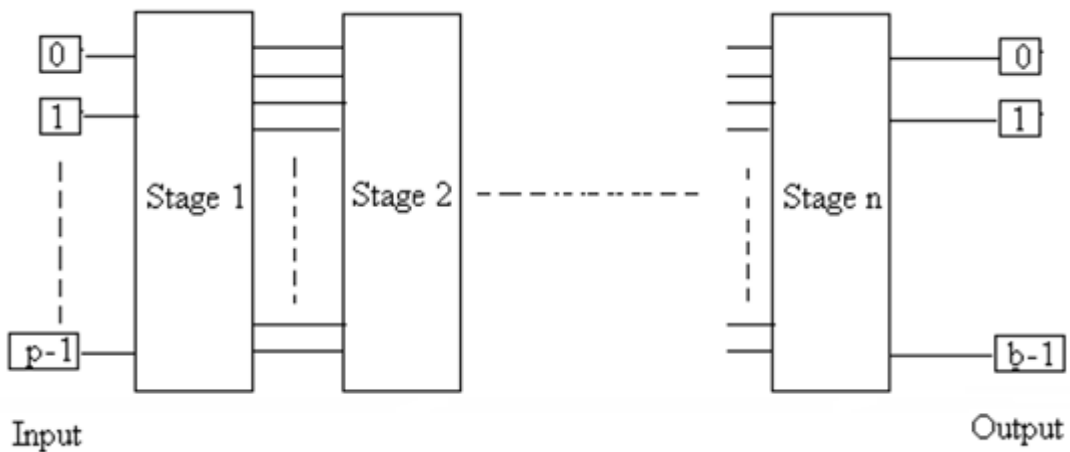


**Figure 1.5: A Bus Network**

### 1.4 Multistage Interconnection Networks

Multistage interconnection networks (MINs) consist of more than one stages of small interconnection elements called switching elements and links interconnecting them. Multistage interconnection networks (MINs) are used in multiprocessing systems to provide cost-effective, high-bandwidth communication between processors and/or memory modules. A MIN normally connects  $N$  inputs to  $N$  outputs and is referred as an  $N \times N$  MIN. The parameter  $N$  is called the size of the network [25].

There are several different multistage interconnection networks proposed and studied in the literature. Figure 1.6 illustrates a structure of multistage interconnection network, which are representatives of a general class of networks. This Figure shows the connection between  $p$  inputs and  $b$  outputs, and connection between these is via  $n$  number of stages.



**Figure 1.6: A Multistage Interconnection Network (MIN)**

A multistage interconnection network is actually a compromise between crossbar and shared bus networks, indicated in the Table 1.1, describing the properties of various types of multiprocessor interconnections networks [8]. Multistage interconnection networks are:

- Attempt to reduce cost
- Attempt to decrease the path length

In a multistage interconnection network, as in a crossbar, switching elements are distinct from processors. Instead messages pass through a series of switch stages.

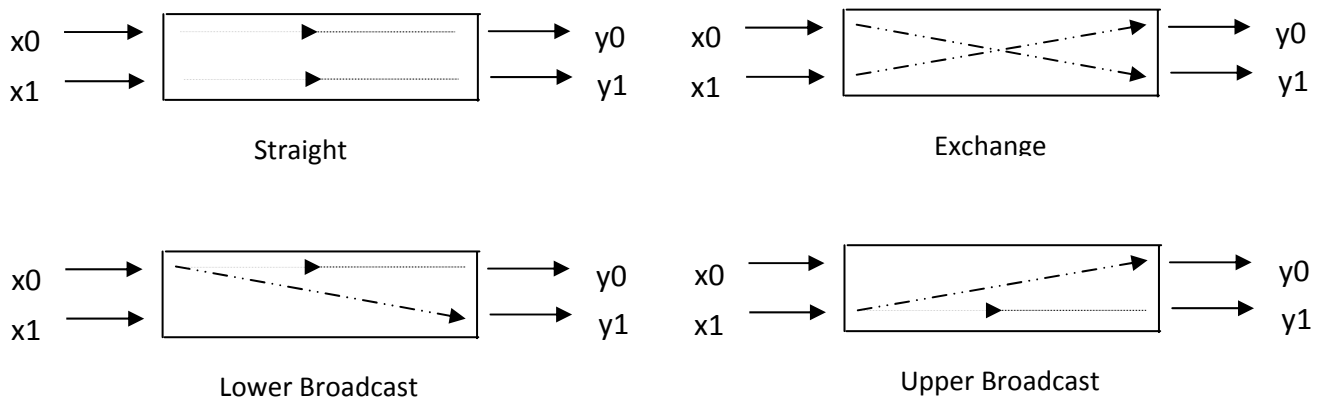
**Table 1.1: Properties of different Networks**

<b>Property</b>	<b>Bus</b>	<b>Crossbar</b>	<b>Multistage</b>
Speed	Low	High	High
Cost	Low	High	Moderate
Reliability	Low	High	High
Configurability	High	Low	Moderate
Complexity	Low	High	Moderate

The network can be constructed from unidirectional or bi-directional switches and links. In a unidirectional MIN, all messages must traverse the same number of wires, and so the cost of sending a message is independent of processor location. In effect, all processors are equidistant. In a bi-directional MIN, the number of wires traversed depends to some extent on processor location, although to a lesser extent than a mesh or hypercube [7].

#### **1.4.1 Switching Elements**

The switching element is the basic element of a multistage interconnection network. It may be viewed as a very small network. These switches are the devices having multiple inputs and multiple outputs [1]. A switch box have any one of the following four states i.e. straight, exchange, upper broadcast and lower broadcast shown in Figure 1.7.

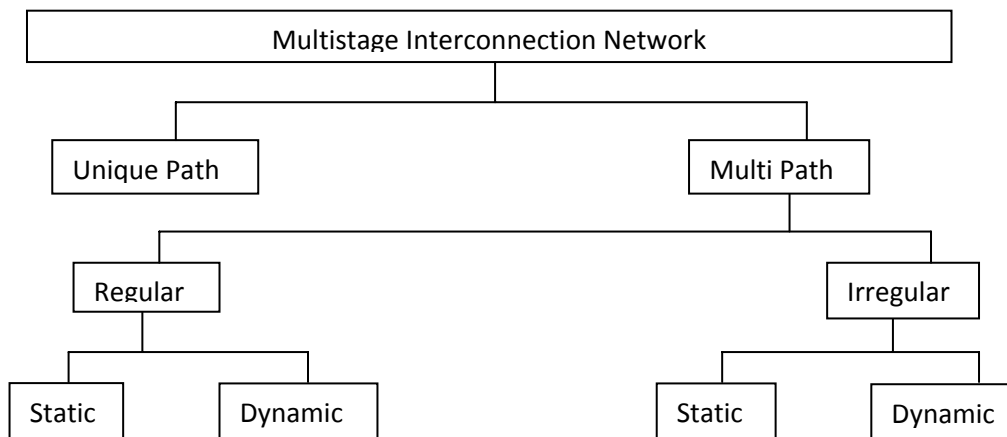


**Figure 1.7: Types of Switching Elements**

## 1.5 Classification of Multistage Interconnection Networks

Multistage interconnection networks can be classified according to different categories. The main classification categories are:

- According to number of paths
- According to switch
- According to control
- According to availability of path



**Figure 1.8: Types of Multistage Interconnection Networks (MINs)**

### 1.5.1 Classification According to Number of Paths

**Unique path networks:** These networks provide unique path between every source and destination. The failure of any switching element along the path disconnects some source-destination pairs. These are not reliable for a large multiprocessor system, as

they cannot tolerate even a single fault. These networks provide poor performance as a source destination connection may be blocked by a previously established connection [19].

**Multi path networks:** These provide more than one path between source and destination. In case, there is a failure of one switching element in the path, the request is routed through some alternative path. Multi path multistage interconnection networks can be static or dynamic. For static networks, backtracking is used if fault is encountered,. In dynamic networks, if a fault is encountered in a particular stage, a switching element in preceding stage will re-route data through an alternative available path [19].

### **1.5.2 Classification According to Switches**

**Regular networks:** Regular multistage interconnection networks have an equal number of switching elements per stage. As a result they may impose equal time delay to all h requests passing through them [22].

**Irregular networks:** Irregular multistage interconnection networks have unequal number of switching elements at each stage and thus they are inherently multi path in nature. For a given source destination pair, multiple paths are available [22].

### **1.5.3 Classification According to Control**

**Flip controlled networks:** Flip controlled multistage interconnection networks have a common control signal for switching in various switching elements at a given stage [8]. These networks are less complicated due to lesser number of control signals but have lesser bandwidth.

**Distributed control networks:** Distributed control multistage interconnection networks have a separate control signal for every switching element. These have higher bandwidth due to selection of source destination pair at a given time and are quite complex [8].

### **1.5.4 Classification According to Availability of Path**

**Blocking networks:** Blocking is the conflict that arises between two paths established between two source and destination pairs. When sources generate connection requests to an identical destination, it is possible that some paths need to pass simultaneously through an identical output of the SE. A path can never share the output with another path at any point in time, and hence blocking arises in the SE. Omega network is a type of blocking network [24].

**Non-blocking networks:** A network is called non-blocking if it is possible to route from any source to any destination, in presence of other established source-destination routes, provided no two sources have same destination. In other words, a network that can handle all possible connections without blocking is called non-blocking network [24].

## 1.6 Routing of MINs

In networks the process of moving data packet from source to destination is called routing. Routing is a key feature of the networks because it enables messages to pass from one computer to another and eventually reach the destination. Each intermediary switch performs routing by passing along the message to the next switch. The routing in multistage interconnection networks take place through the generation of routing tags, which specify a fault free path. Each switch in a MIN performs routing according to the routing tag [6]. There are several objectives of routing strategy:

- Minimize the transmit time
- Minimize the costs
- Maximize the network throughput capability

To minimize the transmit time under conditions of changing load, many control signals or overheads would be sent so that network throughput would be reduced. On the other hand, maximizing the throughput could be done at the expense of packet transmission time.

### 1.6.1 Routing Tag

Routing tag is a way of describing the path through the network. For multistage interconnection networks, these tags are generally expressed as a multidigit binary number expressed by the destination. Each successive digit in this binary code

is used to find out the switch in the next stage along a desired path. For each multistage interconnection network routing tag may be different [6].

### 1.6.2 Types of Routing in MINs

There are basically three types of routing that is commonly used in multistage interconnection networks.

- **Non adaptive routing:** In this method a source learns a fault when a path is attempting to establish reaches the faulty network component. A notice of fault is sent to the source, which then tries next alternative available path. This method has poor performance though it requires little hardware.
- **Adaptive routing:** The adaptive routing can be of following types :
  - (a) **Notification on demand:** with notification on demand, a source maintains a table of faults it encountered in attempting to establish paths and uses this information to guide the future routing.
  - (b) **Broadcast routing:** With broadcast notification of a fault, all the sources are notified of the fault components as they are diagnosed.
  - (c) **Dynamic routing:** A fault free path need not to be specified by a source if the routing tags are modified in response to the faults as a path is followed. The dynamic routing can be accomplished in multistage interconnection networks constructed of switches, which are capable of performing the necessary tag revision.

## 1.7 Performance Parameters

The performance parameters applicable for MINs are:

### 1.7.1 Permutation Passability

A one to one correspondence between a source and a destination is called permutation. Permutation passability means how many input requests occurring simultaneously at the input are able to pass through a given network, and how many of them will successfully mature i.e. will reach their destination [24]. The request always pass from the most suitable path available (generally, the minimum length path), if such path is busy or faulty then the request is pass through an alternate path.

If no alternate path is available then the request has to be simply dropped or said to be having clash. So some of the requests will pass through the most favorable path, others have to be routed through an available alternative path. If no alternative paths are available then some requests cannot be served at all [27].

### **1.7.2. Fault Tolerance**

A fault tolerant multistage interconnection network provides service routing even under the faults. Fault can be permanent or transient in nature. Fault tolerance is a criteria that must be met for the network which has tolerated a given fault or faults. A network is called single fault tolerant if it can tolerate or function in case of a single fault. In general, if any set of  $i$ -faults can be tolerated by a network, then network is called  $i$ -fault tolerant [2].

### **1.7.3. Bandwidth (BW)**

It is the most common performance parameter used in analyzing a synchronous interconnection network. It is defined as the number of processing elements requests honoured per unit of time. In other words, it is the average number of requests honoured per unit time. So bandwidth (BW) also takes into account the memory access conflicts caused by the random nature of the process requests. BW is also defined as the expected number of destination receiving requests in any given cycle. Thus it is the total number of requests matured. A high bandwidth is often desired at reasonably low network cost.

### **1.7.4 Throughput (TP)**

Throughput is the maximum number of traffic accepted by the network per unit time. The average number of packets delivered from source to destination by network in unit time called throughput. It is measured as packets per node per cycle. It can also be defined as the average number of cells delivered by the network per unit time per unit line.

### **1.7.5. Probability of Acceptance (Pa)**

It is defined as the ratio of expected bandwidth to the expected number of requests generated per cycle.

### **1.7.6. Processor Utilization (PU)**

It is the expected percentage of time a processor is active. A processor is said to be active when it is doing internal computation without accessing the global memory.

### **1.7.7. Reliability**

Reliability, of a system is the probability that it will successfully perform its intended operations for a given time under stated operating conditions. In the presence of multiple CPUs if one goes down, the others can be able to take over its work. In case of networks if one switch is fail then the possibility that it will not fail for some time period. It is one of the major design issue for any network [20].

### **1.7.8. Cost Effectiveness**

To estimate the cost of a network, one common method is to calculate the switch complexity with the assumption that the cost of a switch is proportional to the number of gates involved, which is roughly proportional to the number of crosspoints within a switch. For example, a 4 x 4 switch has 16 units of hardware cost whereas a 2 x 2 switch has 4 units. For the multiplexers and demultiplexers, it is roughly assumed that each of  $K \times 1$  multiplexers or  $1 \times K$  demultiplexers has  $K$  units of cost [4].

## **1.8 Thesis Outline**

A survey of existing regular and irregular multi-stage interconnection networks is covered in chapter 2. Chapter 3 deals with problem definition and problem statement. Chapter 4 covers the reliability analysis of different MINs which include SEN, Omega and Cube networks with one and two additional stages. Chapter 5 shows the experimental results with the help of tables and graphs. The last Chapter is regarding the conclusions and future scope of the work.

## **2.1 Introduction**

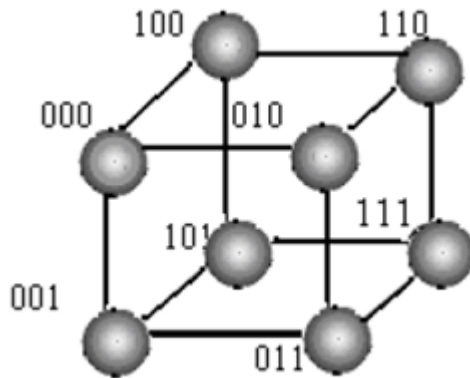
To solve the problem of providing fast, reliable and efficient communication at a reasonable cost in large parallel processing systems, many different networks between the extremes of single bus and the cross bar have been proposed. Such interconnection networks can be constructed from single or multiple stages of switches. In a single stage network, data may have to be passed through the switches several times before reaching the final destination. In multistage network, one pass of multistage stages of switches is usually sufficient. The way input units are connected with the output units, determine the functional characteristics of the network i.e. the allowable interconnections [8].

The single stage network is also called a recirculating network. Data items may have to recirculate through the single stage several times before reaching their final destination. Number of recirculations needed depends upon connectivity in a single stage network. In general, the higher is the hardware connectivity, the lesser is the number of recirculations.

## **2.2 Survey of Regular MINs**

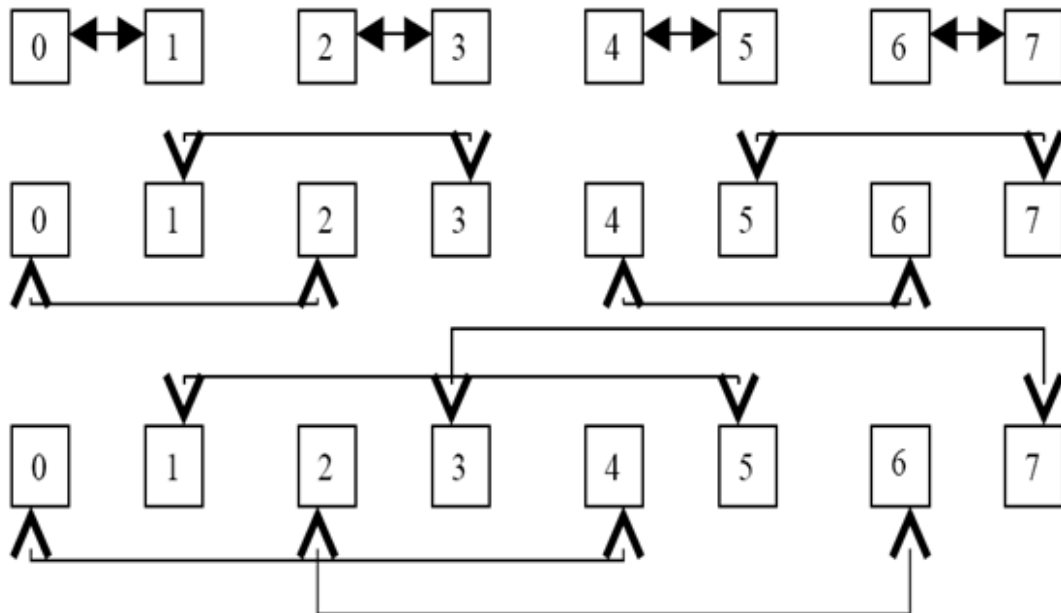
### **(a) Cube interconnection network**

In a cube, vertical lines connect vertices whose addresses differ in most significant bit position. Vertices at both ends of diagonal lines differ in middle bit position. Horizontal lines differ in least significant bit positions [3]. The unit cube concept can be extended to an n-dimensional unit space called n-cube with n bits per vertex. Two-function (straight and exchange) switch boxes are used in constructing multistage cube network. The stages are numbered as 0 at the input end and increased to n-1 at the output end.

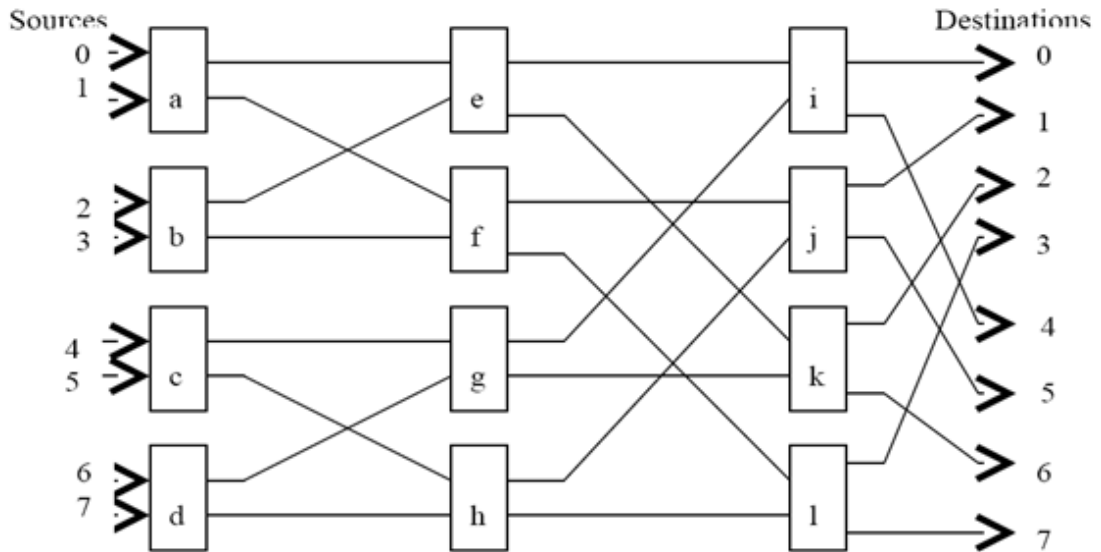


**Figure 2.1: A Three Dimensional Binary Cube**

The implementation of a single stage cube network is given in the Figure 2.2 for 8 nodes. The interconnection of the switching elements, corresponding to three routing functions is given separately in this Figure.



**Figure 2.2: The Recirculating Cube Network for N=8**



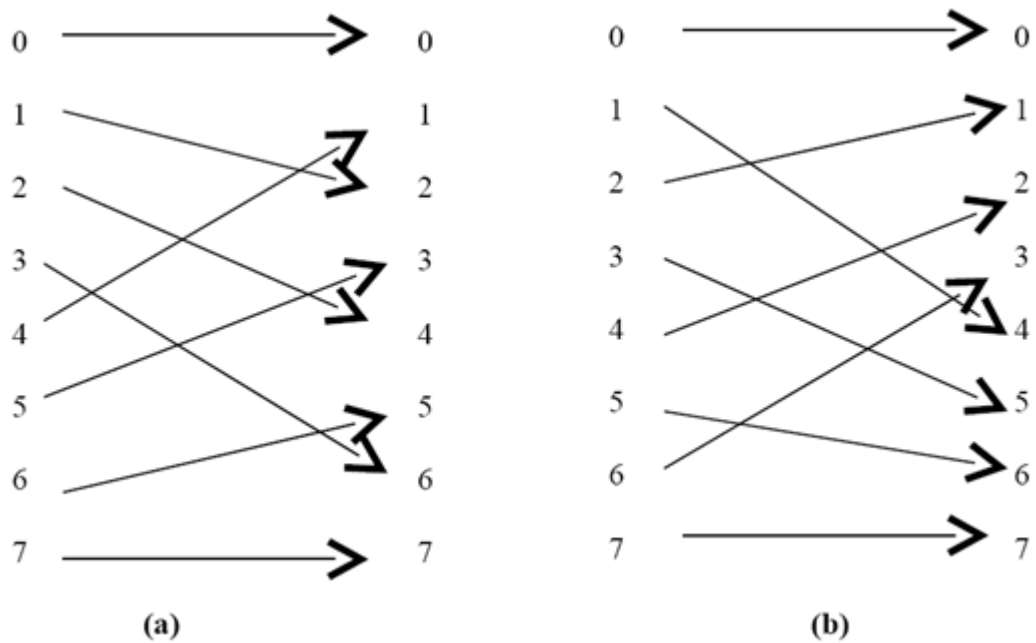
**Figure 2.3: A Multistage Cube Network for N=8**

The same set of cube routing functions,  $c_0$ ,  $c_1$ ,  $c_2$  can also be implemented by using a three stage cube network. Two function switch boxes i.e. straight and exchange are used in construction of multistage cube network. The stages are numbered as 0 at input end and increased to  $n-1$  at output. The stage  $i$  implements  $C_i$  routing function for  $i = 0, 1, 2, \dots, (n-1)$ . So, switch box at stage  $i$  connect an input line to output line that differs from it only at  $i$ th bit position [14].

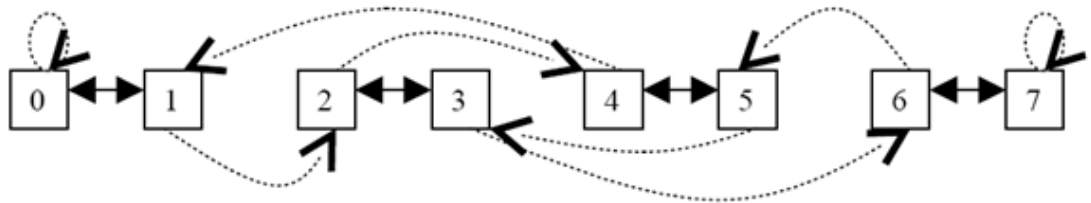
**(b) Shuffle Exchange Network**

Shuffle exchange network is based on two routing functions, Shuffle and Exchange. A perfect shuffle of  $N = 8$  is shown in Figure 2.4(a). Perfect shuffle cuts the deck into two halves from the centre and then intermixes them evenly. Inverse perfect shuffle does the opposite to restore the original ordering as shown in Figure 2.4(b).

These shuffle exchange functions can be implemented as either recirculating network or a multistage network [14]. Figure 2.5 represent a single stage recirculating shuffle exchange network, where solid lines indicate exchange and dashed lines indicate shuffle.



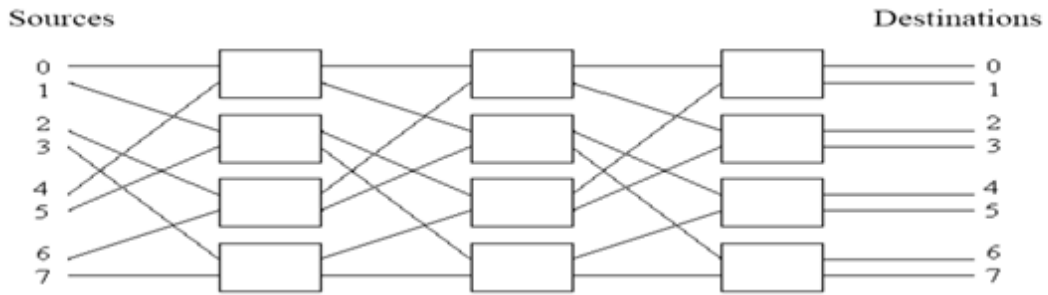
**Figure 2.4: (a) A Perfect Shuffle and (b) The Inverse Perfect Shuffle**



**Figure 2.5: A Shuffle Exchange Recirculating Network for N=8**

**(c) Omega Network**

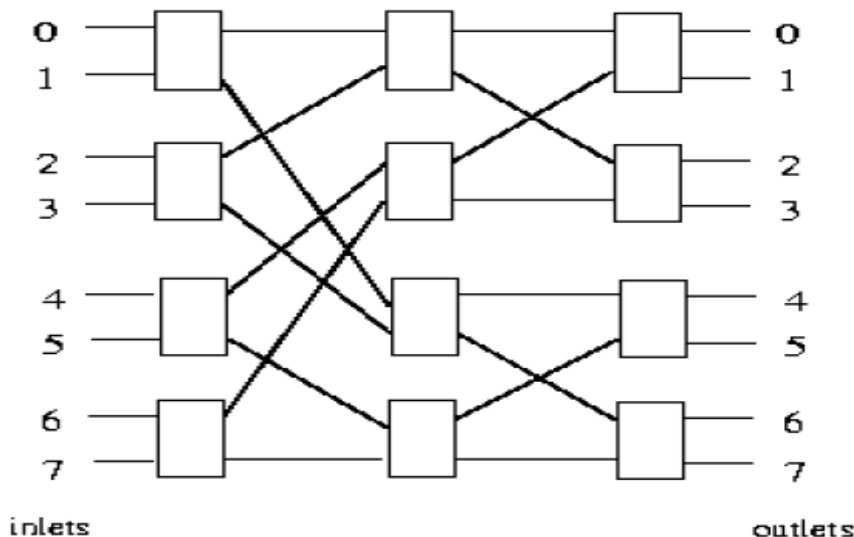
The shuffle exchange has been implemented with multistage Omega network. Figure 2.6 represents Omega network for N= 8. An  $N \times N$  Omega network consists of  $\log_2 N$  identical stages and between two stages there is a perfect shuffle interconnection. Each stage has  $N/2$  switch boxes under independent box control [14].



**Figure 2.6: An Omega Network for N=8**

**(d) Banyan Network**

A banyan network constructed from 2x2 switching elements consists of  $n = \log_2 N$  stages. Banyan networks are commonly used in multistage ATM switches because of their high degree of parallelism, self-routing, modularity, constant delay for all input-output port pairs, in-order delivery of cells, and suitability for VLSI implementation. Their shortcoming remains blocking and throughput limitation. The existence of such conflicts leads to a maximum achievable throughput which is much lower than that obtained with the crossbar switch. An 8x8 banyan network is shown in Figure 2.7. Banyan networks are commonly used in multistage. However, in banyan networks, there is only one path between each input and output port pair, and the edges of such a path are not dedicated. This means that other communicating pairs may share some links of a path connecting an input-output port pair.



**Figure 2.7: A Banyan Network**

## 2.3 Reliability Analysis

Interconnection networks play a major role in the performance of modern parallel computers. These networks can provide the communication in a parallel processing system consisting of a large number of processors that are working together to perform a single overall task. Multistage interconnection networks (MINs) are designed to provide fast and efficient communication at a reasonable cost.

Many different interconnection networks between the extremes of the single bus and the completely connected scheme have been proposed in the literature [14]. These networks provide interconnection between the set of processors (inputs) and the set of memory modules (outputs). Multistage networks are described by three characterizing features: the switch box, the network topology and the control structure. Many switch boxes are used in a multistage network. Illustrated are four states of a switch box: straight, exchange, upper broadcast and lower broadcast.

Reliability, of a system is the probability that it will successfully perform its intended operations for a given time under stated operating conditions. In the presence of multiple CPUs if one goes down, the others can be able to take over its work. In case of networks if one switch is fail then the possibility that it will not fail for some time period. There are mainly three types of reliability measures, which are important to MINs, namely terminal reliability, broadcast reliability and network reliability. Terminal Reliability, generally used as a measure of robustness of a MIN, is the probability of existence of at least one fault free path between a designated pair of input and output terminals.

Broadcast reliability is the probability of a MIN in broadcasting data from a given input terminal to all of the output terminals of the network. Broadcast reliability is important in those applications where broadcasting is practiced. A network is said to have failed when a connection cannot be made from a given input terminal to at least one of its output terminals. Network reliability (all terminal reliability) is the probability that there exists a connection between each input to all outputs. The network is considered to be operational as long as every source can communicate with each destination . These three reliability parameters are developed for one and two extra-stage MINs. Either transmits the inputs straight through itself or has cross

connections. The number of switches per stage, the number of links and the connection between stages are consistent.

## **2.4 Survey of major Research Papers Studied**

### **1. “Reliability analysis of shuffle-exchange network systems”, Indra Gunawan:**

In this paper, SEN with additional stages that provide more redundant paths are analyzed. A common network topology with a  $2 \times 2$  basic building block in a SEN and its variants in terms of extra-stages is investigated. Three types of SENs are compared: SEN, SEN with an additional stage (SEN+), and SEN with two additional stages (SEN+2). Finally, three measures of reliability: terminal, broadcast, and network reliability for the three SEN systems are developed to evaluate the performance of these networks and to assess the effect of additional stages on SEN reliability. After evaluation conclusion is drawn that adding an additional stage to the SEN leads to the most reliable SEN since the SEN+ has the highest terminal, broadcast, and network reliability comparing to the original SEN and SEN with two additional stages (SEN+2).

### **2. “New Bounds on the Reliability of Augmented Shuffle-Exchange Networks”, Bernard L. Menezes and Umesh Bakhru**

The reliability resulting from two forms of redundancy, spatial and temporal, in multistage interconnection networks is examined. The extra-stage shuffle-exchange network (SEN+) which is an example of the multistage interconnection networks is analyzed here. The SEN+ is decomposed into two subnetworks connected by two extreme stages. Given  $k$  random faults in one subnetwork, our problem reduces to estimating the maximum and minimum number of switches in the other subnetwork that must be operational for full access, that is, connection between every input (processor) and output (memory) pair. This investigation results in obtaining analytical estimates for the upper and lower bounds on its reliability which are a substantial improvement over existing ones. One measure of the effectiveness of such schemes is the all terminal reliability,  $R(t)$ . This is the probability that there exists a path between each source and every destination given that switch failures are random and independent events with probability  $1 - r(t)$  (the reliability of a switch is  $r(t)$ ). According to this paper, one possible candidate for a minimal covering set of 12

faults is the 8 x 8 SEN shown in Fig. 3. A possible minimal covering set of 9 faults is contained inside the 8x8 SEN and includes all switches in the 4x4 SEN.

**3. “Extra-Stage Cube Network Reliability Estimation Using Stratified Sampling Monte Carlo Method”, Indra Gunawan, Sellappan Palaniappan, Lim Chooi Sien:**

This paper presents the Extra-Stage Cube interconnection network reliability estimation using Stratified Sampling Monte Carlo (SSMC) method. The parameter of reliability includes terminal reliability, broadcast reliability and network reliability. The SSMC method provides approximate solutions by performing statistical sampling experiments on a computer. This method of reliability prediction is useful when system complexity makes the formulation of exact models essentially impossible. A computer program has been developed and the SSMC method was demonstrated as a good estimator for all the three types of the reliability parameter after compared with the exact reliability and reliability bounds. This paper shows that the results for  $N=256$  for terminal, broadcast and network reliability for extra-stage cube network. The comparison and analysis showed that the Stratified Sampling Monte Carlo method provides satisfied results for terminal reliability, broadcast reliability and network reliability of small size to large size ESC network.

**4. “Multistage Interconnection Network Reliability”, James T. Blake and Kishor S. Trivedi:**

In this paper, we examine the reliability of an unique-path multistage interconnection network (MIN) and a fault-tolerant scheme aimed at improving system reliability. We derive closed-form expressions for the time-dependent reliability of the 8 x 8 and 16 x 16 shuffle-exchange multistage interconnection networks (SEN) and with an additional stage (SEN+). The SEN+ network has an additional stage which is used to increase the reliability of the basic SEN. The focus of the reliability analysis that has been performed on MINs in terms of the average number of switch failures tolerated and mean time to failure, on terminal reliability, a measure often used for packet-switching applications. Results of this paper shows transient reliability analysis of the SEN and SEN+ networks. The lower bound obtained was compared to the exact solution derived for the 16 x 16 SEN+ to verify that it is a close approximation of SEN+ reliability.

**5. “Optimal Realization of Any BPC Permutation on K-Extra-Stage Omega Networks”, Xiaojun Shen:**

An  $N \times N$   $k$ -Omega network is obtained by adding  $k$  more stages in front of an Omega network. An  $N$ -permutation defines a bijection between the set of  $N$  sources and the set of  $N$  destinations. Such a permutation is said to be admissible to a  $k$ -Omega if  $N$  conflict-free paths, one for each source-destination pair defined by the permutation, can be established simultaneously. When an  $N$ -permutation is not admissible, it is desirable to divide the  $N$  pairs into a minimum number of groups (passes) such that the conflict-free paths can be established for the pairs in each group. This paper generalizes their result to a  $k$ -Omega where  $k$  can be any integer between 0 and  $n$ . An  $O(N \log N)$  algorithm is given which realizes any BPC (Bit Permutation Complement) permutation in a minimum number of passes on a  $k$ -Omega. In this paper, result shows an  $O(N \log N)$  algorithm which realizes any BPC permutation on a  $k$ -Omega Network with a minimum number of passes. Many extensions of this result are possible for larger class.

**6. “Reliability analysis of fault-tolerant irregular multi-stage interconnection networks”, Rinkle Aggarwal, Lakhwinder Kaur, Himanshu Aggarwal :**

Multistage Interconnection Networks (MINs) are recognized as cost-effective means to provide programmable data paths between functional modules in multiprocessor system. In the case of irregular networks, the path length varies from any input to any output, in contrast with regular networks, where it is the same. Reliability of a system is the probability that it will perform its intended function satisfactorily for a given time under stated operating conditions. Reliability can be measured in terms of Mean Time to Failure (MTTF). The MTTF of a MIN is defined as the expected time elapsed before some source is disconnected from some destination. The analysis is based on the lower and upper bounds of the network reliability. In this paper three Irregular networks MDOT, FT and MFT have been compared for their cost-effectiveness, fault-tolerance and reliability in terms of MTTF. It can be concluded that although the fault-tolerance and reliability of FT and MFT is much higher than MDOT but at the expense of much higher cost. The reliabilities of these networks decrease marginally with increase in network size. Hence, the network MFT is the best in all respects among these three networks.

**7. “Reliability of a Class of Multistage Interconnection Networks”, Xian Cheng and Oliver C. Ibe:**

This paper considers the reliability of extra-stage interconnection networks. Three types of reliability are analyzed. These are the terminal reliability, which is the probability that at least one fault-free path exists between a given input-output pair the network reliability, which is the probability that at least one fault-free path exists between every input-output pair and the broadcast reliability, which is the probability that at least one fault-free path exists between a given input and all outputs. A recursive expression for the broadcast reliability is obtained. Also, tight bounds are derived for the network reliability. The result obtained in this paper are the exact broadcast reliability of the extra-stage MIN as well as tight bounds for the network reliability. The results shown that the bounds proposed in this paper are very tight at high values of reliability of switching elements. We believe that an SEN+ MIN with a network reliability of less than 0.5 is not a useful network.

**8. “Design and Reliability Analysis of a new Fault-tolerant Multistage Interconnection Network”, Rinkle Aggarwal, Lakhwinder Kaur, Himanshu Aggarwal:**

In this paper a new irregular network IASEN (Irregular Augmented Shuffle Exchange Network) has been proposed, which is derived from the regular ASEN-2 (Augmented Shuffle Exchange Network). ASEN-2 is a multipath network with limited fault tolerance. The reliabilities of the IASEN and ASEN-2 multi-stage interconnection networks have been calculated and compared in terms of the Upper and Lower bounds of Mean Time To Failure (MTTF). It has been observed that the proposed IASEN multistage interconnection network provides much better fault-tolerance by providing more paths between any pair of source destination and better reliability at the expense of little more cost than ASEN-2. An Irregular (IASEN) is designed using existing regular Augmented Baseline Network (ASEN-2) It has comparatively one more stage. IASEN is a dynamically re-routable and provides multiple paths of varying lengths between a source-destination pair. The reliability analysis shows that IASEN has better performance than ASEN-2. Thus the new

network IASEN provides better fault-tolerance and reliability than the existing ASEN-2 with little more cost

**9. “Performance of Multiprocessor Interconnection Networks”, Laxmi N. Bhuyan, Dharma P. Agrawal:**

In this paper three types of interconnection networks possess different hardware features and different system performances are compared. In particular, we will compare the hardware cost and system performance of the three interconnection networks. The number of switching elements used in an  $N \times M$  crossbar is  $N \times M$  in contrast to  $N \log N$  of MINs. Since each bus in the multiple-bus system needs to drive  $N + M$  modules, the bus load is proportional to  $N + M$ , while the bus load of an MIN is one due to the one-to-one connection. All three of these networks require certain types of arbiters to resolve the request conflicts. Once a bus is granted to a memory, only one of the processors that requests the memory can proceed while the others, if any, are delayed. Expandability and reliability are two other very important hardware features. In this context, a multiple-bus system shows its advantages over the other two because of its reconfigurability and multiple-data paths between every processor and memory.

**10. “Broadcast Reliability Evaluation of Multistage Interconnection Networks”, Ashok K. Kulkarni and Jerry L. Trahan:**

In this paper, a simple and efficient algorithm for the reliability evaluation of a shuffle exchange network with extra stage (SENE) by a two step method, also called the sum of disjoint products method is proposed. We derive reliability expressions for cases in which all outputs are to be reached (broadcast reliability) and in which only a subset of outputs of size  $s$  is to be reached ( $s$ -terminal reliability). We introduce the process of generating the broadcast tree terms using simple Boolean algebra. Once we obtain the broadcast tree terms, we generate a sum of disjoint products for the reliability expression by a relatively small number of calculation. Then compare this algorithm with the earlier one for the reliability evaluation of MINs.

## Chapter 3

### Problem Statement

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#### 3.1 Problem Definition

Reliability is one of the major consideration in the design of MINs. Reliability of a system is the probability that it will successfully perform its intended operations for a given time under stated operating conditions. The reliability of the network should be high. The Reliability  $R(t)$  is defined as the probability that the component or system experiences no failures during the time interval zero to  $t$  given that the component or system was repaired to a like new condition. There are three main measures of reliability, terminal, broadcast and network.

Terminal reliability is defined as the probability of successful communication between an input and an output switches. Broadcast reliability is defined as the probability that a single-input switch is able to broadcast data or connected to all the output switches. The ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability

#### 3.2 Problem Statement

In this thesis, the reliability analysis of three multistage interconnection networks—SEN, Omega, and Cube. MINs with one and two additional stages have been done. Additional stage provide redundant path in any MIN.

Three measures of network system reliability namely terminal, broadcast and network reliability are developed to evaluate the performance of these networks and to access the effect of additional stages on these networks reliability. On the basis of these three reliability parameters, comparative analysis of SEN, Omega and Cube have been made.

The objectives of the thesis are:

- To study the existing MINs.
- To calculate the reliability of Regular Multistage Interconnection Networks with one and two additional stages .

- Comparison of Regular MINs on the basis of Reliability.

## Chapter 4

### Reliability Analysis

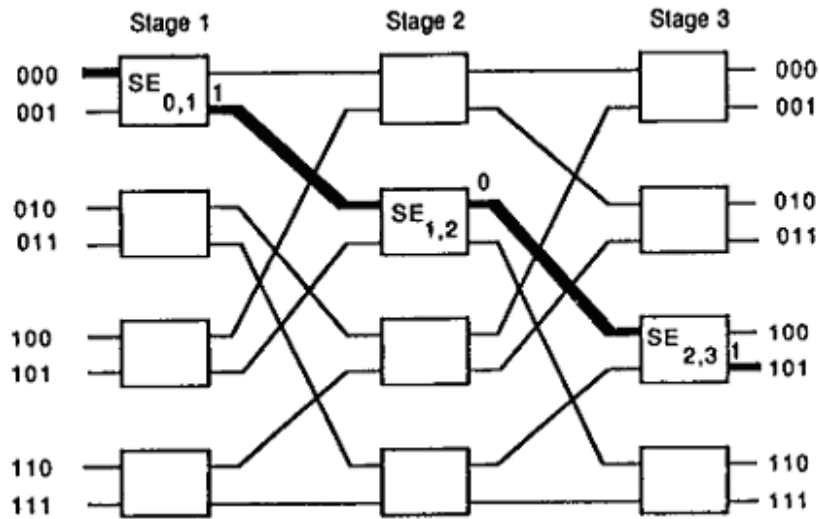
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In this chapter, three reliability parameters: terminal, broadcast, and network reliability for SEN, Omega and Cube networks with one and two additional stages have been evaluated.

#### 4.1 Reliability Analysis of SEN, SEN+, SEN+2

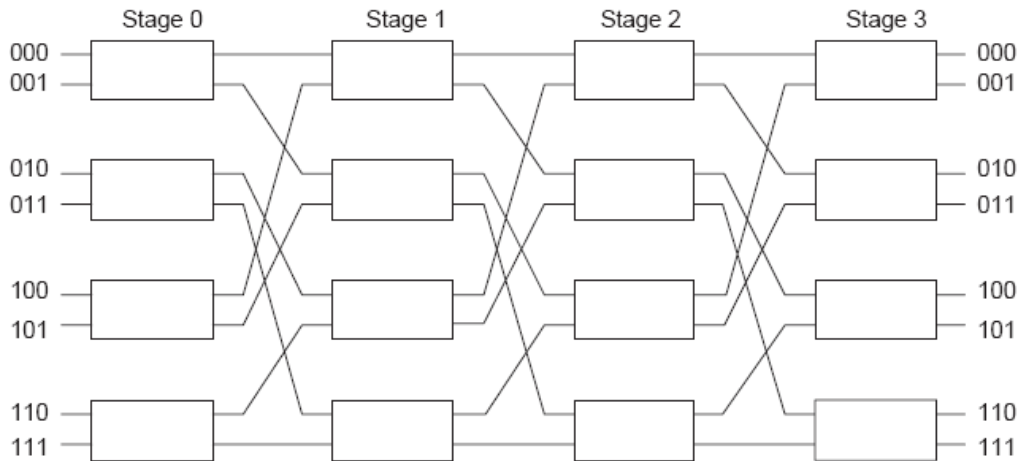
A SEN is a unique path MIN [13,12]. Therefore, there is only a single path between a particular input output pair. In this type of network, all SEs are critical and assumed as series connection. The SE can either transmit the inputs straight through itself or has cross connections. The number of switches per stage, the number of links and the connection between stages are consistent. An eight input / eight-output SEN with three stages, 12 switches (SEs), and 32 links as shown in Figure 4.1.

A SEN+ is an  $N \times N$  SEN with an additional stage. The SEN+ system has  $N$  inputs and  $N$  outputs, with two paths between each source–destination pair. It has  $n = \log_2 N + 1$  stages and each stage has  $N/2$  SEs. In general, the switch complexity for the  $N \times N$  SEN+ is  $N/2(\log_2 N + 1)$ . Thus, the additional cost of the SEN+ is  $N/2$  switches or a fractional increase of  $1/\log_2 N$ , which is small for a large  $N$ . An example of the  $8 \times 8$  SEN+ is demonstrated by Figure 4.2. The addition of an extra-stage to the SEN allows two paths for communication between each source and destination. While the paths in the first and the last stages of the SEN+ are not disjoint, the paths in the intermediate stages do disjoint links traverse. So the path redundancy in the SEN+ is achieved at the expense of an additional stage to the SEN.



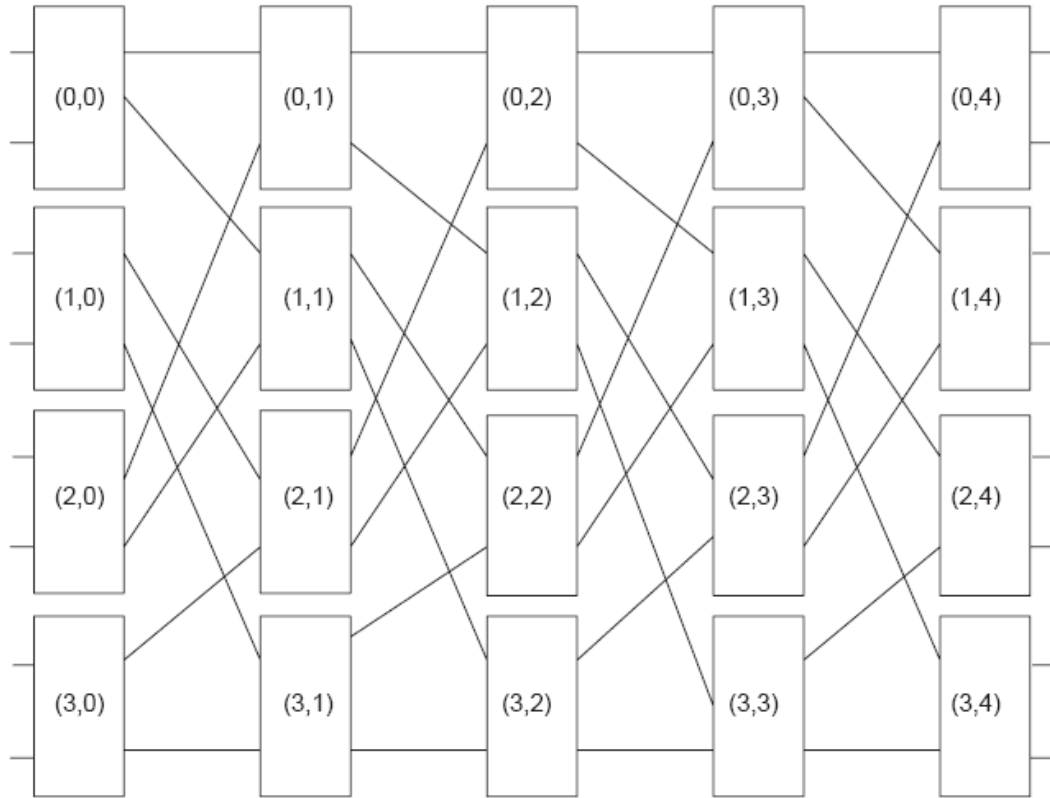
**Figure 4.1: 8x8 Shuffle-Exchange Network (SEN).**

As a comparison to SEN and SEN+, a SEN with two additional stages (SEN+2) is presented [11], and the reliability is evaluated. In general, a SEN+2 consists of  $N$  inputs and  $N$  outputs,  $N/2$  SEs per stage,  $\log_2 N + 2$  stages, and  $(N)(\log_2 N + 3)$  links. The network complexity is defined as the total number of SEs in the MIN, that is,  $(N/2)(\log_2 N + 2)$  which is 20 SEs for an  $8 \times 8$  SEN+2.



**Figure 4.2: 8x8 SEN with an Extra-Stage (SEN+).**

The number of terminal paths between an input and an output switches will be increased to  $2k$  by adding  $k$  extra stages to the SEN. This is also true for broadcast network. The additional  $k$  stages will create  $2k$  broadcast paths between a particular source and all destinations.



**Figure 4.3: 8×8 Shuffle-Exchange Network with two Additional Stages (SEN+2)**

Therefore, a SEN is a  $(2k-1)$  fault tolerant. For the  $8 \times 8$  case, the terminal paths and the broadcast paths of the SEN+ and SEN+2 are 2 and 4 respectively.

#### 4.1.1 Terminal reliability

Terminal reliability is defined as the probability of successful communication between an input and an output switches. In this section, terminal reliability of SEN, SEN+ and SEN+2. As an illustration, the terminal reliability of these three networks is evaluated for the  $8 \times 8$  network. The SEN is a unique-path MIN that has  $N$  input switches and  $N$  output switches and  $n$  stages, where  $n = \log_2 N$ . Each stage consists of  $N/2$  interchange boxes, where each box being controlled individually through routing tags. An  $8 \times 8$  SEN with three stages have 12 SEs and 32 links.

Let  $r$  be the probability of a switch being operational. Since these networks are unique-path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there are  $\log_2 N$  SEs in series for each terminal path. Hence, the terminal reliability of an  $N \times N$  regular network is

$$R_t(\text{Network})=(r)^{\log_2 N}$$

As there is only a single path between a particular input  $S_i$ ,  $i = 1, 2, 3, 4$ , and a particular output in the SEN so the terminal reliability for  $N = 8$  is

$$R_t(\text{SEN})=r^3$$

SEN+ is a two-path MIN derived from the SEN by adding an extra-stage. Figure 4.2 shows an  $8 \times 8$  SEN+ with four stages consisting of 16 SEs and 40 links. Since the SEN+ is a two-path MIN, there are two distinct paths between a particular input and output pair. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $(\log_2 N)-1$  SEs each. Where each path is connecting the input and output SE in series. Hence the terminal reliability of an  $N \times N$  SEN+ is

$$R_t(\text{SEN+})= (r)^2(1-(1-r^{(\log_2 N)-1})^2)$$

By adding an extra-stage to the  $8 \times 8$  SEN, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of the  $8 \times 8$  SEN+ is higher than that of the  $8 \times 8$  SEN. From above equation, the terminal reliability of the SEN+ for  $N = 8$  is

$$R_t(\text{SEN+})= (r)^2(1-(1-r^2)^2) = (r)^2(1-(1+(r^4)-2(r^2))) = (r)^2(2(r^2)-(r^4)) = 2(r)^2 - (r)^6$$

An  $8 \times 8$  SEN+2 having four SEs per stage, five stages, and 48 links as demonstrated in Figure 4.3. It is observed that there are four terminal paths between any pair of input  $S_i$  ( $i = 1, 2, 3$ , and  $4$ ) and output  $D_i$ .

Suppose that the position of a SE  $i$  in stage  $j$  is represented by  $SE_{i,j}$ . Since there are 20 SEs in the  $8 \times 8$  SEN+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from  $SE_{0,0}$ ,  $SE_{1,0}, \dots, SE_{2,4}$ ,  $SE_{3,4}$ . As an example, the terminal reliability between  $SE_{0,0}$  and  $SE_{0,4}$  is examined as shown in Figure 4.3. The terminal reliability of the  $8 \times 8$  SEN with two additional stages for  $N = 8$  is

$$\begin{aligned} R_t(\text{SEN+2}) &= r^2 \sum c_i r^{(2N-i)} (1-r)^i \quad \text{where } i = 0 \text{ to } 8 \\ &= r^{10} + 2r^9(1-r) + 8r^8(1-r)^2 + 8r^7(1-r)^3 + 2r^7(1-r)^2 \\ &\quad + 4r^6(1-r)^3 + 4r^6(1-r)^2 + 4r^5(1-r)^2 \end{aligned}$$

#### 4.1.2 Broadcast reliability

Broadcast reliability is defined as the probability that a single input switch is able to broadcast data or connected to all the output switches. Since SEN is a unique-path MIN that has N inputs and N outputs, there is only one broadcast path in this network. The broadcast reliability of the SEN can be calculated by assuming that the SE in the input stage and the SE in the output stage have to work in order the network to be operational. Then by conditioning on the first stage and listing all possible combinations of paths between an input and all outputs, the broadcast reliability can be computed.

The failure of any switch in a broadcast path will cause the system failure, so from the reliability point of view; all SEs in a broadcast path are critical and can be assumed as in series. Since a broadcast path in the 8×8 SEN consists of seven SEs, the broadcast reliability as a function of the reliability of a SE for N = 8 is

$$R_b(\text{SEN})=r^7$$

As SEN+ is a two-path MIN, there are two broadcast paths from every input to all outputs. It can be observed that by having two broadcast paths, the SEN+ is much more reliable than the SEN that has only one broadcast path. The broadcast reliability of the SEN+ for N = 8 is

$$R_b(\text{SEN+}) = 2(r)^8 + 2(r)^9 - 4(r)^{10} + (r)^{11}.$$

In case of SEN+2 two extra-stages are added to the SEN therefore, there are four broadcast paths in the SEN+2. The broadcast reliability of the SEN with two additional stages for N =8 is

$$R_b(\text{SEN+2}) = r^{15} + 4r^{14}(1-r) + 20r^{13}(1-r)^2 + 32r^{12}(1-r)^3 + 16r^{11}(1-r)^4 + 10r^{11}(1-r)^2 + 12r^{10}(1-r)^3 + 4r^9(1-r)^2.$$

#### 4.1.3 Network reliability

The ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability. Network reliability of the 8×8 SEN, SEN+, and SEN+2 is evaluated in this section. Since SEN is a single-path MIN, the failure of any switch

will cause the system failure. Therefore, from the reliability point of view, there are  $(N/2)(\log_2 N)$  SEs in series. The Cube network uses two-function switch boxes, whereas the Omega network uses four-function switch boxes. The data-flow directions in the two networks are opposite to each other. The reliability of an  $N \times N$  SEN

$$R_n(\text{SEN}) = r^{(N/2)(\log_2 N)}$$

As for  $8 \times 8$  SEN system, the network reliability for  $N = 8$  is

$$R_n(\text{SEN}) = r^{12}$$

The network reliability of the  $8 \times 8$  SEN+ (consists of four stages: stage 0, 1, 2, and 3) is calculated as follows:

1. All SEs in stages 0 and 3 must be working for the system to be operational. For reliability evaluation, it can be assumed that these SEs are in series. Therefore, the reliability for stages 0 and 3 is equal to  $r^8$ .
2. In stage 1, consider the state of each switch as conditional and proceed with all possible combinations through stage 2 that make the input-output connections. A k out of n redundancy system is applied where there is at least k out of n components to function for the system to work.
3. Every SE has only two possible states, working or failed.

Hence, the network reliability of the  $8 \times 8$  SEN+ for  $N = 8$  is derived as

$$\begin{aligned} R_n(\text{SEN+}) &= (r^8) \text{ (the sum of all reliability in the stages 1 and 2),} \\ &= 2r^{12} + 4r^{14} - 8r^{15} + 3r^{16} \end{aligned}$$

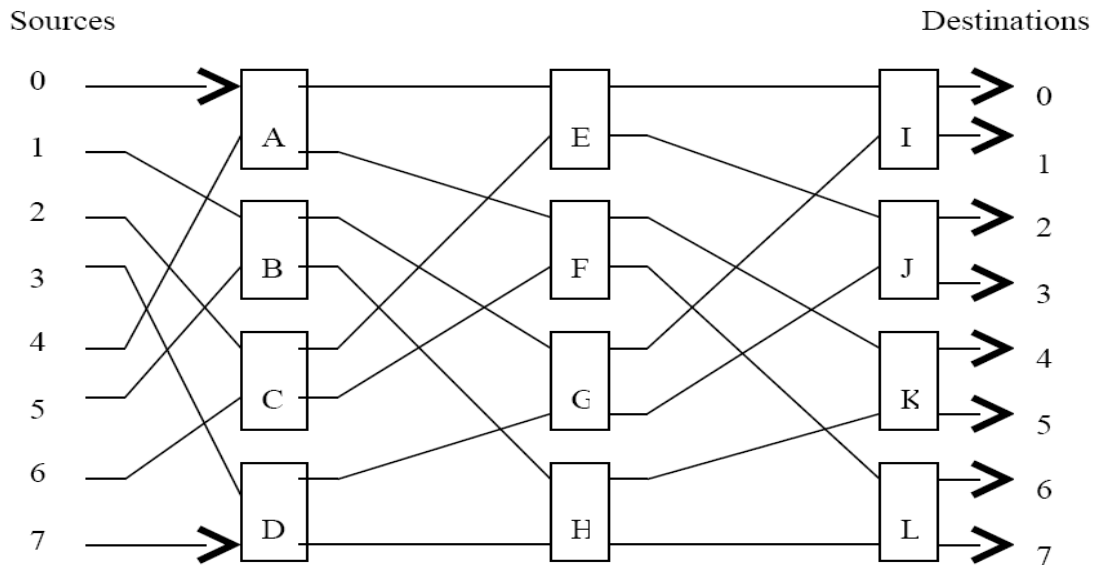
By using equivalent approach as described above for the SEN+, the network reliability of the  $8 \times 8$  SEN+2, where  $N = 8$  with two additional stages is derived as

$$\begin{aligned} R_n(\text{SEN+2}) &= r^{20} + 4r^{19}(1-r) + 36r^{18}(1-r)^2 + 120r^{17}(1-r)^3 + 168r^{16}(1-r)^4 \\ &\quad + 2r^{16}(1-r)^2 + 96r^{15}(1-r)^5 + 20r^{15}(1-r)^3 + 16r^{14}(1-r)^6 \\ &\quad + 14r^{14}(1-r)^4 + 4r^{13}(1-r)^3 \end{aligned}$$

## 4.2 Reliability Analysis of Omega, Omega+ and Omega+2

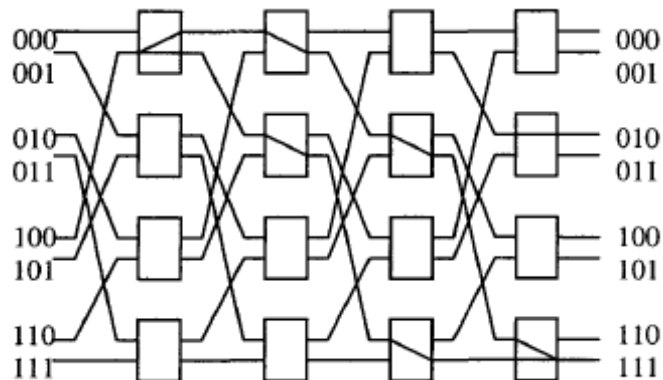
An  $N \times N$  ( $N = 8$ ) Omega network consists of  $n = \log_2 N$  stages of  $2 \times 2$  switching elements (SEs) which provide connections between  $N$  sources and  $N$

destinations. A unique path between any source to a desired destination can be established by properly setting each SE to a state “through” or “cross.” An N-permutation defines N paths by specifying a distinct destination for each of the N sources.



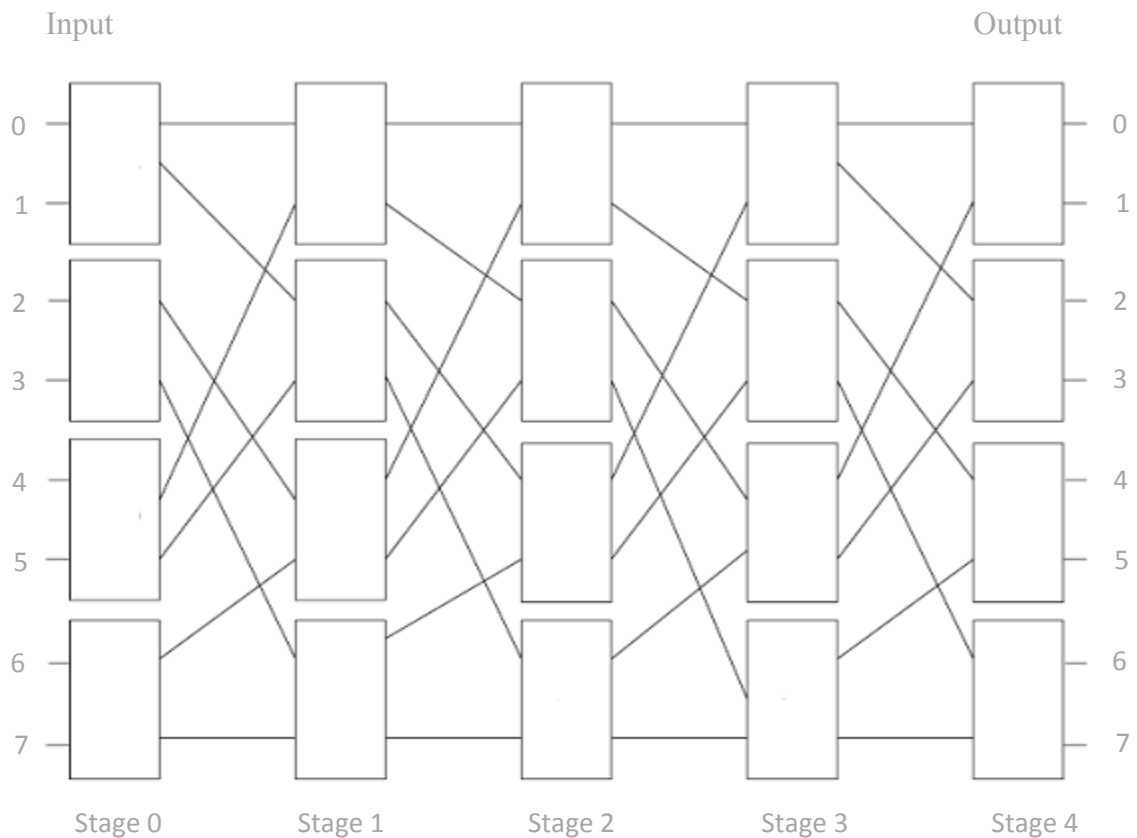
**Figure 4.4: An Omega network for N=8**

Two paths may conflict if they meet at a common output port of the same SE. Figure 4.5 shows an example where path (001, 111) and path (101, 101) conflict in an 8x8 Omega. A k-Omega provides  $2^k$  different paths between any pair of source and destination and therefore provides fault tolerance ability and increases permutation capability. Figure 4.5 shows an example of a Omega with one additional stage.



**Figure 4.5: An Extra-Stage Omega (Omega+) Network for N=8**

An Omega network with two additional stages (Omega+2) is presented in Figure 2.6 and the reliability of an 8×8 network is evaluated. In general, a Omega+2 consists of N inputs and N outputs, N/2 SEs per stage,  $\log_2 N + 2$  stages, and  $(N)(\log_2 N + 3)$  links. The network complexity is defined as the total number of SEs in the MIN, that is  $(N/2)(\log_2 N + 2)$  which is 20 switches for an 8×8 Omega+2.



**Figure 4.6: 8×8 Omega network with two additional stages (Omega+2).**

#### 4.2.1 Terminal reliability

Terminal reliability is defined as the probability of successful communication between an input output pair. In this section, terminal reliability of Omega, Omega+ and Omega+2 has evaluated. The Omega is a unique-path MIN that has N input switches and N output switches and n stages, where  $n = \log_2 N$ . An 8×8 Omega has three stages, 12 SEs and 32 links.

Let  $r$  be the probability of a switch being operational. As Omega is a unique-path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there are  $\log_2 N$  SEs in series for each terminal path. Hence, the terminal reliability of an  $N \times N$  Omega is

$$R_t(\text{Omega}) = (r)^{\log_2 N}$$

As there is only a single path between a particular input  $S_i$ ,  $i = 1, 2, 3, 4$ , and a output in an  $8 \times 8$  Omega so the terminal reliability is

$$R_t(\text{Omega}) = (r)^3.$$

Omega+ is a two-path MIN derived from the Omega by adding an extra-stage. Figure 4.5 shows an  $8 \times 8$  Omega+ with four stages consisting of 16 SEs and 40 links. Since the Omega+ is a two-path MIN, there are two connection paths between a particular input and output. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $\log_2 N - 1$  SEs each. Where, each path is connecting the input and output SE in series. Hence, the terminal reliability of an  $N \times N$  Omega+ is

$$R_t(\text{Omega+}) = (r)^2 (1 - (1 - r^{(\log_2 N) - 1})^2).$$

By adding an extra-stage to an Omega, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of an Omega+ is higher than that of an Omega. From above equation, the terminal reliability of the Omega+ for  $N = 8$  is

$$R_t(\text{Omega+}) = (r)^2 (1 - (1 - r^2)^2) = 2(r)^4 - (r)^6$$

An  $8 \times 8$  Omega+2 consists of eight inputs and eight outputs, four SEs per stage, five stages, and 48 links. It is observed that there are four terminal paths between any pair of input and output.

Suppose that the position of a SE  $i$  in stage  $j$  is represented by  $SE_{i,j}$ . Since there are 20 SEs in the  $8 \times 8$  Omega+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from  $SE_{0,0}$ ,  $SE_{1,0}$ , ...,  $SE_{2,4}$ ,  $SE_{3,4}$ . The terminal reliability of an Omega network with two additional stages for  $N = 8$  is

$$R_t(\text{Omega}+2) = r^{10} + 2r^9(1-r) + 8r^8(1-r)^2 + 8r^7(1-r)^3 + 2r^7(1-r)^2 + 4r^6(1-r)^3 + 4r^6(1-r)^2 + 4r^5(1-r)^2.$$

#### 4.2.2 Broadcast reliability

Broadcast reliability is defined as the probability that a single input switch is able to broadcast data or connected to all the output switches. Since Omega is a unique-path MIN that has N inputs and N outputs, there is only one broadcast path in this network. The broadcast reliability of the Omega can be calculated by assuming that the SE in the input stage and the SEs in the output stage have to work in order the network to be operational. Then by conditioning on the first stage and listing all possible combinations of paths between an input and all outputs, the broadcast reliability can be computed.

The failure of any switch in a broadcast path will cause the system failure, so from the reliability point of view; all SEs in a broadcast path are critical and can be assumed as in series. Since a broadcast path in the 8×8 Omega consists of seven SEs, the broadcast reliability as a function of the reliability of a SE for N = 8 is

$$R_b(\text{Omega}) = (r)^7$$

As Omega+ is a two-path MIN, there are two broadcast paths from every input to all outputs. It can be observed that by having two broadcast paths, the Omega+ is much more reliable than the Omega that has only one broadcast path. The broadcast reliability of the 8×8 Omega+ for N = 8 is

$$R_b(\text{Omega}+) = 2(r)^8 + 2(r)^9 - 4(r)^{10} + (r)^{11}$$

In case of Omega+2 two extra-stages are added to the Omega, there are four broadcast paths in the Omega+2. The broadcast reliability of the Omega with two additional stages for N = 8 is

$$R_b(\text{Omega}+2) = r^{15} + 4r^{14}(1-r) + 20r^{13}(1-r)^2 + 32r^{12}(1-r)^3 + 16r^{11}(1-r)^4 + 10r^{11}(1-r)^2 + 12r^{10}(1-r)^3 + 4r^9(1-r)^2$$

#### 4.2.3 Network reliability

The ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability. Network reliability of the 8×8 Omega, Omega+, and Omega+2 is evaluated in this section. Since Omega is a single-path MIN, the failure of any switch will cause the system failure. Therefore, from the reliability point of view, there are (N/2)(log<sub>2</sub> N) SEs in series. The reliability of an N×N Omega is

$$R_n (\text{Omega}) = (r)^{(N/2)(\log_2 N)}$$

As for 8×8 SEN system, the network reliability for N = 8 is

$$R_n (\text{Omega}) = (r)^{12}$$

The network reliability of the 8×8 Omega+ (consists of four stages: stage 0, 1, 2, and 3) is calculated as follows:

$$\begin{aligned} R_n (\text{Omega}+) &= (r^8) \text{ (the sum of all reliability in the stages 1 and 2)} \\ &= 2r^{12} + 4r^{14} - 8r^{15} + 3r^{16} \end{aligned}$$

It is noted that the network reliability of the 8×8 Omega+ is much higher than that of the 8×8 Omega for all reliability values of the SEs. By using equivalent approach as described above for the Omega+, the network reliability of the 8×8 SEN+2, where N=8 with two additional stages is

$$\begin{aligned} R_n (\text{Omega}+2) &= r^{12} \sum c_i r^{(2N^2-i)} (1-r)^i \quad \text{where } i = 0 \text{ to } 8 \\ &= r^{20} + 4r^{19} (1-r) + 36r^{18} (1-r)^2 + 120r^{17} (1-r)^3 + \\ &\quad 168r^{16} (1-r)^4 + 2r^{16} (1-r)^2 + 96r^{15} (1-r)^3 + 20r^{15} (1-r)^3 + \\ &\quad 16r^{14} (1-r)^6 + 14r^{14} (1-r)^4 + 4r^{13} (1-r)^3 \end{aligned}$$

### 4.3 Reliability Analysis of Cube, Cube+ and Cube+2

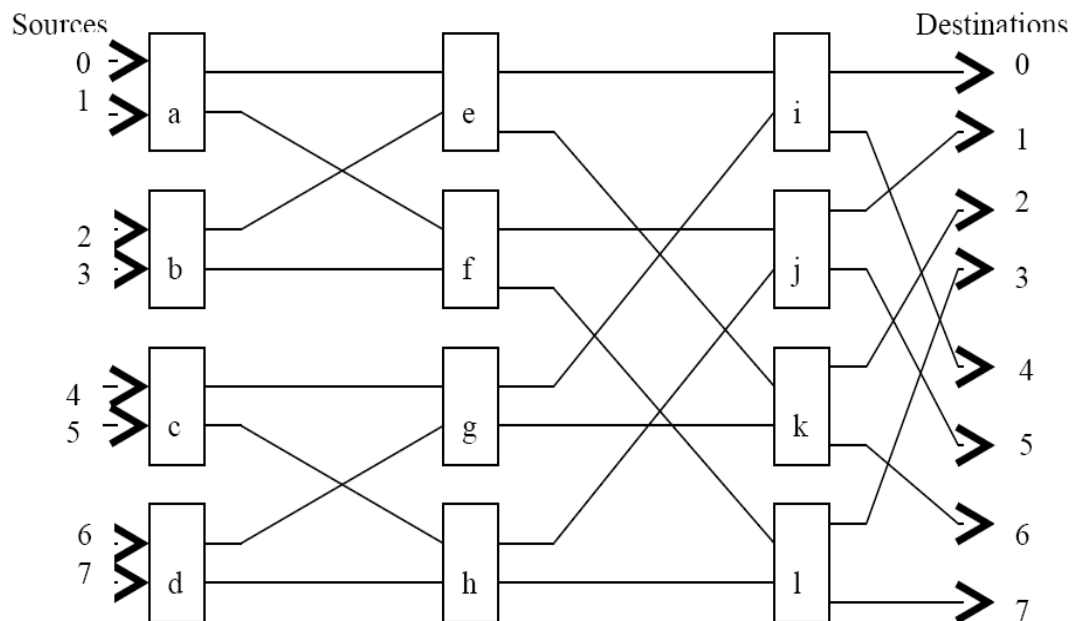
A cube network can be implemented as either a recirculating network or as a multistage network for SIMD machines. A cube network for an SIMD machine with N PEs corresponds to an n cube where n=log<sub>2</sub>N. Formally, an n-dimensional cube network of N PEs is specified by the following n routing functions:

$$C_i(a_{n-1}, \dots, a_1 a_0) = a_{n-1}, \dots, a_{i+1} a_i a_{i-1}, \dots, a_0 \quad \text{for } i=0, 1, 2, \dots, n-1$$

Two-function (straight and exchange) switch boxes are used in constructing multistage cube networks. The stages are numbered as 0 at the input end and

increased to  $n-1$  at the output. This means that switch boxes at stage  $i$  connect at input line to the output line that differs from it only at the  $i$ th bit position. The cube-routing is implemented by a three stage cube network is shown in Figure 4.6.

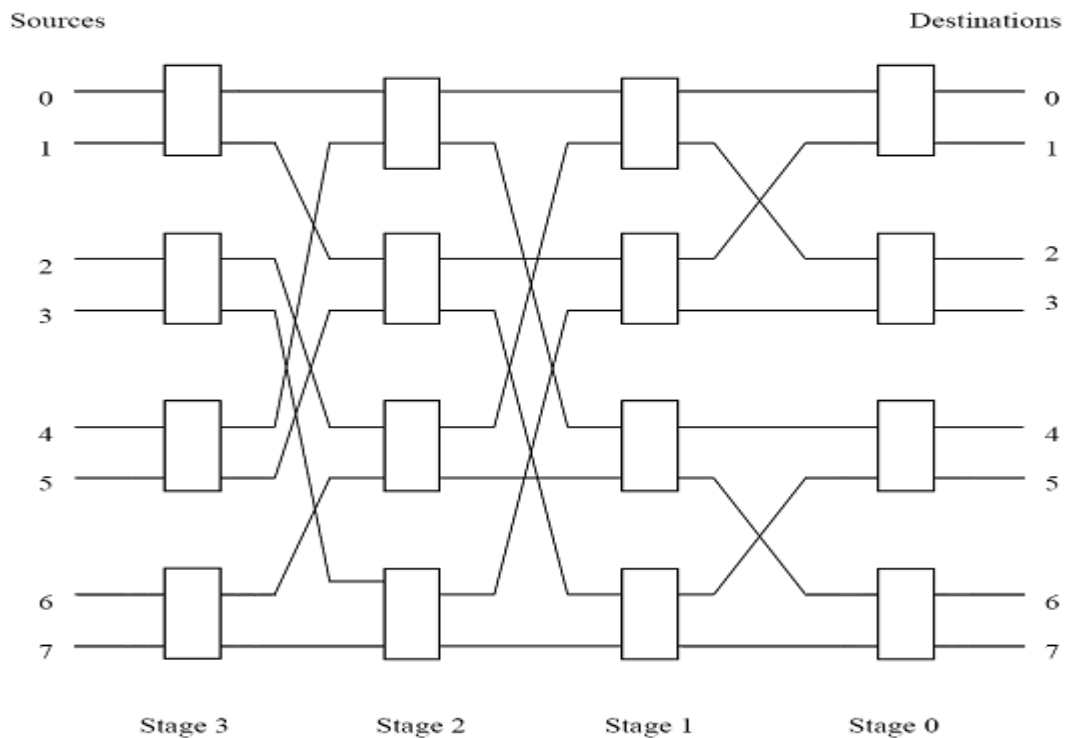
The cube network uses two-function switch boxes, whereas the Omega network uses four-function switch boxes. The data-flow directions in the two networks are opposite to each other. The Omega network can perform one to many connections, while the  $n$ -cube network cannot. To establish the I/O connections zero to five and one to seven. The Omega network can perform this task whereas the  $n$ -cube network cannot. On the other hand, the  $n$ -cube network can connect five to zero and seven to one, but the Omega network cannot.



**Figure 4.7: A Multistage Cube Network for  $N=8$**

The Extra-Stage Cube (ESC) network is formed from the generalized cube by adding an extra stage to the network along with multiplexers and demultiplexers at the input and output stages, respectively, as shown in Figure 4.7. ESC is a 2-Path MIN that provides 2 unique paths from the input side to the output side of the MIN. The redundant paths function as alternate transition path from any input to any output, when any switching element fails. Generally, the number of stages  $n$  is  $\log_2 N + 1$ .

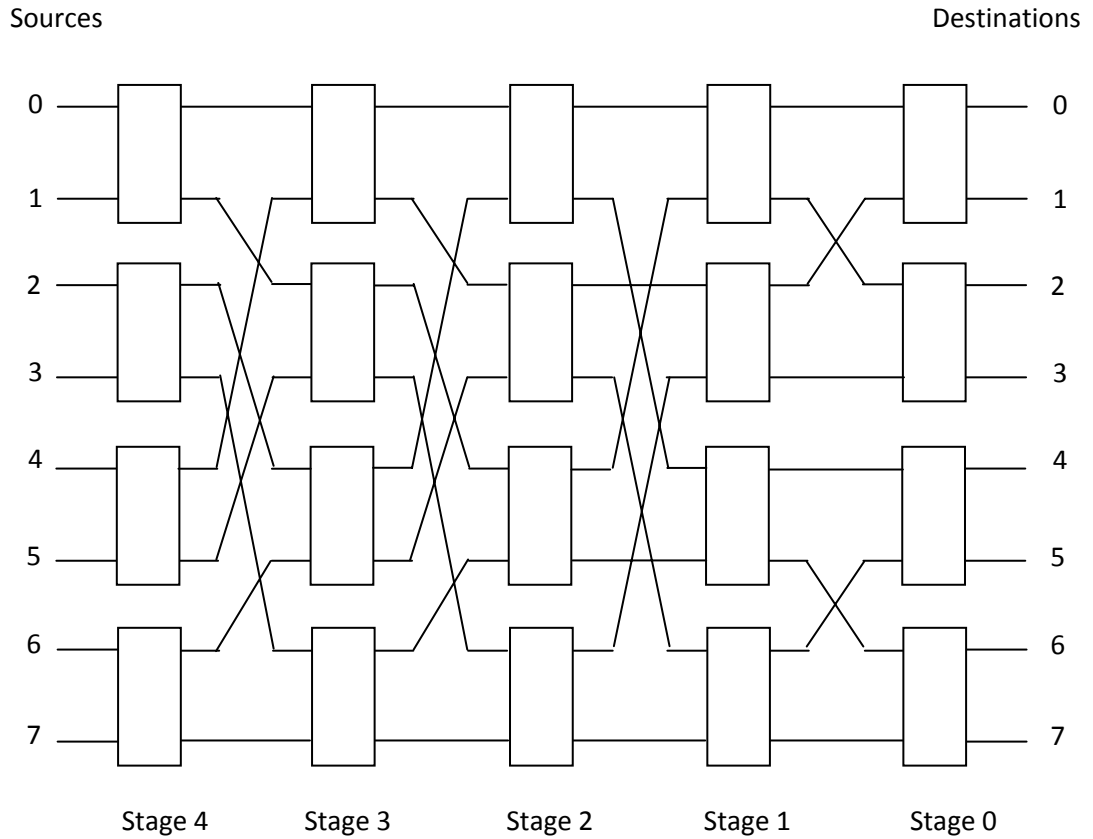
Stage  $n$  is connected like stage 0, where links that vary in the low-order bit are paired. For example, 0(000 in binary bit) is paired with 1(001 in binary bit) where the low-order bit is differing.



**Figure 4.8: Extra Stage Cube Network for N= 8**

A number of assumptions are made to evaluate network reliability of ESC:

1. All failures are statistically independent.
2. All switching elements (SEs) are substantially less reliable than the links. Even though the links can fail, they are generally much more reliable than the SEs.
3. Each SE in the MIN has only two possible states, working or failed. Suppose it is defined: 1 = SE working, 0 = SE failed.
4. The reliability of the SE being in a particular state is known.
5. All SEs are identical and have constant exponential failure rates.
6. SEs cannot be repaired.
7. All SEs at stage 0 and  $n$  must work for the system to be operational.
8. A SE is assumed as failed when it cannot perform any of the connection functions (straight, exchange/swap, upper-broadcast, and lower-broadcast).



**Figure 4.9: 8×8 Cube network with two additional stages (Cube+2)**

### 4.3.1 Terminal reliability

Terminal Reliability, generally used as a measure of robustness of a MIN, is the probability of existence of at least one fault free path between a designated pair of input and output terminals. In this section, terminal reliability of Cube, Cube+ and Cube+2 are compared. As an illustration, the terminal reliability of these three networks is evaluated for the 8×8 network. The Cube is a unique-path MIN that has  $N$  input switches and  $N$  output switches and  $n$  stages, where  $n = \log_2 N$ . Each stage consists of  $N/2$  interchange boxes, where each box being controlled individually through routing tags. An eight-input / eight-output Cube with three stages, 12 SEs and 32 links. Let  $r$  be the probability of a switch being operational. Since Cube is a unique-path MIN, hence the terminal reliability of Cube is same as SEN. The failure of any switch will cause system failure, so from the reliability point of view, there are

$\log_2 N$  SEs in series for each terminal path. Hence, the terminal reliability of an  $N \times N$  Cube is

$$R_t(\text{Cube}) = (r)^{\log_2 N}.$$

It is noted that there is only a single path between a particular input  $S_i$ ,  $i = 1, 2, 3, 4$ , and a particular output  $D_i$  in the  $8 \times 8$  Cube so the terminal reliability for  $N=8$  is

$$R_t(\text{Cube}) = (r)^3$$

Cube+ is a two-path MIN derived from the Cube by adding an extra-stage. Figure 4.6 shows an eight-input/eight output Cube+ with four stages consisting of 16 SEs and 40 links. Since the Cube+ is a two-path MIN, there are two connection paths between a particular input and output. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $(\log_2 N)-1$  SEs each. A lower bound on  $R$  was obtained by noting that as long as 1) none of the switches in the first and last stages had failed, and 2) that at least one of the two complete subnetworks was fault-free, then full access would be guaranteed. So,  $R > r^N (1 - (1 - r^{N'})^2)$ , where  $n$  is the number of stages,  $N = 2^n$  is the number of PE's and  $N' = (N/4)(\log N - 1)$  is the number of switches in each subnet. The upper bound on  $R$  was obtained by observing that full access is lost if a switch in the first or last stage fails or if a pair of switches that occupy corresponding positions in the two subnetworks fail. Since there are many combinations of failed switches which will cause the network to be failed other than such pairs, this consideration alone overestimates  $R$ . Hence,  $R < r^N [1 - (1 - r)^2]^{N'}$ . Our approach is to obtain bounds, on each coefficient,  $c_i$ , of the reliability polynomial,

$$R = r^N \sum_{i=0}^{2N'} c_i r^{2N'-i} (1-r)^i.$$

The  $k_{th}$  term in the polynomial is the probability of an operational network with exactly  $k$  faulty switches and  $c_k$  is the number of ways in which  $k$  switches in the subnets may fail without destroying full access. Where, each path is connecting the input and output SE in series. Hence, using the above equation the terminal reliability of an  $N \times N$  Cube+ is calculated as follows

$$R_t(\text{Cube+}) = (r)^2 (1 - (1 - r^{(N')})^2) \quad \text{Where, } N' = N/4(\log_2 N - 1)$$

By adding an extra-stage to the Cube, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of the 8×8 Cube+ is higher than that of the Cube. From the above equation, the terminal reliability of the Cube+ for N = 8 is

$$\begin{aligned} R_t(\text{Cube+}) &= (r)^2 (1 - (1 - r^4)^2) \\ &= (r)^2(1 - (1+r^8 - 2r^4)) = (r)^2(1 - 1 - r^8 + 2r^4) = 2(r)^6 - (r)^{10} \end{aligned}$$

An 8×8 Cube+2 having four SEs per stage, five stages, and 48 links as demonstrated in. It is observed that there are four terminal paths between any pair of input  $S_i$  ( $i = 1, 2, 3,$  and  $4$ ) and output  $D_i$ . Suppose that the position of a SE  $i$  in stage  $j$  is represented by  $SE_{i,j}$ . Since there are 20 SEs in the 8×8 Cube+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from  $SE_{0,0}, SE_{1,0}, \dots, SE_{2,4}, SE_{3,4}$ . Using the above equation, the terminal reliability of the Cube with two additional stages for N = 8 is:

$$\begin{aligned} R_t(\text{Cube+2}) &= r^4 \sum c_i r^{2N-i} (1-r)^i \quad \text{where } i=0 \text{ to } 8 \\ &= r^4(r^8 + 4r^7(1-r) + 10r^6(1-r)^2 + 6r^5(1-r)^3 + 8r^4(1-r)^2 + 6r^3(1-r)^2 + \\ &\quad 8r^2(1-r)^2) \\ &= r^{12} + 4r^{11}(1-r) + 10r^{10}(1-r)^2 + 6r^9(1-r)^3 + 8r^8(1-r)^2 + \\ &\quad 6r^7(1-r)^2 + 8r^6(1-r)^2 \end{aligned}$$

#### 4.3.2 Broadcast reliability

Broadcast reliability is defined as the probability that a single-input switch is able to broadcast data or connected to all the output switches. As an illustration, the broadcast reliability of Cube, Cube+, and Cube+2 for the 8×8 case are examined. Since Cube is a unique-path MIN that has N inputs and N outputs, there is only one broadcast path in this network. The broadcast reliability of the Cube can be calculated by assuming that the SE in the input stage and the SEs in the output stage have to work in order the network to be operational. Then by conditioning on the first stage and listing all possible combinations of paths between an input and all outputs, the broadcast reliability can be computed.

The failure of any switch in a broadcast path will cause the system failure, so from the reliability point of view; all SEs in a broadcast path are critical and can be

assumed as in series. Since a broadcast path in the 8×8 Cube consists of nine SEs, the broadcast reliability as a function of the reliability of a SE for N = 8 is

$$R_b(\text{Cube}) = (r)^9$$

As the Cube+ is a two-path MIN, there are two broadcast paths from every input to all outputs. It can be observed that by having two broadcast paths, the Cube+ is much more reliable than the Cube that has only one broadcast path. The broadcast reliability of the 8×8 Cube+ for N = 8 is

$$\begin{aligned} R_b(\text{Cube+}) &= [r^9 (1 - (1-r^2)^2)] + [r^8 (1 - (1-r^2)^2)] \\ &= [r^9 (2r^2 - r^4)] + [r^8 (2r^2 - 4r^4)] = 2r^{10} + 2r^{11} - 4r^{12} - r^{13} \end{aligned}$$

In case of Cube+2 two extra-stages are added to the Cube, there are four broadcast paths in the Cube+2. Using the above equation, the broadcast reliability of the 8×8 Cube with two additional stages for N = 8 can be calculated as follows:

$$\begin{aligned} R_b(\text{Cube+2}) &= r^9 \sum c_i r^{(2N-i)} (1-r)^i \quad \text{where } i=0 \text{ to } 8 \\ R_b(\text{Cube+2}) &= r^{17} + 6r^{16} (1-r) + 28r^{15} (1-r)^2 + 18r^{14} (1-r)^3 + 14r^{13} (1-r)^4 \\ &\quad + 18r^{12} (1-r)^2 + 10r^{11} (1-r)^3 \end{aligned}$$

### 4.3.3 Network reliability

The ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability. Since Cube is a single-path MIN, the failure of any switch will cause the system failure. Therefore, from the reliability point of view, there are (N/2) (log<sub>2</sub>N) SEs in series. The reliability of an N×N Cube is

$$R_n(\text{Cube}) = (r)^{2[(N/4)(\log_2 N-1)]}$$

Using the above equation, the network reliability for N = 8 is

$$R_n(\text{Cube}) = (r)^{12}$$

The network reliability of the 8×8 Cube+ (consists of four stages: stage 0, 1, 2, and 3) is calculated as follows:

$$R_n(\text{Cube+}) = (r^8) \text{ (the sum of all reliability in the stages 1 and 2),}$$

$$\begin{aligned}
&= r^8 [1 - (1 - r^4)^2] + [1 - (1 - (2r)^3)^2] = (2r^{12} - r^{16}) + (4r^{11} - 4r^{14}) \\
&= 4r^{11} + 2r^{12} - 4r^{14} - r^{16}
\end{aligned}$$

As the network reliability of the 8×8 Cube+ is much higher than that of the 8×8 Cube for all reliability values of the SEs. By using equivalent approach as described above for the Cube+, the network reliability of the 8×8 Cube+2, where N = 8 with two additional stages is

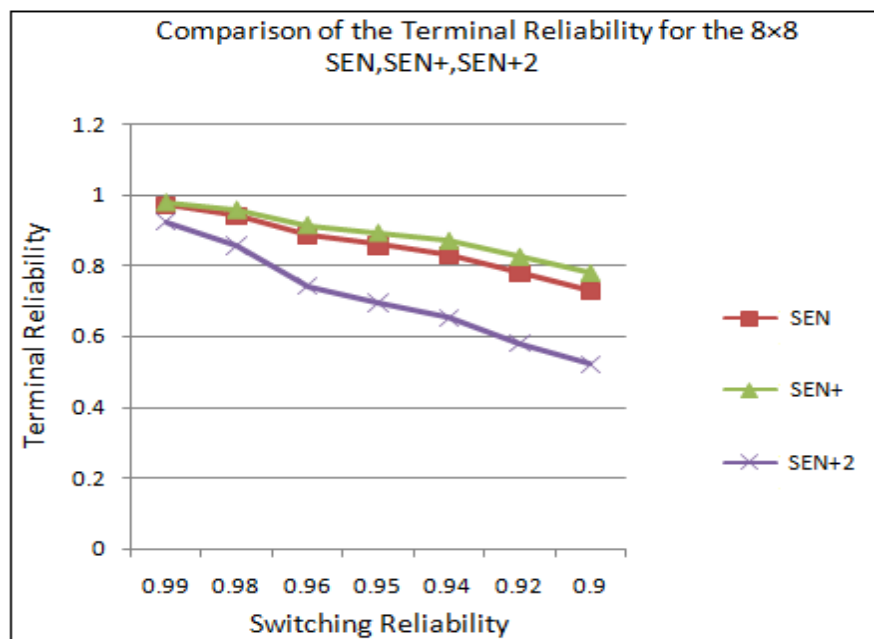
$$\begin{aligned}
R_n (\text{Cube}+2) &= r^{12} \sum c_i r^{(2N-i)} (1 - r)^i \quad \text{where } i = 0 \text{ to } 8 \\
&= r^{20} + 8r^{19} (1 - r) + 42r^{18} (1 - r)^2 + 32r^{17} (1 - r)^3 + 64r^{16} (1 - r)^2 \\
&\quad + 2r^{16} (1 - r)^4 + 92r^{15} (1 - r)^5 + 16r^{15} (1 - r)^3 + 54r^{14} (1 - r)^6
\end{aligned}$$

**5.1 Reliability Analysis of SEN and its Variants**

**5.1.1 Terminal Reliability of SEN, SEN+ and SEN+2**

**Table 5.1: Comparative Terminal reliability of SEN, SEN+ and SEN+2**

Switching Reliability	Terminal Reliability of SEN	Terminal Reliability of SEN+	Terminal Reliability of SEN+2
0.99	0.970299	0.979712	0.924345
0.98	0.941192	0.958894	0.856787
0.96	0.884736	0.915935	0.742687
0.95	0.857375	0.893921	0.694677
0.94	0.830584	0.871628	0.651818
0.92	0.778688	0.826431	0.579273
0.90	0.729000	0.780759	0.520995



**Figure 5.1: Terminal reliability graph of the 8×8 SEN, SEN+, SEN+2**

The comparison of the SEN, SEN+ and SEN+2 for the 8×8 networks is presented in Table 5.1. From Figure 5.1 it is clear that the terminal reliability of the SEN+ is the highest whereas the terminal reliability of SEN+2 is the lowest among these three networks. Therefore, there is not a direct relation between additional paths and increase in terminal reliability because the additional paths may increase the links complexity of the network, leading to a higher failure. Hence, it is concluded that adding one additional stage to the SEN is more efficient way to improve terminal reliability rather than two stages.

**5.1.2 Broadcast Reliability of SEN, SEN+ and SEN+2**

**Table 5.2: Comparative Broadcast reliability of SEN, SEN+ and SEN+2**

<b>SE Reliability</b>	<b>Broadcast Reliability of SEN</b>	<b>Broadcast Reliability of SEN+</b>	<b>Broadcast Reliability of SEN+2</b>
0.99	0.932065	0.950334	0.897863
0.98	0.868126	0.901462	0.809831
0.96	0.751447	0.806766	0.667692
0.95	0.698337	0.761192	0.610132
0.94	0.648478	0.716965	0.559696
0.92	0.557847	0.632844	0.475794
0.90	0.478297	0.554872	0.408837

The results of the broadcast reliability evaluation are summarized in Table 5.2. From Figure 5.2 it is clear that the broadcast reliability of the 8×8 network is the highest in SEN+ and the lowest in SEN+2. Although the number of broadcast paths in the SEN+2 is greater than that of the SEN+, the broadcast reliability of the SEN+2 is the lowest among these three networks. Therefore, it can be concluded that based on the broadcast reliability results, SEN+ is the most reliable network among the three networks.

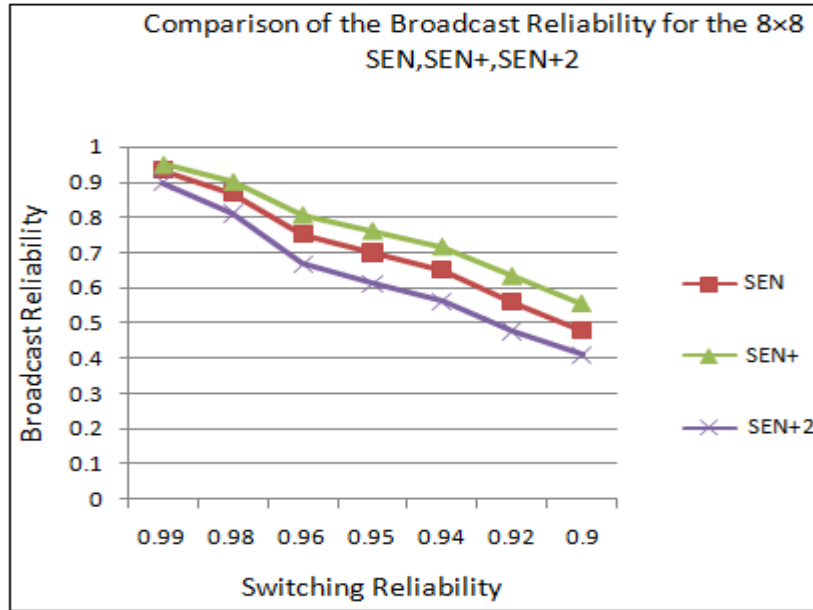


Figure 5.2: Broadcast reliability graph of the 8x8 SEN, SEN+, SEN+2

### 5.1.3 Network Reliability of SEN, SEN+ and SEN+2

Table 5.3: Comparative Network reliability of SEN, SEN+ and SEN+2

SE Reliability	Network Reliability of SEN	Network Reliability of SEN+	Network Reliability of SEN+2
0.99	0.886385	0.921659	0.854252
0.98	0.784717	0.846842	0.733541
0.96	0.612710	0.708630	0.549891
0.95	0.540360	0.645470	0.480112
0.94	0.475920	0.586322	0.421465
0.92	0.367666	0.479906	0.329657
0.90	0.282430	0.388708	0.262133

The comparison of the network reliability evaluation of the SEN, SEN+ and SEN+2 for the 8x8 case is summarized in Table 5.3. From Figure 5.3 it is clear that the 8x8 SEN+ has the highest network reliability and the SEN+2 has the lowest network reliability. This result again proves that adding just one additional stage to the SEN, leads to a SEN+ with the highest network reliability for a SEN.

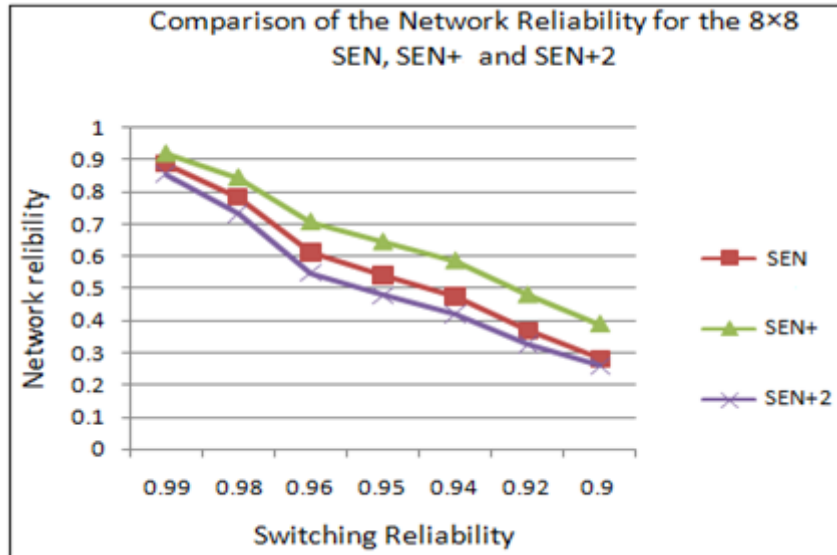


Figure 5.3: Network reliability graph of the 8x8 SEN, SEN+, SEN+2.

## 5.2 Reliability Analysis of Omega and its Variants

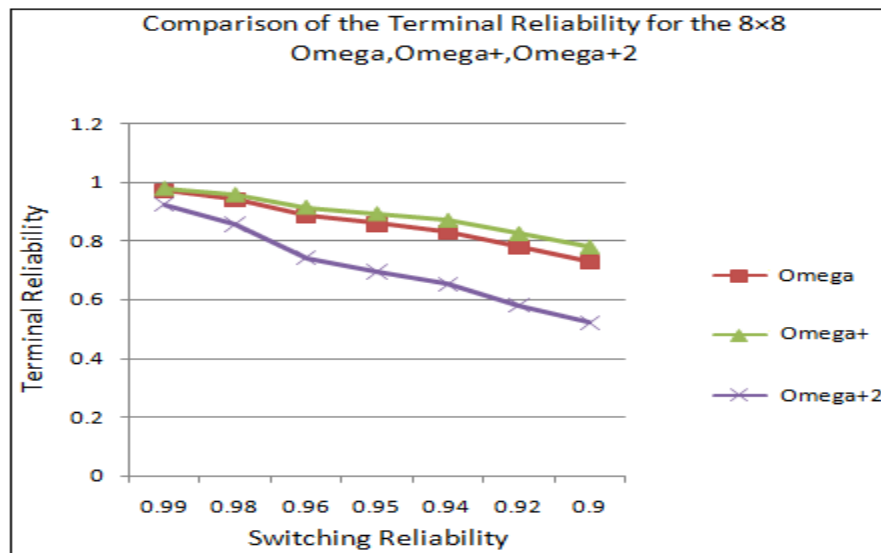
### 5.2.1 Terminal Reliability of Omega, Omega+ and Omega+2

Table 5.4: Comparative Terminal reliability of Omega, Omega+ and Omega+2

Switching Reliability	Terminal Reliability of Omega	Terminal Reliability of Omega+	Terminal Reliability of Omega+2
0.99	0.970299	0.981912	0.920174
0.98	0.941192	0.976894	0.879613
0.96	0.884736	0.917955	0.804785
0.95	0.857375	0.897928	0.764970
0.94	0.830584	0.889160	0.651869
0.92	0.778688	0.845987	0.5969825
0.90	0.729000	0.778547	0.5368793

The data values for terminal reliability of the Omega, Omega+ and Omega+2 networks are presented in Table 5.4. From Figure 5.6 it is clear that the terminal reliability of the Omega+ is the highest whereas terminal reliability of Omega+2 is the

lowest among these three networks. Therefore, that there is not a direct relation between additional paths and increase in the terminal reliability because the additional paths may increase the links complexity of the network, leading to a higher failure. Hence, it can be concluded that adding one additional stage to the Omega is more efficient way to improve terminal reliability rather than two stages.



**Figure 5.4: Terminal reliability graph of the 8x8 Omega, Omega+, Omega+2**

### 5.2.2 Broadcast Reliability of Omega, Omega+ and Omega+2

**Table 5.5: Comparative Broadcast reliability of Omega, Omega+ and Omega+2**

SE Reliability	Broadcast Reliability of Omega	Broadcast Reliability of Omega+	Broadcast Reliability of Omega+2
0.99	0.932065	0.967431	0.897863
0.98	0.868126	0.917946	0.809831
0.96	0.751447	0.863064	0.667692
0.95	0.698337	0.803192	0.610132
0.94	0.648478	0.795960	0.559696
0.92	0.557847	0.679845	0.475794
0.90	0.478297	0.614672	0.408837

The results of the broadcast reliability evaluation are summarized in Table 5.5. From Figure 5.5 it is clear that the broadcast reliability of the network is the highest in Omega+ and the lowest in Omega+2. Although the number of broadcast paths in the Omega+2 is greater than that of the Omega+, the broadcast reliability of the Omega+2 is the lowest among these three networks. Therefore, it can be concluded that based on the broadcast reliability results, Omega+ is the most reliable network among the three networks.

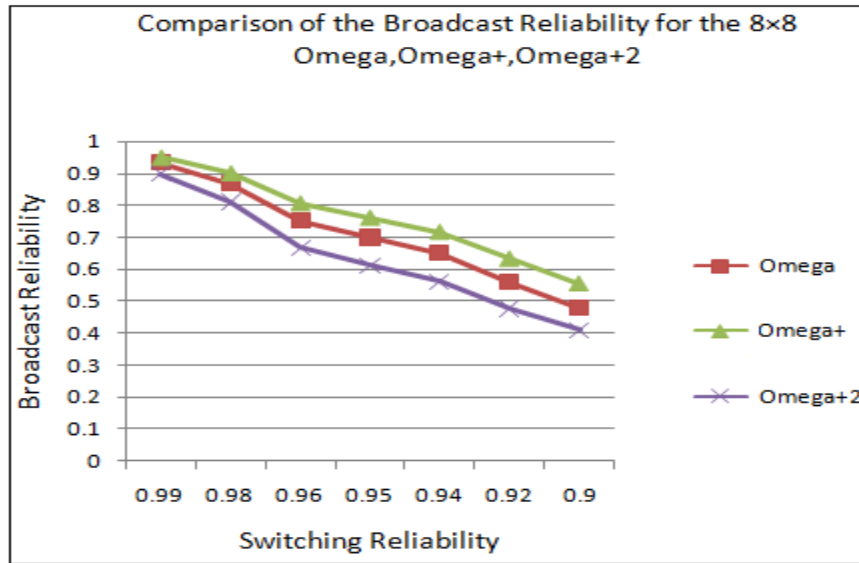


Figure 5.5: Broadcast reliability graph of Omega, Omega+, Omega+2

### 5.2.3 Network Reliability of Omega, Omega+ and Omega+2

Table 5.6: Comparative Network reliability of Omega, Omega+ and Omega+2

SE Reliability	Network Reliability of Omega	Network Reliability of Omega+	Network Reliability of Omega+2
0.99	0.886385	0.941657	0.864754
0.98	0.784717	0.886540	0.756792
0.96	0.612710	0.798653	0.587231
0.95	0.540360	0.724560	0.529061
0.94	0.475920	0.586329	0.457852
0.92	0.367666	0.489506	0.325762
0.90	0.282430	0.425873	0.278923

The comparison of the network reliability evaluation of the Omega, Omega+, and Omega+2 for the 8×8 case is summarized in Table 5.6. From Figure 5.6 it is clear that the 8×8 Omega+ has the highest network reliability whereas the Omega+2 has lowest. This result again proves that adding just one additional stage to the Omega, leads to a Omega+ with the highest network reliability for a Omega.

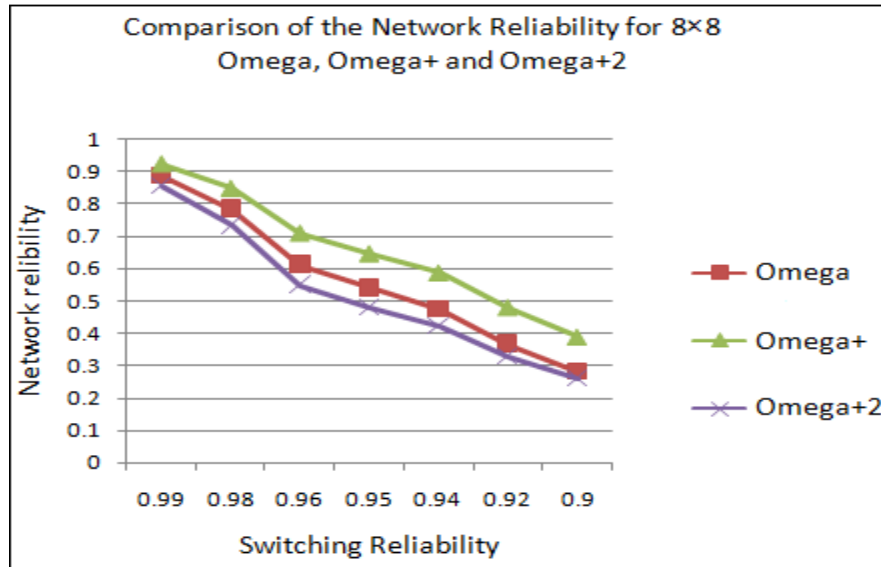


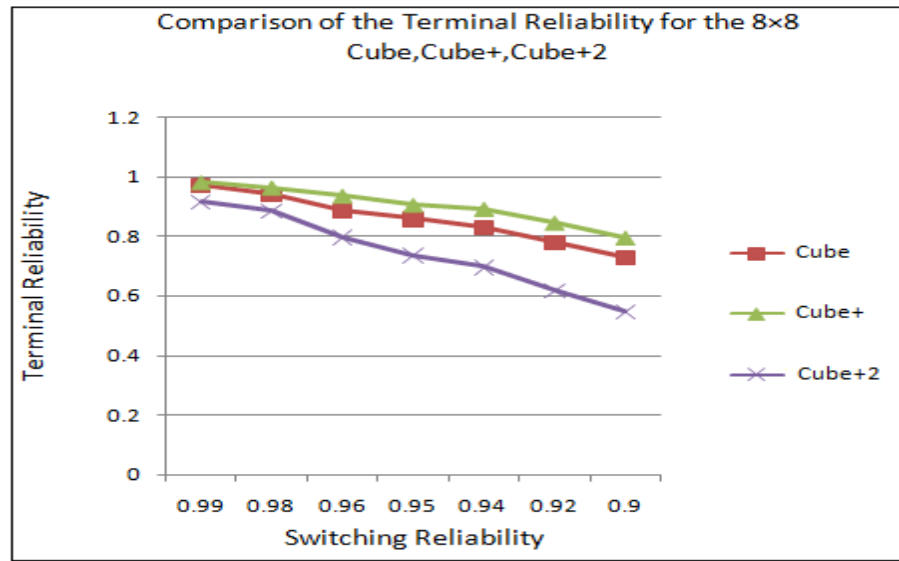
Figure 5.6: Network reliability graph of the 8×8 Omega, Omega+, Omega+2.

### 5.3 Reliability Analysis of Cube and its Variants

#### 5.3.1 Terminal Reliability of Cube, Cube+ and Cube+2

Table 5.7: Comparative Terminal reliability of Cube, Cube+ and Cube+2

Switching Reliability	Terminal Reliability of Cube	Terminal Reliability of Cube+	Terminal Reliability of Cube+2
0.99	0.970299	0.994624	0.9146875
0.98	0.941192	0.987457	0.884962
0.96	0.884736	0.945678	0.796328
0.95	0.857375	0.904567	0.736092
0.94	0.830584	0.891254	0.696488
0.92	0.778688	0.863994	0.619356
0.90	0.729000	0.7933694	0.547899



**Figure 5.7: Terminal reliability graph of the 8×8 Cube, Cube+, Cube+2**

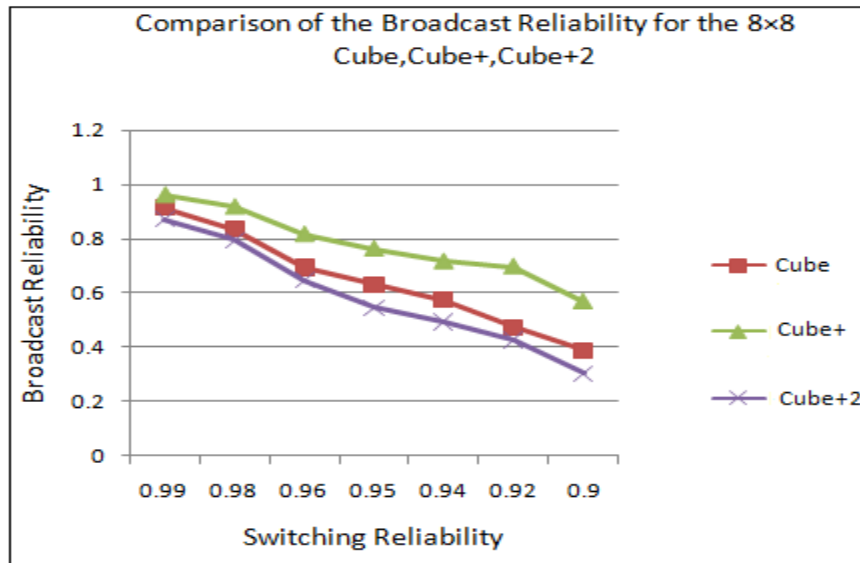
The comparison of the Cube, Cube+, and Cube+2 for the 8×8 networks is presented in Table 5.7. From Figure 5.7 it is clear that the terminal reliability of the Cube+ is the highest and the terminal reliability of Cube+2 is the lowest among these three networks. Therefore, there is not a direct relation between additional paths and the increase in terminal reliability because the additional paths may increase the links complexity of the network, leading to a higher failure. Hence, it can be concluded that adding one additional stage to the Cube is more efficient way to improve terminal reliability rather than an addition of two stages.

### 5.3.2 Broadcast Reliability of Cube, Cube+ and Cube+2

**Table 5.8: Comparative Broadcast reliability of Cube, Cube+ and Cube+2**

SE Reliability	Broadcast Reliability of Cube	Broadcast Reliability of Cube+	Broadcast Reliability of Cube+2
0.99	0.91351725	0.985016	0.869218
0.98	0.83374776	0.956738	0.793560
0.96	0.692534	0.906388	0.647941
0.95	0.6302494	0.882955	0.546218
0.94	0.5729948	0.805873	0.495481

0.92	0.47216136	0.797458	0.427962
0.90	0.38742049	0.689320	0.305962



**Figure 5.8: Broadcast reliability graph of the 8x8 Cube, Cube+, Cube+2**

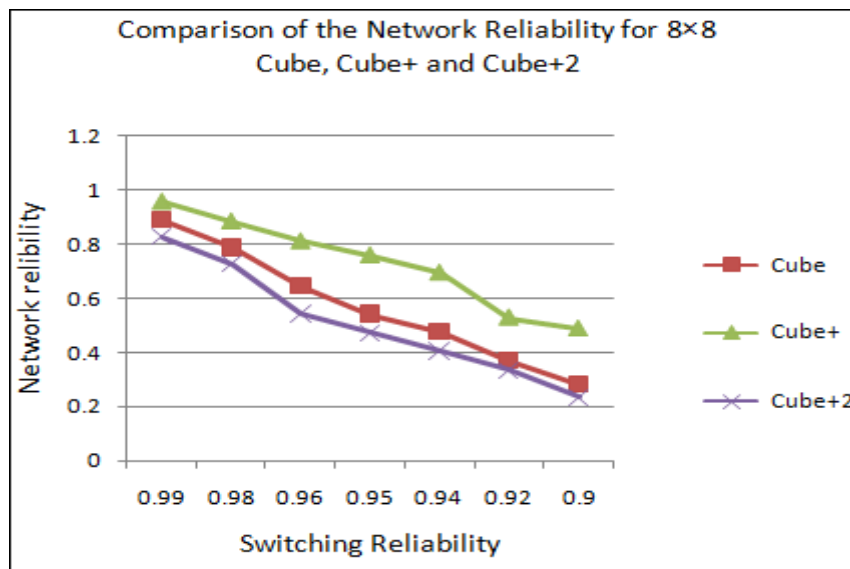
The results of the broadcast reliability evaluation are summarized in Table 5.8. From Figure 5.8 it is clear that the broadcast reliability of the network is the highest in Cube+ and the lowest in Cube+2. Although the number of broadcast paths in the Cube+2 is greater than that of the Cube+, the broadcast reliability of the Cube+2 is the lowest among these three networks. Therefore, it can be concluded that based on the broadcast reliability results, Cube+ is the most reliable network among the three networks.

### 5.2.3 Network Reliability of Cube, Cube+ and Cube+2

**Table 5.9: Comparative Network reliability of Cube, Cube+ and Cube+2**

SE Reliability	Network Reliability of Cube	Network Reliability of Cube+	Network Reliability of Cube+2
0.99	0.886385	0.979124	0.824581
0.98	0.784717	0.914384	0.72684
0.96	0.612710	0.870492	0.543879

0.95	0.540360	0.799125	0.471957
0.94	0.475920	0.73485	0.40579
0.92	0.367666	0.6488159	0.336781
0.90	0.282430	0.498215	0.237438

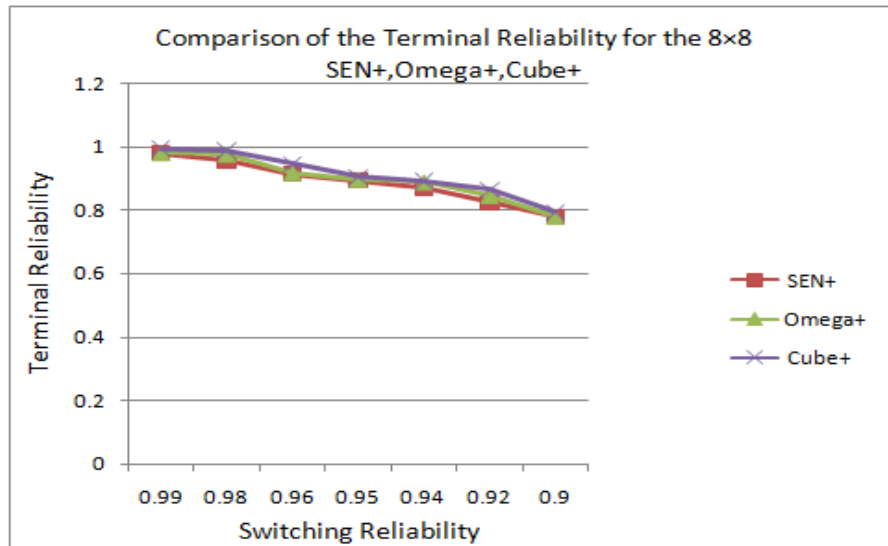


**Figure 5.9: Network reliability graph of the 8×8 Cube, Cube+, Cube+2.**

The comparison of the network reliability evaluation of the Cube, Cube+, and Cube+2 for the 8×8 case is summarized in Table 5.9. From Figure 5.9 it is clear that the Cube+ has the highest network reliability and the Cube+2 has the lowest network reliability. This result again proves that adding just one additional stage to the Cube, leads to a Cube+ with the highest network reliability for a Cube.

## 5.4 Comparative Reliability Analysis of SEN+, Omega+ and Cube+

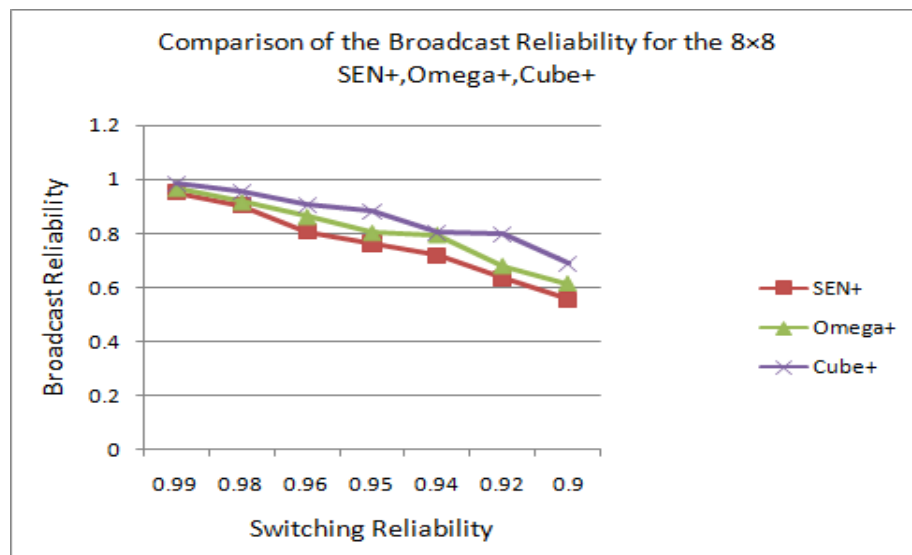
### 5.4.1 Terminal reliability



**Figure 5.10: Terminal reliability graph of the 8x8 SEN+, Omega+ and Cube+.**

The comparison of the SEN+, Omega+, and Cube+ for the 8x8 networks is presented in Figure 5.10. It is clear that the terminal reliability of the Cube+ is the highest whereas terminal reliability of SEN+ is the lowest among these three networks.

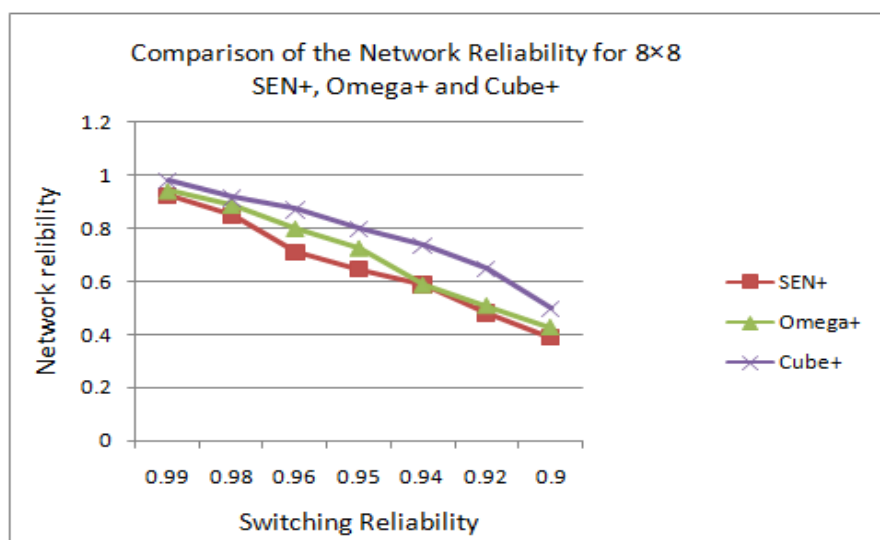
#### 5.4.2 Broadcast reliability



**Figure 5.11: Broadcast reliability graph of the 8x8 SEN+, Omega+ and Cube+.**

The comparison of broadcast reliability graph among these three networks is illustrated by Figure 5.11. It is clear that the broadcast reliability of the  $8 \times 8$  network is the highest in Cube+ and the lowest in SEN+. Therefore, it can be concluded that based on the broadcast reliability results, Cube+ is the most reliable network among these three networks.

### 5.4.3 Network reliability



**Figure 5.12: Network reliability graph of the  $8 \times 8$  SEN+, Omega+ and Cube+.**

From Figure 5.12 it is clear that the  $8 \times 8$  Cube+ has the highest network reliability and the SEN+ has the lowest network reliability. This result again proves that adding just one additional stage to the Cube, leads to a Cube+ with the highest network reliability for a Cube.

#### 6.1 Conclusions

In this thesis Reliability analysis of three regular multistage interconnection networks namely SEN, Omega and Cube has been done. With the addition of one and two extra stages more regular MINs namely SEN+, Omega+, Cube+ and SEN+2, Omega+2, Cube+2 are derived from SEN, Omega and Cube respectively. As measures of network performance, the terminal, broadcast and network reliability of all three networks have been evaluated. From the reliability analysis following conclusions have been made.

- (1) Addition of one stage to any of SEN, Omega or Cube networks provide higher reliability in terms of terminal, broadcast and network, than the addition of two stages in the corresponding network.
- (2) Further comparative analysis among SEN+, Cube+ and Omega+ has shown that Cube+ is most reliable than SEN+ and Omega+.

#### 6.2 Future Scope

The field of regular networks can be further explored in the light of the following suggestions:

A comparative analysis of other regular multistage interconnection networks can be carried out with respect to the other parameters. The designing of more regular networks having better reliability and permutation passibility can be explored. The study can be extended to optical MINs.

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## List of Publications

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1. Sushila Baloda, Rinkle Rani Aggarwal, “Reliability Analysis of Regular Multistage Interconnection Networks with Extra Stages”, **Communicated** in International Journal of Computer Theory and Engineering, International Association of Computer Science and Information Technology (IACSIT), Singapore, July 2010.