Stabilization of Mobile Inverted Pendulum System Using Conventional and Fuzzy PID Controllers

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CERTIFICATE

Certified that the dissertation entitled, "**Stabilization of mobile inverted pendulum system using Conventional and Fuzzy PID controllers**", which is being submitted by **Sankalp paliwal (801451023)** in fulfillment of the requirements for the award of the **M.E. in Electronic Instrumentation & Control,** to Thapar University, Patiala, is a bona-fide record of the candidate's own work carried out by him under my supervision and guidance. The matter contained in this dissertation has not been submitted, neither in part nor in full to any other university or institute for award of any degree.

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ABSTRACT

An **inverted pendulum** is a **pendulum** that has its center of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. The '**mobile inverted pendulum (MIP)'** is a special case of the fundamental inverted pendulum system where the cart is replaced with a two wheeled robotic system which can move two and forth in horizontal direction and can take U-turn about its axis.

This work deals with the stabilization of mobile inverted pendulum using Proportional integral derivative (PID) controllers. A two loop controller scheme has been implemented to stabilize the MIP system. The tuning of the PID controllers has been done using three techniques:

a) Trial and error: In this method PID parameters are tuned by hit and trial method. First tuning of controller1for angle control is done and then the second controller is tuned for position control.
b) Pole placement technique: In this technique the PID parameters are given by placing the dominant closed loop poles at desired locations which are obtained by the LQR design of the system
c) Fuzzy PID: In this method the PID parameters are auto tuned by fuzzy logic. For stabilizing the MIP two fuzzy based PID controllers are used. In one controller position error and its derivative is passed as inputs while in other controller angle error and its derivative is passed as inputs. The outputs in both the controllers are the PID gains proportional K_p, integral K_i and derivative K_d.

The simulation results of all the techniques are compared and it is shown that the performance parameters obtained using Fuzzy PID are better than the pole placement and trial and error technique.

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW: Mobile inverted pendulum (MIP) system is an under actuated system with two degrees of freedom. MIP system has become an important platform for researchers and students for the demonstration of control theory and practice. The system has gained popularity as it is a relatively simple system to design yet it is an unstable and nonlinear system. MIP system is a special case of the fundamental inverted pendulum system where the cart is replaced with a two wheeled robotic system which can move two and forth in horizontal direction and can take U-turn about its axis.

MIP system has been chosen to implement the findings of this report because:

- A progressive system model can be designed. It is a nonlinear system yet it can be linearized for small operating ranges.
- The system relatively easy to design but still it is a nonlinear and unstable system

Diagram of a MIP system is shown in figure 1.1.

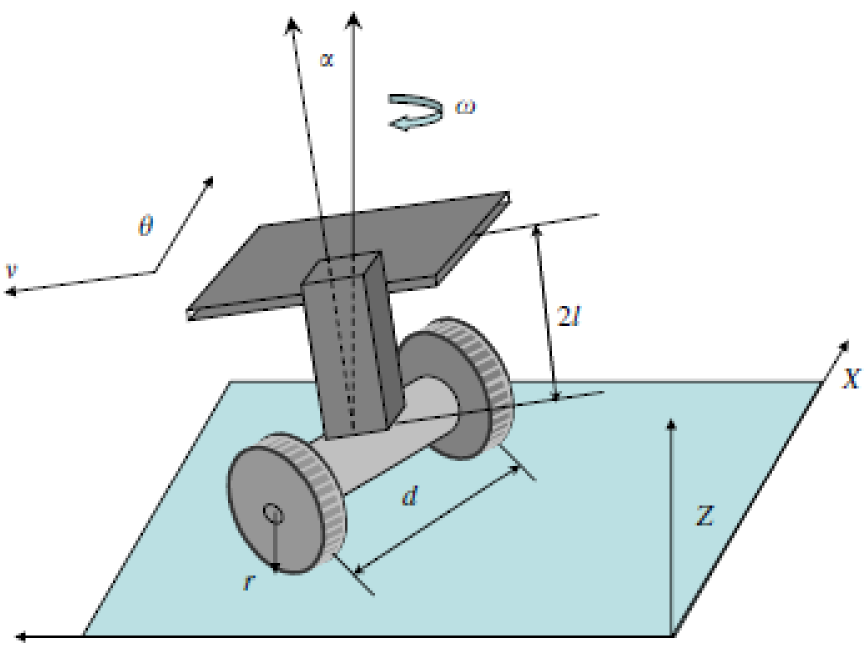

Figure 1.1 Diagram of MIP [2]

Various applications related to mobile inverted pendulum are:

- The development of automatic two wheeled wheel chairs for physically impaired persons [6].
- To build an automatic human transporter [1, 2].
- The development of humanoid robots which can navigate on two wheels and perform various tasks [3].
- To build automatic baggage navigation and transportation system in airports [8].
- Development of war tanks to stabilize the body of the tank and the gun to increase the precision of the shot.

The most common and widely used controller for stabilization of the MIP system is the PID controller. A PID controller is a control loop feedback controller commonly implemented in control systems.

1.2 OBJECTIVES AND SCOPE OF THIS DISSERTATION:

1) Developing the state space model of MIP system.

2) Control the mobile Inverted pendulum using PID controllers.

3) Tune the PID controllers using three different techniques:
   a. Trial and error: In this method PID parameters are tuned by hit and trial method.
   b. Pole placement technique: In this technique the PID parameters are given by placing the dominant closed loop poles at optimum places which are given by the LQR design of the system
   c. Fuzzy PID: In this method the PID values are auto tuned by fuzzy logic controllers. For stabilizing the MIP two fuzzy based PID controllers are used. In one controller position error and its derivative is passed as inputs while in other controller angle error and its derivative is passed as inputs. The outputs in both the controllers are the PID gains K_p, K_i and K_d.
4) Comparing the results obtained using different tuning methods in terms of various performance specifications.

1.3 ORGANISATION OF THIS DISSERTATION:

After giving a brief introduction about the topic, the dissertation is arranged as follows:
Chapter 2 provides a relevant literature survey regarding distinct control techniques for controlling MIP system.
Chapter 3 gives the mathematical modeling and state space representation of MIP system.

Chapter 4 gives the conventional control of MIP system.

Chapter 5 gives the MIP control using Fuzzy PID technique

Chapter 6 shows the simulation results and discussions

Chapter 7 provides the conclusion of entire work done and proposes the future scope.

CHAPTER 2

LITERATURE SURVEY

A mobile inverted pendulum (MIP) robot has become an important platform for researchers and students for the demonstration of control theory and practice. Recently MIP robot has become important tool for city commuting and patrol transporter in the form of Segway transporter [1, 2].

There are various other applications of MIP robots are that they can also be used as humanoid robots which can do various tasks such as boxing bots [2, 4, 5]. MIP system can also be used for baggage transportation and navigation at airports [6].

Stabilizing a MIP means keeping the pendulum in vertical upward position by applying appropriate control signal. Various control methods have been proposed to stabilize the MIP system which includes both the conventional and intelligent controllers and some combination of both of them. The most basic conventional method is state feedback controller [7].

LQR [8] and LQG [9] controllers are also widely used for controlling the MIP system which depends on the dynamic model of the system.

Sliding mode control strategy has also been proposed in the literature for stabilizing the MIP [10, 11, 12]. In sliding mode control the controller is developed by choosing the important sliding constraints by minimizing the quadratic index.

The most important and basic controller used for stabilizing the MIP system widely is the PID controller [13, 14].

There are various methods by which tuning of PID controllers can be done these methods include the trial and error in which the PID constraints are acquired using trial and error method.

The other method is pole placement technique in which the method of dominant poles is used to determine the PID parameters [15, 16]. In pole placement the PID parameters are found placing the dominant poles in desired locations obtained by the LQR design of the system.

In this dissertation a two loop PID control scheme has been implemented for stabilizing the MIP. PID controllers have been used one for position control while the other for angle control.

The tuning of the controller has been done using fuzzy logic [21]. For fine-tuning of PID controllers using fuzzy logic the inputs provided are the error e and derivative of error de/dt and the PID parameters are obtained as the output of the controller.

The RBF network is used as a secondary controller to aid the main PID controllers to accomplish well [24, 25]. The back propagation technique has been designed for the RBF network.

Waypoint tracking control of MIP system has been discussed in the literature [26]. A novel time-invariant controller is designed to meet the position and angle control objectives.

A low order two-channel controller has been proposed for controlling the MIMO linearized model of a MIP system [27].

Kaustubh *et al*. [28] proposed a two controller scheme is for controlling MIP system. First a two level speed controller for controlling the vehicle alignment while controlling the vehicle pitch. The other controller is used for controlling vehicle's position.

Yun-Su *et al*. [29] proposed a Trajectory control of MIP system by dividing the control algorithm into three parts: velocity control, steering control and position control. The robot consists of gyro sensors to measure the inclination angle and rotary encoders for wheel rotation measurement.

Song Hyok *et al*. [30] developed a dynamic surface controller which is based on a nonlinear disturbance observer used to control the MIP system. The controller designed can compensate the external turbulences to improve the system performance considerably.

The trajectory control of MIP system is done using time state controller in the literature [31]. Two controllers are used one for angle control. The other controller is used for converging a trajectory dependent on the time state control form.

An adaptive fuzzy position tracking controller for MIP system is proposed in the literature [32] which stabilized the system in occurrence of system changes and external turbulences.

Fuzzy control is used in [33] to stabilize the MIP system. The fuzzy membership functions for the control of MIP system are achieved by 3 user weights using hit and trial method.

An adaptive motion controller using fuzzy wavelet neural networks (FWNN) for stabilizing MIP system is proposed in the literature [34]. Two FWNN motion controllers are designed to attain station keeping, position and angle control.

Sayid marie *et al.* [35] proposed a fuzzy logic controller is for stabilizing a MIP system using tilt angle. Tilt angle sensing error is minimized by using a sensor combination technique. From the experimental results we can see that the system has the ability to stabilize itself during its movement.

Ching-Chih *et al.* [36] developed an intelligent adaptive back stepping sliding-mode controller using fuzzy basis function networks (FBFN) technique for stabilizing a MIP system. A decoupling technique is designed to decouple the system's active model such that the position controller can be designed employing back stepping and sliding-mode control.

Bature *et al.* [37] proposed an experimental assessment between model based controllers and non-model based controllers in stabilizing the MIP system. A FLC which is a non-model based controller and a LQR which is a model-based controller, and the orthodox controller, Proportional Integral Derivative (PID) were applied on MIP system and results were compared.

Ahmad *et al.* [38] designed a two wheeled wheel chair using fuzzy logic control algorithm. The intelligent design of the system impressionists a double inverted pendulum with three motors, one for each wheel, and one for chair position. A two-level FLC is designed.

Kahani *et al.* [39] developed Fuzzy Linear Quadratic Regulator (FLQR) controller. The stability condition of whole system is shown by employing the Linear Matrix Inequality (LMI) approach.
Zhijun Li *et al.* [40] proposed an adaptive fuzzy logic controller for MIP stabilization which is based on dynamic balance and motion of the MIP system. The controllers designed are based on the physical properties of the MIP system.
An indirect adaptive fuzzy based controller is proposed for controlling the MIP system in the literature [41]. The controller proposed is based on error data based trajectory planner and generates the desired values for the MIP system.
An adaptive output recurrent cerebral model articulation controller (AORCMAC) is proposed in the literature [42] for stabilizing MIP system. The AORCMAC captures the system dynamics and hence it can control the system better and provides a good system dynamic response.

The fuzzy logic control application theory spread out in middle of 70's when Mamdani [43-45] in his research papers proposed a fuzzy logic in process control, in which fuzzy set theory are used for the processing by a computer, and the obtained information can be used in fuzzy control algorithm to control a process without having an explicit model of the process.

The mathematical approximation to Fuzzy Logic was given by Kosko and Wang[46, 47] in their research paper in which they proved that the Fuzzy system are universal approximators and such system can be used for control analysis, in which approximation modeling is done.

Schwatz and Piero [48, 49] provided a mathematical foundation in fuzzy logical system including approximation reasoning. This concept opened a new way in control application problems which revolutionized the market mainly in the field of consumer electronic and electrical product.

By the advent of 1990 hybrid system architectures was considered in attention of the researchers which included fuzzy system and neural network system all together as a single system in which both system best features are employed. Most of the techniques were based on Takagi Sugeno [50] model which is a generalization of fuzzy logic.

CHAPTER 3

STATE SPACE MODELLING OF MOBILE INVERTED PENDULUM SYSTEM

3.1 INTRODUCTION: An **inverted pendulum** is a **pendulum** that has its center of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole [21]. The '**mobile inverted pendulum (MIP)'** is a special case of the fundamental inverted pendulum system where the cart is replaced with a two wheeled robotic system which can move two and forth in horizontal direction and can take U-turn about its axis. MIP is an under actuated system with two degrees of freedom. MIP is an unstable system and unlike conventional inverted pendulum system, it is more difficult to stabilize the pendulum on two wheels.

3.2 MATHEMATICAL MODELLING OF MIP: In order stabilize the MIP system the wheel dynamics and the dynamics of pendulum body must be understood.

3.2.1 WHEEL DYNAMICS:

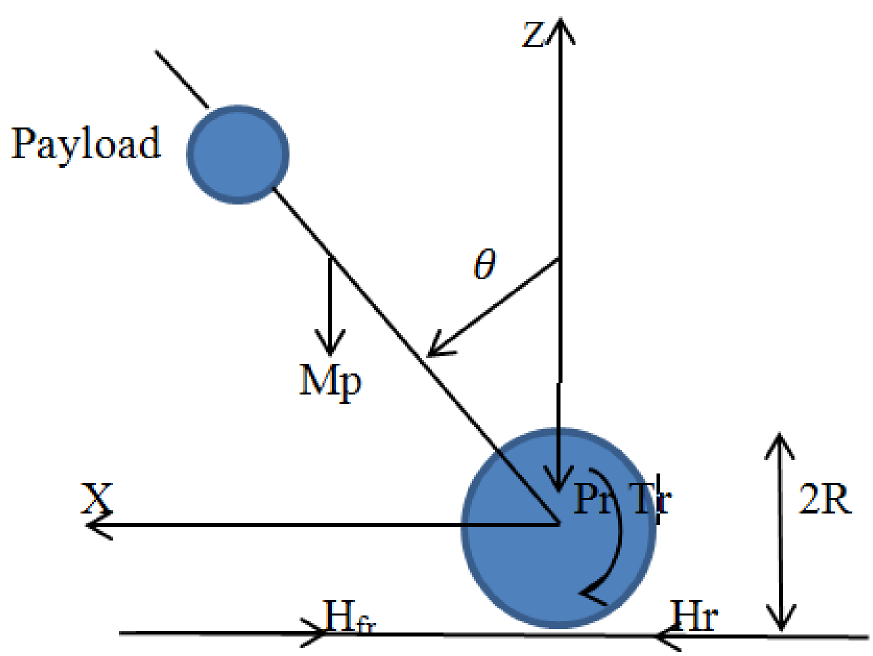

Figure 3.1 Free body diagram of wheel

The equations of the torque generated by the motors of MIP system are given below.

The torque produced by motor is

$$T_m = k_M i \tag{3.1}$$

i= current through motor coil

k_M= motor torque constant

T_m= motor torque

The back EMF produced is given as

$$V_e = k_E \dot{\theta_w} \tag{3.2}$$

V_e= back EMF

k_E= back EMF constant

By Kirchhoff's law sum of voltages is zero. Hence,

$$V_a - Ri - L\frac{di}{dt} - V_e = 0 \tag{3.3}$$

V_a= applied voltage

R= resistance of coil

L= inductance of coil

Neglecting the inductance in eq. 3.3

$$i = -\frac{k_E \dot{\theta_w}}{R} + \frac{V_a}{R} \tag{3.4}$$

Based on newton's law of motion sum of torques is directly proportional to the acceleration of the shaft.

$$\sum M = T_m - T_a = I_w \dot{\theta_w} \tag{3.5}$$

T_a= applied torque

I_w= wheel inertia

$$\frac{k_M i}{I_w} - \frac{T_a}{I_w} = \dot{\theta_w} \tag{3.6}$$

Substituting eq. (3.4) in eq. (3.6) equation of torque can be given as

$$T_R = -\frac{k_M k_E}{R}\left(\dot{\theta_w}\right) + \frac{k_M}{R}(V_R) \tag{3.7}$$

Let $T_a = T_R$

Force on wheel in horizontal direction can be given as

$$\sum F_x = M_w \ddot{x} \tag{3.8}$$

F_x= force on wheel in horizontal direction

M_w= mass of wheel

x = linear displacement in horizontal direction

$$M_w \ddot{x} = H_{fr} - H_r \tag{3.9}$$

H_{fr}= friction force

H_r= horizontal reaction force

Sum of forces around center of mass is given as

$$\sum M_o = I_w \ddot{\theta_w} \tag{3.10}$$

M_o= total moment of force

$$I_w \ddot{\theta_w} = T_R - H_{fr} R_w \tag{3.11}$$

R_w= radius of wheel

Substituting eq. (3.7) in eq. (3.11)

$$I_w\ddot{\theta_w} = -\frac{k_M k_E}{R}\left(\dot{\theta_w}\right) + \frac{k_M}{R}(V_R) - H_{fr}R_w \tag{3.12}$$

$$H_{fr} = -\frac{k_M k_E}{RR_w}\left(\dot{\theta_w}\right) + \frac{k_M}{RR_w}(V_R) - \frac{I_w\ddot{\theta_w}}{R_w} \tag{3.13}$$

Substituting eq. (3.13) in eq. (3.9)

For left wheel,

$$M_w\ddot{x} = -\frac{k_M k_E}{RR_w}\left(\dot{\theta_w}\right) + \frac{k_M}{RR_w}(V_L) - \frac{I_w\ddot{\theta_w}}{R_w} - H_L \tag{3.14}$$

For right wheel,

$$M_w\ddot{x} = -\frac{k_M k_E}{RR_w}\left(\dot{\theta_w}\right) + \frac{k_M}{RR_w}(V_R) - \frac{I_w\ddot{\theta_w}}{R_w} - H_R \tag{3.15}$$

Transforming angle rotation to linear motion eq. (3.14) and eq. (3.15) can be rewritten as

For left wheel,

$$M_w\ddot{x} = -\frac{k_M k_E}{R{R_w}^2}(\dot{x}) + \frac{k_M}{RR_w}(V_a) - \frac{I_w\ddot{\theta_w}}{R_w} - H_L \tag{3.16}$$

For right wheel,

$$M_w\ddot{x} = -\frac{k_M k_E}{R{R_w}^2}(\dot{x}) + \frac{k_M}{RR_w}(V_a) - \frac{I_w\ddot{\theta_w}}{R_w} - H_R \tag{3.17}$$

Adding eq. (3.16) and eq. (3.17)

$$2\left[M_w + \frac{I_w}{R{R_w}^2}\right]\ddot{x} = -\frac{2k_M k_E}{R{R_w}^2}(\dot{x}) + \frac{2k_M}{RR_w}(V_a) - (H_R + H_L) \tag{3.18}$$

3.2.2 BODY DYNAMICS:

Applying newton’s law of motion in x direction in figure (1)

$$\sum F_x = (Mp)\ddot{x} \tag{3.19}$$

F_x= sum of forces in x direction

Mp= mass of body

$\sum F_x$= horizontal reactive force +force leading to linear motion

$$\sum F_x = (H_R + H_L) - (Mp)\left(L\theta cos\theta - L\dot{\theta}^2 sin\theta\right) = Mp\ddot{x} \qquad (3.20)$$

θ= tilt angle of pendulum

L= length of frame

Considering forces in perpendicular direction

∑F↑= horizontal reaction forces+ vertical reaction forces+ force due to linear acceleration

$$\sum F \uparrow = (H_R + H_L)cos\theta + (P_R + P_L)sin\theta - (Mp)(L\ddot{\theta} - Gsin\theta) \qquad (3.21)$$

G= gravitational force

P_R, P_L= perpendicular reaction forces

Sum of moments is

$$\sum moments = I_{eq}\ddot{\theta} \qquad (3.22)$$

I_{eq}= moment of inertia of body

$$I_{eq}\ddot{\theta} = -\{(H_R + H_L)cos\theta + (P_R + P_L)sin\theta\}L - (T_R + T_L) \qquad (3.23)$$

Torque due to each motor

$$T_R + T_L = -\frac{k_M k_E}{RR_w}(\dot{x_R} + \dot{x_L}) + \frac{k_M}{R}(V_R + V_L) \qquad (3.24)$$

Putting eq. (3.24) in eq. (3.23)

$$I_{eq}\ddot{\theta} = -\{(H_R + H_L)cos\theta + (P_R + P_L)sin\theta\}L - \left\{-\frac{k_M k_E}{RR_w}(\dot{x_R} + \dot{x_L}) + \frac{k_M}{R}(V_R + V_L)\right\} \qquad (3.25)$$

Adding eq. (3.25) and eq. (3.21)

$$\left(I_{eq} + (Mp)L^2\right)\ddot{\theta} = -\frac{2k_M k_E}{RR_w{}^2}(\dot{x}) + \frac{2k_M}{RR_w}(V_a) - (Mp)LGsin\theta + (Mp)L\ddot{x}cos\theta \qquad (3.26)$$

Putting the value of $H_R + H_L$ from eq. (3.20) in eq. (3.18)

$$2\left[M_w + \frac{I_w}{RR_w{}^2}\right]\ddot{x} = -\frac{2k_M k_E}{RR_w{}^2}(\dot{x}) + \frac{2k_M}{RR_w}(V_a) - (Mp)\left(L\theta cos\theta - L\dot{\theta}^2 sin\theta\right) - Mp\ddot{x} \quad (3.27)$$

Rearranging eq. (26) and eq. (27) gives nonlinear equations of motion

$$(Ieq + (Mp)L^2)\ddot{\theta} - \frac{2KmKe}{RRw^2}(\dot{x}) + \frac{2Km}{RRw^2}(Va) + MpLGsin\theta = -(Mp)L\ddot{x}cos\theta \qquad (3.28)$$

$$\left[2Mw + \frac{2Iw}{Rw^2} + Mp\right]\ddot{x} + \frac{2KmKe}{RRw^2}(\dot{x}) + (Mp)\left(L\ddot{\theta}cos\theta - L\dot{\theta}^2 sin\theta\right) = \frac{2KM}{RRw}(Va) \qquad (3.29)$$

Equation (3.28) and equation (3.29) contain nonlinear terms which can be linearized by assuming $\theta = \pi + \emptyset$ where $\emptyset$ represents a small angle in vertical direction.

For small angle cos θ=-1, sin θ=-$\emptyset$ and $\dot{\theta}^2 = 0$. Applying these in eq. (3.28) & (3.29) we get the linearized equations as:

$$(Ieq + (Mp)L^2)\ddot{\emptyset} - \frac{2KmKe}{RRw^2}(\dot{x}) + \frac{2Km}{RRw^2}(Va) - MpLG\emptyset = MpL\ddot{x} \qquad (3.30)$$

$$\frac{2KM}{RRw}(Va) = \left[2Mw + \frac{2Iw}{Rw^2} + Mp\right]\ddot{x} + \frac{2KmKe}{RRw^2}(\dot{x}) - MpL\ddot{\emptyset} \qquad (3.31)$$

Rearranging equation (3.30) & (3.31) the state equations of the MIP are obtained

$$\ddot{\emptyset} = \frac{MpL\ddot{x}}{(Ieq+(Mp)L^2)} + \frac{2KmKe}{RRw^2(Ieq+(Mp)L^2)}(\dot{x})$$

$$-\frac{2Km}{RRw^2(Ieq+(Mp)L^2)}(Va) + MpLG\emptyset/(Ieq + (Mp)L^2) \qquad (3.32)$$

$$\ddot{x} = \frac{2Km}{RRw\left[2Mw + \frac{2Iw}{Rw^2} + Mp\right]}(Va) - \frac{2KmKe}{RRw^2\left[2Mw + \frac{2Iw}{Rw^2} + Mp\right]}(\dot{x})$$

$$+MpL\ddot{\emptyset}/\left[2Mw + \frac{2Iw}{Rw^2} + Mp\right] \qquad (3.33)$$

By substituting equation (3.32) in eq. (3.30), and eq. (3.33) in eq. (3.31) state space equation of the system can be given as:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\emptyset} \\ \ddot{\emptyset} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2KmKe(MpLRw - Ieq - Mp^2L^2)}{RRw^2\alpha} & \frac{Mp^2L^2G}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2KmKe(Rw\beta - MpL)}{RRw^2\alpha} & \frac{MpGL}{\beta} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \emptyset \\ \dot{\emptyset} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2Km(MpLRw + Ieq + MpL^2)}{RRw\alpha} \\ 0 \\ \frac{2Km(-Rw\beta + Mpl)}{RRw\alpha} \end{bmatrix} Va$$

Where $\dot{x}$, $\ddot{x}$, $\dot{\emptyset}$, $\ddot{\emptyset}$ are the system states

$$\beta = [2Mw + \frac{2Iw}{Rw^2} + Mp] \qquad \alpha = Ieq\beta + 2MpL^2[Mw + \frac{Iw}{Rw^2}] \tag{3.34}$$

The value of various parameters used are shown in Table 3.1

Table 3.1

Parameter values of MIP [20]

Mass of the body Mp	1
Mass of the wheel Mw	0.03
Length of the pendulum L	0.07
Motor resistance R	3
Radius of the wheel Rw	0.051
Motor torque constant Km	0.0235
Back EMF constant Ke	0.006087

The state space obtained after putting these values is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ (\dot{\emptyset}) \\ (\ddot{\emptyset}) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0414 & 11.0126 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.1121 & 171.484 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \emptyset \\ (\dot{\emptyset}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3470 \\ 0 \\ 1.0412 \end{bmatrix} Va \tag{3.35}$$

The state space equation obtained above is used in the linear representation of the non-linear MIP system. The system above is used in Matlab Simulink as a plant for implementation of controllers to control the system.

CHAPTER 4

CONVENTIONAL PID CONTROL OF MOBILE INVERTED SYSTEM

The stabilization of MIP system has been done using conventional PID controllers which is explained below.

4.1 PID CONTROLLER: A PID controller is a control loop feedback controller commonly used in control systems.

The basic equation of a PID controller is written below:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\, dt + K_d \frac{de(t)}{dt} \tag{4.1}$$

Where e(t) is the error between the reference and the feedback of the system. $u(t)$ is the input given to system. K_p is the proportional gain while K_i is the integral gain and K_d is the derivative gain of the PID controller.

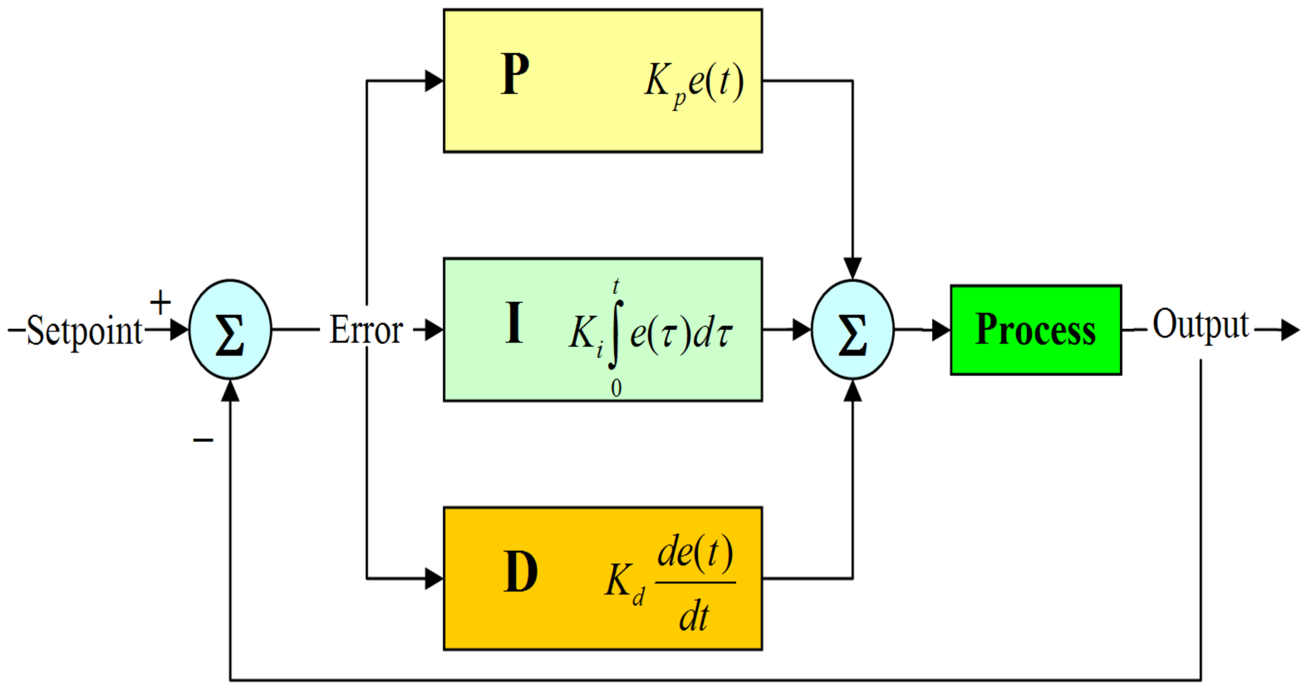

Figure 4.1 Basic block diagram of PID controller

4.2 TWO LOOP PID CONTROL SCHEME FOR MIP STABILIZATION: For stabilizing the MIP a two loop control scheme has been used for stabilizing the MIP as shown in figure 4. PID1 has been used for controlling the position x while PID2 for controlling angle θ.

P1= X(s)/U(s) and P2= $\theta(s)/U(s)$ represents the two transfer functions.

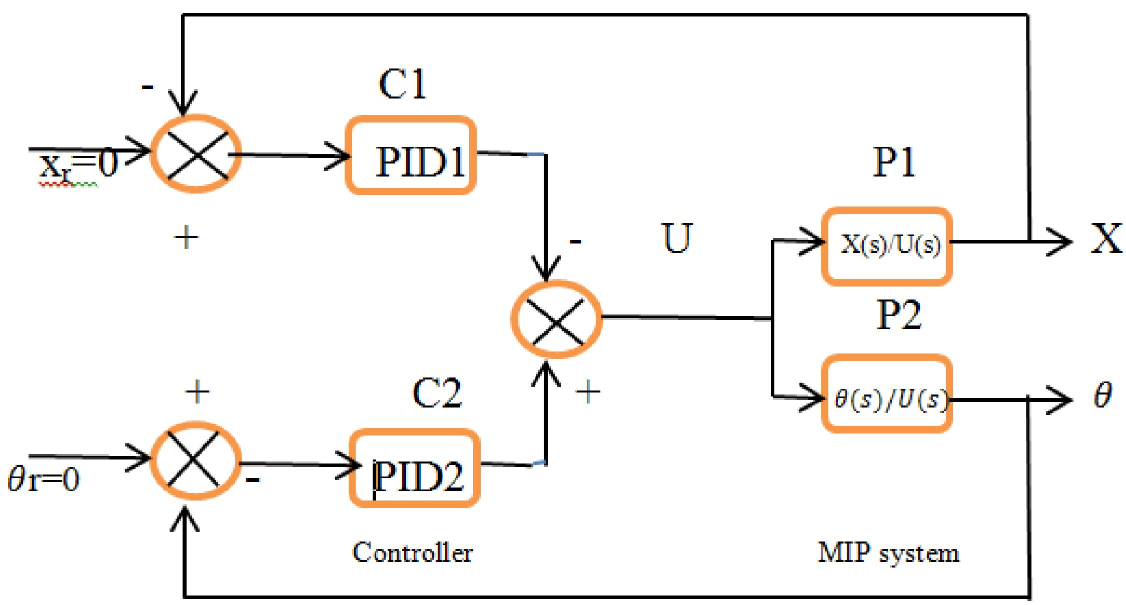

Figure 4.2 Two loop PID controller for MIP system

4.3 TUNING OF PID CONTROLLER: Tuning of PID controller means modification of its control parameters (proportional gain, integral gain, derivative gain) to optimal values for the desired control response.

To stabilize the MIP following tuning methods have been used.

4.3.1 TRIAL AND ERROR METHOD: In this method the PID parameters are obtained by hit and trial. First PID1 is designed for controlling the position x and then the second controller PID2 is designed for controlling the angle θ. The PID parameters obtained are given in Table 4.1.

Table 4.1

PID parameters obtained by trial and error

Controller	Kp	Ki	Kd
PID1	335	20	27
PID2	25	2	20

4.3.2 POLE PLACEMENT TECHNIQUE: In this technique the optimum closed loop poles are placed at preferred points which are attained from LQR technique [16].

The characteristic equation for the system shown in figure 3 is given as

$$1 - P1C1 + P2C2 = 0 \qquad (4.2)$$

Where $C1$ and $C2$ are given as

$$C1 = \frac{K_d^1 s^2 + K_p^1 s + K_i^1}{s} \qquad (4.3)$$

$$C2 = \frac{K_d^2 s^2 + K_p^2 s + K_i^2}{s} \qquad (4.4)$$

Let the characteristic equation be

$$s^5 + p1s^4 + p2s^3 + p3s^2 + p4s + p5 \qquad (4.5)$$

The dominant poles of the two loop PID controller are obtained using LQR design.

LQR is state feedback controller designed to minimize a performance index which takes care of the design constraints [16].

The performance index is given as

$$j = \frac{1}{2}\int_0^\infty \{X^T Q X + U^T R U\}\, dt \qquad (4.6)$$

Here Q is the state weighted matrix and R is the control weighted matrix the performance index j is minimized using Riccati equation given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{4.7}$$

State feedback gain vector K≜[K1,K2,K3,K4] is obtained by

$$K = -R^{-1}B^T P \tag{4.8}$$

For simplicity the state weighted matrix is chosen as Q=diag {q1, q2, q3, q4} and the control weighted matrix is chosen as a scalar vector R=r.

In this thesis Q vector is q1=500q, q2=q3=20q, q4=q and r= 10^n. By trial and error It is found that for optimal results q=100 and n=4.

The closed loop poles are obtained by the Eigen values of A-BK. The four poles obtained are -13.152, -13.0388, -0.6042±0.5945i. The fifth pole is taken as six times the real part of the most dominant pole. By putting these poles in the characteristic equation (4.5) we can obtain the PID parameters.

The PID parameters obtained using pole placement techniques are given in table 4.2.

Table 4.2

PID parameters using pole placement

Controller	Kp	Ki	Kd
PID1	301.41	197.67	35
PID2	15.19	7.065	13.17

The Simulink model for pole placement technique is shown in Figure 4.3

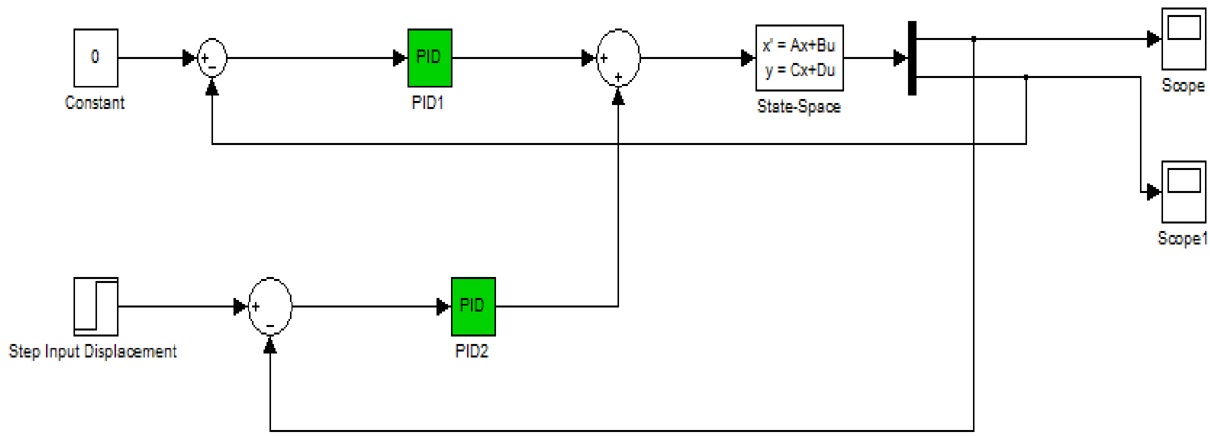

Figure 4.3 Simulink diagram for pole placement technique

In the above Simulink diagram PID1 is the controller for angle control and the reference given to PID1 is a zero constant. The controller PID2 is the controller used for position control and the reference given to PID2 is step input with amplitude of one.

The state space in the diagram is same as given in equation 3.34.

The PID parameters for PID1 and PID2 are obtained by pole placement technique as explained above.

CHAPTER 5

FUZZY PID CONTROL OF MIP SYSTEM

5.1 FUZZY LOGIC CONTROLLER: A fuzzy control system is based on fuzzy logic which is a mathematical system that analyses input values in terms of logical variables and take continuous values between 0 and 1 [22].

In fuzzy logic the input values are mapped by sets of membership functions called fuzzy sets. The process of converting crisp values to fuzzy sets is called fuzzification.

In general there are three stages in a fuzzy logic controller input stage, processing stage and the output stage. The input stage takes inputs from sensors or other inputs like switches etc. The processing stage processes the inputs according to the fuzzy rule base and gives the output to output stage. Finally the output stage converts back the results from processing stage to the control value.

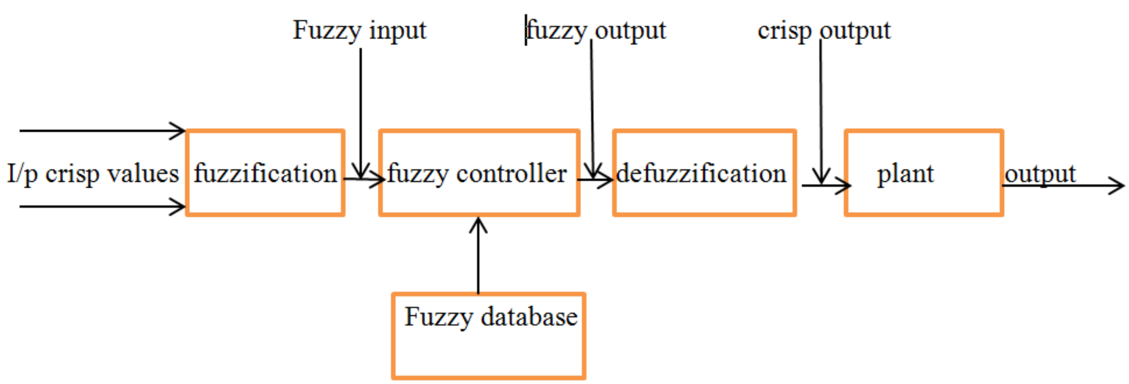

Figure 5.1 Block diagram of fuzzy controller

5.2 TUNING OF PID CONTROLLERS USING FUZZY LOGIC:

For tuning the PID controllers using fuzzy logic controllers the closed loop feedback error e(t) and the derivative of error de(t)/dt are given as inputs to the fuzzy logic controller and the outputs are taken as the PID parameters K_p, K_d and K_i.

The structure of fuzzy PID controller is given in figure 5.2.

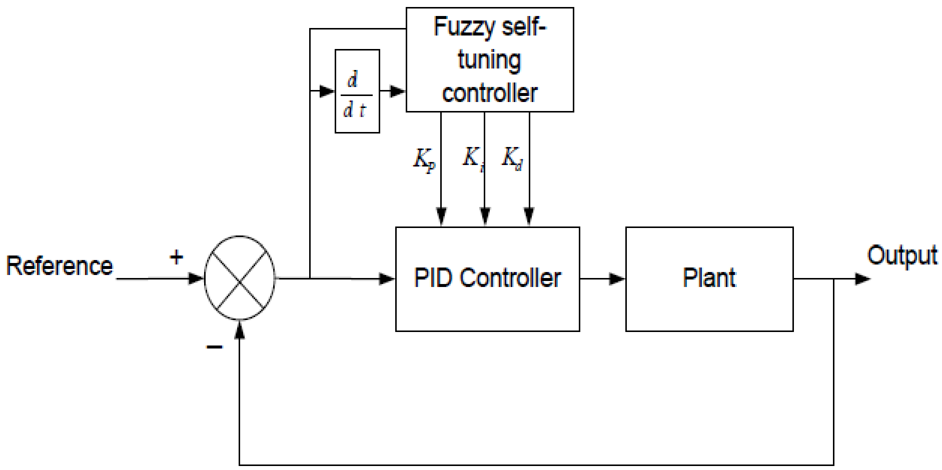

Figure 5.2 Block diagram for fuzzy PID controller [19]

The structure of fuzzy controller used for tuning of PID controller is given in figure 5.3 which shows the inputs and outputs of the fuzzy controller.

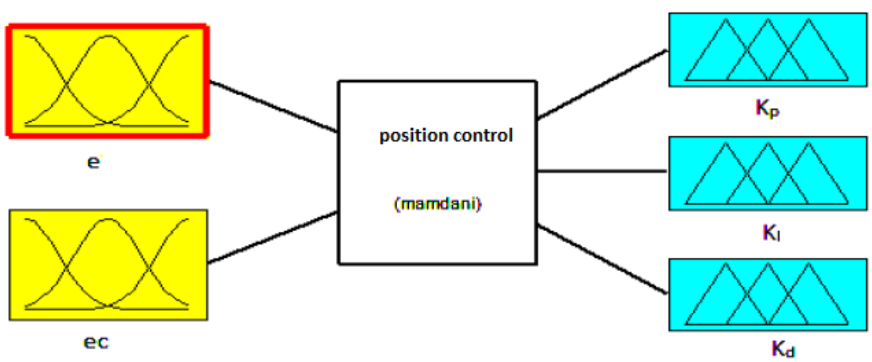

Figure 5.3 Structure of fuzzy controller [19]

For stabilizing the MIP two fuzzy based PID controllers are used. In one controller position error and its derivative is passed as inputs while in other controller angle error and its derivative is passed as inputs. The outputs in both the controllers are the PID gains K_p',K_i' and K_d'.

The output gains are scaled down as follows:

$$K_p' = \frac{Kp - K_{pmin}}{K_{pmax} - K_{pmin}} \tag{5.1}$$

$$K_i' = \frac{Ki - K_{imin}}{K_{imax} - K_{imin}} \tag{5.2}$$

$$K_d' = \frac{Kd - K_{dmin}}{K_{dmax} - K_{dmin}} \tag{5.3}$$

The membership functions for both error e and de/dt are taken as NB (negative big), NS (negative small), Z (zero), PS (positive small), and PB (positive big).

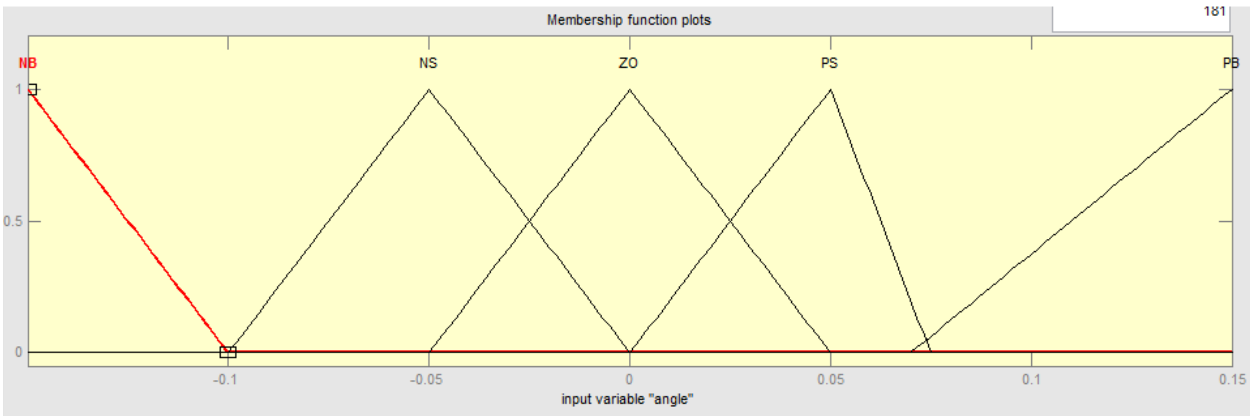

Figure 5.4 Membership function structure of error e

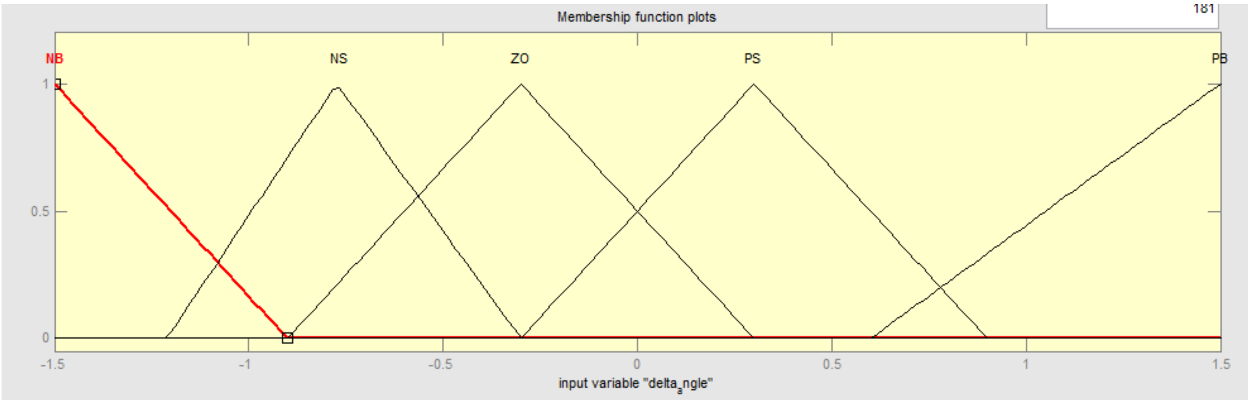

Figure 5.5 Membership function structure of error de/dt

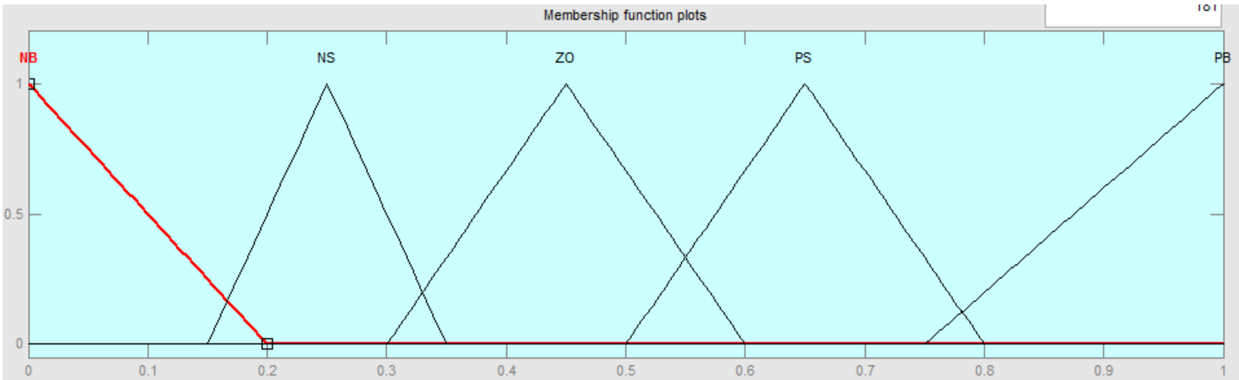

Figure 5.6 Membership function for output sets Kp’,Ki’,Kd’

The fuzzy rule base implemented for this fuzzy controller can be given as

Table 5.1

Fuzzy rule base for fuzzy controller

e \ de/dt	NB	NS	Z	PS	PB
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB

The rule base is defined as follows:

IF error e is NB and derivative of error de/dt is also NB then output variable i.e. K_p’, K_i’, K_d’ will be NB and if e is Z and derivative of error de/dt is NS then output will NS and so on.

The Simulink diagram of the implementation of fuzzy PID control algorithm is shown in Figure 5.7.

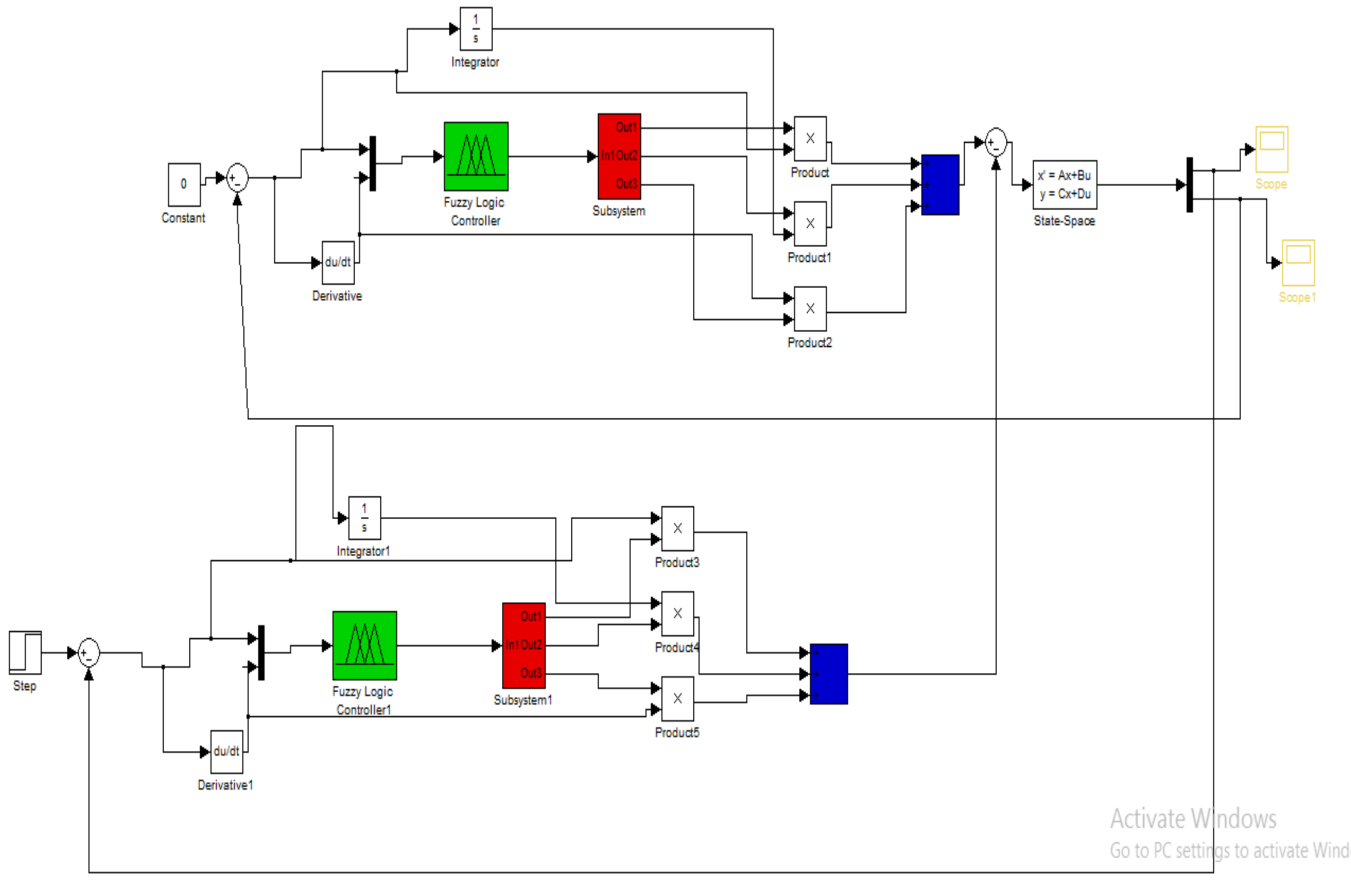

Figure 5.7 Simulink diagram for Fuzzy PID controller

In the above Simulink diagram two fuzzy logic controllers have been used to tune the PID controllers used control angle and position of the MIP system.

The inputs to fuzzy controllers are error and derivative of error and the output of fuzzy controller are PID parameters K_p, K_i and K_d.

The reference for the angle controller is taken as zero constant. The reference for the position controller is a step input with amplitude 1.

CHAPTER 6

RESULTS AND DISCUSSION

In this work PID controllers are used for stabilizing the MIP system. The PID controllers have been tuned with different techniques and there simulation results are compared and discussed in this section.

6.1 TRIAL AND ERROR METHOD: In this method the PID controllers are tuned by hit and trial method.

The simulation results are shown in Figure 6.1 and Figure 6.2

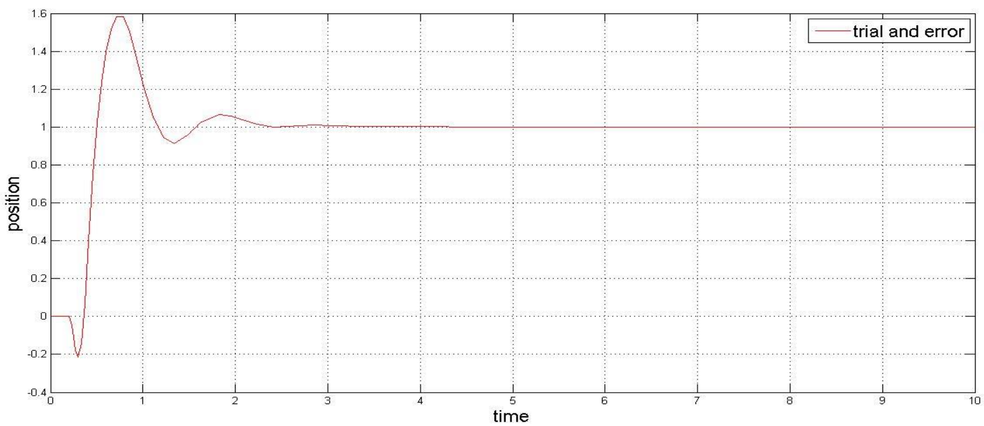

Figure 6.1 Position vs. Time curve

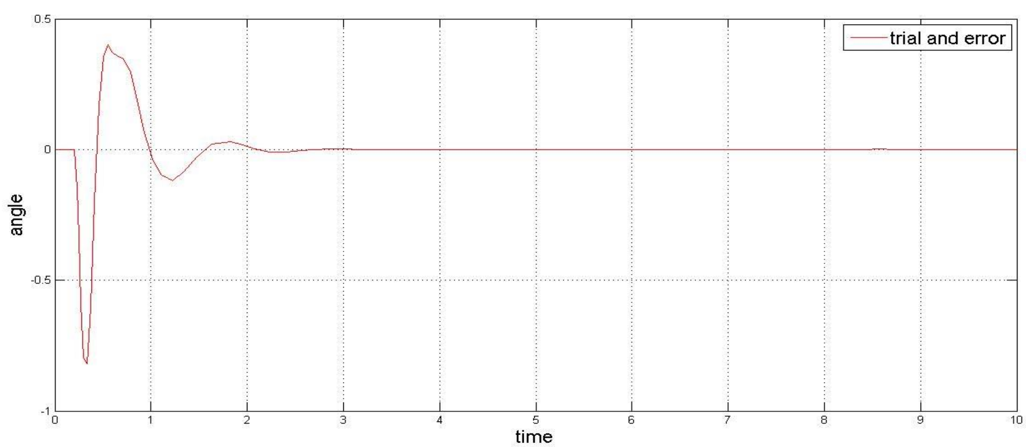

Figure 6.2 Angle vs. Time curve

6.2 POLE PLACEMENT TECHNIQUE: In this technique the dominant closed loop poles are placed at desired locations which are obtained from LQR technique. The poles are obtained by the Eigen values of the matrix A-BK. Where K is the gain matrix obtained by the LQR design.

The simulation results of this technique are shown in Figure 6.3 and Figure 6.4

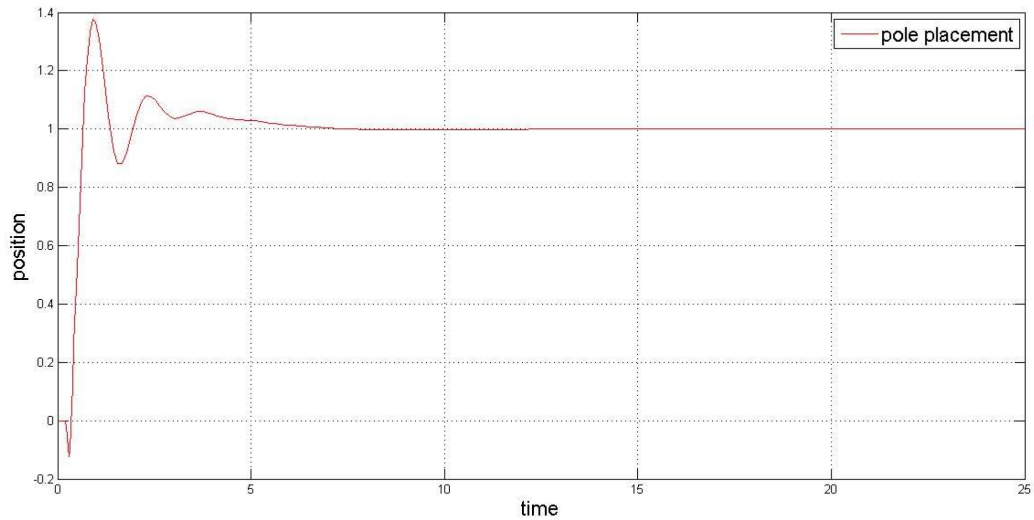

Figure 6.3 Position vs. Time curve

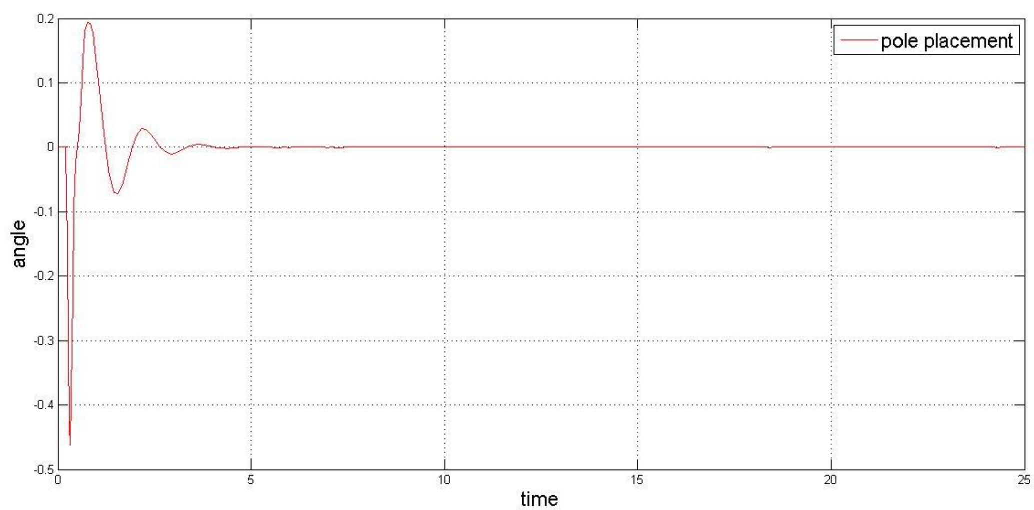

Figure 6.4 Angle vs. Time curve

6.3 FUZZY PID CONTROL: In this method the PID parameters are auto tuned by fuzzy logic controllers. For stabilizing the MIP two fuzzy based PID controllers are used. In one controller position error and its derivative is passed as inputs while in other controller angle error and its derivative is passed as inputs. The outputs in both the controllers are the PID gains K_p', K_i' and K_d'.

The simulation results of fuzzy PID technique are shown in Figure 6.5 and Figure 6.6

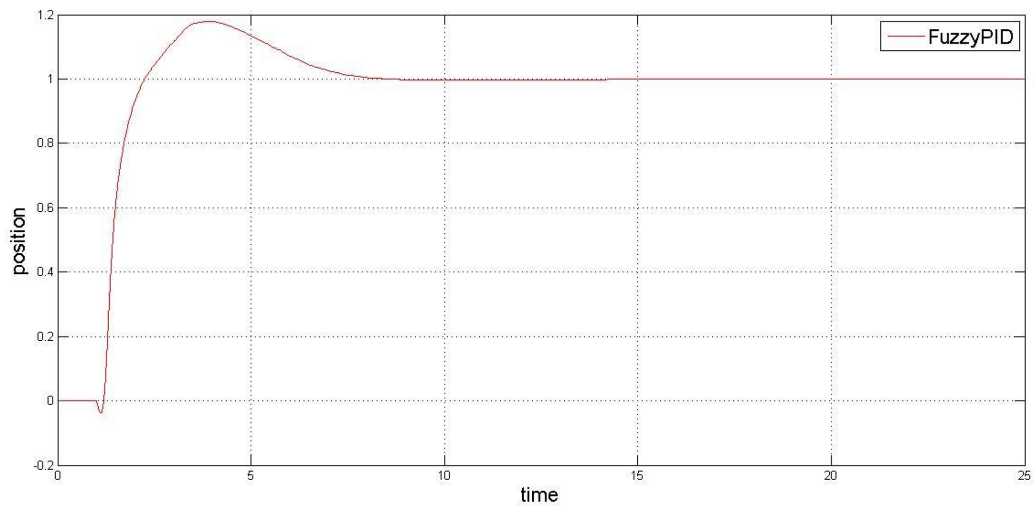

Figure 6.5 Position vs. Time curve

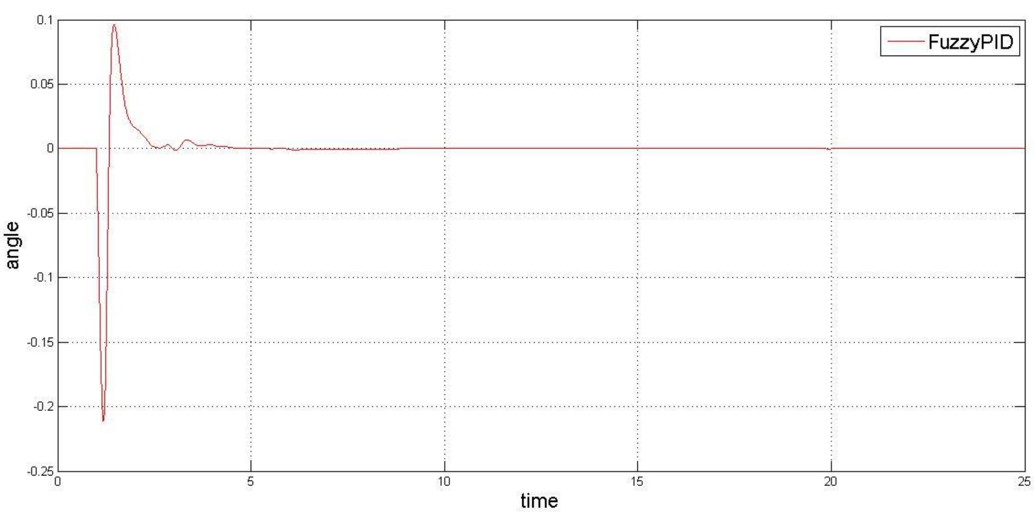

Figure 6.6 Angle vs. Time curve

The comparison of the results of different tuning methods is shown in Figure 6.7 and Figure 6.8

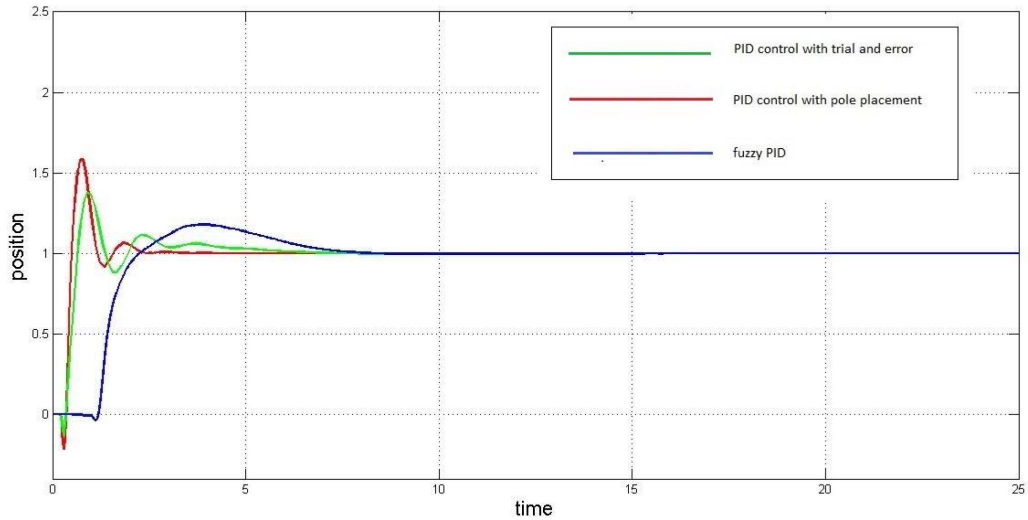

Figure 6.7 Position vs. Time curve

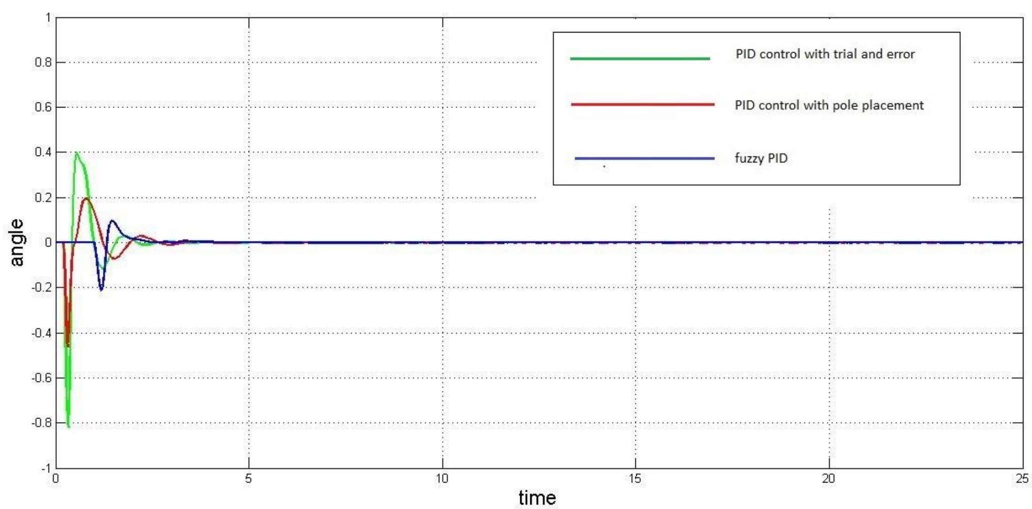

Figure 6.8 Angle vs. Time curve

A comparative analysis of different tuning methods is given in table 6.1 and table 6.2 for position and angle control

Table 6.1

Performance parameters for position curve

Parameters	Trial and error	Pole placement	Fuzzy PID
Rise time tr (sec)	0.35	0.45	1.5
Settling time ts (sec)	2.8	5.4	7.1
Max overshoot (Mp %)	58.38	37.644	17.919
Max undershoot (Mu %)	21.46	12.308	3.8422
Steady state error (e_{ss})	0	0	0

Table 6.2

Performance parameters for angle curve

Parameters	Trial and error	Pole placement	Fuzzy PID
Rise time tr (sec)	0.27	0.15	0.1
Settling time ts (sec)	2.7	4.8	2.5
Max overshoot (Mp %)	49.62	42.74	27.77
Max undershoot (Mu %)	24.22	10.019	6.09
Steady state error (e_{ss})	0	0	0

From Table 6.1 and Table 6.2 it can be seen that the performance of fuzzy PID is much better than trial and error and pole placement technique in terms of max. Overshoot and undershoot in case of both position and angle control.

CONCLUSION AND FUTURE SCOPE

In this dissertation the MIP system has been stabilized using a two loop PID controller scheme. The PID controllers are tuned using conventional as well fuzzy logic method. The performance of Fuzzy PID controller is better than the conventional PID controllers which are tuned by pole placement technique and trial and error method. The simulation results show that the max. Overshoot (Mp) and max. Undershoot (Mu) in case of fuzzy PID are better than trial and error and pole placement technique for position curve. For angle curve rise time (tr), settling time (ts), max. Overshoot (Mp), max undershoot (Mu) parameters of Fuzzy PID are better than other tuning techniques.

For position curve the percentage improvement in Mp for fuzzy PID is 52.3 % with respect to pole placement and 69.3% with respect to trial and error technique. Moreover the percentage improvement for Mu for fuzzy PID is 68.7% with respect to pole placement and 82.09% with respect to trial and error.

For angle curve the percentage improvement in tr for fuzzy PID is 33.3% with respect to pole placement and 62.9% with respect to trial and error. The percentage improvement in ts for fuzzy PID is 47.9% with respect to pole placement and 7.4% with respect to trial and error. Also the percentage improvement in Mp for fuzzy PID is 31.3% with respect to pole placement and 44.03% with respect to trial and error. Finally the percentage improvement in Mu for fuzzy PID is 39.2% with respect to pole placement and 74.8% with respect to trial and error.

FUTURE SCOPE:

1. AI control techniques can be improved so that a robust controller is designed which gives better response. The limitations of Fuzzy PID technique can be eliminated by using combination of two or more AI techniques like combination of neural network and fuzzy logic.
2. The modeling of MIP system under the effect of uncertainties such as friction can be done and the performance of these controllers can be tested on such system.
3. Real time control hardware model of the system can be developed using the controllers designed.

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