DIFFERENT APPROACHES FOR ANALYZING THE FUZZY RELIABILITY OF A MARINE POWER PLANT

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DEDICATED

TO

GOD, MY PARENTS AND MY SUPERVISOR

List of Research Papers

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6. A note on "A novel general approach to evaluating the PCBA for components with different membership function", (Correction of above mentioned paper published in Applied Soft Computing) (**Communicated**)

CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled "**Different Approaches for Analyzing the Fuzzy Reliability of a Marine Power Plant**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of **Dr. Amit Kumar**.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ABSTRACT

One of the important engineering tasks in design and development of a technical system is reliable engineering. It is well known that the conventional reliability analysis, using the probabilities, has been found to be inadequate to handle uncertainty of failure data and modeling. Fuzzy sets theory can be the best tool to solve the above mentioned problem, which is able to analyze the fault events without the need for exact information of fault probability.

In this thesis fuzzy reliability of marine power plant has been analyzed using different approaches and the results are compared. The main topics are fuzzy qualitative analysis of marine power plant using cut set approach, fuzzy reliability analysis using fuzzy number arithmetic operations, fuzzy reliability analysis using *LR* type fuzzy numbers, fuzzy reliability analysis using interval of confidence, fuzzy reliability analysis using with different types of membership functions.

The chapter-wise summary of the thesis is as follows:

Chapter 1 is introductory in nature. This chapter includes basic definitions, operations and concepts used throughout the work. It also presents brief review of the work done in the area of fuzzy reliability analysis.

In **Chapter 2**, to overcome the shortcoming of a fuzzy qualitative analysis of marine power plant existing in literature, a new approach has been developed for the fuzzy qualitative analysis.

In **Chapter 3** using the proposed fuzzy qualitative analysis and arithmetic operations between trapezoidal fuzzy numbers, the fuzzy reliability of marine power plant has been evaluated.

In **Chapter 4** according to real life situations, instead of trapezoidal fuzzy numbers *LR* type fuzzy numbers have been used for analyzing the fuzzy reliability of marine power plant. Also the some expressions existing in literature, for n- ary possibilistic AND, OR and NEG operations has been corrected and the corrected expressions are used for analyzing the fuzzy reliability of marine power plant.

In **Chapter 5** the shortcomings of chapter 3 and chapter 4 have been pointed out and the arithmetic operations between interval of confidence have been used to analyze the fuzzy reliability of marine power plant.

In the last chapter an algorithm has been proposed to find the arithmetic operations between different types of fuzzy numbers and hence using this algorithm the fuzzy reliability of marine power plant has been evaluated.

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Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

There has been tremendous research and development in the field of fuzzy sets and fuzzy logic after pioneering paper by Zadeh [28]. In fact, it made a revolution in the field of uncertainty, which challenged the probability theory. Instead of two-valued logic i.e. success and failure (0 and 1) as used in probability theory, fuzzy set theory advocated whole range of values between 0 and 1. The proposition like *x is a member of A* is not necessarily true in fuzzy set theory. It may be true to some degree. This is described by degree of membership function, which lies between 0 and 1.

As an engineering discipline, reliability is relatively old, but its relevance to modern world and society is still making a great impact. Its growth has been motivated by several factors like complexity and sophistication of systems, organizational awareness to produce and sell high reliability products, apparently conflicting requirement of low cost and high performance product/service. Reliability theory is inter-disciplinary in nature and it studies the dependability of engineering products under specific operating conditions when put into service. Reliability is a measure of the expected capability of an engineering product to operate without failures under specific conditions for a given period of time.

Fuzzy reliability analysis [27] is comparatively a new area of research and interest. Fuzzy reliability analysis is a novel concept in systems engineering as fuzzy sets can capture subjective, uncertain and ambiguous information. This is an alternative reliability theory, which is rooted in fuzzy sets and possibility theory. At any given time, the product may be in operating state to some degree and in failed state in another degree. Also, the behavior of the product with respect to two fuzzy states can be characterized using possibility theory. Fuzzy reliability theories are more meaningful than probabilistic reliability when the number of data or samples available is small or if the data/information is ambiguous, inexact or subjective. It is based on the assumption of fuzzy states and possibility.

The basic definitions used throughout the work are as follows.

1.2 Crisp set theory

In this section, the definitions [2, 9, 10, 19, 25, 27] of crisp set, crisp reliability, the methods to analyze the crisp reliability of series and parallel systems, crisp fault tree and minimal cut set are presented.

Definition 1.1

A crisp set or a classical set A is defined as a collection of distinct and distinguishable objects. The objects are called elements of A. A crisp set A, defined on the universal set X, can also be represented by $A = \{(x, \mu_A(x)); x \in X\}$ where $\mu_A : X \to \{0,1\}$ is called characteristic function defined by

$$\mu_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

Definition 1.2

Crisp reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered.

1.2.1 Crisp reliability of a series system

The components in a set are said to be in series from the reliability point of view if they must all work for a system success or only one need to fail for system failure.

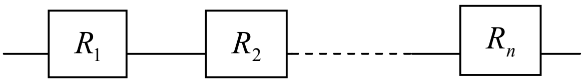

Figure 1.1 Series system

Consider a series system consisting of 'n' components as shown in Figure 1.1. The crisp reliability R_S of the series system is evaluated as follows:

$$R_S = \prod_{i=1}^{n} R_i$$

where R_i represents the reliability of the i^{th} component.

1.2.2 Crisp reliability of a parallel system

The components, in a set, are said to be in parallel from the reliability point of view if only one need to be working for system success or all must fail for system failure.

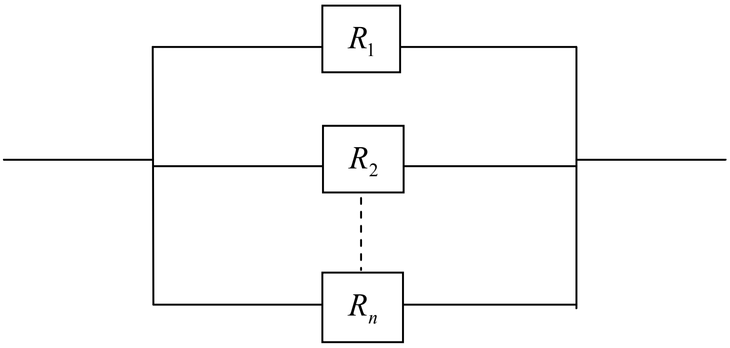

Figure 1.2 Parallel system

Consider a parallel system consisting of 'n' components as shown in Figure 1.2. The crisp reliability R_P of the parallel system is evaluated as follows:

$$R_P = 1 - \prod_{i=1}^{n} (1 - R_i)$$

where R_i represents the reliability of the i^{th} component.

1.2.3 Crisp fault tree

A fault tree is a logic diagram representing various combinations of failure events leading to a major undesired event. Construction of the fault tree begins by selecting the undesired or the top event for analysis. In many cases, this choice is based upon its criticality. Using top-down logic, all immediate cases of the top event are noted along with their logical relationships. These intermediate events are then sub-divided to identify their cases and logical relationships, and so on.

This process continues until the events can no longer be sub-divided or until failure data are no longer available. If no further sub-division is possible, this event is called a basic event. Basic events are represented in the fault tree by a circle. If no failure data are available, then the event is called an undeveloped event. The undeveloped events are represented by the diamond shapes. These events represent initiating conditions which lead to occurrence of the top event.

A fault tree connects all events by logic states, which are essentially AND and OR. The output of an AND gate occurs, if and only if, all input events occur. This describes the intersection of the sets containing all input events of that gate. The output of and OR gate occurs if one of the input event occurs. This describes the union of the sets containing all input events to the gate. In practice, other gates do exist. However, they are simple variations of those listed above.

1.2.4 Minimal cut set

One of the main purposes of representing a fuzzy fault tree in terms of Boolean equations is that these equations can then be used to determine the fuzzy fault tree's associated "minimal cut sets". The minimal cut sets define the "failure modes" of the top event and are usually obtained when a fault tree is evaluated. The formal definition of minimal cut set is defined as: a minimal cut set is a smallest combination of component failures which, if they all occur, will cause the top event to occur. By the definition, a minimal cut set is thus a combination (intersection) of primary events sufficient for the top event. The combination is a "smallest" combination in that all the failures are needed for the top event to occur; if one of the failures in the cut set does not occur, then the top event will not occur (by this combination).

Any fault tree will consist of a finite number of minimal cut sets, which are unique for that top event. The one-component minimal cut sets, if there are any, represent those single failures which will cause the top event to occur. The two-component minimal cut sets represent the double failures which together will cause the top event to occur. For an n-component minimal cut set, all n components in the cut set fail in order for the top event to occur.

The minimal cut set expression for the top event can be written in the general form,

$$T = M_1 + M_2 + \cdots + M_k$$

where T is the top event and M_i are the minimal cut sets. Each minimal cut set consists of a combination of specific component failures, and hence the general n-component minimal cut can be expressed as

$$M_2 = X_1 \times X_2 \times \cdots \times X_n$$

where $X_1, X_2, \cdots, X_n$ are basic component failures on the tree.

To determine the minimal cut sets of a fault tree, the tree is first translated to its equivalent Boolean equations and then either the "top-down" or "bottom-up" substitution method is used. The methods are straightforward and they involve substituting and expanding Boolean expressions. Two Boolean laws, the distributive law and the law of absorption, are used to remove the redundancies. In more complex systems, however, where the system failure modes are not so obvious, the minimal cut set computation provides the analyst with a thorough and systematic method for identifying the basic combinations of component failures which can cause an undesired event.

1.3 Fuzzy set theory

In this section, the definitions [1, 3, 7, 11-16, 19-23, 27-30] of fuzzy set, alpha-cut, support of fuzzy set, convex fuzzy set, fuzzy number, triangular fuzzy number, arithmetic operations between triangular fuzzy numbers, trapezoidal fuzzy number, arithmetic operations between trapezoidal fuzzy numbers, *L-R* type fuzzy number, fuzzy probability, fuzzy reliability, the methods to analyze the fuzzy reliability of series and parallel systems, and fuzzy fault tree are presented.

Definition 1.3

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e $\mu_{\tilde{A}} : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A.

A fuzzy set is a set without a crisp, clearly defined boundary. For the first time, the notion “fuzzy” was used by Zadeh in 1965 [28]. The fuzzy sets theory or the fuzzy logic is based on the idea that each element in a certain system can get one value within the interval 0 to 1. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set. $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more x belongs to A. Therefore, $\tilde{A}$ is completely characterized by the set of ordered pairs.

Definition 1.4

Let $\tilde{A}$ be a fuzzy set and α be a real number in the interval $[0,1]$. The crisp set A_α defined by $A_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$ is called $\alpha-$ **cut** of $\tilde{A}$.

The crisp set $A_{\alpha^+} = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$ is called strong $\alpha-$ **cut** of $\tilde{A}$.

Definition 1.5

The support of a fuzzy set $\tilde{A}$ is the crisp subset of X and is presented as:

$$\text{supp}(\tilde{A}) = \{\, x \in X \mid \mu_{\tilde{A}}(x) > 0 \}.$$

Definition 1.6

A fuzzy set $\tilde{A}$, defined on the universal set X, is said to be convex if

$$\mu_{\tilde{A}}(\alpha_1 x_1 + \alpha_2 x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in X \quad \text{and } \alpha_1, \alpha_2 \geq 0, \quad \alpha_1 + \alpha_2 = 1.$$

Definition 1.7

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$.

3. Its strictly increasing on $[c,a]$ and strictly decreasing on $[b,d]$.

4. $\mu_{\tilde{A}}(x)=1$ for all $x\in[a,b]$.

Eventually, it can be $c=-\infty$ or $c=a$ or $a=b$ or $b=d$ or $d=\infty$.

There are mainly two type of fuzzy number which are described below:

Definition 1.8

A fuzzy number $\tilde{A}=(a_l,a,a_u)$ is called a triangular fuzzy number if its membership function $\mu_{\tilde{A}}$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_l}{a-a_l} & ,a_l\le x\le a,\\[2ex] \dfrac{x-a_u}{a-a_u} & ,a\le x\le a_u.\end{cases}$$

The triangular fuzzy number $\tilde{A}$ has the shape of a triangle as shown in Figure 1.3 given below:

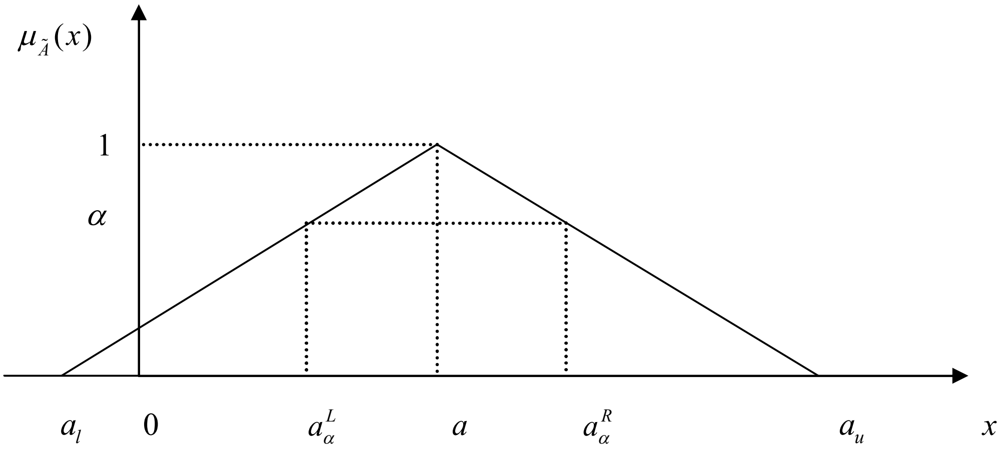

Figure 1.3 A Triangular fuzzy number $\tilde{A}=(a_l,a,a_u)$

Further, the α - cut of the triangular fuzzy number $\tilde{A}=(a_l,a,a_u)$ is the closed interval

$$A_\alpha=\left[a_\alpha^L,a_\alpha^R\right]=\left[a_l+(a-a_l)\alpha\ ,\ a_u+(a-a_u)\alpha\right],\ \alpha\in(0,1].$$

The another representation of triangular fuzzy number is $\tilde{A}=(a-\alpha, a, a+\beta)$ where $a-\alpha=a_l$ and $a+\beta=a_u$ and $\tilde{A}$ can also be written in another form as $\tilde{A}=(a,\alpha,\beta)$.

1.3.1 Arithmetic operations between two triangular fuzzy numbers

Let $\tilde{A}=(a_l,a,a_u)$ and $\tilde{B}=(b_l,b,b_u)$ (or $\tilde{A}=(a,\alpha_1,\beta_1)$ and $\tilde{B}=(b,\alpha_2,\beta_2)$) be two triangular fuzzy numbers, then using the α - cuts, A_α and B_α for $\alpha\in(0,1]$ one can compute $\tilde{A}*\tilde{B}$ where $*$ may be any operation.

Addition ($\oplus$):- $\tilde{A}\oplus\tilde{B}=(a_l+b_l, a+b, a_u+b_u)$

Subtraction (Θ):- $\tilde{A}\,\Theta\,\tilde{B}=(a_l-b_u, a-b, a_u-b_l)$

Scalar Multiplication ($k\tilde{A}$) **:-** $k\tilde{A}=(ka_l,\ ka,\ ka_u)$ if $k>0$ is a scalar and

$k\tilde{A}=(ka_u,\ ka,\ ka_l)$ if $k<0$ is a scalar.

Symmetry (or mirror image) ($-\tilde{A}$) **:-** $-\tilde{A}=(-a_u,-a,-a_l)$.

Multiplication ($\otimes$) :- $\tilde{A}\otimes\tilde{B}=(a',b',c')$ where $a'=\min(a_lb_l, a_lb_u, b_la_u, a_ub_u)$, $b'=ab$ and $c'=\max(a_lb_l, a_lb_u, b_la_u, a_ub_u)$.

Division ($\oslash$) :- $\tilde{A}\oslash\tilde{B}=(a',b',c')$ where $a'=\min\left(\frac{a_l}{b_l},\frac{a_l}{b_u},\frac{a_u}{b_l},\frac{a_u}{b_u}\right)$, $b'=\frac{a}{b}$ and

$$c'=\max\left(\frac{a_l}{b_l},\frac{a_l}{b_u},\frac{a_u}{b_l},\frac{a_u}{b_u}\right),\ b_l>0\ \text{ or }\ b_u<0.$$

Remark – To find arithmetic operations between $\tilde{A}=(a,\alpha_1,\beta_1)$ and $\tilde{B}=(b,\alpha_2,\beta_2)$, first convert $\tilde{A}$ into $\tilde{A}=(a-\alpha_1,a,a+\beta_1)$ and $\tilde{B}$ into $\tilde{B}=(b-\alpha_2,b,b+\beta_2)$ and then use arithmetic operation given in section 1.2 on $\tilde{A}$ and $\tilde{B}$.

Definition 1.9

A fuzzy number $\tilde{A}=(a_l,\underline{a},\bar{a},a_u)$ is called a trapezoidal fuzzy number if its membership function is given by :

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_l}{\underline{a}-a_l} & ,a_l\le x\le \underline{a}, \\ 1 & ,\underline{a}\le x\le \bar{a}, \\ \dfrac{x-a_u}{\bar{a}-a_u} & ,\bar{a}\le x\le a_u.\end{cases}$$

The trapezoidal fuzzy number $\tilde{A}$ is denoted by the quadruplet $\tilde{A}=(\underline{a}-\alpha,\ \underline{a},\ \bar{a},\ \bar{a}+\beta)$ and has the shape of a trapezoid as shown in the Figure 1.4 given below:

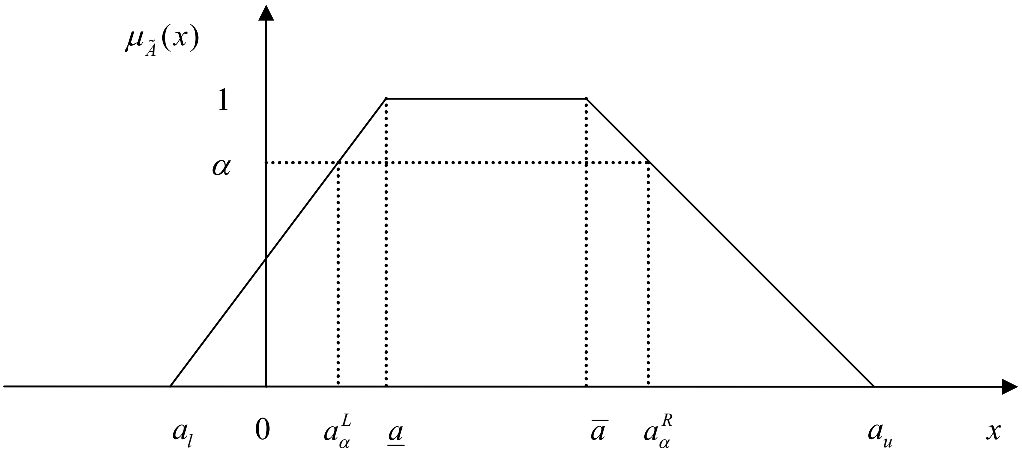

Figure1.4 Trapezoidal fuzzy number $\tilde{A}=(a_l,\underline{a},\bar{a},a_u)$

Further, the α- cut of the trapezoidal fuzzy number $\tilde{A}=(a_l,\underline{a},\bar{a},a_u)$ is the closed interval

$$A_\alpha=\left[a_\alpha^L,a_\alpha^R\right]=\left[a_l+(\underline{a}-a_l)\alpha\ ,\ a_u+(\bar{a}-a_u)\alpha\right],\ \alpha\in(0,1].$$

The another representation of trapezoidal fuzzy number is $\tilde{A}=(\underline{a}-\alpha,\underline{a},\bar{a},\bar{a}+\beta)$ where $\underline{a}-\alpha=a_l$ and $\bar{a}+\beta=a_u$ and $\tilde{A}$ can also be written in another form as $\tilde{A}=(\underline{a},\bar{a},\alpha,\beta)$.

1.3.2 Arithmetic operations between two trapezoidal fuzzy numbers

Let $\tilde{A} = (a_l, \underline{a}, \overline{a}, a_u)$ and $\tilde{B} = (b_l, \underline{b}, \overline{b}, b_u)$ (or $\tilde{A} = (\underline{a}, \overline{a}, \alpha_1, \beta_1)$ and $\tilde{B} = (\underline{b}, \overline{b}, \alpha_2, \beta_2)$) be two trapezoidal fuzzy numbers, then using the α - cuts, A_α and B_α for $\alpha \in (0,1]$ one can compute $\tilde{A} * \tilde{B}$ where $*$ may be any operation.

Addition ($\oplus$) :- $\tilde{A} \oplus \tilde{B} = (a_l + b_l,\ \underline{a} + \underline{b},\ \overline{a} + \overline{b},\ a_u + b_u)$

Subtraction (Θ):- $\tilde{A}\ \Theta\, \tilde{B} = (a_l - b_u, a - b, a_u - b_l)$

Scalar Multiplication ($k\tilde{A}$) :- $k\tilde{A} = (ka_l, k\underline{a}, k\overline{a}, ka_u)$ if $k > 0$ is a scalar and

$k\tilde{A} = (ka_u,\ k\overline{a},\ k\underline{a},\ ka_l)$ if $k < 0$ is a scalar.

Symmetry (or mirror image) ($-\tilde{A}$) :- $-\tilde{A} = (-a_u, -\overline{a}, -\underline{a}, -a_l)$

Multiplication ($\otimes$):- $\tilde{A} \otimes \tilde{B} = (a', b', c')$ where $a' = \min(a_l b_l, a_l b_u, b_l a_u, a_u b_u)$, $b' = ab$

and $c' = \max(a_l b_l, a_l b_u, b_l a_u, a_u b_u)$.

Division ($\oslash$):- $\tilde{A}\ \oslash \tilde{B} = (a', b', c')$ where $a' = \min\left(\frac{a_l}{b_l}, \frac{a_l}{b_u}, \frac{a_u}{b_l}, \frac{a_u}{b_u}\right)$, $b' = \frac{a}{b}$ and

$$c' = \max\left(\frac{a_l}{b_l}, \frac{a_l}{b_u}, \frac{a_u}{b_l}, \frac{a_u}{b_u}\right),\ b_l > 0 \ \text{ or } \ b_u < 0.$$

Definition 1.10

A fuzzy number $\tilde{A}$ in $\Re$ (real line) is said to be a *LR* type fuzzy number if its membership function is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} R\left(\dfrac{x-a}{\beta}\right), & a \le x \le a + \beta, \\ L\left(\dfrac{a-x}{\alpha}\right), & a - \alpha \le x \le a, \\ 0, & \text{otherwise} \end{cases}$$

where $a \in \Re$ is the center, $\alpha > 0$ is the left spread and $\beta > 0$ is the right spread of $\widetilde{A}$. L and R are non-increasing and continuous functions from [0, 1] to [0, 1] satisfying $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. A *L-R* type fuzzy number $\widetilde{A}$ is denoted by $\widetilde{A} = (a, \alpha, \beta)_{LR}$.

1.3.3 Fuzzy probability

Fuzzy probability represents a fuzzy number, between zero and one, assigned to the probability of an event. One can choose different types of membership functions for fuzzy probability.

The belief that an event A is said to occur with probability p can be described in terms of the following membership function:

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x = p \\ 0 & \text{otherwise} \end{cases}$$

The belief that the event A is said to occur with probability $p \in [a,b]$, can be described by the following rectangular membership function:

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x = [a,b] \\ 0 & \text{otherwise} \end{cases}$$

The belief that the event A is said to occur with probability $p \in [a,b]$, in which some portion of $[a,b]$ is more accurate than other portions, can be described by a membership function; the more confident portion is given the value 1 and other portions are given values between [0, 1].

Experts can use different kinds of membership functions to subjectively address the uncertainty of a failure probability. Fuzzy probabilities with triangular membership function, trapezoidal membership function or bell-shaped membership function [16] are most often used fuzzy numbers.

Definition 1.11

The crisp reliability is the ability of the component to keep certain state in some certain situation. Usually, the crisp reliability is described by probability of ‘success’ of the component.

The fuzzy reliability means that the ‘the ability of the component to keep certain state in some certain situation’ is a fuzzy conception, and the fuzzification of the conception is the base of the fuzzy theory. The mathematics base is the fuzzy mathematics.

1.3.4 Fuzzy reliability analysis of a series system

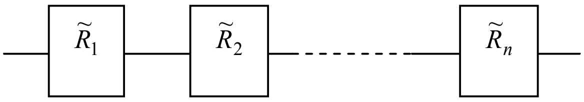

Figure 1.5 Series system

Consider a series system consisting of ‘n’ components as shown in Figure 1.5. The fuzzy reliability $\widetilde{R}_S$ of the series system is evaluated as follows:

$$\widetilde{R}_S = \overset{n}{\underset{i=1}{\otimes}} \widetilde{R}_i ,$$

where $\widetilde{R}_i$ represents the fuzzy reliability of the i^{th} component.

1.3.5 Fuzzy reliability analysis of a parallel system

Consider a parallel system consisting of ‘n’ components as shown in Figure 1.6.

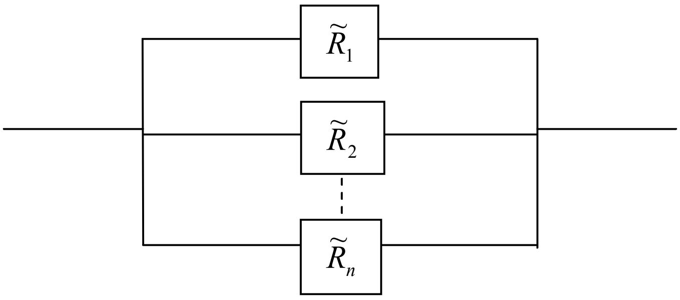

Figure 1.6 Parallel system

The fuzzy reliability $\widetilde{R}_P$ of the parallel system is evaluated as follows:

$$\widetilde{R}_P = 1 \Theta \overset{n}{\underset{i=1}{\otimes}} (1 \Theta \widetilde{R}_i)$$

where $\widetilde{R}_i$ represents the fuzzy reliability of the i^{th} component.

1.3.6 Fuzzy fault tree analysis

In fault tree, probability of all basic events are needed in quantitative analysis and these probabilities are mostly obtained by statistical data or subjective judgmental data based on expert's experience. The data have uncertainty itself because of various affecting factors during the statistical procedure and the limitation of expert's experiences. It is necessary to define fuzzy value in probabilistic space and possibility of failure instead of failure probability.

In fuzzy fault tree analysis method, instead of assuming the input parameter as a random variable it is considered as a fuzzy number and the top event is propagated by fuzzy algebraic operations. Such methods are based on alpha cut method, also known as resolution identity, which are computationally simple and can be applied to a fault tree with repeated events.

1.4 Literature review

In this section the work done in the area of fuzzy reliability analysis has been presented.

For fast technology innovation, the developments of new products are becoming much complicated not only due to its system functioning but also because of its system components. Therefore, system reliability analysis is an important issue for academic research and practice. In the beginning, the idea of reliability of a system or a machine or a person was based on theoretical discussions. Later on, a mathematical shape was given to the concept of reliability of a system. The mathematical development of reliability gave thrust to many ideas in the field of electrical, mechanical and electronics engineering and allied industries.

Fault tree analysis (FTA) is a powerful diagnosis technique and is widely used for demonstrating the root causes of undesired event in system failure. The concept of FTA was developed by Bell telephone laboratory in 1961. It is now widely used in many fields, such as in nuclear reactors, chemical and aviation industries [1, 2, 9, 11, 18-21, 26].

Fault tree models are often used to determine the reliability of large integrated systems whose failure occurs due to failure of components in the systems. Fault-tree is a logical diagram from which the probability of the top event can be determined with known failure probabilities of components in the system or primary events. The probability of the top event is a function of the failure probability of a primary event. Variation in the probability of primary event may result in variation of the top event probability. Quantitative analysis of a fault tree consists of transforming its established logical structure into an equivalent probability form. The failure probability of the top event is calculated from the failure probabilities of the basic events.

In general, the failure probabilities of components are considered as crisp. These crisp values are determined from past occurrences which are very uncertain, since the systems environment change. Moreover, failures of components which have never failed before must be considered. Therefore, some degree of uncertainty exists in such cases.

Singer [24] presented a fuzzy set [28] approach for fault tree and the reliability analysis in which the relative frequencies of the basic events are considered as fuzzy numbers.

Initially, Cai et al. [4] pointed out that there are two fundamental assumptions in the conventional reliability theory, which are as follows:

a. Binary state assumptions: the system is precisely defined as functioning or failing.
b. Probability assumptions: the system behavior is fully characterized in the context of probability measures.

However, because of the inaccuracy and uncertainty in data, the estimation of precise values of probability becomes very difficult in many systems.

Therefore, they [5] presented the following two assumptions:

a. Fuzzy-State assumption: the meaning of the system failure can’t be precisely defined in a reasonable way. At any time the system may be in one of the following two states: fuzzy success state or fuzzy failure state.
b. Possibility assumption: the system behavior can be fully characterized in the context of possibility measures.

Further, Cai et al. [6] has given the following three forms of “fuzzy reliability theories” :

a. PROFUST reliability theory, based on the PRObability assumption and FUzzy-STate assumption.
b. POSBIST reliability theory, based on the POSsibility assumption and BInary STate assumption.
c. POSFUST reliability theory, based on the POSsibility assumption and the FUzzy-STate assumption.

Cheng and Mon [8] used interval of confidence for analyzing the fuzzy system reliability. Through theoretical analysis and computational results they have shown that their proposed approach is more general and simpler than presented in [24].

Chen [7] presented a new method for analyzing the fuzzy system reliability using fuzzy number arithmetic operations and used simplified fuzzy arithmetic operations rather than complicated interval fuzzy arithmetic operations of fuzzy numbers [8] or the complicated extended algebraic fuzzy numbers [24].

Conventionally, it is not always easy to obtain the system reliability for components with different individual failure probability density functions, due to necessary but complicated combination and integration of various probability density functions. For such a problem, and considering fuzzy environment and data imprecision in real life, Mon and Cheng [22] applied fuzzy set theory to solve the inadequacy of the conventional probability in accounting and processing of built-in uncertainties in the probabilistic data and used fuzzy distribution instead of the classical probability distribution for the components and calculate the functions of fuzzy numbers to solve the fuzzy system reliability via non-linear programming.

Chapter 2

FUZZY QUALITATIVE ANALYSIS OF MARINE POWER PLANT USING CUT SET APPROACH

2.1 Introduction

Systematic fuzzy fault tree analysis involves finding out the minimal cut sets before the quantification phase. This will ensure elimination of redundancy or repetition of same events.

Kumar et al. [17] proposed an algorithm to analyze the fuzzy reliability of a marine power plant. In their problem two events are repeating but they have carried out all the calculations without finding out the minimal cut sets which is not mathematically correct to quantify directly at the gate level. To overcome this shortcoming, in this chapter, the qualitative analysis of marine power plant has been studied using minimal cut sets and the work done by Kumar et al [17] has been corrected.

2.2 Algebraic properties of fuzzy sets

The algebraic properties of fuzzy sets have immediate practical importance in relation to fuzzy fault trees. A fuzzy fault tree can be thought of as a pictorial representation of those relationships among fuzzy fault events which cause the top event to occur. In fact, a fuzzy fault tree can always be translated into an entirely equivalent set of fuzzy equations. Thus an understanding the properties of fuzzy sets contributes materially toward the construction and simplification of fuzzy fault trees. Once a fuzzy fault tree has been drawn, it can be evaluated to yield its qualitative and quantitative characteristics. These characteristics cannot be obtained directly from the fuzzy fault tree, but they can be obtained from the equivalent fuzzy equations. In this evaluation process the fuzzy algebraic reduction techniques can be used.

The fuzzy sets $\tilde{X}$ and $\tilde{Y}$ satisfies the following properties:

Mathematical symbolism	**Engineering symbolism**	**Designation**
$(1a)\ \tilde{X}\cap\tilde{Y}=\tilde{Y}\cap\tilde{X}$	$\tilde{X}\otimes\tilde{Y}=\tilde{Y}\otimes\tilde{X}$	Commutative Law
$(1b)\ \tilde{X}\cup\tilde{Y}=\tilde{Y}\cup\tilde{X}$	$\tilde{X}\otimes\tilde{Y}=\tilde{Y}\otimes\tilde{X}$	
$(2a)\ \tilde{X}\cap\left(\tilde{Y}\cap\tilde{Z}\right)=\left(\tilde{X}\cap\tilde{Y}\right)\cap\tilde{Z}$	$\tilde{X}\otimes\left(\tilde{Y}\otimes\tilde{Z}\right)=\left(\tilde{X}\otimes\tilde{Y}\right)\otimes\tilde{Z}$	Associative Law
$(2b)\ \tilde{X}\cup\left(\tilde{Y}\cup\tilde{Z}\right)=\left(\tilde{X}\cup\tilde{Y}\right)\cup\tilde{Z}$	$\tilde{X}\oplus\left(\tilde{Y}\oplus\tilde{Z}\right)=\left(\tilde{X}\oplus\tilde{Y}\right)\oplus\tilde{Z}$	
$(3a)\ \tilde{X}\cap\left(\tilde{Y}\cup\tilde{Z}\right)=\left(\tilde{X}\cap\tilde{Y}\right)\cap\left(\tilde{X}\cap\tilde{Z}\right)$	$\tilde{X}\otimes\left(\tilde{Y}\oplus\tilde{Z}\right)=\left(\tilde{X}\otimes\tilde{Y}\right)\oplus\left(\tilde{X}\otimes\tilde{Z}\right)$	Distributive Law
$(3b)\ \tilde{X}\cup\left(\tilde{Y}\cap\tilde{Z}\right)=\left(\tilde{X}\cup\tilde{Y}\right)\cap\left(\tilde{X}\cup\tilde{Z}\right)$	$\tilde{X}\oplus\tilde{Y}\otimes\tilde{Z}=\left(\tilde{X}\oplus\tilde{Y}\right)\otimes\left(\tilde{X}\oplus\tilde{Z}\right)$	
$(4a)\tilde{X}\cap\tilde{X}=\tilde{X}$	$\tilde{X}\otimes\tilde{X}=\tilde{X}$	Idemponent Law
$(4b)\tilde{X}\cup\tilde{X}=\tilde{X}$	$\tilde{X}\oplus\tilde{X}=\tilde{X}$	
$(5a)\tilde{X}\cap\left(\tilde{X}\cup\tilde{Y}\right)=\tilde{X}$	$\tilde{X}\otimes\left(\tilde{X}\oplus\tilde{Y}\right)=\tilde{X}$	Law of Absorption
$(5b)\tilde{X}\cup\left(\tilde{X}\cap\tilde{Y}\right)=\tilde{X}$	$\tilde{X}\oplus\left(\tilde{X}\otimes\tilde{Y}\right)=\tilde{X}$	
$(6)\left(\tilde{X}'\right)'=\tilde{X}$	$\left(\tilde{X}'\right)'=\tilde{X}$	
$(7a)\left(\tilde{X}\cap\tilde{Y}\right)'=\tilde{X}'\cup\tilde{Y}'$	$\left(\tilde{X}\otimes\tilde{Y}\right)'=\tilde{X}'\oplus\tilde{Y}'$	De Morgan's Law
$(7b)\left(\tilde{X}\cup\tilde{Y}\right)'=\tilde{X}'\cap\tilde{Y}'$	$\left(\tilde{X}\oplus\tilde{Y}\right)'=\tilde{X}'\otimes\tilde{Y}'$	
$(8a)\tilde{X}\cup\left(\tilde{X}'\cap\tilde{Y}\right)=\tilde{X}\cup\tilde{Y}$	$\tilde{X}\oplus\left(\tilde{X}'\otimes\tilde{Y}\right)=\tilde{X}\oplus\tilde{Y}$	
$(8b)\tilde{X}'\cap\left(\tilde{X}\cup\tilde{Y}'\right)=\tilde{X}'\cap\tilde{Y}'=\left(\tilde{X}\cup\tilde{Y}\right)'$	$\tilde{X}'\otimes\left(\tilde{X}\oplus\tilde{Y}'\right)=\tilde{X}'\otimes\tilde{Y}'=\left(\tilde{X}\oplus\tilde{Y}\right)'$	

Table 2.1

2.3 Case study

A marine power plant [25] has two generators G_1 and G_2 one located at the stern and the other at bow. Each generator is connected to its respective micro switch board-1 and micro switch board-2. The distributive switch board receives the supply from the switch boards through cables C_1 and C_2 and respective junction boxes D and E. The two micro switch boards are interconnected through a long cable C_3 and the junction boxes A and B. The schematic diagram is shown in Figure 2.1.

The basic components subjected to failure are

(a) Generators G_1 and G_2.

(b) Micro switch board-1 and Micro switch board-2.

(c) Interconnecting cables C_3 and junction boxes A and B, all treated as one unit.

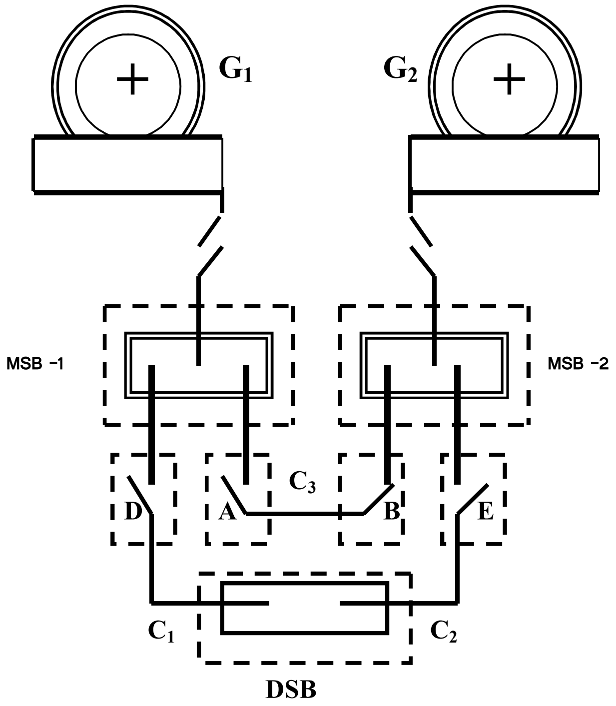

Figure 2.1 Marine power plant

2.3.1 Fault tree

The fault tree for the marine power plant is shown in Figure 2.2.

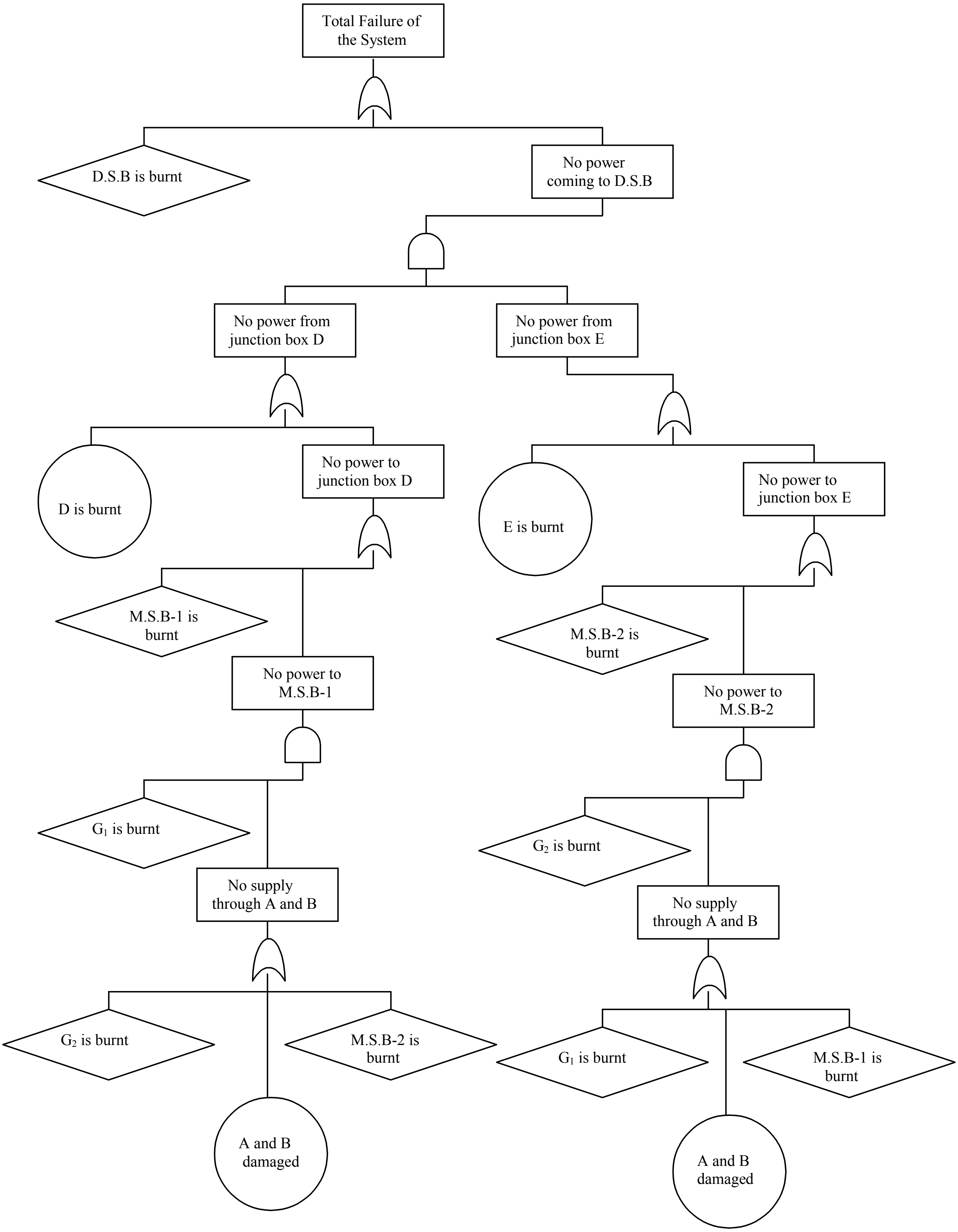

Figure 2.2 Fault tree of marine power plant

2.3.2 Fuzzy fault tree

With the help of fault tree a fuzzy fault tree, shown in Figure 2.3., is developed for analyzing the fuzzy reliability of marine power plant.

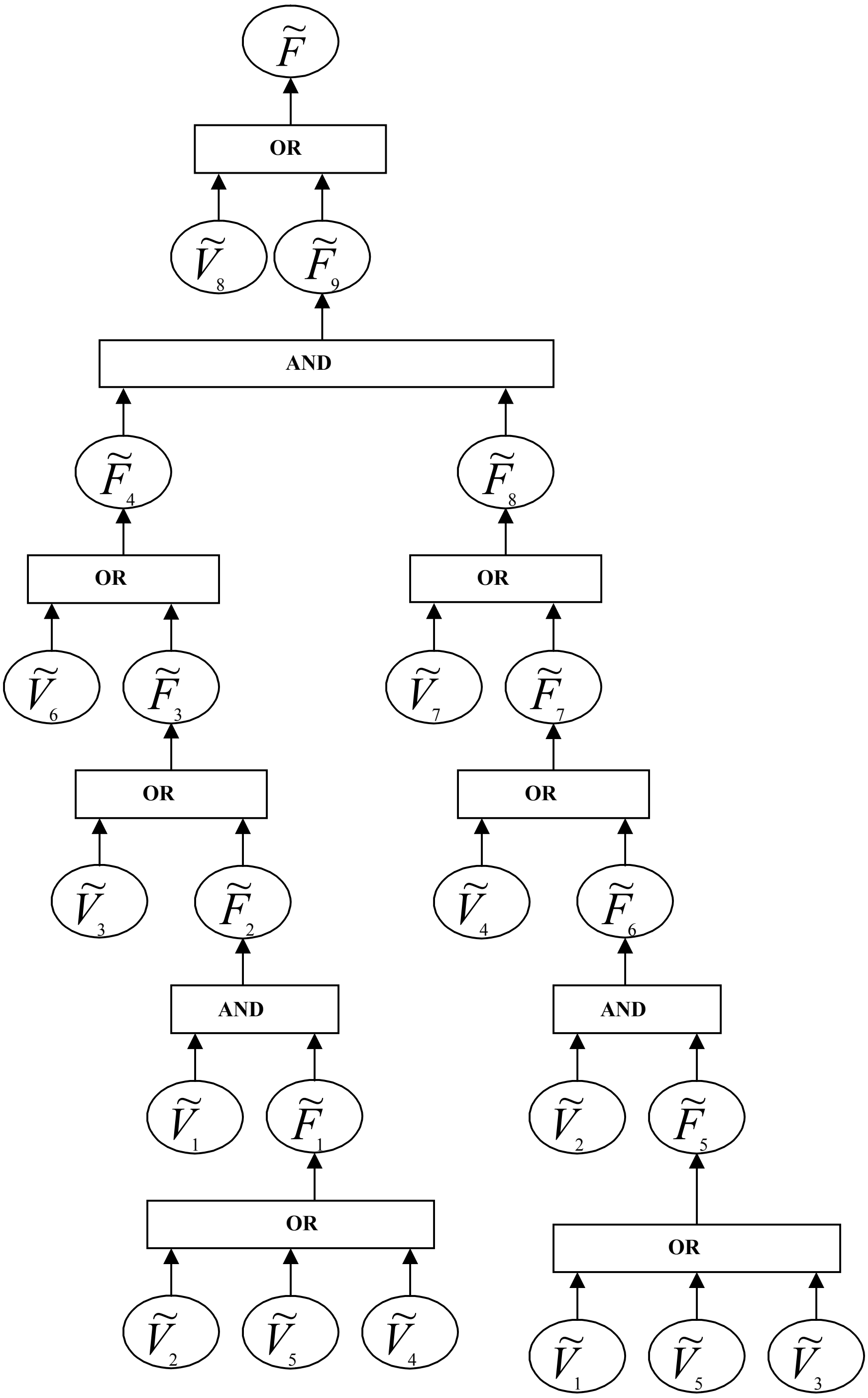

Figure 2.3

2.3.3 Notations

$R_{\tilde{V}_1} =< [(x_1, y_1, z_1, w_1)] >$, represents the fuzzy unreliability of generator G1.

$R_{\tilde{V}_2} =< [(x_2, y_2, z_2, w_2)] >$, represents the fuzzy unreliability of generator G2.

$R_{\tilde{V}_3} =< [(x_3, y_3, z_3, w_3)] >$, represents the fuzzy unreliability of micro switch board-1.

$R_{\tilde{V}_4} =< [(x_4, y_4, z_4, w_4)] >$, represents the fuzzy unreliability of micro switch board-2.

$R_{\tilde{V}_5} =< [(x_5, y_5, z_5, w_5)] >$, represents the fuzzy unreliability of the junction boxes A and B.

$R_{\tilde{V}_6} =< [(x_6, y_6, z_6, w_6)] >$, represents the fuzzy unreliability of the junction box D.

$R_{\tilde{V}_7} =< [(x_7, y_7, z_7, w_7)] >$, represents the fuzzy unreliability of the junction box E.

$R_{\tilde{V}_8} =< [(x_8, y_8, z_8, w_8)] >$, represents the fuzzy unreliability of the distributive switch board.

$R_{\tilde{F}_1} =< [(X_1, Y_1, Z_1, W_1)] >$, represents the fuzzy reliability of event that there is no power through the junction boxes A and B.

$R_{\tilde{F}_2} =< [(X_2, Y_2, Z_2, W_2)] >$, represents the fuzzy reliability of the event that no power is coming to micro switch board-1.

$R_{\tilde{F}_3} =< [(X_3, Y_3, Z_3, W_3)] >$, represents the fuzzy reliability of the event that there is no power to the junction box D.

$R_{\tilde{F}_4} =< [(X_4, Y_4, Z_4, W_4)] >$, represents the fuzzy reliability of the event that no power from the junction box D.

$R_{\tilde{F}_5} =< [(X_5, Y_5, Z_5, W_5)] >$,represents the fuzzy reliability of the event that there is no power through the junction boxes D and E.

$R_{\tilde{F}_6} =< \left[\left(X_6, Y_6, Z_6, W_6 \right) \right] >$, represents the fuzzy reliability of the event that no power is coming to micro switch board-2.

$R_{\tilde{F}_7} =< \left[\left(X_7, Y_7, Z_7, W_7 \right) \right] >$, represents the fuzzy reliability of the event that there is no power to the junction box E.

$R_{\tilde{F}_8} =< \left[\left(X_8, Y_8, Z_8, W_8 \right) \right] >$, represents the fuzzy reliability of the event that no power from the junction box E.

$R_{\tilde{F}_9} =< \left[\left(X_9, Y_9, Z_9, W_9 \right) \right] >$, represents the fuzzy reliability of the event that no power is coming to distributive switch board.

$R_{\tilde{F}}$, represents the fuzzy unreliability of the system.

$\widetilde{R}$, represents the fuzzy reliability of the system.

2.3.4 Proposed algorithm

In this section, a new algorithm has been proposed to evaluate the fuzzy reliability of the marine power plant. The qualitative analysis of the marine power plant will be completed by the construction of the “minimal cut sets” of the fuzzy fault tree. Firstly, code the fuzzy fault tree so that it can be described by a set of fuzzy algebraic equations and then, solve these equations so that the top and intermediate events are individually expressed in terms of minimal cut sets that involve only basic events. The steps of the proposed algorithm are as follows:

Step 1

$$\tilde{F}_1 = \tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5$$

Step 2

$$\begin{aligned} \tilde{F}_2 &= \tilde{V}_1 \otimes \tilde{F}_1 \\ &= \tilde{V}_1 \otimes \left(\tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5 \right) \end{aligned}$$

Step 3

$$\begin{aligned}\tilde{F}_3 &= \tilde{V}_3 \oplus \tilde{F}_2 \\ &= \tilde{V}_3 \oplus \tilde{V}_1 \otimes \left(\tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5\right)\end{aligned}$$

Step 4

$$\begin{aligned}\tilde{F}_4 &= \tilde{V}_6 \oplus \tilde{F}_3 \\ &= \tilde{V}_6 \oplus \tilde{V}_3 \oplus \left(\tilde{V}_1 \otimes \left(\tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5\right)\right)\end{aligned}$$

Step 5

$$\tilde{F}_5 = \tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5$$

Step 6

$$\begin{aligned}\tilde{F}_6 &= \tilde{V}_2 \otimes \tilde{F}_5 \\ &= \tilde{V}_2 \otimes \left(\tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5\right)\end{aligned}$$

Step 7

$$\begin{aligned}\tilde{F}_7 &= \tilde{V}_4 \oplus \tilde{F}_6 \\ &= \tilde{V}_4 \oplus \tilde{V}_2 \otimes \left(\tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5\right)\end{aligned}$$

Step 8

$$\begin{aligned}\tilde{F}_8 &= \tilde{V}_7 \oplus \tilde{F}_7 \\ &= \tilde{V}_7 \oplus \tilde{V}_4 \oplus \left(\tilde{V}_2 \otimes \left(\tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5\right)\right)\end{aligned}$$

Step 9

$$\begin{aligned}\tilde{F}_9 &= \tilde{F}_4 \otimes \tilde{F}_8 \\ &= \left(\tilde{V}_6 \oplus \tilde{V}_3 \oplus \left(\tilde{V}_1 \otimes \left(\tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5\right)\right)\right) \otimes \left(\tilde{V}_7 \oplus \tilde{V}_4 \oplus \left(\tilde{V}_2 \otimes \left(\tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5\right)\right)\right)\end{aligned}$$

Step 10 The top event $\tilde{F}$ is expressed in terms of the basic events by substitution as,

$$\begin{aligned}\tilde{F} &= \tilde{V}_8 \oplus \tilde{F}_9 \\ &= \tilde{V}_8 \oplus \left[\left(\tilde{V}_6 \oplus \tilde{V}_3 \oplus \left(\tilde{V}_1 \otimes \left(\tilde{V}_2 \oplus \tilde{V}_4 \oplus \tilde{V}_5\right)\right)\right) \otimes \left(\tilde{V}_7 \oplus \tilde{V}_4 \oplus \left(\tilde{V}_2 \otimes \left(\tilde{V}_1 \oplus \tilde{V}_3 \oplus \tilde{V}_5\right)\right)\right)\right]\end{aligned}$$

Using the algebraic properties of fuzzy sets,

$$
\begin{aligned}
&= \tilde{V}_8 \oplus \left(\tilde{V}_6 \oplus \tilde{V}_3 \oplus \tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_1\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_4\right) \otimes \left(\tilde{V}_7 \oplus \tilde{V}_4 \oplus \tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_2\tilde{V}_5 \oplus \tilde{V}_2\tilde{V}_3\right) \\
&= \tilde{V}_8 \oplus \tilde{V}_6\tilde{V}_7 \oplus \tilde{V}_4\tilde{V}_6 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_6 \oplus \tilde{V}_2\tilde{V}_5\tilde{V}_6 \oplus \tilde{V}_2\tilde{V}_3\tilde{V}_6 \oplus \tilde{V}_3\tilde{V}_7 \oplus \tilde{V}_3\tilde{V}_4 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_3 \oplus \tilde{V}_2\tilde{V}_3\tilde{V}_5 \oplus \tilde{V}_2\tilde{V}_3 \\
&\quad \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_7 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_4 \oplus \tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_3 \oplus \tilde{V}_2\tilde{V}_3\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_4\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_3\tilde{V}_5 \oplus \\
&\quad \tilde{V}_1\tilde{V}_4\tilde{V}_7 \oplus \tilde{V}_1\tilde{V}_4 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_4 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_4\tilde{V}_5 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_3\tilde{V}_4
\end{aligned}
$$

Since

$\tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_6 = \tilde{V}_1\tilde{V}_2$, $\tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_1\tilde{V}_2\tilde{V}_3\tilde{V}_5 = \tilde{V}_1\tilde{V}_2$ etc. (Using the Absorption law of fuzzy sets)

The minimal cut set for the top event $\tilde{F}$ is expressed as:

$$
\tilde{F} = \tilde{V}_8 \oplus \tilde{V}_1\tilde{V}_2 \oplus \tilde{V}_1\tilde{V}_4 \oplus \tilde{V}_2\tilde{V}_3 \oplus \tilde{V}_3\tilde{V}_4 \oplus \tilde{V}_3\tilde{V}_7 \oplus \tilde{V}_4\tilde{V}_6 \oplus \tilde{V}_6\tilde{V}_7 \oplus \tilde{V}_1\tilde{V}_5\,\tilde{V}_7 \oplus \tilde{V}_2\tilde{V}_5\tilde{V}_6
$$

This expression gives the event combinations which consists of one one-event, seven two-event and two four-event minimal cut sets that result in the total failure of the system. The fuzzy reliability of the system can be obtained now as follows.

The fuzzy unreliability is

$$
\begin{aligned}
R_{\tilde{F}} &= 1\Theta\left(1\Theta R_{\tilde{V}_8}\right)\otimes\left(1\Theta R_{\tilde{V}_1}R_{\tilde{V}_2}\right)\otimes\left(1\Theta R_{\tilde{V}_1}R_{\tilde{V}_4}\right)\otimes\left(1\Theta R_{\tilde{V}_2}R_{\tilde{V}_3}\right)\otimes\left(1\Theta R_{\tilde{V}_3}R_{\tilde{V}_4}\right)\otimes\left(1\Theta\right. \\
&\quad \left. R_{\tilde{V}_3}R_{\tilde{V}_7}\right)\otimes\left(1\Theta\ R_{\tilde{V}_4}R_{\tilde{V}_6}\right)\otimes\left(1\Theta R_{\tilde{V}_6}R_{\tilde{V}_7}\right)\otimes\left(1\Theta R_{\tilde{V}_1}R_{\tilde{V}_5}R_{\tilde{V}_7}\right)\otimes\left(1\Theta R_{\tilde{V}_2}R_{\tilde{V}_5}R_{\tilde{V}_6}\right)
\end{aligned}
$$

which can be represented by Figure 2.4

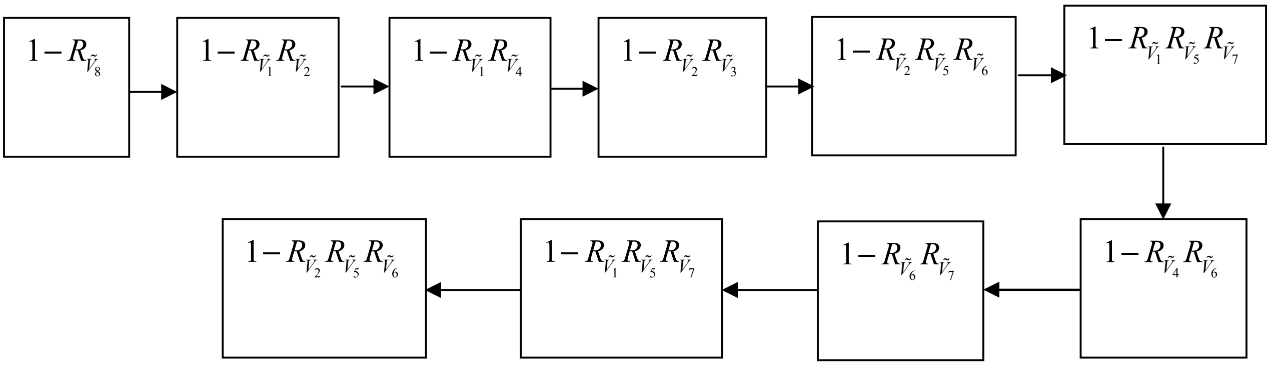

Figure 2.4

The fuzzy reliability of marine power plant is $\tilde{R} = 1 \ominus R_{\tilde{F}}$

$$= \left(1 \ominus R_{\tilde{V}_8}\right) \otimes \left(1 \ominus R_{\tilde{V}_1} R_{\tilde{V}_2}\right) \otimes \left(1\ R_{\tilde{V}_1} R_{\tilde{V}_4}\right) \otimes \left(1 \ominus R_{\tilde{V}_2} R_{\tilde{V}_3}\right) \otimes \left(1 \ominus R_{\tilde{V}_3} R_{\tilde{V}_4}\right) \otimes \left(1 \ominus R_{\tilde{V}_3} R_{\tilde{V}_7}\right) \otimes \left(1 \ominus R_{\tilde{V}_4} R_{\tilde{V}_6}\right) \otimes \left(1 \ominus R_{\tilde{V}_6} R_{\tilde{V}_7}\right) \otimes \left(1 \ominus R_{\tilde{V}_1} R_{\tilde{V}_5} R_{\tilde{V}_7}\right) \otimes \left(1 \ominus R_{\tilde{V}_2} R_{\tilde{V}_5} R_{\tilde{V}_6}\right) \tag{2.1}$$

2.4 Result and discussion

Equation (2.1) obtained by proposed algorithm can be used for the quantitative analysis of marine power plant.

2.5 Conclusion

In this chapter, to overcome the shortcoming of Kumar et al. [17] a new approach for qualitative analysis of marine power plant has been proposed using minimal cut set approach.

Chapter 3

FUZZY RELIABILITY ANALYSIS USING FUZZY NUMBER ARITHMETIC OPERATIONS

3.1 Introduction

In this chapter, fuzzy reliability of marine power plant, studied in previous chapter, has been evaluated using equation (2.1). To evaluate the fuzzy reliability, the reliabilities of the basic events are represented by trapezoidal fuzzy numbers and the arithmetic operations between trapezoidal fuzzy numbers have been used.

3.2 Arithmetic operations

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers of the universe of discourse U with the membership functions $f_{\tilde{A}}$ and $f_{\tilde{B}}$, respectively , where $f_{\tilde{A}} : U \rightarrow [0,1]$ and $f_{\tilde{B}} : U \rightarrow [0,1]$, let x and y be two real numbers in U . The fuzzy number arithmetic operations are given below:

Fuzzy number addition $\oplus$:

$$f_{\tilde{A}\oplus\tilde{B}}(z) = \underset{z=x+y}{\vee}\left(f_{\tilde{A}}(x) \wedge f_{\tilde{B}}(y)\right)$$

Fuzzy number subtraction Θ:

$$f_{\tilde{A}\Theta\tilde{B}} = \underset{z=x-y}{\vee}\left(f_{\tilde{A}}(x) \wedge f_{\tilde{B}}(y)\right)$$

Fuzzy number multiplication $\otimes$:

$$f_{\tilde{A}\otimes\tilde{B}}(z) = \underset{z=xy}{\vee}\left(f_{\tilde{A}}(x) \wedge f_{\tilde{B}}(y)\right)$$

Fuzzy number division$\oslash$:

$$f_{\tilde{A}\oslash\tilde{B}}(z)=\underset{z=x/y}{\vee}\left(f_{\tilde{A}}(x)\wedge f_{\tilde{B}}(y)\right)$$

3.3 Fuzzy reliability analysis of series and parallel systems

In this section the expressions for analyzing the fuzzy reliability of series and parallel systems has been obtained.

3.3.1 Series system

Consider a series system, shown in Figure 3.1, where the reliability of the subsystem P_i is represented by a trapezoidal fuzzy number $\tilde{R}_i$, $\tilde{R}_i=\left(m_i-\alpha_i,m_i,n_i,n_i+\beta_i\right)$, where α_i and β_i are called left and right spreads, respectively and $1\le i\le k$.

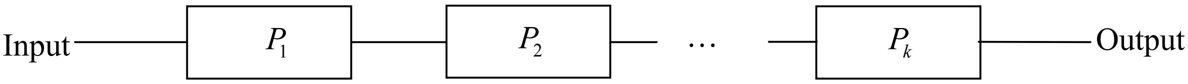

Figure 3.1

The fuzzy reliability of the series system can be evaluated as follows:

$$\begin{aligned}\tilde{R}_1\otimes\tilde{R}_2\otimes\cdots\otimes\tilde{R}_k&=\left(m_1-\alpha_1,m_1,n_1,n_1+\beta_1\right)\otimes\left(m_2-\alpha_2,m_2,n_2,n_2+\beta_2\right)\\&\otimes\cdots\otimes\left(m_k-\alpha_k,m_k,n_k,n_k+\beta_k\right)\\&=\left(\prod_{i=1}^{k}\left(m_i-\alpha_i\right),\prod_{i=1}^{k}m_i,\prod_{i=1}^{k}n_i,\prod_{i=1}^{k}\left(n_i-\beta_i\right)\right)\end{aligned}$$

3.3.2 Parallel system

Consider a parallel system, where the reliability of the subsystem P_i is represented by a trapezoidal fuzzy number $\tilde{R}_i=\left(m_i-\alpha_i,m_i,n_i,n_i-\beta_i\right)$ and $1\le i\le k$. The fuzzy reliability of the parallel system can be evaluated as follows:

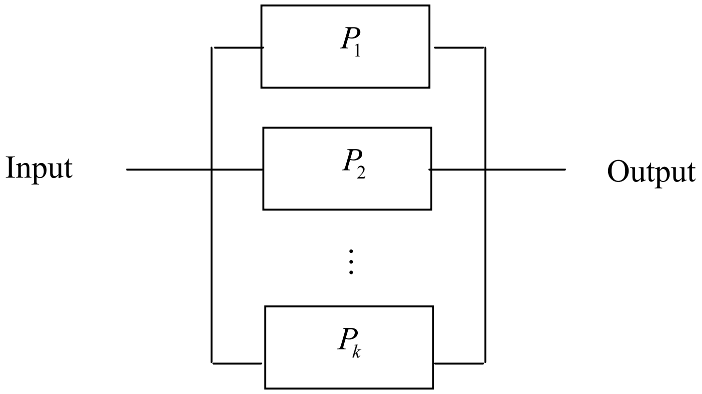

Figure 3.2

$$1\Theta\prod_{i=1}^{k}(1\ \Theta\ \tilde{R}_i\big)= 1\ \Theta\prod_{i=1}^{k}(1\ \Theta(m_i-\alpha_i,\ m_i,\ n_i,\ n_i+\beta_i))$$

$$=1\ \Theta\ \prod_{i=1}^{k}(1-(m_i+\beta_i),1-m_i,1-n_i,1-(n_i-\alpha_i))$$

$$=1\ \Theta\ (1-(1-(m_1+\beta_1),1-m_1,1-n_1,1-(n_1-\alpha_1))$$

$$\otimes(1-(1-(m_2+\beta_2),\ 1-m_2,\ 1-n_2,\ 1-(n_2-\alpha_2))$$

$$\otimes\cdots\otimes(1-(1-(m_k+\beta_k),\ 1-m_k,\ 1-n_k,\ 1-(n_k-\alpha_k))$$

$$=1\ \Theta\left(\prod_{i=1}^{k}[1-(n_i+\beta_i)],\prod_{i=1}^{k}(1-n_i),\prod_{i=1}^{k}(1-m_i),\prod_{i=1}^{k}[1-(m_i-\alpha_i)]\right)$$

$$=(1,1,1)\ \Theta\left(\prod_{i=1}^{k}[1-(n_i+\beta_i)],\prod_{i=1}^{k}(1-n_i),\prod_{i=1}^{k}(1-m_i),\prod_{i=1}^{k}[1-(m_i-\alpha_i)]\right)$$

$$=\left(1-\prod_{i=1}^{k}[1-(m_i-\alpha_i)]\ ,1-\prod_{i=1}^{k}(1-m_i),\prod_{i=1}^{k}(1-n_i),1-\prod_{i=1}^{k}[1-(n_i-\beta_i)]\right)$$

3.4 Data

To explain the proposed approach, the fuzzy reliability of different basic components of marine power plant has been represented by the following trapezoidal fuzzy numbers:

$R_{\tilde{V}_1} =< [(0.003, 0.006, 0.013, 0.026)]>$ $\quad$ $R_{\tilde{V}_5} = < [(0.021, 0.032, 0.053, 0.084)]>$

$R_{\tilde{V}_2} =< [(0.006, 0.008, 0.015, 0.028)]>$ $\quad$ $R_{\tilde{V}_6} = < [(0.041, 0.062, 0.093, 0.134)]>$

$R_{\tilde{V}_3} =< [(0.011, 0.022, 0.033, 0.046)]>$ $\quad$ $R_{\tilde{V}_7} = < [(0.052, 0.083, 0.140, 0.175)]>$

$R_{\tilde{V}_4} =< [(0.012, 0.024, 0.040, 0.059)]>$ $\quad$ $R_{\tilde{V}_8} = < [(0.111, 0.132, 0.163, 0.184)]>$

3.5 Results and discussion

In this section the results of the existing and the proposed approaches are compared.

3.5.1 Existing approach [17]

The fuzzy reliability of marine power plant is

$$\tilde{R} =< [(0.783,0.819,0.860,0.886)]>$$

The membership function representing the fuzzy reliability $\tilde{R}$ is shown in Figure 3.3.

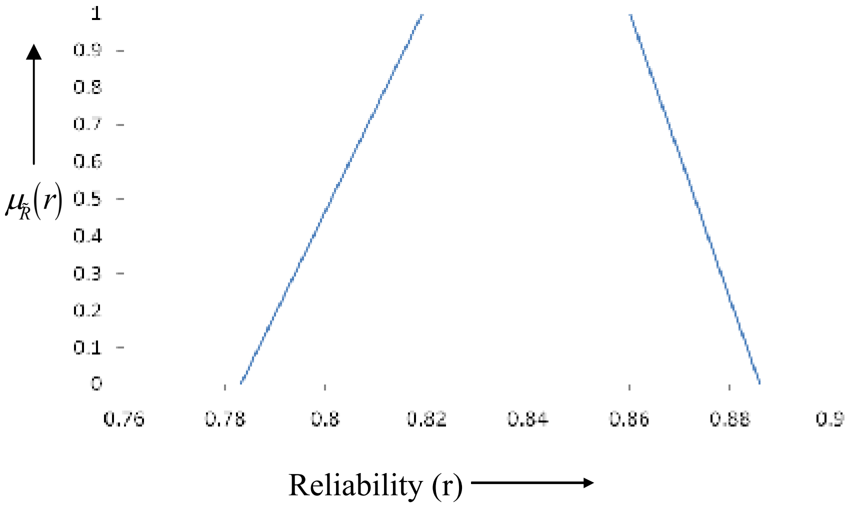

Figure 3.3

The obtained result can be explained as follows:

a) Reliability of marine power plant lies between 0.783 and 0.886.

b) Values of $\mu_{\tilde{R}}(r)$ corresponding to different values of crisp reliabilities $r, r \in [0.783, 0.886]$, can be evaluated as follows:

$$\mu_{\tilde{R}}(r) = \begin{cases} (r - 0.783) \div 0.036, & 0.783 \le r \le 0.819 \\ 1, & 0.819 \le r \le 0.860 \\ (0.886 - r) \div 0.026, & 0.860 \le r \le 0.886 \end{cases}$$

3.5.2 Proposed approach

Using equation (2.1), the arithmetic operations between trapezoidal fuzzy numbers, the assumed data and the proposed algorithm the fuzzy reliability $\tilde{R}$ of the marine power plant is obtained as

$\tilde{R} =$ < [(0.77876, 0.81702, 0.85986, 0.88593)]>

The membership function representing the fuzzy reliability $\tilde{R}$ is shown in Figure 3.4.

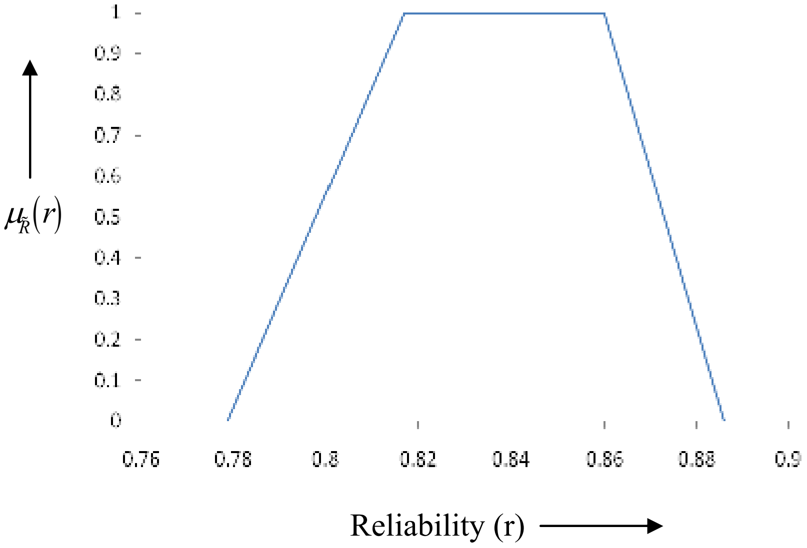

Figure 3.4

The obtained result can be explained as follows:

a) Reliability of marine power plant lies between 0.77876 and 0.88593.

b) Values of $\mu_{\tilde{R}}(r)$ corresponding to different values of crisp reliabilities $r, r \in [0.77876\ ,\ 0.88593]$, can be evaluated as follows:

$$\mu_{\tilde{R}}(r) = \begin{cases} (r - 0.77876) \div 0.03826, & 0.77876 \le r \le 0.81702 \\ 1, & 0.81702 \le r \le 0.85986 \\ (0.88593 - r) \div 0.02607, & 0.85986 \le r \le 0.88593 \end{cases}$$

Since in the existing approach repetition of same components has not been considered before quantitative analysis so, the results of existing approach are not exact.

3.6 Conclusion

In this chapter, using the qualitative analysis of chapter 2 and arithmetic operations between trapezoidal fuzzy numbers, the exact value of fuzzy reliability of marine power plant has been obtained. The results of the proposed approach are compared with the existing approach.

Chapter 4

FUZZY RELIABILITY ANALYSIS USING *LR* TYPE FUZZY NUMBERS

4.1 Introduction

In previous chapter, to evaluate the fuzzy reliability the reliabilities of basic events are represented by trapezoidal fuzzy numbers. In real life problems, it is not always possible to represent the reliabilities by trapezoidal fuzzy numbers. To overcome this shortcoming of the proposed method of previous chapter, in this chapter, the reliabilities of basic components are represented by *LR* type fuzzy numbers. Trapezoidal fuzzy number is a particular type of *LR* type fuzzy numbers, so using the proposed approach of this chapter the results of chapter 2 can also be obtained. Also the expressions proposed by singer [24], for n - ary possibilistic AND, OR and NEG operations has been corrected and the corrected expressions are used for analyzing the fuzzy reliability of marine power plant.

4.2 Arithmetic operations

The *LR* type fuzzy numbers can be denoted by (m,n,α,β). Let $(m_1,n_1,\alpha,\beta)_{LR}$ and $(m_2,n_2,\gamma,\delta)_{LR}$ be two LR type fuzzy numbers and the extended algebraic operations on fuzzy numbers can be defined as follows:

Changing of sign:

$$-(m_1,n_1,\alpha,\beta)_{LR}=(-m_1,-n_1,\alpha,\beta)_{RL} \qquad (4.1)$$

Addition $\oplus$:

$$\left(m_1, n_1, \alpha, \beta\right)_{LR} \oplus \left(m_2, n_2, \gamma, \delta\right)_{LR} = \left(m_1 + m_2, n_1 + n_2, \alpha + \gamma, \beta + \delta\right)_{LR} \qquad (4.2)$$

Subtraction Θ:

$$\left(m_1, n_1, \alpha, \beta\right)_{LR} \Theta \left(m_2, n_2, \gamma, \delta\right)_{LR} = \left(m_1 - m_2, n_1 - n_2, \alpha + \delta, \beta + \gamma\right)_{LR}\ , m_1, m_2, n_1, n_2 \geq 0 \qquad (4.3)$$

Multiplication ⊗ :

$$\left(m_1, n_1, \alpha, \beta\right)_{LR} \otimes \left(m_2, n_2, \gamma, \delta\right)_{LR} = \left(m_1 m_2, n_1 n_2, \alpha m_2 + \gamma m_1, n_2 \beta + n_1 \delta\right)_{LR} \qquad (4.4)$$

4.3 Fuzzy reliability analysis using fuzzy operators

When deriving the AND, OR and NEG operators for the fuzzy reliability analysis, one must keep in mind in constructing fuzzy functions using the operations given above, that they are commutative and associative, but not generally distributive. The possibility (membership) functions of the fuzzy AND, OR and NEG operators can be obtained considering the variables as fuzzy variables and substituting the algebraic operations with the fuzzy operations given in (4.1)-(4.4).

The fuzzy form of the n-ary AND operator function is

$$\tilde{p}_Y = \prod_{i=1}^{n} \tilde{p}_i = \mathrm{AND}(\tilde{p}_1, \ldots, \tilde{p}_n) \qquad (4.5)$$

where $\prod$ denotes fuzzy multiplication. Using Eq. (4.4) for fuzzy multiplication, (4.5) can be written in the following recursive form:

$$\begin{aligned}
\tilde{p}_Y(x) &= (m_Y, n_Y, \alpha_Y, \beta_Y) \\
&= (m_{p_1}, n_{p_1}, \alpha_{p_1}, \beta_{p_1}) \otimes (m_{p_2}, n_{p_2}, \alpha_{p_2}, \beta_{p_2}) \otimes \cdots \otimes (m_{p_n}, n_{p_n}, \alpha_{p_n}, \beta_{p_n}) \\
&= \left(m_{r_{i-1}} m_{p_i}, n_{r_{i-1}} n_{p_i}, m_{r_{i-1}} \alpha_{p_i} + m_{p_i} \alpha_{r_{i-2}}, n_{r_{i-1}} \beta_{p_i} + n_{p_i} \beta_{r_{i-2}}\right)\Big|_{i=n}
\end{aligned}$$

where $m_{r_i}, n_{r_i}, \alpha_{r_i}$ and β_{r_i} denote the following recursive terms, $i = 0, 1, 2, \ldots, n$:

$$m_{r_0} \triangleq m_{p_1},\ m_{r_1} = m_{p_1}, m_{r_2} = m_{p_1} m_{p_2}, \cdots, m_{r_i} = m_{r_{i-1}} m_{p_i}$$

$$n_{r_0} \triangleq n_{p_1},\ n_{r_1} = n_{p_1}, n_{r_2} = n_{p_1} n_{p_2}, \cdots, n_{r_i} = n_{r_{i-1}} n_{p_i}$$

$$\alpha_{r_0} \triangleq \alpha_{p_1}, \alpha_{r_1} = m_{p_1}\alpha_{p_2} + m_{p_2}\alpha_{p_1}, \alpha_{r_2} = m_{r_1}\alpha_{p_3} + m_{p_3}\alpha_{r_1}, \cdots, \alpha_{r_i} = m_{r_{i-1}}\alpha_{p_{i+1}} + m_{p_{i+1}}\alpha_{r_{i-1}}$$

$$\beta_{r_0} \triangleq \beta_{p_1}, \beta_{r_1} = n_{p_1}\beta_{p_2} + n_{p_2}\beta_{p_1}, \beta_{r_2} = n_{r_1}\beta_{p_3} + n_{p_3}\beta_{r_1}, \cdots, \beta_{r_i} = n_{r_{i-1}}\beta_{p_{i+1}} + n_{p_{i+1}}\beta_{r_{i-1}}$$

The fuzzy form of the n-ary OR operation function is:

$$\tilde{p}_Y = 1 - \prod_{i=1}^{\tilde{n}} (1 - \tilde{p}_i) = \text{ OR}(p_1, p_2, \ldots, p_n). \tag{4.6}$$

Using (4.4), expression (4.6) can be written in the following recursive form:

$$\begin{aligned}
\tilde{p}_Y(x) = (1,1,0,0) - ((1,1,0,0) - (m_{p_1}, n_{p_1}, \alpha_{p_1}, \beta_{p_1})&\\
\otimes (1,1,0,0) - (m_{p_2}, n_{p_2}, \alpha_{p_2}, \beta_{p_2})&\\
\otimes \cdots \otimes (1,1,0,0) - (m_{p_n}, n_{p_n}, \alpha_{p_n}, \beta_{p_n})),&\\
= (1,1,0,0) - \left(m_{r_{i-1}} \left(1 - m_{p_i}\right), n_{r_{i-1}} \left(1 - n_{p_i}\right), m_{r_{i-1}}\alpha_{p_i} + \left(1 - m_{p_i}\right)\alpha_{r_{i-2}}, n_{r_{i-1}}\beta_{p_i} + \left(1 - n_{p_i}\right)\beta_{r_{i-2}} \right)&
\end{aligned}$$

where $m_{r_i}, n_{r_i}, \alpha_{r_i}$ and β_{r_i} denote the following recursive terms, $i = 0, 1, 2, \ldots, n$:

$$m_{r_0} \triangleq m_{p_1},\ m_{r_1} = \left(1 - m_{p_1}\right), m_{r_2} = \left(1 - m_{p_1}\right)\left(1 - m_{p_2}\right), \ldots, m_{r_i} = m_{r_{i-1}}\left(1 - m_{p_i}\right)$$

$$n_{r_0} \triangleq n_{p_1},\ n_{r_1} = \left(1 - n_{p_1}\right), n_{r_2} = \left(1 - n_{p_1}\right)\left(1 - n_{p_2}\right), \ldots, n_{r_i} = n_{r_{i-1}}\left(1 - n_{p_i}\right)$$

$$\alpha_{r_0} \triangleq \alpha_{p_1}, \alpha_{r_1} = \left(1 - m_{p_1}\right)\alpha_{p_2} + \left(1 - m_{p_2}\right)\alpha_{p_1}, \alpha_{r_2} = m_{r_1}\alpha_{p_3} + \left(1 - m_{p_3}\right)\alpha_{r_1}, \ldots, \alpha_{r_i} = m_{r_{i-1}}\alpha_{p_{i+1}} + \left(1 - m_{p_{i+1}}\right)\alpha_{r_{i-1}}$$

$$\beta_{r_0} \triangleq \beta_{p_1}, \beta_{r_1} = \left(1 - n_{p_1}\right)\beta_{p_2} + \left(1 - n_{p_2}\right)\beta_{p_1}, \beta_{r_2} = n_{r_1}\beta_{p_3} + \left(1 - n_{p_3}\right)\beta_{r_1}, \ldots, \beta_{r_i} = n_{r_{i-1}}\beta_{p_{i+1}} + \left(1 - n_{p_{i+1}}\right)\beta_{r_{i-1}}$$

4.4 Data

To explain the proposed approach the data presented in chapter 3 is converted into *LR* type fuzzy numbers and then using the arithmetic operations of *LR* type fuzzy numbers the fuzzy reliability of marine power plant has been evaluated.

$$R_{\tilde{V}_1} = (0.006, 0.013, 0.003, 0.013)_{LR} \qquad R_{\tilde{V}_5} = (0.032, 0.053, 0.011, 0.031)_{LR}$$

$$R_{\tilde{V}_2} = (0.008, 0.015, 0.002, 0.013)_{LR} \qquad R_{\tilde{V}_6} = (0.062, 0.093, 0.021, 0.041)_{LR}$$

$$R_{\tilde{V}_3} = (0.022, 0.033, 0.011, 0.013)_{LR} \qquad R_{\tilde{V}_7} = (0.083, 0.140, 0.031, 0.035)_{LR}$$

$R_{\tilde{V}_4} = (0.024,\ 0.040,\ 0.012,\ 0.019)_{LR}$ $\qquad$ $R_{\tilde{V}_8} = (0.132,\ 0.163,\ 0.021,\ 0.021)_{LR}$

4.5 Results and discussion

Using the data and proposed algorithm the fuzzy reliability $\widetilde{R}$ of the marine power plant is obtained as

$$\widetilde{R} = (0.81702,\ 0.85986,\ 0.038260,\ 0.02606)_{LR}$$

Here $L(x)$ and $R(x)$ may be any reference functions according to real life problems. If $L(x) = R(x) = max\ (0,\ 1\text{-}x)$ then the shape of membership function will be same as of chapter 3 i.e. a trapezoidal fuzzy number.

4.6 Conclusion

According to the real life situations, instead of trapezoidal fuzzy numbers the fuzzy reliabilities of different components of marine power plant have been represented by *LR* type fuzzy numbers instead of trapezoidal fuzzy numbers and then using equation (2.1), assumed data and the arithmetic operations between *LR* type fuzzy numbers the fuzzy reliability of marine power plant has been evaluated.

Chapter 5

FUZZY RELIABILITY ANALYSIS USING INTERVAL OF CONFIDENCE

5.1 Introduction

In chapter 3, to evaluate the fuzzy reliability the arithmetic operations between trapezoidal fuzzy numbers have been used but the expression that have been used for the product of two trapezoidal fuzzy numbers gives an approximate result [12]. Similarly, in chapter 4 the arithmetic operations between *LR* type fuzzy numbers gives the exact result only when the values of left and right spreads are small i.e. the results obtained in chapter 3 and 4 are approximate not exact. To obtain exact results, in this chapter, the interval of confidences have been obtained from *LR* type fuzzy numbers and then arithmetic operations between interval of confidences have been used to evaluate the fuzzy reliability of marine power plant.

5.2 Arithmetic operations between interval of confidences

Let $\tilde{A}$ and $\tilde{B}$ be two non negative fuzzy numbers and $A_\alpha = [a_1^{(\alpha)}, a_4^{(\alpha)}]$, $B_\alpha = [b_1^{(\alpha)}, b_4^{(\alpha)}]$, are their interval of confidences corresponding to some value $\alpha \in [0,1]$ then the arithmetic operations between A_α and B_α are defined as follows:

(1) Addition

$$A_\alpha \oplus B_\alpha = \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right] \oplus \left[b_1^{(\alpha)}, b_4^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_3^{(\alpha)} + b_3^{(\alpha)}\right] \tag{5.1}$$

(2) Subtraction

$$A_\alpha \Theta B_\alpha = \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right] \Theta \left[b_1^{(\alpha)}, b_4^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} - b_3^{(\alpha)}, a_3^{(\alpha)} - b_1^{(\alpha)}\right] \tag{5.2}$$

(3) Multiplication

$$A_\alpha \otimes B_\alpha = \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right] \otimes \left[b_1^{(\alpha)}, b_4^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} \cdot b_1^{(\alpha)}, a_3^{(\alpha)} \cdot b_3^{(\alpha)}\right] \tag{5.3}$$

(4) Division

$$A_\alpha \oslash B_\alpha = \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right] \oslash \left[b_1^{(\alpha)}, b_4^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} / b_3^{(\alpha)}, a_3^{(\alpha)} / b_1^{(\alpha)}\right] \tag{5.4}$$

5.3 Error calculations [12]

In this section using the arithmetic operations between interval of confidence the errors occurring in third chapter i.e. due to arithmetic operations between trapezoidal fuzzy numbers used are calculated

The trapezoidal fuzzy numbers that have been used in the proposed fault tree analysis and diagnostic technique requires some computational simplification, especially for product and division operations. For example, the resultant fuzzy number of a product operation (or division operation) on two trapezoidal fuzzy numbers is no longer a trapezoidal fuzzy number. However, this resultant can be approximated to another trapezoidal fuzzy number, using a procedure which is suggested by Kaufmann [12].

Define the following two trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ such that

$\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$

They can be defined in their α - cuts as,

$A_\alpha = \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right]$ and $B_\alpha = \left[b_1^{(\alpha)}, b_4^{(\alpha)}\right]$

where

$$a_1^{(\alpha)} = a_1 + (a_2 - a_1)\alpha,\ a_4^{(\alpha)} = a_4 - (a_4 - a_3)\alpha,\ b_1^{(\alpha)} = b_1 + (b_2 - b_1)\alpha,\ b_4^{(\alpha)} = b_4 - (b_4 - b_3)\alpha$$

Let us define the resultant of the product $\tilde{A} \otimes \tilde{B}$ to be another fuzzy number $\tilde{C} = (c_1, c_2, c_3, c_4)$, which is expressed in terms of its α - cuts as $C_\alpha = \left[c_1^{(\alpha)}, c_4^{(\alpha)}\right]$.

It can be calculated as follows,

$$\begin{aligned} C_\alpha &= A_\alpha B_\alpha \\ &= \left[a_1^{(\alpha)}, a_4^{(\alpha)}\right]\left[b_1^{(\alpha)}, b_4^{(\alpha)}\right] \\ &= \left[a_1^{(\alpha)} b_1^{(\alpha)}, a_4^{(\alpha)} b_4^{(\alpha)}\right] \end{aligned}$$

Therefore,

$$\begin{aligned} C_\alpha &= \left[\left(a_1 + (a_2 - a_1)\alpha\right)\left(b_1 + (b_2 - b_1)\alpha\right), \left(a_4 - (a_4 - a_3)\alpha\right)\left(b_4 - (b_4 - b_3)\alpha\right)\right] \\ &= \Big[a_1 b_1 + \left(a_1 (b_2 - b_1) + b_1 (a_2 - a_1)\right)\alpha + (a_2 - a_1)(b_2 - b_1)\alpha^2, \\ &\qquad a_4 b_4 - \left(a_4 (b_4 - b_3) + b_4 (a_2 - a_3)\right)\alpha + (a_4 - a_3)(b_4 - b_3)\alpha^2\Big] \end{aligned}$$

Now, approximate the product operation $\tilde{A} \otimes \tilde{B}$ by a trapezoidal fuzzy number $\tilde{P}$ such that

$$\tilde{P} = (p_1, p_2, p_3, p_4) = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4)$$

Then the α - cuts of $\tilde{P}$ are given by

$$P_\alpha = \left[p_1^{(\alpha)}, p_4^{(\alpha)}\right],$$

where

$$\begin{aligned} p_1^{(\alpha)} &= a_1 \cdot b_1 + (a_2 \cdot b_2 - a_1 \cdot b_1)\alpha \\ p_4^{(\alpha)} &= a_4 \cdot b_4 - (a_4 \cdot b_4 - a_3 \cdot b_3)\alpha \end{aligned}$$

In order to evaluate this approximation, both the left maximum divergence and the right maximum divergence between the exact $\tilde{C}$ and the approximate $\tilde{P}$ should be defined and computed.

First calculate the right and left divergence $\in_{1\alpha}$ and $\in_{r\alpha}$, respectively, as follows,

$$\begin{aligned}
\in_{1\alpha} &= c_1^{(\alpha)} - p_1^{(\alpha)} \\
&= \left[a_1 b_1 + (a_1(b_2 - b_1) + b_1(a_2 - a_1))\alpha + (a_2 - a_1)(b_2 - b_1)\alpha^2 \right] \\
&\quad - \left[a_1.b_1 + (a_2.b_2 - a_1.b_1)\alpha \right] \\
&= (a_2 - a_1)(b_2 - b_1)(\alpha^2 - \alpha).
\end{aligned}$$

It is clear that the maximum value $\in_{1\alpha}^{\max}$ depends on α only, and occurs for $\alpha = 0.5$, therefore, $\in_{1\alpha}^{\max} = -0.25(a_2 - a_1)(b_2 - b_1)$

Similarly,

$$\begin{aligned}
\in_{1\alpha} &= c_4^{(\alpha)} - p_4^{(\alpha)} \\
&= \left[a_4 b_4 + (a_4(b_4 - b_3) + b_4(a_4 - a_3))\alpha + (a_4 - a_3)(b_4 - b_3)\alpha^2 \right] \\
&\quad - \left[a_4.b_4 + (a_4.b_4 - a_3.b_3)\alpha \right] \\
&= (a_4 - a_3)(b_4 - b_3)(\alpha^2 - \alpha).
\end{aligned}$$

Again, the maximum value $\in_{1\alpha}^{\max}$ depends on α only, and occurs for $\alpha = 0.5$, therefore, $\in_{1\alpha}^{\max} = -0.25(a_2 - a_1)(b_2 - b_1)$

Applying this procedure on given fuzzy numbers representing the fuzzy probability of failure results in a great margin of safety.

5.4 Fuzzy reliability analysis

In this section the expressions for evaluating the fuzzy reliability of series and parallel system has been obtained using interval of confidence.

Consider a system of n components with fuzzy reliabilities $\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n$, and statistical independence of components are assumed. The fuzzy system reliability $\tilde{p}$ is given by

$$\tilde{p} = \tilde{\Phi}\left(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n\right), \tag{5.5}$$

where $\tilde{\Phi}$ is the fuzzy reliability function of the system.

We define α - cuts as:

$$\begin{aligned}
\left(\tilde{p}_1\right)^{\alpha} &= \left[a_{11}^{\alpha}, a_{14}^{\alpha}\right] \\
&= \left[a_{11} + (a_{12} - a_{11})\alpha \ , \ a_{14} + (a_{13} - a_{14})\alpha\right] \\
\left(\tilde{p}_2\right)^{\alpha} &= \left[a_{21}^{\alpha}, a_{24}^{\alpha}\ \right] \\
&= \left[a_{21} + (a_{22} - a_{21})\alpha \ , \ a_{24} + (a_{23} - a_{24})\alpha\right] \\
&\vdots \\
\left(\tilde{p}_n\right)^{\alpha} &= \left[a_{n1}^{\alpha}, a_{n4}^{\alpha}\right] \\
&= \left[a_{n1} + (a_{n2} - a_{n1})\alpha \ , \ a_{n4} + (a_{n3} - a_{n4})\alpha\right].
\end{aligned} \tag{5.6}$$

5.4.1 Series system

The fuzzy reliability of series system can be evaluated as follows:

$$\begin{aligned}
\tilde{p} &= \prod_{i=1}^{n} \tilde{p}_i \\
&= \tilde{p}_1 (\cdot)\, \tilde{p}_2 (\cdot) \cdots (\cdot)\, \tilde{p}_n \\
&= \left[a_{11}, a_{14}\right] \cdot \left[a_{21}, a_{24}\right] \cdot \cdots \cdot \left[a_{n1}, a_{n4}\right] \\
&= \left[a_{11} + (a_{12} - a_{11})\alpha \ , \ a_{14} + (a_{13} - a_{14})\alpha\right] \cdot \left[a_{21} + (a_{22} - a_{21})\alpha \ , \ a_{24} + (a_{23} - a_{24})\alpha\right] \\
&\quad \cdots\cdots \left[a_{n1} + (a_{n2} - a_{n1})\alpha \ , \ a_{n4} + (a_{n3} - a_{n4})\alpha\right] \\
&= \left[\prod_{i=1}^{n} \left[a_{i1} + (a_{i2} - a_{i1})\alpha\right], \prod_{i=1}^{n} \left[a_{i4} + (a_{i3} - a_{i4})\alpha\right]\right]
\end{aligned} \tag{5.7}$$

5.4.2 Parallel system

The fuzzy reliability of the parallel system can be evaluated as follows:

$$\begin{aligned}
\tilde{p} = 1 - \prod_{i=1}^{n} (1 - \tilde{p}_i) &= \left[\left[1,1\right] - \prod_{i=1}^{n} \left[1,1\right] - \left[a_{i1}^{\alpha}, a_{i4}^{\alpha}\right]\right] \\
&= \left[\left[1,1\right] - \prod_{i=1}^{n} \left[1 - a_{i4} + (a_{i4} - a_{i3})\alpha\right], \prod_{i=1}^{n} \left[1 - a_{i1} + (a_{i1} - a_{i2})\alpha\right]\right] \\
&= \left[1 - \prod_{i=1}^{n} \left[1 - a_{i1} + (a_{i1} - a_{i2})\alpha\right], 1 - \prod_{i=1}^{n} \left[1 - a_{i4} + (a_{i4} - a_{i3})\alpha\right]\right]
\end{aligned} \tag{5.8}$$

5.5 Data

The intervals of confidence corresponding to fuzzy reliabilities of basic components of marine power plant are evaluated which are as follows:

$$R_{\tilde{V}_1}^{\alpha} = \left[0.003 + 0.003\alpha, 0.026 - 0.013\alpha\right]$$

$$R_{\tilde{V}_2}^{\alpha} = \left[0.006 + 0.002\alpha, 0.028 - 0.013\alpha\right]$$

$$R_{\tilde{V}_3}^{\alpha} = \left[0.011 + 0.011\alpha, 0.046 - 0.013\alpha\right]$$

$$R_{\tilde{V}_4}^{\alpha} = \left[0.012 + 0.012\alpha, 0.059 - 0.019\alpha\right]$$

$$R_{\tilde{V}_5}^{\alpha} = \left[0.021 + 0.011\alpha, 0.084 - 0.031\alpha\right]$$

$$R_{\tilde{V}_6}^{\alpha} = \left[0.041 + 0.021\alpha, 0.13 - 0.041\alpha\right]$$

$$R_{\tilde{V}_7}^{\alpha} = \left[0.052 + 0.031\alpha, 0.175 - 0.03\alpha\right]$$

$$R_{\tilde{V}_8}^{\alpha} = \left[0.111 + 0.021\alpha, 0.184 - 0.021\alpha\right]$$

The values intervals of confidence corresponding to different basic components of marine power plant has been evaluated at some particular values of α and are shown in Table (5.1) and Table (5.2).

α	$R^{\alpha}_{\tilde{V}_1}$	$R^{\alpha}_{\tilde{V}_2}$	$R^{\alpha}_{\tilde{V}_3}$	$R^{\alpha}_{\tilde{V}_4}$
0.0	$[0.00300, 0.02600]$	$[0.00600, 0.02800]$	$[0.01100, 0.04600]$	$[0.01200, 0.05900]$
0.1	$[0.00330, 0.02470]$	$[0.00620, 0.02670]$	$[0.01210, 0.04470]$	$[0.01320, 0.05710]$
0.2	$[0.00360, 0.02340]$	$[0.00640, 0.02540]$	$[0.01320, 0.04340]$	$[0.01440, 0.05520]$
0.3	$[0.00390, 0.02210]$	$[0.00660, 0.02410]$	$[0.01430, 0.04210]$	$[0.01560, 0.05330]$
0.4	$[0.00420, 0.02080]$	$[0.00680, 0.02280]$	$[0.01540, 0.04080]$	$[0.01680, 0.05140]$
0.5	$[0.00450, 0.01820]$	$[0.00700, 0.02150]$	$[0.01650, 0.03950]$	$[0.01800, 0.04950]$
0.6	$[0.00480, 0.01820]$	$[0.00720, 0.02020]$	$[0.01760, 0.03820]$	$[0.01920, 0.04760]$
0.7	$[0.00510, 0.01690]$	$[0.00740, 0.01760]$	$[0.01870, 0.03690]$	$[0.02040, 0.04570]$
0.8	$[0.00540, 0.01560]$	$[0.00760, 0.01760]$	$[0.01980, 0.03560]$	$[0.02280, 0.04190]$
0.9	$[0.00570, 0.01430]$	$[0.00780, 0.01630]$	$[0.02090, 0.03430]$	$[0.02280, 0.04190]$
1.0	$[0.00600, 0.01300]$	$[0.00800, 0.01500]$	$[0.02200, 0.03300]$	$[0.02400, 0.03300]$

Table 5.1

α	$R_{\tilde{V}_5}^{\alpha}$	$R_{\tilde{V}_6}^{\alpha}$	$R_{\tilde{V}_7}^{\alpha}$	$R_{\tilde{V}_8}^{\alpha}$
0.0	$[0.02100, 0.08400]$	$[0.04100, 0.13400]$	$[0.05200, 0.17500]$	$[0.11100, 0.18400]$
0.1	$[0.02210, 0.08090]$	$[0.04310, 0.12990]$	$[0.05510, 0.17200]$	$[0.11310, 0.18190]$
0.2	$[0.02320, 0.07780]$	$[0.04520, 0.012580]$	$[0.05820, 0.16900]$	$[0.11520, 0.17980]$
0.3	$[0.02430, 0.07470]$	$[0.04730, 0.12170]$	$[0.06130, 0.16600]$	$[0.11730, 0.17770]$
0.4	$[0.02540, 0.07160]$	$[0.04940, 0.11760]$	$[0.06440, 0.16300]$	$[0.11940, 0.17560]$
0.5	$[0.02650, 0.06850]$	$[0.05150, 0.11350]$	$[0.06750, 0.16000]$	$[0.12150, 0.17350]$
0.6	$[0.02760, 0.06540]$	$[0.05360, 0.10940]$	$[0.07060, 0.15700]$	$[0.12360, 0.17140]$
0.7	$[0.02870, 0.06230]$	$[0.05570, 0.10530]$	$[0.07370, 0.15400]$	$[0.12570, 0.16930]$
0.8	$[0.02980, 0.05920]$	$[0.05780, 0.10120]$	$[0.07680, 0.15100]$	$[0.12780, 0.16720]$
0.9	$[0.03090, 0.05610]$	$[0.05990, 0.09710]$	$[0.07990, 0.14800]$	$[0.12990, 0.16510]$
1.0	$[0.03200, 0.05300]$	$[0.06200, 0.09300]$	$[0.08300, 0.14500]$	$[0.13200, 0.16300]$

Table 5.2

5.6 Result and discussion

It is clear from section 5.3 that the obtained results of chapter 3 are not exact. Since the values of left and right spreads are not small so the results obtained in chapter 4 is also approximate not exact.

The exact value of the fuzzy reliability of marine power plant is

$$\tilde{R} = 1 \ominus R_{\tilde{F}}$$

$$= \left(1 \ominus R_{\tilde{V}_8}\right) \otimes \left(1 \ominus R_{\tilde{V}_1} R_{\tilde{V}_2}\right) \otimes \left(1\ R_{\tilde{V}_1} R_{\tilde{V}_4}\right) \otimes \left(1 \ominus R_{\tilde{V}_2} R_{\tilde{V}_3}\right) \otimes \left(1 \ominus R_{\tilde{V}_3} R_{\tilde{V}_4}\right) \otimes \left(1 \ominus R_{\tilde{V}_3} R_{\tilde{V}_7}\right) \otimes (1$$

$$\ominus R_{\tilde{V}_4} R_{\tilde{V}_6}\Big) \otimes \left(1 \ominus R_{\tilde{V}_6} R_{\tilde{V}_7}\right) \otimes \left(1 \ominus R_{\tilde{V}_1} R_{\tilde{V}_5} R_{\tilde{V}_7}\right) \otimes \left(1 \ominus R_{\tilde{V}_2} R_{\tilde{V}_5} R_{\tilde{V}_6}\right)$$

Intervals of confidence for fuzzy reliability $\tilde{R}$ at some particular values of α are shown in Table 5.3 and the membership function of the obtained result is shown in Figure 5.1.

α	$\tilde{R} = 1 \Theta R_{\tilde{F}}$
0.0	$[0.11407, 0.22124]$
0.1	$[0.11657, 0.21727]$
0.2	$[0.11910, 0.21335]$
0.3	$[0.12165, 0.20947]$
0.4	$[0.12422, 0.20564]$
0.5	$[0.12679, 0.20183]$
0.6	$[0.12942, 0.19807]$
0.7	$[0.13207, 0.19436]$
0.8	$[0.13476, 0.19046]$
0.9	$[0.13742, 0.18708]$
1.0	$[0.14012, 0.18352]$

Table 5.3

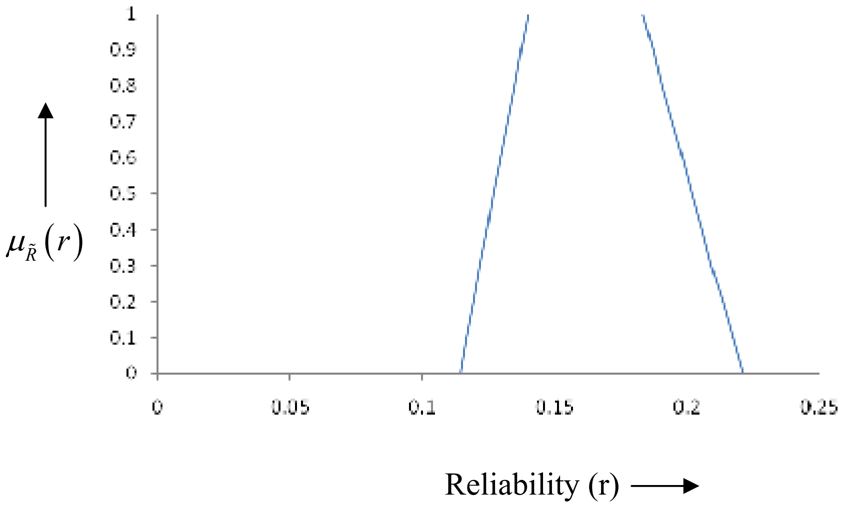

Figure 5.1

5.6 Conclusion

Using the arithmetic operations between interval of confidence the errors occurring in third chapter i.e. due to arithmetic operations between trapezoidal fuzzy numbers are calculated and then using equation (2.1), the interval of confidence, corresponding to assumed data and arithmetic operations between intervals of confidence the exact fuzzy reliability of marine power plant has been evaluated.

Chapter 6

FUZZY RELIABILITY ANALYSIS USING DIFFERENT MEMBERSHIP FUNCTIONS

6.1 Introduction

In previous chapters, arithmetic operations between same types of fuzzy numbers are discussed. Also to analyze the fuzzy system reliability, it is assumed that the reliability of all components of a system follow the same membership function. However, in real life problems, such type of situations rarely occurs. Therefore, there is need of a method by which the fuzzy reliability of systems, having components following different type of membership functions, can be found. In this chapter, a new approach has been introduced to perform various arithmetic operations between different types of fuzzy numbers. The proposed approach has been used to analyze the fuzzy reliability of marine power plant. The fuzzy reliability of basic components has been represented by different types of membership functions.

6.2 Proposed algorithm

In this section, an algorithm has been proposed to perform certain arithmetic operations between different types of fuzzy numbers $\tilde{V}_1, \tilde{V}_2, \ldots, \tilde{V}_n$ defined on the universe of discourse U.

Step 1

Find the intervals for $\tilde{V}_i$ corresponding to certain values of $\alpha \in [0,1]$. Let the interval for $\mu_{\tilde{V}_i}(x) = p, \quad 0 \le p \le 1$ is $\left[x_{i1}, x_{i2}\right]$, where $x_{i1}, x_{i2} \in U$.

Step 2

The interval of confidence of the resultant fuzzy set $\tilde{V}$ corresponding to $\mu_{\tilde{V}}(x) = p$ after addition, multiplication and subtraction can be obtained as follows:

$$\left[\sum_{i=1}^{n} x_{i1}, \sum_{i=1}^{n} x_{i2}\right] \quad , \quad \left[\prod_{i=1}^{n} x_{i1}, \prod_{i=1}^{n} x_{i2}\right] \text{ and } \left[x_{11} - \left(\sum_{i=2}^{n} x_{i2}\right), x_{12} - \left(\sum_{i=2}^{n} x_{i1}\right)\right] \text{ respectively.}$$

Step 3

Draw the membership functions of the resultant fuzzy sets after finding the intervals for certain values of p (including 0 and 1).

6.3 Data

To explain the proposed approach the fuzzy reliability of the basic events of marine power plant has been represented by different types of membership functions.

$$\mu_{R_{\tilde{V}_1}}(x_1) = \begin{cases} \dfrac{x_1 - 0.003}{0.003}, & 0.003 \le x_1 \le 0.006, \\ 1, & 0.006 \le x_1 \le 0.013, \\ \dfrac{0.026 - x_2}{0.013}, & 0.013 \le x_1 \le 0.026, \end{cases} \qquad (\text{trapezoidal type})$$

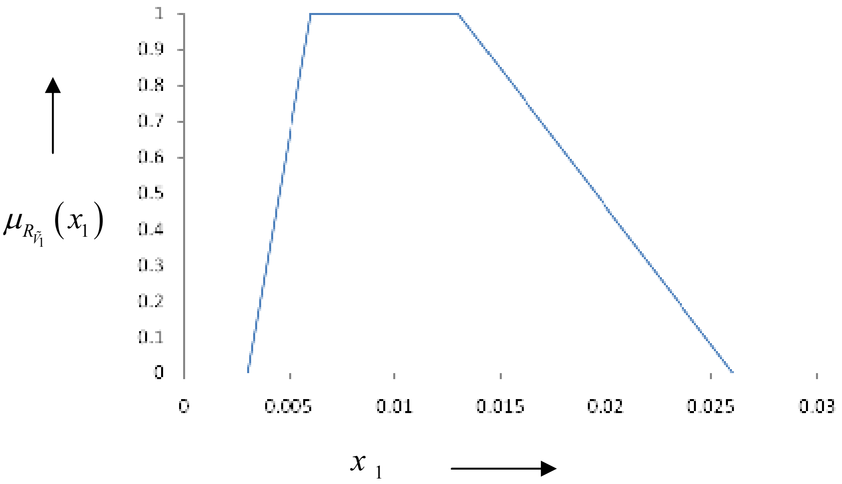

Figure 6.1

$$\mu_{R_{\tilde{V}_2}}(x_2) = \left\{\exp\left(-400(x_2 - 0.02)^2\right), \qquad 0.012 \le x_2 \le 0.028 \qquad (\text{normal type})\right.$$

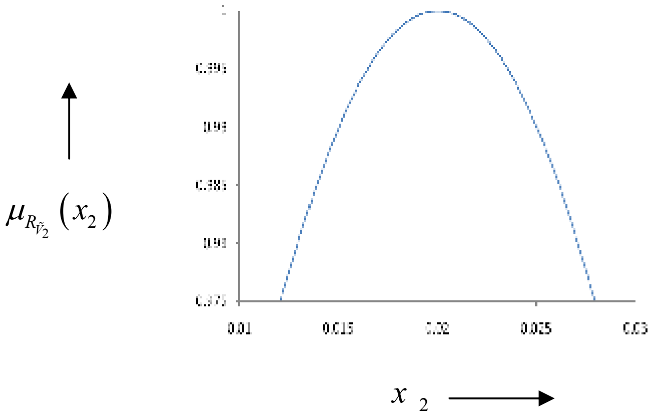

Figure 6.2

$$\mu_{R_{\tilde{V}_3}}(x_3) = \begin{cases} \dfrac{x_3 - 0.011}{0.011}, & 0.011 \le x_3 \le 0.022, \\ 1, & 0.022 \le x_3 \le 0.033, \\ \dfrac{0.046 - x_3}{0.013}, & 0.033 \le x_3 \le 0.046, \end{cases} \qquad (\text{trapezoidal type})$$

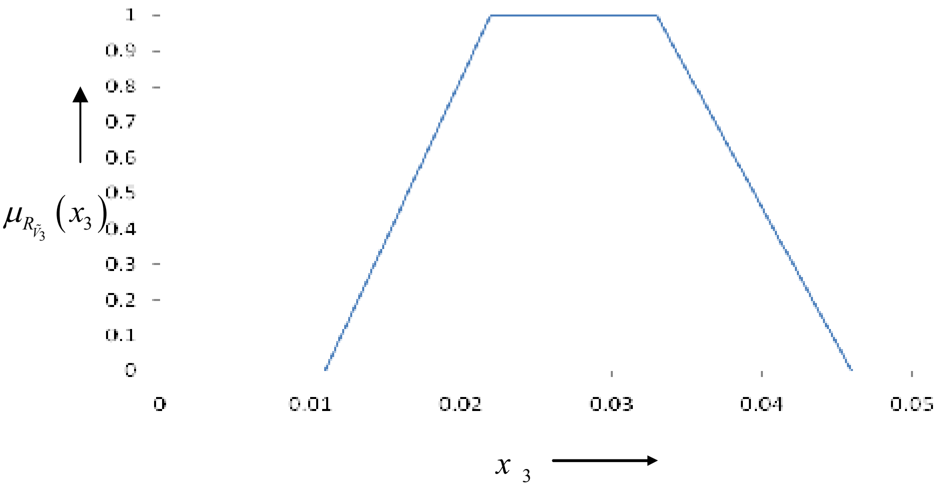

Figure 6.3

$$\mu_{R_{\tilde{V}_4}}(x_4)=\begin{cases}\exp\left[150(x_4-0.032)\right], & 0.012\le x_4\le 0.032\\ \exp\left[-150(x_4-0.032)\right], & 0.032\le x_4\le 0.059\end{cases}\qquad (\text{sharp gamma type})$$

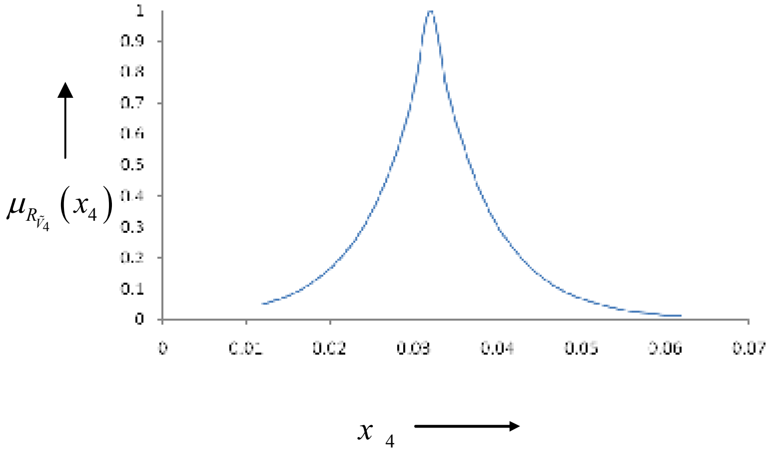

Figure 6.4

$$\mu_{R_{\tilde{V}_5}}(x_5)=\begin{cases}\dfrac{x_5-0.021}{0.011}, & 0.021\le x_5\le 0.032,\\ 1, & 0.032\le x_5\le 0.053,\\ \dfrac{0.084-x_5}{0.031}, & 0.053\le x_5\le 0.084,\end{cases}\qquad (\text{trapezoidal type})$$

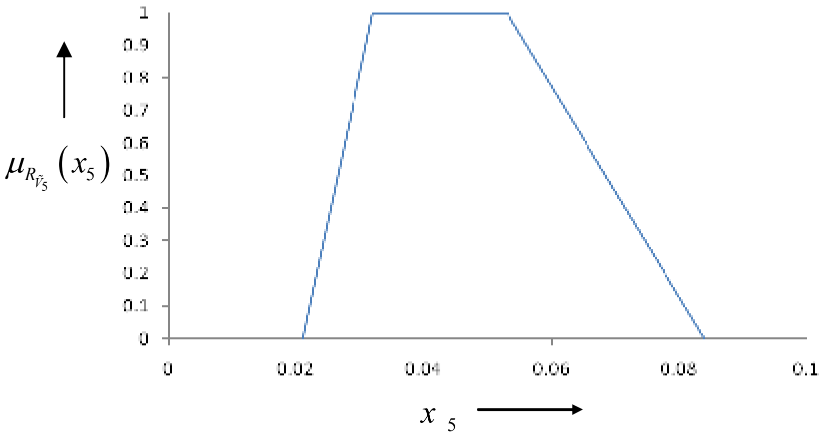

Figure 6.5

$$\mu_{R_{\tilde{V}_6}}\left(x_6\right)=\begin{cases}\dfrac{x_6-0.041}{0.021}, & 0.041\le x_6\le 0.062,\\ 1, & 0.062\le x_6\le 0.093,\\ \dfrac{0.134-x_6}{0.041}, & 0.093\le x_6\le 0.134,\end{cases}\qquad (\text{trapezoidal type})$$

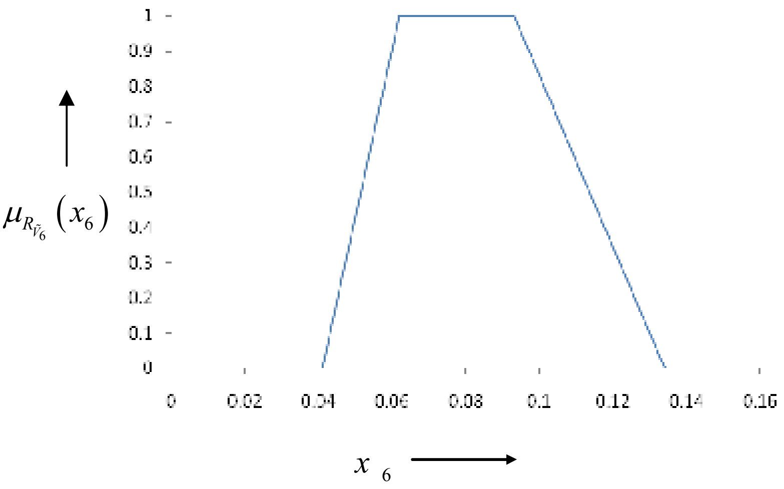

Figure 6.6

$$\mu_{R_{\tilde{V}_7}}\left(x_7\right)=\begin{cases}0, & 0.052\le x_7\le 0.83\\ 1, & 0.083\le x_7\le 0.140\\ 0, & 0.140\le x_7\le 0.175\end{cases}\qquad (\text{rectangular type})$$

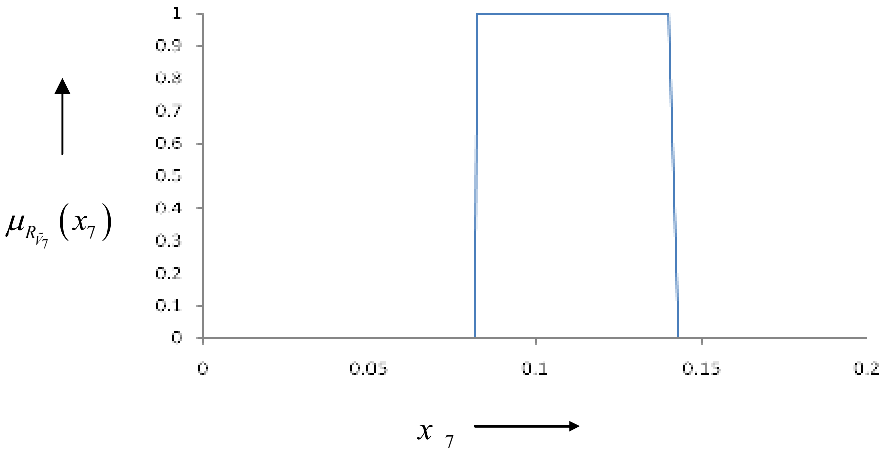

Figure 6.7

$$\mu_{R_{\tilde{V}_8}}(x_8)=\begin{cases}\exp\left[75(x_8-0.148)\right], & 0.111\le x_8\le 0.148 \\ \exp\left[-75(x_8-0.148)\right], & 0.148\le x_8\le 0.184\end{cases} \qquad \text{(sharp gamma type)}$$

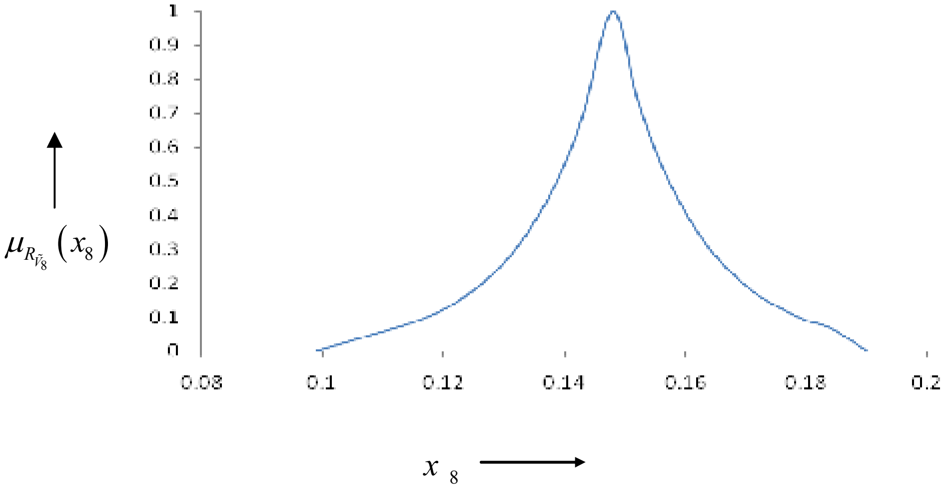

Figure 6.8

where $x_1, x_2, \ldots, x_8$ are the probabilities of failure which belongs to $R_{\tilde{V}_1}, R_{\tilde{V}_2}, \cdots, R_{\tilde{V}_8}$ respectively. The intervals of confidence corresponding to fuzzy reliabilities of different components at some particular values of α are shown in Table 6.1 and Table 6.2.

α	$R_{\tilde{V}_1}$	$R_{\tilde{V}_2}$	$R_{\tilde{V}_3}$	$R_{\tilde{V}_4}$
0.0	$[0.00300, 0.02600]$	$[0.01200, 0.02800]$	$[0.01100, 0.04600]$	$[0.01200, 0.05900]$
0.1	$[0.00330, 0.02470]$	$[0.01424, 0.02576]$	$[0.01210, 0.04470]$	$[0.01665, 0.04735]$
0.2	$[0.00360, 0.02340]$	$[0.01598, 0.02402]$	$[0.01320, 0.04340]$	$[0.02127, 0.04273]$
0.3	$[0.00390, 0.02210]$	$[0.01699, 0.02301]$	$[0.01430, 0.04210]$	$[0.02397, 0.04003]$
0.4	$[0.00420, 0.02080]$	$[0.01771, 0.02229]$	$[0.01540, 0.04080]$	$[0.02589, 0.03811]$
0.5	$[0.00450, 0.01950]$	$[0.01827, 0.02173]$	$[0.01650, 0.03950]$	$[0.02738, 0.03662]$
0.6	$[0.00480, 0.01820]$	$[0.01872, 0.02128]$	$[0.01760, 0.03820]$	$[0.02859, 0.03540]$
0.7	$[0.00510, 0.01690]$	$[0.01911, 0.02089]$	$[0.01890, 0.03690]$	$[0.02962, 0.03438]$
0.8	$[0.00540, 0.01560]$	$[0.01944, 0.02089]$	$[0.01980, 0.03560]$	$[0.03051, 0.03349]$
0.9	$[0.00570, 0.01430]$	$[0.019737, 0.02026]$	$[0.02090, 0.03430]$	$[0.03130, 0.03270]$
1.0	$[0.00600, 0.01300]$	$[0.02000, 0.02000]$	$[0.02200, 0.03300]$	$[0.03200, 0.03200]$

Table 6.1

α	$R_{\tilde{V}_5}$	$R_{\tilde{V}_6}$	$R_{\tilde{V}_7}$	$R_{\tilde{V}_8}$
0.0	$[0.02100, 0.08400]$	$[0.04100, 0.13400]$	$[0.08300, 0.14000]$	$[0.11100, 0.18400]$
0.1	$[0.02210, 0.08090]$	$[0.04310, 0.12990]$	$[0.08300, 0.14000]$	$[0.11730, 0.17870]$
0.2	$[0.02320, 0.07780]$	$[0.04520, 0.12580]$	$[0.08300, 0.14000]$	$[0.12654, 0.16946]$
0.3	$[0.02430, 0.07470]$	$[0.04730, 0.12170]$	$[0.08300, 0.14000]$	$[0.13195, 0.16405]$
0.4	$[0.02540, 0.07160]$	$[0.04940, 0.11760]$	$[0.08300, 0.14000]$	$[0.13578, 0.16022]$
0.5	$[0.02650, 0.06850]$	$[0.05150, 0.11350]$	$[0.08300, 0.14000]$	$[0.13876, 0.15724]$
0.6	$[0.02760, 0.06540]$	$[0.05360, 0.10940]$	$[0.08300, 0.14000]$	$[0.14119, 0.15481]$
0.7	$[0.02870, 0.06230]$	$[0.05570, 0.10530]$	$[0.08300, 0.14000]$	$[0.14324, 0.15276]$
0.8	$[0.02980, 0.05920]$	$[0.05780, 0.10120]$	$[0.08300, 0.14000]$	$[0.14502, 0.15097]$
0.9	$[0.03090, 0.05610]$	$[0.05990, 0.09710]$	$[0.08300, 0.14000]$	$[0.14659, 0.14940]$
1.0	$[0.03200, 0.05300]$	$[0.06200, 0.09300]$	$[0.08300, 0.14000]$	$[0.14800, 0.14800]$

Table 6.2

6.4 Result and discussion

The interval of confidences corresponding to the fuzzy reliability $\tilde{R}$ at some particular values of α are shown in Table 6.3 and the membership function for $\tilde{R}$ is shown in Figure 6.9.

α	$\tilde{R}=1-\tilde{F}$
0.0	$[0.11558, 0.21617]$
0.1	$[0.12240, 0.20808]$
0.2	$[0.13217, 0.19739]$
0.3	$[0.13803, 0.19076]$
0.4	$[0.14230, 0.18580]$
0.5	$[0.14549, 0.18176]$
0.6	$[0.14635, 0.17831]$
0.7	$[0.15098, 0.17526]$
0.8	$[0.15354, 0.17250]$
0.9	$[0.15509, 0.16997]$
1.0	$[0.15689, 0.16763]$

Table 6.3

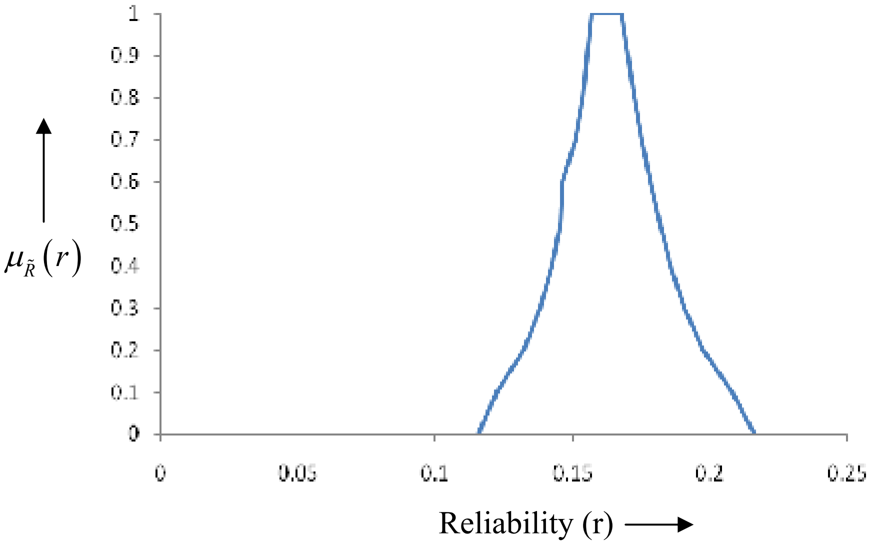

Figure 6.9

Using the proposed algorithm of this chapter the results of the previous chapters can be easily obtained.

6.5 Conclusion

A new algorithm has been proposed to find the arithmetic operations among different types of fuzzy numbers. To explain the proposed algorithm the fuzzy reliabilities of different components of marine power plant has been represented by different types of fuzzy numbers and then using equation (2.1) and the proposed algorithm the fuzzy reliability of marine power plant has been evaluated.

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