

A DISSERTATION REPORT

on

Topology-free Generation of Gouge-free and Minimized
Undercut Tool Path from “Ball Drop” Based Cutter
Location Data

Submitted in partial fulfillment of the requirements for the award of degree of

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in

CAD/CAM

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Certificate

I hereby certify that the thesis entitled "Topology-free Generation of Gouge-free and Minimized Undercut Tool Path from "Ball Drop" Based Cutter Location Data" is an genuine record of my own work carried out in completing the thesis requirement for the degree of **Master of Engineering in CAD/CAM Engineering** at Thapar University, Patiala, under the guidance of Mr. Ajayinder Singh Jawanda, Associate Professor, Mechanical Engineering Department, Thapar University, Patiala.

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(Mukhmeet Singh)

Abstract

One methods used for tool path (TP) planning for CNC machining of a sculptured surface is done using cutter location (CL) from the “Ball drop” method, for STL format of CAD geometry, proposed by **Manos et al. [1]**.

Ball drop method is a fast and robust method to get gouge free cutter location points, as it does not need the topology of the model and its connectivity. This method gives us a gouge-free CL point by finding the position of a ball, representing the end of a ball end cutter, where it will rest on the triangulated STL part surface, when dropped along the tool axis. If these CL points are connected in sequence of the footprint of the path to be followed by the tool tip then the tool path generated using this method would not be gouge-free. This is because the connecting path between CL points is not checked for gouging with the STL part surface. Thus, the final machined geometry would have overcut material and undercut material due to forward and side step of the tool path, scallops and concave fillet due to spherical tool tip radius.

In this thesis work a method, known as ‘TP from CL interpolation’, is developed and validated by simulation and experimentally to generate a gouge-free tool path using cutter location data based on ‘Ball Drop Method’. The advantages of ‘Ball Drop Method’ i.e. its speed and robustness are used to obtain the initial CL data. The implementation of this work is done on ‘PBG (KW) Inc. Milling Lathe CNC’ using ball nose end mill tool.

Complexity is Good but Simple is Genius

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Abbreviations

CAD:	Computer Aided Design
CAM:	Computer Aided Manufacturing
CNC:	Computer Numerical Control
NC:	Numerical Control
STL:	Standard Tessellation Language
CL:	Cutter Location
CC:	Cutter Contact
B-Rep:	Boundary Representation
SCALM:	Single Controlled Axis Lathe Mill
ASCII:	American Standard Code for Information Interchange
3D:	Three Dimensional
RP:	Rapid Prototyping
TPG:	Tool Path Generation

CHAPTER 1

Introduction

As the world is developing day by day, the demand for accuracy in machining of complex geometries is rising; this leads to the development of various tools and machines in the field of NC machining. CAD/CAM technology has been broadly used for the design and manufacturing of the complex and highly finished parts which are used in various kind of industries. Traditionally, to make a product physically, it undergoes the various processes as shown in Figure 1.1.

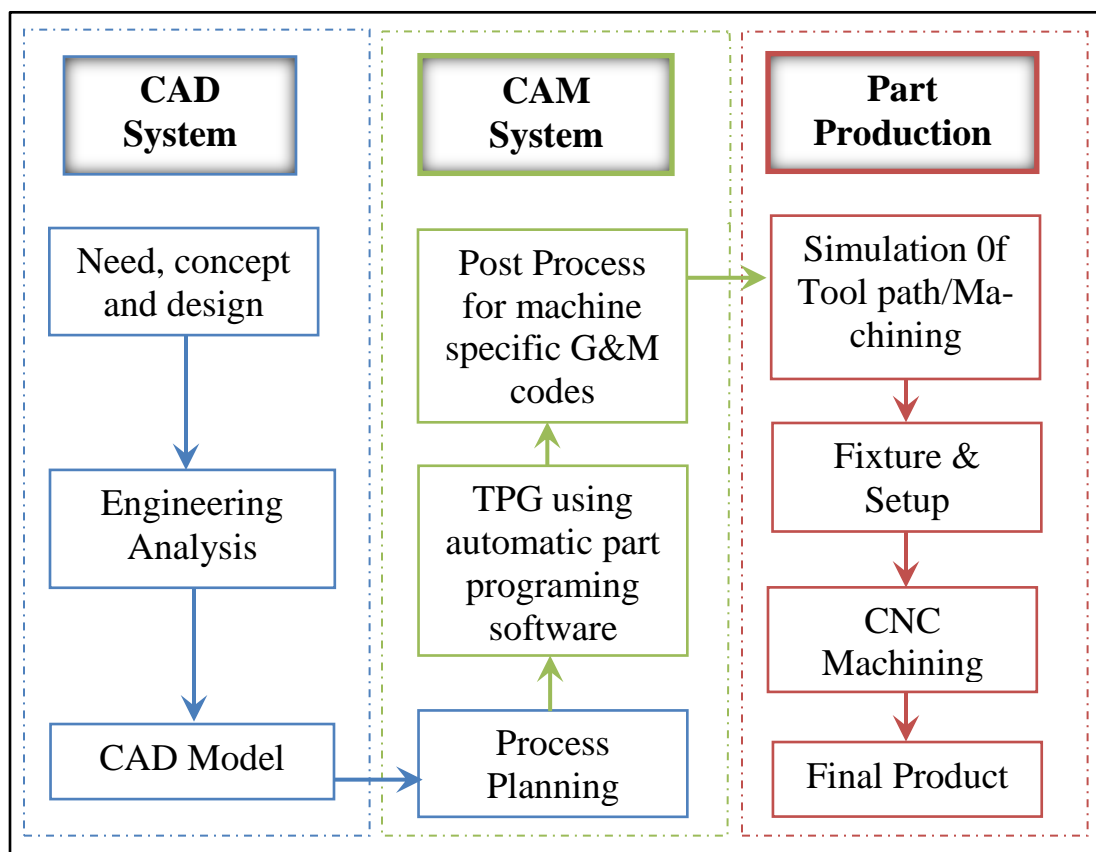


Figure 1.1: Traditional CAD CAM machining approach

After making the CAD model of work piece then tool path is converted into G&M codes in post process using CAM module, it could lead to the loss of data while conversion, especially when the geometric model is made using curves and splines. To overcome this limitation the need of an alternate to post processing is raised, which leads to concept of 'Direct machining and control' (DMAC) proposed by Gay et al. [9].

According to the concept, CAD model is converted into STL file using which integrated Tool Path generation (TPG) is done. This generated tool path is directly fed to CNC controller. The modified approach of CNC machining (referred from [8]) is shown in Figure 1.2.

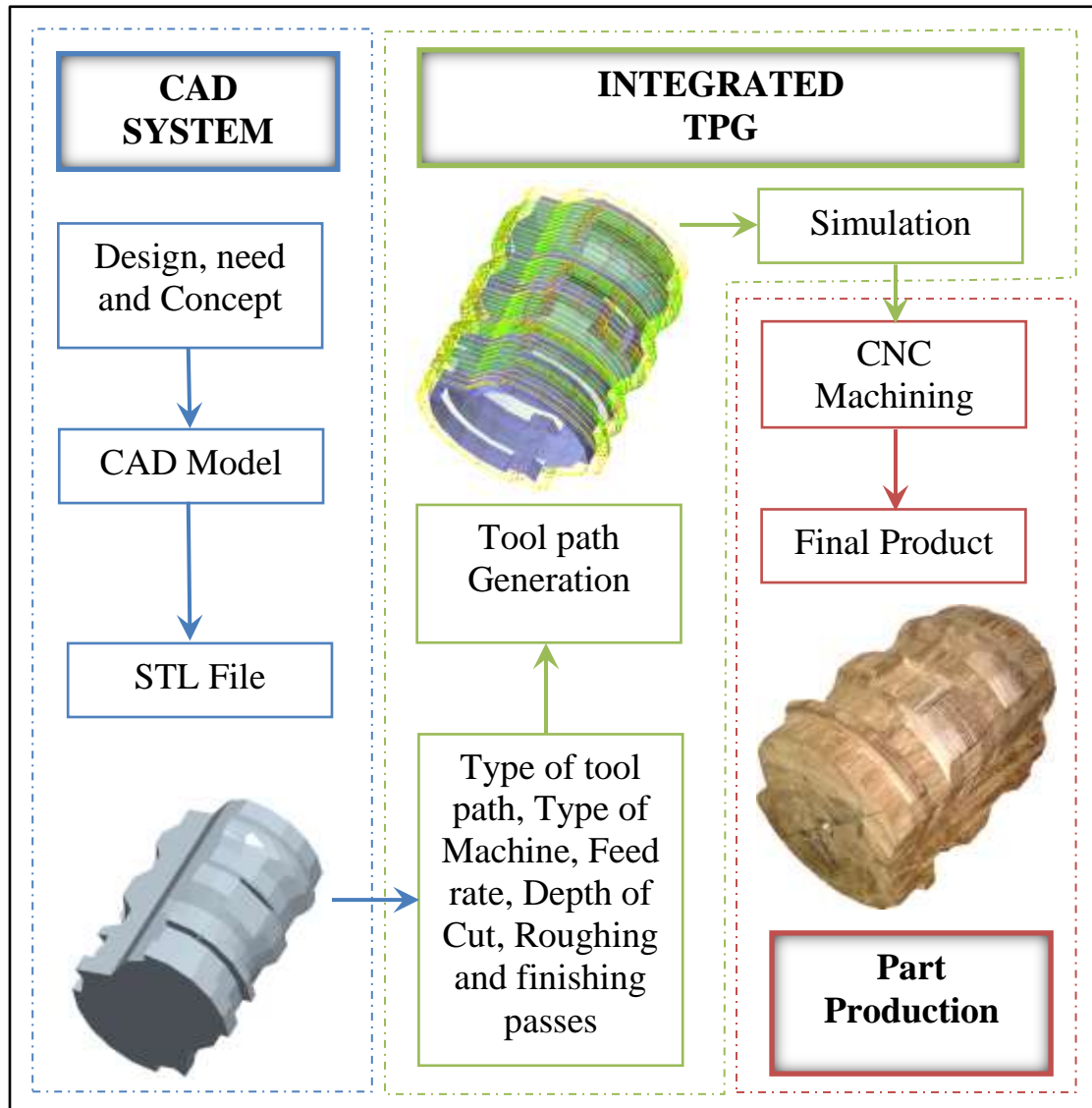


Figure 1.2: Direct Model to Part CNC machining approach

Generally, there are three types of CNC machining i.e. Milling, Tuning and Turn Milling, further subdivided into 3 axis, in which only the three axis are controlled by the controller and 5 axis machining, in which the controller could control extra two axis. The tool follows point to point motion as driven by the tool path footprints. The plan by which a tool should move over the surface of the work piece, which includes Tool path

footprint, tool position, side step, forward step, depth of cut etc. is known as Tool Path Planning. Over the past few decades, a lot of researchers proposed the numerous generalized methods in the field of tool path planning for making a better tool path for various sculptured geometries, which aims the various factors such as accuracy of the geometry, machining time, computational time, surface finish and roughness etc. This work represents an approach to further development in the tool path planning for higher accuracy of the geometry.

1.1 Tool Path Generation

For tool path generation, a pseudo symmetric 3D model of the product is designed using any of the CAD tool. Using the information of the geometry, extracted from the CAD model, the corresponding tool path points are generated and fed to the CNC machine controller as required. The Tool Path for a pseudo symmetric part is as shown in Figure 1.3.

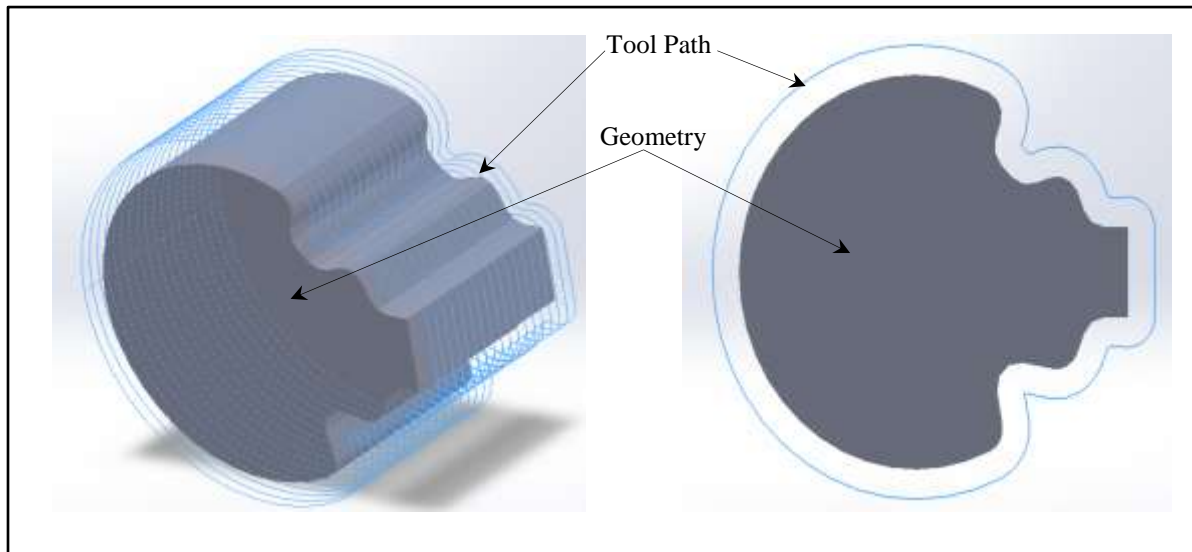


Figure 1.3: Representation of Tool Path

1.1.1 Basic Terminology

The points fed to the controller for the movement of the tool from one position to another are the CL points continuously making a tool path and the surface formed by those points is known as the CL surface. The CC points are the point where the tool's cutting

periphery actually touches the work piece. The CC points vary as the change in the shapes of the geometry occurs. Ideally, the tool derived via CL surface generates a gouge free surface. Figure 1.4 show the different tool position and its corresponding CL and CC points for ball nose end mill.

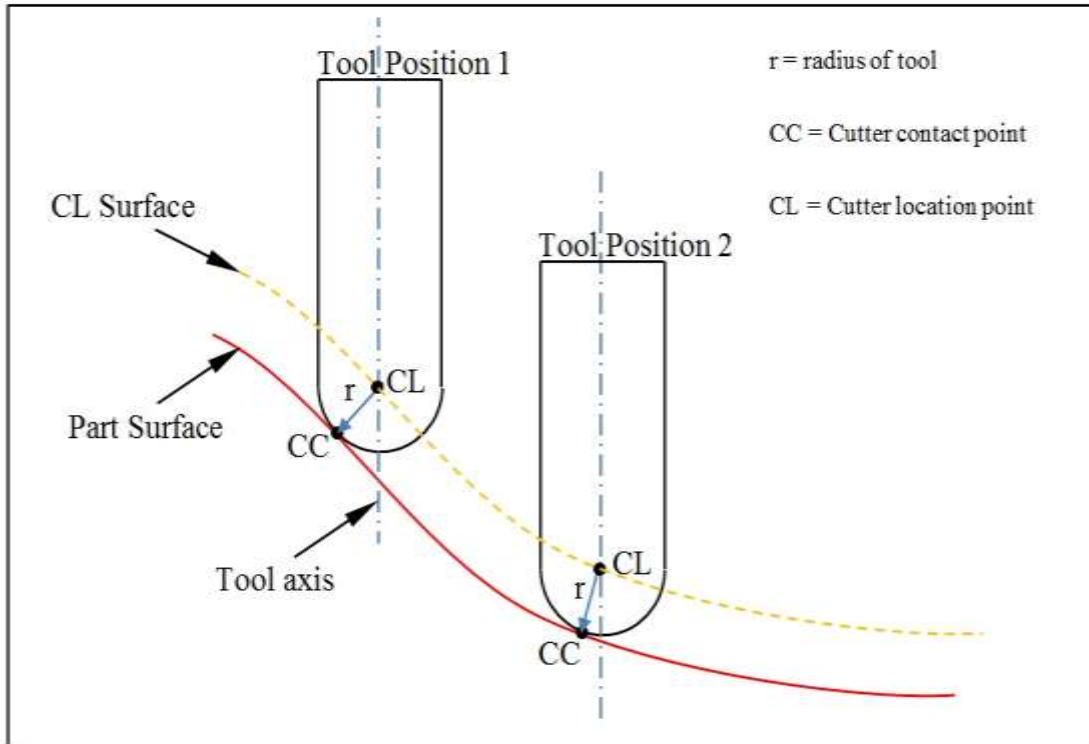


Figure 1.4: Basic Tool Path Terminologies [10]

1.1.2 Tool Path footprint

Tool Path footprint is the shadow of the path followed by the tool over the surface of the geometry. Basically, the tool path footprint decides, how the tool will move and in which direction over the surface of work piece. Tool path footprint is as shown in Figure 1.5.

- a) **Forward step:** Forward step is the distance between two consecutive CL points in machining.
- b) **Side Step:** Side step is the offset distance parallel to previous cutting path followed by the tool. It is also known as path interval or pitch in helical tool path. Side step causes scallops of uncut material to form on the part surface.

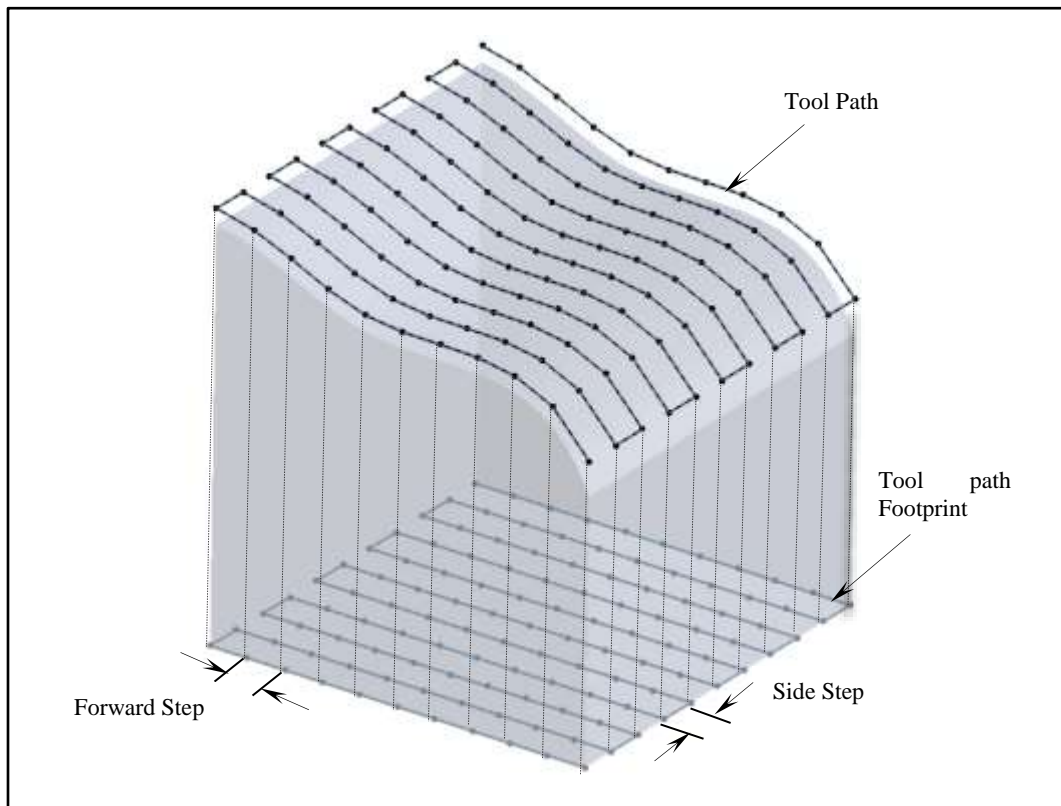


Figure 1.5: Representation of Tool Path and its footprint [11]

1.2 Factors Affecting Tool Path & Work Piece Accuracy

Following are the factors that affect the tool path planning and accuracy of the work piece.

1.2.1 Scallop Height and Machining Time

The machining of a work piece generally done in the three stages:

- a) Roughing
- b) Finishing
- c) Bench work

Nearly 80% of the whole machining time is spend on the Finishing and the bench work, which also includes the grinding and polishing of the work piece for removing the unwanted scallops that are built during the process. Scallop is the key factor that decides

the accuracy and the surface finish of the work piece. Scallop height is the measure of roughness on the surface of work piece.

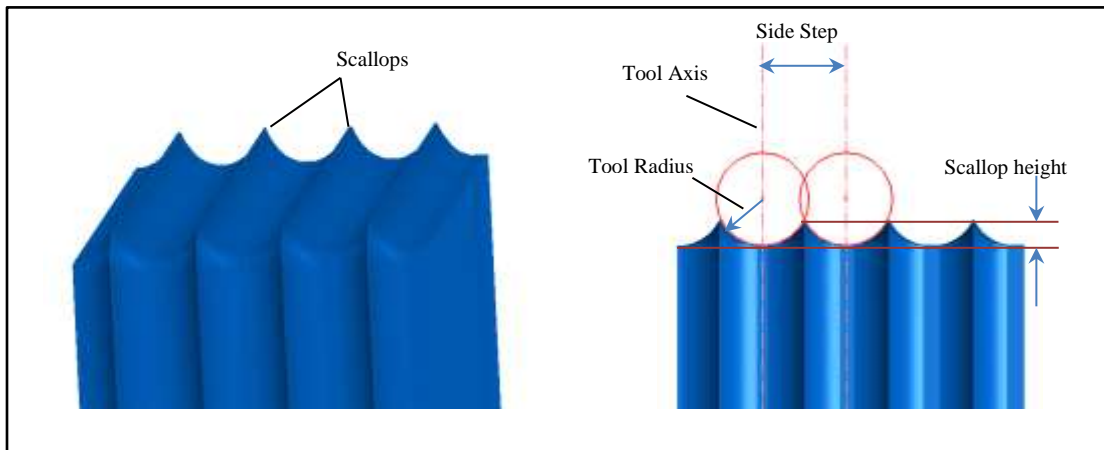


Figure 1.6: Scallops formed on the surface of the work piece

As Figure 1.6 clearly depicts that scallop height is the function of side step taken in the tool path planning. If the side step taken for machining is less the scallop height is low, hence surface finishing is more and vice versa. If we decrease the side step to achieve higher surface finishing, the number of tool paths increases, apparently, which give rise in the overall machining time, which is also not acceptable. So both of the terms, scallop height and machining time, are contrary to each other, if we try to decrease one other will increase, literally.

1.2.2 Overcutting and Undercutting

Generally, there are two kinds of changes that occur in the geometry. Convex and concave, as shown in Figure 1.7.

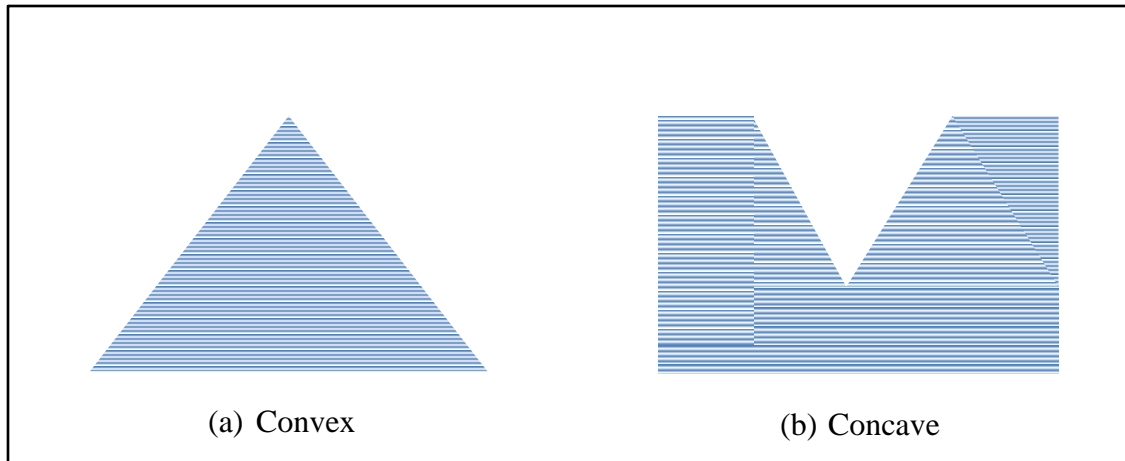


Figure 1.7: Representation of general changes in geometry

If in tool path planning, the fixed increment is given to side step or forward step, there are chances of overcutting of convex and undercutting of concave surfaces, as the tool path is blind to change in geometry, as shown in Figure 1.8

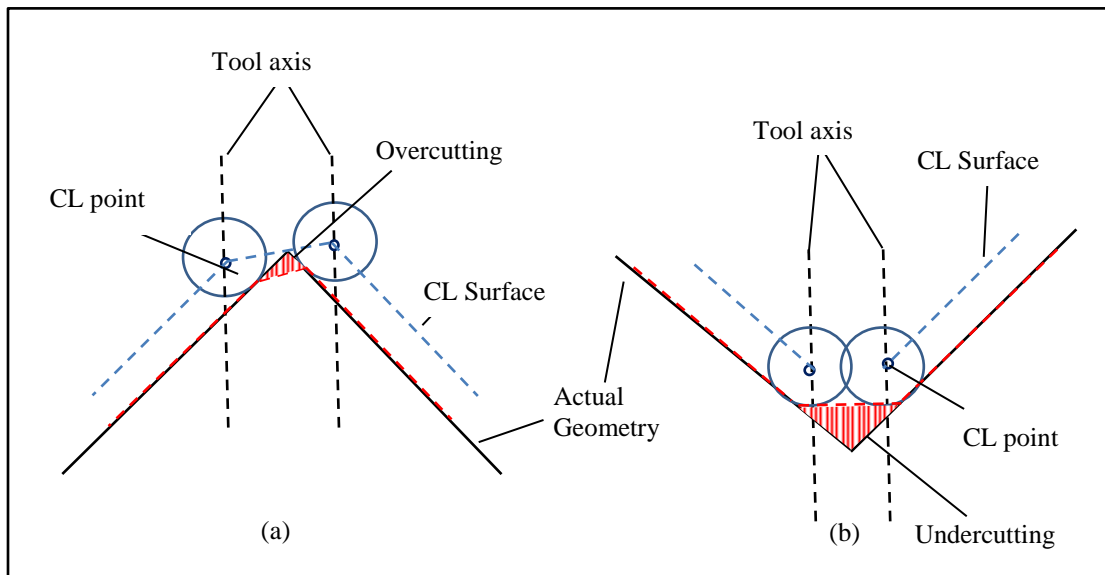


Figure 1.8: Errors in machining. (a) Overcutting of convex surface, (b) Undercutting of concave surface

1.3 Need

Tool path planning for a CNC machining of sculptured surface is done using various methods proposed by researchers. One of the methods is known as “Ball drop” cutter location method using STL format of CAD geometry proposed by **Manos et al. [1]**.

The “Ball drop method” is a robust and fast method as it does not need to refer to the topology of a part i.e. it is blind to changes in the geometry if any. It takes a fixed increment in two axes defining a forward and side step and intersecting the tool axis vector with the STL surface. This leads to an overcutting of convex and undercutting of concave surface intersection, as shown in Figure 1.8

The objective of this work is to define, implement and experimentally verify method to eliminate or minimize such errors using the tool path generated by the “Ball drop method”.

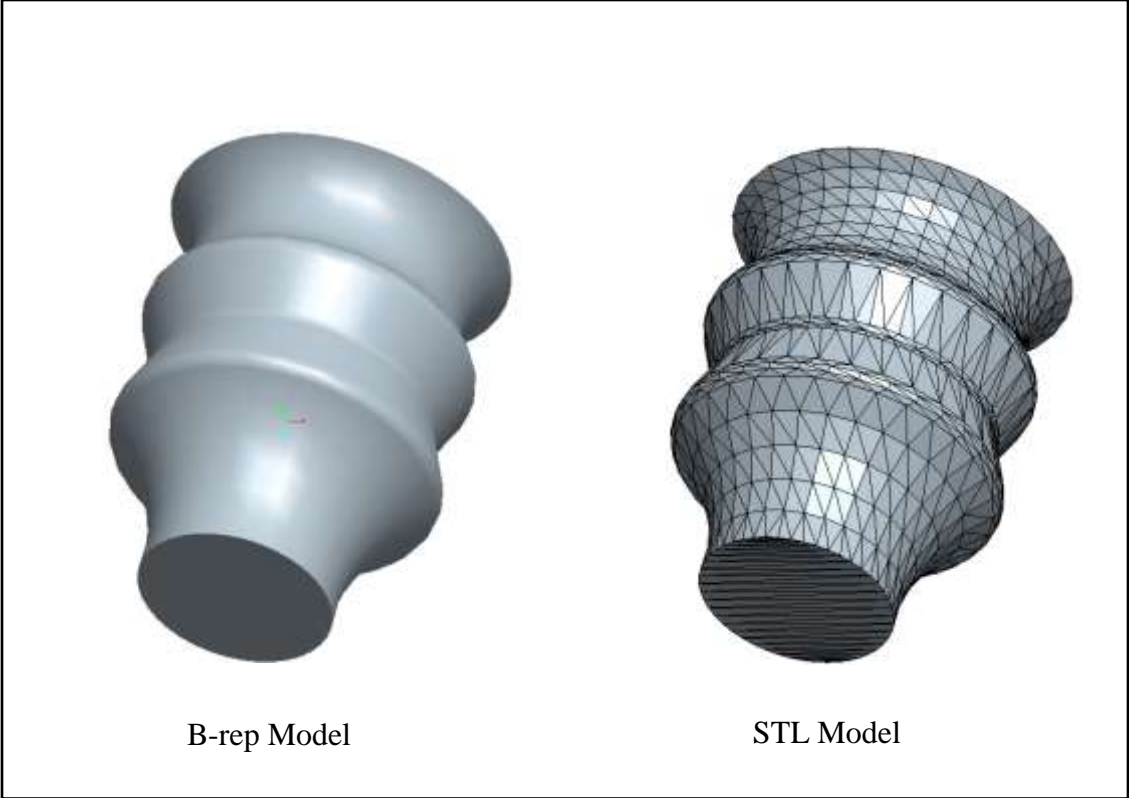


Figure 2.2: Representation of B-rep model and STL file

ASCII format	Binary format
<pre> solid <i>name</i> facet normal $n_i n_j n_k$ outer loop vertex $v1_x v1_y v1_z$ vertex $v2_x v2_y v2_z$ vertex $v3_x v3_y v3_z$ endloop endfacet </pre>	<pre> UINT8[80] – Header UINT32 – Number of triangles for each triangle REAL32[3] – Normal vector REAL32[3] – Vertex 1 REAL32[3] – Vertex 2 REAL32[3] – Vertex 3 UINT16 – Attribute byte count end </pre>

Figure 2.3: Comparison of the structure of ASCII and Binary format of STL file [12]

2.2 Tool path Generation Methods

A lot of researchers have developed various methods for the generation of tool path. Either CC or CL points are used to generate a tool path. Some of the methods that are used to generate a tool path are discussed under this review section.

Iso parametric tool path: In this method, tool path is defined by the isoparametric curves. In this CL points are obtained by offsetting the CC points in the normal direction. This CC points are computed by increasing the parameter of the curve with constant interval. If the interval is large, this may result in undercutting and overcutting and there may also be the gouge problems for model. [13]

Iso Planar (Cartesian) tool path: In this method, tool path is defined by the slicing the surface of actual model or after offsetting the model with an offset distance equals to tool radius. It is of two types, ‘constant Z level’ and ‘one way’ tool path. In the earlier one the plane of slice is parallel to the x-y plane and in later one vertical plane perpendicular to x-y plane is the plane of slice. It is also known as the Cartesian method. [13]

In above tool path generation methods, the side step is kept constant, which will lead to more scallop height at higher slopes of the model. So, for better finishing of the surface it is required to keep the scallop height of the work piece within the tolerance zone, which leads to the development of iso scallop tool path generation methods.

Iso Scallop tool path: In this method, the tool path is generated in such a way that the scallop height of the work piece remains within the tolerance limit defined throughout the surface for uniform machining. This tool path generation approach is also known as adaptive iso planar tool path generation method, as in this method, the slicing planes are selected in such an adaptive way that the number of slicing planes varies according to the slope of the model with respect to the slicing plane. [2]

In this way, Iso Scallop tool path is far better than the iso planar and iso parametric tool paths.

Choi et al. [3] gave the relation between the scallop height and the path interval i.e.

$$L = \frac{|\rho| \sqrt{4(r + \rho)^2(h + \rho)^2 - [\rho^2 + 2r\rho + (h + \rho)^2]^2}}{(r + \rho)(h + \rho)}$$

Where, ‘ ρ ’ is the radius of curvature that is positive for convex and negative for concave surface. ‘ r ’ is radius of the tool, ‘ h ’ is the scallop height and ‘ L ’ is path interval. If the radius of curvature ‘ ρ ’ of surface is large enough as compared to radius of tool or the surface is flat surface, then the surface between two consecutive paths is assumed to be planar and path interval for that surface can be taken as

$$L = 2\sqrt{2rh - h^2}$$

Lai et al. [4] used the above relations for calculating the path interval between every tool path and relate the path interval with the slope of the geometry in z-axis and the surface normal of the XY plane at that particular location. They gave the scallop height variation with the slope of the contact plane for machining a plane surface and developed a tool path generation method which resulted in maintaining the roughness of the surface of work piece within the tolerance zone by keeping scallop height less than the limit and constant throughout.

Ding et al. [5] with the help of the above explained work related to path interval and the slope of geometry for keeping the scallop height constant, developed the adaptive isoplanar tool path generation method and also enhanced the work to remove the redundancy of the tool paths, which was the limitation previous work, by using the concept of ‘isophote’ applied to the surface in the different regions and increased the machining efficiency.

Kim and Yang [6] gave a new approach for constant scallop height tool path generation method. They used a CL surface deformation approach. They computed the deformation vectors by using the slope and the curvature of the offset CL surface and deform the offset CL surface according to the deformation vectors. Then after computing the tool path by slicing the deformed CL surface, the tool path is inversely deformed by the same deformation vectors.

2.3 Tool Positioning Methods

These are the methods that are used to find the position of the tool over the work piece such that the tool's interaction with the work piece gives a gouge free machining. A lot of researchers have developed various methods by which we can find out the tool position; few of them are discussed under this review section.

Tang et al. [7] developed an approach to generate a gouge free the tool path for NC machining in 3-axis. They proposed a concept of offsetting the surface of the model solving the C^0 and C^1 discontinuity. Then the resulting tool path formed by the intersection curves between the offsets and drive planes is gouge free and smooth.

Manos et al. [1] developed a method to generate a gouge free tool position for 3 axis machining, known as 'Ball Drop Cutter Location Method'. On the contrary to the method developed by Tang, in ball drop method there is no offset approach. According to this approach, if a spherical ball, radius equals to tool radius, is dropped onto the surface of the model. The first contact point the ball will give with the surface will be the CC point and the center of the ball at that position will give us gouge free CL point. The axis along which the ball is dropped supposed to be the axis of the tool. They also gave us the mathematical model of this method applicable only for the triangulated surfaces. It includes following four checks:

a) Shadow Check:

In this entire surface of the geometry is scanned and the triangles that lies within the shadow of the tool are considered for the next three checks and rest are dismissed.

The angular range of the shadow of the tool is defined by following expression:

$$\partial = \tan\left(\frac{r}{y}\right)$$

Where ' ∂ ' is half the angle subtended by the shadow of tool from the work piece surface to the origin, 'r' is radius of tool, 'y' is the radius of work piece.

b) Triangle Check:

This check is done to determine whether the tool touches the triangle facet or not with the help of following equation.

$$T_1 + u(T_2 - T_1) = P_1 + (P_2 - P_1)s + (P_3 - P_1)t + \hat{N}r$$

Where T_1 and T_2 are the points on the tool axis, P_1 , P_2 and P_3 are the vertices of the triangle, \hat{N} is the unit normal vector of the triangle, 'r' is the radius of the tool and rest are the parameter whose value varies from 0 to 1.

If a point lie within the triangle then the tool position is saved, otherwise it is discarded and preceded for the next check.

c) Edge Check:

This check is done to determine if the tool touches the edge of the triangle by equating the tool axis equation with every edge of the triangle. The equation used is as below:

$$T_1 + u(T_2 - T_1) = P_1 + (P_2 - P_1)s + \hat{N}r$$

After solving it, we will get a contact point and a tool location point. If contact point is at the edge, we will keep that point else it will be discarded and preceded for the next check.

d) Vertex Check:

This check is done to determine if the tool touches any of the vertexes of the triangle or not, by using the following equation:

$$P_i - [T_1 + u(T_2 - T_1)] = r$$

After solving above equation for u, the CL point is obtained. If there is no real valued solution, that implies tool will not touch the vertex for any value of u. if the tool can touch the vertex, it will give us two solutions, we will chose the solution far away from the axis of the work piece.

Ball drop method is the fastest and robust method to get the gouge free cutter location points, as it does not cater about the topology of the model and its connectivity. In this the tool path vectors are impeded on the STL surface, blindly, with a fixed forward and side step interval. So there is not any consideration of the topology or any connection of the faces or planes with each other, which makes the functionality of this method

faster than the other methods. This method gives us a gouge-free CL point but if these CL points are connected in sequence of the footprint of the path to be followed by the tool tip then the tool path generated using this method would not be gouge-free. This is because the connecting path between CL points is not checked for gouging with the STL part surface.

2.4 Gap in Literature

As observed from the literature survey, ball drop method works with the fixed forward and side step intervals, which gives the uncertainty in machining accuracy of the work piece. At the sharp corners there might be the undercutting or overcutting of the material. The worst case is at the corner where there is a 90° change in the slope of geometry. Figure 2.4 to Figure 2.6 shows the comparison of the STL surface and the machined surface using “Ball Drop Method”^[1] implemented with tool of radius 6.35mm and constant side step interval of 90% of tool radius at different slopes.

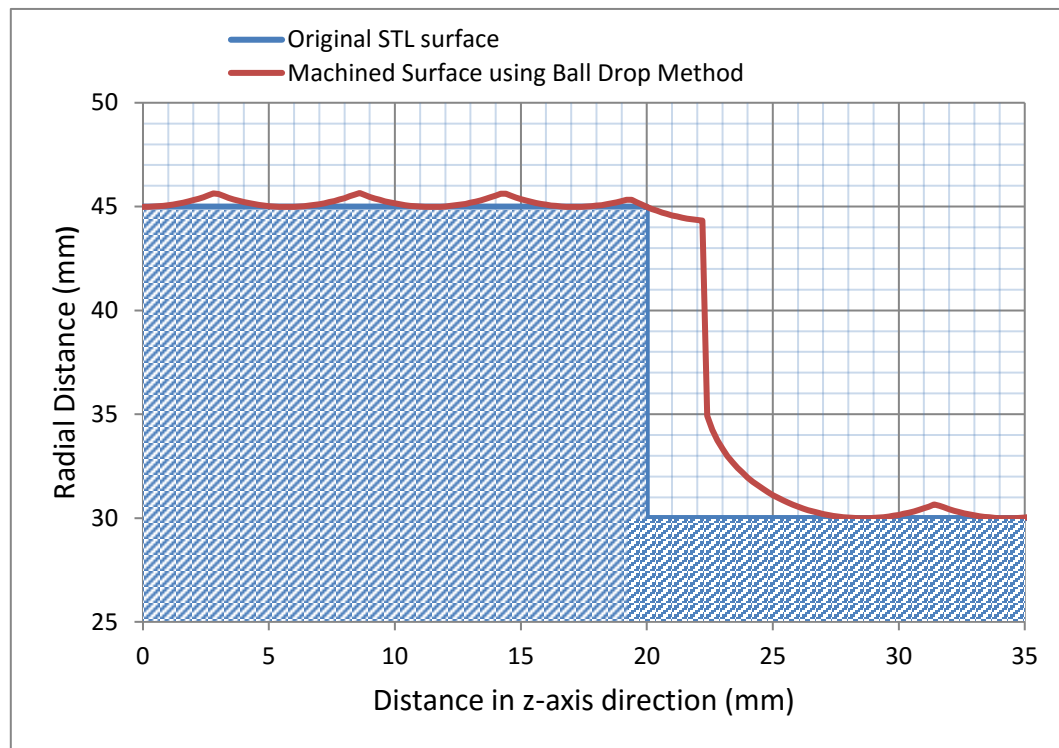


Figure 2.4: Comparison of STL and machined surface at slope of 90°

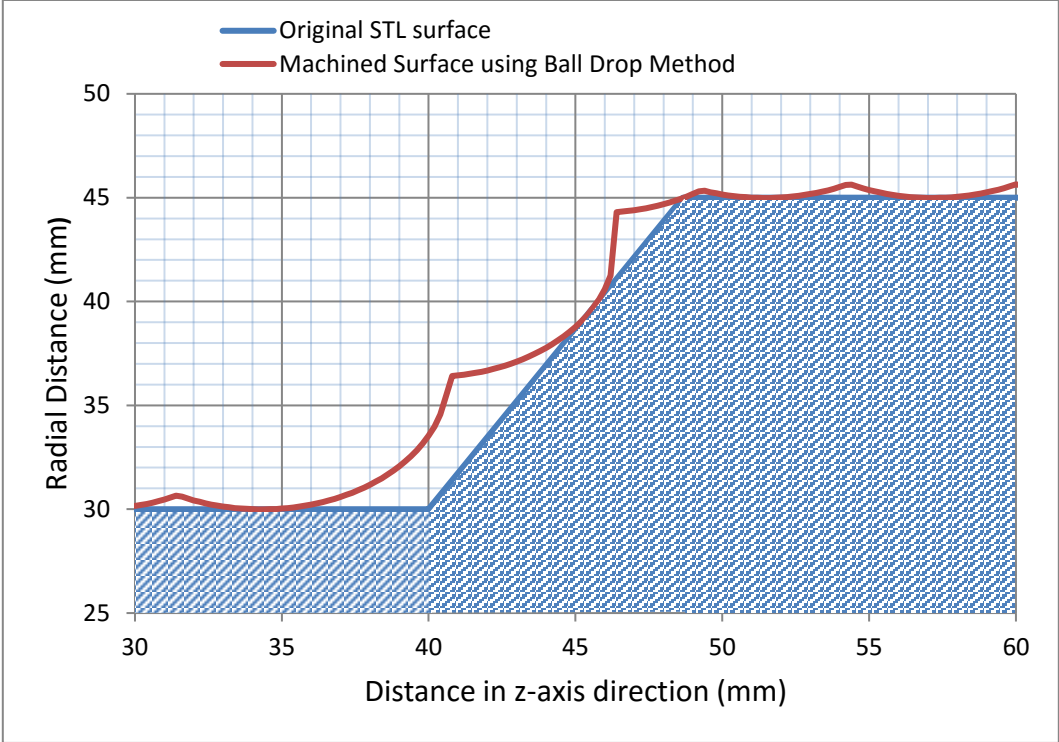


Figure 2.5: Comparison of STL and Machined surface at slope of 60°

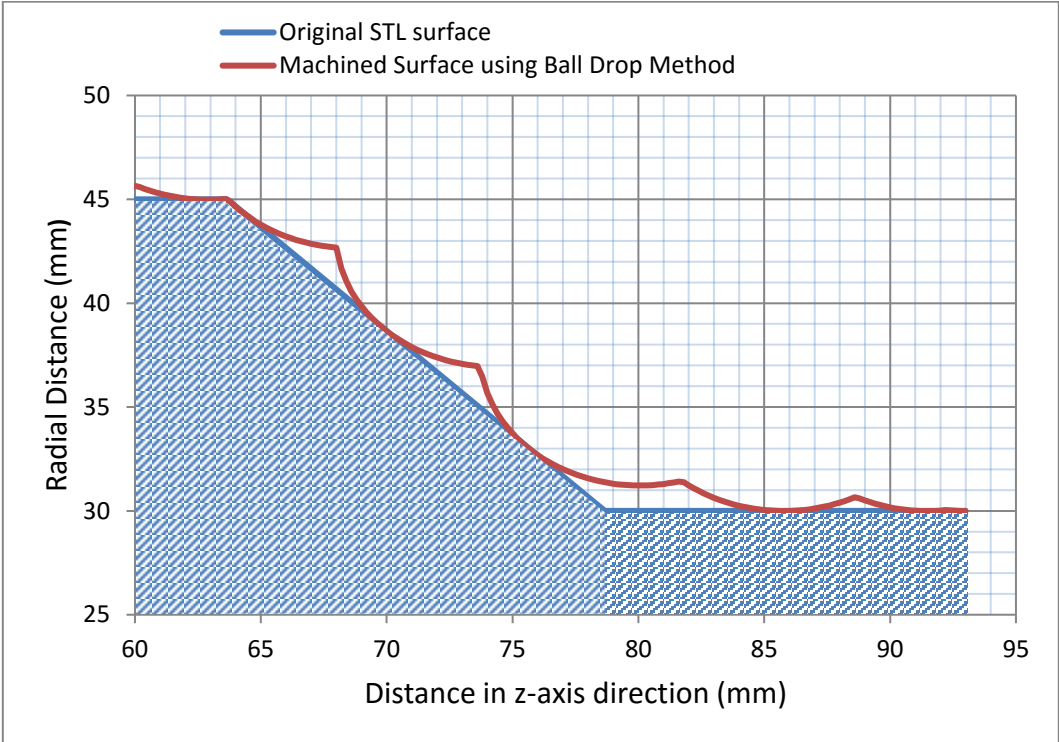


Figure 2.6: Comparison of STL and Machined surface at slope 45°

Above figures clearly depicts the deviation of the machined surface using 'Ball Drop Method' from the actual STL surface. There is a large undercut of the work piece, because of fix side step interval Comparison of actual surface and machined surface using 'Ball Drop Method'. From Figure 2.4 it can be easily seen that when the slope is 90° , there is huge volume of uncut material that left over during the machining of work piece, which can easily be machined with zero deviation from actual geometry. Figure 2.5 & Figure 2.6 shows that at different slopes with constant side step interval, scallop height of the scallops generated at slopes is higher than that of scallop generated at flat surface, which leads the roughness of surface of work piece beyond tolerance limit.

2.5 Objective

Objective of this work is to develop and validate experimentally a method to refine the tool path generated by the previously discussed 'Ball Drop Method', in such a way that the advantages of 'Ball Drop Method' i.e. its robustness and speed, are used and the drawbacks of it could be minimized as discussed in section 2.4 without referring to topology of the model.

CHAPTER 3

Proposed Work

The previous section clearly depicts the problem and objective of this work. This work is basically a refinement method of the already generated tool path using ‘Ball Drop Method’ from STL geometry for better surface accuracy. To fulfill the objective, a method named ‘CL interpolation method’ is proposed. According to this method the consecutive Cutter Location points are interpolated to find out the change of triangles and their slopes to add the CL points at the corners and position of triangle change.

3.1 ‘Five Point Grid’ method for CL Interpolation

‘Five point grid’ method is based on the concept that at a position of change in triangles and their slopes, the CL surface is approximated by the lines generated using consecutive CL points around both the sides of that change. The lines represent the approximation of the slope of triangles on both sides of changing position. If these lines are extended and their intersection is found then that intersection point will give us the position of corner where change in triangles occur. Using that position of the found intersection point, we could generate the CL point dedicated to that particular change by applying ‘Ball drop method’ at that position.

Basically, CL surface made a mesh of CL points over the surface of model offset by a distance equal to tool radius, the CL points are connected continuously to make an effective tool path. As it can be clearly seen from the different view of CL points generated on tool path over the surface of work piece shown in the Figure 3.1, the position of CL points, changes along the changes in the model. The expected CL point can be determined by interpolating between two adjacent CL points, wherever there is change in triangles and their slopes. Lines, made by two CL points on both the side of new expected CL point, are used for interpolating between two adjacent CL points. It make a grid of total five CL points in each direction with expected CL point T_c^* as its center and two-two CL points on each side of that point in both directions, making four sides around the T_c^* in both forward and side feed direction, as shown in Figure 3.2.

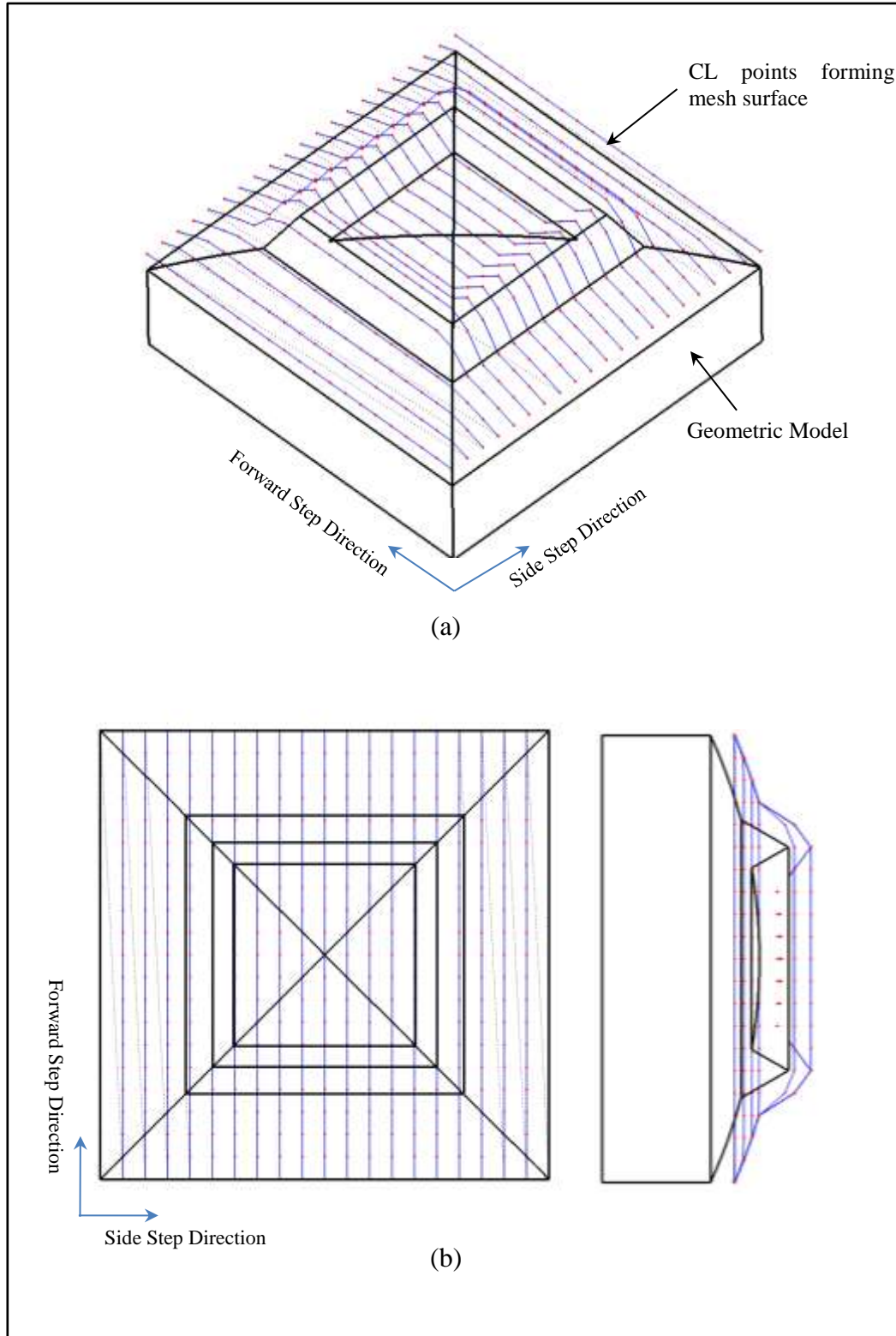


Figure 3.1: Geometric model and mesh made by the CL points offset by tool radius
(a) isometric view, (b) orthographic view

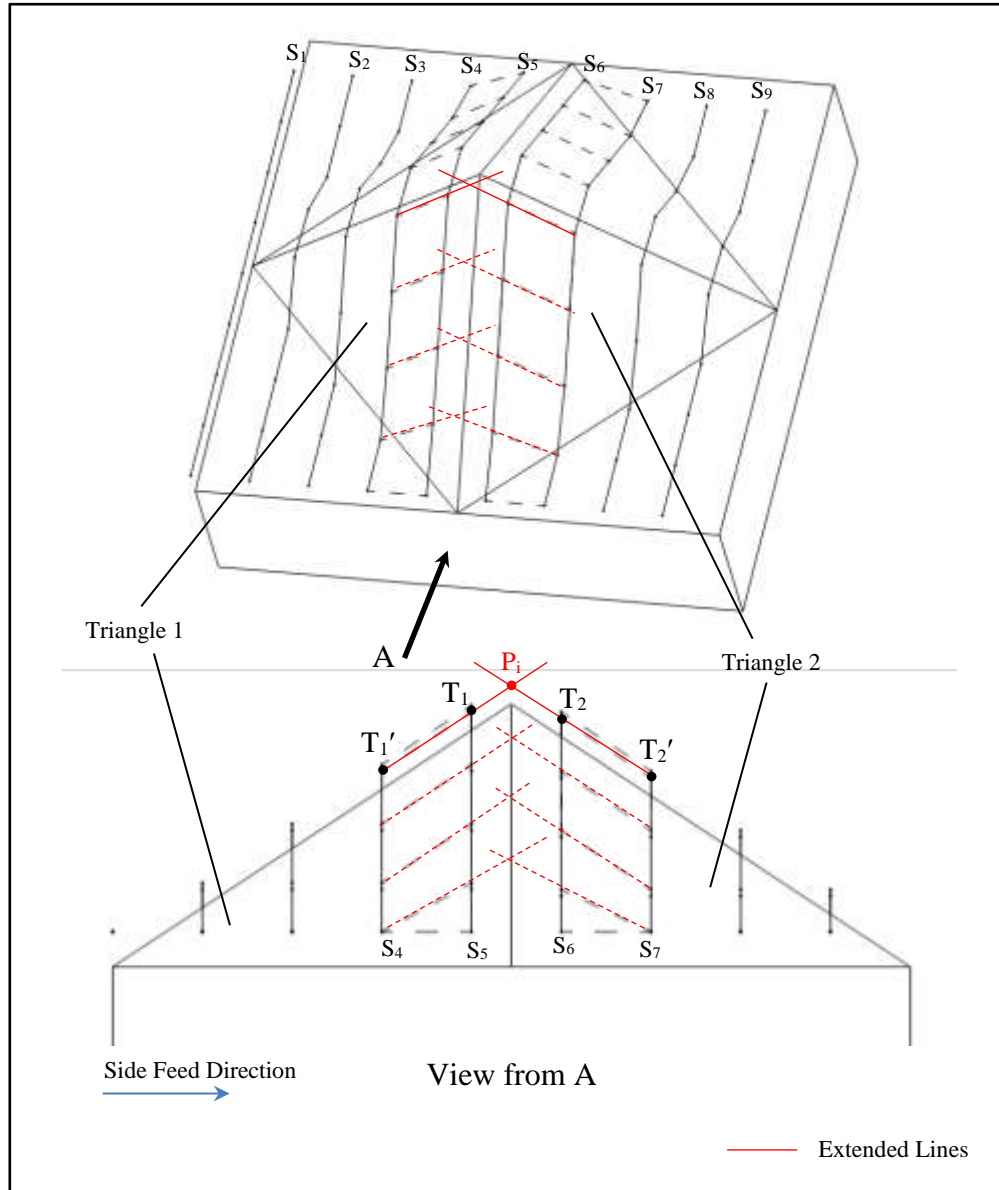


Figure 3.2: Five Points Grid formations in Side feed Direction

Figure 3.2 show the formation of grid in side feed direction with five CL points i.e. T_1' , T_1 , T_2 , T_2' and P_i . Where, T_1' and T_1 are the CL points associated with triangle 1; T_2 and T_2' are the CL points associated with triangle 2 and S_1 to S_9 are the number of paths. The extended lines are made by joining these CL points for each associated triangle 1 and 2. The intersection point (P_i) obtained from extended lines gives the position of

change of triangles. After adding CL points in side feed direction, the same concept is used for forward feed direction.

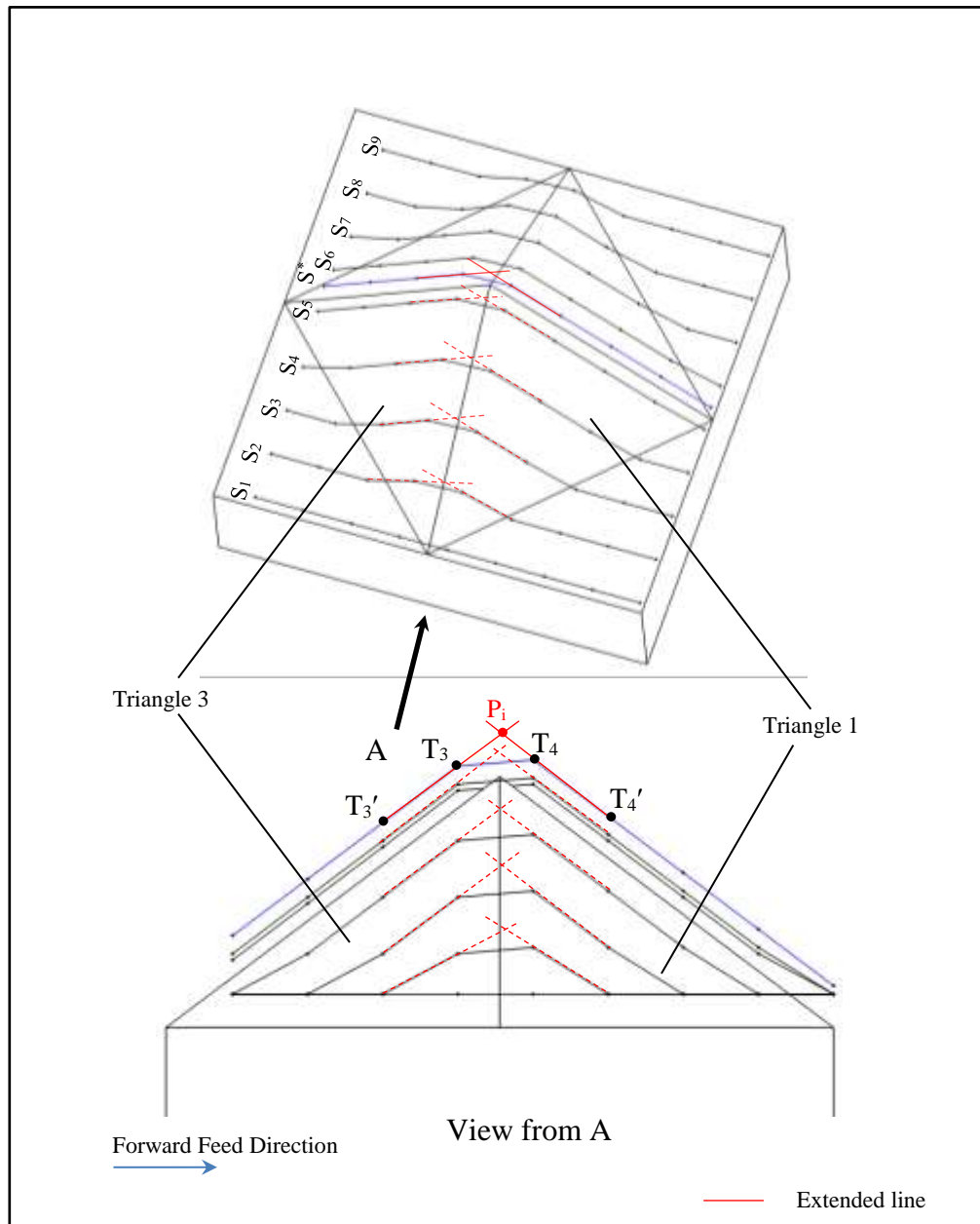


Figure 3.3: Five Points Grid formations in Forward feed Direction

Figure 3.3 show the formation of grid in side feed direction with five CL points i.e. T_3' , T_3 , T_4 , T_4' and P_i . Where, T_3' and T_3 are the CL points associated with triangle 3; T_4 and T_4' are the CL points associated with triangle 1 and S_1 to S_9 are the number of paths and S^* is the added tool path from the interpolation in Side feed direction. The extended

lines are made by joining these CL points for each associated triangle 1 and 3. The intersection point (P_i) obtained from extended lines gives the position of change of triangles. After successfully implementing the Five point Grid concept in both the directions, the final tool path obtained is as shown in Figure 3.4.

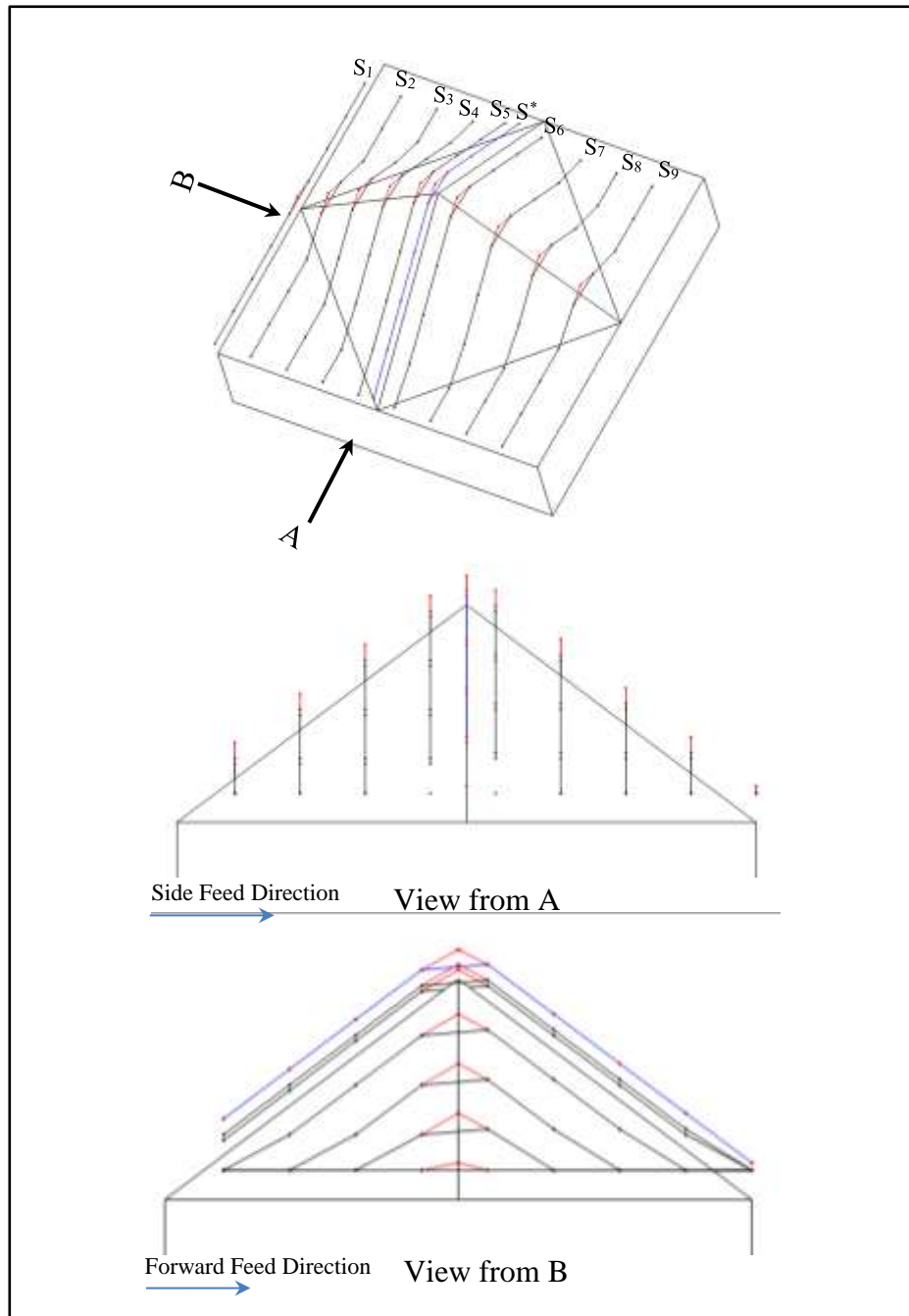


Figure 3.4: Tool path after implementation of Five point Grid Concept

Figure 3.4 clearly depicts the addition of tool path in side feed direction, shown in blue color, at the position of change of triangles and CL points in forward feed direction also increased at the change of triangle. In forward feed direction, at the position of triangle change instead of following the previous path, shown in black, the tool will follow the red path connected by additional CL points at that position.

For finding the T_c^* using this ‘Five point Grid’ concept at the place where any change in triangle occurs, lines made on each side of that point are extended to find the intersection P_i between them. At the point of intersection P_i a new CL point T_c^* is generated using ‘Ball Drop Method’, as shown in Figure 3.5. If the intersection point P_i does not lie in between its adjacent CL points for both directions of T_c^* , the grid is made to converge up to the extent of tolerance limit, defined by user, using ‘Trisection method’ and then intersection of lines is calculated and T_c^* is generated over there. The ‘Trisection method’ is explained ahead in section 3.2.

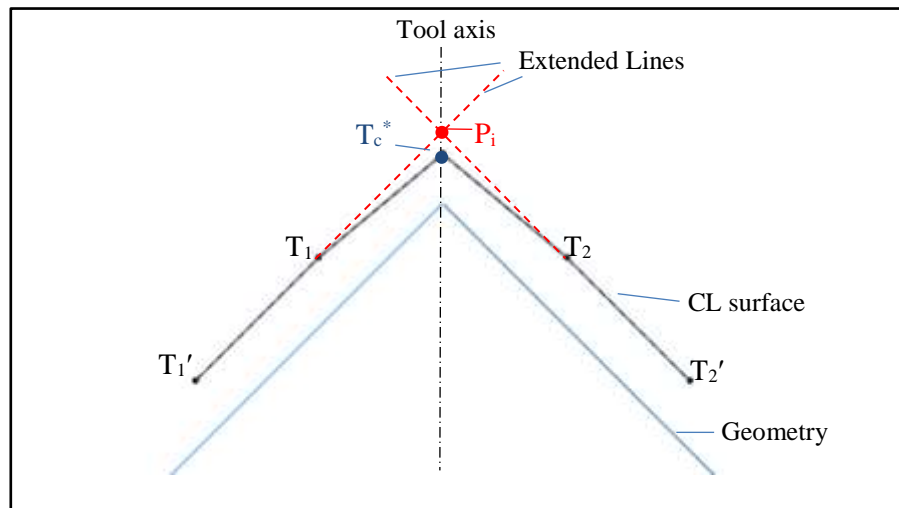


Figure 3.5: Generation of Expected CL point using intersection of lines

Now this ‘Five point grid’ is scanned over the whole surface of the model in both, side feed and forward feed, directions. Wherever the triangle and its slope change it will add an extra CL point T_c^* .

If there is a change in triangles but the slope of triangles does not change then the surface of geometry is considered to be flat and there is no need to find the intersection points generating the lines using adjacent CL points.

3.2 Trisection method

As explained in the previous section, if the intersection of the lines could not fall in grid range as a case shown in Figure 3.6, then grid is made to converge using trisection method.

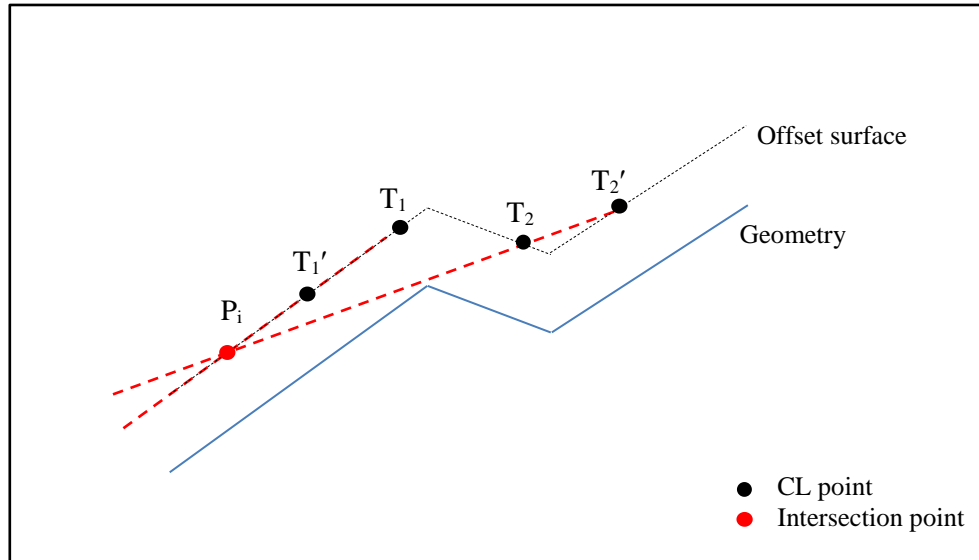


Figure 3.6: Showing a possible case for intersection point not in grid range

Figure 3.6 shows a possible case in which the intersection point P_i does not fall in the range defined by T_1 and T_2 CL points.

In trisection method, the distance between two consecutive CL points, which are interpolated to find out intersection point in between them, is trisected into three equal parts. It will give us two new positions at equal distance from each other. After getting two new trisected positions in between the already got CL points, the CL points T_{sect1} and T_{sect2} at those obtained trisected positions are generated using ‘Ball drop method’ and compared with the model shape position associated with previous CL points and only those points are selected which are relevant. Now after selection a new grid is formed with new CL points and then the intersection of lines is calculated for the position of expected CL point. This newly generated five point grid, using trisection method, is volatile in nature i.e. these trisected CL points are stored in temporary memory. The relevancy of the selection of CL points of new grid depends up on the following cases.

Case 1: When the position of trisected CL points are such that the point T_{sect1} is nearer to triangle that belongs to CL point T_1 and T_{sect2} is nearer to triangle that is associated with CL point T_2 , as shown in Figure 3.7 (a).

Now the points $T_{1'}$ and $T_{2'}$ will be discarded and the points T_1 , T_{sect1} , T_{sect2} and T_2 will make a new volatile grid.

Case 2: When the position of trisected CL points are such that the point T_{sect1} and T_{sect2} is nearer to the triangle belongs to T_1 , then the points T_1 and $T_{1'}$ will be discarded and the points T_{sect1} , T_{sect2} , T_2 and $T_{2'}$ will make a new volatile grid, as shown in Figure 3.7 (b).

Case 3: When the position of trisected CL points are such that the point T_{sect1} and T_{sect2} is nearer to the triangle belongs to T_2 , then the points T_2 and $T_{2'}$ will be discarded and the points $T_{1'}$, T_1 , T_{sect1} , and T_{sect2} will make a new volatile grid, as shown in Figure 3.7 (c).

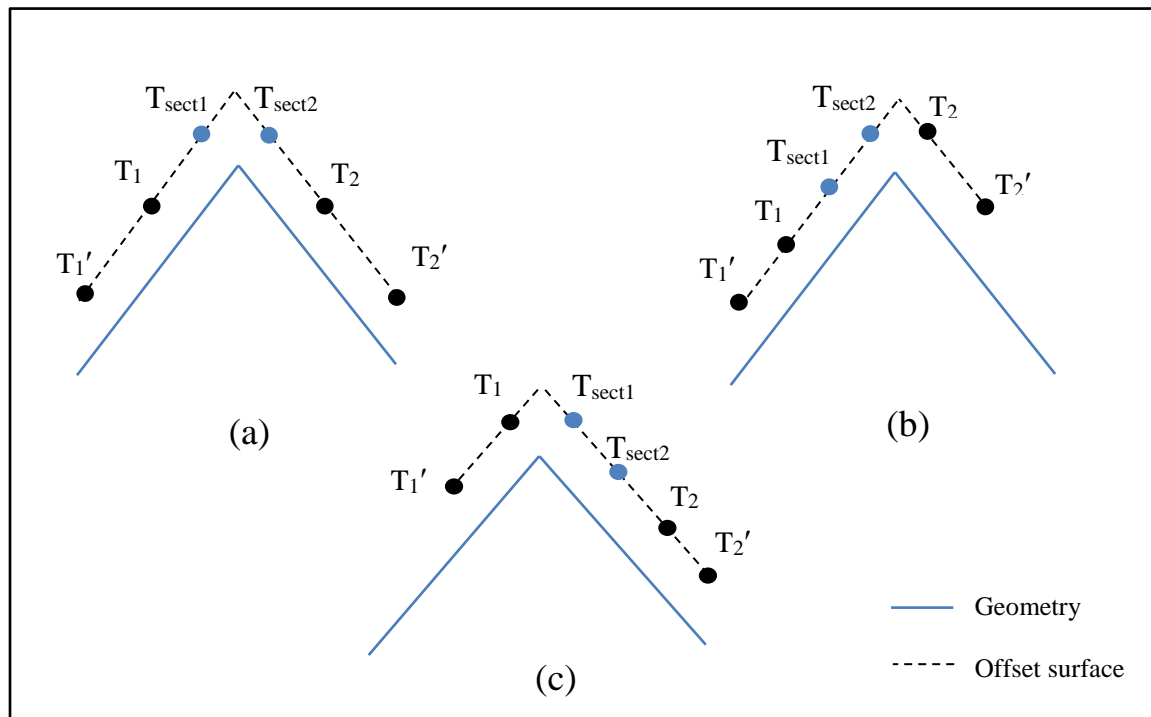


Figure 3.7: Showing different cases for trisection method

For convergence, trisection method is used instead of bisection method because the interpolation between CL points is done by moving with a five point grid in both the directions.

3.3 Side step refinement at slopes

As discussed in section 2.4, the scallop height varies according to change in slope of the geometry. At inclined surfaces scallop height is more as compared to flat surfaces; it can easily be shown in Figure 2.6. This leads to non-uniform scallops generation over the whole surface of model.

To overcome this problem, the numbers of tool paths are increased at slopes, such that the scallop height generated at slopes should be in defined tolerance zone. The path interval, which is calculated from scallop height and tool radius, is made a function of slope throughout the model. So at higher slopes the path interval becomes less accordingly. This is referred from the work proposed by **Lai et al. [4]**.

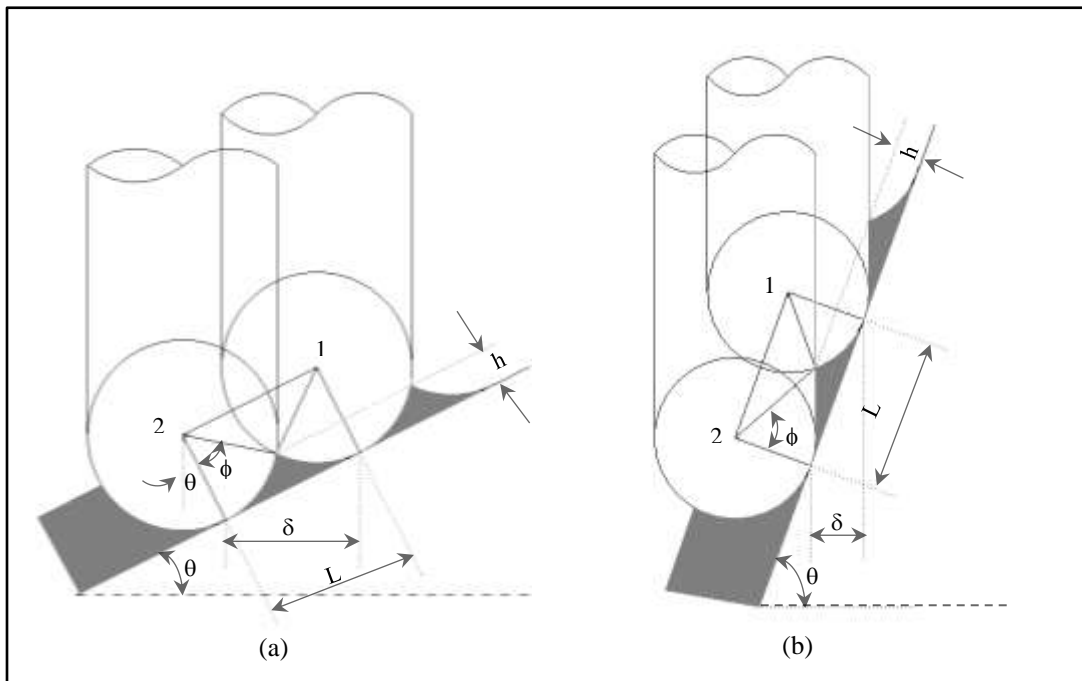


Figure 3.8: Scallop Height determination for plane face (a) for gentle slopes (b) for steep slopes [4]

Figure 3.8 shows the variation of scallop height at different inclination of flat surface. In Figure 3.8(a), when the slope is gentle, the scallops are generated by spherical

periphery of ball nose end mill at both the tool positions and the path interval is given by

$$\delta = 2\sqrt{2rh - h^2} \cos\theta$$

In Figure 3.8(b), when the slope is steep, the scallops are generated by spherical periphery at one tool position but at other tool position the scallop is formed by cylindrical periphery of flank of ball nose end mill. For this kind of scallop the path interval is given by

$$\delta = r(1 - \sin\theta) + h \sin\theta + \sqrt{2rh - h^2} \cos\theta$$

Where, ‘ δ ’ is path interval between two consecutive tool paths i.e. side step, ‘ r ’ is tool radius, ‘ h ’ is allowable scallop height and ‘ θ ’ slope of flat surface. The slope of the geometry is calculated by comparing the surface normal of geometry at that particular point to the surface normal of a flat surface.

The transition between gentle and steep slopes is based on

$$\alpha = \theta + \phi$$

$$\phi = \cos^{-1}\left(\frac{r-h}{r}\right)$$

If ‘ α ’ is less than 90° the slope is gentle, whereas if ‘ α ’ is more than 90° the slope is steep.

3.4 Typical case of a vertical step

Generally, the variation in geometry is classified into two surfaces i.e. Convex and Concave surfaces as shown in Figure 1.7. For identifying the corners and change in geometry for convex and concave surfaces, these can be dealt with above explained concept in this chapter. But if the change in geometry is 90° i.e. a vertical step, making it a special case, ‘Five point grid’ concept will never give intersection of lines within the grid range even after convergence of the grid. So these types of condition need to be solved specially which give rise to the concept of ‘Vertical line approximation’.

According to this concept, the step in the geometry is approximated by a straight vertical line in the CL surface. In this, instead of four CL points, two CL points are used to

interpolate through a step. Whenever there is step change of 90° or nearly 90° in geometry it is found by measuring radial distance between two consecutive CL points. Initially, the line made by joining two consecutive CL points is inclined in a manner, as the CL points are offset to each other. Then offset distance between these two CL points is subdivided using Trisection method, as explained in previous section, till the line formed by CL points is vertically straight, in such a way that tool moves parallel to step offset by tool radius. After achieving this position all the four CL points T_1 , T_{v1} , T_{v2} and T_2 are stored. The explanation of this concept is shown by the Figure 3.9.

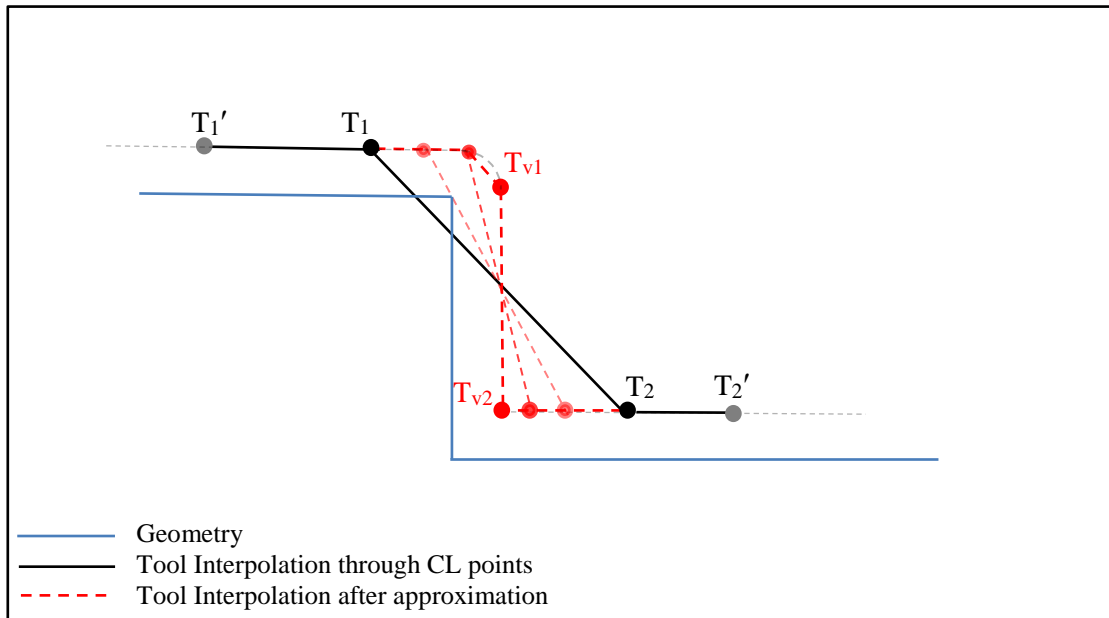


Figure 3.9: Vertical Line Approximation

In above Figure, the subdivision is done by using T_1 and T_2 CL points, which are obtained from already defined CL mesh. T_{v1} and T_{v2} are the CL points that are finally stored after approximation the step by this method.

3.5 Prerequisites

Before implementing the above work the following mentioned things are required beforehand.

- a) As this concept particularly refers 'Ball drop method' [1], so every CL point is generated by this method only, using a STL format geometric model.
- b) The Four CL point pairs used to determine 5th interpolated point must be co-planar. so that the intersection should be possible.
- c) The triangular mesh base from STL format should be the same to the base data that is used to generate the tool path using 'Ball Drop method'
- d) The CAD model created should be pseudo symmetric.

CHAPTER 4

Implementation

The implementation of this work is done on ‘PBG (KW) Inc. Milling Lathe CNC’ using ball nose end mill tool, as shown in Figure 4.1. The material used for work piece is wood. This work is only concerned about the surface accuracy of work piece and tool positioning.



Figure 4.1: PBG (KW) Inc. Milling Lathe Machine

This machine is specially designed for wood working by PBG incorporation, Waterloo, Canada. This CNC machine is for turn milling in which 3 of the axis are controlled by the controller i.e. X axis, Z axis and C axis (rotation). It works on the input of these three axes in the form of the values. So tool path is generated for this machine using a footprint in which the geometry is sliced by the vectors generated through the ring of points made around 360° of geometry with particular degree interval also known as z-parallel tool footprint, as shown in Figure 4.2 and tool location is calculated for the each

vector and is fed to this CNC machine in continuous form. These rings are generated throughout the length of the work piece at particular intervals.

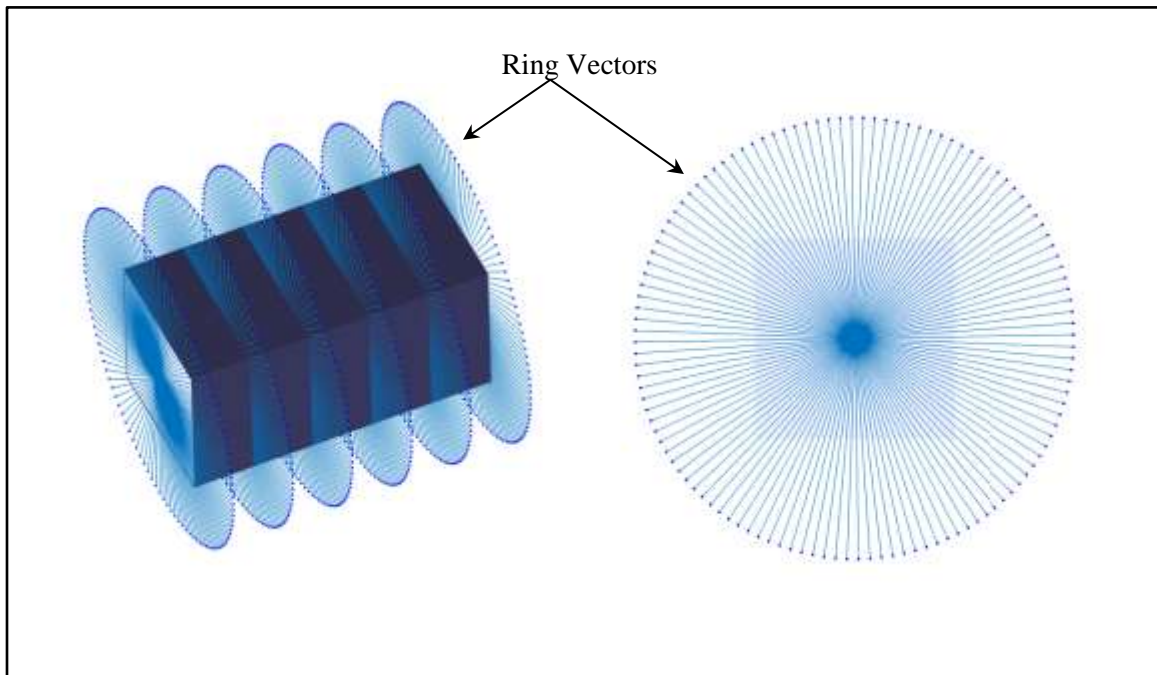


Figure 4.2: Representation of Vectors along the rings around geometry

4.1 Cutter Location Calculation

For calculating the CL points the program is developed using MATLAB technical computing language. The calculation of CL location is staged into two stages:

Stage 1: Deals with the slope of the geometry, which is also a referred work, to form an iso scallop surface throughout the work piece.

Stage 2: Deals with the corners and sharp changes in the work piece to reduce undercutting and overcutting of material.

4.1.1 Stage 1

As this stage refers to control the scallop height formation over the work piece, to maintain this scallop height within the tolerance zone, so it is more an adaptive form of generation of rings instead of rings generated by fix path interval. This stage follows as the following steps

- a) First of all, the maximum path interval for side step is decided from the scallop height defined by the user.
- b) Now ring vectors are generated at the starting point of work piece for whole revolution of 360° .
- c) Using these ring vectors the CL points are calculated for each vector over the ring, by implementing 'Ball drop method'. In this Ball drop method is little modified. Beside of extracting the maximum value of u parameter, the CL point and its associative triangle number is also extracted (which will help in stage 2).
- d) As the triangle numbers are extracted, the slope of triangles with reference to flat surface is calculated for the whole ring and the maximum slope is stored.
- e) Using this maximum slope and path interval for flat surface, the next position of ring is decided, by using the relations given by **Lai et al. [4]** for higher and gentle slopes.
- f) If the maximum slope calculated is different from calculated slope of previous ring and ring is moved from flat surface to the surface with higher slope, then a new ring is added in between both the rings decided by new slope.

These steps are followed for the whole length of work piece. Up to this point the stage 1 is complete.

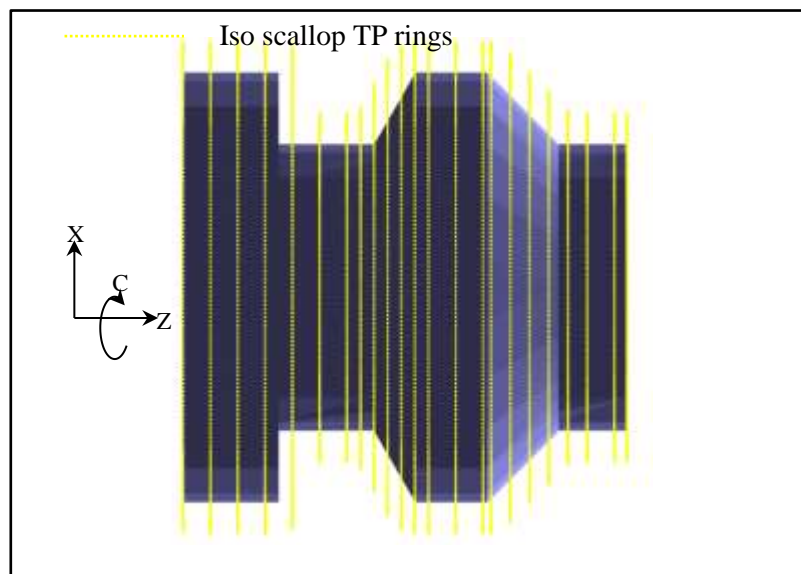


Figure 4.3: Output TP after Stage 1 (iso scallop TP generation)

This gives a uniform grid of CL points within the rings in feed forward direction and iso scallop rings in side step direction, as shown in Figure 4.3. The output of this stage, i.e. CL point data and their associative triangle numbers and u parameter values, is fed to the next stage.

4.1.2 Stage 2

In this stage input taken from stage 1 is processed by using ‘CL interpolation method’. The five point grid concept for turn milling is divided into two direction, side step and forward step.

4.1.2.1 Side step direction

- a) For this direction four consecutive rings are processed together.
- b) The points of the middle rings are scanned for the triangle change and the difference in slope of the triangles w.r.t. each other.
- c) If there is a change in triangle and the slope difference is also more at any point, interpolation between them is done by using ‘CL interpolation method’.
- d) If any intersection is found within those rings, another ring is added at that position with same number of vectors as there are in other rings.
- e) If the intersection points are many then the ring is added by getting the mean of intersection points.

The required rings added, are as shown in Figure 4.4, with green color for the whole geometry. After addition of required rings the CL interpolation method is now applied in forward Step direction.

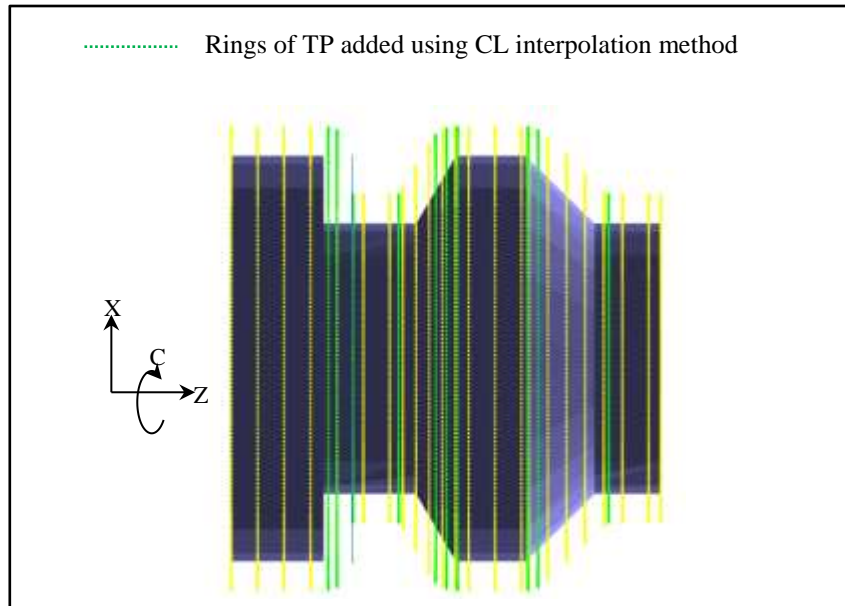


Figure 4.4: Output TP after refinement in Z-axis direction

4.1.2.2 Forward step direction

- In this direction, four consecutive points are moved within the ring.
- The middle points are scanned for change in triangle and their slopes.
- If there is change in triangle and their slope difference is more interpolation between them is done by using 'CL interpolation method'.
- If the intersection is found within those points, another CL point is added in that ring at the point of intersection, using 'Ball drop method'.

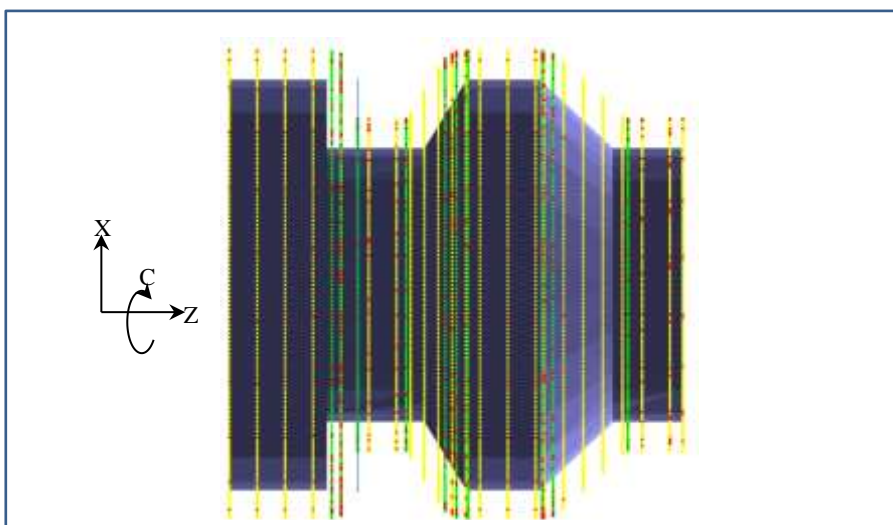


Figure 4.5: Output TP after refinement in C-axis direction

After completing the refinement in forward step direction the CL points are added, shown in red in Figure 4.5. This completes the process of stage 2 and gives the final tool path after CL interpolation.

4.1.2.3 Special Case

- a) Whenever there is step in the geometry, this is dealt specially by recognizing them separately.
- b) Step is recognized by the radial distance between two points. If the radial distance between two middle CL points is more there a step is considered in both the directions.
- c) The CL points are scanned in both the directions, first in side step direction and then in forward step direction. If step is found at any stage, the CL points dedicated to those step are added.

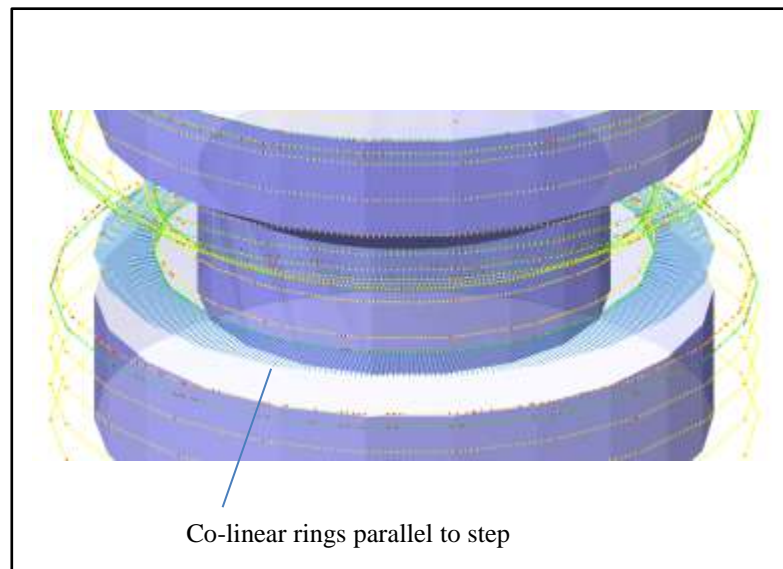


Figure 4.6: Vertical line approximation at step

Both the Stages, Stage 1 and Stage 2, are implemented to generate CL points of a geometry generated by any CAD package in STL format in such a way that it should cater all the corners, edges and slopes in the geometry without referring to any topology of it. This will make the surface of work piece more accurate besides keeping the advantages of original 'Ball drop method', i.e. its robustness and speed, as it is.

CHAPTER 5

Result and Discussion

The above work is implemented and validated by making two models, one for showing the comparison of surface accuracy after machining with original Ball drop method and refined Ball drop method using CL interpolation method, other to show the robustness of this method in case the geometry is complex. The geometric models are generated in Pro-e and are as shown in Figure 5.1.

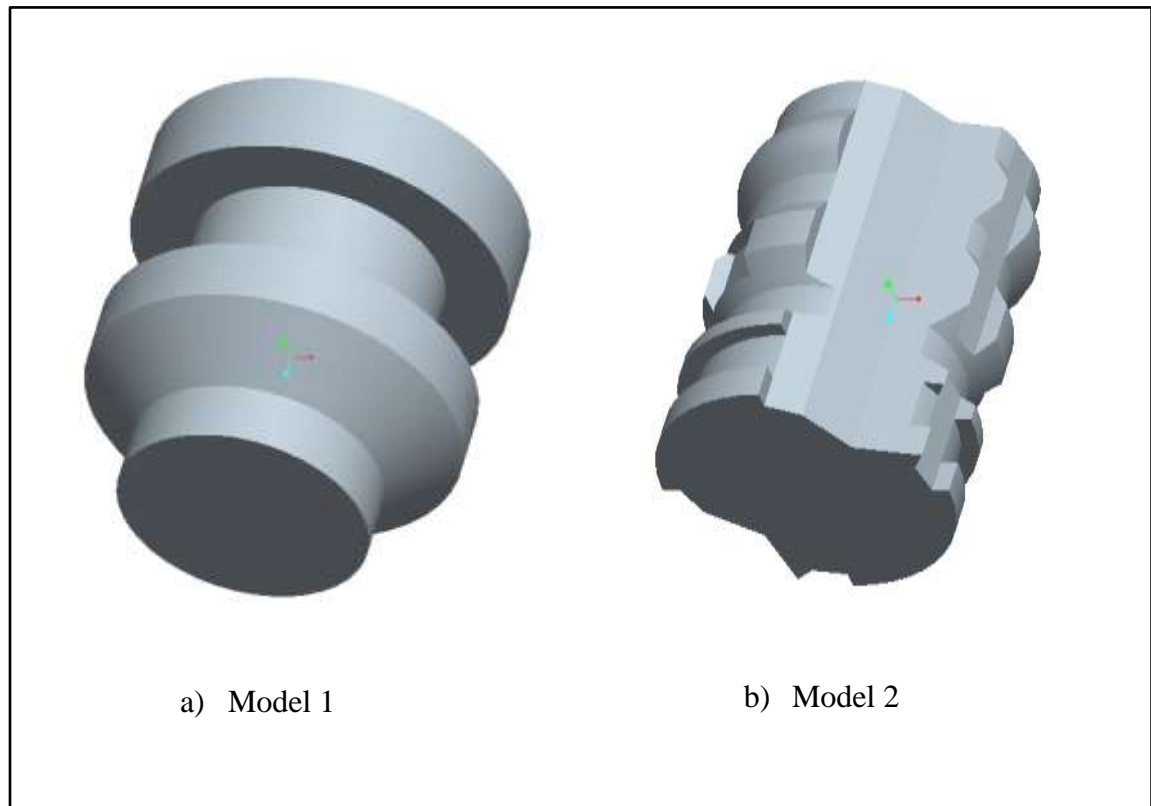


Figure 5.1: Geometric Models used to validate the results

5.1 Model 1

The tool path generated for model 1 is for roughing with ball nose end mill of radius 6.35mm and side step defined by scallop height of 0.68mm. For this, two tool paths are generated with the same conditions, one with original Ball drop method and other with refined ball drop method using CL interpolation approach, to show the comparison of surface accuracy due to both. As roughing is the scaled up version of finishing, due to

this the results obtained after roughing can easily be seen by naked eyes, whereas in finishing the results would be microscopic. But the problem remains the same for both, roughing and finishing.

5.1.1 Comparison of tool paths

After converting the CAD geometry of Model 1 into STL format, both the tool paths are generated which are as shown in Figure 5.2.

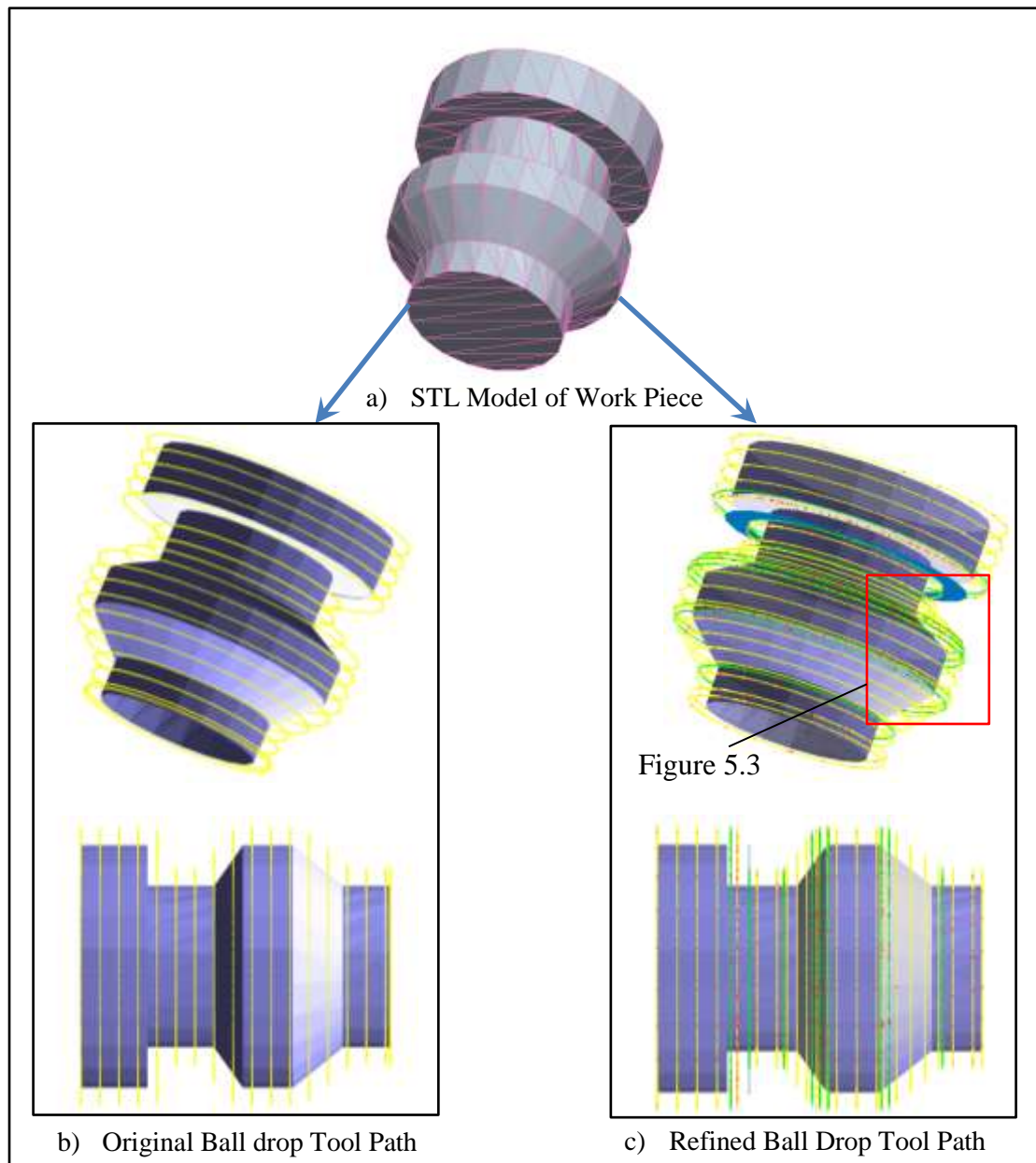


Figure 5.2: Comparison of Tool paths

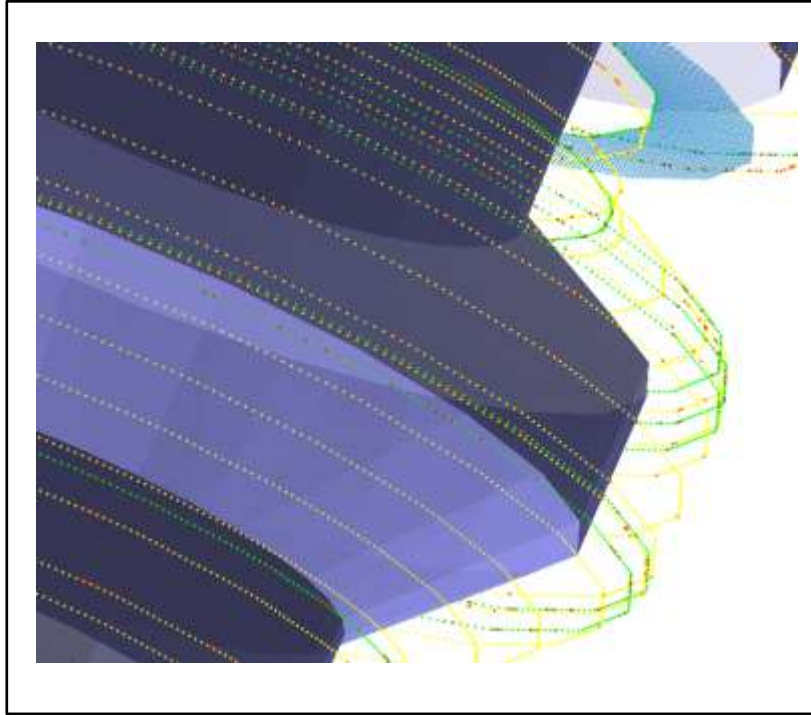


Figure 5.3: Zoomed view of tool path

Figure 5.2a) shows the STL model generated from the CAD geometry. In Figure 5.2b) yellow rings gives tool path of the work piece using original Ball drop method and Figure 5.2c) shows the tool path generated for the same geometry using refined Ball drop method, in which it can clearly be seen the rings, colored green, added where there are changes in geometry, and CL points, colored red, added within the rings where there is change in triangle. So at the position of change in geometry, the CL point data is added but when there is no change i.e. flat surface, the CL points are same as that are with original Ball drop method. The zoomed in view of refined tool path is also shown in Figure 5.3 for better demonstration of tool path,

5.1.2 Simulation results

After generating the tool path with both the methods, the simulation of machining is done using ToolSim Simulator provided by the machine. Simulation results using both the tool paths are shown in Figure 5.4.

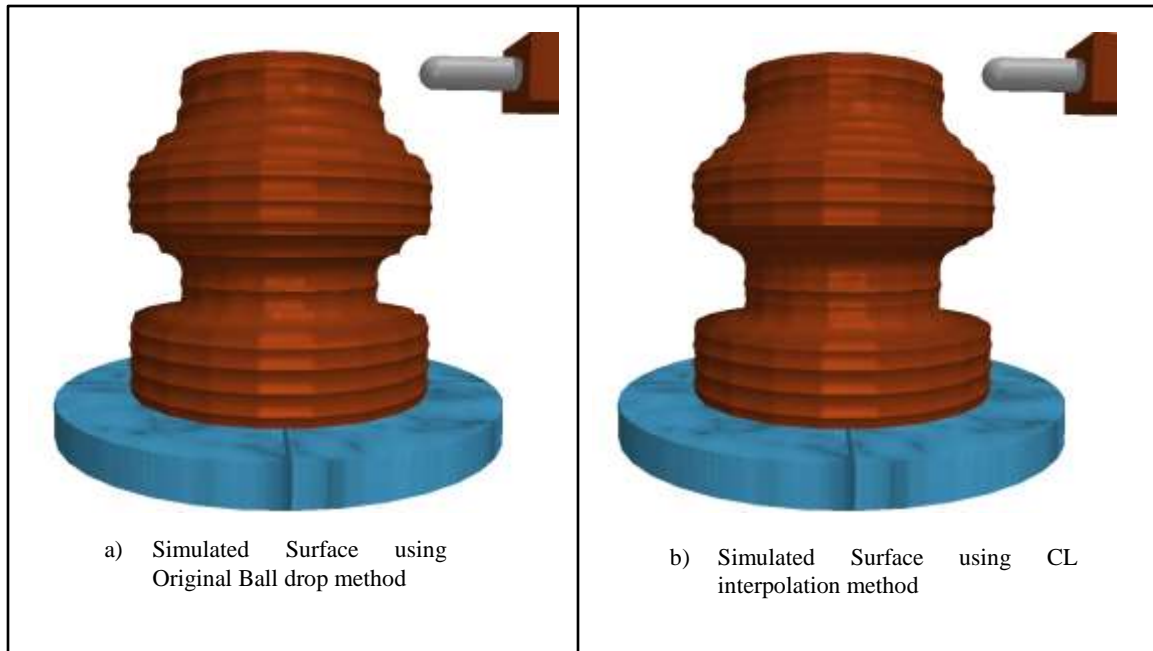


Figure 5.4: Comparison of Simulated Machined surface with Original and Refined Ball Drop Method

Figure 5.4(a) shows the simulated stock of work piece using Original Ball drop method and Figure 5.4(b) shows the simulated stock of work piece using refined Ball drop method after implementing CL interpolation method. These stocks has been saved as object (.obj) file format, the data from object file is read and imported in excel file. After importing data in excel, comparison plot of simulated surface and original STL geometry surface is generated, which is as shown in Figure 5.5.

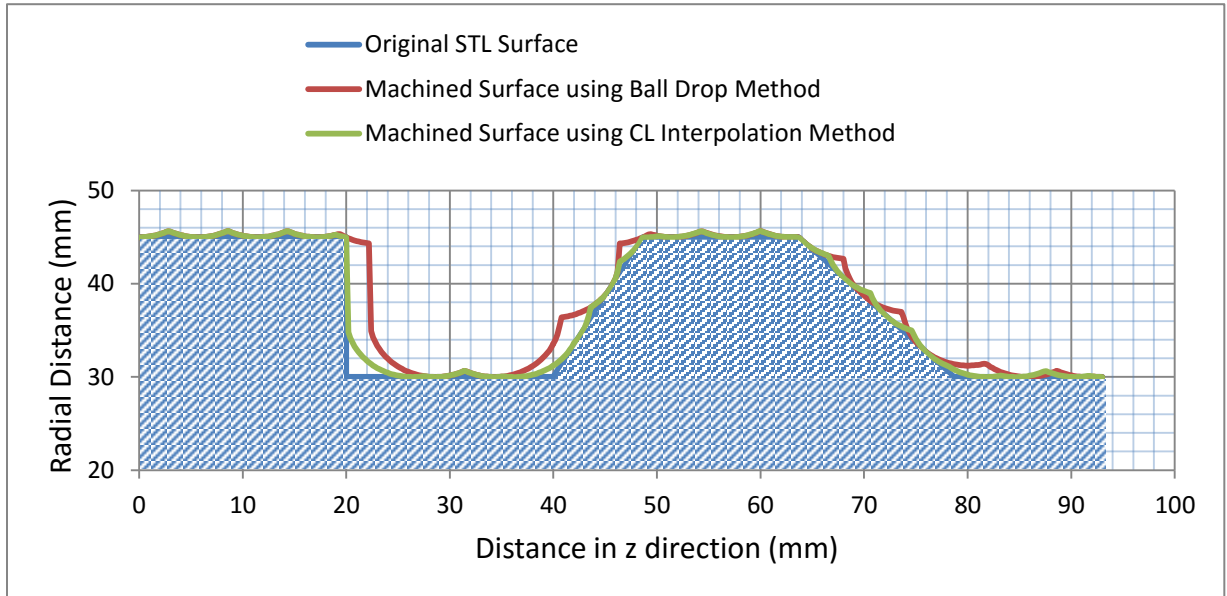


Figure 5.5: Graphical Comparison of Machined and Actual surface

As the graph in Figure 5.5 clearly depicts the deviation of machined surface using CL interpolation method is less as compared to machined surface using original Ball drop method. At change of 90° , original ball drop method has left a huge amount of uncut material. On the other hand, CL interpolation gives the approximate actual geometry; there is no undercutting; only uncut material is due to spherical geometry of Ball nose end mill cutter. At the different slopes, scallop height generated after implementing CL interpolation method is within tolerance zone, which is more in case of original Ball drop method and also at position of change in geometry the undercutting is minimized as compared to original Ball drop method. Whereas at flat surface there is no change of surface accuracy, as for flat portion machined geometries generated using both methods overlap each other.

5.1.3 Physical Demonstration

After simulation, actual machining is done for both the methods. The physical parts are as shown in Figure 5.6.

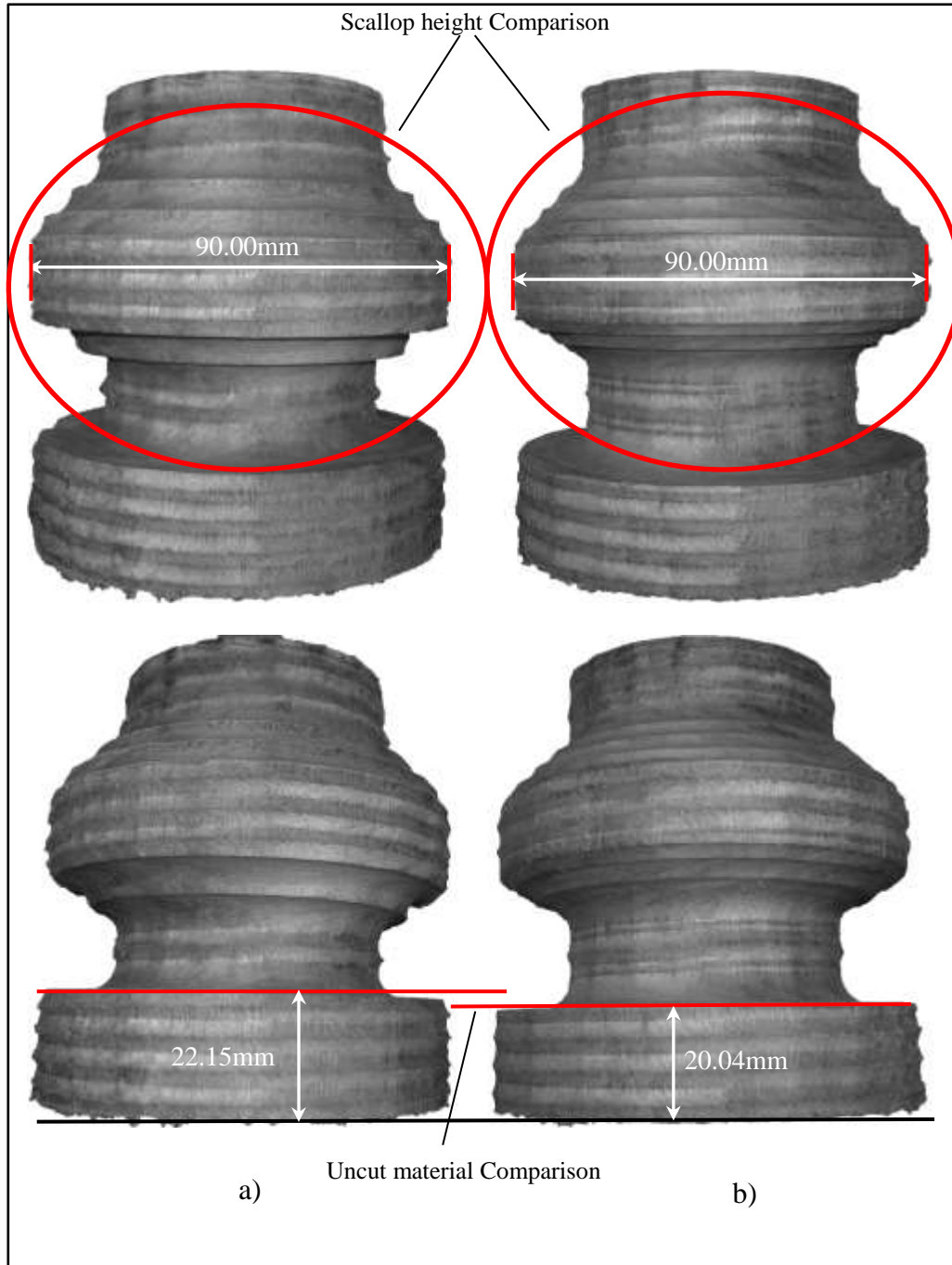


Figure 5.6: Comparison of Physical Models a) using original Ball drop method
b) using CL interpolation method

The Figure 5.6 clearly depicts the difference of machining of model using both the methods. The scallop height of work piece, machined by using CL interpolation method, is constant throughout the surface as compared to work piece machined by using original ball drop method and also the large uncut material left in work piece machined by using original ball drop method as compared to work piece machined by using CL interpolation method when the change in geometry is 90° .

5.2 Model 2

Two tool paths are generated for Model 2, one is for roughing with ball nose end mill of radius 6.35mm and side step defined by scallop height of 0.68mm and other is for finishing with ball nose end mill of radius 1.58mm and side step defined by scallop height of 0.08mm. For this, refined tool paths are generated using CL interpolation method, only. This model is to just demonstrate the robustness of the method to the complexity. The finished tool path for this model is also generated using original Ball drop method with same conditions, just to show the comparison of finished surface with both methods in simulation.

5.2.1 Comparison of tool path

After converting the CAD geometry of Model 2 into STL format, both the tool paths are generated. In Figure 5.7(a) the STL file generated from CAD geometry is shown and referring to this STL file tool paths are generated using CL interpolation method for roughing, as shown in Figure 5.7(b) and also for finishing, as shown in Figure 5.7(c). In both finishing and roughing tool paths rings shown in green color are due to change in geometry or triangle in side feed direction and CL points shown in red color are due to change in geometry or triangle in forward feed direction. The figures clearly depict that at slopes and change in geometry the number of rings or CL points are more as compared to flat surfaces in the geometry, where there is no need to add the CL points. So the final CL mesh generated is fine at the slopes or changes as compared to the flat surface, making a non-uniform mesh all over the surface of work piece for the accuracy defined by scallop height.

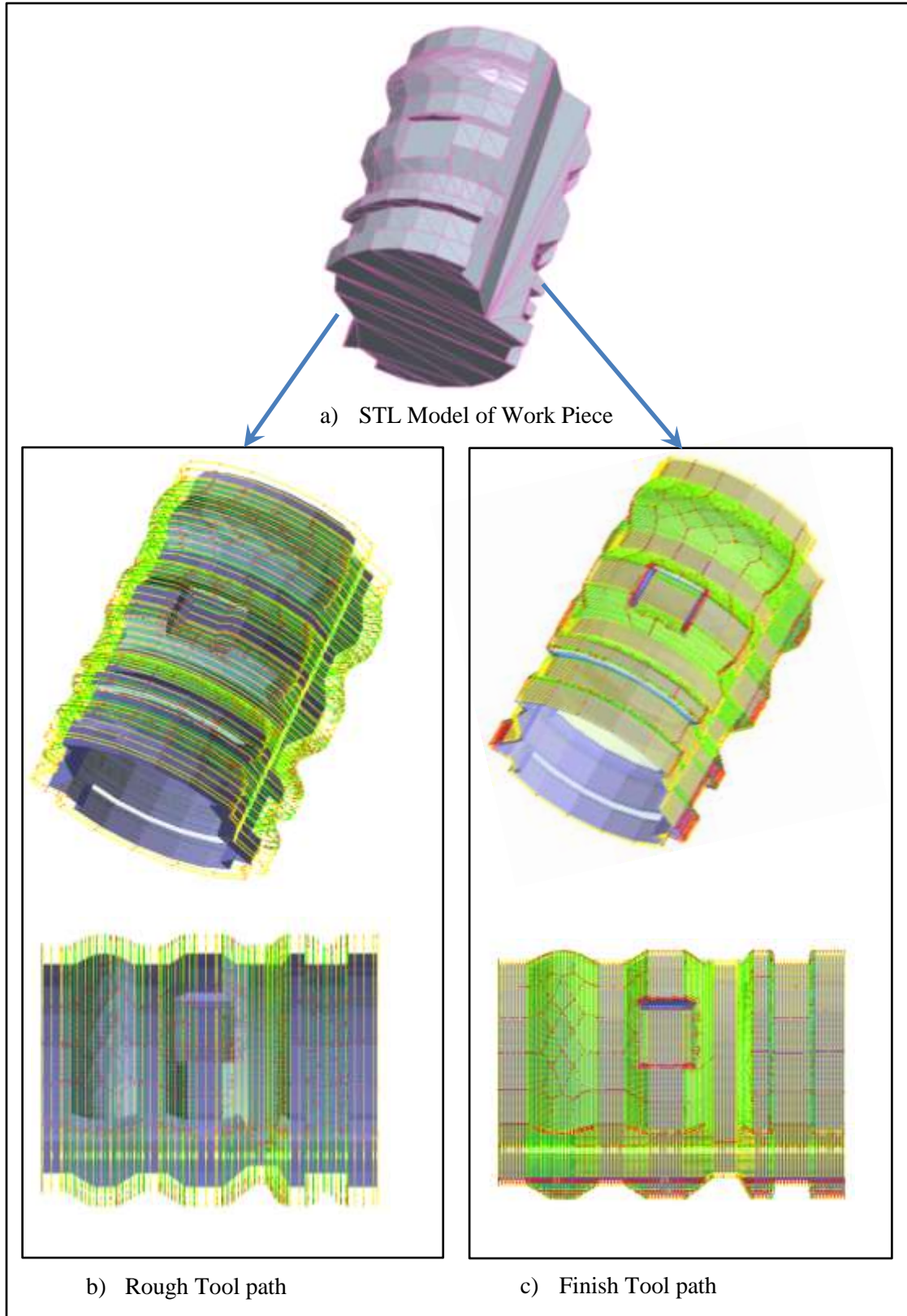


Figure 5.7: Representation of Refined Tool Paths using CL interpolation method

5.2.2 Simulation results

The simulation results for model 2 are obtained by simulating the machining using tool paths generated by both the methods i.e. by original ball drop method as well as the refined ball drop method using CL interpolation method, for finishing only. Simulation results using both the tool paths are shown in Figure 5.8.

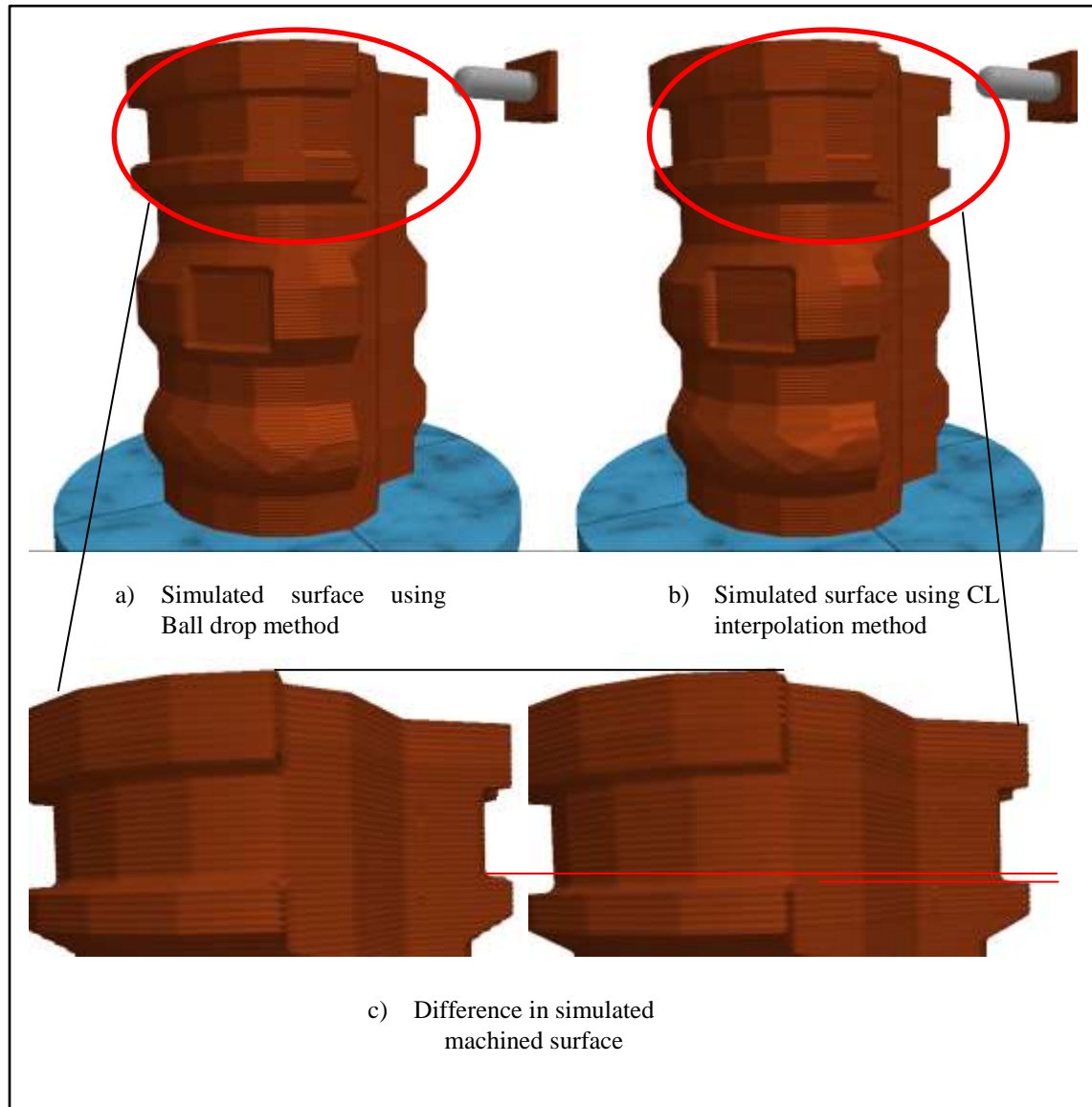


Figure 5.8: Comparison of Simulated Machined surface with Original and Refined Ball Drop Method

Figure 5.8(a) shows the simulated stock of work piece using Original Ball drop method; Figure 5.8(b) shows the simulated stock of work piece using refined Ball drop method

after implementing CL interpolation method and Figure 5.8(c) shows the difference in geometries at change of 90°. These stocks has been saved as object (.obj) file format, the data from object file is read and imported in excel file. After importing data in excel, comparison plot of simulated surface and original STL geometry surface is generated from a particular longitudinal cross section where the change in geometry is more, which is as shown in Figure 5.9.

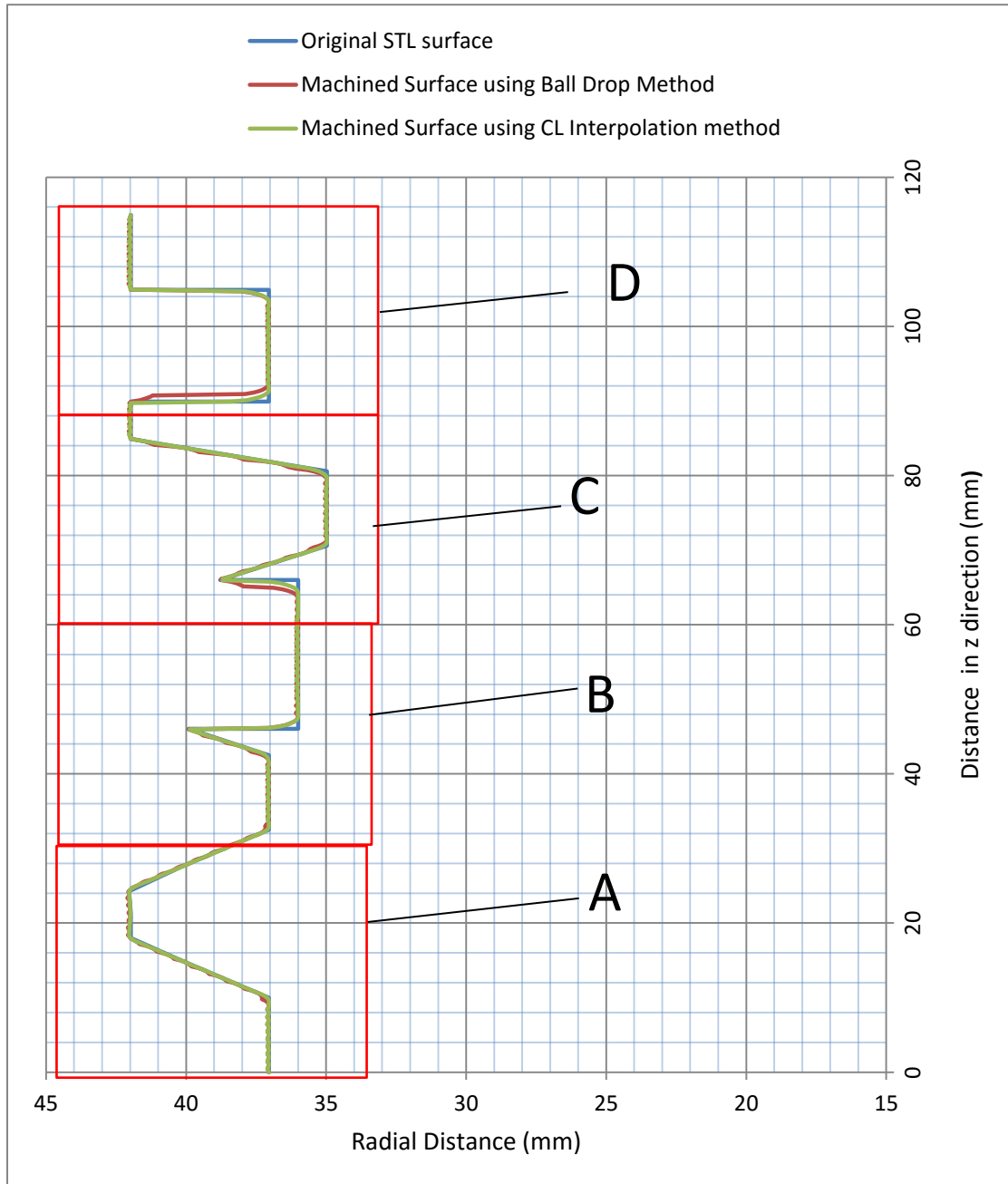
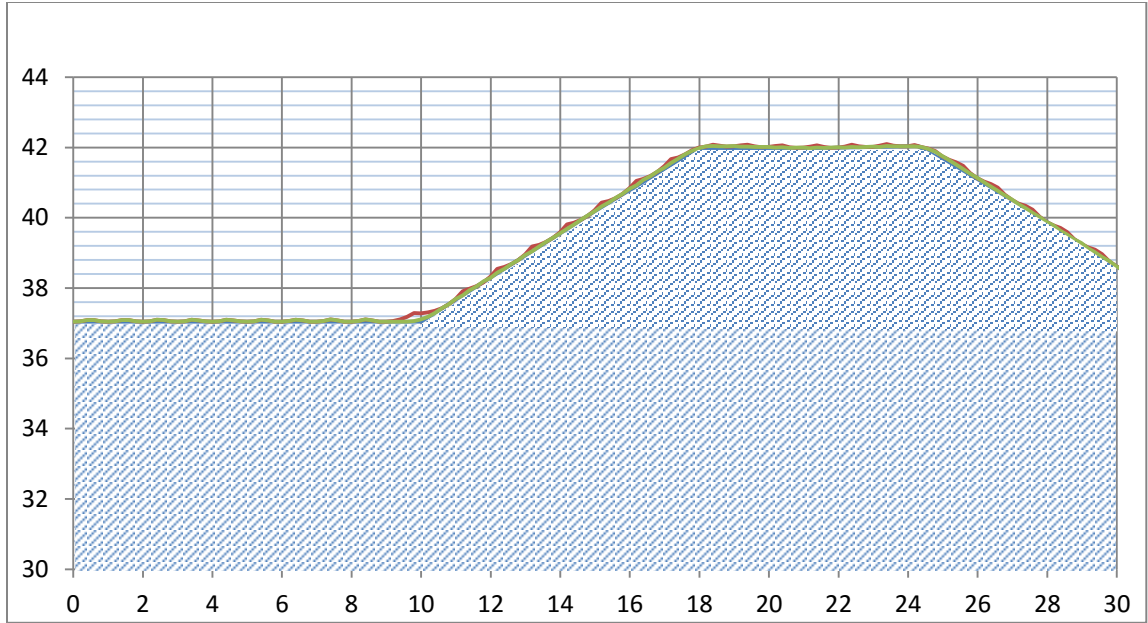
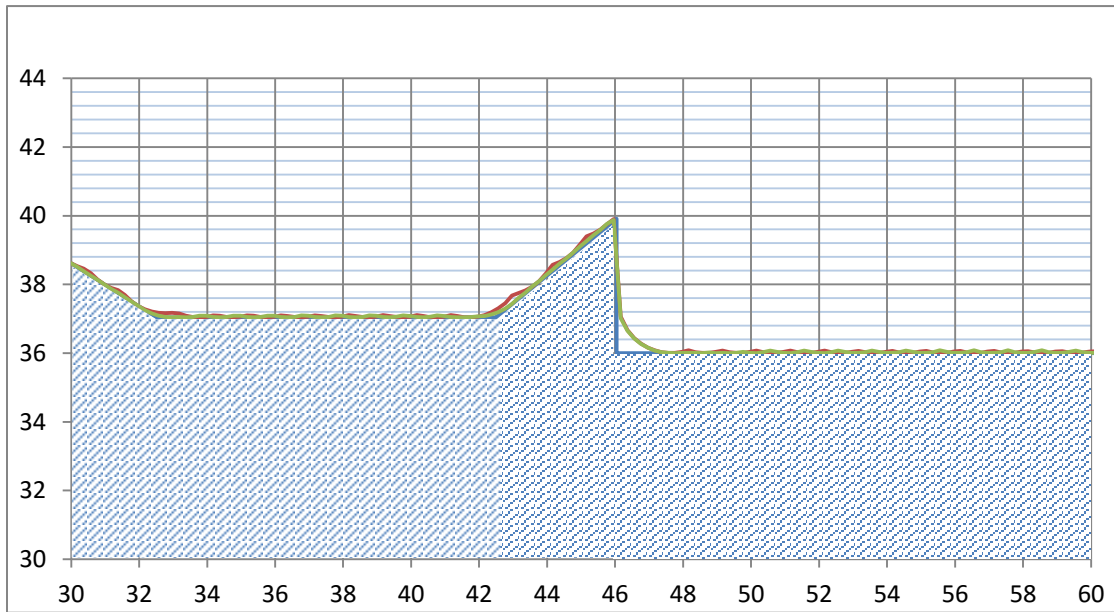


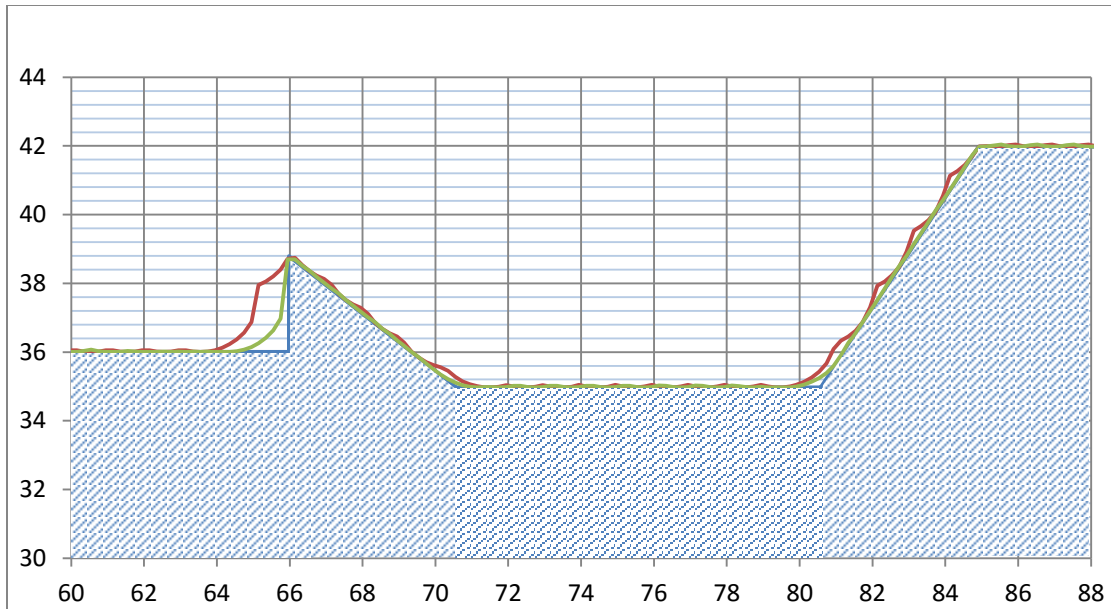
Figure 5.9: Graphical Comparison of Machined and Actual surface



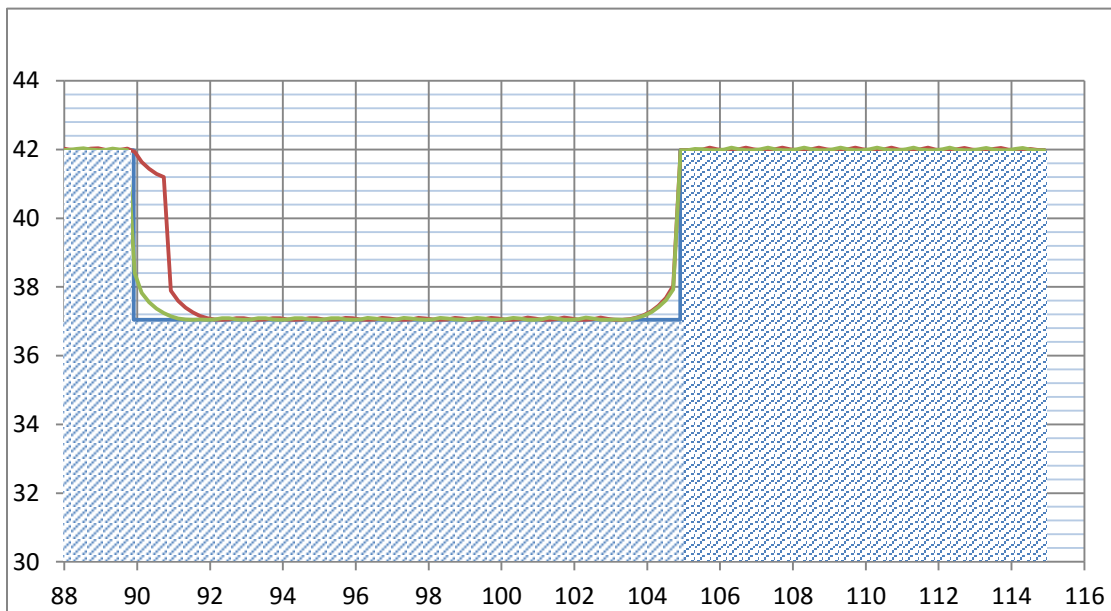
A: Zoom in view of Figure 5.9 from 0-30 mm in z direction



B: Zoom in view of Figure 5.9 from 30-60 mm in z direction



C: Zoom in view of Figure 5.9 from 60-88 mm in z direction



D: Zoom in view of Figure 5.9 from 88-116 mm in z direction

The graph in Figure 5.9 shows the deviation of surfaces after machining using original ball drop method (shown by red color line type) and CL interpolation method (shown by green color line type) from the actual surface obtained from STL of geometry (shown by blue color line type). As the graph is of finished surface, so the changes are microscopic and are shown in zoom in view of that graph in further graphs named A,B

and C at different range in z direction, where the deviation can be clearly seen. In graph B and C when there is a change of 90^0 in geometry there is huge uncut material left by original ball drop method whereas using CL interpolation method approximately exact shape of geometry has been achievable, the only uncut material is due to spherical shape of tool. The deviation of surface at slopes can also be seen. It can also be seen that whenever geometry changes its slope the original ball drop method left a uncut material which can be machined but it is not able to give a that much accuracy but with the CL interpolation method is able to generate tool path which can machine that uncut material (left by Ball drop method) to its minimum extent.

5.2.3 Physical Demonstration

After simulation, actual machining is done using only CL interpolation method. The physical part is as shown in Figure 5.10.

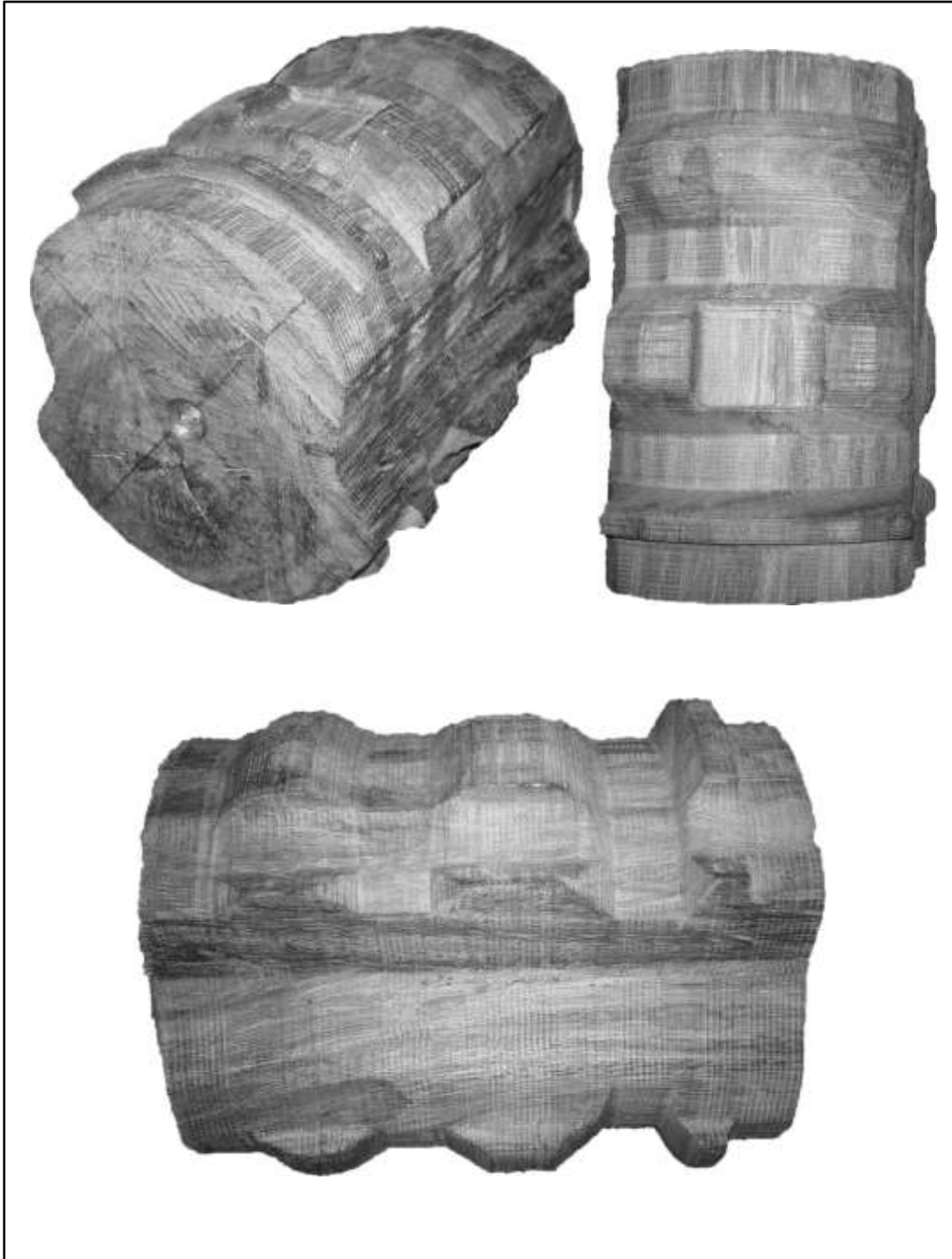


Figure 5.10: Physical demonstration of Model 2 using CL interpolation method

CHAPTER 6

Conclusion and Future Scope

6.1 Conclusion

In this thesis work a method, known as ‘TP from CL interpolation’, has been developed and validated by simulation and experimentally for ‘PBG (KW) Inc. Milling Lathe CNC’ using MATLAB technical programming language. Ball nose end tool with radius of 6.35mm, for roughing and 1.58mm for finishing is used. The tool path is generated in such a way that errors in machining of STL surface done by the tool path, generated by connecting CL points obtained using Ball drop method proposed by Manos et al. [1] are minimized. The accuracy of machined STL surface depends upon the connectivity of CL points with each other.

Comparing the machined surface generated with refined tool path and tool path made by connecting CL points generated by ball drop method with fixed intervals as proposed by Manos et al. [1], it has been seen that the this approach gives better result as compared to original one by eliminating overcutting at convex and minimizing the undercut on concave surface intersections and step. The only uncut material is due to fillet left by ball nose cutter..

6.2 Scope for Further Study

- a) The present work of thesis is confined to 3-axis turn milling machine and ball nose end mill cutter using only z-parallel footprint. The future work includes the further development of this work for different shapes of tool, such as flat end mill, conical and toroidal cutter etc. and also using different footprints such as helical etc.
- b) This work can also be extended for overall time optimization, which includes calculation and machining time.
- c) This work can also further enhanced and optimized for 5-axis CNC milling machines and other machines.
- d) This work can also be enhanced for different work piece other than pseudo symmetric.

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