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THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

THESIS
ON

”THE EFFECT OF GALAXY COMPONENTS IN THE LOPSIDEDNESS OF GALAXIES”

submitted in partial fulfilment of the requirement for the award of the degree of

Masters of science

in mathematics and computing

By

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under the supervision of

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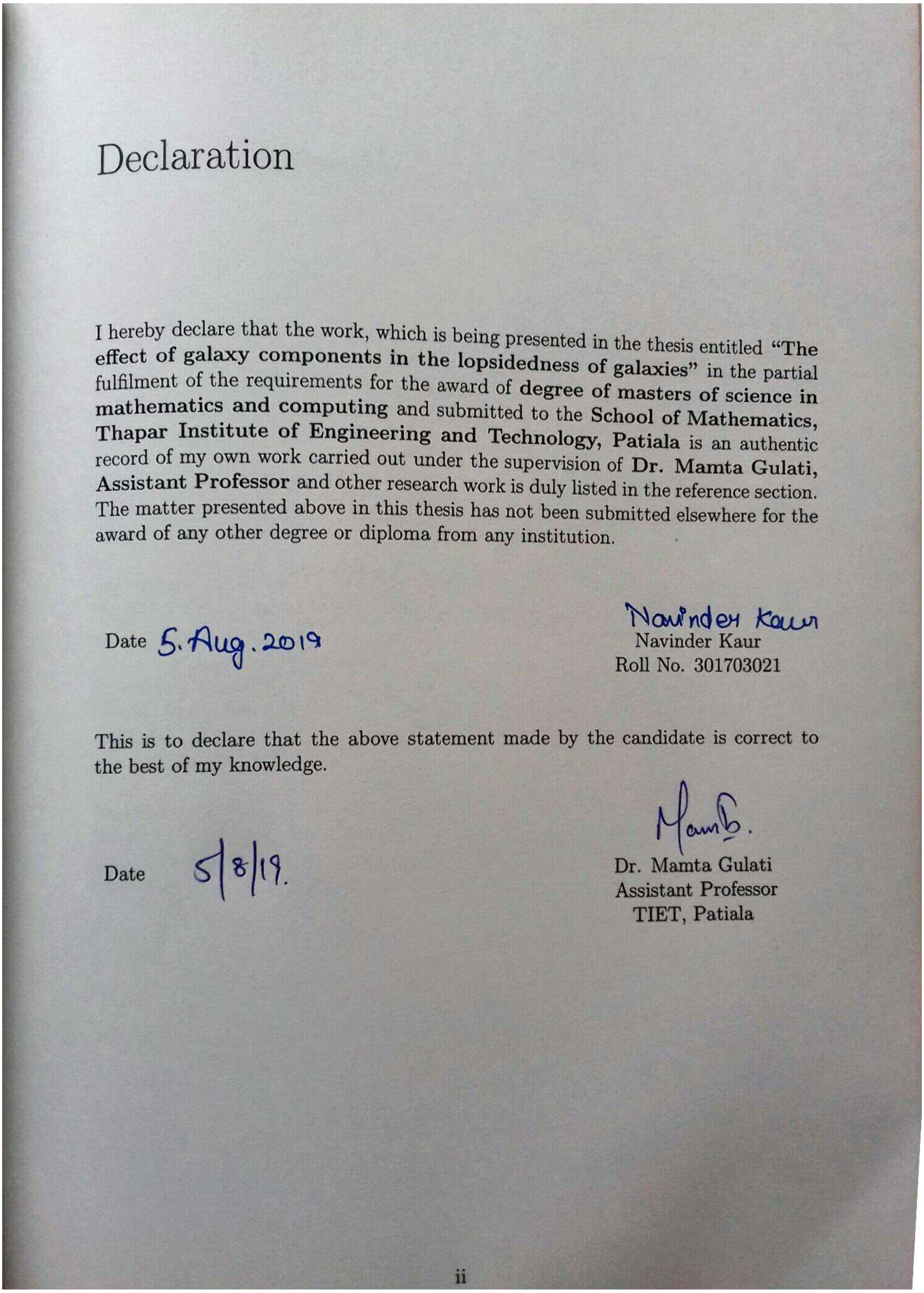
Dedicated to my parents.



Declaration

I hereby declare that the work, which is being presented in the thesis entitled **“The effect of galaxy components in the lopsidedness of galaxies”** in the partial fulfilment of the requirements for the award of **degree of masters of science in mathematics and computing** and submitted to the **School of Mathematics, Thapar Institute of Engineering and Technology, Patiala** is an authentic record of my own work carried out under the supervision of **Dr. Mamta Gulati, Assistant Professor** and other research work is duly listed in the reference section. The matter presented above in this thesis has not been submitted elsewhere for the award of any other degree or diploma from any institution.

Date 5. Aug. 2019

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This is to declare that the above statement made by the candidate is correct to the best of my knowledge.

Date 5/8/19.

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Everything is meaningless without paying regards to those who gave me such a beautiful life. Thank you Mom and Dad for your unconditional love, support, always being there for me, giving me confidence so that I can be myself. Thanks for encouraging me and inspiring me to achieve my goals.

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Abbreviations

i.e : that is
etc: et cetera
w.r.t : with respect to
BH : Black Hole
LHS: left hand side
RHS: right hand side
LSR: local standard of rest

Abstract

This thesis is an account of effect of components of the galaxies on the lopsidedness seen at the centers of the galaxies. First we discuss about galaxies and their types. We give an account of some basic terminology used in the thesis and then we discuss about Milky Way galaxy and its components. A brief account of the observations (lopsided stellar distribution and off-centered dark matter halo) at the center of the galaxies is also given. The stellar orbits are found to follow epicyclic orbits, which we also discuss in this thesis. The implications of off-centered dark matter halo on the orbits of stars is the deviation from circular orbits. The effect of dark matter halo is the off-centered distribution of the stars at the center of the galaxies. This model was first proposed by Prasad and Jog (2017), however their analysis does not have the central black hole in their galaxy model. We explicitly add the black hole in he disk and follow their calculations to give the orbits of the stars. The effect of presence of black hole can be seen in the radial and azimuthal velocities of the stars.

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Chapter 1

Introduction

This chapter serves as the introduction to the work done during the course of this thesis. We first give a brief introduction to galaxies and its types in section 1.1. We then discuss some basic definitions which we will be using in this thesis in section 1.2. A brief account of Milky Way galaxy is given in section 1.3 and in section 1.4 we discuss observations in galaxies that we wish to model. Lastly in section 1.5 we give the plan of the thesis.

1.1 Galaxies and their types

Galaxies are the massive system which are bound by their own gravity. Galaxies are of varying sizes from dwarf to the giant. Dwarf galaxies contains 10^8 stars and the giant galaxies contains 10^{14} stars. These are of varying shapes and sizes. Their diameter vary from 10^3 to 10^6 parsec. Most of the galaxies contain suppermassive black hole at their center. We also belong to one of such galaxy and the name of the galaxy to which we belong is the Milky Way galaxy. There are trillion of galaxies present in the universe and all of them are of different size and shape. Depending upon their shapes the galaxies were classified by Hubble [Binney & Tremaine(2008)] into the following four types

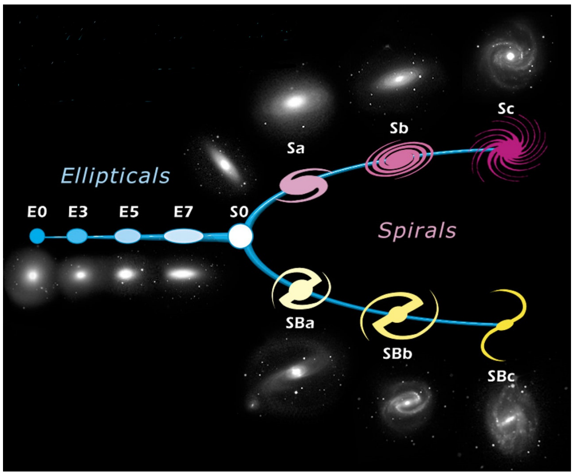

Figure 1.1: Hubble tuning fork diagram showing the different types of galaxies present in the universe. (Image credits: ESA/ Hubble Heritage)

- **Elliptical galaxies**
  As it is clear from the name "elliptical" these are elliptical in appearance. These are the galaxies with mostly old stars. They may vary in shape from totally round to round at the ends and flattened at bottom and top. These galaxies are featureless and smooth in appearance. These galaxies have low amount of gas or dust in them. Due to the less amount of gas and dust in these galaxies there is no recent star formation in these galaxies. One of the example we can see in figure 1.2 of the elliptical galaxy ESO 325-G004.

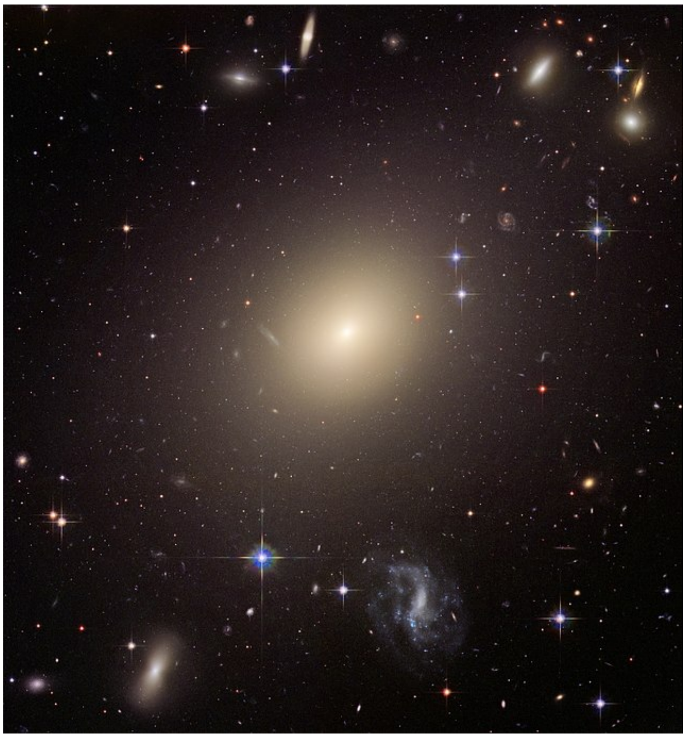

Figure 1.2: Image of elliptical galaxy ESO 325-G004. (Image credits: NASA, ESA and Hubble Heritage)

- **Spiral galaxies**
  However as name suggest spiral, these galaxies contain material distributed along spiral shaped structure. These are disc galaxies. They also contain dust and gas in them. Due to the presence of the dust and gas in these galaxies there is continuous star formation and the stars are formed in the spiral arms of the galaxy. The shape and size of these spiral arms vary from one galaxy to the other but these are always present in the galaxies. Most of the spiral galaxies also contains bulge. Spiral galaxies are again classified into two categories one is "normal" and another is "bared". In bared galaxies there is a bar present at the center and the spiral arms starts from the end points of the bar. Our own galaxy i.e. Milky Way is also a bared spiral galaxy. Example of spiral galaxy is given in figure 1.3 is the Pinwheel galaxy.
- **Lenticular galaxies**
  Lenticular galaxies have the property of both spiral and elliptical galaxies. They have central bulge or bar and disc like spiral galaxies. Like elliptical

Figure 1.3: Image of the Pinwheel galaxy. (Image credits: NASA, ESA and Hubble Heritage)

galaxies these galaxies are smooth, featureless in appearance and have no gas in them. There is no star formation due to lack of gas and dust in them. In figure 1.4 is a Spindle galaxy which is lenticular galaxy.

Figure 1.4: Image of Spindle galaxy. (Image credits: NASA, ESA and Hubble Heritage)

- **Irregular galaxies**
  There are minority of galaxies that had been put in the category of ”irregular galaxies” because these galaxies do not fit into any of the above mentioned category. These called irregular because these galaxies have no obvious shape. These galaxies include spiral galaxies and elliptical galaxies that has been gone

through collision with its nearby galaxies. For example in figure 1.5 we can see NGC 1427A galaxy which is irregular galaxy.

Figure 1.5: Image of irregular galaxies NGC 1427A. (Image credits: NASA, ESA and Hubble Heritage)

1.2 Some definitions

- **Rotational curve and Dark matter**
  It is plot of orbital velocity of visible stars/gas versus their radial distance from center of the galaxy. One such plot [Schweizer et al.(1983)] of galaxy Messier 33 is shown in figure 1.6. It is observed that the curve remain constant with the increasing distance. This contradicts the expected rotational curve in which the orbital velocity decreases with the increases in distance. As there is substaintial difference between expected and observed rotational curve this

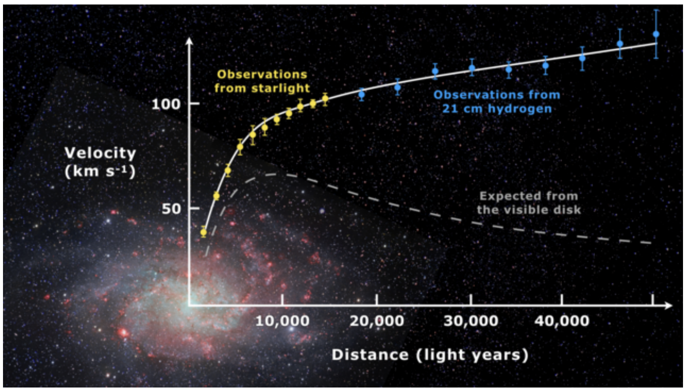

Figure 1.6: Curve of velocity of the visible gas versus the radial distance of galaxy Messier 33. (Image credits: [Schweizer et al.(1983)])

indicates existence of some additional invisible matter. We are able to see matter because of the light produced or reflected by it. This additional matter does nothing of this, which makes it unobservable by standard means. Hence it is named dark matter.

- **Black hole**
  Black hole is like a dead body of a dead star. When the massive star is at end of its life then the gravity crushes the core of the star, this leads to the massive explosion which is known as supernova which in turn leads to the formation of the black hole. These are called stellar mass black holes. We also observe black holes at the centers of galaxies which are suppermassive in nature. The outer boundary of the black hole is known as event horizon and its center is a singularity. The distance from center to the event horizon is known as Schwarzschild's radius.
  The black holes themselves are not visible. However in black hole large mass is concentrated into a small region giving rise to high gravity regions which makes the nearby matter to revolve around it at large velocities. The matter revolving around the black hole get heated up and starts glowing which makes it possible for us to see the black hole. The presence of black hole is indicated by its intraction with electromagnetic radiations such as visible light. Event Horizon Telescope which is the network of the radio telescopes, captured the first image of the supermassive black hole on 10 April 2019 which is at the center of supergiant elliptical galaxy Messier 87, as shown in figure 1.7. The mass of the black hole is ∼7 billion solar masses.

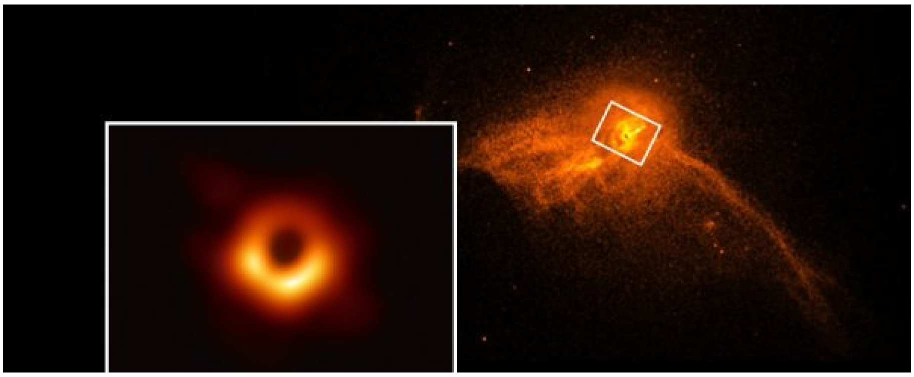

Figure 1.7: Image of the black hole observed at the center of elliptical galaxy M87 by Event Horizon telescope and the glowing outer boundary is formed by the hot gas revolving around the black hole. (Image credit: [Event Horizon Telescope Collaboration et al.(2019)])

1.3 Milky Way

Milky Way galaxy which is also known as the Galaxy, is a huge system with 200 billion stars. As Galaxy is a huge system so there must be force which holds it together, gravity is that force. Milky Way is a spiral galaxy and we are able to say this by looking at the pictures which are taken from the darkest places of the earth.

In these pictures we can see bands of dust and gas spread in the night sky, as we can see in figure 1.8. It has two major spiral arms and three small arms. From one end to the other it measures 6,000 trillion miles. There is a high amount of gas and dust present in these spiral arms which is the raw material for the formation of the new stars. The new stars are formed in cold clouds of gas and dust, the gravity pulls the gas and dust inwards and this is the beginning of the formation of new stars.

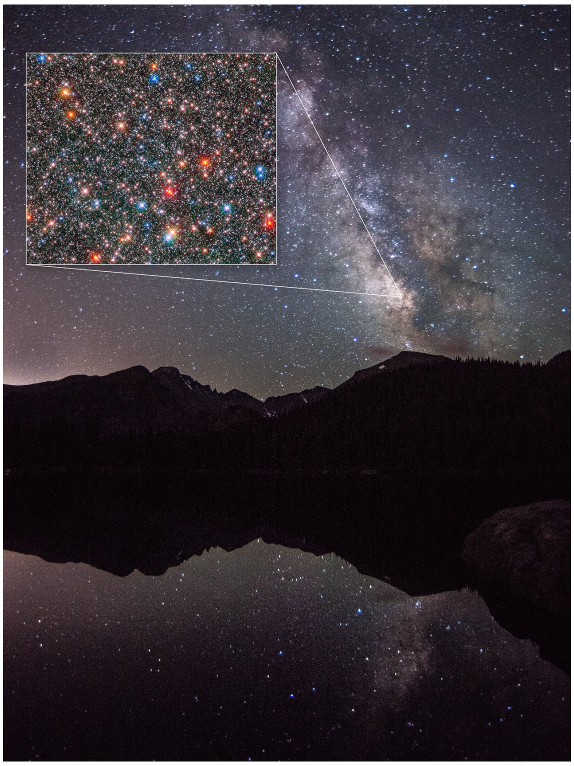

Figure 1.8: Image of Milky Way which is seen in night sky above the silhouetted Rocky Mountains. (Image credit: ESA/Hubble)

1.3.1 Components of Milky Way

Below given a brief account of components of Milky Way galaxy [Choudhuri(2010)]

- One of the main component of the galaxy is galactic disk which is flat and axisymmertic structure and stars lie in this galactic disk.
- There is also a centrally located bulge present in the galaxy which we can clearly see in infrared observations. There are also stars present in the bulge which are believed to be formed at the time of formation of the galaxy. Stars of the bulge are symmetrically distributed about the galactic mid plane.
- In the center of the Milky Way there is a giant black hole which is named Sagittarius A^*.
- Another component of the Galaxy is halo. It is believed that the stars of the halo are the first component that are formed in the galaxy due to their low metallicity.

- There is another important component of the galaxy known as Cluster. These are of two types : Open clusters and Globular clusters. Open clusters are of irregular shape and they contain 10^2 to 10^4 stars in them. Not all of the stars are in the galactic disk many of the stars are in the Globular clusters. Globular clusters are the old clusters and they contain 10^4 to 10^6 stars in them. In our galaxy there are about 150 globular cluster.

1.4 Observations of centers of galaxies

Here we will discuss some features which are observed at the center of the galaxies. Due to the limited resolution of the telescopes we are not able to make observations at the center of many galaxies. But their are two galaxies named NGC 4486B and M31, in whose central observations double peak nuclei are observed which can be seen in figure 1.10 and figure 1.9 respectively.

Double peak observed in these galaxies are different from each other. The double peak observed in NGC 4486B has 5 - 7 times larger separation than the double peak observed in M31. Distance of peaks in NGC 4486B is same from the center whereas in M31 the distance of peaks is not same from the center. In NGC 4486B peaks have almost same central surface brightness which is not in M31. This section is referred from [Amanpreet, 2018]

There are evidence which shows that the central dark matter halo is off-centered in galaxies w.r.t. the disc center. This is seen in simulations [Kuhlen et al.(2013)]. This off-centered dark matter halo was also observed in galaxy M94 [Chemin et al.(2016)] and also in galaxy cluster Abell 3827 [Massey et al.(2015)].

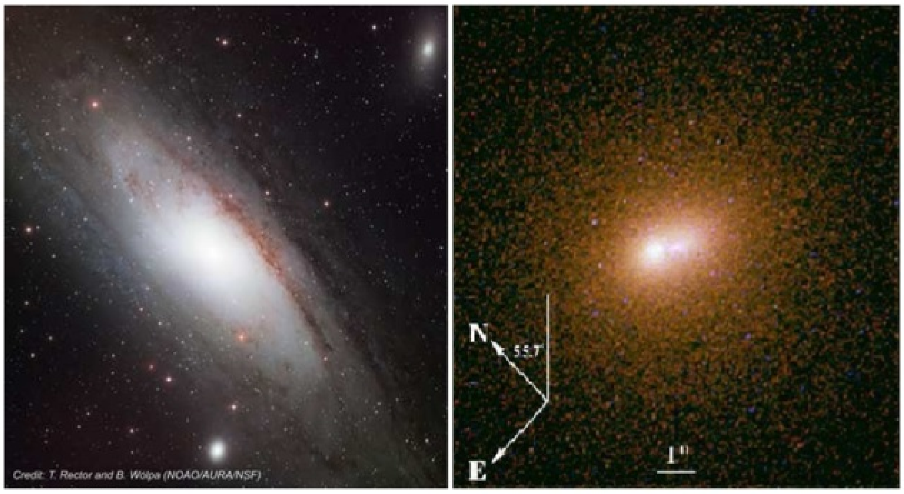

Figure 1.9: Left side image is the full image of the M31 (Image credit: T.Rector and B. Wolpa (NOAO/AURO/NSF) and right side image shows the central few arcsec, we can clearly see the double peak in RHS image

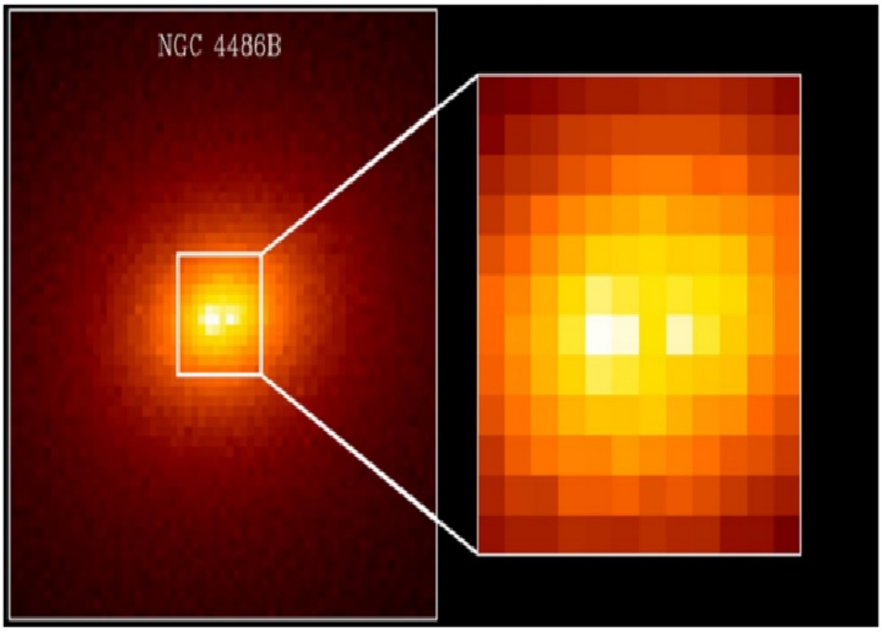

Figure 1.10: Clear image of two peaks which are observed at the center of NGC 4486B. (Image credit: Karl Gebhardt, Tod Lauer and NASA)

1.5 Plan of Thesis

In present chapter we discussed some basic terminologies that we shall be using in the thesis. We also discussed about galaxies, their types and the observations of the center of galaxies. The plan of rest of the thesis is as below

- In **chapter 2**, we discuss about orbits of stars, Epicycle theory and then we give the derivation of orbits of stars and also discuss about Poisson's equation.
- In **chapter 3**, we discuss the implications off-centered distribution of dark matter halo on the orbits of stars. The calculation follows work by Prasad and Jog [2017]. However they have not included the central black hole in their calculations, which is an essential part while discussing the dynamics of the center of galaxies.
- In **chapter 4**, we shall give some concluding remarks along with limitations of the analysis.

Chapter 2

Orbits of stars

2.1 Epicycle theory

The stars appear to be in circular orbits in the galactic disk of the galaxies but in reality it is not so, stars do not have the circular orbits. Like planets have elliptical orbits similarly we observe that the stars also do not move in exact circular orbit as mass of a galaxy is distributed all over it. The epicycle theory helps us to know about the orbits of the stars. This theory tell us that the stars moves in ellipse w.r.t. local standard of rest (LSR). LSR is the circular radius of star approximated using Newton's laws for mass in the interior of the disk w.r.t. the position of star. LSR move around the galactic center. This motion of the stars is known as epicycles i.e. the elliptical path of the stars w.r.t. the LSR is known is epicycle where LSR is the frame of reference which moves in circular orbit around the center of galaxy. A schematic of epicyclic motion of planet around earth is shown in figure 2.1. This section is referred from [Choudhuri(2010)]

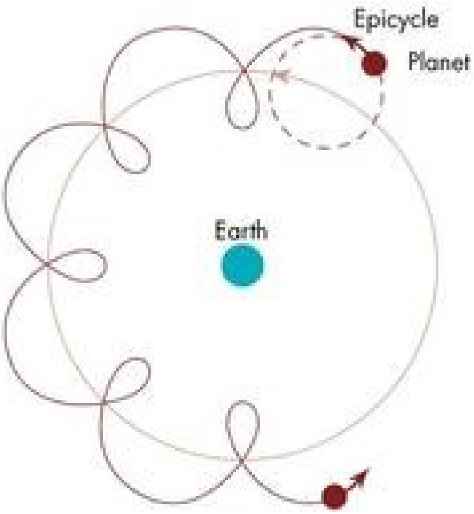

Figure 2.1: Epicyclic motion of planet around the earth

2.2 Derivation of orbits

Assume the star at a distance R_0 from center of galaxy which is moving with angular speed Θ_0. To measure deviations from circular orbits we suppose that star given a sudden small kick in radial direction. Then its motion is given by the below equations

$$r'' - r\theta'^2 = f_r \tag{2.1}$$

$$r^2\theta' = constant \tag{2.2}$$

Here (′) gives derivative w.r.t. time.
Since $\Theta = r\theta'$, where Θ is a speed in θ direction and using

$$f_r = -\frac{\Theta_{circ}^2}{r}$$

and noting that angular momentum of the star did not change with the kick and remains $R_0\theta_0$, as we gave only radial kick, we get

$$r'' = \frac{\Theta^2}{r} - \frac{\Theta_{circ}^2}{r} \tag{2.3}$$

$$r\Theta = R_0\Theta_0 \tag{2.4}$$

Take

$$r = R_0\left(1 + \frac{\xi}{R_0}\right) \tag{2.5}$$

Assuming $\xi \ll R_0$ and keeping the terms linear in ξ, we get

$$\frac{\Theta^2}{r} \approx \frac{\Theta_0^2}{R_0}\left(1 - \frac{3\xi}{R_0}\right) \tag{2.6}$$

and using equation (2.5) and $\Theta_{circ}(r) \approx \Theta_0 - (A+B)\xi$, we get

$$\frac{\Theta_{circ}^2}{r} \approx \frac{\Theta_0^2}{R_0}\left[1 - \frac{2(A+B)}{\Theta_0}\xi - \frac{\xi}{R_0}\right] \tag{2.7}$$

where A and B are the Oort's constants given by

$$A = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] \tag{2.8}$$

$$B = -\frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right] \tag{2.9}$$

R is the distance between star and the galactic center and Θ is the circular speed. [Choudhuri(2010)]
Noting $r'' = \xi''$ and using equation (2.6) , (2.7) and $\frac{\Theta_0}{R_0} = A - B$ in equation (2.3), we get

$$\xi'' + \kappa^2\xi = 0 \tag{2.10}$$

where

$$\kappa = \sqrt{-4B(A-B)}$$

As B is negative and (A-B) is positive, so κ is real. Hence stars moves in simple harmonic motion (SHM) in radial direction w.r.t. circular orbit $r = R_0$. Then the radial velocity $\Pi = r^{'}$ w.r.t. the LSR and is given by

$$\Pi = \Pi_0 \cos \kappa t \tag{2.11}$$

and displacement is given by

$$\xi = \frac{\Pi_0}{\kappa} \sin \kappa t \tag{2.12}$$

Linear velocity is given as

$$\Delta\Theta = -\frac{2\Pi_0\Theta_0}{\kappa R_0} \sin \kappa t \tag{2.13}$$

And the displacement is given as

$$\eta = \frac{2\Pi_0\Theta_0}{\kappa^2 R_0} \cos \kappa t$$

We get

$$\eta = \frac{\Pi_0}{-2B} \cos \kappa t \tag{2.14}$$

by using

$$\frac{\Theta_0}{k^2 R_0} = \frac{1}{-4B}$$

equation (2.12) and equation (2.14) shows that the stars moves in an ellipse w.r.t. LSR and LSR move around the galactic center in circular orbit.

2.3 Poisson's equation

Poisson equation relates the system's mass density to the gravitational potential of the mass distribution in self gravitating systems [Gulati, 2014]. If f is the force acting on a particle due gravitational pull and mass density is given by ρ then f can be written in terms of $\nabla\phi$, where ϕ is the gravitational potential. Gravitational potential is related with density by Poisson equation given below

$$\nabla^2\phi = 4\pi G\rho \tag{2.15}$$

2.4 Conclusion

In this chapter we discussed about the circular orbits and how the stars move in the epicyclic orbits. Then we discribed mathematically the epicycle theory for the motion of stars we then discussed about Poisson's equation which relates density with potential.

Chapter 3

Off-centered distribution of dark matter halo at the center of galaxies

3.1 Introduction

Centers of many galaxies are observed to be lopsided. There are various models invoked to explain such observations at centers of galaxies as discussed in section 1.4. Prasad and Jog [2017] linked the observations of off-centered distribution of dark matter halo to the lopsided distribution of stars at the centers. But in their calculations they did not included the central black hole which is the important component of the galaxies. So we are repeating the steps of their calculation by adding the central black hole to the system.

3.2 Potential due to off-centered halo and disc

Potential due to the off-centered halo and the disc at point A figure 3.1 can be calculated in the terms of disc and halo contributions w.r.t. the center of the disc and additional perturbations. The halo center and the disc center are denoted by C_H and C_D and the offset between the disc center and halo center is denoted by d where $d \ll R$. R_C and R_D are halo core radius and the exponential disc scale length respectively, whereas ϕ is the angle between the radius vector from the center of the disc to the A point and the direction between the halo center and disc center.

Density distribution for pseudo isothermal halo and an exponential disc are as follows

$$\rho(r) = \frac{\rho_0}{1 + \frac{r^2}{R_C^2}} \tag{3.1}$$

where r is the spherical radius of halo.

$$\Sigma(R) = \Sigma_0 \exp\left(-\frac{R}{R_D}\right) \tag{3.2}$$

where R is the distance between disc center and point A.
Potential at A point in the mid plane because of the disc is given as follows

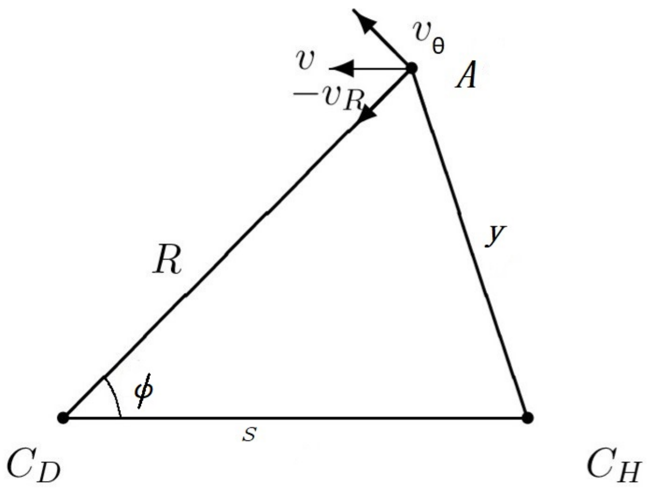

Figure 3.1: Diagram showing center of disk and off-centered halo separated by distance s

[Binney & Tremaine(2008)]

$$\Phi_D = -\pi G \Sigma_0 R[I_0(z)K_1(z) - I_1(z)K_0(z)] \tag{3.3}$$

where $z = R/2R_D$. This is derived by using Poisson's equation given in equation (2.15)
The halo potential at point A when halo is concentric with disc is given by

$$\Phi_H(s=0) = 4\pi G \rho_0 R_C^2 \left[\frac{\log(R_C^2+R^2)}{2} + \frac{R_C}{R}\arctan\left(\frac{R}{R_C}\right) - 1\right] \tag{3.4}$$

Halo potential at A point when halo is off-centered is given below

$$\Phi_H = 4\pi G \rho_0 R_C^2 \Big[\frac{\log(R_C^2+y^2)}{2} + \frac{R_C}{y}\arctan\left(\frac{y}{R_C}\right) - 1] \tag{3.5}$$

Considering for small values of s(≪R), so that

$$y^2 \approx R^2\left(1 - \frac{2s\cos\phi}{R}\right) \tag{3.6}$$

After simplifying other terms, we get

$$\log(R_C^2+y^2) \approx log(R_C^2+R^2) - \frac{2sR\cos\phi}{R_C^2+R^2} \tag{3.7}$$

$$\frac{R_C}{y}\arctan\left(\frac{y}{R_C}\right) \approx \frac{R_C}{R}\arctan\left(\frac{R}{R_C}\right) + \frac{R_C s\cos\phi}{R^2}\arctan\left(\frac{R}{R_C}\right) \tag{3.8}$$

Therefore, total halo potential Φ_H is given as below

$$\Phi_H = \Phi_H(s=0) + 4\pi G \rho_0 R_C^2 s\cos\phi\left[\frac{R_C}{R^2}\arctan\left(\frac{R}{R_C}\right) - \frac{R}{R_C^2+R^2}\right] \tag{3.9}$$

where second terms is the perturbation term Φ_{pert}. Therefore, the total potential is given by,

$$\Phi = \Phi_D + \Phi_H(s = 0) + \Phi_{pert} \tag{3.10}$$

where the first and second term on the r.h.s. gives the potential due to disc and halo with concentric center respectively and the third term gives the perturbation potential due to the offset.

3.3 Perturbation potential with a lopsided form

The perturbation potential is given as

$$\Phi_{pert}(R, \phi) = \Psi(R) \cos\phi \tag{3.11}$$

where

$$\Psi(R) = 4\pi G\rho_0 R_C^2 s \left[\frac{R_C \arctan\left(\frac{R}{R_C}\right)}{R^2} - \frac{R}{R_C^2 + R^2}\right] \tag{3.12}$$

The dimensionless perturbation potential ϵ_{pert} is given below

$$\epsilon_{pert}(R) = \frac{\Psi(R)}{(\Phi_D + \Phi_H)} \tag{3.13}$$

To obtain perturbation in R we use following equation

$$\delta R = -\frac{s\cos\phi_0}{\kappa^2 - \Omega^2} \frac{4\pi G\rho_0 R_C^2}{R_C^2 + R_0^2} \left(\frac{R_C^2}{R_0^2} + \frac{2R_0^2}{R_C^2 + R_0^2} - 3\right) \tag{3.14}$$

If v_ϕ denotes azimuthal velocity and perturbations in it is given by,

$$\begin{aligned}\delta v_\phi = -4\pi G\rho_0 R_C^2 s \frac{\cos\phi_0}{\Omega} \left[\frac{R_C \arctan\left(\frac{R_0}{R_C}\right)}{R_0^3} - \frac{1}{R_0^2 + R_C^2}\right] + 4\pi G\rho_0 R_C^2 s \\ \times \frac{\cos\phi_0 \Omega}{\kappa^2 - \Omega^2} \frac{1}{R_0^2 + R_C^2} \left(\frac{R_C^2}{R_0^2} + \frac{2R_0^2}{R_0^2 + R_C^2} - 3\right)\end{aligned} \tag{3.15}$$

v_R is the radial velocity and perturbation in v_R is given by

$$v_R = \frac{\sin\phi_0}{\kappa^2 - \Omega^2} \frac{4\pi G\Omega\rho_0 R_C^2 s}{R_C^2 + R_0^2} \left(\frac{R_C^2}{R_0^2} + \frac{2R_0^2}{R_C^2 + R_0^2} - 3\right) \tag{3.16}$$

where κ and Ω are the radial and azimuthal frequencies respectively [Tremaine(2001)] given by the following equations

$$\kappa^2 = \frac{GM}{R^3} + \frac{d^2}{dR^2}(\Phi_D) + \frac{3}{R}\frac{d}{dR}(\Phi_D) \tag{3.17}$$

$$\Omega^2 = \frac{GM}{R^3} + \frac{1}{R}\frac{d}{dR}(\Phi_D) \tag{3.18}$$

3.4 Results

3.4.1 Parameters

For plotting the orbits, we take the core radius $R_C = 5$ kpc, exponential disc scale length $R_D = 3.2$ kpc, halo central density $\rho_0 = 0.035\ pc^{-3}$ M$_\odot$, s=350 pc and $\Sigma_0 = 640.9\ pc^{-2}$ M$_\odot$. These values are taken from Galaxy mass model given by [Mera et al.(1998)].

3.4.2 Resulting lopsidedness

In figure 3.2 we plot ϵ_{pert} given by equation (3.13) in which all the components are discussed in previous section for these parameters. Their is a significant amount of perturbation and the perturbation is maximum near the center and decreases as we move away from the center. Figure 3.3 is the plot R versus ϕ at one particular orbit. As we start from $\phi = 0°$, at R=1.5 kpc as radius is higher the orbit is shifted towards the higher radius and as we move back the ϕ goes 180° and the orbit is shifted towards the center so their is the displacement in the orbit of the star which can be clearly seen in the next figure given by figure 3.5, which is the plot in R and ϕ plane. Then the next plot figure 3.4 is plotted for different R and form this we can clearly see that with increasing R the lopsidedness decreases. These plots are similar to the plots of Prasad and Jog [2017] apart from the factor of 2 which is coming in the perturbation and which is carried forward into the orbits. Next we will plot the azimuthal velocity and radial velocity, which are given by figure 3.6 and figure 3.7 respectively. The azimuthal velocity is given by $R\Omega(R) - \delta v_\phi$, where δv_ϕ is given by equation (3.15) and the radial velocity is given by equation (3.16), which is direct radial velocity. These plots differ from the plots given in Prasad and Jog and the possible reason for this is the presence of the central black hole.

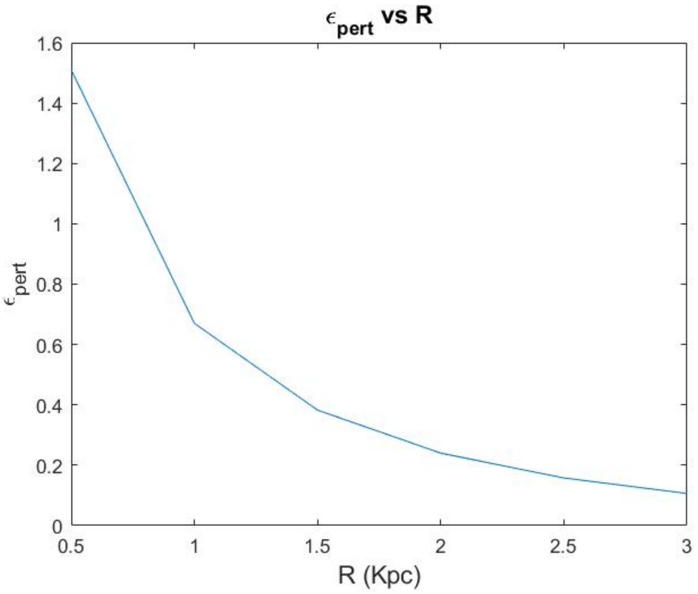

Figure 3.2: Plot of perturbation potential ϵ_{pert} versus the radius R for y=350 pc

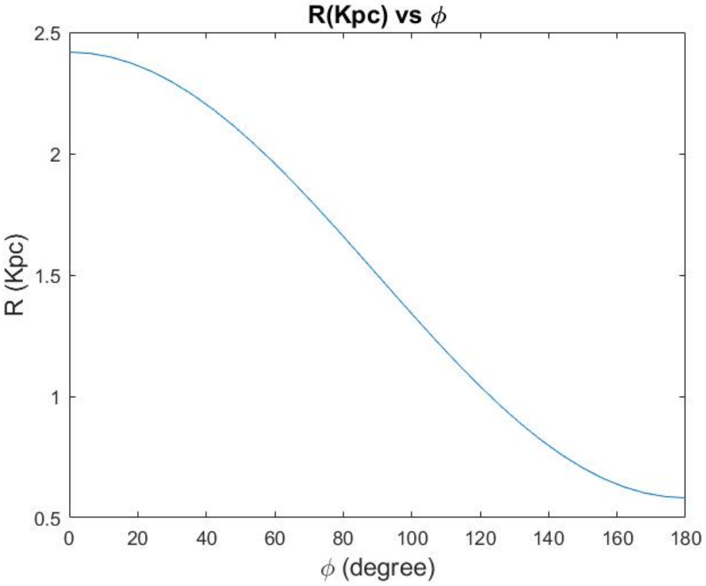

Figure 3.3: Plot of the R vs ϕ at initial R=1.5 kpc

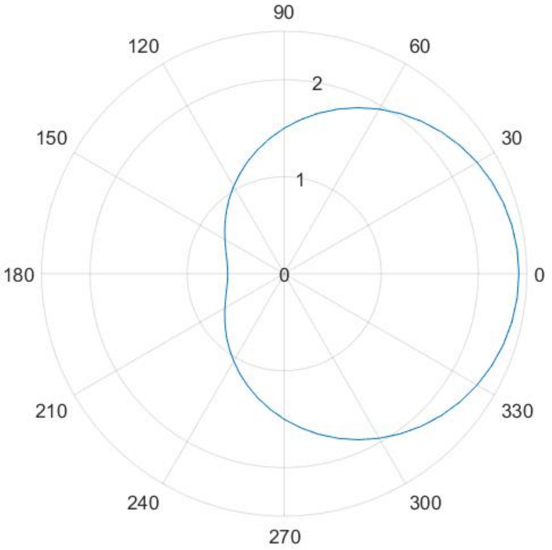

Figure 3.4: Polar plot of R versus ϕ at initial circular orbit R=1.5 kpc and we can see the orbit lopsided, it is short along $\phi = 180°$ and elongated along $\phi = 0°$

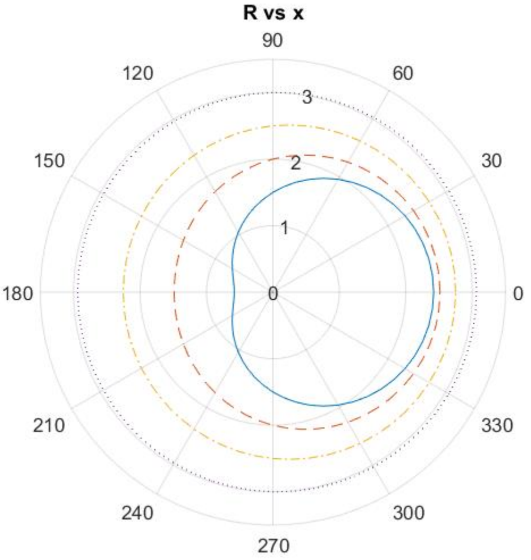

Figure 3.5: Polar plot for different values of R vs ϕ, we plotted this graph for R=1.5, 2, 2.5, 3 kpc and the corresponding perturbed orbits are shown by solid, dashed, dashed-dotted and dotted orbit respectively and we can see it is lopsided more at smaller R and lopsidedness decreases with increase in R

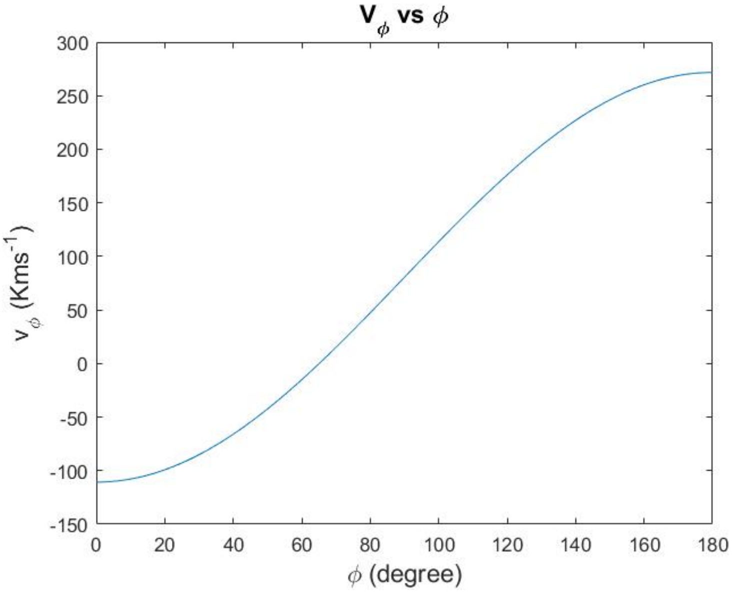

Figure 3.6: Plot of azimuthal velocity v_ϕ vs ϕ

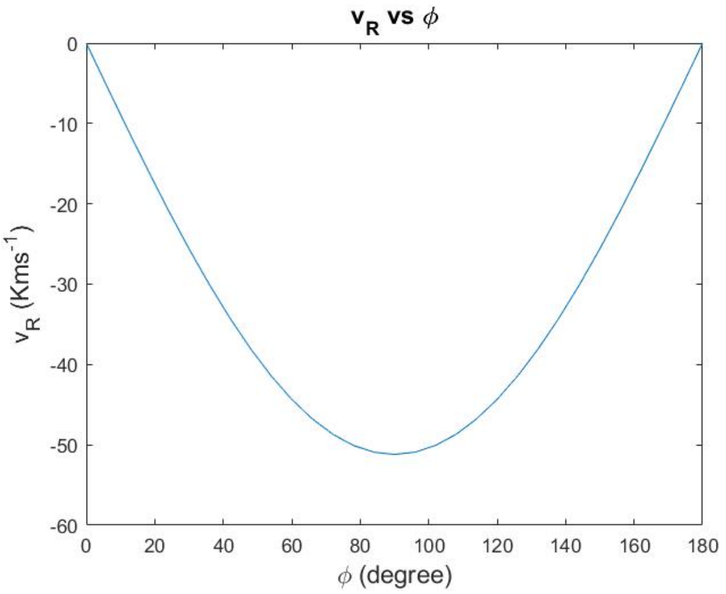

Figure 3.7: Plot of radial velocity v_R vs ϕ

3.5 Conclusion

In this chapter we calculated the orbits of stars perturbed due to the off-centered dark matter halo. The calculation follows Prasad and Jog [2017]. However the authors have not added the central black hole explicitly in their work, which is an integral part of system when studying the dynamics of centers of galaxies. Our calculation takes into account the effect of central black hole into the system. We observe that there is no qualitative change in the the orbit by adding the central black hole as we can see in figure 3.3 because the black hole potential cancel out in the form $\kappa^2 - \Omega^2$ which we can see from equation of κ and Ω given above, so we have no effect of central black hole in (3.14). But there are some change in the azimuthal velocity v_ϕ and radial velocity v_R as seen in figure 3.6 and figure 3.7 respectively. In figure 3.4 we can see that the perturbation decreases in the outer regions of galaxies where there is large perturbations in the inner region.

Chapter 4

Conclusion

In this chapter we offer some concluding remarks and future directions of the work carried out in this thesis. Following are the conclusions from this thesis

- In first chapter, we discussed about galaxies and the off-centered feature found in them. Then we did overview of types of galaxies and then discussed about rotational curve, dark matter and black hole. After that we discussed about Milky Way and also about its components.
- In chapter second we discussed about epicyclic theory in which we come to know why stars do not have circular or elliptical orbits, instead they have epicyclic orbits. After that we derived the orbits of stars in epicyclic approximation.
- In the third chapter, we calculate the stellar orbits perturbed by the presence of off-centered dark matter halo. We plotted the graphs for the perturbation potential, perturbation in R, azimuthal velocity v_ϕ and radial velocity v_R. The calculation largely follow Prasad and Jog [2017]. The orbits seems to give a lopsided distribution of stars in the disc. The major difference in our and Prasad and Jog [2017] work is that we have explicitly added a black hole at the center of the galaxy disc. The effect of black hole is seen in the radial and azimuthal velocity as discussed in section 3.4. Though this model explains the lopsided stellar distribution in disc galaxies we also see double peak stellar distribution in elliptical galaxies given in section 1.4. In this thesis we model disc galaxy but this offset is seen in the elliptical galaxies which are different from disc galaxies. In future, by working on the model of elliptical galaxies we can know the reason of this offset which is seen in elliptical galaxies.

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