Estimation of Power Spectral Density in Different Frequency Bands

Thesis submitted in partial fulfillment of the requirement for the award of degree of

Master of Engineering
in
Electronics Instrumentation and Control

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JULY 2010

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Declaration

*I hereby declare that the report entitled "**Estimation of Power Spectral Density in Different Frequency Bands**" is an authentic record of my own work carried out as requirements for the award of degree of M.E. (Electronic Instrumentation & Control) at Thapar University, Patiala, under the guidance of Dr. Mandeep Singh (AP, EIED) during January to July 2010.*

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Acknowledgment

The real spirit of achieving a goal is through the way of excellence and austere discipline. I would have never succeeded in completing my task without the cooperation, encouragement and help provided to me by various personalities.

With deep sense of gratitude I express my sincere thanks to my esteemed and worthy supervisor, Dr. Mandeep Singh, Assistant Professor, Department of Electrical & Instrumentation Engineering, Thapar University, Patiala, for his valuable guidance in carrying out this work under his effective supervision, encouragement, enlightenment and cooperation.

I shall be failing in my duties if I do not express my deep sense of gratitude towards Mr. Smarajit Ghosh, Professor & Head of the Department of Electrical & Instrumentation Engineering, Thapar University, Patiala who has been a constant source of inspiration for me throughout this work.

I am also thankful to all the staff members of the Department for their full cooperation and help.

Acknowledgement is due, to UGC Assistance for Strengthening Virtual Instrumentation as Special Paper in M.E. at Department of Electrical and Instrumentation, Thapar University, Patiala under innovative program of teaching and research in inter-disciplinary and emerging area, for providing the financial help for purchase of the necessary equipment and software.

My greatest thanks are to all who wished me success especially my parents. Above all I render my gratitude to the ALMIGHTY who bestowed self-confidence, ability and strength in me to complete this work.

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Abstract

Heart Rate (HR) in human being keeps on changing and meets with the requirements of body giving rise to Heart Rate Variability. This variability is therefore indicating of adaptability of heart rate variability (HRV). Heart Rate Variability signal analysis has become a widely employed tool in the diagnosis process of cardiovascular disease and powerful means of observing interplay between the sympathetic and parasympathetic nervous system. Over many years researchers have worked upon various techniques to gauge the variability in the Heart Rate and the selection of sampling frequency plays a critical role in obtaining the Power Spectral density. This work presents an algorithm to obtain the Power Spectral Density of Heart Rate Variability without linear interpolation and with linear interpolation using MATLAB software tool and also helps us to determine the Power Spectral Density of RR interval sequence to quantify Heart Rate Variability (HRV) into well defined and well documented frequency bands. This work is done with 34 healthy subjects taken randomly from MIT-BIH Normal Sinus Rhythm Database (nsrdb), Normal Sinus Rhythm RR interval Database (nsr2db), MIT-BIH Arrhythmia Database (mitdb) with variation in age from 20 years to 76 years and the power is calculated in different frequency bands VLF, LF, HF and the variation in powers is estimated with the help of scatter plots between standard deviation (σ) and the Relative Band Power Difference (RBPD). The obtained plots show the appropriate selection of sampling frequency. The results obtained from the visual inspection of scatter plots and from Pearson's correlated values are same.

Organization of Thesis

The first chapter introduces the heart rate variability, its analysis, measurement techniques and the advantages of frequency domain over time domain.

The second chapter discusses the details of correlation, interpolation and the power spectral density.

The third chapter tells about the work that has been already carried out in this field.

The fourth chapter formulates the problem.

The fifth chapter gives the detailed description of proposed solution with an algorithm and implementation of various tools of application software.

The sixth chapter shows the result obtained in the tabular form and discussion over the result.

In the seventh chapter thesis concluded with future scopes.

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List of abbreviations

HR	Heart Rate
HRV	Heart Rate Variability
ECG (EKG)	Electrocardiogram
PPG	Photoplethysmograph
SA	Sino-Atrial
CNS	Central Nervous System
SDNN	Standard Deviation of all NN Intervals
SDANN	Standard Deviation of the Averages of NN Intervals
RMSSD	Square Root of Mean of Sum of Squares of Intervals
NN	Number of Pairs of Adjacent NN intervals
PSD	Power Spectral Density
TP	Total Power
MI	Myocardial Infarction
AR	Auto-Regressive
FFT	Fast Fourier Transform
MATLAB	Matrix Laboratory
NSRDB	Normal Sinus Rhythm Database
RBPD	Relative Band Power Difference

CHAPTER 1

INTRODUCTION

1.1 Overview:

Heart rate variability (**HRV**) refers to the regulation of the sinoatrial node, the natural pacemaker of the heart by the sympathetic and parasympathetic branches of the autonomic nervous system. Our assumption, when we assess HRV, is that the beat-to-beat fluctuations in the rhythm of the heart provide us with an indirect measure of heart health, as defined by the degree of balance in sympathetic and vagus nerve activity.

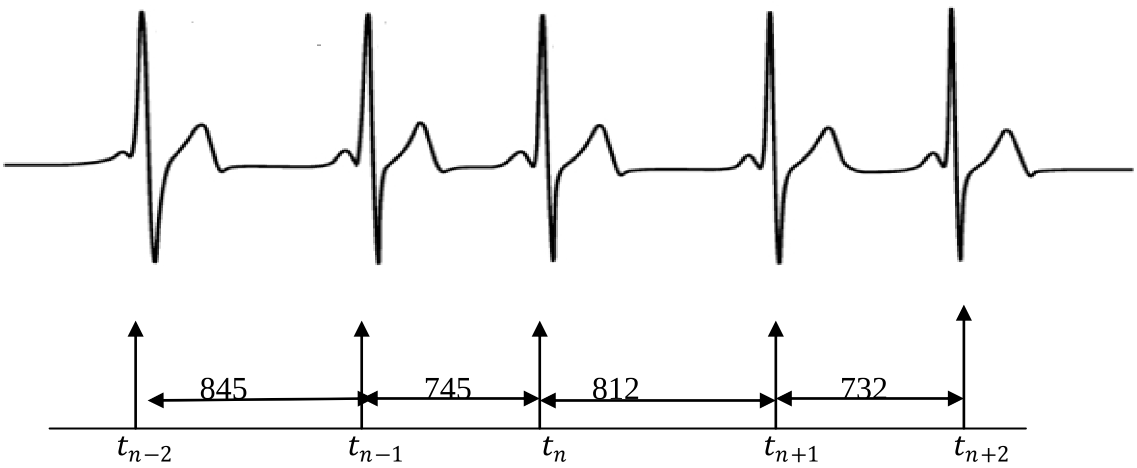


Figure 1.1: Interbeat oscillations between consecutive heart rate

HRV is the phenomenon of oscillation in the interval between consecutive heartbeats as well as the oscillations between consecutive instantaneous heart rates (HR). "Heart rate variability" has become the conventionally accepted term to describe variations of both instantaneous heart rate and RR intervals. HRV is measured by calculating the time between R spikes on ECG trace as shown in figure1.1. To describe oscillation in consecutive cardiac cycles, other terms have been used in the literature, for example, cycle length variability, heart period variability, RR variability, and RR interval tachogram, and they more appropriately emphasize the fact that it is the interval between consecutive beats that is being analyzed rather than the heart rate per sec.

1.2 HRV Analysis:

The heart rate variability analysis is a powerful tool in assessment of the autonomic function. It is accurate, reliable, reproducible, yet simple to measure and process. The source information for HRV is a continuous beat-by-beat measurement of interbeat intervals.

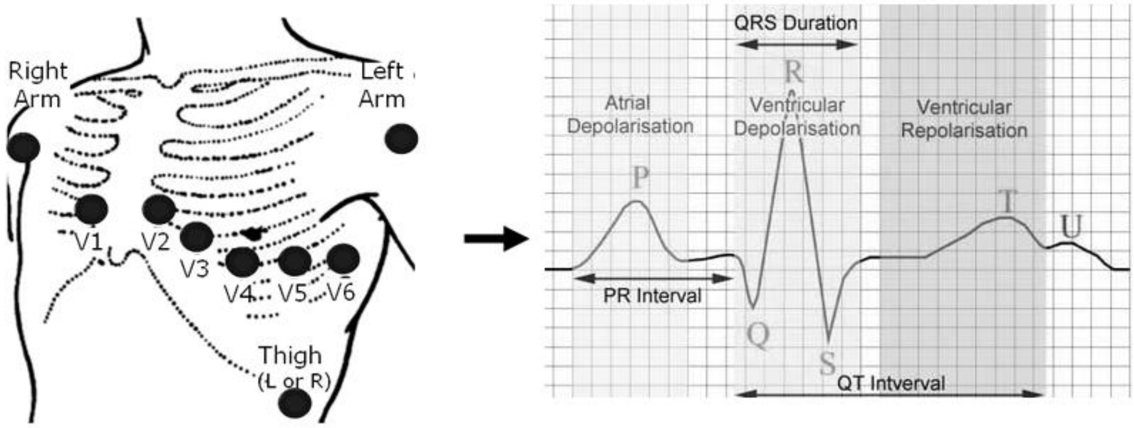


Figure 1.2: Typical ECG trace

1.2.1 The Electrocardiogram (ECG):

The electrocardiograph (ECG or EKG) is considered as the best way to measure interbeat intervals. The bio potentials generated by the muscles of the heart result in the electrocardiogram, abbreviated ECG (sometimes EKG, from the German electrokardiogram). To understand the origin of the ECG, it is necessary to have some familiarity with the anatomy of the heart. Electrocardiography (ECG or EKG) is a Tran’s thoracic interpretation of the electrical activity of the heart over time captured and externally recorded by skin electrodes. It is a non-invasive recording produced by an electrocardiographic device. The etymology of the word is derived from electro, because it is related to electrical activity, cardio, Greek for heart, and graph, a Greek root meaning "to write".

The first little hump is known as the P wave as shown in figure 1.2. It occurs when the atria depolarize (i.e. trigger). The next three waves constitute the QRS complex. They represent the ventricles depolarizing. These three are lumped together because a normal rhythm may not have all three. Many times, we'll only see a R and an S. This is not abnormal. If there are less than three, how do we know which one is which? Well, the R wave is the first wave above the isoelectric line. Then name the waves in relation to the R wave. If it falls before the R wave, it is called the Q wave; after the R wave is the S wave.ECG is an electrical signal measured with special conductive electrodes placed on chest around heart area or limbs. It reflects minute changes in electrical field generated by heart muscle cells originating from its SA node. ECG signal has a very specific and robust waveform simple to detect and analyze. Because of that cardiac rhythm derived from ECG is the best way to detect not only true sinus rhythm but all types of ectopic heartbeats.

Electrodes:

To record an electrocardiogram, a number of electrodes usually five, are affixed to the body of the patients. The electrodes are connected to the ECG machine by the same number of electrical wires. These wires and in a more general sense, the electrodes to which they are connected are usually called leads.

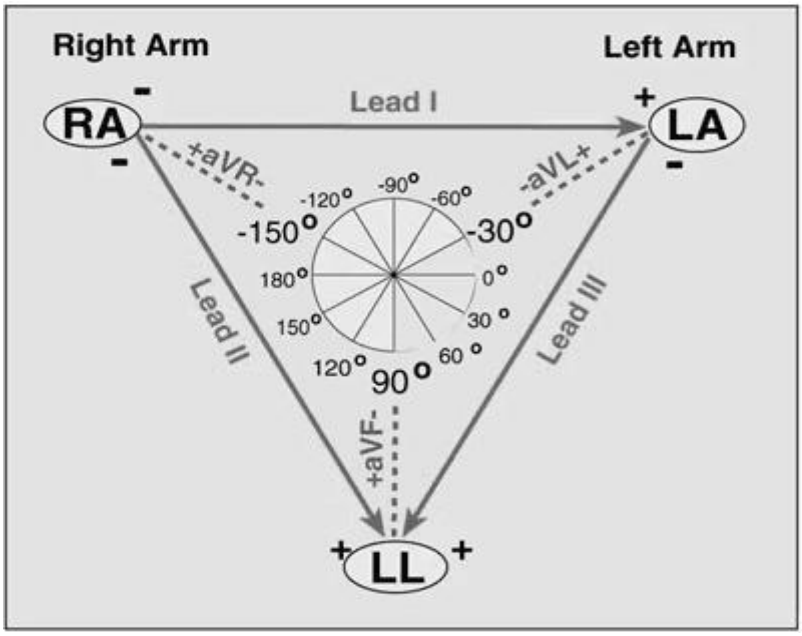


Figure 1.3: The Einthoven Triangle

The electrodes applied to the right leg of the patient, for example it is called RL lead. For recording of the electrocardiogram, two electrodes or one electrode and an inter connected group of electrodes are selected and connected to the input of the recording amplifier. It is somewhat confusing that a particular electrode selected and the ways in which they are connected are also referred to as a lead. For the individual lead wire, as well as the physical connection to the body of the patient, the term electrode will be used. The voltage generated by the pumping action of heart is actually a vector whose magnitude, as well as spatial orientation, changes with time. Because the ECG signal is measured from electrodes applied to the surface of the body, the waveform of this signal is very dependent on the placement of electrodes. The ECG is recorded from a number of different leads, usually 12, to ensure that no important detail of the waveform is missed. Placement of electrodes and names and configuration of the leads have become standardized and used same way throughout the world. The leg selected was the left one, probably because it terminates vertically below the heart. The early electrocardiograph machine employed three electrodes, of which only two were used at a time. With the introduction of the electronic amplifier, an additional connection to the body was needed as a ground reference. Although an electrode could have been positioned almost anywhere on the body for this purpose, it became a convention to use the “free” right leg. By assuming that the ECG potentials at the shoulders are essentially the same as the wrists and the potential at the crotch differ little from those at the either ankle, Einthoven let the points of this triangle represent the electrode position for the three limb leads. This triangle is known as Einthoven Triangle as shown in figure 1.3.

Leads:

In the normal electrode placement four electrodes are used to record the electrocardiogram; the electrode on the right leg is only for the ground reference. Because the input of the ECG recorder has only two terminals, a selection must be made among the available active electrodes. The three bipolar limb lead selection first introduced by Einthoven, are as follows:

Lead I: Left Arm (LA) and Right Arm (RA)

Lead II: Left Leg (LL) and Right Arm (RA)

Lead III: Left Leg (LL) and Left Arm (LA)

These three leads are called bipolar because for each lead the electrocardiogram is recorded from two electrodes and the third electrode is not connected.

Electrical impulses in the heart originate in the sinoatrial node and travel through the intimate conducting system to the heart muscle. The impulses stimulate the myocardial muscle fibres to contract and thus induce systole. The electrical waves can be measured at electrodes placed at specific points on the skin. Electrodes on different sides of the heart measure the activity of different parts of the heart muscle. An ECG displays the voltage between pairs of these electrodes, and the muscle activity that they measure, from different directions, can also be understood as vectors. This display indicates the overall rhythm of the heart and weaknesses in different parts of the heart muscle. It is the best way to measure and diagnose abnormal rhythms of the heart, particularly abnormal rhythms caused by damage to the conductive tissue that carries electrical signals, or abnormal rhythms caused by electrolyte imbalances. In a myocardial infarction (MI), the ECG can identify if the heart muscle has been damaged in specific areas, though not all areas of the heart are covered. The ECG cannot reliably measure the pumping ability of the heart, for which ultrasound-based (echocardiography) or nuclear medicine tests are used.

1.2.2 Pulse Wave Measurement:

The other approach to measure cardiac intervals is a measurement of pulse wave. It is less invasive and simple method of measurement based on photoplethysmograph. (PPG) is a signal reflecting changes in a blood flow detected when infrared light is emitted towards microcirculatory blood vessels. Depending on blood flow volume certain portion of that light is absorbed letting other part to pass or be reflected as shown in figure 1.4. An optical sensor detects a quantity of light passed (or reflected from) the blood flow producing a waveform identifying pulse wave. Such waveform can also be processed to derive beat-by-beat interbeat intervals.

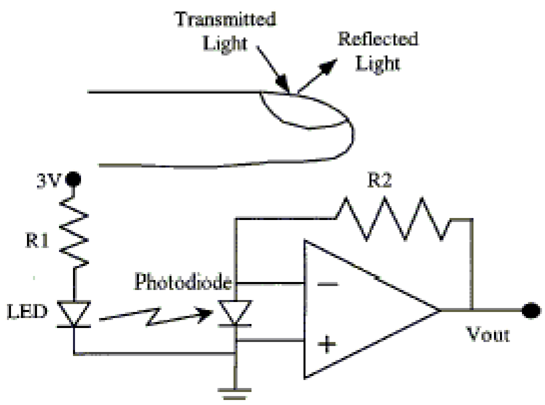


Figure 1.4: Circuit diagram of Photoplethysmograph

Although PPG gives the summary information reflecting both cardiac and blood vessel components of HRV, some research studies showed a significantly high correlation between interbeat interval data measured by both ECG and PPG in short-term steady-state recordings. One of the important issues when measuring either ECG or PPG is the absence of abnormal heartbeat used in interval detection. Only heartbeats originated in SA node can be processed to obtain HRV data. Whether there are ectopic heartbeats (other types of extra systolic heartbeats) or various movement artifacts on ECG (or PPG) considered as heartbeats, they must be excluded from consideration. There are various statistically-based algorithms of detection of such abnormal heartbeats that minimize chances to get contaminated HR recordings. Nevertheless, for the sake of accuracy in HRV analysis it is important to be able to visually verify all heartbeats automatically found, remove abnormal ones and include missing. The Heart Rhythm Scanner has an automatic detection of such movement artifacts and also gives the possibility to manually correct it.

1.3 Physiological Basics of Heart Rate Variability (HRV):

The origin of heartbeat is located in a sino-atrial (SA) node of the heart, where a group of specialized cells continuously generates an electrical impulse spreading all over the heart muscle through specialized pathways and creating process of heart muscle contraction well synchronized between both atriums and ventricles. The SA node generates such impulses about 100-120 times per minute at rest.

1.3.1 Autonomic nervous system:

The autonomic nervous system is a part of the nervous system that non-voluntarily controls all organs and systems of the body as shown in figure 1.5 below. There are two branches of the autonomic nervous system - sympathetic and parasympathetic (vagal) nervous systems that always work as antagonists in their effect on target organs.

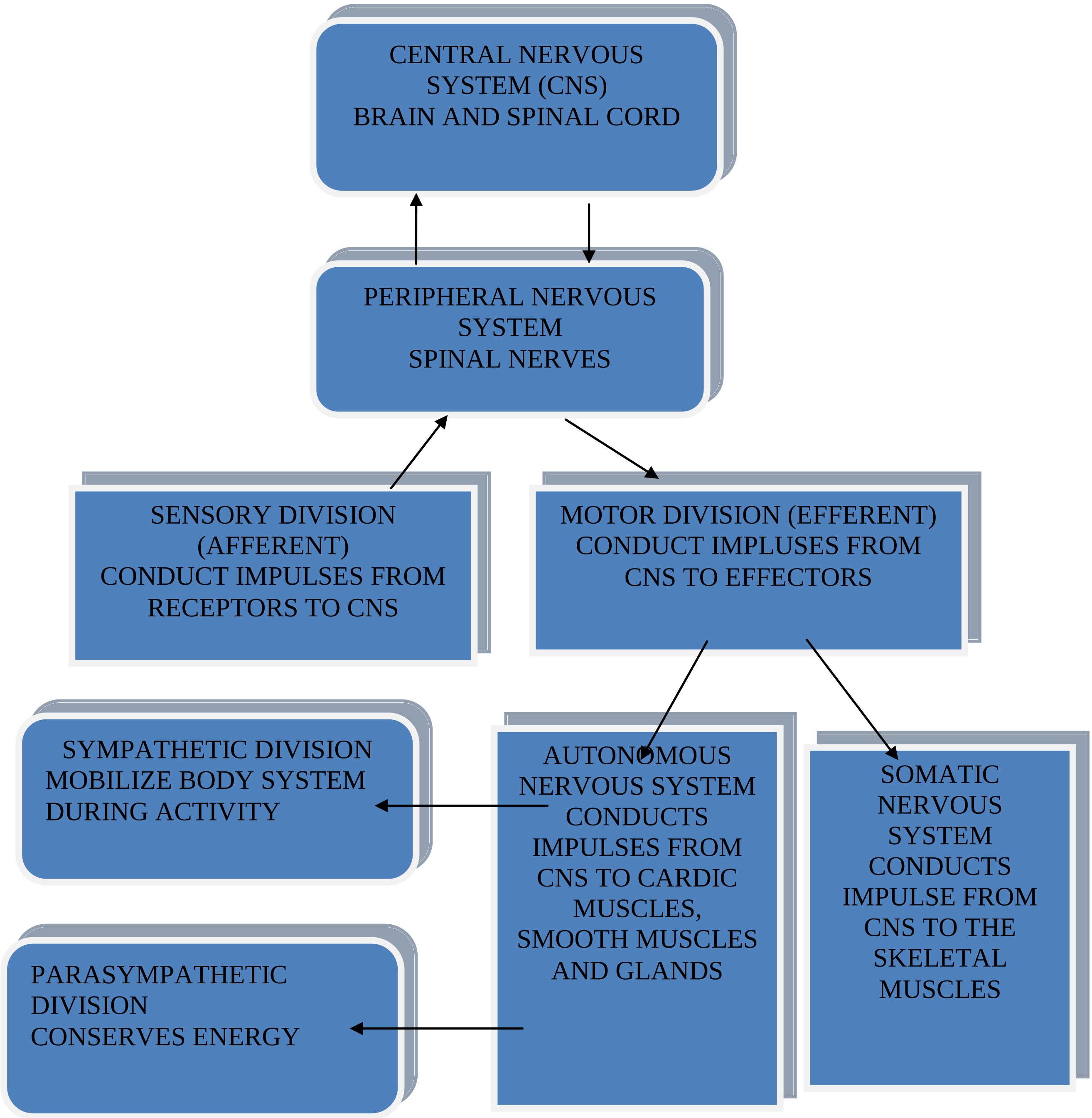


Figure 1.5: Flow Chart for the autonomous nervous system

1.3.2 Sympathetic nervous system:

For most organs including heart the sympathetic nervous system stimulates organ's functioning. An increase in sympathetic stimulation causes increase in HR, stroke volume, etc. The heart response time to sympathetic stimulation is relatively slow. It takes about 5 seconds to increase HR after actual onset of sympathetic stimulation and almost 30 seconds to reach its peak steady level.

1.3.3 Parasympathetic nervous system:

An increase in parasympathetic stimulation causes decrease in HR, stroke volume, etc. The heart response time to parasympathetic stimulation is almost instantaneous. Depending on actual phase of heart cycle it takes just 1 or 2 heartbeats before heart slows down to its minimum proportional to the level of stimulation.

At rest both sympathetic and parasympathetic systems are active with parasympathetic dominance. The actual balance between them is constantly changing in attempt to achieve optimum considering all internal and external stimuli. There are various factors affecting autonomic regulation of the heart, including but not limited to respiration, thermoregulation, humoral regulation (rennin-angiotensin system), blood pressure, cardiac output, etc. One of the most important factors is blood pressure. There are special baroreceptive cells in the heart and large blood vessels that sense blood pressure level and send afferent stimulation to central structures of the ANS that control HR and blood vessel primarily through sympathetic and somewhat parasympathetic systems forming continuous feedback dedicated to maintain systemic blood pressure.

1.4 Measurement of Heart Rate Variability:

There are different methods of HRV analysis as shown in figure 1.6. One of the methods is time domain analysis. This method extracts a few special measures using only the temporal RR interval signals. Another method is spectral analysis. This method interpolates the RR interval at a certain rate and transforms this interval into the frequency domain.

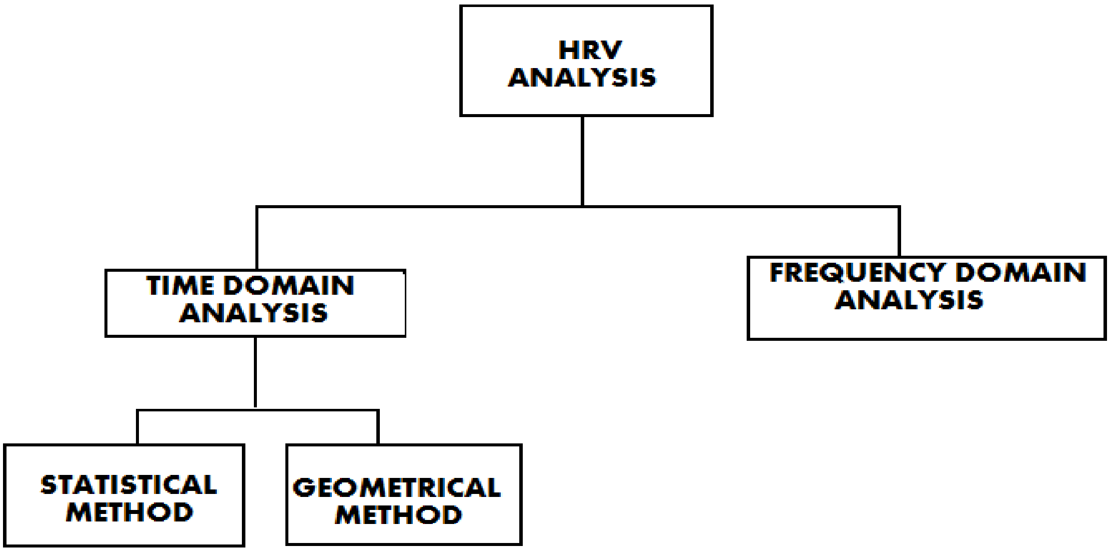


Figure 1.6: HRV analysis techniques.

1.4.1 Time Domain Analysis: The time domain analysis of heart rate variability can be analysed by two methods:

Statistical Method:

Conventionally, heart rate fluctuation has been assessed by calculating indices based on statistical operations on RR intervals (means and variance). The most widely used time domain index is the average heart rate. It is easy to calculate over a suitable length of time. The calculations of other different time domain indices naturally require precise timing of R waves. Time domain analysis can be performed on short electrocardiogram segments (lasting from 0.5 to 5 minutes) as shown in figure 1.7 or on 24-hour electrocardiographic recordings.

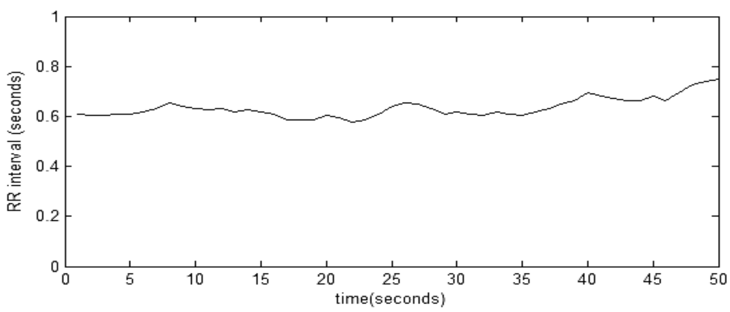


Figure 1.7: Plot of RR interval in time domain

Long-term variability indices mainly reflect slower fluctuation of RR intervals. These indices are calculated from the RR intervals occurring in a chosen time window. An example of a short term variability index is the standard deviation of beat-to-beat RR interval differences within the time window. The standard deviation of all the RR intervals or the difference between maximum and minimum RR interval length, within the window are examples of long term indices. The value of the estimate depends on the record length. The simplest variable to calculate is the standard deviation of the NN intervals (SDNN), that is, the square root of variance. Since variance is mathematically equal to total power of spectral analysis, SDNN reflects all the cyclic components responsible for variability in the period of recording.

Table 1.1 Statistical Measures

Variables	Units	Description
SDNN	ms	Standard deviation of all NN intervals
SDANN	ms	Standard deviation of the averages of NN intervals in all 5 min segments of the entire recording
RMSSD	ms	The square root of the mean of the sum of the squares of differences between adjacent NN interval
SDNN index	ms	Mean of the standard deviations of all NN intervals for all 5 min segments of the entire recording
NN50 count		Number of pairs of adjacent NN intervals differing by more than 50 ms in the entire recording.
pNN50	%	NN50 count divided by the total number of all NN intervals.

In many studies SDNN is calculated over a 24-hour period and thus encompasses short-term HF variations as well as the lowest-frequency components seen in a 24-

hour period. As the period of monitoring decreases, SDNN estimates shorter and shorter cycle lengths. It also should be noted that the total variance of HRV increases with the length of analyzed recording. Thus, on arbitrarily selected ECGs, SDNN is not a well-defined statistical quantity because of its dependence on the length of recording period. In practice, it is inappropriate to compare SDNN measures obtained from recordings of different durations. On the contrary, durations of the recordings used to determine SDNN values (and similarly other HRV measures) should be standardized. As discussed further in this document, short-term 5-minute recordings and nominal 24-hour long-term recordings appear to be appropriate options. The standard deviation is defined as:

$$sdnn = \sqrt{1/N \sum_{j=1}^{N}(x_j - \bar{x})^2} \text{ - } \sqrt{\sum_{j=1}^{N} x_j^2/\mathrm{N} - (\sum_{j=1}^{N} x_j /N)^2}$$

Where x_j is sample point (RR interval), $\bar{x}$ is the mean of all samples points and N is total no of sample points. Other commonly used statistical variables calculated from segments of the total monitoring period include SDANN, the standard deviation of the average NN interval calculated over short periods, usually 5 min, which is an estimate of the changes in heart rate due to cycles longer than 5 min. The SDNN index, the mean of the 5-min standard deviation of the NN interval calculated over 24 h, which measures the variability due to cycles shorter than 5 min. The most commonly used measures derived from interval differences include RMSSD, the square root of the mean squared differences of successive NN intervals. The RMSSD is defined as:

$$rmssd = \sqrt{1/(N-1) \sum_{j=1}^{N-1}(x_{j+1} - x_j)}^2$$

Where, x_j the sample points and N is the total number of samples.

NN50, the number of interval differences of successive NN intervals greater than 50 ms, and pNN50 the proportion derived by dividing NN50 by the total number of NN intervals. The percentage of differences greater than x (pNNx) calculates the percentage of the differences between successive samples that are greater than a given value of x. Typical values for x are 30 ms and 50 ms.

$$pNNx = \sum_{j=1}^{N-1} \Theta((|x_{j+1} - x_j|) - x)$$

$$\Theta(i) = \{1, when\ i \geq 0$$
$$\Theta(i) = 0, otherwise$$

Geometrical Method:

Geometrical methods present RR intervals in geometric patterns and various approaches have been used to derive measures of heart rate variability from them. The triangular index is a measure where the length of RR intervals serves as the x-axis of the plot and the number of each RR interval length serves as the y-axis. The length of the base of the triangle is used and approximated by the main peak of the RR interval frequency distribution diagram. Triangular interpolation approximates the RR interval distribution by a linear function and the baseline width of this approximation triangle is used as a measure of the heart rate variability index. This triangular index had a high correlation with the standard deviation of all RR intervals, but it is highly insensitive to artefacts and ectopic beats, because they are left outside the triangle. This reduces the need for pre-processing of the recorded data. The general approach used in geometrical method is:

(1) A basic measurement of the geometric pattern (for example, the width of the distribution histogram at the specified level) is converted into the measure of HRV,

(2) The geometric pattern is interpolated by a mathematically defined shape (for example, approximation of the distribution histogram by a triangle or approximation of the differential histogram by an exponential curve) and then the parameters of this mathematical shape are used, and

(3) The geometric shape is classified into several pattern-based categories that represent different classes of HRV (for example, elliptic, linear, and triangular shapes of Lorenz plots). Most geometric methods require the RR (or NN) interval sequence to be measured on or converted to a discrete scale that is not too fine or too coarse and permits the construction of smoothed histograms.

1.4.2 Frequency Domain Analysis:

Several methods are available for the analysis of heart rate variability in frequency domain. Power spectral density, using parametric or nonparametric methods, provides basic information of the power (variation) distribution across frequencies. One of the most commonly used PSD methods is the discrete Fourier transform. Frequency-domain measures pertain to HR variability at certain frequency ranges associated with specific physiological processes. Before frequency-domain analysis is performed, all abnormal heartbeats and artifacts must be detected and removed, then cardiotachogram (sequence of RR intervals) must be re-sampled to make it as if it is a regularly sampled signal.

Table 1.2: Selected frequency domain measure of HRV

Variables	Units	Description Analysis of short term recording (5-min)	Frequency Range
VLF	ms^2	Power in very low frequency range	≤0.04Hz
LF	ms^2	Power in low frequency range	0.04-0.15Hz
HF	ms^2	Power in high frequency range	0.15-0.4Hz
LF/ HF		Ratio of LF[ms^2]/HF[ms^2]	

A standard spectral analysis routine is applied to such modified recording and the following parameters evaluated on 5-min time interval: Total Power (TP), High Frequency (HF), Low Frequency (LF) and Very Low Frequency (VLF). When long-term data is evaluated an additional frequency band is derived - Ultra Low Frequency. The HF power spectrum is evaluated in the range from 0.15 to 0.4 Hz. This band reflects parasympathetic (vagal) tone and fluctuations caused by spontaneous respiration known as respiratory sinus arrhythmia. The LF power spectrum is evaluated in the range from 0.04 to 0.15 Hz. This band can reflect both sympathetic and parasympathetic tone. The VLF power spectrum is evaluated in the range from 0.0033 to 0.04 Hz. The figure 1.8 shows the ranges of the frequency bands.

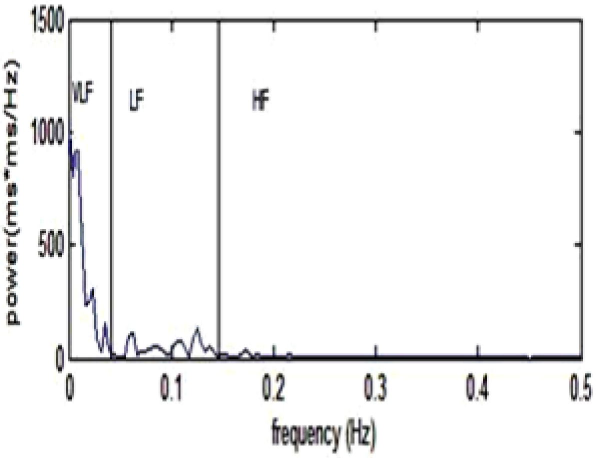


Figure 1.8: Plot of RR interval in frequency domain with specific bands.

The physiological meaning of this band is most disputable. With longer recordings it is considered representing sympathetic tone as well as slower humeral and thermoregulatory effects. There are some findings that in shorter recordings VLF has fair representation of various negative emotions, worries, rumination etc. The total power (TP) is a net effect of all possible physiological mechanisms contributing in HR variability that can be detected in 5-min recordings, however sympathetic tone is considered as a primary contributor. The LF/HF Ratio is used to indicate balance between sympathetic and parasympathetic tone. A decrease in this score might indicate either increase in parasympathetic or decrease in sympathetic tone. It must be considered together with absolute values of both LF and HF to determine what factor contributes in autonomic misbalance.

1.5 Advantage of Frequency Domain Analysis over Time Domain Analysis:

1) The main advantage of spectral analysis of signals is the possibility to study their frequency-specific oscillations.

2) Spectral analysis involves decomposition of the series of sequential RR intervals into a sum of sinusoidal functions of different amplitudes and frequencies.
3) The result can be displayed with the magnitude of variability as a function of frequency (power spectrum).
4) The power spectrum reflects the amplitude of the heart rate fluctuations present at different oscillation frequencies. Methods based on Fast Fourier transformation and autoregressive analysis are most commonly used to transform signals into the frequency domain.

CHAPTER 2
MATHEMATICAL CONCEPTS: A REVIEW

The thesis work made use of same mathematical concepts at many stages. A review of those is presented here as a warm up exercise.

2.1 Interpolation:

In the mathematical subfield of numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points. A different problem which is closely related to interpolation is the approximation of a complicated function by a simple function. Suppose we know the function but it is too complex to evaluate efficiently. Then we could pick a few known data points from the complicated function, creating a lookup table, and try to interpolate those data points to construct a simpler function. Of course, when using the simple function to calculate new data points we usually do not receive the same result as when using the original function, but depending on the problem domain and the interpolation method used the gain in simplicity might offset the error.

2.1.1 Piecewise Constant Interpolation:

The simplest interpolation method is to locate the nearest data value, and assign the same as value shown in figure 2.1. In one dimension, there are seldom good reasons to choose this one over linear interpolation, which is almost as cheap, but in higher dimensional multivariate interpolation, this can be a favourable choice for its speed and simplicity.

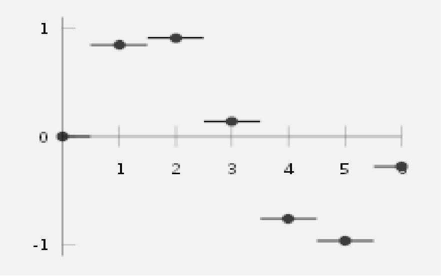


Figure2.1: Piecewise constant Interpolation

2.1.2 Linear interpolation:

Linear interpolation is a method of curve fitting using linear polynomials. It is heavily employed in mathematics (particularly numerical analysis), and numerous application including computer graphics. It is a simple form of interpolation. If the two points are given by the coordinates$(x_{o,}y_o)$ and $(x_{1,}y_1)$, the linear interpolation is the straight line between these points as shown in figure 2.2. For a value x in the interval $(x_{o,}x_1)$, the value y along the straight line is given from the equation.

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

which can be derived geometrically from the figure on the right.

Solving this equation for *y*, which is the unknown value at *x*, gives

$$y = y_0 + (x - x_0)\frac{y_1 - y_0}{x_1 - x_0}$$

the formula for linear interpolation in the interval (x_0, x_1). Outside this interval, the formula is identical to linear extrapolation. The interp1 command interpolates between data points. It finds values at intermediate points, of a one-dimensional function f(x). The description for the Linear Interpolation is as given below:

yi = interp1(x, Y, xi) interpolates to find yi, the values of the underlying function Y at the points in the vector or array xi. x must be a vector. Y can be a scalar, a vector, or an array of any dimension, subject to the following conditions:

- If Y is a vector, it must have the same length as x. A scalar value for Y is expanded to have the same length as x. xi can be a scalar, a vector, or a multidimensional array, and yi has the same size as xi.

- If Y is an array that is not a vector, the size of Y must have the form [n,d1,d2,...,dk], where n is the length of x. The interpolation is performed for each d1-by-d2-by-...-dk value in Y. The sizes of xi and yi are related as follows:

  - If xi is a scalar or vector, size (yi) equals [length(xi), d1, d2, ..., dk].

  - If xi is an array of size [m1,m2,...,mj], yi has size [m1,m2,...,mj,d1,d2,...,dk].

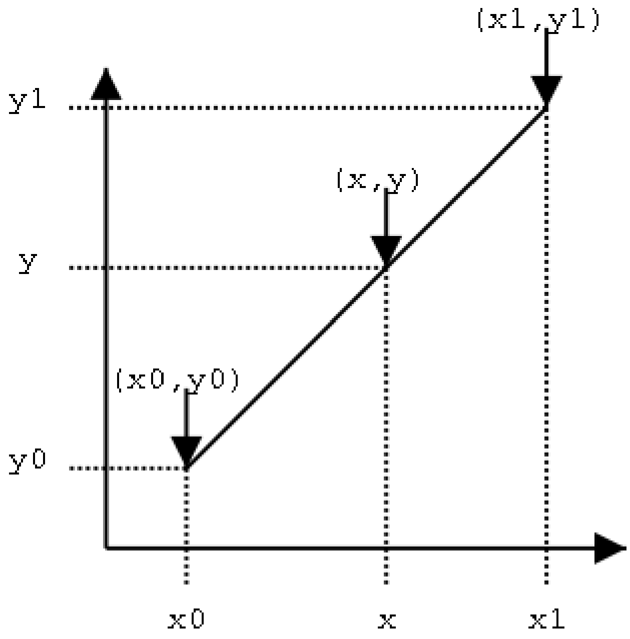


Figure2.2: Linear Interpolation

2.1.3 Polynomial Interpolation:

Polynomial interpolation is a generalization of linear interpolations shown in figure 2.3. Note that the linear interpolant is a linear function. We now replace this interpolant by a polynomial of higher degree.

Consider again the problem given above. The following sixth degree polynomial goes through all the seven points:

$$f(x) = -0.0001521x^6 - 0.003130x^5 + 0.07321x^4 - 0.3577x^3 + 0.2255x^2 + 0.9038x.$$

Substituting x = 2.5, we find that f(2.5) = 0.5965.

Generally, if we have n data points, there is exactly one polynomial of degree at most n−1 going through all the data points. The interpolation error is proportional to the distance between the data points to the power n. Furthermore, the interpolant is a polynomial and thus infinitely differentiable. So, we see that polynomial interpolation solves all the problems of linear interpolation. However, polynomial interpolation also has some disadvantages. Calculating the interpolating polynomial is computationally expensive (see computational complexity) compared to linear interpolation. Furthermore, polynomial interpolation may exhibit oscillatory artifacts, especially at the end points. These disadvantages can be avoided by using spline interpolation.

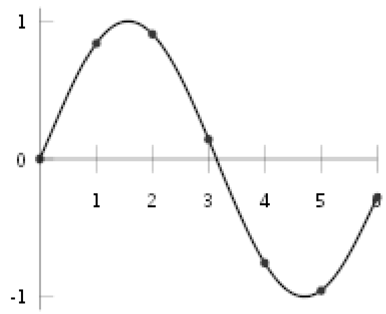


Figure2.3: Polynomial Interpolation

2.1.4 Spline Interpolation:

The linear interpolation uses a linear function for each of intervals $[x_k, x_{k+1}]$. Spline interpolation uses low-degree polynomials in each of the intervals, and chooses the polynomial pieces such that they fit smoothly together. The resulting function is called a Spline. There are different types of Spline interpolation:

a) Cubic Spline
b) Piecewise Spline
c) Quadratic Spline

2.2 Power Spectral Density:

The power spectral density, PSD, describes how the power (or variance) of a time series is distributed with frequency. Mathematically, it is defined as the Fourier Transform of the autocorrelation sequence of the time series.

There are different methods for estimation of power spectral density:

2.2.1 Periodogram :

[Pxx,w] = periodogram(x) returns the power spectral density (PSD) estimate Pxx of the sequence x using a periodogram. The power spectral density is calculated in units of power per radians per sample. The corresponding vector of frequencies w is computed in radians per sample, and has the same length as Pxx.

2.2.2 Burg (spectrum.burg):

Hs = spectrum.burg returns a default Burg spectrum object, Hs, that defines the parameters for the Burg parametric spectral estimation algorithm. The Burg algorithm estimates the spectral content by fitting an auto-regressive (AR) linear prediction filter model of a given order to the signal. Hs = spectrum.burg (order) returns a spectrum object, Hs with the specified order. The default value for order is 4. The units of power/frequency are db/Hz.

2.2.3 Covariance (spectrum.cov):

Hs = spectrum.cov returns a default covariance spectrum object, Hs, that defines the parameters for the covariance spectral estimation algorithm. The covariance algorithm estimates the spectral content by fitting an auto-regressive (AR) linear prediction model of a given order to the signal. The default value for order is 4. The units of power/frequency are db/Hz.

2.2.4 Modified covariance (spectrum.mcov):

Hs = spectrum.mcov returns a default modified covariance spectrum object, Hs, that defines the parameters for the modified covariance spectral estimation algorithm. The modified covariance algorithm estimates the spectral content by fitting an auto-regressive (AR) linear prediction filter model of a given order to the signal. The default value for order is 4. The units of power/frequency are db/Hz.

2.2.5 Thomson multitaper method (MTM) (spectrum.mtm):

Hs = spectrum.mtm returns a default Thompson multitaper spectrum object, Hs that defines the parameters for the Thompson multitaper spectral estimation algorithm, which uses a linear or nonlinear combination of modified periodograms. The periodograms are computed using a sequence of orthogonal tapers (windows in the frequency domain) specified from discrete prolate spheroidal sequences. The units of power/frequency is db/Hz

2.2.6 Welch (spectrum.welch):

Hs = spectrum.welch (WindowName) returns a spectrum object, Hs, using Welch's method with the specified window and the default values for all other parameters.

The units obtained from all the above methods are in radians/hertz and decibel/hertz.

Table 2.1: Comparison of different methods of calculating the PSD

	Burg	Covariance	Modified Covariance	Yule-Walker
Characteristics	Does not apply window to data	Does not apply window to data	Does not apply window to data	Applies window to data
	Minimizes the forward and backward prediction errors in the least squares sense, with the AR coefficients	Minimizes the forward prediction error in the least squares sense	Minimizes the forward and backward prediction errors in the least squares sense	Minimizes the forward prediction error in the least squares sense (also called "Autocorrelation method")
Advantages	High resolution for short data records	Better resolution than Y-W for short data records (more accurate estimates)	High resolution for short data records	Performs as well as other methods for large data records
	Always produces a stable model	Able to extract frequencies from data	Able to extract frequencies from data	Always produces a stable model

		consisting of pure sinusoids	consisting of pure sinusoids	
			Does not suffer spectral line-splitting	
Disadvantages	Peak locations highly dependent on initial phase	May produce unstable models	May produce unstable models	Performs relatively poorly for short data records
	May suffer spectral line-splitting for sinusoids in noise, or when order is very large	Frequency bias for estimates of sinusoids in noise	Peak locations slightly dependent on initial phase	Frequency bias for estimates of sinusoids in noise
	Frequency bias for estimates of sinusoids in noise		Minor frequency bias for estimates of sinusoids in noise	
Conditions for Nonsingularity		Order must be less than or equal to half the input frame size	Order must be less than or equal to 2/3 the input frame size	Because of the biased estimate, the autocorrelation matrix is guaranteed to positive-definite, hence non-singular

An equivalent definition of PSD is the squared modulus of the Fourier transform of the time series, scaled by a proper constant term. Being power per unit of frequency, the dimensions are those of a power divided by Hertz. For instance, typical units of PSD are $mmHg^2/Hz$ for blood pressure signals, ms^2/Hz for interval tachogram, $(beats/min)^2/Hz$ for instantaneous heart-rate signals as shown in figure 2.4. Different algorithms are used for the estimation of PSD. The more popular in the field of cardiovascular signals analysis are those based on the direct computation of the squared modulus of the Fourier transform of the time series (often termed periodogram estimators) through FFT.

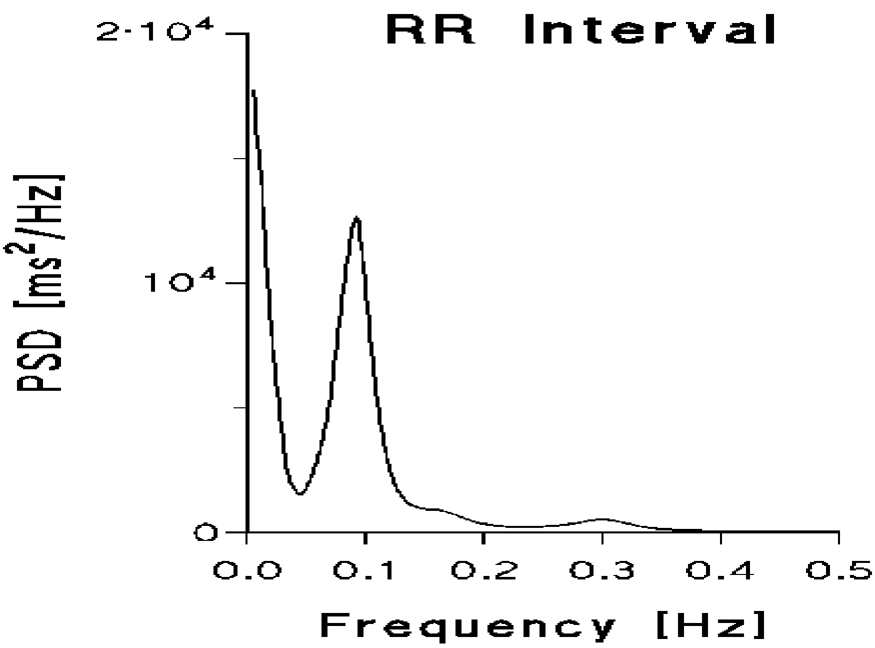


Figure2.4: Power Spectral Density of RR Intervals

2.3 Correlation:

The correlation coefficient for short is a measure of the degree of linear relationship between two variables, usually labelled X and Y. While in regression the emphasis is on predicting one variable from the other, in correlation the emphasis is on the degree to which a linear model may describe the relationship between two variables. In regression the interest is directional, one variable is predicted and the other is the predictor; in correlation the interest is non-directional, the relationship is the critical aspect. The computation of the correlation coefficient is most easily accomplished with the aid of a statistical calculator. The correlation coefficient may take on any value between plus and minus one.

$$-1.00 \leq r \leq +1.00$$

The sign of the correlation coefficient (+, -) defines the direction of the relationship, either positive or negative. A positive correlation coefficient means that as the value of one variable increases, the value of the other variable increases; as one decreases the other decreases. A negative correlation coefficient indicates that as one variable increases, the other decreases, and vice-versa. Taking the absolute value of the correlation coefficient measures the strength of the relationship. A correlation coefficient of r=.50 indicates a stronger degree of linear relationship than one of r=.40. Likewise a correlation coefficient of r=-.50 shows a greater degree of relationship than one of r=.40. Thus a correlation coefficient of zero (r=0.0) indicates the absence of a linear relationship and correlation coefficients of r=+1.0 and r=-1.0 indicate a perfect linear relationship.

2.3.1 Types of Correlation:

In Research Methodology of the Management, Correlation is broadly classified into six types as follows:

Positive Correlation

Negative Correlation

Perfectly Positive Correlation

Perfectly Negative Correlation

Zero Correlation

Linear Correlation

Table 2.2: Range of coefficients of correlation

Types of Correlation	Description	Range
Positive Correlation	When two variables move in the same direction then the correlation between these two variables is said to be Positive Correlation. When the value of one variable increases, the value of other value also increases at the same rate.	+0.75 to +1
Negative Correlation	In this type of correlation, the two variables move in the opposite direction. When the value of a variable increases, the value of the other variable decreases.	-0.75 to -1
Perfectly Positive Correlation	When there is a change in one variable, and if there is equal proportion of change in the other variable say Y in the same direction, then these two variables are said to have a Perfect Positive Correlation.	+1
Perfectly Negative Correlation	Between two variables X and Y, if the change in X causes the same amount of change in Y in equal proportion but in opposite direction, then this correlation is called as Perfectly Negative Correlation.	-1
Zero Correlation	When the two variables are independent and the change in one variable has no effect in other variable, then the correlation between these two variables is known as Zero Correlation.	0
Linear Correlation	If the quantum of change in one variable has a ratio of change in the quantum of change in the other variable then it is known as Linear correlation. .	number between 0 and 1

2.4 Methods of Determining Correlation:

We shall consider the following most commonly used methods.(1) Scatter Plot (2) Karl Pearson's coefficient of correlation (3) Spearman's Rank-correlation coefficient.

2.4.1 Scatter Plot:

Scatter plot shows the relationship between two variables by displaying data points on two dimensional graphs. The variables that might be considered an explanatory variable are plotted on x-axis, and the response variable is plotted on y-axis. Scatter plot are especially useful when there is a large number of data points. They provide the following information about the relationship between two variables:

Strength

Shape – linear, curved etc.

Direction – positive or negative

Presence of outliers

A correlation between the variables results in the clustering of data points along the line. The scatter plot is shown in figure 2.5:

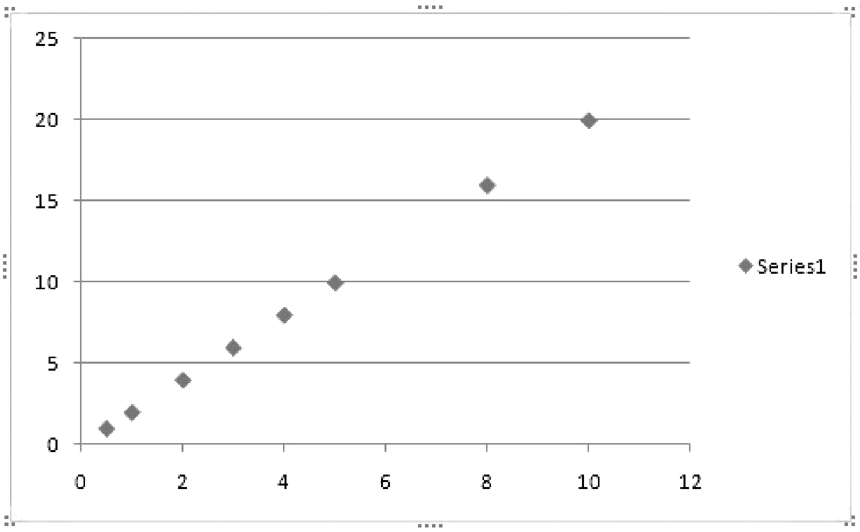


Figure 2.5: Basic Scatter Plot

Advantages of Scatter plot:

- Shows a trend in the data relationship
- Retains exact data values and sample size
- Shows minimum/maximum and outliers

Disadvantages of Scatter plot:

- Hard to visualize results in large data sets
- Flat trend line gives inconclusive results
- Data on both axes should be continuous

2.4.2 Spearman's Rank Correlation Coefficient:

The Spearman correlation coefficient is often thought of as being the Pearson correlation coefficient between the ranked variables. In practice, however, a simpler procedure is normally used to calculate ρ. The n raw scores X_i, Y_i are converted to ranks x_i, y_i and the differences $d_i = x_i - y_i$ between the ranks of each observation on the two variables are calculated.

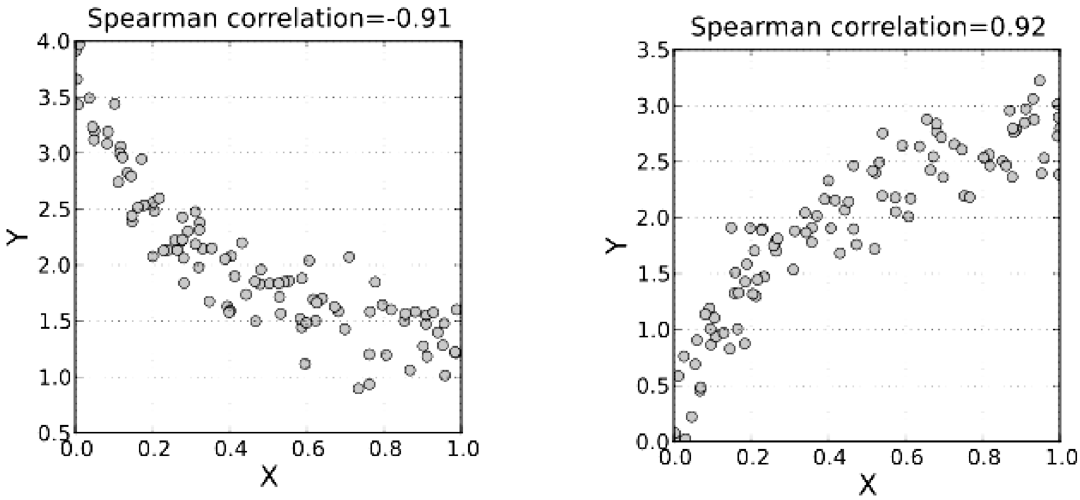
If there are no tied ranks, then ρ is given by:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}.$$

If tied ranks exist, Pearson's correlation coefficient between ranks should be used for the calculation:

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}.$$

One has to assign the same rank to each of the equal values. It is an average of their positions in the ascending order of the values. The plots of negative and positive Spearman correlation are shown in figure2.6 (a) and fugure2.6 (b) respectively.



(a) (b)

Figure 2.6 (a): Negative Spearman correlation coefficient and Figure 2.6 (b) Positive Spearman correlation cofficient

Advantage of Spearman's coefficient of correlation:

The advantage with this measure is that it is much easier to use since it does not matter which way we rank the data, ascending or descending. We may assign rank 1 to the smallest value or the largest value, provided we do the same thing for both sets. So the only requirement of this method is that data should be ranked.

2.4.3 Karl Pearson's Coefficient of Correlation:

It gives the numerical expression for the measure of correlation. It is noted by 'r '. The value of 'r' gives the magnitude of correlation and sign denotes its direction. To compute the correlation values the assumptions are the following:

- linear relationship between x and y
- continuous random variables
- both variables must be normally distributed
- x and y must be independent of each other

Termed Pearson's Product Moment Correlation Coefficient:

- Good with metric data.
- Probably the most popular correlation coefficient.
- It is required that both variables involved be normally distributed.

- Represents quantitatively the extent to which scores on two variables occupy the same relative position.

The formula for the correlation is:

$$R=(N\sum xy-(\sum x)(\sum y))/\sqrt{[N\sum x^2-(\sum x)^2][N\sum y^2-(\sum y)^2]}$$

Where:

N = number of pairs of scores

$\sum xy$ = sum of products of paired scores

$\sum x$= sum of x scores

$\sum y$ = sum of y scores

$\sum x^2$ = sum of squared x scores

$\sum y^2$ = sum of squared y scores

We use the symbol **r** to stand for the correlation. Through the magic of mathematics it turns out that r will always be between -1.0 and +1.0. If the correlation is negative, we have a negative relationship; if it's positive, the relationship is positive.

When the value of coefficient of correlation is exactly equals to -1 then the correlation is perfectly negative as shown in figure 2.7:

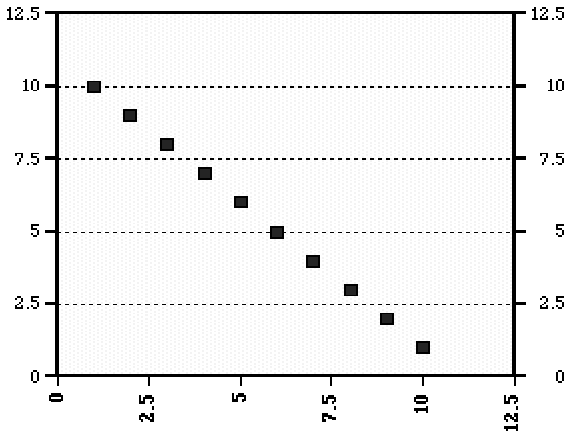


Figure 2.7: Perfect negative correlation

When the value of coefficient of correlation is exactly equals to +1 then the correlation is perfectly positive as shown in figure 2.8:

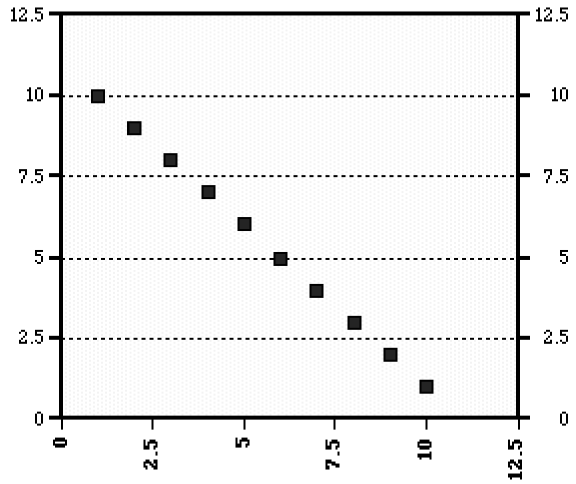


Figure 2.8: Perfect positive correlation

When the value of coefficient of correlation is exactly equals to 0 then there is no correlation as shown in figure 2.9:

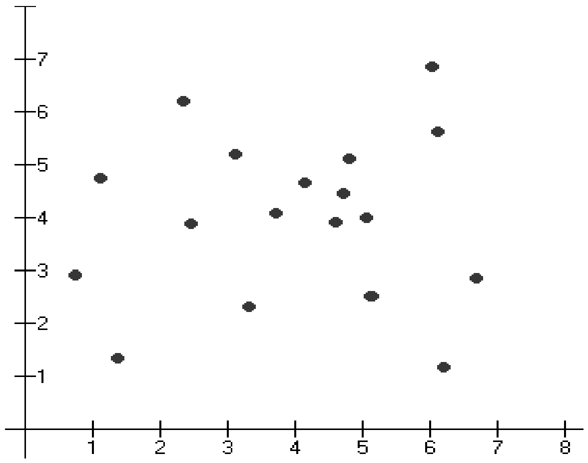


Figure 2.9: No correlation

CHAPTER 3
LITERATURE REVIEW

3.1 Heart rate variability. Frequency domain analysis:

In 1992, Ori.Z et al. evaluated that the frequencies lower than 0.04 Hz in view of reports that the VLF portion of the spectrum (0.01-0.04 Hz) reflected a purer form of sympathetic activity despite the potential applicability to clinical problems, only very little was known about the physiologic basis of the VLF and ULF bands. However, it was important to note that meaningful determinations of VLF and ULF power may be difficult because decreases in frequency to such low levels are associated with an increasing propensity to violate the rules governing power spectral determinations. It was also noteworthy that the reliability of spectral power determinations diminishes with decreases in the power of the signal and of the signal-to-noise ratio.

3.2 Effects of the lower frequency limit on determining high frequency power of heart rate fluctuations:

In 1992, Dan sapoznikov et al. studied the effects of the lower frequency limit on determining high frequency power of heart rate fluctuations and parasympathetic autonomic nervous system activity reflected in the high frequency (hf) range of heart rate fluctuations. The proper selection of the limit of this high frequency range is important for following parasympathetic activity under different physiologic or pathologic conditions. A low cut-off frequency of 0.15 hz allowed power from lower frequencies components which are not solely connected with the Parasympathetic system to enter into calculations of hf activity. The error in hf power estimation is responsible for the masking of true value of hf power.

3.3 Spectral Analysis of Heart Rate without Resampling:

In 1993, GB Moody had focussed on the Standard methods of estimating the power spectral density (PSD) of irregularly sampled signals such as instantaneous heart rate (HR) require resampling at uniform intervals. The Lomb periodogram was a means of obtaining PSD estimates directly from irregularly sampled time series. GB Moody compared Fourier, autoregressive, and Lomb PSD estimates from synthetic, real, and noise-corrupted real heart rate time series, and examined systematic differences among these estimates. An algorithm was presented for obtaining a heart rate time series suitable for Lomb PSD estimation from an RR interval time series with included ectopic beats and erroneous measurements.

3.4 High Resolution R-R Interval Measurement:

In 1993, T Harel et al. focussed on the R-R interval measurement from digitized ECG signal has an inherent error due to the finite sampling frequency. This error showed distortion of heart rate variability (HRV) power spectrum. Results of implementation of an algorithm designed to increase the resolution of R-R interval measurement are reported. The algorithm first estimated the R-R interval; with a finite resolution accuracy. Then the interval truncation error caused by the finite sampling frequency is estimated. The algorithm was evaluated on inartificial signal, generated by the IPFM model. The results of the simulation indicated that the algorithm increases the resolution of R-R interval measurement even in the presence of noise. Measurement of R-R interval with high resolution may improve the reliability of HRV power spectrum estimation especially when dealing with low sampling frequency of the ECG signal.

3.5 Spectrum analysis of heart rate variability:

In 1995, X. Wang et al. had investigated that the respiratory influence on the heart rate variability. The experiment took ECG and respiratory signals from ten normal healthy subjects. Matrixes of tests were conducted in different postures (sitting and standing) and with different breathing patterns (paced and natural). Spectrum analysis was used on the interbeat interval signal (IBI) to obtain the heart rate spectrum. In the paced breathing condition, the relation between the magnitude of respiration in the IBI spectrum and the breathing frequency was found to be exponential. In natural breathing (sitting posture) the relation was weaker than in paced breathing. X . Wang also found that there was no significant difference in the relation between sitting data and standing data during either paced or natural breathing.

3.6 Effect of age on long-term heart rate variability:

In 1997, Vikram K. Yeragani et al. investigated the effect of age on the long term heart rate variability. Previous studies showed that there was inverse relation between the age and spectral power of heart rate variability in the various frequency band. Vikram K. Yeragani et al. proved that there was significant negative correlation between the age and frequency bands: (very low frequency band (0.003-0.04), low frequency band (0.04-0.15) high frequency band (0.15-0.4)) and positive correlation between age and LF/HF ratios. The sleep ultra low frequency band is not affected by age but awake ultra low frequency has negative correlation with age.

3.7 Insights from the study of heart rate variability:

In 1999, P.K. Stein et al.studied that the indices of heart rate variability (HRV) provide a window onto autonomic modulation of the heart. HRV indices, determined in either the time or frequency domain, are closely related and reflect parasympathetic, mixed sympathetic and parasympathetic and circadian rhythms. The decreased HRV had predictive value for mortality among healthy adults. Decreased HRV had mixed predictive success in congestive heart failure. Reduced HRV identifies diabetic patients with autonomic neuropathy. HRV in combination with other risk, e.g. ejection fraction, can identify cardiac patients at especially high risk of mortality. Many but not all interventions associated with increased HRV are also associated with better survival rates.

3.8 Standardized tests of heart rate variability:

In 2001, Marcus W. Agelink et al. studied the effect of age, gender and heart rate on heart rate variability. The method used was very simple, well tolerated and economical. It allows adequate quantitative estimation of cardiac autonomic nervous system regulation in a manner largely independent of patient cooperation. Marcus W. Agelink et al. showed that the HRV indices of all tests are inversely age dependent (except LF/HF ratio) the spectral bands of the 5-minute resting study (except for the LF/HF ratio), the coefficient of variation , and the E/I ratio during deep breathing, and the max/min 30:15 ratio, are independent of HR (unlike the remaining HRV indices studied). The women up to the age of 55 years have a higher heart rate compared with men in the same age group, and young and middle-aged women have a significantly lower LF power and LF/HF ratio compared with age matched men, whereas no significant gender differences are seen for the absolute HF power.

3.9 Comparison of Detrended Fluctuation Analysis and Spectral Analysis for Heart Rate Variability in Sleep and Sleep Apnea:

In 2003 Thomas Penzel et al. studied the effect of sleep stages and sleep apnea on autonomic activity by analyzing heart rate variability (HRV). The spectral parameters VLF, LF, HF, and LF/HF confirmed increasing parasympathetic activity from wakefulness and which was reduced in patients with sleep apnea. Discriminance analysis was used on a person and sleep stage basis to determine the best method for the separation of sleep stages and sleep apnea severity. Using spectral parameters 69.7% of the apnea severity assignments and 54.6% of the sleep stage assignments were correct, while using scaling analysis these numbers increased to 74.4% and 85.0%, respectively. Thomas Penzel concluded that changes in HRV are better quantified by scaling analysis than by spectral analysis.

3.10 Sampling frequency of the RR interval time series for spectral analysis of heart rate variability:

In 2004 D. Singh et al. focused on the sampling frequency of RR interval series for spectral analysis of heart rate variability. Previous studies showed that the time series was sampled using large number of sampling frequencies and the sampling rate of RR interval series must be selected with consideration to mean and minimum RR intervals. D. Singh et al concluded that the choice of sampling frequency was arbitrary, generally choose 4Hz. This was an appropriate sampling rate for the study of autonomic regulation, since it enabled us to compute reliable spectral estimates between DC and 1Hz, which represented the frequency band within which the autonomic nervous system had a significant response.

3.11 Heart rate variability: a review

In 2006 U. Rajendra et al. investigated Heart rate variability (HRV) is a reliable reflection of the many physiological factors modulating the normal rhythm of the heart. In fact, they provide a powerful means of observing the interplay between the sympathetic and parasympathetic nervous systems. It shows that the structure generating the signal is not only simply linear, but also involves nonlinear contributions. Heart rate (HR) is a nonstationary signal; its variation may contain indicators of current disease, or warnings about cardiac diseases. The indicators may be present at all times or may occur at random—during certain intervals of the day. It is strenuous and time consuming to study and pinpoint abnormalities in voluminous data collected over several hours. Hence, HR variation analysis (instantaneous HR against time axis) has become a popular non-invasive tool for assessing the activities of the autonomic nervous system. Computer based analytical tools for in-depth study of data over day long intervals can be very useful in diagnostics. Therefore, the HRV signal parameters, extracted and analyzed using computers, are highly useful in diagnostics. U. Rajendra had discussed the various applications of HRV and different linear, frequency domain, wavelet domain, nonlinear techniques used for the analysis of the HRV.

3.12 Time Series Calculation of Heart Rate Using Multi Rate FIR Filters:

In 2007 MR Risk et al. presented a method to extract the spectral analysis of heart rate variability, based on the Fourier transform, needed even sampled data. The objectives of this study were to develop an interpolation method based on multi rate FIR filters, and then to implement this method for parallel processing machines. A total of three data sets were used: simulated heart rate with an IPFM model, autonomic blockage database (both pharmacological and postural), and long term Holter studies (recordings of 24 hours). Spectral analysis, for the three data sets, was processed for both interpolation using FIR filters and cubic splines, the results showed that for low frequency band the difference was 47±131 ms2, for the high frequency band the difference was 3±48 ms2. The presented method of time series calculation, using FIR filters, proved to be equivalent for both simulated and real data, and is suitable for parallel programming implementation.

3.13 Time and Frequency Domain Analysis of Heart Rate Variability and their Correlations in Diabetes Mellitus

In 2008, P. T. Ahamed Seyd et al. studied the changes in ANS (autonomous nervous system) activity are quantified by means of frequency and time domain analysis of R-

R interval variability. Electrocardiograms (ECG) of 16 patients suffering from Diabetes Mellitus (DM) and of 16 healthy volunteers were recorded. Frequency domain analysis of extracted normal to normal interval (NN interval) data indicates significant difference in very low frequency (VLF) power, low frequency (LF) power and high frequency (HF) power, between the DM patients and control group. Time domain measures, standard deviation of NN interval (SDNN), root mean square of successive NN interval differences (RMSSD), successive NN intervals differing more than 50 ms (NN50 Count), percentage value of NN50 count (pNN50), HRV triangular index and triangular interpolation of NN intervals (TINN) also show significant difference between the DM patients and control group.

3.14 Improved Power Spectrum Estimation for RR-Interval Time Series:

In 2009, B. S. Saini et al. had studied that the RR interval series is non-stationary and unevenly spaced in time. For estimating its power spectral density (PSD) using traditional techniques like FFT, required resampling at uniform intervals. The researchers had used different interpolation techniques as resampling methods. All these resampling methods introduced the low pass filtering effect in the power spectrum. The lomb transform was a means of obtaining PSD estimates directly from irregularly sampled RR interval series, thus avoiding resampling. In this work, the superiority of Lomb transform method had been established over FFT based approach, after applying linear and cubicspline interpolation as resampling methods, in terms of reproduction of exact frequency locations as well as the relative magnitudes of each spectral component.

3.15 A review of measurement and analysis of heart rate variability:

In 2009, Dipali Banal et al. found that Electrocardiogram (ECG) is one of the most important physiological parameter that gives correct assessment of heart function. QRS complex is a prominent waveform in an ECG that provides the basis for analyzing heart rate variability (HRV). HRV refers to the beat-to-beat alterations in heart rate. Commercial devices these days provide preset computerized measurement of HRV, thus providing the cardiologist a simple tool for both research and clinical learning. To obtain meaningful data from the ECG, a noise free inter-beat interval

(IBI) time series is required to be extracted. This is realized using standard peak detection algorithms packed with data acquisition hardware and software. Dipali Banal et al. described various QRS detection methods used to derive HRV and also reviewed various time and frequency domain HRV parameters. The significance and meaning of these different measures of HRV are a potential area of research.

CHAPTER 4

PROBLEM FORMULATION

Measures of heart rate variability are increasingly being employed in applications ranging from basic investigations of central regulation of autonomic state, to studies of fundamental links between psychological processes and physiological functions, to evaluations of cognitive development and clinical risk. The classical time domain heart period signal samples, i.e. RR intervals, are defined as:

$$In = t_{n-}t_{n-1},$$

where tn is the time instant at which the nth QRS peak occurs. From figure 4.1, it becomes immediately clear that the RR intervals are nonequispaced in time.

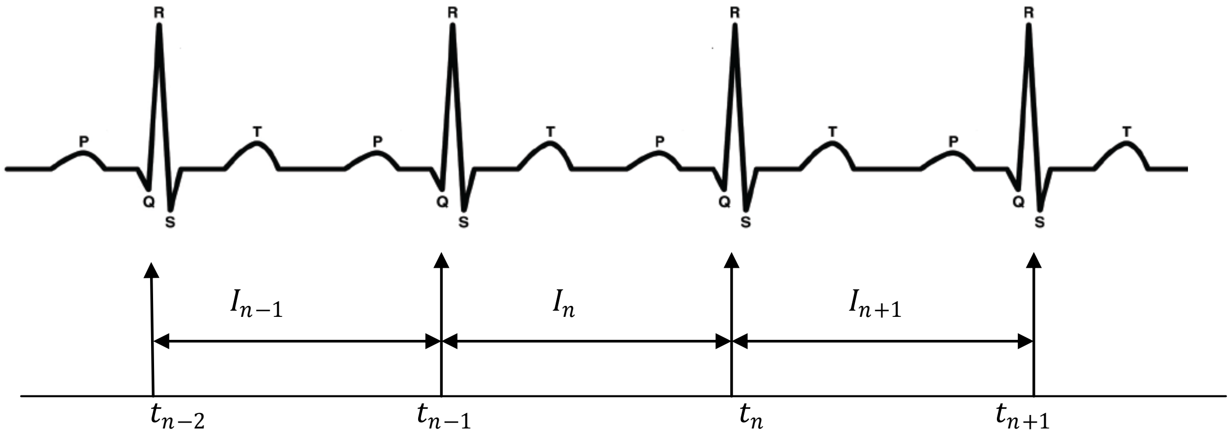


Figure 4.1: ECG and cardiac event series

4.1 Procedure for determining Power Spectral Density:

This is well known that the Power spectral Density of RR interval series can be estimated with a number of analytical expressions. The different researchers have explained different methods to find out the heart rate variability in frequency domain but the problem is that the previous papers tell us only that the frequency domain analysis is done by non parametric fast Fourier transform (FFT) method, parametric method and about the software e.g. Preprocessing of the recorded data was done using Pan and Tompkinson's algorithm and heart rate computation were done using AcqKnowledge and MATLAB software and they do not explain anything regarding the implementation. Till now, studies only mention the frequency band ranges i.e. VLF (0.003-0.04), LF (0.04-0.15), HF (0.15-0.4) but no research explains the whole process of finding the powers in these frequency bands. The present aim is defining an appropriate procedure for finding the powers in the frequency bands.

4.2 Resampling:

Earlier the time series had been resampled using wide range of sampling frequencies i.e.1, 2, 2.17, 3, 3.12 Hz etc [16]. But many research papers has showed that the choice of resampling frequency should be arbitrary say 4 hertz, there is no consistent approach for explaining all these estimations. The present work investigated the resampling frequency.

4.3 Interpolation:

Most of the DSP algorithms are designed for equispaced samples. These algorithms thus cannot be applied to RR beat signals as these signals are not equispaced. Many researchers "assumed" the samples to be equispaced if the deviation of intervals from RR mean is small i.e. in the interval spectrum method as sampling is function of beat number [17]. Secondly many research papers has showed different techniques to find out the power spectral density without interpolation by using the parametric methods i.e. auto regressive approach and Lomb periodogram but the problem with the auto regressive approach is that it is very tedious because for this technique the order for the model should be exactly known and there is not any consistent approach for finding the order [18] and Lomb periodogram results in ectopic beats and erroneous measurements. In the present work, we explore the application of linear interpolation at 2Hz and 4Hz sampling frequency for finding the Power Spectral Density in ms^2/Hz.

CHAPTER 5
PROPOSED SOLUTION AND IMPLEMENTATION

5.1 Database:

The RR intervals of 34 normal persons are taken from physiobank. PhysioBank is a large and growing archive of well-characterized digital recordings of physiologic signals and related data for use by the biomedical research community. PhysioBank currently includes databases of multi-parameter cardiopulmonary, neural, and other biomedical signals from healthy subjects and patients with a variety of conditions with major public health implications, including sudden cardiac death, congestive heart failure, gait disorders, sleep apnea, and aging. In a normal heart rhythm, the sinus node generates an electrical impulse which travels through the right and left atrial muscles producing electrical changes which is represented on the electrocardiogram (ECG) by the p-wave. The electrical impulse then continues to travel through specialized tissue known as the atrioventricular node, which conducts electricity at a slower pace. This will create a pause (PR interval) before the ventricles are stimulated. This pause is helpful since it allows blood to be emptied into the ventricles from the atria prior to ventricular contraction to propel blood out into the body. The ventricular contraction is represented electrically on the ECG by the QRS complex of waves. This is followed by the T-wave which represents the electrical changes in the ventricles as they are relaxing. The cardiac cycle after a short pause repeats itself, and so on.

The followings are the databases:

1 MIT-BIH Normal Sinus Rhythm Database (nsrdb)
2 Normal Sinus Rhythm RR interval Database (nsr2db)
3 MIT-BIH Arrhythmia Database (mitdb)

Example: Data is taken from physionet of 28 years old female with time duration of 5 min and the figure 5.1 shows the variation in magnitudes of RR intervals.

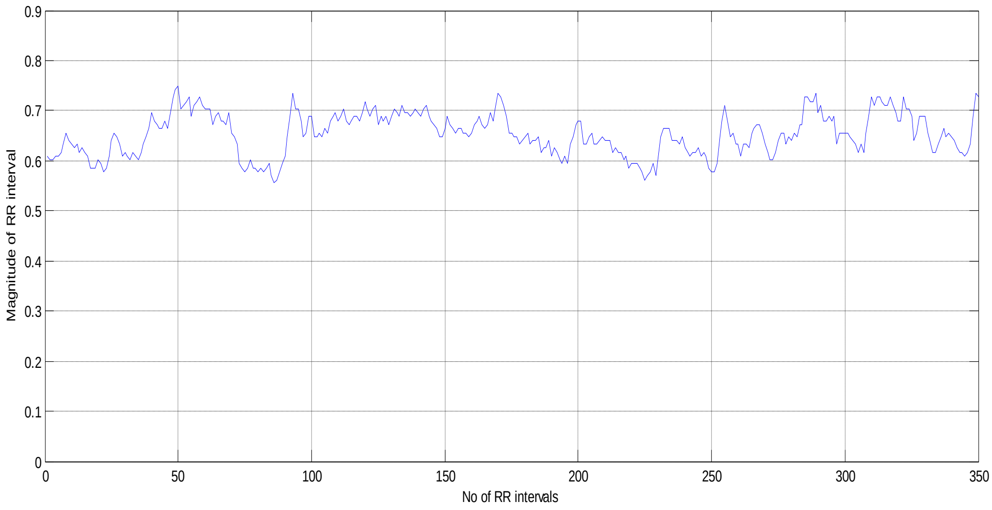


Figure 5.1: Plot of RR interval

5.2 Matlab:

The MATLAB software tool is used for the spectral analysis of heart rate variability as the MATLAB is a software package for high performance numerical computation and visualization. It provides an interactive environment with hundreds of built-in-functions for technical computation, graphics, and animations. It also provides easy extensibility with its own high- level programming language. The name MATLAB stands for Matrix Laboratory. MATLAB’s built-in function provide excellent tools for linear algebra computations, data analysis, signal processing, optimization, numerical solution of ordinary differential equations (ODEs), and many other types of scientific computations. There are also several optional toolboxes available for the developers of MATLAB. These toolboxes are collections of functions written for special applications such as symbolic computation, image processing, statistics, control system design neural network etc. MATLAB simply loves matrices and matrix operations. The built-in functions are optimized for vector operations. Consequently, vectorized commands or codes run much faster in MATLAB. The present work involves lengthy and cumbersome computations which can be carried out in any computer language like C/C++. However the built in tools and functions available in MATLAB like plot, hann, FFT, mean, Interpolation, correlation etc make the coding much easier. Consequently work is done in MATLAB for fast coding and easier verification.

5.3 Conversion of Data:

The data available on the physionet is in seconds, we have firstly converted it into milliseconds by multiplying it with 1000 as shown in the figure 5.2. Mathematically,

$$Y1 = X*1000$$

Where, X is the RR interval series.

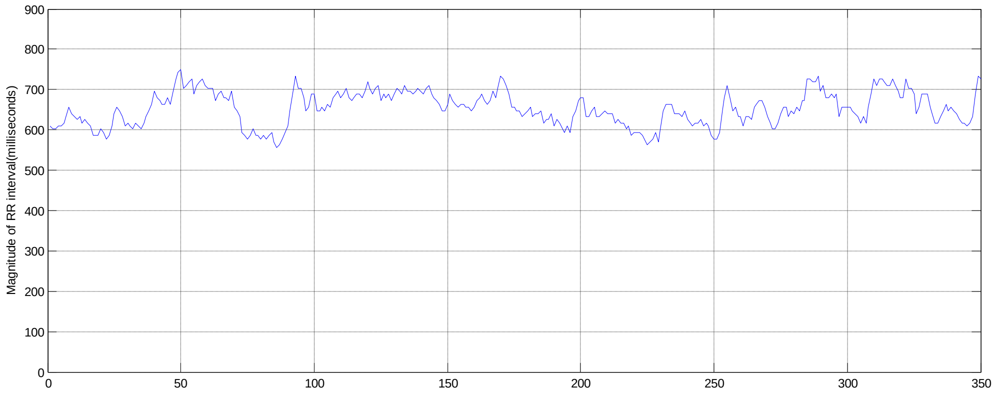


Figure 5.2: RR intervals in milliseconds

5.4 Normalization of RR Interval:

Normalization is the process of efficiently organizing data in a database. There are two goals of the normalization process: eliminating redundant data and ensuring data dependencies make sense. Both of these are worthy goals as they reduce the amount of space a database consumes and ensure that data is logically stored as shown in the figure 5.3. The normalization of RR interval can be done with the help of mathematical equation:

$$A = mean(Y1(:));$$

$$B = (Y1(;1) – A)$$

where, A is the mean of RR series.

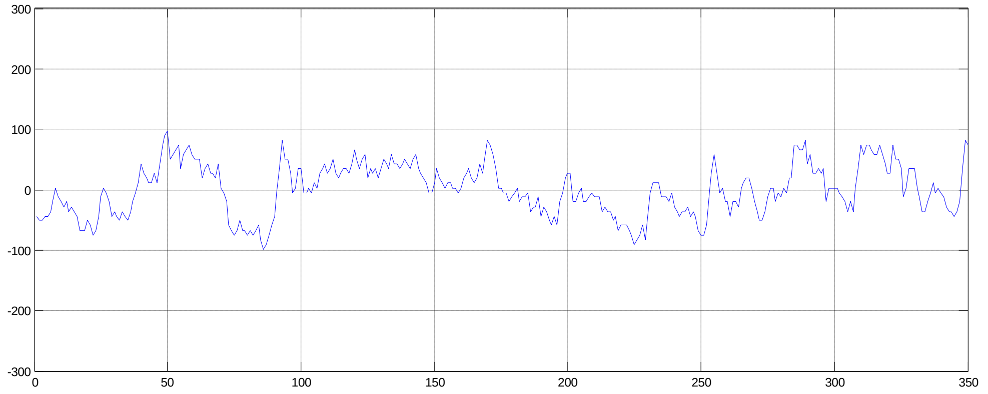


Figure 5.3: Plot after normalization process

5.5 Hann Window:

The Hann function is typically used as a window function in digital signal processing to select a subset of a series of samples in order to perform a Fourier transform. Using a finite-length discrete signal in the time domain in FT calculations is to apply a rectangular window to the infinite-length signal. This does not cause a problem with the transient signals which are time-bounded inside this window. But, what happens if a continuous time signal like a sine wave is of interest? If the length of the window (i.e., the time record of the signal) contains an integral number of cycles of the time signal, then, periodicity introduced by discretization makes the windowed signal exactly same as the original. In this case, the time signal is said to be periodic in the time record. On the other hand, there is a difficulty if the time signal is not periodic in the time record, especially at the edges of the record (i.e., window). If the DFT or FFT could be made to ignore the ends and concentrate on the middle of the time record, it is expected to get much closer to the correct signal spectrum in the frequency domain. This may be achieved by a multiplication by a function that is zero at the ends of the time record and large in the middle. This is known as windowing. A few examples of common window functions are (n = 0, 1,2, … (N-1)):

Rectangular: $W(n) = 1$

Hanning: $W(n) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi n}{N}\right)$

Hamming: $W(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right)$

It should be realized that, the time record is tempered and perfect results shouldn't be expected. For example, windowing reduces spectral leakage but does not totally eliminate it. It should also be noted that, windowing is introduced to force the time record to be zero at the ends as shown in figure therefore transient signals which occur (starts and ends) inside this window do not require a window. They are called self-windowed signals, and examples are impulses, shock responses, noise bursts, sine bursts, etc. The resulting vector is multiplied element-by-element with the sampled signal vector before applying the FFT as shown in figure 5.4. The representation of code is:

Han= B(i) * w(k)

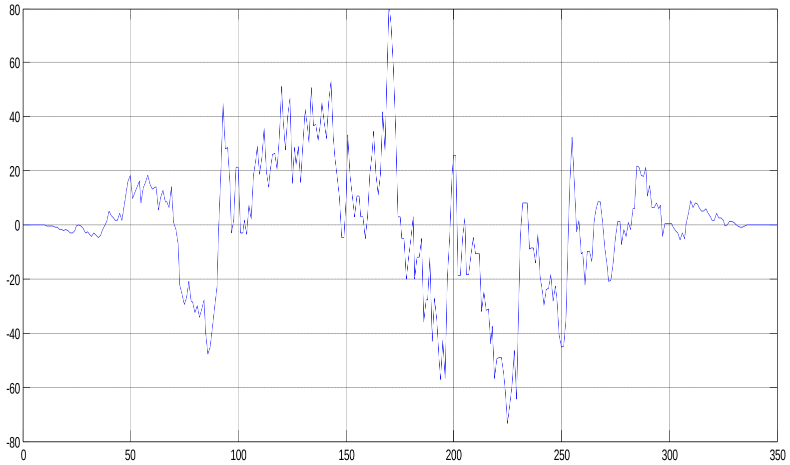


Figure 5.4: Time Series Multiplied by Hanning Window

The advantage of the Hann window is very low aliasing, and the tradeoffs is slightly decreased resolution (widening of the main lobe). If the Hann window is used to sample a signal in order to convert to frequency domain, it is complex to reconvert to the time domain without adding distortions.

5.6 Sampling Frequency:

The sampling rate, sample rate, or sampling frequency defines the number of samples per second (or per other unit) taken from a continuous signal to make a discrete signal. For time-domain signals, the unit for sampling rate is 1/s. The inverse of the sampling frequency is the sampling period or sampling interval, which is the time between samples. The concept of sampling frequency can only be applied to samplers in which samples are taken periodically. Some samplers may sample at a non-periodic rate. The common notation for sampling frequency is f_s which stands for frequency (subscript) sampled. Previously it was considered that RR interval is equispaced if the deviation of RR intervals from RR mean is small. In this case , the sampling frequency was taken as the reciprocal of the average of RR interval. According to the Nyquist theorem the Nyquist rate is the minimum sampling rate required to avoid aliasing, equal to twice the highest frequency. Mathematically,

Fmax = Fs/2

where ,

Fmax is the maximum frequency

Fs = Sampling frequency directly taken by taking the reciprocal of average of RR interval, as the data is considered equispaced.

5.7 Fast Fourier Transform:

The transformation from the time domain to the frequency domain (and back again) is based on the Fourier transform and its inverse, which are defined as

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$$

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df$$

Here, *s(t), S(ω), and f* are the time signal, the frequency signal and the frequency, respectively, and $j = \sqrt{-1}$. We, the physicists and engineers, sometimes prefer to write the transform in terms of angular frequency $\omega = 2\pi f$, *as*

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt$$

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega$$

which, however, destroys the symmetry. To restore the symmetry of the transforms, the convention

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt$$

$$s(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega$$

is sometimes used. The FT is valid for real or complex signals, and, in general, is a complex function of ω (or *f*). The FT is valid for both periodic and non-periodic time signals that satisfy certain minimum conditions. Almost all real world signals easily satisfy these requirements (It should be noted that the Fourier series is a special case of the FT).

- FT is defined for continuous time signals.
- In order to do frequency analysis, the time signal must be observed infinitely.

Under these conditions, the FT defined above yields frequency behavior of a time signal at every frequency, with zero frequency resolution. Stepwise calculation of the fast Fourier transform is;

C = 2^nextpow2(N)

where,

N is the length of the RR interval series nextpow2 is the next highest power of 2. This function is useful for optimizing FFT operations, which are most efficient when sequence length is an exact power of two. For e.g. For any integer n in the range from 513 to 1024, nextpow2(n) is 10.

$$Y = \text{fft (hann, C)}/N$$

$$Z = \text{abs}(Y)$$

where abs is the absolute value of values obtained after applying fft.

The results obtained after applying the fft are the combination of real and imaginary values i.e. complex numbers. When the object is real and has a signed data type, the absolute value of the most negative value is problematic since it is not representable. In this case, the absolute value saturates to the most positive value representable by the data type. The plot shows the variation of amplitude with respect to frequency, the amplitude is taken on the Y-axis and frequency is on the X-axis as shown in figure 5.5.

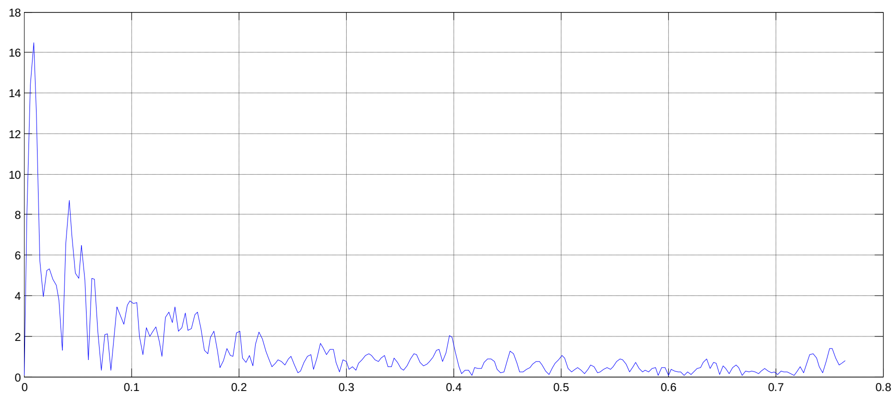


Figure 5.5: Amplitude Spectrum

5.8 Power Spectral Density:

Power spectral density (PSD), describes how the power of a signal or time series is distributed with frequency. Power spectral density function (PSD) shows the strength of the variations (energy) as a function of frequency. In other words, it shows at which frequencies variations are strong and at which frequencies variations are weak. The unit of PSD is energy per frequency (width) and you can obtain energy within a specific frequency range by integrating PSD within that frequency range.

Computation of PSD is done directly by the method called FFT or computing autocorrelation function and then transforming it.

$$Power = Z (:)^2$$

The plot obtained as shown in figure 5.6:

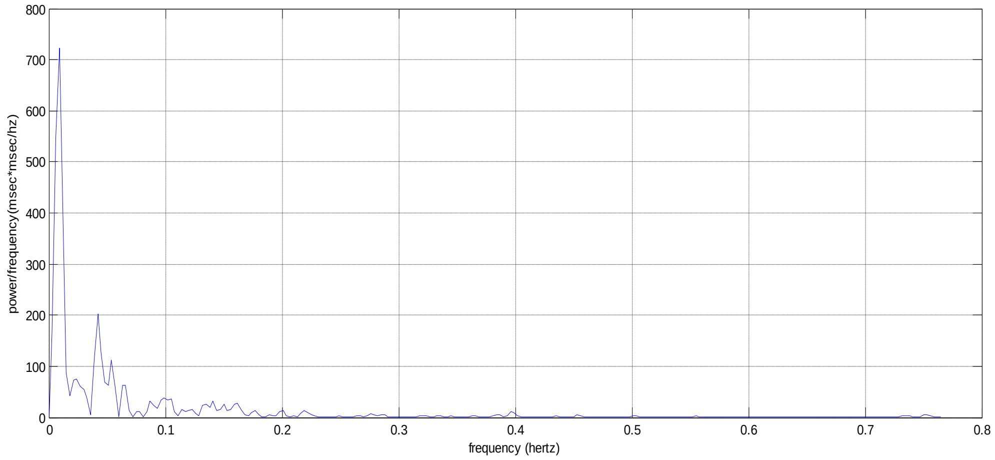


Figure 5.6: Power spectral density obtained by Fourier transform of RR interval time series

Table 5.1: Representative table of first 350 RR intervals of 28 years old Female(database 16273) for calculating power in specific bands

X	Conversion into millisecond	Normalization	Hann Window	FFT	Square Magnitude	Frequency Fs=1.53
0.602	602	-44.017	-0.0036	0.0569	0.0086	0
0.602	602	-51.017	-0.0165	7.9507	168.147	0.003
0.609	609	-51.017	-0.0372	14.3697	549.257	0.006
0.609	609	-44.017	-0.057	16.4817	722.582	0.009
0.617	617	-44.017	-0.0891	12.9926	449.03	0.012
0.633	633	-36.017	-0.105	5.701	86.4549	0.015
0.656	656	-20.017	-0.0794	3.9409	41.3123	0.018
0.641	641	2.9829	0.0154	5.2434	73.1312	0.021
0.633	633	-12.017	-0.0787	5.3156	75.1592	0.024

0.625	625	-20.017	-0.1618	4.8083	61.4978	0.027
0.633	633	-28.017	-0.2738	4.5106	54.1202	0.03
0.617	617	-20.017	-0.2327	3.7811	38.03	0.033
0.625	625	-36.017	-0.491	1.3092	4.5594	0.036
0.617	617	-28.017	-0.4426	6.621	116.609	0.039
0.609	609	-36.017	-0.6527	8.7138	201.974	0.042
0.586	586	-44.017	-0.9068	6.9425	128.206	0.045
0.586	586	-67.017	-1.5572	5.1113	69.4928	0.048
0.586	586	-67.017	-1.7441	4.868	63.036	0.051
0.602	602	-67.017	-1.9413	6.4897	112.03	0.054
0.594	594	-51.017	-1.6358	4.785	60.9037	0.057
0.578	578	-59.017	-2.0839	0.8371	1.864	0.06
0.586	586	-75.017	-2.9038	4.8709	63.1102	0.063
0.609	609	-67.017	-2.8319	4.8167	61.7141	0.066
0.641	641	-44.017	-2.0227	2.1871	12.7239	0.069
0.656	656	-12.017	-0.5984	0.3278	0.2858	0.072
0.648	648	2.9829	0.1604	2.0895	11.6132	0.075
0.633	633	-5.0171	-0.2906	2.1208	11.9639	0.078
0.609	609	-20.017	-1.2449	0.3412	0.3097	0.081
0.617	617	-44.017	-2.9321	2.1009	11.7407	0.084
0.609	609	-36.017	-2.5634	3.4237	31.1801	0.087
0.602	602	-44.017	-3.3396	3.0201	24.2615	0.09
0.617	617	-51.017	-4.1174	2.5986	17.9628	0.093
0.609	609	-36.017	-3.0858	3.5396	33.3261	0.096
0.602	602	-44.017	-3.996	3.7585	37.5756	0.099
0.617	617	-51.017	-4.8987	3.5977	34.4287	0.102
0.633	633	-36.017	-3.6518	3.6601	35.6342	0.105
0.648	648	-20.017	-2.1396	1.9998	10.6376	0.108
0.664	664	-5.0171	-0.5645	1.0861	3.1375	0.111
0.695	695	10.982	1.2989	2.4249	15.6416	0.114
0.68	680	41.982	5.2118	1.973	10.355	0.117
0.672	672	26.982	3.5115	2.1834	12.6808	0.12

0.664	664	18.982	2.5865	2.4397	15.8328	0.123
0.664	664	10.982	1.565	1.7283	7.9459	0.126
0.68	680	10.982	1.6347	0.9972	2.645	0.129
0.664	664	26.982	4.1906	2.933	22.8826	0.132
0.695	695	10.982	1.7779	3.1696	26.7238	0.135
0.727	727	41.982	7.077	2.6639	18.8768	0.138
0.742	742	73.982	12.9737	3.4578	31.8047	0.141
0.75	750	88.982	16.218	2.2654	13.6509	0.144
0.703	703	96.982	18.3551	2.4064	15.4037	0.147
0.711	711	49.982	9.8148	3.1416	26.2529	0.15
0.719	719	57.982	11.8032	2.3038	14.1174	0.153
0.727	727	65.982	13.9132	2.3833	15.1092	0.156
0.688	688	73.982	16.1469	3.0707	25.0813	0.159
0.711	711	34.982	7.8968	3.1959	27.1681	0.162
0.719	719	57.982	13.5276	2.364	14.8655	0.165
0.727	727	65.982	15.8993	1.2901	4.4271	0.168
0.711	711	73.982	18.3997	1.1238	3.3594	0.171
0.703	703	57.982	14.874	1.9645	10.2662	0.174
0.703	703	49.982	13.2168	2.2297	13.2244	0.177
0.703	703	49.982	13.6155	1.3122	4.5802	0.18
0.672	672	49.982	14.0179	0.4291	0.4898	0.183
0.688	688	18.982	5.478	0.7878	1.6508	0.186
0.695	695	34.982	10.3818	1.3883	5.1266	0.189
0.68	680	41.982	12.8059	1.0563	2.968	0.192
0.68	680	26.982	8.455	1.0197	2.7657	0.195
0.672	672	26.982	8.6811	2.1492	12.2869	0.198
0.695	695	18.982	6.2675	2.2553	13.5302	0.201
0.656	656	41.982	14.2179	0.8986	2.1478	0.204
0.648	648	2.9829	1.0357	0.7085	1.3351	0.207
0.633	633	-5.0171	-1.7851	1.0304	2.8243	0.21
0.594	594	-20.017	-7.2951	0.5155	0.7069	0.213
0.586	586	-59.017	-22.021	1.6245	7.0195	0.216

0.578	578	-67.017	-25.59	2.2208	13.1188	0.219
0.586	586	-75.017	-29.303	1.8392	8.9976	0.222
0.602	602	-67.017	-26.768	1.2818	4.3706	0.225
0.586	586	-51.017	-20.828	0.8981	2.1454	0.228
0.586	586	-67.017	-27.954	0.5019	0.6701	0.231
0.578	578	-67.017	-28.549	0.6471	1.1138	0.234
0.586	586	-75.017	-32.626	0.8124	1.7555	0.237
0.578	578	-67.017	-29.746	0.7483	1.4893	0.24
0.586	586	-75.017	-33.968	0.587	0.9165	0.243
0.594	594	-67.017	-30.947	0.8581	1.9587	0.246
0.57	570	-59.017	-27.783	1.0113	2.7204	0.249
0.555	555	-83.017	-39.827	0.5932	0.9362	0.252
0.562	562	-98.017	-47.905	0.2104	0.1177	0.255
0.578	578	-91.017	-45.303	0.2748	0.2009	0.258
0.594	594	-75.017	-38.015	0.6888	1.2619	0.261
0.609	609	-59.017	-30.438	1.022	2.7782	0.264
0.648	648	-44.017	-23.097	1.081	3.1086	0.267
0.688	688	-5.0171	-2.6778	0.3719	0.3679	0.27
0.734	734	34.982	18.9854	0.9393	2.347	0.273
0.703	703	80.982	44.6756	1.6608	7.3372	0.276
0.703	703	49.982	28.021	1.4402	5.5176	0.279
0.68	680	49.982	28.4671	1.0969	3.2003	0.282
0.648	648	26.982	15.608	1.3567	4.8961	0.285
0.656	656	-5.0171	-2.9467	1.357	4.8983	0.288
0.688	688	2.9829	1.7783	0.7129	1.3517	0.291
0.688	688	34.982	21.164	0.2401	0.1533	0.294
0.648	648	34.982	21.4713	0.8348	1.8537	0.297
0.648	648	-5.0171	-3.1232	0.7285	1.4116	0.3
0.656	656	-5.0171	-3.1669	0.3839	0.392	0.303
0.648	648	2.9829	1.9087	0.4848	0.6252	0.306
0.664	664	-5.0171	-3.2536	0.3413	0.3099	0.309
0.656	656	10.982	7.2166	0.6575	1.1498	0.312

0.68	680	2.9829	1.9854	0.8513	1.9278	0.315
0.688	688	26.982	18.1881	1.0281	2.8118	0.318
0.695	695	34.982	23.8748	1.1365	3.4359	0.321
0.68	680	41.982	29.0027	1.0478	2.9203	0.324
0.688	688	26.982	18.864	0.8222	1.7983	0.327
0.703	703	34.982	24.7446	0.7308	1.4205	0.33
0.68	680	49.982	35.7624	0.9022	2.1649	0.333
0.672	672	26.982	19.5243	1.0526	2.9474	0.336
0.68	680	18.982	13.8878	0.4988	0.6618	0.339
0.688	688	26.982	19.9548	0.4739	0.5974	0.342
0.688	688	34.982	26.1461	0.9308	2.3045	0.345
0.68	680	34.982	26.4183	0.6978	1.2954	0.348
0.695	695	26.982	20.5846	0.4062	0.4389	0.351
0.719	719	41.982	32.3475	0.3365	0.3012	0.354
0.703	703	65.982	51.3359	0.5287	0.7436	0.357
0.688	688	49.982	39.2593	0.8721	2.0233	0.36
0.703	703	34.982	27.7344	1.1323	3.4104	0.363
0.711	711	49.982	39.9887	1.1016	3.2277	0.366
0.672	672	57.982	46.8038	0.6939	1.2807	0.369
0.688	688	18.982	15.4568	0.5208	0.7216	0.372
0.68	680	34.982	28.728	0.6198	1.022	0.375
0.688	688	26.982	22.3431	0.7314	1.4229	0.378
0.672	672	34.982	29.2033	0.9671	2.4877	0.381
0.688	688	18.982	15.9725	1.303	4.5159	0.384
0.695	695	49.982	42.7028	0.7555	1.5184	0.39
0.688	688	41.982	36.1322	1.1708	3.6464	0.393
0.711	711	34.982	30.3237	2.05	11.1789	0.396
0.695	695	57.989	50.6118	1.9361	9.9709	0.399

From the above calculations the power can be calculated:

VLF(0.003-0.04) = 2439 ms^2

LF(0.04-0.15) = 1350.16 ms^2

HF(0.15-0.4) = 387.22 ms^2

5.9 Linear Interpolation:

The above calculations are done without interpolation, now the whole calculations are repeated with interpolation. There are different methods of the interpolation but we prefer linear interpolation.

Linear interpolation is a method of curve fitting using linear polynomials. It is heavily employed in mathematics (particularly numerical analysis), and numerous application including computer graphics. It is a simple form of interpolation. If the two points are given by the coordinates$(x_{o,}y_o)$ and $(x_{1,}y_1)$, the linear interpolation is the straight line between these points. For a value x in the interval $(x_{o,}x_1)$, the value y along the straight line is given from the equation.

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

which can be derived geometrically from the figure on the right.

Solving this equation for *y*, which is the unknown value at *x*, gives

$$y = y_0 + (x - x_0)\frac{y_1 - y_0}{x_1 - x_0}$$

which is the formula for linear interpolation in the interval (x_0, x_1). Outside this interval, the formula is identical to linear extrapolation. The code for the linear interpolation in MATLAB is given as under:

Y1=interp1(y,x,t)

where, x is the RR interval series

y is the addition of values of x column-wise

t is the range with which the two samples are equi spaced.

In the calculation the difference between two intervals is taken 0.5 and 0.25; accordingly this sampling frequency is taken 2 hertz and 4 hertz. After interpolation the whole process is repeated as same in the above for without interpolated cases. Brief description of all the steps in the form of algorithm is given below:

Step1: Interpolate the RR interval with the linear interpolation method.

Y1'= interp1(y,x,t)

Step2: Conversion of data into milliseconds

Y1=Y1'*1000

Step3: Normalised the above obtained data with the help of mathematical equation:

A = mean(Y1(:));

B = (Y1(;1) – A)

Step4: Apply hann window:

W (k) = (1-cos(2∏x/(N-1))/2

where x is the variation from one to length of data and

N is the length of data

The resulting vector is multiplied element by element with the sampled signal vector before applying DFT (discrete Fourier transform)

Han= B(i)*W(k)

Step5: Maximum frequency is the half of sampling frequency,

f(max) = fs/2

i.e. if the sampling frequency is of 2Hz then maximum frequency is 1Hz and if the sampling frequency is of 4Hz then maximum frequency is 2Hz.

Step6: Apply the DFT on the values obtained from Step4:

Mathematically, $X(k) = \sum_{j=1}^{N} x(j)\, w_N^{(j=1)(k-1)}$

$$X(j) = (\sum_{k=1}^{N} X(r) w_N^{-(j-1)(k-1)}$$

Where w_N =$e^{(-2\prod i)/N}$ is the Nth root of unity

C = 2^nextpow2(N)

Where $w_N = e^{(-2\Pi i)/N}$ is the Nth root of unity

$$C = 2^\wedge nextpow2(N)$$

where, N is the length of the RR interval series, nextpow2 is the next highest power of 2.

$$Y = fft\ (hann,\ C)/N$$

$$Z = abs(Y)$$

where abs is the absolute value of values obtained after applying fft.

Step7: Then the squaring magnitude of each component computed and each frequency component was multiplied by 2.66 to correct for hann window to obtain power.

$$Power = Z\ (:)^\wedge 2$$

Step8: Plot the results as the power on Y-axis and frequency on X-axis with sampling frequency of 2 hertz and 4 hertz.

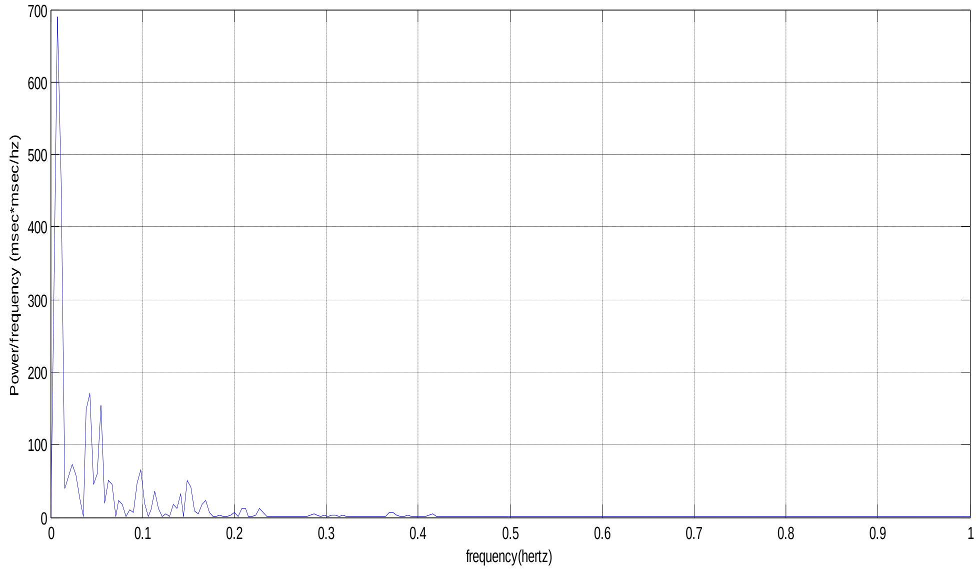


Figure 5.7: Power spectral density obtained by Fourier transform of RR interval time series (Fs=2Hz)

Table 5.2: Representative table of first 350 RR intervals with interpolation(Fs=2 Hz) of 28 years old female (16273) calculating power in specific bands:

X	Y	Intepolation Y'	Y'* 1000	Normlization	Hann Window	FFT	Square - Magnitude	Frequen-cy
0.609	0.61	0.609	609	-46.227	-0.0022	0.0042	0	0
0.602	1.211	0.6032	603.176	-52.051	-0.01	10.361	285.554	0.0039
0.602	1.813	0.602	602	-53.227	-0.0229	16.097	689.284	0.0078
0.609	2.422	0.6054	605.414	-49.813	-0.0382	13.192	462.978	0.0118
0.609	3.032	0.609	609	-46.227	-0.0553	3.8159	38.7325	0.0157
0.617	3.649	0.61	610.011	-45.216	-0.0779	4.5954	56.1738	0.0196
0.633	4.282	0.6165	616.494	-38.733	-0.0908	5.2621	73.6559	0.0235
0.656	4.938	0.6287	628.652	-26.575	-0.0814	4.6608	57.7831	0.0275
0.641	5.578	0.6445	644.5	-10.727	-0.0416	3.1788	26.8791	0.0314
0.633	6.211	0.652	651.969	-3.2582	-0.0156	0.4436	0.5234	0.0353
0.625	6.836	0.6406	640.596	-14.631	-0.0846	7.4854	149.044	0.0392
0.633	7.469	0.6343	634.277	-20.951	-0.1441	7.9944	170.002	0.0431
0.617	8.086	0.6279	627.893	-27.334	-0.2206	4.145	45.7026	0.0471
0.625	8.711	0.6285	628.463	-26.764	-0.2504	4.7738	60.6195	0.051
0.617	9.328	0.6293	629.344	-25.883	-0.2779	7.5927	153.348	0.0549
0.609	9.938	0.6173	617.307	-37.92	-0.4629	2.7229	19.7222	0.0588
0.586	10.52	0.6237	623.707	-31.52	-0.4342	4.3704	50.8068	0.0627
0.586	11.11	0.6198	619.827	-35.4	-0.5464	4.1046	44.8152	0.0667
0.586	11.69	0.6133	613.302	-41.925	-0.7206	0.3537	0.3327	0.0706
0.602	12.29	0.6022	602.249	-52.978	-1.0082	2.9779	23.5882	0.0745
0.594	12.89	0.586	586	-69.227	-1.4516	2.6114	18.1397	0.0784
0.578	13.46	0.586	586	-69.227	-1.592	0.5048	0.6779	0.0824
0.586	14.05	0.586	586	-69.227	-1.7388	1.9391	10.0015	0.0863
0.609	14.66	0.597	597.022	-58.205	-1.5907	1.568	6.5397	0.0902
0.641	15.30	0.5978	597.785	-57.442	-1.702	4.1971	46.8584	0.0941
0.656	15.96	0.5879	587.938	-67.289	-2.1547	4.9564	65.345	0.098
0.648	16.61	0.5799	579.925	-75.302	-2.5981	2.6512	18.6972	0.102

0.633	17.24	0.5881	588.077	-67.15	-2.4895	0.8134	1.76	0.1059
0.609	17.85	0.607	606.961	-48.266	-1.9177	1.9596	10.2141	0.1098
0.617	18.46	0.6313	631.265	-23.962	-1.0179	3.6939	36.2955	0.1137
0.609	19.07	0.648	647.974	-7.2529	-0.3287	2.1925	12.7864	0.1176
0.602	19.68	0.6542	654.163	-1.0636	-0.0513	0.6977	1.2948	0.1216
0.617	20.29	0.648	648	-7.227	-0.3703	1.2195	3.9557	0.1255
0.609	20.90	0.6361	636.133	-19.094	-1.0376	0.4161	0.4606	0.1294
0.602	21.50	0.6185	618.521	-36.706	-2.1113	2.5481	17.2715	0.1333
0.617	22.12	0.6123	612.345	-42.882	-2.6065	2.1783	12.6214	0.1373
0.633	22.75	0.6151	615.148	-40.079	-2.5704	3.447	31.6051	0.1412
0.648	23.40	0.6086	608.628	-46.599	-3.1485	0.7726	1.5877	0.1451
0.664	24.07	0.6028	602.814	-52.413	-3.7255	4.3696	50.7884	0.149
0.695	24.76	0.6125	612.454	-42.773	-3.1942	3.9637	41.7915	0.1529
0.68	25.44	0.6129	612.895	-42.332	-3.317	1.7665	8.3003	0.1569
0.672	26.11	0.6066	606.636	-48.591	-3.9901	1.1991	3.8247	0.1608
0.664	26.78	0.6045	604.48	-50.747	-4.362	2.5124	16.7904	0.1647
0.664	27.44	0.6166	616.635	-38.592	-3.4684	2.9987	23.9191	0.1686
0.68	28.12	0.6293	629.259	-25.968	-2.4376	1.5315	6.2388	0.1725
0.664	28.78	0.6411	641.136	-14.091	-1.3802	0.359	0.3429	0.1765
0.695	29.48	0.6529	652.892	-2.3354	-0.2384	0.5375	0.7685	0.1804
0.727	30.21	0.6657	665.74	10.5126	1.1178	0.9589	2.446	0.1843
0.742	30.95	0.688	688.042	32.8148	3.6303	0.6486	1.119	0.1882
0.75	31.70	0.6874	687.412	32.1848	3.7015	0.3097	0.2552	0.1922
0.703	32.40	0.678	678.048	22.8207	2.7262	0.9932	2.6238	0.1961
0.711	33.11	0.6721	672.095	16.8683	2.0914	1.6559	7.2942	0.2
0.719	33.83	0.6661	666.072	10.8453	1.3945	0.6462	1.1108	0.2039
0.727	34.56	0.664	664	8.773	1.169	2.0704	11.4023	0.2078
0.688	35.25	0.6679	667.865	12.6376	1.7438	2.0887	11.6051	0.2118
0.711	35.96	0.6796	679.647	24.4196	3.487	0.7306	1.4199	0.2157
0.719	36.68	0.6683	668.313	13.0863	1.9325	0.7286	1.412	0.2196
0.727	37.40	0.6783	678.297	23.0705	3.5209	0.8743	2.0335	0.2235
0.711	38.11	0.7005	700.51	45.2827	7.1377	2.1593	12.4028	0.2275

0.703	38.82	0.7225	722.548	67.3213	10.953	1.5779	6.6225	0.2314
0.703	39.52	0.7351	735.066	79.8391	13.400	0.3524	0.3303	0.2353
0.703	40.22	0.7437	743.675	88.4477	15.305	0.6134	1.0007	0.2392
0.672	40.89	0.749	749.008	93.781	16.722	0.3011	0.2412	0.2431
0.688	41.58	0.7228	722.828	67.6012	12.414	0.3504	0.3265	0.2471
0.695	42.28	0.7053	705.284	50.0572	9.4622	0.5049	0.6782	0.251
0.68	42.96	0.7109	710.91	55.683	10.82	0.6268	1.045	0.2549
0.68	43.64	0.7165	716.482	61.2549	12.25	0.7196	1.3772	0.2588
0.672	44.31	0.722	722.015	66.7882	13.728	0.4573	0.5564	0.2627
0.695	45.00	0.7243	724.332	69.1049	14.592	0.3335	0.2959	0.2667
0.656	45.66	0.6959	695.948	40.7206	8.83	0.3705	0.3652	0.2706
0.648	46.31	0.6996	699.646	44.4186	9.8865	0.2198	0.1285	0.2745
0.633	46.94	0.7127	712.658	57.4309	13.114	0.4518	0.5429	0.2784
0.594	47.53	0.7182	718.221	62.9942	14.752	0.9968	2.643	0.2824
0.586	48.12	0.7237	723.732	68.5048	16.446	1.3322	4.7206	0.2863
0.578	48.70	0.7224	722.432	67.2048	16.533	1.1764	3.6815	0.2902
0.586	49.28	0.7112	711.18	55.9531	14.100	0.8028	1.7141	0.2941
0.602	49.89	0.7054	705.401	50.1742	12.946	0.9793	2.5509	0.298
0.586	50.47	0.703	703	47.773	12.617	0.3535	0.3323	0.302
0.586	51.06	0.703	703	47.773	12.909	1.0265	2.8029	0.3059
0.578	51.64	0.703	703	47.773	13.204	0.872	2.0224	0.3098
0.586	52.22	0.6853	685.332	30.1049	8.508	0.5441	0.7874	0.3137
0.578	52.80	0.6769	676.914	21.6872	6.2647	0.8355	1.8567	0.3176
0.586	53.39	0.6882	688.241	33.0144	9.7445	0.8262	1.8159	0.3216
0.594	53.98	0.6933	693.27	38.0432	11.469	0.7471	1.4848	0.3255
0.57	54.55	0.6878	687.754	32.5271	10.013	0.7951	1.6818	0.3294
0.555	55.11	0.68	680	24.773	7.7853	0.3741	0.3722	0.3333
0.562	55.67	0.68	680	24.773	7.9449	0.2221	0.1312	0.3373
0.578	56.25	0.6744	674.417	19.1897	6.2786	0.3951	0.4153	0.3412
0.594	56.84	0.6818	681.829	26.6018	8.8769	0.5601	0.8345	0.3451
0.609	57.45	0.6889	688.936	33.709	11.469	0.4881	0.6337	0.349
0.648	58.10	0.6592	659.21	3.9834	1.3815	0.456	0.5531	0.3529

0.688	58.78	0.6505	650.502	-4.7246	-1.669	0.3138	0.2619	0.3569
0.734	59.52	0.641	640.962	-14.265	-5.1359	0.2529	0.1702	0.3608
0.703	60.22	0.6222	622.214	-33.013	-12.105	0.7149	1.3594	0.3647
0.703	60.93	0.593	593.031	-62.196	-23.222	1.5564	6.4437	0.3686
0.68	61.61	0.5862	586.205	-69.022	-26.234	1.5901	6.7253	0.3725
0.648	62.25	0.5793	579.287	-75.94	-29.374	0.9601	2.4517	0.3765
0.656	62.91	0.5836	583.556	-71.671	-28.2	0.3966	0.4184	0.3804
0.688	63.60	0.5945	594.532	-60.695	-24.298	0.7842	1.6359	0.3843
0.688	64.28	0.596	596.021	-59.207	-24.1	1.0161	2.7461	0.3882
0.648	64.93	0.586	586	-69.227	-28.655	0.6744	1.2099	0.3922
0.648	65.58	0.5853	585.35	-69.878	-29.401	0.2148	0.1227	0.3961
0.656	66.24	0.5784	578.429	-76.798	-32.838	0.2214	0.1304	0.4

From the above calculations the power can be calculated:

VLF (0.003-0.04) =1840.6 ms^2

LF (0.04-0.15) = 980.9 ms^2

HF (0.15-0.4) = 321.79 ms^2

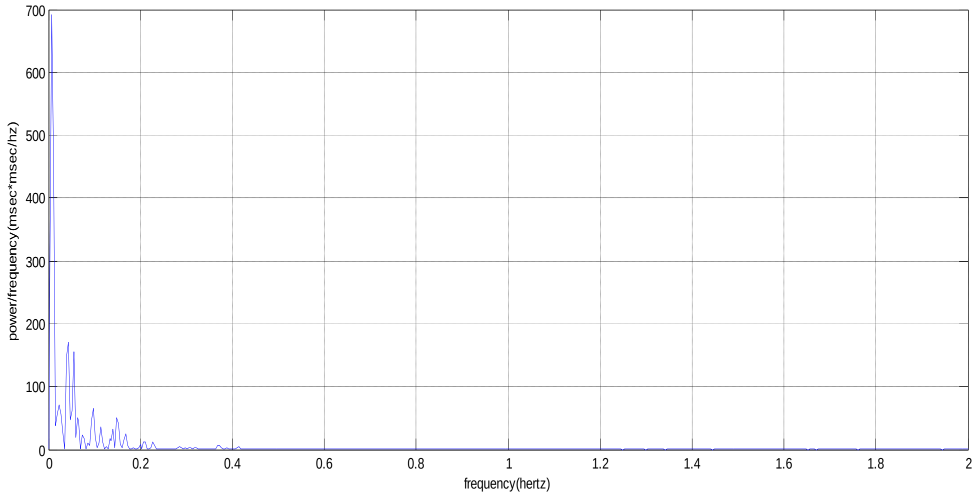


Figure 5.8: Power spectral density obtained by Fourier transform of RR interval time series (Fs=4hz)

Table 5.3: Representative table of first 350 RR intervals with interpolation(Fs=4 Hz) of 28 years old female(16273) calculating power in specific bands:

X	Y	Interpolation Y'	Y'* 1000	Normalization	Hann window	FFT	Square - Magnitude	Freq -uecy
0.609	0.61	0.609	609	-46.289	-0.0006	0.0819	0.0179	0
0.602	1.211	0.6061	606.1	-49.201	-0.0024	10.3376	284.263	0.0039
0.602	1.813	0.6032	603.2	-52.113	-0.0056	16.1208	691.2853	0.0078
0.609	2.422	0.602	602	-53.289	-0.0102	13.1931	462.9957	0.0117
0.609	3.032	0.602	602	-53.289	-0.0159	3.7299	37.0062	0.0157
0.617	3.649	0.6025	602.5	-52.749	-0.0227	4.5679	55.5032	0.0196
0.633	4.282	0.6054	605.4	-49.875	-0.0292	5.1956	71.8043	0.0235
0.656	4.938	0.6083	608.3	-47.002	-0.0359	4.6148	56.6486	0.0274
0.641	5.578	0.609	609	-46.289	-0.0448	3.1618	26.5916	0.0313
0.633	6.211	0.609	609	-46.289	-0.0553	0.4734	0.5961	0.0352
0.625	6.836	0.61	610	-45.278	-0.0654	7.4649	148.2289	0.0391
0.633	7.469	0.6133	613.3	-42.036	-0.0723	7.9933	169.9564	0.0431
0.617	8.086	0.6165	616.5	-38.795	-0.0783	4.1652	46.1485	0.047
0.625	8.711	0.6223	622.3	-32.956	-0.0771	4.8388	62.2805	0.0509
0.617	9.328	0.6287	628.7	-26.637	-0.0715	7.6326	154.9623	0.0548
0.609	9.938	0.6357	635.7	-19.554	-0.0597	2.7383	19.9448	0.0587
0.586	10.524	0.6445	644.5	-10.789	-0.0372	4.3839	51.1206	0.0626
0.586	11.11	0.6533	653.3	-2.0238	-0.0078	4.1108	44.9501	0.0665
0.586	11.696	0.652	652	-3.3203	-0.0143	0.3869	0.3982	0.0705
0.602	12.297	0.6461	646.1	-9.1797	-0.0438	2.9834	23.6761	0.0744
0.594	12.891	0.6406	640.6	-14.694	-0.0773	2.6235	18.3078	0.0783
0.578	13.469	0.6374	637.4	-17.853	-0.103	0.4883	0.6342	0.0822
0.586	14.055	0.6343	634.3	-21.013	-0.1325	1.9344	9.9531	0.0861
0.609	14.664	0.6311	631.1	-24.196	-0.1661	1.5825	6.6613	0.09
0.641	15.305	0.6279	627.9	-27.396	-0.204	4.2201	47.3733	0.0939
0.656	15.961	0.6253	625.3	-29.986	-0.2415	4.9856	66.1178	0.0978

0.648	16.61	0.6285	628.5	-26.826	-0.2329	2.6592	18.8096	0.1018
0.633	17.242	0.6316	631.6	-23.667	-0.2209	0.8327	1.8444	0.1057
0.609	17.852	0.6293	629.3	-25.945	-0.2598	2.0064	10.7081	0.1096
0.617	18.469	0.6229	622.9	-32.428	-0.3474	3.7211	36.8309	0.1135
0.609	19.078	0.6173	617.3	-37.982	-0.4343	2.1821	12.6662	0.1174
0.602	19.68	0.6205	620.5	-34.782	-0.4237	0.704	1.3183	0.1213
0.617	20.297	0.6237	623.7	-31.582	-0.409	1.2487	4.1478	0.1252
0.609	20.907	0.6231	623.1	-32.221	-0.4429	0.3875	0.3994	0.1292
0.602	21.508	0.6198	619.8	-35.463	-0.5164	2.5397	17.1576	0.1331
0.617	22.125	0.6166	616.6	-38.709	-0.5961	2.2128	13.0248	0.137
0.633	22.758	0.6133	613.3	-41.987	-0.6829	3.4686	32.0038	0.1409
0.648	23.407	0.61	610	-45.266	-0.7763	0.8436	1.8931	0.1448
0.664	24.071	0.6022	602.2	-53.04	-0.9578	4.3887	51.2343	0.1487
0.695	24.766	0.5924	592.4	-62.852	-1.1936	3.9525	41.5553	0.1526
0.68	25.446	0.586	586	-69.289	-1.382	1.7136	7.811	0.1566
0.672	26.118	0.586	586	-69.289	-1.4497	1.1327	3.4128	0.1605
0.664	26.782	0.586	586	-69.289	-1.5191	2.4915	16.5126	0.1644
0.664	27.446	0.586	586	-69.289	-1.59	3.0137	24.1586	0.1683
0.68	28.125	0.586	586	-69.289	-1.6625	1.5456	6.3545	0.1722
0.664	28.789	0.5904	590.4	-64.923	-1.6271	0.4051	0.4366	0.1761
0.695	29.485	0.597	597	-58.267	-1.5239	0.4647	0.5745	0.18
0.727	30.211	0.6012	601.2	-54.138	-1.4763	0.9124	2.2142	0.184
0.742	30.953	0.5978	597.8	-57.505	-1.6335	0.6384	1.0841	0.1879
0.75	31.703	0.5944	594.4	-60.872	-1.7997	0.2982	0.2366	0.1918
0.703	32.407	0.5879	587.9	-67.351	-2.0709	1.021	2.7727	0.1957
0.711	33.118	0.581	581	-74.272	-2.3731	1.6876	7.5756	0.1996
0.719	33.836	0.5799	579.9	-75.364	-2.5005	0.6545	1.1395	0.2035
0.727	34.563	0.5833	583.3	-71.951	-2.4771	2.075	11.4529	0.2074
0.688	35.25	0.5881	588.1	-67.212	-2.3994	2.0822	11.5328	0.2114
0.711	35.961	0.5975	597.5	-57.77	-2.1371	0.7114	1.3461	0.2153
0.719	36.68	0.607	607	-48.328	-1.8514	0.7464	1.4819	0.2192
0.727	37.407	0.6188	618.8	-36.504	-1.4473	0.8519	1.9306	0.2231

0.711	38.118	0.6313	631.3	-24.024	-0.9851	2.1394	12.1751	0.227
0.703	38.821	0.6423	642.3	-13.031	-0.5524	1.5565	6.4442	0.2309
0.703	39.524	0.648	648	-7.315	-0.3203	0.3213	0.2746	0.2348
0.703	40.227	0.6537	653.7	-1.5985	-0.0723	0.6147	1.0052	0.2387
0.672	40.899	0.6542	654.2	-1.1257	-0.0525	0.3019	0.2424	0.2427
0.688	41.586	0.6511	651.1	-4.2074	-0.2025	0.3382	0.3043	0.2466
0.695	42.282	0.648	648	-7.289	-0.3617	0.4787	0.6095	0.2505
0.68	42.961	0.6421	642.1	-13.223	-0.6761	0.5855	0.912	0.2544
0.68	43.641	0.6361	636.1	-19.156	-1.0089	0.694	1.281	0.2583
0.672	44.313	0.6284	628.4	-26.932	-1.4603	0.4339	0.5007	0.2622
0.695	45.008	0.6185	618.5	-36.768	-2.0516	0.3664	0.3571	0.2661
0.656	45.664	0.6091	609.1	-46.185	-2.6508	0.3874	0.3992	0.2701
0.648	46.313	0.6123	612.3	-42.944	-2.5343	0.2096	0.1169	0.274
0.633	46.946	0.6156	615.6	-39.702	-2.4081	0.4456	0.5282	0.2779
0.594	47.539	0.6151	615.1	-40.141	-2.5013	0.9495	2.3981	0.2818
0.586	48.125	0.6119	611.9	-43.425	-2.779	1.2949	4.4603	0.2857
0.578	48.703	0.6086	608.6	-46.661	-3.0655	1.1634	3.6005	0.2896
0.586	49.289	0.6057	605.7	-49.568	-3.3419	0.8016	1.7091	0.2935
0.602	49.891	0.6028	602.8	-52.475	-3.6294	0.9749	2.5284	0.2975
0.586	50.477	0.6064	606.4	-48.913	-3.4693	0.3936	0.4121	0.3014
0.586	51.063	0.6125	612.5	-42.835	-3.1146	1.0582	2.9785	0.3053
0.578	51.641	0.6162	616.2	-39.115	-2.9148	0.8857	2.0866	0.3092
0.586	52.227	0.6129	612.9	-42.394	-3.2365	0.5468	0.7954	0.3131
0.578	52.805	0.6096	609.6	-45.673	-3.5711	0.85	1.9221	0.317
0.586	53.391	0.6066	606.6	-48.653	-3.8949	0.8512	1.9273	0.3209
0.594	53.985	0.6037	603.7	-51.565	-4.2252	0.759	1.5323	0.3249
0.57	54.555	0.6045	604.5	-50.809	-4.2601	0.7954	1.6828	0.3288
0.555	55.11	0.6106	610.6	-44.732	-3.8367	0.3714	0.367	0.3327
0.562	55.672	0.6166	616.6	-38.654	-3.3906	0.2222	0.1314	0.3366
0.578	56.25	0.6229	622.9	-32.349	-2.9011	0.3635	0.3516	0.3405
0.594	56.844	0.6293	629.3	-26.03	-2.3861	0.5579	0.8279	0.3444
0.609	57.453	0.6354	635.4	-19.932	-1.867	0.4914	0.6422	0.3483

0.648	58.102	0.6411	641.1	-14.153	-1.3544	0.4551	0.551	0.3523
0.688	58.789	0.6469	646.9	-8.3753	-0.8186	0.3295	0.2887	0.3562
0.734	59.524	0.6529	652.9	-2.3975	-0.2393	0.247	0.1623	0.3601
0.703	60.227	0.6589	658.9	3.6266	0.3695	0.7493	1.4933	0.364
0.703	60.93	0.6657	665.7	10.450	1.0867	1.5877	6.705	0.3679
0.68	61.61	0.6769	676.9	21.601	2.292	1.5979	6.7919	0.3718
0.648	62.258	0.688	688	32.752	3.5451	0.9481	2.3912	0.3757
0.656	62.914	0.6929	692.9	37.637	4.155	0.3829	0.3899	0.3796
0.688	63.602	0.6874	687.4	32.122	3.6161	0.7852	1.6401	0.3836
0.688	64.289	0.6819	681.9	26.608	3.0537	1.0327	2.8366	0.3875
0.648	64.938	0.678	678	22.758	2.6622	0.717	1.3674	0.3914
0.648	65.586	0.6751	675.1	19.784	2.3582	0.2599	0.1796	0.3953
0.656	66.242	0.6721	672.1	16.806	2.0412	0.2412	0.1548	0.3992
0.648	66.891	0.6691	669.1	13.795	1.7068	0.4302	0.4922	0.4031

From the above calculations the power can be calculated:

VLF (0.003-0.04) =1834.92 ms^2

LF (0.04-0.15) = 987.62 ms^2

HF (0.15-0.4) = 322.19 ms^2

Although the shapes of graphs in all the three cases are same there is significant shift in the frequency axis of graph. Accordingly there is significant difference in powers associated with frequency bands. This can be easily visualized from calculations given above. We studied 34 different subjects and the variation of age is from 20 years to 76 years. The powers in all frequency bands of 34 persons can be calculated as mentioned above in an example of 28 years old female (16273).

CHAPTER 6

RESULTS AND DISCUSSION

One of the objectives of this was to make procedure for determining the Power Spectral Density of RR interval sequence to quantify Heart Rate Variability (HRV) into well defined and well documented frequency bands. We have chosen to represent the Power Spectral Density bands in ms^2/Hz. There are variety of other units reported by several researchers like db/Hz, radian/Hz etc. The second objective was to know the effect of interpolating unequally spaced samples into uniformly spaced signals as we have seen in figures 5.6, 5.7, 5.8. The general shape of this images are same, however significant difference in frequency shift. Therefore the power associated with each band also changed. Having observed this we wished to know whether the interpolation was to be applied only where there is large variation in sampling time. The table given below shows band power in VLF, LF, HF bands for 34 subjects.

Table 6.1: Age-wise calculations of power in VLF, LF, HF bands:

Database	Sex	Age	Frequency Bands	Without Inter polation Power(ms^2)	With Inter polation Power (ms^2) and Fs=2 Hz	With Inter polation Power (ms^2) and Fs= 4 Hz
16795	F	20	VLF	23852.6	26840.3	26905
			LF	7958.3	8009.6	8046
			HF	1893	1386.4	1407.3
113	F	24	VLF	2758.8	2955.64	2973.44
			LF	4373.54	3873.51	3887.32
			HF	8720	5611.88	5619.89
16773	M	26	VLF	24789	32251.8	32384
			LF	2682	3576.8	3590
			HF	631	665	673
16273	F	28	VLF	2439	1840.6	1834.92

			LF	1350.16	980.9	987.62
			HF	387.22	321.79	322.19
nsr050	M	29	VLF	621.57	521	518.89
			LF	2091.95	1715.99	1726.58
			HF	153.24	107.3	108.92
16786	F	32	VLF	6769.5	7419.5	7468
			LF	1748.8	1916	1918.1
			HF	1627	1455.8	1451
19093	M	34	VLF	3287.67	3637.87	3624.88
			LF	3692.6	3896.32	3904.85
			HF	1519.86	1316.82	1315.29
16539	F	35	VLF	3117.05	2094	2109.93
			LF	4264.01	2683.01	2670.52
			HF	803.96	452.66	457
16420	F	38	VLF	3859.45	2790.59	3551.36
			LF	1705.37	1223.54	1238.96
			HF	408.24	259.66	299
115	F	39	VLF	3084.9	3273.21	3307.42
			LF	4256	4372.73	4383.21
			HF	6001.2	3854.87	3830.01
nsr051	M	40	VLF	2278	2859.68	2881.75
			LF	5233.14	6172	6185.14
			HF	2034.68	2045	2034.32
16483	M	42	VLF	2130.55	1445.54	1446.64
			LF	1285.01	963.46	965.52
			HF	147	86.84	89.3
17052	F	45	VLF	11623.2	14437	14884
			LF	2238	1735.4	1729

			HF	506.3	385.6	393.6
14046	M	46	VLF	1832	1402	1405
			LF	2094	1561	1562
			HF	1186	706	709
119	M	51	VLF	1863.59	1763.75	1785
			LF	537.5	297.81	301.54
			HF	213	110	112.33
112	M	54	VLF	255.99	179.44	181.63
			LF	62	46.37	46.65
			HF	52.31	30.72	31.19
234	F	56	VLF	54.49	43.15	41
			LF	38.42	28.79	29.45
			HF	268.69	165.14	163.84
nsr020	F	58	VLF	4180	2936	2956.45
			LF	1661	1241	1152.5
			HF	200.41	131.57	129.86
205	M	59	VLF	68.41	52.52	52.31
			LF	51.39	38.6	38.19
			HF	159	97.01	96.58
nsr036	F	60	VLF	1240.89	1472	1472.68
			LF	1317	923	917.81
			HF	351	240.31	239.14
209	M	62	VLF	130.24	115.68	117
			LF	212.84	160.72	160
			HF	806.74	558	560.44
123	F	63	VLF	6051	4277	4276
			LF	24348	18828	18809
			HF	11942	5432	5380

nsr041	F	64	VLF	5811.7	4331	4325
			LF	3490	2545.33	2546.68
			HF	222.9	118.48	117.34
nsr014	F	65	VLF	2264.63	3281	370.22
			LF	1217.63	1407.09	1428.84
			HF	339	366.72	367.8
nsr043	M	66	VLF	23202.7	32587.7	32611.6
			LF	3080.3	4090.5	4101.5
			HF	559.3	565.9	568.4
201	M	68	VLF	448	462	471
			LF	15554	13650	13242
			HF	27575	16782	16257
116	M	68	VLF	236.3	269.68	270.29
			LF	72.06	98.74	97.89
			HF	153.2	152.44	151.87
117	M	69	VLF	1038	882	877
			LF	442	351	352
			HF	975	295	294
nsr030	F	70	VLF	552.4	842	844
			LF	343.65	400.91	402.68
			HF	88.38	91.17	92.82
nsr017	M	71	VLF	1194	1673	1663
			LF	836	1002	987
			HF	742	728	725.17
114	M	72	VLF	2335.6	2925.5	2935.4
			LF	4665.5	5895.9	6001
			HF	19775	11771	11905
nsr016	M	73	VLF	23848	28928	28977

			LF	489	1449	1459
			HF	469	393.9	397.9
nsr015	M	74	VLF	628.33	682	694.11
			LF	339	435.9	437.53
			HF	117	131.59	132.88
nsr007	M	76	VLF	16544	17676	17707
			LF	763.3	2556	2549
			HF	560.4	672	666

We have chooses the standard deviation as the parameter to represent the variation in data by we have formulated mathematical expression given below as parameter of difference occurring on application of interpolation. This parameter will be named as relative band power difference (RBPD).

$$\boldsymbol{\delta} = \sqrt{(\boldsymbol{P_{VL1}} - \boldsymbol{P_{VL2}})^2 + (\boldsymbol{P_{L1}} - \boldsymbol{P_{L2}})^2 + (\boldsymbol{P_{H1}} - \boldsymbol{P_{H2}})^2} / (\boldsymbol{P_{VL2}} + \boldsymbol{P_{L2}} + \boldsymbol{P_{H2}})$$

Here we have calculated the Relative Band Power Difference (δ_{W-I2}) for signal without interpolation and signal with interpolation having sampling frequency of 2 Hz. The results and scatter plot are given below:

Table6.2: Calculation of standard deviation and RBPD (δ_{W-I2}) in frequency bands:

Database	Age	Sex	Standard Deviation σ	Average (RR interval)	δ_{W-I2}
16795	20	F	0.097	0.893	0.0837
113	24	F	0.068	1.04	0.2536
16773	26	M	0.093	0.76	0.2059
16273	28	F	0.0417	0.65	0.2251
nsr050	29	M	0.032	0.52	0.1672
16786	32	F	0.057	0.8	0.0642
19093	34	M	0.06	0.888	0.0512

16539	35	F	0.0617	0.72	0.3663
16420	38	F	0.045	0.645	0.2767
115	39	M	0.065	0.95	0.1877
nsr051	40	M	0.076	0.8073	0.0997
16483	42	M	0.034	0.653	0.3045
17052	45	F	0.0811	0.77	0.1728
14046	46	M	0.045	0.62	0.2279
119	51	M	0.037	0.7	0.128
112	54	M	0.015	0.7	0.3166
234	56	F	0.013	0.64	0.441
nsr020	58	F	0.0405	0.652	0.305
205	59	M	0.011	0.66	0.3494
nsr036	60	F	0.029	0.584	0.1786
209	62	M	0.023	0.636	0.3047
123	63	F	0.116	1.2	0.3055
nsr041	64	F	0.0478	0.67	0.2515
nsr014	65	F	0.0373	0.75	0.2048
nsr043	66	M	0.105	0.85	0.2535
201	68	M	0.1508	0.6778	0.3548
116	68	M	0.015	0.759	0.0805
117	69	M	0.0308	1.191	0.4605
nsr030	70	F	0.0311	0.7415	0.2217
nsr017	71	F	0.048	0.799	0.149
114	72	M	0.099	1.098	0.394
nsr016	73	F	0.089	0.763	0.168
nsr015	74	M	0.031	0.704	0.089
nsr007	76	M	0.0711	0.722	0.101

Scatter plot for the above results are given below:

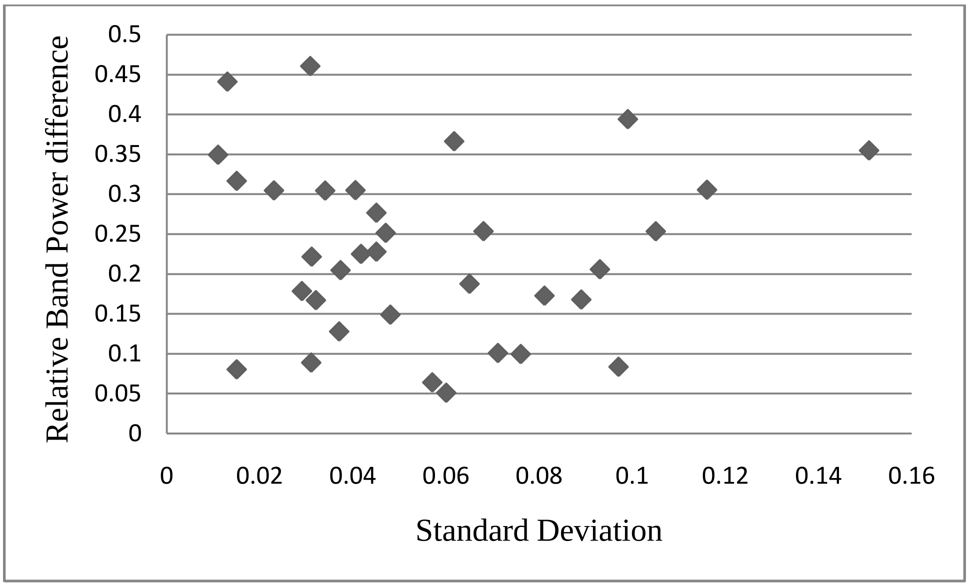


Figure 6.1: The variation of power Relative Band Power Difference (δ_{W-I2}) with the standard deviation

The average value of Relative Band Power Difference is **0.2277**. Directly from the scatter plots we observe that the two variables are independent and the change in one variable has no effect in other variable. Now the correlation between the power deviation and the standard deviation is calculated with the help of expression:

$$R=(N\sum xy-(\sum x)(\sum y))/\sqrt{[N\sum x^2-(\sum x)^2][N\sum y^2-(\sum y)^2]}$$

where,

N = number of persons

$\sum xy$ = sum of products of pairs

$\sum x$= sum of x

$\sum y$ = sum of y

$\sum x^2$ = sum of squared x

$\sum y^2$ = sum of squared y

And in the MATLAB the correlation can be easily find with the help of expression:

$$r=corr2(x, y)$$

where r is the correlated value.

The correlated value comes out **-0.0372**

Similarly, we can calculate the Relative Band Power Difference (δ_{I2-I4}) for signal with interpolation having sampling frequency of 2 Hz and 4 Hz. The results and scatter plot are given below:

Table 6.3: Calculation of standard deviation and RBPD (δ_{I2-I4}) in frequency bands:

Database	Age	Sex	Standard Deviation σ	Average (RR interval)	δ_{I2-I4}
16795	20	F	0.097	0.893	0.0021
113	24	F	0.068	1.04	0.0019
16773	26	M	0.093	0.76	0.0037
16273	28	F	0.0417	0.65	0.003
nsr050	29	M	0.032	0.52	0.0049
16786	32	F	0.057	0.8	0.0045
19093	34	M	0.06	0.888	0.0017
16539	35	F	0.0617	0.72	0.0039
16420	38	F	0.045	0.645	0.0498
115	39	M	0.065	0.95	0.0037
nsr051	40	M	0.076	0.8073	0.0025
16483	42	M	0.034	0.653	0.0015
17052	45	F	0.0811	0.77	0.0263
14046	46	M	0.045	0.62	0.0012
119	51	M	0.037	0.7	0.0102
112	54	M	0.015	0.7	0.0087
234	56	F	0..013	0.64	0.0129
nsr020	58	F	0.0405	0.652	0.0218
205	59	M	0.011	0.66	0.005
nsr036	60	F	0.029	0.584	0.0023
209	62	M	0.023	0.636	0.0034
123	63	F	0.116	1.2	0.0019

nsr041	64	F	0.0478	0.67	0.00088
nsr014	65	F	0.0373	0.75	0.0047
nsr043	66	M	0.105	0.85	0.00071
201	68	M	0.1508	0.6778	0.022
116	68	M	0.015	0.759	0.0033
117	69	M	0.0308	1.191	0.0034
nsr030	70	F	0.0311	0.7415	0.0022
nsr017	71	F	0.048	0.799	0.0054
114	72	M	0.099	1.098	0.0082
nsr016	73	F	0.089	0.763	0.0016
nsr015	74	M	0.031	0.704	0.0097
nsr007	76	M	0.0711	0.722	0.0015

Scatter plot for the above results are given below:

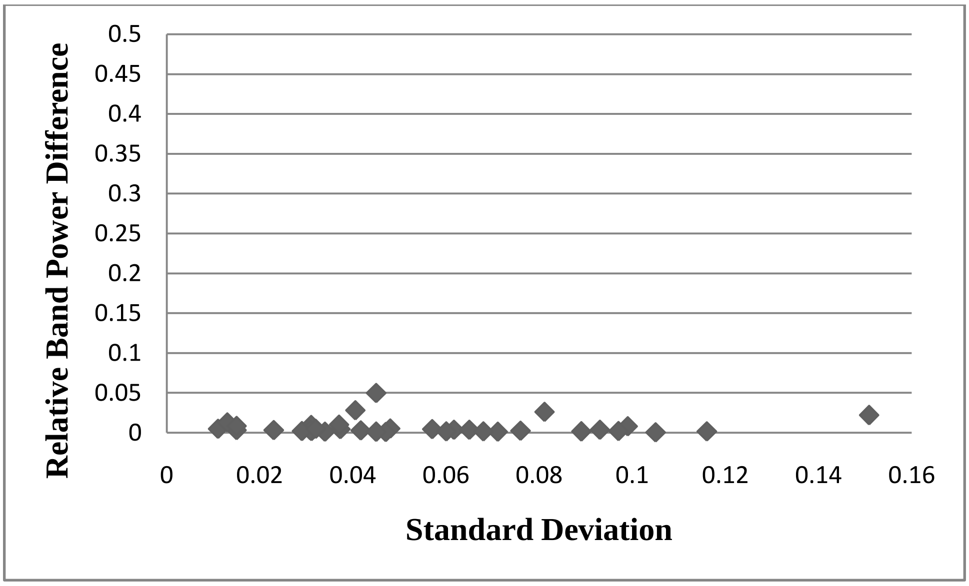


Figure 6.2: The variation of Relative Band Power Difference (δ_{I2-I4}) with the standard deviation

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The average value of Relative Band Power Difference is **0.0072**. Directly from the scatter plots we observe that the two variables are independent and the change in one variable has no effect in other variable. From the calculation the correlation between the Relative Band power Difference and the standard deviation comes out **0.0349.**

CHAPTER 7
CONCLUSION AND FUTURE SCOPE

7.1 CONCLUSION:

Having done with this exercise we wanted to know the appropriate procedure for the estimation of powers in different frequency bands, effect of interpolation and resampling frequency. Different researchers used different methods to calculate the powers in the frequency bands but in this present work we estimated the powers by squaring the Fourier transform of time series as it is most appropriate method in the field of cardiovascular signals. Effect of interpolation on the values of powers can be directly analysed from the scatter plot obtained above in figure 6.1. This plot shows that there is large difference in the values of power calculated without interpolation and with interpolation. We tried the above samples with the resampling frequency of 2 Hz and 4 Hz. The average value of Relative Band Power Difference is very low i.e.**0.0072**. So, we can safely compute that whether the resampling frequency is 2 Hz or 4 Hz, the different in the values of powers of Heart Rate Variability in specific frequency bands are minute. This result can directly analyse from the figure 6.2. Although the Relative Band Power Difference is low, out of courtesy we want to know about the correlation and the correlated value for the interpolated value with resampling frequency of 2 Hz and 4 Hz comes out **0.0349.** This correlated value is very low, so there is no correlation between the values of powers calculated in specific bands.

7.2 Future Scope:

In this present work, we have done the analysis with 34 subjects and for the short term recording of RR interval i.e. 5 minutes. The results obtained are satisfactory but the correlated values are not exactly zero. For the more precise results this work can be extended with more number of subjects.

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