

**STUDY OF GROUND STATE BARYON
 $J^P = \frac{1}{2}^+$ OCTET AND $J^P = \frac{3}{2}^+$ DECUPLET
USING STATISTICAL METHODS**

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Amanpreet Kaur



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TIET, PATIALA-147004

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Dedicated to
My Family
for their endless support

CERTIFICATE

This is to certify that the thesis entitled “**STUDY OF GROUND STATE BARYON $J^P = \frac{1}{2}^+$ OCTET AND $J^P = \frac{3}{2}^+$ DECUPLET USING STATISTICAL METHODS**” being submitted by **Ms. Amanpreet Kaur** for the fulfillment of the requirements for the award of Degree of Doctor of Philosophy in the School of Physics and Materials Science, Thapar Institute of Engineering and Technology, Patiala, is a record of the candidate’s own work carried out by her under my supervision. The matter presented in this thesis has not been submitted in part or full for the award of any degree in any university or institute.

Supervisor



Dr. Alka Upadhyay

Associate Professor

School of Physics and Materials Science

Thapar Institute of Engineering and Technology (TIET)

Patiala- 147004

Punjab (India)

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(Amanpreet Kaur)

Publications

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Abstract

In this thesis, various low energy properties of ground state baryonic systems have been studied using the statistical model. Various properties like masses, magnetic moments, spin distribution, of spin $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet particles are analyzed in this work. This thesis is dedicated to some of the basic unanswered questions like role of sea (including $s\bar{s}$) in determining properties of baryons to study internal structure of ground state baryons. The calculations are performed at the energy scale of 1GeV . Therefore, the importance of the strange quark contributions is explicitly studied in the present work. Here, baryonic structure is considered to be consisting of valence quarks and sea limited to a few number of quark-antiquark pairs multiconnected non perturbatively through gluons. Statistical model is used to find the answers of some of the queries by using the statistical techniques. The thesis is organized into the following six chapters.

- **Chapter 1**, gives a brief account of some theoretical concepts of standard model and quark model of particles, group theory, symmetry breaking and QCD. The young tableaux method is explained to understand the valence picture of baryonic system. This is done by decomposing products of $SU(N)$ representations to irreducible ones. The chapter also outlines a brief experimental as well as theoretical literature review of the various static properties at high energies.
- **Chapter 2**, contains introduction of the work. It begins with the statistical model applied to study the static properties of spin $1/2^+$ and $3/2^+$ baryonic systems. The statistical model is based on the assumption that hadronic structure can be assumed to be made up of quark-gluon Fock states. The Fock states can be single gluon and multi-gluon states along with quark-antiquark pairs. The hadronic structure consists of two parts, one is q^3 valence part and other

is sea containing quark-gluon Fock states multi-connected nonperturbatively through gluons. A wave function for baryon decuplet is constructed with inclusion of sea containing quark gluon Fock states. The statistical model used here aimed at calculating the relative probabilities in flavor, spin and color space. The probabilities in flavor are computed by using principle of detailed balance while the probabilities in spin and color is estimated using relative multiplicities in the form of ratios. The present work uses two different types of statistical approaches namely Model *C* and *D* (Model assuming sea with suppressed multiplicity). Chapter 2 also discusses the various low energy properties like magnetic moments, masses, spin distribution in brief.

- In **chapter 3**, we study the $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryon octet and decuplet masses by analyzing their mass formulae. These formulae are function of constituent quark masses and spin spin interaction terms for the quarks inside the baryons. The coefficients in the mass formulae is estimated by the statistical model, where the baryonic structure is considered to be consisting of valence quarks and “sea” containing $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs and gluons, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ particles. The statistical approach and detailed balance principle are used in combination to compute the unknown coefficients $[(a_8; a_{10}; b_1; b_8; b_{10}; c_8; d_8); (a_0; b_1; b_8; d_1; d_8)]$ of the wave functions leading to scalar, vector and tensor contribution to the masses of spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ particles, respectively. The results are compared with the available theoretical and experimental data. In case of octet, since *D* model is giving better match with experimental data of masses and that too, major contribution comes from scalar plus tensor sea that dominates the contribution from vector sea by 66%. Similarly, for decuplet, *C* model gives better match with experimental data and hence scalar plus tensor sea contribution dominates the vector sea by 99.5%.

- In **chapter 4**, the statistical model in combination with detailed balance principle is applied to find magnetic moments of octet and decuplet particles incorporating strange sea. The $s\bar{s}$ pairs in the sea is generated via the basic quark mechanism but suppressed by the strange quark mass factor $m_s > m_{u,d}$. Formalism suggest construction of a suitable wave function for baryon octet and decuplet having specific color, flavor and spin space. Here, valence q^3 and a sea combines in a way to reproduce desired quantum numbers i.e. spin $1/2$, color singlet and flavor 8 for $J^P = \frac{1}{2}^+$ and spin $3/2$, color singlet and flavor 10 for $J^P = \frac{3}{2}^+$ particles. The magnetic moment operator is applied to the total wave function on flavor and spin space. Principle of detail balance and statistical approach is applied in combination to find the relative probabilities in spin, flavor and color states. Here, we have incorporated the effect of (a) quark effective masses, (b) quark effective charges and (c) both i.e quark effective mass plus effective charge, to compute the magnetic moments for $J^P = \frac{1}{2}^+$ particles. The results are compared with different theoretical approaches and experimental data. Our results of magnetic moments for $J^P = \frac{1}{2}^+$ particles with quark effective mass are closer to experimental data than with quark effective charge.

- **Chapter 5** deals with importance of strangeness in determining other baryonic properties like strangeness suppression, $SU(3)$ symmetry breaking for magnetic moments of baryons and role of strange and non-strange sea quark gluon effects. We have also computed $\bar{d} - \bar{u}$ asymmetry and average number of partons for all octet and decuplet particles despite non-availability of any data for comparison (in case of decuplets). To understand the existence of strange quarks in sea, we have defined strangeness suppression factor as $\lambda_s = \frac{2(s\bar{s})}{(u\bar{u}+d\bar{d})}$. The value for the strangeness suppression factor is in good agreement with both the values determined in exclusive reactions and in high-energy produc-

tion for baryon octet. Symmetry breaking is applied on valence and sea quarks resulting in altered values of octet magnetic moments. $SU(3)$ symmetry breaking effects is because of difference in mass between the strange and non strange quarks. The strange mass corrections are parameterized through the symmetry breaking parameter “r”. Here “r” depends on the ratio of mass of s and u quark. Very less contribution of the strange quark to the magnetic moments is seen in the calculated values. The reason being the higher mass of strange quark. The contribution of strange quark is an order of magnitude less than the up and down quarks. The results with $SU(3)$ symmetry breaking is not in much concurrence with the experimentally observed values. Plausibly, at energies 1GeV , the results are better (with $SU(3)$ symmetry) due to conservation of $SU(3)$ symmetry where $s \sim u \sim d$ is applicable. Statistical model and detailed balance principle are used in combination to study the strange quark importance to the magnetic moment of decuplets. To check the contributions from strange and non-strange quark sea, strange sea is being added. We have put the mass reduction coefficient $(1 - C_l)^{n-1}$ where $C_l = \frac{2M_s}{M_B - 2M_s}$ (M_B = mass of baryon, M_s = mass of strange quark, n is the total number of partons), to the decuplet to accommodate $s\bar{s}$ pairs in the sea.

- **chapter 6**, provides a summary and brief outlook.

1

Introduction and Overview

1.1 High Energy Physics

The challenge of high energy physics is to discover what our world is made up and how it works. Particle physics helps us to explore the undiscovered universe from the smallest particles called “quarks”. Many questions are still unanswered despite advances in our knowledge like: what is mass and why there is more matter than antimatter? Though no single experiment can address all of these questions, but still Large hadron collider has been able to produce higgs boson or evidence of some other mass-generating mechanism. The T2K and MINOS experiments have been able to detect neutrinos which are the most elusive of the elementary particles because they have hardly any mass and are difficult to detect. Particle physics is the science which aims at the basic constituents of matter and their interactions.

The search for the simplest, most basic objects has led to the study of matter at very small distances. The forms of matter on these short distance scales are called particles. Interactions of the particles must be described by quantum mechanics and the present state of experimentation has allowed the study of physics at distance scales down to about 10^{-16} cm. These studies typically require high energies; in fact, the energies in such particle processes are frequently large as compared to the masses of the particles involved, implying relativistic motion. As a consequence, the production of new particles through the interactions is typical, since the large kinetic energy carried by the interacting particles can provide the energy for the creation of additional particles.

Particle physics is also called high energy physics as many elementary particles in nature do not prevail in normal conditions, they can be created by energetic collisions with other particles. These collisions are performed in scattering experiments, where, the kinetic energy of the collision of two particles can be used to create new particles. Further, only high energies are able to probe small structures and its substructures as suggested by the De Broglie relation, $\lambda \sim 1/p$ (λ is wavelength and p is momentum). To resolve distances of 10^{-15} m i.e. on the scale of the atomic nucleus, requires energies at the scale of GeV . This is the energy scale of high-energy physics. In general, to unravel the constituent structure of particle, say proton, we need a probe with simple structure. Such a probe can be lepton of high energy and scattering with $q^2 = \lambda_{QCD}^2$. This kind of electron-proton scattering is called deep inelastic scattering, indicating that momentum transfer is large. The function of momentum transfer which shows the interaction between quarks and gluons in QCD is defined as coupling constant in $\alpha_s(\mu)$ QCD and is given by:

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2N_q) \ln\left(\frac{\mu^2}{\lambda_{QCD}^2}\right)}$$

where N_q is the number of quark flavor and μ is the energy-scale parameter known

as the renormalization point. Electron-proton scattering is important because it established the fact that proton has light constituents which interact in a point-like fashion. These constituents were identified as light quarks and gluons which are needed to understand the spectrum of hadronic states.

Particle physics is based on experiments searching the most elementary constituents of matter. This means that it involves probing at very small length scales (order of 10^{-15} m or less) and the masses involved are of order of 10^{-27} kg. In general, the standard system of units in physics is the International System of Units (*SI*). In particle physics we use a system of units called Natural Units, obtained by setting $c = \hbar = 1$. Their application can simplify notations of particle physics. Since particle physicist is concerned with particles both in relativistic and quantum mechanics, a multitude of \hbar 's and c 's will encumber the large equations if natural units are not adopted. The preferred length unit in particle physics is the femtometer (or fermi), where $1 \text{ fm} = 10^{-15} \text{ m}$. For example, the radius of proton is 1.0 fm and $1 \text{ GeV}^{-1} = 0.1975 \text{ fm}$. Energies are measured in *GeV*, or giga-electron volts ($1 \text{ GeV} = 1.6 \times 10^{-10} \text{ J}$). The choice of *GeV* is made because the rest mass energy of a nucleon is approximately $1 \text{ GeV}/c^2$.

1.2 The Standard Model

Particle physics is concerned with the basic constituents of matter either experimentally or theoretically. The success arises from a succession of pioneering experiments that would provide crucial hints for the theorists. The development of new accelerators and detectors in the last few decades in parallel with theoretical ideas has led to development of viable theories and models. The Standard Model (SM) is a gauge quantum field theory (QFT) based on $SU(3)_{\text{Color}} \times SU(2)_{\text{weak isospin}} \times U(1)_{\text{hypercharge}}$ group. Here, $SU(3)_{\text{Color}}$ part is gauge group of strong interactions between quarks whereas $SU(2) \times U(1)$ group combines to give the electroweak interactions. Each

group represents a gauge interaction, with associated gauge boson, for the three fundamental interactions of particles. This model specifies the basic particles and their interaction [1]. It was created in the 1960-1970s, and postulates that the matter in the whole universe is made up of elementary particles which interact by fundamental forces. The particles can be divided into two general categories: fermions having half-integral spin and bosons with integral spin. Fermions can further be categorized into quarks and leptons. Six quarks and six leptons are listed in the standard model. The quarks are named as up, down, strange, charm, bottom and top. The leptons are named as electron, the electron-neutrino, the muon, muon-neutrino, tau and tau-neutrino. The second group i.e. bosons are associated with the interacting fields and contains photons, gluons, W^+ , W^- and Z^0 bosons and the recently discovered Higgs boson (spin-0 particle).

Fundamental constituents of the matter interact through the fundamental forces. Four fundamental forces in nature are: strong, electromagnetic, weak and gravitation [2, 3]. These fundamental forces are mediated by the exchange of a boson. The strong force mediator is called gluon, electromagnetic forces are mediated by the photon, weak forces by intermediate vector bosons, W and Z and gravitational force is mediated by graviton. The strong interaction between quarks $\simeq 100$ times $>$ electromagnetic interaction at distances of the order of the diameter of a proton. On the other hand, the strong coupling between quarks becomes weak at smaller distances, and the quarks start to behave like free particles. This effect is known as asymptotic freedom. Both weak and electromagnetic interactions affect leptons and hadrons and are mediated by 1^- particles.

1.2.1 Beyond the Standard Model

Despite its great predictive power, the standard model has many perceived limitations. Its shortcomings have led to study of extended theories, commonly called as the physics Beyond the Standard Model (BSM). Gravity is not included in this

model. This fourth and weakest force of nature does not seem to have any impact on the subatomic interactions which the Standard Model explains. Another shortcoming of the Standard Model is that it does not describe the observed matter-antimatter asymmetry, usually called baryon asymmetry, which comprises one of the greatest mysteries of the universe. Also, the Standard model does not explain the origin of particle masses. So although the Standard Model accurately describes the phenomena within its domain, it is still incomplete. Perhaps it is only a part of a bigger picture that includes new physics hidden deep in the subatomic world or in the dark recesses of the universe.

1.3 Symmetries and Group Theory

Symmetries and groups plays an important role in particle physics. The presence of symmetries leads to physical simplicity at classical and quantum levels. Every symmetry of nature leads to a conservation law; or conversely we can say that every conservation law reveals an underlying symmetry. Symmetries are an essential part in the formulation of quantum field theories which helps in describing the elementary particles and their generations. Group theory is the branch of physics which underlies the treatment of symmetry and symmetry is defined as the operation which do not change the state of system. Groups help us to formulate symmetries. And groups are represented by matrices. These matrices are in one to one correspondence with each element of the group. All the theoretical approaches involving groups assumes that the particles belong to decomposition of some basic group ($SU(N)$ in our case) to irreducible representations, hence, forming a multiplet. Therefore, they have the properties like the mass, spin and parity. The most common groups in elementary particle physics are the groups of unitary matrices, $U(N)$, i.e. the collection of all unitary $N \times N$ matrices, where $N= 1, 2, 3, \dots$ and N^2 are the generators of the group $U(N)$. $N^2 - 1$ are the generators for $SU(N)$ due to the unimodularity condition

and unit determinant i.e. $U^\dagger U = U U^\dagger = 1$. It is $SU(2)$ in isospin invariance and $SU(3)$ in the eightfold way.

1.3.1 Special Unitary Groups ($SU(2)$ and $SU(3)$)

The special unitary group $SU(2)$ is the special unitary group of all 2×2 unitary matrices with determinant equal to one. The simplest example of a multiplet of $SU(2)$ is a two-dimensional representation where an object can exist in two possible states namely (u, d) or up and down. For example, the proton and neutron is represented in such a way in $SU(2)$ isospin group. The generators for $SU(2)$ are denoted by $J_i = \frac{1}{2}\sigma_i$ where $i = 1, 2, 3$, σ_i are called the Pauli matrices, in the fundamental representation.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (1.1)$$

The generators will give the commutation relation:

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

where ϵ_{ijk} is total antisymmetric Levi-Civita tensor and $\epsilon_{123} = 1$. The basis for this representation are eigen vectors of σ_3 i.e. column vectors

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.2)$$

describing a spin $1/2$ particle of spin projection up ($m = +\frac{1}{2}$ or \uparrow) and spin projection down ($m = -\frac{1}{2}$ or \downarrow) along the z-axis, respectively. The Pauli matrices σ_i are hermitian, and the transformation matrices $U(\theta_i) = e^{-i\theta^i\sigma_i/2}$ are unitary. The set of all unitary 2×2 matrices is called as the group $U(2)$.

SECTION 1.3: SYMMETRIES AND GROUP THEORY

The set of special unitary 3×3 matrices with $\det U = 1$ form a subgroup of $SU(3)$. $SU(3)$ symmetry group is higher symmetry group compared to $SU(2)$ group. This group incorporates isospin, hypercharge together and contains $SU(2)_T \times U(1)_Y$ as a subgroup. A computation of the $SU(3)$ breaking is the mass splitting within the multiplet, e.g. $(m_{\Delta^+} - m_{\Sigma^{*+}})/(m_{\Delta^+} + m_{\Sigma^{*+}}) \approx -0.06$. There exist two different $SU(3)$ symmetries which are appropriate for the strong interactions: First is the $SU(3)$ color symmetry for quark and gluon dynamics and other one is the $SU(3)$ flavor symmetry for light quarks. In this representation, the generators are $3^2 - 1 = 8$ linearly independent hermitian 3×3 matrices. They are denoted by λ_a , where $a = 1, 2, \dots, 8$, known as Gell-Mann matrices.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The normalization for them is written as:

$$\text{tr}(\lambda_a \lambda_b) = 2\delta_{ab}$$

and satisfy the commutation relation:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc} \frac{\lambda_c}{2}.$$

where f_{abc} are the real numbers and are called the structure constants of $SU(3)$. So, in general, $SU(2)$ describes the isospin symmetry of the weak interaction and $SU(3)$ describes the color symmetry of the strong interaction.

1.4 Quark Model

The interactions of neutrons and protons are studied at low energies (few tens of MeV) but in 1947, new types of hadrons, not present in ordinary matter, were discovered in cosmic rays by groups from the universities of Bristol and Manchester, which require high energies, in accordance with Einsteins mass-energy relation $E = mc^2$, and as intense beams of particles of higher energies became available at accelerator laboratories, more and more hadrons were discovered. Later in 1960s, several other hadrons were known, and some theoretical framework was needed to interpret these states of particles for any progress. So, the quark model came into picture.

In 1964, Gell-Mann and Zweig [4, 5] proposed independently a model in which all particles were made up of more elementary ones, called quarks. The quarks are basic constituents of hadrons, having a fractional charge $2/3$, $-1/3$ and $-1/3$, in units of e and follows the Fermi statistics. The u , d and s correspond to vectors which form a three-dimensional representation of $SU(3)$. When symmetry breaks and reduced to the $SU(2)$ subgroup, the space of states breaks into two invariant subspaces, a two-dimensional corresponding to iso-doublet (u, d) and isosinglet (s). In this way the whole spectrum of existing particles was organized in Meson and Baryon multiplets, according to the Irreps of $SU(3)$, but a particle, named Δ^{++} ,

could not be classified because it resulted from three uuu quarks, violating the Pauli principle. In order to solve the problem, one possibility was the hypothesis that each quark possesses another internal degree of freedom: color charge. The idea of color was introduced in mid 1960s by MooYoung Han and Yoichiro Nambu [6] as well as Oscar W Greenburg [7]. Quarks come in color red, green or blue. The color symmetry is, just like the flavor symmetry, an $SU(3)$ symmetry. Hadrons can exist if they are colorless i.e. if they are invariant under an $SU(3)$ color transformation. For the known quarks we have:

$$u_{rgb} = (u_r, u_g, u_b)$$

$$d_{rgb} = (d_r, d_g, d_b)$$

$$s_{rgb} = (s_r, s_g, s_b)$$

$$c_{rgb} = (c_r, c_g, c_b)$$

$$b_{rgb} = (b_r, b_g, b_b)$$

Here, r, g, b denotes red, green and blue color. According to this model proton was composed of a red u quark, a blue d quark and a green u quark. A neutron had as constituents a green u quark, a red d quark, and a blue d quark. The five types of flavor quarks correspond to five different color triplets. Later, existence of top quark was confirmed in 1995 by the CDF [8] and DO [9] experiments at fermilab. Hence, the top and bottom quark together forms the third generation of quarks. The quark model was emerged to describe the regularities seen in the hadron spectrum. This model also describes the dynamics of hadrons at high energy. Baryons is a bound state of three quarks and mesons constitute one quark and one antiquark.

Once the quark structure of hadrons got studied, it became obvious to search for the dynamics obeyed by the quarks responsible for structure of hadrons and hadronic reactions. For this, a beam of structure less particles such as leptons was applied

to probe inside the hadron, (e.g., proton). Study of hadronic structure with higher resolution is possible with higher energies and larger momentum transfers. The electromagnetic form factors are helpful in understanding of the internal structure of composite hadrons like the nucleon, as they contain the information about the distributions of charges and currents. The understanding of hadron form factors, especially for the nucleons and the pions, gives the information about their electromagnetic structure. Large and small distances can be studied by varying momentum transfer, which allows one to study hadronic physics. De-Broglie wavelength of an electron becomes much shorter than the size of a typical nucleus at sufficiently high energies in GeV range. In such cases, the scattering result is dominated by the charge distributions within individual nucleons. The primary interest of scattering at these energies shifts to the structure of nucleon rather than that of nucleus.

The presence of quarks as real particles was seriously doubted because of failure of all attempts to detect free quarks, or any other fractionally charged particles. These doubts were subsequently removed in two ways. Firstly, a series of experimental results showed, the dynamical effects of individual quarks in the proton with the scattering of high-energy electrons from protons, in 1968. Secondly, a detailed theory of strong interactions was constructed which was called as quantum chromodynamics (QCD). This theory successfully described the experimental results and tried to explain why isolated free quarks could not be observed. As a result of these developments, the quark hypothesis is now universally accepted. Although only three quarks were initially proposed, six are now known to exist.

We now study the simplest multiplets for composite states. States made up of a colored quark and a colored antiquark can come in many states that fall into two multiplets, one with one state (singlet) and one with eight states (octet). Symbolically, it can be written as:

$$3 \otimes \bar{3} = 1 \oplus 8$$

This is the $SU(3)$ vector addition. For a state made of two quarks the multiplets

SECTION 1.4: QUARK MODEL

are:

$$3 \otimes 3 = \bar{3} \oplus 6$$

Since these multiplets are made of identical objects, they have a given permutation symmetry. The $\bar{3}$ is anti-symmetric, the 6 symmetric. A state made of three quarks having three flavors and three colors can form the following multiplets:

$$\begin{aligned} 3 \otimes 3 \otimes 3 &= (6 \oplus \bar{3}) \otimes 3 \\ &= 10_S \oplus 8_{MS} \oplus 8_{MA} \oplus 1_A \end{aligned}$$

Here, S , A denotes symmetric and anti-symmetric states respectively and MS , MA denotes mixed symmetric and mixed anti-symmetric states respectively. Thus, baryons appear as octets and decuplets whereas mesons appear as singlets and octets. In terms of weight diagram, the baryon octet and decuplet are represented in the figure 1.1 and 1.2.

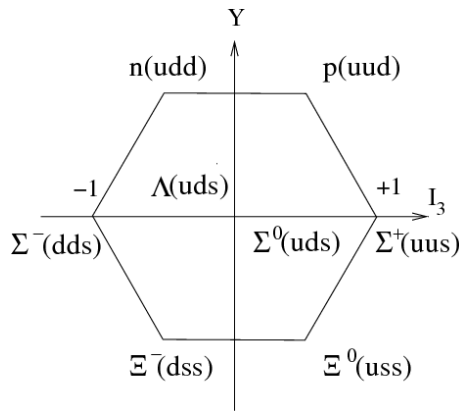


Figure 1.1: $J^P = \frac{1}{2}^+$ Octet

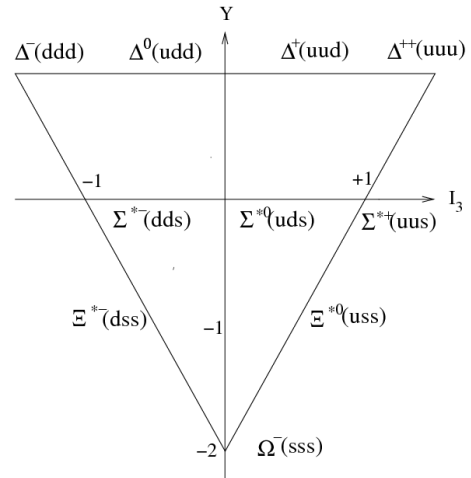


Figure 1.2: $J^P = \frac{3}{2}^+$ Decuplet

1.5 Young Tableaux

A Young tableau or Young diagram describes the symmetry of a collection of an integer identical particles, each of which can be in one of several available states. Young Tableaux method/technique is applicable for arbitrary $SU(N)$, hence, making it easy to deduce the dimensions of the irreducible representations arising from products of other representations of the group. For the study of general case of $SU(N)_f$, utilization of this method is convenient. The bound state of three quarks form a basis for some representation of the $SU(2)_{spin}$ and $SU(3)_{flavor/color}$ group. The fundamental representation of a $SU(N)$ group (single quark) is denoted by a box,

$$\square = \text{dimension } N \quad (1.3)$$

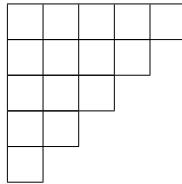
In $SU(2)$,

$$\square = 2 \text{ or } 2^* \quad (1.4)$$

but in $SU(3)$,

$$\square = 3 \text{ and } \begin{array}{c} \square \\ \square \\ \square \end{array} = 3^* \quad (1.5)$$

So, a Young tableau represents left-justified rows where any row is not longer than the row on top of it, e.g.



Here, any column cannot have more than N boxes. The fundamental representations are same in $SU(2)$ but are different for $SU(3)$ and higher groups. Only one row denotes symmetric representation whereas column is associated with antisymmetric representations. So, the decomposition rule for a three particle system in terms of

irreducible representations can be obtained as:

$$\square \otimes \square \otimes \square = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array}_A \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \square & \\ \hline \end{array}_M \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline & \square \\ \hline & \\ \hline \end{array}_M \oplus \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}_S \quad (1.6)$$

The corresponding dimension d of each representation can be obtained by the following ratio: $d = num/den$. We start writing the number N in the top left box of the Young tableau for the numerator. Then, moving to the right, write the number increased by a unit at each step. Moving to the bottom, write the number decreased by a unit at each step. The numerator is obtained by the product of the entries in each box. For denominator, we start writing in each box the number of boxes to its right plus the number of boxes beneath it plus a unit (the hook length). The

denominator is obtained by the product of the entries in each box. E.g.

6	4	1
4	2	
3	1	
1		

. So,

$$\text{dimension } d = \frac{N(N+1)(N+2)(N-1)N(N-2)(N-1)(N-3)}{6 \times 4 \times 4 \times 2 \times 3}$$

An advantage of this method is that each Young tableau with n boxes defines an irreducible representation of the group of permutations of n objects (S_n) and therefore belonging to a definite symmetry type. In eq. (1.6) the labels A , M , S refers respectively to the antisymmetric, mixed and symmetric character of each representation. The $SU(3)$ flavor or color states are constructed using the scheme given in (1.6); the resulting combinations and symmetry types yield:

$$\text{Flavor or Color : } SU(3) : 3 \otimes 3 \otimes 3 = 1_A \oplus 8_M \oplus 8_M \oplus 10_S \quad (1.7)$$

$$\text{Spin and Flavor/Color : } SU(6) : 6 \otimes 6 \otimes 6 = 20_A \oplus 70_M \oplus 70_M \oplus 56_S \quad (1.8)$$

Here, the 1_A representation is realized for color. The baryons must be colorless, that means that the color part must be a singlet, the $SU(3)$ singlet is completely antisymmetric by exchange of particles so that to fulfill the Pauli's principle the non-

color part of the $3q$ wave function must be symmetric, this means, in particular, that the spin-flavor and the spatial part of the $3q$ wave function must be of the same symmetry type. The low-lying baryons together make up the ground-state 56_S representation in equation (1.8), in which the orbital angular momenta between the quark pairs are zero (so that the spatial part of the state function is trivially symmetric). The 70 and 20 require some excitation of the spatial part of the state function in order to make the overall state function symmetric.

1.6 Quantum Chromodynamics

Quantum Chromodynamics (QCD) is a quantum field theory of the strong interactions. This theory involves renormalizable Lagrangian. With its unmatched richness after quantization and latent non-abelian $SU(3)$ color symmetry, as revealed by a spectrum of different behavior over a range of energy scales, i.e. from confinement to asymptotic freedom, this theory becomes an exception at the classical extent. Fundamental constituents of QCD are quarks having spin $\frac{1}{2}$ and gluons having spin 1 which interact with the quarks as well as among themselves. The four-component Dirac fields $q_c^f(x)$ denotes the quarks and the quark masses are shown by m_f . The QCD Lagrangian is written as:

$$L_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G_{\mu\nu,a}G_a^{\mu\nu}$$

with $q_f^x = (q_f^{red}, q_f^{green}, q_f^{blue})$. Each of the flavors (u, d, s, c, b, t) has a distinct mass which is denoted by m_f , the mass parameter in the Lagrangian. This Lagrangian is invariant under local transformations in color space. The gauge covariant derivative is given by:

$$(D_\mu)_{ab} = (\delta_{ab})_\mu + ig_s(t^c A_\mu^c)_{ab},$$

SECTION 1.7: FUNDAMENTAL SYMMETRIES AND THEIR BREAKING

where the gluon fields $A_\mu^c(x)$ are eight independent vector bosons belonging to the adjoint representation of $SU(3)_c$ to fulfill local gauge invariance. The gluonic field-strength tensor $G_{\mu\nu}^{a(x)}$ is given by:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c,$$

Quantum number namely color is assigned to each of the quark fields in the Lagrangian and is denoted by the indices i, j, \dots . These indices can take the colors red, green or blue. Origin of strong interactions is caused by this so-called color charge. The associated interaction strength is calculated by the coupling parameter g .

In 1973, Wilczek, Gross and Politzer [10,11] found that QCD is an asymptotic free theory which means that the higher the energy scale μ , smaller the coupling $g(\mu)$. This fact results in the possibility of perturbatively expanding QCD matrix elements in powers of coupling parameter, g . This also means that the quarks behave like free particles at high energies. On the contrary, the coupling g is large at low energies (below 1 GeV), which renders a perturbative treatment of QCD impossible. Since QCD has the non-Abelian nature, it give rise to gluon self interactions. The basic scheme of this theory is to make the $SU(3)$ color symmetry a local, rather than just a global symmetry.

1.7 Fundamental Symmetries and their Breaking

Symmetries can be exact, approximate, or broken. Symmetry is exact in nature if a number of fields, under a given symmetry, transform like a multiplet. The particles in that field should have same mass, for example, proton and neutron would have the same mass under exact symmetry as they form a doublet under an isospin symmetric group. Masses of proton and neutron vary by 1 MeV that leads to an isospin symmetry and because of this small symmetry, it is called an approximate.

With respect to group theory, symmetry breaking implies that the initial symmetry group breaks to one of its subgroups. Symmetry breaking are of two different types: “explicit” and “spontaneous” symmetry breaking.

1.7.1 Explicit and Spontaneous Symmetry Breaking

Explicit symmetry breaking is due to application of an external force which disturbs the equation of motion of the system, thereby, breaks the symmetry. However, such external forces are not required in spontaneous symmetry breaking, that is why the term spontaneous is applicable. For example, the rotational symmetry of a rod about its axis. An application of external force to one end of the rod implies that the symmetry can be broken and hence, the rod releases its rotational symmetry on bending. This is termed as explicit symmetry breaking. The spontaneous symmetry breaking is realized when a force is applied in the longitudinal direction of the rod.

So, a symmetry is said to be spontaneously broken if ground state of the system is no longer invariant under the full group of the Hamiltonian. It is important to note two properties of spontaneously broken system. Firstly, the direction of bending of rod is arbitrary and there are many ground states (degenerate) that are equally well occupied. Other property to note is that the different directions of bending (the different ground states) and the original rotational symmetry are relatable. Hence, a symmetry is said to be spontaneously broken if the ground state of the physical system is not invariant under the group of the Hamiltonian (which manages the dynamics of the physical system). In general, the physical system as a whole changes but the underlying laws do not change. An example of the spontaneous symmetry breaking is the Higgs mechanism.

1.7.2 Chiral Symmetry Breaking and Goldstone Theorem

Chiral symmetry is a type of continuous global symmetry of Lagrangian in QCD. This is an internal symmetry of right and left handed spinors. It has importance in low energy hadronic physics, since its spontaneous breaking generates Goldstone bosons having zero spin, negative parity, zero baryon number and unit isospin called pions. Therefore, existence of pions is justified by a broken approximate chiral symmetry. In that case, u and d quarks have small but non-zero masses. The transition from the fundamental to the effective scale happens via a phase transition due to spontaneous symmetry breaking generating Goldstone boson. Since the masses of light quarks are small with respect to Λ_{QCD} , let us set the values of these quark masses zero in the first approximation and moreover, make the masses of heavy quarks, m_c , m_b and m_t to be infinity. At low energies, the quarks are categorized into light u , d , s and heavy quarks c , b , t , by fulfilling a hierarchical ordering:

$$m_u, m_d, m_s < 1\text{GeV} \leq m_c, m_b, m_t$$

The heavy quarks are treated as static at 1 GeV . Here, by using the three light quarks, we approximate the QCD Lagrangian as these quarks are the only active degrees of freedom of QCD for the low-energy scale that we are interested in. The light quark masses are small when compared with the hadronic scales, for example the nucleon mass is 939 MeV . Therefore, our study of low-energy QCD begins with massless quarks $m_u = m_d = m_s = 0$, called as the chiral limit. In this limit, QCD Lagrangian LQCD becomes invariant under the following group of (space time independent) transformations which act on the three flavor indices (u, d, s):

$$q = q_L + q_R$$

$$q_R = \frac{1}{2}(1 + \gamma_5)q, q_L = \frac{1}{2}(1 - \gamma_5)q$$

The above group of transformations is $SU(3)_R \times SU(3)_L$ and the resulting symmetry of the QCD Lagrangian is called chiral symmetry of QCD. In QCD, chiral symmetry

is the relevant symmetry that is spontaneously broken, and the associated order parameter is the quark-antiquark condensate.

For example, consider the case of weak decays of pions. In the Fermi theory, weak interaction Hamiltonian is represented by sum of axial and vector currents. Because of parity, weak decay of pion is controlled by the axial current between pion and vacuum. The pion mass is small compared to hadronic scales, the axial current is approximately conserved. On the one hand, meson mass spectrum does not reflect the axial-vector symmetry but on the other side, weak decay of pion seems to be consistent with the partial conservation of axial current. This leads to the conclusion that axial current is spontaneously broken. The finite value of u , d and s quark masses in the QCD lagrangian explicitly break the chiral symmetry, resulting in divergences of symmetry currents. Chiral symmetry breaking (CSB) is a non perturbative phenomenon, which is known to govern the low energy properties of hadrons. The effective chiral Lagrangians have been proposed before the advent of QCD and the phenomenon of CSB and Nambu-Goldstone theorem was established more than 40 years ago.

1.8 Phenomenological Models

Phenomenological models are theoretical models that relate directly with the measurable quantities. In particular, it applies general principles and methods to calculate the quantities that can directly be compared with the observations. Due to this kind of application a large number of unknown parameters have turn up in these extended models leading us to go beyond the standard model. So, phenomenologist frames a model by keeping in mind all the interactions possible between particles, the best way to measure these unknown parameters.

Constituent quark models assume baryons to be made up of three constituent quarks in non-relativistic limits which calculates the magnetic moment of baryons. Further

calculations on baryons were done by adding sea quark contributions and quark orbital momentum effects. C. P. Singh [12] et. al., calculated the masses of charmed and b-quark hadrons by including the flavor and spin dependent hyperfine interactions between two quarks and a quark and an antiquark in the non relativistic additive quark model. The results were matching with the available data.

Another important approach is the chiral constituent quark model (χCQM) [13,14] which is useful in the nonperturbative regime of QCD. This model employs the effective interaction Lagrangian approach. The phenomenon of chiral symmetry breaking, quark-antiquark excitations and its spontaneous breaking are also implemented in this approach. Chiral quark model is used to determine the magnetic moment for the low energy and excited states having spin $1/2^-$ and $3/2^-$ resonances respectively, as well as their transitions. The quark cluster model [15] is applied to the baryon-baryon interaction in terms of quark-quark interaction. Quark exchange and quark-quark interaction between the two nucleons describes the short-range repulsion and spin-orbit forces in the nucleon-nucleon potential. The properties of baryons are an essential tool to understand hadronic structure. The structure and properties of baryons are examined by one of the most peculiar property called magnetic moments. Continuous efforts are being made theoretically and experimentally, to understand these properties of the hadrons.

Other models studying properties of baryons are, the relativistic quark model (RQM) [16, 17], QCD-based quark model (QCDQM) [18], effective mass scheme (EMS) [19,20], light cone QCD sum rule (LCQSR) [21], QCD sum rule (QCDSR) [22–25], Skyrme model [26, 27] etc. Some meson cloud models like the meson convolution models, the cloudy bag model and chiral models [28, 29] studies the peculiarity of Fock states in the baryonic wave functions. Several attempts have been made to study the higher Fock state importance by taking the help of the constituent quark model. A small number of higher Fock components was introduced by Riska and coworkers, which were fitted to reproduce the experimental data [30].

Isgur and Karl Model [31] suggested that the interaction between quarks are expressed in form of a harmonic oscillator incorporated with hyperfine interactions and anharmonic perturbation. Several potential models [32–35] have used and gone beyond Isgur and Karls model to study the wave functions and spectrum of baryons. This attempt was made to rectify the flaws in the non relativistic model. An example being the relativized quark model which was first applied to mesons [36] and then applied to baryons [37]. In this type of model, the Schrodinger wave equation is solved in Hilbert space which was made up of valence quarks with finite spatial extent.

So, numerous advanced techniques/models have been developed to analyze the properties of hadrons. These properties are studied with and without strangeness effects. These predictions motivated experimentalist to determine more properties of existing particles as well as search new particles.

1.9 Experimental Updates

Studying the hadronic internal structure in the nonperturbative regime of quantum chromodynamics (QCD) is, so far, a challenging area theoretically and experimentally. Low energy properties like magnetic moments, masses, charge radii can provide valuable perception of the nonperturbative aspects of QCD and underlying dynamics. In the last few years, exclusive scattering processes like deep virtual meson production (DVMP) or deep virtual Compton scattering (DVCS) has proven to be a successful way to probe the structure of hadrons.

Heavy-ion collision experiments taking place in FSI at EMC [38], CBM at FAIR [39], etc., aims towards the study of matter in the presence of medium as well as free space. Valence quarks of proton should be surrounded by a sea which constitutes quark-antiquark pairs as these valence quarks carry only a small fraction of its spin. This was suggested in the deep inelastic scattering (DIS) experiments [40–42]

by the measuring the proton's polarized structure functions. So, the information provided by these experiments becomes more relevant if we include the detailed structure of hadron. The study of flavor distribution functions in the New Muon Collaboration [43,44] and E866 experiments [45–47] reveals that the flavor structure is not limited to only u and d quarks for the nucleon.

Experimental information of baryons on the decays, properties and reactions has accumulated since the emergence of the first accelerators. Due to the upcoming information from these experiments, hadrons are the current and interesting topic of research. At present, experiments are planned or are taking place in Laboratories like TJNAF (USA) in photo production reactions (CLAS), CERNSPS (Europe) with hadron beams (COMPASS), LNF (Italy) in kaonic atoms (SIDDHARTA), GSI (Germany) in pp collisions (HADES) etc. More experiments are scheduled in CERN-SPS (NA-62), J-PARC or GSI (pion beam).

Despite of the many mysteries, these experiments have provided a useful data which helped to explore the hadronic structure. The data from the experiments have created a substantial interest in understanding the particle physics. Overall results and predictions help in attaining a clear picture of the baryon structure, hence, motivating experiments for further investigation.

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2

Statistical Model and Applications

Many calculations have been performed in the last few years with the aim of understanding the baryon spectrum. In order to understand baryon spectroscopy, the low energy properties should be essentially understood. A simple yet unique concept of particle physics, i.e. quark model suggest that all baryons are made of three quarks and mesons composed of a quark and an antiquark.

A new state of matter called “pentaquarks” ($uudc\bar{c}$) have been observed at LHCb [1] bringing a revolution in the study of baryon spectroscopy. The search from the LHCb was motivated by the prediction made by theoretical approaches. Besides this, the prediction of lifetime of heavy baryon of spin $3/2$ at CMS [2] aids in understanding baryonic properties in a better way. Lots of advancement have been seen both theoretically and experimentally for studying and exploring the baryonic proper-

ties since the octet magnetic moments were predicted by Coleman and Glashow [3] about fifty years ago. These predictions motivated theorist and experimentalist to measure masses, baryon octet magnetic moments, spin distributions which are low energy properties [4]. European Muon Collaboration [5] and Spin Muon Collaboration [6] provides insight into the spin structure of the nucleon. The experimental information about decuplet baryons is limited because they have short lifetimes, so at present experimental data of Δ^{++} , Δ^+ , Ω^- are available [7–10]. The advancements in the experimental facilities at CDF [11] have become a subject of motivation to study baryon properties and hence its structure in the non perturbative Quantum Chromodynamics (QCD).

The aim of our thesis is to investigate baryon properties in the statistical model with certain assumptions. The model applicability is understood for fundamental properties of baryons like magnetic moments, masses and spin distributions. To calculate the low energy properties of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles, we assume the baryon to be comprised of a valence part and a virtual sea consisting of quark-antiquark pairs multiconnected by gluons. Certain modifications to the model are suggested which expand the limits of applicability of the statistical model in conjugation with principle of detailed balance and illustrate its strengths.

2.1 Wave-Function For Baryons

In non-relativistic quark model, baryons are considered as bound colorless system of three quarks which are fermionic particles with flavor, spin and color degrees of freedom. The structure of baryon constitutes two parts i.e valence part q^3 and other is sea part which consist of quark-antiquark pairs muticonnected through gluons [12–16]. A q^3 state in the baryon are in 1, 8 and 10 color states which means that sea should also be in the corresponding states to make a color singlet baryon. Taking into consideration the relevant degrees of freedom, the valence part of the

SECTION 2.1: WAVE-FUNCTION FOR BARYONS

baryonic wave function is formed and written as:

$$\Psi = \Phi(|\phi\rangle|\chi\rangle|\psi\rangle)(|\xi\rangle) \quad (2.1)$$

where $|\phi\rangle, |\chi\rangle, |\psi\rangle$ and $|\xi\rangle$ denote flavor, spin, color and space q^3 wave functions and their contribution make total wave function antisymmetric in nature. Here, in order to get the total antisymmetric wave function of the baryon, flavor $|\phi\rangle$, space ($|\xi\rangle$) and spin $|\chi\rangle$ are contributing their symmetric part, where as the color $|\psi\rangle$ contributes its anti symmetric part to obtain the anti symmetrisation of the total wave function. Properties of baryonic systems are studied using possible combination of q^3 and sea such as to give spin 1/2 (3/2) , flavor octet or decuplet and color singlet (1) state.

Sea considered here is in S-wave state with spin (0,1,2) and color (1,8, $\overline{10}$) and is assumed to be flavorless. Let $H_{0,1,2}$ and $G_{1,8,\overline{10}}$ denote spin and color sea wave functions, which satisfy $\langle H_i|H_j\rangle = \delta_{ij}$, $\langle G_k|G_l\rangle = \delta_{kl}$. In general, different possible states for valence and sea particles can be written as:

$$\mathbf{Spin} : qq\bar{q} : 2 \otimes 2 \otimes 2 = 4_S \oplus 2_{ms} \oplus 2_{ma}$$

$$gg \text{ case} : 1 \otimes 1 = 0_s \oplus 1_a \oplus 2_s$$

$$\mathbf{Color} : qq\bar{q} : 3 \otimes 3 \otimes 3 = 1_a \oplus 8_{ms} \oplus 8_{ma} \oplus 10_s$$

$$gg \text{ case} : 8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \overline{10}_a \oplus 27_s$$

$$ggg \text{ case} : 8 \otimes 8 \otimes 8 = 120_s \oplus 168_{ms} \oplus 168_{ma} \oplus 56_a$$

$$\mathbf{Flavor} : qq\bar{q} : 3 \otimes 3 \otimes 3 = 1_a \oplus (8_{ms} \oplus 8_{ma})_{Octet} \oplus (10_s)_{Decuplet}$$

Here, subscripts s and a denotes symmetry and antisymmetry under the exchange

of any two valence quarks $q_1 \leftrightarrow q_2$ or gluons and m_s, m_a are used to show mixed symmetric and mixed anti symmetric states under quark permutations $q_1 \leftrightarrow q_2$ or two gluons. We have assumed in our model that gluon and $q\bar{q}$ carry same quantum numbers. Here, color anti symmetrization is leading to the anti symmetrization of total wave function having qqq as valence quarks and gluons as sea. Color is obtained provided $qqq = \mathbf{1}_a \oplus 8_{ms} \oplus 8_{ma} \oplus 10_s$ is combined with $\mathbf{1}_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \overline{10}_a \oplus 27_s$ of gluons. We have not taken the higher multiplicities of color combinations. In general, we have assumed the whole system to be fermionic and a proper anti symmetrization can only be achieved from all of its constituents, here, we assume all quarks, antiquarks and gluons inside the baryon. For the spin part, since quarks have spin 1/2, the total spin S and the third component S_Z of the quark systems can be calculated by the addition of angular momentum. For baryons, the spin states of the qqq system is coupled to give total spin of $S = \frac{3}{2}$ i.e combining three $S = \frac{1}{2}$ states. All possible combinations results in $2^3 = 8$ states are as follows:

$$|S, S_Z\rangle = |((\frac{1}{2} \otimes \frac{1}{2})_{s_{12}} \otimes \frac{1}{2})_{s, S_Z}\rangle | \frac{3}{2}, \frac{3}{2} \rangle = \uparrow\uparrow\uparrow$$

$$| \frac{3}{2}, -\frac{3}{2} \rangle = \downarrow\downarrow\downarrow$$

$$| \frac{3}{2}, \frac{1}{2} \rangle = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

$$| \frac{3}{2}, -\frac{1}{2} \rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$

$$| \frac{1}{2}, \frac{1}{2} \rangle_+ = \frac{1}{\sqrt{6}}(2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$| \frac{1}{2}, \frac{1}{2} \rangle_- = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$| \frac{1}{2}, -\frac{1}{2} \rangle_+ = -\frac{1}{\sqrt{6}}(2 \downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow)$$

$$| \frac{1}{2}, -\frac{1}{2} \rangle_- = -\frac{1}{\sqrt{2}}(\downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow)$$

SECTION 2.1: WAVE-FUNCTION FOR BARYONS

In general, we get symmetric wave function when (symmetric-symmetric) or (antisymmetric-antisymmetric) wave-functions combines and we get antisymmetric wave function when (symmetric-antisymmetric) wave function combinations takes place. Therefore, taking these conditions into consideration, a total flavor-spin-color-space wave function for $J^P = \frac{1}{2}^+$ octets and $J^P = \frac{3}{2}^+$ decuplets with all possibilities of quark-gluon Fock states is constructed.

2.1.1 Baryon $J^P = \frac{1}{2}^+$ Octet

The possible combinations of valence q^3 valence and sea wave-functions resulting in spin 1/2, flavor octet (8) and color singlet (1) state thereby maintaining the anti symmetrization of the total wave-function are as follows [17]:

$$\begin{aligned} \text{Octet} = & \Phi_1^{(1/2)} H_0 G_1, \Phi_8^{(1/2)} H_0 G_8, \Phi_{10}^{(1/2)} H_0 G_{\overline{10}}, \\ & \Phi_1^{(1/2)} H_1 G_1, \Phi_8^{(1/2)} H_1 G_8, \Phi_{10}^{(1/2)} H_1 G_{\overline{10}}, \\ & \Phi_8^{(3/2)} H_1 G_8, \Phi_8^{(3/2)} H_2 G_8 \end{aligned} \tag{2.2}$$

$H_{0,1,2}$ and $G_{1,8,\overline{10}}$ denote spin and color sea wave functions. Here, $\Phi_8^{(3/2)}$ wave-function can give spin resultant 1/2 only if sea-part is having spin either one or two. All other possibilities like $H_2 G_1, H_2 G_{\overline{10}}$ are excluded because they do not give color singlet states. Contributions from states like $H_0 G_{27}, H_2 G_{27}$ are ignored due to higher multiplicities and less contribution. A single gluon Fock state in the sea with spin 1 and color octet state will contribute to only $H_1 G_8$ component of sea. So, the total flavor-spin-color of a spin up $J^P = \frac{1}{2}^+$ baryon octet consisting of valence

quarks and sea can be written as:

$$\begin{aligned}
\text{Octet} = |\Phi_{1/2}^{(\uparrow)}\rangle &= \frac{1}{N} [\Phi_1^{(1/2\uparrow)} H_0 G_1 + a_8 (\Phi_8^{(1/2)} \otimes H_0)^\uparrow G_8 + \\
&a_{10} \Phi_{10}^{(1/2\uparrow)} H_0 G_{\overline{10}} + b_1 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_1 + b_8 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_8 + \\
&b_{10} (\Phi_{10}^{(1/2)} \otimes H_1)^\uparrow G_{\overline{10}} + c_8 (\Phi_8^{(3/2)} \otimes H_1)^\uparrow G_8 + \\
&d_8 (\Phi_8^{(3/2)} \otimes H_2)^\uparrow G_8]
\end{aligned} \tag{2.3}$$

where

$$N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2 \tag{2.4}$$

Where N is the normalization constant. Here, a_8 , a_{10} , b_1 , b_8 , b_{10} , c_8 and d_8 are the coefficients that represent each baryon in octet. Also, these coefficients represents the possibilities with which every color, flavor and spin of the octet baryons is obtained. Let λ denotes symmetry under interchange of any two quarks in q^3 wave-function whereas ρ denotes anti-symmetry for $q_1 \rightarrow q_2$. So,

$$\Phi_1^{(\frac{1}{2})} = \Phi(8, \frac{1}{2}, 1_c) = F_S \psi_1^A$$

$$\text{where, } F_S = \frac{1}{\sqrt{2}} (\phi^\lambda \chi^\lambda + \phi^\rho \chi^\rho)$$

Here, $\Phi_1^{(\frac{1}{2})}$ is for spin $\frac{1}{2}$ color singlet and 8 represents the flavor part. It can be written as a product of two functions F_S and ψ_1^A (contributing for color of baryons being anti-symmetric in nature) and F_S denotes flavor and spin of the q^3 wave function. The subscripts S and A represents the symmetry and anti-symmetry of wave-function. For F_S to be symmetric, ϕ and χ should either be both symmetric

SECTION 2.1: WAVE-FUNCTION FOR BARYONS

(λ) or antisymmetric (ρ) in nature.

$$\Phi_8^{(\frac{1}{2})} = \Phi(8, \frac{1}{2}, 8_c) = \frac{1}{\sqrt{2}}(F_{MS}\psi_8^\rho - F_{MA}\psi_8^\lambda)$$

$$\text{where, } F_{MS} = \frac{1}{\sqrt{2}}(\phi^\rho\chi^\rho - \phi^\lambda\chi^\lambda)$$

$$\Phi_{10}^{(\frac{1}{2})} = \Phi(8, \frac{1}{2}, 10_c) = F_A\psi_{10}^S$$

$$\text{where, } F_A = \frac{1}{\sqrt{2}}(\phi^\lambda\chi^\rho - \phi^\rho\chi^\lambda)$$

$$\Phi_8^{(\frac{3}{2})} = \Phi(8, \frac{3}{2}, 10_c) = F_{A'}\chi^{\frac{3}{2}}$$

$$\text{where, } F_{A'} = \frac{1}{\sqrt{2}}(\phi^\lambda\psi^\rho - \phi^\rho\psi^\lambda)$$

In above equations, $\Phi_8^{(\frac{1}{2})}$, $\Phi_{10}^{(\frac{1}{2})}$ represents the wave-functions with spin 1/2 whereas $\Phi_8^{(\frac{3}{2})}$ is for spin 3/2 respectively. Each wave-function is a composition of two parts such that one is symmetric and other is anti-symmetric. Also, the first three terms in the eq. (2.3) are calculated by combining q^3 wave function with spin 0 (scalar sea) and next three terms are result of coupling q^3 with spin 1 (vector sea). The last two terms is the outcome of coupling with spin 2 (tensor sea). We name the sea as scalar, vector and tensor sea with Fock states having spin 0, 1 and 2 respectively.

The seven coefficients ($a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8$) in equation (2.4) are to be determined statistically from above wave-function to study the low energy properties. To calculate the various baryonic properties like magnetic moments, spin distribution, suitable operator \hat{O} is applied on the wave function. This operator depends on quark flavor and spin and does not depend on color and space.

$$\begin{aligned}
\langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle &= \frac{1}{N^2} [\langle \Phi_1^{(1/2\uparrow)} | \hat{O} | \Phi_1^{(1/2\uparrow)} \rangle + \sum_{i=8,10} a_i^2 \langle \Phi_i^{(1/2\uparrow)} | \hat{O} | \Phi_i^{(1/2\uparrow)} \rangle + \\
&\sum_{i=1,8,10} b_i^2 \langle \Phi_{bi}^{(1/2\uparrow)} | \hat{O} | \Phi_{bi}^{(1/2\uparrow)} \rangle + 2b_8 c_8 \langle \Phi_{b8}^{(1/2\uparrow)} | \hat{O} | \Phi_{c8}^{(1/2\uparrow)} \rangle + \\
&c_8^2 \langle \Phi_{c8}^{(1/2\uparrow)} | \hat{O} | \Phi_{c8}^{(1/2\uparrow)} \rangle + d_8^2 \langle \Phi_{d8}^{(1/2\uparrow)} | \hat{O} | \Phi_{d8}^{(1/2\uparrow)} \rangle]
\end{aligned}$$

Any operator $\hat{O} = \sum_i \hat{O}_f^i \sigma_Z^i$ where \hat{O}_f^i depends upon on the flavor of i^{th} quark and σ_Z^i is the spin projection operator of i^{th} quark. $\langle \hat{O}_f^i \rangle^{\lambda\lambda} = \langle \phi^\lambda | \hat{O}_f^i | \phi^\lambda \rangle$, $\langle \sigma_Z^i \rangle^{\rho\uparrow\rho\uparrow} = \langle \chi^{\rho\uparrow} | \hat{O}_f^i | \chi^{\rho\uparrow} \rangle$ and $\langle \hat{O}_f^i \rangle^{\lambda\rho} = \langle \phi^\lambda | \hat{O}_f^i | \phi^\rho \rangle$ where λ denotes the symmetric wave-function and ρ denotes the antisymmetry of the wave-function. The wave function mentioned above is rewritten in terms of parameters a , b , c , d and e .

$$\begin{aligned}
\langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle &= \frac{1}{N^2} [a \sum_i [\langle \hat{O}_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle \hat{O}_f^i \rangle^{\rho\rho} \langle \sigma_Z^i \rangle^{\rho\uparrow\rho\uparrow} + 2 \langle \hat{O}_f^i \rangle^{\lambda\rho} \langle \sigma_Z^i \rangle^{\lambda\uparrow\rho\uparrow}] \\
&\quad + b \sum_i (\langle \hat{O}_f^i \rangle^{\lambda\lambda} + \langle \hat{O}_f^i \rangle^{\rho\rho}) (\langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle \sigma_Z^i \rangle^{\rho\uparrow\rho\uparrow}) \\
&\quad + c \sum_i [\langle \hat{O}_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\rho\uparrow\rho\uparrow} + \langle \hat{O}_f^i \rangle^{\rho\rho} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} - 2 \langle \hat{O}_f^i \rangle^{\lambda\rho} \langle \sigma_Z^i \rangle^{\lambda\uparrow\rho\uparrow}] + \\
&\quad d [\sum_i \langle \hat{O}_f^i \rangle^{\lambda\lambda} + \langle \hat{O}_f^i \rangle^{\rho\rho}] + e [\sum_i (\langle \hat{O}_f^i \rangle^{\rho\rho} - \langle \hat{O}_f^i \rangle^{\lambda\lambda}) \langle \sigma_Z^i \rangle^{\lambda\uparrow\frac{3}{2}\uparrow} + 2 \langle \hat{O}_f^i \rangle^{\lambda\rho} \langle \sigma_Z^i \rangle^{\rho\uparrow\frac{3}{2}\uparrow}]
\end{aligned}$$

Suitable expressions are obtained from the eigen values coming from the above defined operators. For instance, by applying the spin operator on $J^P = \frac{1}{2}^+$ baryon wave-function, symmetric part gives $\frac{2}{3}$ and operating on antisymmetric wave-function gives 0.

$$\begin{aligned}
\langle \chi^{\lambda\uparrow} | \sigma_z^i | \chi^{\lambda\uparrow} \rangle &= \langle \frac{1}{\sqrt{6}} (\uparrow\downarrow + \downarrow\uparrow) \uparrow - 2 \uparrow\uparrow\downarrow | \sigma_z^1 | \frac{1}{\sqrt{6}} (\uparrow\downarrow + \downarrow\uparrow) \uparrow - 2 \uparrow\uparrow\downarrow \rangle = \frac{2}{3} \\
\langle \chi^{\rho\uparrow} | \sigma_z^i | \chi^{\rho\uparrow} \rangle &= \langle \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \uparrow | \sigma_z^1 | \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \uparrow \rangle = 0
\end{aligned}$$

SECTION 2.1: WAVE-FUNCTION FOR BARYONS

To calculate the low energy properties like magnetic moments and spin distribution, it is suitable to write contributions from scalar, vector and tensor sea in the form of two parameters α, β [17]. The generalized expressions are written as:

$$\alpha = \frac{1}{N^2} \left[\frac{4}{9} \right] (2a + 2b + 3d + \sqrt{2e}) \quad (2.5)$$

$$\beta = \frac{1}{N^2} \left[\frac{1}{9} \right] (2a - 4b - 6c - 6d + 4\sqrt{2e}) \quad (2.6)$$

where coefficients are defined as:

$$a = \frac{1}{2} \left[1 - \frac{b_{1^2}}{3} \right], b = \frac{1}{4} \left[a_8^2 - \frac{b_8^2}{3} \right],$$

$$c = \frac{1}{2} \left[a_{10}^2 - \frac{b_{10}^2}{3} \right], d = \frac{1}{18} [5c_8^2 - 3d_8^2], e = \frac{\sqrt{2}}{3} b_8 c_8$$

So, α and β can be written as:

$$\alpha = \frac{2(6 + 3a_8^2 - 2b_1^2 - b_8^2 + 4b_8 c_8 + 5c_8^2 - 3d_8^2)}{27(1 + a_8^2 + a_{10}^2 + b_1^2 + b_{10}^2 + b_8^2 + c_8^2 + d_8^2)}$$

$$\beta = \frac{(3 - 9a_{10}^2 - 3a_8^2 - b_1^2 + 3b_{10}^2 + b_8^2 + 8b_8 c_8 - 5c_8^2 + 3d_8^2)}{27(1 + a_8^2 + a_{10}^2 + b_1^2 + b_{10}^2 + b_8^2 + c_8^2 + d_8^2)}$$

The importance of these two parameters lies in the fact that the parameters are directly related to number of spin up ($n(q \uparrow)$) and spin-down ($n(q \downarrow)$) baryons. If $\Delta q = (n(q \uparrow)) + (n(\bar{q} \uparrow)) + (n(q \downarrow)) + (n(\bar{q} \downarrow))$ where ($q = u, d, s$) then the quark spin polarizabilities Δu and Δd can be directly related to the parameters α and β . It is estimated from the wave-function in equation (2.3) that $\Delta u = 3\alpha$ and $\Delta d = -3\beta$. The other important low energy properties are directly related to the polarized quark spin distributions. Thus, it becomes important to define all the properties in terms of α and β . Also, as there are no antiquarks or s quarks in the wave function so, $(n(\bar{q} \uparrow)) - (n(\bar{q} \downarrow)) = 0$ and $\Delta s = 0$.

This α and β are defined in terms of square of the coefficients. So, they are not complex and have been calculated from the probabilities. Hence, these coefficients are the real numbers, which are obtained from various combinations of spin, color and flavor multiplicities giving us the desired quantum numbers of the baryons. The baryon magnetic moments can be expressed in terms of the quark magnetic moments (μ_u, μ_d, μ_s) and parameters α and β as [17]: $\mu_p = 3(\mu_u\alpha - \mu_d\beta)$, $\mu_n = 3(\mu_d\alpha - \mu_u\beta)$, $\mu_\Lambda = \frac{1}{2}(\alpha - 4\beta)(\mu_u + \mu_d + (2\alpha + \beta)\mu_s)$ and spin distributions in terms of α and β are written as [17]: $I_1^P = \frac{1}{6}(4\alpha - \beta)$, $I_1^N = \frac{1}{6}(\alpha - 4\beta)$, $I_1^\Lambda = \frac{1}{4}(\alpha - 2\beta)$. Expressions of magnetic moments and spin distribution for other baryons have been mentioned in tables 4.2 and 5.7, respectively.

2.1.2 Baryon $J^P = \frac{3}{2}^+$ Decuplet

For decuplet, the feasible combinations of valence q^3 and sea wave-functions yielding spin 3/2, flavor decuplet (10) and color singlet (1) state thereby maintaining the anti-symmetrization of the total wave-function are [18]:

$$\Phi_1^{(3/2)} H_0 G_1, \Phi_1^{(3/2)} H_1 G_1, \Phi_8^{(1/2)} H_1 G_8, \Phi_1^{(3/2)} H_2 G_1, \Phi_8^{(1/2)} H_2 G_8 \quad (2.7)$$

The total wave-function of a spin up baryon decuplet can be written as:

$$\begin{aligned} |\Phi_{3/2}^{(\uparrow)}\rangle = \frac{1}{N} [& a_0 \Phi_1^{(3/2\uparrow)} H_0 G_1 + b_1 (\Phi_1^{(3/2)} \otimes H_1)^\uparrow G_1 + \\ & b_8 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_8 + d_1 (\Phi_1^{(3/2)} \otimes H_2)^\uparrow G_1 + \\ & d_8 (\Phi_8^{(1/2)} \otimes H_2)^\uparrow G_8] \end{aligned} \quad (2.8)$$

where

$$N^2 = a_0^2 + b_1^2 + b_8^2 + d_1^2 + d_8^2 \quad (2.9)$$

where N is the normalization constant. Here, a_0 , b_1 , b_8 , d_1 and d_8 are the coefficients that represent each baryon in decuplet. Also, these coefficients represents

SECTION 2.1: WAVE-FUNCTION FOR BARYONS

the possibilities with which every color, flavor and spin of the decuplet baryons is obtained. So, these values will vary for all decuplet members as each baryon has different mass and quark content. $\Phi_1^{(3/2)}$ in equation 2.8 denotes a function with spin 3/2 and color singlet. This function can be written as combination of F_s (denotes flavor and spin) and ψ_1^A (represents color of baryons and is anti-symmetric). For F_s to be symmetric, ϕ (flavor) and χ (spin) should be symmetric in nature. Similarly, other functions like $\Phi_1^{1/2}$ and $\Phi_8^{1/2}$ denotes a function with spin 1/2 and flavor 10 with color singlet and octet respectively.

The first term in the eq.(2.8) is obtained by combining q^3 wave function with spin 0 (scalar sea). Second and third term in eq. (2.8) are obtained by coupling q^3 with spin 1 (vector sea) i.e.:

$$(\Phi_1^{(3/2)} \otimes H_1)^\dagger \equiv \phi_{b_1}^{(3/2\uparrow)} \psi_1^A, \quad (2.10)$$

$$(\Phi_8^{(1/2)} \otimes H_1)^\dagger \equiv \phi_{b_8}^{(1/2\uparrow)} \psi_1^{MS}, \quad (2.11)$$

where

$$\phi_{b_1}^{(3/2\uparrow)} = \sqrt{\frac{3}{5}} H_{1,0} F_S^{(3/2\uparrow)} - \sqrt{\frac{2}{5}} H_{1,1} F_S^{(1/2\uparrow)} \quad (2.12)$$

$$\phi_{b_8}^{(1/2\uparrow)} = \sqrt{\frac{3}{5}} H_{1,0} F_S^{(3/2\uparrow)} - \sqrt{\frac{3}{5}} H_{1,1} F_S^{(1/2\uparrow)} \quad (2.13)$$

Here, b_1 is the coefficient associated with color combination of valence quarks having color singlet with sea having color singlet whereas b_8 is the coefficient associated with color combination of valence quarks having color octet with sea quarks having color octet in vectorial sea. The q^3 wave functions in eq.(2.10-2.11) for a flavor decuplet baryon can be written as:

$$\Phi_1^{(3/2)} \equiv \Phi(10, 3/2, 1) = F_S \psi_1^A \quad (2.14)$$

where

$$F_S = \phi^\lambda \chi^\lambda \quad (2.15)$$

and

$$\Phi_1^{(1/2)} \equiv \Phi(10, 1/2, 1) = F_{MS} \psi_1^A \quad (2.16)$$

$$\Phi_8^{(1/2)} \equiv \Phi(10, 1/2, 8) = F_A \phi_8^S \quad (2.17)$$

where

$$F_{MS} = \phi^\lambda \chi^{MS} \quad (2.18)$$

$$F_A = \frac{1}{\sqrt{2}} (\chi^\lambda \psi^\rho - \chi^\rho \psi^\lambda) \quad (2.19)$$

Here, superscripts S and A denote symmetry and antisymmetry and λ , MS denotes mixed symmetry under quark permutations $q_1 \leftrightarrow q_2$. Each wave-function in the above equations is a combination of symmetric and antisymmetric term so that total wave-function becomes antisymmetric in nature.

The final two terms are result of coupling with spin 2 (tensor sea). Their expressions can be written as:

$$(\Phi_1^{(3/2)} \otimes H_2)^\uparrow \equiv \phi_{d1}^{(3/2\uparrow)} \psi_1^A, \quad (2.20)$$

$$(\Phi_8^{(1/2)} \otimes H_2)^\uparrow \equiv \phi_{d8}^{(1/2\uparrow)} \phi_8^S, \quad (2.21)$$

where

$$\begin{aligned} \phi_{d1}^{(3/2\uparrow)} = & \sqrt{\frac{1}{5}} H_{2,0} F_S^{(3/2\uparrow)} - \sqrt{\frac{2}{5}} H_{2,1} F_S^{(1/2\uparrow)} \\ & + \sqrt{\frac{2}{5}} H_{2,2} F_S^{(1/2\downarrow)} \end{aligned} \quad (2.22)$$

SECTION 2.2: PRINCIPLE OF DETAILED BALANCE

$$\begin{aligned}
\phi_{d_8}^{(1/2\uparrow)} &= \sqrt{\frac{1}{5}} H_{2,0} F_A^{(3/2\uparrow)} - \sqrt{\frac{2}{5}} H_{2,1} F_A^{(1/2\uparrow)} \\
&\quad + \sqrt{\frac{2}{5}} H_{2,2} F_A^{(1/2\downarrow)}
\end{aligned} \tag{2.23}$$

Here, d_1 is the coefficient associated with color combination of valence quarks having color singlet with another color singlet of sea whereas d_8 is the coefficient associated with color combination of valence quarks having color octet with sea quarks having color octet in tensor sea. Wave functions $\phi_{b_1}^{(3/2\uparrow)}, \phi_{b_8}^{(1/2\uparrow)}, \phi_{d_1}^{(3/2\uparrow)}, \phi_{d_8}^{(1/2\uparrow)}$ are written by taking coupling between spin of flavor part and sea part of q^3 wave-function. The parameter a_0 comes from a spin 3/2 of q^3 state coupled to spin 0 (scalar) of sea, b_1, b_8 comes when spin 3/2 and 1/2 of q^3 state is coupled to spin 1 (vector) of sea and d_1, d_8 corresponds coupling of spin 3/2 to spin 2 (tensor) of sea.

2.2 Principle of Detailed Balance

The principle of detailed balance was suggested to study the insights of effects in the nucleon and the $d\bar{d}$ and $u\bar{u}$ asymmetry. This principle is free from any parameters. It assumes the proton as an ensemble of quark-gluon gas in dynamical balance, where partons combine and split through processes such as $g \Leftrightarrow q\bar{q}$, $g \Leftrightarrow gg$, $q \Leftrightarrow qq$. As proposed by Zhang et al. [19–21], this model calculates the equilibrium of quark gluon Fock states present inside hadrons. The detailed balance principle demands equality between arriving in from one substate and leaving it. The hadron is treated to be consisting of a set of quark gluon Fock states and is expressed as:

$$|B\rangle = \sum_{i,j,l,k} C_{i,j,l,k} |(q), (i, j, l), (k)\rangle \tag{2.24}$$

where q represents the valence quarks of the baryon, i is the number of quark-antiquark $u\bar{u}$ pairs, j is the number of quark-antiquark $d\bar{d}$ pairs, l is the number of $s\bar{s}$ pairs and k is the number of gluons in sea. The probability to find a quark-gluon

Fock states is:

$$\rho_{i,j,l,k} = |C_{i,j,l,k}|^2, \quad (2.25)$$

and $\rho_{i,j,l,k}$ satisfies the normalization condition,

$$\sum_{i,j,l,k} \rho_{i,j,l,k} = 1 \quad (2.26)$$

Assumption of detailed balance principle is that every two sub ensembles balance with each other in a way:

$$\rho_{i,j,l,k}|(q), (i, j, l, k)\rangle \stackrel{balance}{\rightleftharpoons} \rho_{i',j',l',k'}|(q), (i', j', l', k')\rangle$$

$$\rho_{i,j,l,k}N|(q), (i, j, l, k)\rangle \rightarrow |(q), (i', j', l', k')\rangle \equiv \rho_{i',j',l',k'}N|(q), (i', j', l', k')\rangle \rightarrow |(q), (i, j, l, k)\rangle$$

$$\frac{\rho_{i,j,l,k}}{\rho_{i',j',l',k'}} \equiv \frac{N|(q), (i', j', l', k')\rangle \rightarrow |(q), (i, j, l, k)\rangle}{N|(q), (i, j, l, k)\rangle \rightarrow |(q), (i', j', l', k')\rangle} \quad (2.27)$$

where $N(A \rightarrow B)$ implies the number of processes that transfer from A to B. The transfer between two Fock states has two ways: go-out rate and come in rate. These rates are proportional to the number of partons splitting and number of partons recombining respectively. The calculation of probabilities includes various sub processes like $g \Leftrightarrow q\bar{q}$, $g \Leftrightarrow gg$, $q \Leftrightarrow qq$. For example, detailed balance principle is applied to Σ^{*0} to calculate probability of occurrence of quark gluon Fock states and can be written as :

1. When $q \Leftrightarrow qq$ is considered: The general expression of probability can be written as:

$$|uds, i, j, l, k - 1\rangle \stackrel{(3+2i+2j+2l)}{\rightleftharpoons} |uds, i, j, l, k\rangle \quad (2.28)$$

SECTION 2.2: PRINCIPLE OF DETAILED BALANCE

Using equation (2.27),

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{1}{k} \quad (2.29)$$

Here $\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}}$ represents the probability ratio of the two processes.

2. When both the processes $g \Leftrightarrow gg$ and $q \Leftrightarrow qg$ are considered:

$$|uds, i, j, l, k-1\rangle \xrightarrow[(3+2i+2j+2l)k + \frac{k(k-1)}{2}]{3+2i+2j+2l+k-1} |uds, i, j, l, k\rangle \quad (2.30)$$

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{3 + 2i + 2j + 2l + k - 1}{(3 + 2i + 2j + 2l)k + \frac{k(k-1)}{2}} \quad (2.31)$$

3. When $g \Leftrightarrow q\bar{q}$ is considered: The processes $g \Leftrightarrow u\bar{u}$, $g \Leftrightarrow d\bar{d}$, $g \Leftrightarrow s\bar{s}$ are involved here. The transition probabilities involving $g \Leftrightarrow q\bar{q}$ depend upon the valence quark content and differ in all baryons. As the mass of strange quark is large, the generation of $s\bar{s}$ pair from gluons is restricted by applying a mass reduction coefficient on total number of strange quark-antiquark pair i.e. $k(1 - C_l)^{n-1}$ which is limited by gluon free energy distribution. Here, k is the number of gluons and n is the number of partons present in the Fock state i.e. $n = 3 + 2i + 2j + l + 2k$. Free energy is defined as the energy available for gluons or $q\bar{q}$ pairs. The subprocess $g \Leftrightarrow s\bar{s}$ is active only when it satisfies the condition that gluons should have energy larger than at least two times the mass of strange quark because strange quark has non-negligible mass. So taking $(1 - C_l)^{n-1}$ as the mass reduction factor for generating $s\bar{s}$ pairs from a gluon and by using the detailed balance model, the strange quark contribution to the baryon can be calculated. Therefore, the splitting and recombination for the processes involving $g \Leftrightarrow q\bar{q}$ undergoes $SU(3)$ symmetry breaking in sea where q is for some heavier quark flavor. Here, $C_{l-1} = \frac{2M_s}{M_B - 2(l-1)M_s}$, M_s is the

mass of s quark and M_B is the mass of baryon.

$$|q, g\rangle \xrightleftharpoons[2 \times 1]{1(1-C_0^3)} |q, s\bar{s}\rangle$$

$$|q, u\bar{u}g\rangle \xrightleftharpoons[1 \times 2]{1(1-C_0^5)} |q, u\bar{u}s\bar{s}\rangle$$

Generalizing to a number of gluons “ k ” and quark-antiquark pairs “ i ”, “ j ” and “ l ” for proton.

$$|q, i, j, l-1, k\rangle \xrightleftharpoons[l(l+1)]{k(1-C_{l-1}^{n-1})} |q, i, j, l, k\rangle$$

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l-1,k}} = \frac{k(1-C_{l-1}^{n-1})}{l(l+1)}$$

Suppose, initially no $s\bar{s}$ is present and that the generation of one $s\bar{s}$ pair require gluon to have sufficient energy, then the condition becomes:

$$|q, i, j, 0, k\rangle \xrightleftharpoons[l(l+1)]{k(1-C_0^{n-2l-1})} |q, i, j, 1, k-1\rangle$$

$$\frac{\rho_{i,j,1,k-1}}{\rho_{i,j,0,k}} = \frac{k(1-C_0)^{1-2l-1}}{l(l+1)}$$

$$|q, i, j, 1, k-1\rangle \xrightleftharpoons[2(2+1)]{(k-1)(1-C_1^{n-2l})} |q, i, j, 2, k-2\rangle$$

Similar treatment is applied till all gluons have been converted into strange quark-antiquark pairs.

$$|q, i, j, k-1, 1\rangle \xrightleftharpoons[k(k+1)]{1(1-C_{k-1}^{n-k-2})} |q, i, j, k, 0\rangle$$

$$\frac{\rho_{i,j,1,0}}{\rho_{i,j,0,l}} = \frac{(k(k-1)(k-2)\dots 1)(1(1-C_0)^{n-2l-1})(1-C_1)^{n-2l})\dots(1-C_{l-1})^{n-k-2}}{k!(k+1)!}$$

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Generalizing it to “k” number of gluons, the ratio becomes [22]:

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l+k,0}} = \frac{(k)(k-1)\dots 1(1-C_0)^{n-2l-1}\dots(1-C_{l-1})^{n+k-2}}{(l+1)(l+2)\dots(l+k)(l+k+1)} \quad (2.32)$$

The importance of reduction coefficient $(1 - C_l)^{n-1}$ can be realized by considering doubly strange baryon and single strange baryon with $s\bar{s}$ pairs in sea. It is observed with the calculations, that the value of reduction coefficient is more in doubly strange baryon as compared to single strange baryon when we take one $s\bar{s}$ pair or two $s\bar{s}$ pair in sea. However, the value of number of $s\bar{s}$ pairs has been restricted to two due to its large mass and limited free energy of gluon undergoing the subprocess $g \Leftrightarrow s\bar{s}$. Detailed balance principle when applied to different baryons gives different results because sea content will split and recombine with quark content which is different for every baryon. Detailed balance principle and Statistical model are used in combination to compute the coefficients in eq. (2.3) and (2.8).

The expressions of probabilities in terms of $\rho_{0,0,0,0}$ for Σ^{*0} :

$$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{1}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!} \quad (2.33)$$

Similar expressions of probabilities for other octet and decuplet particles can be written in form of $\rho_{0,0,0,0}$ and are shown below in table 2.1. The probabilities we are talking in this work is chances of particular baryon to have certain number of quarks or gluon Fock states because finally these Fock states will combine in a specific way to give a particular states of flavor, spin or color quantum numbers. In general, $\rho_{i,j,k,l}$ is the probability to find the quark gluon Fock states. Also, $\rho_{0,0,0,0}$ implies that we do not have any sea quarks and gluons in baryons.

Table 2.1: Expressions for probabilities in terms of $\rho_{0,0,0,0}$ for $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles:

Table 2.2: Octet

$J^P = \frac{1}{2}^+$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}}$
p	$\frac{2}{i!(i+2)!j!(j+1)!(l+k)!(l+k)!}$
n	$\frac{2}{i!(i+1)!j!(j+2)!(l+k)!(l+k)!}$
λ^0	$\frac{1}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!}$
Σ^+	$\frac{2}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!}$
Σ^0	$\frac{1}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!}$
Σ^-	$\frac{2}{i!i!j!(j+2)!(l+k)!(l+k+1)!}$
Ξ^0	$\frac{2}{i!(i+1)!j!j!(l+k)!(l+k+2)!}$
Ξ^-	$\frac{2}{i!(i)!j!(j+1)!(l+k)!(l+k+2)!}$

Table 2.3: Decuplet

$J^P = \frac{3}{2}^+$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}}$
Δ^{++}	$\frac{3}{i!(i+3)!j!j!(l+k)!(l+k)!}$
Δ^+	$\frac{2}{i!(i+2)!j!(j+1)!(l+k)!(l+k)!}$
Δ^0	$\frac{2}{i!(i+1)!j!(j+2)!(l+k)!(l+k)!}$
Δ^-	$\frac{3}{i!i!j!(j+3)!(l+k)!(l+k)!}$
Σ^{*+}	$\frac{2}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!}$
Σ^{*0}	$\frac{1}{i!(i+1)!j!(j+1)!(l+k)!(l+k+1)!}$
Σ^{*-}	$\frac{2}{i!i!j!(j+2)!(l+k)!(l+k+1)!}$
Ξ^{*0}	$\frac{2}{i!(i+1)!j!j!(l+k)!(l+k+2)!}$
Ξ^{*-}	$\frac{2}{i!i!j!(j+1)!(l+k)!(l+k+2)!}$
Ω^-	$\frac{3}{i!i!j!j!(l+k)!(l+k+3)!}$

The normalization condition $\sum_{i,j,k,l} \rho_{i,j,k,l} = 1$ gives the individual probabilities of baryon octets and decuplets. The entire list of probabilities of various Fock states i.e $\rho_{i,j,k,l}$'s are shown in table 2.4 for all baryon members as well.

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Table 2.4: The values of probabilities of all Fock states i.e $\rho_{i,j,l,k}$'s for one of the singly strange baryon decuplet particle.

i	j	l	$\rho_{i,j,l,k}$	k=0	k=1	k=2
0	0	0	uds	0.15454	0.09922	0.00599
0	0	1	uds $s\bar{s}$	0.07727	0.03247	0.00910
0	1	0	uds $d\bar{d}$	0.07727	0.03692	0.00157
1	0	0	uds $u\bar{u}$	0.07727	0.036921	0.00157
0	1	1	uds $d\bar{d}s\bar{s}$	0.03863	0.01148	0.00321
1	0	1	uds $u\bar{u}s\bar{s}$	0.03863	0.01148	0.00321
1	1	0	uds $u\bar{u}d\bar{d}$	0.03863	0.01373	0.00041
1	1	1	uds $u\bar{u}d\bar{d}s\bar{s}$	0.01931	0.00405	0.00113
1	2	0	uds $u\bar{u}d\bar{d}d\bar{d}$	0.00643	0.00170	0.00003
0	2	0	uds $d\bar{d}d\bar{d}$	0.01287	0.00457	0.00013
0	2	1	uds $d\bar{d}d\bar{d}s\bar{s}$	0.00643	0.00135	0.00037
2	0	0	uds $u\bar{u}u\bar{u}$	0.01287	0.00457	0.00013
2	1	0	uds $u\bar{u}d\bar{d}$	0.00643	0.00170	0.00003
2	0	1	uds $u\bar{u}u\bar{u}s\bar{s}$	0.00643	0.00135	0.00037

From the table 2.4, it is well observed that in case of strange sea, Fock states with a single gluon state shows a remarkable decrease in probability due to possibility of a gluon going into strange quark anti-quark pair. The number of $s\bar{s}$ pairs have been limited to two because of heavy mass of strange quark and limited free energy of gluon as strange anti strange pairs are generated from subprocess $g \leftrightarrow s\bar{s}$.

2.3 Statistical Model and its Assumptions

In literature, statistical models provided intuitive appeal, giving an amazing success in describing the parton structure functions for baryons [23]. The importance of statistical model lies in the fact that, no additional parameters are required, the

equations can be written in terms of the most common statistical quantities or parameters. The hadrons made up of valence quarks and gluons in terms of sea have been modeled diversely to interpret its observed properties like magnetic moments, spin distributions, quadrupole moments etc. The model here assumes that baryon consist of valence quarks surrounded by a sea containing quark-antiquark pairs and gluons and the quantum number like spin, charge, color, flavor are conserved by the constituent Fock states. The domains of validity and stability of the results obtained can be checked by calculating maximum number of baryonic parameters or properties within these models.

In the instanton model [24], the quark-antiquark sea in a nucleon results from a scattering of a valance quark off a non perturbative vacuum fluctuation of the gluon field, instanton. In the instanton induced interaction described by 't Hooft effective lagrangian, the flavor of the produced quark-antiquark is different from the flavor of the initial valance quarks. *J. P Singh et. al.* [25] in statistical model, constructed the Fock states of nucleon having definite spin and color quantum numbers and specific symmetry properties. They studied the ratio of the magnetic moments of the nucleons, quarks contribution to the spin of the nucleons and the ratio of $SU(3)$ matrix elements for the axial current, in this model. The most interesting point to note here is that the properties are directly linked with the probabilities associated with each of the Fock state in definite spin-color-flavor space. Later, *M. Batra et. al.* [22, 26] studied the static properties like magnetic moments, distribution of spin among quarks, axial form-factors etc. of $J^P = \frac{1}{2}^+$ octet baryons using statistical model. The strange and non-strange quark-gluon Fock states are studied for the low energy properties of nucleonic system. Also, on the basis of statistical modeling, the contribution from various parts of sea (scalar, vector ,tensor) has been analyzed [27]. The hyperons are also checked against the impact of $SU(3)$ breaking on the different properties of baryons. Symmetry breaking corrections are applied and its effects are studied with respect to spin distribution among quarks and axial vector form factors.

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In our formalism, we have extended the statistical model [25], by assuming three quarks in valence and few quark antiquark pairs and gluons to be in sea part. The statistical model in conjugation with principle of detail model is worth studying few other properties like masses, magnetic moments of baryon octet and decuplet particles. We statistically decompose quark-gluon Fock states $|q^3, i, j, l, k\rangle$ of a baryon to a set of states in which the valence and sea part will have definite spin and color quantum numbers and different possibilities of spin, color and flavor are chosen so that they lead to desired property of baryon. The statistical model provides us the feature to check the effect of individual contributions of the different Fock states of sea.

The wave function in eq. (2.3) and (2.8) can also be written in the form of $\Phi_{val}\Phi_{sea}$ and the unknown parameters $(a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8)$, $(a_0, b_1, b_8, d_1, d_8)$ by a factor $\sum n_{\mu\nu}^* c_{sea}$ such that total wave function becomes $|\Phi_{\frac{3}{2}}^\dagger\rangle = \sum_{\mu,\nu} (n_{\mu\nu}^* c_{sea}^{\mu\nu}) \Phi_{val} \Phi_{sea}$ where μ and ν have values 0, 1, 2 and 1, 8, $\overline{10}$ respectively. All $n'_{\mu,\nu}$ s are calculated from multiplicities of each Fock state in spin and color space. These multiplicities are shown in the form of $\rho_{p,q}$ where relative probability for valence part should have spin p and sea to have spin q such that the resultant should come out as 1/2 or 3/2 and color singlet.

Calculation of these probabilities helps to find common factor “c” for every combination of valence and sea which is multiplied with multiplicity factor (n) for each Fock state. The detailed calculation of common parameter “c” is mentioned in section 2.3.1. Each unknown parameter in the equation of wave function will have a definite value of $\sum n_{\mu\nu} c_{sea}$ depending on the Fock states ($|gg\rangle, |u\bar{u}g\rangle, |d\bar{d}g\rangle$) etc.: [26]

$$\begin{aligned}
 a_0^2 = & (n_{01} c_{sea})_{|gg\rangle} + (n_{01} c_{sea})_{|u\bar{u}g\rangle} + (n_{01} c_{sea})_{|d\bar{d}g\rangle} \\
 & + (n_{01} c_{sea})_{|s\bar{s}g\rangle} + \dots
 \end{aligned}
 \tag{2.34}$$

$$\begin{aligned}
b_1^2 &= (n_{11}c_{sea})_{|gg\rangle} + (n_{11}c_{sea})_{|u\bar{u}g\rangle} + (n_{11}c_{sea})_{|d\bar{d}g\rangle} \\
&\quad + (n_{11}c_{sea})_{|s\bar{s}g\rangle} + \dots
\end{aligned}
\tag{2.35}$$

$$\begin{aligned}
d_1^2 &= (n_{21}c_{sea})_{|gg\rangle} + (n_{21}c_{sea})_{|u\bar{u}g\rangle} + (n_{21}c_{sea})_{|d\bar{d}g\rangle} \\
&\quad + (n_{21}c_{sea})_{|s\bar{s}g\rangle} \\
&\quad + \dots
\end{aligned}
\tag{2.36}$$

Combinations for other unknown parameters can be written in a similar way. These calculations will give the value of a factor “nc” for Fock states which has a significant role in determining the magnetic moments. A detailed comparison of the all such probabilities on a purely statistical basis is given for octet and decuplet in the following sections. The approach given below is named as “Model C”. Model C aims at finding relative probabilities of the Fock states in color, spin and flavor states. To check the stability of results obtained, under variation in correlation, we introduce a modification in primary approach and repeat calculations of all properties in modified approach as well. Another approach used in our calculations is by suppressing the contribution of states with higher multiplicities. This approach namely “Model D” finds the probabilities of Fock states by suppressing the contribution of states with higher multiplicities. We will discuss both the approaches for both octet and decuplet.

2.3.1 Model C

$J^P = \frac{1}{2}^+$ **Octet**

The method is based on the counting multiplicities in spin and color states for all possible sets of Fock states in valence and sea. And then defining these multiplicities in the form of suitable ratios. We decompose the Fock states $|uud, i, j, k\rangle$ in terms of

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spin and color space. The ratios of probabilities in spin and color space for various decompositions are carried out, for octet, in a way [25]:

1. Consider the decomposition of state $|q^3, 0, 0, 2\rangle$ or $|gg\rangle$ sea (two gluons in the sea). On comparing such probabilities.

$$\frac{\rho_{\frac{1}{2}, 0_s}}{\rho_{\frac{1}{2}, 1_a}} = \frac{\left(\frac{4}{8}\right) \cdot \left(\frac{1}{9}\right) \cdot 1}{\left(\frac{4}{8}\right) \cdot \left(\frac{3}{9}\right) \cdot \left(\frac{2}{6}\right)} = 1 \quad (2.37)$$

$$\frac{\rho_{\frac{1}{2}, 0_s}}{\rho_{\frac{3}{2}, 2_s}} = \frac{\left(\frac{4}{8}\right) \cdot \left(\frac{1}{9}\right) \cdot 1}{\left(\frac{4}{8}\right) \cdot \left(\frac{5}{9}\right) \cdot \left(\frac{2}{20}\right)} = 2 \quad (2.38)$$

$$\frac{\rho_{\frac{3}{2}, 1_a}}{\rho_{\frac{3}{2}, 2_s}} = \frac{\left(\frac{4}{8}\right) \cdot \left(\frac{3}{9}\right) \cdot \left(\frac{2}{12}\right)}{\left(\frac{4}{8}\right) \cdot \left(\frac{5}{9}\right) \cdot \left(\frac{2}{20}\right)} = 1 \quad (2.39)$$

$$\frac{\rho_{\frac{1}{2}, 1_a}}{\rho_{\frac{3}{2}, 2_s}} = \frac{\left(\frac{4}{8}\right) \cdot \left(\frac{3}{9}\right) \cdot \left(\frac{2}{6}\right)}{\left(\frac{4}{8}\right) \cdot \left(\frac{3}{9}\right) \cdot \left(\frac{2}{12}\right)} = 2 \quad (2.40)$$

All the terms on *r.h.s* are multiplicities expressed in the form of $\rho_{p,q}$ where valence quarks has spin p and “sea” carry spin q such that resultant spin is $1/2$. Here, subscripts s and a stands for symmetry and antisymmetry, respectively. The first term in numerator or denominator in r.h.s is the relative probability for the valence quarks to have spin p , the second term is for Fock state of gluons to have spin q and the third term is same for p and q to have resultant spin i.e. $1/2$. For example, for the first factor in numerator of equation (2.37), there are total eight possibilities to couple three spin $\frac{1}{2}$ states (for valence part), four of which have spin $\frac{1}{2}$ so resultant is $\left(\frac{4}{8}\right)$ i.e. $qqq : 2 \otimes 2 \otimes 2 = 4_S \oplus 2_{ms} \oplus 2_{ma}$. Similarly, for the second factor, $\frac{1}{9}$, there are total 9 possibilities to couple two spin 1 gluons, only one of which have spin 0 i.e. $1 \otimes 1 = 0_s \oplus 1_a \oplus 2_s$. The last factor is the probability for coupling the two spin states to give total spin $\frac{1}{2}$.

Similarly, we can compare the probabilities for the q^3 valence and gg such that they

are in different color substates yielding a color singlet baryon.

$$\frac{\rho_{1,1_s}}{\rho_{8,8_s}} = \frac{\rho_{1,1_s}}{\rho_{8,8_a}} = \frac{1}{2} \quad (2.41)$$

$$\frac{\rho_{1,1_s}}{\rho_{10,\overline{10}}} = \frac{(\frac{1}{27}) \cdot (\frac{1}{64}) \cdot (1)}{(\frac{10}{27}) \cdot (\frac{10}{64}) \cdot (\frac{1}{100})} = 1 \quad (2.42)$$

Here, for the first factor $(\frac{1}{27})$ in numerator of equation (2.42), there are total 27 possibilities to couple three valence quarks in color states i.e. $3 \otimes 3 \otimes 3 = 1_a \oplus 8_{ms} \oplus 8_{ma} \oplus 10_s$, one of which have color singlet. Similarly, for the second factor $(\frac{1}{64})$ (for sea part) i.e. $8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \overline{10}_a \oplus 27_s$, there are total 64 possibilities to couple two gluons in sea, out of which one will have color singlet. The last factor is the probability for coupling the two color singlet states to give total color singlet. The product of probabilities in spin and color spaces can be written in terms of one common factor “c” as;

$$\rho_{\frac{1}{2},0_s}[\rho_{1,1_s}, \rho_{8,8_s}] = 2c(1, 2) \quad (2.43)$$

$$\rho_{\frac{1}{2},1_a}[\rho_{8,8_a}, \rho_{10,\overline{10}}] = 2c(2, 1) \quad (2.44)$$

$$\rho_{\frac{3}{2},1_a}[\rho_{8,8_a}] = 2c \quad (2.45)$$

$$\rho_{\frac{3}{2},2_s}[\rho_{8,8_s}] = 2c \quad (2.46)$$

No contribution to $H_0 G_{\overline{10}}$ and $H_1 G_1$ sea is seen from gg states as H_0 and G_1 are symmetric whereas H_1 and $G_{\overline{10}}$ are antisymmetric under the exchange of the two gluons, thereby, making these combinations in terms of wave functions antisymmetric and hence unacceptable for a bosonic system. Equating the sum of these partial probabilities to value of probabilities $\rho_{i,j,l,k}$ i.e ρ_{0002} taken from table I of Ref. [21]

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gives the unknown parameter c as:

$$\begin{aligned}
 & 2c + 4c + 4c + 2c + 2c + 2c \\
 & = 16c = 0.081887 \\
 & \Rightarrow c_{0002} = 0.005118
 \end{aligned}$$

Similarly, the combined probabilities in spin and color space for other Fock states can be written as [25]:

- (i) $|g, q\bar{q}\rangle$ and $|u\bar{u}, d\bar{d}\rangle$ sea, symmetry consideration is not needed. Here, we have assumed that $q\bar{q}$ carries the same quantum numbers as that of a gluon due to the sub processes $g \Leftrightarrow q\bar{q}$. This gives the relative probability density in color space as $\frac{\rho_{11}}{\rho_{88}} = \frac{1}{4}$. The ratio $\frac{\rho_{11}}{\rho_{10\bar{10}}}$ and the relative densities in spin space remain the same as in (1). Proceeding as in the previous case, the products of densities in spin and color spaces come out as:

$$\begin{aligned}
 & \rho_{\frac{1}{2},0}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{3}{2},1}[\rho_{8,8}]; \rho_{\frac{3}{2},2}[\rho_{8,8}] \\
 & = 0.00344[(1, 4, 1); (1, 4, 1); 2; 2] \text{for } |g, u\bar{u}\rangle \\
 & = 0.00517[(1, 4, 1); (1, 4, 1); 2; 2] \text{for } |g, d\bar{d}\rangle \\
 & = 0.00366[(1, 4, 1); (1, 4, 1); 2; 2] \text{for } |u\bar{u}, d\bar{d}\rangle.
 \end{aligned}$$

(ii) $|gg, q\bar{q}\rangle$ and $|q\bar{q}q\bar{q}, g\rangle$ sea,

$$\rho_{\frac{1}{2},0_s}[\rho_{1,1_a}, \rho_{8,8_a}, \rho_{10,\overline{10}_a}]; \rho_{\frac{1}{2},1_a}[\rho_{1,1_a}, \rho_{8,8_a}, \rho_{10,\overline{10}_a}]; \rho_{\frac{1}{2},1_s}[\rho_{1,1_s}, \rho_{8,8_s}, \rho_{10,\overline{10}_s}];$$

$$\rho_{\frac{3}{2},1_a}[\rho_{8,8_a}]; \rho_{\frac{3}{2},1_s}[\rho_{8,8_s}]; \rho_{\frac{3}{2},2_a}[\rho_{8,8_a}]$$

$$= 0.00051[(1, 8, 2); (1, 8, 2); (2, 16, 4); 4; 8; 4]$$

for $|gg, u\bar{u}\rangle$

$$= 0.00076[(1, 8, 2); (1, 8, 2); (2, 16, 4); 4; 8; 4]$$

for $|gg, d\bar{d}\rangle$

$$= 0.00007[(1, 8, 2); (1, 8, 2); (2, 16, 4); 4; 8; 4]$$

for $|u\bar{u}u\bar{u}\rangle, g\rangle$

$$= 0.00025[(1, 8, 2); (1, 8, 2); (2, 16, 4); 4; 8; 4]$$

for $|d\bar{d}d\bar{d}\rangle, g\rangle$.

(iii) $|u\bar{u}\rangle, d\bar{d}, g\rangle$ sea,

$$\rho_{\frac{1}{2},0}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}]; \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}]; \rho_{\frac{3}{2},1}[\rho_{8,8}]; \rho_{\frac{3}{2},2}[\rho_{8,8}]$$

$$= 0.00048[(1, 8, 2); (3, 24, 6); 12; 8].$$

(iv) $|ggg\rangle$ sea, The wave function for this sea should be completely symmetric under the exchange of any two gluons. Among the product spin function, the total spin $S = 0$ is completely antisymmetric and one $S = 1$ is completely

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symmetric.

$$\begin{aligned} & \rho_{\frac{1}{2},0_a}[\rho_{1,1_a}, \rho_{8,8_a}]; \rho_{\frac{1}{2},1_s}[\rho_{1,1_s}, \rho_{8,8_s}]; \rho_{\frac{3}{2},1_s}[\rho_{8,8_s}] \\ & = 0.00534[(1, 2); (1, 2); 1]. \end{aligned}$$

$J^P = \frac{3}{2}^+$ **Decuplet**

The ratios of probabilities in spin and color space for various decompositions are carried out, for decuplet, in a way:

1. Consider the decomposition of state $|q^3, 0, 0, 0, 2\rangle$ or $|gg\rangle$ sea: Different cases of probability ratios for spin of valence and sea can be written as:

$$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{1}{2},2}} = \frac{(\frac{4}{8}) \cdot (\frac{3}{9}) \cdot (\frac{4}{6})}{(\frac{4}{8}) \cdot (\frac{5}{9}) \cdot (\frac{4}{10})} = 1 \quad (2.47)$$

$$\frac{\rho_{\frac{3}{2},0}}{\rho_{\frac{3}{2},1}} = \frac{(\frac{4}{8}) \cdot (\frac{1}{9}) \cdot (\frac{4}{4})}{(\frac{4}{8}) \cdot (\frac{3}{9}) \cdot (\frac{4}{12})} = 1 \quad (2.48)$$

$$\frac{\rho_{\frac{3}{2},0}}{\rho_{\frac{3}{2},2}} = \frac{(\frac{4}{8}) \cdot (\frac{1}{9}) \cdot (\frac{4}{4})}{(\frac{4}{8}) \cdot (\frac{5}{9}) \cdot (\frac{4}{20})} = 1 \quad (2.49)$$

$$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{3}{2},1}} = \frac{(\frac{4}{8}) \cdot (\frac{3}{9}) \cdot (\frac{4}{6})}{(\frac{4}{8}) \cdot (\frac{3}{9}) \cdot (\frac{4}{12})} = 2 \quad (2.50)$$

$$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{3}{2},0}} = \frac{(\frac{4}{8}) \cdot (\frac{3}{9}) \cdot (\frac{4}{6})}{(\frac{4}{8}) \cdot (\frac{1}{9}) \cdot (\frac{4}{4})} = 2 \quad (2.51)$$

Similar probability ratios can be calculated for color spaces finally giving a color singlet baryon and can be expressed as:

$$\frac{\rho_{1,1}}{\rho_{8,8_s}} = \frac{(\frac{1}{27}) \cdot (\frac{1}{64}) \cdot (1)}{(\frac{16}{27}) \cdot (\frac{8}{64}) \cdot (\frac{1}{64})} = \frac{1}{2} = \frac{\rho_{1,1}}{\rho_{8,8_a}} \quad (2.52)$$

$$\frac{\rho_{1,1}}{\rho_{10,\overline{10}}} = \frac{\left(\frac{1}{27}\right) \cdot \left(\frac{1}{64}\right) \cdot (1)}{\left(\frac{10}{27}\right) \cdot \left(\frac{10}{64}\right) \cdot \left(\frac{1}{100}\right)} = 1 \quad (2.53)$$

These are the probabilities to find the valence quarks in spin 3/2 and color singlet states with sea. To compute the common parameter “c” the product of probabilities in spin and color spaces can be written in terms of common factor “c” as;

$$\rho_{\frac{1}{2},1}[\rho_{8,8_a}, \rho_{10,\overline{10}}] = c(2, 1) \quad (2.54)$$

$$\rho_{\frac{1}{2},2}[\rho_{1,1}, \rho_{8,8_s}] = c(1, 2) \quad (2.55)$$

$$\rho_{\frac{3}{2},0}[\rho_{1,1}, \rho_{8,8_s}] = c(1, 2) \quad (2.56)$$

$$\rho_{\frac{3}{2},1}[\rho_{8,8_a}] = 2c \quad (2.57)$$

$$\rho_{\frac{3}{2},2}[\rho_{1,1}, \rho_{8,8_s}] = c(1, 2) \quad (2.58)$$

Values present on the *r.h.s* of the above eqs. are the multiplicities for a particular Fock state. There is no contribution from $H_0G_{\overline{10}}$, H_1G_1 and $H_2G_{\overline{10}}$ as they creates an antisymmetric sea under the exchange of two gluons, making the wave functions antisymmetric and hence unacceptable for a bosonic system (gg). Equating the sum of all these partial probabilities to value of probabilities $\rho_{i,j,l,k}$ i.e ρ_{0002} , ρ_{2000} , ρ_{0200} taken from table 2.4 for Σ^{*0} gives the unknown parameter c as:

$$\begin{aligned} 2c + c + c + 2c + c + 2c + 2c + c + 2c \\ = 14c = 0.00599579 \\ \Rightarrow c_{0002} = 0.000428271 \end{aligned}$$

and other values can be computed and written as $c_{2000} = 0.000919879$, $c_{0200} = 0.000919879$. Similar decompositions can be done for other Fock states as well as shown in tables 2.5 and 2.6. Both the tables are for $J^P = \frac{3}{2}^+$ decuplet particles. Table 2.6 is applicable for Fock states with symmetry conditions.

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Table 2.5: Computed probability ratios for various Fock states in spin and color space for spin $J^P = \frac{3}{2}^+$ decuplet particles.

Probability Ratio \rightarrow	$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{1}{2},2}}$	$\frac{\rho_{\frac{3}{2},0}}{\rho_{\frac{3}{2},1}}$	$\frac{\rho_{\frac{3}{2},0}}{\rho_{\frac{3}{2},2}}$	$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{3}{2},1}}$	$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{3}{2},0}}$	$\frac{\rho_{1,1}}{\rho_{8,8}}$	$\frac{\rho_{1,1}}{\rho_{10,10}}$
Fock States \downarrow							
$ gg\rangle$	1	1	1	2	2	$\frac{1}{2}$	1
$ u\bar{u}g\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ d\bar{d}g\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ s\bar{s}g\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ u\bar{u}d\bar{d}\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ u\bar{u}s\bar{s}\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ s\bar{s}d\bar{d}\rangle$	1	1	1	2	2	$\frac{1}{4}$	1
$ d\bar{d}d\bar{d}\rangle$	1	1	1	2	2	$\frac{1}{2}$	1
$ u\bar{u}u\bar{u}\rangle$	1	1	1	2	2	$\frac{1}{2}$	1

Table 2.6: Computed probability ratios for various Fock states in spin and color space for spin $J^P = \frac{3}{2}^+$ decuplet particles.

Probability Ratio \rightarrow	$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{1}{2},2}}$	$\frac{\rho_{\frac{3}{2},1}}{\rho_{\frac{3}{2},2}}$	$\frac{\rho_{\frac{1}{2},1}}{\rho_{\frac{3}{2},1}}$	$\frac{\rho_{\frac{3}{2},1}}{\rho_{\frac{3}{2},0}}$	$\frac{\rho_{1,1}}{\rho_{8,8}}$	$\frac{\rho_{1,1}}{\rho_{10,10}}$
Fock States \downarrow	S,A	S,A	S,A	S,A	S,A	S,A
$ d\bar{d}d\bar{d}g\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ d\bar{d}s\bar{s}g\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ u\bar{u}gg\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ d\bar{d}gg\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ s\bar{s}gg\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ u\bar{u}u\bar{u}g\rangle$	2,1	2,1	2,2	1	$\frac{1}{8}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{2}$
$ u\bar{u}d\bar{d}g\rangle$ (no symmetry condition)	$\frac{3}{2}$	$\frac{3}{2}$	-	3	$\frac{1}{8}$	$\frac{1}{2}$

2.3.2 Model D

Model D is the modification to the basic approach (Model C) and it assumes that sea containing large number of gluons have relatively smaller probabilities and hence their multiplicities have been suppressed over the rest of valence particles with limited quarks. Model “D” is assumed to be a special case of Model “C”. In order to check the predicting power of statistical approach, we have modified the relative probabilities by suppressing the contribution of states coming from higher multiplicities. Here, relative probabilities are divided with their respective spin and color multiplicities to achieve the suppression. This modification is based on the phenomenological ground [26] stating, that the higher the multiplicities, lower will be the associated probabilities. This probability factor is additional to the previously incorporated factors in the probabilities. With this new input, we decompose Fock states as follows.

$$J^P = \frac{1}{2}^+ \text{ Octet}$$

The decomposition on the basis of spin and color space for various sea can be written as:

(i) $|gg\rangle$ sea

$$\begin{aligned} & \rho_{\frac{1}{2},0_s}^1[\rho_{1,1_s}, \rho_{8,8_s}]; \rho_{\frac{1}{2},1_a}^1[\rho_{1,1_a}, \rho_{8,8_a}]; \rho_{\frac{3}{2},1_a}^3[\rho_{8,8_a}]; \rho_{\frac{3}{2},2_a}^3[\rho_{8,8_s}] \\ & = 2d[(2, \frac{1}{16}); (\frac{1}{48}, \frac{1}{50}); \frac{1}{192}; \frac{1}{320}] \end{aligned}$$

(ii) $|q\bar{q}\rangle$ and $|u\bar{u}d\bar{d}\rangle$ sea

$$\begin{aligned} & \rho_{\frac{1}{2},0}^1[\rho_{1,1}, \rho_{8,8}, \rho_{10,10}]; \rho_{\frac{1}{2},1}^1[\rho_{1,1}, \rho_{8,8}, \rho_{10,10}]; \rho_{\frac{3}{2},1}^3[\rho_{8,8}]; \rho_{\frac{3}{2},2}^3[\rho_{8,8}] \\ & = 2d[(2, \frac{1}{8}, \frac{1}{50}); (\frac{1}{24}, \frac{1}{150}); \frac{1}{96}; \frac{1}{160}] \end{aligned}$$

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(iii) $|ggq\bar{q}\rangle$ and $|q\bar{q}q\bar{q}\rangle$ sea

$$\begin{aligned} & \rho_{\frac{1}{2},0_s}[\rho_{1,1_a}, \rho_{8,8_a}, \rho_{10,\overline{10}_a}]; \rho_{\frac{1}{2},1_s}[\rho_{1,1_a}, \rho_{8,8_a}, \rho_{10,\overline{10}_a}]; \rho_{\frac{3}{2},1_a}[\rho_{8,8_a}]; \rho_{\frac{3}{2},1_s}[\rho_{8,8_s}]; \rho_{\frac{3}{2},2_a}[\rho_{8,8_a}] \\ & = 2d[(1, \frac{1}{8}, \frac{1}{50}); \frac{1}{32}; \frac{1}{32}; \frac{1}{160}] \end{aligned}$$

(iv) $|gu\bar{u}d\bar{d}\rangle$ sea

$$\begin{aligned} & \rho_{\frac{1}{2},0_s}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}]; \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}]; \rho_{\frac{3}{2},2}[\rho_{8,8}] \\ & = 2d[(\frac{1}{2}, \frac{1}{16}, \frac{1}{100}); (\frac{1}{2}, \frac{1}{16}, \frac{1}{100}); \frac{1}{160}] \end{aligned}$$

(v) $|ggg\rangle$ sea

$$\begin{aligned} & \rho_{\frac{1}{2},1_a}[\rho_{1,1_a}, \rho_{8,8_a}]; \rho_{\frac{1}{2},1_s}[\rho_{1,1_s}, \rho_{8,8_s}]; \rho_{\frac{3}{2},1_s}[\rho_{8,8_s}] \\ & = 2d[(1, \frac{1}{32}); (1, \frac{1}{3}, \frac{1}{96}); \frac{1}{384}] \end{aligned}$$

$J^P = \frac{3}{2}^+$ **Decuplet**

The decompositions of Fock states with this new input for $J^P = \frac{3}{2}^+$ particles is shown as:

1. $|gq\bar{q}\rangle, |u\bar{u}d\bar{d}\rangle, |u\bar{u}s\bar{s}\rangle, |d\bar{d}s\bar{s}\rangle$ sea, symmetry consideration is not needed.

$$\begin{aligned} \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}] &= c(1, 4, 1) \\ &= d(\frac{1}{3}, \frac{1}{48}, \frac{1}{300}) \end{aligned} \tag{2.59}$$

$$\begin{aligned} \rho_{\frac{1}{2},2}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}] &= c(1, 4, 1) \\ &= d(\frac{1}{5}, \frac{1}{80}, \frac{1}{500}) \end{aligned} \tag{2.60}$$

$$\begin{aligned} \rho_{\frac{3}{2},0}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}] &= c(1, 4, 1) \\ &= d(\frac{1}{2}, \frac{1}{32}, \frac{1}{200}) \end{aligned} \tag{2.61}$$

$$\rho_{\frac{3}{2},1}[\rho_{8,8}] = 4c = \frac{d}{96} \quad (2.62)$$

$$\begin{aligned} \rho_{\frac{3}{2},2}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\overline{10}}] &= c(1, 4, 1) \\ &= d\left(\frac{1}{10}, \frac{1}{160}, \frac{1}{1000}\right) \end{aligned} \quad (2.63)$$

Summing and equating all the partial probabilities to ρ_{1001} , ρ_{0101} , ρ_{0011} , ρ_{1100} , ρ_{1010} , ρ_{0110} we get values of d as: 0.030115253, 0.030115253, 0.026488907, 0.031513051, 0.031513051, 0.031513051 respectively. Similar numbers can be obtained for other Fock states as well.

Computed values of coefficients of $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ wave functions in the two models i.e. C and D are shown in table 2.7 and 2.8, respectively. These coefficients carry the utmost importance in themselves because these coefficients are identified with over all probability of Fock states.

Table 2.7: Coefficients of $J^P = \frac{1}{2}^+$ particles

Particle	C Model								D Model							
	a_0	a_8	a_{10}	b_1	b_8	b_{10}	c_8	d_8	a_0	a_8	a_{10}	b_1	b_8	b_{10}	c_8	d_8
p	0.155	0.097	0.022	0.024	0.134	0.035	0.067	0.056	0.435	0.022	0.003	0.142	0.014	0.002	0.003	0.001
n	0.155	0.097	0.022	0.024	0.134	0.035	0.067	0.056	0.435	0.022	0.003	0.142	0.014	0.002	0.003	0.001
Λ	0.192	0.089	0.016	0.023	0.123	0.033	0.061	0.048	0.435	0.018	0.002	0.112	0.011	0.001	0.002	0.001
Σ^+	0.157	0.080	0.017	0.019	0.113	0.031	0.056	0.047	0.400	0.018	0.002	0.109	0.011	0.001	0.002	0.001
Σ^0	0.190	0.082	0.017	0.020	0.113	0.030	0.056	0.047	0.436	0.018	0.002	0.112	0.011	0.001	0.002	0.001
Σ^-	0.157	0.080	0.017	0.019	0.113	0.031	0.056	0.047	0.400	0.018	0.002	0.109	0.011	0.001	0.002	0.001
Ξ^0	0.163	0.073	0.015	0.017	0.102	0.028	0.051	0.044	0.392	0.016	0.002	0.094	0.009	0.001	0.002	0.001
Ξ^-	0.157	0.080	0.017	0.019	0.113	0.031	0.056	0.047	0.400	0.018	0.002	0.109	0.011	0.001	0.002	0.001

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Table 2.8: Coefficients of $J^P = \frac{3}{2}^+$ particles

Particle	C Model					D Model				
	a_0	b_1	b_8	d_1	d_8	a_0	b_1	b_8	d_1	d_8
Δ^{++}	0.166	0.0005	0.0002	0.002	0.0002	0.334	0.030	0.006	0.064	0.007
Δ^+	0.120	0.0006	0.0002	0.002	0.0002	0.298	0.033	0.006	0.067	0.007
Δ^0	0.121	0.0006	0.0002	0.002	0.0002	0.298	0.033	0.006	0.067	0.007
Δ^-	0.166	0.0005	0.0002	0.002	0.0002	0.334	0.030	0.006	0.064	0.007
Σ^{*+}	0.108	0.0004	0.0002	0.002	0.0002	0.264	0.020	0.005	0.059	0.006
Σ^{*0}	0.161	0.0005	0.0002	0.002	0.0002	0.316	0.030	0.006	0.064	0.007
Σ^{*-}	0.108	0.0004	0.0002	0.002	0.0002	0.264	0.020	0.005	0.059	0.006
Ξ^{*0}	0.136	0.0005	0.0002	0.002	0.0003	0.309	0.034	0.007	0.071	0.008
Ξ^{*-}	0.136	0.0005	0.0002	0.002	0.0003	0.309	0.034	0.007	0.071	0.008
Ω^-	0.241	0.0001	0.0002	0.002	0.0002	0.395	0.009	0.004	0.053	0.005

2.4 Baryonic Properties at Low Energy

The full resolution of internal structure of baryons is being studied extensively over the globe in the form of experiments and theoretical developments over the past 50 years. The relativistic and non-perturbative nature of its constituents makes the study of baryon more challenging and interesting. The presence of quark-gluon interaction in valence quarks implies that $q\bar{q}$ pairs can be produced perturbatively by gluons emitted from valence quarks itself [28]. The sea could be consisting of light-quarks as well as heavy ones and plays a peculiar role in understanding the concept of hadronic structure. The distribution of \bar{u} and \bar{d} in sea have large asymmetry according to observations in deep inelastic scattering [29] and Drell-Yan experiments [30, 31]. The baryons made of valence quarks and gluonic degrees of freedom in terms of “sea” have been modeled diversely to interpret its observed properties like

magnetic moments, spin distributions, quadrupole moments etc. The domains of validity and stability of the results obtained can be checked by calculating maximum number of baryonic parameters or properties within these models. Low energy properties of hadrons are an important tool to learn the hadronic structure. Various nucleonic parameters have been calculated theoretically using chiral quark soliton model [32,33], Skyrme model [34,35], sum rules etc [36–38]. Here, we will look into the details of all the low energy observable using phenomenological models.

2.4.1 Masses

In this section, we look at the baryon mass spectrum resulting from interactions inside the baryon. Many of the baryon mass splitting observed till date are produced by the splitting among the quark masses. Much research has been devoted in describing baryon masses which also includes masses of baryons with heavy quarks, often with considerable success [39–45]. Morpurgo [46], by the use of field theory and the non relativistic quark model, gave a general parameterizations of the masses and magnetic moments of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles. His method (GPM), shows the mass operators in terms of flavor-dependent terms are proportional to powers of the strange-quark projection operator P^λ and Pauli spin operators σ . Several authors have studied the heavy baryon masses in the non-relativistic quark model that includes the spin and flavor dependant hyperfine interaction between two quarks and between a quark and an antiquark and many other different techniques [47]. Study of baryon mass spectrum has been a subject of increasing interest due to related experiments at Fermilab, CERN etc. [4]. Also several models including non relativistic quark model (NRQM) [40], chiral perturbation theory (ChPT) [48], hyper central model [49] have evaluated the data of light and heavy baryon mass spectrum having nice agreement with experimental information available. C. P. Singh [40] et. al., calculated the masses of charmed

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and b-quark hadrons in the non relativistic additive quark model by incorporating the spin and flavor dependant interaction between two quarks and a quark and an antiquark, which was in agreement with the available data.

Masses of heavy and light baryons are computed using, $\text{Mass} = M_{quark} + \text{Spin-Spin Correction}$, where M_{quark} is the mass obtained using scalar quarks and the spin-spin correction term. In order to understand the mass spectrum, spin-spin interaction term has a pivotal role in the mass formulae. The spin dependent terms are not small, so they are not just a perturbation. It is worth to mention that we are considering S-wave quark-antiquark systems here. The parameters required for these calculations are completely determined by previously studied properties, using the wave functions, so the results are entirely predictive. The spin-spin interaction contribution is also responsible for $\Sigma - \Lambda$ mass splitting in case of baryons [39]. The reason for this splitting is that, in case of Σ^0 , the quarks are in symmetric state which means that quark pair (u,d) must also be symmetric in spin state, such that $\vec{S}_u \cdot \vec{S}_d = \frac{1}{4}$. For Λ , the quark pair (u,d) has isospin zero which means total spin is zero (antisymmetric state), such that $\vec{S}_u \cdot \vec{S}_d = \frac{-3}{4}$.

2.4.2 Magnetic Moments

Magnetic moments are properties of baryons observed at low energies and long distances. They are important as they constitute information about the internal structure. All the constituents of baryon (valence+sea), by experiencing the same magnetic field, contributes to the magnetic moments. The electromagnetic properties of baryons, when in static position, arise from the respective constituent quarks. In general, in the absence of orbital motion and sea, the magnetic moment of the baryons is defined as the sum of constituent quark magnetic moments. Thus, for

baryons at ground state, the magnetic moment operator is written as:

$$\hat{\mu} = \sum_i \mu_i \sigma_i \quad (2.64)$$

where $\mu_i = \frac{e_i}{2m}$, μ_i represents magnitude of quark magnetic moments for $i = u, d, s$ and e_i represents the quark charge. σ_i is the Pauli matrix representing the spin term. Thus, the magnetic moment of baryonic system (B) can be parameterized as:

$$\langle B | \mu | B \rangle = \sum_{i=u,d,s} \langle B | \frac{e_i \sigma_i}{2m} | B \rangle$$

Magnetic moment of a baryonic system can also be defined in terms of quark polarizations:

$$\mu(B) = \sum_{q=u,d,s} \Delta q^B \mu_q = \Delta u \mu_u + \Delta d \mu_d + \Delta s \mu_s$$

where $\Delta u, \Delta d, \Delta s$ are the quark polarizations and μ_u, μ_d, μ_s are the magnetic moments of quarks u, d and s , respectively.

2.4.3 $\bar{d} - \bar{u}$ Asymmetry

The basic idea of flavor asymmetry is simple. Flavor asymmetry means that when sea $q\bar{q}$ pairs are generated flavor blindly by gluon splitting, then \bar{u} quarks have more probability to annihilate than \bar{d} quarks. This is because of presence of more u quarks than d quarks in the proton, thereby leading to asymmetry.

An investigation [50] of the light antiquark asymmetry in the nucleon stated that asymmetries in the up and down sea do not exceed 1% due to perturbative processes. Thus, for a large $\bar{d} - \bar{u}$ asymmetry, a non-perturbative origin for an appreciable fraction of these light antiquarks is needed. Field and Feymann [51] suggested that the presence of extra $u\bar{u}$ in the sea can lead to suppression of extra $g \rightarrow u\bar{u}$ via Pauli blocking. Zhang et al. [19–21] calculated the flavor asymmetry in sea using

principle of detailed balance in case of proton. The model generated $\bar{d} - \bar{u} = 0.124$ which was in agreement with the measurements of E866/NuSea [31, 52, 53] result of 0.118 ± 0.012 . We have computed $\bar{d} - \bar{u}$ asymmetry for all other baryons using the relation:

$$\bar{d} - \bar{u} = \left(\left[\sum_{j=0, k=0}^{j=2, k=3} \rho_{0,j,0,k} \right] - \left[\sum_{i=0, k=0}^{i=2, k=3} \rho_{i,0,0,k} \right] \right) \quad (2.65)$$

where i is the number of quark-antiquark $u\bar{u}$ pairs, j is the number of quark-antiquark $d\bar{d}$ pairs, l is the number of $s\bar{s}$ pairs and k is the number of gluons in sea. The values for flavor asymmetry for all $J^P = \frac{1}{2}^+$ octet, $J^P = \frac{3}{2}^+$ decuplet members are calculated and shown in the following chapters.

2.4.4 Spin Distributions

A well known and important problem for physicists over last 20 years is the proton spin puzzle: How proton's spin is built from constituent quarks and gluons? Deep inelastic experiments and European Muon Collaboration predicted very less contribution by quark's intrinsic spin to total spin of proton. This was contrary to the results of non-relativistic quark model which assumes that whole spin of the proton comes from its constituent quarks. The whole story led the phenomenologist as well as experimentalist to think beyond the already known facts and non-relativistic quark model. Various experiments [54, 55] measured the spin structure function (g_1) at $x = 0.1$ to $x = 0.01$. One of the possible solution to proton spin puzzle can be from anomalous gluon effect arising from the axial anomaly [56]. A polarized gluon is preferred to split into a quark antiquark pairs with helicities antiparallel to the gluon spin. Thus a positive gluon spin component ΔG can give rise to negative sea quark polarization. The lattice calculation indicates that sea polarization is almost independent of light quark flavors. This $SU(3)$ flavor symmetry implies that it is indeed the axial anomaly, which is independent of light quark masses, that accounts

for the bulk of helicity contribution of sea quarks. This anomaly allows spin carried by the gluons to mix with the spin carried by quarks, thus modifying the structure of quark sea. So, at low energies, a proton is a system of three massive constituent quarks interacting self-consistently with cloud of virtual pions and condensates generated from spontaneous breaking of chiral symmetry between left and right handed quarks. On the other other hand, when probed at high resolution, the structure of proton seems to be a combination of three valence quarks plus sea of quark-antiquark pairs and gluons. Thus, we conclude that nucleonic spin is distributed among gluons, valence and sea quarks plus their angular momenta. The individual spin polarization of quarks is defined as, $\Delta q = n(q \uparrow) - n(q \downarrow) + n(\bar{q} \uparrow) - n(\bar{q} \downarrow)$ for $q = u, d, s$, where $n(q \uparrow)$ is the number of spin-up and $n(q \downarrow)$ is the number of spin down quarks of flavor q for both quarks and anti-quarks. In addition, total spin distributions of baryon is also calculated by applying the operator $I_1^B = \sum_i \frac{1}{2} e_i^2 \sigma_Z^i$ where e_i and σ_Z^i are the charge of quark and spin projection operator respectively, to the baryonic wave function.

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3

Masses of $J^P = \frac{1}{2}^+$ Octet and $J^P = \frac{3}{2}^+$ Decuplet Baryons

3.1 Introduction

Mass is an important and essential property to understand matter. Unraveling mass has been a central affair in subject since its introduction in 1687 by Issac Newton [1]. However, understanding the origin of mass of particles constituting matter is still a hot topic among theoreticians and experimentalist for the detailed exploration of universe. The masses of elementary particles like leptons, quarks, gauge bosons can be generated with the Higgs boson proposed in the 1960s in the Brout-Englert-Higgs mechanism [2–4]. The presence of higgs boson was confirmed by its observation in the ATLAS [5] and the CMS [6] experiments at CERNs Large Hadron Collider in

2012 .

Whenever elementary particles are considered theoretically or experimentally, the “Standard Model” is appealed and it is suggested that this model describes the properties like magnetic moments, masses, spin and many more. However, since the introduction of the concept of quarks, the precise theoretical values for the masses of the baryons, mesons and leptons have not yet been calculated, in the last forty years. The light baryons, especially the nucleons, have been extensively studied in all aspects following a reason that they need modest energies and that the target is very abundant allowing good statistics. The masses of octet and decuplet baryons have been studied and analyzed by several authors using different theoretical models. Masses of baryons have been studied by several models including non relativistic quark model (NRQM) [7, 8], MIT-Bag model [9], Skyrme model [10]. El Naschie [11–14] has proposed a topological theory from which the masses of the proton and the electron and pions, is calculated with great accuracy. Sidharth [15], in an another example, has suggested a model of the π^0 meson in which an electron and a positron encircle their center of mass. Masses are calculated using the effective theory with combined $1/N_c$ and chiral expansions for non strange baryons [16]. Lichtenberg [17] predicted exotic hadron masses using a quark diquark model. A semi empirical formula [18] is used for the color-hyperfine mass splitting in mesons and then generalizing the formula to baryons [19]. Chiral $SU(3)$ quark mean field model [20] is used to study the modification in masses and magnetic moments of $J^P = \frac{1}{2}^+$ baryons in the presence of hot and dense isospin symmetric strange baryonic matter. When the masses of the quarks increases, the mass spectrum becomes more and more dense and one can see that a number of excited states lie below the threshold corresponding to a quark pair creation. $SU(2)$ isospin multiplet members have same mass to 5 MeV , however, u, d, s flavor symmetry is broken by mass differences of order 100 MeV . The ground state masses and the positive and negative parity excited state masses of doubly heavy Ξ baryons has been calculated by using

a hypercentral constituent quark model [21].

To study the structure of octet and decuplet baryons we need to study the wave functions of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles as shown in equations (2.3) and (2.8). In this chapter, we calculate baryon octet and decuplet masses by analyzing their structure using statistical model. The mass formulae, used here, are function of constituent quark masses and spin-spin interaction terms for the quarks inside the baryons. The coefficients in the mass formulae is estimated by the statistical model, where the baryonic structure is considered to be consisting of valence quarks and “sea” containing $u\bar{u}, d\bar{d}, s\bar{s}$ pairs and gluons, for $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ octet and decuplet particles, respectively. The statistical approach and detailed balance principle are used in combination to compute the unknown coefficients $[(a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8); (a_0, b_1, b_8, d_1, d_8)]$ of the wave functions leading to scalar, vector and tensor contribution to the masses of spin $\frac{1}{2}$ and spin $\frac{3}{2}$ particles.

3.2 Mass Formulae

The hadronic mass spectrum is an essential ingredient in the theoretical study of the physics involving strong interactions. The mass spectrum of mesons and baryons are studied in various models and aspects. The hadron mass formula is discussed in the quark-counting aspect which shows that the free quark picture gives the Gell Mann-Okubo formula [22]. In quark models, the baryons are bound state of three-quark states (q^3) and there are various different model calculations for their masses. Empirical mass formulae [23] for the baryon octet and decuplet are the functions of one integer variable assigned to each member of the baryon sets and charge state of the baryon. These formulae are independent of any specific model. Further, the gross features of simple quark model has helped in unraveling the detailed properties of mass spectra of baryons and mesons. For example, even though the particles

have same quark content, how we determine the different masses of $\Lambda^{(\frac{1}{2})}$ (1115.683), $\Sigma^{0(\frac{1}{2})}$ (1192.642) and $\Sigma^{*0(\frac{3}{2})}$ (1382.8). The difference in spin spin interactions among quarks is the answer to this query. We assume that the mass of hadron arises from the constituent quark masses plus the spin-spin interaction energies of quarks for a meson and a baryon and can be written as [24]:

$$M_{meson} = a_m + m_i + m_j + bm_0^2 \frac{S_i \cdot S_j}{m_i m_j} \quad (3.1)$$

$$M_{baryon} = a_b + \sum_i m_i + \frac{bm_0^2}{3} \sum_{i < j} \frac{S_i \cdot S_j}{m_i m_j} \quad (3.2)$$

where a_m , a_b , b are parameters with the dimension of mass and m_i , m_j are the masses of respective quarks (antiquarks), m_0 is a scale factor which we shall take to be the mass of the lightest quark, i.e. $m_0 = m_u = m_d$. The spin dependent term includes a contribution of each pair that is directly proportional to the expectation value of $S_i \cdot S_j$ and inversely proportional to the product of the constituent quark masses $m_i m_j$. For the non spin-dependent terms, we might expect the mass equal to sum of the constituent quark masses. With the help of the wave functions of baryon octet and decuplet described, equation (3.2) gives the masses of the hadrons in terms of the parameters.

To find the masses of the baryons in $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet, we need to evaluate equation (3.2) for each baryon, by finding the expectation values for spin operators for each quark pair within the respective baryon. The spin interaction term we need to evaluate for these baryons, which are made of up of, say, u , d , s is,

$$\frac{\vec{S}_u \cdot \vec{S}_d}{m_u^2} + \frac{\vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s}{m_u m_s} \quad (3.3)$$

The hyperfine term $\sum_{i < j} S_i \cdot S_j$ are two-body operators which involve spin couplings

between two quarks. The eigen values for $S_u \cdot S_d$ are $1/4$ and $-3/4$ for spin triplet ($I = 1$) and singlet ($I = 0$) states respectively, results in evaluation of other terms by considering $\vec{J} = \vec{S}_u + \vec{S}_d + \vec{S}_s$.

$$\text{Then, } \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = \frac{1}{2}[J^2 - (S_u^2 + S_d^2 + S_s^2)] - \vec{S}_u \cdot \vec{S}_d \quad (3.4)$$

$$\text{For } J^P = \frac{1}{2}^+, \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = -1(\text{symmetric}) \quad (3.5)$$

$$\vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = 0(\text{antisymmetric})$$

$$\text{For } J^P = \frac{3}{2}^+, \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = \frac{1}{2}(\text{symmetric}) \quad (3.6)$$

$$\vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = \frac{3}{2}(\text{antisymmetric})$$

These values are applicable for all the baryon $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles relative to their quark content. The mass operator given in equation (3.3) is applied to the terms of the wave function in equation (2.3) and (2.8). The eigen values of spin operator term given in equation (3.5) and (3.6) is then used to obtain the relations in terms of parameters with the dimension of mass. It can be shown for Σ^{*0} as:

$$\begin{aligned} |\Phi_{3/2}^{(\uparrow)}\rangle &= \frac{1}{N}[a_0\Phi_1^{(3/2\uparrow)}H_0G_1 + b_1(\Phi_1^{(3/2)} \otimes H_1)^\uparrow G_1 + \\ & b_8(\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_8 + d_1(\Phi_1^{(3/2)} \otimes H_2)^\uparrow G_1 + \\ & d_8(\Phi_8^{(1/2)} \otimes H_2)^\uparrow G_8]b_1\Phi_1^{(3/2)}H_1G_1 \end{aligned}$$

The first term in above equation is obtained by combining q^3 wave function with spin 0 (scalar sea), second and third term are obtained by coupling q^3 with spin 1

(vector sea). When the mass operator is applied on first term,

$$\begin{aligned} \langle a_0 \Phi_1^{(3/2)} H_0 G_1 | \widehat{O} | a_0 \Phi_1^{(3/2)} H_0 G_1 \rangle &= \langle a_0 (F_s \psi_1^A) H_0 G_1 | \widehat{O} | a_0 (F_s \psi_1^A) H_0 G_1 \rangle \\ &= a_0 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} [\frac{1}{4 m_i^2} + \frac{1}{2 m_i m_j}]) \end{aligned}$$

Here, as the spin of F_s and H_0 is symmetric so, the mass operator applied on this term gives $S_u \cdot S_d = 1/4$ and $\vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = \frac{1}{2}$. Similarly, mass operator is applied on other terms of wave functions of $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ states and are displayed in tables 3.1 and 3.2. The spin-spin interactions mentioned in the mass formulae are internal to hadron and are responsible for mass spectrum. These terms are not small, so they are not just a perturbation, they play a vital role in calculation of masses and other properties of hadrons.

Table 3.1: Derived mass relation for $J^P = \frac{1}{2}^+$ octet particles.

Term	After applying operator on each term
$a_0 \Phi_1^{(1/2)} H_0 G_1$	$a_0 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{2}}) [\frac{1}{4 m_i m_j} - \frac{1}{m_i m_j} - \frac{3}{4 m_i m_j}])$
$a_8 \Phi_8^{(1/2)} H_0 G_8$	$a_8 (a_b + \sum_i m_i + \sum_{i,j} (-\frac{b m_0^2}{3} [\frac{1}{4 m_i m_j} - \frac{1}{m_i m_j}]))$
$a_{10} \Phi_{10}^{(1/2)} H_0 G_{\overline{10}}$	$a_{10} (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{2}}) [-\frac{3}{4 m_i m_j} - \frac{1}{4 m_i m_j} + \frac{1}{m_i m_j}])$
$b_1 (\Phi_1^{(1/2)} \otimes H_1) G_1$	$b_1 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}) [-\frac{3}{4 m_i m_j} + \frac{1}{4 m_i m_j} - \frac{1}{m_i m_j}])$
$b_8 (\Phi_8^{(1/2)} \otimes H_1) G_8$	$b_8 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{12}}) [\frac{1}{4 m_i m_j} - \frac{1}{m_i m_j} + \frac{3}{4 m_i m_j}] + (\frac{1}{\sqrt{12}} - \frac{1}{\sqrt{6}}) [-\frac{3}{4 m_i m_j} + \frac{1}{4 m_i m_j} - \frac{1}{m_i m_j}])$
$b_{10} (\Phi_{10}^{(1/2)} \otimes H_1) G_{\overline{10}}$	$b_{10} (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}) [\frac{1}{4 m_i m_j} - \frac{1}{m_i m_j} + \frac{3}{4 m_i m_j}])$
$c_8 (\Phi_8^{(3/2)} \otimes H_1) G_8$	$c_8 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{2} - \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}}) [\frac{3}{4 m_i m_j}])$
$d_8 (\Phi_8^{(3/2)} \otimes H_2) G_8$	$d_8 (a_b + \sum_i m_i + \sum_{i,j} \frac{b m_0^2}{3} (\frac{1}{\sqrt{5}} - \sqrt{\frac{3}{20}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{20}}) [\frac{1}{4 m_i m_j} - \frac{1}{m_i m_j}])$

SECTION 3.2: MASS FORMULAE

Table 3.2: Derived mass relation for $J^P = \frac{3}{2}^+$ decuplet particles.

Term	After applying operator on each term
$a_0 \Phi_1^{(3/2)} H_0 G_1$	$a_0(a_b + \sum_i m_i + \sum_{i,j} \frac{bm_0^2}{3} [\frac{1}{4m_i^2} + \frac{1}{2m_i m_j}])$
$b_1 \Phi_1^{(3/2)} H_1 G_1$	$b_1(a_b + \sum_i m_i + \sum_{i,j} \frac{bm_0^2}{3} [\sqrt{\frac{3}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}] - \sqrt{\frac{2}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]])$
$b_8 \Phi_8^{(1/2)} H_1 G_8$	$b_8(a_b + \sum_i m_i + \sum_{i,j} \frac{bm_0^2}{3} [\sqrt{\frac{3}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}] - \sqrt{\frac{2}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]])$
$d_1 \Phi_1^{(3/2)} H_2 G_1$	$d_1(a_b + \sum_i m_i + \sum_{i,j} \frac{bm_0^2}{3} [\sqrt{\frac{1}{5}} [\frac{1}{4m_i^2} + \frac{1}{2m_i m_j}] - \sqrt{\frac{2}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]] + \sqrt{\frac{2}{5}} [\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]))$
$d_8 \Phi_8^{(1/2)} H_2 G_8$	$d_8(a_b + \sum_i m_i + \sum_{i,j} \frac{bm_0^2}{3} [\sqrt{\frac{1}{10}} [\frac{1}{m_i^2} - \frac{1}{m_i m_j}] - \sqrt{\frac{2}{10}} [\frac{1}{m_i^2} - \frac{1}{m_i m_j}]] + \sqrt{\frac{2}{10}} [\frac{1}{m_i^2} - \frac{1}{m_i m_j}]))$

In tables 3.1 and 3.2, m_i, m_j are the constituent quark masses of the respective quarks in the baryon, a_b, b are the parameters with the dimension of mass and m_0 is a scale factor equal to mass of the lightest quark, i.e $m_0 = m_u$ or $m_0 = m_d$. The set of different Fock states ($|gg\rangle, |u\bar{u}g\rangle, |d\bar{d}g\rangle$) are the same for all baryon $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet members but the probability distribution is different due to mass inherited from flavor leading to different values of unknown parameters (a_0, a_8, a_{10}, b_1) etc. By substituting the eigen values of spin operator term in the baryon wave functions the masses of particles are obtained and the

detailed calculation for the proton and Σ^{*0} is shown below:

$$\begin{aligned}
Mass_{(proton)} = & 0.43(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} - \frac{3}{4m_u m_u}])) + \\
& 0.022(a_b + m_u + m_u + m_d + (-\frac{bm_0^2}{3}[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d}])) + \\
& 0.003(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[- \frac{3}{4m_u m_u} - \frac{1}{4m_u m_u} + \frac{1}{m_u m_d}])) + \\
& 0.142(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[- \frac{3}{4m_u m_u} + \frac{1}{4m_u m_u} - \frac{1}{m_u m_d}])) + \\
& 0.014(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{12}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} + \frac{3}{4m_u m_u}])) + \\
& (\frac{1}{\sqrt{12}} - \frac{1}{\sqrt{6}})[- \frac{3}{4m_u m_u} + \frac{1}{4m_u m_u} - \frac{1}{m_u m_d}])) + \\
& 0.0023(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} + \frac{3}{4m_u m_u}])) + \\
& 0.0035(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{2} - \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}})[\frac{3}{4m_u m_u}])) + \\
& 0.0014(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{5}} - \sqrt{\frac{3}{20}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{20}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d}]))
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
Mass_{(\Sigma^{*0})} = & 0.161(a_b + m_u + m_d + m_s + \frac{bm_0^2}{3}[\frac{1}{4m_u m_u} + \frac{1}{m_u m_s}])) + 0.0005(a_b + m_u + \\
& m_d + m_s + \frac{bm_0^2}{3}(\sqrt{\frac{3}{5}} - \sqrt{\frac{2}{5}})[\frac{3}{4m_u m_u} + \frac{3}{m_u m_s}])) + 0.00024(a_b + m_u + m_d + m_s) + \\
& 0.0025(a_b + m_u + m_d + m_s + \frac{bm_0^2}{3}\sqrt{\frac{1}{5}}[\frac{1}{4m_u m_u} + \frac{1}{m_u m_s}])) + 0.00027(a_b + m_u + m_d + m_s)
\end{aligned} \tag{3.8}$$

3.3 Estimation of Hadron Masses

The baryon masses are calculated in the literature using various models and taking the inputs of constituent quark masses. The constituent quark masses are defined as effective masses of quarks bound inside a baryon and are model based parameters, so we allow suitable range (in MeV) to them and try to fit these parameters to the available $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet masses, using *Mathematica* 7.0. Here, the input in the mass formulae are the coefficients calculated statistically from the statistical model assuming sea quark-gluon Fock states to be in specific flavor, spin and color states. The computed values of model parameters for proton are:

$$m_u = m_0 = 290MeV, m_d = 340MeV, b = 600, a_b = 220 \quad (3.9)$$

where a_b, b are parameters with the dimension of mass and m_u, m_d are the masses of u and d quarks, respectively and m_0 is a scale factor which we take to be the mass of the lightest quark. Now substituting these values of parameters in equation (3.7), we determine the mass of proton in the D model with all sea contributions:

$$Mass_{(proton)} = 937.6MeV$$

The mass of the proton can be calculated with other modifications such as in C model or with individual sea contributions. Model *D* find the probabilities of Fock states by suppressing the contribution of states with higher multiplicities. Model *D* is assumed to be a special case of Model *C*. Similarly, the model parameters for other baryons in octet and decuplet are calculated and are shown in the form of specific range below. For example, the model parameters for Σ^{*0} in $J^P = \frac{3}{2}^+$ decuplet can be written as:

$$m_u = m_0 = 260MeV, m_d = 310MeV, m_s = 450MeV$$

$$b = 600, a_b = 200$$

The procedure of calculation described in the former section gives the masses of the $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons and are presented in tables 3.3 and 3.4. The set of parameters, for octet, in our model are: $b= 600$ to 630 , $m_u= 250$ to 300 MeV , $m_d= 300$ to 340 MeV , $m_s= 450$ to 550 MeV and $a_b= 200$ to 230 . The set of parameters, for $J^P = \frac{3}{2}^+$ decuplet, in our model are: $b = 600$ to 640 , $m_u= 200$ to 260 MeV , $m_d= 250$ to 330 MeV , $m_s= 400$ to 500 MeV and $a_b = 200$ to 240 . The masses of spin $1/2$ and $3/2$ particles are computed in both C and D model with different sea contributions and are shown in the table 3.3 and 3.4, respectively. For calculating the contribution from the pure scalar sea, the following assumptions were made: $a_0 \neq 0$ and $b_1, b_8, d_1, d_8 = 0$, for the vector: $b_1, b_8 \neq 0$ and $a_0, d_1, d_8 = 0$ and similarly for the tensor $d_1, d_8 \neq 0$ and $a_0, b_1, b_8 = 0$. For the case of the scalar plus tensor sea: $a_0, d_1, d_8 \neq 0$ and $b_1, b_8 = 0$, for decuplet and similarly for octet. Also, to check the validity of our results, we have checked Gell-Mann-Okubo mass formula for octet, equal spacing rule for decuplet and electromagnetic mass splittings, with the results from our model. The values obtained for masses (in MeV) from our model are shown below every formula:

1. $\frac{N+\Xi}{2} = \frac{3\Lambda+\Sigma}{2}$
 $1129.40 \sim 1132.39;$
2. $\Delta - \Sigma^* = \Sigma^* - \Xi^* = \Xi^* - \Omega$
 $150.69 \sim 147.87 \sim 136.5$
3. $\Delta^+ - \Delta^{++} = n - p - (\Sigma^+ + \Sigma^- - 2\Sigma^0)$
 $0.08 \sim 0.08$
4. $\Delta^0 - \Delta^+ = \Sigma^{*0} - \Sigma^{*+} = n - p$
 $0.75 \sim 1.05 \sim 2.25$
5. $\Delta^- - \Delta^0 = \Sigma^{*-} - \Sigma^{*0} = \Xi^{*-} - \Xi^{*0} = n - p + (\Sigma^+ + \Sigma^- - 2\Sigma^0)$
 $1.08 \sim 4.5 \sim 4.34 \sim 4.42$

SECTION 3.3: ESTIMATION OF HADRON MASSES

Table 3.3: Masses of $J^P = \frac{1}{2}^+$ octet particles

Particle	C Model (<i>MeV</i>)		D Model (<i>MeV</i>)		Data (<i>MeV</i>) [25]
	With scalar sea	With (scalar+tensor) sea	With (scalar+tensor) sea	With (scalar+vector +tensor) sea	
p	1044.48	1053.17	857.29	937.6	938.27
n	1036.47	1045.7	938.8	939.85	939.56
Λ	1175.6	1187.53	1061.27	1113.5	1115.683
Σ^+	1249.53	1261.37	1141.98	1183.67	1189.37
Σ^0	1261.67	1239.81	1115.83	1189.05	1192.642
Σ^-	1226.15	1237.78	1165.86	1196.6	1197.449
Ξ^0	1469.13	1469.13	1315.99	1314.89	1314.86
Ξ^-	1390.72	1403.2	1267.03	1321.21	1321.71

Table 3.4: Masses of $J^P = \frac{3}{2}^+$ decuplet particles

Particle	C Model (<i>MeV</i>)		D Model (<i>MeV</i>)		Data (<i>MeV</i>) [25]
	With (scalar+tensor) sea	With (scalar+vector +tensor) sea	With scalar sea	With (scalar+tensor) sea	
Δ^{++}	1231.57	1230.62	1234.17	1207.44	1232.0
Δ^+	1231.87	1230.7	1234.87	1210.5	1232.0
Δ^0	1226.58	1231.45	1229.44	1206.91	1232.0
Δ^-	1233.52	1232.53	1236.67	1204.38	1232.0
Σ^{*+}	1383.04	1382.17	1385.67	1364.76	1382.8
Σ^{*0}	1383.88	1383.22	1385.56	1367.32	1383.7
Σ^{*-}	1388.62	1387.72	1391.33	1369.76	1387.2
Ξ^{*0}	1531.46	1531.09	1621.9	1523.58	1531.80
Ξ^{*-}	1536.7	1535.8	1539.2	1516.11	1535.0
Ω^-	1668.37	1668.04	1670.0	1650.7	1672.45

3.4 Conclusion

We have calculated the masses of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles using the mass formulae by taking the effect of constituent quark masses and spin spin interaction between the quarks and then applying the statistical approach. The detailed analysis is based on the calculation of masses within different approaches namely *C* and *D* model and further analyzing them by including the individual contributions coming from scalar, vector and tensor sea. To appreciate the importance and validity of sea with spin, various sea contributions are presented in table 3.3 and 3.4. This individual analysis of various sea and their possible contributions shows

the importance of scalar, vector and tensor sea in finding the masses of octet and decuplet particles.

It can be very well seen from table 3.3, that the results of masses from contribution of scalar sea in masses from C model is showing a deviation of 5% to 11% while combination of scalar-tensor contribution shows a deviation of 3% to 12%, when compared with experimental values available for $J^P = \frac{1}{2}^+$ particles. On the other hand, results from D model are deviating 0.03% to 0.47%, in case of total sea contribution, which shows that masses from total sea is providing good match with PDG data. Similarly, it can be seen from table 3.4, for decuplet, that contribution from total sea in case of C model is providing a good match with experimental masses available as compared with individual contributions from sea.

In case of octet, D model is giving better match with experimental data of masses and that too, major contribution comes from scalar plus tensor sea that dominates the contribution from vector sea by 66%. Similarly, for decuplet, C model gives better match with experimental data and hence scalar plus tensor sea contribution dominates the vector sea by 99.5%. Hence, for the calculation of masses for both $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles, scalar plus tensor sea contribution dominancy can be easily seen. The comparison of our data from table 3.3 and 3.4, for different cases with the corresponding experimental results shows that the results are not justified by ignoring contribution from any sea parameter completely. Each term of sea has non-negligible dependance on masses of octet and decuplet particles. It is also interesting to note that the set of constituent quark masses are different for every particle in $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet. We expect that the masses of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons are getting suppressed with the dynamism of the sea which is contributed by either the vectorial contribution or the spin spin interactions between the quarks. Also, the particles with two or three heavy (strange) quarks leads to less or negligible contribution of the spin-spin

interaction term to the overall masses of particles and hence will be less significant.

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4

Magnetic Moments of $J^P = \frac{1}{2}^+$ Octet and $J^P = \frac{3}{2}^+$ Decuplet Baryons

4.1 Introduction

The static properties of baryons such as magnetic moments, masses, charge radii etc. plays significant role in the study of internal structure of baryons in particle physics. Magnetic moment is one of the important quantity in understanding the properties of light baryons. It can also provide valuable understanding of mechanism of strong interaction at low energies, i.e., in the non perturbative aspects of QCD (Quantum Chromo Dynamics). Continuous theoretical efforts are being made to investigate the magnetic moments, and the calculations have benefited a lot from the information provided by the experiments.

In 1933, Frisch and Stern [1] performed the first measurement of the magnetic moment of the proton and obtained the evidence for the internal structure of the nucleon. However, forty years later the quark structure of the nucleon was directly observed in deep inelastic electron scattering experiments but we still lack a quantitative theoretical understanding of these properties including the magnetic moments. Since Coleman and Glashow [2] predicted the magnetic moments of the baryon octet, there has been a lot of progress in both the theoretical and experimental verification for the baryon magnetic moments. The measurements of the baryon decuplet magnetic moments were reported for Δ^{++} [3] and Ω^- [4].

A number of theoretical models for calculating the magnetic moments of the $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons have been put forth over the past few decades. The approaches include a non relativistic quark model, which calculates the magnetic moments of the baryons as the sum of its constituent quark magnetic moments, QCD string approach [5], hypercentral model [6] and chiral quark model with exchange currents [7]. These studies indicate the growing interest in this field.

Further improvements were made by including effects such as sea quark contributions [8], quark orbital momentum effects [9], $SU(3)$ symmetry breaking effects [10–13]. Sogami and Ohyamaguchi [14] presented a concept of effective mass to calculate magnetic moments of baryons and later, Bains and Verma [15] used the concept of effective mass and screened charge of quarks to calculate magnetic moments. The subject of magnetic moments is challenging to understand because this phenomenon of baryons is contributed from the valence quarks magnetic moments as well as from other effects such as contributions from pion cloud, relativistic effects, etc.

The method of QCD sum rules [16, 17] has been successful to calculate the octet baryon magnetic moments. Predictions grounded on a number of theoretical formalisms have been done to calculate the various properties of octet baryons. The Cloudy bag model [18], approach of chiral perturbation theory [19], chiral quark-

soliton model [20], phenomenological quark model [21], Lorentz covariant chiral quark model [22], an unquenched quark model [23], a spin-flavor symmetry based parametrization method of quantum chromodynamics [24], a covariant and confining Nambu-Jona-Lasinio model [25], chiral constituent quark model (χCQM) [26] are few to name. Recently, the strength of the pion cloud contributions to the octet, form factors [27] are constrained by the masses and magnetic moments of the baryon octet.

The experimental information on the $J^P = \frac{3}{2}^+$ decuplet baryons is limited due to their short life time. Predictions based on a number of theoretical formalisms have been made to calculate the magnetic moment of decuplet baryons. The relativistic quark model (RQM) [28, 29], QCD-based quark model (QCDQM) [30], effective mass scheme (EMS) [31], light cone QCD sum rule (LCQSR) [32], QCD sum rule (QCDSR) [33–35], Skyrme model [36, 37], chiral quark soliton model (CQSM) [38–40], chiral perturbation theory (χPT) [41, 42], lattice QCD (LQCD) [43–45] etc. are few to name. The magnetic moments of decuplet baryons are calculated using the momentum projection technique of Peierls and Yoccoz in the non-scaling color dielectric model(CDM) [46].

Recently, it has been shown that chiral quark model [47] with configuration mixing when coupled with the quark sea polarization and orbital angular momentum through the Cheng-Li mechanism is able to give an excellent fit to the octet and decuplet magnetic moments. Discrete methods are suggested in literature [8] which have, studied the nucleonic properties like magnetic moments, weak decay coupling ratios, spin distributions by considering the contributions from sea as well. These methods describe baryon as composites of the valence quarks and sea containing gluons and quark-antiquark pairs.

In order to understand the implications of sea quarks and to make our analysis more responsive, it would be interesting to examine the effect of the quark effective masses

and charges on the magnetic moment. We are applying the statistical approach in addition to the detailed balance principle looking into the effect of effective quark charges and masses where we have successfully retrieved the magnetic moments for all the $J^P = \frac{1}{2}^+$ particles. Here, the contribution of the sea quark is quite significant when compared with the the valence quarks and its effects have been analyzed with the effects of effective charges and masses of the quarks present.

4.2 Magnetic Moments of $J^P = \frac{1}{2}^+$ Octet Particles

Out of many approaches and models such as the lattice QCD, quark-diquark model, chiral constituent model, potential model, QCD sum rules etc., available for the three body systems, we employ here the statistical approach to study the baryons in $J^P = \frac{1}{2}^+$ state. Earlier, *M. Batra et. al.* [48, 49] studied the magnetic moments of $J^P = \frac{1}{2}^+$ octet baryons using statistical model by analyzing the strange and non-strange quark-gluon Fock states for the nucleonic system. Also, statistical model is applied to a system with strange quarks in the valence part i.e. lambda and other hyperons.

Magnetic moments are low energy and long distance phenomena. The magnetic moments of $J^P = \frac{1}{2}^+$ octet baryons are computed using the spin-flavor wave functions of the constituting quarks. Here, we have incorporated the effect of (a) quark effective masses, (b) quark effective charges and (c) both *i.e.* quark effective mass plus effective charge, to compute the magnetic moments.

The magnetic moment of the hadron is precisely a function of its structure and of its properties such as spin, flavor, charge and the effective masses of the bound quarks. In general, the magnetic moments of baryons are obtained by applying the following

SECTION 4.2: MAGNETIC MOMENTS OF $J^P = \frac{1}{2}^+$ OCTET PARTICLES

magnetic moment operator on the baryonic wave function [eq. (2.3)]:

$$\hat{\mu} = \sum_i \mu_i^{eff} \sigma_i$$

where $\mu_i^{eff} = \frac{e_i}{2m_{eff}}$, μ_i^{eff} represents magnitude of effective quark magnetic moments for $i = u, d, s$ and e_i represents the quark charge. σ_i is the Pauli matrix representing the spin term. The values of quark magnetic moments used in our approach are $:\mu_u = 1.852\mu_N, \mu_d = -0.972\mu_N, \mu_s = -0.613\mu_N$.

In general, the meaning of the constituent quark mass refers to the energy that the quarks carry inside the color singlet hadrons, and we call it the effective masses of the quarks [50]. Accordingly, the effective mass varies from system to system of hadronic states. In order to study the effect of quark confinement on the magnetic moments, the effective quark masses can be considered. In conformity with additivity assumption, the simplest way to incorporate this adjustment is to first express M_q in the magnetic moment operator in terms of M_B , the mass of the baryon obtained additively from the quark masses, which then is replaced by $M_B + \Delta M$, ΔM being the mass difference between the experimental value and M_B . This leads to the following mass adjusted magnetic moments of constituent quarks [51, 52]:

$$\begin{aligned} \mu_u^{eff} &= 2[1 - (\Delta M/M_B)]\mu_N, \\ \mu_d^{eff} &= -[1 - (\Delta M/M_B)]\mu_N, \\ \mu_s^{eff} &= -M_u/M_s[1 - (\Delta M/M_B)]\mu_N \end{aligned} \tag{4.1}$$

where M_B is the mass of the baryon calculated by adding the quark masses and ΔM is the difference between the experimental mass value and M_B . The baryon magnetic moments calculated after incorporating this effect would be referred to as baryon magnetic moments with effective quark masses. Thus, the magnetic moment

of baryonic system (B) can be parameterized as:

$$\langle B|\mu^{eff}|B\rangle = \sum_{i=u,d,s} \langle \Psi_B | \frac{e_i \sigma_i}{2m_{eff}} | \Psi_B \rangle \quad (4.2)$$

where Ψ_B is the baryonic wave function. In general, [8]

$$\begin{aligned} \langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle &= \frac{1}{N^2} [\langle \Phi_1^{(1/2\uparrow)} | \hat{O} | \Phi_1^{(1/2\uparrow)} \rangle + \sum_{i=8,10} a_i^2 \langle \Phi_i^{(1/2\uparrow)} | \hat{O} | \Phi_i^{(1/2\uparrow)} \rangle + \\ &\sum_{i=1,8,10} b_i^2 \langle \Phi_{bi}^{(1/2\uparrow)} | \hat{O} | \Phi_{bi}^{(1/2\uparrow)} \rangle + 2b_8 c_8 \langle \Phi_{b8}^{(1/2\uparrow)} | \hat{O} | \Phi_{c8}^{(1/2\uparrow)} \rangle \quad (4.3) \\ &+ c_8^2 \langle \Phi_{c8}^{(1/2\uparrow)} | \hat{O} | \Phi_{c8}^{(1/2\uparrow)} \rangle + d_8^2 \langle \Phi_{d8}^{(1/2\uparrow)} | \hat{O} | \Phi_{d8}^{(1/2\uparrow)} \rangle] \end{aligned}$$

The detailed calculation is shown in section 2.1.1. The magnetic moment relations obtained after applying operator for $J^P = \frac{1}{2}^+$ particles in terms of parameters α , β and quark effective masses ($\mu_u^{eff}, \mu_d^{eff}, \mu_s^{eff}$) are shown in first column of table 4.2. We repeat our computations by varying the effective masses of quarks (in MeV) from 370 to 390 for u and d quarks and 500 to 530 for strange quark, to have an idea about the most suitable set of effective quark masses that yields the quark magnetic moments and hence magnetic moments of baryons. As the values of effective masses are model dependent so the magnetic moments of quarks are also model dependent and we have to take their values compatible with the constituent quark masses. Also, the masses used for the above calculations, for all baryons (in case of 2 gluons), are shown in detail, in the table 4.1.

SECTION 4.2: MAGNETIC MOMENTS OF $J^P = \frac{1}{2}^+$ OCTET PARTICLES

Table 4.1: Table showing the values of individual effective quark masses and their experimental masses for every baryon in octet.

Baryon octet	M_B (MeV)	Experimental mass (MeV) [53]
p (uud)	370+370+378	938.27
n (udd)	370+386+386	939.56
Λ^0 (uds)	370+370+530	1115.683
Σ^+ (uus)	370+370+500	1189.37
Σ^0 (uds)	370+370+510	1192.642
Σ^- (dds)	380+380+530	1197.449
Ξ^0 (uss)	370+500+500	1314.86
Ξ^- (dss)	370+500+500	1321.71

In addition to the variation of the quark masses, charge also get affected. For example, charge of the baryon quark, when probed by a soft photon, get screened due to presence of quarks in its vicinity. This situation is similar to the shielding of the nuclear charge of the helium atom due to its outer electron cloud. So, we have calculated magnetic moments with effective charge also. Thus, the magnetic moment of baryonic system (B) can be parameterized as:

$$\mu_B = \sum_{i=u,d,s} \langle \Psi_B | \frac{e_i^{(eff)} \sigma_Z^i}{2m_{eff}} | \Psi_B \rangle \quad (4.4)$$

We have considered the linear dependance of effective charge on the charge of the shielding quarks. Thus, the effective charge e_a of quark a for the baryon $B(a, b, c)$ can be written as [54, 55]:

$$e_a^B = e_a + \alpha_{ab}e_b + \alpha_{ac}e_c \quad (4.5)$$

where e_a is the bare charge of quark a . Taking $\alpha_{ab} = \alpha_{ba}$ and applying isospin

symmetry, we have

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta'$$

$$\alpha_{us} = \alpha_{ds} = \alpha'$$

$$\alpha_{ss} = \gamma'$$

The screened quark charges for baryons in terms of the parameters α', β', γ' (*not to be misunderstood with statistical parameters i.e. α, β*) can be expressed as:

$$\begin{aligned} e_u^p &= \frac{2}{3}(1 + \frac{1}{2}\beta'), e_d^p = -\frac{1}{3}(1 - 4\beta'), \\ e_u^n &= \frac{2}{3}(1 - \beta'), e_d^n = -\frac{1}{3}(1 - \beta'), \dots \end{aligned} \tag{4.6}$$

Substituting these values of effective charge and applying the magnetic moment operator to wave function as in equation (4.3) and using the unknown parameters in effective charge relations i.e. $\alpha' = 0.248$, $\beta' = 0.025$ and γ' varying from 0.018 to 0.029, as input, we determine the magnetic moments and are shown in table 4.2.

Table 4.2: Magnetic moments of $J^P = \frac{1}{2}^+$ octet baryons with (a) effective quark masses, (b) quark effective charge and (c) both i.e quark effective mass plus quark effective mass. The results of magnetic moments with various modifications are shown taking two and three gluons in sea respectively.

Baryon octet magnetic moments	Magnetic moments (μ_N)						Exp. Results [53]
	With effective quark mass		With effective quark charge		With effective quark mass+ effective quark charge		
	(2 gluons)	(3 gluons)	(2 gluons)	(3 gluons)	(2 gluons)	(3 gluons)	
$\mu_p = 3(\mu_u^{eff}\alpha - \mu_d^{eff}\beta)$	2.79	2.29	1.81	1.73	2.74	2.62	2.79
$\mu_n = 3(\mu_d^{eff}\alpha - \mu_u^{eff}\beta)$	-1.83	-1.50	-0.85	-0.84	-1.38	-1.37	-1.91
$\mu_\Lambda = \frac{1}{2}(\alpha - 4\beta)(\mu_u^{eff} + \mu_d^{eff}) + (2\alpha + \beta)\mu_s^{eff}$	-0.634	-0.60	-0.41	-0.36	-0.59	-0.52	-0.613
$\mu_{\Sigma^+} = 3(\mu_u^{eff}\alpha - \mu_s^{eff}\beta)$	2.464	2.11	1.99	1.78	3.0	2.77	2.458
$\mu_{\Sigma^0} = \frac{3}{2}(\mu_u^{eff}\alpha + \mu_d^{eff}\alpha - 2\mu_s^{eff}\beta)$	0.74	0.64	0.58	0.52	0.86	0.8	0.791 [37]
$\mu_{\Sigma^-} = 3(\mu_d^{eff}\alpha - \mu_s^{eff}\beta)$	-0.974	-0.82	-0.83	-0.75	-1.29	-1.17	-1.160
$\mu_{\Xi^0} = 3(\mu_s^{eff}\alpha - \mu_u^{eff}\beta)$	-1.388	-1.203	-0.933	-0.84	-1.36	-1.23	-1.250
$\mu_{\Xi^-} = 3(\mu_s^{eff}\alpha - \mu_d^{eff}\beta)$	-0.615	-0.53	-0.422	-0.403	-0.57	-0.55	-0.6507

Table 4.3: Table showing the final results of magnetic moments of octet particles calculated using statistical model by taking effective quark mass plus effective quark charge.

Baryon octet	Magnetic moments (μ_N) with effective quark mass+ effective quark charge	Exp. Results [53]
p	2.74	2.79
n	-1.38	-1.91
Λ^0	-0.59	-0.613
Σ^+	3.0	2.458
Σ^0	0.86	0.791 [50]
Σ^-	-1.29	-1.160
Ξ^0	-1.36	-1.250
Ξ^-	-0.57	-0.6507

The results shown in table 4.2 are found to be better with two gluons than three gluons. Our results of magnetic moments are showing error percentage upto 2-30% when compared with experimental data.

4.3 Magnetic Moments of $J^P = \frac{3}{2}^+$ Decuplet Particles

Similarly, we apply magnetic moment operator ($\hat{O} = \mu_i \sigma_i$) to decuplet wave function (eq 2.8) to calculate the magnetic moments. The magnetic moment operator

Table 4.4: Expressions obtained after applying magnetic moment operator to baryon decuplet are shown.

Baryon	$\langle \Phi_{3/2}^\dagger \widehat{O} \Phi_{3/2}^\dagger \rangle$
Δ^{++}	$a_0^2(15\mu_u) + b_1^2(11\mu_u) + b_8^2(11\mu_u) + d_1^2(3\mu_u) + d_8^2(\frac{3}{2}\mu_u)$
Δ^+	$a_0^2(30\mu_u + 15\mu_d) + b_1^2(22\mu_u + 11\mu_d) + b_8^2(22\mu_u + 11\mu_d) + d_1^2(8\mu_u + \mu_d) + d_8^2(4\mu_u + \frac{1}{2}\mu_d)$
Δ^0	$a_0^2(30\mu_d + 15\mu_u) + b_1^2(22\mu_d + 11\mu_u) + b_8^2(22\mu_d + 11\mu_u) + d_1^2(8\mu_d + \mu_u) + d_8^2(4\mu_d + \frac{1}{2}\mu_u)$
Δ^-	$a_0^2(15\mu_d) + b_1^2(11\mu_d) + b_8^2(11\mu_d) + d_1^2(3\mu_d) + d_8^2(\frac{3}{2}\mu_d)$
Σ^{*+}	$a_0^2(30\mu_u + 15\mu_s) + b_1^2(22\mu_u + 11\mu_s) + b_8^2(22\mu_u + 11\mu_s) + d_1^2(8\mu_u + \mu_s) + d_8^2(4\mu_u + \frac{1}{2}\mu_s)$
Σ^{*0}	$a_0^2[5(\mu_u + \mu_d + \mu_s)] + b_1^2[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] + b_8^2[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] + d_1^2(\mu_u + \mu_d + \mu_s) + d_8^2[\frac{1}{2}(\mu_u + \mu_d + \mu_s)]$
Σ^{*-}	$a_0^2(30\mu_d + 15\mu_s) + b_1^2(22\mu_d + 11\mu_s) + b_8^2(22\mu_d + 11\mu_s) + d_1^2(8\mu_d + \mu_s) + d_8^2(4\mu_d + \frac{1}{2}\mu_s)$
Ξ^{*0}	$a_0^2(30\mu_s + 15\mu_u) + b_1^2(22\mu_s + 11\mu_u) + b_8^2(22\mu_s + 11\mu_u) + d_1^2(8\mu_s + \mu_u) + d_8^2(4\mu_s + \frac{1}{2}\mu_u)$
Ξ^{*-}	$a_0^2(30\mu_s + 15\mu_d) + b_1^2(22\mu_s + 11\mu_d) + b_8^2(22\mu_s + 11\mu_d) + d_1^2(8\mu_s + \mu_d) + d_8^2(4\mu_s + \frac{1}{2}\mu_d)$
Ω^-	$a_0^2(15\mu_s) + b_1^2(11\mu_s) + b_8^2(11\mu_s) + d_1^2(3\mu_s) + d_8^2(\frac{3}{2}\mu_s)$

depends on the flavor and spin of the i^{th} quark.

$$\begin{aligned}
\langle \Phi_{3/2}^\dagger | \widehat{O} | \Phi_{3/2}^\dagger \rangle = \frac{1}{N^2} [& a_0^2 \langle O_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} + b_1^2 \langle O_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} + b_8^2 \langle O_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} \\
& + d_1^2 \langle O_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow} + d_8^2 \langle O_f^i \rangle^{\lambda\lambda} \langle \sigma_Z^i \rangle^{\lambda\uparrow\lambda\uparrow}]
\end{aligned}
\tag{4.7}$$

where $O_f^i = \frac{e_i}{2m_i}$, for magnetic moments and λ denotes the symmetric wave-function.

The expressions for magnetic moment is written in terms of unknown parameters (a_0, b_1, b_8, d_1, d_8) as shown in table 4.4. These parameters are calculated in the statistical model and are shown in table 4.5. In all the expressions, μ_u, μ_d, μ_s are the magnetic moments of u, d, s quark, respectively.

Table 4.5: The table for “nc” shows the calculated coefficients in the wave function of baryon decuplet particles.

States	H_0G_1	H_1G_1	H_1G_8	H_2G_1	H_2G_8
$ gg\rangle$	0.000428271	0	0.000004461	0.000085654	0.0000053534
$ u\bar{u}u\bar{u}\rangle$	0.000919879	0	0.000009582	0.000183976	0.0000114985
$ d\bar{d}d\bar{d}\rangle$	0.000919879	0	0.000009582	0.000183976	0.0000114985
$ s\bar{s}s\bar{s}\rangle$	0.000919879	0	0.000009582	0.000183976	0.0000114985
$ u\bar{u}g\rangle$	0.000659309	0	0.000027471	0.000263724	0.0000329654
$ d\bar{d}g\rangle$	0.000659309	0	0.000027471	0.000263724	0.0000329654
$ s\bar{s}g\rangle$	0.000579918	0	0.000024163	0.000231967	0.0000289959
$ u\bar{u}d\bar{d}\rangle$	0.000689911	0	0.000028746	0.000275964	0.0000344955
$ u\bar{u}s\bar{s}\rangle$	0.000689911	0	0.000028746	0.000275964	0.0000344955
$ d\bar{d}s\bar{s}\rangle$	0.000689911	0	0.000028746	0.000275964	0.0000344955
$ u\bar{u}d\bar{d}g\rangle$	0.00005677	0.00008516	0.000007097	0.000045418	0.0000056773
$ u\bar{u}s\bar{s}g\rangle$	0.00004744	0.00007116	0.000005930	0.000037950	0.0000047438
$ d\bar{d}s\bar{s}g\rangle$	0.00004744	0.00007116	0.000005930	0.000037950	0.0000047438
$ u\bar{u}u\bar{u}g\rangle$	0.00001892	0.00003785	0.000003154	0.000022709	0.0000028386
$ d\bar{d}d\bar{d}g\rangle$	0.00001892	0.00003785	0.000003154	0.000022709	0.0000028386
$ s\bar{s}s\bar{s}g\rangle$	0.00005322	0.00010643	0.000008869	0.000063859	0.0000079824
$ u\bar{u}gg\rangle$	0.00000652	0.00001304	0.000001086	0.000007822	0.0000009777
$ d\bar{d}gg\rangle$	0.00000652	0.00001304	0.000001086	0.000007822	0.0000009777
$ s\bar{s}gg\rangle$	0.00003761	0.00007522	0.000006268	0.000045134	0.0000056418
$ 0\rangle$	0.15454	-	-	-	
Total	0.161989536	0.000510899	0.000241126	0.002516263	0.000274684

We substitute the values of coefficients from table 4.5 or 2.8 to the magnetic moment expression of Σ^{*0} ,

$$\begin{aligned} \mu_{\Sigma^{*0}} = & a_0^2[5(\mu_u + \mu_d + \mu_s)] + b_1^2[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] + b_8^2[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] \\ & + d_1^2(\mu_u + \mu_d + \mu_s) + d_8^2[\frac{1}{2}(\mu_u + \mu_d + \mu_s)] \end{aligned} \quad (4.8)$$

$$\begin{aligned} \mu_{\Sigma^{*0}} = & (\frac{1}{N})[(0.161989536)[5(\mu_u + \mu_d + \mu_s)] + (0.000510899)[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] \\ & + (0.000241126)[\frac{11}{3}(\mu_u + \mu_d + \mu_s)] + (0.002516263)(\mu_u + \mu_d + \mu_s) \\ & + (0.000274684)[\frac{1}{2}(\mu_u + \mu_d + \mu_s)]] \end{aligned} \quad (4.9)$$

Substituting the of μ_u, μ_d, μ_s , we can compute magnetic moments for Σ^{*0} . Similarly, magnetic moments of remaining decuplet particles are calculated and are shown in table 4.6. To appreciate the importance of sea with spin, modifications in the model is done by choosing sea to be contributing through scalar, vector or tensor coefficients as shown in table 4.7. Here, sea with spin 0, 1, 2 are called scalar, vector and tensor sea respectively. These coefficients are directly related to probabilities of quark-gluon Fock states in spin, color and flavor space. Here, sea is found to be dynamic for scalar and tensor part unlike baryon octets where tensor part seems to be less dominating because of quark spin flip processes [56].

Table 4.6: Comparison of computed magnetic moments (in terms of μ_N) of baryon decuplet with other models and experimental data.

Particle	C Model	D Model	SQM	QCDQM [30]	χ QM [13]	CQSM [39, 40]	Sum rules [57]	Data
Δ^{++}	5.47	4.70	5.56	5.689	5.30	4.85	4.52	4.52 ± 0.50 [58]
Δ^+	2.68	2.32	2.73	2.778	2.58	2.35	2.12	$2.7_{-1.3}^{+1.0} \pm 1.5 \pm 3$ [59]
Δ^0	-0.09	-0.14	-0.09	-0.134	-0.13	-0.14	-0.29	-
Δ^-	-2.87	-2.47	-2.92	-3.045	-2.85	-2.63	-2.69	-
Σ^{*+}	3.03	2.62	3.09	2.933	2.88	2.47	2.63	-
Σ^{*0}	0.26	0.22	0.27	0.137	0.17	-0.02	0.08	-
Σ^{*-}	-2.52	-2.19	-2.56	-2.659	-2.55	-2.52	-2.48	-
Ξ^{*0}	0.61	0.46	0.63	0.424	0.47	0.09	0.44	-
Ξ^{*-}	-2.15	-1.81	-2.31	-2.307	-2.25	-2.40	-2.27	-
Ω^-	-1.82	-1.63	-1.84	-1.970	-1.95	-2.29	-2.06	-2.02 ± 0.05 [53]
$\frac{\mu_{\Delta^{++}}}{\mu_p}$	1.96	1.68	2.0	-	-	-	-	1.62 [58]
$\frac{\mu_{\Omega^-}}{\mu_{\Lambda^0}}$	2.96	2.65	3.0	-	-	-	-	3.16 [60]

Table 4.7: Table showing various modifications in magnetic moments by using C model (in terms of μ_N) of decuplet particles using statistical model.

Particle	With (scalar+vector +tensor) sea	With vec- tor sea	With scalar- tensor sea	Exp. Results
Δ^{++}	5.47	4.09	5.47	4.52 ± 0.50
Δ^+	2.68	2.007	2.69	$2.7_{-1.3}^{+1.0} \pm 1.5 \pm 3$
Δ^0	-0.09	-0.068	-0.097	-
Δ^-	-2.87	-2.13	-2.88	-
Σ^{*+}	3.03	2.28	3.04	-
Σ^{*0}	0.26	0.19	0.26	-
Σ^{*-}	-2.52	-1.87	-2.53	-
Ξ^{*0}	0.61	0.45	0.61	-
Ξ^{*-}	-2.15	-1.63	-2.15	-
Ω^-	-1.82	-1.34	-1.82	-2.02 ± 0.05

The magnetic moment ratios $\frac{\mu_{\Delta^{++}}}{\mu_p}$ and $\frac{\mu_{\Omega^-}}{\mu_{\Lambda^0}}$ have been experimentally determined [58, 60]. To check the validity of our model we have calculated these ratios with values of $\mu_{\Delta^{++}}$ and μ_{Ω^-} obtained from our model and are shown in table 4.6. Our model has been able to produce the results which are verified by the sum rules given by Coleman-Glashow [61] for baryon decuplet magnetic moments i.e

$$\mu_{\Sigma^{*0}} = \frac{1}{2}\mu_{\Sigma^{*+}} + \frac{1}{2}\mu_{\Sigma^{*-}}$$

$$\mu_{\Delta^-} + \mu_{\Delta^{++}} = \mu_{\Delta^0} + \mu_{\Delta^+}$$

In our model,

$$\mu_{\Sigma^{*0}} = 0.26; \quad \frac{1}{2}\mu_{\Sigma^{*+}} + \frac{1}{2}\mu_{\Sigma^{*-}} = 0.26 \quad (4.10)$$

$$\mu_{\Delta^-} + \mu_{\Delta^{++}} = 2.6; \quad \mu_{\Delta^0} + \mu_{\Delta^+} = 2.6 \quad (4.11)$$

4.4 Conclusion

Statistical models provide simplicity in describing various properties of the baryonic states which includes the “sea”. Baryonic structure is considered to be consisting of valence quarks and sea limited by a few number of quark-antiquark pairs multiconnected non-perturbatively through gluons. Magnetic moments of baryon $J^P = \frac{1}{2}^+$ octet particles and $J^P = \frac{3}{2}^+$ decuplet particles are calculated in statistical model. The relevant operator is applied on wave functions with $(a_0, a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8)$ and $(a_0, b_1, b_8, d_1, d_8)$ coefficients to calculate their magnetic moments.

In the present chapter, the interaction between quarks within the baryons are considered for the calculation of magnetic moments through the inclusion of effective mass and effective charge of the constituent quarks within the baryon. Effective

quark masses and effective quark charges for quarks u, d , and s are calculated using fixed inputs for baryon masses (PDG) and statistical parameters (α, β) as input in the respective formulae. These effective masses and effective charges of u, d and s are acting as an input to the magnetic moments of $J^P = \frac{1}{2}^+$ baryon octet. Our results give a good match with the experimental values specifically when calculated with quark effective mass (with 2 gluons) whereas magnetic moments deviate when quark effective charge is considered, as seen in table 4.2. The results suggest that the obtained magnetic moments are small as compared to PDG results because of large effective mass of the quarks in baryons. There could be the small momentum transfer by a soft gluon in the measurement, that affects the effective quark masses hence leading to their sensitivities in calculating the magnetic moments of baryons. Though in all the cases, the contribution of quark sea is quite significant. Here, each Fock states has a certain probability which determines the coefficients coming in the wave function, used to calculate the effect of effective quark charges or effective quark masses to the magnetic moments with different range of parameters. The parameters discussed here are highly dependent on coefficients $a_0, b_1, b_8 \dots$ of the octet wave function.

Magnetic moments of $J^P = \frac{3}{2}^+$ decuplet baryons are calculated in two approaches i.e. model C and model D where model C aims at finding relative probabilities of the Fock states in color, spin and flavor space whereas model D finds the probabilities of Fock states by suppressing the contribution of states with higher multiplicities. The magnetic moments obtained from C model seems to deviating upto 5% from simple quark model and D model upto 25% when compared with SQM in table 4.6. The explicit numerical values from scalar (spin 0), vector (spin 1) and tensor (spin 2) sea contributions to the magnetic moment of $J^P = \frac{3}{2}^+$ decuplet baryons are also calculated, by suppressing higher multiplicities contributions. This analysis provide the basis to understand the extent to which the sea quarks contribute to the structure of the baryon. For calculating the contribution from pure scalar sea,

following assumptions were made: $a_0 \neq 0$ and $b_1, b_8, d_1, d_8 = 0$, for vector: $b_1, b_8 \neq 0$ and $a_0, d_1, d_8 = 0$ and similarly for tensor $d_1, d_8 \neq 0$ and $a_0, b_1, b_8 = 0$. For the case of scalar plus tensor sea: $a_0, d_1, d_8 \neq 0$ and $b_1, b_8 = 0$. Table 4.7 shows the extent to which sea contributions from scalar-tensor and vector effects the magnetic moments of decuplet members. For instance, when vector sea is neglected in statistical model (C model), the magnetic moments vary by 10-20% from experimental data as shown in table 4.7 whereas the data varies 10-30% when the calculation is done with only vector sea and not scalar-tensor sea. It is well observed, that only scalar-tensor sea is enough to retrieve the experimentally known magnetic moments and therefore we conclude scalar-tensor dominance over vector sea. Also, as the experimental data of magnetic moments is available for $\Delta^{++}, \Delta^+, \Omega^-$, it is interesting to compare our results with experimental information as well as with the results of other theoretical models. From table 4.6, it is clearly evident that these values are preserved in our model and other magnetic moments are compared with some theoretical models matching well within the error percentage of 10-20%.

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5

Strange and Non-Strange Sea Quark-Gluon Effects for Baryons

5.1 Introduction

Interest in the concept of strangeness has grown exponentially, since it was introduced decades ago. Measurements with strange hadrons can provide important information on several topics in hadronic physics, for example, the nucleon tomography and quark orbital momentum, attainable by studying the Generalized Parton Distribution (GPDs) and the Transverse Momentum dependent parton Distribution functions (TMDs) and the hadron spectroscopy.

The contribution of strange quarks to baryon properties has been of great interest

too. Many achievements and much progress have been realized in both theoretical and experimental fields. Presence of $s\bar{s}$ pairs in the nucleon is evident by the measurements [1] performed in deep-inelastic scattering of electron-nucleon through neutrino-induced charm production and polarization effects. The production of ϕ exceeds in $\bar{p}p$ annihilation [2] above the naive OZI rule, indicates the presence of $s\bar{s}$ component in the proton [3]. Our information on the presence of strange quark in the nucleons comes from neutrino DIS and charged-lepton semi-inclusive DIS (SIDIS) experiments. Strange quark contribution to the nucleon form factors is the topic of interest because it comes purely from sea quarks. Lots of experiments from different collaborations have measured the contributions precisely [4–6]. There are also many theoretical discussions on the strange form factors [7, 8]. In 2002, the strange form factors were studied in perturbative chiral quark model (PCQM) [9]. The analysis of strangeness production and propagation in particle research experiments [10], and the investigation of the possible “strange” phases in the interior of neutron stars [11] helps in understanding the dynamics of hadrons with strangeness. Strange particles such as kaons (antikaons) are the lightest mesons made up of one antistrange (strange) quark, and one up or down quark (antiquark). Considering the fact that kaons are would be Goldstone bosons, the study of their interaction with other hadrons and particularly with nucleons allows for the investigation of the non-perturbative character of QCD at low energies. This allows to test the scales and symmetries of QCD in this energy regime, such as chiral symmetry and its partial restoration in dense and/or hot matter [12]. This permits a better understanding of the nature of newly discovered states with the strange degree of freedom that whether they can be understood as dynamically generated states via hadron-hadron scattering processes. Recently, experiments like SAMPLE at MIT-Bates [13], G0 at JLab [14], A4 at MAMI [15] and HAPPEX at JLab [16] have given indications of strangeness contribution in the nucleon. These experiments have provided considerable insight on the role played by strange quarks in the charge, current and spin

structure of the nucleon.

Theoretical models which explain the flavor asymmetry ($\bar{d} - \bar{u}$) provide predictions like that the $s(x)$ and $\bar{s}(x)$ distributions are different, e.g. in the meson cloud model [17], chiral soliton model [18]. The observation of the ($\bar{d} - \bar{u}$) flavor asymmetry in the light quark sea of the proton [19–21] has been one of the important topic in hadronic physics over the past two decades, leading to a major reexamination of our understanding of the quark structure of the nucleon. In particular, the measurement revealed the importance of 5-quark Fock state components of the nucleons wave function, and the crucial role played by chiral symmetry breaking. This asymmetry had been anticipated by Thomas [22] a decade earlier, and has subsequently been studied using various non perturbative models [23, 24].

Many theoretical trials have been done to describe the origin of the nucleon sea and its antiquark asymmetry [23, 25]. It is assumed that the sea is generated by gluon splitting into $u\bar{u}$ and $d\bar{d}$ pairs. Field and Feymann [26] suggested that the presence of extra $u\bar{u}$ in the sea can lead to suppression of extra $g \rightarrow u\bar{u}$ via Pauli blocking. However, a subsequent calculation [27] found that the effects of Pauli blocking are very small, and this result has also been confirmed by another calculation [28], believing that there must be a non perturbative origin.

Another attempt to understand the sea flavor asymmetry of the proton is from a pure statistical consideration named as principle of detailed balance [29] as explained in section 2.2. In this chapter, Statistical model and detailed balance principle are used to calculate $\bar{d} - \bar{u}$ asymmetry, strange quark importance to magnetic moments, $SU(3)$ symmetry breaking and spin distribution of octet particles. A significant role is played by the the constituents like quarks, anti quarks and gluons in the sea, in determining the properties of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles which leads to refine the data and understanding of the dynamics inside the baryons on the basis of sea consisting of gluons and quarks having strangeness, therefore, checking

the consistency of statistical model in form of certain modifications.

5.2 Flavor Asymmetry

The $\bar{d} - \bar{u}$ asymmetry in the baryon, also called the antiquark flavor asymmetry, is defined as the difference between antiquarks \bar{u} and \bar{d} in the baryon. Several experiments during the last few years have predicted that there exist a flavor asymmetry in the light antiquark distributions in the baryons specifically nucleon. For a compilation of experiments which have measured the $\bar{d} - \bar{u}$ asymmetry, see Ref. [23]. On phenomenological grounds, meson cloud model and chiral quark model have also been able to confirm the values observed experimentally. In our work, the detailed balance model is used in congruence with the statistical effects to look into $\bar{d} - \bar{u}$ asymmetry. The model generated $\bar{d} - \bar{u} = 0.124$ which is in concurrence with the predictions of E866/NuSea result [21] of 0.118 ± 0.012 . We have computed $\bar{d} - \bar{u}$ asymmetry for all other octet and decuplet baryons using the relation:

$$\bar{d} - \bar{u} = \left(\left[\sum_{j=0, k=0}^{j=2, k=3} \rho_{0, j, 0, k} \right] - \left[\sum_{i=0, k=0}^{i=2, k=3} \rho_{i, 0, 0, k} \right] \right) \quad (5.1)$$

where i is the number of quark-antiquark $u\bar{u}$ pairs, j is the number of quark-antiquark $d\bar{d}$ pairs, l is the number of $s\bar{s}$ pairs and k is the number of gluons in the sea. The values for flavor asymmetry are shown in table 5.1 and 5.2 followed by list of average number of partons for all $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet members. The total number of intrinsic partons inside every baryon of octet can be calculated as [29, 30]:

$$N' = u_{val} + d_{val} + s_{val} + u\bar{u}_{sea} + d\bar{d}_{sea} + s\bar{s}_{sea} + g$$

and from the normalization condition, $\sum_{i,j,l,k} \rho_{i,j,l,k} = 1$, we can find the average number of partons for all baryons.

The importance of $SU(3)$ symmetry and its breaking has been discussed for baryon octets [31]. Due to the limited experimental information on $SU(3)$ symmetry breaking in decuplets, we have restricted ourselves to analyse $\bar{d} - \bar{u}$ asymmetry. This data may be useful for experimentalist to investigate further. Due to the variation in the individual contributions coming from sea containing either $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs motivates us to study the $\bar{d} - \bar{u}$ asymmetry in the valence and sea quarks. The anti-quark flavor asymmetry is calculated for all $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles to justify their importance in the sea and is shown in tables 5.1 and 5.2, respectively. It is worth to mention that our calculations are performed in non-relativistic frame.

Table 5.1: $\bar{d} - \bar{u}$ asymmetry and average number of partons for $J^P = \frac{1}{2}^+$ octet baryons.

$J^P = \frac{1}{2}^+$ octets	\bar{u}	\bar{d}	$\bar{d} - \bar{u}$	Data	Avg. no. of partons
p	0.33	0.46	0.134	0.124 [21]	4.84
n	0.46	0.33	-0.134	-	4.84
Λ^0	0.44	0.44	0	0	4.73
Σ^+	0.31	0.71	0.39	0.410 [32]	4.87
Σ^0	0.44	0.44	0	0	4.74
Σ^-	0.71	0.31	-0.39	-	4.87
Ξ^0	0.43	0.70	0.26	0.27 [32]	4.85
Ξ^-	0.70	0.43	-0.26	-	4.85

Table 5.2: $\bar{d} - \bar{u}$ asymmetry and average number of partons for $J^P = \frac{3}{2}^+$ decuplet baryons.

$J^P = \frac{3}{2}^+$ octets	\bar{u}	\bar{d}	$\bar{d} - \bar{u}$	Data [33]	Avg. no. of partons
Δ^{++}	0.19	0.59	0.4	0.109	4.68
Δ^+	0.27	0.39	0.12	0.036	4.63
Δ^0	0.39	0.27	-0.12	0.036	4.63
Δ^-	0.59	0.19	-0.4	0.109	4.68
Σ^{*+}	0.44	0.44	0	0.371	4.52
Σ^{*0}	0.39	0.39	0	0.000	4.32
Σ^{*-}	0.66	0.29	-0.37	0.371	4.53
Ξ^{*0}	0.39	0.65	0.26	0.216	4.42
Ξ^{*-}	0.65	0.39	-0.26	-0.216	4.42
Ω^-	0.59	0.59	0	0.000	4.45

More experimental data for the decuplet baryons are needed in order to be able to compare with our predictions. It is observed from our calculations that there exist simple relations between the flavor asymmetries, e.g., the excess of \bar{d} over \bar{u} in the proton is equal to the excess of \bar{u} over \bar{d} in the neutron, and similar relations for the Δ and the singly strange or doubly strange particles.

The sea quark asymmetries calculated for various $J^P = \frac{1}{2}^+$ octet baryons having different flavor in valence part are listed in Table 5.3. This difference in flavor of valence part will justify for differences in these asymmetries. These values can confirm the mechanism we proposed to explain the sea quark asymmetry in the baryons. It can be observed from Table 5.3 that the sea quark asymmetries are enhanced by the difference of corresponding valence quark numbers and suppressed by the sum of valence quark numbers. When the valence quark numbers $u_v > d_v$,

Table 5.3: Sea quark asymmetry values for different u , d valence quark numbers for $J^P = \frac{1}{2}^+$ octet particles.

Asymmetry values d valence quark number	u valence quark number			
	0	1	2	3
0	0	0.26 (Ξ^0)	0.39 (Σ^+)	0.4 (Δ^{++})
1	-0.26 (Ξ^-)	0 (Λ^0), (Σ^0)	0.13 (p), 0.12 (Δ^+)	-
2	-0.39 (Σ^-)	-0.13 (n), -0.12 (Δ^0)	-	-
3	-0.4 (Δ^-)	-	-	-

the sea quarks \bar{u} are easier to annihilate because of the existence of more u valence quarks and it leads the sea quark asymmetry. On the other hand, for a large number of total valence quarks $[u_v + d_v]$, there is a suppression in the relative difference of valence quarks, that weakens the quark asymmetries even if $[u_v - d_v]$ remains the same [34].

As described earlier in chapter 2, Zhang et al. [35] calculated probabilities in flavor space based on statistical approach without introducing any external parameter to find the $d\bar{d}$ and $s\bar{s}$ asymmetry of the sea. Here, we have extended that approach by including the Fock states with $s\bar{s}$ pairs in the sea.

5.3 Importance of $SU(3)$ Symmetry and its Breaking

Flavor breaking in the baryonic sea is because the mass of strange quark is much higher than up and down quarks. Strange quark mass restricts the exchange of gluon into $s\bar{s}$ pair. Thus, we incorporate $SU(3)$ breaking effects in sea for spin 1/2 hyperons. The hyperons are checked against the impact of $SU(3)$ symmetry breaking for their magnetic moments. Strange quark mass breaks the $SU(3)$ symmetry in the structure of hyperon. The strange mass corrections are parameterized through the symmetry breaking parameter “ r ”. Here “ r ” is the ratio of mass of s and u

quark. Symmetry breaking corrections are applied to valence and sea quarks and its effect lead to modified values of magnetic moments of octet hyperons. The magnetic moment operator accompanied by symmetry breaking parameter “ r ” is applied on wave function in equation (2.3). In the case of breaking in $SU(3)$ for $J^P = \frac{1}{2}^+$, the values obtained have the form of seven coefficients and symmetry breaking parameter “ r ”, due to which the expressions of magnetic moments are modified in terms of “ r ”. Symmetry breaking is applied to the magnetic moments calculated with effective masses (with 2 gluons), in terms of the parameter $r = m/m_s$ [36] where m is the effective mass of u and d quarks. Certainly, “ r ” includes a direct dependence of m_s and m_1 plus a constant of proportionality. Here $m_1 = m_u$ or m_d under isospin symmetry. The value of “ r ” lies in the range 0.70 to 0.78, depending upon the respective effective quark masses. The best-fit value obtained for “ r ” yields the theoretical results for magnetic moments. The results shown under the third column in table 5.4 are the magnetic moments with effective quark masses (with 2 gluons), previously shown in second column of table 4.2.

Table 5.4: Magnetic moments of particles with $SU(3)$ symmetry breaking, $SU(3)$ symmetry and comparison with experimental data available for octet hyperons.

Baryon octet magnetic moments	Magnetic moments (μ_N)		
	With SU(3) symmetry breaking	With SU(3) symmetry	Exp. Re- sults [38]
Λ^0	-0.44	-0.634	-0.613
Σ^+	2.40	2.464	2.458
Σ^0	0.691	0.745	0.791 [37]
Σ^-	-1.018	-0.974	-1.160
Ξ^0	-1.063	-1.388	-1.250
Ξ^-	-0.35	-0.615	-0.6507

5.4 Importance of Strangeness in sea for $J^P = \frac{3}{2}^+$ Decuplet Members

Significance of strange sea over non-strange sea is significant specifically for baryons having strange quarks. To check the contributions from strange and non-strange quark sea, strange sea is being added. The strange quarks contribute through the quark sea generated by $g \rightarrow s\bar{s}$. Here, our aim is to see the importance of the sea in the relative probabilities having strange and non strange quark contents, in the Fock states. For this purpose, we have applied the mass reduction coefficient $(1 - C_l)^{n-1}$ where $C_l = \frac{2M_s}{M_B - 2M_s}$ (M_B = mass of baryon, M_s = mass of strange quark, n is the total number of partons), to the $J^P = \frac{3}{2}^+$ decuplet baryons to accommodate strange quark-antiquark pairs in the sea. Using $M_s=101 \text{ MeV}$, a proper mass reduction coefficient is applied to the sea, to see the importance of strange sea vs non-strange sea. Hidden strangeness can be said to be generated via the process where quarks can emit gluon and gluon can emit $s\bar{s}$ pair i.e. $q \rightarrow qg \rightarrow qs\bar{s}$. The accomodatability of strange quark-antiquark is limited upto two in the Fock states. Table 5.5 provide the results for magnetic moments with strange and non-strange sea and their comparison with the experimental results. We see in table 5.5 that the strange quark contributions are also very negligible for non-strange particles but the value comes out to be different from zero when the strange sea is included in the sea.

Table 5.5: Computed magnetic moments (in terms of μ_N) of baryon decuplet in strange sea vs non-strange sea.

Particle	Magnetic Moments with strange sea ($g \rightarrow u\bar{u}, d\bar{d}, s\bar{s}$)	Magnetic Moments with non-strange sea ($g \rightarrow u\bar{u}, d\bar{d}$)	Exp. results [38]
Δ^{++}	5.47	5.48	4.52 ± 0.50
Δ^+	2.69	2.70	$2.7_{-1.3}^{+1.0} \pm 1.5 \pm 3$
Δ^0	-0.097	0.091	-
Δ^-	-2.88	-2.88	-
Σ^{*+}	3.04	3.02	-
Σ^{*0}	0.26	0.26	-
Σ^{*-}	-2.53	-2.51	-
Ξ^{*0}	0.61	0.59	-
Ξ^{*-}	-2.15	-2.12	-
Ω^-	-1.82	-1.72	-2.02 ± 0.05

Here, the importance of reduction coefficient $(1 - C_l)^{n-1}$ can be seen with the data shown in the following table, which clearly distinguishes between doubly strange baryon and singly strange baryon to accommodate $s\bar{s}$ pairs in sea. The same trend can be observed for baryons with higher $s\bar{s}$ pairs in sea.

Fock state	Value of constraint $(1 - C_l)^{n-1}$
uds $u\bar{u}s\bar{s}g$	0.271
uss $u\bar{u}s\bar{s}g$	0.317
uds $s\bar{s}s\bar{s}g$	0.2504
uss $s\bar{s}s\bar{s}g$	0.3060

This kind of reduction coefficient is proven to be helpful to understand the strange behavior of the sea in various baryons. It has been noticed in general that, the

strange sea dominates more over non-strange sea quarks for strange baryon particles ($\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega^-$) as compared to non strange decuplet members ($\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$).

We are also interested in measuring how often, compared with pair of light quarks, strange quarks are made. For this purpose, we have defined the strangeness suppression factor as $\lambda_s = \frac{2(s\bar{s})}{(u\bar{u}+d\bar{d})}$. This ratio shows the presence of strange quarks in sea. So, we have calculated the strangeness suppression factor for all particles in $J^P = \frac{1}{2}^+$ octet state in the framework of principle of detailed balance. The calculation is not performed for $J^P = \frac{3}{2}^+$ particles due to non availability of theoretical or experimental data. The calculation of strangeness suppression factor includes various sub processes where quark and gluon can emit $q\bar{q}$ and gluons respectively, for example $g \Leftrightarrow q\bar{q}$, $g \Leftrightarrow gg$, $q \Leftrightarrow qq$. The value for the strangeness suppression factor for all baryon octet have been calculated and presented in table 5.6. In specific, the results obtained for proton have been compared with the unquenched model [39] and the ref. [40] results and found to be in good agreement.

Table 5.6: Strangeness suppression for all particles in octet are shown in this table. The values in the second row with single * shows the results from Unquenched quark model (UQM) [39] and with double ** are the results from ref. [40] for strangeness suppression in proton.

Baryon octet	$\frac{s\bar{s}}{d\bar{d}}$	$\frac{u\bar{u}}{d\bar{d}}$	$\lambda_s = \frac{2(s\bar{s})}{(u\bar{u}+d\bar{d})}$
p	0.32/0.26*/0.22**	0.71/0.57*/0.74**	0.38/0.34*/0.29**
n	0.46	1.40	0.38
Λ^0	0.28	1.00	0.29
Σ^+	0.19	0.43	0.27
Σ^0	0.28	1.00	0.28
Σ^-	0.45	2.26	0.27
Ξ^0	0.17	0.62	0.21
Ξ^-	0.28	1.61	0.21

5.5 Spin Contributions from Individual Quarks to the Baryons

Great efforts have been already spent on understanding the spin and flavor content of the baryons. It is still a matter of concern to understand that how quarks carry the nucleon spin. The $SU(6)$ model helps in explaining the spin distribution of the Λ but the predictions from the quark model are contradictory from the deep-inelastic experiments, which claims the spin content from u and d quarks to be 40% and rest part may come from strange quark. Further, the Δq^Λ is related to the fragmentation function measured experimentally [41]. Here, Δq^Λ is the fraction of the spin of the Λ carried by the spin of quarks and antiquarks of flavor q .

The individual spin polarization due to quarks for hyperons are calculated and compared with data of other available model. In application, the individual spin polarization is defined as, $\Delta q = n(q \uparrow) - n(q \downarrow) + n(\bar{q} \uparrow) - n(\bar{q} \downarrow)$ for $q = u, d, s$, where $n(q \uparrow)$ is the number of spin-up and $n(q \downarrow)$ is the number of spin down quarks of flavor q for both quarks and anti-quarks. In addition, total spin distributions of baryon is also determined by applying the operator $I_1^B = \frac{1}{2} \sum_i e_i^2 \sigma_Z^i$ where e_i and σ_Z^i are the charge of quark and spin projection operator respectively, to the baryonic wave function. The results from statistical model for hyperons and comparison with other models are shown in table 5.7. All the properties in table 5.7 are defined in terms of parameters α and β .

Table 5.7: Spin distribution from individual quarks and total spin distribution computed in statistical model.

Baryon	Quark spin polarizations and distribution	Calculated Values	Chiral Quark Soliton Model [42]
Λ	$\Delta u = \frac{\alpha}{2} - 2\beta$	-0.02	-0.03
	$\Delta d = \frac{\alpha}{2} - 2\beta$	-0.02	-0.03
	$\Delta s = 2\alpha + \beta$	0.70	0.74
	$I_1^\Lambda = \frac{1}{4}(\alpha - 2\beta)$	0.041	0.027
Σ^+	$\Delta u = 3\alpha$	0.91	0.98
	$\Delta d = 0$	-7.40×10^{-17}	-0.02
	$\Delta s = -3\beta$	-0.21	-0.29

	$I_1^{\Sigma^+} = \frac{2}{3}\alpha - \frac{1}{6}\beta$	0.191	-
Σ^0	$\Delta u = \frac{3}{2}\alpha$	0.46	0.48
	$\Delta d = \frac{3}{2}\alpha$	0.46	0.48
	$\Delta s = -3\beta$	-0.22	-0.29
	$I_1^{\Sigma^0} = \frac{5}{12}\alpha - \frac{1}{6}\beta$	0.117	-
Σ^-	$\Delta u = \frac{1}{6}\alpha - \frac{1}{6}\beta$	0.03	-0.02
	$\Delta d = 3\alpha$	0.91	0.98
	$\Delta s = -3\beta$	-0.22	-0.29
	$I_1^{\Sigma^-} = \frac{1}{6}\alpha - \frac{1}{6}\beta$	0.0389	-
Ξ^0	$\Delta u = -3\beta$	-0.22	-0.29
	$\Delta d = 0$	0	-0.02
	$\Delta s = 3\alpha$	0.95	0.98
	$I_1^{\Xi^0} = \frac{1}{2}\alpha - \beta$	0.0838	-
Ξ^-	$\Delta u = -3\beta$	-0.2	-0.020
	$\Delta d = \alpha - 4\beta$	0.017	-0.29
	$\Delta s = 3\alpha$	0.95	0.98
	$I_1^{\Xi^-} = \frac{1}{6}\alpha - \frac{1}{6}\beta$	0.0404	-

5.6 Discussion and Conclusion

Thus, principle of detailed balance is considerably successful in finding and explaining the flavor asymmetry of $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles. The same principle with addition of $s\bar{s}$ pairs in the sea calculates the $\bar{d} - \bar{u}$ asymmetry. A reduction coefficient $(1 - C_l)^{n-1}$, is applied here by limiting the maximum number of $s\bar{s}$ pairs due to momenta and mass of s -quark which limits the gluons to have larger free energy.

The strangeness suppression factor λ_s , discussed in this chapter suggests the strange quark contribution for all baryons in sea. It also suggests the importance of strangeness in sea. Though the data for this factor is available only for proton, but we have calculated this strangeness suppression factor for all the $J^P = \frac{1}{2}^+$ particles in our model. Hence, this suppression factor suggests that $s\bar{s}$ sea accomodability enhances for particles with higher strangeness in their valence part.

The calculations having strange quark ($m_s = 101\text{MeV}$) involve an application of mass reduction coefficient and are used for studying magnetic moments of decuplet

baryons. The comparison of our calculations and results for different cases with the experimental results available, shows that although the strange quark contribution in sea is negligible, yet its effects is visible through the calculation of magnetic moments.

The calculated values of magnetic moments of hyperons with $SU(3)$ breaking is compared when magnetic moments with $SU(3)$ symmetry is taken and can be seen in table 5.4. The listed value shows that the strange quark contribution to the magnetic moment due to its mass is an order of magnitude smaller than the up and down quarks. Therefore, very less contribution from the heavy quarks is noticed comparable with the light quarks contribution. The results with $SU(3)$ symmetry breaking is not in much agreement with the experimentally observed values. Plausibly, at energies $1GeV$, the results are better in column 3 of table 5.4 due to conservation of $SU(3)$ symmetry where $s \sim u \sim d$ is applicable. Energies of $1GeV$ are considered to be low energies in high energy physics. We study all the properties such as magnetic moments and spin distribution at such low energies.

The individual spin polarization for u and d quarks matches with the theoretical results. Further, to check the contributions from strange and non-strange quark polarized spin densities, strange sea is included. We see in table 5.7 that strange and non-strange quark spin contributions are giving comparable results with ref. [42]. Hyperons are different from that of nucleons because of presence of strange quark in valence part leading to larger breaking. This motivates us to throw some light on the spin and flavor structure of strange particles of octet. Our data is found to be consistent with ref. [42] for the individual spin polarization of strange quark Δs but it shows deviation for Δu , Δd in few cases. The strange sea seems favoring the data in ref. [42], thereby, seems to the proof of the fact that gluons undergo quark-antiquark pair annihilation and creation.

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Summary and Outlook

The main objective of this thesis is to study low energy properties of ground state baryon $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet using statistical methods. These low energy properties include masses, magnetic moments, spin distribution and $\bar{d} - \bar{u}$ asymmetry. Baryon is a microscopic system with complicated structure, which can not be simply described by just three quarks, rather it contains $q\bar{q}$ pairs and gluons in sea part. The dominant role, in determining the low energy properties of these ground state baryons, is played by these constituent quarks and gluons. The phenomenological models like the QCD-based quark model (QCDQM), chiral constituent quark model, effective mass scheme (EMS), QCD sum rules (QCDSR), Skyrme model etc, study these low energy properties by assuming limited number of $q\bar{q}$ and gluon content in the mesons and baryons apart from the constituent quarks.

At low energies QCD is non perturbative and ungovernable, hence models plays a major role in the study of baryonic properties with distinctive ability, accuracy and validity at different energy scale.

QCD allows existence of rich sea of virtual quarks, antiquarks and gluons by the valence quarks. Also, this sea plays a crucial role in determining the importance of other heavy quark like strangeness in sea, even if strange quarks are not present in the valence part of baryons. Since a fully dynamical calculation with several constituents in the sea is a difficult task, the statistical interpretations are helpful in analyzing the properties like masses, spin distribution and magnetic moments of the ground state baryons. Statistical models provide intuitive appeal and physical simplicity, that have made success in describing the structure of baryons. This interest is due to rapid growth and advancements in the experimental facilities at different energy scales. For example, the experimental facilities at CDF and BASE have become a subject of motivation to study baryon properties and hence its structure in the non perturbative Quantum Chromodynamics (QCD). We have used a statistical model in which a baryon is taken as an ensemble of quark-gluon Fock states. These Fock states can be single gluon or multi-gluon states along with quark-antiquark pairs. These quarks and gluons have to be understood as 'intrinsic' partons of the baryon. The detailed balance principle in combination with the statistical models have been useful to determine the low energy properties.

An antisymmetric total wave function for $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet has been constructed for a three-quark valence and a sea with definite spin and color quantum numbers and the respective expansion coefficients have been determined. The sea is taken to be flavorless but with the spin and color quantum numbers which, when combined with the corresponding quantum numbers of three-quark valence part makes baryon total spin 1/2 or 3/2 and colorless system respectively. We have used the approximations in which a quark in the valence part is not anti symmetrized with an identical quark in the sea, and have treated quarks and gluons

as non relativistic particles moving in S-wave motion. The probabilities in flavor are computed by using principle of detailed balance while the probabilities in spin and color is estimated using relative multiplicities in the form of ratios in the statistical model. With the respective wave functions for octet and decuplet baryons, a proper statistical approach is applied to calculate the masses, magnetic moments, spin distributions, strange and non-strange sea quark-gluon effects.

Mass is an important and essential property to understand matter. The masses are calculated in statistical model by using the mass formulae which is a function of constituent quark masses and spin spin interaction terms between the quarks. The masses of particles are then calculated using the wave functions of respective baryons and operating them with relevant mass operators. Masses have been analyzed from the sea perspective by taking the contributions from scalar, vector and tensor sea and also some of their combinations. It is observed that major contribution to the masses of baryons comes from scalar and tensor sea, that dominate the vector sea by 66% and 99.5%. Hence, in the calculation of masses for both $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles, scalar plus tensor sea contribution dominancy can be easily seen. In general, the sea is found to be dynamic for the scalar and tensor in both octet and decuplet. Here, the spin-spin interaction term dominates for vectorial sea and hence it suppresses the overall contribution to the masses from vector sea. It has also been analyzed that the particles with two or three heavy strange quarks has little or negligible contribution from the spin-spin interaction term to the overall masses of the particles and hence will be less significant.

The magnetic moments of the $J^P = \frac{1}{2}^+$ baryon are studied in the framework of the statistical model along with principle of detailed balance, in which the effect of “sea” is taken into account via inclusion of quark effective mass and quark effective charge. Our results for the magnetic moments of $J^P = \frac{1}{2}^+$ baryons are comparable with the experimental values, specifically when calculated with quark effective mass (with 2

gluons) whereas the values deviate when we include quark effective charge, as seen in table 4.2. Further, magnetic moments are calculated for baryon $J^P = \frac{3}{2}^+$ particles. The magnetic moments obtained from C model seems to deviating upto 5% from simple quark model and D model upto 25% when compared with SQM in table 4.6. Also, to check the validity of our approach few modifications were made i.e. the values are studied in Model C and Model D and also from scalar, vector and tensor sea. It has been observed that when vector sea is neglected in statistical model (C model), the magnetic moments deviate by 10-20% from experimental data whereas the data deviate by 10-30% when the calculation is performed with only vector sea and not scalar-tensor sea. It is well observed, that only scalar-tensor sea is enough to retrieve the experimentally known magnetic moments and therefore we conclude the dominance of scalar-tensor over vector sea. The available experimental information on magnetic moment of $\Delta^{++}, \Delta^+, \Omega^-$ are well preserved within our approach and magnetic moments for other particles are compared with few theoretical models in table 4.6 which is giving a match within the error percentage of 10-20% (see table 4.6).

In the final part of the thesis, the importance of strange and non-strange sea quark-gluon for baryons is also seen by studying their effects on these properties. The antiquark flavor asymmetry, $\bar{d} - \bar{u}$ have been computed along with the total number of intrinsic partons inside every baryon octet and decuplet. It can be observed from the results that, there exist simple relations between the flavor asymmetries, e.g. the excess of \bar{d} over \bar{u} in the proton is equal to excess of \bar{u} over \bar{d} in the neutron, and similarly for the Δ , singly strange or doubly strange particles. The isospin symmetry leads to these relations among the flavor asymmetries of octet baryons. Moreover, importance of $SU(3)$ symmetry and its breaking is seen for spin 1/2 strange baryons, for the magnetic moment by using a symmetry breaking parameter “r”. This parameter plays a significant role by providing the basis to understand the extent to which sea quarks contribute to the structure of baryon. The results with $SU(3)$

symmetry breaking is not in good agreement with the experimentally observed values. Plausibly, at energies 1GeV , the results are better due to conservation of $SU(3)$ symmetry where $s \sim u \sim d$ is applicable. The listed values in table 5.4 show that the strange quark contribution to the magnetic moment due to its mass is almost an order of magnitude smaller than the up and down quarks, leading to a very small contribution from the heavy quarks when compared with the contribution coming from the light quarks.

Importance of strangeness in sea is studied by using a mass reduction coefficient $(1 - C_l)^{n-1}$ due to higher mass of strange quark. The magnetic moments of decuplet particles are checked with and without strange sea and the results indicate very little but significant contribution of strange quarks-antiquark pairs particularly for non-strange ground state baryon particles. In addition to this, to appreciate the strange quarks in the sea, the strangeness suppression factor for all particles in $J^P = \frac{1}{2}^+$ octet state has been applied in the framework of principle of detailed balance. The strangeness suppression factor discussed in table 5.6 suggests the strange quark contribution for all baryons in the sea. It also shows the importance of strangeness in the sea. This suppression factor suggests that $s\bar{s}$ sea accommodability enhances for particles with higher strangeness in their valence part.

The uniqueness of our model lies in the fact that our framework is working well for all $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet particles. To conclude, the statistical model has estimated the sea contributions to the ground state properties of the baryons. The present statistical model results are able to give a qualitative and quantitative description of the results and further endorse the fact that sea quarks provide valuable information about baryons. The future experiments for other $J^P = \frac{3}{2}^+$ baryons will not only provide a direct method to determine the sea quark contributions to the magnetic moments but also impose important constraints on the statistical parameters. We can also extend our knowledge of low energy properties by calculating charge radii and quadrupole moments of baryons. The statistical model here is ap-

plied to $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons but this model can also be extended to a system of pentaquarks and excited baryons as well for higher angular momentum. This extension can be done by constructing the suitable wave function for pentaquarks. Pentaquarks are bound state of a meson and a baryon. Also, parton distribution functions of baryons can be studied in statistical model.

The present results can be very useful for the predictions of decuplet magnetic moments not only in free space but also in the presence of hadronic media created in heavy ion collision experiments focused at structural study of baryons, such as FAIR, RHIC and NICA. The present results can be of relevance for the Baryon Antibaryon Symmetry Experiment (BASE) at LHC.