EQUILIBRIUM STRUCTURE OF ROTATIONALLY, TIDALLY AND MAGNETICALLY DISTORTED POLYTROPIC MODELS OF STARS

A THESIS

Submitted in fulfilment of the

Requirement for the award of the degree of

Masters Of Science, in

Physics

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July, 2016

Candidate's Declaration

I hereby certify that the work which is being presented in the thesis entitled ' EQUILIBRIUM STRUCTURE OF ROTATIONALLY, TIDALLY AND MAGNETICALLY DISTORTED POLYTROPIC MODELS OF STARS' in fulfilment of the requirement for the award of the Degree Of Master of Physics, submitted in the School of Physics and Material Science (SPMS) of the University is an authentic record of my own work carried out during a period from August 2015 to June 2016 under the supervision of Dr. A. K. Lal.

The matter embodied in this present thesis has not been submitted by me for the award of any other degree.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Dedicated To
My Parents

ACKNOWLEDGEMENTS

It gives me great pleasure to place on record my sense of sincere gratitude to Dr.A.K.Lal , Associate Professor and Head of School of Mathematics and Computer Applications, Thapar University, whose keen interest in the field gave wings to my imagination and made me challenge my own limits. I am forever in gratitude for his meticulous guidance, understanding spirit , constant encouragement and inspiration, support all through the pursuance of this work , without which the completion of this project would not have been possible.

I also place on record my deep appreciation and indebtedness to Dr.O.P.Pandey , for strongly supporting my vision and granting me the permission to work with Dr. A. K. Lal in the branch of Astrophysics.

I would like to express my deep gratitude to Dr. Manoj K Sharma for his immense cooperation and efforts for permitting me to work with Dr. A .K. Lal.

I also take this opportunity to express my sincere thanks to all the faculty members and the staff of School of Mathematics, Thapar University, whose names may all not be enumerated. Their contributions are sincerely appreciated and gratefully acknowledged.

I owe my work to my parents for being my pillars of strength. It is because of their love, continuous support and blessings that I have gathered the courage to manifest my vision. I owe my undying determination to my mother, who has always been a great mentor to my soul.

Above all, I express my earnest gratitude towards the Great Almighty, the author of knowledge and wisdom, for His infinite grace and love.

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ABSTRACT

The current thesis takes into account the problems of investigating the collective effects of rotational, tidal and magnetic distortions up on the equilibrium structure of a binary star system. This study has practical significance in the branch of astrophysics which requires to comprehend the challenges regarding the stability of a binary star system and its corresponding variations arising because of tidally and magnetically distorted non-synchronously rotating stars along with the stars in binary and multiple system.

In theoretical models, most of the observed stars are essentially considered as self-gravitating spheres of gases in both hydrostatic and thermal equilibrium. Theoretical analysis of these kinds of problems which deal with the investigation of the equilibrium structure of stars, have been thoroughly carried out to comprehend the intriguing environment of the inner stellar structure.

Some of such gaseous spheres are observed to be single while many are found in clusters. The single stars rotate about their own axis of rotation.

Stars with a single companion are termed as binary stars while those with multiple companions are termed as multiple star. Such types of stars rotate about their own axis besides revolving around each other. If we take into account the equilibrium model of a distinct isolated star, which does not rotate, as a gaseous sphere, then the corresponding model describing the equilibrium structures of rotating stars would be rotationally distorted star. Likewise the equilibrium model of the stars present in the form of a cluster would be tidally distorted gaseous sphere provided it does not rotate but in the presence of rotation, the model would be a rotationally and tidally distorted gaseous sphere.

Complex analytical studies have been discussed in literature to reveal the equilibrium structure of rotationally and tidally distorted stars. Generally, magnetic forces have been neglected while obtaining Roche equipotential surface of stars because its impact was presumed to be very minute in comparison with centrifugal and gravitational forces.

However, magnetic field is also one of the factors causing the asphericity in stars. These stellar magnetic fields are considered to enact a vital role in controlling and thus affecting the stability of a star or a system of stars.

By taking into account the major significance of magnetic distortion, on the structure of a star or a system of stars, an effort in this direction has been made in the present work to theoretically (and computationally) inspect significant aspects of magnetic distortion acting upon the equilibrium assemblies of asynchronously rotationally and tidally distorted gaseous spheres, that furthermore requires examinations as an attempt to gain deeper understanding of stellar structure.

Although many efforts have been made to develop analytical expressions in the form of series, but to retrieve closed form solutions from these has not been possible. It is also observed that no significant practical result can be extracted from these analytical solutions.

It thus seems encouraging to bring into picture the detailed series expansion of the distortion parameters r_{ψ}, u, v, w, f_p *and* f_t etc. which are required essentially in establishing the equilibrium structure of rotationally , tidally and magnetically distorted stars. In today's era it seems to be an excellent proposition by keeping in view of the availability of fast computing machines. Hence arises the need to develop an efficient and reliable computational method which could directly evaluate these distortion parameters without depending on their explicit expression in the form of series.

The thesis consist of 3 chapters. First chapter is introductory in nature. It comprises of brief discussion on the astrophysical importance of the equilibrium structures of rotationally, tidally and magnetically distorted binary stellar models. It also comprises of a brief inspection of the literature on this explicit topic.

Chapter two aligns with the brief description about the binary stars and Roche equipotential surfaces. It describes in detail the effect of inclusion of magnetic forces upon the mass transfer that takes place in a binary system and how it causes stellar mass reduction. It provides us an insight into the methodology employed by Kippenhahn and Thomas to understand the stellar environment in more depth and breadth. Phenomenon of magnetic braking has been discussed to emphasise on the significance of inclusion of magnetic distortion on the stellar grounds, which were not considered to an appreciable extent to be included in the past stellar studies. The chapter concludes with the introduction to the equipotential surfaces of the polytropic model of rotationally, tidally and magnetically distorted binary stars.

Chapter three is the final chapter of my thesis. It involves the numerical computation, the methods and approach employed to obtain the stellar parameters. Certain conclusions based upon the present study have been extracted. The chapter concludes on the final note on the discussion of the importance of the present work along with the limitations observed in our approach, and finally the future scope of our present work in astrophysics

Chapter One

INTRODUCTION

This chapter is of introductory nature. It involves the discussion regarding the astrophysical importance of the analytical research and understanding the concept of evaluating the influence of rotational, tidal and magnetic distortion on the equilibrium non-synchronous structure of binary stars. It contains a summarised review of the works related to the present topic of discussion. It also brings upon the focus towards the basic equations which play a vital role I determining the equilibrium structures of the stellar gaseous spheres. The next section explains the averaging technique of Kippenhahn and Thomas that has a very crucial significance in current work. This technique is put forward to determine the equilibrium structures of rotationally, tidally and magnetically distorted binary stars.

1.1 Astrophysical Significance of Determining the Equilibrium Structure of Rotationally, Tidally and Magnetically Distorted Binary Stars

A star is a delicate balance between the gravitational force (pulling it inwards) and the outward pressure force (expanding the star outwards). Unlike other physical experiments where a theoretical idea can be instantaneously assessed, the studies on a star (or a system of stars) takes into picture a variety of simultaneous theories, approximation techniques and computational skills in order to draw strongly valid conclusions about the stellar environment. This quandary arises as a result of the complex features of the system formed at such an extreme energy scale due to the nuclear collisions and also because of the unmapped and unexplored landscapes of the stellar environment we study.

Theoretically a star is analysed as a fundamentally self-gravitating (due to the inward pull) sphere of highly dense molecular clouds, existing in both hydrostatic as well as thermal equilibrium.

Stars being the most widely observed astronomical objects, represent the most essential building blocks of our galaxies. Through stellar studies astrophysicists have made successful efforts to determine the age, composition as well as the distribution of theses gaseous spheres in the known galaxies. The analysis of these collected data help us in tracing the very origin, stellar dynamics and stellar evolution of that particular galaxy.

The stellar evolution is fundamentally responsible for the production and universal distribution of heavy elements found in our periodic table, such as C, N, O and their characteristics deeply linked to that of the planetary systems that may coalesce with them. As a consequence of this, the study of the stellar world becomes integral to the field of astronomy.

Majority of the observed stars spin about their individual axis. The deformation of equilibrium structures of such spinning stars is caused by tides, provided the star is stationary. Inhomogeneities on the surface of the stars provide us with a rare opportunity of studying the stellar structures and stellar magnetic fields for various kinds of stars. Surface inhomogeneities are developed differently depending upon the velocity with which a star rotates. The magnetic fields hence play an important part at various stages involved in the stellar formation and hence evolution. Magnetic fields are majorly responsible for the loss in the angular momentum of the new stars. Magnetic fields also act as the dominant energy source for a wide range of dynamic phenomenon (for instance solar flares, X-ray emission, star spots) which normally occur at the gradations (layers) of a star. But these fields pose a difficulty due to the challenge of their detection and their direct modelling.

Astrophysicists have spent years to observe and analyse the binary star systems where the primary component of star is considered a lot more massive as opposed to the secondary component. In majority of the cases, both the components rotate about their own axis. Moreover, both the components revolve about their shared centre of mass. These can thus be classified into synchronous or non-synchronous binary systems depending on the orbital and spin angular velocity. With this consideration the current thesis tries to further explore some characteristics of the challenges associated with the equilibrium structures of the non-synchronously, rotationally, tidally and magnetically distorted binary stars. Astrophysicists assess the rotational and tidal effects on the equilibrium structure of stars by estimating the real equipotential surfaces of the star by Roche equipotentials, in an applied manner.

1.2 Literature Review

Most researchers in the past have studied the stellar equilibrium structures, by supposing the star or a system of stars, to be undistorted. Since then many authors have worked towards the problem of how to find out the equilibrium structures of rotationally and tidally distorted model of a star. In a series, Chandrashekhar **[2]** proposed a first order analysis to investigate the rotational, tidal and binary stars problem. Well along authors in **[12,13]** have recommended a rational approach of how to evaluate the impact of rotation along with tidal deformations on the equilibrium structures of the stars or a system of stars by estimating the real equipotential surfaces of the stars by Roche Equipotentials . Lal et al **[16]** have considered the equilibrium structures of tidally along with rotationally distorted primary components of the binary stars by considering the impact of variation of mass within the interior of the star. A perturbation theory to compute the effect of gradual differential rotation on the adiabatic non-radial models of stellar oscillations has been developed earlier. Careful evaluation of results and the effects of Coriolis force along with the ellipticity, in parallel, by means of the perturbation technique has been performed.

Hamiltonian operator that was created upto the second order in eigen-frequencies and upto first order in eigen-functions.

Kopal **[14]** established a coordinate system, called the Roche coordinates, to investigate the problem of rotating stars in the binary systems. Mohan and Saxena **[23]** used the averaging technique along with Kopal's result on Roche equipotentials to find the cumulative effects of rotation and tidal distortions on the equilibrium structures and oscillations of the polytropic model of stars.

Complex analytical studies have been discussed in literature to find the equilibrium structure of rotationally and tidally distorted stars. In 1972, Kopal introduced the concept of Roche equipotentials to analyze such problems. After that many authors **[14]**, Mohan and Singh **[23]**, **[16,17,21]**, have addressed themselves to this problem. Roche approximation has been employed to formulate the potential of a rotating star and a star in a binary system. While this technique takes into account the centrifugal, Coriolis and gravitational forces, it doesn't account for the effect of magnetic force. Generally, the magnetic force has been neglected while obtaining Roche equipotential surface of stars because its impact is anticipated to be very little in comparison to those of centrifugal and gravitational forces.

However, magnetic field is also one of the factors causing the asphericity in stars. Many efforts have been made in literature to incorporate the effect of magnetic fields and the consequent Lorentz force into the equation of stellar structure. Authors **[4,5,8]** have studied the effects of magnetic field on the equipotential surfaces of the stars. It was difficult to analyze the situation owing to the presence of the non-radial component of the Lorentz Force. While many authors have formulated this problem by taking only the toroidal component of the magnetic field, Djurašević **[4]** studied the critical surfaces in closed binary system by considering the radial component of the magnetic field – magnetic pressure.

The significance of Roche Limit geometry study in binary systems for different mass ratio is determined by our knowledge that close binaries, where one or both components have reached their Roche limit, are observed in the sky. Several authors have discussed about the Roche limit and the point of contact of distorted stars. In a previous work **[16]**, Lal analyzed the impact of Coriolis force on the shapes of rotating stars and stars in binary system. Nevertheless, the shape of Roche equipotentials in the existence of magnetic field could not be tackled satisfactorily. A productive search direction could be the analysis of such an effect of magnetic field on the shapes and equilibrium structures of rotationally and tidally distorted stars.

Following the assumptions of **[4]** and the approach of **[16]** , an effort has been made in the current work to evaluate the impact of magnetic pressure on the shapes of Roche equipotential surfaces of rotationally and tidally distorted stars. The location of their point of contact has also been critically analysed.

1.3 Elementary Equations Determining the Equilibrium Structure of a Star

All the stars in a galaxy are not of the same mass, composition and age. As a result each one of them has a uniquely fluctuating internal structure. Many stellar structure models have been developed to portray the stellar interior structure in detail and as a result helps the astronomers to predict the colour, luminosity, energy production and the future of a star or a system of stars.

From the data collected through the observations of the stars, in various regions of the electromagnetic spectrum and particularly the precise observations of our Sun’s pulsation modes as well as neutrinos provide the data required to fabricate models of the stellar interior. Hence astronomers have found a way to know about the interior of the star without physically investigating it.

1.3 (a) Mathematical Models

Mathematical models of the interior of a star is constructed by employing the information obtained from the stellar surface and the understanding of the behaviour of gases in diverse conditions. The mathematical models in the most basic sense is a set of equations which describe in what way certain mechanisms take place layer by layer in a star. Fortunately enough the stellar interior is absolutely (downright) gaseous all the way to the centre. Hence the equations are less tedious and thus relatively simpler to work with.

The essential physics of gases can be put forward in 3 parameter :

1. Temperature: It is a measure of the random motion (the average kinetic energy) of the gaseous particles. The particles have higher random kinetic energy at higher temperatures.
2. Pressure: The magnitude of force per unit area. Thus a hot gas expands to create pressure on its surroundings.
3. Mass Density: It is a measure of amount of mass per unit volume. Gaseous materials can be compacted to lower volumes and greater densities.

The synergy of these parameters to explain the material under study, is expressed by the equation of state. It relates the pressure, density and temperature.

The set of primary equations of the equilibrium structure of the stars in hydrostatic and thermal equilibrium are ascertained in related work. They relate to the apparent challenge to find equilibrium structure of stellar models. Let **P** and **ρ** denote the pressure and density at a point, correspondingly, at a distance ***r*** from the sphere's center..

Mass Conservation

Let M(***r***) be the mass enclosed within radius ***r***. So the mass confined within the shell from ***r*** to ***r* + *dr*** is expressed as

$$M(\boldsymbol{r} + \boldsymbol{dr}) - M(\boldsymbol{r}) = \rho(\boldsymbol{r})\, dV \qquad (1.1a)$$

where

r is the radial coordinate of the star

M(r) is the stellar mass enclosed in a star of radius r

P(r) is the density of the star

$dV = 4\pi r^2 dr$, is the prescribed volume of the shell of the star. We can write the LHS of equation (1.1a) as

$$\frac{dM(r)}{dr}\, dr = \rho(r) 4\pi r^2 \mathrm{dr} \qquad (1.1b)$$

Eliminating dr from the numerator and denominator from the LHS , we get the following

$$\frac{dM}{dr} = \rho(r) 4\pi r^2 \qquad (1.1c)$$

Conservation Of Momentum (Hydrostatic Equilibrium)

When a star is in Hydrostatic equilibrium, the gravitational force at any point must be equal to the pressure. To explain this, we assume a shell of radius r. Let the mass per unit area in a particular shell be ρdr. Also the weight per unit area is –gρdr. This weight (gravity) must be balanced by an equal amount of pressure force, experience from one end of the shell to the other. For a shell element with radius ranging from r to r + dr, the area element would be dA.

Thus a coordinate system, in which the radial component grows towards outer side, is established. Consequently the force of pressure acting inwards in a star of radius r is positive whereas the pressure force upon the outer side of radius r + dr is negative.

Thus

$$P(r)dA - P(r + dr)dA = [P(r) - P(r + dr)]dA = -\frac{dP}{dr}\,drdA \qquad (1.2a)$$

The gravitational force, owing to the spherical symmetry, points towards center of star, hence it is negative.

$$-\frac{GM_r dm}{r^2} \qquad (1.2b)$$

where dm is the mass of the small element taken into account.

We know $dm = \boldsymbol{\rho}(r)dV = \boldsymbol{\rho}(r)dAdr$ (1.2c)

Substituting this value of dm in equation (1.2b), we obtain the following

$$-\frac{G\rho(r)dAdrM_r}{r^2} \qquad (1.2d)$$

Using Newtons's second law , we've the following expression

$$\rho(r)dA\frac{d^2r}{dt^2} = -\frac{dP}{dr}dA - \frac{G\rho(r)dAM_r}{r^2} \qquad (1.2e)$$

after cancelling the factors dr and dA and obtain

$$\rho(r)\frac{d^2r}{dt^2} = -\frac{dP}{dr} - \frac{G\rho(r)M_r}{r^2} \qquad (1.2f)$$

For the stellar system is in hydrostatic equilibrium, the cumulative forces must disappear, i.e.,

$$\frac{dP}{dr} - \frac{G\rho(r)M_r}{r^2} = 0$$

Or

$$\frac{dP}{dr} = \frac{G\rho(r)M_r}{r^2}$$

Energy Conservation

Stars emit energy through radiation. This reduction needs to be cancelled out by the energy emitted by the interior nuclear reactions occurring at the core.

Let L(r) be the energy flow across the gaseous sphere of radius r. The total loss in energy from the shell of radius r to r + dr is

$$L(r + dr) - L(r) = \frac{dL(r)}{dr}\,dr \qquad (1.3a)$$

Let us suppose ε is the amount of energy generated per kg. Then the entire energy produced in this shell will be given as

$$dE = \varepsilon\rho(r)4\pi r^2 dr \qquad (1.3b)$$

Thermal equilibrium can be maintained only when the radiation loss is same as energy gain (from nuclear fusion). Thus

$$\frac{dL(r)}{dr}dr = \varepsilon\rho(r)4\pi r^2 dr \qquad (1.3c)$$

Or

$$\frac{dL(r)}{dr} = \varepsilon\rho(r)4\pi r^2 \qquad (1.3d)$$

In astrophysical problems, where the thermal characteristics of a model are neither investigated nor are required, comes the role of equilibrium structure of the stars, which is evaluated through the use of equations (1.1c and 1.2h) with a prescribed set of equations and the boundary conditions.

- At the center r = 0 , M(r) = 0
- At the surface r = 0 , M(r) = 0 ; P = 0 or P_s ; **ρ = 0 or ρ_s**

Numerous studies (theoretical as well as numerical) about the equilibrium structures of stars are found in the **[2, 17, 18, 28, 35] .**

1.4 Averaging Technique of Kippenhahn and Thomas

A complete knowledge of the formation, structure and the evolution of the stars incorporating effect of rotation and magnetic field is still a major problem persisting in the modern astrophysics. Under a broad spectrum the effects of rotation on a star and a system of stars, have been understood thoroughly in the past **[9, 10, 12]**. There are two prominent ways in which stellar rotation affects the structure and hence the evolution of a star. The first affect leads to the departure of a star from its spherical symmetry owing to the addition of the centrifugal forces. Secondly, it produces secular as well as dynamical instabilities which produce a coupled effect as a direct consequence of which redistribution of the angular momentum throughout the star takes place. This very redistribution of the angular momentum of the entire star will bound to affect the stellar structure (interior) through the rotational distortions along with the associated mass motion which in turn causes the chemical fusing which is bound to have a great effect on their evolution.

Numerous procedures to compute the rotating stellar models were created in the past, carrying their own benefits as well as disadvantages. Initially **[2]** gave the first order perturbation technique to understand the impacts of rotation and tidal distortions. The initial order perturbation technique proposed by Chandrasekhar has been continued to second by **[5,6,7]** and even to the third order.

To inspect the impact of rotational and tidal distortion on the equilibrium structures of the star, Kippenhahn and Thomas **[13]** put forward the model of the topologically equivalent spherical surface, in consistence with real equipotential surfaces of rotationally and tidally distorted stellar models. It encompasses outcomes of rotation and tidal distortions required for fundamental stellar structure equation. This method differs in approach because it incorporates the net potential (gravitational and centrifugal) and peak distances are considered as 2 distinct variables. Hence numerous physical quantities, for instance density, become functions of net potentials itself. By considering a potential field, additional quantities depending upon peak distance are averaged over equipotential surfaces. Thus problem reduces to a one dimensional space.

Kippenhahn and Thomas defined several quantities for these topologically equivalent like $\bar{f}$,$\bar{g}$ etc.to indicate specific mean of quantities f, g, correspondingly for an actual equipotential surface.

Suppose ψ denotes the net potential on any random point P(x,y,z) . Then ψ(x,y,z) = constant , is an equipotential surface. Let V_ψ be stellar volume confined by the surface and S_ψ be the surface area of this equipotential surface.

For a function f(x,y,z) we designate $\bar{f}$ as its average over ψ . It can be mathematically related as

$$\bar{f} = \frac{1}{S_\psi}\int f d\sigma \qquad (1.4)$$

Here dσ indicates small surface element of ψ. Evidently $\bar{f}$ is determined by ψ which could be derived from equation (1.3) for every ψ .

Another variable defined by Kippenhahn and Thomas is r_ψ in resemblance with the radius of the star. It is given by the following relation

$$V_\psi = \frac{4}{3}\pi r_\psi^3 \qquad (1.5)$$

The surface area is given as $$S_\psi = \int d\sigma \qquad (1.6)$$

To incorporate the force of gravity acting on a star Kippenhahn and Thomas defined a function g(x,y,z) by

$$g = \frac{d\psi}{dn} \tag{1.7}$$

This function g corresponds to the stellar gravity.

If we take two adjacent surfaces say ψ and $\psi + d\psi$ (= constant) , the the distance dn between these two surfaces is not a constant in general (i.e. it is variant at various points on the surface). They employed equation (1.6) to find averages i.e. $\bar{g}$ and $\overline{g^{-1}}$ by the use of the following

$$\bar{g} = \frac{1}{S_\psi}\int \frac{d\psi}{dn}\, d\sigma$$

$$\overline{g^{-1}} = \frac{1}{S_\psi}\int \left(\frac{d\psi}{dn}\right)^{-1} d\sigma \tag{1.8}$$

$\bar{g}$ and $\overline{g^{-1}}$ are both functions of ψ itself and symbolize the value of g and g^{-1} respectively over the topologically equivalent spherical surface. Volume dV_ψ between the surface ψ and $\psi + d\psi$ is

$$dV_\psi = \int dn d\sigma = \int \left(\frac{d\psi}{dn}\right)^{-1} dn = S_\psi\, \overline{g^{-1}} d\psi \tag{1.9}$$

Non-Dimensional parameters defined for this approach are as followed :-

$$u = \frac{S_\psi}{4\pi r_\psi^2}$$

$$v = \frac{\bar{g}\, r_\psi^2}{GM_\psi}$$

$$w = \frac{\overline{g^{-1}} GM_\psi}{r_\psi^2}$$

(1.10)

M_ψ indicates mass contained in the equipotential surface.

Equations (1.3) and (1.8) are entirely mathematical logic statements that were used to obtain gravitational fields of stars which were rotationally and tidally distorted. When hydrostatic equilibrium is achieved the equipotential surfaces become equi-pressure as well as equi-density surfaces. Hence for an equipotential surface, the pressure P_ψ and density ρ_ψ remain same.

With the help of such ideas along with the equation (1.4), mass of the equipotential surface ψ and $\psi + d\psi$ is followed

$$dM_\psi = dV_\psi \rho_\psi = 4\pi r_\psi^2 \rho_\psi dr_\psi \tag{1.11}$$

We obtain
$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \tag{1.12}$$

From (1.10) and (1.11) we obtain

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = \left(\frac{dV_\psi}{d\psi}\right)^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi \overline{g^{-1}} \;\; \rho_\psi} \tag{1.13}$$

By the use of (1.10)

$$d\psi = \frac{GM_\psi dM_\psi}{4\pi r_\psi^4 \rho_\psi \, uw} \tag{1.14}$$

$\frac{dP_\psi}{d\psi} = -\rho_\psi$ is the equation for hydrostatic equilibrium. This can be modified and written in the following form

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \tag{1.15}$$

where $f_p = \frac{1}{uw} = \frac{4\pi r_\psi^4}{GM_\psi} \frac{1}{\overline{g^{-1}} S_\psi}$

f_p is once again dependent on value of ψ itself. Thus if ψ is recognized, we can determine equipotential surface and in turn we can derive the values of S_ψ , r_ψ , $\bar{g}$ and $\overline{g^{-1}}$ for individual equipotential surface from geometry of the equipotentials . Mass M_ψ is dependent upon the density profile ρ_ψ and could be obtained from integration of equation (1.12).

In a similar manner, other structural equations have been derived by Kippenhahn and Thomas by involving influence of tidal and rotational distortions on the equilibrium structure of stellar system.

For chemically homogenous spherical domains, the rate of generation of the nuclear energy (ε) depends upon two factors namely, density (ρ_ψ) and temperature T_ψ and as a result is constant on an equipotential surface. Thus suppose L_ψ be the energy which emits per unit second across given equipotential surface ψ (= constant) , then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \qquad (1.16)$$

From equation (1.12) , we observe that

$$\frac{dL_\psi}{dM_\psi} = 4\pi r_\psi^2 \rho_\psi \varepsilon \qquad (1.17)$$

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 ac T_\psi^3 r_\psi^4} f_T \qquad (1.18)$$

From (1.11) , we may express the above equation as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa \rho_\psi L_\psi}{16\pi^2 ac T_\psi^3 r_\psi^2} f_T \qquad (1.19)$$

Here $f_T = \frac{1}{u^2 vw}$ (1.20)

So equations (1.12),(1.15),(1.16) and (1.18) define the fundamental stellar equations which govern the equilibrium structure of stars distorted by rotational as well as tidal forces.

These can be formulated as :

1. $\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi$ (1.21a)
2. $\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p$ (1.21b)
3. $\frac{dL_\psi}{dM_\psi} = \varepsilon$ (1.21c)
4. $\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 ac T_\psi^3 r_\psi^4} f_T$ (1.21d)

where $f_p = \frac{1}{uw}$ and $f_T = \frac{1}{vwu^2}$

These complex appearing equations can be reduced to the normal set of equations, which are employed in investigating the equilibrium structures, by setting each of the distortion parameters (u,v,w) equal to one.

The boundary conditions required to be satisfy the above mentioned equations are as follow:

$$M_\psi = 0\,, L_\psi = 0 \tag{1.22a}$$

At center $r_\psi = 0$, $M_\psi = M_0$, $L_\psi = L_{\psi s}$, P_ψ=0, $T_\psi = 0$ or $P_\psi = P_{\psi s}$ and $T_\psi = T_{\psi s}$

At free surface $\qquad r_\psi = R_\psi \qquad$ (1.22b)

M_0 is considered to be the net mass of the binary stellar system and $L_{\psi s}$, $P_{\psi s}$, $T_{\psi s}$ are basically the values of L_ψ, T_ψ, P_ψ at outermost equipotential surface.

1.5 Improved Expression For Roche Equipotentials Including The Effects Of Magnetic Forces

In a binary star system, we have a primary component which is exceptionally huge than secondary component, that is assumed to be a point mass. Also, stars in a binary system rotate at their own axis and revolve about the axis passing through their common center of mass.

Following Kopal's work , the masses of major and minor components are assumed to be M_0 and M_1 respectively, such that $M_0 >> M_1$. The interior configuration of the massive star is approximated by Roche Model, that considers the total star mass is focussed at its center and this point mass is wrapped by transitory envelope where density profile is inversely proportional to the squared distance from its center. Such an estimation is plausible for majority of stars which are in the main and the post-main sequence stages **[2].**

Generally, it is indicated that magnetic field has negligible impact upon the shape and structure of the equipotential surfaces. Besides, there were several challenges resulting owing to the presence of non-radial component of the Lorentz force. Thus, magnetic pressure which is the radial component of Lorentz force is used in present work to formulate the potential equation. Here effect of magnetic field on the critical surfaces of RTD stars following the assumptions of **[4]** in which the author studied the problem of determination of equipotential surfaces. Djuraševi formulated the problem by including the effects of radiation due to both the components of the binary star system and neglected the magnetic field **[4]**. Contrary to this, we have considered only the magnetic field and neglected any radiation effects in this study. So effect of magnetic field only due to the primary component is taken into account while obtaining potential equation. The magnetic effect due to secondary component could not be considered as it is assumed to be a point mass star.

If the position of stars is taken in a rectangular system of Cartesian coordinates and the gravitational center of primary star is taken to be at origin and that of the secondary to be at a distance D from primary, then the center of gravity C of the binary system lies on x-axis having coordinates (d_1, 0, 0) where

$$d_1 = \frac{M_1 D}{(\mathrm{M}_1 + \mathrm{M}_{0)}}$$

D is distance by which the two stars (primary M_0 and secondary M_1) are separated.

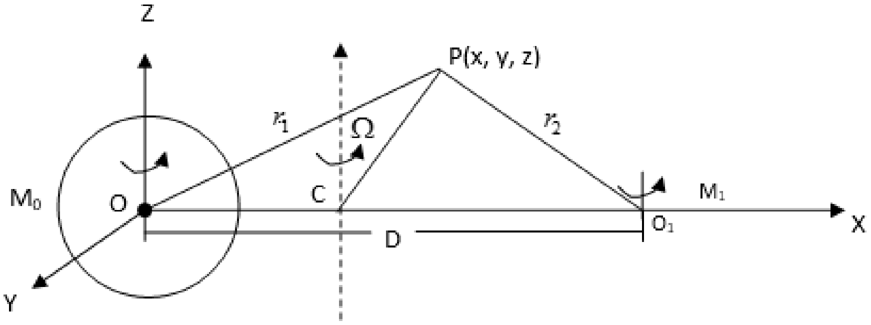

Fig 1.1: Axis of reference for a binary system

Z-Axis is considered to be orthogonal to the plane of trajectory of the 2 stars. Let Ω indicates angular velocity of revolution for a system around a line collateral to z-axis passing through the center of gravity C of the system.

In a 2 star system, the primary star rotates about axis OZ by means of angular velocity Ω_1 in addition to revolving about an axis collateral to z-axis passing via shared centre of mass C having angular velocity Ω. Point P will experience the magnetic pull or force besides the gravitational and centrifugal forces, if it lies within the primary component of 2 star system.

In the Fig.1.1, $r_1 = \sqrt{x^2 + y^2 + z^2}$, $r_2 = \sqrt{(D-x)^2 + y^2 + z^2}$ and $r = \sqrt{(x-d_1)^2 + y^2 + z^2}$.

These indicate separation of a point P (x, y, z) from centre of gravity of principal star (point O), mid-point of gravity of subordinate star (point O_1) and the mid-point of gravity of the whole system (point C) respectively.

Using fundamentals of classical dynamics and following assumptions of Djurašević, the potential at a point P (x, y, z) which experience effect of magnetic forces besides the gravitational and centrifugal forces, is given by

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}\left(\vec{\Omega}\times\vec{r}\right).\left(\vec{\Omega}\times\vec{r}\right) - \left(\frac{1}{2}\right)^{1/4} \frac{kT_1}{2\mu m_H}\left[1-\sqrt{1-\left(\frac{R_1}{r_1}\right)^2}\,\right]^{1/4}\left(5+4\frac{P_m}{P_g}\right) \tag{1.23}$$

where the initial 2 expressions corresponds to gravitational potential, 3rd term is because of centrifugal force and the last term is due to the magnetic pressure.

As discussed in Djurašević **[4]**, k is the Boltzmann constant, T_1 is effective temperature of the primary component, μ is the mean molecular weight, m_H is the atomic mass unit, R_1 is radius of primary star, P_m is magnetic pressure and P_g is the gas pressure. On applying Binomial approximation to the last term of equation (1.23), we get

$$\frac{kT_1}{2\sqrt{2}\mu m_H}\left(5+4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}} \tag{1.24}$$

The third term is the cross product of the angular velocity which is along z- axis and $\vec{r}$. If $\vec{r} = x\hat{i} + yj + zk$ and $\vec{\Omega} = \Omega k$, then the term becomes

$$\vec{\Omega} \times \vec{r} = \Omega^2\ [(x-d)^2 + y^2] \tag{1.25}$$

Now the modified expression of potential can be written as

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}\Omega^2\left[(x-d)^2 + y^2\right] - \frac{kT_1}{2\sqrt{2}\mu m_H}\sqrt{\frac{R_1}{r_1}}\left(5+4\frac{P_m}{P_g}\right)$$

Following Kopal (1972), equation (1.23) has been written in a non-dimensional form as:

$$\psi^* = \frac{1}{r^*} + q\left(\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right) + n(1-\nu^2)r^{*2} - \frac{\alpha}{\sqrt{r^*}} \tag{1.26}$$

where $\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$ is non-dimensional way of potential ψ , $r^* = (r_1/D)$ is the non-dimensional form of r$_1$, $q = \frac{M_1}{M_0}$ is the mass ratio and is known as tidal parameter

n = $\frac{(q+1)}{2}$ (for synchronous rotation) and n $\neq \frac{(q+1)}{2}$ (for asynchronous rotation). n is also known as rotational parameter.

$$\alpha = \left(\frac{D}{GM_0}\right)\frac{kT_1}{2\sqrt{2}\mu m_H}\left(5+4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}} \text{ is the magnetic parameter.}$$

$\lambda = sin\theta cos\varphi$, $\mu = sin\theta sin\varphi$ *and* $\vartheta = cos\theta$ are the spherical polar coordinates. And $\frac{P_g}{P_m}$ is the plasma parameter.

Now assuming the composition of the star to be dominated by ionized hydrogen, μ = ½, and substituting for the value of Boltzmann constant k = 1.38 × 10^{-23} m^2 kg s^{-2} K^{-1} , the value of α varies between zero and one. Like tidal and rotational parameters, α is also non-dimensional parameter. The effects of magnetic field on the binary system can be examined by changing the value of this parameter. For $\frac{P_g}{P_m} = 0$, the effect due to magnetic pressure vanishes and then the equipotential surfaces will be defined by gas pressure gradient, gravitational and centrifugal forces only. However, the modifications in the Roche model due to gas pressure are negligible therefore they can be neglected. If α = 0, then the resulting expression becomes

$$\psi^* = \frac{1}{r^*} + q\left(\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right) + n(1-\nu^2)r^{*2} \tag{1.27}$$

Equation (1.27) is the same as obtained in **[14].** Setting q = 0, (1.26) takes the form of a magnetically rotating stellar model.

Equation (1.26) becomes the case of pure tidal distortion on setting α = 0 and n = 0.

From computational point of view, equation (1.26) can also be written using Legendre Polynomials $P_J(\lambda)$ as: $$\psi^* = \frac{1}{r^*} + q + q\sum_{j=2}^{\infty} r^{*j} P_j(\lambda) + n(1-\nu^2)r^{*2} - \frac{\alpha}{\sqrt{r^*}} \tag{1.28}$$

To find the shape of Roche equipotential surfaces defined by setting Ψ^*, r* has to be found for a given value of Ψ^* and specified values of θ and φ, r* cannot be computed explicitly. Kopal thus attained a series expansion for r* in expressions of other parameters. Following **[14]** and **[23]** and taking non-dimensional variable $r_0 = \frac{1}{\psi^* - q}$ as our first approximation to the distance of equipotential surface from the center, several iterations are carried out to obtain the value of r* in terms of r_0.

The expression of r* is given as:

$$\begin{aligned} r^{*} = {} & r_0[1-\alpha r_0^{1/2}+\frac{\alpha}{2}r_0+\{qP_2+n(1-v^2)\}r_0^3+\left\{\frac{-7}{2}\alpha(qP_2+n(1-v^2))\right\}r_0^{7/2}+qP_3r_0^4+\left\{\frac{-9}{2}\alpha qP_3\right\}r_0^{9/2}+ \\ & qP_4r_0^5+\left\{\frac{-11}{2}\alpha qP_4\right\}r_0^{11/2}+\{qP_5+3P_2^2q^2+6qn(1-v^2)P_2+3n^2(1-v^2)^2\}r_0^6+\left\{\frac{-13}{2}aqP_5\right\}r_0^{13/2}+ \\ & \{qP_6+7P_3P_2q^2+7qn(1-v^2)P_3\}r_0^7+\left\{\frac{-15}{2}\alpha qP_6\right\}r_0^{15/2}+\{qp_7+8P_4P_2q^2+4P_3^2q^2+8qn(1-v^2)P_4\}r_0^8+ \\ & \left\{\frac{-17}{2}\alpha qP_7\right\}r_0^{17/2}+\{qP_8+9P_5P_2q^2+9P_4P_3q^2+9qn(1-v^2)P_5\}r_0^9+\left\{\frac{-19}{2}\alpha qP_8\right\}r_0^{19/2}+ \\ & \{qP_9+10P_6P_2q^2+10P_5P_3q^2+5P_4^2q^2+10qn(1-v^2)P_6\}r_0^{10}+...] \end{aligned} \tag{1.29}$$

Following Kopal **[14],** the volume V_ψ , radius r_ψ and surface area S_ψ for equipotential surface Ψ* are obtained as

$$V_\psi=\frac{4}{3}\pi r_0^3D^3[1-3\alpha r_0^{1/2}+\frac{5}{2}\alpha^2r_0+2nr_0^3-11\alpha nr_0^{7/2}+\left\{\frac{12}{5}q^2+\frac{8}{5}qn+\frac{32}{5}n^2\right\}r_0^6+\frac{15}{7}q^2r_0^8+2q^2r_0^{10}+..] \tag{1.30}$$

$$r_\psi=r_0D\,[1-\alpha r_0^{1/2}+\frac{1}{2}\alpha^2r_0+\frac{2}{3}nr_0^3-\frac{7}{3}\alpha nr_0^{7/2}+\left\{\frac{4}{5}q^2+\frac{8}{15}qn+\frac{76}{45}n^2\right\}r_0^6+\frac{5}{7}q^2r_0^8+\frac{2}{3}q^2r_0^{10}+..] \tag{1.31}$$

And,

$$S_\psi=4\pi r_0^2D^2[1-2\alpha r_0^{1/2}+2\alpha^2r_0+\frac{4}{3}nr_0^3-6\alpha nr_0^{7/2}+\left\{\frac{7}{5}q^2+\frac{14}{15}qn+\frac{56}{15}n^2\right\}r_0^6+\frac{9}{7}q^2r_0^8+\frac{11}{9}q^2r_0^{10}+..] \tag{1.32}$$

In all the above expressions ((1.29), (1.30), (1.31), (1.32)) terms upto second order of efficiency in n, q and α are retained. On substituting α = 0, these reduce to respective expressions found by the previous work in **[12, 19, 20].**

With the help of the adapted formulation expressions for Roche Equipotentials in addition to aforementioned factors that merge these effects namely gravitational, centrifugal and magnetic forces, we obtain the values of the new distortion parameters *u, v, w,* f_p, f_T, f_1, f_2. f_3 .

$$u = \left[1 + 4\alpha^2 r_0 - 12\alpha r_o^{\frac{7}{2}} n - \left(\frac{1}{5}q^2 + \frac{94}{9}n^2 + \frac{2}{15}\,nq\right) r_0^6 - \left(\frac{q^2}{7}\right) r_o^8 - \left(\frac{1}{9}q^2\right) r_0^{10}\right] \tag{1.33}$$

$$v = \left[1 - 5\alpha^2 r_0 - r_0^3\left(\frac{4}{3}n + 3\alpha^2\right) - \frac{35}{3}\alpha^2 r_0^4 - \left(\frac{7}{5}q^2 + \frac{88}{45}n^2 + \frac{14}{15}nq\right) r_0^6 - \frac{6}{7}q^2 r_0^8 - \frac{4}{9}q^2 r_0^{10}\right] \tag{1.34}$$

$$w = \left[1 - 3\alpha^2 r_0^1 - \alpha^4 r_0^2 - 7\alpha^2 r_0^4 + \frac{4}{3}n r_0^6 + \left(q^2 + \frac{64}{15}nq + \frac{4}{9}n^2\right) r_0^6 + \frac{4}{7}q^2 r_0^8 + \frac{1}{3}q^2 r_0^{10}\right] \tag{1.35}$$

$$f_p = \left[1 - \alpha^2 r + \alpha^2 r^2(\alpha^2 + 7r^2) - \frac{4}{3}n r^3 - \left(\frac{4}{5}q^2 + \frac{84}{9}n^2 + \frac{62}{15}nq\right) r^6 - \frac{3}{7}q^2 r^8 - \frac{2}{9}r^{10} q^2\right] \tag{1.36}$$

$$f_T = \left[1 + \alpha^4 r^2 + 3\alpha^2 r^3 + -7\alpha^2 r^4 - \left(\frac{4}{5}q^2 + \frac{12}{5}n^2 + \frac{134}{15}nq\right) r^6 - \frac{4}{7}r^8 q^2 - \frac{1}{3}q^2 r^{10}\right] \tag{1.37}$$

$$f_1 = r_0^2\left[1 + \frac{4}{3}n r_0^5 + \left(\frac{36}{5}q^2 + \frac{884}{45}n^2 + \frac{24}{5}nq\right) r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} - \frac{3}{2}\alpha + \alpha^2 r_0\right] \tag{1.38}$$

$$f_2 = \frac{1}{r_0^2}\left[1 + \left(\frac{16}{5}q^2 - \frac{20}{9}n^2 - \frac{4}{5}nq\right) r_0^6 + \frac{32}{7}q^2 r_0^8 + \frac{34}{9}q^2 r_0^{10} + \alpha n r_0^{\frac{1}{2}}\left(2 - \frac{35}{6}r_0^3 - \frac{3}{2}r_0^{\frac{1}{2}}\right) + \alpha^2 r_0(7r_0^3 - 1)\right] \tag{1.39}$$

$$f_3 = \frac{1}{r_0^2}\left[1 + +2\alpha r_0^{\frac{1}{2}} - \frac{3}{2}\alpha r_0 + \alpha^2 r_0^2(\alpha^2 - 7r_0^2) - \frac{35}{6}\,\alpha n r_0^{\frac{7}{2}} + \frac{4}{3}n r_0^3 + \left(\frac{16}{5}q^2 + \frac{431}{45}n^2 - \frac{94}{5}nq\right) r_0^6 + \frac{31}{7}q^2 r_0^8 + \frac{17}{3}q^2 r_0^{10}\right] \tag{1.40}$$

where $r_\psi^* = \frac{r_\psi}{D}$ is non dimensional form of r_ψ and terms upto the 2nd order of exactness in n and q. terms upto r_0^{10} in r_ψ.

Thus the modified equations are :

$$\frac{dM_\psi}{dr_\psi} = 4\pi D^3 r_0^3 \rho_\psi f_1 \tag{1.40 a}$$

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{Dr_0^2}\,\rho_\psi f_2 \tag{1.40 b}$$

$$\frac{dL_\psi}{dM_\psi} = 4\pi\varepsilon D^3 \rho_\psi r_0^2 f_1 \tag{1.40 c}$$

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi \rho_\psi}{16\pi^1 DacT_\psi^3 r_0^2} f_3 \tag{1.40 d}$$

Here f_1 f_2 and f_3 are some functions of the distortion parameters which incorporate the effect of magnetic forces along with gravitational and centrifugal upon equilibrium structure of rotationally, tidally and magnetically distorted stellar model.

In an ideal scenario i.e. no distortion $f_1=f_2=f_3=0$

Explicit expressions for $f_1=\frac{dr_\psi}{dr_0}\frac{r_\psi^2}{D^3}$, $f_2 = \frac{f_p}{r_\psi^2}\frac{dr_\psi}{dr_0}D$ and $f_3 = \frac{f_1}{r_\psi^2}\frac{dr_\psi}{dr_0}D$

Here too expressions upto 2nd order of exactness in n and q and uptp $(r_o)^{10}$ in r_o are retained. These are given as follow :

$$f_1= r_0^2\left[1-\frac{3}{2}\alpha+\alpha^2 r_0+\frac{4}{3}nr_0^5+\left(\frac{36}{5}q^2+\frac{884}{45}n^2+\frac{24}{5}nq\right)r_0^6+\frac{55}{7}q^2r_0^8+\frac{26}{3}q^2r_0^{10}\right] \tag{1.41}$$

$$f_2=\frac{1}{r_0^2}\left[1+\alpha n r_0^{\frac{1}{2}}\left(2-\frac{35}{6}r_0^3-\frac{3}{2}r_0^{\frac{1}{2}}\right)+\alpha^2 r_0(7r_0^3-1)+\left(\frac{16}{5}q^2-\frac{20}{9}n^2-\frac{4}{5}nq\right)r_0^6+\frac{32}{7}q^2r_0^8+\frac{34}{9}q^2r_0^{10}+\right] \tag{1.42}$$

$$f_3=\frac{1}{r_0^2}\left[1+2\alpha r^{\frac{1}{2}}-\frac{3}{2}\alpha r_0+\alpha^2 r_0^2(\alpha^2-7r_0^2)-\frac{35}{6}\alpha n r_0^{\frac{7}{2}}+\frac{4}{3}nr_0^3+\left(\frac{16}{5}q^2+\frac{431}{45}n^2-\frac{94}{5}nq\right)r_0^6+\frac{31}{7}q^2r_0^8+\frac{17}{3}q^2r_0^{10}\right] \tag{1.43}$$

and

$$r_0 = r_\psi^{*^2}\left[1+2\alpha r_\psi^{*^{\frac{1}{2}}}-\alpha^2 r_\psi^*-\frac{2}{3}nr_\psi^{*^3}+\frac{7}{3}\alpha n r_\psi^{*^{\frac{7}{2}}}-\left(\frac{4}{5}q^2+\frac{8}{15}qn+\frac{76}{45}n^2\right)r_\psi^{*^6}-\frac{5}{7}q^2r_\psi^{*^8}-\frac{2}{3}q^2r_\psi^{*^{10}}+\dots\right] \tag{1.44}$$

where r_ψ^* is the non-dimensional value of radius for the theoretically considered topologically corresponding equipotential exterior. Effect of magnetic forces arises in above terms through α. Boundary condition now take a different expression as

At center $r_0 = 0,\ M_\psi = 0$

At surface $r_0 = r_{0s}$, $M_\psi = M_0$, $P_\psi = 0\ or\ P_{\psi s}$

M_o is the entire bulk matter of the stellar model, $P_{\psi s}$ is the value of P_ψ on the outermost equipotential surface ψ^*= constant.

At surface $r_o = r_{os}$.

1.6 Proposed Work

The general issue to find the configuration of equilibrium structure of stars deformed because of rotational, tidal and magnetic forces is of huge significance in the field of astronomy. Challenging issues like thus facilitate the enhanced comprehension of the nature and environment of the internal configuration of rotating, tidal and magnetically deformed stars. At present no general approach is available through which an exact determination of the cumulative effects of rotation, tidal along with magnetic disturbances on the equilibrium structures of binary stars. Many attempts were made thus far in this very direction to explore some specific aspects of this problem in certain approximate ways. Currently the puzzle of determining these effects upon the structure and stability of a binary star system is thus not satisfactory solved. By keeping in mind its astronomical significance, there is still a lot of scope and need for further investigations in this direction. We have thus attempted to investigate and put forward certain aspects of this problem.

The accurate mathematical approach and investigation of the problem of determination of the cumulative effects of rotational, tidal and magnetic distortions upon the stellar structure is extremely complex and time consuming. Thus an attempt has been made to investigate the problem at hand with reliable and reasonable simplifying assumptions.

Thus we have supposed the distortions to a limited extent such that the nonconformity of the shape of the distorted binary system, from its actual (undistorted) spherical symmetry is not exceptionally huge. Care has been taken to neglect the distortion effects beyond second order of exactness in all three distortions involved.

Hence it is under these assumptions that we have put efforts to develop certain techniques which can further be employed to include, in a reasonably approximate way, the combined influence of rotation, tidal and magnetic distortions on the equilibrium structure of non-synchronously rotating binary stars.

CHAPTER TWO

EFFECT OF ROTATIONAL, TIDAL AND MAGNETIC DISTORTION ON THE EQUILIBRIUM STRUCTURE OF NON-SYNCHRONOUSLY ROTATING BINARY STARS

2.1 Binary Stars

A binary star system includes two stars of different masses orbiting around their shared barycenter. These play a vital role in the modern astrophysics because by computing their orbits, the possibility to determine the stellar masses of their component gaseous spheres, becomes high. Consequently many other crucial parameters of stars such as their radius, density profile etc. can be estimated, though indirectly. Their studies have been successfully able to find an empirical mass-luminosity relationship (abbreviated as MLR) which in turn estimates the masses of single stars in our galaxy.

With the evolution of the stars which appear in the main sequence chain, the size of the stars increases at a certain point. At and beyond a certain point, when a star surpasses its Roche lobe, the stellar material will stream onto the binary companion, generally via accretion discs. Roche lobe is defined as a distinctively shaped (tear drop) region outlining a binary star system, which is bound to the star the star's gravitational pull. Any substance lying beyond the star's Roche lobe may either escape the binary system entirely, orbit both the stars or descend onto the binary component, as a subject to what its initial energy, position and momentum were.

To visualize it simply, let us take a close binary system wherein one of the component stars has expanded (increased in size).

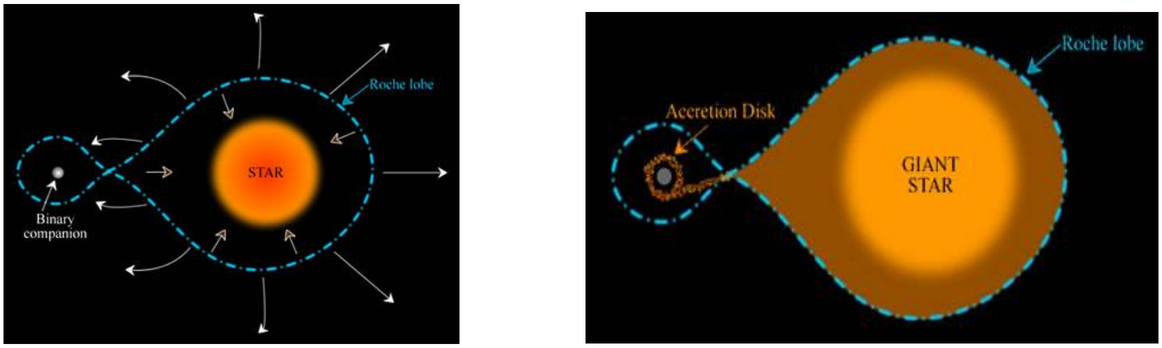

Fig 2.1

The above representation depicts that when the star saturates its Roche lobe, its stellar material will outflow towards its binary component. This phenomenon is termed as Roche-lobe overflow. It takes place through the internal Lagrangian point, which is the mathematical point through which this mass transfer becomes possible. It is defined as the physical point where the gravitational forces of the two stars cancel each other. It accounts for numerous astronomical phenomenon like novae, X-ray binary system etc..

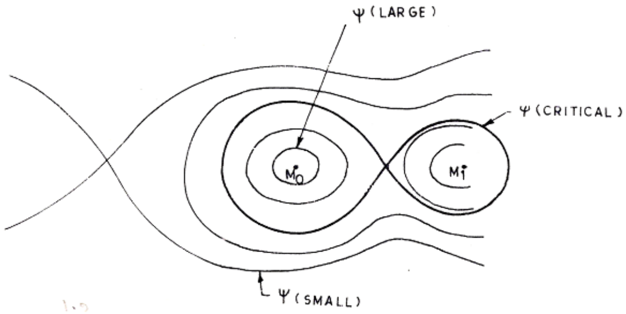

Fig 2.2 Roche equipotential surface in two dimension

There are two types of mass transfers.

1. **Conservative Mass Transfer :** This takes place when no expelled substance exits a binary system. During this mass transmission, system's orbital element may transform because of the transfer of angular momentum between a star and its binary counterpart. Let us assume a stellar environment with total mass $M=M_1 + M_2$ (M_1 mass of the primary component while M_2 of the secondary component). Eccentricity of the system be e and the total angular momentum $J = M_1 M_2 \sqrt{\frac{Ga(1-e^2)}{M_1 M_2}}$, will aslo be conserved. Here G is universal gravitational constant.

   So J=0,M=0 and $M_1= -M_2$. Hence if M1 is the star which is shredding its mass, then $\dot{M}_1$ is negative. Thus a conclusion can be drawn that when the mass exhausting star is more giant than the stellar path would suffer a definite shrink and thus period reduces. On the other hand if the star is not as colossal as its significant binary counterpart, the stellar trajectory will be widened with rise in period.

2. **Non-Conservative Mass Transfer:** In this the stellar mass along with the angular momentum is neglected from system. Orbital Angular Momentum of any two body arrangement is defined as J $=\left(\frac{M_1 M_2}{M_1+M_2}\right) a^2 \Omega = \left(\frac{q}{(1+q)^2}\right) M a^2 \Omega$, where, $\left(\frac{M_1 M_2}{M_1+M_2}\right) a^2 = I$ is the Moment of inertia. $\Omega = \frac{2\pi}{P}$ will be the angular speed and P the orbital period.

2.2 Effect of Magnetic Fields on The Mass Reduction and Transmission

At the present time it has convinced astrophysical community that close binaries possessing a relatively cool F-K type star exhibit enhanced magnetic activities. The close binaries with orbital periods less than 5-6 days possess certain characteristics because of the rapid rotations. Several authors **[19, 22, 24]** have examined the primary and secondary components in a closed binary to exhibit various time dependent magnetic properties, which gives rise to the variations observed in the brightness of the light curves, cyclic variations in the binary system's orbital period, radiations of X-rays, UV and Infrared.

2.3 Generations of Poloidal and Toroidal Magnetic Fields

We are aware that the dynamo mechanism is most probable source of a massive scale magnetic production in the stars possessing convective layers. The differential rotation observed amongst the core which radiates and convective cover, curves up field and leads to the distortion of the poloidal field thereby producing an extra toroidal field component, thereby generating a Lorentz force which works against the shear caused by the very poloidal field itself. The influence of magnetic fields upon the mass and angular momentum transmission and mass reduction of both binary companions are extremely convincing. In a close binary system, wherein the orbital angular momenta and spin are coalesced very strongly, the stellar spin down imposes a reduction on the orbital period of the binary system even in the absence of mass transfer. Magnetic field here becomes the significant factor upon which the value of coupling constant depends and may become prominent if the field is compelling (or fierce) enough. An additional outcome which accompanies magnetic field is the alteration of the rotation via providing a torque to the star by the mass overflow.

2.4 Magnetic Braking

Although the main result of mass exchange is the transfer of mass, generally from a primary binary component to a secondary one in a whole contact phase, but along with it magnetic braking is a prominent and usual occurrence for entire closed binary system. The role of loss of angular momentum has been explained and concluded to be extremely essential for the birth plus future evolution of closed binaries. The magnetized stellar winds are directed towards the outer stellar surface of the active star but these stellar winds get twisted (distorted) because of the rapid stellar rotation. The charged entities present within stellar gust are confined within stellar magnetic field. These are further pulled collateral to the magnetic lines of forces. This very process marks the transfer of angular momentum of the star, via the magnetic field, to the charged particles. After escaping a star's surface, these stellar winds are pulled down by the strong magnetic fields. This pulling down of the charged particles in the stellar winds decelerate rotation of the star. As an example, in close binaries where synchronicity is expected between the rotational and orbital periods, the decrease of rotational angular momentum takes place at the cost of OAM. Consequently the period drops i.e. constituents spin up followed by their movements towards each other to produce a single rapidly spinning star. It has been proved that closed binaries are magnetically exceptionally dynamic. Generally it is accepted that these exhaust their mass plus AM through the magnetized gust. The constituents however are not separated through a distinct layer. As a result, the astrophysicists expect the magnetic field interactions among the 2 constituents to be highly intense and thus their impact upon the angular momentum loss is significant enough because of the formation of magnetic loops among the magnetic fields arising at the surface of either or both binary components.

The magnetic torque generated from the magnetic field in the stellar gust is dependent upon the magnitude of magnetic field. Nevertheless, the in depth investigation of this notion along with its conception and with experimentally verified data remains one of the major challenge for astrophysicists for the future.

The stellar wind contains the charged particles which depart from the star radially. At the external stellar surface the tangential and rotational surface velocity components of the gaseous spheres are equal. But it is expected that the velocities of the very same particles tend to reduce to much slower velocities, at large distances in outer space. However it has been observed that in case of the Sun, the measured speeds of the particles is of the order 1-10 km/s. This is 10^2 - 10^3 times steadier than the expected speeds. The reason for this exception is that the charged particles not only flow radially outwards but also follow the twisted open field lines.

Thus it is evident that it is in fact the field's magnetic energy for every unit of volume which ought to be larger than the particle's very own kinetic energy, which verifies that a charged particle's trajectory is dominated not by the gravitational field but rather the direction of magnetic field lines.

When one or more magnetic field lines of a star are exposed and recombined through the constituent star, then these charged particles may either hit (collide) the particles which are directed by the companion star along the magnetic field lines in the same way as the primary star or may fall into the environment of the secondary companion star (fig 2.1 and 2.2).
The intriguing part is that these particles carry the same angular momentum of the stars themselves. Keeping this in mind it is easier to analyze that it is because of the rapid rotation of the star that bending of magnetic field lines takes place. Now these bent lines of field form a curvature which in turn is responsible for producing a force which acts upon the stellar plasma enveloping the binary star system.

The effect of magnetic braking is negligible when the stellar magnetic poles coincide with the rotational poles as a virtue of which the angular momentum of the binaries is reduced. However when the field is aligned either at or near the equatorial plane, then the magnetic braking would be highly intense. Thus excess amount of angular momentum would be neglected. Through observations astrophysicists have indicated that magnetic braking is very prominent in massive late F-K type stars. The presence of tidal (gravitational) interactions among the binaries results in the faster rotation of the component gaseous spheres (10-100 times) in comparison to the rotation of a single star. Thus we expect that the production of magnetic field lines at the stellar surface of the binaries o be highly intense and as compared to that of a single star.

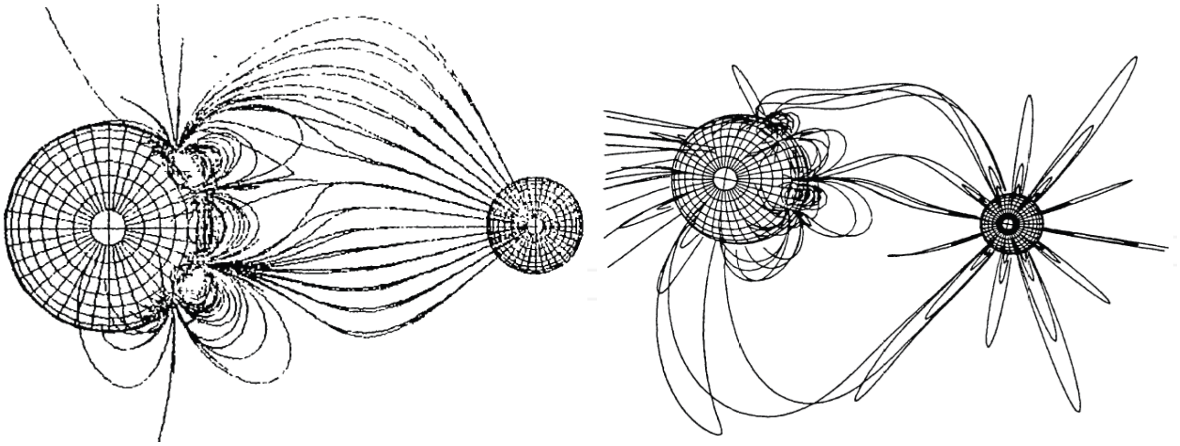

Fig1.4 : The formation of magnetic loops in a binary system at a large scale **[32,33]**

2.5 Equilibrium Structures of Rotationally, tidally and magnetically deformed non synchronous Binary model of gaseous spheres

There has been a very limited study of the role which magnetic fields play in the stellar formation and evolution up till now. But the inclusion of the role of magnetic fields is extremely important . Primarily because we have enough evidences backing up the significant influence of the interstellar magnetic fields to begin the onset of gravitational collapse in the molecular clouds. By neglecting the magnetic effects, astrophysicists have in practice departed from the baseline presented by **[4,31, 34]**.

The differential equations which govern the equilibrium configuration of rotationally, tidally and magnetically deformed binary model of non-synchronous stars in an explicit non-dimensional form is : -

$$\frac{d}{dr_o}\left[A(r_0,n,q,\alpha)\frac{d\varphi_\psi}{dr_0}\right] = -\frac{\varepsilon^2}{k^2}\varphi_\psi^N r_0^2 f_1(n,q,\alpha,r_0) \tag{2.1}$$

Where

$$A(r_0,n,q,\alpha) = \frac{r_0^2}{f_2} = \left\{1 + 2\alpha r_0^{\frac{1}{2}}(1-n) - \frac{16}{3}nr_0^3 - \left\{\frac{24}{5}q^2 + \frac{4}{15}nq + \frac{8}{5}n^2\right\}r_0^6 - 6q^2r_0^8 - \frac{46}{9}q^2r_0^{10} + \cdots\right\} \tag{2.2}$$

$$B(r_0,q,n,\alpha) = f_1 = r_0^2\left[\left(1-\frac{3}{2}\alpha\right) + \alpha^2 r_0 + \frac{4}{3}nr_0^5 + \left(\frac{36}{5}q^2 + \frac{884}{45}n^2 + \frac{24}{5}nq\right) + \frac{55}{7}q^2r_0^8 + \frac{26}{3}q^2r_0^{10}\right] \tag{2.3}$$

In the (2.2 and 2.3) terms upto 2nd order of exactness in n, q and upto r_0^{10} in r_0 are retained. Rest are neglected.

In the absence of magnetic parameter α the terms $A(r_0,n,q,\alpha)$ and $B(r_0,q,n,\alpha)$ diminish to their resultant expressions when the impacts of magnetic forces are unaccounted for.

After solving equation (2.1) as per which the boundary conditions become

At the center : $r_0 = 0$, $\varphi_\psi = 1, \frac{d\varphi_\psi}{dr_0} = 0$

At the surface : $r_0 = r_{os}$, $\varphi_\psi = \frac{1}{\varphi_\psi}$

r_{os} indicates r_o at the outer surface and both these quantities are non-dimensional.

To obtain the numerical solution of equation (2.1) it is integrated by keeping in mind to choose appropriate values of $\frac{1}{\varphi_0^2}$,K,n,α with specific boundary equations. The integration of equation is carried on till φ_ψ equals $\frac{1}{\varphi_0}$. The value of r_0 at which φ_ψ becomes $\frac{1}{\varphi_0}$, establishes the outermost free surface r_{os} of the binary model. Once we obtain the solutions to the equation (2.1) , we find the value of φ_ψ for a number of non-dimensional independent variable r_0 ranging from 0 to r_{os}.

2.6 Computation of Surface Area, Volume and Other Physical Parameters of Rotationally , Tidally and Magnetically Distorted Polytropic Model of Stars

In the field of astronomy, the term polytrope refers to the solution of Lane-Emden equation. In this the pressure and the density of the star in consideration are dependent on each other in the following form: $\mathbf{P = K}\boldsymbol{\rho}^{\frac{(\boldsymbol{n}+\mathbf{1})}{\boldsymbol{n}}}$**,** where P is stellar pressure, ρ is stellar density and K is constant of proportionality and n is a constant termed as poytropic index. Polytropic models have been extensively employed in existing works for describing the inner stellar structure of real observable gaseous spheres. It is defined as the model where the amount of supplied heat (dq) is in direct proportion to the immediate transformation in temperature (dt) such that $\frac{dq}{dt}$=1. For any polytropic model, pressure term P and the density **ρ** at an arbitrary point in the model is given by the relation P=$P_c\theta^{N+1}$ and **ρ**=$\rho_c\theta^N$, where P_c , ρ_c are the central stellar pressure and density specific to the model and θ ($0\leq \theta \leq 1$) is the parameter which depends upon the distance of the selected point from the center. N is a polytropic index of proposed model. For different practical cases of stellar structure problems N lies between 0-5. The polytropic index is basically the degree of the central concentration of the given stellar system i.e. higher value of N would lead to a high central condensation. Interestingly a polytropic model with N=5, is extremely condensed at the mid-point of model with stellar radius ranging to infinity. In contrast if the value of N = 0, then the stellar model has a consistent structure in which stellar density is even across the framework.

Rational solutions of the above equation are feasible for N = 0, 1 and 5. Chandrasekhar and several other authors have investigated and obtained numerical solutions of Lane Emden equation (2.1) that satisfies the above mentioned boundary conditions. After deriving the solution of Lane-Emden equation , the values of different stellar factors for stars with inner structures as a polytrope with index N may thus be obtained .

2.7 Equilibrium Structures of Rotationally, Tidally and Magnetically Distorted Polytropic Models of Stars

Let us assume that the polytropic model is under the influence of rotation, tidal and magnetic field, so the corresponding configuration develops a rotationally, tidally and magnetically distorted polytropic model. In line with the existing work to calculate such structures, we may approximate the equipotential surfaces of such a distorted model by employing the modified Roche equipotental surfaces.

For polytropic models, the equations governing hydrostatic equilibrium configuration of rotationally, magnetically and tidally can be combined to yield equation (2.1)

$$\frac{d}{dr_o}\left[A(r_0,n,q,\alpha)\frac{d\varphi_\psi}{dr_0}\right] = -\frac{\varepsilon^2}{k^2}\varphi_\psi^N r_0^2 f_1(n,q,\alpha,r_0)$$

Here A= $\frac{r_o^2}{f_2}$ and B =f$_1$

where f$_1$ and f$_2$ are distortion parameters derived in equations (1.41) and (1.42) respectively.

With regards to the boundary conditions P_ψ and ρ_ψ must be maximum at the center and zero at the surface. This requires θ_ψ to be maximum at the center and zero at the surface.

In the case of no i.e. for f$_1$ = f$_2$ = 1 equation (star) reduces to Lane-Emden equation, thus governing the equilibrium structure of an undistorted polytropic model of index N in a non-dimensional form. The quantity l defined in above equation is of the dimension of length. If we set r_ψ = lε , then ε is a non-dimensional variable which is exclusively defined for the topologically equivalent spherical model. It corresponds to the Emden variable ε for an undistorted spherical polytropic model.

At the outer surface ε_μ= ε .

As a matter of fact

$$\frac{D}{l} = \frac{D\varepsilon_\mu}{l\varepsilon_\mu} = \frac{D}{R_\psi}\varepsilon_\mu = \frac{1}{K}\varepsilon_\mu \qquad (2.4)$$

where ε_μ is ε itself, at external surface of the non-distorted model of polytropes.

In this section , we present expressions for obtaining the volume, surface area and the shape of a rotationally, tidally and magnetically distorted polytropic model. By following the approach of Mohan and Saxena, the radius r_ψ of the topologically equivalent spherical surface ψ ,volume and surface area enclosed by a rotationally, tidally, magnetically distorted polytropic model is given by :

$$r_\psi = r_0 D\,[1-\alpha r_0^{1/2} + \frac{1}{2}\alpha^2 r_0 + \frac{2}{3} n r_0^3 - \frac{7}{3}\alpha n r_0^{7/2} + \left\{\frac{4}{5}q^2 + \frac{8}{15}qn + \frac{76}{45}n^2\right\} r_0^6 + \frac{5}{7}q^2 r_0^8 + \frac{2}{3}q^2 r_0^{10} + ..] \qquad (2.5)$$

$$V_\psi = \frac{4\pi}{3}\pi r_{os}^3 \left(\frac{l\varepsilon_\mu}{K}\right)^3 \Bigg[1 - 3\alpha r_{os}^{\frac{1}{2}} - 11\alpha n r_{os}^{\frac{7}{2}} + \frac{5}{2}\alpha^2 r_{os} + 2n r_{os}^3 + \left\{\frac{12}{5}q^2 + \frac{8}{5}qn + \frac{32}{5}n^2\right\} r_{os}^6 + \frac{15}{7}q^2 r_{os}^8 + 2q^2 r_{os}^{10} + \cdots.\Bigg] \qquad (2.6)$$

$$S_\psi = 4\pi r_{os}^2 \left(\left(\frac{l\varepsilon_\mu}{K}\right)^2\right) \Bigg[1 - 2\alpha r_{os}^{\frac{1}{2}} - 6\alpha n r_{os}^{\frac{7}{2}} + 2\alpha^2 r_{os} + \frac{4}{3} n r_{os}^3 + \left\{\frac{7}{5}q^2 + \frac{14}{15}nq + \frac{56}{15}n^2\right\} r_{os}^6 + \frac{9}{7}q^2 r_{os}^8 + \frac{11}{9}q^2 r_{os}^{10}\Bigg] \qquad (2.7)$$

In the absence of magnetic parameter α the expressions A (r_0,n,q,α) and B(r_0,q,n,α) reduce to their corresponding expressions when the effects of magnetic force are not explicitly considered. The relations (2.6 and 2.7) determine the volume and surface area of the binary stars, distorted due to rotation, tidal and magnetic forces, upto the exactness of second order in n and q and upto $r_{os}^{10}\ in\ r_{os}$ are retained. However if need arises to determine the volume and surface area of some inner equipotential surface of the distorted model , then we need to replace r_{os} by the appropriate value of r_o for that surface in the above relations .

CHAPTER THREE

Concluding Observations

3.1 Numerical Computations

For obtaining the numerical solutions equation (2.1) has been numerically integrated by using 4th order Ranga-Kutta method for the specified values of input parameters. A series solution similar to one variable for undistorted polytropic models **[2]** has been developed to begin the integration at points near the center. The series solution is given by :

$$\theta_\psi = 1 - \frac{k^2}{6} r_0^2 + \frac{Nk^4}{120} r_0^4 - \frac{2nk^2}{15} r_0^5 - \frac{k^6 N(8N-5)}{3x*5040} r_0^6 + \left[\frac{K^8 N\left(122N^2-183N-70\right)}{9x*362880} - \frac{k}{36}^2 \left(3q^2 + 2nq + 8n^2\right)\right] r_0^8 + \cdots \tag{3.1}$$

Numerical integration of equation (3.1) has been carried forward using Ranga-Kutta method of the order four. Using the step length of h= 0.005, numerical integration was continued till θ_ψ first becomes zero. Once the outermost radius is obtained r_{os} is known, the corresponding volume, surface area and shape of the distorted model can be obtained as equations (3.1).

Numerical results obtained from different parameters are tabulated in Table (3.1 and 3.2). The results for the volume and surface area are given in the table (3.3 and 3.4) and these show that the values of different polytropic indices N=1.5 and N= 3.0.

<u>Table 3.1:</u> Values of r_{os} for various types of rotationally, tidally and magnetically distorted polytropic models with polytropic index N = 1.5

<table>
<tr><th colspan="6">N=1.5 $\varepsilon_{\mu} = 3.65375$</th></tr>
<tr><th colspan="6">Magnetic Effect</th></tr>
<tr><td>Model. No</td><td>n</td><td>q</td><td>α</td><td>K</td><td>r_{os}</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1.</td><td>0.0</td><td>0.0</td><td>2</td><td>1.0</td><td>0.5050278</td></tr>
<tr><td>2.</td><td>0.0</td><td>0.0</td><td>2</td><td>0.5</td><td>0.2404211</td></tr>
<tr><td>3.</td><td>0.0</td><td>0.0</td><td>10</td><td>1.0</td><td>0.5050278</td></tr>
<tr><td>4.</td><td>0.0</td><td>0.0</td><td>0.005</td><td>1.0</td><td>0.5050278</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><th colspan="6">Rotational and Magnetic Effects</th></tr>
<tr><td>5.</td><td>0.1</td><td>0.0</td><td>2</td><td>1.0</td><td>0.4976845</td></tr>
<tr><td>6.</td><td>0.05</td><td>0.0</td><td>10</td><td>1.0</td><td>0.50131806</td></tr>
<tr><td>7.</td><td>0.1</td><td>0.0</td><td>2</td><td>1.0</td><td>0.497684</td></tr>
<tr><td>8.</td><td>0.2</td><td>0.0</td><td>5</td><td>1.0</td><td>0.4906336</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><th colspan="6">Tidal and Magnetic Effects</th></tr>
<tr><td>9.</td><td>0.0</td><td>0.05</td><td>2</td><td>0.5</td><td>0.240421</td></tr>
<tr><td>10.</td><td>0.0</td><td>0.1</td><td>2</td><td>0.5</td><td>0.240427</td></tr>
<tr><td>11.</td><td>0.0</td><td>0.2</td><td>2</td><td>0.5</td><td>0.2404198</td></tr>
<tr><td>12.</td><td>0.0</td><td>0.8</td><td>2</td><td>0.5</td><td>0.2403999</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><th colspan="6">Rotational, Tidal and Magnetic Effects (Non-Synchronous)</th></tr>
<tr><td>13.</td><td>0.05</td><td>0.1</td><td>2</td><td>0.5</td><td>0.2402267</td></tr>
<tr><td>14.</td><td>0.1</td><td>0.05</td><td>2</td><td>0.5</td><td>0.2400332</td></tr>
<tr><td>15.</td><td>0.5</td><td>0.5</td><td>2</td><td>0.5</td><td>0.2384905</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><th colspan="6">Rotational, Tidal and Magnetic Effects (Synchronous)</th></tr>
<tr><td>16.</td><td>0.55</td><td>0.1</td><td>2</td><td>0.5</td><td>0.2383085</td></tr>
<tr><td>17.</td><td>0.55</td><td>0.1</td><td>1</td><td>0.5</td><td>0.2383085</td></tr>
<tr><td>18.</td><td>075</td><td>0.5</td><td>2</td><td>0.5</td><td>0.2375456</td></tr>
<tr><td>19.</td><td>0.6</td><td>0.2</td><td>2</td><td>0.5</td><td>0.2881181</td></tr>
<tr><td>20.</td><td>0.525</td><td>0.05</td><td>2</td><td>0.5</td><td>0.2384030</td></tr>
</table>

<u>Table 3.2:</u> Values of r_{os} for various types of rotationally, tidally and magnetically distorted polytropic models with polytropic index N = 3.0

N=3.0 $\varepsilon_{\mu} = 6.89685$					
Model. No	n	q	α	K	r_{os}
Magnetic Effect					
1.	0.0	0.0	2	1.0	0.2934737
2.	0.0	0.0	2	0.5	0.1428448
3.	0.0	0.0	10	1.0	0.2934737
4.	0.0	0.0	0.005	1.0	0.2934737
Rotational and Magnetic Effects					
5.	0.1	0.0	2	1.0	0.2925625
6.	0.05	0.0	10	1.0	0.2925625
7.	0.1	0.0	2	1.0	0.2930169
8.	0.2	0.0	5	1.0	0.2934681
Tidal and Magnetic Effects					
9.	0.0	0.05	2	0.5	0.1428448
10.	0.0	0.1	2	0.5	0.1428448
11.	0.0	0.2	2	0.5	0.1428448
12.	0.0	0.8	2	0.5	0.1428443
Rotational, Tidal and Magnetic Effects (Non-Synchronous)					
13.	0.05	0.1	2	0.5	0.1428191
14.	0.1	0.05	2	0.5	0.1427934
15.	0.5	0.5	2	0.5	0.1425885
Rotational, Tidal and Magnetic Effects (Synchronous)					
16.	0.55	0.1	2	0.5	0.1425631
17.	0.55	0.1	1	0.5	0.1425631
18.	075	0.5	2	0.5	0.1424609
19.	0.6	0.2	2	0.5	0.1425376
20.	0.525	0.05	2	0.5	0.1425759

<u>Table 3.3:</u> Volume and Surface areas of rotationally, tidally and magnetically distorted polytropic models with polytropic index N = 1.5

N=1.5 $\boldsymbol{\varepsilon_\mu = 3.65375}$							
Undistorted Case							
Model No	n	q	α	K	r_{os}	Volume × 10^{-3}	Surface ×10^{-2}
0	0.0	0.0	0.0	1.0	1.000000	2.0432	1.6776
Magnetic Effect							
Model. No	n	q	α	K	r_{os}	Volume × 10^{-3}	Surface × 10^{-2}
1.	0.0	0.0	2	1.0	0.5050278	0.047013	0.940305
2.	0.0	0.0	2	0.5	0.2404211	0.010500	0.373159
3.	0.0	0.0	10	1.0	0.5050278	2.788046	37.564310
4.	0.0	0.0	0.005	1.0	0.5050278	0.026038	0.424846
Rotational and Magnetic Effects							
5.	0.1	0.0	2	1.0	0.4976845	0.039753	0.861070
6.	0.05	0.0	10	1.0	0.50131806	0.043239	0.899476
7.	0.1	0.0	2	1.0	0.497684	2.602410	35.702840
8.	0.2	0.0	5	1.0	0.4906336	0.489828	7.294993
Tidal and Magnetic Effects							
9.	0.0	0.05	2	0.5	0.240421	0.010500	0.373159
10.	0.0	0.1	2	0.5	0.240427	0.010500	0.373158
11.	0.0	0.2	2	0.5	0.2404198	0.010500	0.373158
12.	0.0	0.8	2	0.5	0.2403999	0.010502	0.373132
Rotational, Tidal and Magnetic Effects (Non-Synchronous)							
13.	0.05	0.1	2	0.5	0.2402267	0.010320	0.371042
14.	0.1	0.05	2	0.5	0.2400332	0.010141	0.368941
15.	0.5	0.5	2	0.5	0.2384905	0.008779	0.352696
Rotational, Tidal and Magnetic Effects (Synchronous)							
16.	0.55	0.1	2	0.5	0.2383085	0.008614	0.350729
17.	0.55	0.1	1	0.5	0.2383085	0.002357	0.186208
18.	075	0.5	2	0.5	0.2375456	0.007982	0.343034
19.	0.6	0.2	2	0.5	0.2881181	0.008453	0.348784
20.	0.525	0.05	2	0.5	0.2384030	0.008695	0.351705

Table 3.4: **Volume and Surface areas of rotationally, tidally and magnetically distorted polytropic models with polytropic index N = 3.0**

N=3.0 $\boldsymbol{\varepsilon_\mu = 6.89685}$							
Undistorted Case							
Model No	n	q	α	K	r_{os}	Volume × 10^{-3}	Surface × 10^{-2}
0	0.0	0.0	0.0	1.0	1.000000	1.37474	5.9774
Magnetic Effect							
Model. No	n	q	α	K	r_{os}	Volume × 10^{-3}	Surface × 10^{-2}
1.	0.0	0.0	2	1.0	0.2934737	0.023770	0.607923
2.	0.0	0.0	2	0.5	0.1428448	0.005151	0.307827
3.	0.0	0.0	10	1.0	0.2934737	18.751490	133.844100
4.	0.0	0.0	0.005	1.0	0.2934737	2.018588	25.153830
Rotational and Magnetic Effects							
5.	0.1	0.0	2	1.0	0.2925625	0.034452	0.512032
6.	0.05	0.0	10	1.0	0.2925625	0.022558	0.595551
7.	0.1	0.0	2	1.0	0.2930169	0.023158	0.601693
8.	0.2	0.0	5	1.0	0.2934681	0.384669	5.241022
Tidal and Magnetic Effects							
9.	0.0	0.05	2	0.5	0.1428448	0.005151	0.307827
10.	0.0	0.1	2	0.5	0.1428448	0.005151	0.307827
11.	0.0	0.2	2	0.5	0.1428448	0.005151	0.307827
12.	0.0	0.8	2	0.5	0.1428443	1.183123	7.970107
Rotational, Tidal and Magnetic Effects (Non-Synchronous)							
13.	0.05	0.1	2	0.5	0.1428191	0.005117	0.307455
14.	0.1	0.05	2	0.5	0.1427934	0.005083	0.307083
15.	0.5	0.5	2	0.5	0.1425885	0.004816	0.304139
Rotational, Tidal and Magnetic Effects (Synchronous)							
16.	0.55	0.1	2	0.5	0.1425631	0.004783	0.303772
17.	0.55	0.1	1	0.5	0.1425631	0.007016	0.256821
18.	0.75	0.5	2	0.5	0.1424609	0.004651	0.302319
19.	0.6	0.2	2	0.5	0.1425376	0.004750	0.303408
20.	0.525	0.05	2	0.5	0.1425759	0.004799	0.303955

3.2 Conclusion and Discussion of Results

The values of volume and surface area, tabulated in Table 3.3 and 3.4, of rotationally, magnetically and tidally deformed binary model have been compared with the corresponding results of Mohan and Saxena **[23]** which did not take the magnetic deformation into account. Our results show a significant variation in the corresponding volume and surface area of the star in the binary system.

When compared with the undistorted model, the obtained values of volume and surface area have been observed to reduce to a significant extent. The cause of this decrease is attributed to the inclusion of three distortion parameters namely rotational, tidal and magnetic forces. The undistorted model is an ideal model. But with inclusion of real effects, there is a significant variation in the corresponding values of stellar volume and surface area.

When the magnetic parameter $\alpha = 10$, for N= 1.5, and no tidal and rotational deformations are taken into account, the stellar volume increases by 9.41% while the surface area increases by 14.5%. For $\alpha = 2$ and rotational parameter $n = 0.05$, the stellar volume increases by 2.79 % while the surface increases by 11.38%. With $\alpha = 10$ for N= 3.0, again there is an appreciable increase in the volume and surface area.

However, with the addition of combined effects of rotational, tidal and magnetic forces, there is a significant reduction in the stellar volume and surface area.

On the basis of aforementioned argument, we may suggest that a stellar magnetic field produces magnetosphere, which extends outward into the space enveloping it. This magnetosphere contains within it charged particles which are restrained in stellar gust and move along magnetic lines of forces only. Rotation of the star leads to the rotation of magnetosphere, by dragging along the charged particle. This may result the transfer of angular momentum from a star to its binary component. Hence it might be responsible for the gradual yet prominent decrease in the rates of rotation of the star, as it collapses due to emission of matter in the form of stellar wind.

Through the earlier works and conclusions in addition to our study it could be suggested that when the stellar matter is emitted, because of the drag force produced by magnetosphere, it causes an enormous decrease in the stellar volume and surface area. In an ideal condition, if the emission and transfer of mass from one binary component to the other continues, then one of the component (star) would gradually vanish while the other approaches to be a Red Giant, by gaining stellar mass of its binary component.

The value of θ_ψ remains approximately same as that obtained by **[23]**.

The expressions obtained in (2.5), (2.6) and (2.7) can further be applied to determine the equilibrium structures of rotationally, tidally and magnetically distorted stellar models. The techniques proposed in the present thesis can easily be incorporated into any software available commonly for computing the equilibrium structures of distorted stars.

The analysis and investigation carried out and presented in the thesis to formulate the equations of equipotential surfaces however does not include the effects of radiation pressure or effects of Coriolis forces. Further studies can be carried out by including these effects to obtain more accurate analysis of the distortions.

3.3 Scope for Future Work

The present challenge is the determination of stellar structure of a system of non-synchronous binary stars, which are distorted due to the rotational, tidal and magnetic forces, as it is extremely difficult to obtain analytical solutions to this tricky problem which is applicable to almost all the models with unique type of distortions. The relevance of these problems of determining stellar structure is of peculiar interest to the field of astronomy as it is with their help alone that astronomers and astrophysicist can explore and approximate the stellar properties of a binary system without physically approaching the stellar environment, which is next to impossible. Our present work is concerned with the investigation of these problems by supposing that the rotational, tidal and magnetic effects, distorting the stellar structure, are subsequently so minor such that the terms upto second order are successfully retained but any other higher order term of distortion parameter n and q can be neglected with ease. However it would be useful to further extend the approach developed in the second chapter in such a manner so that the inclusion of higher order terms for both n and q may be pertained to the astronomical problems with comparatively larger values of magnetic, rotational and tidal distortion parameters α, n and q, respectively.

Our present work involves the approximation of the equipotential surface of rotationally, tidally and magnetically distorted binary star system by the Roche equipotential approach. The limitation of this is that this approach is valid for highly centrally condensed types of stars, for instance it is applicable for theoretical models of real observable stars in late stages of evolution. But cases in which the central condensation is not as large, there this method of approach and approximation of equipotential surface is not justified to an appreciable extent. Hence it would be a positive construct to see whether a model of such nature with approximation of actual equipotential surfaces via Roche equipotential surfaces could be improved and advanced upon.

From the astrophysical significance , it would be beneficial to advance the developed technique in the present work to explore the equilibrium structures along with the evolutionary tracks of various realistic stellar models of such distorted stars by employing the approach used in chapter two.

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