

A Thesis
On

**Probability Oscillations And Its Importance For Non –Zero θ_{13}
Value**

Submitted in the partial fulfilment of requirement for the award of
the
Degree of
Master of Science (Physics)

Submitted by

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July 2012

Dedicated
To
My Family

Certificate

This is certify that the report entitled “ Probability Oscillation And Its Importance For Non-Zero θ_{13} Value” submitted by Ms. Paramjot Kaur (301004012) of M.Sc (Physics), Thapar University, Patiala was carried out by her under my supervision. She has not submitted this material for credit towards any other degree at Thapar University, Patiala or any other university.

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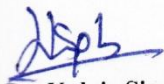


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Date: 16, July, 2012 .

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ABSTRACT

According to standard model, neutrinos are massless but from the beyond standard model facts it has proved that neutrinos has some mass. Neutrino oscillations are sign of massive neutrinos and they oscillate in vacuum as well as in matter. A probability oscillation in vacuum differs from matter because of the interactions of neutrinos with matter and their densities. We study these differences in a quantum field theoretical formalism and by comparing vacuum and matter probability oscillation curves. Our main focus in the present thesis is to present a detailed analysis of these probabilities with the change in the energies, length and mixing angles. We also emphasize the role of unknown neutrino third mixing angle θ_{13} which is proven to be a non zero value in the recent advances . The effect of non-zero θ_{13} changes various other results and its accurate determination is important and hence it is important to see the effect on probabilities with large and small values of θ_{13} and also the effects on other parameters like CP violation, mass hierarchy etc.

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Natural Units	$\hbar=c=1$
Mass Mc^2/c^2	GeV
length $\hbar c/Mc^2$	$\text{GeV}^{-1}=0.1975 \text{ fm}$
Time $\hbar c/Mc^3$	$\text{GeV}^{-1}=6.59*10^{-25} \text{ s}$

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CHAPTER 1

1. INTRODUCTION

1.1 MOTIVATION IN HIGH ENERGY PHYSICS:

High energy physics plays a very important role in today's life. With the help of high energy physics we are dealing with the small particles which are present in atoms. With this we can visualize the particles by which an atom is affected. By high energy physics, we are now familiar with the leptons, quarks, neutrinos etc. Like neutrinos are the most abundant particles present. Today, scientists are starting to open up a completely new window by using another elementary particle, the neutrino, instead of the photon. In the sun, an enormous number of neutrinos are produced in the fusion process when four hydrogen atoms transform into one helium atom. Despite the large number of neutrinos, an average of only about one of these will interact with a person's body during a lifetime. That's all we know is just because of high energy physics. Neutrinos are the elementary particle of light used to investigate the universe. This new field called neutrino astronomy.

The status of the visible phenomena in the context of scientific research has changed due to the progress in the elementary particle physics in recent decades. Particular emphasis on the pre-eminence of the visible regime, are affected by this development, which is in an empiricist position in philosophy of science. The constructive empiricist's emphasis on the scientist's aims makes it essential for her to provide a satisfactory motivation for scientific inquiry. However in the case of elementary particle physics, it seems very difficult to achieve on an empiricist basis.

For the motivation of theory development, which challenges the status of the experimental process as the primary driving force behind scientific progress, the intra-theoretical questions arise. This devaluation of the visible phenomena is potentially dangerous for empiricist positions in philosophy of science. Philosophical positions which put much emphasis on a privileged role of visible phenomena may look less convincing, if the role of visible phenomena decreases in fundamental physics.

1.2 IMPORTANCE OF ELEMENTARY PARTICLE PHYSICS

Particle physics has a huge importance in our life. It gives us many different new particles from which we can discover new matters, particles, anti-particles and their properties. New discoveries and particles are discussed as follows:

- Higgs bosons.
- Dark matters.
- Muon collider.

- Massive neutrinos.
- INO.

1.2.1 HIGGS BOSONS

The Higgs boson is elementary particle, which is yet undiscovered, The Higgs field is a particularly simple one –from every direction, it has the same properties and in important aspects it is indistinguishable from empty space . The Higgs' interactions with other particles are taken into account for instance, particles call vector bosons can travel with the grain, in which case they move easily for large distances and may be observed as photons - that is, particles of light that we can see or record using a camera; or against, in which case their effective range is much shorter, and we call them W or Z particles. These play a central role in the physics of nuclear reactions, such as those occurring in the core of the sun.

The Higgs field enables us to view these apparently unrelated phenomenon as two sides of the same coin; both may be described in terms of the properties of the same vector bosons. When particles of matter such as electrons or quarks (eltravel through the grain, they are constantly flipped "head-over-heels". this forces them to move more slowly than their natural speed, that of light, by making them heavy. We believe the Higgs field responsible for endowing virtually all the matter we know about with mass.

1.2.2 DARK MATTER

Astrophysicists now know that 80% of the matter in the universe is `dark matter', composed of neutral and weakly interacting elementary particles that are not part of the Standard Model of particle physics. I will summarize the evidence for dark matter. I will explain why I expect dark matter particles to be produced at the CERN LHC. We will then need to characterize the new weakly interacting particles and demonstrate that they are the same particles that are found in the cosmos.

1.2.3 MUON COLLIDERS

Even if the Large Hadron Collider (LHC) has barely began functioning properly for a couple of years, particle physicists are already thinking about the next generation of elementary particle splitters, and one particular concept appears to have captured their imagination – muon colliders.

1.2.4 MASSIVE NEUTRINOS

It was predicted first that neutrinos has no mass. They are massless particles but after long researches it is proved that neutrinos has some mass because neutrinos are oscillating from one mass eigen state to another. Its oscillation just possible if it has some mass. Neutrinos are massive particles.

1.2.5 INO (INDIA- BASED NEUTRINO OBSERVATORY)

INO collaboration is progressing well in India. It has presently about 55 members from about fifteen Institution and Universities. INO is a multi-institutional collaboration which aims to

build an underground laboratory for pure Science research, especially in Neutrino Physics .INO is a proposed particle physics research project to primarily study atmospheric neutrinos. This is one of the biggest experimental particle physics project undertaken in India. The project is expected to be complete in 2015 at an estimated cost of 250 million dollars. The location of the site was supposed to be southwest of Masinagudi in the Nilgiri hills of the South India. INO is looking for the study of the problem via stimulations. The geometry of the detector is stimulated on the computer and interaction are fed in. Different processes like interaction of the neutrinos with the detector materials are studied and this is used to determine which are more promising processes to study. The project includes construction of an iron calorimeter detector, called ICAL, that will be the world's most massive detector when completed. ICAL at INO holds the key to understanding several fundamental issues regarding the nature and interactions of neutrinos.

1.3 Elementary Particles - The smallest constituents of matter and energy, which don't seem to be made from combinations of smaller particles. All of these particles are like zoo of other particles. It aims to determine the fundamental laws that control the make-up of matter and the physical universe. The science of this study is called Particle Physics, Elementary Particle Physics or High Energy Physics (HEP).

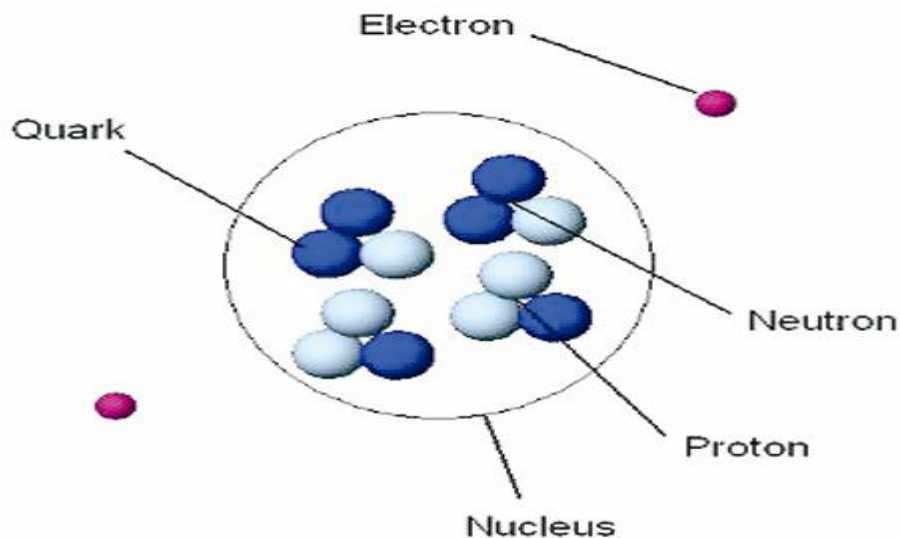


FIG 1.1 PARTICLES IN AN ATOM

1.4 PARTICLES CLASSIFICATION

The classification of the particles on the basis of which statistics they obey:

Fermi-Dirac statistics- This statistics apply to those particles restricted by the Pauli exclusion principle. Particles obeying the Fermi-Dirac statistics are known as fermions. Leptons and quarks are fermions.

Bose-Einstein statistics- This statistics apply to all particles not covered by the exclusion principle, and such particles are known as bosons. The number of bosons in a given quantum state is not restricted.

Fermions - Fermions are particle that have a particle spin equal to a half-integer value ($-1/2$, $1/2$, $3/2$, etc.). These particles make up the matter that we observe in our universe.

Quarks - A class of fermion. Quarks are the particles that make up hadrons, such as protons and neutrons. Quarks also have antimatter counter parts called antiquarks .There are 6 distinct types of quarks:

Up(u), Charm(c) , Top(t) , Down(d) , Strange(s) , Bottom(b)

NAME OF THE QUARKS	SPIN	ELECTRIC CHARGE
UP (u)	$1/2$	$2/3$
CHARM (c)	$1/2$	$+2/3$
TOP (t)	$1/2$	$+2/3$
DOWN (d)	$1/2$	$-1/3$
STRANGE (s)	$1/2$	$-1/3$
BOTTOM (b)	$1/2$	$-1/3$

Table 1.1: Fundamental Quarks and its spin and charge.

With the combination of quarks, they form heavier particles called baryons, and quarks and antiquarks combine to form mesons. Protons and neutrons, are examples of baryons. Positive and negative kaons are examples of mesons.

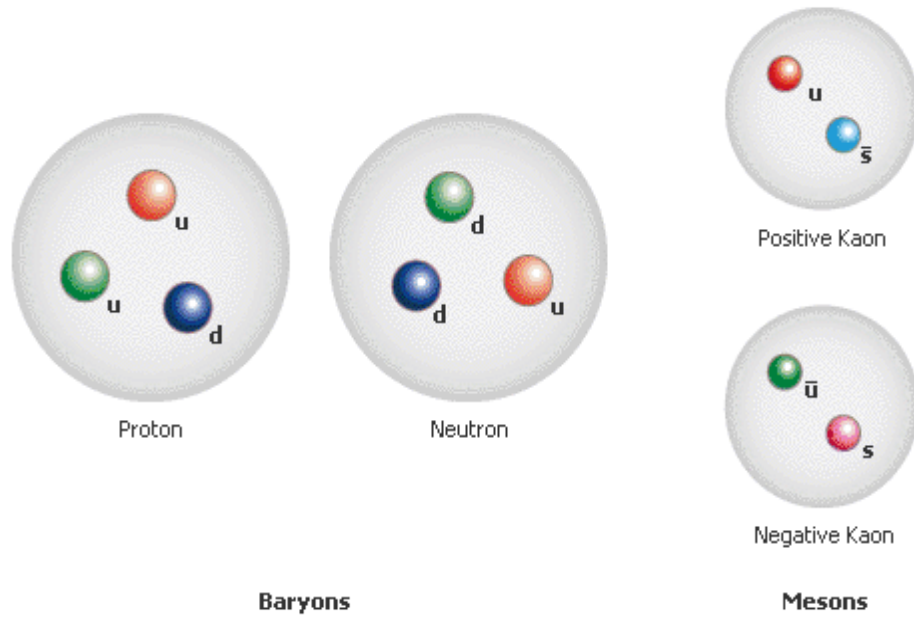


Fig 1.2: combinations of quarks to make baryons and mesons.

Leptons – It is a class of fermion. There are 6 types of leptons:

Electron(e) , Electron Neutrino(ν_e) , Muon(μ) , Muon Neutrino(ν_μ) , Tau(τ) , Tau Neutrino(ν_τ)

We all are familiar with electron, muon and tau are heavier than electron. The muon and tau are rarer than electron but has negative charge same as electron has. Leptons are much lighter than quarks.

NAME OF LEPTONS	GENERATION	LEPTON NUMBER	SPIN	ELECTRIC CHARGE
ELECTRON(e)	FIRST	1	$\frac{1}{2}$	-1
ELECTRON NEUTRINO(ν_e)	FIRST	1	$\frac{1}{2}$	0
MUON(μ)	SECOND	1	$\frac{1}{2}$	-1
MUON NEUTRINO(ν_μ)	SECOND	1	$\frac{1}{2}$	0
TAU(τ)	THIRD	1	$\frac{1}{2}$	-1
TAU NEUTRINO(ν_τ)	THIRD	1	$\frac{1}{2}$	0

Table 1.2: fundamental leptons ,its spin and charge.

All these sets of particles also include their anti-particles, or we can say that their complementary opposites. These make up matter and anti-matter. The anti-particles are same as the particles (like same in mass) but has different properties than particles. When a particle and anti particle meets they annihilate and librates large amount of energy. Anti-matter can be produced using accelerators.

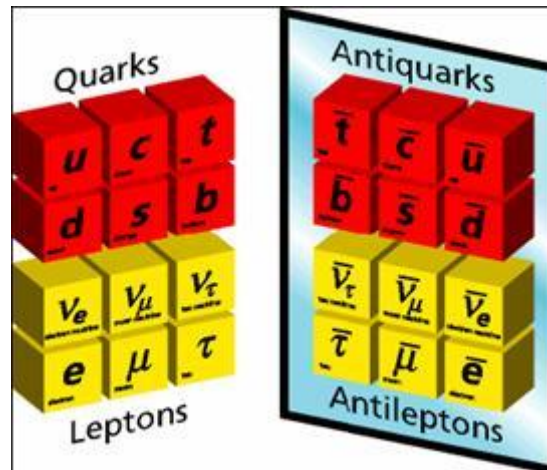


Fig 1.3: Particles And Antiparticles.

1.5 STANDARD MODEL

In particle physics, standard model describes the universe in terms of matter and force. Matter is fermions and force is boson. Standard model describes approximately 200 particles and their interactions using 17 fundamental particles. The Standard Model was Sheldon Glashow's discovery in 1960.

We know that the universe is started off with big bang. Big bang has enormously high energy and temperature concentrated in very small volume. The Universe immediately started to expand at a furious rate and some of the energy was converted into pairs of particles and antiparticles with mass. In the very dense phase, particles and antiparticles collided and annihilated each other into photons, leaving just a tiny fraction of matter , which is carry on in the Universe.

The Standard Model includes the electromagnetic, strong and weak forces and all their carrier particles, and explain that forces which act on all the matter particles. Gravity is not part of the Standard Model, which is most familiar in our life.

Three Generations of Matter (Fermions)				
	I	II	III	
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name →	u up	c charm	t top	γ photon
Quarks	4.8 MeV $-\frac{1}{3}$ $\frac{1}{2}$ d down	104 MeV $-\frac{1}{3}$ $\frac{1}{2}$ s strange	4.2 GeV $-\frac{1}{3}$ $\frac{1}{2}$ b bottom	0 0 1 g gluon
	< 2.2 eV 0 $\frac{1}{2}$ ν_e electron neutrino	< 0.17 MeV 0 $\frac{1}{2}$ ν_μ muon neutrino	< 15.5 MeV 0 $\frac{1}{2}$ ν_τ tau neutrino	91.2 GeV 0 1 Z weak force
	0.511 MeV -1 $\frac{1}{2}$ e electron	105.7 MeV -1 $\frac{1}{2}$ μ muon	1.777 GeV -1 $\frac{1}{2}$ τ tau	80.4 GeV ±1 1 W[±] weak force
Leptons				Bosons (Forces)

Fig1.4 : Standard Model

Electromagnetic force: The electromagnetic force affects any electrically charged fundamental particle. It's the same force that makes lightning strike and different poles of bar magnets attract each other.

Weak force: The weak force is responsible for radioactive decay. It actually makes neutrons turn into protons, and other things, and every type of matter particle experiences it. and for interactions of neutrinos and other leptons with matter. The weak force is a billion times weaker than the strong force.

Strong force: The strong force is only felt by quarks. It behaves like elastic, because when we pull two quarks, the stronger the strong force gets between them.

Besides the three generations of quarks and leptons, the standard model also contains the gauge bosons i.e. photons, gluons, W- and Z-bosons, carriers of weak forces, which are for the transfer the interactions between the quarks and leptons. The carriers of the gravitational field are called gravitons and are unique in having a spin of 2. The Standard Model groups two major theories—quantum electroweak theory and quantum chromodynamics theory that describes the interactions between all known particles in terms of quantum field theory (QFT).

KNOWN FORCES (Bosons)

FORCE	PARTICLE /QUANTUM	RELATIVE STRENGTH	MASS (GeV)	RANGE (meters)
Strong nuclear	Gluon	1	0.14 (?)	10^{-15}
Electromagnetic	Photon	7×10^{-3}	None	Infinite
Weak nuclear	W^+, W^- & Z bosons	10^{-5}	80-90	10^{-17}
Gravitation	gravitron (tentative)	6×10^{-39}	None	Infinite

Table1.3: Fundamental Forces.

1.6 BEYOND STANDARD MODEL

Although all experimental evidence confirms the predictions of the Standard Model, many physicists find this model to be unsatisfactory due to its many undetermined parameters, many fundamental particles, the non-observation of the Higgs boson and other more theoretical considerations such as the hierarchy problem. There are many speculative theories beyond the Standard Model that attempt to rectify these deficiencies.

1.6.1 Grand unification

One extension of the Standard Model attempts to combine the electroweak interaction with the strong interaction into a single 'grand unified theory' (GUT). Such a force would be spontaneously broken into the three forces by a Higgs-like mechanism. The most dramatic prediction of grand unification is the existence of X and Y bosons, which cause proton decay. However, the non-observation of proton decay at Super-Kamiokande rules out the simplest GUTs, including SU(5) and SO(10). Before the unification point, forces are indistinguishable and have symmetry. After the unification point, the forces act differently and the symmetry is broken.

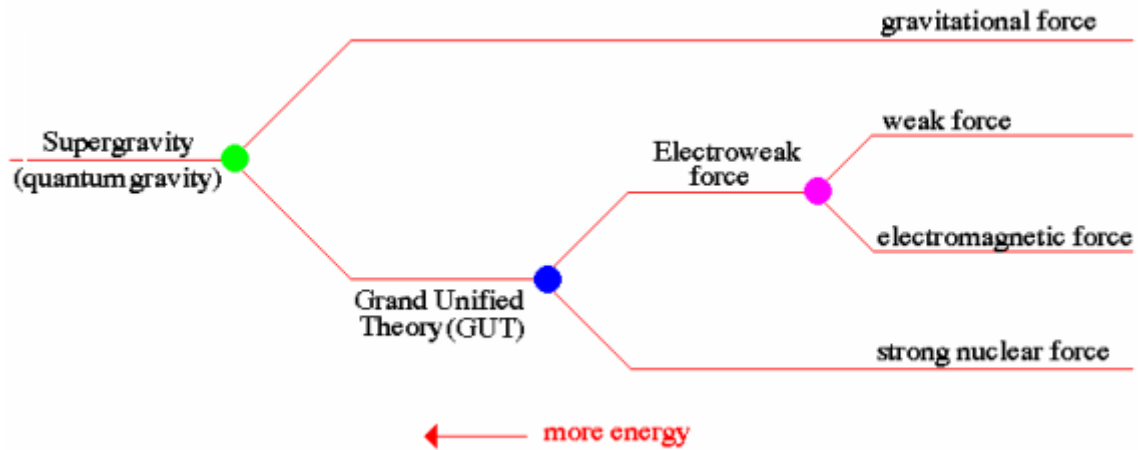


FIG 1.5: Grand Unification .

1.6.2 Supersymmetry

Supersymmetry extends the Standard Model by adding an additional class of symmetries to the Lagrangian. These symmetries exchange fermionic particles with bosonic ones. Such a symmetry predicts the existence of supersymmetric particles, abbreviated as sparticles, which include the sleptons, squarks, neutralinos, and charginos. Each particle in the Standard Model would have a superpartner whose spin differs by $1/2$ from the ordinary particle. Due to the breaking of supersymmetry, the sparticles are much heavier than their ordinary counterparts; they are so heavy that existing particle colliders would not be powerful enough to produce them. However, some physicists believe that sparticles will be detected when the Large Hadron Collider at CERN begins running.

1.6.3 String theory

String Theory is a model of physics where all "particles" that make up matter are composed of strings (measuring at the Planck length) that exist in an 11-dimensional universe. These strings vibrate at different frequencies that determine mass, electric charge, color charge, and spin. A string can be open (a line) or closed in a loop (a one-dimensional sphere, like a circle). As a string moves through space it sweeps out something called a world sheet. String theory predicts 1- to 10-branes (a 1-brane being a string and a 10-brane being a 10-dimensional object) that prevent tears in the "fabric" of space using the uncertainty principle (E.g., the electron orbiting a hydrogen atom has the probability, albeit small, that it could be anywhere else in the universe at any given moment). String theory proposes that our universe is merely a 4-brane, inside which exist the 3 space dimensions and the 1 time dimension that we observe. The remaining 6 theoretical dimensions either are very tiny and curled up (and too small to affect our universe in any way) or simply do not/cannot exist in our universe (because they exist in a grander scheme called the "multiverse" outside our known universe).

Some predictions of the string theory include existence of extremely massive counterparts of ordinary particles due to vibrational excitations of the fundamental string and existence of a massless spin-2 particle behaving like the graviton.

1.6.4 Technicolor

Technicolor theories try to modify the Standard model in a minimal way by introducing a new QCD-like interaction. This means one adds a new theory of so called Techniquarks, interacting via so called Technigluons. The main idea is that the Higgs-Boson is not an elementary particle but a bound state of these objects.

1.6.5 Preon theory

According to preon theory there are one or more orders of particles more fundamental than those (or most of those) found in the Standard Model. The most fundamental of these are normally called preons, which is derived from "pre-quarks". In essence, preon theory tries to do for the Standard Model what the Standard Model did for the particle zoo that came before it. Most models assume that almost everything in the Standard Model can be explained in terms of three to half a dozen more fundamental particles and the rules that govern their interactions. Interest in preons has waned since the simplest models were experimentally ruled out in the 1980s.

1.6.6 Acceleron theory

Accelerons are the hypothetical subatomic particles that integrally link the newfound mass of the neutrino and to the dark energy conjectured to be accelerating the expansion of the universe. In theory, neutrinos are influenced by a new force resulting from their interactions with accelerons. Dark energy results as the universe tries to pull neutrinos apart.

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2. Neutrinos: Sources, Experiments, Detectors And Interactions

2.1 HISTORY OF NEUTRINOS

Neutrinos are one of the fundamental particles which make up the universe. They are also one of the least understood. Neutrinos do not carry electric charge, which means that they are not affected by the electromagnetic forces that act on electrons. A neutrino is an electrically neutral, weakly interacting elementary subatomic particle with a small but non-zero mass.

Neutrino was first discovered in 1930 by Wolfgang Pauli to explain that why the electrons were not emitted with the full reaction energy of nuclear transition in beta decay. Neutrinos are created as a result of certain types of radioactive decay, or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atoms. There are three types, or "flavors", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos. We can classify the three neutrinos by their masses, and call them ν_1 , ν_2 , ν_3 , from lightest to heaviest, neutrino-1, neutrino-2, neutrino-3. We'll call this the mass classification. Each type also has a corresponding antiparticle, called an antineutrino.

Most neutrinos passing through the Earth emanate from the Sun. About 65 billion (6.5×10^{10}) solar neutrinos per second pass through every square centimeter perpendicular to the direction of the Sun in the region of the Earth.

Neutrinos are of three types:

- a) Electron Neutrino.
- b) Muon Neutrino.
- c) Tau Neutrino.



Fig 2.1 : Neutrino Flavors

Similarly, the anti-neutrinos are $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$. Electron neutrino, muon neutrino and tau neutrino means that they have mass of electron, muon, tau respectively in it. Mass of three of them are different. Mass of electron neutrino is much more than that of muon and tau neutrino.

Neutrinos can be divided into two categories, low energy and high energy. It is an arbitrary division, the way in which detectors are built. The low energy neutrinos are mainly produced in nuclear processes, like the fusion reactions in the sun or in the centre of an

exploding Supernova. The high energy neutrinos are mainly produced in high energy particle collisions producing short lived mesons, decaying to neutrinos and other particles.

In a particle physics scale the low energy neutrinos have energies in the 10^{th} of MeV (Mega electron Volts), whilst the high energy neutrinos have energies above 10^{th} of GeV (Giga electron Volts).

2.2 NEUTRINO MASSES

According to the Standard Model of particle physics, neutrinos should be massless – just like the photons that make up light – but in reality they do have a very small mass. What the Standard Model failed to take into account is the fact that neutrinos undergo something known as oscillations, or mixing. Neutrinos come in three “flavours”: electron neutrinos, mu neutrinos and tau neutrinos (as well as a corresponding antiparticle for each). When a neutrino is created, for example in the Sun during nuclear fusion, it has a specific flavour. Over time the neutrino can change flavour, but only if its mass is non-zero. All of the neutrinos created in the Sun are electron neutrinos. However, by the time they reach the Earth, we detect equal amounts of each flavour of neutrino; this is how we know that neutrinos must have mass.

2.3 NEUTRINO VELOCITIES

Neutrinos are elementary particles that travel close to the speed of light, but are very difficult to detect because they are not electrically charged. In fact, in the time it takes you to read this sentence, thousands of billions of neutrinos will have passed through your body – and you won't have felt a thing.

The OPERA collaboration has claimed that muon neutrinos with mean energy of 17.5 GeV travel 730 km from CERN to the Gran Sasso at a speed exceeding that of light by about 7.5 km/s or 25 ppm. However, we show that such superluminal neutrinos would lose energy rapidly via the bremsstrahlung of electron-positron pairs. For the claimed superluminal neutrino velocity and at the stated mean neutrino emissions en route, causing the beam to be depleted of higher energy neutrinos

2.4 NEUTRIONS AND ITS OSCILLATION

A phenomenon in which a neutrino in one of the three known flavor states (electron neutrino, mu neutrino, or tau neutrino) becomes a mixture of flavor states that changes back and forth periodically as the neutrino travels through space; it will occur if neutrinos have mass and if each flavor state is a mixture of different mass states.

Neutrino oscillation is a quantum mechanical phenomenon predicted by Bruno Pontecorvo whereby a neutrino created with specific lepton flavor (electron, muon or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates. Neutrino oscillation is of theoretical and experimental interest since observation of the phenomenon implies that the

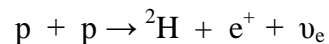
neutrino has a non-zero mass, which is not part of the original Standard Model of particle physics

2.5 THE SOURCES OF NEUTRINOS

The neutrinos in the universe come from weak interactions (like beta decays in atomic nuclei). Three rivers can be distinguished: the neutrinos from space, the neutrinos from the earth, the neutrinos from mankind activity. But there are many types of neutrinos origins, which can be quite arbitrarily classified in five sources:

2.5.1 Solar neutrino oscillation

The first experiment to detect the effects of neutrino oscillation was Ray Davis's Homestake Experiment in the late 1960s, in which he observed a deficit in the flux of solar neutrinos with respect to the prediction of the Standard Solar Model, using a chlorine-based detector. This gave rise to the *solar* neutrino problem. Many subsequent radiochemical and water Cherenkov detectors confirmed the deficit. Solar neutrinos have energies below 20 MeV and travel an astronomical unit between the source in the Sun and detector on the Earth. At energies above 5 MeV, solar neutrino oscillation actually takes place in the Sun through a resonance known as the MSW effect .They come along with the process of thermonuclear fusion inside the stars (our sun or any other star in the universe). Their energy is quite weak (some MeV) and they can travel in a long and quite way. They come from different nuclear reactions whose main reaction (85% of the solar neutrinos come from it)is:



p is a proton, H is a deuterium nucleus, e is an anti-electron and the last one is a neutrino. Depending on the nuclear reaction concerned, the neutrino has not the same energy.

2.5.2 Neutrinos from mankind activity

These are high energy neutrinos produced by the particles accelerators and low energy neutrinos coming out of nuclear reactors. The first ones, whose energy can reach about 100 GeV, are produced to study the structure of the nucleons (protons and neutrons composing the atomic nuclei) and to study the weak interaction. The second ones are here although we did not ask for them. They are an abundant product made by the nuclear reactions inside the reactors cores (a standard nuclear plant radiate about $5 \cdot 10^{20}$ neutrinos per second) and their energy is around 4 MeV. They have been the first to be detected and the first to be used to put some limits on the neutrino oscillation.

2.5.3 Neutrinos from the earth

Our great old planet has kept since its birth many radioactive atomic nuclei. This is what we call "natural radioactivity". This radioactivity is quite important, badly known and its main contribution could be to keep in fusion the matter under the crust of the earth. The power coming from this natural radioactivity is estimated at about 20.000 Giga Watts (about 20.000 nuclear plants!) and the neutrinos coming from this radioactivity are numerous: about 6

millions per second and per cm^2 . But those neutrinos, despite of their quantity, are often locally drowned in the oceans of neutrinos coming from the nuclear plants.

2.5.4 Neutrinos from cosmic rays

When a cosmic ray (proton coming from somewhere in space) penetrates the atmosphere, it interacts with an atomic nucleus and this generates a particles shower. Under the same principle which guides the neutrinos production at CERN, some neutrinos are created: they are called "atmospheric neutrinos". Some experiments like Kamiokande and Super-Kamiomande in Japan have tried to see the oscillations of the neutrinos inside those particle showers. The results in 1998 seem positive.

2.5.5 Neutrinos from the Big-Bang

The "standard" model of the Big-Bang predicts, like for the photons, a cosmic background of neutrinos. Those neutrinos, nobody has never seen them. They are yet very numerous: about 330 neutrinos per cm^3 . But their energy is theoretically so little (about 0.0004 eV), that no experiment, even very huge, has been able to detect them.

2.5.6 Beam neutrino oscillation

Neutrino beams produced at a particle accelerator offer the greatest control over the neutrinos being studied. Many experiments have taken place which study the same neutrino oscillations which take place in atmospheric neutrino oscillation, using neutrinos with a few GeV of energy and several hundred km baselines. The MINOS experiment recently announced that it observes consistency with the results of the K2K and Super-K experiments

2.5.7 Reactor neutrino oscillation

Many experiments have searched for oscillation of electron anti-neutrinos produced at nuclear reactors. A high precision observation of reactor neutrino oscillation has been made by the KamLAND experiment since 2002. Neutrinos produced in nuclear reactors have energies similar to solar neutrinos, a few MeV. The baselines of these experiments have ranged from tens of meters to over 100 km.

2.5.8 Supernova Neutrinos

The source of extraterrestrial neutrinos was observed during 10 seconds in 1987 when a star in the Large Magellanic Cloud exploded as a supernova, which was later named SN1987. The neutrinos from the inner part of the collapse reached the earth after a journey of 170,000 years, a few hours before the arrival of light. The neutrinos were able to travel more or less directly from the central collapse in the inner part of the star, but the effect of the explosion was not visible at the star surface until later. About 25 neutrino interactions were observed by the detectors at Kamiokande (Japan), Baksan (Soviet union) and IMB (USA) during 10 seconds. This observation of neutrinos from the sun and the supernova represented a new kind of astronomy since the neutrinos give us information from processes deep inside objects hidden from visible light or photons in general.

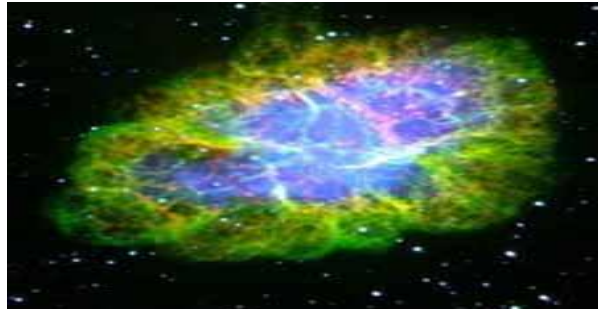


Fig 2.2: Supernova Explosion

2.6 TYPES OF NEUTRINO OSCILLATIONS

Neutrino oscillation can be of different types like solar neutrinos, atmospheric neutrinos etc.

2.6.1 SOLAR NEUTRINO

Solar neutrinos are somewhat more complicated because of the matter effects that the neutrinos experience from the production region until they exit the sun. As the neutrino propagates in matter, the electron neutrino plays a special role due to the forward scattering of the electron neutrino on the electrons in the matter coming from W-boson exchange, i.e. the charged current interaction.

EXPERIMENTS FOR SOLAR NEUTRINOS

ABBREVIATION	FULL NAME	TYPE	TYPE OF DETECTOR	LOCATION	OPERATIONS
GALLEX	Gallium Experiment	ν_e	Radiochemical	Gran Sasso, Italy	1991-1997
HOMESTAKE-CHLORINE	Homestake – chlorine Experiment	ν_e	Radiochemical	Homestake Mine, South Dakota	1967-1998
SNO	Sudbury Neutrino Observatory	ν_e, ν_μ, ν_τ	Cherenkov	Creighton Mine Ontario	1999-2006
BOREXINO	Boron Experiment	ν_e	Scintillation	Gran Sasso, Italy	May 2007

DOUBLE CHOOZ	Double Chooz Reactor Neutrino	ν_e	Scintillation	Chooz France	2011
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Table 2.1: Experiments For Solar Neutrinos

2.6.2 ATMOSPHERIC NEUTRINOS

A cosmic ray is a radiation of high energy particles arriving at the Earth from the Universe. These cosmic ray particles are mostly high energy protons, about 5% are helium nuclei and a still smaller fraction of heavier nuclei. The energy spectrum of these particles extends to very high energies, although the flux of these particles decreases rapidly with increasing energy. These particles, once they enter into the Earth's atmosphere, interact with the nuclei (most of which are nitrogen or oxygen) in high altitude atmosphere. In these high energy nuclear interactions, many π mesons, and less abundantly K mesons, are produced. Since these mesons are unstable, they decay into other particles. For example, a π^+ decays into a muon (μ^+) and a ν_μ . The produced muon (μ^+) is also unstable and decays into a positron (e^+), an anti- ν_μ and a ν_e . A similar decay process occurs for π^- and K mesons. In this manner, neutrinos are produced when a cosmic ray particle enters the atmosphere. These neutrinos are called atmospheric neutrinos.

EXPERIMENTS FOR ATMOSPHERIC NEUTRINOS

ABBREVIATION	FULL NAME	TYPE	TYPE OF DETECTOR	LOCATION	OPERATION
KAMIOKANDE	Kamioka nucleon Decay Experiment	ν_e	Cherenkov	Kamioka, Japan	1986-1995
NEVOD	Cherenkov water detector NEVOD	ν_μ	Cherenkov	Moscow, Russia	1993
SUPER-K	Super-kamiokande	ν_e, ν_μ	Cherenkov	Kamioka, Japan	1996
MINOS	Main Injector Neutrino Oscillation Search	ν_e, ν_μ	Scintillation	Illinois and Minnesota, United States	2005
ICE CUBE	Ice cube Neutrino Detector	ν_e, ν_μ, ν_τ	Cherenkov	South Pole, Antarctica	2006

Table 2.2: Experiments For Atmospheric Neutrinos

2.6.3 SOLAR NEUTRINO PROBLEM

The Sun is fuelled by nuclear fusion reactions. In stars like our Sun, protons (or hydrogen nuclei) are fused into alpha-particles (or helium nuclei). Neutrinos rarely interact with anything and so these neutrinos escape the Sun, and we are able to detect some of them on the Earth. The light produced from fusion in the core of the Sun is unable to escape directly, therefore the light we receive only provides us with information about the outer regions of the Sun. Neutrinos therefore allow us to 'see' into the Sun's core.

In 1968 John Bahcall and Raymond Davis, Jr. attempted to prove that the Sun generated its energy via nuclear fusion of protons. They did so by trying to detect some of the neutrinos emitted by the Sun (so called, solar neutrinos). To detect the neutrinos they built a large tank in a mine in Lead, South Dakota, USA and filled it with cleaning fluid (C_2Cl_4). They then measured the Ar^{37} produced by interactions between the electron-neutrinos and the chlorine.

They did indeed detect the solar neutrinos, and thus prove that the Sun was powered by nuclear fusion. However they detected only around one third of the neutrinos they expected to. The discrepancy was so large that it couldn't be explained as a result of the error on the experiment. The Solar Neutrino Problem or The Mystery of the Missing Neutrinos was born.

2.7 NEUTRINO DETECTORS

A neutrino detector is a physics apparatus designed to study neutrinos. Because neutrinos are only weakly interacting with other particles of matter, neutrino detectors must be very large in order to detect a significant number of neutrinos. Neutrino detectors are often built underground to isolate the detector from cosmic rays and other background radiation. The field of neutrino astronomy is still very much in its infancy – the only confirmed extraterrestrial sources so far are the Sun and supernova SN1987A. Neutrino observatories will "give astronomers fresh eyes with which to study the universe."

Various detection methods have been used. Super Kamiokande is a large volume of water surrounded by phototubes that watch for the Cherenkov radiation emitted when an incoming neutrino creates an electron or muon in the water. The Sudbury Neutrino Observatory is similar, but uses heavy water as the detecting medium. Other detectors have consisted of large volumes of chlorine or gallium which are periodically checked for excesses of argon or germanium, respectively, which are created by neutrinos interacting with the original substance. MINOS uses a solid plastic scintillator watched by phototubes, Borexino uses a liquid pseudocumenes scintillator also watched by phototubes while the proposed NOvA detector will use liquid scintillator watched by avalanche photodiodes.

2.7.1 THREE FAMILIES OF DETECTORS

Detectors for solar neutrinos:

Solar neutrinos have an energy between 0 and 20 MeV, depending of the type of solar nuclear reaction they come. Underground, undersea or under the ice, the detectors made for them detect either the Cerenkov light emitted when a neutrino interact with the water (like Kamiokande or Super-Kamiokande) either the transformation of atoms under neutrino interaction, the remaining atom being radioactive: Chlorine 37 coming from Argon in the Homestake experiment, or Germanium 71 coming from Gallium like in GALLEX experiment.

Detectors near nuclear plants:

The anti-neutrinos coming out of nuclear reactors are emitted in great quantity and have a mean energy of 4 MeV. The neutrino detector uses the inverse beta decay reaction (anti-neutrino + proton --> neutron + anti-electron) to detect anti-neutrinos. It detects the photons emitted when the neutron is absorbed by matter and when the anti-electron coming from the neutrino interaction annihilates with an electron of matter. This detection type was used by Reines et Cowan experiment for the first detection of neutrino in 1956, by BUGEY, by CHOOZ, etc..

Detectors with neutrino beam:

Nowadays, neutrinos generated by accelerators have energy of some 10 MeV to some 100 GeV. The detectors in this case identify the particles coming out of the high energy neutrino interaction with a proton, a neutron or an electron of the detector matter. The neutrino beams are produced using a proton beam coming from an accelerator and sent against a Beryllium target, then filtered through a great amount of dense matter (lead, concrete, iron, earth...). This detection type was used by the Brookhaven experiment which discovered the ν_{μ} neutrino in 1962, by CHARM II experiment in 1974, by NOMAD or CHORUS experiments in 1995, etc...

2.8 DETECTORS RUNNING

2.8.1 MACRO Detector

In this detector they perform an indirect search for weakly interacting massive particles (WIMPs) using the MACRO detector to look for neutrino-induced upward-going muons resulting from the annihilation of WIMPs trapped in the Sun and Earth. The search is conducted in various angular cones centered on the Sun and Earth to accommodate a range of WIMP masses. No significant excess over the background from atmospheric neutrinos is seen. They set experimental flux limits on the upward-going muon fluxes from the Sun and the Earth. These limits are used to constrain neutralino particle parameters from supersymmetric theory, including those suggested by recent results from DAMA-NaI.

2.8.2 MINOS Detectors

The MINOS experiment started detecting neutrinos from the NuMI beam in February 2005. On 30 March 2006, the MINOS collaboration announced that the analysis of the initial data, collected in 2005, is consistent with neutrino oscillations, with the oscillation parameters

which are consistent with Super-K measurements. Our data are consistent with quantum-mechanical oscillations of neutrino flavor with mass splitting

$$|\Delta m^2| = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2 \text{ (68\% C.L.)}$$

And mixing angle

$$\sin^2(2\theta) > 0.90 \text{ (90\% C.L.)}$$

Our data disfavor two alternative explanations for the disappearance of neutrinos in flight: namely, neutrino decays into lighter particles and quantum decoherence of neutrinos, at the 3.7 and 5.7 standard-deviation levels, respectively.

2.8.3 Neutrino beams from muon storage rings

High-intensity high-energy neutrino beams could be produced by exploiting a very intense future muon source, and allowing the muons to decay in a storage ring containing a long straight section. Taking the parameters of muon source designs that are currently under study, the characteristics of the neutrino beams that could be produced are discussed and some examples of their physics potential given. It is shown that the neutrino and antineutrino beam intensities may be sufficient to produce hundreds of charged current interactions per year in a detector on the far side of the Earth.

2.8.4 LSND Experiment

Dec. 4, 2001 -- A collaboration of university scientists and researchers working at Los Alamos National Laboratory has published the final paper from the Liquid Scintillator Neutrino Detector (LSND) experiment. The results, based on six years of data collection, strengthen previously published, but controversial LSND results and provide further evidence of neutrino oscillation and mass. A search for $\nu_\mu \rightarrow \nu_e$ oscillations has been conducted with the LSND apparatus using ν_μ from π^+ decay in flight. Two analyses observe a total of 40 beam-on high-energy (60–200 MeV) electron events consistent with the $\nu_e \rightarrow e^+ X$ inclusive reaction. This number is significantly above the 21.9 ± 2.1 events expected from the ν_e contamination in the beam and the beam-off background. If interpreted as an oscillation signal, the observed oscillation probability of $(2.6 \pm 1.0 \pm 0.5) \times 10^{-3}$ is consistent with the previously reported $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation evidence from LSND.

2.8.5 Fréjus Experiment

The Fréjus nucleon-decay detector has been operated between 19-02-1984 and 13-09-1988. A search for neutrino oscillations is performed. Atmospheric neutrino interactions have been studied with the Fréjus proton decay detector using a total fiducial sensitivity of 1.56 kT yr. The atmospheric neutrino interaction sample has been compared to a Monte Carlo simulation which includes neutrino oscillations.

The $\nu_e (\bar{\nu}_e) \varepsilon / \nu_\mu, (\bar{\nu}_\mu), \nu_e (\bar{\nu}_e) \varepsilon / \nu_\tau, \nu_\mu (\bar{\nu}_\mu) \varepsilon / \nu_\tau (\bar{\nu}_\tau)$ oscillations channels have been studied using the ratio of electron to muon charged current events. Three independent analyses have been performed and no evidence for neutrino oscillations has been found.

2.9 Interactions Of Neutrinos

Neutrinos interactions are as follows:

- Neutral current interaction.
- Charge current interaction.
- Leptonic interaction.

Neutral Current Interaction:

Neutral current nucleon elastic scattering occurs when a neutrino transfers momentum to a nucleon via a Z^0 boson: $\nu N \rightarrow \nu N$. This interaction has been used to investigate weak interactions, probe nucleon structure, study neutrino oscillations, and has been proposed to measure the spectrum of neutrinos from supernovae. In neutral weak interaction, these couplings are given in terms of the electromagnetic and weak couplings by the electroweak unification theory. Neutral current interaction is sensitive to all three flavors with equal cross section.

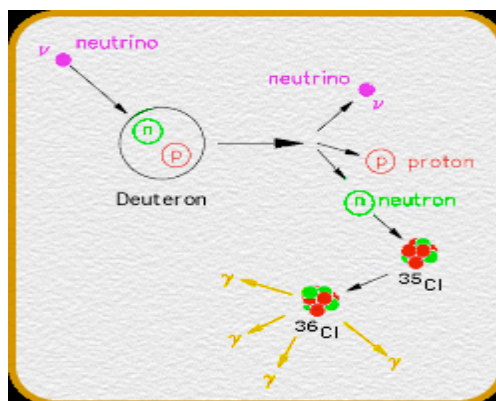
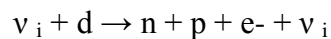
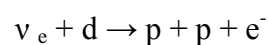


Fig 2.3 :Neutral Current Interaction.

Charge Current Interaction:

The charge current interaction operates only between the electron neutrinos and electrons in matter. If we follow the neutrino line from left lower corner before scattering to the right upper corner after scattering, during that time it propagates through virtual massive particle W boson.



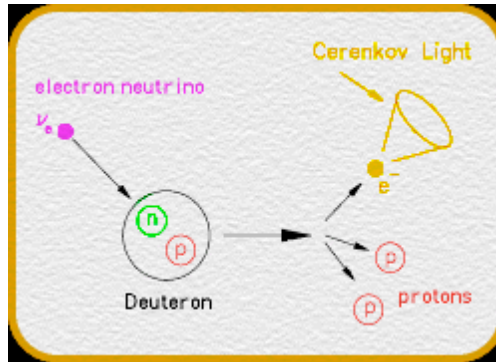


Fig 2.4:Charge Current Interaction.

Electronic Interaction:

It is helpful to start with the scattering of neutrinos from effectively massless pointlike fermions, such as neutrino-electron scattering. Although this interaction is of limited practical interest for accelerator oscillation experiments, the calculation of pointlike scattering will serve multiple purposes as we begin to explore more complicated cross section phenomenology. Sensitive to all three neutrino flavors, but ν_e sensitivity dominates by a factor of 6.

$$\nu_i + e^- \rightarrow \nu_i + e^-$$

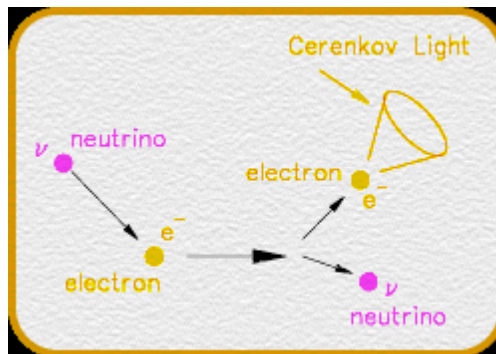


Fig 2.5:Electronic Interaction.

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3 Neutrino Oscillations in Matter and the Importance of θ_{13}

The neutrino is an elementary particle. The neutrino is a particle with no electric charge and which only interacts with matter via the weak nuclear force. In recent years it has been discovered that neutrinos have a small mass, debunking the earlier assumption that it was massless.

Neutrinos are of three types

NEUTRINOS	ν_e	ν_μ	ν_τ
CHARGED PARTNERS	electron (e)	muon (μ)	tau (τ)

Table 3.1 Neutrino Particles.

When a neutrino interacts with matter, it can either continue as a neutrino after the interaction ("neutral current interaction") or create the corresponding charged particle ("charge current interaction"). The electron neutrino creates an electron, the muon neutrino a muon, and the tau neutrino a tau lepton.

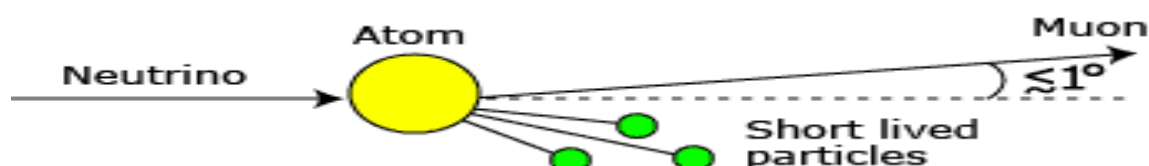


Fig 3.1 : A muon neutrino interacting with an atom producing a muon and a shower of short-lived particles.

They proceed in high-energy collisions and travelling essentially at the speed of light, and unaffected by magnetic fields, neutrinos meet the basic requirements for astronomy.

Their unique advantage arises from a fundamental property: they are affected only by the weakest of nature's forces (but for gravity) and are therefore essentially unabsorbed as they travel from atmosphere to human body. Since neutrinos has mass and mixed with each other.

Oscillations between neutrinos can represented easily with the help of mass eigenstates of neutrinos that are ν_1, ν_2 and ν_3 . The three flavour of neutrinos has three flavour eigenstates i.e. ν_e, ν_μ and ν_τ .

3.1 Neutrino Mixing

Assuming the mixing terms are relatively small as in the quark sector, then each of the flavor eigenstates nearly overlaps with one of the mass eigenstates and only involves minor quantities of the others. We can associate ν_e predominantly with ν_1 , ν_μ with ν_2 and ν_τ with ν_3 . So when we speak about the "mass" of the electron neutrino, we're really speaking about the mass of ν_1 and the same applies to the muon and tau neutrino "masses". In particle physics the symmetries have great importance. The symmetry SU(2) is very important group. It makes only doublet's. The symmetry SU(3) makes the triplet group.

The symmetry SU(2) doublet's are like:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\begin{aligned} |\nu_e(t=0)\rangle &= |\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \\ |\nu_\mu(t=0)\rangle &= |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \end{aligned}$$

3.2 Theory Of Neutrino Flavors

Neutrino oscillations has been discovered by various scientists, experimentally and theoretically with different approaches like classical treatment to quantum mechanical treatment of wave packet and then in theory. Neutrinos has different flavour eigen states i.e. ν_e, ν_μ, ν_τ with their mass eigen states ν_1, ν_2, ν_3 with their respective masses m_1, m_2, m_3 . Neutrinos has momentum P. The mixing matrix U relates the flavor eigen states and mass eigen states.

In quantum field theory , Schrodinger equation for the dependence of the flavor of states is:

$$i \frac{\partial |\Psi(t)\rangle}{\partial t} = H |\Psi(t)\rangle \quad (3.1)$$

where H is the total Hamiltonian.

The general solution for equation (3.1) is:

$$|\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle \quad (3.2)$$

At $t=0$, which is initial time of the state i.e. $|\Psi(0)\rangle$

We will apply now this general formalism to a neutrino beam. The initial state is the state of the flavor neutrino ν_l ($l = e, \mu, \tau$) and mass eigen states can be related by unitary matrix U as:

$$|\nu_l\rangle = \sum_{i=1}^3 U_{li}^* |\nu_i\rangle \quad (3.3)$$

As per the above equation we have

$$|\Psi(0)\rangle = |\nu_l\rangle$$

Taking into account of Hamiltonian operator as following:

$$H|\nu_i\rangle = E_i|\nu_i\rangle$$

Where, we know

$$E_i = \sqrt{p_i^2 + m_i^2}$$

Energy in neutrino states are relate with their momentum p with $p_i=p$ and mass m_i with $m_i^2/p^2 \ll 1$.

$$E_i \cong p + m_i^2 / 2p$$

Where energy difference between two neutrino flavor states will be:

$$E_i - E_j = \Delta m_{ji}^2 / 2p \quad (3.4)$$

Where $\Delta m_{ji}^2 = m_i^2 - m_j^2$

As $E \approx p$,

$$\therefore E_i - E_j = \frac{\Delta m_{ji}^2}{2E} \quad (3.5)$$

The left-handed neutrino at the time $t \geq 0$, is:

$$|\nu_l\rangle_t = e^{-iHt} |\nu_l\rangle = \sum_{i=1}^3 e^{-iE_i t} U_{li}^* |\nu_i\rangle \quad (3.6)$$

Similarly for the right handed anti- neutrinos at the time $t > 0$:

$$|\bar{\nu}_l\rangle_t = e^{-iHt} |\bar{\nu}_l\rangle = \sum_{i=1}^3 e^{-iE_i t} U_{li} |\bar{\nu}_i\rangle \quad (3.7)$$

The neutrinos flavor oscillation gives the amplitude of transition $\nu_l \rightarrow \nu_{l'}$ during the time t and with different energies E_1, E_2, E_3 at time t is written as:

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_{i=1}^3 U_{il'} e^{-iE_i t} U_{li}^* \quad (3.8)$$

For the anti-neutrinos, the amplitude of transition $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ during the time t is:

$$A(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) = \sum_{i=1}^3 U_{il'}^* e^{-iE_i t} U_{li} \quad (3.9)$$

Probability for the neutrinos will be:

$$P(\nu_l \rightarrow \nu_{l'}) = |\langle \nu_{l'} | \nu(t) \rangle|^2$$

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{i=1}^3 U_{il'} e^{-iE_i t} U_{li}^* \right|^2 \quad (3.10)$$

Similarly for anti- neutrinos will be:

$$P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) = \left| \sum_{i=1}^3 U_{il'}^* e^{-iE_i t} U_{li} \right|^2 \quad (3.11)$$

Another way to find the probability with amplitude is:

$$\langle \nu_l | e^{-iHt} | \nu_{l'} \rangle = \sum_{i,i'} U_{l,i'} \langle \nu_l | e^{-iHt} | \nu_{l'} \rangle U_{l'i}^* = \sum_{i=1}^3 U_{l'i} e^{-iEt} U_{li}^* \equiv A(\nu_l \rightarrow \nu_{l'})$$

We assume that t is the time in which neutrino from the production time to detection time will be detected with length L from source to detector.

$$t \cong L$$

$$(E_i - E_j)t = \frac{\Delta m_{ji}^2 L}{2E} \quad (3.12)$$

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{i=1}^3 U_{l'i} e^{-i \frac{\Delta m_{ji}^2 L}{2E}} U_{li}^* \right|^2 \quad (3.13)$$

We know that

$$\sum_i U_{l'i} U_{li}^* = \delta_{ll'}$$

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \delta_{ll'} + \sum_{i \neq j} U_{il'} (e^{-i \frac{\Delta m_{ji}^2 L}{2E}} - 1) U_{li}^* \right|^2 \quad (3.14)$$

The transition probability $\nu_l \rightarrow \nu_{l'}$ is given as:

$$P(\nu_l \rightarrow \nu_{l'}) = \sum_{i,k} U_{l'i} U_{l'k}^* U_{li}^* U_{lk} e^{-i \frac{\Delta m_{ji}^2 L}{2E}} \quad (3.15)$$

$$\begin{aligned} &= \sum_i |U_{l'i}|^2 |U_{li}|^2 + 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk} e^{-i \frac{\Delta m_{ji}^2 L}{2E}}) \\ &= \sum_i |U_{l'i}|^2 |U_{li}|^2 = \delta_{ll'} - 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk}) \\ &= \delta_{ll'} - 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk} + 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk} e^{-i \frac{\Delta m_{ji}^2 L}{2E}}) \\ &= \delta_{ll'} - 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk} (1 - e^{-i \frac{\Delta m_{ki}^2 L}{2E}})) \end{aligned} \quad (3.16)$$

An mathematical identity for any complex a and b , $\operatorname{Re}(ab) = \operatorname{Re}(a)\operatorname{Re}(b) - \operatorname{Im}(a)\operatorname{Im}(b)$

$$\begin{aligned} P(\nu_l \rightarrow \nu_{l'}) &= \delta_{ll'} - 2 \operatorname{Re} \sum_{i>k} (U_{l'i} U_{l'k}^* U_{li}^* U_{lk}) (1 - \cos \frac{\Delta m_{ki}^2 L}{2E}) + 2 \\ &\sum_{i>k} \operatorname{Im}(U_{l'i} U_{l'k}^* U_{li}^* U_{lk}) \sin \frac{\Delta m_{ki}^2 L}{2E} \end{aligned} \quad (3.17)$$

3.2.1 Two flavor oscillation probability in vacuum:

In the two flavor neutrino oscillation case, it has two flavor eigen states ν_e and ν_μ and it has two mass eigen states ν_1 and ν_2 with respective masses m_1 and m_2 . Both the neutrinos has momentum P . The mixing matrix U relate both the flavor eigen state and mass eigen state. Where as a matrix equation can be written as :

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

And

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\begin{aligned} |\nu_e(t=0)\rangle &= |\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \\ |\nu_\mu(t=0)\rangle &= |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \end{aligned}$$

At time $t=t$,

$$\begin{aligned} |\nu_\mu(t=t)\rangle &= |\nu_\mu(t)\rangle = -\sin \theta |\nu_1\rangle e^{-\frac{iE_1 t}{\hbar}} + \cos \theta |\nu_2\rangle e^{-\frac{iE_2 t}{\hbar}} \\ &= -\sin \theta |\nu_1\rangle e^{-\frac{-i(p+\frac{m_1^2}{2p})t}{\hbar}} + \cos \theta |\nu_2\rangle e^{-\frac{-i(p+\frac{m_2^2}{2p})t}{\hbar}} \end{aligned}$$

Where

$$\begin{aligned} E_1 &= (p^2 + m_1^2)^{1/2} \quad \text{and} \quad E_2 = (p^2 + m_2^2)^{1/2} \\ P(\nu_e \rightarrow \nu_\mu) &= |\langle \nu_\mu | \nu_e(t) \rangle|^2 \\ \langle \nu_\mu | &= \cos \theta \langle \nu_1 | + \sin \theta \langle \nu_2 | \\ P(\nu_e \rightarrow \nu_\mu) &= |\langle \nu_\mu | \nu_e(t) \rangle|^2 \\ &= e^{iz - iz} \sin^2 \theta \cos^2 \theta (1 - e^{\frac{i\Delta m^2}{2p}x}) (1 - e^{-\frac{i\Delta m^2}{2p}x}) \end{aligned}$$

For neutrinos to be relativistic, the substitution $p = E_\nu$ and $x = L$ can be made:

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= \sin^2 \theta \cos^2 \theta (1 - e^{\frac{i\Delta m^2}{2E_\nu}L}) (1 - e^{-\frac{i\Delta m^2}{2E_\nu}L}) \\ &= (4\sin^2 \theta \cos^2 \theta / 4) (e^{\frac{i\Delta m^2}{2E_\nu}L} - 1) (e^{-\frac{i\Delta m^2}{2E_\nu}L} - 1) \\ &= (\sin^2 2\theta / 4) (1 - e^{\frac{i\Delta m^2}{2E_\nu}L} - e^{-\frac{i\Delta m^2}{2E_\nu}L} + 1) \\ &= (\sin^2 2\theta / 4) (2 - e^{\frac{i\Delta m^2}{2E_\nu}L} - e^{-\frac{i\Delta m^2}{2E_\nu}L}) \end{aligned}$$

We know $\cos \theta = e^{i\theta} - e^{-i\theta} / 2$

$$\begin{aligned} \therefore P(\nu_e \rightarrow \nu_\mu) &= (\sin^2 2\theta / 4) (2 - 2\cos \frac{\Delta m_{ji}^2}{2E} L) \\ &= \sin^2 2\theta / 2 (1 - \cos \frac{\Delta m_{ji}^2}{2E} L) \\ &= \sin^2 2\theta (1 - \cos 2 \frac{\Delta m_{ji}^2}{4E} L) \\ &= \sin^2 2\theta \sin^2 (\frac{\Delta m_{ji}^2}{4E} L) \end{aligned}$$

For experimental convenience, we using the dimensionless quantities with appropriate vales of \hbar 's and c 's to get the value of probability in two flavor eigen states.

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 (\frac{\Delta m_{ji}^2}{4E} \frac{c^4}{\hbar c} L)$$

Where \hbar is Planck constant .

And c is velocity of light.

We get

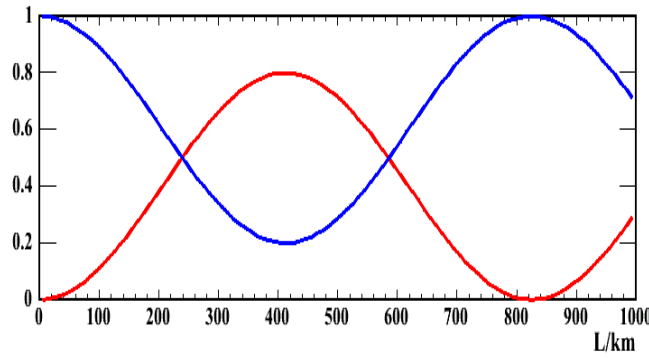
$$P(\nu_e \rightarrow \nu_\mu)(L,E) = \sin^2 2\theta \sin^2\left(1.27 \Delta m^2 \frac{L}{E_\nu}\right) \quad (3.19)$$

The corresponding two flavor survival probability is:

$$P(\nu_e \rightarrow \nu_e)(L,E) = 1 - \sin^2 2\theta \sin^2\left(1.27 \Delta m^2 \frac{L}{E_\nu}\right) \quad (3.20)$$

With different mixing angles between mass eigen states θ and length with short or long baselines and having energy E produced by source measured with different values.

Let us take $\Delta m=0.003\text{eV}^2$, $\sin^2 2\theta=0.8$ and $E_\nu=1\text{ GeV}$



Plot3.1: Two flavor survival probability of electron to electrons neutrinos.

3.2.2 Three Flavor Probability Oscillations in Vacuum:

In case of three flavor neutrino oscillations, three flavor eigen states and three mass states are taken and related through unitary matrix U . These three flavor and mass eigen states are written as follows:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

The unitary 3×3 mixing matrix for neutrinos and introduce the parameters i.e. three mixing angles and one phase, which characterize it as:

$$\begin{aligned} |i\rangle & \quad (i = 1, 2, 3) \\ \langle i|k\rangle & = \delta_{ik} \end{aligned}$$

The first Euler rotation experimented at the angle θ_{12} around the vector $|3\rangle$ produces new orthogonal and normalized vectors as:

$$\begin{aligned} |1\rangle^{(1)} & = c_{12} |1\rangle + s_{12} |2\rangle \\ |2\rangle^{(1)} & = -s_{12} |1\rangle + c_{12} |2\rangle \end{aligned}$$

$$|3\rangle^{(1)} = |3\rangle$$

Here $c_{12} = \cos \theta_{12}$ and $s_{12} = \sin \theta_{12}$,

$$|\nu\rangle^{(1)} = U^{(1)} |\nu\rangle$$

Where

$$|\nu\rangle^{(1)} = \begin{pmatrix} |1^{(1)}\rangle \\ |2^{(1)}\rangle \\ |3^{(1)}\rangle \end{pmatrix} \quad \text{and} \quad |\nu\rangle = \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix}$$

$$U^{(1)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Second rotation is at the angle θ_{13} around vector the $|2\rangle^{(1)}$, which introduces the CP phase δ ,

$$|1\rangle^{(2)} = c_{13}|1\rangle^{(1)} + s_{13}e^{-i\delta}|3\rangle^{(1)}$$

$$|2\rangle^{(2)} = |2\rangle^{(1)}$$

$$|3\rangle^{(2)} = -s_{13}e^{-i\delta}|1\rangle^{(1)} + c_{13}|3\rangle^{(1)}$$

In the matrix $|\nu\rangle^{(2)} = U^{(2)}|\nu\rangle^{(1)}$

where

$$U^{(2)} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}$$

Similarly rotation around vector $|1\rangle^{(2)}$ at the angle θ_2

$$|1\rangle^{\text{mix}} = |1\rangle^{(2)}$$

$$|2\rangle^{\text{mix}} = c_{23}|2\rangle^{(2)} + s_{23}|3\rangle^{(2)}$$

$$|3\rangle^{\text{mix}} = -s_{23}|2\rangle^{(2)} + c_{23}|3\rangle^{(2)}$$

$$|\nu^{\text{mix}}\rangle = U^{(3)}|\nu\rangle^{(2)}$$

$$U^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$|\nu^{\text{mix}}\rangle = U|\nu\rangle \quad \text{Where } U = U^{(3)}U^{(2)}U^{(1)}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{12}c_{23}e^{i\delta} & c_{12}s_{23} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

The CP violation is effected by the phase δ which can take values from 0 to 2π . The mixing angles are parameters for three neutrino oscillations, in which all real parts of the quadratic products of elements of the mixing matrix present in the three-neutrino oscillation probabilities is given as: $\text{Re}(U_{l'i} U_{1'k}^* U_{li}^* U_{lk})$. It is shown below that probability for neutrino oscillations depend on three mixing angle and two mass squared difference:

$$\text{Re}(U_{l'i} U_{1'k}^* U_{li}^* U_{lk}) = (U_{22} U_{21}^* U_{12}^* U_{11}) \text{ for } l' = 2, l = 1, i = 2, k = 1$$

$$= (c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta}) (-c_{23}s_{12} - s_{13}s_{23}c_{12}e^{-i\delta}) (c_{13}s_{12})(c_{13}c_{12})$$

$$= -\frac{1}{4} c_{13}^2 \sin 2\theta_{12} [\sin 2\theta_{12} (c_{23}^2 - s_{23}^2 s_{13}^2) + \cos 2\theta_{12} \sin 2\theta_{23} s_{13} \cos \delta_{13}]$$

$$\begin{aligned} \text{Re}(U_{1i} U_{1k}^* U_{li}^* U_{lk}) &= (U_{23} U_{22}^* U_{13}^* U_{12}) \\ &= (c_{13} s_{23}) (c_{23} c_{12} - s_{13} s_{23} s_{12} e^{-i\delta}) (s_{13} e^{i\delta}) (c_{13} s_{12}) \\ &= -c_{13}^2 s_{13} s_{12} s_{23} (s_{13} s_{23} s_{12} - c_{23} c_{12} \cos \delta_{13}) \end{aligned}$$

$$\begin{aligned} \text{Re}(U_{1i} U_{1k}^* U_{li}^* U_{lk}) &= (U_{23} U_{21}^* U_{13}^* U_{11}) \\ &= (c_{13} s_{23}) (-c_{23} s_{12} - s_{13} s_{23} c_{12} e^{-i\delta}) (s_{13} e^{i\delta}) (c_{13} c_{12}) \\ &= -c_{13}^2 s_{13} c_{12} s_{23} (c_{23} s_{12} e^{i\delta} + s_{13} s_{23} c_{12}) \end{aligned}$$

Probability Oscillation of neutrino flavors are like:

Probability	i, k	$Re(U_{li} U_{lk}^* U_{li}^* U_{lk})$
$P(\nu_e \rightarrow \nu_{\bar{\mu}})$	i=2,k=1	$-\frac{1}{4} c_{13}^2 \sin 2\theta_{12} [\sin 2\theta_{12} (c_{23}^2 - s_{23}^2 s_{13}^2) + \cos 2\theta_{12} \sin 2\theta_{23} s_{13} \cos \delta_{13}]$
	i=3,k=2	$c_{13}^2 s_{13} s_{12} s_{23} (s_{13} s_{23} s_{12} - c_{23} c_{12} \cos \delta_{13})$
	i=3,k=1	$c_{13}^2 s_{13} c_{12} s_{23} (c_{23} s_{12} e^{i\delta} + s_{13} s_{23} c_{12})$

Table 3.2: Probability Oscillation Neutrino Flavor.

The survival probability oscillations of neutrinos are:

$P(\nu_e \rightarrow \nu_e)$	i=2, k=1	$\frac{1}{4} c_{13}^4 \sin^2 2\theta_{12}$
	i=3, k=2	$\frac{1}{4} s_{12}^2 \sin^2 2\theta_{13}$
	i=3, k=1	$\frac{1}{4} c_{12}^2 \sin^2 2\theta_{13}$

Table 3.3: Survival Probability Oscillations Of Neutrinos.

Characteristic values of L and E for various neutrino sources and experiments and the corresponding ranges of Δm^2 to which they can be most sensitive.

Experiment	L(m)	E(MeV)	$\Delta m^2(\text{eV}^2)$
Solar	10^{10}	1	10^{-10}
Atmospheric	$10^4 - 10^7$	$10^2 - 10^5$	$10^{-1} - 10^{-4}$
Reactor	SBL $10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL $10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL 10^2	$10^3 - 10^4$	>0.1
	LBL $10^5 - 10^6$	10^4	$10^{-2} - 10^{-3}$

Table 3.4: Experimental Ranges Of Parameters.

3.2.3 PARAMETERS EFFECTING NEUTRINO OSCILLATIONS PROBABILITY

The parameters which effect the neutrino oscillations through which the oscillations probability can measure easily. These parameters are

- Length
- Mass
- Mixing Angle
- Energy

i. LENGTH

The distance between source and detector from which the oscillations between neutrinos can measure with its particular energy, mass and mixing angle.

We had defined the oscillation length λ_{osc} :

$$\lambda_{osc} = 4\pi E / \Delta m^2$$

Length can measure with base lines that can be of short base line or long base line. Different experiments do work with different base line.

SHORT BASE LINE EXPERIMENTS

This is the two-detector technique. The full apparatus would consist of 3 modules of identical structure, but for the length. One module (140 planes) would be located at 130 m from the proton target and act as near detector. Two more modules (320 planes each) would be placed in a hall at 880 m from the proton target and act as far detector. The quantity to measure is the ratio of electron-like to muon-like events in the near and far detector.

Typical values of L for LSND are 30 m. This L would be perfectly matched by experiments using neutrino beams produced by the PS at CERN or by the Booster at FNAL. The neutrino beam is produced by the CERN Proto Synchrotron (PS). The energy of the PS proton beam would be 19.2 GeV and the existing 50m long decay tunnel would be used.

LONG BASE LINE NEUTRINO EXPERIMENTS

- K2K-

In this experiment, which has a baseline of 250 km, the beam from KEK is directed to Super-K detector. K2K can, in principle, observe oscillations by the detection of quasi-elastic events. But the flux is too low to give any useful information on μ 13.

- **MINOS**

This is a high-statistics experiment under construction with beam from Fermilab directed to SOUDAN mine, with a baseline of 731 km. The beam is broad band with neutrino energy varying from 1 to 10 GeV.

- **OPERA**

this experiment will be located at Gran Sasso and will receive ν_μ beam from CERN, which corresponds to a baseline of 730 km. OPERA is an emulsion experiment which seeks to detect τ s produced in ν_τ interactions. For the best-fit parameters given by atmospheric neutrino analysis, 10 clean events are expected in five years time..

- **ICARUS**

This experiment will be located at Gran Sasso and will receive ν_μ beam from CERN, which corresponds to a baseline of 730 km. ICARUS uses liquid argon as detector material and has the capability to identify both electrons and muons. It can search for $\nu_\mu \rightarrow \nu_\tau$ by observing the semi-leptonic decays (into electrons or muons) of the τ s. Twelve clean τ events are expected in five years time. The good electron identification capability enables it to detect $\nu_\mu \rightarrow \nu_e$ oscillations but the event rates at higher energies are rather small.

- **J2K**

This is a very high statistics experiment that is expected to start taking data in 2009. At the same time the high intensity proton synchrotron at Japan Hadron Facility (JHF) is completed. In this experiment, the high intensity ν_μ beam from JHF is directed to Super-Kamiokande (SK) detector 295 km away. The neutrino flux is about 100 times the flux of beam from KEK. The number of ν_μ charged current events expected, in the case of no oscillations, is about 3100 per year.

- **BNL to Homestake**

This recently proposed experiment is capable of achieving all the goals of long baseline neutrino experiments. In this experiment ν_μ beam from Brookhaven is to be aimed at 0.5 megaton water Cerenkov detector located at Homestake or some other suitable site with a baseline of about 2500 km. This proposal is a realization of the super beam experiments proposed by Richter [9] with very long baselines and very high statistics. The expected number of events in case of no oscillations is about 3000 per year.

- ii. **MASS**

The neutrinos are sensitive to the mass square difference therefore m_1^2 shows the mass of electron neutrino and m_2^2 shows the mass of muon neutrino and m_3^2 shows the mass of tau

neutrino. The two independent mass square differences: $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{31}^2 = m_3^2 - m_1^2$. WLOG, we take Δm_{21}^2 to be the smaller mass square difference (known as the solar mass squared difference, since it governs the oscillation of solar neutrinos) and Δm_{31}^2 to be the larger mass squared difference (known as the atmospheric mass squared difference, since it governs the oscillations of atmospheric neutrinos). Solar neutrino data require Δm_{21}^2 to be positive. Data from atmospheric neutrino as well as accelerator neutrino experiment constrains only magnitude of m_{31}^2 but not its sign. Determination of sign (Δm_{31}^2) is also called mass hierarchy determination in the limit where the lightest mass eigenstates is essentially mass less.

If sign of (Δm_{31}^2) is positive

$$\Delta m_{31}^2 > 0 \dots \dots \dots \text{Normal Mass Hierarchy}$$

then we have following pattern $m_3 \gg m_2 \gg m_1$, this is referred to as Normal hierarchy.

If sign (Δm_{31}^2) is negative

$$\Delta m_{31}^2 < 0 \dots \dots \dots \text{Inverted Mass Hierarchy}$$

the mass pattern is $m_2 \geq m_1 \gg m_3$. This is referred to as Inverted hierarchy.

HIERARCHY

The neutrino mass hierarchy can be determined, in principle, by measuring a phase in the disappearance oscillation probability in vacuum, without relying on the matter effect, using a single channel. This phase is not the same for the normal and inverted neutrino mass spectra. The key feature of the method is to detect advancement (normal) or retardation (inverted) of the phase of the atmospheric-scale oscillation relative to the solar-scale oscillation.

It was recently realized that three-flavor effects could peculiarly modify the development of spectral splits induced by collective oscillations, for supernova neutrinos emitted during the cooling phase of a protoneutron star. It systematically explore this case, explaining how the impact of these three-flavor effects depends on the ordering of the neutrino masses. In inverted mass hierarchy, the solar mass splitting gives rise to instabilities in regions of the (anti)neutrino energy spectra that were otherwise stable under the leading two-flavor evolution governed by the atmospheric mass splitting and by the 1–3 mixing angle. As a consequence, the high-energy spectral splits found in the electron (anti)neutrino spectra disappear, and are transferred to other flavors. Imperfect adiabaticity leads to smearing of spectral swap features. In normal mass hierarchy, the three-flavor and the two-flavor instabilities act in the same region of the neutrino energy spectrum, leading to only minor departures from the two-flavor treatment

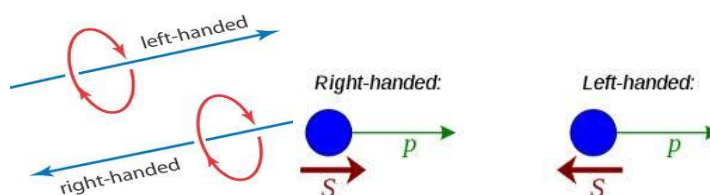


Fig 3.2 Neutrino's Helicity

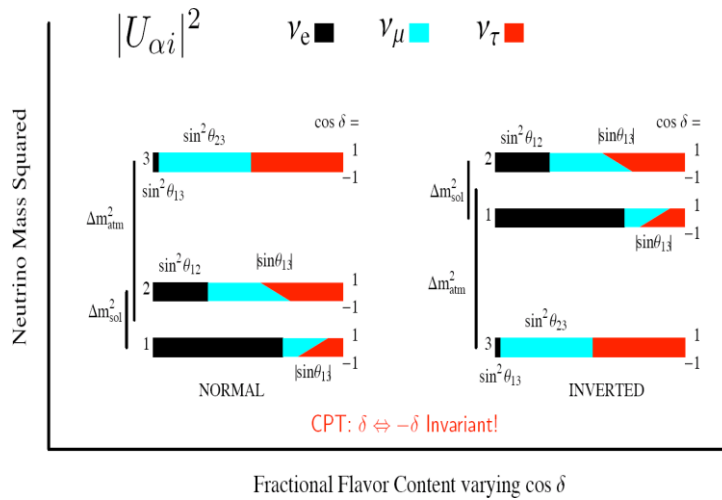


Fig 3.3 Mass Hierarchy

iii. MIXING ANGLES

The mixing of three neutrinos is parameterised by three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$. The two angles that have been measured are big, and the third is very small. This is a big surprise, because with quarks the corresponding angles are all small.

The big angles correspond to strong mixing and hint that CP violation by neutrinos might explain the universe's imbalance of matter and antimatter. If we measure a nonzero θ_{13} , it will tell us something about all of these things, but then we have to sort it out, this is according to Double Chooz. A measurable θ_{13} could be used to optimize the accelerator experiments in the works, NOvA the US and T2K in Japan which themselves can estimate θ_{13} , but not as cleanly as a reactor experiment and to lay out the next steps in neutrino research. But if θ_{13} is ultimately found to be zero, then we cannot extract a CP -violating phase.

Neutrino mixing angles may explain the large values required by solar and atmospheric neutrino oscillations. Implementation of such mechanism in the Standard Model and many of its extensions to amplify the solar angle, the atmospheric or both requires (at least two) quasi-degenerate neutrino masses, but is not always possible. In neutrino mixing angles can easily be driven to large values at low energy as they approach infrared pseudo-fixed points at large mixing. In addition, quasi-degeneracy of neutrino masses is not always required.

If the mixing angle is small, the eigenstates are almost pure flavour eigenstates and vice versa. The flavour eigenstate ν_e generated in the Sun thus breaks up into its two mass eigenstates ν_1 and ν_2 we then follow the latter on their way towards the Earth. We see what role their possible masses, or rather the mass difference, do play during propagation. It should be noted that if a neutrino mass is not zero, different neutrino types will possibly have different masses and on an increasing scale, as is the case for the corresponding charged

particles e , μ and τ . For this reason, any mass difference will be close to the mass of the heaviest neutrino, since the other masses are presumably much smaller, hence negligible.

iv. ENERGY

To suggest that the presence of a quantum-gravity-induced length can be explored by using neutrino oscillation probabilities. Neutrinos seem ideally suited for this investigation because they can propagate freely over large distances and can therefore pile up minimal length effects beyond detectable thresholds. To determine the modified survival probability in a scenario with a minimal length and find deviations from the classical behaviour for high energies. We find that for the currently available experimental statistics, the deviations from the standard oscillations only allow for a bound of $\ell^{-1} \geq 10\text{GeV}$ from MINOS data. On the other hand, oscillations of high-energy neutrinos emitted by galactic and extragalactic sources are strongly suppressed, leading to a possible observation of quantum gravity effects at neutrino telescopes such as IceCube and ANTARES.

The probability formula for two-flavor oscillation is depend on $\sin^2 2\theta$, which governs the strength of the mixing if the two states. And it depends on Δm^2 which is the difference in mass^2 between the two states. Finally, it depends on the ratio of the neutrino energy, E , to the neutrino travel distance, L . High energy neutrinos that travel a short distance (small L/E) are unlikely to change flavors compared to low energy neutrinos that travel a long distance (large L/E).

3.3 Probability Oscillation in Matter

When neutrinos propagate in matter, the interactions with matter affect their properties. These effects can be coherent and incoherent.

In coherent interactions, the medium remains unchanged and it is possible to have interference of scattered and unscattered neutrino waves which enhances the effect. Coherence further allows one to decouple the evolution equation of the neutrinos from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter.

The evolution equation for n ultrarelativistic neutrinos propagating in matter written in the mass basis can be casted in the following form:

$$i \frac{d\vec{v}}{dx} = H \vec{v} \tag{3.3.1}$$

$$H = H_m + U^{\nu} V U^{\nu}$$

Where H_m is the Hamiltonian for the kinetic energy.

$$H_m = 1/2 (\text{diag}(m_1^2, m_2^2, \dots, m_n^2))$$

V is the effective potential that describes the coherent forward interactions of the neutrinos with matter in the interaction basis. U^{ν} is the $n \times n$ submatrix of the unitary V^{ν} matrix corresponding to the n ultrarelativistic neutrino states.

The state $|\psi(t)\rangle$ can be expanded over the total system of states of flavor neutrinos ν_i , with momentum p ,

$$|\psi(t)\rangle = \sum \mathbf{a}_i(t) |\nu_i\rangle \quad (3.3.2)$$

Here

$$|\nu_i\rangle = \sum_i U_{ii}^* |\nu_i\rangle$$

$$\mathbf{H}_0 |\nu_i\rangle = \mathbf{E}_i |\nu_i\rangle,$$

where

$$E_i = \sqrt{p_i^2 + m_i^2} \cong p + \frac{m_i^2}{2E} \quad (3.3.3)$$

and $a_i(t) = \langle \nu_i | \psi(t) \rangle$ is the amplitude of probability to find ν_i in state which is described by $|\psi(t)\rangle$.

\therefore

$$i \frac{\partial \mathbf{a}_i(t)}{\partial t} = \sum_i \langle \nu_i | \mathbf{H}_0 | \mathbf{a}_i(t) \rangle \quad (3.3.4)$$

Where,

$$\langle \nu_i | \nu_j \rangle = U_{ij} \quad (3.3.5)$$

For the free Hamiltonian in the flavor representation we have the following expression:

$$\langle \nu_j | \mathbf{H}_0 | \nu_i \rangle = \sum_l U_{jl} E_l U_{li}^* \cong p + \sum_l U_{jl} \frac{m_l^2}{2E} U_{li}^* \quad (3.3.6)$$

Therefore neutrino evolution equation is:

$$i \frac{\partial \mathbf{a}(t)}{\partial t} = \mathbf{U} \frac{\mathbf{m}^2}{2E} \mathbf{U}^\dagger \mathbf{a}(t) \quad (3.3.7)$$

By introducing a function we can solve above equation:

$$\mathbf{a}'(t) = \mathbf{U}^\dagger \mathbf{a}(t) \quad (3.3.8)$$

With the use of equation (3.3.7), and (3.3.8) following equation can be satisfied as:

$$i \frac{\partial}{\partial t} \mathbf{a}'(t) = \frac{\mathbf{m}^2}{2E} \mathbf{a}'(t) \quad (3.3.9)$$

The solution of equation has the form:

$$\mathbf{a}'(t) = e^{-i \frac{\mathbf{m}^2}{2E} (t-t_0)} \mathbf{a}'(t_0) \quad (3.3.10)$$

Where $\mathbf{a}'(t_0)$ is the wave function at the initial time t_0 .

$$\mathbf{a}(t) = \mathbf{U} e^{-i \frac{\mathbf{m}^2}{2E} (t-t_0)} \mathbf{U}^\dagger \mathbf{a}(t_0). \quad (3.3.11)$$

the initial state $|\Psi(t_0)\rangle$ is the state of the flavor neutrino ν_1

$$|\Psi(t_0)\rangle = |\nu_1\rangle. \quad (3.3.12)$$

$\Delta m_{ii}^2 = m_i^2 - m_1^2$ and $L = (t-t_0)$ which is the distance between the source and detector points.

The most important effect in propagation of neutrinos in matter is coherent forward elastic scattering of neutrinos with which state of matter does not change. Earlier for both normal and inverted hierarchies, Earth matter effects have been studied for long base lines. CC interaction can give contribution to the process of elastic scattering of ν_e on electrons.

Moreover, neutral current interactions occur for all flavors, leads to addition of an extra term in Hamiltonian for flavor oscillation probability. For the low energy, an effective Hamiltonian of the neutrino interaction obtained from the diagonal matrix element.

$$\langle \mathbf{p} \text{mat} | \mathbf{H}_I^{CC} | \mathbf{p} \text{mat} \rangle \quad (3.3.13)$$

Where
$$\mathbf{H}_I^{cc} = \frac{G_F}{\sqrt{2}} 2\bar{\nu}_{eL}(x)\gamma_\alpha \nu_{eL}\bar{e}(x)\gamma^\alpha(1-\gamma_5)e(x) \quad (3.3.14)$$

Where G_F is the weak Fermi coupling constant.

$$G_F = 2 \times \text{energy} \times \text{density}.$$

The dimensions and size of the Fermi constant, which make the weak interaction “weak” at low energies, have their origin in the propagator associated with the exchange of the W boson. In the electroweak theory, G_F can be expressed in terms of M_W and an overall weak coupling constant, g_w , as:

$$G_F = \frac{\sqrt{2}}{8} \left(\frac{g_w}{M_W} \right)^2$$

therefore, g_w is a coupling of O(1) and roughly the same size as the electromagnetic coupling constant in this unified theory of the two interactions.

and the vector
$$| \mathbf{p} \text{mat} \rangle = | \mathbf{p} \rangle | \text{mat} \rangle. \quad (3.3.15)$$

Now substituting the Hamiltonian we obtain:

$$\begin{aligned} & \langle \mathbf{p} \text{mat} | \frac{G_F}{\sqrt{2}} 2\bar{\nu}_{eL}(x)\gamma_\alpha \nu_{eL}\bar{e}(x)\gamma^\alpha(1-\gamma_5)e(x) | \mathbf{p} \text{mat} \rangle \\ &= \langle \mathbf{p} | \langle \text{mat} | \frac{G_F}{\sqrt{2}} 2\bar{\nu}_{eL}(x)\gamma_\alpha \nu_{eL}\bar{e}(x)\gamma^\alpha(1-\gamma_5)e(x) | \mathbf{p} \rangle | \text{mat} \rangle \end{aligned} \quad (3.3.16)$$

But the pseudo vector as for unpolarised matter $\langle \text{mat} | \bar{e}(x)\gamma^\alpha\gamma_5e(x) | \text{mat} \rangle = 0$

Also
$$\langle \text{mat} | \bar{e}(x)\gamma^\alpha e(x) | \mathbf{p} \rangle | \text{mat} \rangle = \langle \text{mat} | \bar{e}(x)e(x) | \mathbf{p} \rangle | \text{mat} \rangle \delta_{\alpha 0} = n_e(x)\delta_{\alpha 0}$$

where $n_e(x)$ is the number density at the point x.

$$\langle \mathbf{p} | \bar{\nu}_{eL}(x)\gamma_\alpha \nu_{eL} | \mathbf{p} \rangle = 1 \quad (3.3.17)$$

Substituting all these values, $\mathbf{H}_I^{mat}(t) = \sqrt{2}G_F n_e(t)\beta$, for the effective CC interaction Hamiltonian of neutrinos in matter. Using $\beta_{\nu_e, \nu_e} = 1$, all other elements of matrix β are equal to zero for ultra relativistic neutrinos.

$$x \sim t$$

Let us now consider the NC interaction. Induced by the Z^0 exchange, the Hamiltonian of NC interactions of neutrinos with electrons and nucleons has the form:

$$H_I^{NC}(x) = 2 \frac{G_F}{\sqrt{2}} \sum \bar{\nu}_{iL}(x)\gamma^\alpha \nu_{iL}(x) j_\alpha^{NC}(x) \quad (3.3.18)$$

Where $j_\alpha^{NC}(x)$ is the sum of electron and nucleon (quark) neutral current. For the vector part of effective hadrons neutral current,

$$v_{\alpha}^{NC(N)}(\mathbf{x}) = \frac{1}{2} \bar{N}(\mathbf{x}) \gamma_{\alpha} \tau_3 N(\mathbf{x}) - 2 \sin^2 \theta_w \bar{\mathbf{p}}(\mathbf{x}) \gamma_{\alpha} \mathbf{p}(\mathbf{x}) \quad \text{As } N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{and } \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.3.19)$$

where θ_w is the weak angle. Similarly from (3.3.19) we can write for the effective part of electron current:

$$v_{\alpha}^{NC(n)}(\mathbf{x}) = \left(-\frac{1}{2}\right) \bar{\mathbf{n}}(\mathbf{x}) \gamma_{\alpha} \mathbf{n}(\mathbf{x}) \quad (3.3.20)$$

$$v_{\alpha}^{NC(e)}(\mathbf{x}) = \left(-\frac{1}{2} + 2 \sin^2 \theta_w\right) \bar{\mathbf{e}}(\mathbf{x}) \gamma_{\alpha} \mathbf{e}(\mathbf{x}) \quad (3.3.21)$$

For the corresponding matter matrix elements we have

$$\langle \mathbf{mat} | v_{\alpha}^{NC(e)}(\mathbf{x}) | \mathbf{mat} \rangle = \left(-\frac{1}{2} + 2 \sin^2 \theta_w\right) \mathbf{n}_e(\mathbf{x}) \delta_{\alpha 0} \quad (3.3.22)$$

$$\langle \mathbf{mat} | v_{\alpha}^{NC(p)}(\mathbf{x}) | \mathbf{mat} \rangle = \left(\frac{1}{2} - 2 \sin^2 \theta_w\right) \rho_p(\mathbf{x}) \delta_{\alpha 0} \quad (3.3.23)$$

And

$$\langle \mathbf{mat} | v_{\alpha}^{NC(n)}(\mathbf{x}) | \mathbf{mat} \rangle = -\frac{1}{2} \rho_n(\mathbf{x}) \delta_{\alpha 0} \quad (3.3.24)$$

For the neutral matter, $\mathbf{n}_e(\mathbf{x}) = \rho_p(\mathbf{x})$, we conclude that the contributions of electron and proton NC to the effective Hamiltonian cancel each other. Thus

$$\langle \mathbf{mat} | j_{\alpha}^{NC}(\mathbf{x}) | \mathbf{mat} \rangle = -\frac{1}{2} \rho_n(\mathbf{x}) \delta_{\alpha 0} \quad (3.3.25)$$

In the three flavor neutrinos only $\nu_e - e$ CC interaction gives a contribution to the effective Hamiltonian. Thus the evolution equation of neutrino has the form:

$$i \frac{\partial \mathbf{a}(t)}{\partial t} = \left(U \frac{m^2}{2E} U^{\dagger} + \sqrt{2} G_F n_e(t) \beta \right) \mathbf{a}(t)$$

Similarly for the antineutrinos effective Hamiltonian differs in sign from the neutrino-electron interactions, thus

$$\bar{H}_I^{mat}(\mathbf{x}) = -\sqrt{2} G_F n_e(t) \beta$$

Here we are more concerned with matter effects of constant density.

The total Hamiltonian of neutrino in matter

$$H = U \frac{m^2}{2E} U^{\dagger} + \sqrt{2} G_F n_e \beta \quad (3.3.26)$$

Where

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

then the total effective Hamiltonian is :

$$H = \frac{1}{2} Tr H + H^m \quad (3.3.27)$$

Here $\frac{1}{2} \text{Tr H} = \frac{m_1^2 + m_2^2}{4E} + \frac{1}{2} \sqrt{2} G_F n_e$

and H^m is the traceless part of Hamiltonian.

$$H^m = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta + A & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A \end{pmatrix}$$

Where

$$A = 2\sqrt{2} G_F n_e E$$

$$H^m = U^m E^m U^{m\dagger} \quad (3.3.28)$$

$$U^m = \begin{pmatrix} \cos \theta^m & \sin \theta^m \\ -\sin \theta^m & \cos \theta^m \end{pmatrix} \quad (3.3.29)$$

And

$$E^m = \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} \quad (3.3.30)$$

Where

$$E_{1,2}^m = \pm \frac{1}{4E} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} \quad (3.3.31)$$

From equation, we find that the mixing angle θ^m is given as:

$$\cos 2\theta^m = \frac{\Delta m^2 \cos 2\theta - A}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}$$

$$\sin 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}$$

And

$$\tan 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$

Three expressions for atmospheric neutrinos for normal hierarchy are given as follows:

$$P_{\mu\tau\text{matter}} = (\text{Cos}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * (\text{Sin}[(1.27 * (\Delta m_{31} + A + \Delta m_{31}^m) * L)/E])^2$$

$$+ (\text{Sin}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * (\text{Sin}[(1.27 * (A + \Delta m_{31} - \Delta m_{31}^m) * L)/E])^2$$

$$- (\text{Cos}[2\theta_{23}])^2 * (\text{Sin}[2\theta_{13m}])^2 * (\text{Sin}[\theta_{23}])^2 * (\text{Sin}[(1.27 * \Delta m_{31}^m) * L)/E])^2$$

$$P_{\mu\mu\text{matter}} = (1 - \text{Cos}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * (\text{Sin}[(1.27 * (A + \Delta m_{31} - \Delta m_{31}^m) * L)/E])^2$$

$$P_{\mu e\text{matter}} = (\text{Sin}[2\theta_{13}^m])^2 * (\text{Sin}[\theta_{23}])^2 * (\text{Sin}[(1.27 * \Delta m_{31}^m) * L)/E])^2$$

And for inverted hierarchy

$$P_{\mu\tau\text{matter}} = (\text{Cos}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * (\text{Sin}[(1.27 * (A - \Delta m_{31} + \Delta m_{31}^m) * L)/E])^2$$

$$+ (\text{Sin}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * \left(\text{Sin} \left[\frac{1.27 * (A + \Delta m_{31} - \Delta m_{31}^m) * L}{E} \right] \right)^2$$

$$- (\text{Cos}[2\theta_{23}])^2 * (\text{Sin}[2\theta_{13m}])^2 * (\text{Sin}[\theta_{23}])^2 * (\text{Sin}[(1.27 * \Delta m_{31}^m) * L)/E])^2$$

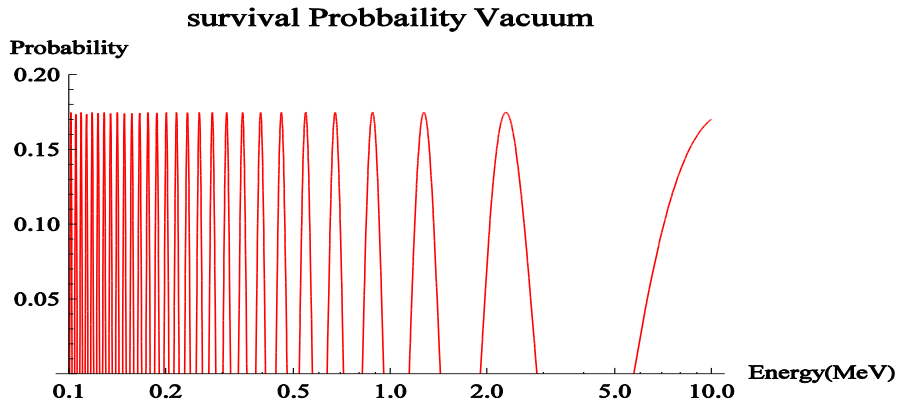
$$P_{\mu\mu\text{matter}} = (1 - \text{Cos}[\theta_{13}^m])^2 * (\text{Sin}[2\theta_{23}])^2 * (\text{Sin}[(1.27 * (A - \Delta m_{31} - \Delta m_{31}^m) * L)/E])^2$$

$$P_{\mu e\text{matter}} = (\text{Sin}[2\theta_{13}^m])^2 * (\text{Sin}[\theta_{23}])^2 * (\text{Sin}[(1.27 * \Delta m_{31}^m) * L)/E])^2$$

3.4 Probability Oscillation Graphs: Vacuum And Matter

3.4.1 Graph For Survival Probability Oscillation In Vacuum

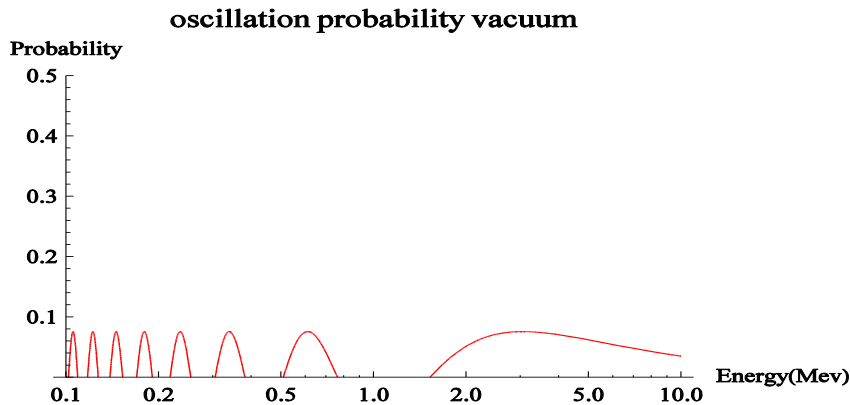
To study the probability oscillation in vacuum we taking the parameters from KAMLAND data. In this Length is 175000 km and mass square difference is $\Delta m^2_{21}=8.1 \times 10^{-5} \text{eV}^{-2}$. The mixing angles are $\theta_{12}=32^\circ$; $\theta_{13}=9.217^\circ$. We plot the graph between probability and Energy by using the probability expression $P(\nu_e \rightarrow \nu_e) = 1 - [\sin^2 2\theta_{12} \cos^4 \theta_{13}] \sin^2 \frac{\Delta m^2_{21} L}{2E}$. Energy is along x-axes and probability is along y-axes. In this plot electron to electron survival probability in vacuum is shown.



Plot 3.2: Probability vs Energy for solar neutrinos in long base line.

3.4.2 Graph For Probability Oscillation In vacuum

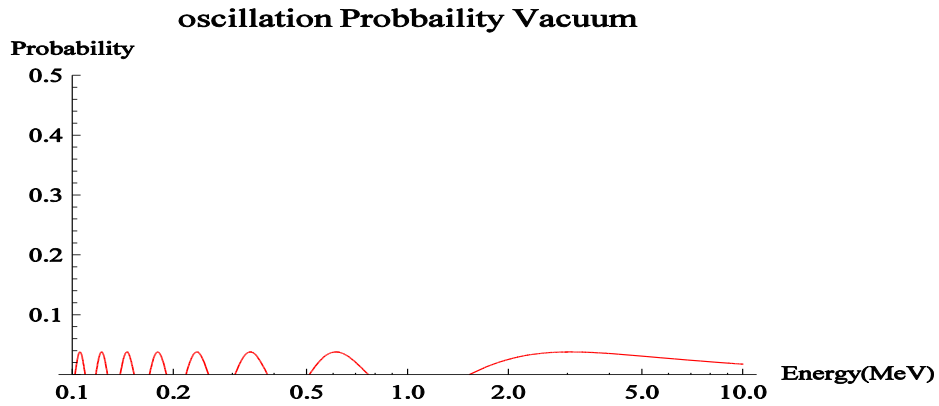
To study the probability oscillation in vacuum, we take data from DOUBLE CHOOZ experiment. The mass square difference is $\Delta m^2_{32}=2.4 \times 10^{-5} \text{eV}^{-2}$. The mixing angles are $\theta_{12}=34^\circ$, $\theta_{23}=39^\circ$, $\theta_{13}=9^\circ$. The length is long baseline i.e. Length=200,000. We plot the graph between probability and energy(MeV) of probability oscillation in vacuum from electron to muon. Energy is along x-axes and probability is along y-axes.



Plot 3.3: Probability vs Energy for vacuum from electron to muon in long baseline.

3.4.3 Graph for probability oscillation from muon to tau in vacuum

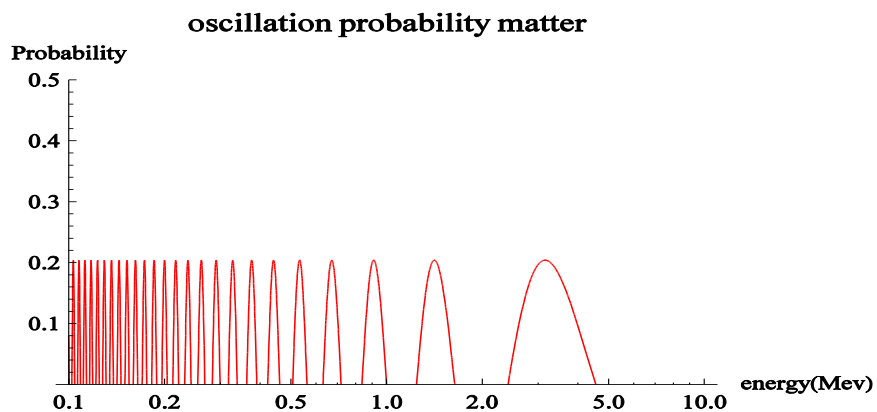
Taking the data from DUOBLE CHOOZ experiment. To study the Probability oscillation in vacuum from muon to tau. The mass square difference is $\Delta m_{32}^2 = 2.4 \times 10^{-5} \text{ eV}^2$, $\Delta m_{31}^2 = 2.4 \times 10^{-5} \text{ eV}^2$. The CP violation phase is $\delta_{13} = \pi/2$. The mixing angles are $\theta_{12} = 34^\circ, \theta_{23} = 39^\circ, \theta_{13} = 9^\circ$. The length taken is long baseline i.e. Length = 200000. The plot is between probability and energy.



Plot 3.4 : Probability vs Energy in vacuum for muon to tau oscillation in for long base line.

3.4.4 Graph for probability oscillation in matter

To study the Probability oscillation in matter we take data from Daya Bay, RENO and Double Chooz. The constant Density = 2.9 ($q = 2 \times \text{energy} \times \text{Density} / \Delta m_{12 \text{vacuum}}^2$). The mixing angles are $\theta_{12 \text{matter}} = \sin^{-1} \left(0.5 \sqrt{\sin^2 2\theta_{12} / \cos^2 2\theta_{12} - q + \sin^2 2\theta_{12}} \right)$, $\theta_{12 \text{vacuum}} = 34^\circ$, $\theta_{12} = 32.3^\circ$. The mass square difference is $\Delta m_{12 \text{matter}} = \Delta m_{12 \text{vacuum}} \sqrt{\cos^2 \theta_{12} - q + \sin^2 2\theta_{12}}$, $\Delta m_{12 \text{vacuum}} = 7.5 \times 10^{-5} \text{ eV}^2$. The length taken is 200000. Probability oscillation in matter is $P_{\text{matter}} = \sin^2[\theta_{12 \text{matter}}] \sin^2[1.27 \Delta m_{12 \text{matter}} L/E]$. This plot is between probability and energy in long base line. Probability is along y-axes and energy is along x-axes.

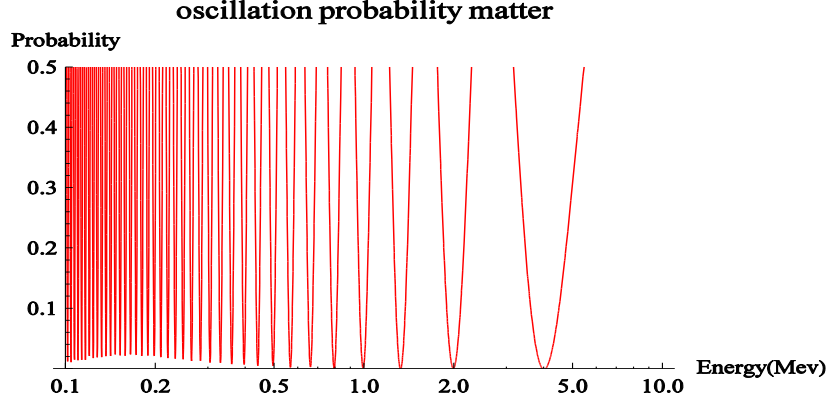


Plot 3.5 : probability vs energy from electron to muon in matter in long base line.

3.4.5 Graph for probability oscillation in matter from muon to tau.

To study the Probability oscillation from muon to tau in matter. The constant density taken in matter oscillation to find its probability is $2.9 (q = 2 * \text{energy} * \text{Density} / \Delta m_{12 \text{vacuum}}^2)$. The mixing angles are $\theta_{12} = 34^\circ, \theta_{23} = 39^\circ, \theta_{13} = 9^\circ$. The mass square difference is $\Delta m_{12 \text{matter}} =$

$\Delta m_{12\text{vacuum}} \sqrt{\cos^2 \theta_{12} - q + \sin^2 2\theta_{12}}$, $\Delta m_{12\text{vacuum}}=7.5*10^{-5}\text{ev}^{-2}$ $\Delta m_{31}=2.4*10^{-5}\text{eV}^{-2}$. The length taken is 200000. The plot is between probability oscillation from muon to tau in matter for long base line. Energy is along x-axes and probability is along y-axes.



Plot 3.6 : Probability vs energy from muon to tau in matter for long base line.

3.5 Comparison Between Matter And Vacuum

Probability oscillation undergoes significant changes in matter and vacuum. Probability oscillation in matter increases than vacuum in hierarchy dependence case but after some time it drops down with length both in vacuum and matter.

Matter effects in $P_{\mu\tau}$:

In vacuum we have

$$P_{\mu\tau}^{\text{vac}} = \cos^4 \theta_{13} \sin^2 2\theta_{23} \sin^2 (1.27\Delta_{31}L/E)$$

$$= \cos^2 \theta_{13} \sin^2 2\theta_{23} \sin^2 (1.27\Delta_{31}L/E) - \cos^2 \theta_{23} P_{\mu e}^{\text{vac}}$$

Including the matter effects changes this to:

$$P_{\mu\tau}^{\text{mat}} = \cos^2 \theta_{13}^m \sin^2 \theta_{23} \sin^2 [1.27(\Delta_{31} + A + \Delta_{31}^m)L/2E] + \sin^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 [1.27$$

$$(\Delta_{31} + A - \Delta_{31}^m)L/2E] - \cos^2 \theta_{23} P_{\mu e}^{\text{mat}}$$

Compared to $P_{\mu e}^{\text{mat}}$, the matter dependent mass eigen states here have a more complicated dependence on the ν_{μ} and ν_{τ} flavor content. Labelling the vacuum mass eigenstates as ν_1 , ν_2 , ν_3 , in the approximation where $\Delta_{21} = 0$, ν_1 can be chosen to be almost entirely ν_e and ν_2 to have no ν_e component. Inclusion of the matter term A leaves ν_2 untouched but gives a nonzero matter dependent mass to ν_1 . As the energy increases, the ν_e component of ν_1^m decreases and the ν_{μ} , ν_{τ} components increase such that at resonance energy they are 50%. Similarly, increasing energy increases the ν_e component of ν_3^m so that at resonance it becomes 50%. Thus in the resonance region, all three matter dependent mass eigenstates ν_1^m , ν_2^m , and ν_3^m contain significant ν_{μ} and ν_{τ} components. We seek ranges of energy and path

lengths for which there are large matter effects in $P_{\mu\tau}$, i.e, for which $\Delta P_{\mu\tau} = P_{\mu\tau}^{mat} - P_{\mu\tau}^{vac}$ is large. We show that this occurs for two different sets of conditions, leading in one case to a decrease from a vacuum maximum and in another to an increase over a broad range of energies.

From plots we studied that the probability oscillation in matter is fast rather than the vacuum. Neutrino oscillates in matter affected by the density of the matter but in vacuum, neutrino oscillates itself. There is no effect of density in vacuum. Any probability also changes because of the length that depend on long base line and short base line. Energy range changes the probability oscillation in matter and vacuum.

3.6 Non Zero θ_{13} Effects

Neutrinos are devious little particles. They can oscillate between three neutrino types, or "flavors," changing their identity on the fly. It has measured one of the key descriptors of the neutrino's flavor-changing behaviour—a number called theta 13. That number, known as a mixing angle, describes the probability that an electron neutrinos antiparticle, the electron antineutrino, will oscillate into another flavor over a relatively short distance. Two other neutrino oscillation parameters, or mixing angles, have already been measured, but theta 13 is relatively small compared with the other two and has proved harder to pin down.

The experiment is not even fully built yet. The new estimate, which falls within previous limits set by other experiments, establishes that theta 13 is not equal to zero, and in fact is relatively large compared with what was plausible in light of other recent results. A zero value for theta 13 would mean that electron neutrinos would not appear in beams of muon neutrinos or, in the Daya Bay case, that electron antineutrinos would not disappear by the time they reached the far detectors. Another reactor experiment, called KamLAND, has also registered the disappearance of antineutrinos over much larger distances, where the oscillation is described by the mixing angle theta 12, rather than theta 13.

There have been recent indications, but none of the other results were significant enough to match it. The Daya Bay group claims better than 5-sigma evidence in support of a nonzero value for theta 13. 5 sigma, or five standard deviations, implies that the finding has only a one-in-several-million chance of being caused by a statistical fluke.

Solar and long-baseline reactor neutrino experiments have measured the mass-mixing parameters ($\Delta m_{21}^2, \theta_{12}$) in the $\nu_e \rightarrow \nu_e$ channel, while atmospheric and long-baseline accelerator (LBL) experiments have measured ($\Delta m_{23}^2, \theta_{23}$) in the $\nu_\mu \rightarrow \nu_\mu$ channel. Conversely, short-baseline reactor experiments, mainly sensitive to ($\Delta m_{21}^2, \theta_{13}$), have set upper—but not lower—bounds on the mixing angle θ_{13} . The two data sets mainly sensitive to δm^2 and to Δm^2 provided two separate hints in favor of $\theta_{13} > 0$ which, in combination, disfavoured the null hypothesis $\theta_{13} = 0$ at 90% C.L.

T2K and NOVA is studying that the magnitude of a θ_{13} signal at a new reactor neutrino experiment is affected only by the uncertainty of the value of Δm_{32}^2 , which is currently

bounded by $2.48 < \Delta m_{32}^2 < 3.18 \times 10^{-3} \text{eV}^2/c^4$. On the other hand, the ability of an accelerator experiment to measure θ_{13} is also affected by the uncertainty in θ_{23} , $0.36 < \sin^2(\theta_{23}) < 0.637$, and the uncertainty in the CP violating phase δ , $0 < \delta < 2\pi$. Thus a precise measurement of θ_{13} by both reactor and accelerator experiments could be used to constrain θ_{23} and/or δ .

To compare experiments, it is common to quote a single number as the θ_{13} limit, but this requires a few assumptions. As a consequence, a large variety of numbers are quoted as the CHOOZ limit, such as $\sin^2(2\theta_{13}) < 0.10, 0.11, 0.14, 0.15, 0.20$. One cause for this is the time-dependence of the Δm_{32}^2 measurement of Super-K and MINOS. There is also no unique method of picking the value of Δm_{32}^2 to use. The “best fit” Δm_{32}^2 value is often chosen, the PDG has elected to use the one sigma low value of Δm_{32}^2 where the larger value of θ_{13} is achieved, and they obtain $\sin^2(2\theta_{13}) < 0.19$. The accelerator

Experiments have additional ambiguities and degeneracy’s in interpreting a θ_{13} limit from θ_{23} , δ and the mass hierarchy.

3.6.1 Value Of θ_{13} Is Large Or Small

Two of the angles are known to be large $\theta_{12} \sim 34^\circ$ and $\theta_{23} \sim 45^\circ$. It is highly unlikely that only the remaining angle θ_{13} is extremely small, the reason for small θ_{13} is , if symmetry exists such that θ_{13} vanishes at symmetry limit. This is the neutral reason for small θ_{13} .

T2K and MINOS suggested that the θ_{13} is large. If it is confirmed, it gives tough time for these symmetries which aims at explaining small θ_{13} . Unless a large correction to symmetry limit is shown to be naturally induced. In case of small θ_{13} , baselines of about 2500 to 5000 km is the optimal choice for the CP violation measurement with E_μ as low as 12 GeV can be considered. On the other side, for large θ_{13} , the lower threshold and the backgrounds reconstructed at lower energies allow in fact for muon energies as low as 5 to 8 GeV at considerably shorter baselines, such as Fermilab to Homestake. If θ_{13} is large, it is setups aiming at exploring lepton CP violation through neutrino oscillation. If $\sin^2 2\theta_{13} \sim 10^{-4}$, baselines between 4000 and 5000 km are preferred with $E_\mu \sim 20 - 25$ GeV, which corresponds more to the choice of L_1 .

The suggested large value of θ_{13} , still need to be confirmed and assumed, stimulates to examine other possibilities such as reactor-accelerator combined method, combining different accelerator measurement, or the one adding one more new facility which may or may not require an extensive cost. Concerning non oscillation searches, the evidence for $\theta_{13} > 0$ provides small but “guaranteed” contributions of the third neutrino mass m_3 to both single beta and double beta decay searches, which need to be accounted for in detailed analyses. From a more theoretical viewpoint, relatively large values for θ_{13} will certainly trigger new ideas for model building.

The CP sensitivity achievable by the reactor-accelerator combined method was examined in detail. The results obtained in this analysis indicate that combining data taken and to be taken by all planned experiments is not enough to guarantee discovery of CP violation in δ_ℓ coverage of more than 30% in most of the allowed region of θ_{13} even though one assumes

that the mass hierarchy is known. It should be noticed, as observed, that the CP sensitivity is severely damaged by the parameter degeneracy due to unknown neutrino mass hierarchies. For a recent overview of the parameter degeneracy.

Living in the world with large θ_{13} is that one can achieve the same goal by less expensive “all in one” setting. More concretely, a single detector (such as a megaton water Cherenkov detector) assuming prior existence of intense neutrino beam. Timely enough the LOI of Hyper-Kamiokande (Hyper-K) project has just appeared. With intense neutrino beam from 1.66 MW proton driver at J-PARC it is demonstrated that Hyper-K with 0.56 megaton fiducial mass has a superb performance for discovery of CP violation, which covers most of the region of θ_{13} allowed at 3σ CL.

If θ_{13} is small, it allows the clean measurement of CP violation but difficult to measure mass hierarchy which such short range of baseline and another important goal of future neutrino experiments. If θ_{13} is large, however, the mass hierarchy can be determined by high-statistics observation of atmospheric neutrinos. The sensitivity to the mass hierarchy depend very much on which octant θ_{23} lives but it can be carried out by ~ 3 – 10 years running of Hyper-K for large θ_{13} . It should also having other capabilities such as proton decay discovery is of crucial importance.

3.6.2 Large- θ_{13} perturbation theory of neutrino oscillation

If theoretical treatment of neutrino oscillation well oiled when a large value of θ_{13} close to the Chooz limit is established then “ $\sqrt{\theta}$ perturbation theory” ($\theta \equiv \Delta m_{21}^2/\Delta m_{31}^2$) has been formulated by assuming $s_{13} \sim \sqrt{\theta} \sim 0.18$, which roughly corresponds to the Chooz limit. By doing so one can systematically compute the large- θ_{13} corrections to the Cervera et al. formula. While large corrections arise in certain limited region of energy and baseline, one can show on general ground that the correction terms are of order $\sim \theta^2 \sim 10^{-3}$. Therefore, the large- θ_{13} correction to the degenerate solutions obtained with the Cervera et al. formula, generally speaking, is not sizable. An important factor will

be whether the goal becomes further limits on a small value of θ_{13} , or more precise measurements of a non-zero value. Statistical precision better than $\delta(\sin^2(2\theta_{13})) < 0.01$ can be imagined, but experience with systematic errors and backgrounds must be weighed along with the capabilities and needs of accelerator experiments. Ideas already exist for more ambitious reactor experiments to study θ_{13} further.

3.6.3 Reactor Neutrino experiments

Important general features of reactor experiments are the effects of luminosity on the sensitivity, detector design, scintillator stability, calibration, backgrounds and systematic errors. The neutrino oscillation sensitivity for a reactor neutrino experiment comes from measuring a smaller number of neutrinos than would be expected if $\theta_{13} = 0$, and measuring an energy distribution consistent with $\bar{\nu}_e$ disappearance due to oscillations. It can be known by the rate test and shape test. The effective “luminosity” for a reactor experiment can be expressed in GW-ton-years, or the product of the reactor’s thermal power times the size of the detector times the length of time the detectors operate.

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Summery and Conclusion

We are studying the phenomenological theory of neutrino oscillation in vacuum and matter, the basic mechanism of neutrino mass generation and the corresponding structure of lepton mixing matrix. One of the spectacular possibilities of such models is that of 'grand unification' of the standard model interactions, viz. the strong and electro-weak interactions. Such models are often based on larger 'gauge groups' into which the gauge symmetries of the standard model would fit into. There are many models today which predict and accommodate the observed masses and mixing.

Within the framework of the standard three neutrino scenario, we derive an exact and simple formula of the oscillation probability $P(\nu_e \rightarrow \nu_\mu)$ in matter with constant density by using a new method. From this formula, it is found that the matter effects can be separated from the pure CP violation effects. Furthermore, the oscillation probability can be written in the form, $P(\nu_e \rightarrow \nu_\mu) = A \cos \delta + B \sin \delta + C$, in the standard parameterization without any approximation. . We have studied neutrino oscillations in constant matter within the framework of the three neutrino scenario. We also demonstrate that the approximate formula in high-energy can be easily reproduced. Neutrino oscillations are sign for small masses of neutrinos. In standard model neutrinos are considered to be massless, hence this observation points to the possibility of physics beyond standard model. Thus the precise determinations of neutrino oscillations are important for formulating theories beyond standard model. In this work, we study the phenomenology of two and three-flavor neutrino oscillations in vacuum and matter, focusing in particular on the prospects of recent experiments. We analysed various experiments that are currently going on, for their potential to determine these parameters precisely. After discussing the details about the detectors and reactors working on neutrino oscillations, we have given a theoretical formalism on the basis of parameters of solar and atmospheric neutrino experiments for three generation case. We discuss in the thesis the three-flavour effects appearing in $\nu_e \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_e$, and $\nu_\mu \rightarrow \nu_\tau$ oscillations through their plots. The plots have been compared with the experimental papers and found to be consistent with them.

Due to the importance of θ_{13} for CP violation and the mass hierarchy, a potential long-term program of reactor neutrino measurements lies ahead of us. Results from Double Chooz, Daya Bay, RENO, and later Angra, will be used to determine the value of upgrades, additional detectors, and new projects. An important factor will be whether the goal becomes further limits on a small value of θ_{13} , or more precise measurements of a non-zero value.

CP violation probability is strongly dependent on θ_{13} parameter. No experiments are possible now to test CP violation via neutrino oscillations, since beams of both neutrino and antineutrino with the same flavor would be needed. However, in the future such beams might be available. If we know the θ_{13} value then we can estimated about CP violation (CPV)

probability on δ_{CP} in order to suggest new experiments to measure CPV for neutrinos moving in matter. The effect of non-zero θ_{13} changes various other results and its accurate determination is important and hence it is important to see the effect on probabilities with large and small values of θ_{13} and also the effects on other parameters like CP violation, mass hierarchy etc.