

# **Designing and Analysis of Advance Fractional Order Controller for Pitch Control of Aircraft System**

*A dissertation submitted in partial fulfilment of the  
requirements for the award of degree of*

**Master of Engineering  
in  
Electronic Instrumentation and Control**



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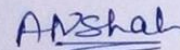
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## DECLARATION

I hereby certify that the work is being presented in this thesis work entitled "Designing and Analysis of Advance Fractional Order Controller for Pitch Control of Aircraft System" in partial fulfilment of award of degree of Master of Engineering in Electronics Instrumentation & Control submitted in Electrical & Instrumentation Engineering Department, Thapar University, Patiala is an authentic record of my own work carried under the supervision of Dr. Gagandeep Kaur, Assistant Professor, Department Of Electrical & Instrumentation Engineering, Thapar University, Patiala, Punjab.

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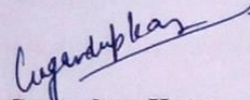


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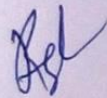
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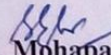
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## ABSTRACT

The combination of nonlinear dynamics, modelling uncertainties and parameters variation in characterizing are major problem in aircraft. Three rotations Roll, Yaw and Pitch can be controlled by using Aileron, rudder and elevator. The control system of aircraft can be divided into two parts which are longitudinal and lateral control. Pitch control is a longitudinal problem which is used to design autopilot. To achieve better result, for such critical control scheme, Fractional Order Proportional Integral Derivative control FOPID, Fractional Order Of [Proportional Integral Derivative] FO[PID] control are proposed in this thesis work. These controllers are compared with conventional order controller. Zeigler-Nichols Tuning proves to be more effective in giving accurate results. The main function of designing fractional order controller is to determine the two parameter  $\lambda$  and  $\mu$  apart from the usual tuning parameters i.e.  $K_p$ ,  $K_i$  and  $K_d$  of controller. Both the parameters  $\lambda$  and  $\mu$  are in fraction order controller, increase the robustness of the system and gives an optimal control because  $\lambda$  is in the integral action and  $\mu$  is in derivative action so it improve the response in fraction.

From the results and graphs of FOPID controller gives better response than conventional order controller. There is improvement in control parameters with rise time improved by 0.0792 seconds, peak overshoot by 0.1873 %, settling time improved by 0.099 seconds, ISE,IAE and ITAE is 0.0068% ,0.008% ,0.0751 % decrease respectively . The control parameters of FO[PID] controller is improved as compared to FOPID controller with rise time 0.0791 seconds, peak overshoot by 0.0353%, settling time with 0.0989 seconds and ISE , IAE by 0.1605% ,0.0035% decrease respectively. There is a improvement in the control parameters of FOPID controller as compared to conventional PID controller with rise time 0.1583 seconds, peak overshoot by 0.2226%, settling time with 0.1979 seconds, ISE, IAE and ITAE by 0.1673% , 0.0117% 0.0322% decrease.

As first order disturbance in the system, the FOPID Controller gives better response then conventional order controller. The value of rise time and Settling time is 1.3698 and 0.7232 second decrease, IAE values 0.0028% decrease but ISE and ITAE values 0.3113 and 0.5708 are increase respectively. As compare to FO[PID] control to conventional order PID controller the rise time is decreased by 1.4843 second, settling time is decreased by 1.3583 second and ISE, IAE and ITAE is increased by 0.2804% , 0.4474% and 0.3215%.

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## **LIST OF SYMBOLS AND ABBREVIATION**

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PID	Proportional Integral Derivative
PI	Proportional Integral
$K_c$	Controller Gain
$K_p$	Proportional Gain
$K_i$	Integral Gain
$K_d$	Derivative Gain
$T_i$	Integral Time
$T_d$	Derivative Time
$P_{cr}$	Ultimate Period of Oscillator
FPID	Fractional Proportional Integral Derivative
FOPID	Fractional order Proportional Integral Derivative
FO[PID]	Fractional order of Proportional Integral Derivative
$T_s$	Settling Time
$M_p$	Peak Overshoot
ISE	Integral Square Error
IAE	Integral Absolute Error
ITAE	Integral Time Absolute Error
MIMO	Multiple Input Multiple Output
X	X directional force
M	Mass of aircraft
g	Gravity force
$\theta, \Phi, \delta_e$	Orientation of aircraft in earth system & elevator deflection angle
L	Aerodynamics moment components for Roll Axis
M	Aerodynamics moment components for Pitch Axis
N	Aerodynamics moment components for Yaw Axis
p	Angular Axis about to Roll Axis
q	Angular Axis about to Pitch Axis
r	Angular Axis about to Yaw axis
u	Velocity component for Roll Axis
v	Velocity component for Pitch Axis
w	Velocity component for Yaw Axis

$\alpha$	Angle of attack
$\beta$	Angle of sideslip
$\lambda$	Integral Order
$\mu$	Derivative Order

# CHAPTER 1

## INTRODUCTION

---

### 1.1 Overview

Aircraft and missile systems are encumbered with control systems whose tasks are to provide stability, disturbance attenuation and reference signal tracking. An aircraft in flight is free to rotate in three dimensions which are roll, pitch and yaw. In roll rotation aircraft rotates about an axis running from nose to tail. In pitch rotation, aircraft rotates its nose up or down about an axis running from wing to wing and in yaw rotation, aircraft rotates its nose left or right about an axis running up and down. This rotation is controlled by using aileron, rudder and elevator.

The control system of the aircraft is further divided into two parts i.e. longitudinal control and lateral control. In longitudinal control, the elevator controls pitch and the motion of the aircraft. Pitch control is a longitudinal problem. This work presents on design an autopilot that controls the pitch of the aircraft. A pilot relief mechanism that assist in maintaining an attitude, heading, altitude or flying to navigation or landing reference is known as autopilot.

The combination of modelling uncertainties, nonlinear dynamics and parameters variations in characterizing an aircraft and its operating environment are the major problems in any flight mechanism. In this work there is study of the control strategies required to address the complex longitudinal dynamic characteristics in this aircraft. There are many research works done to control the stability of the air craft but it is still an open work to resolve this problem.

The development of Pitch control schemes for pitch angle of the aircraft by using fractional order controller. Fractional order controller are further divided into fractional order PID control and fractional order of PID control. The performance of all control strategy, the pitch angle of aircraft longitudinal dynamic can be investigated. Simulation is developed within Simulink and Matlab for all fractional order strategy. For better performance with the all fractional order models it is required to compare all the parameters which are discuss in this project work.

## 1.2 Objective of Thesis

The main objective of the proposed work are given as

- i. The main objective of this dissertation is to control the pitch of Aircraft machine by using different types of fractional order controller.
- ii. Design and implementation of conventional order controller, Fractional order Proportional Integral Derivative FOPID, fractional order of Proportional Integral Derivative FO[PID] controller as well as fractional order Proportional Integral Derivative FOPID, fractional order of Proportional Integral Derivative FO[PID] with disturbance.
- iii. Evaluate the performance of conventional and intelligent controllers. For the best result, it is required to compare all the parameters , time response analysis and find out the best values for integral order and derivative order for the fractional order system.

## 1.3 Organization of the Thesis

The brief introduction of the organization of the thesis is shown as below.

- |           |  |
|-----------|--|
| Chapter 1 | An introduction of the thesis is given.  |
| Chapter 2 | The relevant literature review regarding control of the Aircraft pitch control and intelligent techniques are discussed. |
| Chapter 3 | An overview of the controller design methodologies are explained.  |
| Chapter 4 | An introduction about Aircraft pitch control is given.   |
| Chapter 5 | The mathematical modelling of Aircraft pitch control is drawn.   |
| Chapter 6 | It shows the results are shown and discussion is done about control system.  |
| Chapter 7 | The conclusion of the entire thesis work is done and the future scope kept for further result.                           |
| Chapter 8 | It shows the originality of the thesis.  |

**CHAPTER 2****LITERATURE REVIEW**

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The following section describes the literature survey that is relevant with the work carried out for this thesis work.

H.Bulter et.al., (1989), used an adaptive time-optimal position controller for a direct-drive DC motor with the design based on the model reference adaptive approach is presented. The high- acceleration torque of new DC motors with permanent magnets permits a direct coupling of the load to the motor axis, avoiding the use of a transmission with its inherent disadvantages (such as backlash and friction). However, the direct coupling induces a large sensitivity of the motor behavior to load variations; therefore, a fixed, linear controller cannot provide an acceptable response under varying load conditions. Because of variations of load inertia, the desired response and the reference model in the adaptive controller have to be adjusted to the motor capabilities. This is achieved by estimating the load inertia by means of a least-squares method and adjusting the reference model accordingly. The controller is tested on a direct-drive motor, and the results are compared with those obtained with a fixed proportional-integral-derivative controller.[1]

Schlegel Milos et.al.,(2006), performed the three-parameter of PID controller is the most popular industrial controller. Only two more parameters arise after considering the integrator and derivator of arbitrary real order. Such a generalized controller is called fractional-order PID controller (FPID). Moreover, these two parameters named the order of integrator and derivation have a clear physical understanding. In this paper the robustness regions method for traditional PID controller was discussed. The aim of this paper is to present a generalized robustness regions method for the FPID controller. The proposed procedure allows to define several frequency domain closed loop requirements simultaneously - gain and phase margins, limitations of sensitivity functions, proper bandwidth. The final example shows that the FPID controller ensures the fulfilment of stricter closed-loop requirements than the traditional PID.[2].

Serdar Hamamci et.al.,(2007), proposed that the technical note presents a solution to the problem of stabilizing a given fractional-order system with time delay using fractional-order  $PI^\lambda D^\mu$  controllers. It is based on determining a set of global stability regions in the space corresponding to the fractional orders and in the range of (0, 2) and then choosing the biggest global stability region in this set. This method can be also used to find the set of stabilizing controllers that guarantees pre-specified gain and phase margin requirements. The algorithm is simple and has reliable result which is illustrated by an example, and, hence, is practically useful in the analysis and design of fractional-order control systems. [3].

Varsha Bhamhani et.al.,(2008), has presented comparative experimental study on coupled-tank liquid level control using fractional order PI control and integer order PI control. Tuning of integer order PI control and fractional order PI control has been done using Ziegler and Nichols tuning method. Experimental results confirmed that fractional order PI controller is a promising controller in terms of percentage overshoot and system response in liquid-level control in face of nonlinearities introduced by pumps, valves and sensors. [4]

Hyo Sung Ahn et.al., (2008), implemented a strategy to tune a fractional order integral and derivative controller satisfying gain and phase margins. The closed-loop system designed has a feature of robustness to gain variations with step responses exhibiting a nearly iso-damping property. This paper aims to apply the tuning procedure proposed to temperature control at selected points in Quanser's heat flow experimental platform. From the comparison with the traditional PI/PID controller based on Ziegler Nichol's tuning method, the effectiveness and validity of the proposed methodologies are illustrated [5].

Li Meng et.al., (2009) , developed the fractional-order PID controller provides more adjustable parameters in the controller optimization than conventional PID controller. Therefore, FOPID is designed to achieve more goals, and multi-objective optimization based genetic algorithm is adopted. To solve this problem, a multi-objective optimization design method is proposed in this paper. Not only the robust performance, but also frequency angle margin, overshoot and rise time are all taken as

the objectives to optimize. Then a variant of NSGA-II reach the optimal solution. This method can obtain uniformly distributed Pareto-optimal solutions and have good convergence and excellent robustness. The satisfactory solution is selected in Pareto optimum solution set according to the system requirement, which provides an effective tool for trade-off among the performance of quickness, stability and robustness. Simulation results support the superiority and effectiveness of the proposed method.[6]

Arijit Biswas, et.al.,(2009), implemented Differential evolution (DE) has recently emerged as a simple yet very powerful technique for real parameter optimization. This article describes an application of DE to the design of fractional-order proportional–integral–derivative (FOPID) controllers involving fractional-order integrator and fractional-order differentiator. FOPID controllers' parameters are composed of the proportionality constant, integral constant, derivative constant, derivative order and integral order, and its design is more complex than that of conventional integer-order proportional–integral–derivative (PID) controller. Here the controller synthesis is based on user-specified peak overshoot and rise time and has been formulated as a single objective optimization problem. In order to digitally realize the fractional-order closed-loop transfer function of the designed plant, Tustin operator-based continuous fraction expansion (CFE) scheme was used in this work. Several simulation examples as well as comparisons of DE with two other state-of-the-art optimization techniques (Particle Swarm Optimization and binary Genetic Algorithm) over the same problems demonstrate the superiority of the proposed approach especially for actuating fractional-order plants. The proposed technique may serve as an efficient alternative for the design of next-generation fractional-order controllers. [7].

Ying Luo et.al., (2010), proposed two fractional order proportional integral controllers are proposed and designed for a class of fractional order systems. For fair comparison, the proposed fractional order proportional integral (FOPI), fractional order [proportional integral] (FO[PI]) and the traditional integer order PID (IOPID) controllers are all designed following the same set of the imposed tuning constraints, which can guarantee the desired control performance and the robustness of the designed controllers to the loop gain variations. This proposed design scheme offers a

practical and systematic way of the controllers design for the considered class of fractional order plants. From the simulation and experimental results presented, both of the two designed fractional order controllers work efficiently, with improved performance comparing with the designed stabilizing integer order PID controller by the observation. Moreover, it is interesting to observe that the designed FO[PI] controller outperforms the designed FOPI controller following the proposed design schemes for the class of fractional order systems considered. [8].

N.Wahid et.al.,(2011), represents the investigation into the development of hybrid control scheme for pitch control of aircraft system. Proportional-integral-derivative (PID) and PID-type fuzzy logic controller are used in this investigation to control the pitch angle of aircraft system. The dynamic modelling of system begins with a derivation of suitable mathematical model to describe the longitudinal motion of an aircraft. The reason behind this research is to investigate which approach provides the best performance base on time response specification and disturbances rejection for an autopilot of longitudinal dynamic in pitch aircraft. Through the simulation in Matlab and Simulink results shows that, the PID-type fuzzy logic controller perform the best performance compared to classical controller technique, Proportional-Integral-Derivative (PID) [9].

Abhinav Gautam et.al.,(2011), has proposed the PID controller is a well known controller which is used in most control applications. Around 90% control applications use PID controller as the controlling element. The tuning of PID controller is mostly done using Zeigler-Nichols tuning method. But there are some inherent drawbacks of Ziegler-Nichols based tuning. For the optimal tuning of controller, the tuned values have to be changed using computer simulation to meet the process needs. In PID controller the derivative and the integral order are in integer. fractional order PID (FOPID) is a special kind of PID controller whose derivative and integral order are fractional rather than integer. The key challenge of designing FOPID controller is to determine the two key parameters  $\lambda$  (integral order) and  $\mu$  (derivative order) apart from the usual tuning parameters of PID using different tuning methods. Both  $\lambda$  and  $\mu$  are in fraction which increases the robustness of the system and gives an optimal control. This paper proposes a novel tuning method for tuning  $\lambda$  and  $\mu$  of FOPID using genetic algorithms. [10]

Rinku Singhal et.al.,(2012), has presented Conventional PID controller is one of the most widely used controllers in industry, but the recent advancement in fractional calculus has introduced applications of fractional order calculus in control theory. One of the prime applications of fractional calculus is fractional order PID controller and it has received a considerable attention in academic studies and in industrial applications. Fractional order PID controller is an advancement of classical integer order PID controller. In many a cases fractional order PID controller has outperformed classical integer order PID controller. This research paper, studies the control aspect of fractional order controller in speed control of DC motor. A comparative study of classical PID controller and fractional order PID controller has been performed. [11].

Sneha D. Joshi et.al.,(2013), presented Control and modeling of nonlinear systems like Autonomous Underwater Vehicle (AUV) is a challenge for researchers. The modelling and controller scheme designed is based on fractional order system. Fractional order proportional integral derivative (FOPID) controller is designed and applied to fractional order depth and steering systems of AUV and it is compared with classical integer order PID Controller[42].

**CHAPTER 3****CONTROLLER DESIGN METHODOLOGIES**

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**3.1 Introduction**

The control system engineers are responsible to control a portion of an environment known as a system or plant to produce desired products for the mankind. The basic knowledge of system always played vital role in effective controlling of the system. A control engineer should have the basic knowledge of different engineering principles like electrical, mechanical and chemical etc.

Control system can be categorized as open loop and close loop feedback control and feedback control systems can be further categorized a single input single output known as SISO and multiple input multiple output known as MIMO, which is known as Multivariable System. An open loop control system is structured to meet the desired output by using a reference signal that operates the actuators that directly control the process output. The close loop control systems consists output of the feedback controller. The controller processes the error signal which is the difference of current output and desired output to find the desired output. [11]

**3.2 Controller types**

There are many types of control structures available. It is very important to select desired control structure as per the requirements. Main categorized control structures are listed [12].

- (i) Feedback Control Structure
- (ii) Feedforward Control Structure
- (iii) Feedback + Feedforward Control Structure
- (iv) Cascade Control Structure

**3.2.1 Feedback Control Structure**

Feedback control is a mechanism which regulated the controlled variable by taking negative feedback from the output and taking regulatory action through the controller and changing the manipulated variable accordingly.

### ***3.2.2 Feedforward Control Structure***

The basic concept of the feed forward controller is to measure the disturbances  $d(t)$  and take corrective action before the disturbance upset the process  $g(t)$  input that means the corrective action will take place before the error  $e(t)$  occurred.

### ***3.2.3 Feedback and Feedforward Control Structure***

In some systems the disturbance  $d(t)$  can be predicted. This feedforward structure gives good results, but in a process  $g(t)$  where there is always some unknown cause and source of disturbances which affects the system working, so for that's why only this kind of systems i.e. feedforward structure is not sufficient for that one should use feedback structure along with feedforward structure.

### ***3.2.4 Cascade Control Structure***

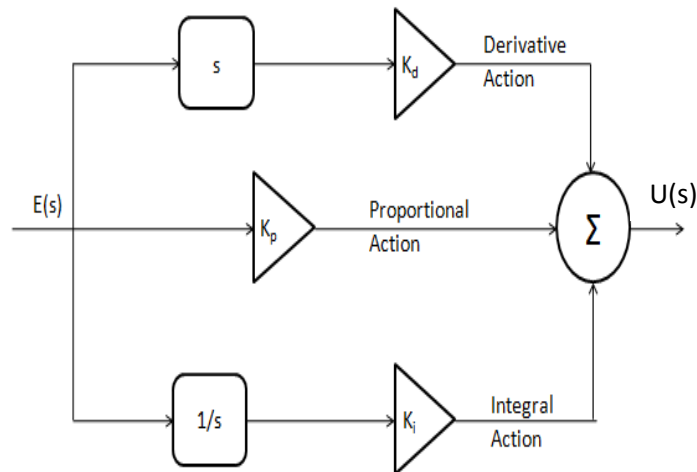
In a cascade control, there are two or more than two controllers of which one controller's output drives the set point of another controller. The controller driving the set point is called the primary, outer or master controller. The controller receiving the set point is called the secondary, inner or slave controller. The cascade control is beneficial only if the dynamics of the inner loop are fast compared to those of the outer loop.

## **3.3 PID Controller**

The proportional-integral-derivative (PID) controllers have been the most commonly used controller in process industries for more than 50 years even though significant developments have been made in advanced control theory. A PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs [13].

PID control is a name commonly divided into three-term control. The PID Controller calculation involves three separate constant parameters [14].

- Proportional Term
- Integral Term
- Derivative Term



**Figure 3.1:** Integer PID Controller

PID Controller can be describe by the following equation 3.1

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} + K_i \int_0^t e(t) dt \quad (3.1)$$

The PID refers to the first letters of the names of the individual terms that makes with the standard three term controllers. These are P stand for Proportional Term, I for the Integral Term and D indicate Derivative Term in the controller.

The theoretical basis for analyzing the performance of PID control is considerable aided by the simple representation of an Integrator by the Laplace transform  $1/s$  and a Differentiator using.

### 3.3.1 Characteristics of PID Controller

A proportional Controller ( $K_p$ ) used to reduce the rise time and steady state error. An integral control ( $K_i$ ) eliminates the steady-state error but it makes the transient response poor. A derivative control ( $K_d$ ) increase the stability of the system also reducing the overshoot and improving the transient response. Effects of each of controllers  $K_p$ ,  $K_i$ ,  $K_d$  on a closed loop systems are summarized in the Table shown in 3.1 [16]

These correlations may not be accurate exactly, because  $K_p$ ,  $K_i$ , and  $K_d$  are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the Table should only be used as a reference when you are determining the values for  $K_i$ ,  $K_p$  and  $K_d$  [17].

**Table 3.1** Effects of independent P, I, and D tuning

<b>Controller Response</b>	<b>Rise Time</b>	<b>Overshoot</b>	<b>Settling Time</b>	<b>Steady State Error</b>	<b>Stability</b>
Increasing $K_p$	Decrease	Increase	Small Increase	Decrease	Degrade
Increasing $K_i$	Small Decrease	Increase	Increase	Large Decrease	Degrade
Increasing $K_d$	Decrease	Decrease	Decrease	Minor Change	Improve

### 3.4 Performance of Indices

The optimal control can't be defined precisely in general. A solution, which is optimum for one particular application for given set of conditions, may not be optimal for another problem. In such case, defining performance criterion and minimize in the value of such performance index, optimal constants can be obtained as required for economic and practical problem. One of such index is cost function, which is used by most of the control engineers. Such performance indicates help us in assessing the quality of a control system [15].

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#### 3.4.1 Performance of the Index (P.I.)

A performance index is a quantitative measure of the performance of the system. A system is considered an optimal control system when the system parameters are adjusted so that the index reaches an extreme value, commonly a minimum value.

- **ISE (Integral of Square Error)** A suitable fitness function (Performance index) is the integral of square error.

$$\text{ISE} = \int_0^{\infty} e^2(t) dt \quad (3.2)$$

- **IAE(Integral of Absolute Error)** Another fitness function is the integral of the absolute error as follow:

$$\text{IAE} = \int_0^{\infty} |e(t)| dt \quad (3.3)$$

This criterion penalized large errors less heavily and small errors more heavily compared to ISE.

- **ITAE(Integral of Time Multiplied by Absolute Value of Error):** It is achieved by adding time weighting to IAE and defined as:

$$\text{ITAE} = \int_0^{\infty} t |e(t)| dt \quad (3.4)$$

This generally leads to systems with reasonable transient characteristics. It is comparable in many respects to ITAE but is not mathematically analytic. This also has a property that initial large errors gets weighted lightly and late occurring error gets penalized heavily.

### 3.5 Limitations of PID Controller

Several limitations are associated with PID controller [18]. Some of the limitations of PID controllers are as follow

- Feedback controller
- Linearity
- Noise in derivative

#### 3.5.1 Feedback Controller

The fundamental difficulty with PID control is that it is a feedback system, with constant parameters. It has no direct knowledge of the process, and thus overall performance is reactive and a compromise.

### 3.5.2 *Linearity*

PID controller is linear in nature this is another limitation of the controller. Thus performance of PID controllers in non-linear systems is variable. For example, in temperature control, a common use case is active heating with a heating element but passive cooling which are heating off, but no cooling, so overshoot can only be corrected slowly. It cannot be forced downward. In this case the PID should be tuned to be over damped, to prevent or reduce overshoot, though this reduces performance and it increases settling time.

### 3.5.3 *Noise in Derivative*

A problem with the derivative term is if the small amounts of measurement or process noise has been there but there is cause of large amounts of change in the output. It is often helpful to use filter the measurements with a low-pass filter in order to remove higher-frequency noise components. However, low-pass filtering and derivative control can cancel each other out, so reducing noise by instrumentation is a much better choice [18].

## 3.6 Tuning of PID Controller

The industrial control companies took the opportunity to develop new PID controller methods for use with the new ranges of controller technology when analog control was being replaced by digital processing hardware. Appearing Consequently, the Ziegler–Nichols methods became the focus of research and have since, because of better understood. New versions of the Ziegler–Nichols procedures were introduced, notably the Astrom and Hagglund relay experiment. In many applications, the implicit under damped closed-loop performance inherent in the original Ziegler–Nichols design rules was found to be unacceptable. The result was an extensive development of the rule-base for PID controller tuning. O’Dwyer has published summaries of a large class of the available results. Continuing competitive pressures in industry have led to a constant need for continual improvements in control loop performance. One result of these trends is that industry is much better at being able to specify the type of performance that a control system has to deliver [19].

### 3.7 Ziegler-Nichols Tuning

Ziegler and Nichols, both of them are the employees of Taylor Instruments. They described simple mathematical procedures in the form of the open loop and close loop methods, for tuning PID controllers. Their methods were used for non-first order plus dead time situations, and involved intense manual calculations. With improved optimization software, most manual methods such as these are no longer used. However, even with computer aids, the following two methods are still employed today, and are considered among the most common. There are two methods of Ziegler-Nichols tuning which are

- i. Open loop method
- ii. Closed loop method

These methods are explained in the following sections [20].

#### 3.7.1 Open Loop Method

The Ziegler-Nichols open-loop method is also referred to as a process reaction method, because it tests the open-loop reaction of the process to a change in the control variable output. This basic test requires that the response of the system be recorded, preferably by a plotter or computer. Once certain process response values are found, they can be plugged into the Ziegler-Nichols equation with specific multiplier constants for the gains of a controller with either P, PI, or PID actions. The response is characterized by two parameters, L the delay time and T the time constant. These are found by drawing a tangent to the step response at its point of inflection and noting its intersections with the time axis and the steady state value. The plant model is therefore

$$G(s) = \frac{Ke^{-sL}}{Ts + 1} \quad (3.5)$$

Where

G(s) = Transfer function of the process

K = Gain of the process

T = Time constant of the process

**Table 3.2** Ziegler-Nichols recipe- open loop method

<b>Controller Type</b>	$K_p$	$T_i = \frac{K_p}{K_i}$	$T_d = \frac{K_d}{K_p}$
<i>P</i>	$\frac{T}{L}$	$\infty$	0
<i>PI</i>	$0.9 \frac{T}{L}$	$\frac{L}{0.3}$	0
<i>PID</i>	$1.2 \frac{T}{L}$	$2L$	$0.5L$

In the Table 3.2, proportional gain, integral time and derivative time for different controller types are derived using ultimate gain and ultimate period using Ziegler-Nichols open loop method.

**3.7.2 Close Loop Method**

The Ziegler-Nichols closed-loop tuning method allows to use the ultimate gain value,  $K_{cu}$ , and the ultimate period of oscillation,  $P_u$ , to calculate  $K_p$ . You can obtain the controller constants  $K_p$ ,  $T_i$ , and  $T_D$  in a system with feedback.

The steps for tuning a PID controller via the close loop method are as follows:

- Step1** Remove the integral and derivative actions.
- Step2** Adjust the proportional gain by increasing or decreasing until at which sustained oscillation occurs.
- Step3** Record the gain ultimate gain value  $K_{cu}$  and ultimate period of oscillation  $P_u$

The controller gains are now specified as follows

**Table 3.3** Ziegler Nichols recipe- closed method

<b>Type of controller</b>	$K_p$	$T_i$	$T_d$
<i>P</i>	$0.5 K_{cu}$	$\infty$	0
<i>PI</i>	$0.35 K_{cu}$	$\frac{P_u}{1.2}$	0
<i>PID</i>	$0.6 K_{cu}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

In the above Table 3.3 proportional gain, integral time and derivative time for different controller types are derived using ultimate gain and ultimate period using Ziegler-Nichols second method.

### **3.8 Fractional order controller**

Fractional order controller is described by fractional order differential equations. Fractional calculus allows the derivatives and integrals to be any real number. Expanding derivatives and integrals to fractional orders can adjust control system's frequency response directly and continuously. This great flexibility makes it possible to design more robust control system [21]. The fundamental advantage of FOC is that the fractional order integrator weights history using a function that decays with a power-law tail. The effect is that the effects of all time are computed for each iteration of the control algorithm. This creates a 'distribution of time constants,' the upshot of which is there is no particular time constant, or resonance frequency, for the system.

Fractional order control shows promise in many controlled environments that suffer from the classical problems of overshoot and resonance, as well as time diffuse applications such as thermal dissipation and chemical mixing. Fractional controller can be designed in both frequency and time domain [22].

#### ***3.8.1 Fractional order calculus***

Fractional order calculus has gained acceptance in last couple of decades. J Liouville made the first major study of fractional calculus in 1832. In 1867, A. K. Grunwald worked on the fractional operations. G. F. B. Riemann developed the theory of fractional integration in 1892. Fractional order mathematical phenomena allow us to describe and model a real object more accurately than the classical "integer" methods. Earlier due to lack of available methods, a fractional order system was used to be approximated as an integer order model. But at the present time, there are many available numerical techniques which are used to approximate the fractional order derivatives and integrals. A typical example of a non-integer (fractional) order system is the voltage current relation of a semi-infinite loss transmission line [23].

The past decade has seen an increase in research efforts related to fractional calculus and use of fractional calculus in control system. For a control loop perspective there are four situations like

1. Integer order plant with integer order controller.
2. Integer order plant with fractional order controller.
3. Fractional order plant with integer order controller.
4. Fractional order plant with fractional order controller.

Fractional order control enhances the dynamic system control performance

### 3.8.1.1 Fractional Order Calculus: Mathematical model

Fractional order calculus is an area where the mathematicians deal with derivatives and integrals from non-integer orders. Gamma function is simply the generalization of the factorial for all real numbers. The definition of the gamma function is given by

$$\Gamma(x) = \int_0^{\infty} z^{x-1} e^{-z} dz \quad (3.6)$$

$$\Gamma(x) = (x - 1) ! \quad (3.7)$$

Differ integral Operator

Differ integral operator is denoted by  ${}_a D_t^\alpha$ . It is the combination of differentiation and integration operation commonly used in fractional calculus. Reimann- Liouville definition for is

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (dz)^{-\alpha} & \alpha < 0 \end{cases} \quad (3.8)$$

Here  ${}_a D_t^\alpha$  is the fractional order. a and t are the limits. There are two commonly used definitions for general Differintegral  ${}_a D_t^\alpha$ .

- (1) Grunwald - Letnikov
- (2) Riemann- Liouville

Grunwald – Letnikov Definition

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\left(\frac{t-a}{h}\right)} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (3.9)$$

${}_a D_t^\alpha$  is a flooring-operator here

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+2)\Gamma(\alpha-j+1)} \quad (3.10)$$

Riemann- Liouville Definition

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (3.11)$$

The condition for above equation is  $n-1 < \alpha < n$ .  $\Gamma(\cdot)$  is called the gamma function.

Laplace Transform of Differ integral Operator

Differ integral operator is denoted by  ${}_a D_t^\alpha f(t)$  and the Laplace transform of Differ integral operator is represented as

$$L [ {}_a D_t^\alpha f(t) ] = \int_0^\infty e^{-st} {}_a D_t^\alpha f(t) dt \quad (3.12)$$

$$L [ {}_a D_t^\alpha f(t) ] = s^\alpha F(s) - \sum_{m=0}^{n-1} s(-1)^j {}_0 D_t^{\alpha-m-1} f(t) \quad (3.13)$$

### 3.8.2 Properties of fractional Calculus

The main properties of fractional derivative and integrals are following [24]

- If  $f(t)$  is an analytical function of  $t$ , its fractional derivative  ${}_0 D_t^\alpha f(t)$  is an analytical function of  $z$  and  $\alpha$ .
- For  $\alpha = n$ , where  $n$  is an integer, the operation  ${}_0 D_t^\alpha f(t)$  gives the same result as classical differentiation of integer order  $n$ .
- For  $\alpha = 0$  the operation  ${}_0 D_t^\alpha f(t)$  is the identity operator:

$${}_0 D_t^0 f(t) = f(t) \quad (3.14)$$

- Fractional differentiation and fractional integration are linear operations

$${}_0 D_t^\alpha a f(t) + b g(t) = a {}_0 D_t^\alpha f(t) + b {}_0 D_t^\alpha g(t) \quad (3.15)$$

- The additive index law (semi group property)

$${}_0 D_t^\alpha {}_0 D_t^\beta f(t) = {}_0 D_t^\beta {}_0 D_t^\alpha f(t) = {}_0 D_t^{\alpha+\beta} f(t) \quad (3.16)$$

Holds under some reasonable constraints on the function  $f(t)$ .

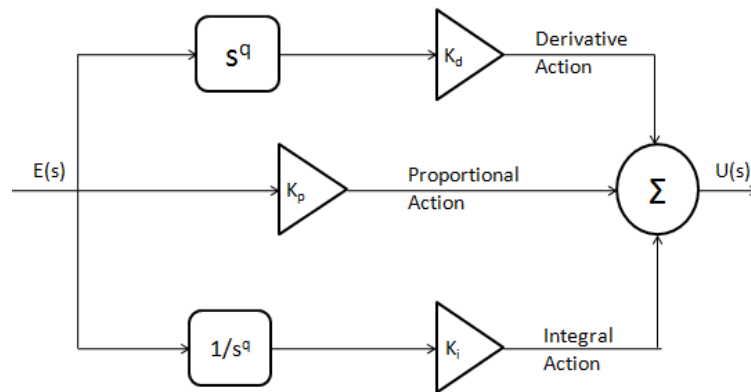
### 3.9 Fractional Order Controller

#### 3.9.1 Fractional Order PID Controller

One of the primary controllers is PID controller, which is widely used. Fractional controller is denoted by  $PI^\lambda D^\mu$  was proposed by Igor Podlubny in 1997 [25], here  $\lambda$  and  $\mu$  have non-integer values. Figure 3.2 shows the block diagram of fractional order PID controller.

The transfer function for conventional PID controller is

$$G_{PID}(s) = \frac{u(s)}{e(s)} = K_c \left( 1 + \frac{1}{\tau_i s} + \tau_d s \right) \quad (3.17)$$



**Figure 3.2:** Fractional order PID control

Transfer function for fractional order PID controller is

$$G_{FOPID}(s) = \frac{u(s)}{e(s)} = K_c \left( 1 + \frac{1}{\tau_i s^\lambda} + \tau_d s^\mu \right) \quad (3.18)$$

Where  $\lambda$  and  $\mu$  are an arbitrary real numbers,  $K_p$  is amplification (gain),  $T_i$  is integration constant and  $T_d$  is differentiation constant. Taking  $\lambda=1$  and  $\mu=1$ , a classical PID controller is obtained. For further practical digital realization, the derivative part has to be complemented by first order filter. The filter is used to remove high frequency noise.

$$G_{FOPID}(s) = \frac{u(s)}{e(s)} = K_c \left( 1 + \frac{1}{\tau_i s^\lambda} + \frac{\tau_d s^\mu}{\frac{\tau_d}{N} s + 1} \right) \quad (3.19)$$

The  $PI^\lambda D^\mu$  controller is more flexible and gives an opportunity to better adjust the dynamics of control system. Its compact and simple but the analog realization of fractional order system is very difficult.

Intuitively, the FOPID has more degree of freedom than the conventional PID. It can be expected that the FOPID can provide better performance with proper choice of controller parameters. However, with more parameters to be tuned, the associated optimization problem will be more difficult to deal with. It is motivated to develop a systematic procedure for the FOPID optimization to achieve a certain performance. The fractional order  $PI^\lambda D^\mu$  generalizes the conventional integer order PID controller and expands it from point to plane. This expansion could provide much more flexibility in PID control design.

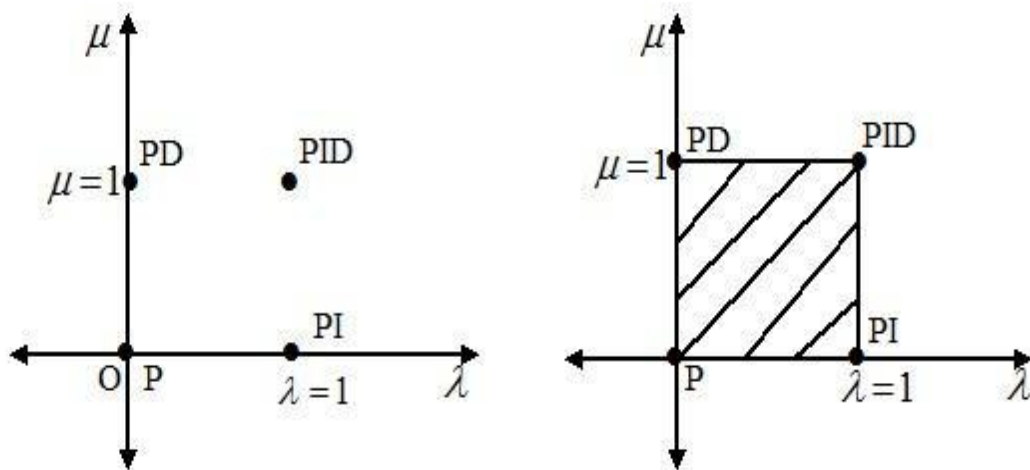


Figure 3.3: PID controller with fractional Order

As shown in the figure 3.3 Riemann-Liouville definition, fractional order systems have an infinite dimension. To realize fractional order controllers perfectly, all the past inputs should be memorized. Several proper approximation methods by finite differential or difference equation were proposed in recent researches, such as Sampling Time scaling, Short memory principle, Tustin Taylor Expansion, Lagrange function interpolation method. Because fractional order systems have an infinite dimension, the digital realization of FOPID keeps somewhat difficulty. The above FOPID controller can be approximated using different discrete methods, which is given by

$$G(z) = K_p + K_i W_i(z) + K_d W_d(z) \tag{3.20}$$

Where  $W_i(z)$  is the discrete approximation equation of fractional order integral  $s^{-\lambda}$ ,  $W_d(z)$  is the discrete approximation equation of  $s^{\mu}$ . The higher the order of approximation equation, the closer the discrete model is approximates the real fractional order systems [26].

## CHAPTER 4

# AIRCRAFT PITCH CONTROL

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### 4.1 Introduction

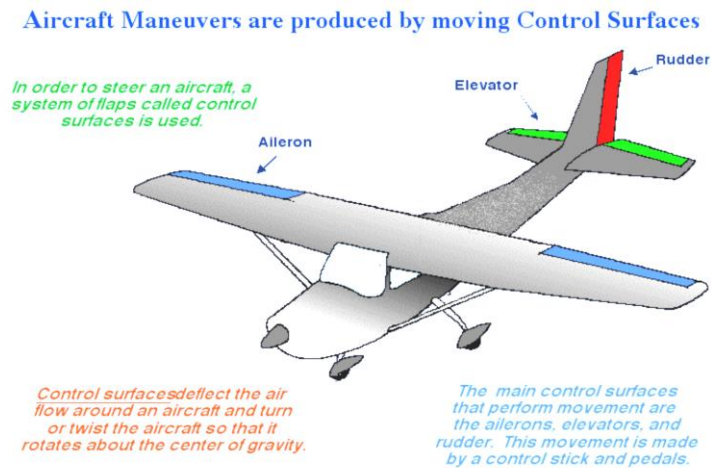
The first aircraft machine invented by the Wright brothers and with their unique invention, mankind can watch the beautiful creation of the nature from the upside from the earth. Day by day this machine enter in the various department and help in the different sector to reduce the work. To get noise rejection, stability, and reference signal tracking aircraft and missiles are usually loaded with a control system. Today's with high performance in military, commercial and general aviation aircraft required the development of many technologies those are structure, propulsion, aerodynamics, materials, and flight controls. An aircraft in flight is free to rotate in three dimensions i.e. pitch, yaw and roll. In pitch rotation, aircraft rotates its nose up or down about an axis running from wing to wing. In yaw rotation, aircraft rotates its nose left or right about an axis running up and down and in roll rotation aircraft rotates about an axis running from nose to tail. The axes are alternatively designated as lateral, longitudinal and vertical. These axes move with the vehicle and rotate relative to the Earth along with the craft.

These rotations are produced by torques or moments about the principal axes. On an aircraft, these are produced by means of moving control surfaces, which vary the distribution of the net aerodynamic force about the vehicle's centre of gravity. Elevators moving flaps on the horizontal tail which produce pitch. A rudder on the vertical tail produces yaw, and ailerons flaps on the wings which move in opposing directions produce roll. On a spacecraft, the moments are usually produced by a reaction control system consisting of small rocket thrusters used to apply asymmetrical thrust on the vehicle.

### 4.2 Principal Axes

There are mainly three types of Principal Axes which are listed Below.

- (i) Normal Axis or Yaw Axis: This axis is drawn from top to bottom , and perpendicular to the other two axes. This axis is parallel to the fuselage station.



**Figure 4.1:** Aircraft exercises produced by moving control surface [29]

- (ii) Lateral Axis or Transverse Axis or Pitch Axis: This axis running from the pilot's left to right in piloted aircraft, and parallel to the wings of a winged aircraft. This axis is parallel to the buttock line.
- (iii) Longitudinal Axis or Roll Axis: This axis drawn through the body of the vehicle from tail to nose in the normal direction of flight, or the direction the pilot faces. This axis is parallel to the waterline.

As shown in figure 4.1 these axes are represented by the letters X, Y and Z in order to compare them with some reference frame, usually named x, y, z. Normally this is made in such a way that the X is used for the longitudinal axis but there are other possibilities to do it.[30]

#### **4.2.1 Normal axis**

A yaw motion is a movement of the nose of the aircraft from side to side. The pitch axis is perpendicular to the yaw axis and is parallel to the body of the wings with its origin at the centre of gravity and directed towards the right wing tip. A pitch motion is an up or down movement of the nose of the aircraft. The roll axis is perpendicular to the other two axes with its origin at the centre of gravity, and is directed towards the nose of the aircraft. A rolling motion is an up and down movement of the wing tips of the aircraft. The rudder is the primary control of yaw.[27, 30]

### The Rudder Controls the Yaw Angle

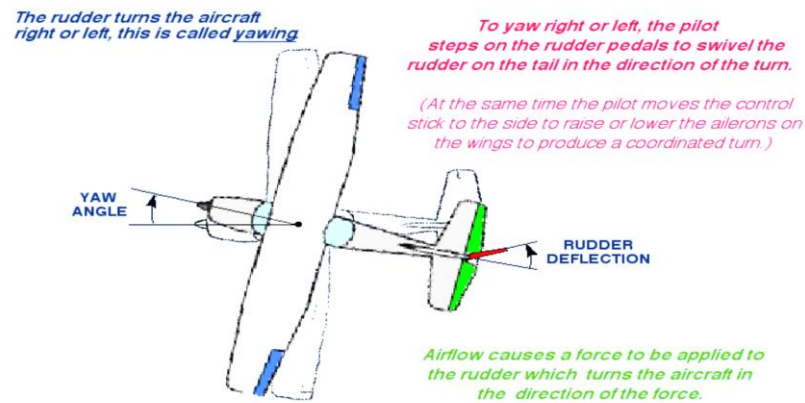


Figure 4.2: The rudder controls of the Yaw angle[29]

#### 4.2.2 Lateral axis

The lateral axis also called transverse axis passes through the plane from wingtip to wingtips. Rotation about this axis is called pitch. Pitch changes the vertical direction the aircraft's nose is pointing. The elevators are the primary control of pitch.[27]

### The Elevator Controls the Pitch Angle

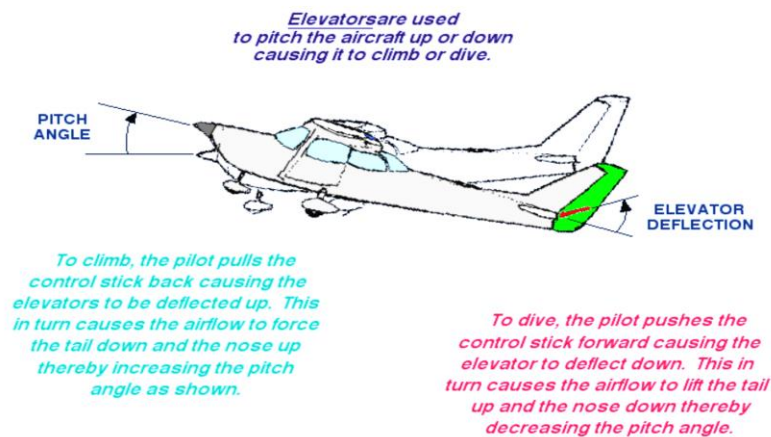


Figure 4.3: The elevator control of Aircraft pitch control[29]

Elevators are flight control surfaces which are usually at the rear of an aircraft, to control the aircraft's longitudinal attitude by changing the pitch balance, the angle of attack and the lift of the wing. The elevators are usually hinged to a fixed or adjustable rear surface making as a whole a tail plane or horizontal stabilizer. There may be also the only pitch control surface present which is sometimes located at front are found in early airplanes or integrated in a rear "all-moving tail plane" also called a slab elevator or stabilator.[30]

#### ***4.2.2.1 Elevator's effect***

The wing in which elevators are hinged provides the horizontal stabilization. It is mostly located at the rear end of an aircraft which has the opposite effect to the main wing while flying. The horizontal stabilizer usually creates a downward force which balances the nose down moment due to the airplane's center being located in front of the centre of lift, and other moments due to the effects of drag and engine thrust [31]. In balanced condition, the setting of the elevator determines the airplane's trim speed - a given elevator position has only one lift coefficient and one speed at a given altitude at which the aircraft will maintain a constant, unaccelerated condition. In control mode, the elevators change the aircraft trim and make the aircraft nose-up or nose-down. The elevators decrease or increase the downward force created by the rear wing [43]. To shoot up the downward force which is produced by up elevator forces the tail down and the nose up. The shoot up wing angle of attack causes a greater lift to be produced by the wing and more drag demanding more power to keep the speed or climb. To reduce the downward force at the tail which is produced by down elevator allows the tail to rise and the nose to lower. The decrease in angle of attack reduces the lift demanding more speed also adding power or going to a descent to produce the required lift [31].

Supersonic aircraft have stabilators because early supersonic flight research revealed that shock waves generated on the rear part of a tail plane rendered hinged elevators ineffective. Delta winged aircraft combine ailerons and elevators and their respective control inputs into one control surface, called an elevon [31].

#### ***4.2.2.2 Elevator's location***

Elevators are part of the tail at the rear of an aircraft. In some aircraft, pitch-control surfaces are in the front, ahead of the wing in a two-surface aircraft. This type of configuration is called a canard known as duck or a tandem wing. Early three surface aircraft had front elevators modern three surface aircraft may have both front and rear elevators [31].

The development of automatic control system has played an important role in the growth of civil and military aviation. Modern aircraft include a variety of automatic control system that aids the flight crew in navigation, flight management

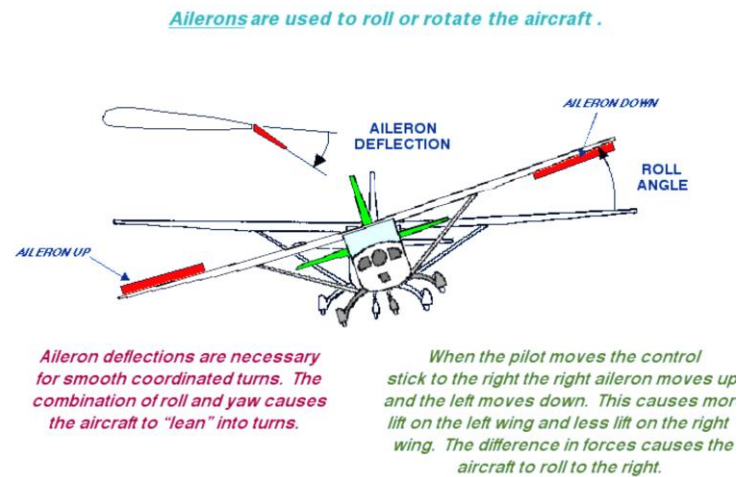
and augmenting the stability characteristic of the airplane[46]. For this situation an autopilot is designed that control the pitch of aircraft that can be used by the flight crew to lessen their workload during cruising and help them land their aircraft during adverse weather condition in the real situation [33]. The autopilot is an element within the flight control system. It is a pilot relief mechanism that assists in maintaining an attitude, heading, altitude or flying to navigation or landing references [34]. Designing an autopilot requires control system theory background and knowledge of stability derivatives at different altitudes and Mach numbers for a given airplane [35]. Lot of works has been done in the past to control the pitch of an aircraft for the purpose of flight stability and yet this research still remains an open issue in the present and future works [36, 37, 38, 39 and 40]

Pitch is controlled by the rear part of the tail plane's horizontal stabilizer being hinged to create an elevator. By moving the elevator control backwards the pilot moves the elevator up which is a position of negative camber and the downwards force on the horizontal tail is increased. The angle of attack on the wings increased so the nose is pitched up and lift is generally increased. In micro-lights and hang gliders the pitch action is reversed and the pitch control system is much simpler, so when the pilot moves the elevator control backwards it produces a nose down pitch and the angle of attack on the wing is reduced. The pitch angle of an aircraft is controlled by adjusting the angle and therefore the lift force of the rear elevator. The aerodynamic forces which are lift and drag as well as the aircraft's inertia are taken into account. This is a third order, nonlinear system which is linearized about the operating point.[27]

#### ***4.2.3 Longitudinal axis***

The longitudinal axis passes through the plane from nose to tail. Rotation about this axis is called bank or roll. Bank changes the orientation of the aircraft's wings with respect to the downward force of gravity. The pilot changes bank angle by increasing the lift on one wing and decreasing it on the other. This differential lift causes bank rotation around the longitudinal axis. The ailerons are the primary control of bank. The rudder also has a secondary effect on bank.[27]

## The Ailerons Control the Roll Angle



**Figure 4.4:** The ailerons control of the roll angle[29]

These axes are related to the principal axes of inertia, but are not the same. They are geometrical symmetry axes, regardless of the mass distribution of the aircraft. In aeronautical and aerospace engineering intrinsic rotations around these axes are often called Euler angles, but this conflicts with existing usage elsewhere. The calculus behind them is similar to the Frenet-Serret formulas. Performing a rotation in an intrinsic reference frame is equivalent to right-multiply its characteristic matrix and this matrix that has the vector of the reference frame as columns by the matrix of the rotation.

A conventional fixed-wing aircraft flight control system consists of flight control surfaces, the respective cockpit controls, connecting linkages, and the necessary operating mechanisms to control an aircraft's direction in flight. Aircraft engine controls are also considered as flight controls as they change speed.[27]

### 4.3 Flight Dynamics

Flight dynamics is the study of the performance, stability, and control of vehicles flying through the air or in outer space. It is concerned with how forces acting on the vehicle influence its speed and attitude with respect to time. Flight dynamics is the science of air vehicle orientation and control in three dimensions. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as roll, pitch and yaw

Aircraft engineers develop control systems for a vehicle's orientation (attitude) about its center of mass. The control systems include actuators, which exert forces in various directions, and generate rotational forces or moments about the center of gravity of the aircraft, and thus rotate the aircraft in pitch, roll, or yaw. For example, a pitching moment is a vertical force applied at a distance forward or aft from the center of gravity of the aircraft, causing the aircraft to pitch up or down.

Roll, pitch and yaw refer, to rotations about the respective axes starting from a defined equilibrium state. The equilibrium roll angle is known as wings level or zero bank angle, equivalent to a level heeling angle on a ship. Yaw is known as "heading".

A fixed-wing aircraft increases or decreases the lift generated by the wings when it pitches nose up or down by increasing or decreasing the angle of attack (AOA). The roll angle is also known as bank angle on a fixed-wing aircraft, which usually "banks" to change the horizontal direction of flight. An aircraft is usually streamlined from nose to tail to reduce drag making it typically advantageous to keep the sideslip angle near zero, though there are instances when an aircraft may be deliberately "side slipped" for example a slip in a fixed-wing aircraft. [27]

**CHAPTER 5****MODELLING OF AIRCRAFT PITCH CONTROL**

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**5.1 Introduction**

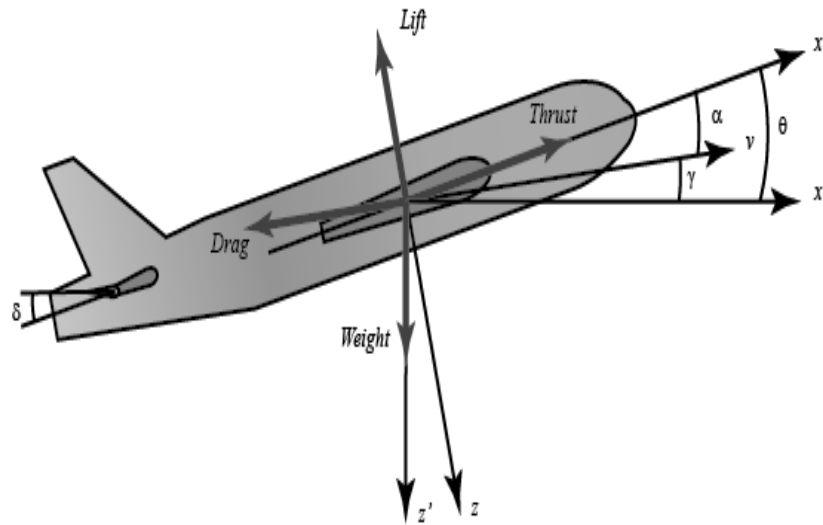
Control of dynamic systems with modern sophistication and complexities has often been an important research area due to the difficulties in uncertainties, modelling, and nonlinearities, particularly when there is a constant change in system dynamics. The response of the dynamic nonlinear plant cannot be tracked into a desired pattern with a linear controller. So, a changing parameter of dynamic controller in a plant is critically important to control in a strategic way [41]. The dynamic controller of aircraft required continuous monitoring from different sensors the aircraft system provided to measure the attitude, heading, altitude or flying to navigation or landing reference.

Pitch is defined as a rotation around the lateral or transverse axis, which is parallel to the wings, and is measured as the angle between the direction of speed in a vertical plan and the horizontal line. Changes of pitch are caused by the deflection of the elevator, which rises or lowers the nose and tail of the aircraft. When the elevator is raised defined as negative value, the force of the airflow will push the tail down. Hence, the nose of the aircraft will rise and the altitude of the aircraft will increase. One of the targets of a pitch control system is to control or help a pilot to control an aircraft to keep the pitch attitude constant so that make the aircraft can return to desired attitude in a reasonable length of time after a disturbance of the pitch angle or make the pitch follow a given command as quickly as possible [42].

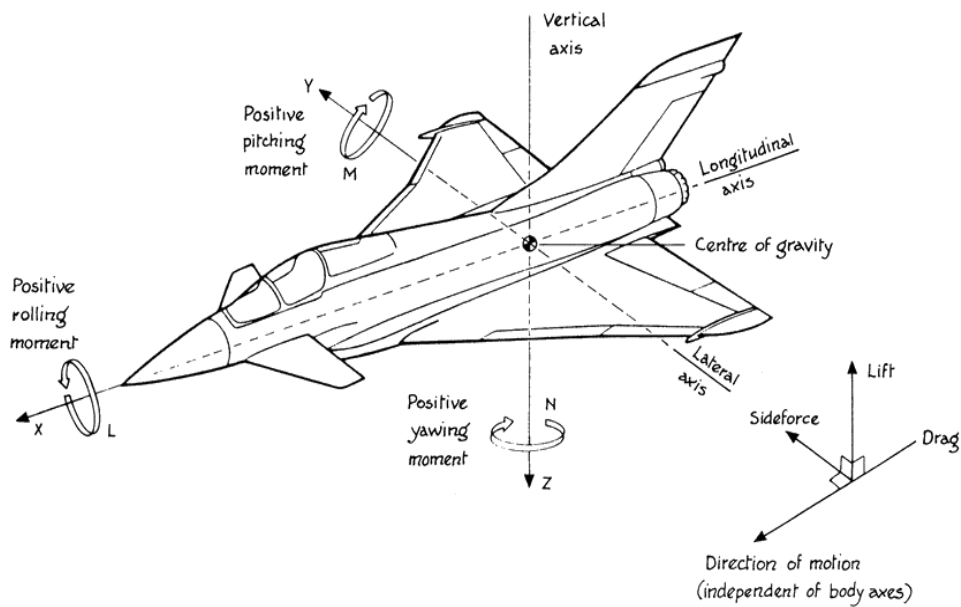
**5.2 Modelling of a pitch control**

Effective control can be achieved after proper modelling mode even after applying different inputs to the aircraft. The equation for governing motion of an aircraft can be divided into two groups to reduce the complexity of analysis. These two groups are longitudinal and lateral equations. Figure 5.1 represents the pitch control system. In this figure  $X_b$ ,  $Y_b$  and  $Z_b$  represent the aerodynamic force components as well as  $\theta$ ,  $\Phi$

and  $\delta_e$  represent the elevator deflection angle and the orientation of aircraft pitch angle in the earth axis system.



**Figure 5.1:** Aircraft pitch control system



**Figure 5.2:** Different forces, moments and velocity components of the aircraft system

Figure 5.2 shows the forces, moments and velocity components of the aircraft system. The roll, pitch and yaw axis of the represent as L, M and N term. Also the term p, q and r represent the angular rates about roll, pitch and yaw axis. The term u, v and w represent the velocity components of roll, pitch and yaw axis and  $\alpha$  and  $\beta$  are represents the angle of attack and sideslip. The data from General Aviation Aeroplane [43] is used in the system for analysis and modelling.

For the thrust and drag are cancel out and lift and weight balance out each other, the aircraft is assumed as the steady state cruise at constant attitude and velocity as well as under any circumstance, the change in pitch angle does not change the speed of an aircraft. These two assumptions are need to be considered before continuing with the modelling process.

**Table 5.1** Longitudinal stability derivative parameters

Longitudinal Derivatives	Components		
	Dynamic Pressure and Dimensional Derivative $Q = 36.81 \text{ lb/ft}^2$ , $QS = 6771 \text{ lb}$ , $QS_{\bar{c}} = 38596 \text{ ft.lb}$ . $(\bar{c}/2u_0) = 0.016\text{s}$		
	X-Force( $S^{-1}$ )	Z-Force( $S^{-1}$ )	Pitching Moment( $FT^{-1}$ )
Rolling Velocities	$X_u = -0.045$	$Z_u = -0.369$	$M_u = 0$
Yawing Velocities	$X_w = 0.036$ $X_{\dot{w}} = 0$	$Z_w = -2.02$ $Z_{\dot{w}} = 0$	$M_w = -0.05$ $M_{\dot{w}} = -0.051$
Angle Of Attack	$X_\alpha = 0$ $X_{\dot{\alpha}} = 0$	$Z_\alpha = -355.42$ $Z_{\dot{\alpha}} = 0$	$M_\alpha = -8.8$ $M_{\dot{\alpha}} = -0.8976$
Pitching Rate	$X_q = 0$	$Z_q = 0$	$M_q = -2.05$
Elevator Deflection	$X_{\delta_e} = 0$	$Z_{\delta_e} = -28.15$	$M_{\delta_e} = -11.874$

The longitudinal stability derivatives parameters used are denoted in Table 5.1. Equation (5.1), (5.2) and (5.3) are represent the dynamic equations for the Aircraft pitch control.

$$X - mgS_\theta = m(\dot{u} + qv - rv) \quad (5.1)$$

$$Z + mgC_\theta C_\Phi = m(\dot{w} + pv - qu) \quad (5.2)$$

$$M = I_y \dot{q} + rq (I_x - I_z) + I_{xz} (p^2 - r^2) \quad (5.3)$$

Where

$m$  = mass of aircraft

$g$  = gravity force

$\theta, \Phi, \delta_e$  = Orientation of aircraft in earth system & elevator deflection angle

L = aerodynamics moment components for Roll Axis

M= aerodynamics moment components for Pitch Axis

N= aerodynamics moment components for Yaw Axis

p= Angular Axis about to Roll Axis

q= Angular Axis about to Pitch Axis

r= Angular Axis about to Yaw axis

u= Velocity component for Roll Axis

v= Velocity component for Pitch Axis

w= Velocity component for Yaw Axis

$\alpha$ = angle of attack

$\beta$  = angle of sideslip

Above Equations should be linearized using small disturbance theory. The equations are replaced by a variable or reference value plus a perturbation or disturbance ,

$$\begin{array}{lll}
 u = u_0 + \Delta u & p = p_0 + \Delta p & X = X_0 + \Delta X \\
 v = v_0 + \Delta v & s = s_0 + \Delta s & r = r_0 + \Delta r \\
 M = M_0 + \Delta M & Z = Z_0 + \Delta Z & \delta = \delta_0 + \Delta \delta \\
 & w = w_0 + \Delta w & 
 \end{array}$$

For convenience, the reference flight condition is assumed to be symmetrical and propulsive forces are assumed to remain constant. For that The values of  $u_0 = v_0 = M_0 = p_0 = s_0 = Z_0 = w_0 = X_0 = r_0 = \delta_0 = 0$ . After the linearization the following equations are found.

$$\left( \frac{d}{dt} - X_u \right) \Delta u - X_w \Delta w + (g \cos \theta_0) \Delta \theta = X_{\delta_e} \Delta \delta_e \quad (5.4)$$

$$-Z_u \Delta u + \left[ (1 - Z_w) \frac{d}{dt} - Z_w \right] \Delta w - \left[ (u_o + Z_q) \frac{d}{dt} - g \sin \theta_0 \right] \Delta \theta = Z_{\delta_e} \Delta \delta_e \quad (5.5)$$

$$-M_u \Delta u - \left( M_w \frac{d}{dt} + M_w \right) \Delta w + \left( \frac{d^2}{dt^2} - M_q \frac{d}{dt} \right) \Delta \theta = M_{\delta_e} \Delta \delta_e \quad (5.6)$$

After substituting longitudinal stability derivative parameters from Table 5.1 to the above equations (5.4), (5.5) and (5.6) , the following transfer function for the change in the pitch rate to the change in elevator deflection angle is shown in the following equation (5.7)

$$\frac{\Delta q(s)}{\Delta \delta_e(s)} = \frac{-(M_{\delta e} + M_{\alpha} Z_{\delta e} / u_0)s - (M_{\alpha} Z_{\delta e} / u_0 - M_{\delta e} Z_{\alpha} / u_0)}{s^2 - (M_q + M_{\alpha} + Z_{\alpha} / u_0)s + (Z_{\alpha} M_q / u_0 - M_{\alpha})} \quad (5.7)$$

The transfer function of the change in pitch angle to the change in elevator angle can be obtained from the change in pitch rates to the change in elevator angle in the following way.

$$\Delta q = \Delta \dot{\theta} \quad (5.8)$$

$$\Delta q(s) = s\Delta \theta(s) \quad (5.9)$$

$$\frac{\Delta \theta(s)}{\Delta \delta_e(s)} = \frac{1}{s} \cdot \frac{\Delta q(s)}{\Delta \theta(s)} \quad (5.10)$$

Thus, the transfer function of the Aircraft pitch control system is represented as the following equation (5.11) and (5.12)

$$\frac{\Delta \theta(s)}{\Delta \delta_e(s)} = \frac{1}{s} \cdot \frac{-(M_{\delta e} + M_{\alpha} Z_{\delta e} / u_0)s - (M_{\alpha} Z_{\delta e} / u_0 - M_{\delta e} Z_{\alpha} / u_0)}{s^2 - (M_q + M_{\alpha} + Z_{\alpha} / u_0)s + (Z_{\alpha} M_q / u_0 - M_{\alpha})} \quad (5.11)$$

After evaluating all values from the Table 5.1 , the transfer function of the Aircraft pitch control is

$$\frac{\Delta \theta(s)}{\Delta \delta_e(s)} = \frac{11.732 s + 22.3}{s^3 + 4.9376 s^2 + 12.89 s} \quad (5.12)$$

The transfer function can also be represented in state-space form and output equation as state by equation (5.13) and (5.14)

$$\begin{bmatrix} \Delta \dot{\alpha} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} -4.9676 & -12.8900 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 11.7320 \\ 22.3000 \end{bmatrix} [\Delta \delta_e] \quad (5.13)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + [0] \quad (5.14)$$

## CHAPTER 6

### RESULTS AND DISCUSSION

Aircraft pitch control system using fractional order controller is concluded for its working under normal conditions and when some unexpected input disturbances. The system considered for that the mathematical modelling is done.

The mathematical model of for Aircraft pitch control is represented in terms of state space model as

$$\begin{bmatrix} \Delta \dot{\alpha} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} -4.9676 & -12.8900 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 11.7320 \\ 22.3000 \end{bmatrix} [\Delta \delta_e] \quad (6.1)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + [0] \quad (6.2)$$

This is also represented in terms of transfer function given as

$$G_p(s) = \frac{11.7320 s + 22.3}{s^3 + 4.9376 s^2 + 12.89 s} \quad (6.3)$$

Where,

$G_p(s)$  is the transfer function for Aircraft pitch control

#### 6.1 Aircraft pitch control using conventional PID controller

Aircraft pitch control is controlled through for conventional PID controller is used. The transfer function for Aircraft pitch control is as shown in equation 6.3. Conventional PID controller tuning is done with Ziegler-Nichols tuning method. With using Ziegler- Nichols method for Aircraft pitch control Transfer function to get constant value of  $K_p$ ,  $K_i$  and  $K_d$ . The PID controller parameters found out are:

Proportional Gain ( $K_{p1}$ ) = 4.15

Integral Gain ( $K_{i1}$ ) = 0.04

Derivative Gain ( $K_{d1}$ ) = 0.9

In conventional order PID controller, the value of  $\lambda$  is 1 and the value of  $\mu$  is 1. The step response of the system has been recorded.

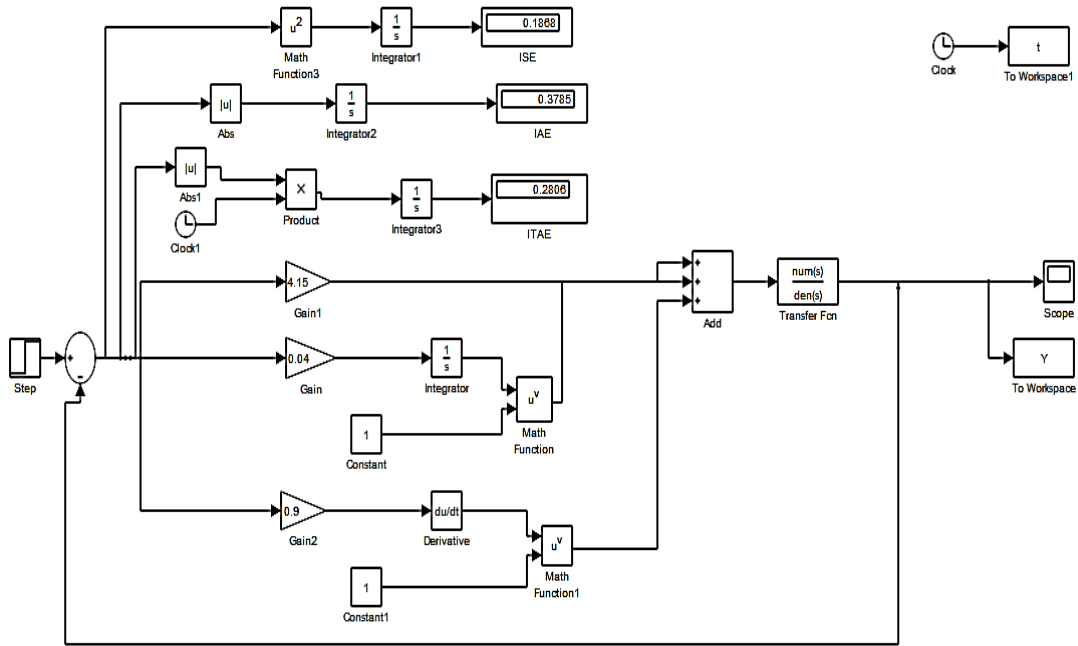


Figure 6.1: Aircraft pitch control with conventional PID controller

The above figure 6.1 shows the Aircraft pitch control with conventional order PID controller. The system step is tested for input with feedback in simulink model. ISE, IAE and ITAE are calculated from this model.

6.1.1 Conventional PID controller

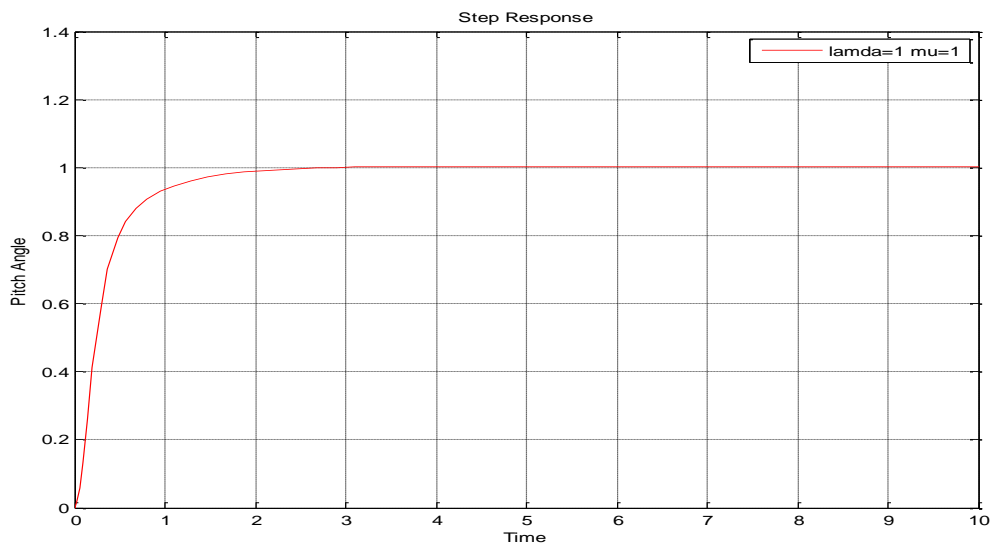


Figure 6.2: Step response of aircraft pitch control using conventional PID controller

It can be seen from the above figure 6.2 that the step response is improved by fractional order PID controller and fractional  $u^v$  order of PID controller.

**Table 6.1** Different parameter with conventional order PID controller

Serial No	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	0.8792	0.3342	1.0990	0.1868	0.3785	0.2806

In the above the Table 6.1 , different control parameters are shown. From the above Table, it can be seen that settling time, peak overshoot, integral square error (ISE) , integral absolute error (IAE), integral time absolute error (ITAE).

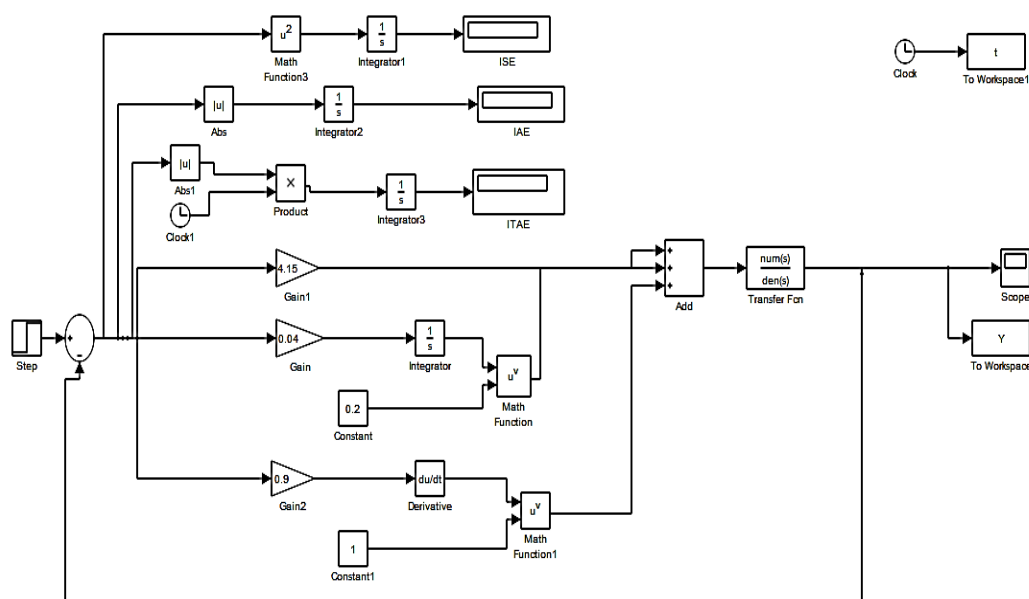
In the next section ,fractional order PID controller for Aircraft pitch control has been introduced.

### 6.2 Aircraft pitch control using fractional order PID control

Fractional order PID controller is used as primary controller. For changes in the values of  $\lambda$  and  $\mu$ , output response is observed.  $\lambda$  is the integral order function and  $\mu$  is the derivative order function.

Transfer function of fractional PID Controller is

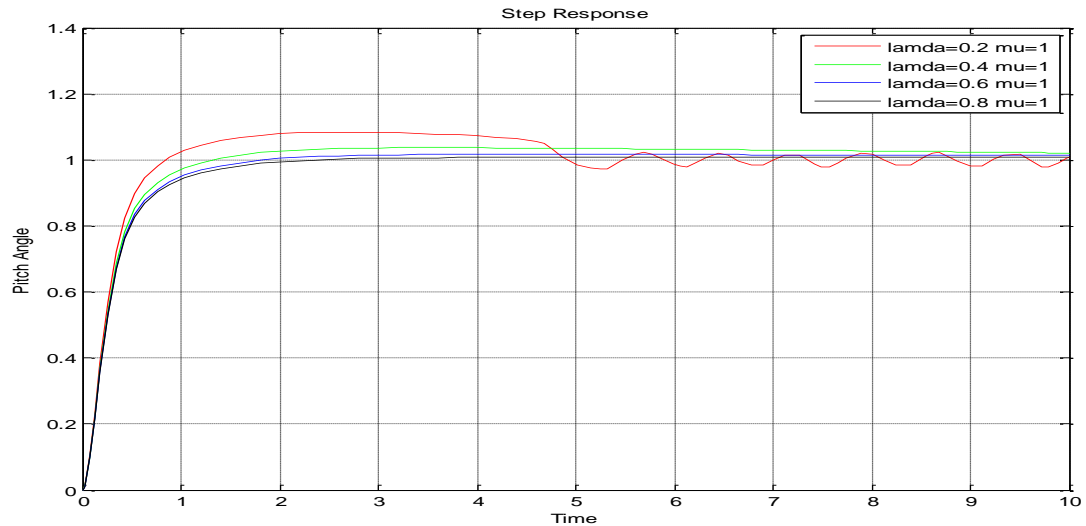
$$G_{FOPID}(s) = \frac{u(s)}{E(s)} = K_c \left( 1 + \frac{1}{\tau_i s^\lambda} + \tau_d s^\mu \right) \tag{6.4}$$



**Figure 6.3:** Aircraft pitch control with fractional PID controller

The figure 6.3 is the simulink block for Aircraft pitch control using fractional order PID controller. In a fractional order PID controller, variation in integral order ( $\lambda$ ) and derivative value ( $\mu$ ), different step response come.

### 6.2.1 With varying values of integral order $\lambda < 1$ and derivative order $\mu = 1$



**Figure 6.4:** Step response for aircraft pitch control using FOPID for different values of  $\lambda < 1$

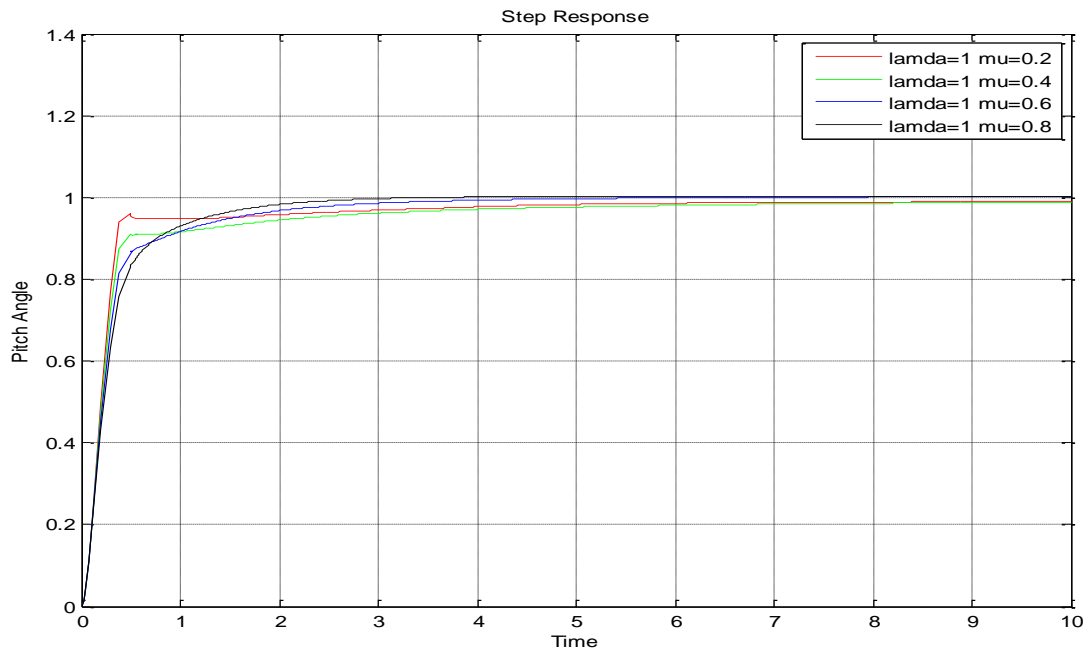
The above figure 6.4 shows the step response of Aircraft pitch control using fractional order PID Controller for integral order  $\lambda < 1$  and derivative order  $\mu = 1$ .

**Table 6.2** Different parameters with varying values of integral order  $\lambda < 1$  and derivative order  $\mu = 1$

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Second)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Absolute Error (ITAE)
1	$\lambda=0.2$ $\mu=1$	3.7135	8.3987	4.6419	0.1879	0.6017	1.2940
2	$\lambda=0.4$ $\mu=1$	0.6363	3.7101	0.7954	0.1864	0.5633	1.5
3	$\lambda=0.6$ $\mu=1$	0.7571	1.6751	0.9463	0.1854	0.4527	0.8315
4	$\lambda=0.8$ $\mu=1$	0.7570	0.7524	0.9462	0.1862	0.4000	0.4548

It can be seen from the above Table 6.2 that with the increased variation in the integral order  $\lambda$ , control parameters are remain same.

### 6.2.2 With value of integral order $\lambda=1$ and varying derivative order $\mu<1$



**Figure 6.5:** Step response for aircraft pitch control using FOPID for different values of  $\mu<1$

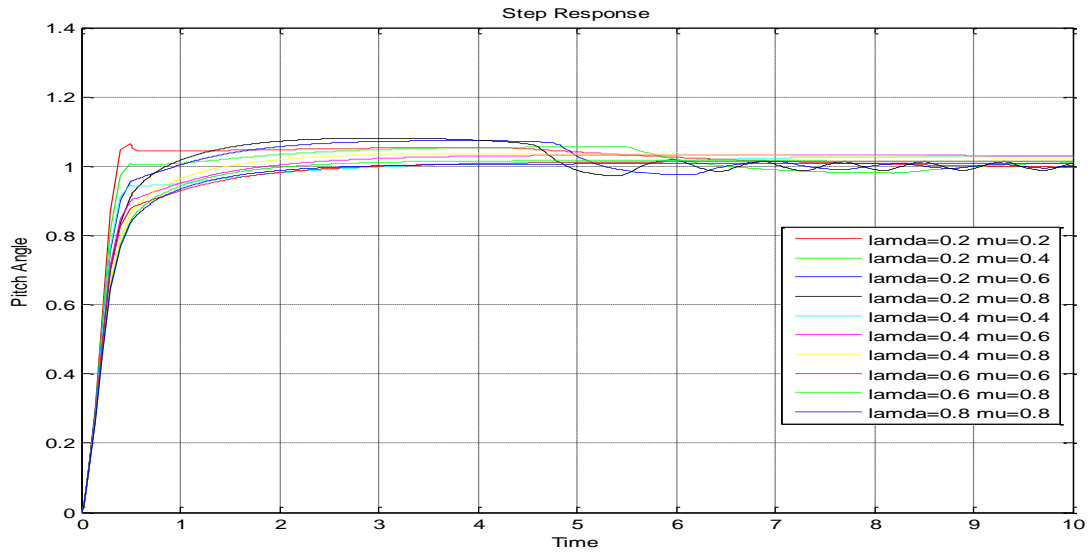
The above figure 6.5 shows the step response of air craft pitch control using fractional order PID controller for integral order  $\lambda=1$  and derivative order  $\mu<1$

**Table 6.3** Different parameters with value of integral order  $\lambda=1$  and derivative order  $\mu<1$

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1$ $\mu=0.2$	0.2307	-	0.2882	0.1531	0.3856	0.7483
2	$\lambda=1$ $\mu=0.4$	1.8150	-	2.2687	0.1684	0.4970	0.8304
3	$\lambda=1$ $\mu=0.6$	1.1268	0.4460	1.4085	0.1716	0.4040	0.3841
4	$\lambda=1$ $\mu=0.8$	0.9361	0.3158	1.1701	0.1786	0.3784	0.2862

It can be seen from the above Table 6.3 that with the increase in the values of derivative order  $\mu$ , control parameters are improved.

### 6.2.3 With varying values of integral order $\lambda < 1$ and derivative order $\mu < 1$



**Figure 6.6:** Step response for aircraft pitch control using FOPID for different values of integral order  $\lambda < 1$  and derivative order  $\mu < 1$

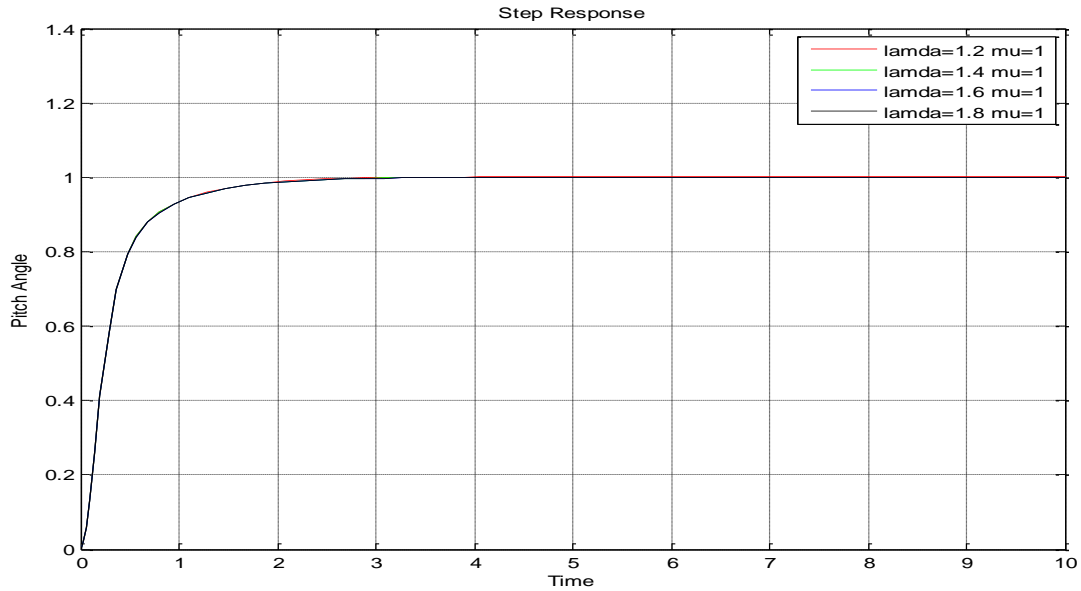
The above figure 6.4 shows the step response of aircraft pitch control using fractional order PID Controller for integral order  $\lambda < 1$  and derivative order  $\mu < 1$

**Table 6.4** Different Parameters with varying values of integral order  $\lambda < 1$  and derivative order  $\mu < 1$

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=0.2 \mu=0.2$	2.9476	7.8377	3.6845	0.1609	0.5790	1.584
2	$\lambda=0.2 \mu=0.4$	4.5127	6.0903	5.6408	0.1552	0.4665	1.2500
3	$\lambda=0.2 \mu=0.6$	3.9278	7.2007	4.9097	0.1654	0.5061	1.074
4	$\lambda=0.2 \mu=0.8$	3.7430	8.0362	4.6788	0.1765	0.5462	1.094
5	$\lambda=0.4 \mu=0.4$	0.8819	2.9063	1.1023	0.1575	0.4417	1.211
6	$\lambda=0.4 \mu=0.6$	0.8132	3.2946	1.0165	0.1675	0.5163	1.4920
7	$\lambda=0.4 \mu=0.8$	0.6667	3.5726	0.8334	0.1771	0.5458	1.5080
8	$\lambda=0.6 \mu=0.6$	0.9659	1.4928	1.2073	0.1683	0.4244	0.7561
9	$\lambda=0.6 \mu=0.8$	0.8020	1.6184	1.0025	0.1766	0.4414	0.8198
10	$\lambda=0.8 \mu=0.8$	0.9358	0.7246	1.1698	0.1778	0.3952	0.4493

It can be seen from the above Table 6.4 that with the increase in the values of  $\mu$ , control parameters are improved.

#### 6.2.4 With varying values of integral order $\lambda \in (1,2)$ and derivative order $\mu=1$



**Figure 6.7:** Step Response For Aircraft pitch control Using FOPID For different values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$

The above figure 6.7 shows the step response of Air Craft Pitch Control using fractional order PID Controller for integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$ .

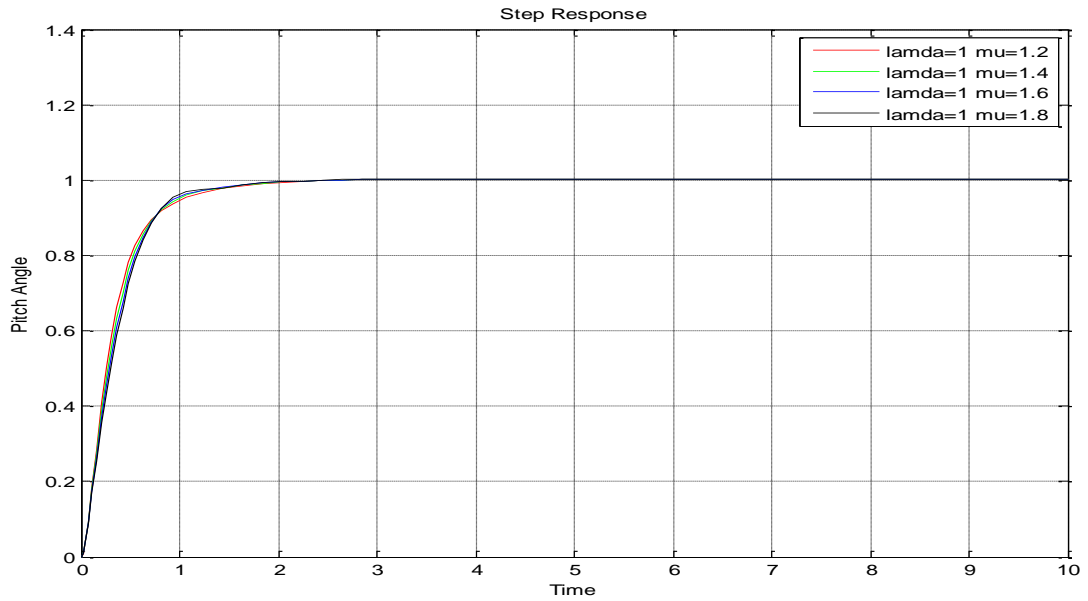
**Table 6.5** Different parameters with varying values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2$ $\mu=1$	0.8000	0.1469	1.0000	0.1872	0.3703	0.2055
2	$\lambda=1.4$ $\mu=1$	0.9220	0.0640	1.1525	0.1873	0.3675	0.1749
3	$\lambda=1.6$ $\mu=1$	0.9220	0.0277	1.1525	0.1874	0.3666	0.1630
4	$\lambda=1.8$ $\mu=1$	0.9220	0.0120	1.1526	0.1874	0.3664	0.1571

It can be seen from the above Table 6.5 that with the increase in the values of integral order  $\lambda$ , control parameters are remain same.

### 6.2.5 With value of integral order $\lambda=1$ and different values of derivative order

$$\mu \in (1,2)$$



**Figure 6.8:** Step response for aircraft pitch control using FOPID for value of integral order  $\lambda=1$  & different values of derivative order  $\mu \in (1,2)$

The above figure 6.8 shows the step response of Aircraft pitch control using fractional order PID controller for integral order  $\lambda=1$  & derivative order  $\mu \in (1,2)$

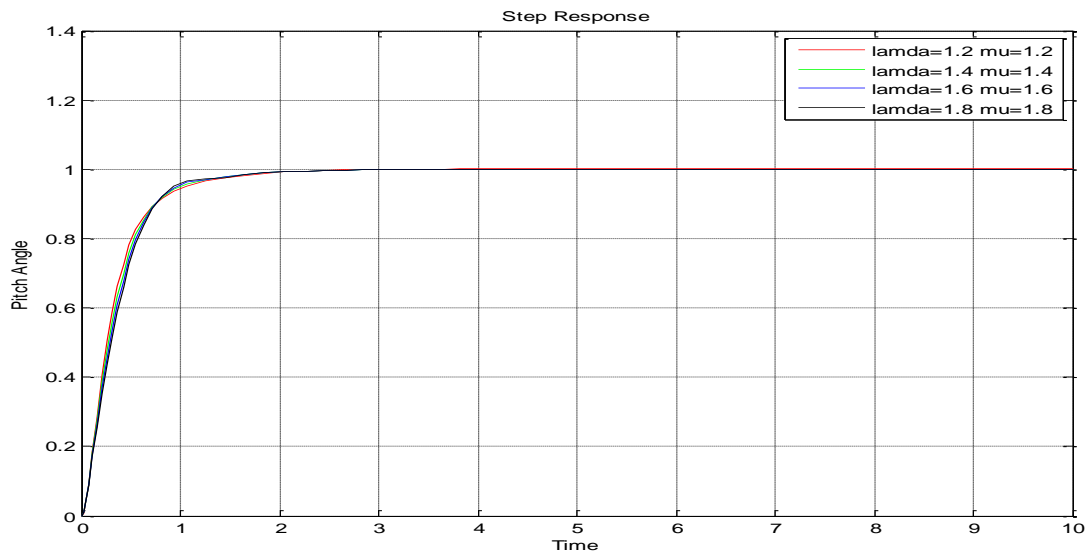
**Table 6.6** Different parameters of integral order  $\lambda=1$  and different values of derivative order  $\mu \in (1,2)$

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1$ $\mu=1.2$	0.8000	0.3363	1.0000	0.1948	0.3816	0.2741
2	$\lambda=1$ $\mu=1.4$	0.7739	0.3413	0.9673	0.2026	0.3877	0.2743
3	$\lambda=1$ $\mu=1.6$	0.6836	0.3466	0.8545	0.2094	0.3938	0.2767
4	$\lambda=1$ $\mu=1.8$	0.7173	0.3526	0.8967	0.2159	0.4007	0.2808

It can be seen from the above Table 6.6 that with the increase in the values of derivative order  $\mu$ , control parameters are remain same.

### 6.2.6 With varying values of integral order $\lambda \in (1,2)$ and derivative order $\mu \in (1,2)$

#### 6.2.6.1 Values of integral order $\lambda$ and derivative order $\mu$ are same ( $\lambda = \mu$ )



**Figure 6.9:** Step response for aircraft pitch control using FOPID for integral order  $\lambda$  and derivative order  $\mu$  are same

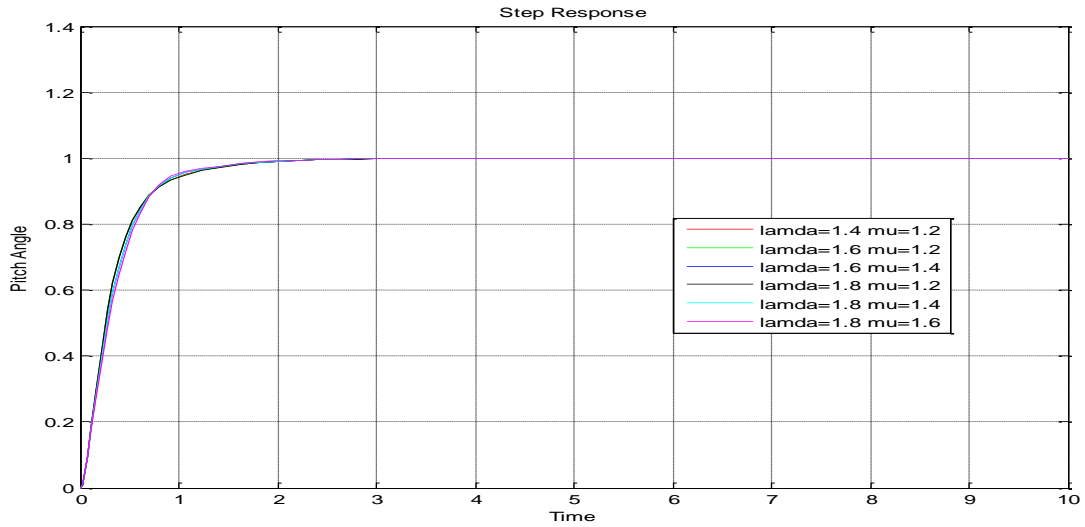
The above figure 6.9 shows the step response of aircraft pitch control using fractional order PID Controller for integral order  $\lambda$  and derivative order  $\mu$  are same.

**Table 6.7** Different parameters with integral order  $\lambda$  and derivative order  $\mu$  are same

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2, \mu=1.2$	0.8000	0.1477	1.0000	0.1951	0.3725	0.1961
2	$\lambda=1.4, \mu=1.4$	0.7744	0.0657	0.9680	0.2030	0.3746	0.1610
3	$\lambda=1.6, \mu=1.6$	0.6840	0.0293	0.8550	0.2098	0.3791	0.1474
4	$\lambda=1.8, \mu=1.8$	0.7182	0.0131	0.8977	0.2163	0.3852	0.1435

It can be seen from the above Table 6.7 that with the same values of integral order  $\lambda$  & derivative order  $\mu$ , control parameters are almost same.

### 6.2.6.2 Values of integral order $\lambda$ is greater than derivative order $\mu$ ( $\lambda > \mu$ )



**Figure 6.10:** Step response for aircraft pitch control using FOPID for different values of integral order  $\lambda$  is greater than derivative order  $\mu$

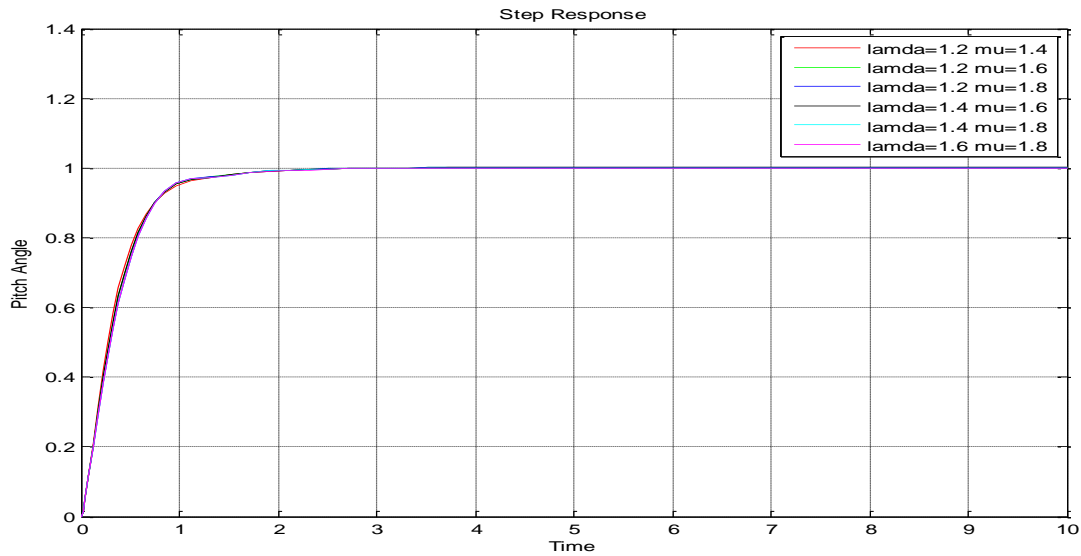
The above figure 6.10 shows the step response of aircraft pitch control using fractional order PID controller for integral order  $\lambda$  is greater than derivative order  $\mu$ .

**Table 6.8** Different parameters with Values of integral order  $\lambda$  is greater than derivative order  $\mu$  ( $\lambda > \mu$ )

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.4$ $\mu=1.2$	0.8000	0.0644	1.0000	0.1952	0.3691	0.1636
2	$\lambda=1.6$ $\mu=1.2$	0.8000	0.0279	1.0000	0.1953	0.3680	0.1507
3	$\lambda=1.6$ $\mu=1.4$	0.7745	0.0286	0.9681	0.2030	0.3734	0.1476
4	$\lambda=1.8$ $\mu=1.2$	0.8000	0.0121	1.0000	0.1953	0.3677	0.1457
5	$\lambda=1.8$ $\mu=1.4$	0.7745	0.0124	0.9681	0.2031	0.3730	0.1424
6	$\lambda=1.8$ $\mu=1.6$	0.6840	0.0127	0.8550	0.2098	0.3787	0.1420

It can be seen from the above Table 6.8 that with the different values of integral order  $\lambda$  & derivative order  $\mu$ , control parameters are increased.

### 6.2.6.3 Values of integral order $\lambda$ is less than derivative order $\mu$ ( $\lambda < \mu$ )



**Figure 6.11:** Step response for Aircraft pitch control using FOPID for different values of integral order  $\lambda$  is less than derivative order  $\mu$

The above figure 6.11 shows the step response of aircraft pitch control using fractional order PID Controller for integral order  $\lambda$  is less than derivative order  $\mu$ .

**Table 6.9** Different parameters combination values of integral order  $\lambda$  is less than derivative order  $\mu$  ( $\lambda < \mu$ )

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2$ $\mu=1.4$	0.7743	0.1503	0.9678	0.2029	0.3781	0.1945
2	$\lambda=1.2$ $\mu=1.6$	0.6839	0.1531	0.8548	0.2096	0.3840	0.1953
3	$\lambda=1.2$ $\mu=1.8$	0.7179	0.1562	0.8973	0.2161	0.3905	0.1979
4	$\lambda=1.4$ $\mu=1.6$	0.6840	0.0671	0.8550	0.2097	0.3804	0.1613
5	$\lambda=1.4$ $\mu=1.8$	0.7181	0.0687	0.8976	0.2162	0.3869	0.1632
6	$\lambda=1.6$ $\mu=1.8$	0.7181	0.0300	0.8977	0.2163	0.3856	0.1490

It can be seen from the above Table 6.9 that with the different values of integral order  $\lambda$  & derivative order  $\mu$ , control parameters are almost same.

### 6.3 Aircraft pitch control using fractional order of PID control:

In fractional order of PID controller values of  $\lambda = 1$  and the value of  $\mu = 1$ . Change in values of  $\lambda$  which is in the power of transfer function causes fractional change in the transfer function of the system.  $\lambda$  is the integral order and  $\mu$  is the derivative order. Tuning of primary controller is done using Ziegler-Nichols tuning method and tuning of controller is done by using auto tuning function in simulink. Using Ziegler- Nichols method for Aircraft pitch control transfer function to get constant value of  $K_p$ ,  $K_i$  and  $K_d$ . The PID controller parameters found out are

Proportional Gain ( $K_{p1}$ ) = 4.15

Integral Gain ( $K_{i1}$ ) = 0.04

Derivative Gain ( $K_{d1}$ ) = 0.9

Transfer function of fractional order of PID Controller is

$$G_{FO[PID]}(s) = \frac{u(s)}{E(s)} = K_c \left( 1 + \frac{1}{\tau_i s^\lambda} + \tau_d s^\mu \right) \quad (6.5)$$

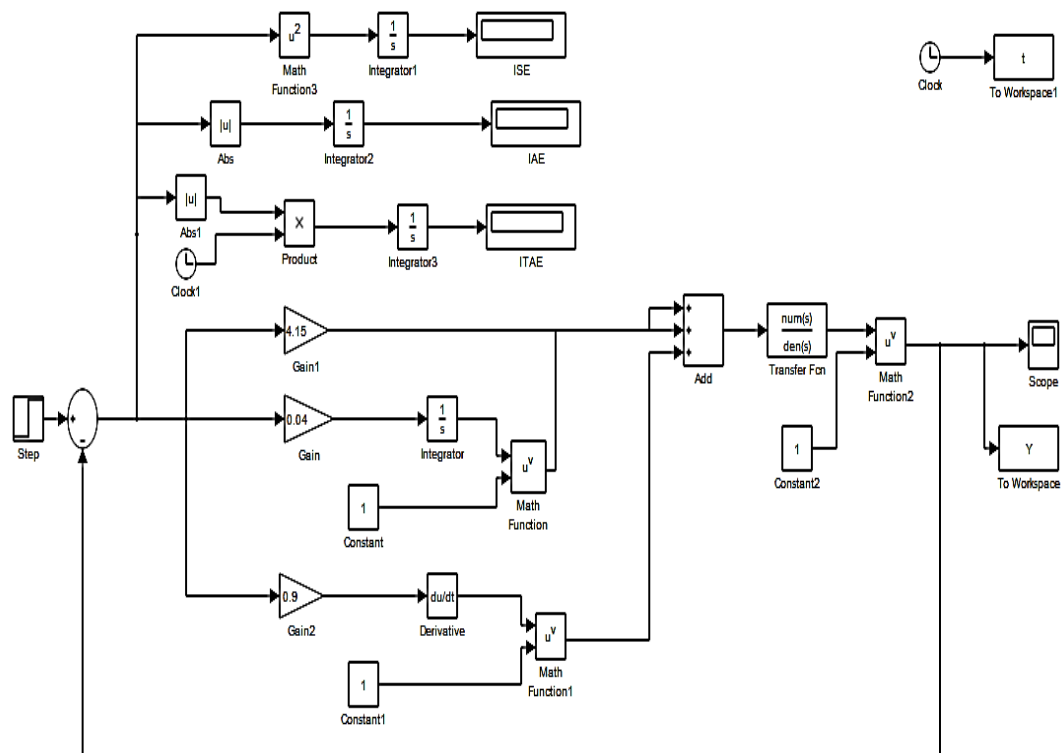
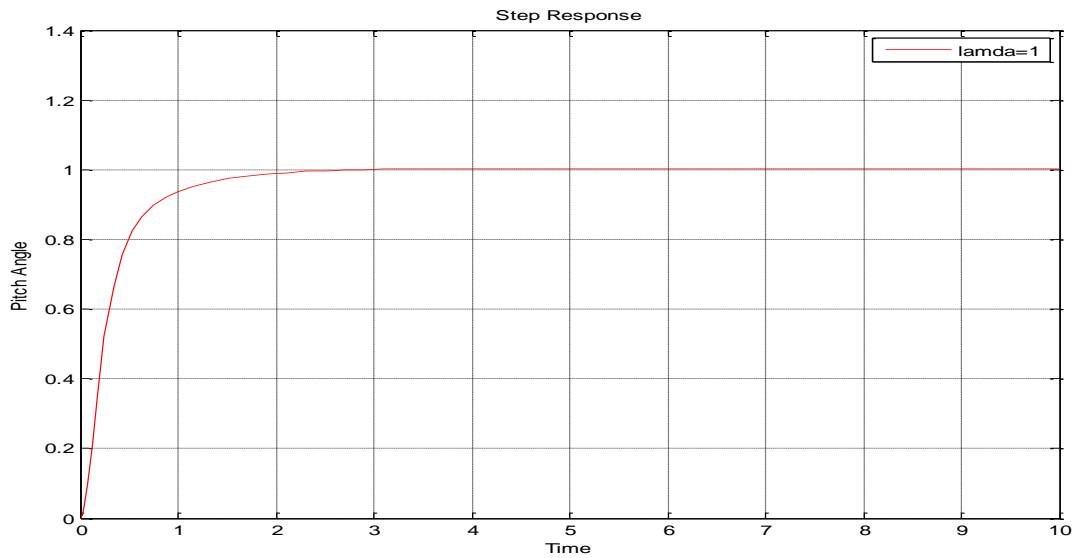


Figure 6.12: Aircraft pitch control simulink with fractional order of PID controller

### 6.3.1 Aircraft pitch control for fractional order of PID controller with value of $\lambda = 1$



**Figure 6.13:** Step response of Aircraft pitch control with fractional order of PID controller for  $\lambda = 1$

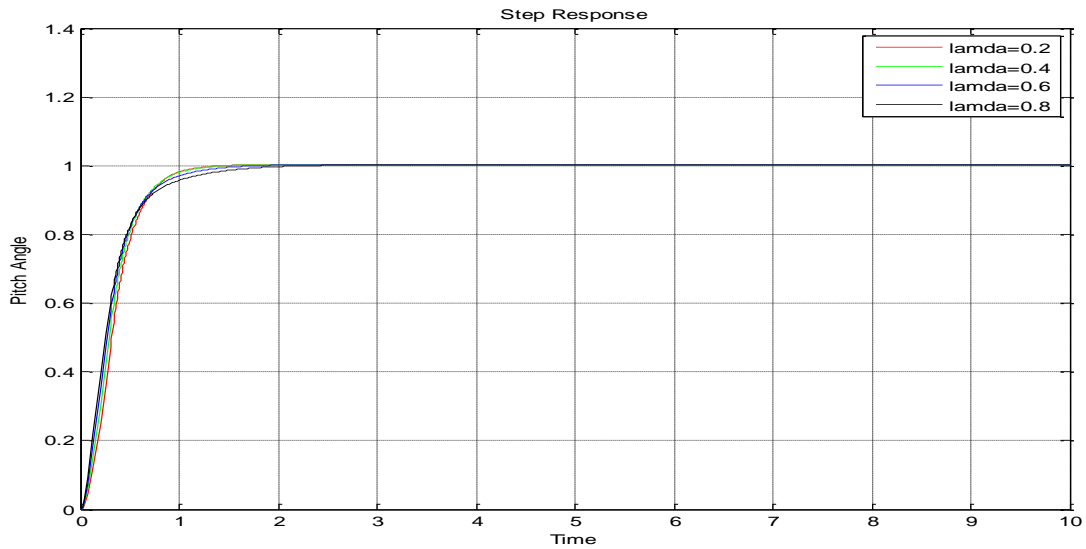
The above figure 6.13 shows the step response of Aircraft pitch control with fractional order of PID controller where value of  $\lambda$  is 1

**Table 6.10** Different Parameters with fractional order of PID controller values of  $\lambda$  is 1

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 1$	0.8000	1.0000	0.3342	0.1868	0.3784	0.2805

In the above Table 6.10, shows the settling time, rise time, peak overshoot, integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE) with taking value of  $\lambda = 1$ .

### 6.3.2 Aircraft pitch control for fractional order of PID controller values of $\lambda$ is less than 1 ( $\lambda < 1$ )



**Figure 6.14:** Step response of Aircraft pitch control with fractional order of PID controller for  $\lambda < 1$

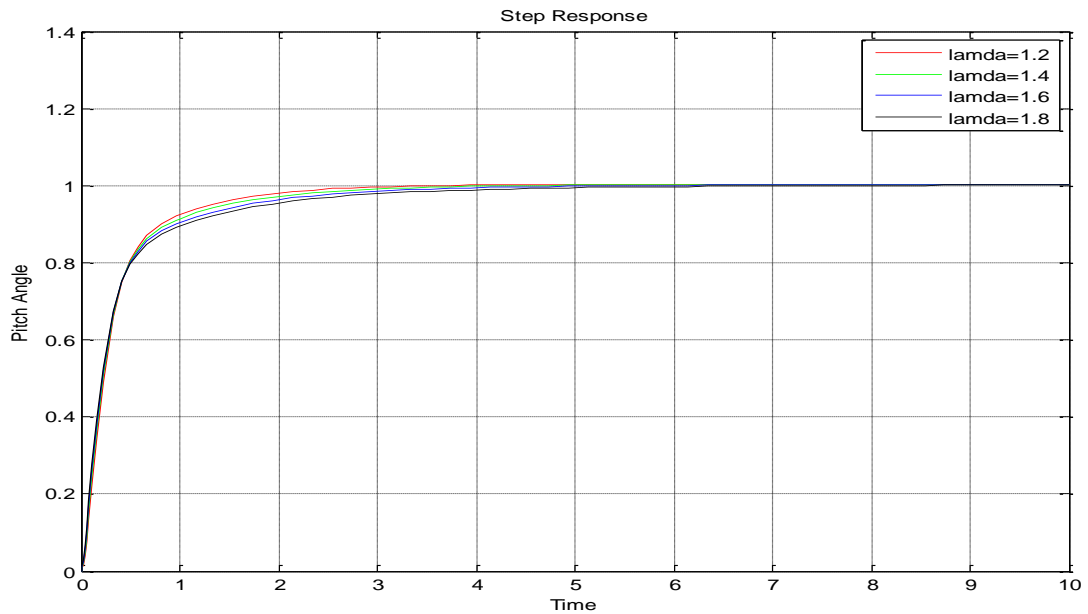
The above figure 6.14 shows the step response of Aircraft pitch control with fractional order of PID Controller for  $\lambda < 1$

**Table 6.11** Different parameters for fractional order of PID controller where values of  $\lambda$  is less than 1

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 0.2$	0.6504	0.3656	0.8130	0.2433	0.3935	0.2597
2	$\lambda = 0.4$	0.6481	0.3378	0.8101	0.2219	0.3721	0.2458
3	$\lambda = 0.6$	0.6912	0.3343	0.8640	0.2058	0.3633	0.2469
4	$\lambda = 0.8$	0.7209	0.1116	0.9011	0.0952	0.3668	0.2484

It can be seen from the Table 6.11 that with the increase the value of  $\lambda$ , control parameters are improved.

### 6.3.3 Aircraft pitch control for fractional order of PID controller values of $\lambda$ is greater than 1 ( $\lambda > 1$ )



**Figure 6.15:** Step response of Aircraft pitch control with fractional order of PID controller for  $\lambda > 1$

The above figure 6.15 shows the step response of Aircraft pitch control with fractional order of PID controller for value of  $\lambda$  is greater than 1

**Table 6.12** Different parameters for fractional order of PID controller values of  $\lambda > 1$

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 1.2$	0.9391	0.3436	1.1739	0.1805	0.3909	0.3071
2	$\lambda = 1.4$	1.1200	0.3468	1.4000	0.1765	0.4079	0.3335
3	$\lambda = 1.6$	1.2295	0.2978	1.5368	0.1743	0.4301	0.3622
4	$\lambda = 1.8$	1.4400	0.1655	1.8000	0.1733	0.4561	0.4161

It can be seen from the Table 6.11 that with the increase the value of  $\lambda$ , control parameters are improved.

### 6.4 Aircraft pitch control with conventional PID with disturbance

The transfer function of the aircraft control system is represented as follow

$$G_p (s) = \frac{11.7304 s + 22.578}{s^3 + 4.9676 s^2 + 12.941 s} \tag{6.4}$$

$$G_d (s) = \frac{1}{0.1s + 1} \tag{6.5}$$

Equation 6.5 is the first order system disturbance. In the system with the disturbance the value of  $K_p$ ,  $K_i$  and  $K_d$  are as follow.

Proportional Gain ( $K_{p2}$ ) = 1.4966

Integral Gain ( $K_{i2}$ ) = 1.2588

Derivative Gain ( $K_{d2}$ ) = 0.3147

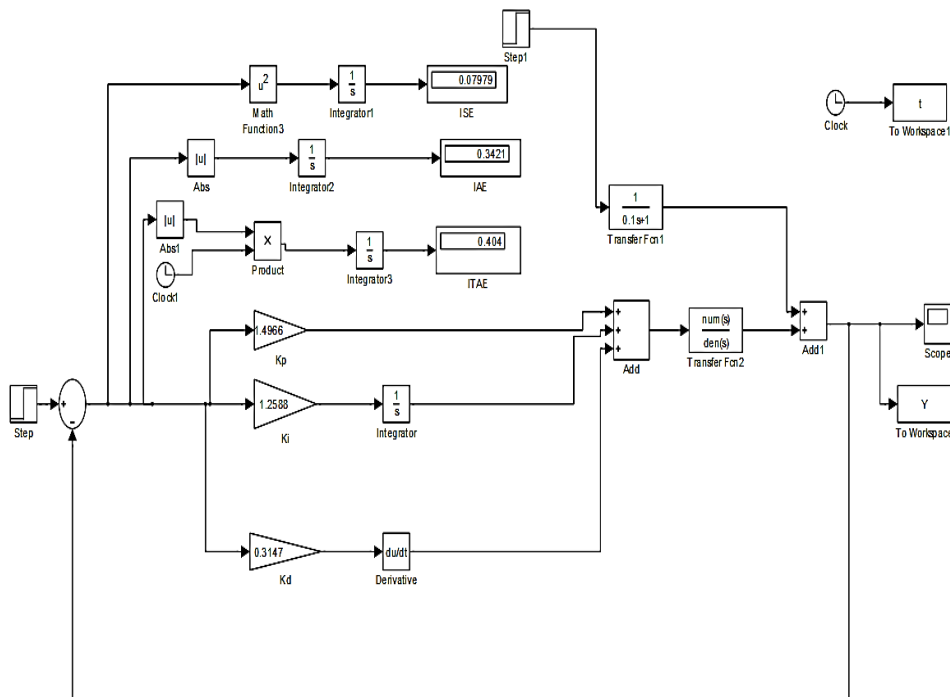
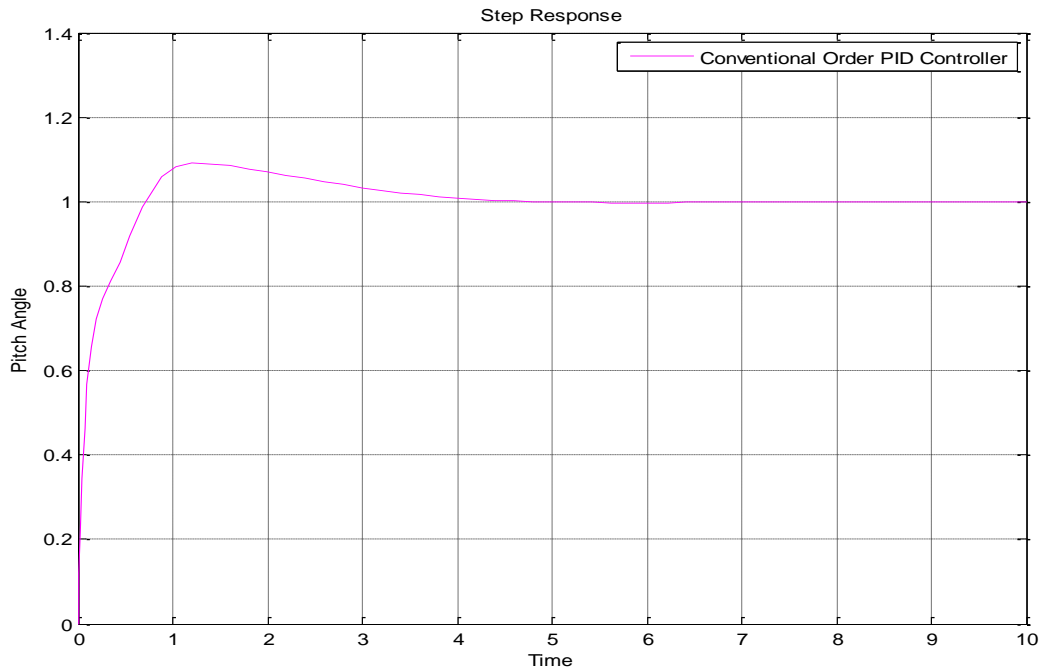


Figure 6.16: Aircraft pitch control with Conventional PID with disturbance

The above figure 6.16 shows the Aircraft pitch control with conventional PID with disturbance Controller. The first order disturbance appears in the system. Step signal is consider as input. Unity feedback is used in this simulink model. ISE, IAE and ITAE are calculated from this model.

### 6.4.1 Conventional PID controller with Disturbance



**Figure 6.17:** Step response of Aircraft pitch control using conventional PID controller

It can be seen from the above figure 6.17 that the step response is improved by FOPID, FO[PID] controller with disturbance.

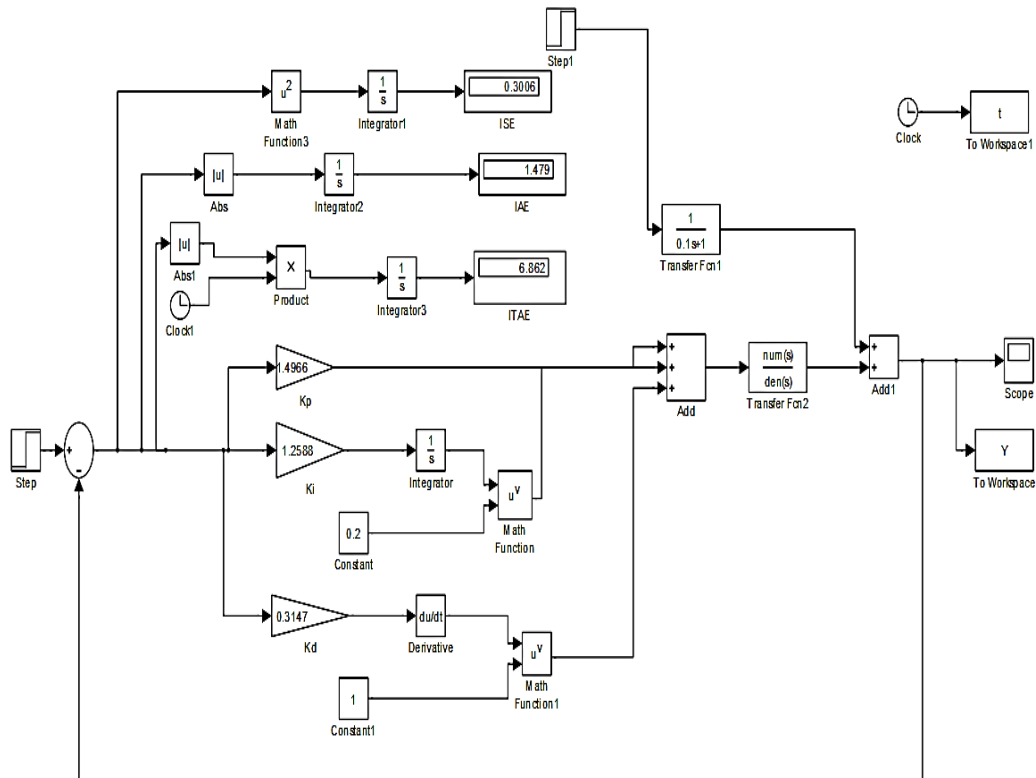
**Table 6.13** Different parameter using conventional order PID controller with disturbance

Serial No	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	1.9195	9.0681	2.3993	0.0797	0.3421	0.404

In the above Table 6.13, shows the settling time, rise time, peak overshoot, integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE) with conventional order PID controller with disturbance.

## 6.5 Aircraft pitch control using fractional order PID with disturbance

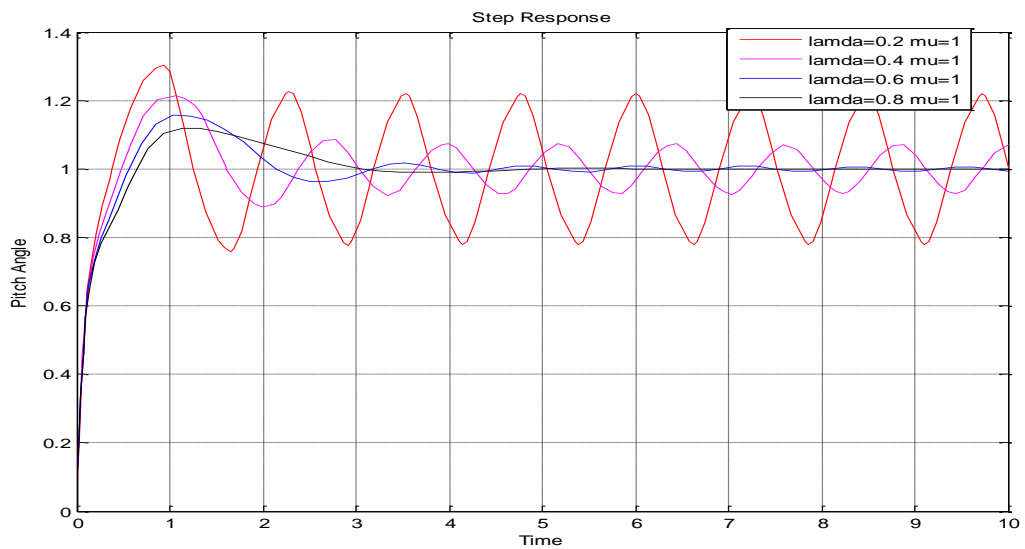
Fractional Order PID controller is used as primary controller. For different values of  $\lambda$  and  $\mu$ , output response is taken.  $\lambda$  is the integral order and  $\mu$  is the derivative order. Figure 6.18 shows the model of fractional PID controller with first order disturbance.



**Figure 6.18:** Aircraft pitch control with fractional PID controller with disturbance

The above figure 6.18 is the simulink block when external unknown input disturbance of Aircraft pitch control which has fractional order PID controller. In this controller, for different values of integral order ( $\lambda$ ) and derivative value ( $\mu$ ), different step response come out.

### 6.5.1 With varying values of Integral order $\lambda < 1$ and derivative order $\mu=1$ with disturbance



**Figure 6.19:** Step response for Aircraft pitch control using FOPID for different value of  $\lambda < 1$  with disturbance

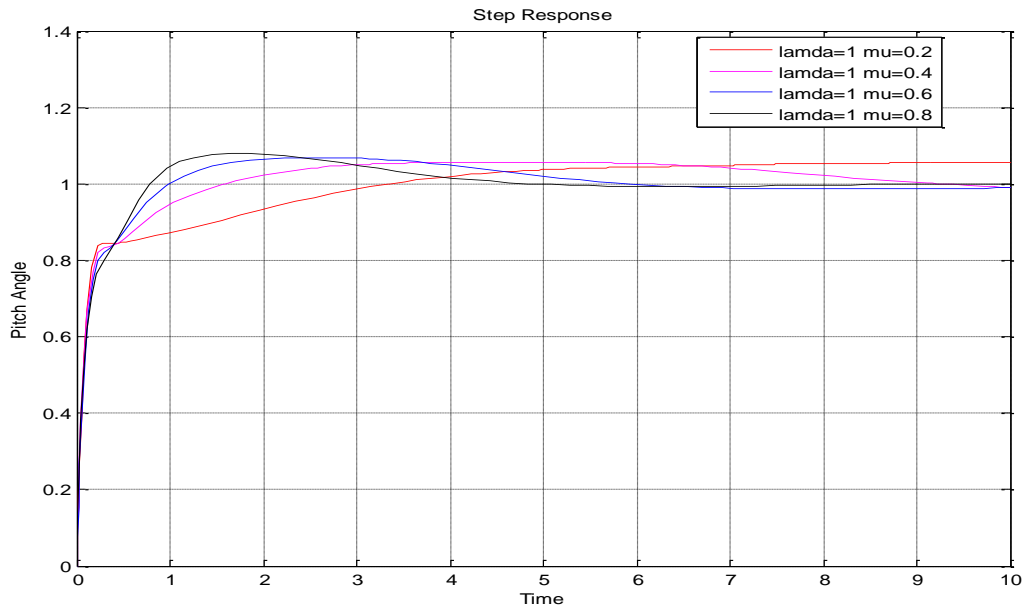
The above figure 6.19 shows the step response of Aircraft pitch control of fractional order PID controller with disturbance for values of  $\lambda$  is less than 1 and  $\mu=1$ .

**Table 6.14** Different parameters for Aircraft pitch control using FOPID for different value of  $\lambda < 1$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Absolute Error (ITAE)
1	$\lambda=0.2$ $\mu=1$	7.9607	30.3198	9.9509	0.3006	1.479	1.2940
2	$\lambda=0.4$ $\mu=1$	8	21.4546	10	0.1141	0.7045	2.472
3	$\lambda=0.6$ $\mu=1$	1.4371	15.8465	1.7964	0.0851	0.3738	0.5513
4	$\lambda=0.8$ $\mu=1$	1.8370	11.9990	2.2963	0.08157	0.3371	0.3498

It can be seen from the above Table 6.14 that with the increase in the values of  $\mu$ , control parameters are improved.

### 6.5.2 With value of integral order $\lambda=1$ and varying derivative order values $\mu<1$ with disturbance



**Figure 6.20:** Step response with values of integral order  $\lambda=1$  and derivative order  $\mu<1$  with disturbance

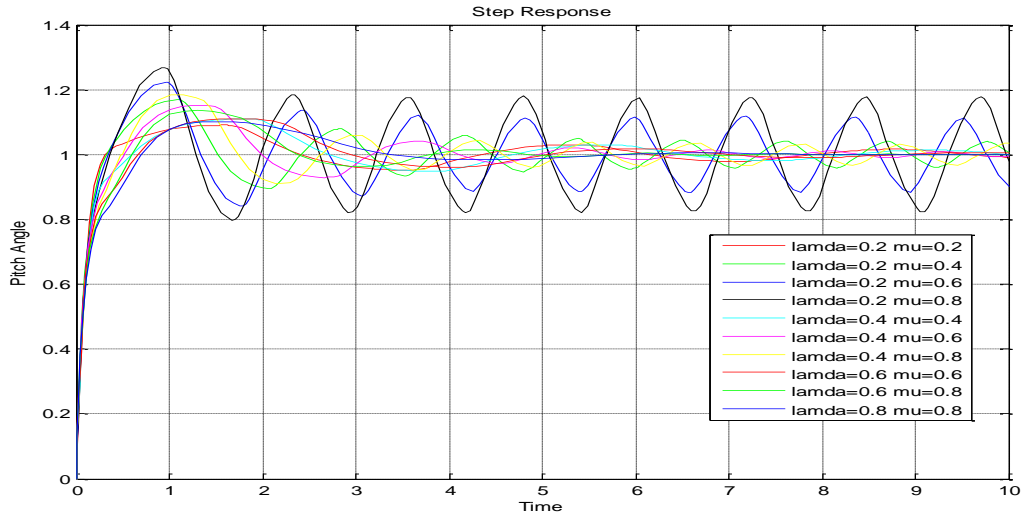
The above figure 6.20 shows the step response of Aircraft pitch control with disturbance using fractional order PID Controller for  $\lambda=1$  and  $\mu<1$

**Table 6.15** Different parameters with value of integral order  $\lambda=1$  and derivative order  $\mu<1$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1$ $\mu=0.2$	8.8	5.5288	10	0.0922	0.6293	2.375
2	$\lambda=1$ $\mu=0.4$	5.0594	5.6393	6.3243	0.07912	0.5082	1.6200
3	$\lambda=1$ $\mu=0.6$	3.0385	6.8149	3.0385	0.07579	0.4281	1.051
4	$\lambda=1$ $\mu=0.8$	2.2639	7.8476	2.8298	0.07648	0.3627	0.5652

It can be seen from the above Table 6.15 that with the increase in the values of  $\mu$ , control parameters are improved.

**6.5.3 With varying values of integral order  $\lambda < 1$  and derivative order  $\mu < 1$  with disturbance**



**Figure 6.21:** Step Response for Aircraft pitch control using FOPID for different values of integral order  $\lambda < 1$  and derivative order  $\mu < 1$

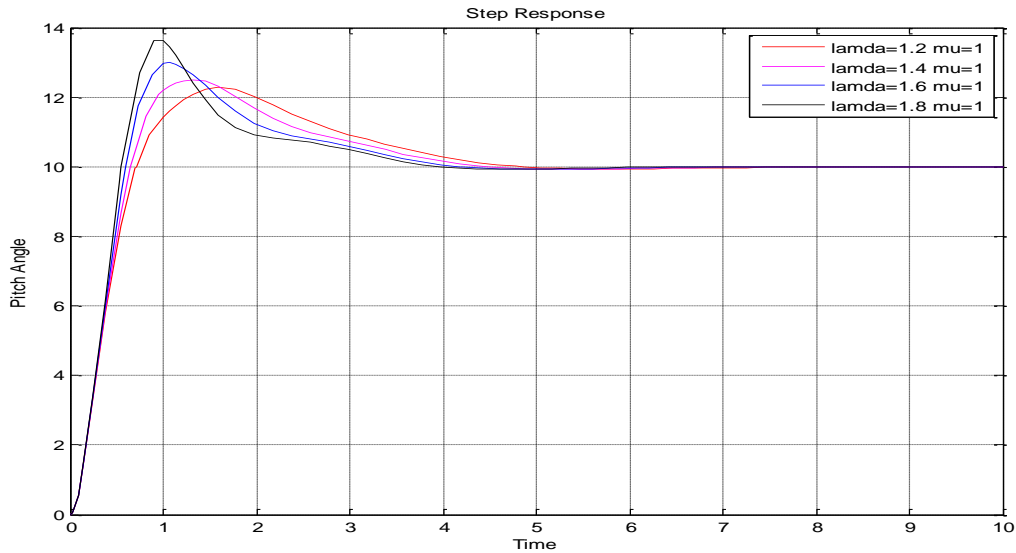
The above figure 6.21 shows the step response of Air Craft Pitch controller with disturbance using fractional order PID Controller for  $\lambda < 1$  and  $\mu < 1$ .

**Table 6.16** Different parameters with varying values of  $\lambda < 1$  and  $\mu < 1$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=0.2 \mu=0.2$	1.5330	9.0369	1.9163	0.05998	0.3484	0.3928
2	$\lambda=0.2 \mu=0.4$	3.8533	16.9339	4.8166	0.0806	0.5137	1.571
3	$\lambda=0.2 \mu=0.6$	8	22.3051	10	0.1357	0.8815	3.638
4	$\lambda=0.2 \mu=0.8$	7.9130	26.7435	9.8913	0.2191	1.224	5.514
5	$\lambda=0.4 \mu=0.4$	3.1692	10.1317	2.8868	0.0696	0.4049	0.9972
6	$\lambda=0.4 \mu=0.6$	2.3095	15.1317	2.8868	0.0779	0.4049	0.79
7	$\lambda=0.4 \mu=0.8$	2.4512	18.4747	3.0640	0.09041	0.5154	1.391
8	$\lambda=0.6 \mu=0.6$	1.9565	11.0388	2.4456	0.07554	0.4002	0.7951
9	$\lambda=0.6 \mu=0.8$	1.5706	13.5345	1.9632	0.07968	0.3584	0.4704
10	$\lambda=0.8 \mu=0.8$	1.9559	10.1695	2.4448	0.07756	0.3596	0.5135

It can be seen from the above Table 6.16 that with the increase in the values of  $\mu$ , control parameters are improved

**6.5.4 With varying values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$  with disturbance**



**Figure 6.22:** Step response for Aircraft pitch control using FOPID for different values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$  with disturbance

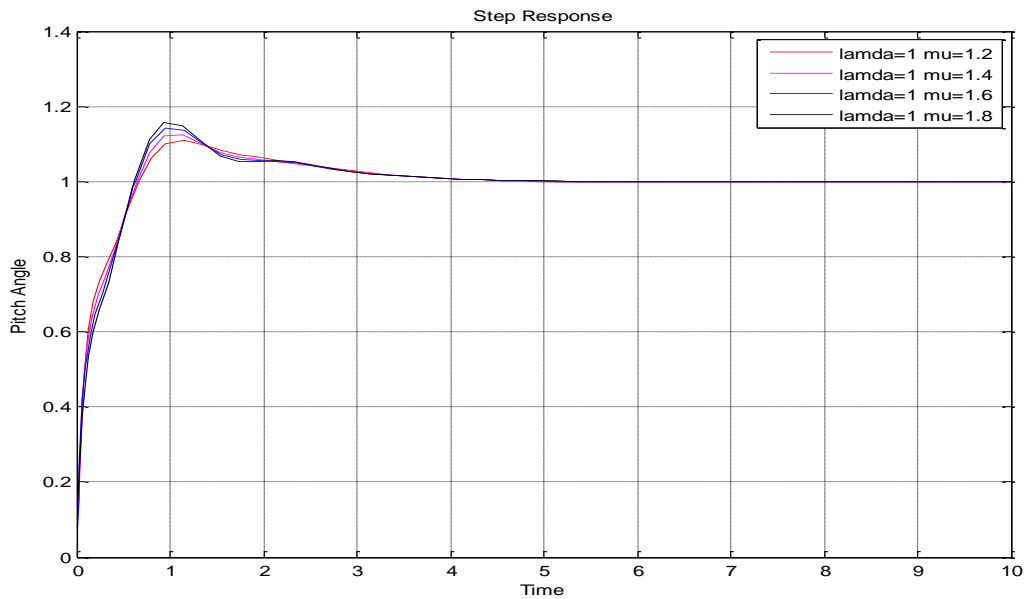
The above figure 6.22 shows step response of aircraft pitch control with disturbance using fractional order PID Controller for  $\lambda \in (1,2)$  and  $\mu=1$ .

**Table 6.17** Different parameters with varying values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu=1$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2 \mu=1$	1.4626	2.4626	0.07865	0.3485	0.3485	0.4843
2	$\lambda=1.4 \mu=1$	1.5588	5.2499	1.9485	0.07780	0.3527	0.5897
3	$\lambda=1.6 \mu=1$	0.5497	4.1055	0.6871	0.07735	0.3478	0.6378
4	$\lambda=1.8 \mu=1$	0.5499	3.2855	0.6874	0.07698	0.3378	0.644

It can be seen from the above Table 6.17 that with the increase in the values of  $\lambda$ , control parameters are decreased.

### 6.5.5 With values of integral order $\lambda=1$ and varying derivative order $\mu \in (1,2)$ with disturbance



**Figure 6.23** With value of integral order  $\lambda=1$  and derivative order  $\mu \in (1,2)$  with disturbance using disturbance

The above figure 6.23 shows the step response of aircraft pitch control with disturbance using fractional order PID controller for  $\lambda=1$  and  $\mu \in (1,2)$

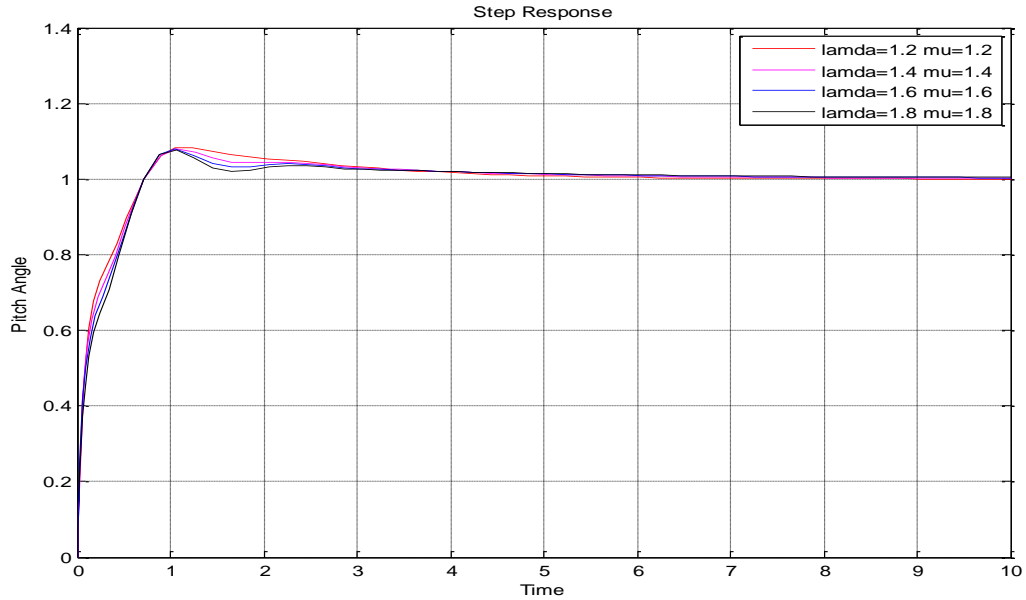
**Table 6.18** Different parameters with value of integral order  $\lambda=1$  and derivative order  $\mu \in (1,2)$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1$ $\mu=1.2$	1.7243	10.9156	2.1554	0.0848	0.3451	0.3663
2	$\lambda=1$ $\mu=1.4$	1.7123	12.4439	2.1403	0.09117	0.3568	0.3616
3	$\lambda=1$ $\mu=1.6$	1.7203	14.1403	2.1503	0.09858	0.3718	0.3670
4	$\lambda=1$ $\mu=1.8$	1.8651	15.8237	2.3314	0.1069	0.3884	0.3761

It can be seen from the above Table 6.18 that with the increase in the values of  $\mu$ , control parameters are remain same.

**6.5.6 With varying values of integral order  $\lambda \in (1,2)$  and derivative order  $\mu \in (1,2)$  with disturbance.**

**6.5.6.1 Values of integral order  $\lambda$  and derivative order  $\mu$  are same ( $\lambda = \mu$ )**



**Figure 6.24:** Step response for Aircraft pitch control with disturbance in FOPID for value of integral order  $\lambda$  and derivative order  $\mu$  are same

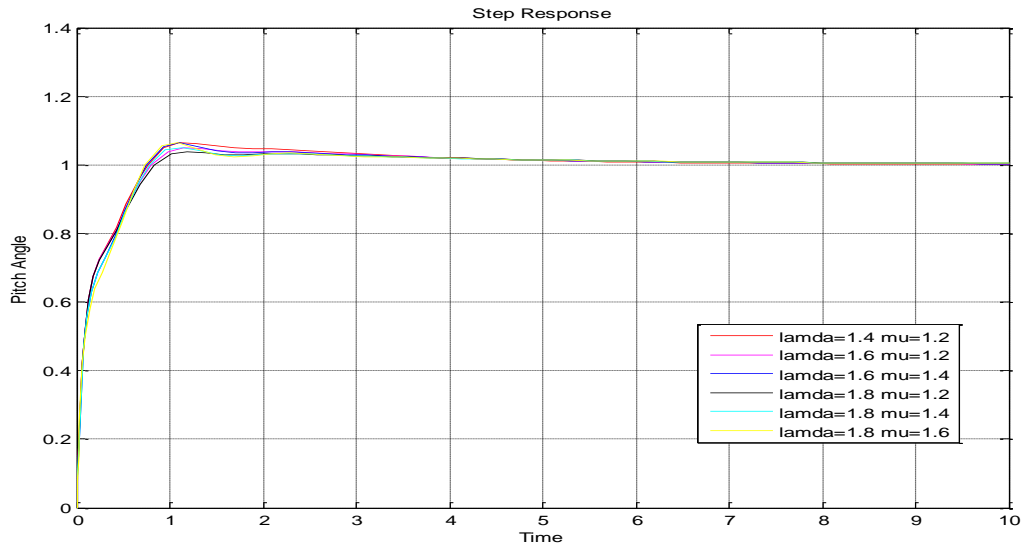
The above figure 6.24 shows the step response of aircraft pitch control with disturbance using fractional order PID controller for  $\lambda = \mu$ .

**Table 6.19** Different parameters with values of integral order  $\lambda$  and derivative order  $\mu$  are same

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2, \mu=1.2$	1.7887	8.2766	2.2359	0.08328	0.355	0.4755
2	$\lambda=1.4, \mu=1.4$	1.1686	8.0207	1.4608	0.08802	0.3681	0.567
3	$\lambda=1.6, \mu=1.6$	1.0065	7.8823	1.2581	0.09376	0.3747	0.6117
4	$\lambda=1.8, \mu=1.8$	1.0075	7.7574	1.2591	0.01003	0.328	0.3269

It can be seen from the above Table 6.19 that with the same values of  $\lambda$  &  $\mu$ , control parameters are increased.

### 6.5.6.2 Values of integral order $\lambda$ is greater than derivative order $\mu$ ( $\lambda > \mu$ )



**Figure 6.25 :** Step response for aircraft pitch control using FOPID with disturbance for values of integral order  $\lambda$  is greater than derivative order  $\mu$

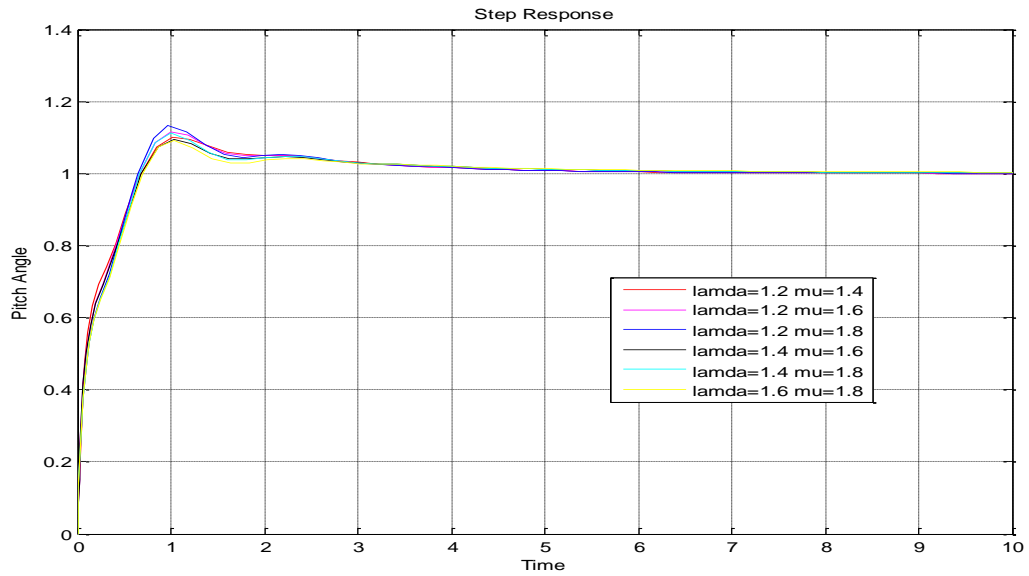
The above figure 6.25 shows the step response of aircraft pitch with disturbance control using fractional order PID controller for  $\lambda > \mu$ .

**Table 6.20** Different parameters with values of integral order  $\lambda$  is greater than derivative order  $\lambda > \mu$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.4$ $\mu=1.2$	1.1892	6.3759	1.4865	0.0822	0.3571	0.5689
2	$\lambda=1.6$ $\mu=1.2$	0.4289	4.9201	0.5361	0.0815	0.3506	0.6089
3	$\lambda=1.6$ $\mu=1.4$	1.0386	6.4187	1.2983	0.0871	0.3608	0.6054
4	$\lambda=1.8$ $\mu=1.2$	0.5409	3.7042	0.6761	0.0810	0.3393	0.6112
5	$\lambda=1.8$ $\mu=1.4$	0.9049	5.0872	1.1312	0.0864	0.3488	0.6067
6	$\lambda=1.8$ $\mu=1.6$	0.8684	6.4498	1.0856	0.0929	0.3623	0.6137

It can be seen from the above Table 6.20 that with the different values of  $\lambda$  &  $\mu$ , control parameters are almost remain same.

### 6.2.6.3 Values of integral order $\lambda$ is less than derivative order $\mu$ ( $\lambda < \mu$ )



**Figure 6.26 :** Step response for aircraft pitch control using FOPID with disturbance for different values of integral order  $\lambda <$  derivative order  $\mu$

The above figure 6.26 shows the step response of aircraft pitch control with disturbance using fractional order PID controller for values of  $\lambda$  greater than  $\mu$ .

**Table 6.21** Different parameters combination values of integral order  $\lambda$  is less then derivative order  $\mu$   
 $\lambda < \mu$  with disturbance

Serial No	Different Values of $\lambda$ (integral order) and $\mu$ (derivative order)	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda=1.2$ $\mu=1.4$	1.6026	9.9350	2.0033	0.0893	0.3669	0.4767
2	$\lambda=1.2$ $\mu=1.6$	1.7536	11.6614	0.9920	0.0964	0.3817	0.4833
3	$\lambda=1.2$ $\mu=1.8$	1.8979	13.2747	2.3724	0.1044	0.3982	0.4935
4	$\lambda=1.4$ $\mu=1.6$	1.1402	9.6010	1.4252	0.0948	0.3824	0.5732
5	$\lambda=1.4$ $\mu=1.8$	1.1229	11.1103	1.4036	0.1026	0.3986	0.5841
6	$\lambda=1.6$ $\mu=1.8$	0.9869	9.2888	1.2336	0.1013	0.3906	0.6237

It can be seen from the above Table 6.21 that with the different values of  $\lambda$  &  $\mu$ , control parameters are increased.

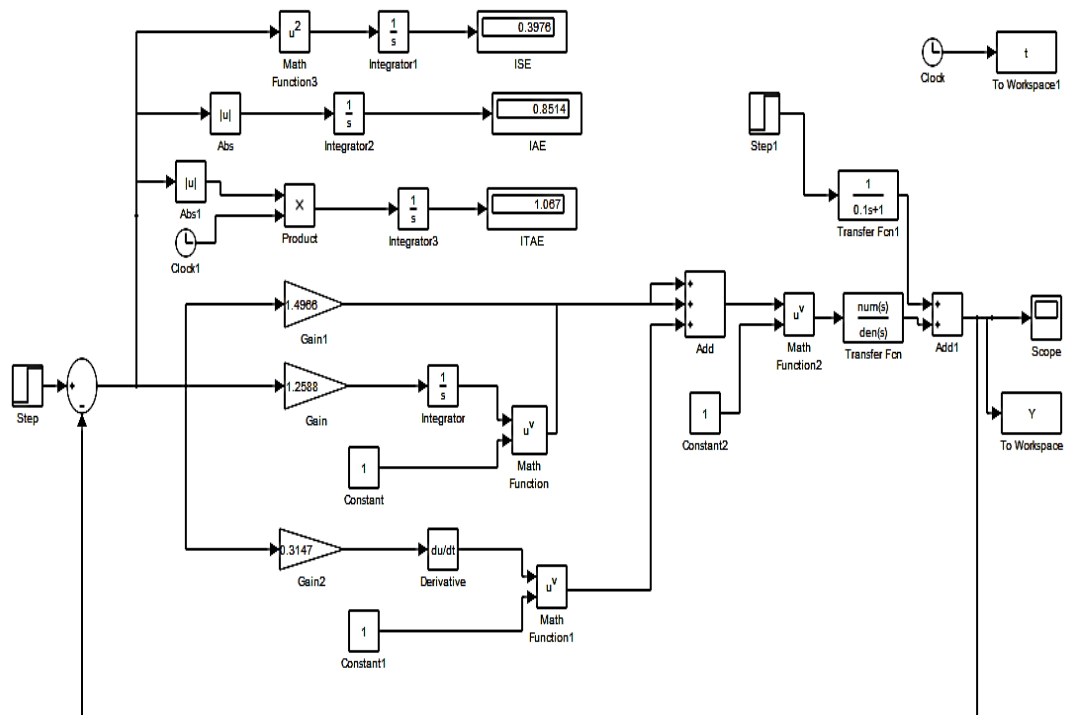
## 6.6 Aircraft pitch control using fractional order of PID control with disturbance:

For Fractional order of PID Controller used as conventional order PID Controller. After taking different values of the power  $\lambda$ , output response is taken.  $\lambda$  is the integral order and  $\mu$  is the derivative order which is taken as 1. Tuning of controller is done using Ziegler-Nichols tuning method and tuning of controller is done by using auto tuning function in simulink. With using Ziegler- Nichols method for Aircraft pitch control transfer function to get constant value of  $K_p$ ,  $K_i$  and  $K_d$ . The PID controller parameters found out are

Proportional Gain ( $K_{p2}$ ) = 1.4966

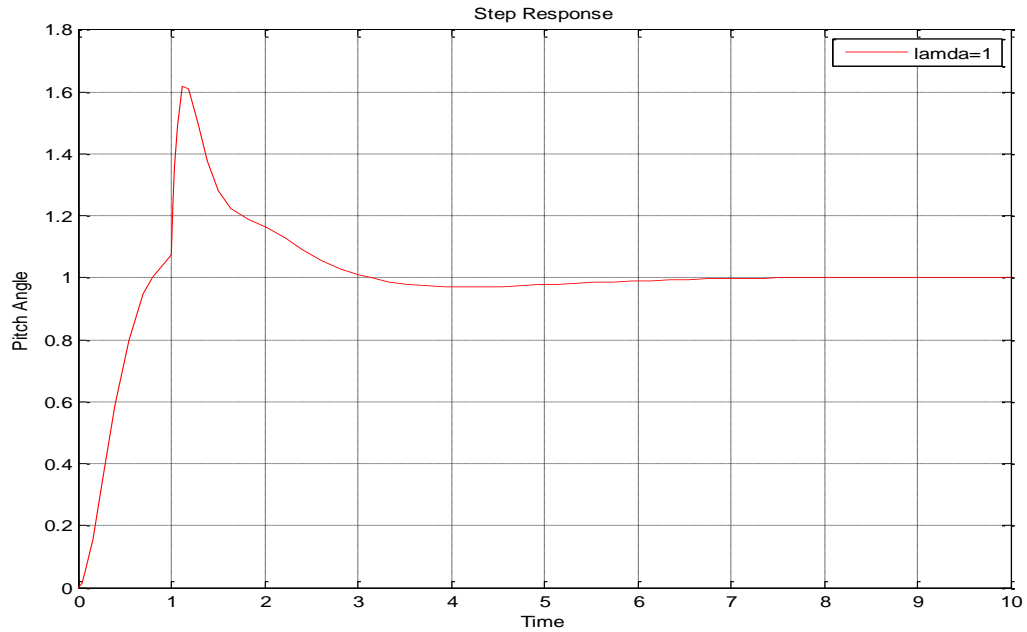
Integral Gain ( $K_{i2}$ ) = 1.2588

Derivative Gain ( $K_{d2}$ ) = 0.3147



**Figure 6.27:** Fractional order of PID controller with disturbance

### 6.6.1 Aircraft pitch control for conventional PID controller with disturbance



**Figure 6.28:** Step Response of Aircraft pitch control with FOPID controller with introducing disturbance for  $\lambda = 1$

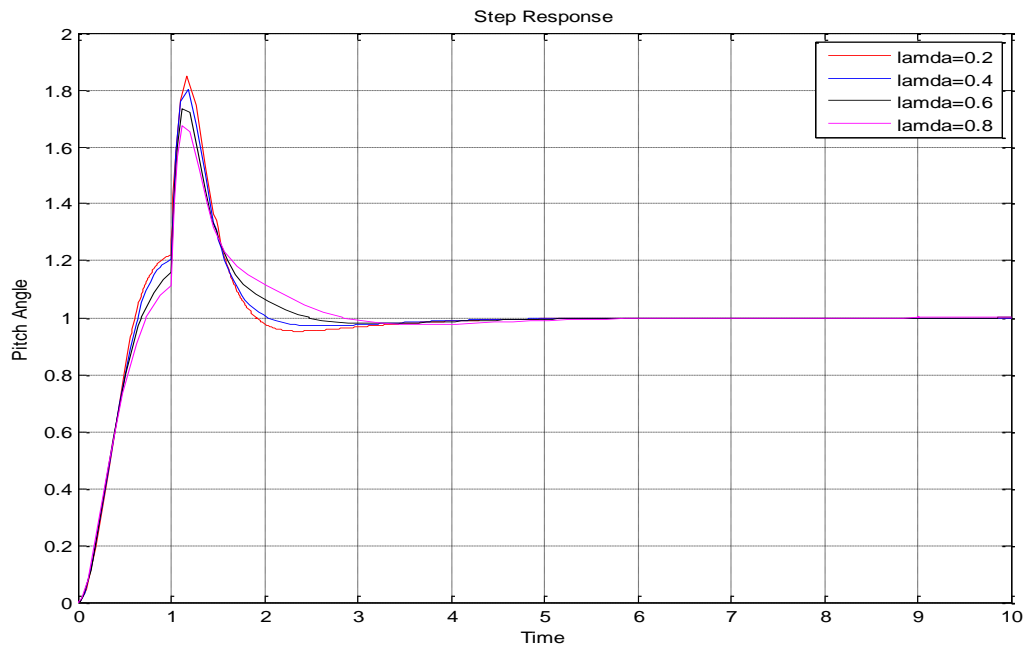
The above figure 6.28 shows the step response of Aircraft pitch control using fractional order of PID controller with disturbance for  $\lambda = 1$

**Table 6.22** Different parameters of fractional order of PID Controller with disturbance values of  $\lambda = 1$

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 1$	1.0147	27.9912	1.2484	0.3976	0.8514	1.067

In the above Table 6.22, shows the settling time, rise time, peak overshoot, integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE) with taking value of  $\lambda = 1$ .

### 6.6.2 Aircraft pitch control for fractional order of PID controller with disturbance values of $\lambda < 1$



**Figure 6.29:** Step response of Aircraft pitch control with fractional order of PID controller with disturbance for  $\lambda < 1$

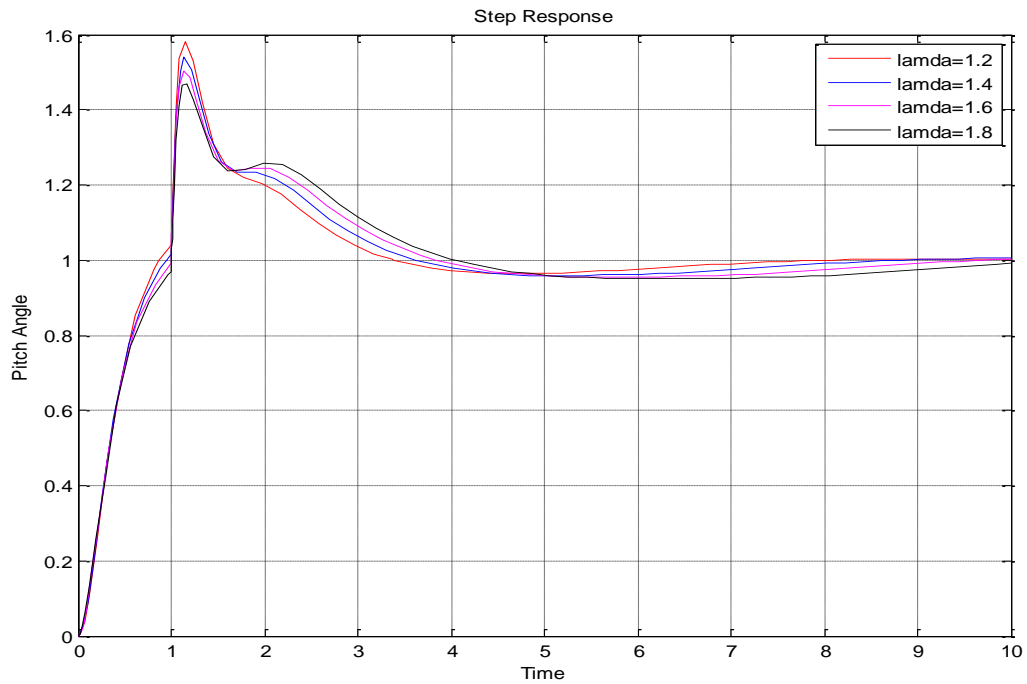
The above figure 6.29 shows the step response of Aircraft pitch control with fractional order of PID controller with disturbance for  $\lambda < 1$

**Table 6.23** Different Parameters for fractional order of PID controller with disturbance values of  $\lambda < 1$

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 0.2$	1.4259	84.7344	1.7824	0.4898	0.835	0.8099
2	$\lambda = 0.4$	1.4352	80.3847	0.0410	0.3601	0.7895	0.7255
3	$\lambda = 0.6$	1.6381	73.4319	2.0477	0.4299	0.7744	0.7331
4	$\lambda = 0.8$	1.7853	67.3786	2.2316	0.4074	0.7994	0.8510

It can be seen from the table 6.23 that with the increase the value of  $\lambda$ , control parameters are increased.

### 6.6.3 Aircraft pitch control for fractional order of PID controller with disturbance values of $\lambda > 1$



**Figure 6.30:** Step Response of Aircraft pitch control with fractional order of PID Controller with disturbance for  $\lambda > 1$

The above figure 6.30 shows the step response of Aircraft pitch control with fractional order of PID controller with disturbance for  $\lambda > 1$

**Table 6.24** Different Parameters for fractional order of PID controller with disturbance values of  $\lambda < 1$

Serial No	Values of power $\lambda$	Rise Time (Sec)	Peak Overshoot (%)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	$\lambda = 1.2$	2.2180	58.2760	2.7724	0.3947	0.9222	1.395
2	$\lambda = 1.4$	2.3240	53.9721	2.9050	0.3955	0.9980	1.7610
3	$\lambda = 1.6$	2.6119	50.4472	3.2648	0.3995	1.077	2.169
4	$\lambda = 1.8$	2.7151	46.9284	3.3939	0.4051	1.158	2.644

It can be seen from the Table 6.24 that with the increase the value of  $\lambda$ , control parameters are increased.

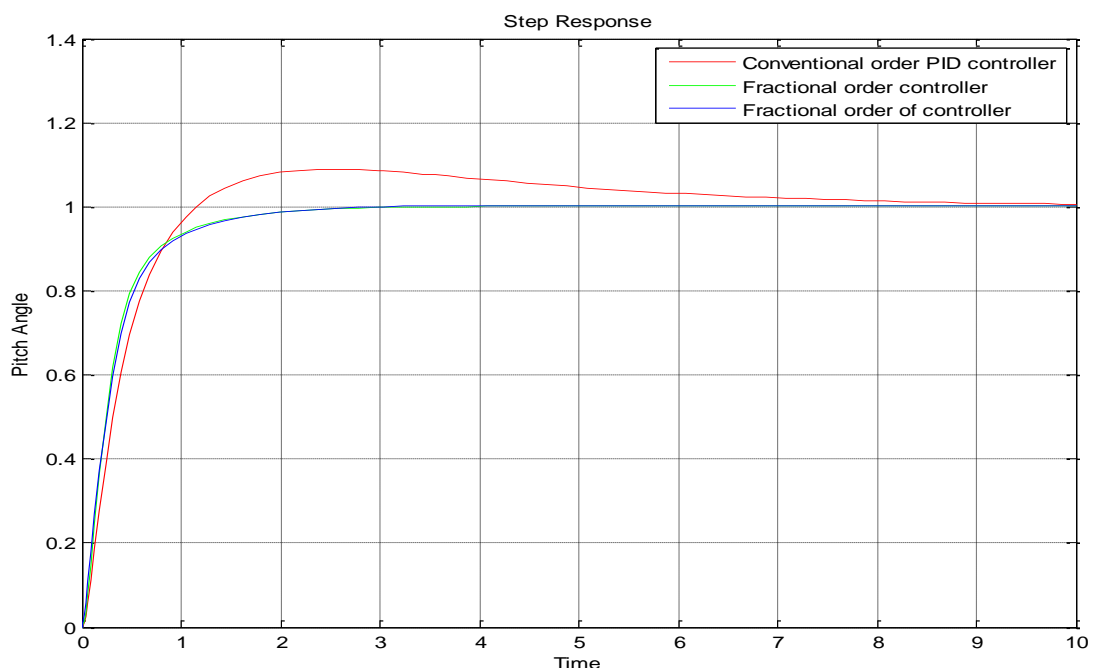
## 6.7 Comparison of different Control Technique

Response of Aircraft pitch control is taken with different control scheme and techniques:

### 6.7.1 PID Parameters without disturbance

After changing different values of integral order  $\lambda$  and derivative order  $\mu$ , it gives different values in rise time, peak overshoot, settling time as well as integral square error (ISE), integral absolute value (IAE) and integral time absolute error (ITAE). After analysis all the value the best values are as below

- Conventional PID controller with  $\lambda = 1$  and  $\mu = 1$
- Fractional order PID controller with  $\lambda = 1.2$  and  $\mu = 1$
- Fractional order of PID Controller with value of power  $\lambda = 0.8$



**Figure 6.31** Step response of Aircraft pitch control using different control technique

The above figure 6.31 shows the step response for Aircraft pitch control using different control techniques. The different controllers are conventional order PID controller, fractional order PID controller and fractional order of PID controller.

**Table 6.25** Comparison of parameters for different control technique for PID parameters

Serial No	Control Strategy	Different Values of $\lambda$ (integral order) & $\mu$ (derivative order)	Rise Time (Sec)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	Conventional PID controller	-	3.8792	4.6785	0.2529	0.7473	1.599
2	Fractional order PID Controller	$\lambda=1.2$ $\mu=1$	3.7096	4.6370	0.1872	0.369	0.2839
3	Fractional Order of PID Controller	Power of $\lambda$ =0.8	0.7209	0.9011	0.1794	0.288	0.2833

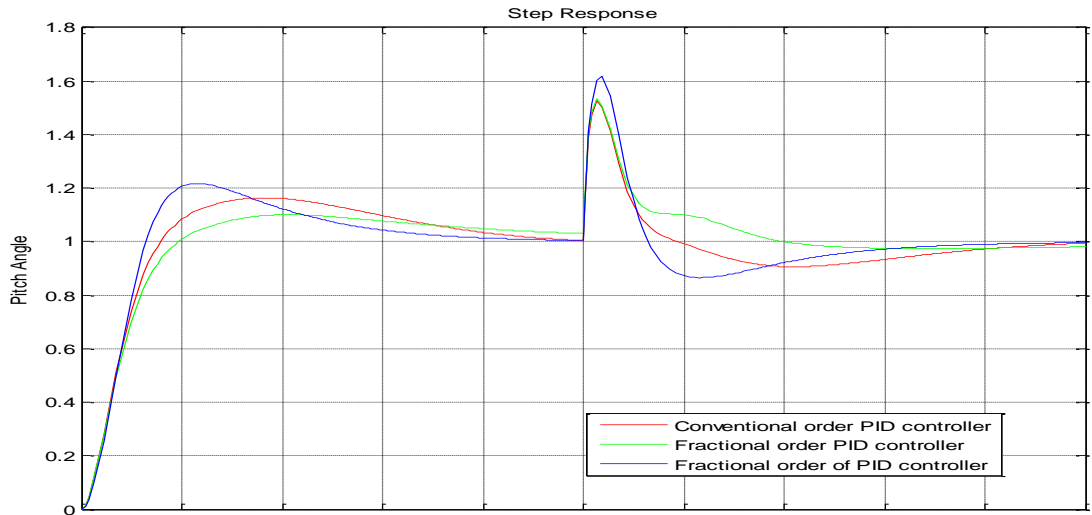
From the graph 6.31 and Table 6.25, shows the response of conventional order PID controller, fractional order PID controller and fractional order of PID controller. As shown in the Table 6.25 the fractional order of PID controller FO[PID] shows the better response than other types of controller.

### 6.7.2 PID Parameters with using disturbance

As first order disturbance in the system, it gives different values when changing the values of integral order and derivative order part.

After analysis all the value the best values are as below

- Conventional PID controller with  $\lambda = 1$  and  $\mu = 1$
- Fractional order PID controller with  $\lambda = 1.8$  and  $\mu = 1.2$
- Fractional order of PID Controller with value of power  $\lambda = 0.4$



**Figure 6.32** Step response of Aircraft pitch control using different control technique

The above figure 6.32 shows the step response for Aircraft pitch control using different control techniques. The different controllers are conventional order PID controller, fractional order PID controller and fractional order of PID controller.

**Table 6.26** Comparison of parameters for different control technique for PID parameters with disturbance

Serial No	Control Strategy	Different Values of $\lambda$ (integral order) & $\mu$ (derivative order)	Rise Time (Sec)	Settling Time (Sec)	Integral Square Error (ISE)	Integral Absolute Error (IAE)	Integral Time Absolute Error (ITAE)
1	Conventional PID controller	-	1.9195	2.3993	0.0797	0.3421	0.7255
2	Fractional order PID Controller	$\lambda=1.8$ $\mu=1.2$	0.5497	1.6761	0.3810	0.3393	0.6112
3	Fractional Order of PID Controller	$\lambda=0.4$	0.4352	1.0410	0.3601	0.3095	0.4045

From the graph 6.32 and Table 6.26, shows the response of conventional order PID controller with disturbance, Fractional order PID controller and fractional order of PID controller. As shown in the table that fractional order of PID controller FO[PID] shows the better response than other types of controller.

**CHAPTER 7****CONCLUSION AND FUTURE SCOPE**

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**7.1 Conclusion**

This thesis work discusses a process to control the pitch of Aircraft system. There are many controllers architecture available in the control system like feedback control system, feedforward control system, P type controller, I type controller, PI controller, PID Controller etc. But among all controller PID controller is widely and versatile controller used in the industries. As per the survey 90% of the process plants use this type of controller. But in recent advancement in fractional calculus has introduced applications of fractional order PID controller and it has received a considerable attention an academic studies and in industrial applications. Fractional order proportional integral derivative controller is an advancement of classical integer order PID controller. In many a cases fractional order PID controller has outperformed classical integer order PID controller. This dissertation, studies the control aspect of fractional order controller in aircraft pitch system.

A comparative study is done using different control techniques. First the conventional Proportional Integral Derivative Controller is implemented. A this time controller gives high overshoot and high settling time. So after that fractional order controller is applied for Proportional Integral Derivative controller system. During fractional order Proportional Integral Derivative controller different values of integral order and derivative order has been taken. The step responses has been recorded. After taking different values of integral order and derivative order, it is found that fractional order controller has the better response then conventional order controller. Fractional order of controller is also new type of controller which are widely used nowadays. In this controller the value of power has been changed and recorded the response. It is also found that fractional order of controller gives better response than fractional order controller.

## 7.2 Future Scope

In the future scope, genetic algorithm based online optimization technique can be implemented to improve the control performance. For tuning of controller and fractional order controller, better tuning method can be used

## CHAPTER 8

### CHECK FOR ORIGINALITY

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The dissertation report presented here has been checked for its originality using online plagiarism checker “Paper Rater”, available at [http://www.paperrater.com/plagiarism\\_checker](http://www.paperrater.com/plagiarism_checker). Various theoretical concepts are explained as per the references from different technical books which I studied during my engineering graduation and post graduation studies. Thanks to all those who are already present in my references text.

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