Thermal-Structural Analysis of a Functionally Graded Rotating Disc with Variable Thickness

A

Dissertation

Submitted in partial fulfilment of the requirement for the award of degree of

MASTER OF ENGINEERING
IN
CAD/CAM ENGINEERING

Submitted

By

Pardeep Kumar

(Roll No. 801281015)

Under Supervision of

Mr. Kishore Khanna

Assistant Professor

Mechanical Engineering Department

MECHANIAL ENGINEERING DEPARTMENT
THAPAR UNIVERSITY
PATIALA-147004, PUNJAB

July 2014

CERTIFICATE

This is to certify that the work in this dissertation report entitled **Thermal-Structural Analysis of a Functionally Graded Rotating Disc with Variable Thickness**, submitted in partial fulfillment of requirement for the award of **Master of Engineering Degree** in **CAD/CAM Engineering** in Mechanical Engineering Department of Thapar University, Patiala, is an authentic record of work carried out by me under the guidance of **Mr. Kishore Khanna**, Assistant Professor, Mechanical Engineering Department, Thapar University, Patiala.

The matter embodied in this report has not been submitted in part or full to any other university or institute for the award of any degree.

Pardeep Kumar

Dated:. 18/07/2014

Pardeep Kumar

This is to certify that the above declaration made by the student concerned is correct to the best of my knowledge and belief.

(Mr. Kishore Khanna)
Assistant Professor
Mechanical Engineering Department
Thapar University, Patiala.

(Dr. Ajay Batish)
Professor Head, MED
Thapar University, Patiala.

(Dr. S.K. Mohapatra)
Dean, Academic Affairs
Thapar University, Patiala.

Acknowledgement

I express my sincere gratitude to my guide **Mr. Kishore Khanna**, Assistant Professor Mechanical Engineering Department, Thapar University, Patiala, for his invaluable guidance, proper advice and constant encouragement during the course of my dissertation work.

I would also express my gratitude to **Dr. Ajay Batish**, Head, Mechanical Engineering Department for his inspirational guidance and whole hearted corporation which has helped me to submit this dissertation.

I thank the entire faculty and staff of Mechanical Engineering Department, Thapar University, for their help and moral support.

PARDEEP KUMAR

Abstract

In the present thesis work, the thermal-structural analysis is performed to determine the stresses, strains and displacement in an FGM disc brake with variable thickness and by varying the speed. For the purpose of the investigation, four discs, made of composite containing silicon carbide particles (SiC_p) in a matrix of pure aluminium, with varying thickness along the radial distance are considered. The uniform composite disc in which the value of stresses, strains and displacement are minimum, further investigated with varying particle content along the radial distance. And then disc made of constant thickness and uniform composite by varying speed is also analyzed. The material of the disc is assumed to be orthotropic.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Introduction of Disc Brake

A disc brake is a device which decreases rotational velocity of the wheel by the friction produced by pressing brake pads in contradiction of a brake disc with a set of calipers. The brake disc is commonly fabricated of cast iron, in some cases it made of composites. Disc brake is coupled to the wheel and/or the axle. Brake pads are made of frictional material which is connected with a device called a brake caliper. Brake pads are pushed towards each other mechanically, hydraulically, pneumatically or electromagnetically. When disc and brake pads make contact with each other which slows down rotation of the wheel. Brakes transform motion to heat, and if the brakes do not dissipate heat, they become less effective, and the phenomenon is called as brake fade [33].

1.2 Brake Disc

The disc is a part of disc brake system which is connected with wheel of the vehicles. It is made of grey iron, which is one form of cast iron. The disc brake has different types of designs. Some are hollow join with fins or vanes, some are purely solid and some discs with holes. Due to more power and heavy weight of the vehicles these days ventilated disc works good in vehicles. Ventilated disc brakes are hollow with a plate on each side. The air enter the center to cool the plates. The ventilated disc easily helps to dissipate the generated heat into the air of all vehicles those are heavily loaded front disc [36].

Disc brakes with holes drilled through disc provide high performance to vehicles. This is famous as cross-drilling and was firstly finished in the 1960s on racing cars. Now a days disc with cross drilled holes are used on some braking component, but it is not used in racing vehicles due to cracks produces under severe conditions [36].

Slots are provided to disc to remove dust and gas by machining into the disc. Slotting is the ideal technique in most racing environments to eliminate gas and water and to deglaze brake pads. But some discs prefer both drilling and slotting. Due to fast wear of brake pads, slotted disc are commonly not used on standard vehicles. This elimination of material is useful to race automobiles since it retains the pads soft and avoids vitrification of their faces. To avoiding thermal stress, cracking and warping, sometimes disc attached in a half loose way to hub with coarse fins. Due to this disc can expand in a controlled symmetrical way and less undesirable heat transfer to the hub [36].

Drilled or slotted disc work effectively in wet situations because the holes or slots avoid a film of water building up between the pads and the disc. Cracks may produce in cross-drilled disc at the position of holes due to metal fatigue [36].

1.2.1 Classification of brakes

There are two main classes of brakes according to the direction of acting force [35]:

(i) Radial brake
(ii) Axial brake

(i) Radial brake: According to its name the force acts in radial direction to the drum as shown in figure 1.1. Further radial brakes are of two types: external brakes and internal brakes [35].

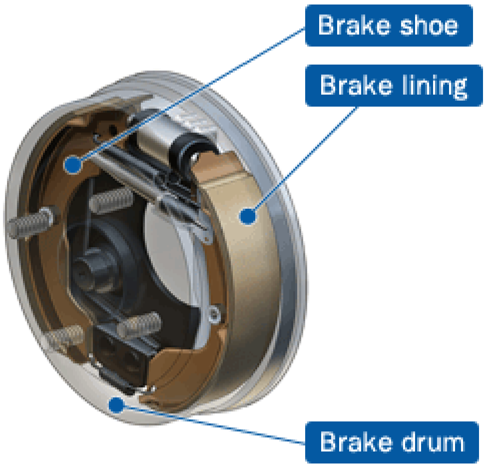


Figure 1.1: Radial brake (Drum brake) [37].

(ii) Axial brake: In these brakes the force acting on the brake is only in the axial direction as shown figure 1.2. e.g. disk brakes, cone brakes [35].

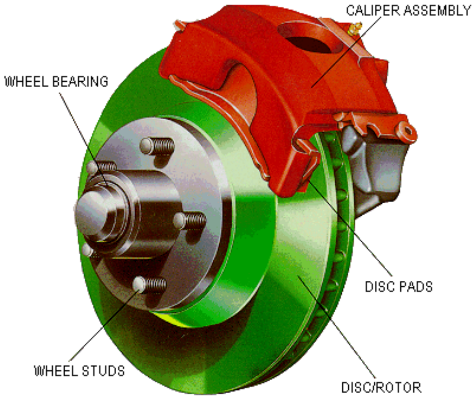


Figure 1.2: Axial Brake (Disc brake) [38]

1.2.2 Disc brake

A cast iron disk brake connected with wheel hub through bolt and a stationary housing called caliper. The caliper is joined to some fixed part of the vehicle similar to the axle casing or the stub axle as is cast in two parts each part having a piston. A friction pad is coupled with piston and friction pad make direct contact with disc. Piston also connected with retaining pins and spring. Each housing got drilled passage in the caliper for fluid to enter and leave. The passages are also attached to extra one for bleeding. Each cylinder holds rubber-sealing ring between the cylinder and piston as shown in the figure 1.3 [33].

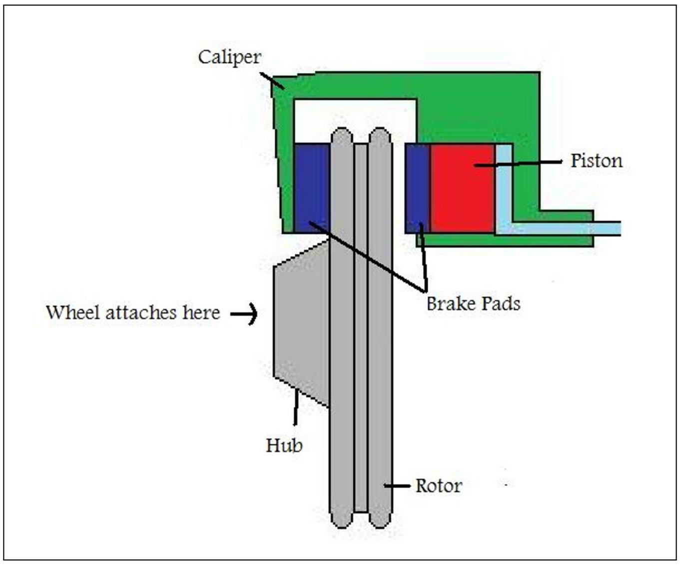


Figure 1.3: Disc brake [39]

The main components of the disc brake are [33]:

1. Brake pads
2. Caliper
3. Rotor

When force is applied to brakes, hydraulically actuated pistons move the pads to make contact with the rotating disc, by putting equal and opposite force. Because of friction produced between disc and friction pads, all the kinetic energy of the vehicle is transformed into heat and vehicle stops after some distance. After releasing the brakes the friction pad which is connected to hydraulically actuated pistons come its original position with the help of rubber-sealing ring [33].

Swinging caliper disc brake

There are two frictional pads in brake system. One is fixed to the caliper and other moves with the help of piston and rubber-sealing ring. This pad is pressed by pressure fluid when brake applied. As equal pressure applied on both pads result friction produced which helps to stop the vehicle. After the caliper move to its original position by swinging about the pin [33].

Sliding caliper disk brake

In this type of disc brake there are two piston, one piston is directly pressed by pressure fluid whereas the other pad pressed indirectly with the caliper [33].

1.3 Disc Damage Mode

There are three damage modes of disc brakes: scarring, cracking, warping or excessive rusting. Sometime service shops changed out the discs entirely for little problem in disc brake where the prize of a new disc may be lower than the cost of the labour to maintain the old disc. Mechanically this is needless except the discs have reached manufacturer's minimum suggested thickness, which would create it risky to use them, or fin rusting is severe (ventilated discs only). Best vehicle manufacturers prefer brake disc skimming (US: turning) as a result for lateral run-out, vibration topics and brake noises. The machining method is completed in a brake lathe, which eliminates a very thin layer off the disc surface to clean off minor loss and bring back uniform thickness. Machining the disc as compulsory will maximize the mileage out of the present discs on the vehicle [34].

1.3.1 Scarring

When brake pads are not changed punctually and they reach the end of their service life then scarring (US: Scoring) can occur as shown in Figure 1.4. The pad retainer rivets or the pad's steel backing plate (for glued pads) bear directly the disc's wear surface because the frictional material is reduced by braking. It putting scratches on the disc and reduced breaking power. Normally a moderately scarred / scored disc, which worked suitably with existing brake pads, will be similarly usable with new pads. When the scarring is deeper but not extreme, it can be fixed by removing a layer of the disc's surface. But only can be done a less number of time because as the disc has a minimum valued safe thickness. The minimum thickness value is normally cast into the disc during developing on the hub or the edge of the disc [34].

Figure 1.4: Scarring in disc brake [40]

1.3.2 Cracking

Cracking is produced commonly in drilled disc as shown in figure 1.5. Small cracks form around the drilled holed near the corner of the disc because of the discś uneven rate of expansion in hard environment. Here are two reason for making the disc with drilled holes one is that average owner of the vehicles will prefer the good look of the vehicles. The second is that due to drilled holes in disc reduce the unsprang weight of the brake assembly.

There is one engineering assumption saying that extra mass of the disc brake absorb racing temperatures and stresses . A brake disc is a heat sink, but the loss of heat sink mass may be balanced by increased surface area to radiate away heat. It is known that the disc brake is a heat sink, due to the loss of heat sink mass the surface area of disc is increased. Because of that Small hairline cracks may produce in any cross drilled metal disc and disc will fail catastrophically. Any repair is not possible for the cracks, and if cracking becomes deep, the disc must be changed. These cracks produce due to the occurrence of low cycle fatigue as a result of repeated hard braking [34].

Figure 1.5: Cracking in disc brake [41]

1.3.3 Rusting

A certain amount of surface rust is normal in disc because the discs are normally made from cast iron as shown in figure 1.6. Which vehicles use regularly they got very clean brake pads and disc, but a vehicle that stored for long period can develop rust in the touching area. It reduce the braking power of the vehicle until the rusted layer is not removed. Now days vented brake discs may invented severe rust corrosion inside the airing slots, compromising the power of the structure and needing replacement [34].

Figure 1.6: Rusting in disc brake [42]

1.4 Composite Materials

Composite materials are the combination of two or more constituent materials which have different chemical and physical properties and after combination produce a material with different characteristics. Composite materials are made in the laboratory and change their strength from the mixture of materials except from the interlocking of a uniform set of atoms. The composite material has the number of characteristic properties like low weight, high temperature performance, high stiffness, high strength, high hardness and conductivity and good corrosion resistance. These all properties are not possible for a monolithic material [2].

The properties are dependent on the following parameters [2] :

(i) Properties of the individual constituents.
(ii) Relative amounts of the constituents.
(iii) Size and shape of the constituents (i.e. Morphology).
(iv) Degree of bonding between constituents.
(v) Orientation of the various constituents.

1.4.1 Classification of composite materials

Composites are classified on the basis of geometry of the reinforcement and the type of matrix as shown in figure 1.7 [1].

Classification Based On Reinforcement Geometry :- According to the reinforcement geometry, composites are classified into three groups namely [1]:

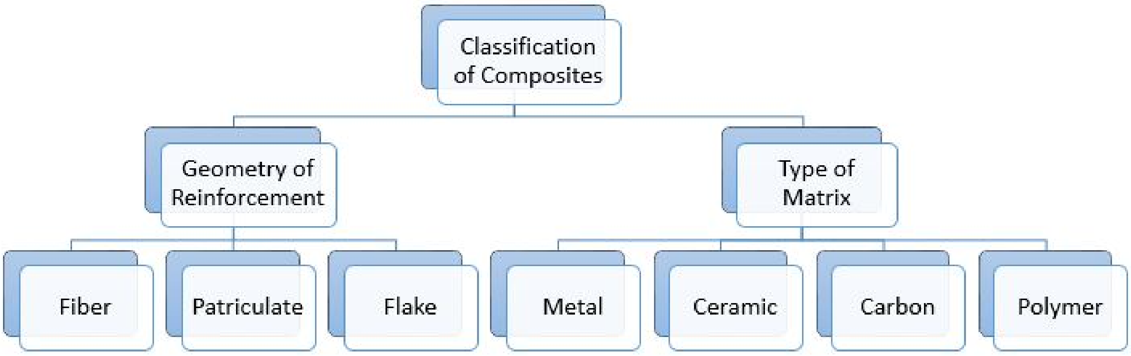


Figure 1.7: Classification of composite materials

(i) Particulate composites

(ii) Flake composites

(iii) Fiber composite

(i) Particulate composites: Particulate composite are strengthened by matrices which are composed of alloy particles and ceramics particles as shown in figure 1.8. There are some advantages of Particulate composite such as oxidation resistance, improved strength and increased operating temperature [1].

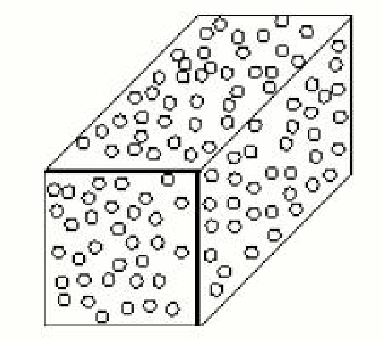


Figure 1.8: Particulate composite [43]

Examples - silicon particles in aluminum matrix, aluminum particles in rubber matrix

(ii) Flake composites: Flake composites are made of flake shaped reinforcement such as silica, silver, glass, mica etc as shown in figure1.9. Flake composites have advantages like higher strength and low cost [1].

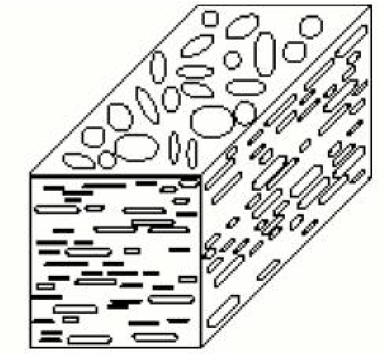
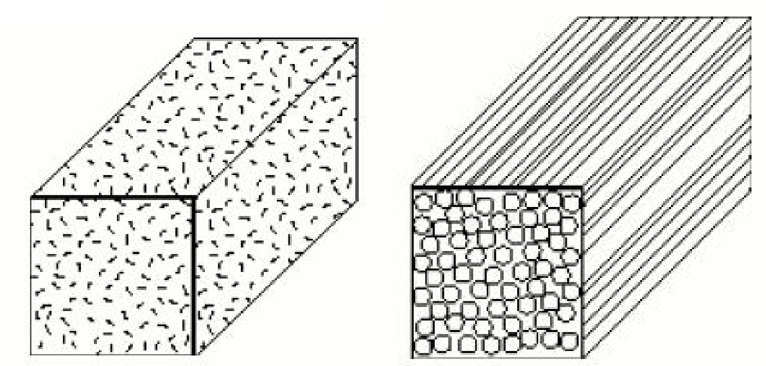


Figure 1.9: Flake composite [43]

(iii) Fiber composites: Fiber composites are made of short (discontinuous) or long (continuous) fibers as shown in figure.1.10 (a,b). Examples - resins epoxy, aluminum and ceramics, calcium-alimino silicate [1].



(a) (b)

Figure 1.10: (a) Short fiber composite (b) Long fiber composite [43]

1.4.2 Classification bassed on matrix

Based on the type of matrix, Composites may be classified as [2]:

(i) Polymer matrix composite (PMC)
(ii) Metal matrix composite (MMC)
(iii) Ceramic matrix composite (CMC)
(iv) Carbon-carbon composite

(i) Polymer matrix composite (PMC): These are the most common advanced composites consisting of polymer (epoxy, polyester, urethane etc) reinforced by thin diameter fibers (graphite, boron etc). Graphite/Epoxy composites are approximately five times stronger than steel on a weight for weight basis.They are commonly employed due to their low cost, high strength and simple manufacturing. Main drawbacks of polymer Laminate Composites (PMCs) include low operating temperature, high coefficient of thermal and moisture expansion and low elastic properties in certain directions. However, their advantages are high strength, low cost, high chemical resistance and good insulating property [2].

(ii) Metal matrix composite (MMC): It consists of metal matrix such as aluminum, magnesium and titanium etc. reinforced with fibers such as carbon, silicon carbide etc. Metals are reinforced to increase or decrease their properties to suit the design needs. For example, the elastic stiffness and strength of metals can be increased, while large coefficients of thermal expansion and thermal and electric conductivities of metal can be reduced by addition of fibers such as silicon car bid e. MMCs are mainly used to provide advantages over monolithic alloy/metals such as steel and aluminum. These advantages include higher specific strength and specific modulus achieved by incorporation of reinforcement in low density metal matrix such as aluminum and titanium. MMCs possesses several advantages over polymer matrix composite such as higher elastic properties, higher service temperature, insensitive to moisture, higher electric and thermal conductivity, better fatigue and flow resistance. However, the drawbacks of MMCs over PMCs include higher processing temperature and higher densities [2].

iii) Ceramic matrix composite (CMC): Ceramic matrix composites (CMCs) have a ceramic matrix such as alumina, calciumalumino-silicate reinforced by fibers such as carbon or silicon car bide. Their main advantages include high strength and hardness, high service temperature, chemical inertness and low density. CMCs are finding extensive applications in high temperature areas where MMCs and PMC s cannot be used [2].

iv) Carbon-Carbon Composite: Carbon-carbon composites have carbon fiber reinforced in matrix of carbon. Carboncarbon composites are used in very high temperature environments up to 6000^oF (3315^oC) and are 20 times stronger and 30% lighter than graphite fibers. Their advantages include ability to withstand high temperatures, low creep at high temperature, low density, good tensile and compressive strength, high fatigue resistance, high thermal conductivity and high coefficient of friction. Their disadvantages include high cost, low shear strength and susceptibility to oxidations at high temperature. These composites find application in space shuttle nose cone, aircraft brakes, mechanical fasteners etc [2].

1.4.3 Parameters for selection of composites

For selecting a composite material for a particular application, the following parameters are to be considered [1]:

(i) Strength
(ii) Toughness
(iii) Formadibility
(iv) Weldeability
(v) Corrosion
(vi) Wear Resistance
(vii) Affordability

1.5 Functionally Graded Material

Functionally graded materials (FGMs) are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs possess a number of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors. A number of reviews dealing with various aspects of FGM have been published in recent years. Proceedings of the international symposiums on FGM also shed light on the most recent research in these materials, their manufacturing, mechanics, thermal properties, and applications [3].

At present, FGMs are usually associated with particulate composites where the volume fraction of particles varies in one or several directions. One of the advantages of a monotonous variation of volume fraction of constituent phases is the elimination of stress discontinuity that is often encountered in laminated composites and accordingly, avoiding delamination-related problems. FGM may also be developed using fiber-reinforced layers with a volume fraction of fibers that is coordinate dependent, rather than constant, producing the optimal set of properties or response [3, 4].

1.5.1 Types of FGM

(i) Thin FGM

(ii) Bulk FGM

(i) Thin FGM: Thin FGM are relatively thin sections or thin surface coating. This type of FGM is made by Plasma Spraying, Self-propagating High temperature Synthesis (SHS), Physical or Chemical Vapour Deposition (PVD/CVD) [5].

(ii) Bulk FGM: The bulk Functionally Graded Material are volume of materials which need more labour intensive processes. Bulk FGM is manufactured using solid freeform technology, centrifugal casting method, powder metallurgy technique [5].

1.5.2 Areas of application of FGM

Some applications of functionally graded materials are listed below [5]:

(a) **Aerospace**
Examples :- Space plane body, rocket engine component etc.

(b) **Medicine**
Examples :- Artificial teeth and bones

(c) **Defense**
Examples :- Armour plates and bullet-proof vests

(d) **Energy**
Examples :- Protective coating on turbine blades in gas turbine engine

(e) **Optoelectronics**
Examples :- Graded refractive index materials and in audio-video discs magnetic storage media

Chapter 2

Literature Review

Finite Element Method is a very versatile tool which can be used for analysis of composites and FGM. Number of researchers has used this tool for the analysis of different composites and FGM. A brief review of such work is discussed in the following paragraphs.

You *et al.* [6] determined the stresses and deformations in elastic plastic rotating disc with new created numerical method by changing thickness and density. The governing equation was solved by Runga Kutta method. Stress function was used to develop the governing equation from equilibrium equation. Finite element method was used to compare the result computed by existing technique.

Eraslan and Orcan [7] investigated elastic plastic deformation of an exponentially varying thickness of disc. The investigation was carried out using Tresca yield criteria. It was analyzed that when the thickness of disc decrease with increasing the value of the parameter n then the angular velocity also increased. By allowing elastic plastic deformation and by choosing appropriate choice of the thickness parameter efficient and economic design is achieved.

Singh and Ray [8] carried out a study on the creep behavior of a rotating disc made of isotropic functionally graded materials. The disc under investigation was made of a composite containing silicon carbide particles in a matrix of pure aluminium. The steady state creep behavior had been described by Norton's law. It was concluded that in an isotropic rotating FGM disc with linearly decreasing particle content from the inner to the outer radius, the steady-state creep response is significantly superior compared to that in a disc with the same particle content distributed uniformly.

Gupta *et al.* [11] examined the steady state creep in a rotating disc made of isotropic aluminium silicon carbide particulate composite. The creep behavior of the composite had been described by Sherby's constitutive model. The creep parameters in the law had been determined using regression equations developed on the basis of available experimental results. The radial and tangential stresses and steady state creep rates in the disc had been calculated and presented for various combinations of material parameters (like particle size and particle content) and temperatures. The study revealed that for given operating conditions, the strain rates in the disc can be controlled by selecting optimum particle content and/or particle size of the reinforcement.

Gupta *et al.* [12] understood the creep behavior of a rotating disc made of isotropic composite containing varying amounts of silicon carbide in the radial direction in the presence of a thermal gradient. The thermal gradient experienced by the disc was the result of braking action as estimated by FEM analysis. The creep behavior of the disc under stresses developing due to rotation had been calculated following Sherby's law and related with that of a similar disc following Norton's law. The presence of thermal gradient and a linear particle gradient separately or their simultaneous presence result in a significant decrease in steady state creep rates as compared to that in a composite disc with the same average particle content (20 vol %) distributed uniformly and operating under isothermal condition.

Calliouglu *et al.* [13] analytically investigated the elastic-plastic stress analysis of a curvilinearly orthotropic rotating annular disc for strain-hardening material behavior. To be able to see the separation of the plastic region, a few angular velocities were taken into consideration for such an analysis. Radial and circumferential stress components were obtained to increase angular velocity. It was seen that the magnitudes of the circumferential stress components were higher than those of the radial stress components. The magnitudes of the residual stress component of the circumferential stress and plastic flow were the highest at the inner surface. The radial displacements in both the elastic and plastic solutions calculated analytically had higher values at the inner surface than those of the outer surface for all the angular velocities.

You *et al.* [14] determined steady-state creep of thick-walled cylindrical vessels made of functionally graded materials subjected to internal pressure. Taking material parameters involved in Norton's law to be the functions of the radial coordinate, a simple and accurate method was developed from the strain rate-stress relations, Norton's law, deformation compatibility condition and equilibrium equation of axisymmetric, plane strain problems. The proposed approach was employed to calculate stresses and creep strain rates in the thick-walled cylindrical vessels. How variations of material parameters along the radial direction affect the stresses in the vessels was examined.

Singh [15] examined steady state creep in a rotating disc of anisotropic aluminum silicon carbide whisker composite. The creep behavior was described by Norton's power law. Stress and strain rate distributions for anisotropic discs had been calculated and compared with those obtained for isotropic disc. It was concluded that the radial strain rate which always remained compressive for the isotropic composite (a = 1.0) and anisotropic disc (a = 1.3), became tensile in the middle region of the disc when the anisotropy parameter a = 0.7. Also if it was reduced from 1.3 to 0.7 the variation of tensile strain rate in the tangential direction remained similar but the magnitude reduces by five orders of magnitude. The study revealed that anisotropy introduced significant change in the strain rates although its effect on the resulting stress distribution may be relatively small.

Bayat *et al.* [16] presented elastic solutions for axisymmetric rotating disks made of functionally graded material with variable thickness. The material properties and disk thickness profile were assumed to be represented by two power-law distributions. For hollow disk, based on the form of the power-law distribution for the mechanical properties of the constituent components and the thickness profile function, both analytical and semi-analytical solutions were given under free-free and fixed-free boundary conditions. Only semi-analytical solution was presented for the solid disk. The effects of the material grading index and the geometry of the disk on the stresses and displacements were investigated. It was found that a functionally graded rotating disk with parabolic or hyperbolic convergent thickness profile has smaller stresses and displacements compared with that of uniform thickness. It was observed for the solid functionally graded disk with parabolic thickness profile that the maximum radial stress is not at the centre like uniform thickness disk. Results of this paper suggested that a rotating functionally graded disk with parabolic concave or hyperbolic convergent thickness profile could be more efficient than the one with uniform thickness.

Hojjati and Hassani [17] used theoretical and numerical methods for stress-strain analysis of rotating discs with non-uniform thickness and density. An elastic-linear hardening material was assumed. For theoretical solution a technique called variable material properties (VMP) theory was employed. A numerical solution was based on solving the governing differential equation using Runge-Kutta's method for elastic and plastic regimes. Finite element modelling of the problem had also been carried out using commercially available software. The results of the three methods were presented and compared and generally showed good agreement. The suggested VMP method provided reliable means for complex discs for which there were no exact solutions. Although a specific disc profile had been studied here, the VMP method showed no restriction for the solution of any other disc profiles.

Pankaj [18] analyzed elastic plastic transitional stresses in an isotropic disc under internal pressure by using Seth's transition theory. Disc was made of compressible material and having variable thickness. It was concluded that disc made of compressible material with variable thickness, yields at a higher pressure as compared to disc made of compressible material. A flat disc made of incompressible material yields at internal surface at higher pressure as compared to disc made of compressible material. Circumferential stress was maximum at the outer surface of the disc having variable thickness.

Zenkour [19] considered two composite structures of functionally graded material (FGM) solid discs. An accurate analytical solution presented for the rotating structures subjected to different boundary condition at the outer surface of the disk. The composite structures were composed of three-layer sandwich solid disks with faces made of different isotropic materials and core made of FGM. The stresses and displacement distributions was smooth through the radial direction of the composite disks. The circumferential stress for some structures of the clamped and free FG disks had an interior maximum.

Afsar and Go [21] performed the finite element analysis of thermoelastic field in a thin circular functionally graded material (FGM) disk subjected to a thermal load and an inertia force due to rotation of the disk. Due to symmetry, the FGM disk was assumed to have exponential variation of material properties in radial direction only. Based on the two dimensional thermoelastic theories, the axisymmetric problem was formulated in terms of a second order ordinary differential equation which was solved by finite element method. The thermoelastic field in an FGM disk is significantly was influenced by the temperature distribution profile, radial thickness, angular speed, and inner and outer surface temperature difference. Thus, the thermoelastic field in an FGM disk can be controlled and optimized by controlling these parameters.

Deepak *et al.* [22] investigated the creep behavior of rotating discs made of functionally graded materials with linearly varying thickness. The discs under investigation were made of composite containing silicon carbide particles in a matrix of pure aluminum. The creep behavior of the composite had been described by threshold stress based creep law by assuming a stress exponent of 5. The study was indicated that with increase in particle gradient in the disc, the radial stress increases throughout the disc, whereas the tangential and effective stresses increase near the inner radius but decrease near the outer radius. The steady state strain rates in the composite disc, having gradient in the distribution of reinforcement, were significantly lower than that observed in a disc having uniform distribution of reinforcement.

Loghman *et al.* [23] analyzed the time-dependent creep stress of rotating disk made of Al-SiC composite using Mendelson's method of successive elastic solution. All mechanical and thermal properties except Poisson's ratio were radial dependent based on volume fraction percent of SiC reinforcement. The material creep behavior was defined by Sherby's constitutive model using Pandey's experimental results on Al-SiC composite. Using equations of equilibrium, stress strain, and strain displacement, a differential equation, containing creep strains, for displacement was found. It was concluded that the uniform distribution of SiC reinforcement does not considerably influence on stresses. However, the minimum and most uniform distribution of circumferential and effective thermoelastic stresses belongs to composite disk of aluminum with 0% SiC at inner surface and 40% SiC at outer surface.

Sharma *et al.* [24] examined a functionally graded thermo elastic disc. Finite element method (FEM) is used to find stresses and strains. Conclusion is came from this paper that the stress, strain and displacement of FGM circular disc for fixed angular velocity get significantly changed due to constant temperature variation, logarithmic thermal variations and non-heat isentropic conditions. By controlling thermal variations, radial thickness and temperature difference at inner and outer surface of the disc could be modeled and improved the field in FGM disc.

Ali and Mostefa [26] analyzed the thermo-mechanical behavior of the dry contact between the brake disc and pads during the braking period. The modeling is based on the ANSYS 11.0. This paper shows more effective cooling is done in ventilated disc. When thermal stresses act with mechanical stress which increase total displacement of the disc, Von Mises stress and contact pressures of the pads which result crack produce and fracture of the bowl and wear of the disc and pads. To calculate the contact pressure distribution in pads, the stress produced in the disc and the displacement, thermal-structural analysis in FEA (Ansys) is used.

Collignon *et al.* [27] studied premature failure which act as main problem in the life of the truck disc brake. The main aim of this study was to increase the lifecycle of the disc brake of the truck. Due to this the safety and reliability of these components upgraded. Two aspects of disc brake design and material were studied in this paper. The behavior of a grey cast iron brake disc joined with two semi-metallic pads under extreme situations studied to know the different damage modes. The hoop stress-hoop strain cycle was used to calculate the temperature evolution during braking. The radial micro cracks were produced in disc brake when residual hoop tensile stresses appeared during cooling. These all the calculations done with a coupled numerical experimental thermo- mechanical approach. Finally, microstructural studies of a bench-tested disc used to validate the modelling results. Radial micro cracks and metallurgical phase transformations were detected.

Hosseini and Livani [28] research was regarding thermoelastic creep behavior of the FGM disks with variable thickness is done in this paper. The governing differential equations (Rate type) are developed and mathematically solved in the form of rate of strain. The thermal and structural properties of the base metal are also measured as a function of temperature. The small deformation theory has been used in this paper.

The clamp-free boundary condition used to analyze the creep behavior of the disc. The thermomechanical properties through the radius direction of disc is required to attain more accurate results throughout the creep behavior analysis of FG rotating disks. An FG disk with variable thickness displays a more appropriate creep behavior in comparison. Thermal loading value has a significant influence on creep rates of FG rotating disk.

Shelar *et al.* [29] had analyzed the thermal convection using an analytical method, and the velocity distribution, temperature contours and Nusselt's number. The heat dissipation and thermal performance of ventilated brake discs strongly depends on the aerodynamic characteristics of the air flow through the rotor passages. Then numerical models for different rotors, pillar post rotors and vane rotors were produced and numerical simulations were conducted to determine the wanted parameters. The heat flux rate, air flow rate, velocity distributions, temperature contours, and pressure distributions were carried out with the help of Fluent and Gambit.

The simulation results were indicated that an increase of vane numbers drastically improves the thermal performance by 63.5%, if the vane numbers increase from 32 to 60 at an angular velocity of 44 rad/s, by 67.9% at an angular velocity of 88 rad/s, and 69.2% at an angular velocity of 120 rad/s.

Bayat *et al.* [30] piece of work was related to the magneto-thermo-mechanical behavior of a functionally graded magneto-elastic material (FGMM) with variable thickness of rotating disk. The material properties of disc fluctuate along the radius according to a power law. The disc put under the thermal load that are hyperbolic in nature and the thickness profile of the disk positioned in a constant magnetic field.

There are some mechanical properties are studied in this paper these properties are geometric non linearity on the mechanical and thermal stresses, grading index and magnetic field. When the value of the grading index n (n = 2) in FGMM rotating hollow disks due to magneto-mechanical load (fixed-free) with hyperbolic convergent thickness profile, then the value of the radial stress is not at the inner surface of the disc. The radial stresses in FGMM hollow disks with inconstant thickness are all the time tensile. The position of the maximum stress in FGMM disks changes when any variation in grading index n will occur.

Tiwari *et al.* [31] investigated the structural fields of the solid disc brake during short and emergency braking with structural material. Transient structural analysis of the rotor disc brake made with structural steel had used to improve braking efficiency and provide greater stability to vehicle. Hybrid composite material was used due to its good material properties such as lighter weight than the cast iron and had good Young's modulus Yield strength and density properties.

The effects of the friction material properties were observed on the contact ratio of friction surfaces. The thermal expansion coefficient and the elastic modulus were the larger influential properties. It had observed that the orthotropic disc brakes can give better brake performance than the isotropic due to uniform and mild pressure distributions.

2.1 Gaps in Literature

Literature available on rotating FGM disc revels that a large amount of work related to, stress analysis in FGM disc of constant thickness, has been done. However, a less amount of work has been carried out on stress analysis in rotating FGM disc with non uniform thickness and further study needs to be done.

Chapter 3

Research Problem

3.1 Identification of Problem

Rotating discs have large number of applications in all engineering field, for example turbines, compressors, computer disc, high-speed gears, fly wheels, shrink fits and braking system of automobiles . In most of these applications discs have to work under very high temperature and high stresses because of the high speed. So, becomes necessary to carry out FE analysis of a rotating disc. Further the researchers have found out new material called Composite such as, aluminum matrix reinforced with silicon particles. These materials have good mechanical properties such as high thermal stability, high specific modulus and toughness [22]. In percent days, functionally graded material have become a good choice of many researcher because of its changing properties with changing locations [22, 25].

In the present age all the complex calculations are carried out with the help of computer. Due to the presence of finite element packages, all the analytical equation are solved with the help of different software. Because of the large number of application, the analysis of rotating disc becomes as importent area of research.

Therefore, it is decided to do work pertaining to **thermal-structural** analysis of rotating FGM disc of non-linearly varying thickness with the help of finite element software (**ANSYS**). The FGM disc made of silicon carbide particles (SiC_p) in matrix of pure aluminum.The particle content of SiC_p in the disc decreases linearly from inner to outer radius

3.2 Objective of Research Work

The purpose of the present work is to observe the effect of particle content, varying speed, varying thickness of disc on radial stress, tangential stress, radial strain, tangential strain and displacement in a FGM rotating disc.

3.3 Methodology

To achieve the above stated objective the work include the development of:-

(i) Model of FGM disc with varying thickness in ANSYS.

(ii) Transient Thermal and Transient Structure analysis are done to obtain mechanical properties of FGM disc .

(iii) Validation of numerical model with available analytical results.

(iv) Analysis of the results to draw appropriate conclusions.

Chapter 4

Thermal-Structural Analysis Formulation

4.1 A Brief History

The finite element method is a numerical technique that can be very useful to obtain approximate solutions to many applications in engineering. There are many method to analyze the problems such as steady, transient, linear, or nonlinear problems in stress analysis, heat transfer, fluid flow and electromagnetism. R. Courant, who developed the Ritz method of numerical analysis and variational calculus to find approximate solutions to vibration systems in 1943. This was the initialization of the Finite Element Analysis (FEA) [9].

In the early 70s, the use of FEA was limited and very expensive, mainly in the field of automotive, defense, aeronautics and nuclear industries. Due to the rapid growth of the computer manufacture company the cost of computers decreases. Then computer became to use in every field. Due to increase in the power of computer, current FEA models have commonly much more number of elements and it improved accuracy. Present day supercomputers are able to calculate more perfect results in very small amount of time [9].

Thermal-structural analysis is carried out to predict the variation of stress, strain and displacement in FGM disc of variable thickness. Modulus of Elasticity , density, shear modulus, thermal conductivity and specific heat are varying through the radial direction.

4.2 Coupled Thermal-Structural Analysis

The combination of analysis from different engineering studies which relate to solve a global engineering problem is known as a sequentially coupled physics analysis. Coupled analysis means the input of one FE analysis depends upon the results from another FE analysis.

Each different analysis environment build individually so they can be used to find the coupled physics solution. It is easy to share the geometry of first physics environment with other coupled environment.

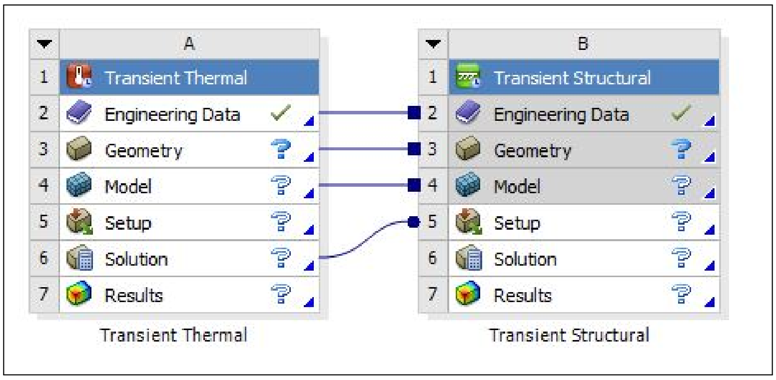


Figure 4.1: Thermal-structural analysis

4.3 Generation of the Geometry

The main purpose of FEA is to generate mathematically the behavior of a real engineering system. In other words, the analysis must be an accurate mathematical model of a physical prototype. The prototype contains all the nodes, elements, constraints and boundary condition.

In ANSYS terminology, the term model generation means to create the nodes and elements that show three dimensional model of the real system. In other words, it is the process of defining the geometric structure of nodes and elements of the model. Different approaches used to model generation in ANSYS are:

(i) Solid modeling in ANSYS

(ii) Using direct generation.

(iii) Importing a model from other CAD software

In this present study, the modeling is done by first method that is solid modeling using ANSYS.

4.3.1 Solid modeling of FGM disc

In order to optimize the design, it is very important to create a model of disc brake like a real system. This involves understanding the forces and constraints on the disc brake, the materials

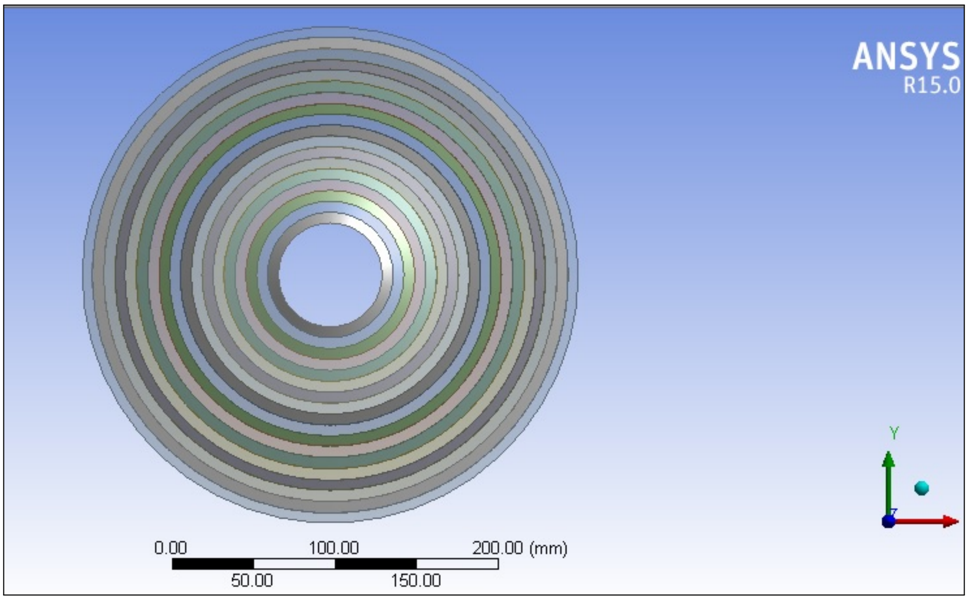


Figure 4.2: Modeling of FGM disc

of the system, and the drawing of the system under consideration. The first stage is to study the Ansys Workbench and then generate the model in design modeler module. the modeling of disc is done layer by layer, then provided material and thermal properties to every layer.

4.4 Mesh Generation

One of the most critical features of FEA is mesh generation. Solver can take long time due to too many cells and give inaccurate results for wrong meshing. ANSYS Meshing technique helps to obtain the right mesh for every analysis. ANSYS Meshing technique has been constructed on the powers of stand-alone, class-leading meshing tools. The strongest features of these distinct tools is to produce some of the most powerful meshing. The highly automated meshing environment makes it simple to generate one of the following mesh types:

(i) Tetrahedral
(ii) Hexahedral
(iii) Prismatic inflation layer
(iv) Hexahedral inflation layer
(v) Hexahedral core
(vi) Body fitted Cartesian
(vii) Cut cell Cartesian

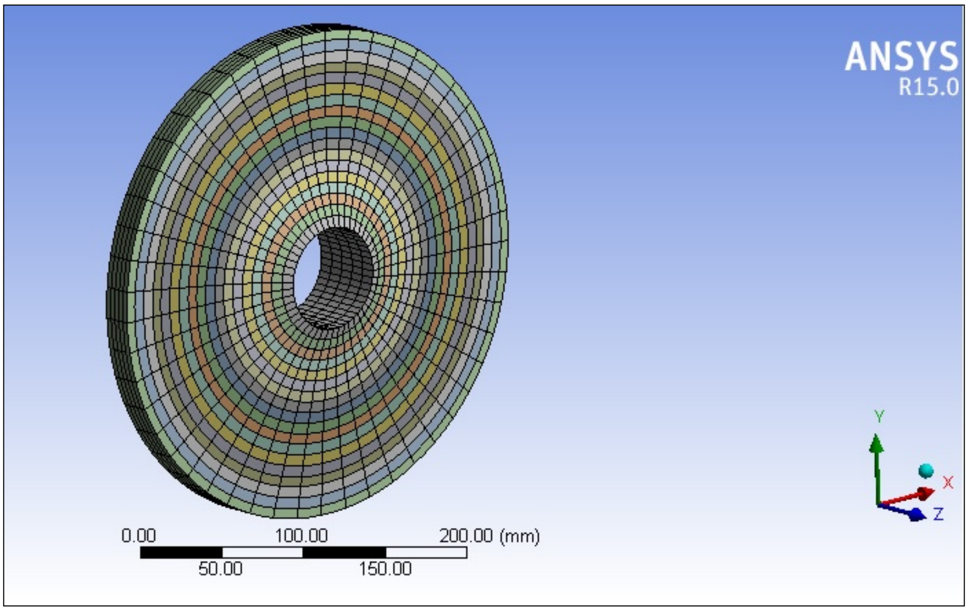


Figure 4.3: Mesh geometry

4.5 Material Properties

The detailed behavior of materials under the influence of flow conditions, such as pressure or temperature, can have a critical effect on the accuracy of CFD predictions. ANSYS Fluent software provides a wide range of material modeling options to ensure nothing stands in the way of achieving the highest fidelity solutions possible.

The material properties provided in ANSYS depends upon the analysis done. Types of material properties :

(i) Linear or nonlinear

(ii) Isotropic, orthotropic, or anisotropic

(iii) Constant temperature or temperature-dependent

In the present work material properties of FGM disc are taken from two research papers [20] and [32].

4.6 Boundary Conditions

The set of condition to indicate the behavior of the solution to a set of differential equations at the boundary of its domain. Boundary conditions play important role to find the solution of many engineering problems.

In the present work boundary condition of FGM disc are taken from research papers [12]. Boundary conditions are used in Thermal-structural analysis of FGM disc as given in table 4.1.

Table 4.1: Boundary conditions

RPM	15600-15000
Heat Flux, (KW/m^2)	130
Convection coefficient, (W/m^2K)	25
Ambient temperature, (K)	303

4.7 Solution

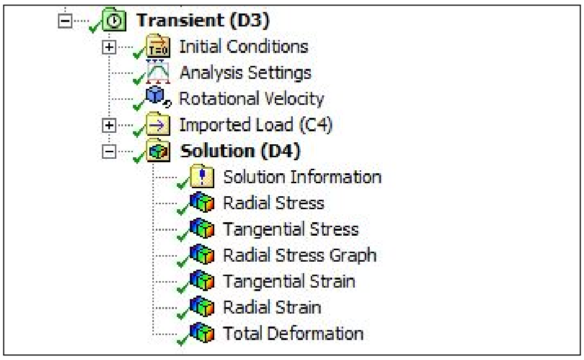
After prepossessing (prepossessing include generation of model, mesh generation, material properties, boundary condition and solver setting) the analysis is ready to begin the solution phase of the ANSYS session. There are three type of analysis :

(i) Static

(ii) Transient(Time-dependent)

(iii) Submodeling and substructuring

In case of structural analysis there are many types of analysis included such as Static analysis, Modal analysis, Buckling analysis, Harmonic analysis, Explicit dynamic analysis etc.



- **Transient (D3)**
  - Initial Conditions
  - Analysis Settings
  - Rotational Velocity
  - Imported Load (C4)
  - **Solution (D4)**
    - Solution Information
    - Radial Stress
    - Tangential Stress
    - Radial Stress Graph
    - Tangential Strain
    - Radial Strain
    - Total Deformation

Figure 4.4: Solution

4.8 Post-Processing

Post-processing tools is used to create animations, important meaningful graphics, and analysis report that make it easy to show present the results of the engineering problems. There are many post-processing features such as contour plots, vector plots, shaded surface, transparent surface, path lines etc. Ansys has good feature to export data to third party CAE packages for additional analysis. It is also possible that solution of thermal and structure analysis share with the other Ansys module.

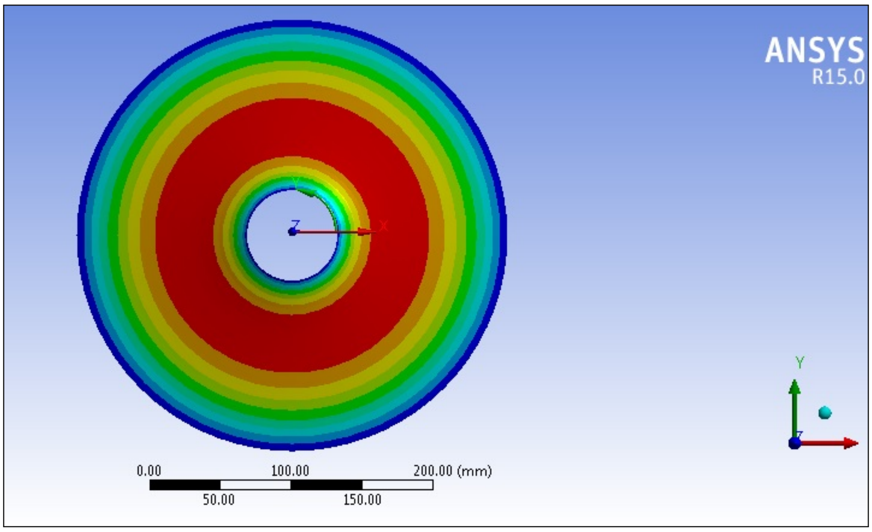


Figure 4.5: Results

Chapter 5

Validation

It is essential to check the accuracy of the result of thermal-structural analysis before performing all the analysis. For this purpose, the radial stress in FGM rotating disc have been calculated and compared with available publishes results [10]. The dimensions, material properties and boundary conditions of the FGM disc are given in Table A.1 of Appendix.

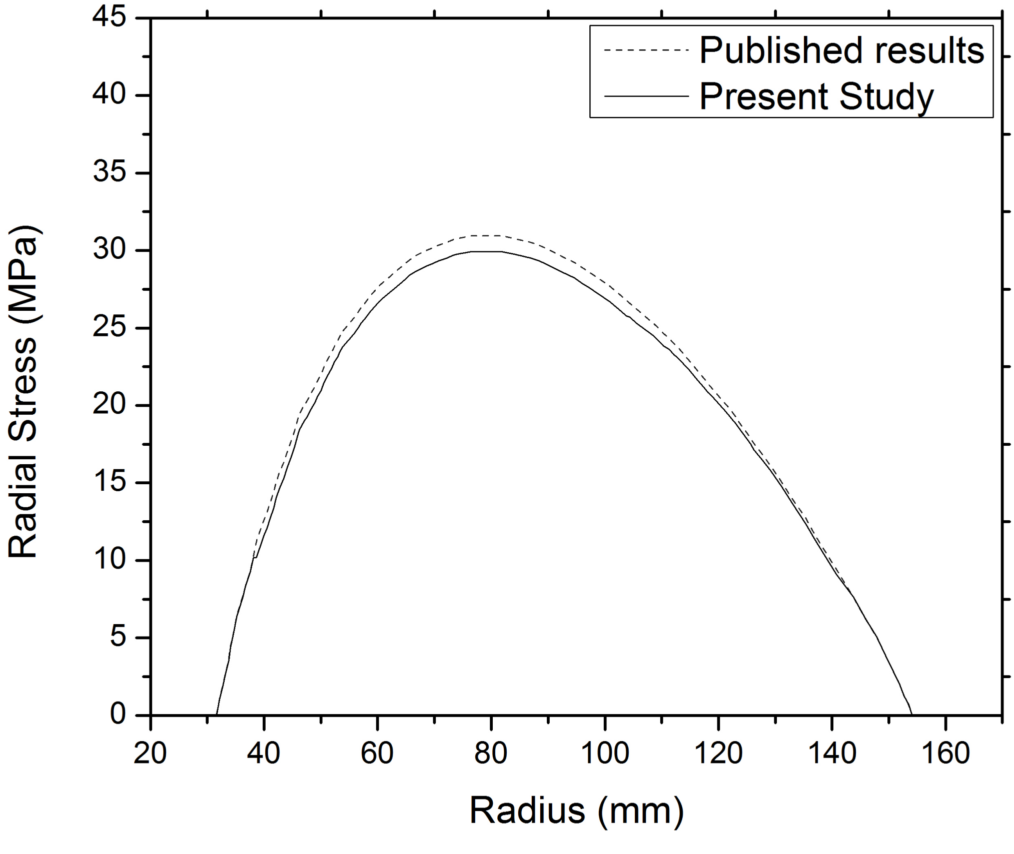


Figure 5.1: Comparison of radial stress (present study) with published result [10]

The graph shows good agreement between the two results, which validate the present analysis.

Chapter 6

Results and Discussions

The main purpose of the present work is to study the effect of different geometry of disc, effect of varing speed and by varying the vol % of SiC. The detailed discussion of the result shown is presented in the following section :

6.1 Disc Profile

To investigate the thermal-structural analysis of the disc brake, the ANSYS simulation is obtained in 4 different types of disc. The geometry of these disc are different as shown in Figure 6.1.

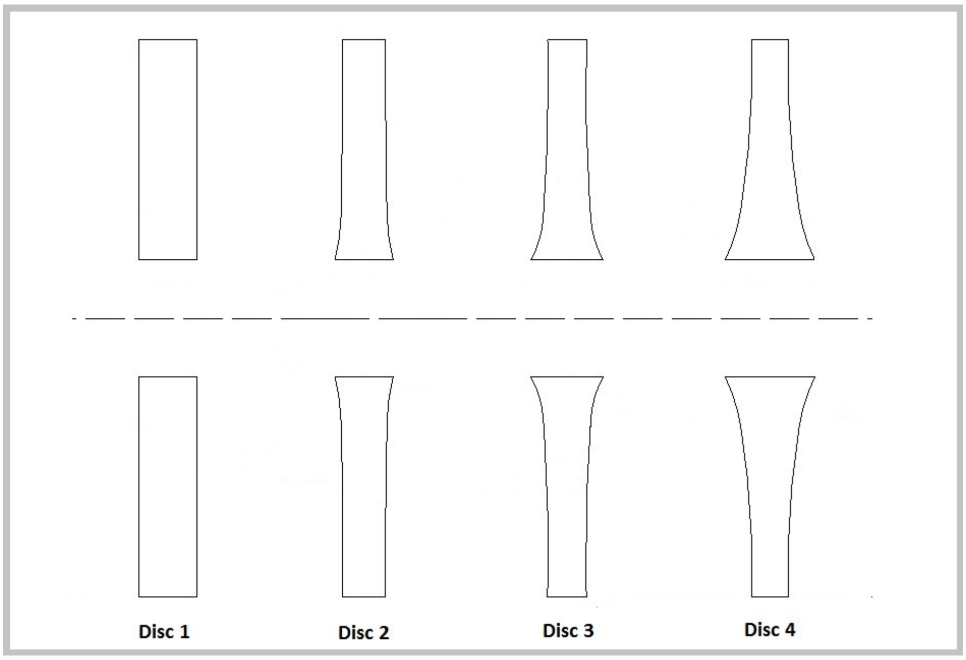


Figure 6.1: Different disc profiles

The inner radius is assumed as 31.8 mm and the outer radius is 152. While the variation of thickness of the disc is also shown in Figure 6.2. In Disc 1 the thickness is constant and in other disc as we go from inner radius to outer radius the thickness decreases non-linearly, but keeping the total volume constant. All these disc are made of Functionally Graded Material that is Aluminum Silicon-Carbide. In simple word, the material properties of all disc are same. These disc are composed of 80% Aluminum and 20% Silicon-Carbide

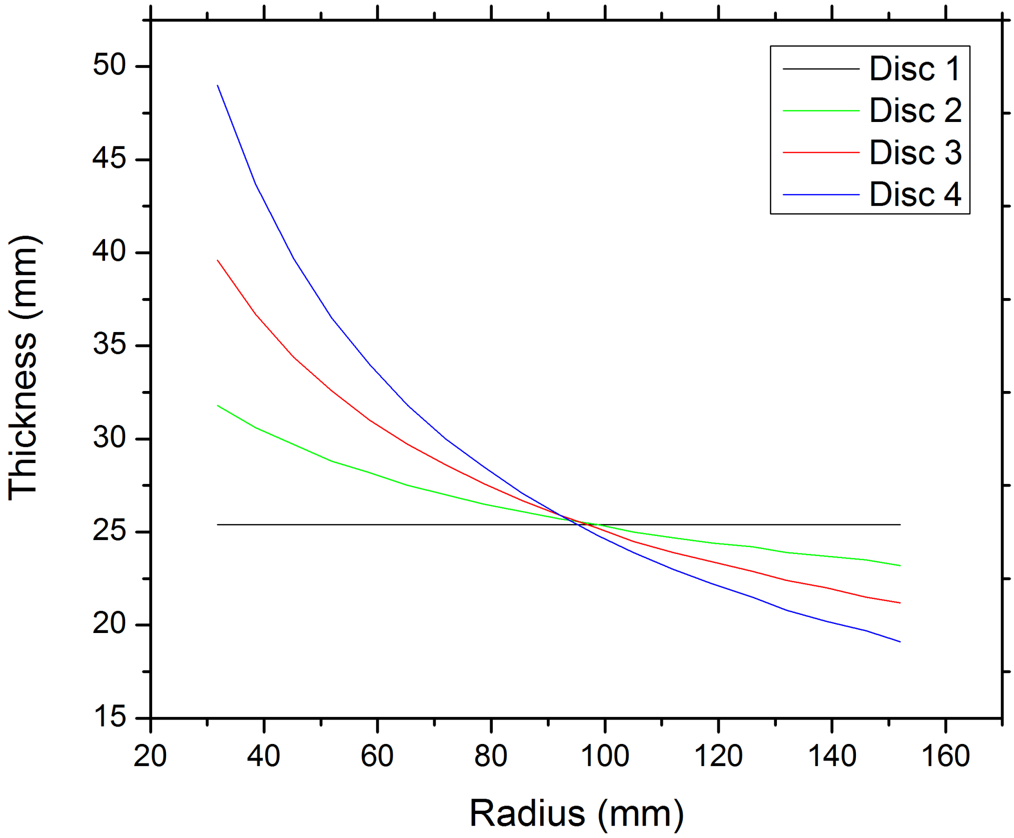


Figure 6.2: Variation of thickness

The Material properties such as Thermal Conductivity, Density, Specific Heat, Modulus of Elasticity, Shear Modulus and Possion Ratio for all disc are same as given in Table A.1 of the Appendix.

6.2 Effect of Varying Thickness

Effect of varying thickness on tangential stress

The variation of tangential stress with the radius due to the effect of varying thickness is shown in Figure 6.3. The tangential stress decreases significantly from inner to outer radius in all the discs. However, decrease observed at the inner radius is higher as compared to decrease at the outer radius.

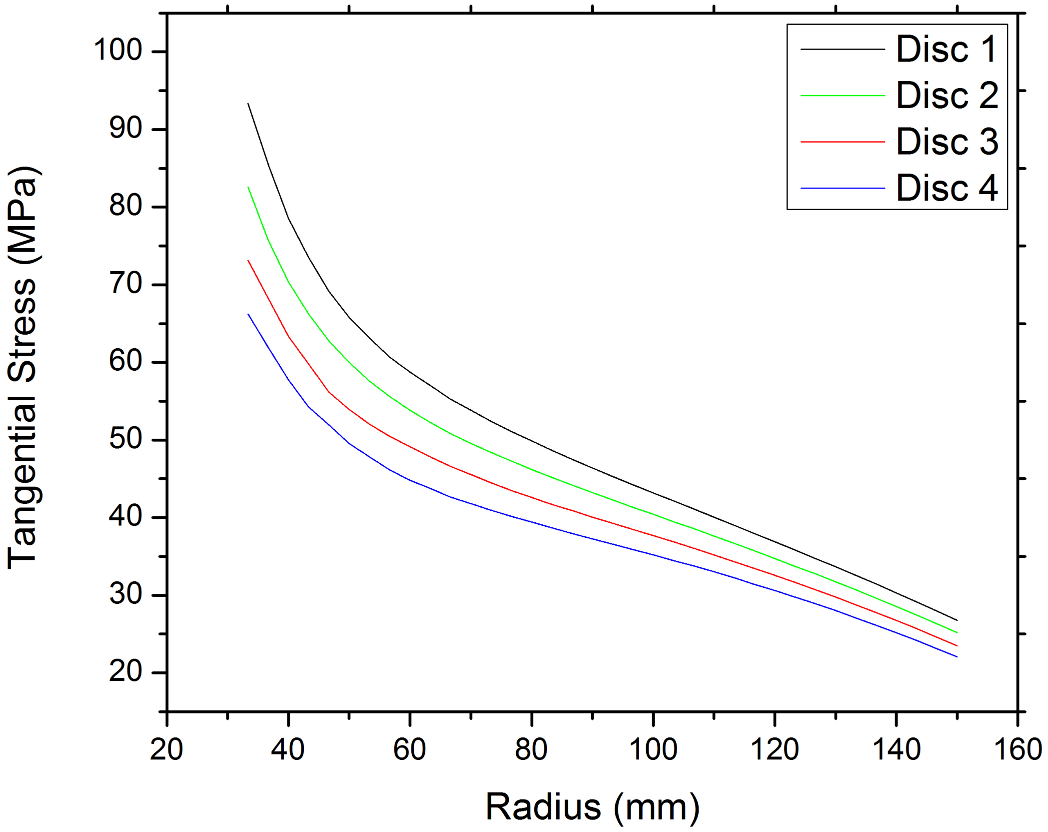


Figure 6.3: Effect of varying thickness on tangential stress

The decreased notified is 33.12 MPa and 4.71 MPa , respectively at the inner and outer radius of the variable thickness Disc 4, when compared to flat Disc 1.

Effect of varying thickness on radial stress

The variation of radial stress with the radius of all four disc are depicted in Figure 6.4. The figure reveals that some where near the middle region of the disc the the radial stress is observed higher in constant thickness disc when compared with those observed in non-uniform thickness discs. The maximum radial stress value is observed as 29.57 MPa at radius 70 mm in the constant thickness disc. It is also detected minimum value of radial stress in non-uniform thickness Disc 4.

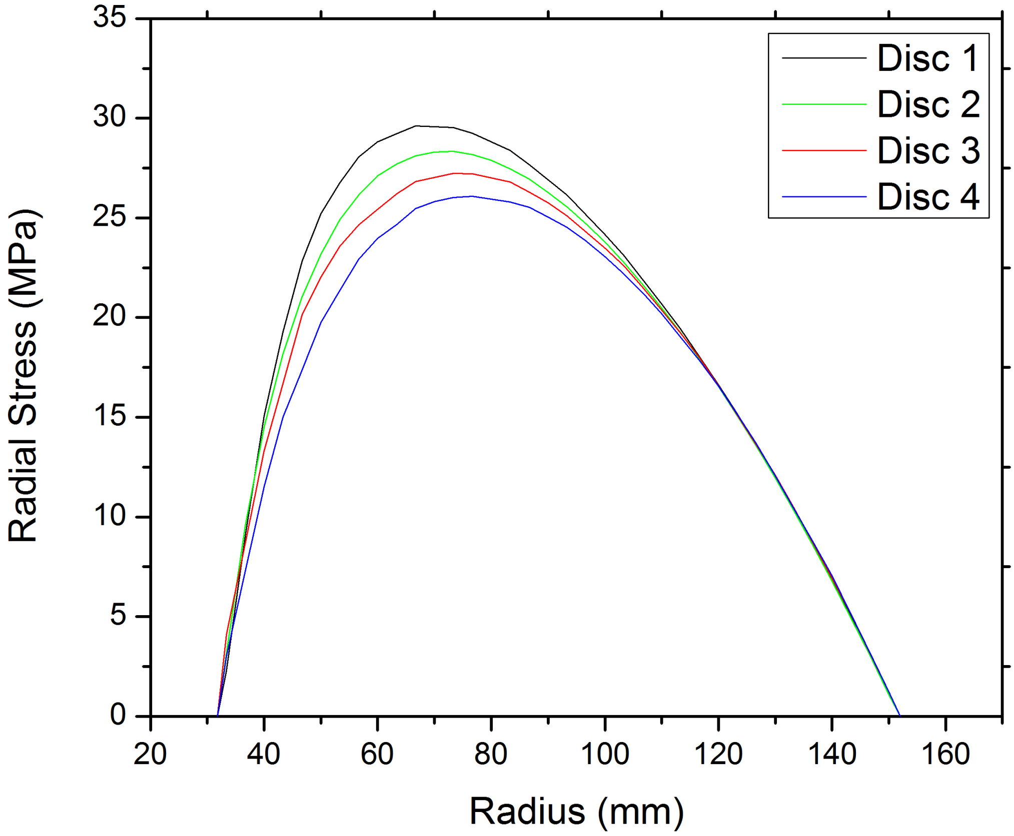


Figure 6.4: Effect of varying thickness on radial stress

Effect of varying thickness on tangential strain

Figure 6.5 illustrates the effect of the varying thickness on tangential strain with radius. It is observed that the tangential strain is maximum in constant thickness Disc 1 and minimum in variable thickness Disc 4. In constant thickness Disc 1, the tangential strain changes from 0.86×10^{-3} to 0.24×10^{-3}, respectively inner to outer radius. In variable thickness Disc 4, the tangential strain changes from 0.62×10^{-3} to 0.20×10^{-3}, respectively inner to outer radius. The decreasing pattern is observed from inner radius to outer radius for all discs.

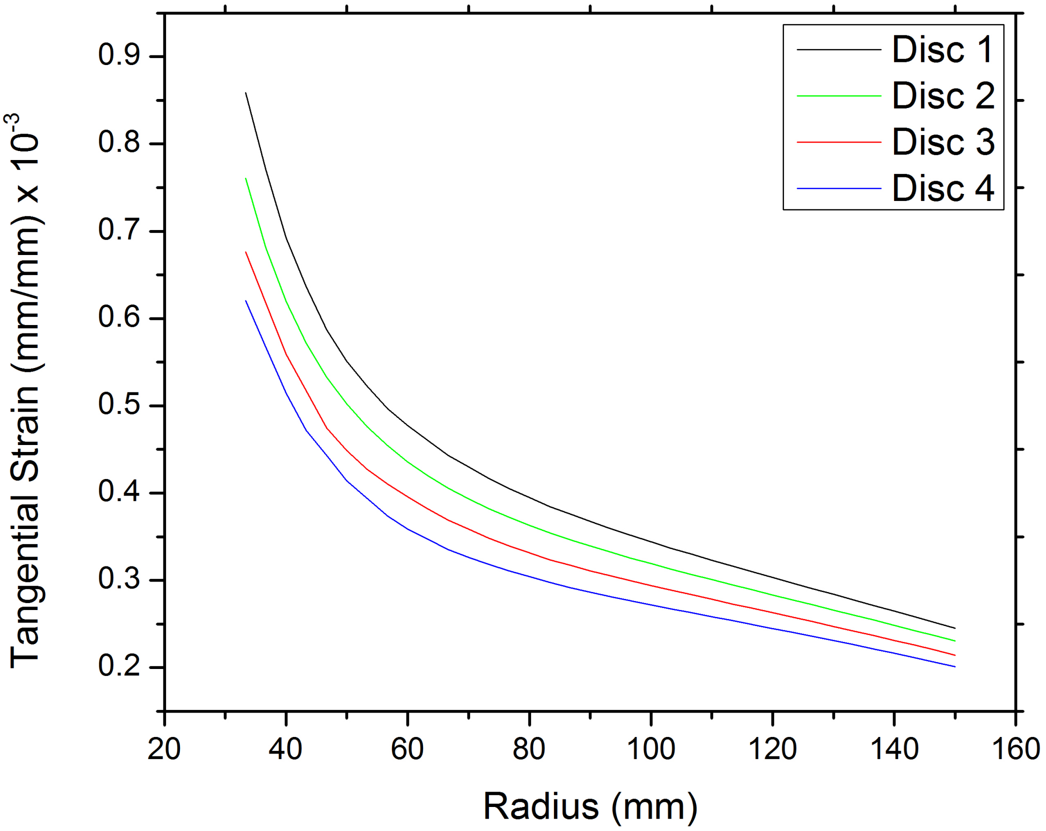


Figure 6.5: Effect of varying thickness on tangential strain

Effect of varying thickness on radial strain

Variation of radial strain with respect to radius is shown in Figure 6.6. As the radius of the disc increases, the radial strain first increases, reaches a maximum value and then decreases. It is observed that for minimum and maximum values of radius, radial strain are compressive in nature. Maximum value of radial strain as obtained from Figure 6.6 is 0.15×10^{-3} and corresponding value of radius is 75mm.

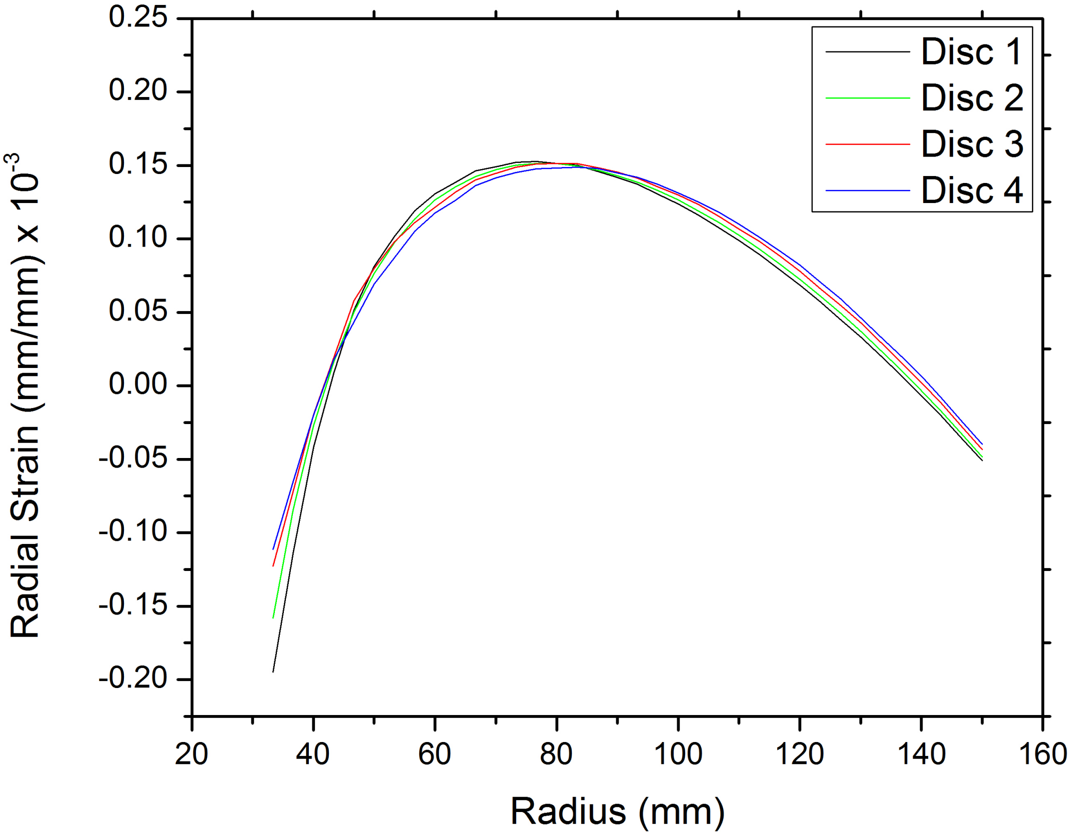


Figure 6.6: Effect of varying thickness on radial strain

Effect of varying thickness on displacement

The displacement along the radius due to the effect of varying thickness is shown in Figure 6.7. By increase the thickness from Disc 1 to Disc 4 the displacement decreases over the entire radius. The value of the displacement increases from inner radius to outer radius in all discs. The maximum difference in the magnitude of displacement corresponding to Disc 1 to Disc 4 is noticed as 0.0071 mm at inner radius and 0.0065 mm, at the outer radius.

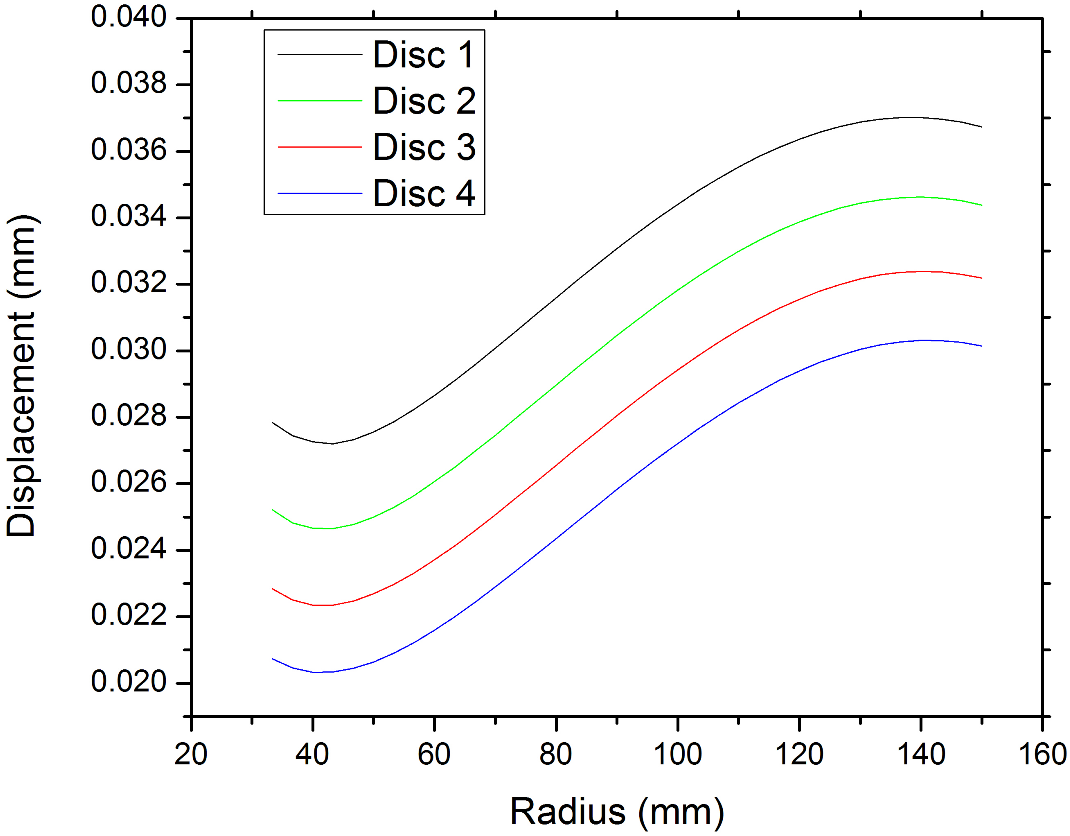


Figure 6.7: Effect of varying thickness on displacement

6.3 Effect of Reinforcement

The effect of variation in reinforcement at different radii is observed by changing the particles distribution SiC contents from inner to outer radius. All the analysis is done on Disc 4 to check the effect of variation in reinforcement. Geometry of the Disc 4 is shown in Figure 6.1.

To analyze the behavior of Radial stress, Tangential stress, Radial strain, Tangential strain and displacement, five disc have been considered which are :

(i) **Disc 4(a) :** having uniform particle distribution of **20 vol%** throughout
(ii) **Disc 4(b) :** vary from inner radius **24.6 vol%** to outer radius **18 vol%**
(iii) **Disc 4(c) :** vary from inner radius **30 vol%** to outer radius **16 vol%**
(iv) **Disc 4(d) :** vary from inner radius **36.4 vol%** to outer radius **14.2 vol%**
(v) **Disc 4(e) :** vary from inner radius **43.8 vol%** to outer radius **12.5 vol%**

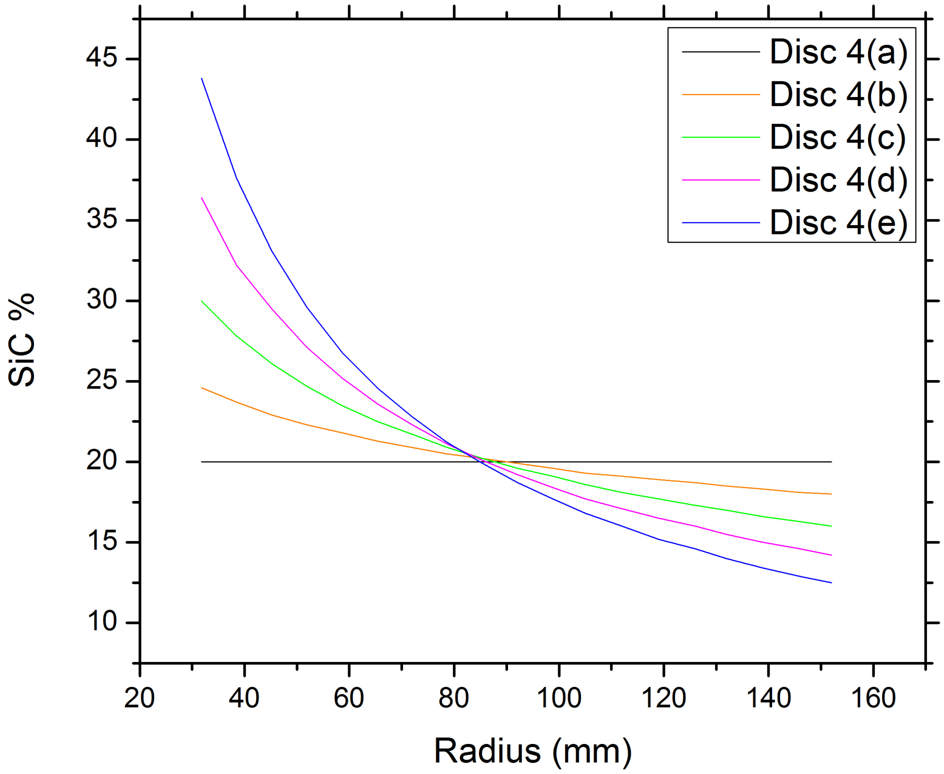


Figure 6.8: Variation of reinforcement

Effect of varying particle content on tangential stress

The variation of tangential stress along radius due to varying the particle content in all FGM disc is shown in Figure 6.9. In all discs the value of tangential stress is maximum at the inner radius and minimum at the outer radius. When increase the particle content of SiC from inner to outer radius then the value of the tangential stress decrease at inner radius but increase at outer radius. The maximum difference in the magnitude of tangential stress corresponding to Disc 4(a) to Disc 4(e) is noticed as 10.37 MPa at inner radius and 3.21 MPa, at the outer radius. Tangential stress remain same at radius around the 50 mm in all the five discs.

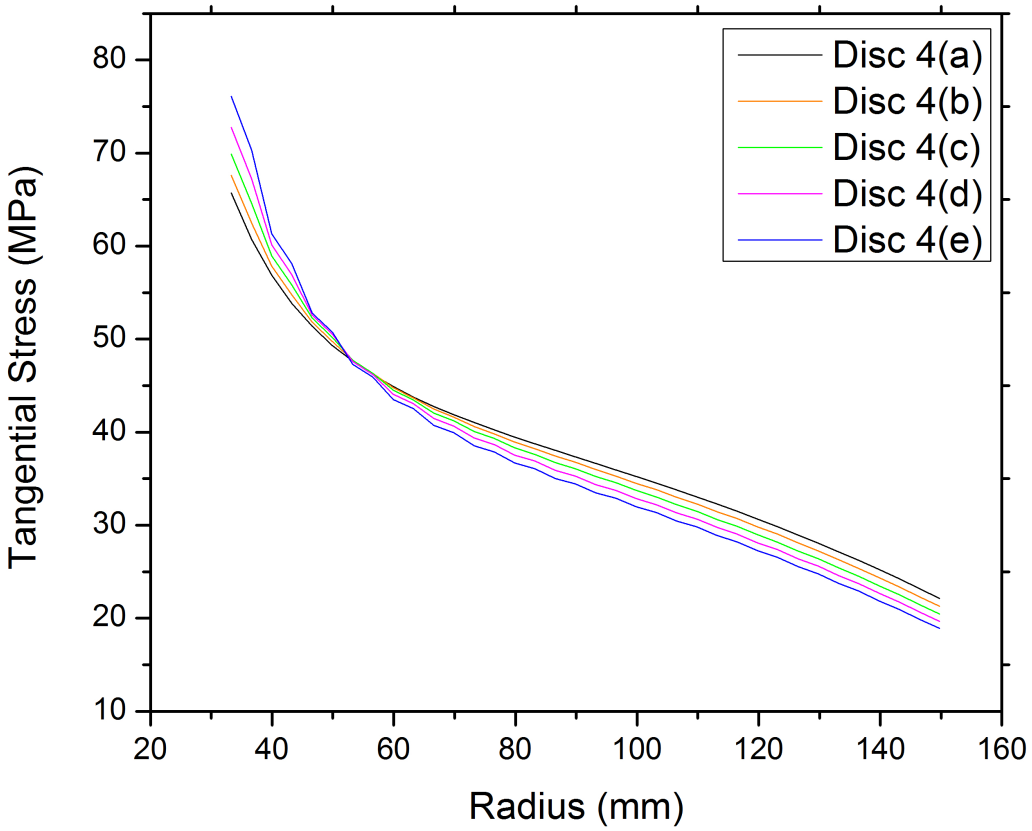


Figure 6.9: Effect of varying particle content on tangential stress

Effect of varying particle content on radial stress

The variation of radial stress due to varying the particle content are shown in Figure 6.10. The figure shows maximum value in the middle region of all FGM discs and minimum at inner and outer radius of discs. The minimum value of radial stress is observed in Disc 4(a) and the maximum in Disc 4(e).

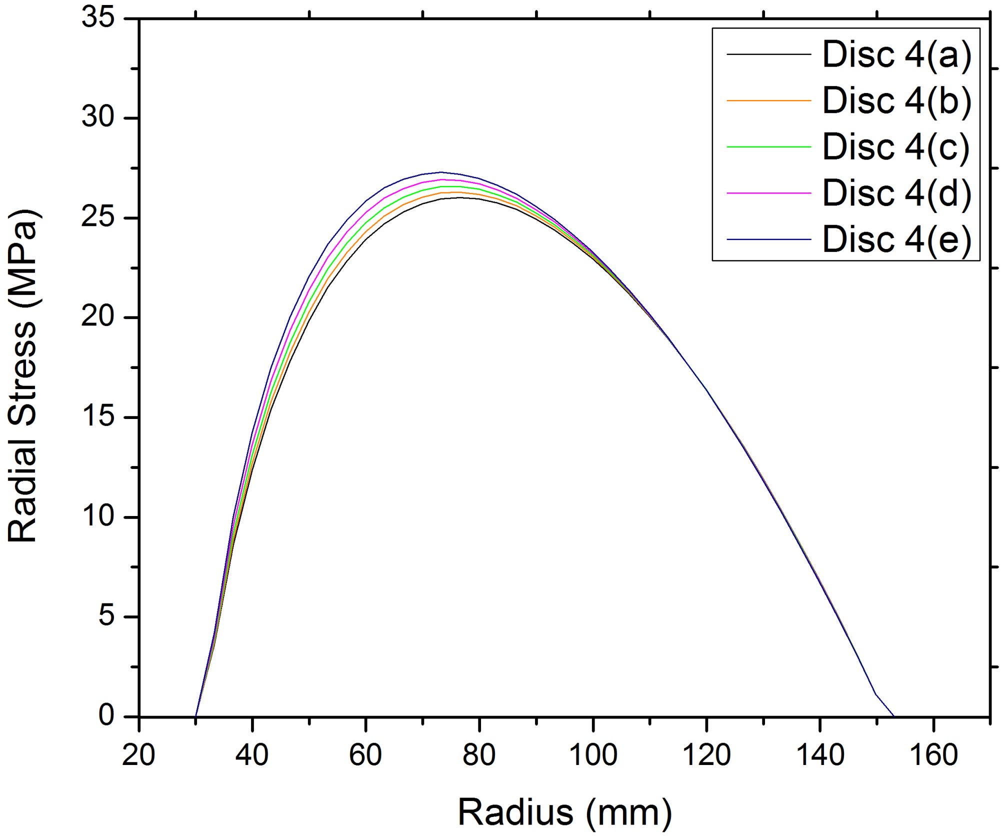


Figure 6.10: Effect of varying particle content on radial stress

Effect of varying particle content on tangential strain

Figure 6.11 presents tangential strain along the radius due to the effect of the particle content. It clearly shows that the tangential strain is maximum for non-FGM Disc 4(a) and minimum for FGM Disc 4(e). It is also observed tangential strain decrease from inner radius to outer radius.

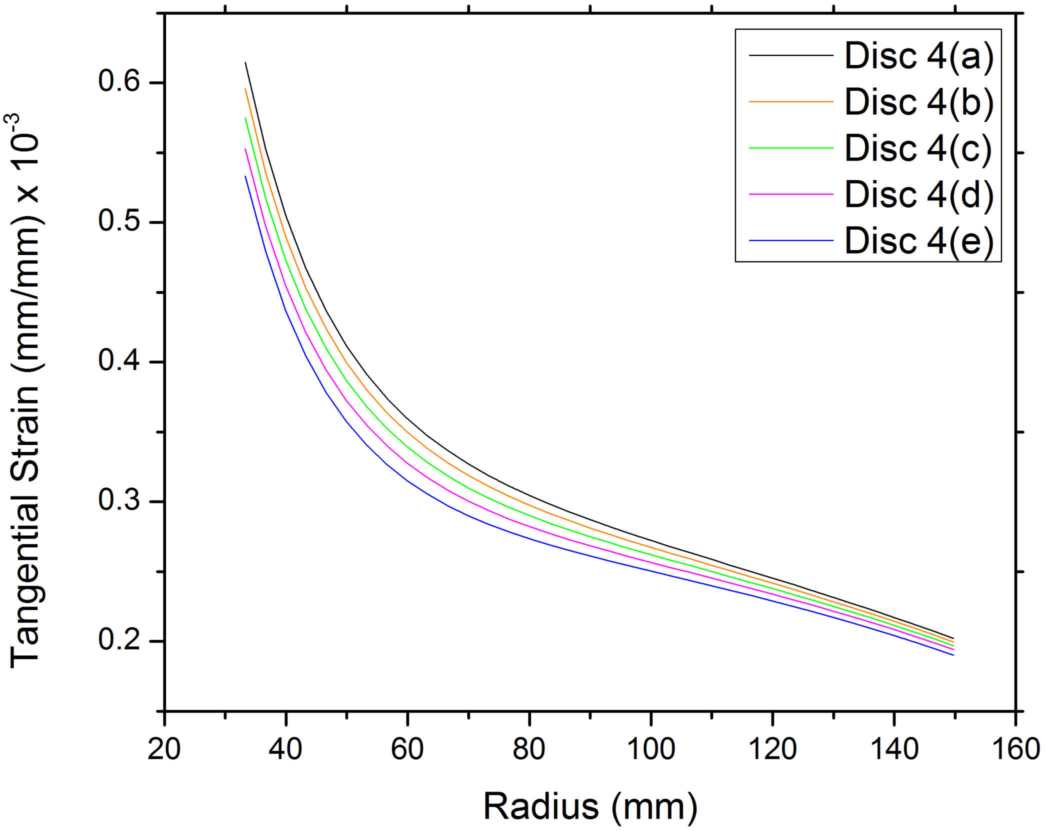


Figure 6.11: Effect of varying particle content on tangential strain

Effect of varying particle content on radial strain

The variation of radial strain due to the effect of particle content is shown in Figure 6.12. It is observed from the graph that the radial strain compressive near the inner and outer radius of all FGM discs. At the inner and outer radius, the compressive radial strain is observed highest for Disc 4(a) and lowest for Disc 4(e).It can also be seen at the middle region the radial strain become tensile.

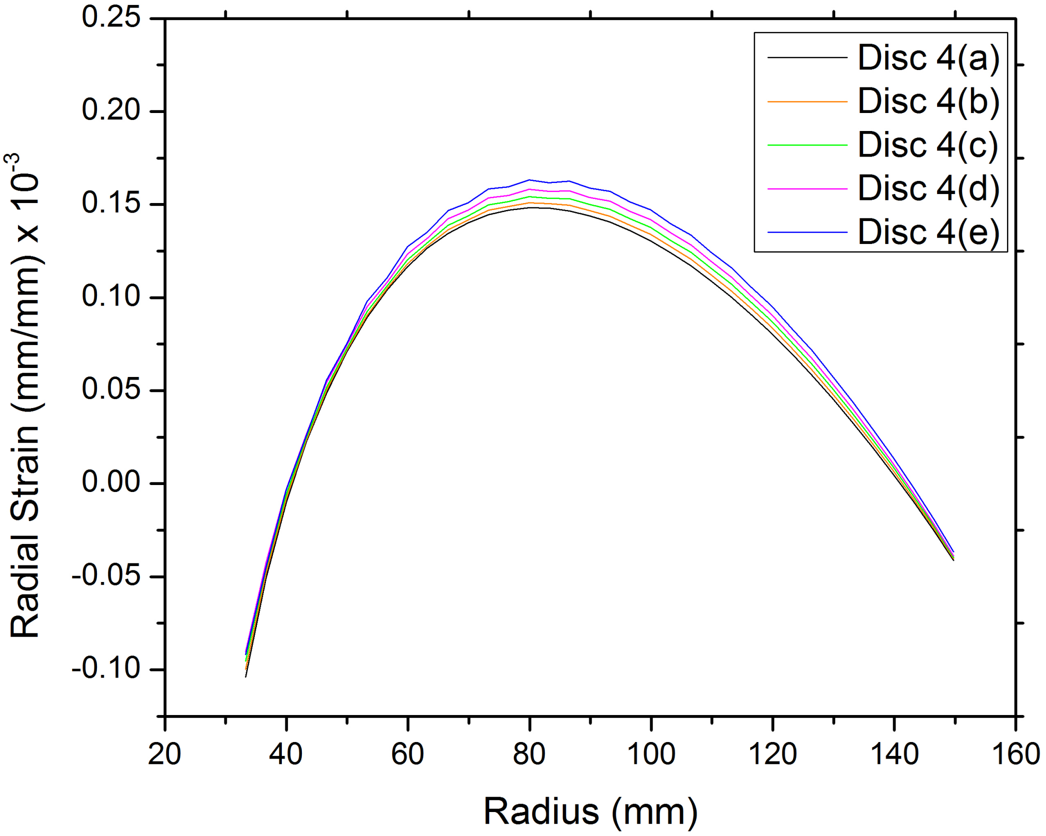


Figure 6.12: Effect of varying particle content on radial strain

Effect of varying particle content on displacement

The displacement along the radius due to the effect of reinforcement are shown in Figure 6.13. By increase the particle content from Disc 4(a) to Disc 4(e) the displacement decreases over the entire radius. The value of the displacement increases from inner radius to outer radius in all FGM discs. The maximum difference in the magnitude of displacement corresponding to Disc 4(a) to Disc 4(e) is noticed as 0.0029 mm at inner radius and 0.00156 mm, at the outer radius.

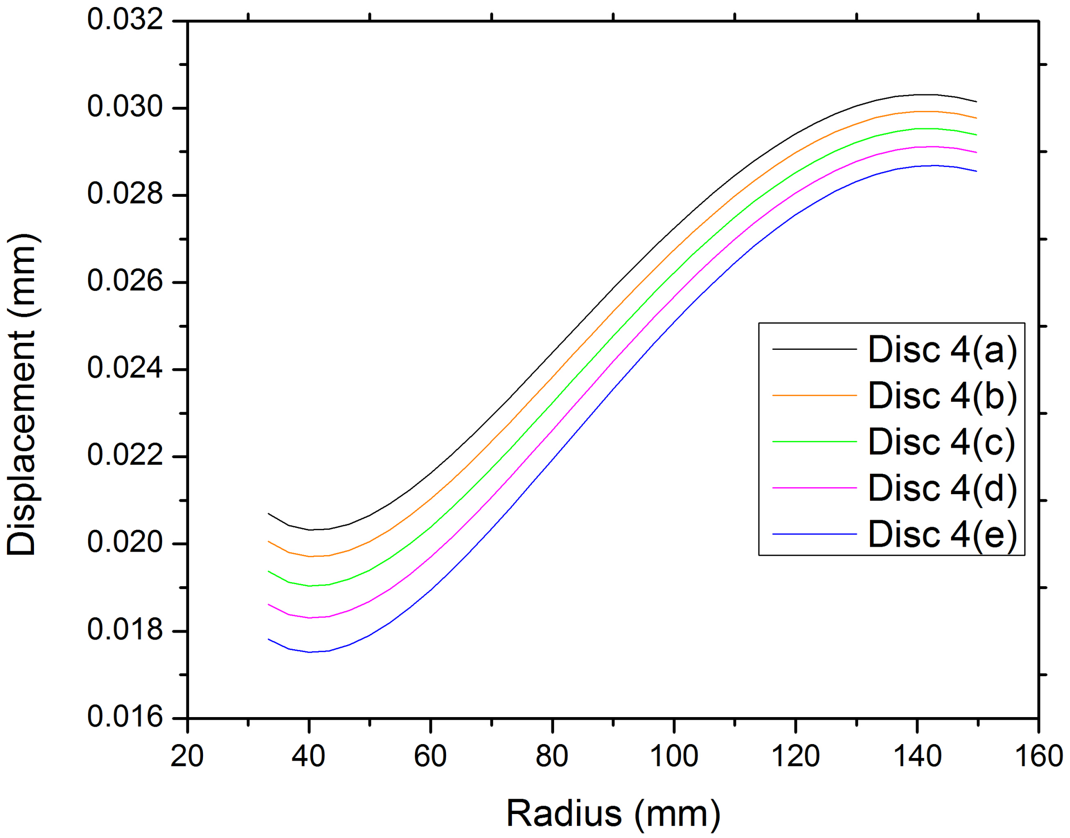


Figure 6.13: Effect of varying particle content on displacement

6.4 Effect of Varying Speed

Angular speed is one of the factor which effect the stresses, strain and displacement. Figures 6.14 to 6.18 show the effect of varying speed when it varies from 14000 rpm to 17000 rpm in four step. The geometry, material properties like Modulus of Elasticity, Shear Modulus, density etc. and boundary conditions of Disc 1 are reported in Table A.2 of the Appendix.

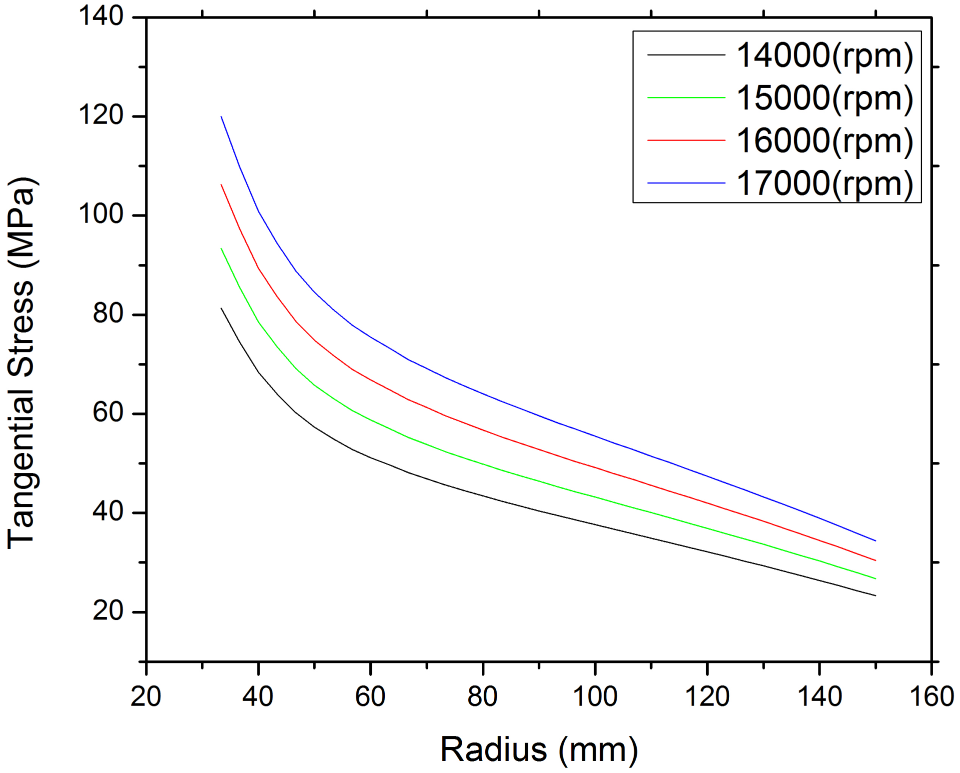


Figure 6.14: Effect of varying speed on tangential stress

Figure 6.14 presented the effect of varying speed on tangential stress of the Disc 1. It shows that at inner radius tangential stress maximum and minimum at outer radius. It can also be seen that as the speed of the disc increases, the overall value of tangential stress also increases.

Effect of varying speed on radial stress

Figure 6.15 shown the variation of radial stress due to the effect of varying speed. It is observed from the graph that the radial strain is compressive near the inner and outer radius of all FGM discs. It also indicates the minimum value of radial stress at the inner and outer radius, maximum value at middle of the FGM discs. The radial stress is highest for disc which operated at 17000 rpm and lowest at 14000 rpm.

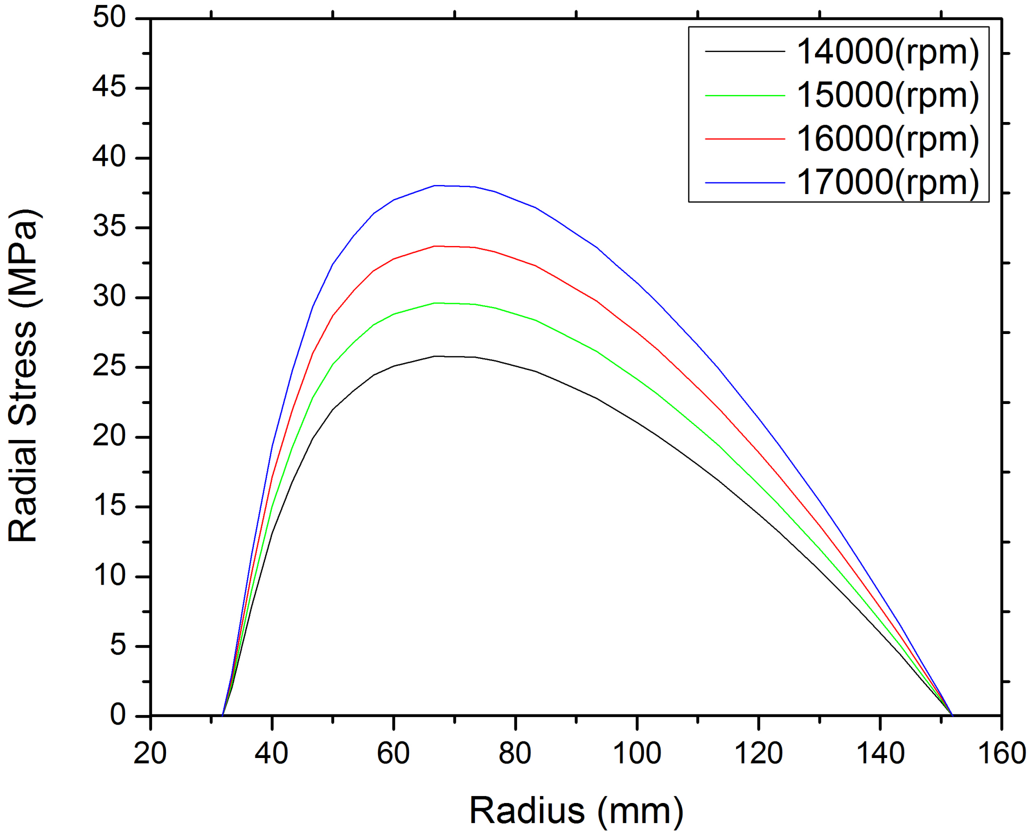


Figure 6.15: Effect of varying speed on radial stress

Effect of varying speed on tangential strain

Figure 6.16 present tangential strain along the radius due to the effect of varying speed. It can be seen that the tangential strain is maximum for disc which operated at 17000 rpm and minimum at 14000 rpm. It is also observed tangential strain decrease, maximum value to minimum value from inner radius to outer radius.

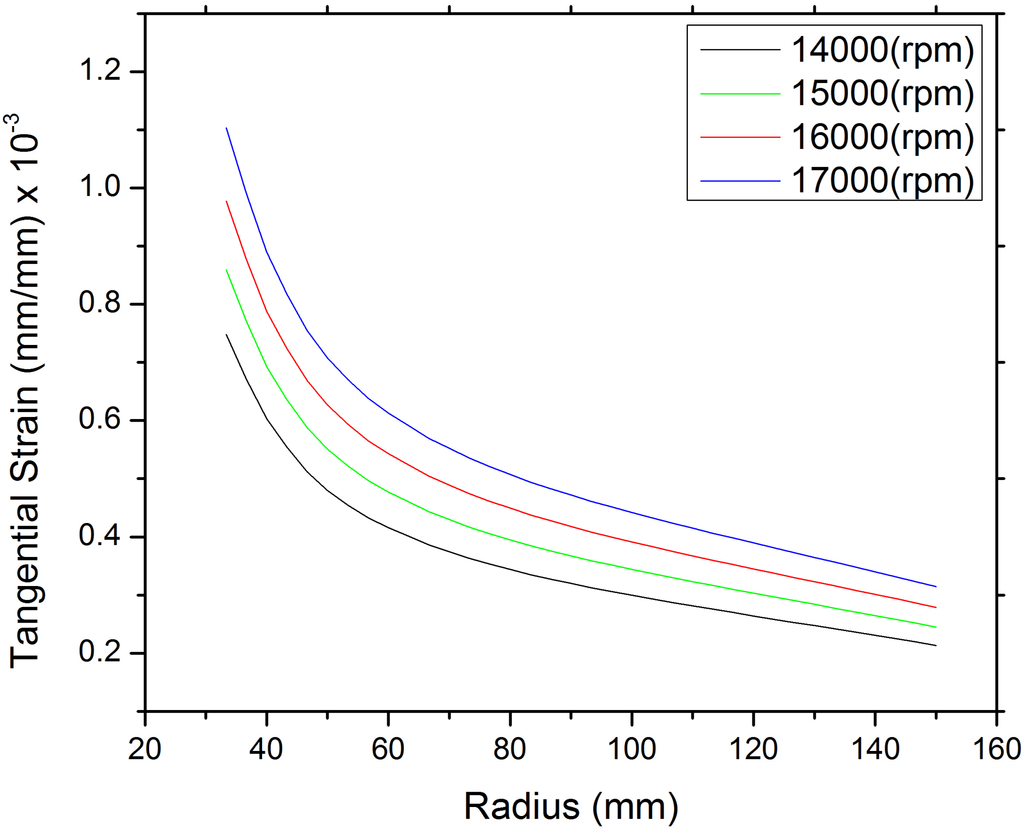


Figure 6.16: Effect of varying speed on tangential strain

Effect of varying speed on radial strain

The variation of radial strain due to the effect of varying speed is shown in Figure 6.17. It is observed from the graph that the radial strain compressive near the inner and outer radius at all RPM. At the inner and outer radius the radial strain is observed compressive in nature. It can be seen that the radial strain maximum for disc which operate at 17000 rpm and minimum at 14000 rpm. It is also observed the radial strain at the middle region become tensile.

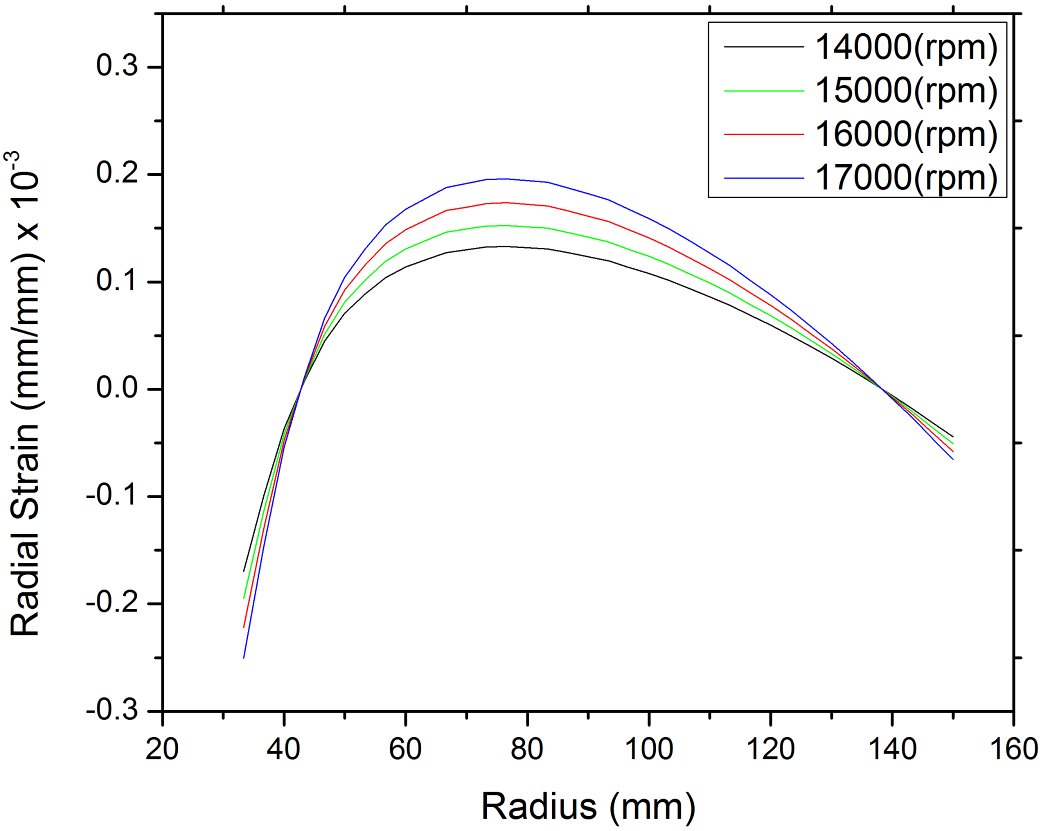


Figure 6.17: Effect of varying speeed on radial strain

Effect of varying speed on displacement

The displacement along the radius due to the effect of varying speed are shown in Figure 6.18. By increase the speed of disc from 14000 rpm to 17000 rpm the displacement increase over the entire radius. The value of the displacement increases from inner radius to outer radius in all disc. The maximum difference in the magnitude of displacement corresponding to disc operated at 14000 rpm to 17000 rpm is noticed as 0.011 mm at inner radius and 0.015 mm, at the outer radius.

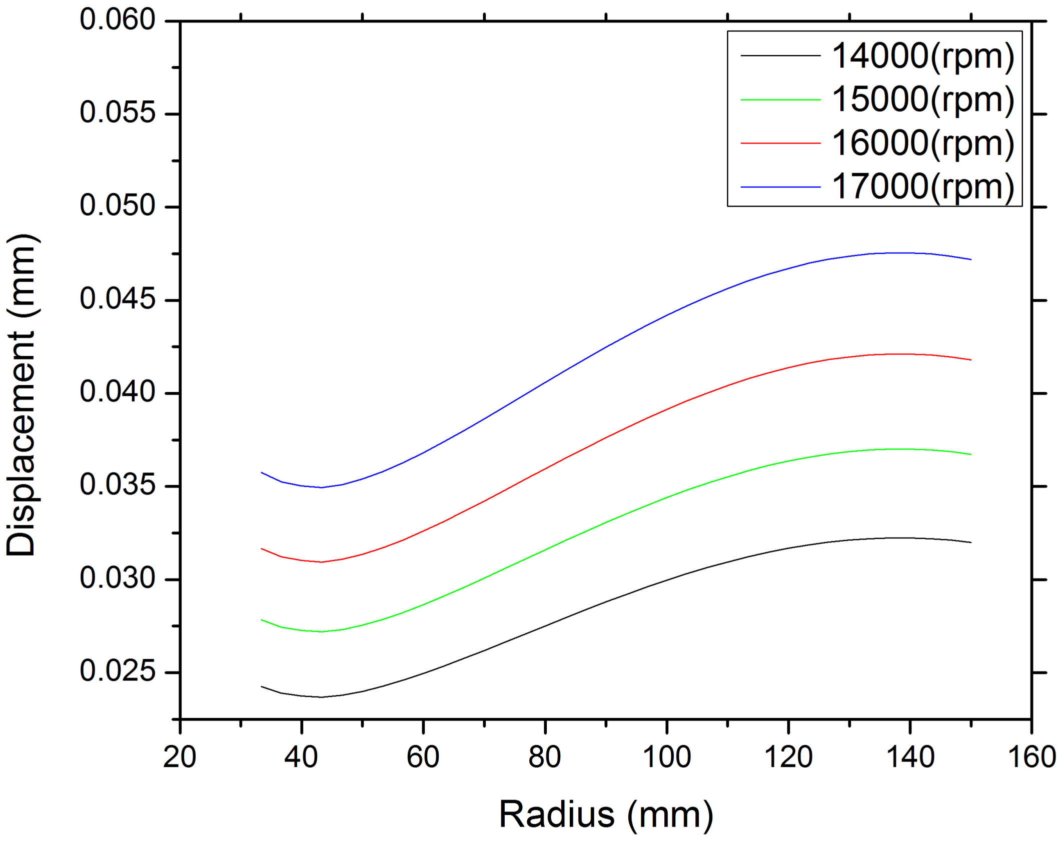


Figure 6.18: Effect of varying speed on displacement

Chapter 7

Conclusions and Future Scope of Work

7.1 Conclusions

Based on achieved results from thermal-structural analysis, some statements from the present thesis work can be brief as follows:

(i) The values of the tangential stress, radial stress, tangential strain and displacement are observed significantly lower in linearly varying thickness disc when related with constant thickness disc.

(ii) The Tangential stress increases near the inner radius and decrease toward the outer radius due to the effect of increase particle content. It is also found the value of radial stress increase with increase in particle content. As the particle content increase at the inner radius, the tangential stress also increase but strain and displacement observed lower.

(iii) By increasing the speed of the disc, tangential stress, radial stress, tangential strain, radial strain and displacement increases along the whole radius.

7.2 Future Scope of Work

The present study carried out in this thesis may be further extended in the following directions:

(i) The similar kind of stress-strain analysis can be carried out for other thickness profile of the disc.

(ii) The analysis can also be extended for some other material also.

(iii) The stress-strain analysis can also be extended for varying temperature from inner to outer radius.

Appendix A

A Sample Appendix

Table A.1: Dimensions, material properties and boundary conditions for validation [10]

Dimensions	Inner Radius = 31.8 mm Outer Radius = 152 mm Disc Thickness = 5 mm
Material Properties	Density ρ = 2.04 g/cm^3 Thermal Conductivity K = 124.08 (W/mK) Specific Heat,c = 0.79 (J/gK) Elastic Modulus,E = 108026.33 (MPa) Shear Modulus G = 43213.49 (MPa) Poisson Ratio = 0.24
Boundary Conditions	Disc Speed RPM = 15600-15000 Convection coefficient = 25 (W/m^2K) Ambient temperature = 303 (K)

Table A.2: Dimensions, material properties, boundary conditions for varying speed

Dimensions	Inner Radius = 31.8 mm Outer Radius = 152 mm Disc Thickness = 25 mm
Material Properties	Density ρ = 2.04 g/cm^3 Thermal Conductivity K = 124.08 (W/mK) Specific Heat,c = 0.79 (J/gK) Elastic Modulus,E = 108026.33 (MPa) Shear Modulus G = 43213.49 (MPa) Poisson Ratio = 0.24
Boundary Conditions	Convection coefficient = 25 (W/m^2K) Ambient temperature = 303 (K)

References

[1] L. J. Broutman and R. H. Krock, *Composite materials*, vol. 2. Academic Pr, 1974.

[2] G. Dieter, “Mechanical metallurgy, mcgracw-hill,” 1976.

[3] V. Birman, “Stability of functionally graded hybrid composite plates,” *Composites Engineering*, vol. 5, no. 7, pp. 913–921, 1995.

[4] V. Birman, “Stability of functionally graded shape memory alloy sandwich panels,” *Smart Materials and Structures*, vol. 6, no. 3, pp. 278–285, 1997.

[5] S. Suresh and A. Mortensen, *Fundamentals of functionally graded materials*. No. LMM-CHAPTER-1998-001, The Institute of Materials, 1998.

[6] L. You, Y. Tang, J. Zhang, and C. Zheng, “Numerical analysis of elastic–plastic rotating disks with arbitrary variable thickness and density,” *International Journal of Solids and Structures*, vol. 37, no. 52, pp. 7809–7820, 2000.

[7] A. Eraslan and Y. Orcan, “Elastic–plastic deformation of a rotating solid disk of exponentially varying thickness,” *Mechanics of Materials*, vol. 34, no. 7, pp. 423–432, 2002.

[8] S. Singh and S. Ray, “Creep analysis in an isotropic FGM rotating disc of Al-SiC composite,” *Journal of materials processing technology*, vol. 143, pp. 616–622, 2003.

[9] S. Moaveni, *Finite element analysis: theory and application with ANSYS*. Pearson Education India, 2003.

[10] V. Gupta, H. Chandrawat, S. Singh, and S. Ray, “Creep behavior of a rotating functionally graded composite disc operating under thermal gradient,” *Metallurgical and Materials Transactions A*, vol. 35, no. 4, pp. 1381–1391, 2004.

[11] V. Gupta, S. Singh, H. Chandrawat, and S. Ray, “Steady state creep and material parameters in a rotating disc of Al-SiC composite,” *European Journal of Mechanics-A/Solids*, vol. 23, no. 2, pp. 335–344, 2004.

[12] V. Gupta, S. Singh, H. Chandrawat, and S. Ray, “Modeling of creep behavior of a rotating disc in the presence of both composition and thermal gradients,” *Journal of Engineering Materials and Technology*, vol. 127, no. 1, pp. 97–105, 2005.

[13] H. Çallıoğlu, M. Topcu, and A. R. Tarakcılar, “Elastic–plastic stress analysis of an orthotropic rotating disc,” *International journal of mechanical sciences*, vol. 48, no. 9, pp. 985–990, 2006.

[14] L. You, H. Ou, and Z. Zheng, “Creep deformations and stresses in thick-walled cylindrical vessels of functionally graded materials subjected to internal pressure,” *Composite Structures*, vol. 78, no. 2, pp. 285–291, 2007.

[15] S. Singh, “One parameter model for creep in a whisker reinforced anisotropic rotating disc of Al-SiC composite,” *European Journal of Mechanics-A/Solids*, vol. 27, no. 4, pp. 680–690, 2008.

[16] M. Bayat, M. Saleem, B. Sahari, A. Hamouda, and E. Mahdi, “Analysis of functionally graded rotating disks with variable thickness,” *Mechanics Research Communications*, vol. 35, no. 5, pp. 283–309, 2008.

[17] M. Hojjati and A. Hassani, “Theoretical and numerical analysis of rotating discs of non-uniform thickness and density,” *International Journal of Pressure Vessels and Piping*, vol. 85, no. 10, pp. 694–700, 2008.

[18] T. Pankaj, “Elastic-plastic transition stresses in an isotropic disc having variable thickness subjected to internal pressure,” *Integritet i vek konstrukcija*, vol. 9, no. 2, pp. 125–132, 2009.

[19] A. Zenkour, “Stress distribution in rotating composite structures of functionally graded solid disks,” *Journal of Materials Processing Technology*, vol. 209, no. 7, pp. 3511–3517, 2009.

[20] S. S. Kumar, V. S. Bai, K. Rajkumar, G. Sharma, T. Jayakumar, and T. Rajasekharan, “Elastic modulus of Al-Si/SiC metal matrix composites as a function of volume fraction,” *Journal of Physics D: Applied Physics*, vol. 42, no. 17, pp. 1704–1715, 2009.

[21] A. Afsar and J. Go, “Finite element analysis of thermoelastic field in a rotating FGM circular disk,” *Applied Mathematical Modelling*, vol. 34, no. 11, pp. 3309–3320, 2010.

[22] D. Deepak, V. Gupta, and A. Dham, “Creep modeling in functionally graded rotating disc of variable thickness,” *Journal of Mechanical science and Technology*, vol. 24, no. 11, pp. 2221–2232, 2010.

[23] A. Loghman, A. G. Arani, A. Shajari, and S. Amir, “Time-dependent thermoelastic creep analysis of rotating disk made of Al-SiC composite,” *Archive of Applied Mechanics*, vol. 81, no. 12, pp. 1853–1864, 2011.

[24] J. Sharma, D. Sharma, and S. Kumar, “Stress and strain analysis of rotating FGM thermoelastic circular disc by using FEM,” *International Journal of pure and applied mathematics*, vol. 74, no. 3, pp. 339–352, 2012.

[25] R. M. Mahamood, E. T. Akinlabi, M. Shukla, and S. Pityana, “Functionally graded material: an overview,” *World Congress on Engineering*, vol. 3, pp. 533–541, 2012.

[26] B. Ali and B. Mostefa, “Thermo mechanical modeling of disc brake contact phenomena,” *FME Transactions*, vol. 41, no. 1, pp. 59–65, 2013.

[27] M. Collignon, A.-L. Cristol, P. Dufrénoy, Y. Desplanques, and D. Balloy, “Failure of truck brake discs: A coupled numerical–experimental approach to identifying critical thermomechanical loadings,” *Tribology International*, vol. 59, pp. 114–120, 2013.

[28] S. Hosseini Kordkheili and M. Livani, “Thermoelastic creep analysis of a functionally graded various thickness rotating disk with temperature-dependent material properties,” *International Journal of Pressure Vessels and Piping*, vol. 111, pp. 63–74, 2013.

[29] K. A. Shelar, S. B. Barve, M. R. Chopade, and P. E. Chaudhari, “Thermal analysis of automotive brake rotor,” 2014.

[30] M. Bayat, M. Rahimi, M. Saleem, A. Mohazzab, I. Wudtke, and H. Talebi, “One-dimensional analysis for magneto-thermo-mechanical response in a functionally graded annular variable-thickness rotating disk,” *Applied Mathematical Modelling*, 2014.

[31] A. Kumar, A. K. Tiwari, P. Yadav, H. S. Yadav, and S. B. Lal, “Finite element analysis of disc brake by ANSYS Workbench,” 2014.

[32] G. K. Reddy, N. Chikkanna, H. Monohar, and B. U. Goud, “Experimental evaluation of thermal transport characteristics of MMCs for heat sink material for electronics cooling,” *Journal of Physics D: Applied Physics*, vol. 42, no. 17, pp. 1504–1511, 2014.

[33] “Bike advice.” http://www.bikeadvice.in/disk-brake.

[34] “Disc brake.” http://en.wikipedia.org/wiki/Disc-damage-modes.

[35] “Machine design 2.” http://nptel.ac.in/courses/Machine.Design.

[36] “Disc brake.” http://en.wikipedia.org/wiki/Discbrake.

[37] “Drum brake.” http://www.subaruforester.org/rotor-shafts.

[38] “Disc brake.” http://corymmoore.wordpress.com/disc-brakes.

[39] “Disc brake.” http://auto.howstuffworks.com/brakes.

[40] “Audi chassis.” http://www.audi-sport.net/audi-chassis/brakes.

[41] “Cracked disc.” http://www.cliosport.net/Cracked-brake-disc.

[42] “Rusted brake.” http://www.dreamstime.com/sport-brake-disc.

[43] “Composite materials.” http://www.efunda.com/composites.