**A dissertation report on**

# MATHEMATICAL VALIDATION OF EIGEN VALUES AND EIGEN VECTORS OF DIFFERENT HUMAN BODY SEGMENTS IN SITTING POSTURE

Submitted in partial fulfilment of the requirement for the award of degree of

**MASTER OF ENGINEERING**
**in**
**CAD/CAM & ROBOTICS**


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# CERTIFICATE

This is to certify that the work which is being presented in this thesis entitled "**Mathematical Validation of Eigen Values and Eigen Vectors of Different Human Body Segments in Sitting Posture**" in partial fulfilment of the requirement for the award of degree of Master of Engineering in CAD/CAM & Robotics in the Mechanical Engineering Department, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. S. P. Nigam, Visiting Professor, and Dr. Ashish Singla, Assistant Professor, Mechanical Engineering Department, Thapar University, Patiala. The matter embodied in this report has not been submitted in part or full to any other university or institute for the award of any degree.

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This is to certify that the above declaration made by the student concerned is correct to the best of our knowledge and belief.

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**(Harry Bansal)**

# ABSTRACT

An 11 degrees-of-freedom multi-body vibratory sitting model with backrest of a human being is considered for vertical as well as horizontal mode of vibrations. Dimensional and Inertial properties of human body are determined from primary inputs of anthropometric data available of 50$^{th}$ percentile Indian male. Mathematical analysis for evaluating the horizontal and vertical stiffness is developed, assuming the shape of all the segments as truncated ellipsoid. Equations of motion are formulated from different segment combinations of model. Using MATLAB, Eigen values and Eigen vectors of different body segments have been computed and found in close agreement with published literature.

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# LIST OF SYMBOLS

| | |
|---|---|
| $\rho$ | Density |
| $E$ | Modulus of Elasticity |
| $l$ | Length |
| $t$ | Time |
| $g$ | Acceleration due to gravity |
| $\omega_n$ | Natural frequency |
| $\omega$ | Frequency in radian |
| $f$ | Frequency in Hertz |

# CHAPTER 1

# INTRODUCTION

## 1.1 Vibrations

Vibrations can be defined as oscillations of mass about a fixed point. Vibration is a form of mechanical wave and, like all waves, it transfers energy but not matter. Vibration needs a mechanical structure through which it needs to travel. This structure might be part of a machine, vehicle, tool, or even a person, but if a mechanical coupling is lost, then the vibration will no longer propagate [1].

The common examples are locomotives, diesel engines mounted on unsound foundations, whirling of shafts, etc. The main causes of vibration are as follows:

(i) Unbalanced forces in the machine.

(ii) Dry friction between the two mating surfaces.

(iii) External excitations.

(iv) Earthquakes.

(v) Winds.

Vibrations are occasionally "desirable", e.g. the motion of a tuning fork, mobile phones or the cone of a loudspeaker is desirable vibration, necessary for the correct functioning of the various devices.

More often, vibrations are undesirable, wasting energy and creating unwanted sound– noise. For example, the vibrational motion of engines, electric motors, or any mechanical device in operation are typically unwanted. Such vibrations can be caused by imbalances in the rotating parts, uneven friction, the meshing of gear teeth, etc. Careful designs usually minimize unwanted vibrations. In examination of the impact of vibration upon the human body, there are several characteristics of the vibration that must be taken into account. Direction, rotation, frequency, magnitude, point of entry, and duration are all factors, which play a part in determining how vibration is transmitted throughout the body, and hence, how the body changes as a result.

The magnitude of vibration is usually expressed in terms of acceleration, velocity and displacement. The International Organization for Standardization (ISO) has provided the following guide lines relating the magnitude to comfort for passengers on public transport as tabulated in Table 1.1.

Table 1.1 ISO Guidelines on magnitude and comfort level

| **Magnitude of vibration ($m/s^2$)** | **Comfort level** |
|---|---|
| <0.5 | Comfortable |
| 0.5–1 | Little uncomfortable |
| 0.8–1.6 | Uncomfortable |
| 1.25–1.5 | Very Uncomfortable |
| >2 | Highly Uncomfortable |

Elimination or reduction of the undesirable vibrations can be obtained by one or more of the following methods:

(i) Removing the causes of vibrations.

(ii) Putting the screens if noise is objectionable.

(iii) Placing the machinery on proper isolators.

(iv) Using shock absorbers.

(v) Using dynamic vibration absorbers.

## 1.2 Human-Body Vibrations

The human body is made up of organ systems composed of different tissues. When it comes in contact with a source of vibration it reacts as a set of linked masses. Each tissue mass has its own natural frequency. Smaller structures tend to resonate at higher frequencies and larger masses tend to resonate at lower frequencies. Biologically, the situation is by no means simpler, especially when psychological effects are included. In considering the response of man to vibrations and shocks it is necessary, however, to take into account both mechanical and psychological effects. In this thesis, the forces was kept on human response to vibration, and vibration transmission within the human body. The human body is a complex system. Thus, the analysis of such a system and the resultant response, will also be complex which has calculated in this course. However, the basic principle can be understood by examining a simple system consisting of one element. In many circumstances the complex response of the body can be approximated with reasonable accuracy by a simple system.

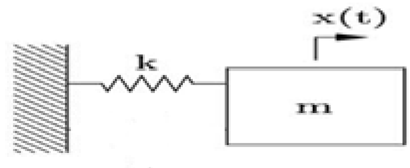



Fig. 1.1: Spring-mass system [7].

The magnitude of vibration, and the natural frequency of the system depends on these properties:

acceleration ($\ddot{\mathbf{x}}$) × mass (**M**)

displacement (**x**) × stiffness (**K**)

Any forces acting on the mass will produce an acceleration of the mass, which will result in an equal and opposite force (mass x acceleration). If the system is displaced and released, it will oscillate freely. Thus one can write a force equilibrium equation for the system as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \tag{1.1}$$

## 1.3 Vibration Axis

Vibration can occur in any direction. The six orthogonal co-ordinate systems are defined in different international standards (e.g. ISO 2631:1974, 1974, 1985) for the expression of magnitude and assessment of exposure in different directions. The co-ordinate includes three translational (‘vertically *z*- axis’, ‘laterally *y* axis’, and ‘fore and aft *x*-axis’), and three rotational (‘pitch about *y*-axis’, ‘yaw about *z*-axis’, and ‘roll about *x*-axis’) directions.

Some biodynamic studies of human body also require the orientation of human body in different planes. Though, the three perpendicular planes of human body include: the *x*-*z* plane, *x*-*y* plane, *y*-*z* plane are known as Sagittal, Transverse or Horizontal, and Frontal plane respectively.

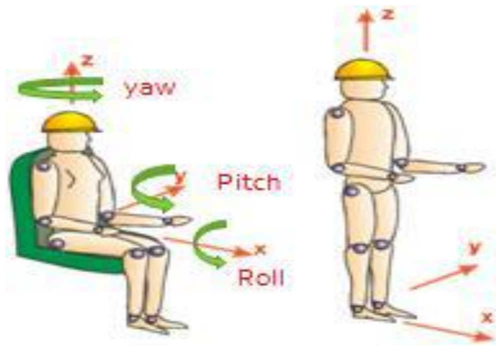



Fig. 1.2: (a) Sitting posture.      (b) Standing posture [7].

## 1.4 Types of Vibrations

### 1.4.1 Free Vibrations

Free vibrations occur, when a mechanical system is set off with an initial input and then allowed to vibrate freely. Examples of this type of vibration are pulling a child back on a swing and then letting go or hitting a tuning fork and letting it ring. The mechanical system will then vibrate at one or more of its "natural frequency" and damp down to zero. The motion of the system is described by a single coordinate x(t) and hence it has one degrees-of-freedom (DOF). The equation of motion for the free vibration of an undamped single degrees-of-freedom system can be rewritten as :

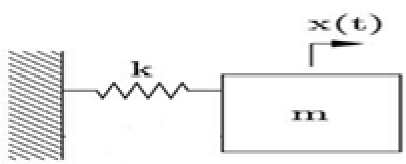



Fig. 1.3: Free Vibration [7].

$$m\ddot{x}(t) + kx(t) = 0 \qquad (1.2)$$

Dividing through by m, the equation can be written in the form

$$\ddot{x}(t) + \omega_n^2 x(t) = 0 \qquad (1.3)$$

in which $\omega_n = (k/m)^{1/2}$ s a real constant.

### 1.4.2 Forced Vibrations

Forced vibrations occur, when an alternating force or motion is applied to a mechanical system. Examples of this type of vibration include a shaking washing machine due to an imbalance, transportation vibration (caused by truck engine, springs, road, etc.), or the vibration of a building during an earthquake. In forced vibration, the frequency of the vibration is the frequency of the force or motion applied, with order of magnitude being dependent on the actual mechanical system.

To start the investigation of the mass-spring-damper system, it is assumed that the damping is negligible and that there is no external force applied to the mass (i.e. free vibration). The force applied to the mass by the spring is proportional to the amount the spring is stretched "x".

$$F = -k * x \tag{1.4}$$

Consider a viscously damped single degrees-of-freedom spring mass system shown in Fig. subjected to a harmonic function F(t)= $F_o$ Sin wt where $F_o$ is the force amplitude and $\omega$ is the circular frequency of the forcing function.

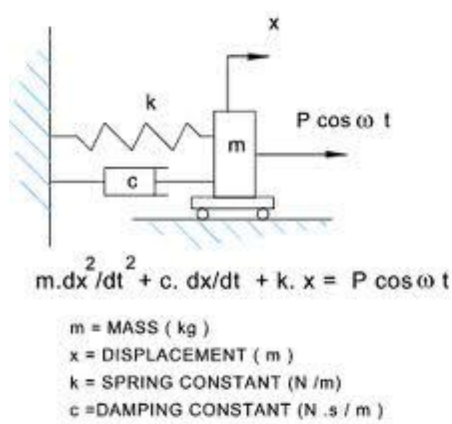



Fig. 1.4: Forced Vibrations [7].

The equations of motion of the system is:

$$\ddot{x} + (c/m)\,\dot{x} + (k/m)\,x = (F_o/m) \text{ Sin wt} \tag{1.5}$$

### 1.4.3 Natural Frequency and Resonance

All material bodies have a natural frequency, which to some extent can be compared to the natural frequency of a swinging pendulum. When a body is exposed to a frequency vibration that coincides with its own natural frequency, it will vibrate strongly.

The various parts and organs of the human body have different natural frequencies. This means that the body does not vibrate uniformly, but rather different parts behave exactly different to one another. External vibrations with frequencies of about 6 Hz are amplified through resonance in the abdomen (or central torso) by up to 200%. Certain vibrations are amplified in the spine by up to 240%. The resonance phenomenon leads to a greater load on the body and thereby a greater risk of injury.

## 1.5 Whole-body vibrations

In the work setting, there are basically three types of vibrations that are significant to the human body. These include whole body vibration, segmental vibration and hand-arm vibrations. The most common is whole body vibration.

### 1.5.1 Whole-body Vibrations

In this case, vibration enters the body through the buttocks and the feet and to a lesser extent through the hands. Describing the consequences of shaking the whole human body- a complex, active, intelligent structure is not a simple matter. The National Board of Occupational Safety and Health has compiled the known effects of humans being over-exposed to whole-body vibrations.

### 1.5.2 Segmental Vibrations

Segmental vibrations are the result of mechanical vibrations that comes in contact with body parts and the effect becomes localized.

### 1.5.3 Hand-arm Vibrations

Hand-arm vibrations are vibration transmitted into workers' hands and arms. This can come from use of hand-held power tools (such as grinders or road breakers) or hand-guided equipment (such as powered lawnmover) and is the cause of significant ill health (painful and disabling disorders of the blood vessels, nerves and joints).

Needless to say, there are also vibrations that are positive. For instance, vibrations that inform drivers about the movement of their vehicle, that they are driving over a zebra crossing or that the right front wheel has a puncture. For safety reasons, such vibrations considers with a sense of the road which cannot be dampened.

"Whole-body vibrations" are those vibrations which affects the whole of the exposed person, i.e. the vibration affects all parts of the body. It is usually transmitted through seat surfaces, backrests, and through the floor, although it can also be important for those standing or lying, such as for those traveling on commuter trains or patients being transported by ambulance. Most whole-body vibration exposures are associated with transportation where vehicle drivers or passengers are exposed to mechanical disturbances and impacts while traveling. Whole-body vibration can affect comfort, performance, and health, depending on the magnitude, waveform, and exposure times. People are most sensitive to whole-body vibration within the frequency range of 0.5 to 80 Hz, although many measurements include higher frequencies.

Biodynamic and physiological experiments have shown that seated WBV exposure can affect the spine by mechanical overloading and excessive muscular fatigue, supporting the epidemiologic findings of a possible causal role of WBV in the development of (low) back troubles. The review also mentions that it is estimated that 4-7% of the working population in the EU is exposed to potentially harmful whole-body vibration.

## 1.6 Theory of Whole Body Vibrations

- Every object (or mass) has a resonant frequency.
- When an object is vibrated at its resonance frequency, the maximum amplitude of its vibration will be greater than the original amplitude (i.e. the vibration is amplified).
- Vibrations in the frequency range of 0.5 Hz to 80 Hz have significant effects on the human body.
- Individual body members and organs have their own resonant frequencies and do not vibrate as a single mass, with its own natural frequency. This causes amplification or attenuation of input vibrations by certain parts of the body due to their own resonant frequencies.
- The most effective resonant frequencies for vertical vibration lie between 4 and 8 Hz.
- Vibrations between 4 and 6 Hz set up resonances in the trunk with amplification of up to 200%.
- Whole body vibration may create chronic stresses and sometimes even permanent damage to the affected organs or body parts.

## 1.7 Measurement system

- Hand held vibration meter (accelerometer)
- Frequency of the vibration (cycles/second) measured in Hertz (Hz)
- Intensity of vibration measured in:

– Amplitude/displacement (m)

– Velocity (m/s)

– Acceleration ($m/s^2$)

– Jerk (rate of change of acceleration – $m/s^3$)

- G – force of gravity ( 9.81 $m/s^2$)
- Power Spectral Density (PSD) – the power at discrete frequencies within a selected bandwidth.
- Root mean square acceleration (RMS) – the total energy across the entire frequency range.

## 1.8 Biodynamic Response Functions

The biodynamic response of the seated human body subjected to vibration has widely been assessed in terms of the different biodynamic response functions: driving point mechanical impedance (DPMI), apparent mass (APMS) and seat-to-head transmissibility (STHT). While the first two functions have often been used interchangeably to describe "to the body" force-motion relationship as a frequency function at the human seat interface ("to the body" transfer function), the third function refers specifically to the transmission of motion through the body.

Mechanical impedance is the inverse of mechanical admittance or mobility. The mechanical impedance is a function of the frequency of the applied force and can vary greatly over frequency. At resonant frequencies, the mechanical impedance will be lower, meaning less force is needed to cause a structure to move at a given velocity. The simplest example of this is when a child pushes another on a swing. For the greatest swing amplitude, the frequency of the pushes must be near the resonant frequency of the system.

$$Z(w)=F(w)/\,v(w) \tag{1.6}$$

Where, F is the force vector, 'v' is the velocity vector, Z is the impedance matrix and $\omega$ is the angular frequency.

Mechanical impedance is the ratio of a potential (e.g. force) to a flow (e.g. velocity) where the arguments of the real (or imaginary) parts of both increase linearly with time. Examples of potentials are: force, sound pressure, voltage, temperature. Examples of flows are: velocity, volume velocity, current, heat flow.

### 1.8.1 Driving-point mechanical impedance

The driving-point mechanical impedance, DPMI, is defined as: The complex ratio of applied periodic excitation force at frequency, f, to the resulting vibration velocity at that frequency, measured at the same point and in the same direction as the applied force.

### 1.8.2 Apparent Mass

For the human body the force required to accelerate the supporting surface is a complex function of frequency. This function is known as apparent mass.

### 1.8.3 Transmissibility

Transmissibility is the ratio of output to input.

Transmissibility (T) = output/input

### 1.8.4 Seat-to-head Transmissibility

This function is defined as the ratio of output responses (such as head) to input excitation (such as seat) at different points. It is frequently called the acceleration or displacement ratio and unit less function.

$T>1$ means amplification and maximum amplification occurs when forcing frequency and natural frequency of the system coincide. The transmissibility is used in calculation of passive have compensation efficiency.

The aim of this introductory section is to provide a brief outline of the characteristics of vibration, the different ways in which human may be exposed to it and the response of human to vibration.

## 1.9 Eigen Values and Eigen Vectors

It is assumed that calculator is being used that can handle matrices, or a program like MATLAB. Many problems present themselves in terms of an eigenvalue problem:

$$\mathbf{Av} = \lambda\mathbf{v} \qquad \textbf{(1.7)}$$

In this equation '*A*' is an n*n matrix, '*v*' is a non-zero n*1 vector and $\lambda$ is a scalar (which may be either real or complex). Any value of $\lambda$ for which this equation has a solution is known as an eigenvalue of the matrix '*A*'. It is sometimes also called the characteristic value. The vector, *v*, which corresponds to this value is called an eigenvector. The eigenvalue problem can be rewritten as:

$$\mathbf{A}\cdot\mathbf{v}-\lambda\cdot\mathbf{v} = 0$$
$$\mathbf{A}\cdot\mathbf{v}-\lambda\cdot\mathbf{I}\cdot\mathbf{v} = 0$$
$$(\mathbf{A}-\lambda\cdot\mathbf{I})\cdot\mathbf{v} = 0 \qquad (1.8)$$

If **v** is non-zero, this equation will only have a solution if

$$|\mathbf{A}-\lambda\cdot\mathbf{I}| = 0 \qquad (1.9)$$

This equation is called the characteristic equation of A, and is an $n^{th}$ order polynomial in $\lambda$ with *n* roots. These roots are called the eigenvalues of A. It will only deal with the case of n distinct roots, though they may be repeated. For each eigenvalue there will be an eigenvector for which the eigenvalue equation is true. In this thesis, eigen values and eigen vectors has determined by using MATLAB.

## 1.10 Vibration Analysis

The analysis of a vibrating system, depends upon many factors such as: the initial conditions, and external excitations. The four steps of a vibration analysis of a physical system are:

1. Preparation of a Mathematical Modelling
2. Formulation of Governing Equations
3. Solution of the Equations of motion
4. Physical Interpretation of results.

## 1.11 ANALYTICAL WORKS

The human body is a very sophisticated dynamic system whose mechanical properties vary from moment to moment and from one individual to another. In the past few decades, plenty of mathematical models have been developed on the basis of diverse field measurements to describe the biodynamic responses of human beings. The main objective of the study of human response to vibration is the definition of an engineering model of the response of the body. Modeling should involve the identification, and inclusion of variables of greatest importance. A biodynamic model has defined relationships between one or more inputs as independent variables (input variable) and one or more outputs as dependent variables. A model is intended to represent the responses in terms of forces and movements of specific people within a specific range of vibrating conditions. A model cannot represent all the functions of the system but represent one or more aspect of the system.[20]

These models could be achieved by different modeling techniques: such as lumped-parameter (LP) models, finite element (FE) models, and multi-body models.

## 1.12 Modelling Techniques

The human body is a very sophisticated dynamic system whose mechanical properties vary from moment to moment and from one individual to another. In the past few decades, plenty of mathematical models have been developed on the basis of diverse field measurements to describe the biodynamic responses of human beings. According to different modeling techniques, these models can be grouped as lumped-parameter models, finite element (FE) models, and multibody models.

### 1.12.1 Lumped-parameter model (LPM)

Lumped-parameter models consider the human body as several concentrated masses interconnected by springs and dampers. This type of model is simple to analyze and easy to validate with experiments. However, the disadvantage is the limitation to one-directional analysis. Coermann (1962) measured human DPM impedance in sitting and standing situations, and suggested a one-DOF model. Suggs et al. (1969) developed a two-DOF model based on measurements using a mechanical simulator to account for the dynamics of the human body when seated under the influence of vibration.

### 1.12.2 Finite Element Model (FEM)

The FE models hypothesize that the human body consists of numerous finite elements whose element properties were mainly obtained from experiments on human corpses. With the incorporation of certain powerful FE programs, this kind of model could be exploited to study the biodynamic responses as well as to assess injury in humans, as long as accurate input human properties are used. However, how well tests on human corpses represent the live human needs further verification [10].

### 1.12.3 Multi-body Model (MB Model)

Multibody human models are made of several rigid bodies interconnected by pin (two-dimensional) and/or ball and socket (three-dimensional) joints, and can be further separated into kinetic and kinematic models. The kinematic models are mainly concerned with the motion of each segment, and are often used in the study of human exercise and injury assessment in a vehicle crash. To investigate the biodynamic behavior due to external excitation, torsional/linear springs and dampers are constantly attached between two body segments, and form kinetic models. That is, kinetics is interested in the study of forces associated with motion, while kinematics is a study of the description of motion, including considerations of space and time.

To investigate the biodynamic behavior due to external excitation, torsion as well as springs and dampers are constantly attached between two body segments, and form kinetic models. That is, kinetics is interested in the study of forces associated with motion, while kinematics is a study of the description of motion, including considerations of space and time. The most important difference between a lumped mass model and a multi-body model is that bodies in a multi-body formulation can be connected by various joint types, due to which the number of degrees of freedom between the elements can be constrained.

# CHAPTER 2

# LITERATURE REVIEW

Whole-body vibration experiments have been extensively performed for a wide range of applications like in driving an automobile, sitting in an erect position, etc. In the past years, a lot of work has been done to investigate and analyze the whole body vibrations in sitting posture. A review of the literature is presented here.

## 2.1 LITERATURE

- **J. A. Bartz and C. R. Gianotti (1975)** developed a digital computer program to calculate dimensional and inertial properties of the human body. The program has been designed so that the user may either select a data set from a program library, or compute a data set from a geometric man-model. From primary program inputs of standing height, seated height, and weight, the program compute body segment link lengths, contact surface dimensions, masses, and moments of inertia from input sets of anthropometric data. Overall validity of the formulation and techniques has been established by comparing computed results with measurements on the human body reported by various investigators.
- **S. P. Nigam and M. Malik (1987)** presented an analysis for evaluating the mass and stiffness values of the model with certain assumptions. As an illustration of the modeling procedure, a 15-degree-of-freedom model of a male body is considered. The computed natural frequencies of the model are found to be within the range of available experimental values.
- **P. E. Boileau and S. Rakheja (1998)** performed measurements for seated subjects with feet supported and hands held in a driving position. Variations in the seated posture, backrest angle, and nature and amplitude of the vibration excitation are introduced within a prescribed range of likely conditions to illustrate their influence on the driving-point mechanical impedance of seated vehicle drivers. Within the 0.75-10 Hz frequency range and for excitation amplitudes maintained below $4 m/s^2$, a four-degree-of-freedom linear driver model is proposed for which the parameters are estimated to satisfy both the measured driving-point mechanical impedance and the seat-to-head transmissibility characteristics defined from a synthesis of published data for subjects seated erect without backrest support.

- **Younggun Cho and Yong-San Yoon (2001)** studied the biomechanical model of a human on a seat with backrest for evaluating the vehicular ride quality. In describing the human body motion, four biomechanical models are discussed: 1 DOF (degree-of-freedom) model mainly describes the z-axis motion of the hip, and 2 and 3 DOF models describe the z-axis motion of the hip and head while 9 DOF model we proposed includes the motions of the floor, hip, back, and head to describe the whole-body vibration in a sitting posture with backrest support. To validate the proposed model, we measured the accelerations at the hip, back, and head of 10 subjects with the floor under vertical vibration excitation. From this measurement, three transmissibilities for each subject were obtained. We also measured the parameters including the material property of seat cushion, joint positions of human body, and the contact positions between human body and seat, while the other parameters such as stiffness and damping at the hip, back, and head were determined by matching the model transmissibilities to the experimental ones.

- **Bauchau *et al.* (2003)** presented a framework for the development of robust time integration schemes for nonlinear, flexible multibody systems. The proposed schemes are designed to meet four specific requirements: nonlinear unconditional stability of the scheme, a rigorous treatment of both geometric and material nonlinearities, exact satisfaction of the constraints, and the presence of high frequency numerical dissipation. Specific algorithms are presented for rigid bodies, cables, beams, shells, and elasto-dynamics. The kinematic nonlinearities are treated in a rigorous manner for all elements, and the material nonlinearities can be handled when the constitutive laws stem from the existence of a strain energy density function. The treatment of the constraint equations associated with the six lower pair joints are presented as well.

- **N. Nawayseh and M. J. Griffin (2003)** investigate the nonlinearity in their biodynamic responses and quantify the response in directions other than the direction of excitation. Twelve males were exposed to random vertical vibration in the frequency range 0.25–25Hz at four vibration magnitudes (0.125, 0.25, 0.625, and 1.25ms$^{-2}$ r.m.s.). The subjects sat in four sitting postures having varying foot heights so as to produce differing thigh contact with the seat (feet hanging, feet supported with maximum thigh contact, feet supported with average thigh contact, and feet supported with minimum thigh contact). Forces were measured in the vertical, fore-and-aft, and lateral directions on the seat and in the vertical direction at the footrest. The characteristic non-linear response of the human body with reducing resonance frequency at increasing vibration magnitudes was seen in all postures, but to a lesser extent with minimum thigh contact. Appreciable forces in the fore-and-aft direction also showed non-linearity, while forces in the lateral direction were low and showed no consistent trend.

- **Kim *et al.* (2005)** developed a biomechanical model of the human body for evaluating the vibration transmissibility and dynamic response to vertical vibrations in sitting posture. The biomechanical model consists of several lumped masses connected by linear spring and dampers. This model was selected from eight structural model after fitting measured data of apparent mass, vertical and rotational vibration transmissibilities to the head of five individuals on a vertical exciter vibrating under random signals.

- **Ya Huang and Michael J. Griffin (2006)** investigated the effects of vibration magnitude and voluntary periodic muscle activity on the apparent mass resonance frequency using vertical random vibration in the frequency range 0.5–20 Hz. Each of 14 subjects was exposed to 14 combinations of two vibration magnitudes (0.25 and 2.0ms$^{-2}$ (rms)) in seven sitting conditions: two without voluntary periodic movement, and five with voluntary periodic movement. Three conditions with voluntary periodic movement significantly reduced the difference in resonance frequency at the two vibration magnitudes compared with the difference in a static sitting condition. Without voluntary periodic movement (condition A:upright), the median apparent mass resonance frequency was 5.47 Hz at the low vibration magnitude and 4.39 Hz at the high vibration magnitude. With voluntary periodic movement (C: back-abdomen bending), the resonance frequency was 4.69 Hz at the low vibration magnitude and 4.59 Hz at the high vibration magnitude.

- **Cho-Chung Liang and Chi-Feng Chiang (2006)** studied the lumped-parameter models for seated human subjects without backrest support under vertical vibration excitation has been carried out. As part of the study, all models have been analyzed systematically, and validated by the synthesis of various experimental data from published literature. Based on the analytical study and experimental validation, the four degree-of-freedom (DOF) model is developed. A simple model that captures the essential dynamics of a seated human exposed to whole body vibration.

- **Rutzel *et al.* (2006)** analysed and averaged the data measurements from subjects with respect to certain attributes (anthropometrics, posture, excitation intensity, etc.). The use of a modal description as an alternative method is presented and its contribution to biodynamic modelling is discussed. Modal description is not limited to just one biodynamic function: The method holds for all transfer functions. This is shown in terms of the apparent mass and the seat-to-head transfer function. The advantages of modal description are illustrated using apparent mass data of six male individuals of the same mass percentile. From experimental data, modal parameters such as natural frequencies, damping ratios and modal masses are identified which can easily be used to set up a mathematical model. Following the phenomenological approach, this model will provide the global vibration behavior relating to the input data.

- **Hinz *et al.* (2007)** provided the possibility of predicting the forces acting on the disc or endplates of lumbar vertebrae. Due to the complex structure of the human body, complex dynamic models based on human anatomy are required to adequately reflect the dynamic properties of the body. Based on experimental results the influence of posture and stature on the biodynamic behaviour of human subjects is described. To reflect the biodynamic response of different occupational groups of workers exposed to whole-body vibration, an existing model was adapted to five typical different postures.

- **T. C. Gupta (2007)** developed a 15 degrees of freedom lumped parameter vibratory model of human body for vertical mode vibrations, using anthropometric data of the 50th percentile US male. The mass and stiffness of various segments are determined from the elastic moduli of bones and tissues and from the anthropometric data available, assuming the shape of all the segments is ellipsoidal. The damping ratio of each segment is estimated on the basis of the physical structure of the body in a particular posture. Damping constants of various segments are calculated from these damping ratios. The human body is modeled as a linear spring-mass-damper system. The optimal values of the damping ratios of the body segments are estimated, for the 15 degrees of freedom model of the 50th percentile US male, by comparing the response of the model with the experimental response.

- **Cho-Chung LIANG and Chi-Feng CHIANG (2008)** investigated in describing the motions of a seated body, two multibody models representative of the automotive postures, one with and the other without a backrest support. Both models were modified to suitably represent the different automotive postures with and without backrest supports, and validated by various experimental data. On the basis of the analytical study and the experimental validation, the fourteen-degrees-of-freedom model proposed in this research was found to be best fitted to the test results.

- **Patra *et al.* (2008)** studied the biodynamic responses of seated humans within three different body mass ranges are characterized under different magnitudes of vibration and three different sitting postures in an attempt to define reference values of apparent mass for applications in mechanical-equivalent model development and anthropodynamic manikin design. Laboratory measurements were performed with adult male subjects of total body mass in the vicinity of 55, 75 and 98 kg (nine subjects for each mass group) seated with and without an inclined back support and exposed to three different magnitudes of noise vertical vibration in the frequency range between 0.5 and 20 Hz. The mean magnitude responses of subjects within three mass groups were compared with idealized ranges defined in ISO 5982 and DIN 45676 for no back support condition. The results revealed significant differences between the mean measured and standardized magnitudes, suggesting that the current standardized values do not describe the biodynamic responses of seated occupant of different masses even for the back not supported condition.

- **Santos *et al.* (2008)** tested different biomechanical responses before and after 60 min of sitting, with and without vertical WBV, on four different days. Postures adopted while sitting and the simulated WBV exposure corresponded to large mining load haul dump (LHD) vehicles as measured in the field. Twelve males performed trials of standing balance on a force plate and a sudden loading test to assess back muscle reflex response, using surface electromyography (EMG). This latter test also allowed to assess if any muscle fatigue occurred as a result of the exposure. Though WBV may not have elicited any effects, new findings have emerged concerning the effect of sitting on muscle fatigue and balance. It was shown that sustaining trunk sitting postures corresponding to mining vehicle operators generate back muscle fatigue. Unexpectedly, standing balance was also improved.

- **Wang *et al.* (2008)** experimented which involve 12 male subjects exposed to three magnitudes of whole-body vertical random vibration (0.25, 0.5, 1 m/s$^2$ rms acceleration) in the 0.5–15 Hz frequency range, and seated with three back support conditions (none, vertical and inclined), and two different hands positions (hands in lap and hands on the steering wheel). The vertical apparent mass and seat-to-head transmissibility responses were acquired during the experiments, where the head acceleration was measured using a light and adjustable helmet-strap mounted accelerometer. The results showed that both the measured responses show good agreements in the primary resonances, irrespective of the back support condition, while considerable differences between the normalized apparent mass and seat-to-head transmissibility could be seen in the secondary resonance range for the two back supported postures. The seat-to-head transmissibility responses are further shown to be relatively sensitive to back supported postures compared with that of apparent mass responses.

- **Rakheja *et al.* (2010)** performed to identify the most probable ranges of biodynamic responses of the human body to whole-body vibration. These include the driving-point biodynamic responses of the body seated with and without a back support while exposed to fore-aft, lateral and vertical vibration and those of the standing body to vertical vibration, and seat-to-head vibration transmissibility of the seated body. The proposed ranges are expected to serve as reasonable target functions in various applications involving coupled human-system dynamics in the design process, and potentially for developing better frequency-weightings for exposure assessments.

- **Naser Nawayseh and Michael. J. Griffin (2011)** calculated different subjects sitting in four postures both with and without a rigid vertical backrest while exposed to four magnitudes of random fore and aft vibration. The Power absorbed by the body at the supporting seat surface when there was no backrest showed a peak around 1Hz and another peak between 3 and 4Hz. The measurements of absorbed power are consistent with backrests being beneficial during exposure to low frequency fore-and-aft vibration but detrimental with high frequency fore-and-aft vibration.

- **Dong *et al.* (2011)** investigated the basic characteristics of the three axis mechanical impedances distributed at the fingers and palm of the hand subjected to vibrations along three orthogonal directions(x, y and z). Seven subjects participated in the experiment on a novel three-dimensional (3-D) hand-arm vibration test system equipped with a 3-D instrumented handle. The total impedance of the entire hand-arm system was obtained by performing a sum of the distributed impedances. Two major resonances were observed in the impedance data in each direction.

- **Marek A. Ksiazek and Daniel Ziemianski (2012)** presented for the analytical synthesis of an optimal vibration isolation system of a human body including its sensitivity to vibration. The optimal active suspension of a seat has been obtained analytically for a human body represented alternatively by apparent mass weighted by standard frequency domain curves depicting discomfort levels. The analytical functions describing the optimal vibration isolation systems (OVISs) have been obtained in the general form for random stationary acceleration excitations, general forms of apparent mass M(s) of the sitting human body and a selected criterion of isolation.

- **Bazil Basri and Michael J. Griffin (2012)** investigated the discomfort arising from whole-body vertical vibration when sitting on a rigid seat with no backrest and with a backrest inclined at $0^\circ$(upright), $30^\circ$, $60^\circ$, and $90^\circ$. Within each of these five postures, 12 subjects judged the discomfort caused by vertical sinusoidal whole-body vibration relative to the discomfort produced by a reference vibration. The locations in the body where discomfort was experienced were determined for each frequency at two vibration magnitudes. At frequencies around 5 and 6.3 Hz there was less vibration discomfort when sitting with an inclined backrest.

## 2.2 Summary:

From above discussion, it is known that there is a more work has done on experimental apparatus than on Analytical methods. It is true that experiments provide more realistic approach compared to analytical approach especially in case of human body which is known to be the world's complex machine. It has been found that a lot of work is still to be done in Analytical techniques to become comparable against experimental techniques. There is a table provided below to compare the models with their posture, degrees-of-freedom and techniques.

Table 2.1 Comparison of Literature w.r.t Posture, DOF and Modelling scheme used

| | **Author** | **Posture** | **DOF** | **Technique** |
|---|---|---|---|---|
| Analytical | J. A. Bartz and C. R. Gianotti (1975) | Standing | _ | Digital program |
| | S. P. Nigam and M. Malik (1987) | Standing | 15 | LPM |
| | Kim *et al.* (2005) | Seating | 8 | LPM |
| | Cho-Chung Liang Chi-Feng Chiang (2006) | Seating | | LPM |
| | T. C. Gupta (2007) | Standing | 15 | LPM |
| | Cho-Chung Liang Chi-Feng Chiang (2008) | Seating | 14 | Multibody |
| | Marek A. Ksiazek and Daniel Ziemianski (2012) | Seating | 9 | LPM |
| Experimental | P. E. Boileau S. Rakheja (1998) | Seating | 4 | Excitation frequency |
| | Younggun Cho and Yong- San Yoon (2001) | Seating | 1,2,3,9 | Excitation frequency |
| | Bauchau *et al.* (2003) | Seating | 6 | Multibody |
| | N. Nawayseh M. J. Griffin (2003) | Seating | | Excitation frequency |

| | | | | |
|---|---|---|---|---|
| | Ya Huang and Michael J. Griffin (2006) | Seating | 7 | Excitation frequency |
| | Cho-Chung Liang Chi-Feng Chiang (2006) | Seating | 4 | Excitation frequency |
| | Rutzel *et al.* (2006) | Standing | 7 | Excitation frequency |
| | Hinz *et al.* (2007) | Standing/ Seating | 6 | Excitation frequency |
| | Cho-Chung Liang Chi-Feng Chiang (2008) | Seating | 14 | Excitation frequency |
| | Patra *et al.* (2008) | Seating | _ | Excitation frequency |
| | Santos *et al.* (2008) | Standing/ Seating | _ | Excitation Frequency |
| | Wang *et al*. (2008) | Seating | _ | Excitation frequency |
| | Rakheja *et al.* (2010) | Seating | _ | Excitation frequency |
| | Naser Nawayseh and Michael. J. Griffin(2011) | Seating | 7 | Excitation frequency |
| | Dong *et al.* (2011) | Standing/ Seating | 7 | Excitation frequency |
| | Bazil Basri Michael J. Griffin (2012) | Seating | 6 | Excitation frequency |

## 2.3 Gaps in Literature

- Lumped Parameter Model based on experimental study does not exactly reproduce results of other experimental studies.
- Lumped-parameter model is simple to analyze and easy to validate with experiments. However, the disadvantage is the limitation to one dimensional analysis.

- FEM provides an accurate result only with lesser degrees of freedom (DOF) against LPM. For higher DOF with good accuracy, Multi-body model is normally preferred.
- From above discussion, it is known that vertical movement of human body is only considered in exposure to vertical vibration, however further research can be done in respect to horizontal movement. The rotational motion of arms can also be considered to determine the results with more accuracy.

## 2.4 Methodology

To fill some of the above mentioned gaps, research is done to develop an approach for determining frequency values.

This thesis carries out a thorough survey of literature on the lumped parameter models for seated human subjects exposed to vertical vibration. There are thirteen models in total listed in this study, from one to eleven degrees of freedom (DOFs), and including both linear and nonlinear systems. An analytical study is also carried out for each model to derive the system equations of motion (EOMs). Solution techniques used in this study include Multi-body approach in frequency domain for linear models and in time domain for nonlinear ones. With the experimental validation of each model by a synthesis of measured data under a prescribed range of conditions mostly close to the modeling.

# Chapter 3

# Free Body Vibrations in Sitting Posture using Lumped Parameter Modelling

## 3.1 Introduction

Human beings are most sensitive to whole body vibrations under low frequency vibration excitation in a sitting posture, therefore, biodynamic responses of a model has been found to be the topic of research with ever increasing trends towards the high speed vehicles. A number of experimental studies have been reported for the study of human responses to vibration but investigations found are with some limitations. An alternative is to approximate the human body by a mathematical model by considering different masses, springs and effect of bones and tissues in the body. Nigam and Malik [2] have developed a mathematical approach to compute natural frequencies of a vibratory model of a human body and also able to find the truncation factor joining two different segments. Liang and Chiang [10] carried out a thorough survey of literature on 13 different DOF lumped parameter models for seated human subjects exposed to vertical vibration.

## 3.2 Lumped Parameter Model

The lumped element model (also called lumped parameter model, or lumped component model) simplifies the description of the behaviour of spatially distributed physical systems into a topology consisting of discrete entities that approximate the behaviour of the distributed system under certain assumptions. It is useful in mechanical multibody systems, heat transfer, acoustics, etc.

Mathematically speaking, the simplification reduces the state space of the system to a finite number, and the partial differential equations (PDEs) of the continuous (infinite-dimensional) time and space model of the physical system into ordinary differential equations (ODEs) with a finite number of parameters.

The simplifying assumptions in this domain are:

(i) all objects are rigid bodies.

(ii) all interactions between rigid bodies take place via kinematic pairs, springs and dampers.

The anthropometric human body model by Nigam and Malik consisting of 15 body segments and 14 joints is adopted in this study as shown in Fig. 3.1. The anthropometric data provides a natural choice to be identified as the lumped masses of a vibratory model. The values of stiffness and mass of each segment is estimated on the basis of physical structure of segment.

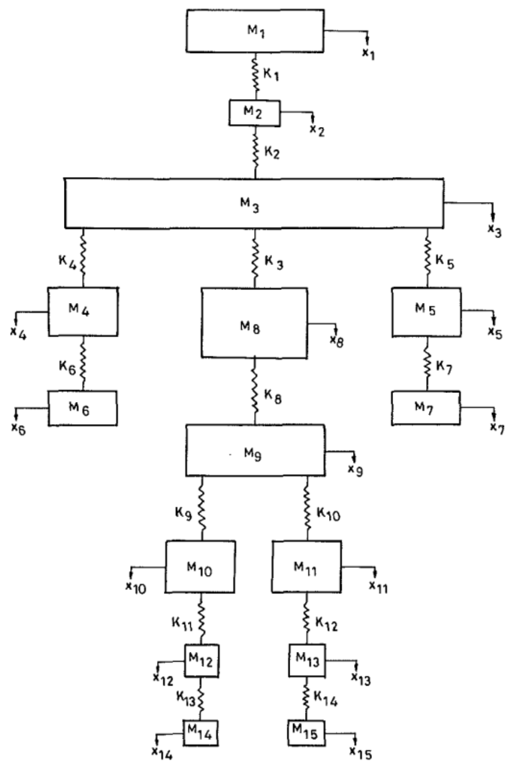
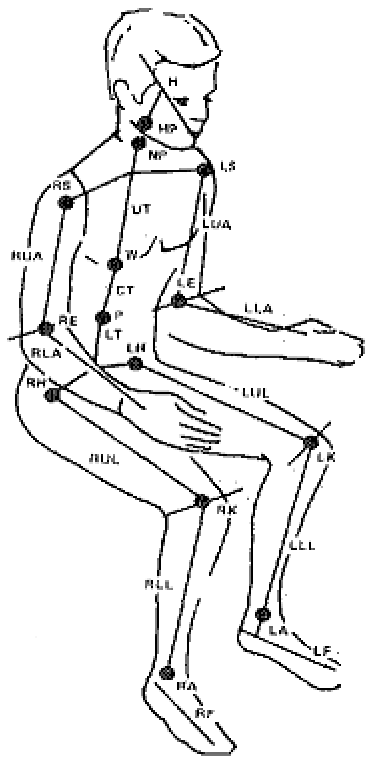



Fig. 1.1: Fifteen segment model [2].

The primary assumptions involved in the development of the model are as follows:

1) For purposes of computing segment dimensions and inertial properties, the body is approximated by 15 rigid body segments connected by 14 joints, in the manner illustrated in fig. 3.2 below:




RUL,LLL-Right Upper and Lower leg, RUA,RLA-Right Upper and Lower arm, UT-Upper Torso, CT-Central Torso, LT-Lower Torso, H-Head, N-Neck, LF,RF-Left and Right foot

Fig. 3.2: Human Body Model [1].

2) Modeling is not restricted only to vertical excitation, horizontal vibrations plays a vital role in the results of natural frequency sitting in an erect state.

3) The different body segments are approximated with simple geometrical shapes. In this model, ellipsoids are used to represent each segment suggested by Bartz and Gianotti.

4) The density of each segment is assumed to be uniform and taken as 1.062 $\times 10^{-3}$ kg/cm$^3$ as volume and mass of human body segments are found to be balancing against each other.

## 3.3 Ellipsoidal Segment

In view of the assumptions, ellipsoids are considered to be best to satisfy the geometrical shapes of different body segments. The mass of an individual segment become proportional to its volume and is expressed as a fraction of total body mass. $a_i$, $b_i$, and $c_i$ are the semi-axes of an $i^{th}$ ellipsoid as defined in fig. 3.3 below and M is the total body mass.

**Nomenclature**

| | |
|---|---|
| $a_i$, $b_i$, $c_i$ | semiaxes of the ellipsoid |
| $d_i$ | half length of the truncated ellipsoid |
| $E$ | elastic modulus of the ellipsoidal segment = $(E_b \, x E_t)^{1/2}$ |
| $E_b$ | Elastic modulus of bones =2.26 $x10^{11}$ dyne/$cm^2$ |
| $E_t$ | Elastic modulus of tissues= 7.5 $x10^4$ dyne/$cm^2$ |
| $f$ | Natural (resonant) frequency of vibratory model |
| $i$ | subscript for ellipsoidal segments |
| $K_i$ | stiffness of spring elements |
| $M_i$ | mass of ellipsoidal segment and of mass element in vibratory model |
| $S_i$ | axial stiffness of ellipsoidal segment |
| $t_r$ | truncation factor, $1-(d_i/c_i)$ |
| $X$ | generalized displacement of vibratory model |
| $\ddot{X}$ | generalized acceleration of vibratory model |

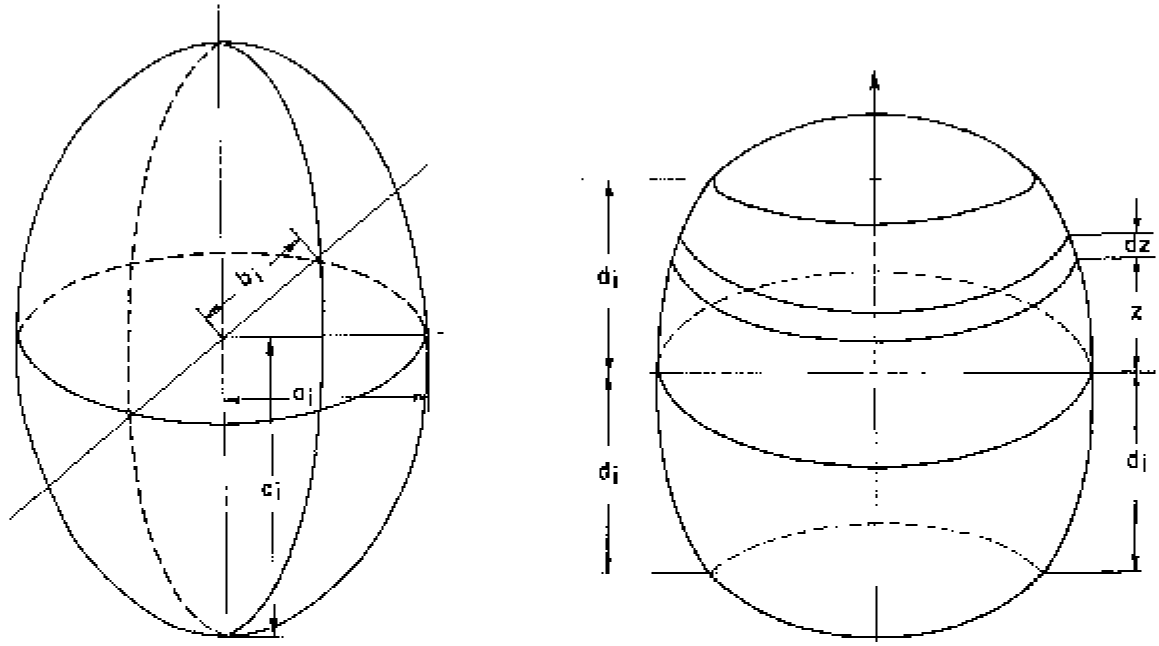



Fig. 3.3: (a) An Ellipsoidal segment[2]. (b) Truncated ellipsoid[2].

## 3.4 Vibratory Model

The stiffness and mass values of the various spring and dashpot elements of the vibratory model are obtained using parallel/series combination of the body segments. The equations of motion of the vibratory model to calculate natural frequency of the free vibration are:

$$\mathbf{M\ddot{x}+Kx= 0} \tag{3.1}$$

where [M] and [K] are mass and stiffness matrices, respectively.

To illustrate the use of above methodology for the mathematical model of a human body, example of a $50^{th}$ percentile male of mass 54kg in sitting posture is considered as shown in Table 3.1

Table 3.1 Anthropometric data of 50th percentile Male [3]

| S. No. | Dimensional Data | | Measurement (cm) |
|---|---|---|---|
| 1. | L1 | Standing Height | 164.8 |
| 2. | L2 | Shoulder Height | 57.6 |
| 3. | L3 | Armpit Height | 41.7 |
| 4. | L4 | Waist Height | 18.8 |
| 5. | L5 | Seated Height | 83.7 |
| 6. | L6 | Head Length | 18.7 |
| 7. | L7 | Head Breadth | 14.7 |
| 8. | L8 | Head to Chin Height | 24.3 |
| 9. | L9 | Neck Circumference | 33.7 |
| 10. | L10 | Shoulder Breadth | 38.0 |
| 11. | L11 | Chest Depth | 20.1 |
| 12. | L12 | Chest Breadth | 29.8 |
| 13. | L13 | Waist Depth | 19.2 |
| 14. | L14 | Waist Breadth | 25.9 |
| 15. | L15 | Buttock Depth | 22.5 |
| 16. | L16 | Hip Breadth Standing | 33.1 |
| 17. | L17 | Shoulder to Elbow Length | 31.2 |
| 18. | L18 | Forearm  Hand Length | 38.9 |
| 19. | L19 | Biceps Circumference | 24.4 |
| 20. | L20 | Elbow Circumference | 27.3 |
| 21. | L21 | Forearm Circumference | 22.8 |
| 22. | L22 | Wrist Circumference | 15.6 |
| 23. | L23 | Knee Height Seated | 51.9 |
| 24. | L24 | Thigh Circumference | 46.7 |
| 25. | L25 | Upper leg Circumference | 42.4 |
| 26. | L26 | Knee Circumference | 35.1 |
| 27. | L27 | Calf Circumference | 31.9 |
| 28. | L28 | Ankle Circumference | 23.9 |
| 29. | L29 | Ankle Height Outside | 6.1 |
| 30. | L30 | Foot Breadth | 9.4 |
| 31. | L31 | Foot Length | 24.8 |
| 32. | L32 | Neck Height | 4.3 |
| 33. | L33 | Upper Leg Length | 27.9 |

Dimensions of the ellipsoids are calculated taking the help of Nigam and Malik methodology.

Table 3.2 Formulae for Statistical dimensions of the ellipsoids representing body segments [2]

| S.No. | Body Segments | Formulae | | | | | | Mass Elements (kg) | |
|---|---|---|---|---|---|---|---|---|---|
| | | $a_i$ (cm) | | $b_i$ (cm) | | $c_i$ (cm) | | | |
| 1. | Head | $L_7/2$ | 7.35 | $L_7/2$ | 7.35 | $L_6/2$ | 9.35 | $M_1$ | 2.247 |
| 2. | Neck | $L_9/2\pi$ | 5.36 | $L_9/2\pi$ | 5.36 | $L_{32}$ | 4.3 | $M_2$ | 0.549 |
| 3. | Upper Torso | $L_{12}/2$ | 14.9 | $L_{11}/2$ | 10.05 | $L_{17}/2$ | 15.6 | $M_3$ | 10.395 |
| 4. | Central Torso | $L_{14}/2$ | 12.95 | $L_{13}/2$ | 9.60 | $(L_{17}+L_{18})/4$ | 17.525 | $M_8$ | 9.695 |
| 5. | Lower Torso | $L_{16}/2$ | 16.55 | $L_{15}/2$ | 11.25 | $L_{18}/4$ | 9.725 | $M_9$ | 8.058 |
| 6. | Upper Arms | $L_{19}/2\pi$ | 3.88 | $L_{19}/2\pi$ | 3.88 | $L_{17}/2$ | 15.6 | $M_4M_5$ | 1.046 |
| 7. | Lower Arms | $L_{21}/2\pi$ | 3.63 | $L_{21}/2\pi$ | 3.63 | $L_{18}/2$ | 19.45 | $M_6M_7$ | 1.138 |
| 8. | Upper Legs | $L_{25}/2\pi$ | 6.74 | $L_{33}$ | 27.9 | $(L_2-L_{17}-L_{23})/2$ | 6.745 | $M_{10}M_{11}$ | 5.648 |
| 9. | Lower legs | $L_{27}/2\pi$ | 5.08 | $L_{25}/2\pi$ | 5.07 | $(L_{23}-L_{29})/2$ | 22.9 | $M_{12}M_{13}$ | 2.624 |
| 10. | Feet | $L_{30}/2$ | 4.7 | $L_{31}/2$ | 12.4 | $L_{29}/2$ | 3.05 | $M_{14}M_{15}$ | 0.791 |

## 3.5 Effect of Truncation Factor

At this stage, it is worthwhile to look into the effect of truncation factor which determines the amount of truncation in the ellipsoidal segments [2]. As shown in Fig. 3.3 (b), the truncation factor provides the base of contact-effect value which effects the stiffness values in parallel/series combination of different segments. It is assumed to be same for all the segments.

$$d_i = 95\ \%\ \text{of}\ c_i \tag{3.2}$$

Therefore,

Truncation factor for each ellipsoidal segment, $t_r = 0.05$;

$$I = \ln(1.95/0.05)$$

$$= 3.66$$

Expression for axial vertical stiffness of the ellipsoid is:

$$S_{vi} = \pi E a_i b_i / (c_i I_i) \tag{3.3}$$

Expression for horizontal stiffness of the ellipsoid is:

$$S_{hi}=_{\Pi}Ea_ic_i/(b_iI_i) \tag{3.4}$$

Stiffness values of segments were calculated as shown in table below:

Table 3.3 Stiffness values of the ellipsoid segments

| S.No | Vertical segment stiffness, $S_{vi}$ (kN/m) | | Horizontal segment stiffness,$S_{hi}$(kN/m) |
|---|---|---|---|
| 1. | $S_1$ | 645.93 | 1045.29 |
| 2. | $S_2$ | 747.22 | 480.72 |
| 3. | $S_3$ | 1073.14 | 2585.66 |
| 4. | $S_4$ | 107.98 | 1744.02 |
| 5. | $S_5$ | 107.98 | 1744.02 |
| 6. | $S_6$ | 75.61 | 2174.44 |
| 7. | $S_7$ | 75.61 | 2174.44 |
| 8. | $S_8$ | 793.07 | 2642.92 |
| 9. | $S_9$ | 2140.37 | 1599.42 |
| 10. | $S_{10}$ | 3119.12 | 182.30 |
| 11. | $S_{11}$ | 3119.12 | 182.30 |
| 12. | $S_{12}$ | 125.73 | 2560.14 |
| 13. | $S_{13}$ | 125.73 | 2560.14 |
| 14. | $S_{14}$ | 2136.22 | 129.24 |
| 15. | $S_{15}$ | 2136.22 | 129.24 |

Moving further, stiffness of different horizontal and vertical springs connecting segments were calculated. The values of horizontal stiffness allowed us to increase degree-of-freedom that provides the stability in calculating the values of a mathematical model.

Table 3.4 Spring stiffness connecting elements

| S.No | Spring Element Designation | Combination of the ellipsoid elements forming the spring | Vertical spring stiffness,($k_{vi}$), (kN/m ) | | Horizontal spring Stiffness($k_{hi}$),(kN/m) |
|---|---|---|---|---|---|
| 1 | $K_1$ | H | $S_1$ | 645.93 | 1045.29 |
| 2 | $K_2$ | N,UT | $S_2$*$S_3$/($S_2$+$S_3$) | 440.50 | 405.36 |
| 3 | $K_3$ | UT,CT | $S_3$*$S_8$/($S_3$+$S_8$) | 456.04 | 1306.99 |
| 4 | $K_4$ | UT,RUA | $S_3$*$S_4$($S_3$+$S_4$) | 98.11 | 1041.52 |
| 5 | $K_5$ | UT,LUA | $S_3$*$S_5$/($S_3$+$S_5$) | 98.11 | 1041.52 |
| 6 | $K_6$ | RLA | $S_6$ | 75.61 | 2174.44 |
| 7 | $K_7$ | ILA | $S_7$ | 75.61 | 2174.44 |
| 8 | $K_8$ | CT,LT | $S_8$*$S_9$/($S_8$+$S_9$) | 578.66 | 996.42 |
| 9 | $K_9$ | LT,RUL | $S_9$*$S_{10}$/($S_9$+$S_{10}$) | 1269.34 | 163.65 |
| 10 | $K_{10}$ | LT,LUL | $S_9$*$S_{11}$/($S_9$+$S_{11}$) | 1269.34 | 163.65 |
| 11 | $K_{11}$ | RUL,RLL | $S_{10}$*$S_{12}$/($S_{10}$+$S_{12}$) | 120.86 | 170.18 |
| 12 | $K_{12}$ | LUL,LLL | $S_{11}$*$S_{13}$/($S_{11}$+$S_{13}$) | 120.86 | 170.18 |
| 13 | $K_{13}$ | RF | $S_{14}$ | 2136.23 | 129.24 |
| 14 | $K_{14}$ | LF | $S_{15}$ | 2136.23 | 129.24 |

## 3.6 Lumped Parameter Modelling

Lumped-parameter models consider the human body as several concentrated masses interconnected by springs and dampers. This type of model is simple to analyze and easy to validate with experiments. This 9 DOF lumped parameter model is developed in order to study the response of seated human body to the vertical and horizontal whole body vibration. Model consists of five segments connected by series and parallel combination of mass interconnected by six vertical and horizontal springs with total mass of 54 kg. The parameters such as mass and stiffness parameters, are described in Table 3.5 for their respective representation in the models. Model represent seated human subject without backrest in an erect posture.

Table 3.5 Symbolic representation of LPM model

| Symbol | Representation |
|---|---|
| $m_1$ | Upper Leg, lower leg, feet (left and right) |
| $m_2$ | Lower Torso (Pelvic) |
| $m_3$ | Central Torso |
| $m_4$ | Upper Torso (including both hands) |
| $m_5$ | Head and Neck |
| $k_{2v}$, $k_{2h}$ | Pelvic vertical and horizontal springs |
| $k_{1h}$, $k_{1v}$ | Upper leg vertical and horizontal springs |
| $k_{21}$ upto $k_{54}$ | Respective springs between body segments |
| $K_{4h}$,$k_{4v}$ | Back vertical and horizontal springs |

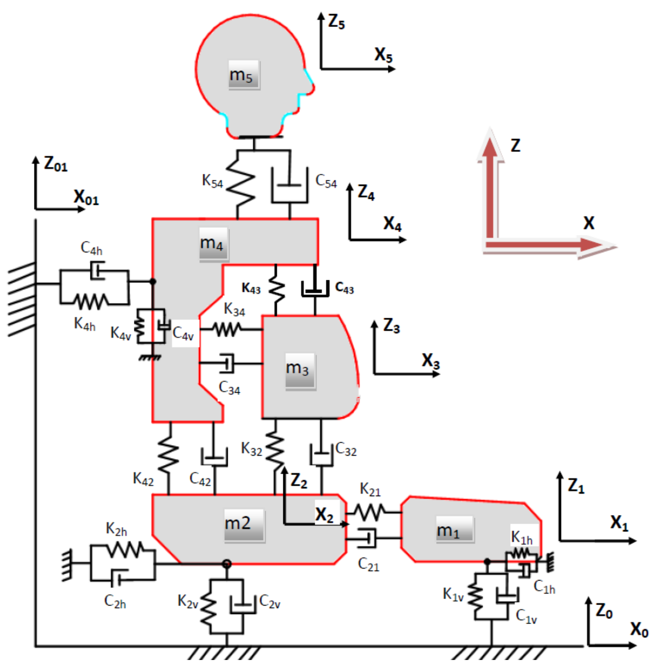



Fig. 3.2: Nine DOF Lumped parameter model [20].

The mass and stiffness values of 15 DOF model are modified to reduce the model to multi-body 11-dof through series and parallel combination as shown in Table 3.6 and 3.7.

Table 3.6 Stiffness values of LPM model

| S.No | Spring element designation | Spring stiffness, ($k_i$), (kN/m) |
|---|---|---|
| 1. | $K_{34} = k_{3h}$ | 1306.99 |
| 2. | $K_{2h} = S_{9h}$ | 1599.42 |
| 3. | $K_{21} = k_{9h+}k_{10h}$ | 327.28 |
| 4. | $K_{1h}=(k_{11h}*k_{13h})/(k_{11h}+k_{13h})+(k_{14h}*k_{12h})/(k_{14h}+k_{12h})$ | 146.91 |
| 5. | $K_{54} = (k_{1v}*k_{2v})/(k_{1v}+k_{2v})$ | 261.90 |
| 6. | $K_{43} = k_{3v}$ | 456.04 |
| 7. | $K_{32} = k_{8v}$ | 578.66 |
| 8. | $K_{42} = (S_{3v}*S_{9v})/(S_{3v}+S_{9v})$ | 714.77 |
| 9. | $K_{2v} = S_{9v}$ | 2140.37 |
| 10. | $K_{1v} = (K_{9v}*k_{11v}*K_{13v})/(k_9*k_{11}+k_{11}*k_{13}+k_{13}*k_9)+ (k_{10}*k_{12}*k_{14})/(k_{10}*k_{12}+k_{12}*k_{14}+k_{10}*k_{14})$ | 209.86 |
| 11. | $K_{4h} = S_{9h}$ | 2642.92 |
| 12. | $K_{4v} = S_{9v}$ | 793.07 |

Table 3.7 Mass Values of LPM model

| S.No | Mass designation | Final mass of elements (kg) |
|---|---|---|
| 1. | $m_5= M_1+M_2$ | 2.80 |
| 2. | $m_4 = M_3+M_4+M_5+M_6+M_7$ | 10.34 |
| 3. | $m_3 = M_8$ | 9.70 |
| 4. | $m_2 = M_9$ | 8.06 |
| 5. | $m_1= M_{10}+M_{11}+M_{12}+M_{13}+M_{14}+M_{15}$ | 18.13 |

Equations of motions are formulated using slippage term concept which is an offshoot of Freebody diagram.

**Formulation of Slippage term:**

Inertia force term(for a mass) + spring force term(corresponding to the end of the spring attached to the mass under consideration) – Slippage term(corresponding to the far end of the spring) =0 e.g:

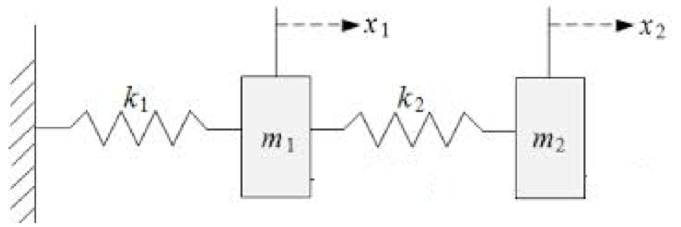



Fig. 3.5: 2-DOF spring-mass system[8].

$$m_1\ddot{x}_1 + k_1x_1 + k_2x_1 - k_2x_2 = 0 \tag{3.5}$$

$$m_1\ddot{x}_2 + k_2x_2 - k_2x_1 = 0 \tag{3.6}$$

The advantage of slippage term concept is that one need not draw the free body diagrams for a complex multi-degree freedom body.

## 3.7 Equations of Motion

Equations of motion are:

$$-\omega^2 * m_1 * z_1 + k_{1v} * z_1 = 0 \tag{3.7}$$

$$-\omega^2 * m_1 * x_1 + k_{1h} * x_1 + k_{21} * x_1 - k_{21} * x_2 = 0 \tag{3.8}$$

$$-\omega^2 * m_2 * z_2 + k_{2v} * z_2 + k_{32} * z_2 - k_{32} * z_3 + k_{42} * z_2 - k_{42} * z_4 = 0 \tag{3.9}$$

$$-\omega^2 * m_2 * x_2 + k_{2h} * x_2 + k_{21} * x_2 - k_{21} * x_1 = 0 \tag{3.10}$$

$$-\omega^2 * m_3 * z_3 + k_{32} * z_3 - k_{32} * z_2 + k_{43} * z_3 - k_{43} * z_4 = 0 \tag{3.11}$$

$$-\omega^2 * m_3 * x_3 + k_{34} * x_3 - k_{34} * x_4 = 0 \tag{3.12}$$

$$-\omega^2 * m_4 * z_4 + (k_{42} + k_{43} + k_{4v} + k_{54}) * z_4 - k_{42} * z_2 - k_{43} * z_3 - k_{54} * z_5 = 0 \tag{3.13}$$

$$-\omega^2 * m_4 * x_4 + (k_{34} + k_{4h}) * x_4 - k_{34} * x_3 = 0 \tag{3.14}$$

$$-\omega^2 * m_5 * z_5 + k_{54} * z_5 - k_{54} * z_4 = 0 \tag{3.15}$$

## 3.8 Generation of Mass and Stiffness Matrices

From above these 9 equations, Mass and Stiffness matrix is generated to calculate the Eigen Values and Eigen Vectors in seated posture with backrest in MATLAB environment. Further work is done to calculate the Seat-to-head Transmissibility (STHT), which is further validated to other experimental results.

```
m=[18.13    0    0    0    0    0    0    0     0;
    0   18.13    0    0    0    0    0    0     0;
    0       0  8.06   0    0    0    0    0     0;
    0       0    0  8.06   0    0    0    0     0;
    0       0    0    0  9.70   0    0    0     0;
    0       0    0    0    0  9.70   0    0     0;
    0       0    0    0    0    0  14.76  0     0;
    0       0    0    0    0    0    0   14.76  0;
    0       0    0    0    0    0    0    0  2.80];

k= [209860  0      0      0      0      0       0      0      0;
    0      474190  0  -327280    0      0       0      0      0;
    0       0   3433800   0   -578660   0   -714770    0      0;
    0     -327280  0   1926700   0      0       0      0      0;
    0       0   -578660   0    1034700  0   -456040    0      0;
    0       0       0     0      0   1306990    0  -1306990   0;
    0       0   -714770   0    -456040  0   2225780    0   -261900;
    0       0       0     0      0  -1306990    0   3949910   0;
    0       0       0     0      0      0   -261900    0   261900];
```

# CHAPTER 4

# Free Body Vibrations in Sitting Posture using Multibody Dynamics

## 4.1 Introduction

The prediction of the dynamic behaviour of multi body mathematical model/system involves the formulation of the governing equations of motion and the evaluation of their kinematic and dynamic characteristics. The contact-impact phenomenon is found to be most important and complex in mathematical modeling because vibration transmits through these contacts. The model development in this work considers the geometry of contacting bodies earlier provided by Gianotti, Nigam & Malik [1,2]. The model is based on the anthropometric data of 50$^{th}$ Indian male body given by Chakraborty [3].

It has been observed that multi body model gives more accurate results than LPM model and FEM model. In FEM model, it is not easy to provide high degrees of freedom and LPM model includes only one-dimensional motion. Multi-body model provides natural frequency of 2 different axis considering rotational joints of arms.

The purpose of this study is to estimate and validate the natural frequency of different body parts through multi body approach with the natural frequencies obtained through other experimental investigations in the literature. In the present study, values of mass, stiffness and the truncated values of ellipsoid are determined by the method proposed by Nigam and Malik. Refinement is made by adding a spring connecting Upper Torso and Lower Torso segments. This is based on the fact that the lower torso gets support from the backbone and these two elements make a spring combination. In order to study the dynamic response of the model, equations of motions are derived.

The anthropometric human body model by Nigam and Malik consisting of 15 body segments and 14 joints is adopted in this study as shown in fig. 3.1. The anthropometric data provides a natural choice to be identified as the lumped masses of a vibratory model. The values of stiffness and mass of each segment is estimated on the basis of physical structure of segment.

## 4.2 Vibratory Model

The stiffness and mass values of the various spring and dashpot elements of the vibratory model are obtained using parallel/series combination of the body segments. The equations of motion of the vibratory model to calculate natural frequency of the free vibration are:

$$\mathbf{M\ddot{x}+Kx=0} \tag{4.1}$$

where [M] and [K] are mass and stiffness matrices, respectively.

To illustrate the use of above methodology for the mathematical model of a human body, example of a $50^{th}$ percentile male of mass 54kg in sitting posture with backrest is considered as shown above in Table 3.1.

Dimensions of the ellipsoids are calculated taking the help of Nigam and Malik methodology.

## 4.3 Multi-body Modelling

Multibody human models are made of several rigid bodies interconnected by pin (two-dimensional) or ball and socket (three-dimensional) joints, and can be further separated into kinetic and kinematic models. The kinematic models are mainly concerned with the motion of each segment, and are often used in the study of human exercise and injury assessment in a vehicle crash. To investigate the biodynamic behavior due to external excitation, springs and dampers are constantly attached between two body segments, and form kinetic models. That is, kinetics is interested in the study of forces associated with motion, while kinematics is a study of the description of motion, including considerations of space and time. The prediction of the dynamic behavior of multi-body mechanical systems involves the formulation of the governing equations of motion and the evaluation of their kinematic and dynamic characteristics.

In this model, a human body in sitting posture without backrest is modeled as a mechanical system composed of several rigid bodies interconnected by translational springs and different joints. Multi-body model treats human body as 7 rigid segments in 2-D sagittal plane, with a total mass of 54 kg and has 11 DOF. It is assumed that, the arms are connected to upper torso by ball and socket joint. The whole body is allowed to move in vertical and horizontal direction with two arms also allowed in rotational motion [20].

Eleven DOF multi-body model without backrest considers joints $J_3$ and $J_4$ as vertical springs and $J_2$ as horizontal spring. Joints $J_1$ and $J_6$ are considered as pin joint, while $J_5$ considered as ball and socket joint.

Table 4.1 Symbolic representation of multi-body model

| Symbol | Representation |
|---|---|
| m1 | Upper Leg, lower leg, feet (left and right) |
| m2 | Lower Torso (Pelvic) |
| m3 | Central Torso |
| m4 | Upper Torso |
| m5 | Head and Neck |
| m6 | Upper arms |
| m7 | Lower arms |
| k2v, k2h | Pelvic vertical and horizontal springs |
| k1h, k1v | Upper leg vertical and horizontal springs |
| k21 upto k54 | Respective springs between body segments |
| l1, l2 | Length of upper arm and lower arm (from anthropometric data) |
| $\theta 1, \theta 2$ | Angle of upper arm and lower arm with upper torso |
| $K_{4h}$,$k_{4v}$ | Back vertical and horizontal springs |

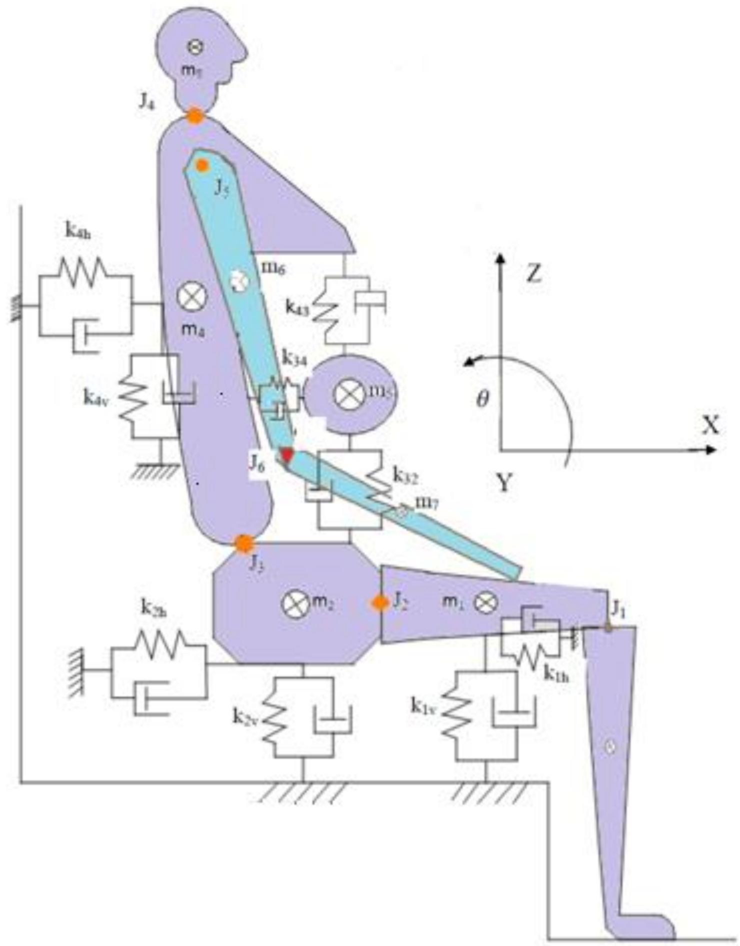



Fig. 4.1: Eleven DOF multi body model [20].

For this model, the analytical approach proposed by Lagrange, for obtaining the equations of motion in generalized coordinates, is used. In this model, the term 'generalized coordinate' is used in order to more easily characterize kinetic energy. For this model, the generalized coordinates or the degrees-of-freedom are: [$x_1$, $x_2$ ,$x_3$, $x_4$, $z_1$, $z_2$, $z_3$, $z_4$, $z_5$, $\theta 1$, $\theta 2$].

## 4.4 Lagrangian Approach

Lagrange's equations are differential equations that consider the energies of the system and the work done instantaneously in time and can be used for systems with greater degrees-of-freedom in more than one direction. Compared with other methods of obtaining equations of motion, the method offers the following potential advantages:

(i) Accelerations do not have to be determined, only velocities are used. This considerably simplifies the kinematics. Also, since velocities are squared, difficulties with algebraic sign are often avoided.
(ii) Required number of equations is automatically obtained.
(iii) Same basic method is used whatever set of co-ordinates might be chosen, which allows us to work with the most convenient set of co-ordinates.

The formula for Lagrange's equations for conservative systems is given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i \qquad \forall\ i = 1,2,3 \ldots . n \tag{4.2}$$

where,

T= Total kinetic energy of a system

V= Total potential energy of the system

$Q_i$= Generalized non potential forces or moments.

Before proceeding to derive the equations of motions of the model it is convenient to relate the Cartesian and polar coordinates of the upper arm ($m_6$) and lower arm ($m_7$). Assuming the center of mass for both upper and lower arm at the center. The following relation can be made:
The displacements will be:

$$x_6 = x_4 + \frac{l_1}{2}\sin\theta_1 \tag{4.3}$$

$$x_7 = x_4 + l_1\sin\theta_1 + \frac{l_2}{2}\sin\theta_2 \tag{4.4}$$

$$z_6 = z_4 + \frac{l_1}{2}\cos\theta_1 \tag{4.5}$$

$$z_7 = z_4 + l_1\cos\theta_1 + \frac{l_2}{2}\cos\theta_2 \tag{4.6}$$

The first derivative (velocity) of the equations will be:

$$\dot{x}_6 = \dot{x}_4 + \frac{l_1}{2}\dot{\theta}_1\cos\theta_1 \tag{4.7}$$

$$\dot{x}_7 = \dot{x}_4 + l_1\dot{\theta}_1 \cos\theta_1 + \frac{l_2}{2}\dot{\theta}_2 \cos\theta_2 \tag{4.8}$$

$$\dot{z}_6 = \dot{z}_4 - \frac{l_1}{2}\dot{\theta}_1 \sin\theta_1 \tag{4.9}$$

$$\dot{z}_7 = \dot{z}_4 - l_1\theta_1 \sin\theta_1 - \frac{l_2}{2}\theta_2 \sin\theta_2 \tag{4.10}$$

Both displacement and velocity relation will be helpful in deriving the system equations of motion. The equations for the Kinetic Energy, Potential Energy are derived.

The equation for the kinetic energy of the system is:

$$T= \tfrac{1}{2} m_1\dot{z}_1^2 + \tfrac{1}{2} m_1\dot{x}_1^2 + \tfrac{1}{2} m_2\dot{z}_2^2 + \tfrac{1}{2} m_2\dot{x}_2^2 + \tfrac{1}{2} m_3\dot{z}_3^2 + \tfrac{1}{2} m_3\dot{x}_3^2 + \tfrac{1}{2} m_4\dot{z}_4^2 + \tfrac{1}{2} m_4\dot{x}_4^2 + \tfrac{1}{2} m_5\dot{z}_5^2 + \tfrac{1}{2} m_5\dot{x}_5^2 + \tfrac{1}{2} m_6(\dot{z}_4 2 + \dot{x}_4 2 + l_1^2/4\, \dot{\theta}1^2 + \dot{x}_4\dot{\theta}1 l_1 cos(\theta 1) - \dot{z}_4\dot{\theta}1 l_1 sin(\theta 1)) + \tfrac{1}{2} m_7(\dot{z}_4^2 + \dot{x}_4^2 + 2\dot{x}_4\dot{\theta}1 l_1 cos(\theta 1) - 2\dot{z}_4\dot{\theta}1 l_1 sin(\theta 1) + \dot{x}_4\dot{\theta}2 l_2 cos(\theta 2) + l_1^2\dot{\theta}1^2 + (l_2^2/4)\dot{\theta}2^2 - \dot{z}_4\dot{\theta}2 l_2 sin(\theta 2) + l_1 l_2\dot{\theta}1\dot{\theta}2 cos(\theta 2 - \theta 1)) \tag{4.11}$$

The equation for the potential energy of the system is:

$$V= \tfrac{1}{2} k_{1v} (z_1-z_0)^2 + \tfrac{1}{2} k_{1h} (x_1-x_0)^2 + \tfrac{1}{2} k_{2v} (z_2-z_0)^2 + \tfrac{1}{2} k_{2h} (x_2-x_0)^2 + \tfrac{1}{2} k_{21} (x_2-x_1)^2 + \tfrac{1}{2} k_{32} (z_3-z_2)^2 + \tfrac{1}{2} k_{42} (z_4-z_2)^2 + \tfrac{1}{2} k_{43} (z_4-z_3)^2 + \tfrac{1}{2} k_{34} (x_3-x_4)^2 + \tfrac{1}{2} k_{4v} (z_4-z_1)^2 + \tfrac{1}{2} k_{4h} (x_4-x_1)^2 + \tfrac{1}{2} k_{54} (z_5-z_4)^2 + m_6 g l_1/2\, (1-\cos(\theta 1)) + m_7 g l_1(1-\cos(\theta 1)) + m_7 g l_2/2(1-\cos(\theta 2)) \tag{4.12}$$

The non-potential forces in the system includes externally applied excitation forces and damping forces. So, it is equal to zero.

Eqn. (4.2) is used to derive the equations of motions of a multi-body model.

The mass and stiffness values of 15 dof model are modified to reduce the model to multi-body 11-dof through series and parallel combination as shown in table 4.2 and 4.3.

Table 4.2 Stiffness values of Multi-body model

| S.No. | Spring element designation | Spring stiffness, ($k_i$), (kN/m) |
|---|---|---|
| 1. | $K_{34} = k_{3h}$ | 1306.99 |
| 2. | $K_{2h} = S_{9h}$ | 1599.42 |
| 3. | $K_{21} = k_{9h+}k_{10h}$ | 327.28 |
| 4. | $K_{1h}=(k_{11h}*k_{13h})/(k_{11h}+k_{13h})+(k_{14h}*k_{12h})/(k_{14h}+k_{12h})$ | 146.91 |
| 5. | $K_{54} = (k_{1v}*k_{2v})/(k_{1v}+k_{2v})$ | 261.90 |
| 6. | $K_{43} = k_{3v}$ | 456.04 |
| 7. | $K_{32} = k_{8v}$ | 578.66 |
| 8. | $K_{42} = (S_{3v}*S_{9v})/(S_{3v}+S_{9v})$ | 714.77 |
| 9. | $K_{2v} = S_{9v}$ | 2140.37 |
| 10. | $K_{1v} = (K_{9v}*k_{11v}*K_{13v})/(k_9*k_{11}+k_{11}*k_{13}+k_{13}*k_9)+ (k_{10}*k_{12}*k_{14})/(k_{10}*k_{12}+k_{12}*k_{14}+k_{10}*k_{14})$ | 209.86 |
| 11. | $K_{4h} = S_{9h}$ | 2642.92 |
| 12. | $K_{4v} = S_{9v}$ | 793.07 |

Table 4.3 Mass Values of Multi-body model

| S.No | Mass designation | Final mass of elements (kg) |
|---|---|---|
| 1. | $m_5= M_1+M_2$ | 2.80 |
| 2. | $m_4 = M_3$ | 14.76 |
| 3. | $m_3 = M_8$ | 9.70 |
| 4. | $m_2 = M_9$ | 8.06 |
| 5. | $m_1= M_{10}+M_{11}+M_{12}+M_{13}+M_{14}+M_{15}$ | 18.13 |

By Lagrangian approach, equations of motions are generated, mass and stiffness matrices are obtained using the model parameters.

## 4.5 Equations of Motion

Equations of motion are:

$$- \omega^2*m_1*z_1 + k_{1v}*z_1 = 0 \tag{4.13}$$

$$- \omega^2 * m_1 * x_1 + k_{1h} * x_1 + k_{21} * x_1 - k_{21} * x_2 = 0 \tag{4.14}$$

$$- \omega^2 * m_2 * z_2 + k_{2v} * z_2 + k_{32} * z_2 - k_{32} * z_3 + k_{42} * z_2 - k_{42} * z_4 = 0 \tag{4.15}$$

$$- \omega^2 * m_2 * x_2 + k_{2h} * x_2 + k_{21} * x_2 - k_{21} * x_1 = 0 \tag{4.16}$$

$$- \omega^2 * m_3 * z_3 + k_{32} * z_3 - k_{32} * z_2 + k_{43} * z_3 - k_{43} * z_4 = 0 \tag{4.17}$$

$$- \omega^2 * m_3 * x_3 + k_{34} * x_3 - k_{34} * x_4 = 0 \tag{4.18}$$

$$- \omega^2 * m_4 * z_4 + (k_{42} + k_{43} + k_{54} + k_{4v}) * z_4 - k_{42} * z_2 - k_{43} * z_3 - k_{54} * z_5 = 0 \tag{4.19}$$

$$- \omega^2 * m_4 * x_4 - \omega^2 * m_6 * (l_1/2) * \theta 1 - m_7 * l_1 * \theta 1 * \omega^2 - m_7 * (l_2/2) * \theta 2 * \omega^2 +$$

$$(k_{34} + k_{4h}) * x_4 - k_{34} * x_3 = 0 \tag{4.20}$$

$$- \omega^2 * m_5 * z_5 + k_{54} * z_5 - k_{54} * z_4 = 0 \tag{4.21}$$

$$1/2 * m_6 * (- \omega^2 * x_4 * l_1 - \omega^2 * (l_1 * l_1/2) * \theta 1) + 1/2 * m_7 * (-2 * \omega^2 * l_1 * x_4 - 2 * \omega^2 * l_1 * l_1 * \theta 1 - \omega^2 * l_1 * l_2 * \theta 2)$$
$$+ m_6 * g * l_1/2 * \theta 1 + m_7 * g * l_1 * \theta 1 = 0 \tag{4.22}$$

$$1/2 * m_7 * (- \omega^2 * l_2 * x_4 - \omega^2 * l_2 * l_2/2 * \theta 2 - \omega^2 * l_1 * l_2 * \theta 1) + m_7 * g * l_2/2 * \theta 2 = 0 \tag{4.23}$$

## 4.6 Generation of Mass and Stiffness Matrices

From these 11 equations, mass and stiffness matrix is generated to calculate the Eigen Values and Eigen Vectors in MATLAB environment.

```
m=[18.13   0     0     0     0     0     0      0     0     0      0;
     0  18.13   0     0     0     0     0      0     0     0      0;
     0      0 8.06    0     0     0     0      0     0     0      0;
     0      0     0  8.06   0     0     0      0     0     0      0;
     0      0     0     0   9.7   0     0      0     0     0      0;
     0      0     0     0     0   9.7   0      0     0     0      0;
     0      0     0     0     0     0   14.76  0     0     0      0;
     0      0     0     0     0     0     0     14.76 0   0.33 0.71;
     0      0     0     0     0     0     0      0   2.80  0      0;
     0      0     0     0     0     0     0   1.036  0   0.27   0.14;
     0      0     0     0     0     0     0    0.44  0   0.14   0.09];



 k=[209860  0      0       0       0      0     0      0      0     0     0;
     0   474190   0    -327280   0      0     0      0      0     0     0;
     0       0  3433800    0  -578660   0 -714770  0      0     0     0;
```

```
0  -327280   0     1926700  0     0    0     0     0    0     0;
0      0 -578660     0   1034700  0 -456040  0     0    0     0;
0      0     0       0      0  1306990 0  -1306990 0    0     0;
0      0  -714770    0  -456040  0   2225780 0 -261900  0     0;
0      0     0       0      0 -1306990 0   3949910 0    0     0;
0      0     0       0      0    0  -261900  0  261900  0     0;
0      0     0       0      0    0     0     0    0    10.1  0;
0      0     0       0      0    0     0     0    0     0    4.33];
```

# CHAPTER 5

# Transmissiblity in Forced Body Vibrations under Multibody Model

## 5.1 Introduction

A mechanical or structural system is often subjected to external forces or external excitations. The external forces may be harmonic, non-harmonic but periodic, non-periodic but having a defined form or random. The response of the system to such excitations or forces is called Forced Response. The response of a system to a harmonic excitation is called Harmonic Response. The non-periodic excitations may have a long or short duration. The response of a system to suddenly applied non-periodic excitations is called Transient Response. The sources of harmonic excitations are unbalance in rotating machines, forces generated by reciprocating machines, and the motion of the machine itself in certain cases [8].

## 5.2 Forced Body Vibrations

Forced vibration is known as when a time-varying disturbance such as acceleration, displacement or velocity (considered as force) is applied to a mechanical system. The disturbance can be a periodic, steady-state input, a transient input, or a random input. The periodic input can be a harmonic or a non-harmonic disturbance. Examples of these types of vibration include a shaking washing machine due to an imbalance.

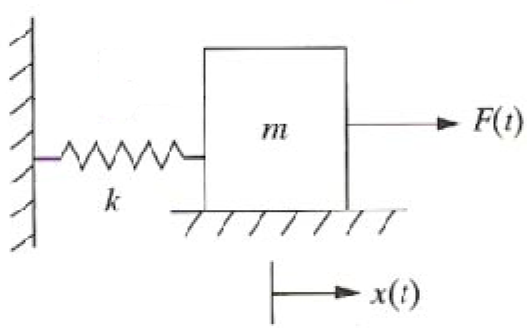



Fig. 5.1: Forced body Vibrations[8].

Equation of motion:

$$m\ddot{x} + kx = F(t) \tag{5.1}$$

## 5.3 Transmissibility

Transmissibilty is defined as the ratio of output responses to input excitation (such as seat, foot) at different points. It is also called as the acceleration or displacement ratio.

For seat-to-head transmissibility, the expression is:

$$T_{STH}(f) = \frac{a_{head}(f)}{a_{seat}(f)} \tag{5.2}$$

where $T_{STH}(f)$ = Seat-to-head transmissibility,

## 5.4 Forced Body Vibratory Model

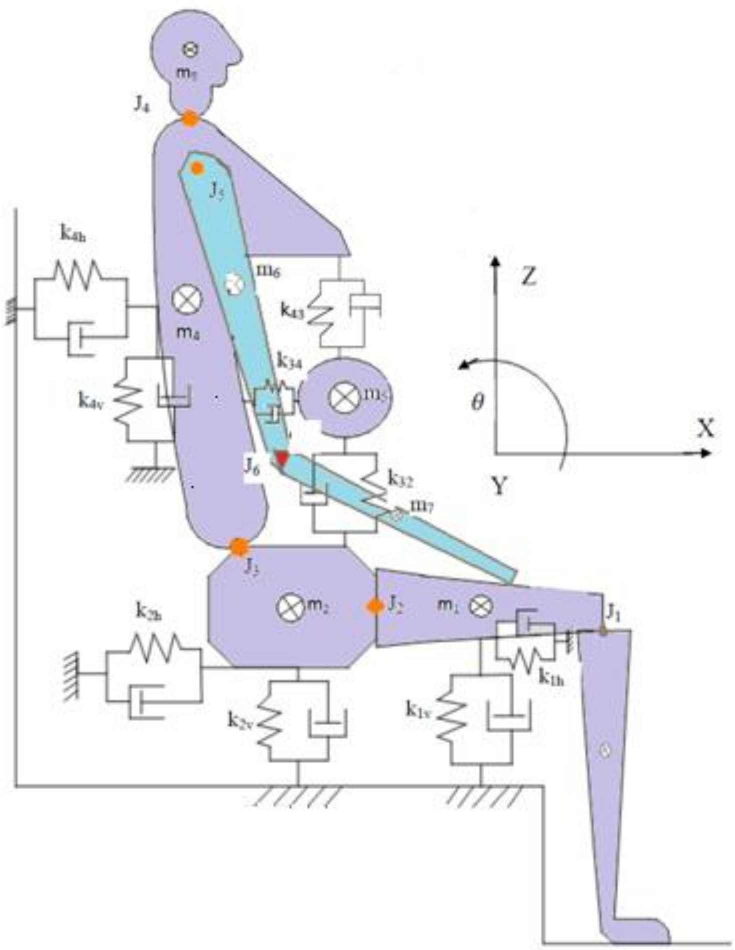



Fig. 5.2: 11-DOF Multi Body Model [20].

In considering the above Multi-body model, now the model is assumed that Force is applied at the foot and in the other case, Force is applied at the seat (or pelvic) to calculate the Seat-to-head Transmissibilty (STHT) and Foot-to-head Transmissibilty. Equations of motion are modified to apply the force in vertical direction at the foot and pelvic. Eq. (4.13) is modified to Eq. (5.3) to apply the force at the foot in vertical direction. '$A_o$' is the acceleration applied at the foot, the value of which is assumed 0.8m/s$^2$ which can be varied to other numbers to note down the Transmissibility at different acceleration values.

$$-\omega^2 * m_1 * z_1 + k_{1v} * z_1 = A_o * \mathrm{Sin}\,\omega t \tag{5.3}$$

Rest of the equations are remained as same as with free vibrations.

In the other case, Eq. (4.15) is modified to Eq. (5.4) to apply the force at the pelvic in vertical direction. '$A_o$' is the acceleration applied at the foot, the value of which is assumed to be 0.8m/s$^2$ and can be varied to different values.

$$-\omega^2 * m_2 * z_2 + k_{2v} * z_2 + k_{32} * z_2 - k_{32} * z_3 + k_{42} * z_2 - k_{42} * z_4 = A_o * \mathrm{Sin}\,\omega t \tag{5.4}$$

Rest of the equations are remained as same as with free vibrations.

Calculating the mass and stiffness matrix of the Multibody model, applying the force at different desired points, transmitted acceleration values at head are generated and Transmissibilty in terms of acceleration and displacement is calculated through MATLAB programming.

Through MATLAB programming, different transmitted acceleration and displacement value graphs at 11 different points can be seen by applying the force at different 11 points.

# CHAPTER 6

# Results and Discussions

## 6.1 RESULTS OF LPM MODEL

### 6.1.1 Natural Frequencies

$f_1$=17.12 Hz, $f_2$=24 Hz, $f_3$=35 Hz, $f_4$=43.08 Hz,

$f_5$=48.68 Hz, $f_6$=62.27 Hz, $f_7$=78.33 Hz, $f_8$=91.26 Hz, $f_9$=106.74 Hz,

### 6.1.2 Eigen Vectors

Values of Eigen Vector are:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.08 | -0.98 | 0.00 | -0.00 | 0.00 |
| 0.98 | -0.09 | -0.18 | -0.09 | 0.00 | -0.00 | -0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -1.00 | -0.18 | -0.00 | -0.00 | 0.00 |
| -0.15 | 0.45 | -0.49 | -0.39 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.91 | 0.57 | 0.00 |
| -0.14 | -0.62 | -0.37 | -0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.41 | -0.82 | 0.00 |
| 0.04 | 0.64 | -0.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |

## 6.2 RESULTS OF MULTIBODY MODEL

### 6.2.1 Natural Frequencies

$f_1$=17.12 Hz, $f_2$=24 Hz, $f_3$=35 Hz, $f_4$=43.08 Hz,

$f_5$=48.68 Hz, $f_6$=76.92 Hz, $f_7$=78.33 Hz, $f_8$=98.91 Hz, $f_9$=107.78 Hz,

$\theta_1$=1.129, $\theta_2$=1.163.

### 6.2.2 EIGEN VECTORS

Values of Eigen Vector are:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 |
| 0.00 | 0.00 | 0.00 | -0.00 | -0.00 | 0.08 | 0.00 | 0.98 | -0.00 | 0.00 | 0 |
| -0.00 | 0.00 | 0.00 | 0.95 | -0.00 | -0.00 | 0.20 | 0.00 | 0.09 | -0.17 | 0 |
| 0.00 | 0.00 | 0.00 | -0.00 | -0.00 | -0.99 | -0.00 | 0.18 | -0.00 | 0.00 | 0 |
| 0.00 | 0.00 | 0.00 | -0.12 | 0.00 | 0.00 | -0.37 | 0.00 | 0.39 | -0.49 | 0 |
| 0.06 | -0.02 | -0.00 | 0.00 | 0.25 | -0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.00 | 0.00 | 0.00 | -0.25 | 0.00 | -0.00 | 0.75 | 0.00 | 0.00 | -0.33 | 0 |
| -0.11 | -0.02 | -0.00 | 0.00 | 0.10 | 0.00 | -0.00 | -0.00 | 0.00 | 0.00 | 0 |
| -0.00 | 0.00 | 0.00 | 0.06 | -0.00 | 0.00 | -0.50 | -0.00 | -0.91 | -0.78 | 0 |
| -0.76 | -0.49 | 0.60 | 0.00 | -0.73 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | 0 |
| -0.62 | 0.87 | 0.80 | 0.00 | 0.61 | 0.00 | -0.00 | -0.00 | 0.00 | 0.00 | 0 |

## 6.3 DISCUSSION

These 11 frequency values provides the value of free vibrations in sitting posture in z and x direction. The values of $\theta$ provides the values of motion in rotation (3-D). However, the degrees-of-freedom of a model can still be increased but that will increase the complexity of calculations. This approach give a brief introduction to the use of Eigen values and Eigen vectors to study the vibrating systems with MATLAB programming.

Analyzing a system in terms of its Eigen values and Eigen vectors greatly simplifies system analysis, and gives important insight into system behavior. For example, once the Eigen values and eigenvectors of the system above have been determined, its motion can be completely determined simply by knowing the initial conditions and solving one set of algebraic equations.

The validity of the proposed modeling may be well assessed on the basis of a comparison of the computed stiffness and frequency values with the other available experimental data studies. A comparison may be made with some experimentally observed resonance peaks as listed in other papers [12]. These values of natural frequency correspond to discrete segments of a human body. A resonance peak can be seen at 33.60 Hz in fig. 6.1 which corresponds to the resonance peak of a segment of human body model, again another peak at 48.70 Hz suggests resonance peak of a different segment, similarly, plotting the different graphs of Transmissibilty applying the force at different segments, resonance peaks can be seen at different natural frequency points. The calculated values of $\theta$ lying between 1 and 1.5 shows the resonance peak of arms. The frequency values shows the movement in both z as well as in x direction.

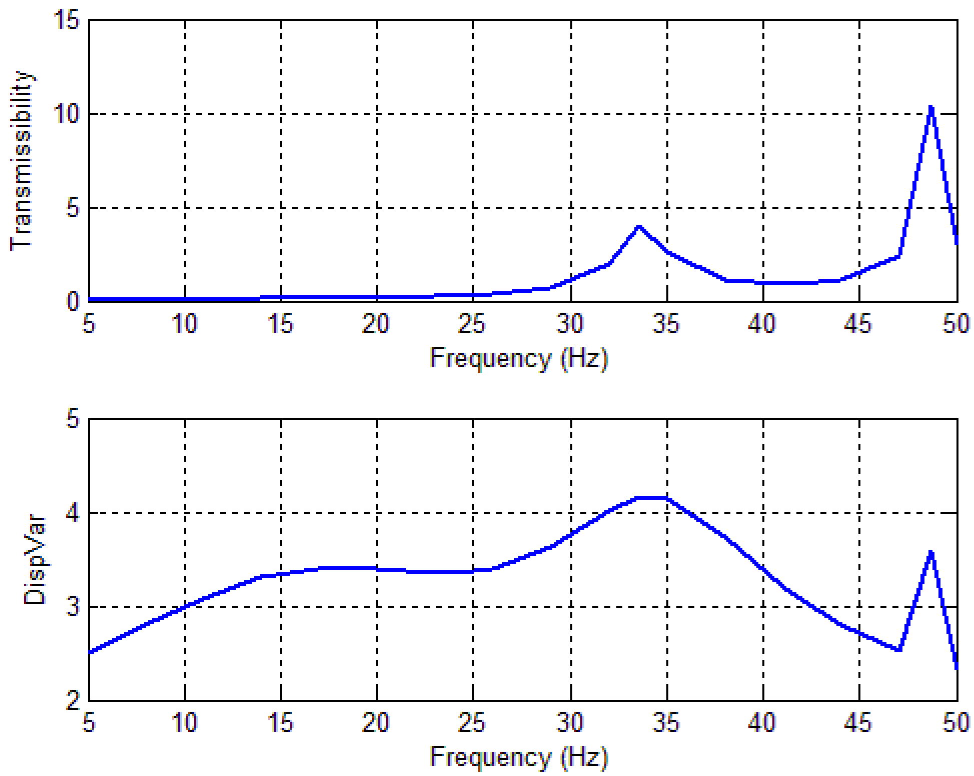



Fig. 6.1: Transmissibilty and Displacement values in seat-to-head transmissibility.

Force is applied in the frequency range of 5 to 50Hz at intervals and on natural frequency positions, Clearly, the resonance peaks can be seen. Comparing with other experimental vibratory models, it is found that the natural frequency of human body lies in the region of below 100Hz [9].

## 6.4 Comparison of Results between LPM and Multibody Model

Table 6.1 Comparison of frequency values between LPM and Multibody model

| S.No. | Frequency values (Hz) | |
|---|---|---|
| | **LPM** | **Multibody** |
| 1 | 17.12 | 17.12 |
| 2 | 24 | 24.01 |
| 3 | 33.6 | 33.6 |
| 4 | 41.7 | 41.7 |
| 5 | 48.68 | 48.7 |
| 6 | 68.27 | 63.2 |
| 7 | 78.36 | 78.36 |
| 8 | 91.26 | 83.92 |
| 9 | 106.54 | 106.37 |
| $\theta 1$ | | 1.129 |
| $\theta 2$ | | 1.163 |

The modeling procedure presented in this paper involves several simplifying assumptions. The foremost being the approximation of the body segments to ellipsoidal bodies, the approximation of segment elastic modulus as a geometric mean of bone and tissue elastic modulii, and converting the 15 DOF model into 11 DOF multi-body model with motion in multiple axis by the series and parallel combination of stiffness. Nevertheless, with the adopted approximations the proposed modeling procedure seems to give a fairly good estimate of the low order mode natural frequencies. Multi-body dynamics using Lagrangian approach provides the additional 2-DOF frequency values which is used to find the rotation of arms. Addition to that, even in the Upper Torso, the frequency values through the Multi-body dynamics are more reliable than Lumped Parameter model, as multi-body model considers the effect of rotation of arms in Upper Torso.

Over the same frequency range, the prediction accuracy of multi-body model found to be much more than other models. Comparison of the vertical seat-to-head vibration transmissibility characteristics computed from other experimental models with those defined mean value, lower limit and upper limits of different subject's response data sets over the frequency range of 20-40 Hz. In general, the effect of vibration magnitude on mean STH transmissibility in erect posture is higher than others postures, at body resonance frequency.

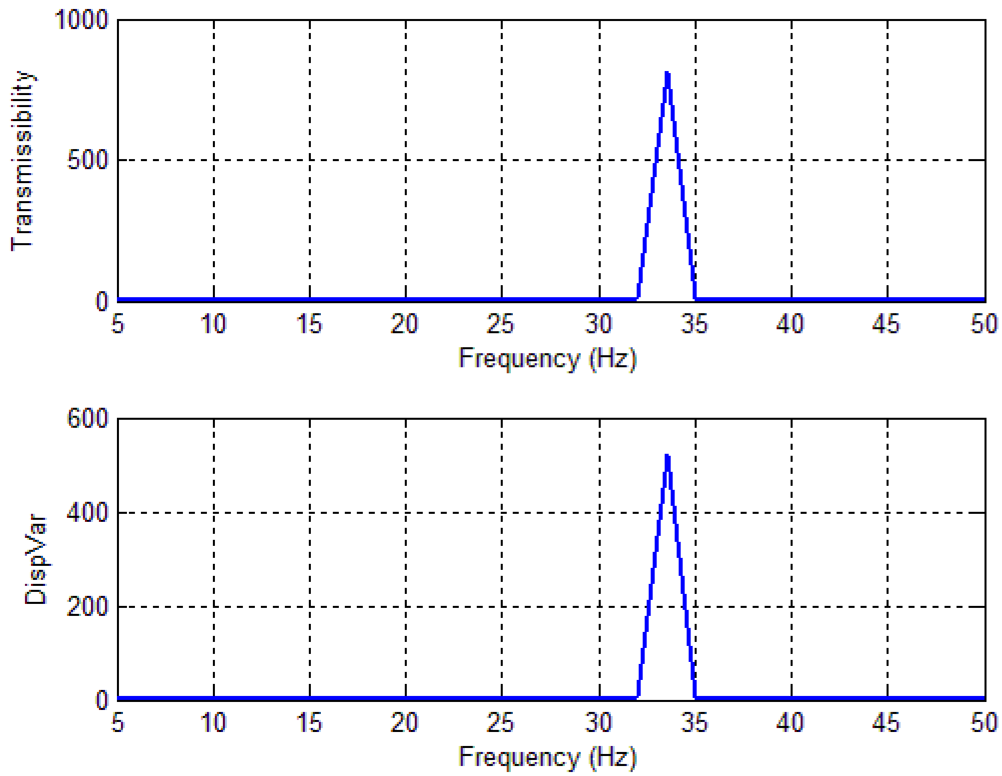



Fig. 6.2: Transmissibilty and Displacement values in foot-to-head transmissibility.

# CHAPTER 7

# Conclusions and Scope for Future Work

Human body is subject to whole body vibration when a person is travelling in vehicles or operators working on vibrating platforms. Analyses of whole body vibration involve study of biodynamics responses on human body.

1. To understand human body vibration problem.
2. To prepare a generalize program in MATLAB.
3. To evaluate bio-dynamics responses of human body model.
4. To calculate Eigen values and Eigen vectors of different segments based on Indian data.
5. To study cooperation of stiffness and mass values with respect to human body models.
6. Validation of Seat-to-head Transmissibility(STHT) values with other experimental values.

## 7.1 CONCLUSIONS

I conclude my thesis by saying that maximum effect of whole body vibration in vertical direction on human body ranges at 0.5 to 80 Hz frequency, and the best resonance peaks seen at 5 to 50Hz., so designer should avoid the 80 Hz frequency at the time of designing the vibrating platform or body. We validate our data with U.S Male data [12]. Human Body Vibration is a subject of bio-mechanical engineering, the thesis work gives us opportunity to enhance our knowledge in multidisciplinary branch of engineering. The Thesis has created interest to understand. The vast international literature available in these area. The biodynamic response plotted by making programs in MATLAB. Therefore, an interest was developed in programming effect of vibration on Whole body is at about 80 Hz [16].

1) This work presents a 9 DOF lumped parameter model for free vibration of human body in sitting posture with backrest. The undamped spring-mass model has been developed. Mass and stiffness of various body segments have been evaluated through anthropometric data using methodology adopted by Nigam & Malik [2].

2) The results have been compared with the values available in the literature and it was found that the Transmissibilty evaluated in the present work are comparable with the resonant frequency peaks obtained from the experimental data [20].

3) In comparison to the experimental values, frequencies calculated in this study seem to be on higher side, as damping has not been included in the free vibration analysis[2].

4) The Eigen Vectors indicate the presence of motions in the different body segments at the fundamental and higher natural frequencies in fore and aft and vertical directions, which match with the physical observation as well.

## 7.2 FUTURE SCOPE

Further research can be done on the mathematical modeling by experimental procedures i.e. by calculating natural frequency from experimental set-up and further analyzing the modeling and calculating the stiffness values. Damping, though inherently present in a human body, has been ignored in the present study. The undamped resonant frequencies are important parameters of any vibratory system as they signify the location of the frequency response peaks of an externally excited system. As such a particular posture of a human body does not seem to be a restriction for present modeling as long as an anthropomorphic model for the posture can be worked out. However, refinements in the model are being sought to account for damping and consider other body postures to make modeling approach more realistic and general [2].

1) The mass of a seated body has considerable influence on certain biodynamic response functions. Therefore, further research can be conducted on the influences of different mass values.

2) The research done is on Indian Male data, further results can be compared to this mathematical approach.

3) Nevertheless, with the adopted approximations the proposed modeling procedure seems to give a fairly good estimate of the low order mode natural frequencies.

4) Further research can be done on different DOF models, but the future work should more concentrate on validation of stiffness values.

## 7.3 FUTURE SCOPE OF BIO-DYNAMIC RESPONSE STUDY

It is apparent that vibration affects human health, performance, activities and comfort. As discussed in literature review, many attempts have been carried out to create comfortable condition under vibration environment which needs better understanding of human response to vibration. In bio-dynamic response studies both experimental and analytical works are conducted to create comfortable, luxury, good performance and healthy environment. Now a days, even though, both experimental and analytical research works have continued, researchers are more resembled to analytical work. Since analytical work (model development) can potentially save the time to conduct experiment, save energy, save setup cost and totally eliminate the ethical problem associated with subjects take part in experiment. Adding this, advanced computation of sophisticated system become possible with the help of powerful softwares, so more representative human body model can be developed using lumped parameter, multi-body and finite element methods in order to better describe human body motion under whole body vibration better than before.

Although considerable effort have been paid on experimental and analytical response studies in this thesis, more accurate results may be possible if some improvements or modifications to the current model are obtained.

1) Experimental data sets can be used for model validation, optimal seat design and comfort analysis over full range of frequency if head may be replaced by other means of measuring head acceleration such as helmet.

2) The seat geometry and inclination can be improved by introducing more flexibility in back support portion. The backrest, for instance, can be modeled in different inclination which normally available in vehicles. In addition adding seat cushion may enable to represent most vehicles real seat conditions.

3) Representing vertical seat vibration by sinusoidal motion is fairly well. But, to make the analysis more accurate introducing random input is essential.

4) This study was totally limited to vertical, horizontal and rotation axis, which dominate the response of human body. However, adding other directions influence in the response study and model validation may be better represent the realistic condition. In this study, all horizontal model parameters are estimated from literature, if these axis experimental values will be conducted, obtaining optimized value of each parameter will be possible.

5) In case of forced vibrations, which help us to find the force at seat though interface, it may be possible to validate all models in this study, with mechanical impedances (driving point mechanical impedance and apparent mass). These may increase the accuracy of the model.

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