STUDY OF TWO-UNIT COLD STANDBY SYSTEMS WITH REGENERATIVE POINT TECHNIQUE

Dissertation submitted in partial fulfillment of the requirement for

the award of the degree of

Master of Science

in

Mathematics and Computing

Submitted by

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July, 2011.

DEDICATED

TO

MY PARENTS, GOD AND GUIDE

CERTIFICATE

I hereby certify that the work which is being presented in the dissertation entitled **"Study of Two-Unit Cold Standby Systems with Regenerative Point Technique"** in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications (SMCA), Thapar University, Patiala is an authentic record of my own work carried out under the supervision of **Dr. Jitender Kumar and Dr. Rajesh Kumar Gupta.**

The matter presented in this dissertation has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of our knowledge.

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ABSTRACT

The whole range of our work reported in this dissertation is covered into three chapters which are described as:

Chapter I:

A review of literature pertaining to the theory of reliability and modeling is given in this chapter. This also includes some basic concepts such as probability distributions, stochastic process, profit function, etc.

Chapter II:

This chapter has been designed with a view to study a two-unit cold standby system with single server in which units are identical. Initially one unit is operative and the others as cold standby.The unit fails completely directly from normal mode. There is a single server who comes immediately at the complete failure of the unit for doing its repair. Various Reliability characteristic such as Reliability, Mean Time to System Failure(MTSF), Time Dependent Availability, Steady State Availability, Time-Dependent Busy Period, Steady State Busy Period Analysis, Time-Dependent Expected number of visits, Steady State Expected number of visits by server are obtained and finally the profit function is determine by using semi-Markov process and regenerative point technique.

Chapter III:

Here we study the extension of the work reported in previous chapter with the separate assumption that the repair process is divided into two stages. In the first stage, the repairing process of the unit is started but it does not get completed; instead the process is completed in the second stage. The elapsed time between two stages is called the waiting time. By using regenerative point technique some reliability characteristic of interest are also obtained.

List of Contents

Notations and Symbols

$q_{ij}(t), Q_{ij}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state visiting any other regenerative state in (0,t]

$g_i(t), G_i(t)$ pdf and cdf of repair time of the failed unit in phase $i = 1, 2...$

$A_i(t)$ probability that the system is up at instant t, given that system entered i to state i at t=0

$\phi_i(t)$ cdf of the first passage time from regenerative state i to a failed state

$B_i(t)$ Probability that the server is busy at an instant t given that system entered into the regenertive state i at t=0

$M_i(t)$ Probability that the system initially up in the regenerative state i is up at time t without passing through any other regenerative state

$m_{ij}(t)$ Contribution to mean sojourn time in state S_i

μ_i Mean time in state S_i

L.S.T Laplace stieltjes Transform in state S_i

L.T Laplace Transform

$**$ Symbol for L.S.T.

$*$ Symbol for L.T.

i.e. $q^*_{ij}(s) = \int_0^\infty e^{-st} q_{ij}(t)dt$

Ⓡ Symbol of stieltjes convolution

e.g. $A(t)(s)B(t) = \int_0^t B(t-u)dA(u)$

© Laplace Convolution, e.g.

$f(t)©g(t) = \int_0^t g(t-u)f(t)dt$

pdf Probability density function

cdf Cumulative density function

CHAPTER I

INTRODUCTION

The study of reliability theory and analysis of reliability models, generally classified into three categories - Probabilistic analysis, Statistical analysis and Optimization. The present dissertation confines only to the probabilistic analysis of reliability models. During last few decades scholars and reliability engineers have concentrated themselves on the analysis of various repairable and non repairable systems. Consequently, a large number of research papers are available in the literature on such systems. Various reliability characteristics have been obtained under different sets of assumptions regarding operating conditions and probability laws for failure and repair of units.

The operating efficiency and performance of a system under stated conditions can be measured only through reliability but failures are inevitable may be because of human error, natural calamities, working stress and mechanical failure of the system etc. These possibilities cannot be ruled out completely but can be reduced to a limit through some in-time precaution to make the system considerably more reliable. Keeping in view the importance of the subject, the system reliability has been engaged the attention of a large number of researchers. The probabilistic theory grew up in World War II. In [1953], Epstein and Sobel began work in the field of life testing. After [1956], system maintainability problems were also considered besides reliability. Gaver [1963] was the first who generalized repair time distribution to analysis his model. An excellent account of the early development of the mathematical theory of reliability has been given by Barlow and Proschan [1965].

Branson and Shah [1971] applied semi-Markov method when repair time distribution was general with exponential failure time. Srinivasan and Gopalan [1973] highlighted the regenerative point technique for analyzing a two-unit system with warm standby and single repair facility. Nakagawa [1976] analyzed the system with replacement of the unit at certain level of damage. Arora [1977] obtained reliability of several standby priority systems. Gopalan and Marathe [1978] evaluated the availability of one server two dissimilar unit system with slow switch.

Ramamurthy and Jaiswal [1982] analyzed a two dissimilar unit cold standby system with allowed down time. Kochar [1983] carried out reliability analysis and investment in electric motors for irrigation. Murari and Goyal [1984] made a comparison of two-unit cold standby reliability models with three types of repair facility. Gopalan and Ramesh [1986] analyzed one-server two-unit parallel system subject to degradation. Singh [1989] evaluated the profit of two-unit cold standby system with random appearance and disappearance time of the service facility. Gupta and Bansal [1991] studied profit analysis of a two-unit priority standby system subject to degradation. Tuteja and Malik [1992] discussed single-unit reliability models with different types of repair policies by using regenerative point technique. Yang and Dhillon [1995] analyzed a general standby system with constant human error and arbitrary system repair rates. Mokaddis et al. [1997] discussed a two-unit warm standby system subject to degradation. The concept of on-line repair of single-unit system has been introduced by Ahlawat [1998]. Arora and Kumar [2000] discussed the system analysis and maintenance management for the coal handling system in a paper plant.

Grewal [2003] has proposed reliability models of non-identical units with different repair policies of server. Kadyan et al. [2004] has made a stochastic analysis of non-identical units reliability models with priority and different modes of failure. Chander and Singh [2005] have evaluated profit and reliability of an electric supply system. Chander and Bhardwaj [2007] have analyzed reliability models for 2-out-of-3 redundant system with general distribution of repair and waiting time. Malik and Nandal [2008] carried out MTSF and Profit of a system with the provision of a spare unit. Malik et al. [2008] discussed stochastic analysis of an operating system with two types of inspection subject to degradation. Chander and Singh [2009] developed reliability model for a 2-out-of-3 redundant system subject to degradation. Kumar et al.[2010] discussed cost benefit analysis of a two unit parallel system subject to degradation after repair.

However, these studies are not exhaustive in nature and further investigations are required to fill up the gap.

RELIABILITY ENGINEERING

Reliability engineering is the discipline of ensuring that a system (or a device in general) will perform its intended function (s) when operated in a specified manner for a specified length of time. Reliability engineering is performed throughout the entire life cycle of a system, including development, test, production and operation. Reliability may be defined in several ways:

- The idea that something is fit for purpose with respect to time;
- The capacity of a device or system to perform as designed;
- The resistance to failure of a device or system;
- The ability of a device or system to perform a required function under stated conditions for a specified period to time;

The probability that a functional unit will perform its required function for a specified interval under stated conditions. Reliability engineers rely heavily on statistics, probability theory, and reliability theory. Many engineering techniques are used in reliability engineering, such as reliability prediction, Weibull analysis, thermal management, reliability testing and accelerated life testing. Because of the large number or reliability techniques, their expense, and the varying degrees of reliability required for different situations, most projects develop a reliability program plan to specify the reliability tasks that will be

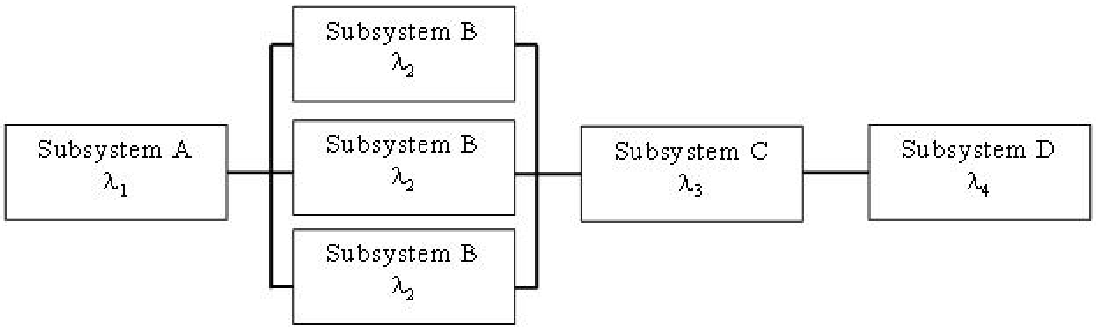

Figure 1: A Reliability Block Diagram

performed for that specific system. The function of reliability engineering is to develop the reliability requirements for the product, establish an adequate reliability program, and perform appropriate analysis and tasks to ensure the product will meet its requirements. These tasks are managed by a reliability engineer, who has additional reliability specific education and training. Reliability engineering is closely associated with maintainability engineering and logistics engineering. Many problems from other fields, such as security engineering, can also be approached using reliability engineering techniques.

Reliability theory is the foundation of reliability engineering. For engineering purposes, reliability is defined as:

The probability that a device(unit) will perform its intended function adequately for a given period of time under stated conditions or environment.

Mathematically, if T is time till the failure of a unit occurs, this may be expressed as,

$$R(t) = Pr(T > t) = \int_t^{\infty} f(x)dx$$

Where f(x) is the failure probability density function and t is the length of the period of time (which is assumed to start from time zero). Reliability engineering is concerned with four key elements of this definition:

• First, reliability is a probability. This means that there is always some chance for failure. Reliability engineering is concerned with meeting the specified probability of success, at a specified statistical confidence level. Since it is a probability, its numerical value is always between one and zero, i.e.

$$R(0) = 1, R(\infty) = 0$$

And $R(t)$ is a non-increasing function between these limits.

• Second, reliability is predicated on "intended function:" Generally, this is taken to mean operation without failure. However, even if no individual part of the system fails, but the system as a whole does not do what was intended,then it is still charged against the system reliability. The system requirements specification is the criterion against which reliability is measured.

•Third, reliability applies to a specified period of time. In practical terms, this means that a system has a specified chance that it will operate without failure before time t. Reliability engineering ensures that components and materials will meet the requirements during the specified time. Units other than time may sometimes be used. The automotive industry might specify reliability in terms of miles, the military might specify reliability of a gun for a certain number of rounds fired. A piece of mechanical equipment may have a reliability rating value in terms of cycles of use.

• Fourth, reliability is restricted to operation under stated conditions. This constraint is necessary because it is impossible to design a system for unlimited conditions.

RELIABILITY PROGRAM PLAN

Many tasks, methods, and tools can be used to achieve reliability. Every system requires a different level of reliability. A commercial airliner must operate under a wide range of conditions, the consequences of failure are grave, but there is a correspondingly higher budget. A pencil sharpener may be more reliable than an airliner, but has a much different set of operational conditions, mild consequences of failure, and correspondingly lower budget.

A reliability program plan is used to document exactly what tasks, methods, tools, analysis and tests are required for a particular system. For complex systems, the reliability program plan is a separate document. For simple systems, it may be combined with the systems engineering management plan. The reliability program plan is essential for a successful reliability program and is developed early during system development. It specifies not only what the reliability engineer does, but also the tasks performed by others. The reliability program plan is approved by top program management.

RELIABILITY REQUIREMENTS

System reliability parameters

Requirements are specified using reliability parameters. The most common reliability parameter is the mean time between failures (MTBF), which can also be specified as the failure rate or the number of failures during a given period. These parameters are very useful for systems that are operated on a regular basis, such as most vehicles, machinery and electronic equipment. Reliability increases as the MTBF increases. The MTBF is usually specified in hours; but can also be used with any unit of duration such as miles or cycles.

In other cases, reliability is specified as the probability of mission success. For example, reliability of a scheduled aircraft flight can be specified as a dimensionless probability or a percentage.

A special case of mission success is the single-shot device or system. These are devices or systems that remain relatively dormant and only operate once. Examples include automobile airbags, thermal batteries and missiles. Single-shot reliability is specified as a probability of success, or is subsumed into a related parameter. Single-shot missile reliability may be incorporated into a requirement for the probability of hit.

In addition to system level requirements, reliability requirements may be specified for critical subsystems. In all cases, reliability parameters are specified with appropriate statistical confidence intervals.

It is a general practice to model the early failure rate with an exponential distribution. This less complex model for the failure distribution has only one parameter: the constant failure rate.

DESIGN FOR RELIABILITY

Reliability must be "designed in" to the system. During system design, the top-level reliability requirements are allocated to subsystems by design engineers and reliability engineers working together. Reliability design begins with the development of a model. Reliability models use block diagrams and fault trees to provide a graphical means of evaluating the relationships between different parts of the system. These models incorporate predictions based on parts-count failure rates taken from historical data. While the predictions are often not accurate in an absolute sense, they are valuable to assess relative differences in design alternatives.

One of the most important design techniques is redundancy. This means that if one part of the system fails, there is an alternate success path, such as a backup system. For example, an automobile brake light might use two light bulbs. If one bulb fails, the brake light still operates using the other bulb. Redundancy significantly increases system reliability, and is often the only viable means of doing so. However, redundancy is difficult and expensive, and is therefore limited to critical parts of the system. Another design technique, physics of failure, relies on understanding the physical processes of stress, strength and failure at a very detailed level. Then the material or component can be re-designed to reduce the probability of failure. Another common design technique is component derating: Selecting components whose tolerance significantly exceeds the expected stress, as using a heavier gauge wire that exceeds the normal specification for the expected electrical current of the system. Another design technique, physics of failure, relies on understanding the physical processes of stress, strength and failure at a very detailed level. Then the material or component can be re-designed to reduce the probability of failure. Many tasks, techniques and analysis are specific to particular industries and applications. Commonly these include:

- Built-in test (BIT)
- Failure mode and effects analysis (FMEA)
- Reliability simulation modeling
- Thermal analysis
- Fault tree analysis
- Sneak circuit analysis
- Weibull analysis
- Electromagnetic analysis

• Statistical interference

Results are presented during the system design reviews and logistics reviews. Reliability is just one requirement among many system requirements. Engineering trade studies are used to determine the optimum balance between reliability and other requirements and constraints.

FAILURE RATE

Failure rate is the frequency with which a system or component fails, expressed for example in failures per hour. It is often denoted by the Greek letter λ (lambda) and is important in reliability theory. In practice, the reciprocal rate MTBF is more commonly expressed and used for high quality components or systems.

Failure rate is usually time dependent, and an intuitive corollary For example, as an automobile grows older, the failure rate in its fifth year of service may be many times greater than its failure rate during its first year of service-one simply does not expect to replace an exhaust pipe, overhaul the brakes, or have major power plant-transmission problems in a new vehicle. So in the special case when the likelihood of failure remains constant with respect to time (for example, in some product like a brick or protected steel beam), failure rate is simply the inverse of the mean time between failure (MTBF), expressed for example in hours per failure. MTBF is an important specification parameter in all aspects of high importance engineering design - such as naval architecture, aerospace engineering, automotive design, etc. - in short, any task where failure in a key part or the whole of a system needs be minimized and severely curtailed, particularly where lives might be lost if such factors are not taken into account. These factors account for many safety and maintenance practices in engineering and industry practices and government regulations, such as how often certain inspections and overhauls are required on an aircraft. A similar ratio used in the transport industries, especially in railways and trucking is

'Mean Distance Between Failure', a variation which attempts to correlate actual loaded distances to similar reliability needs and practices. Failure rates and their projective manifestations are important factors in insurance, business, and regulation practices as well as fundamental to design of safe systems throughout a national or international economy.

Failure Rate in the Discrete Sense

In words appearing in an experiment, the failure rate can be defined as "The total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions".

Here failure rate $\lambda(t)$ can be thought of as the probability that a failure occurs in a specified interval,given no failure before time t. It can be defined with the aid of the reliability function or survival function R(t), the probability of no failure before time t, as:

$$\lambda = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} = \frac{R(t) - R(t + \triangle t)}{\triangle t R(t)}$$

where t_1 (or t) and t_2 are respectively the beginning and ending of a specified interval of time spanning .Note that this is a conditional probability, hence the R(t) in the denominator.

Failure Rate in the Continuous Sense(Instantaneous Hazard rate)

By calculating the failure rate for smaller intervals of time , the interval becomes infinitesimally small. This results in the hazard function, which is the instantaneous failure rate at any point in time:

$$r(t) = \lim_{\triangle t \longrightarrow 0} \frac{R(t) - R(t + \triangle t)}{\triangle t R(t)}$$

Continuous failure rate depends on a failure distribution, F(t), which is a cumulative distribution function that describes the probability of failure prior to time t,

$$P(T \leq t) = F(t) = 1 - R(t), t \geq 0.$$

Where T is the failure time. The failure distribution function is the integral of the failure density function, f(x),

$$F(t) = \int_0^t f(x)dx$$

Now, the hazard function can be defined as

$$r(t) = \frac{R(t + \triangle t) - R(t)}{R(t) \triangle t}$$

$$\text{r(t)}=\frac{f(t)}{R(t)}$$

There are many failure distributions. A common failure distribution is the exponential failure distribution.

$$F(t) = \int_0^t \lambda e^{-\lambda x}dx = 1 - e^{-\lambda t},$$

which is based on the exponential density function. This leads to a constant hazard rate. For other distributions, such as the Weibull distribution, log-normal distribution or bathtub curve, the hazard function is not constant, which means that the failure rate varies with time.

Units of Failure Rate

Failure rates can be expressed using any measure of time, but hours is the most common unit in practice. Other units, such as miles, revolutions, etc., can also be used in place of “time” units.

Failure rates are often expressed in engineering notation as failures per million, or 10^6, especially for individual components, since their failure rates are often very low.

SYSTEM CONFIGURATIONS

By system, we mean an arbitrary device having several units/sub systems/components assuming that their reliabilities are known which help us to predict the reliability of whole system. It is now important that the system structure be known. Various system structures have been considered as follows:

Series Configuration

A system having n-units is said to have series configuration if the failure of an arbitrary unit, say i^{th} unit causes the entire system failure. The examples of the series configurations are :-

(i) The aircraft electronic system consists of a sensor sub system, a guidance subsystem, computer subsystem and the fire control subsystem. These systems can only operate successfully if all these operate simultaneously.

(ii) Deepawali or Christmas glow bulb, where if one bulb fails the whole lead fails. The block diagram of a series system is shown in fig. 1.2.

Let $R_i(t)$be the reliability of i^{th} component, then the system reliability is given by

$$R(t) = Pr[T > t] = Pr[min(T_1, T_2, T_3, \cdots T_n) > t]$$

$$= \prod_{i=1}^{n} P[T_i > t] = \prod_{i=1}^{n} R_i(t)$$

Where T_i is the life time of the i^{th} unit of the system. The system hazard rate, therefore is

$$r(t) = \sum_{i=1}^{n} r_i(t)$$

Where $r_i(t)$ is the instantaneous failure rate of ith unit.

Parallel Configuration

In this configuration, all the units are connected in parallel i.e. the failure of the system occurs only when all the units of system fail. For example, four engined aircraft which is still able to fly with only two engines working. Block diagram representing a parallel configuration is shown in fig. 1.3. Suppose $R_i(t)$ and T_i be the reliability of i^{th} components and the life time of the i^{th} unit in time t respectively, then the system reliability is given by

$$R(t) = Pr[T > t]$$

$$= Pr[max(T_1, T_2, T_3, \cdots T_n) > t]$$

$$= 1 - P[T_1 \leq t, T_2 \leq t, T_3 \leq t, ...T_n \leq t]$$

If the units function independently, then

$$R(t) = 1 - [1 - R_1(t)][1 - R_2(t)][1 - R_3(t)]...[1 - R_n(t)]$$
$$= 1 - \prod_{i=1}^{n}[1 - R_i(t)]$$

Standby Redundant Configuration

To assure high reliability of a system, redundancy is incorporated. In redundant system more units than the required are used so that when failures occur in a system, it does not stop functioning. In standby redundant system with n units, only one unit is on-line at a time. When it fails, it is replaced manually or automatically by a standby unit. This process continues until all (n-1) standby units have been exhausted. For example, consider a cinema hall in a city where power supply is irregular. In order to ensure uninterrupted supply of power apart from the regular source of supply, a generator is kept as standby. The generator is switched on as and when the main supply is resumed. A block diagram of such system is shown in fig. 1.4.

Gnedenko et al. (1969) classified the standby units as follows :-

a) If the off-line unit can fail and is loaded in exactly the same way as the operating unit. It is called the hot standby unit.

b) If the off-line unit can fail and can diminish the load, it is called warm standby unit. The probability of failure for a warm standby is less than the failure of operative unit.

c) If the off-line unit cannot fail and is completely unloaded, it is called cold standby.

Reliability R(t) of an n-unit standby system at any time instant t is given as

$$R(t) = Pr[\sum_{i=1}^{n} T_i > t]$$

where

T_i is the life time of i^{th} unit and all the n units are independent.

k-out-of-n-Configuration

In many problems, the system operates if atleast k-out-of-n-units function e.g. a bridge supported by n cables, k of which are necessary to support the maximum load. If each of n-units are identical with the same reliability $R_0(t)$ (say), then the system reliability becomes

$$R(t) = \sum_{i=k}^{n} n_{C_i} R_0^i(t)[1 - R_0(t)]^{n-1}$$

Series (k = n) and parallel (k = 1) system are subclasses of k-out-of-n structure.

There are many other configurations as series-parallel, parallel series, mixed-parallel, etc. some of them are depicted in Fig. 1.5 to Fig. 1.7.

TRANSFORMS AND CONVOLUTIONS

Laplace Transforms

A transform is merely a mapping or function from one space to another. While it may be very difficult to solve certain equations directly for a particular function of interest, it is often easier to solve a corresponding equation in terms of a transform of the function and then invert the transform to obtain the function. One particular transform, that is, very useful for solving some types of differential equations as well as certain integral equations, is the Laplace transform (L.T.).

Let f(t) be a function of positive real variable t.Then the Laplace transform of f(t) is denoted by$f^*(s)$ and defined as

$$L[f(t)] = f^*(s) = \int_0^\infty e^{-st} f(t)dt$$

For the range of value of s for which the integral exists. Here f(t) is called an inverse Laplace transform of $f^*(s)$and we write $f(t) = L^{-1}f^*(s)$. The following are some important properties of Laplace transform:-

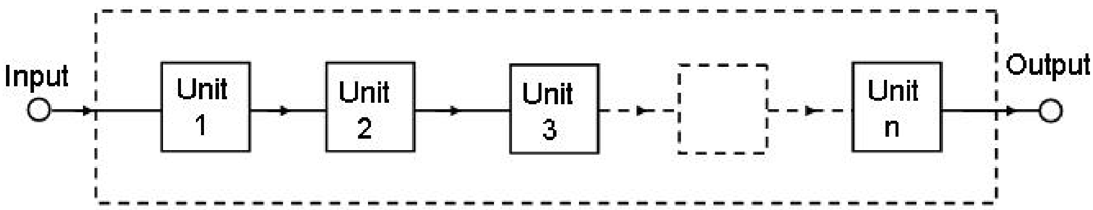

Figure 2: Series Configuration

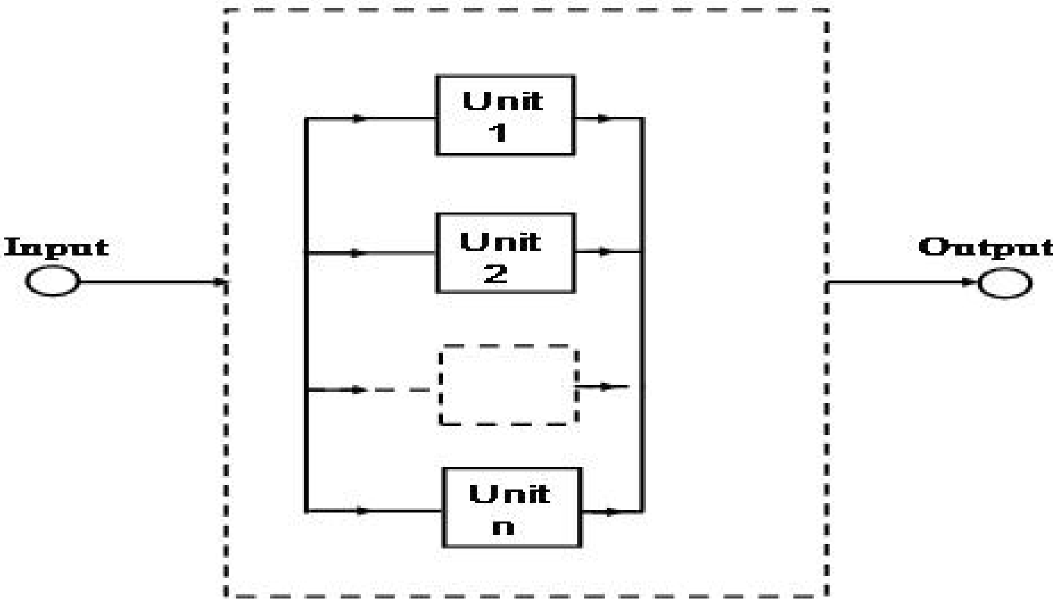

Figure 3: Parallel Configuration

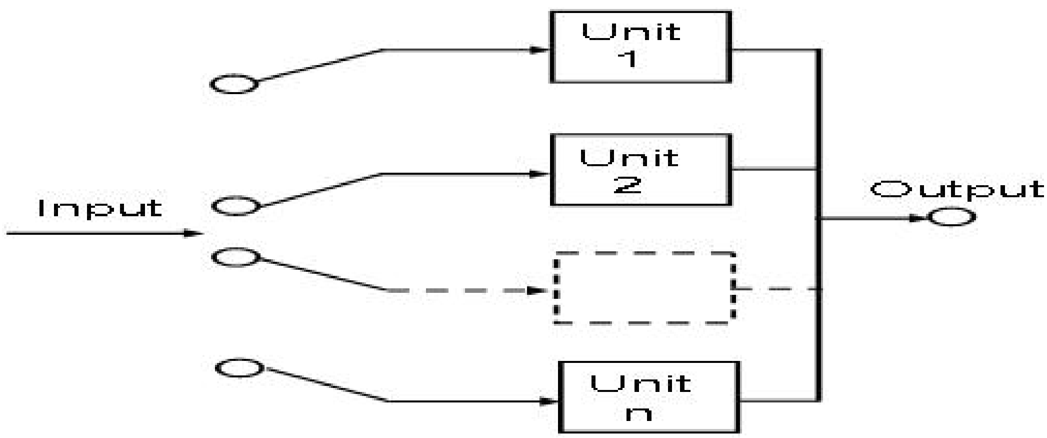

Figure 4: Standby Redundant Configuration

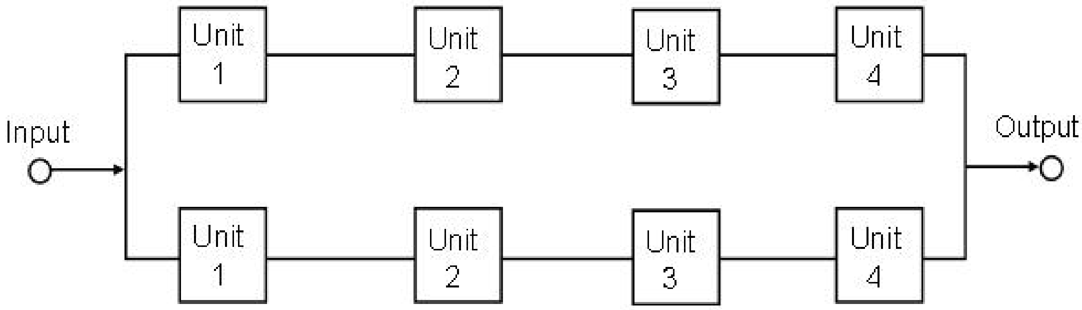

Figure 5: Series Parallel Configuration

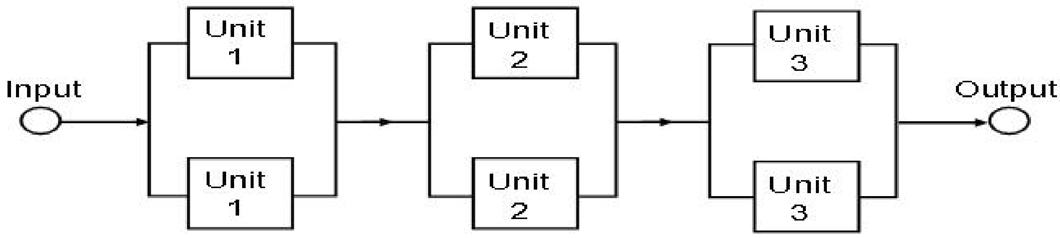

Figure 6: Parallel Series Configuration

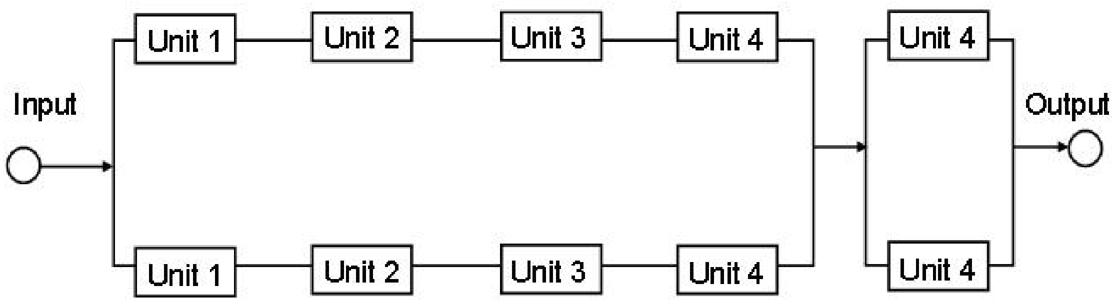

Figure 7: Mixed Parallel Configuration

$$i) L[\sum_{i=1}^{n} c_i f_i(t)] = \sum_{i=1}^{n} c_i f_i^*(s)$$

$$ii) L[t^n f(t)] = \frac{(-1)^n d^n f^*(s)}{ds^n}$$

$$iii) L[\int_o^t f(u)du] = \frac{f^*(s)}{s}$$

$$iv) \lim_{t \longrightarrow 0} f(t) = \lim_{s \longrightarrow \infty} sf^*(s) (\text{initial value theorem})$$

$$v) \lim_{t \longrightarrow \infty} f(t) = \lim_{s \longrightarrow 0} sf^*(s) (\text{final value theorem})$$

$$vi) \lim_{s \longrightarrow \infty} sf^*(s) = 0$$

$vii) \lim_{s \longrightarrow 0} f^*(s) = 1$ if $f^*(s)$ is L.T. of a p.d.f.

Laplace Stieltjes Transforms

Let X be a non-negative random variable with distribution function

$$F(x) = Pr[X \leqslant x]$$

then Laplace Stieltjes transform of $F(x)$is defined for$s > 0$ by

$$F^{**}(s) = \int_0^\infty e^{-sx} dF(x)$$

under certain regular conditions, we have

$$F^{**}(s) = s\int_0^\infty e^{-sx} F(X)dX = sF^*(s) \text{ and}$$

$$F^{**}(s) = \int_0^\infty e^{-sx} f(X)dX = f^*(s)$$

Where

$$f(x) = \frac{dF(x)}{dx}$$

CONVOLUTION

Let $f(t)$ and $g(t)$ be two real valued non-negative continuous functions t, then the integral

$$\int_o^t f(t-u)g(u)du = \int_0^t g(t-u)f(u)du$$

$$= f(t) \copyright g(t)$$

$$= L^{-1}[f^*(s)g^*(s)]$$

is called Laplace convolution of the functions $f(t)$ and $g(t)$. If $F(t)$ and $G(t)$ be two real valued distribution functions defined for $t \geq 0$, the resulting convolution is again a distribution and integral

$$\int_0^t F(t-u)dG(u) = \int_0^t G(t-u)dF(u) = F(t)\circledR G(t)$$

is known as Stieltjes convolution of F(t) and G(t).

MEAN SOJOURN TIME IN A STATE

The expected time taken by the system in a particular state before transiting to any other state in known as mean sojourn time or mean survival time in that state. If T_i be the sojourn time in state i, then the mean sojourn time in state i is

$$\mu_i = \int_0^\infty Pr(T_i > t)dt$$

FIRST PASSAGE TIME

Suppose that a system starts with the state j, then time taken to reach a given state k for the first time from state j is called first passage time. In general, first passage time is a measure of how long it takes to reach a given state from another state.

MEAN TIME TO SYSTEM FAILURE (MTSF)

It is defined as the expected time for which the system is in operation before it completely fails.

Let f(t) be the probability density function of life time of the system, then we have

MTSF $= E(T) = \int_0^\infty tf(t)dt = \int_0^\infty R(t)d(t)$

Also $\lim_{s \longrightarrow 0} R^*(s) = \int_0^\infty R(t)dt$

MTSF $= \lim_{s \longrightarrow 0} R^*(s)$

Let $\phi_0(t)$ be the cumulative distribution function of the first passage time from initial state to a failed state, then

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$$

from above equations, we have

MTSF $= \lim_{s \longrightarrow 0} R^*(s) = \lim_{s \longrightarrow 0} \frac{1 - \phi_0^{**}(s)}{s}$

Where

$R * (s)$ and $\phi^{**}(s)$ are respectively the Laplace transform and Laplace Stieltjes transform of R(t) and $\phi_0(t)$.

AVAILABILITY

Availability is well established in the literature of stochastic modeling and optimal maintenance. Barlow and Proschan [1975] define availability of a repairable system as “the probability that the system is operating at a specified time t” and in reliability theory, the term availability has the following meanings.

The degree to which a system, subsystem or equipment is operable and in a committable state at the start of a mission, when the mission is called for at an unknown, i.e. a random time. Simply put, availability is the proportion of time a system is in a functioning condition. In general, we may categorize this measure as:-

a) Instantaneous Availability

This is the probability that the system will be able to operate within the tolerances at a given instant of time t(say). Let this probability be denoted by A(t).

Let X(t) = 1 if the system is operable at time t and X(t) = 0 when it is not operable.The availability A(t) of the system at time t is given by

$$A(t) = Pr[X(t) = 1|X(0) = 1]$$

b) Average (Interval) Availability

It is the expected fraction of a given interval of time that the system will be able to operate within tolerances.Suppose the given interval of time is (0, t] then interval availability H(0, t] of this interval is given by :-

$$H(0,t] = \frac{1}{t}\int_0^t A(u)du = \frac{\mu_{up}t}{t}$$

When $\mu_{up}t$ = expected up time of the system during (0, t].

c)Steady-state(Limited Interval) Availability

The long run or steady-state availability is defined as the proportion of the time during which an equipment is available for use.

Mathematically, it is the limiting value of the point wise availability when t become finitely large i.e.

$$A = \lim_{t \longrightarrow \infty} A(t)$$

RELIABILITY AND AVAILABILITY

The availability function A(t)is defined as the probability that the equipment is operating at time t. Although this definition appears to be very similar to the reliability function R(t),the two have different meaning. While reliability places emphasis on failure - free operation up to time t, availability is concerned with the status of the equipment at time t. The availability function does not say anything about the number of failures that occurred during time t.This means that two equipments A and B can have different number of failures in a given time interval and can still have the same availability.

MAINTAINABILITY

Maintainability is the probability that the system will be restored to operational effectiveness within a specified time when the maintenance action is taken in accordance with prescribed conditions. Maintenance is one of the effective ways of increasing the reliability of a system. Maintenance action can be classified in several categories:preventive maintenance, corrective maintenance and repair maintenance.

Preventive maintenance includes actions such as lubrication, replacement of a nut or screw of some part of the system, refueling, cleaning, etc. It is designed to minimize the limit that the system will spend in degraded states, it is a sort of repair that is done before a unit actually fails. Corrective maintenance deals with the system performance when it gives wrong result and it involves minor repairs that may creep up between inspections.

Repair maintenance is also concerned with increasing the system availability. In order to increase availability, failed unit upon failure is returned to operation by sending it to a repair facility if available, otherwise waits for repair. There may be two types of repair policies:-

a) Repeat repair policy

Due to certain reasons the repair of a failed unit has to be stopped. When the repair is begun again, it is started all over again.

b) Resume repair policy

The repair of failed component is terminated before completion due to one reason or the other. When it begins again, it is started from the stage where it was prior to the termination of repair.

BUSY PERIOD OF THE REPAIRMAN/SERVER WITH THE SYSTEM

Let B(t) be the probability that a repairman/server is busy with the system in the interval

(0, t], then in the long run the total fraction of time for which a repairman is busy is given by:-

$$B = \lim_{t \longrightarrow \infty} B(t)$$

EXPECTED NUMBER OF VISITS BY THE SERVER

Let N(t) be a random variable representing the number of times, the repairman has visited the system in the interval (0, t], then the expected number of visits by the repairman to the system in (0, t], is E[N(t)] and in the long run this number per unit time is given by

$$N = \lim_{t \longrightarrow \infty} \frac{E[N(t)]}{t}$$

PROFIT ANALYSIS

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost.The major items contributing to the total cost are research and development, production, spares and maintenance. How the cost of these individual items varies with reliability shown in fig.1.8. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production.The profit function in time t is given by:-

P(t) = Expected revenue in (0, t] - Expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time,in infinite time span is expressed as

$$\lim_{t \longrightarrow \infty} \frac{P(t)}{t}$$

i.e. profit per unit time = total revenue per unit time - total cost per unit time.

Considering the various costs, the profit equation is given as

$$P = K_1 A_0 - K_2 B_0 - K_3 N_0$$

Where

P = Profit per unit time incurred to the system

K_1 = Revenue per unit up time of the system

A_0 = Total fraction of time for which the system is up

K_2 = Cost per unit time for which server is busy

B_0 = Total fraction of time for which the server is busy

K_3 = Cost per visit by the server

N_0 = Expected number of visits per unit time for the server

DISTRIBUTIONS

In the concept of reliability theory,the failure times, the repair times, inspection times,waiting time, etc. are random variables. A random variable is completely characterized by the distribution function which is defined in terms of probability parameters,the distribution function F(x) is defined as :

$$F(x) = P[X \leq x]$$

$$= \int_{-\infty}^{x} f(t)dt$$

Here$f(t)$ is known as the probability density function.

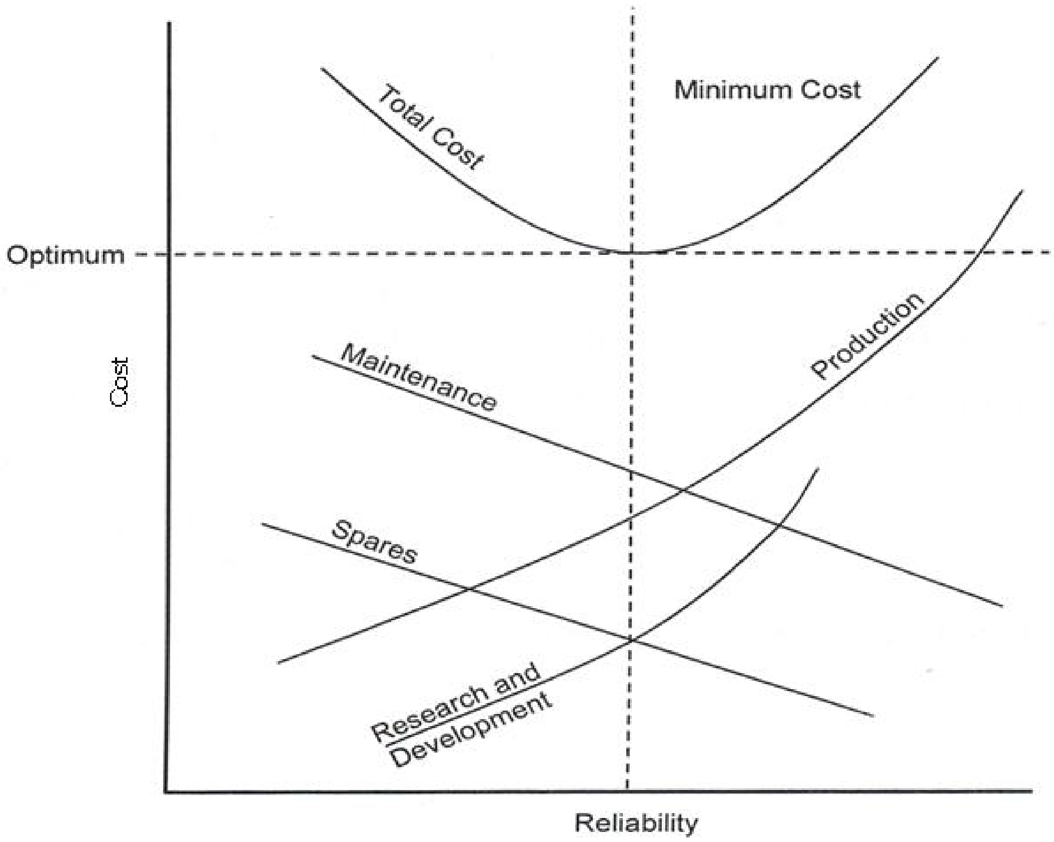

Figure 8: Reliability vs. Cost

EXPONENTIAL DISTRIBUTION

a) Characterization

Probability density function

The probability density function (pdf) of an exponential distribution has the form

$$f(x;\lambda) \quad = \quad \left\{ \begin{array}{ll} (\lambda e^{-\lambda x}), & x \geq 0 \\ \\ 0, & x < 0 \end{array} \right\}$$

Where $\lambda > 0$ is a parameter of the distribution, often called the rate parameter. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim Exp(\lambda)$.

Cumulative distribution function

The cumulative distribution function is given by

$$F(x;\lambda) = \quad = \quad \left\{ \begin{array}{ll} (1 - e^{-\lambda x}), & x \geq 0 \\ \\ 0, & x < 0 \end{array} \right\}$$

Parameters	$\lambda > 0$ rate or inverse scale (real)
Support	$[0, \infty)$
Probability density function (pdf)	$\lambda e^{-\lambda x}$
Cumulative distribution function (cdf)	$1 - e^{-\lambda x}$
Mean	λ^{-1}
Variance	λ^{-2}
Moment-generating function (mgf)	$\left(1 - \frac{t}{\lambda}\right)^{-1}$
Characteristic function	$\left(1 - \frac{it}{\lambda}\right)^{-1}$

Table No. 1.1

b) Occurrence and applications

The exponential distribution is used to model Poisson processes, which are situations in which an object initially in state A can change to state B with constant probability per unit time λ.The time at which the state actually changes is described by an exponential random variable with parameter λ . Therefore, the integral from 0 to T over f is the profitability that the object is in state B at time T.

The exponential distribution may be viewed as a continuous counterpart of the geometric distribution, which describes the number of Bernoulli trials necessary for a discrete process to change state.In contrast, the exponential distribution describes the time for a continuous process to change state.

In real-world scenarios, the assumption of a constant rate (or probability per unit time) is rarely satisfied. For example, the rate of incoming phone calls differs according to the time of day. But if we focus on a time interval during which the rate is roughly constant, such as from 2 to 4 p.m. during work days, the exponential distribution can be used as a good approximate model for the time until the next phone call arrives. Similar caveats apply to the following examples which yield approximately exponentially distributed variables:

- The time until a radioactive particle decays, or the time between beeps of a geiger counter;
- The number of dice rolls needed until you roll a six 11 times in a row
- The time it takes before your next telephone call
- The time until default (on payment to company debt holders) in reduced form credit risk modeling

Reliability theory also makes extensive use of the exponential distribution.Because of the memoryless property of this distribution,it is well-suited to model the constant hazard rate portion of the bathtub curve used in reliability theory. It is also very convenient because it is so easy to add failure rates in a reliability model. The exponential distribution is however not appropriate to model the overall lifetime of organisms or technical devices, because the "failure rates" here are not constant: more failures occur for very young and for very old systems.

(c) Properties

Mean and variance

The mean or expected value of an exponentially distributed random variable X with rate parameter is given by

$$\mathrm{E[X]} = \frac{1}{\lambda}$$

The variance of X is given by

$$\mathrm{Var[X]} = \frac{1}{\lambda^2}.$$

Memorylessness

Exponential distribution plays an important role in reliability theory. Besides a number of mathematical properties it has a very important memoryless property. This means that if a random variable T is exponentially distributed, its conditional probability obeys

$$P(T > s + t \mid T > s) = P(T > t) \text{for all } s, t \geq 0.$$

This says that the conditional probability that we need to wait, for example, more than another 10 seconds before the first arrival, given that the first arrival has not yet happened after 30 seconds, is no different from the initial probability that we need to wait more than 10 seconds for the first arrival. This is often misunderstood: the fact that

$$P(T > 40|T > 30) = P(T > 10)$$

does not mean that the events $T > 40$ and $T > 30$ are independent. To summarize: "memorylessness" of the probability distribution of the waiting time T until the first arrival means

(Right) $P(T > 40 \mid T > 30) = P(T > 10)$.

It does not mean

(Wrong)$P(T > 40 \mid T > 30) = P(T > 40)$.

(That would be independence. These two events are not independent.) Another example of this property is; an electric fuse (assuming it can not melt partially) whose failure life distribution is practically unchanged as long as it has not yet failed. The exponential distributions and the geometric distributions are the only memoryless probability distributions. The exponential distribution also has a constant hazard function.

ERLANG DISTRIBUTION

Erlang distribution is a continuous probability distribution with wide applicability primarily due to its relation to the exponential and Gamma distributions. The Erlang distribution was developed by A.K. Erlang to examine the number of telephone calls which might be made at the same time to the operators of the switching stations. This work on telephone traffic engineering has been expanded to consider waiting times in queuing systems in general. The distribution is now used in the field of stochastic processes.

Characterization

Probability density function

The probability density function (pdf) of an erlang distribution has the form;-

$$f(x;\lambda) = \left\{ \begin{array}{cc} \dfrac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} & for\ x, \lambda \geq 0 \\ 0 & \text{x}, \lambda < 0 \end{array} \right\}$$

The parameter k is called the shape parameter and the parameter λ is called the rate parameter.

Parameters	$k > 0$ shape (integer) $\lambda > 0$ rate(real)
Support	$x[0;\infty)$
Probability density function (pdf)	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$
Cumulative distribution function(cdf)	$\frac{\gamma(k,\lambda x)}{(k-1)!} = 1 - \sum_{n=0}^{k-1} e^{-\lambda x}(\lambda x)^n$
Mean	$\frac{k}{\lambda}$
Variance	$\frac{k}{\lambda^2}$
Moment-generating function (mgf)	$\left(1 - \frac{t}{\lambda}\right)^{-k}$ for $t < \lambda$
Characteristic function	$\left(\frac{(1-it)}{\lambda}\right)^{-k}$

Table No. 1.2

Overview

The distribution is a continuous distribution, which has a positive value for all real numbers greater than zero, and is given by two parameters: the shape k, which is a non-negative integer, and the rate λ which is a non-negative real number. When the shape parameter k=1, the distribution simplifies to the exponential distribution. The Erlang distribution is a special case of Gamma distribution where the shape parameter k is an integer. In the Gamma distribution, this parameter is a real.

WEIBULL DISTRIBUTION

The Weibull distribution (named after Waloddi Weibull) is a continuous probability distribution with the probability density function

$$f(x;k,\lambda) = \left\{ \begin{array}{cc} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{\left(\frac{-x}{\lambda}\right)^{k}} & for\ x \geq 0 \\ 0 & \text{x}, \lambda < 0 \end{array} \right\}$$

where $k > 0$is the shape parameter and $\lambda > 0$ is the scale parameter of the distribution.

The Weibull distribution is often used in the field of life data analysis due to its flexibility. It can mimic the behavior of other statistical distributions such as the normal and the exponential. If the failure rate decreases over time, then $k < 1$. If the failure rate is constant over time, then k = 1. If the failure rate increases over time, then $k > 1$. When k = 1, then the Weibull distribution reduces to the exponential distribution. An understanding of the failure rate may provide insight as to what is causing the failures:

• A decreasing failure rate would suggest that defective items fails early and the failure rate decreases over time as they fall out of the population.

• A constant failure rate suggests that items are failing from random events.

•An increasing failure rate suggests "wear out" parts are more likely to fail as time goes on.

Parameters	$k > 0$ shape (real) $\lambda > 0$scale (real)
Support	$x\varepsilon[0;\infty)$
Probability density function (pdf)	$\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{\left(\frac{-x}{\lambda}\right)^{k}}$
Cumulative distribution function (cdf)	$1 - e^{\left(\frac{-x}{\lambda}\right)^{k}}$

Table No. 1.3

Weibull distribution is most commonly used in life data analysis, though it has found other applications as well.

STOCHASTIC PROCESS

A stochastic process is a family of random variables $\{X(t) \mid t\varepsilon T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set T. Both the parametric set and state space can be independently either discrete or continuous. In stochastic process $\{X(t), t\varepsilon T\}$, where X(t), t and T respectively, the state space, parameter (generally taken to be time) and the index set if T is a countable set as $T = \{0, 1, 2, 3......\}$, then the stochastic process is said to be a discrete parameter process and if $T = \{t : -\infty <$

$t < \infty\}$, the stochastic process is said to be a continuous parametric process. The state space is classified as discrete if it is finite or countable and continuous if it consists of an interval on the real line. In the present study, we have only dealt with discrete state space continuous time parameter stochastic processes.

MARKOV PROCESS

If $\{X(t), t \varepsilon T\}$ is a stochastic process such that given the value of X(s), the value of $X(t), t > s$ do not depend on the values of $X(u), u < s$ i.e. for $t > s,\ i\ \varepsilon\ s$.

$$Pr\{X(t) = i|X(u), 0 \leq u \leq s\} = Pr\{X(t) = i|X(s)\}$$

Then the process $\{X(t), t \varepsilon T\}$ is a Markov process.

MARKOV CHAIN

A discrete parameter Markov process is known as a Markov Chain. The stochastic process $X_n, n = 0, 1, 2, ...$ is called a Markov chain, if, for $j, k, j_1, j_2, ...j_{n-1} \varepsilon N$,

$$Pr[X_n = \frac{k}{X_{n-1}} = j, X_{n-2} = j_1, ..., X_0 = j_{n-1}]$$

$$= Pr\{X_n = k|x_{n-1} = j\}$$

$$= p_{jk}(say)$$

The conditional probability p_{jk} is called transition probability from the state j at$(n-1)^{th}$ trial to the state k at n^{th} trial. If the transition probability p_{jk} is independent of n, the Markov Chain is said to be homogenous; and if it is dependent of n, the chain is said to be non-homogeneous.

SEMI-MARKOV PROCESS

In the above, assume that the process is time homogeneous, i.e.

$$Pr\{X_{n+1} = j, t_{n+1} - t_n \leq t|X_n = i\} = Q_{ij}(t), i, j \varepsilon s,$$

is independent of n, then there exist limiting transition probabilities.

$$p_{ij} = \lim_{t \longrightarrow \infty} Q_{ij}(t) = Pr\{X_{n+1} = j | X_n = i\}$$

then

$\{X_n, n = 0, 1, 2, ...\}$ constitute a Markov Chain with state space $E = \{0, 1, 2, ...\}$

and transition probability matrix (t.p.m.)

$$P = [p_{ij}]$$

The continuous parameter stochastic process Y(t) with state space E defined by

$$Y(t) = X_n, t_n < t < t_{n+1}$$

is called a semi-Markov process. The Markov Chain Xn is said to be an embedded Markov chain of the semi-Markov process.

In other words, we define the semi-Markov process is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state before a transition occurs, is a random variable depending upon the last transition made. Thus at transition instants the semi-Markov process behaves just like a Markov process. However, the times at which transitions occur are governed by a different probability mechanism.

REGENERATIVE PROCESS

Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If t1, t2... are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t1, t2...\}$ is called regenerative process.

CHAPTER II

STUDY OF A RELIABILITY MODEL OF TWO-UNIT COLD STANDBY SYSTEM WITH SINGLE STAGE REPAIR POLICY

In the field of reliability several probabilistic models of two-unit standby systems have been discussed by obtaining reliability and/or economic measures. Many authors such as Murari and Goyal(1984),Goel, Sharma and Gupta (1985), Singh(1989) have discussed systems operating under different environments Srinivasan and Gopaln (1973) studied a Two-unit system with warm standby and a single repair facility. Murari and Goyal in (1984) made a comparison of two-unit cold standby reliability models.In model-**I** the server always remains with the system; where as, in model-**II** the server takes some time to reach the system after the failure of the unit.

Here we review the research work based on model-**I** discussed by Murari and Goyal(1984) as mentioned above in which there is a two-unit cold standby system with single server.Initially one unit is opeartive and the other is cold standby.The units fails completely directly from normal mode.There is a single server who attains the system immediately when unit fails.By using regenerative point technique the various reliability characteristics such as mean time to system failure(MTSF),steady state availability,busy period analysis,Expected number of visits by the server and the profit function is defined for calculating the profit of the model by using the above mentioned characteristics.

System Descriptions and Assumption

1. The system consists of Two-units initially one-unit is operative which fails completely directly from normal mode and the other unit as cold standby.

2. There is a single server who attends the system immediately at the complete failure of the unit.

3. The server cannot leave the system during repair.

4. Units works as new after repair.

5. The failure time of the unit is exponentially disturbed; while distribution of repair is arbitrary.

6. All random variables are mutually independent.

Notation

O	Operative.
Cs	Cold Standby.
λ	Constant failure rate.
F_{ur}	Failed unit under repair.
F_{UR}	Failed unit under repair continuously from previous state.
F_{wr}	Failure unit waiting for repair g(t)/G(t).

States of the System

up states

$S_0 = $ (o, Cs) $S_1 = (F_{UR}, F_{wr})$

down states

$S_2 = (F_{UR}, F_{wr})$

States S_0 and S_1 are regenerative while state S_2 is failed and non regenerative.

Possible transitions between states along with rates and cdfs time are shown in table 2.1

From	S_0	S_1	S_2
S_0	-	λ	-
S_1	$g(t)$	-	λ
S_2	-	$g(t)$	-

Table 2.1

where Transition probabilities are

$$d(Q_{01}(t)) = \lambda e^{-\lambda t} dt$$

$$d(Q_{10}(t)) = e^{-\lambda t} g(t) dt$$

$$d(Q_{12}(t)) = \lambda e^{-\lambda t} G(t) dt$$

$$d(Q_{21}(t)) = g(t) dt$$

$$d(Q_{11}^{(2)}(t)) \;=\; d(Q_{12}(t)dt) \text{ © } dQ_{21}(t)dt$$

$$= (\lambda \; e^{\lambda t} \; \overline{G(t)} \text{ © } g(t))dt$$

letting $t \to \infty$, using $Q_{ij}(\infty) = p_{ij}$, we get

$p_{01} = 1, p_{10} = g{*}{*}(\lambda), p_{12} = 1 - g{*}{*}(\lambda) = p_{11}^{(2)}$

$p_{21} = g{*}{*}(0) = 1.$

Transition probabilities and Mean sojourn times

Simple probabilistic consideration yield . The following expressed for non-zero elements p_{ij} by taking distributions exponentially

$$p_{01} = 1 \qquad p_{10} = \frac{\theta}{\lambda + \theta} \qquad p_{12} = \frac{\lambda}{\lambda + \theta}$$

$$p_{21} = 1 \qquad p_{11}^{(2)} = \frac{\lambda}{\lambda + \theta}$$

The mean sojourn times μ_i in the state S_i are

$\mu_0 = \int_0^\infty P(T > t)dt = \frac{1}{\lambda} = M_0$

$\mu_1 = \frac{1}{\lambda + \theta} = M_1, \mu_2 = \frac{1}{\theta}$

It can be easily verified that

$$p_{01} = 1 = p_{10} + p_{12} = p_{10} + p_{11}^{(2)} = p_{21}$$

and

$$\mu_0 = m_{01} \qquad \mu_1 = m_{10} + m_{12} \qquad \mu_2 = m_{21}$$

Reliability Analysis

The equations determining the reliability of the system. Hence, we have

$$R_0(t) = M_0(t) + q_{01}(t) \text{©} R_1(t)$$

$$R_1(t) = M_1(t) + q_{10}(t)©R_0(t) \tag{2.1}$$

By using Laplace transform technique, we can solve for $R_0^*(s)$ and is given by

$$R_0^*(s) = \frac{M_0^*(s) + q_{01}^*(s)M_1^*(s)}{1 - q_{01}^*(s)q_{10}^*(s)} \tag{2.2}$$

The steady-state reliability of the system given by

$$R_0 = \lim_{s \to 0} sR_0^*(s) = \lim_{t \longrightarrow \infty} R_0(t) \tag{2.3}$$

Availability Analysis

The equations determining the availability of the system. Hence, we have

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) \tag{2.4}$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11}^{(2)}©A_1(t)$$

By taking Laplace transforms of the above equations and solving for $A_0^*(s)$ we get;

$$A_0^*(s) = \frac{N_1(s)}{D(s)} \tag{2.5}$$

where

$N_1(s) = M_0^*(s)[1 - q_{11}^*(s)] + q_{01}^*(s)M_1^*(s)$

$D(s) = 1 - q_{11}^{*(2)}(s) - q_{01}^*(s)q_{10}^*(s)$

The steady-state availability of the system given by

$$A_0 = \lim_{s \longrightarrow 0} sA_0^*(s) = \lim_{t \longrightarrow \infty} A_0(t) \tag{2.6}$$

Busy Period Due To Repair

$B_i(t)$ is defined as the probability that the system is busy at epoch t starting from state $S_i \in E$. we have the following recursive relation:

$$B_0(t) = q_{01}(t)©B_1(t)$$

$$B_1(t) = w_1(t) + q_{10}(t)©B_0(t) + q_{11}^{(2)}©B_1(t) \tag{2.7}$$

By taking Laplace transforms of the above equations and solving for $B_0^*(s)$ we get;

$$B_0^*(s) = \frac{N_2(s)}{D(s)} \tag{2.8}$$

where

$N_2(s) = q^*_{01}(s)w^*_1(s)$

$D(s) = 1 - q^{*(2)}_{11}(s) - q^*_{01}(s)q^*_{10}(s)$

The steady-state of the above busy period is given by

$$B_0 = \lim_{s \longrightarrow 0} sB^*_0(s) = \lim_{t \longrightarrow \infty} B_0(t) \tag{2.9}$$

Expected Number Of Visits By The Server

The following equations determining the expected number of visits:

$$N_0(t) = Q_{01}(t)Ⓡ[1 + N_1(t)]$$

$$N_1(t) = Q_{10}(t)ⓇN_0(t) + Q^{(2)}_{11}(t)ⓇN_1(t) \tag{2.10}$$

By taking L.S.T of the above equation and solving for $N^*_0(s)$.we get

$$N^{**}_0(s) = \frac{N_3(s)}{D(s)} \tag{2.11}$$

where

$N_3(s) = Q^{**}_{01}(s)[1 - Q^{**(2)}_{12}(s)]$

$D(s) = 1 - q^{*(2)}_{11}(s) - q^*_{01}(s)q^*_{10}(s)$

The steady-state of the busy period due o server is given by:

$$N_0 = \lim_{s \longrightarrow 0} sN^{**}_0(s) = \lim_{t \longrightarrow \infty} N_0(t) \tag{2.12}$$

Profit Analysis

Profit incurred to the system in time dependent state is given by:

$P(t) = K_0A_0(t) - K_1B_0(t) - K_2N_0(t)$

Profit incurred to the system in steady state is given by:

$P = K_0A_0 - K_1B_0 - K_2N_0$

Where

K_0=fixed revenue per unit up time of the system

K_1=fixed cost per unit time for which server is busy

K_2=fixed cost per visit by the server.

Hence by using the above results for a particular case by assuming all distributions exponential ,we can obtain the expected profit incurred to the system.

Numerical Results

In this section,some of the results obtained for the above model are illustrated with a numerical example,we assume that

$f(t) = \lambda e^{-\lambda t} \qquad g(t) = \theta e^{-\theta t}$

From equation (2.2),the time-dependent reliability is given by:

$$R_0^*(t) = \sum_{1=1}^{2} \frac{(s_i + \theta + 2\lambda)e^{s_i t}}{\prod_{j=1;i\neq j}^{2} (s_i - s_j)}$$

where $s_i^{'}$ are the roots of the equation.

s$^2 + (2\lambda + \theta) + \lambda^2 = 0$

Hence the mean time to failure of the system is calculated using the relation

$$\text{MTSF}=R_0^*(0) = \int_0^{\infty} R(t)dt$$

$$\text{MTSF}=\frac{(\theta + 2\lambda)}{\lambda^2}$$

Now from equation (2.5) the time-dependent availability of the system is given by:

$$A_0^*(t) = \sum_{1=1}^{2} \frac{[(s_i + \lambda)(s_i + \lambda + \theta)^2]e^{s_i t}}{\prod_{j=1;i\neq j}^{2} (s_i - s_j)}$$

where $s_i' s$ are the roots of the equation given below:

$$\lambda s^2 + s(2\lambda^2 + 2\theta\lambda) + \lambda(\lambda + \theta)^2 = 0$$

In case steady-state availability of the system given by:

$$A_0 = \frac{\theta(\lambda + \theta)}{\lambda(\lambda + \theta) + \theta^2}$$

From equation (2.8) the time-dependent busy period analysis of system is given by:

$$B_0^*(t) = \sum_{1=1}^{2} \frac{[(s_i + \lambda)(s_i + \lambda + \theta)^2]e^{s_i t}}{\prod_{j=1;i\neq j}^{2} (s_i - s_j)}$$

where $s_i's$ are the roots of the equation given below:

$$\theta s^2 + 2\theta s(2\lambda^2 + 2\theta\lambda) + \lambda\theta(\lambda + \theta)^2 = 0$$

From equation (2.9) the steady state busy period due to repair of the system is given by:

$$B_0 = \frac{\lambda(\lambda + \theta)}{\lambda(\lambda + \theta) + \theta^2}$$

The expected no of visits can be calculated from the equation (2.11) as:

$$N_0^*(t) = \sum_{i=1}^{2} \frac{[(s_i + \theta)(s_i + \lambda)(s_i + \lambda + \theta)]e^{s_i t}}{\prod_{j=1; i \neq j}^{2} (s_i - s_j)}$$

where $s_i's$ are the roots of the equation given below:

$$\lambda s^2 + s(2\lambda^2 + 2\theta\lambda) + \lambda(\lambda + \theta)^2 = 0$$

The steady-state expected no of visit is given by:

$$N_0 = \frac{\lambda\theta^2}{\lambda(\lambda + \theta) + \theta^2}$$

CHAPTER III

STUDY OF A RELIABILITY MODEL OF TWO-UNIT COLD STANDBY SYSTEM WITH TWO STAGE REPAIR POLICY

In previous chapter we have studied a two-unit cold standby reliability model with single stage of repair. But in case, if it may not possible to repair the unit at first stage then repair is completed in second stage.

Keeping this in view here we study a reliability model investigated by Khaled M. El-Said and Mohamed S. El-Sherbeny (2010).The repair process is divided into two stage.In the first stage,the repairing process of the unit is started but it does not completed,and the repairing process is completed in the second stage.The elasped time between two stages is called the waiting time.Expressions for the various measures of system effectiveness such as the time-dependent availability steady-state availability and busy period analysis are obtained. Using these above measures,profit is calculated.Numerical value for the time-dependent availability,steady-state availability,mean time to failure, and profit function are obtained .

Notation

$h(t), H(t)$ p.d.f. and c.d.f. of the waiting time

$g(t), G(t)$ p.d.f. and c.d.f. of the repair time of a repairman

$f(t), F(t)$ p.d.f. and c.d.f. of the failure time

$A_i(t)$ Probability that the system is up at time t given that the system entered regenerative state i at t = 0

$R_i(t)$ Probability that the system is up (0, t] given that the system entered regenerative state i at t = 0

$B_i(t)$ Probability that repairman is busy at time t given that the system entered regenerative state i at t = 0