ECONOMIC ORDERING POLICY OF INVENTORY MODEL WITH EXPONENTIAL DECLINING DEMAND CONSIDERING PRODUCTION AND NON PRODUCTION PERIODS FOR SINGLE VENDOR AND MULTIPLE BUYERS

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CERTIFICATE

This is to certify that the thesis entitled ***" Economic Ordering Policy Of Inventory Model With Exponential Declining Demand Considering Production And Non production periods For Single Vendor And Multiple Buyers "****, being presented in partial fulfillment of the requirements for the award of the degree of Master of Science in the School of Mathematics, Thapar University, Patiala, is a bonafide work carried out under the supervision of* ***Dr. Mahesh Kumar Sharma.***

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ABSTRACT

In a production inventory system , the manufacturer produces the items at a particular production rate, dispatches the order quantities to the customers in specific intervals and stores the excess inventory for the subsequent deliveries. In the real life situations inventory manager has to hold the thousand of items in the inventory and also in a competitive market the aim of the vendor and the buyer is to maximize their profit, so in general an integrated policy is required by the vendor and the buyer. In the real life, decay and deterioration occur in some of the products such as fruits, milk products, vegetables and medicines. Various models have been developed in literature for deteriorating item with constant demand. However, in case of the items like food grains, fashion apparels and electronic equipments etc. which have a fixed shelf life and which decreases with time during the end of the season, an inventory model with exponential declining demand rate has been proposed.

The present work has been divided into three chapters.

In Chapter 1, the introduction of some inventory models have been discussed and the literature related to the topic has been given briefly. In Chapter 2, Economic Ordering policy of deteriorated item for Vendor and Buyer (Yang and Wee (2000)) has been considered in which constant demand rate is replaced by the exponential declining function of time and the same approach used by yang and wee (2000) is applied to obtain the optimal solution. A numerical example is presented to demonstrate the model and sensitivity analysis of various parameters is carried out. In Chapter 3, the inventory model considered in chapter 2 is extended for single vendor and multiple buyers with exponential declining demand rate. Numerical example is also mentioned in the support of the model.

Contents

Chapter 1

Introduction

The inventory of a particular company whether it can be small or large, can be defined as the physical stock of goods, units or economic resources that are stored or reserved for the smooth, efficient and effective functioning of business. There are many companies that can have variety of inventories, which consists of many small items such as paper pads, pencils, and paper clips, and fewer big items such as trucks, machines, and computers. The inventory of a particular company is completely dependent to the business in which that particular company is engaged such as a tennis shop has an inventory of tennis rackets, shoes, and balls, a television manufacturer has parts, subassemblies, and finished TV sets in its inventory, a theater has an inventory of seats, a restaurant has an inventory of tables and chairs, and a public accounting firm has an inventory of accountants. If the inventory is less in some firms or might the case it may be left without inventories then a particular customer would have to wait until their orders were filled from a source or were produced. However, in real life customer will not like to wait for long period of time as there is lot of competition in the market. So he might end with purchasing the items from some other company. Another reason for maintaining stock is the price fluctuation of some raw material, so that it would be profitable for a buyer to procure a sufficient quantity of raw material at lower price and use it whenever needed. It is the belief of some researchers, that in order to attract more customers, it is necessary to maintain the inventory on display. To determine the levels of the inventory which is to be maintained , the inventory system or management should have the set of policies, that when should the stock be replenished. Inventory model helps a firm in determining the economic order quantity, and the frequency of ordering, to keep goods or services flowing to the customer without interruption or delay. For example wholesalers and retailers need to maintain inventories of goods to be available for the costumers to purchase.

Inventory modeling deals with determining the level of commodity that business or organization must maintain to ensure smooth operation. The basis for the decision is a model that balances the cost of the capital resulting from holding too much inventory against the penalty cost resulting from inventory shortage. The major factors which affect the solution is the nature of the demand which can be deterministic or probabilistic.

1.1 General Inventory Models

Inventory problems consists of placing and receiving orders of given sizes at some intervals. From this point, an inventory policy always answers two questions:

- How much to order?
- When to order ?

To answer to the first question determines the economic order quantity(EOQ) by minimizing the following cost model:

- Total inventory cost = Purchasing cost + Set up cost + Holding cost + Shortage cost

All these costs must be expressed in terms of desired order quantity and the time between orders

1. Purchasing cost is based upon the price per unit of the item.It may be constant or it may be offered at a discount that depends on size of order
2. Set up cost represents the fixed charge incurred when an order is placed.This cost is independent of size of order

3. Holding cost represents the cost of maintaining inventory in the stock.It includes the interest on capital as well as cost of storage ,maintenance and handling.
4. Shortage cost is the penalty incurred when we run out of stock.

The answer to the second question depends on the type of inventory system with which we are dealing.New orders are placed when the inventory level drops to a prespecified level ,called Reorder point.

1.1.1 Classic Economic Order Quantity Model (Taha,(1992))

Classical inventory model deals with single-item model. The aim of this model is to determine economic order quantity, y which minimizes the total cost of an inventory system when demand is constant with instantaneous order replenishment and when no shortage is allowed. The model is developed under following assumptions:

1. This model deals with single item.
2. The demand rate is known and constant.
3. Quantity discounts are not available.
4. The ordering cost is constant.
5. Shortages are not allowed and lead time is known and is constant.
6. The inventory holding cost per inventory unit per time is known and constant.

let
y = Order quantity(number of units)
D = Demand rate(units per unit time)
t= Ordering cycle length (time units)
using these definitions the inventory level follows the pattern depicted in figure below

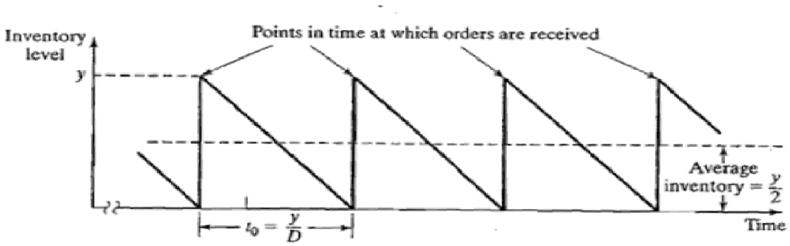


Figure 1.1: Inventory Pattern in the classic EOQ Model

An order of size y units is placed and received instantaneously when the inventory level is zero.The stock is then depleted uniformly at a constant demand rate D.The ordering cycle for this pattern is

$$t = \frac{y}{D}$$

units

The resulting average inventory level is given as

Average inventory level = $\frac{y}{2}$ units

The resulting average inventory level is given as $\frac{y}{2}$ units.

The model requires two cost parameters with

K = setup cost of an item(dollar per order)

h = the holding cost(dollars per inventory unit per unit time).

The total cost per unit time(TCU) is

TCU(y) = setup cost per unit time + Holding cost per unit time

$$= \frac{setup\ cost + Holding\ cost\ \ per\ cycle\ t}{t}$$

$$= \frac{K + h\left(\frac{y}{2}\right)t}{t}$$

$$= \frac{K}{\left(\frac{y}{D}\right)} + h\frac{y}{2}$$

The optimum solution is found when we minimize TCU(y) with respect to y.Here we assume y to be continuous.Then the necessary condition for finding the optimal value of y is

$$\frac{dTCU(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0$$

The solution of the equation gives the EOQ ,y^* as

$$y^* = \sqrt{\frac{2KD}{h}}$$

Thus the optimum inventory policy for the proposed model is summarized as order $y^* = \sqrt{\frac{2KD}{h}}$ units every $t^*= \frac{y^*}{D}$ time units.

It is very difficult to receive new orders instantly in real life. What happens in real life is that a positive lead time may occur between the placement and receipt of an order.In case of a lead time ,reorder point occurs when an inventory level drops to LD units.

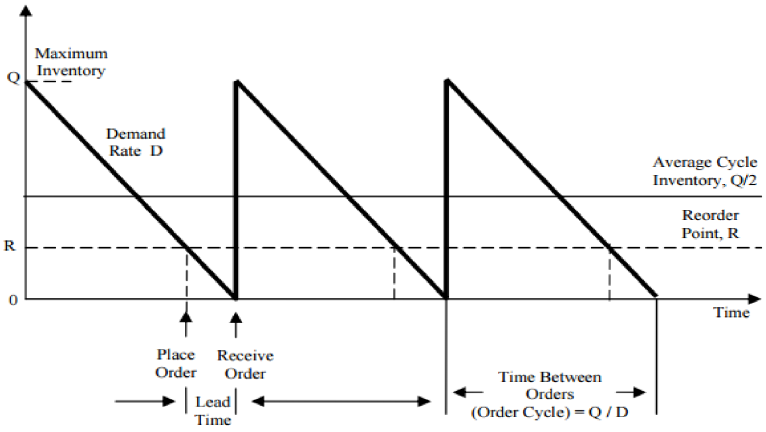


Figure 1.2: Reorder Point in classic EOQ Model

In the above figure 1.2 L is assumed as the lead time. It is also assumed that the lead time L is less than the cycle length t^* which is not in general.

1.1.2 EOQ Model With Price Breaks (Taha,(1992))

In EOQ model with price breaks items may be purchased at a discount if the size of the order y exceeds a given limit q and the unit purchasing price c is given as

$$c = \begin{cases} c_1, & y \leqslant q \\ c_2, & y > q, \end{cases}$$

where, $c_1 > c_2$

Hence,

Purchasing cost per unit time is

$$T = \begin{cases} \frac{c_1 y}{t_0} = Dc_1, & y \leqslant q \\ \frac{c_2 y}{t_0} = Dc_2, & y > q \end{cases} \tag{1.1}$$

Total cost per unit time is

$$TCU(y) = \begin{cases} TCU_1(y) = Dc_1 + \frac{KD}{y} + \frac{hy}{2}, & y \leqslant q \\ TCU_2(y) = Dc_2 + \frac{KD}{y} + \frac{hy}{2}, & y > q, \end{cases} \tag{1.2}$$

The functions $TCU_1(y)$ and $TCU_2(y)$ are represented below and the minima occurs at,

$$y_m = \sqrt{\frac{2KD}{h}}$$

The cost function TCU(y) starts on the left with TCU_1 and drops to TCU_2 at the price break point q which lies in zone I. Optimum value y^* depends where q, lies with zones I, II and III given in the Figure 1.3 as $(0, y_m)$, (y_m, Q) and (Q, ∞), respectively. The value of $Q(> y_m)$ is obtained from

$$TCU_2(Q) = TCU_1(Y_m)$$

$$c_2 D + \frac{KD}{Q} + \frac{hQ}{2} = TCU_1(y_m)$$

which simplifies to

$$Q^2 + \left(\frac{2(c_2 D - TCU_1(y_m))}{h}\right) Q + \frac{2KD}{h} = 0 \tag{1.3}$$

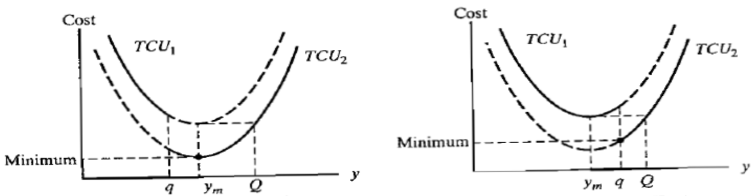
Figure 1.3 shows the optimum quantity y^* is

$$y^* = \begin{cases} y_m, & \text{if } q \text{ is in zones I or III} \\ q, & \text{if } q \text{ is in zone II,} \end{cases}$$

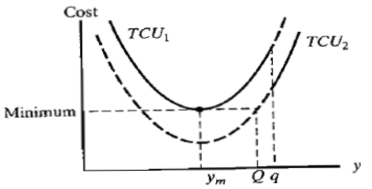
Steps for determining y^* are

Step1. $y^* = y_m$, if q is in zone I or zone III. Otherwise go to step2.

Step2. Determine$Q(> y_m)$ from equation (1.3). If q is in zone II, y^*=q.



Case 1: q falls in zone I, $y^* = y_m$ Case 2: q falls in zone II, $y^* = q$



Case 3: q falls in zone III, $y^* = y_m$

Figure 1.3: EOQ price breaks graph

1.1.3 An Inventory Model for deteriorating items for Single Vendor and Single Buyer (Yang and Wee(2000))

In a competitive market environment,there are suppliers, producers, distributors and retailers. A buyer usually has the advantage to decide on the number of deliveries when an order is made. The optimal number of deliveries chosen by the buyers may not be the best for the vendor. If they both need to minimmize the overall integrated cost then the number of deliveries should be decided in the cooperation with the vendor. Supply chain coordination is the major issue in in supply chain management research. To overcome this situation the integration approach has been preffered and is researched for many years. In this work they developed an economic ordering policy for deteriorating items in which production rate and demand rate are constant with some assumptions. They proved that integrated approach is better than independent decision approach through sensitivity analysis and results in an impressive reduction in cost of both.

In this section the formulation of the model for single vendor and single buyer for constant demand has been shown.

To formulate the model following assumptions and notations have been considered

Assumptions

1. Constant production rate .
2. Constant demand rate.
3. Shortage is not allowed.
4. Rate of deterioration is considered to be constant and is applied for single item.
5. Consideration of deteriorated units is done only after they have been received into inventory.
6. Deteriorated units can't be replaced or repaired.
7. The cost of carrying is applied to the good units only.
8. Single vendor and single buyer are considered.
9. Multiple deliveries per order are considered.
10. There is only one production cycle per order.

Notations

P = production rate

K = Constant demand

I_{t_1} = Change of inventory level with time t_1 during production period

I_{t_2} = Change of inventory level with time t_2 during non-production period

T_1 = the production period in each cycle

T_2 = the non production period in each cycle

T = Time length of cycle

γ = The constant rate of deterioration

n = Number of deliveries per order

$I_{p_c}(t)$ = Inventory level of the vendor

$I_b(t)$ = Buyer's inventory level

C_{o_b} = Cost of ordering for the buyer, per order

C_{s_v} = Cost of setup for the vendor, per production cycle

C_{c_b} = Carrying cost of inventory for the buyer, per time and per unit

C_{c_v} = Carrying cost of inventory for the vendor, per time and per unit

C_b = cost of deteriorated unit for the buyer

C_v = cost of deteriorated unit for the buyer

K_{o_b} = The incoming control cost for the buyer, per delivery

K_{o_v} = The transportation charge for the vendor, per delivery

TC_b = Buyer's total cost function, who buys goods from the vendor

TC_v = Vendor's total cost function

TC = The integrated total cost function including TC_v and TC_b

Formulation

Yang and Wee (2000) developed a model whose aim is to calculate the optimum profit for the items which has constant demand and constant rate of deterioration. The differential equation $I_{p_c(t)}$ which tells us about the inventory level for a vendor as follows :

$$\frac{dI_1}{dt_1} = (P-K) - \gamma I_{t_1} \quad 0 \leqslant t_1 \leqslant T_1 \tag{1.4}$$

$$\frac{dI_2}{dt_2} = -K - \gamma I_{t_2} \quad 0 \leqslant t_2 \leqslant T_2 \tag{1.5}$$

From Spiegel (1960) and using the boundary conditions

$I_1(0) = I_2(T_2) = 0$, the solutions of the above differential equation obtained are :

$$I_{t_1} = \frac{P-K}{\gamma}\left(1 - \exp(-\gamma t_1)\right); \;\; 0 \leqslant t_1 \leqslant T_1 \tag{1.6}$$

$$I_{t_2} = \frac{-K}{\gamma}\left(1 - \exp(\gamma(T_2 - t_2)\right); \;\; T_1 \leqslant t_2 \leqslant T_2 \tag{1.7}$$

From the boundary conditions $I_1(T_1) = I_2(0)$ following expression is obtained,

$$(P-K)\left(1 - \exp(-\gamma T_1)\right) = -K\left(1 - \exp(\gamma T_2)\right) \tag{1.8}$$

Value of γ is assumed to be very small , as deterioration rate cannot be zero. So by using taylor's expansion following expression is obtained,

$$(P-K)T_1(1 - \frac{1}{2}\gamma T_1) = KT_2(1 + \frac{1}{2}\gamma T_2) \tag{1.9}$$

From Misra (1975) the production period of each cycle is

$$T_1 \approx \frac{K}{P-K}T_2\left(1 + \frac{1}{2}\gamma T_2\right) \tag{1.10}$$

Total length of time cycle is obtained as,

$$T = T_1 + T_2 \tag{1.11}$$

$$T = \frac{K}{P-K}T_2\left(1 + \frac{1}{2}\gamma T_2\right) + T_2 \tag{1.12}$$

The final expression of total length of the the time cycle is,

$$T = \frac{T_2}{P-K}\left(P + \frac{1}{2}K\gamma T_2\right) \tag{1.13}$$

Inventory function for a buyer for n deliveries per order is given by :

$$\frac{dI_b(t)}{dt} = -K - \gamma I_b(t) \tag{1.14}$$

On solving above equation inventory level of the buyer is,

$$I_b(t) = \frac{K}{\gamma}\left(\exp\left(\gamma(\frac{T}{n} - t)\right) - 1\right); \;\; 0 \leqslant t \leqslant \frac{T}{n} \tag{1.15}$$

The maximum inventory $I_b(t)$ of the buyer at t=0 is found to be ,

$$I_b(t=0) = \frac{K}{\gamma}\left(\exp(\frac{\gamma T}{n}) - 1\right) \tag{1.16}$$

Now the total cost function for the buyer TC_b is given as follows

$$TC_b = \frac{1}{T}(C_{o_b} + nK_{o_b}) + C_{c_b}\frac{n}{T}\int_0^{\frac{T}{n}} I_b(t)dt + C_b\frac{n}{T}\left(I_b(0) - \frac{T}{n}K\right) \tag{1.17}$$

Using the equations (1.15) and (1.16) in equation (1.17) we get

$$TC_b \approx \frac{1}{T}(C_{o_b} + nK_{o_b}) + \frac{C_{c_b}TK}{2n}\left(1 + \frac{\gamma T}{3n}\right) + \frac{C_bTK\gamma}{2n} \tag{1.18}$$

The total cost function of the vendor is;

$$\begin{aligned} TC_v &\approx \frac{1}{T}\big(C_{sv} + nK_{0v}\big) + \frac{C_{c_v}}{T}\Bigg(\int_0^{T_1} I_{t_1}dt_1 \\ &+ \int_0^{T_2} I_{t_2}dt_2 - n\int_0^{\frac{T}{n}} I_b(t)dt\Bigg) \\ &+ C_v\frac{1}{T}\left(PT_1 - KT - n(I_b(0) - (\frac{T}{n})K)\right) \end{aligned} \tag{1.19}$$

Using equations (1.6), (1.7), (1.15) and (1.16) in equation (1.19) we get,

$$\begin{aligned} TC_v &\approx \frac{1}{T}\big(C_{s_v} + nK_{o_v}\big) + \frac{C_{c_v}}{T}[\frac{(P-K)T_1^2}{2}(1 - \frac{\gamma T_1}{3} \\ &+ \frac{KT_2^2}{2}(1 + \frac{\gamma T_2}{3}) - \frac{KT^2}{2n}(1 + \frac{\gamma T}{3n})] \\ &+ \frac{C_v}{T}(PT_1 - TK - \frac{K\gamma T^2}{2n}) \end{aligned} \tag{1.20}$$

The total cost function TC for the single vendor and single buyer at constant demand rate is given by adding equation (1.18) and (1.20)

$$\begin{aligned} TC &\approx \frac{1}{T}(C_{o_b} + nK_{o_b}) + \frac{C_{c_b}TK}{2n}\left(1+\frac{\gamma T}{3n}\right) + \frac{C_bTK\gamma}{2n} \\ &+ \frac{1}{T}(C_{s_v} + nK_{o_v}) + \frac{C_{c_v}}{T}[\frac{(P-K)T_1^2}{2}(1-\frac{\gamma T_1}{3} \\ &+ \frac{KT_2^2}{2}(1+\frac{\gamma T_2}{3}) - \frac{KT^2}{2n}(1+\frac{\gamma T}{3n})] \\ &+ \frac{C_v}{T}(PT_1 - TK - \frac{K\gamma T^2}{2n}) \end{aligned} \tag{1.21}$$

1.2 Literature Review

The term inventory signifies the goods or materials that are used by a company for the purpose of the sale and production. The basic concern which involves the inventory theory is to maintain the optimum investment and to apply the effective control system in order to minimize the total inventory cost. Management of inventory is useful because it helps the firm to address the two important issues,firstly it helps them in the maintainance of inventory for the smooth production and secondly it helps in the minimization of the investment for the enhancement of the profit in the inventory. Initially Harris (1915) studied on the inventory models. Later on, Wilson (1934) generalised the model which was proposed by Harris (1915) and he gave the formula to obtain (EOQ) i.e Economic Order Quantity. Later on, many models for the single and multiple items were developed, based on different assumptions. For instance, constant lead time, variable demand rate and partial backordering etc. Ghare and Schrader (1963) were the first who analysed in the decaying inventory problems. A simple economic order quantity model was established by them. In their study they concluded that if there is consumption of deteriorating items then it is closely relative to the negative exponential function of time. Dave and Patel (1981) also developed an inventory model for the deteriorating items with the time propotional demand. In this process they considered an EOQ model in which the demand rate was changing linearly with time and deterioration was assumed to be the constant fraction of the onhand inventory and also the proposed model was verified with the help of numerical example and sensitivity analysis. Furthermore, Wee (1993) proposed the model in which they considerd an economic ordering policy for the deteriorating items with the partial backordering. In this model they formulated an economic production plan for the deteriorating items with the partial backordering. They also illustrated their theory with the help of two numerical examples and further concluded that policy of this model leads to lower cost. After a period of time Wee (1995) developed a modified model in which the replenishment policy of the deteriorated item was considered in which the demand rate declines exponentially over a fixed peiod of time. Continuing their study Wee (1997) also developed a model on the replenishment policy for the items with a price dependent demand and with a varying rate of deterioration. This model was also justified with the help of a numerical example and the sensitivity analysis. Chang and Dye (1999) developed an EOQ model for the deteriorating items with the time varying demand and the partial backlogging. In this model they assumed that the shortages are either completely backlogged or are completely lost, their main focus was to notice the effects of the rate of backlogging and on the decision of the economic order quantity. Abad et al (1996) presented a generalised model for the dyanamic pricing and lot sizing by considering the viewpoint of the reseller who sells perishable goods. A simple solution procedure was used by them for solving the optimization problem. Later on, by continuing his work Abad et al.(2001) again proposed the model on the optimal price and order size for a reseller under the partial backlogging. Some more work was done in this research area by Shah and Raykundaliya (2010) who considered the model for Retailer's pricing ordering strategy for the weibull distribution under the trade credit in the declining market. The purpose of this model was to determine the optimal selling price and the quantity ordered in order to maximize the profit of the retailer. This model was supported with the numerical example and sensitivity analysis with respect to various parameters was carried out. Singh and Pattnayak (2013) formulated an EOQ model for deteriorating items by considering the variable deterioration , linear demand and partial backlogging. In this model the shortages were allowed and backlogging rate is variable and is dependent on the waiting time for the next replenishment. The model was developed with an objective to develop an optimal policy which minimizes the average total cost. Considering the work of the researchers on the deteriorated items in the inventory models, Goyal and Giri (2001) has provided the recent trends in the modelling of the deteriorated inventory. In their work they have presented a review of the advancement of the deteriorating inventory literature since the 1990. Tripathy and Mishra (2013) has also discussed the inventory model with deteriorating items considering the time dependent holding cost. In order to find the optimal solution to their problem they have used the technique of the truncated taylor series. Finally the proposed model has been verified with the numerical example and the sensitivity analysis. Yang and Wee (2000) later on, developed a model on economic ordering policy of deteriorated items for vendor and buyer. Through the results from the numerical example they suggested that integerated approach is better than the independent one. After that Yang and Wee (2002) formulated another model for the single vendor and multiple buyers production policy for the deteriorated items by taking the constant production rate and constant demand rate. This proposed model was verified with the help of numerical example , in which they considered two buyers. This model also concluded that the integrated approach is better than the independent approach . Furthermore they also suggested that if there are more than two buyers then the heuristic approach will serve the purpose. By considering the heuristic approach of Yang and Wee (2002), Ghiami and Williams (2015) developed a two echelon production inventory model for the deteriorating items by considering constant demand rate and constant rate of production. They referred to the physical inventory and echleon stock of vendor during the production and the non production periods. The model formulated by Ghiami and Williams (2015) resulted in the better optimal soluion than Yang and Wee (2002) as there is a relaxation of huge surplus. Rau et al.(2003) developed a single supplier, single buyer and single manufacturer model, in which he assumed the production rate to be significantly large than the demand, similar to which was considered by Yang and Wee (2002). Therefore they dropped the part of manufacturer's production period. Most of the times the demand of the item is considered to be constant but in realistic situation this will change in accordance with time. In order to show the effect of time Ouyang and Cheng (2005) developed a model for deteriorated items with exponentially decreasing demand. In this model the shortages were allowed and they were partially backorderd, and also the the backlogging rate was variable and it completely dependent on the waiting time for next replenishment with constant rate of deterioration in order to find the optimal solution of the problem. They also verified the convexity of the total cost function and later on the they justified their model with the help of numerical example and sensitivity analysis. In the future research Dash et al (2014) considered the total optimal cost of an inventory with the exponential declining demand rate with constant rate of deterioration and the time varying holding cost. The formulated model has been verified with the numerical example and sensivity analysis. This type of inventory model is useful in the industries where the demand rate depends upon the time varying holding cost.

1.2.1 Present Work

Based on the literature survey , an attempt has been made in the present work to study the model given by yang and wee (2000), in which the constant demand rate is relaced by the exponential declining demand rate. In Chapter 2, Economic Ordering policy of deteriorated item for Vendor and Buyer (Yang and Wee (2000)) has been considered in which constant demand rate is replaced by the exponential declining function of time and the same approach used by yand and wee (2000) is applied to obtain the optimal solution. A numerical example is presented to demonstrate the model and sensitivity analysis of various parameters is carried out.

In Chapter 3, the inventory model considered in chapter 2 is extended for single vendor and Multiple Buyers with Exponential Declining Demand rate. Numerical example is also mentioned in the support of the model.

Chapter 2

An Inventory Model For Deteriorating Items With Exponential Declining Demand Considering The Production And Non Production Periods For Single Vendor And Single Buyer

2.1 Introduction

In chapter 1, an inventory model for economic ordering policy of deteriorated item for vendor and buyer (Yang and Wee(2000)) with constant demand rate has been considered. Keeping this in view, in this chapter we developed an inventory model for the deteriorating items having a time-dependent exponential declining demand rate considering the production and non production periods for both the single vendor and single buyer.

In the competitive environment a buyer usually has the advantage to decide on the number of deliveries when an order is made. The optimal number of deliveries chosen by the buyers may not be the best for the vendor. If they both need to minimmize the overall integrated cost then the number of deliveries should be decided in the cooperation with the vendor. Supply chain coordination is the major issue in in supply chain management research. To overcome this situation the integration approach has been preffered. In this work an inventory model developed by yang and wee (2000) has been considered in which the exponential demand rate is considered. Justification of this model is also given through numerical example and sensitivity analysis.

2.1.1 Assumptions and Notations

We have considered the same assumptions and notations from the model as proposed by yang and wee (2000) in chapter 1 except the constant demand, which is replaced by exponential demand rate.
The demand rate in this model is deterministic and is an exponential declining function of time, given by :

$d(t) = K \exp(-\beta t)$ where $\beta > 0$,

K = constant

β = rate of change of demand rate

2.2 Formulation of the model

Following Yang and Wee (2000), we considered an inventory model for deteriorating items with exponential declining demand considering the production and non production period for single vendor and single buyer. The inventory differential equation are given as follows

$$\frac{dI_{t_1}}{dt_1} = (P - d(t_1)) - \gamma I_{t_1}; \quad 0 \leqslant t_1 \leqslant T_1 \tag{2.1}$$

$$\frac{dI_{t_2}}{dt_2} = -d(t_2) - \gamma I_{t_2}; \quad 0 \leqslant t_2 \leqslant T_2 \tag{2.2}$$

$$d(t_1) = K\exp(-\beta t_1); \quad 0 \leqslant t_1 \leqslant T_1$$

$$d(t_2) = K\exp(-\beta t_2); \quad 0 \leqslant t_1 \leqslant T_2$$

From Spiegel (1960) and the boundary condition $I_1(0) = I_2(T_2) = 0$

The inventory level that changes with time t_1 for the vendor during the production period is given as below

$$I_{t_1} = \frac{P}{\gamma} - \frac{K}{(\gamma-\beta)}\exp(-\beta t_1) + \left(\frac{K}{\gamma-\beta} - \frac{P}{\gamma}\right)\exp(-\gamma t_1) \tag{2.3}$$

The inventory level that changes with time t_2 for the vendor during the non production period is as follows

$$I_{t_2} = \frac{K}{\gamma-\beta}\left(\exp[\gamma(T_2 - t_2) - \beta T_2] - \exp(-\beta t_2)\right) \tag{2.4}$$

From the boundary condition $I_1(T_1) = I_2(0)$ and Taylor's series expansion and also assuming γ, β to be $<< 1$ and $\beta << \gamma$ and by using taylor series expansion we get the following expression

$$\frac{P}{\gamma} - \frac{K}{(\gamma-\beta)}\left(1 - \beta T_1 + \frac{(\beta T_1)^2}{2} + ...\right) + \left(\frac{K}{\gamma-\beta} - \frac{P}{\gamma}\right)\left(1 - \gamma T_1 + \frac{(\gamma T_1)^2}{2} + ...\right)$$

$$= \frac{K}{\gamma-\beta}\left(1 + (\gamma-\beta)T_2 + \frac{((\gamma-\beta)T_2)^2}{2} + ... - 1\right)$$

on neglecting higher order terms of T_1 and T_2 we get

$$\left(\frac{K}{\gamma-\beta}(\beta-\gamma)+P\right)T_1 = KT_2\left(1+\frac{(\gamma-\beta)T_2}{2}\right)$$

Following Misra (1975), and from above equation the production period in each cycle is given as follows

$$T_1 \approx \frac{K}{P-K}T_2\left(1+\frac{1}{2}(\gamma-\beta)T_2\right) \tag{2.5}$$

So the total time period i.e $T = T_1 + T_2$ is given by following relation

$$T \approx \frac{T_2}{P-K}\left(P+\frac{1}{2}K(\gamma-\beta)T_2\right) \tag{2.6}$$

Now for a buyer with n deliveries per order the differential equation is given as follows

$$\frac{dI_b(t)}{dt} = -K\exp(-\beta t) - \gamma I_b(t);\ \ 0 \leqslant t \leqslant \frac{T}{n} \tag{2.7}$$

On solving equation (2.7) the inventory level for the buyer is obtained as

$$I_b(t) = -\frac{K}{\gamma-\beta}\left(\exp(-\beta t)\right) + \frac{K}{\gamma-\beta}\left(\exp(\gamma-\beta)\frac{T}{n}\right)\exp(-\gamma t) \tag{2.8}$$

The maximum inventory level for the buyer is given as

$$I_b(t=0) = \frac{K}{\gamma-\beta}\left(\exp(\gamma-\beta)\frac{T}{n}-1\right) \tag{2.9}$$

Total demand during the cycle period $[0,T]$ is

$$\int_0^T d(t)dt = \int_0^T K\exp(-\beta t)dt = \frac{K}{\beta}\Big(1-\exp(-\beta T)\Big) \tag{2.10}$$

The total demand during the cycle period $[0,T/n]$ is given as

$$\int_0^{\frac{T}{n}} d(t)dt = \int_0^{\frac{T}{n}} K\exp(-\beta t)dt = \frac{K}{\beta}\Big(1-\exp(-\frac{\beta T}{n})\Big) \tag{2.11}$$

The total cost function TC_v for the vendor is (yang and wee (2000))

$$\begin{aligned} TC_v &\approx \frac{1}{T}\big(C_{sv}+nK_{0v}\big)+\frac{C_{c_v}}{T}\Bigg(\int_0^{T_1} I_{t_1}dt_1 \\ &+ \int_0^{T_2} I_{t_2}dt_2 - n\int_0^{\frac{T}{n}} I_b(t)dt\Bigg) \\ &+ C_v\frac{1}{T}\Bigg(PT_1-\int_0^{T} d(t)dt - n\Bigg(I_b(0)-\int_0^{\frac{T}{n}} d(t)dt\Bigg)\Bigg) \end{aligned} \tag{2.12}$$

With the help of taylor series expansion the value of the integrals with the exponential declining demand rate in the above equation are given below

$$\int_0^{T_1} I_{t_1}dt_1 = \frac{T_1^2}{2}(-K+P)+\frac{T_1^3}{6}\Big(-P\gamma+K(\gamma+\beta)\Big) \tag{2.13}$$

$$\int_0^{T_2} I_{t_2}dt_2 = \frac{KT_2^2}{2}\Big(1-\beta T_2\Big)+\frac{KT_2^3}{6}(\gamma+\beta) \tag{2.14}$$

$$\int_0^{\frac{T}{n}} I_b(t)dt = \frac{KT^2}{2n^3}(n-\beta T)+\frac{KT^3}{6n^3}(\gamma+\beta) \tag{2.15}$$

By using taylor series expansion in the equations (2.9), (2.10) and (2.11) the result obtained is below:

$$I_b(0) = \frac{KT}{n} + \frac{KT^2}{2n^2}(\gamma - \beta) \tag{2.16}$$

$$\int_0^T d(t)dt = KT\left(1 - \frac{\beta T}{2}\right) \tag{2.17}$$

$$\int_0^{\frac{T}{n}} d(t)dt = \frac{KT}{n}\left(1 - \frac{\beta T}{2n}\right) \tag{2.18}$$

using the equations (2.13) ,(2.14), (2.15), (2.16), (2.17) and (2.18) in the equation (2.12), the total cost function for the vendor with the exponential declining demand rate is as follows

$$\begin{aligned} TC_v &\approx \frac{1}{T}\big(C_{sv} + nK_{0v}\big) + \frac{C_{c_v}}{T}\left(\frac{T_1^2}{2}(-K+P)\right. \\ &+ \frac{T_1^3}{6}\big(-P\gamma + K(\gamma+\beta)\big) \\ &+ \frac{KT_2^2}{2}\big(1-\beta T_2\big) + \frac{KT_2^3}{6}(\gamma+\beta) \\ &- \left.\frac{KT^2}{2n^2}(n-\beta T) - \frac{KT^3}{6n^2}(\gamma+\beta)\right) \\ &+ \frac{C_v}{T}\left(PT_1 - KT\big(1-\frac{\beta T}{2}\big) - \frac{KT^2\gamma}{2n}\right) \end{aligned} \tag{2.19}$$

The total cost function for the buyer is given as (yang and wee (2000))

$$\begin{aligned} TC_b = \frac{1}{T}(C_{o_b} + nK_{o_b}) + C_{c_b}\frac{n}{T}\int_0^{\frac{T}{n}} I_b(t)dt \\ +C_b\frac{n}{T}\left(I_b(0) - \int_0^{\frac{T}{n}} d(t)dt\right) \end{aligned} \tag{2.20}$$

Using the equations (2.15), (2.16) and (2.18) in equation (2.20) the total cost function for the buyer is

$$\begin{aligned} TC_b \approx\ & \frac{1}{T}(C_{o_b}+nK_{o_b})+C_{c_b}\frac{n}{T}\left(\frac{KT^2}{2n^3}(n-\beta T)\right. \\ +\ & \left.\frac{KT^3}{6n^3}(\gamma+\beta)\right)+\frac{C_b}{T}\left(KT+\frac{K(\gamma-\beta)T^2}{2n}\right. \\ -\ & \left.KT\big(1-\frac{\beta T}{2n}\big)\right) \end{aligned} \tag{2.21}$$

Total cost function TC for the vendor and the buyer is the sum of the cost function of vendor and buyer as follows $TC = TC_b + TC_v$

By adding the equations (2.19) and (2.21) the total cost of the formulated inventory model is

$$\begin{aligned} TC \approx\ & \frac{1}{T}\big(C_{sv}+nK_{0v}\big)+\frac{C_{c_v}}{T}\left(\frac{T_1^2}{2}(-K+P)+\frac{T_1^3}{6}\big(-P\gamma+K(\gamma+\beta)\big)\right. \\ +\ & \left.\frac{KT_2^2}{2}\big(1-\beta T_2\big)+\frac{KT_2^3}{6}(\gamma+\beta)-\frac{KT^2}{2n^2}(n-\beta T)-\frac{KT^3}{6n^2}(\gamma+\beta)\right) \\ +\ & \frac{C_v}{T}\left(PT_1-KT\big(1-\frac{\beta T}{2}\big)-\frac{KT^2\gamma}{2n}\right)+\frac{1}{T}(C_{o_b}+nK_{o_b}) \\ +\ & C_{c_b}\frac{n}{T}\left(\frac{KT^2}{2n^3}(n-\beta T)+\frac{KT^3}{6n^3}(\gamma+\beta)\right) \\ +\ & \frac{C_b}{T}\left(KT+\frac{K(\gamma-\beta)T^2}{2n}-KT\big(1-\frac{\beta T}{2n}\big)\right) \end{aligned} \tag{2.22}$$

2.3 Solution Procedure

The solution procedure given by Yang and Wee (2000) has been used which is as follows

The aim is to minimize the total cost of both the buyer and the vendor. For that, the value of n is to be find to minimize the total cost. As number of deliveries per order is a discrete variable, so it is easy to find the value of n by the following method.

1. For a range of n-values, find the partial derivative of TC from equation (2.22) with respect to T_2 and equate it to zero. For each find the minimizing T_2 value by $T_2(n^*)$.
2. Obtain the optimal value of n, denoted by n^*, such that $TC(T_2(n^*-1), n^*-1) \geqslant TC(T2(n^*), n^*) \leqslant TC(T_2(n^*+1), n^*+1)$
3. By above condition, find the optimal value of T_1 and T from equation (2.5) and (2.6).
4. Find production quantity, PT_1.
5. .Find the delivery quantity $\frac{K}{\gamma-\beta}(\exp\frac{(\gamma-\beta)T}{n} - 1)$

2.4 Numerical Example

The above model is explained with th the help of the numerical example. The parameters that we took are given below

$P = 2 \times 10^6$ units per year
$K = 5 \times 10^5$ units per year
$\gamma = 0.1$ per year
$\beta = 0.08$ per year
$C_{ob} = 2000$ per order
$C_{sv} = 100000$ per cycle
$K_{0b} = 500$ per delivery
$K_{0v} = 1000$ per delivery
$C_{cb} = 60$ per unit per year
$C_{cv} = 40$ per unit per year
$C_b = 600$ per unit
$C_v = 400$ per unit

Using the solution procedure given above the example has been solved and results are given in below tables.

Table 2.1: Optimal solution on n

n	$T_1(10^{-4})$	$T_2(10^{-4})$	$T(10^{-4})$	$TC_v \times 10^6$	$TC_b \times 10^5$	$TC \times 10^6$
1	157	470	627	1.4004	19.189	3.3193
2	192	523	715	1.9243	10.888	3.0131
3	183	548	731	2.1368	7.7848	2.9153
4	188	564	752	2.2566	6.1716	2.8738
5	192	575	767	2.3369	5.1868	2.8556
6^*	195	584	779	2.3957	4.5354	2.8492^*
7	197	592	789	2.4417	4.0797	2.8496
8	200	599	799	2.4797	3.7463	2.8543
9	202	605	807	2.5124	3.4947	2.8619
14	211	632	843	2.6327	2.8738	2.9201
15	212	637	849	2.6521	2.8172	2.9338
17	215	646	861	2.6883	2.7390	2.9622
20	217	650	867	2.7380	2.6845	3.0065
21	221	663	884	2.7535	2.6768	3.0211
$22^{\#}$	222	667	889	2.7687	$2.6744^{\#}$	$3.0362^{\#}$
23	224	671	895	2.7836	2.6758	3.0512
26	228	683	911	2.8267	2.6978	3.0946

* integrated optimal solution on n which minimize TC.

$^{\#}$ buyer's optimal solution of n which minimizes TC_b

Percentage of integrated cost reduction $(PICR) = \frac{TC(n^{\#})-TC(n^{*})}{TC(n^{\#})} = 6.15\%$

Table 2.1.1: The vendor and Buyer's cost

Cost items	$n^{\#} = 22$	$n^{*} = 6$	cost \$
$TC_b \times 10^5$	2.6744	4.5354	+1.861
$TC_v \times 10^6$	2.7687	2.3957	-0.373
$TC \times 10^6$	3.0362	2.8492	-0.187

* integrated optimal solution on n which minimize TC.

$^{\#}$ buyer's optimal solution of n which minimizes TC_b

+ represent the increment and - represent the decrement in the cost.

2.4.1 Discussion on the numerical example

From table (2.1.1) it is clear that the buyer follows the integrated policy and agrees on 6 deliveries instead of his original optimal value 22 and will lead to an increase in the cost of $\$ + 1.861 \times 10^5$. On the flip side of the inventory the vendor will have a cost saving of $\$0.373 \times 10^5$. The percentage of integrated cost reduction is 6.15%,so here vendor is the winner. So it logical that vendor will provide some incentive to the buyer to accept the integrated policy of nine deliveries. In order to attract the buyers ,vendor will provide some discount to the buyers due to integrated approach so that he can continue his relation with the buyer in the future.

2.4.2 Sensitivity Analysis

Since we have formulated the model while considering the exponential declining demand and also we have determined the optimal value of n which minimizes the total cost. This optimal value which considers the perspective of both the vendor and buyer is denoted by n^{*} while the n which minimizes the total cost of the buyer is denoted by $n^{\#}$. In this model we have considered the pair of parameters as $Q = \{(C_{ob},C_{sv}),\ (K_{0b},K_{0v}),\ (C_{cb},C_{cv}),\ (C_b,C_v),\gamma,\beta,P,K)\}$ denoted by n^* and $TC(n^*)$,respectively.Finally the sensitivity analysis is given below

Table 2.2: Sensitivity analysis when K is changed by 10%

$K(10^5)$	3.5	4	4.5	5	5.5	6	6.5
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.5249	2.6279	2.7358	2.8492	2.9687	3.0950	3.2290
$n^{\#}$	22	22	22	22	22	22	22
$TCn^{\#}(10^6)$	2.6896	2.8001	2.9154	3.0362	3.1629	3.2965	3.4377
$PICR$	6.12%	6.14%	6.16%	6.15%	6.13%	6.11%	6.07%
$(if\beta = 0)PICR$	7.88%	5.75%	5.63%	5.36%	5.22%	4.96%	4.22%

Table 2.3: Sensitivity analysis when P is changed by 10%

$P(10^6)$	1.4	1.6	1.8	2.0	2.2	2.4	2.6
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.7575	2.7910	2.8219	2.8492	2.8731	2.8940	2.9124
$n^{\#}$	22	22	22	22	22	22	22
$TCn^{\#}(10^6)$	2.9314	2.9703	3.0054	3.0362	3.0629	3.0861	3.1064
$PICR$	5.93%	6.03%	6.10%	6.15%	6.19%	6.22%	6.24%
$(if\beta = 0)PICR$	4.36 %	4.86%	5.15%	5.36%	5.51%	5.63%	5.72%

Table 2.4: Sensitivity analysis when C_b and C_v are changed by 10%

C_b	420	480	540	600	660	720	780
C_v	280	320	360	400	440	480	520
n^*	6	6	6	6	6	6	
$TCn^*(10^6)$	2.6174	2.6947	2.7719	2.8492	2.9265	3.0038	3.0811
$n^{\#}$	22	22	22	22	22	22	22
$TCn^{\#}(10^6)$	2.7859	2.8693	2.9527	3.0362	3.1196	3.2030	3.2865
$PICR$	6.04%	6.08%	6.12%	6.15%	6.18%	6.21%	6.66%
$(if\beta = 0)PICR$	5.42%	5.40%	5.38%	5.36%	5.34%	5.32%	5.31%

Table 2.5: Sensitivity analysis when C_{cb} and C_{cv} are changed by 10%

C_{cb}	42	48	54	60	66	72	78
C_{cv}	28	32	36	40	44	48	52
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.6554	2.7200	2.7846	2.8492	2.9138	2.9785	3.0431
$n^\#$	22	22	22	22	22	22	22
$TCn^\#(10^6)$	2.8296	2.8984	2.9673	3.0362	3.1050	3.1739	3.2427
$PICR$	6.153%	6.155%	6.157%	6.159%	6.159%	6.159%	6.159%
$(if \beta = 0) PICR$	5.23%	5.27%	5.32%	5.36%	5.39%	5.43%	5.47%

Table 2.6: Sensitivity analysis when γ is changed by 10%

γ	0.07	0.08	0.09	0.10	0.11	0.12	0.13
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.6987	2.7489	2.7990	2.8492	2.8994	2.9496	2.9997
$n^\#$	22	22	22	22	22	22	22
$TCn^\#(10^6)$	2.8793	2.9316	2.9839	3.0362	3.0884	3.1407	3.1930
$PICR$	6.27%	6.23%	6.19%	6.15%	6.11%	6.08%	6.05%
$(if \beta = 0) PICR$	5.42%	5.40%	5.35%	5.36%	5.34%	5.32%	5.30%

Table 2.7: Sensitivity analysis where K_{ob} and K_{ov} are changed by 10%

K_{ob}	350	400	450	500	550	600	650
K_{ov}	700	800	900	1000	1100	1200	1300
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.8144	2.8260	2.8376	2.8492	2.8608	2.8724	2.8840
$n^\#$	22	22	22	22	22	22	22
$TCn^\#(10^6)$	2.9252	2.9622	2.9992	3.0362	3.0731	3.1101	3.1471
$PICR$	3.78%	4.59%	5.38%	6.15%	6.90%	7.64%	8.36%
$(if \beta = 0) PICR$	2.97%	3.78%	4.58%	5.36%	6.12%	6.85%	7.58%

Table 2.8: Sensitivity analysis where C_{ob} and C_{sv} are changed by 10%

C_{ob}	1400	1600	1800	2000	2200	2400	2600
C_{sv}	70 000	80 000	90 000	100 000	110 000	120 000	130 000
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.4550	2.5864	2.7178	2.8492	2.9781	3.1095	3.2409
$n^\#$	22	22	22	22	22	22	22
$TCn^\#(10^6)$	2.6932	2.8075	2.9218	3.0362	3.1505	3.2626	3.3769
$PICR$	8.84%	7.87%	6.98%	6.15%	5.47%	4.69%	4.027%
$(if \beta = 0) PICR$	8.02%	7.07%	6.18%	5.36%	4.60%	3.89%	3.24%

Table 2.9: Sensitivity analysis when β is changed by 10%

β	0.056	0.064	0.072	0.08	0.088	0.096	0.104
n^*	6	6	6	6	6	6	6
$TCn^*(10^6)$	2.7680	2.7951	2.8222	2.8492	2.8763	2.9033	2.9303
$n^\#$	22	22	22	22	22	22	22
$TCn^\#(10^6)$	2.9429	2.9740	3.0051	3.0362	3.0672	3.0983	3.1293
$PICR$	5.94%	6.015%	6.08%	6.15%	6.22%	6.29%	6.35%

where * represents the integrated optimal solution on n which minimize TC.

$^\#$ shows the optimal buyer's solution which minimizes cost of buyer.

In above tables, $PICR$ is percentage of integrated cost reduction=$[TC(n^\#)$-$TC(n^*)]/TC(n^\#)$, {}, base column.

2.4.3 Observations from the sensitivity Analysis

The observation drawn from analysis is as follows.

1. The range of *PICR* (percentage of integrated cost reduction) varies from 3.78% to 8.84% .Hence by changing the subsequent parameters we found large fluctuations in PICR while considering the exponential declining demand.
2. The average value of *PICR* is $\sim$6.15
3. The values of *PICR* with the constant n^* changes more frequently by changing the parameter of subset (K_{0b}, K_{0v}) and (C_{ob}, C_{sv}). It keeps on increasing with respect to (K_{0b}, K_{0v}) and decreases with the increase in values of (C_{ob}, C_{sv})
4. It is less sensitive to the parameters of subset (C_{cb}, C_{cv}) ,K and γ
5. The value of *PICR* is less sensitive to the subset of parameter (C_b, C_v) , and it keeps on increasing with the increase in its values.
6. The value of *PICR* increases minutely with the increasing the value of γ and vice versa is also true.
7. The value of *PICR* increases as the value of (C_{cb}, C_{cv}) increases.
8. Production rate is also directly proportional to the values *PICR*, as the production rate increases *PICR* also increases.
9. *PICR* increases by increase in the values of the parameter β.
10. With the increase in the values of parameter K, *PICR* also increases.

2.5 Conclusion

In the proposed model, an inventory model is developed which investigates the optimal order quantity of the on-hand inventory due to an exponential declining demand rate. The items like food grains, fashion apparels and electronic equipments etc. have fixed shelf-life which decreases with time during the end of the season and the storage period in which the demand, deterioration, and holding cost depend upon the time and shortage is not allowed. This model is solved analytically by minimizing the total inventory cost. Finally, the proposed model has been verified by the numerical example along with sensitivity analysis and it has been observed that integrated approach is better than the independent one.

Chapter 3

An Inventory Model For Deteriorating Items With Exponential Declining Demand Considering The Production And Non Production Periods For Single Vendor And Multiple Buyers

3.1 Introduction

In chapter 2, we developed an inventory model for the deteriorating items having a time-dependent exponential declining demand rate considering the production and non production periods for both the single vendor and single buyer by considering the model of yang and wee (2000). Keeping this in view, the inventory model considered in chapter 2 is extended to develop an inventory model for deteriorating items with exponential declining demand for single vendor and multiple buyers. Further the model is supported with numerical example and sensitivity analysis.

3.1.1 Assumptions

1. The inventory system consists of single vendor and N_1 buyers.
2. The demand rate is declining exponentially
3. Constant production rate .
4. Shortages are not allowed.
5. Rate of deterioration is considered to be constant and is applied for single item.
6. Consideration of deteriorated units is done only after they have been received into inventory.
7. Deteriorated units can't be replaced or repaired.
8. The cost of carrying is applied to the good units only.
9. Single vendor and single buyer are considered.
10. Multiple deliveries per order are considered.
11. There is only one production cycle per order.

3.1.2 Notations

Following notations are used in the formulation of the model

N_1 = Number of buyers in the inventory system.

$d_i(t)$ = Anual demand rate for the i_{th} buyer, where $d_i(t) = K_i exp(-\beta_i t)$

K_i = Constant demand, for i buyers.

β_i = Rate of change of demand rate for i buyers

n_i = Number of orders during the cycle time T for the i_{t_h} buyer

$I_{bi}(t)$ = Inventory level for the i_{th} buyer at any time t , $0 \leqslant t \leqslant \frac{T}{n_i}$

P = production rate

I_{t_1} = Change in the inventory level with time t_1 during production period

I_{t_2} = Change in the inventory level with time t_2 during non-production period

T_1 = the production period in each cycle

T_2 = the non production period in each cycle

T = Time length of cycle

γ= Constant rate of deterioration

n = Number of deliveries per order

$I_{p_c}(t)$ = Inventory level of the vendor

$I_b(t)$ = The buyer's inventory level

C_{o_b} = Cost of ordering for the buyer, per order

C_{s_v} = Cost of setup for the vendor, per production cycle

C_{c_b} = Carrying of the inventory for the buyer, per time and per unit

C_{c_v} =Carrying cost of the inventory for the vendor, per time and per unit

C_b = cost of deteriorated unit for the buyer

C_v = cost of deteriorated unit for the buyer

K_{o_b} = The incoming control cost for the buyer, per delivery

K_{o_v} = The transportation charge for the vendor, per delivery

TC_b = Buyer's total cost function who buys goods from the vendor

TC_v = Vendor's total cost function

TC = The integrated total cost function including TC_v and TC_b

3.2 Formulation of the model

Following Yang and Wee (2000) the differential equations for single vendor and multiple buyers are given below

$$\frac{dI_{t_1}}{dt_1} = (P - \sum_{i=1}^{N_1} d_i(t_1)) - \gamma I_{t_1} \quad 0 \leqslant t_1 \leqslant T_1 \tag{3.1}$$

$$\frac{dI_{t_2}}{dt_2} = -\sum_{i=1}^{N_1} d_i(t_2) - \gamma I_{t_2} \quad 0 \leqslant t_2 \leqslant T_2 \tag{3.2}$$

$$\sum_{i=1}^{N_1} d_i(t_1) = \sum_{i=1}^{N_1} K_i \exp(-\beta_i t_1); \quad 0 \leqslant t_1 \leqslant T_1$$

$$\sum_{i=1}^{N_1} d_i(t_2) = \sum_{i=1}^{N_1} K_i \exp(-\beta_i t_2); \quad 0 \leqslant t_1 \leqslant T_2$$

From Spiegel (1960) and the boundary condition $I_1(0) = I_2(T_2) = 0$

The inventory level that changes with time t_1 for the vendor during the production period is given as below:

$$I_{t_1} = \frac{P}{\gamma} - \sum_{i=1}^{N_1} \frac{K_i}{(\gamma - \beta_i)} \exp(-\beta_i t_1) + \sum_{i=1}^{N_1} \left(\frac{K_i}{\gamma - \beta_i} - \frac{P}{\gamma} \right) \exp(-\gamma t_1) \tag{3.3}$$

$$I_{t_2} = \sum_{i=1}^{N_1} \frac{K_i}{\gamma - \beta_i} \left(\exp[\gamma(T_2 - t_2) - \beta_i T_2] - \exp(-\sum_{i=1}^{N_1} \beta_i t_2) \right) \tag{3.4}$$

From the boundary condition $I_1(T_1) = I_2(0)$ and Taylor's series expansion and also assuming γ, β to be $<< 1$ and $\beta << \gamma$ and by using taylor series expansion we get the following expression

$$\frac{P}{\gamma} - \sum_{i=1}^{N_1} \frac{K_i}{(\gamma - \beta_i)} \left(1 - \beta_i T_1 + \frac{(\beta_i T_1)^2}{2} + \ldots \right) + \sum_{i=1}^{N_1} \left(\frac{K_i}{\gamma - \beta_i} - \frac{P}{\gamma} \right) \left(1 - \gamma T_1 + \frac{(\gamma T_1)^2}{2} + \ldots \right)$$
$$= \sum_{i=1}^{N_1} \frac{K_i}{\gamma - \beta_i} \left(1 + (\gamma - \beta_i) T_2 + \frac{((\gamma - \beta_i) T_2)^2}{2} + \ldots - 1 \right)$$

on neglecting higher order terms of T_1 and T_2 we get

$$\left(\frac{\sum_{i=1}^{N_1} K_i}{\gamma - \sum_{i=1}^{N_1} \beta_i} (\sum_{i=1}^{N_1} \beta_i - \gamma) + P \right) T_1 = \sum_{i=1}^{N_1} K_i T_2 \left(1 + \frac{(\gamma - \sum_{i=1}^{N_1} \beta_i) T_2}{2} \right)$$

Following Misra (1975), and from above equation the production period in each cycle is

$$T_1 \approx \frac{\sum_{i=1}^{N_1} K_i}{P - \sum_{i=1}^{N_1} K_i} T_2 \left(1 + \frac{1}{2}(\gamma - \sum_{i=1}^{N_1} \beta_i) T_2\right) \tag{3.5}$$

So the total time period i.e $T = T_1 + T_2$ is given by following relation

$$T \approx \frac{T_2}{P - \sum_{i=1}^{N_1} K_i} \left(P + \frac{1}{2}\sum_{i=1}^{N_1} K_i (\gamma - \sum_{i=1}^{N_1} \beta_i) T_2\right) \tag{3.6}$$

The differential equation for the i_{th} buyer for n deliveries per order is given as

$$\frac{dI_{bi}(t)}{dt} = -\sum_{i=1}^{N_1} K_i \exp(-\beta_i t) - \gamma \sum_{i=1}^{N_1} I_{bi}(t) \quad 0 \leqslant t_1 \leqslant \frac{T}{n_i} \tag{3.7}$$

On solving the above differential equation the final equation is given as below

$$I_{bi}(t) = -\sum_{i=1}^{N_1} \frac{K_i}{\gamma - \beta_i}\left(\exp(-\beta_i t)\right) + \sum_{i=1}^{N_1} \frac{K_i}{\gamma - \beta_i}\left(\exp(\gamma - \beta_i)\frac{T}{n_i}\right)\exp(-\gamma t) \tag{3.8}$$

At time t = 0 the maximum invetory level of the buyer is

$$I_{bi}(0) = \sum_{i=1}^{N_1} \frac{K_i}{\gamma - \beta_i}\left(\exp(\gamma - \beta_i)\frac{T}{n_i} - 1\right) \tag{3.9}$$

The total demand during the cycle period [0 ,T] is calculated below

$$\int_0^T d_i(t)dt = \int_0^T K_i \exp(-\beta_i t)dt = \sum_{i=1}^{N_1} \frac{K_i}{\beta_i}\Big(1 - \exp(-\beta_i T)\Big) \tag{3.10}$$

The total demand during the cycle period $[0, T/n_i]$ is given as

$$\int_0^{\frac{T}{n_i}} d_i(t)dt = \int_0^{\frac{T}{n_i}} K_i \exp(-\beta_i t)dt = \sum_{i=1}^{N_1} \frac{K_i}{\beta_i}\Big(1 - \exp(-\frac{\beta_i T}{n_i})\Big) \tag{3.11}$$

The total cost function of the vendor for the N_1 buyers is;

$$\begin{aligned} TC_v \; = \; & \frac{1}{T}\big(C_{sv} + \sum_{i=1}^{N_1} n_i K_{0v}\big) + \sum_{i=1}^{N_1} \frac{C_{c_v}}{T}\Big(\int_0^{T_1} I_{t_1} dt_1 \\ + \; & \int_0^{T_2} I_{t_2} dt_2 - n\int_0^{\frac{T}{n_i}} I_{b_i}(t)dt\Big) + \frac{C_v}{T}\Big(PT_1 \\ - \; & \sum_{i=1}^{N_1}\int_0^T d_i(t)dt - \sum_{i=1}^{N_1} n_i\Big(I_{b_i}(0) - \int_0^{\frac{T}{n_i}} d_i(t)dt\Big)\Big) \end{aligned} \tag{3.12}$$

With the help of taylor series expansion the value of the integrals with the exponential declining demand rate in the above equation are given below

$$\int_0^{T_1} I_{t_1} dt_1 = \frac{T_1^2}{2}(-\sum_{i=1}^{N_1} K_i + P) + \frac{T_1^3}{6}[-P\gamma + \sum_{i=1}^{N_1} K_i(\gamma + \sum_{i=1}^{N_1} \beta_i)] \tag{3.13}$$

$$\int_0^{T_2} I_{t_2} dt_2 = \frac{\sum_{i=1}^{N_1} K_i T_2^2}{2}\Big(1 - \sum_{i=1}^{N_1} \beta_i T_2\Big) + \frac{\sum_{i=1}^{N_1} K_i T_2^3}{6}(\gamma + \sum_{i=1}^{N_1} \beta_i) \tag{3.14}$$

$$\begin{aligned} \int_0^{\frac{T}{n_i}} I_{b_i}(t)dt = & \frac{\sum_{i=1}^{N_1} K_i T^2}{2\sum_{i=1}^{N_1} n_i^3}(\sum_{i=1}^{N_1} n_i - \sum_{i=1}^{N_1} \beta_i T) \\ & + \frac{\sum_{i=1}^{N_1} K_i T^3}{6\sum_{i=1}^{N_1} n_i^3}(\gamma + \sum_{i=1}^{N_1} \beta_i) \end{aligned} \tag{3.15}$$

By using taylor series expansion in the equations (3.9), (3.10) and (3.11) the result obtained is below:

$$I_{b_i}(0)=\frac{\sum_{i=1}^{N_1}K_iT}{\sum_{i=1}^{N_1}n_i}+\frac{\sum_{i=1}^{N_1}K_iT^2}{2\sum_{i=1}^{N_1}n_i^2}(\gamma-\sum_{i=1}^{N_1}\beta_i) \tag{3.16}$$

$$\int_0^T d_i(t)=\sum_{i=1}^{N_1}K_iT\left(1-\frac{\sum_{i=1}^{N_1}\beta_iT}{2}\right) \tag{3.17}$$

$$\int_0^{\frac{T}{n_i}} d_i(t)dt=\sum_{i=1}^{N_1}\frac{K_iT}{n_i}\left(1-\frac{\sum_{i=1}^{N_1}\beta_iT}{2\sum_{i=1}^{N_1}n_i}\right) \tag{3.18}$$

using the equations (3.13), (3.14), (3.15), (3.16), (3.17) and (3.18) in the equation (3.12), we get the total cost function for the vendor with the exponential declinind demand rate is

$$\begin{aligned}
TC_v \;\approx\;& \frac{1}{T}\big(C_{sv}+\sum_{i=1}^{N_1}n_iK_{ov}\big)+\frac{C_{c_v}}{T}\bigg(\frac{T_1^2}{2}(-\sum_{i=1}^{N_1}K_i+P)\\
+\;& \frac{T_1^3}{6}\big(-P\gamma+\sum_{i=1}^{N_1}K_i(\gamma+\sum_{i=1}^{N_1}\beta_i)\big)\\
+\;& \frac{\sum_{i=1}^{N_1}K_iT_2^2}{2}\big(1-\sum_{i=1}^{N_1}\beta_iT_2\big)+\frac{\sum_{i=1}^{N_1}K_iT_2^3}{6}(\gamma+\sum_{i=1}^{N_1}\beta_i)\\
-\;& \frac{\sum_{i=1}^{N_1}K_iT^2}{2\sum_{i=1}^{N_1}n_i^2}(\sum_{i=1}^{N_1}n_i-\sum_{i=1}^{N_1}\beta_iT)-\frac{\sum_{i=1}^{N_1}K_iT^3}{6\sum_{i=1}^{N_1}n_i^2}(\gamma+\sum_{i=1}^{N_1}\beta_i)\bigg)\\
+\;& \frac{C_v}{T}\bigg(PT_1-\sum_{i=1}^{N_1}K_iT\big(1-\frac{\sum_{i=1}^{N_1}\beta_iT}{2}\big)-\frac{\sum_{i=1}^{N_1}K_iT^2\gamma}{2\sum_{i=1}^{N_1}n_i}\bigg)
\end{aligned} \tag{3.19}$$

The total cost function for the N_1 buyers is given as follows

$$\begin{aligned}
TC_b=\frac{1}{T}(C_{o_b}+\sum_{i=1}^{N_1}n_iK_{o_b})+C_{c_b}\sum_{i=1}^{N_1}\frac{n_i}{T}\int_0^{\frac{T}{n_i}}I_{b_i}(t)dt\\
+C_b\sum_{i=1}^{N_1}\frac{n_i}{T}\left(I_{b_i}(0)-\int_0^{\frac{T}{n_i}}d_i(t)dt\right)
\end{aligned} \tag{3.20}$$

Substituting the values from the equations (3.15), (3.16) and (3.18) in the equation (3.20) the total cost TC_b with the exponential declinind demand rate is

$$\begin{aligned} TC_b &\approx \frac{1}{T}(C_{o_b}+\sum_{i=1}^{N_1} n_i K_{o_b})+C_{c_b}\frac{\sum_{i=1}^{N_1} n_i}{T}\Bigg(\frac{\sum_{i=1}^{N_1} K_i T^2}{2\sum_{i=1}^{N_1} n_i^3}(\sum_{i=1}^{N_1} n_i-\beta_i T) \\ &+ \frac{\sum_{i=1}^{N_1} K_i T^3}{6\sum_{i=1}^{N_1} n_i^3}(\gamma+\beta_i\Bigg)+\frac{C_b}{T}\Bigg(\sum_{i=1}^{N_1} K_i T \\ &+ \frac{\sum_{i=1}^{N_1} K_i(\gamma-\beta_i)T^2}{2\sum_{i=1}^{N_1} n_i}-\sum_{i=1}^{N_1} K_i T\big(1-\frac{\sum_{i=1}^{N_1}\beta_i T}{2\sum_{i=1}^{N_1} n_i})\Bigg) \end{aligned} \tag{3.21}$$

Total cost function TC for the vendor and the multiple buyers is the sum of the cost function of vendor and buyer as follows $TC = TC_b + TC_v$

By adding the equations (3.19) and (3.21) the total cost of the formulated inventory model is

$$\begin{aligned} TC \approx{}& \frac{1}{T}\big(C_{sv}+\sum_{i=1}^{N_1} n_i K_{0v}\big)+\frac{C_{c_v}}{T}\Bigg(\frac{T_1^2}{2}(-\sum_{i=1}^{N_1} K_i+P)+\frac{T_1^3}{6}\big(-P\gamma+\sum_{i=1}^{N_1} K_i(\gamma+\sum_{i=1}^{N_1}\beta_i)\big) \\ &+\frac{\sum_{i=1}^{N_1} K_i T_2^2}{2}\big(1-\sum_{i=1}^{N_1}\beta_i T_2\big)+\frac{\sum_{i=1}^{N_1} K_i T_2^3}{6}(\gamma+\sum_{i=1}^{N_1}\beta_i)-\frac{\sum_{i=1}^{N_1} K_i T^2}{2\sum_{i=1}^{N_1} n_i^2}(\sum_{i=1}^{N_1} n_i-\sum_{i=1}^{N_1}\beta_i T) \\ &-\frac{\sum_{i=1}^{N_1} K_i T^3}{6\sum_{i=1}^{N_1} n_i^2}(\gamma+\sum_{i=1}^{N_1}\beta_i)\Bigg)+\frac{C_v}{T}\Bigg(PT_1-\sum_{i=1}^{N_1} K_i T\big(1-\frac{\sum_{i=1}^{N_1}\beta_i T}{2}\big)-\frac{\sum_{i=1}^{N_1} K_i T^2\gamma}{2\sum_{i=1}^{N_1} n_i}\Bigg) \\ &+\frac{1}{T}(C_{o_b}+\sum_{i=1}^{N_1} n_i K_{o_b})+C_{c_b}\frac{\sum_{i=1}^{N_1} n_i}{T}\Bigg(\frac{\sum_{i=1}^{N_1} K_i T^2}{2\sum_{i=1}^{N_1} n_i^3}(\sum_{i=1}^{N_1} n_i-\sum_{i=1}^{N_1}\beta_i T) \\ &+\frac{\sum_{i=1}^{N_1} K_i T^3}{6\sum_{i=1}^{N_1} n_i^3}(\gamma+\sum_{i=1}^{N_1}\beta_i)\Bigg)+\frac{C_b}{T}\Bigg[\sum_{i=1}^{N_1} K_i T+\frac{\sum_{i=1}^{N_1} K_i(\gamma-\sum_{i=1}^{N_1}\beta_i)T^2}{2\sum_{i=1}^{N_1} n_i} \\ &-\sum_{i=1}^{N_1} K_i T\big(1-\frac{\sum_{i=1}^{N_1}\beta_i T}{2\sum_{i=1}^{N_1} n_i})\Bigg] \end{aligned} \tag{3.22}$$

3.3 Numerical Example

We have considered two buyers n_1 and n_2 and rest of the parameters are shown below

$P = 2 \times 10^6$ units per year
$K_1 = 5 \times 10^5$ units per year
$K_2 = 4 \times 10^5$ units per year
$\gamma = 0.1$ per year
$\beta_1 = 0.02$ per year
$\beta_2 = 0.02$ per year
$C_{ob} = 2000$ per order
$C_{sv} = 100000$ per cycle
$K_{0b} = 500$ per delivery
$K_{0v} = 1000$ per delivery
$C_{cb} = 60$ per unit per year
$C_{cv} = 40$ per unit per year
$C_b = 600$ per unit
$C_v = 400$ per unit

The above example is solved by using the solution procedure mentioned in the chapter 2 and final results are shown in the below table

Table 3.1: optimal solution of n

n_1	n_2	$T_1(10^{-4})$	$T_2(10^{-4})$	$T(10^{-4})$	$TC_v \times 10^6$	$TC_b \times 10^5$	$TC \times 10^6$
1	1	187	228	415	2.1845	28.717	5.0562
1	2	233	285	518	2.1716	19.339	4.1055
1	3	266	325	591	2.2713	13.789	3.6502
1	7	285	348	633	2.6703	6.2643	3.2968
1	8	318	388	706	2.6644	6.0476	3.2692
1^*	9^*	323	394	717	2.7098	5.5629	3.2641^*
1	10	327	399	726	2.7461	5.1840	3.2645
1	11	331	404	735	2.7800	4.8875	3.2688
1	22	361	441	802	3.0354	3.7006	3.4054
1	28	374	457	831	3.1375	3.6186	3.4994
$1^\#$	$29^\#$	377	460	837	3.1535	$3.6184^\#$	$3.5154^\#$
1	30	379	462	841	3.1691	3.6218	3.5313
2	2	233	284	517	2.3678	18.210	4.1887
2	10	326	398	724	2.7622	5.5268	3.3148
2	20	357	436	793	3.0105	3.9068	3.4012
3	3	258	315	573	2.5010	13.765	3.8776
3	4	270	330	600	2.5479	11.959	3.7438
3	15	345	421	766	2.9175	4.5546	3.7330
4	4	345	421	766	2.4526	13.708	3.8234
4	9	318	388	706	2.7701	6.7317	3.4433
4	15	345	421	766	2.9320	4.7607	3.4080
5	5	288	352	640	2.6760	9.7376	3.6497

* integrated optimal solution on n which minimize TC.

$^\#$ buyer's optimal solution of n which minimizes TC_b

Percentage of integrated cost reduction ($PICR$) = $\frac{[TC(n_1^\#,n_2^\#)-TC(n_1^*,n_2^*)]}{TC(n_1^\#,n_2^\#)} = 7.14\%$

Table 3.1.1: The vendor and Buyer's cost

Cost items	$(n_1^{\#}, n_2^{\#}) = (1,29)$	$(n_1^{*}, n_2^{*}) = (1,9)$	cost $
$TC_b \times 10^5$	3.6184	5.5629	+1.9445
$TC_v \times 10^6$	3.1535	2.7098	-0.4437
$TC \times 10^6$	3.5154	3.2641	-0.2513

3.3.1 Discussion on the numerical example

In the table 3.1 , the optimal solution is generated for the integrated as well as independent inventory policy. If both the vendor and multiple buyers follow the integrated policy instead of independent policy,then order is made on $(n_1 = 1, n_2 = 9)$ instead of $(n_1 = 1, n_2 = 29)$. From the table (3.1.1) we can see that optimum cost for both the integrated and independent policy is $ 3.2641×10^6 and $ 3.5154×10^6 respectively. If both follow integrated policy then vendor gets the benefit of $ 0.4437×10^6 and buyer has the loss of $ 1.9445×10^5. So in order to continue his partnership with both the buyers the vendor will provide some amount of discount to them so that they can work together as a unit rather than independently.

3.3.2 Sensitivity Analysis

The sensitivity analysis is calculated by increasing or decreasing the assumed parameters by 10%

Table 3.2: Sensitivity analysis when C_b is changed by 10%

C_b	420	480	540	600	660	720	780
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2064	3.2256	3.2449	3.2641	3.2834	3.3026	3.3219
$(n_1^{\#}, n_2^{\#})$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^{\#}(10^6)$	3.4926	3.5002	3.5078	3.5154	3.5230	3.5305	3.5381
$PICR$	8.19%	7.84%	7.49%	7.14%	6.80%	6.40%	6.11%

Table 3.3: Sensitivity analysis when C_v is changed by 10%

C_v	280	320	360	400	440	480	520
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.0788	3.1406	3.2023	3.2641	3.3259	3.3877	3.4495
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.2661	3.3492	3.4323	3.5154	3.5985	3.6815	3.7646
$PICR$	5.73%	6.22%	6.70%	7.14%	7.57%	7.98%	8.37%

Table 3.4: Sensitivity analysis when C_{c_b} is changed by 10%

C_{c_b}	42	48	54	60	66	72	78
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.1850	3.2114	3.2377	3.2641	3.2905	3.3169	3.3432
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4902	3.4986	3.5070	3.5154	3.5238	3.5322	3.5406
$PICR$	8.74%	8.20%	7.67%	7.14%	6.62%	6.09%	5.57%

Table 3.5: Sensitivity analysis when C_{c_v} is changed by 10%

C_{c_v}	28	32	36	40	44	48	52
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.0993	3.1543	3.2092	3.2641	3.3191	3.3740	3.4289
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.2815	3.3594	3.4374	3.5154	3.5934	3.6713	3.7493
$PICR$	5.55%	6.10%	6.63%	7.14%	7.63%	8.09%	8.54%

Table 3.6: Sensitivity analysis when γ is changed by 10%

γ	0.07	0.08	0.09	0.10	0.11	0.12	0.13
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.1285	3.1737	3.2189	3.2641	3.3093	3.3545	3.3996
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.3704	3.4188	3.4671	3.5154	3.5637	3.6120	3.6603
$PICR$	7.17%	7.16%	7.15%	7.14%	7.13%	7.12%	7.12%

Table 3.7: Sensitivity analysis when P is changed by 10%

$P(10^6)$	1.4	1.6	1.8	2.0	2.2	2.4	2.6
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	2.9770	3.0569	3.1622	3.2641	3.3561	3.4376	3.5093
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.1644	3.2722	3.3978	3.5154	3.6199	3.7116	3.7920
$PICR$	5.92%	6.57%	6.93%	7.14%	7.28%	7.38%	7.45%

Table 3.8: Sensitivity analysis when K_{o_b} is changed by 10%

K_{o_b}	350	400	450	500	550	600	650
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2431	3.2501	3.2571	3.2641	3.2711	3.2781	3.2852
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4620	3.4798	3.4976	3.5154	3.5332	3.5510	3.5688
$PICR$	6.32%	6.60%	6.87%	7.14%	7.41%	7.68%	7.94%

Table 3.9: Sensitivity analysis when K_{o_v} is changed by 10%

K_{o_v}	700	800	900	1000	1100	1200	1300
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2221	3.2361	3.2501	3.2641	3.2781	3.2922	3.3062
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4085	3.4442	3.4798	3.5154	3.5510	3.5866	3.6222
$PICR$	5.46%	6.04%	6.60%	7.14%	7.68%	8.20%	8.72%

Table 3.10: Sensitivity analysis when C_{o_b} is changed by 10%

C_{o_b}	1400	1600	1800	2000	2200	2400	2600
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2557	3.2585	3.2613	3.2641	3.2669	3.2697	3.2725
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.5083	3.5106	3.5130	3.5154	3.5178	3.5201	3.5225
$PICR$	7.20%	7.18%	7.16%	7.14%	7.13%	7.11%	7.09%

Table 3.11: Sensitivity analysis when C_{s_v} is changed by 10%

C_{s_v}	70000	80000	90000	100000	110000	120000	1300000
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	2.8434	2.9837	3.1239	3.2641	3.4044	3.5446	3.6848
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.1592	3.2780	3.3967	3.5154	3.6341	3.7528	3.8715
$PICR$	9.99%	8.97%	8.03%	7.14%	6.32%	5.54%	4.82%

Table 3.12: Sensitivity analysis when K_1 is changed by 10%

K_1	350000	400000	450000	500000	550000	600000	650000
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.1754	3.2011	3.2304	3.2641	3.3029	3.3479	3.4000
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4236	3.4507	3.4811	3.5154	3.5544	3.5991	3.6505
$PICR$	7.24%	7.23%	7.20%	7.14%	7.07%	6.97%	6.86%

Table 3.13: Sensitivity analysis when K_2 is changed by 10%

K_2	280000	320000	360000	400000	440000	480000	520000
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.1904	3.2123	3.2368	3.2641	3.2947	3.3291	3.3678
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4395	3.4624	3.4876	3.5154	3.5462	3.5805	3.6188
$PICR$	7.24%	7.22%	7.19%	7.14%	7.09%	7.02%	6.93%

Table 3.14: Sensitivity analysis when β_1 is changed by 10%

β_1	0.014	0.016	0.018	0.02	0.022	0.024	0.026
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2104	3.2283	3.2462	3.2641	3.2820	3.2999	3.3118
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4519	3.4731	3.4942	3.5154	3.5365	3.5577	3.5788
$PICR$	6.99%	7.04%	7.09%	7.14%	7.16%	7.19%	7.46%

Table 3.15: Sensitivity analysis when β_2 is changed by 10%

β_2	0.014	0.016	0.018	0.02	0.022	0.024	0.026
(n_1^*, n_2^*)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)	(1,9)
$TCn^*(10^6)$	3.2104	3.2283	3.2462	3.2641	3.2820	3.2999	3.3118
$(n_1^\#, n_2^\#)$	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)	(1,29)
$TCn^\#(10^6)$	3.4519	3.4731	3.4942	3.5154	3.5365	3.5577	3.5788
PICR	6.99%	7.04%	7.09%	7.14%	7.16%	7.19%	7.46%

3.3.3 Observations from the sensitivity Analysis

1. While considering the example with single vendor and two buyers it was found that the range of *PICR* (percentage of integrated cost reduction) varies from 4.82% to 9.99% .
2. The average value of *PICR* is ∼7.14
3. *PICR* keeps on decreasing with the increse in the values of parameters C_{c_b} and C_b. Huge fluctuations are noted in *PICR* with the 10% change in the values of C_{c_b}.
4. *PICR* is less sensitive with the change in parameters γ, C_{o_b}, β_1, β_2.
5. With the increase in the values of the parameters K_{o_b} , C_{c_v} and K_{o_v} *PICR* also increases.
6. There is little decrease in the values of *PICR* with the increase in the values of parameter K_1 and K_2.
7. Huge fluctuations are recorded in *PICR* while considering the change in the parameters C_{s_v} . It keeps on decresing with the increase in the value of C_{s_v} .
8. There is decrease in the values of *PICR* with the increase in the values of parameter *P*.

3.4 Conclusion

In this work, an inventory model for the single vendor and multiple buyers with the exponential declining demand rate is considerd Finally with the help of numerical example we have concluded that the integerated approach is better than the independent one for the smooth functioning of the business.

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