A Thesis
On

Spectral Anomalies and Spectral Switching With Aperture-Lattice

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CERTIFICATE

This is to certify that the report entitled " **Spectral Anomalies and Spectral Switching with Aperture - Lattice**" submitted by **Mr. Dimple Singla (301004002)** of M.Sc (Physics), Thapar University, Patiala was carried out by him under my Supervision. I certify that the matter reported in the thesis is of candidate's own record and not submitted to any other university in any part or full form for the award of such degree.

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Dimple Singla

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CHAPTER – 1

1 . Introduction:

The phenomenon of diffraction-induced anomalous spectral behaviour attracted significant attention for both theoretical [1-6] and experimental investigations [7-12]. When light falls on an aperture we get some points of zero intensity in the diffracted intensity pattern. We can not determine phase at those points. These points are called phase singularity [1, 2]. At the neighbourhood of the phase singular points light shows different interesting behaviour, e.g., wave front dislocation, spectral anomaly etc.

The diffraction-induced spectral shift of light and hence spectral switching can be achieved with fully or partially coherent light, with different pulses and beams and for different aperture. It can be achieved with both monochromatic and polychromatic light. The research with polychromatic light may be divided into two major catogeries, i.e., fully coherent and partially coherent. In a landmark paper, Gbur et.al, have showed in 2002 that spectral switching is possible for spatially fully coherent polychromatic light [8,9]. They theoretically showed that spectral changes takes place in the neighborhood of phase singularities near the focus of a covering spatially fully coherent polychromatic wave diffracted through an aperture. Their prediction for spectral switching has been experimentally verified by Popescu et.al in the same year [10]. These spectral changes are proved to be the characteristic feature of polychromatic field near phase singularity [16]. A part from this fully coherent light has also been investigated [17-24]. Spectral shifts of partially coherent light are induced by coherence and diffractioin Pu et.al have shown that rapid spectral change i.e., spectral switching may occur for partially coherent light and these predictions are experimentally verified by Kandpal et.al.[19,20].

Results of investigations as explained above have generated renewed interest in the field. The polarization singularity as well as the Lissajous a singularity has also been investigated for two-colour vectyor field [25]. Earlier, Palma et. Al. have widely studied the spectral shifts for Gaussian-Schell model (GSM) beams, which are partially coherent in nature, due to propagation through both free space and an aperture lens [26,27]. Recently, Lu et.al. have shown that the spectral switching for GSM beams propagating through an aperture lens in dependent on the truncation parameter, coherence parameter and Fresnel number [28]. Similar investigation with a multimode laser (using Hermite-Gaussian beams) instead of a fundamental mode laser also shows spectral switching [29]. The diffraction of GSM beam through an astigmatic lens has also showed spectral switching in the vicinity of the intensity minimum in the geometric focal plane [30].

The Fraunhofer diffraction of spatially coherent polychromatic beam through single slit also exhibits drastic spectral change near the dark lines of the diffraction pattern. This has been proved both theoretically and experimentally [31]. Young's double slit experiment for both partial coherent [32] and fully coherent light [33] are other notable investigations along this direction. Both investigations confirm spectral switching at the near zone of dark fringes. Though most investigations on the anomalous spectral behaviour have been carried out for light beams, ultra-short pulses [34-36] also have drawn attention. For example, spectral behaviour as well as the spectral switching of diffracted chirped Gaussian pulses in near field [37] and far field have been studied recently [38-39]. An important issue pertaining to spectral switching is whether such switching could be made tuneable as well. Recently in an elegant investigation, Cosh-Gaussian beam has been introduced in the atmospheric optical communication links. In this Communication, we present an investigation to highlight the possibility of tunable spectral switching in the far field of an aperture with a chirped cosh-Gaussian pulse by varying the cosh parameter of the pulse. This seems to be promising because in addition to the chirp, the cosh parameter should provide another significant tool in controlling spectral switching and also spectral shift.

The possibility of tunable spectral switching in the far field with a chirped cosh-Gaussian pulse by varying its cosh parameter has been recently reported [40]. The switching frequency and both blue and red shifted frequencies can be also tuned by varying cosh parameter. Also tenability can be achieved by using Super Gaussian pulses [41]. The phenomena of spectral switching can be used in data processing and terrestrial communication.

1.1 Definition of the problem

Spectral switching has been investigated for defined aperture (circular, rectangular, elliptical, young's double slit, grating). Spectral switching with multiple apertures with different orientation in plane will be our point of interest because they can help potential applications in data procession. Thus we purpose to investigate anomalous spectral behaviour and spectral switching induced by aperture lattice.

1.2 Objectives:-

(1) To investigate anomalous spectral behaviour and spectral switching induced by aperture lattice.

(2) To investigate the influence of aperture number on spectral switching.

References:

1. M. S. Soskin and M. V. Vasnetsov, Progress in Optics, Ed. E. Wolf, (Elsevier,2001)42, 219.
2. M. S. Soskin and M. V. Vasnetsov, Pure Appl. Opt. 7, (1998) 301-311
3. J. F. Nye and M. V. Berry, Proc. R. Soc. London A 336, (1974)165.
4. F. J. Wright, 1979, Structural Stability in Physics, Ed. W. Guettinger and H. Eikemier (Berlin: Springer) p141.
5. J. M. Vaughan and D. V. Willetts, Optics Commun., 30, (1979) 236
6. N. B. Baranova, A. V. Mamaev, N. F. Pilipetskii, V.V. Shkunov and B. Ya Zel'dovich, J. Opt. Soc. Am.-B, 73 (1983) 525
7. J. F. Nye, Natural Focusing and the Fine Structure of Light (Institute of Physics,Bristol and Philadelphia, 1999).
8. G Gbur, T. D. Visser and E. Wolf, Phys. Rev. Lett., 88, (2002) 013901.
9. G Gbur, T. D. Visser and E. Wolf, J. Opt. Soc. Am A, 19, (2002) 1694.
10. G. Popescu and A. Doariu, Phys. Rev. Lett., 88, (2002) 183902.
11. E. Wolf, Phs. Rev. Lett., 56, (1986) 1370
12. E. Wolf, Phs. Rev. Lett., 63, (1989) 2220
13. Z, Dacic and E. Wolf, J. Opt. Soc. Am A, 5, (1988) 1118
14. J.T. Foley, Optics Commun., 75 (1990) 347
15. A, Wasan, H.C. Kandpal, D.S.Mehta, J.S. Vaishya and K.C.Joshi, Optics Commun., 121 (1995) 89
16. M. V. Berry, New J. Phys 4 (2002) 66.
17. J. Pu, H. Zhang and S. Nemoto, Optics Commun., 162 (1999) 57
18. J. Pu, and S. Nemoto, IEEE J. Quantum Elec., 36 (2000)1407
19. H. C. Kandpal, J. Opt. A: Pure Appl. Opt., 3 (2001) 296

20. H. C. Kandpal, S. Anand and J. S. Vashya, IEEE J. Quantum Elec., 38(2002) 336
21. F. Gori, M. Santarsiero, R. Borghi and S. Vicalvi, J. Mod. Opt. 45 (1998) 539.
22. S. A. Ponomarenko, J. Opt. Soc. Am A, 18, (2001) 150.
23. G Gbur and T. D. Visser, Opt. Commn., 222 (2003) 117.
24. G Gbur, T. D. Visser and E. Wolf, Opt. Commn., 239 (2004) 15.
25. D. A. Kessler and I. Freund, Opt. Lett., 28 (2003) 111.
26. C. Palma and G. Cincotti , J. Opt. Soc. Am A, 14, (1997) 1885.
27. C. Palma and G. Cincotti , Opt. Lett., 22 (1997) 671.
28. Bai-da Lu and Liu-Zhan Pan,IEEE J. Quantum Electron., 38 (2002) 340
29. Liu-Zhan Pan and Bai-da Lu, IEEE J. Quantum Electron., 39 (2003)1334
30. G. Zhao, X. Xiao and B. Lu, Chin. Phys., 13 (2004) 2064
31. J.Pu, C.Cai, X. Hu, and X. Liu, Chin. Phys. Lett., 22 (2005) 2259.
32. L. Pan and B. Lu, IEEE J. Quantum Electron., 37 (2001)1377
33. J.Pu and C.Cai, Chin. Phys. Lett., 21 (2004) 1268.
34. Z Liu and D Fan, Pure Appl. Opt. 6 (1997) L43-48
35. H. Hwang , G Yang and P Han, Jpn. J. Appl. Phys, 41 (2002) 3683.
36. H. Hwang , G Yang and P Han, Opt. Eng. 42 (2003) 686
37. Liu-Zhan Pan and Bai-da Lu, Chinese Physics, 13 (2004) 637
38. Liu-Zhan Pan and Bai-da Lu, Optik, 115 (2004) 57 .
39. S. Sartania, Z. Cheng, M. Lenzner, G. Tempea, Ch. Spielmann, F. Krausz and K. Ferencz, Opt. Lett., 22 (1997) 1562.
40. S Jana S. Konar, Opt. Commun. 276 (2006) 24-31.
41. S Jana S. Konar, Opt. Commun. 281 (2008) 4829-34.

CHAPTER – II

2. Methodology:

We here discuss the methodology with example of a Gaussian laser pulse that incidents on a rectangular aperture (at z = 0 plane) having widths 2a and 2b along x and y directions respectively. The initial field of the incident pulse is given by

$$E(x_0, y_0, 0, t) = \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right) A(t) \qquad \ldots(1)$$

Where, w_0 is the pulse waist and A(t) is the temporal profile of the pulse having the form [16]

$$A(t) = \exp\left(-\frac{t^2(1+ic)}{2T^2}\right) \exp(-i\,\omega_0 t) \qquad \ldots(2)$$

where, T is the pulse duration, c is the chirp parameter and ω_0 is the central frequency of the pulse. The field given by Eq.(1), can be described in the space-frequency domain by Fourier transformation as

$$E(x_0, y_0, 0, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x_0, y_0, 0, t) \exp(i\omega t) dt = \exp\left(-\frac{x_0^2 + y_0^2}{\omega_0^2}\right) F(\omega) \quad \ldots(3)$$

Where, $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t) \exp(i\omega t) dt$ represents the Fourier spectrum of the pulse at the incident plane and ω is the frequency. The initial power spectrum at the incident plane z = 0 can be easily obtained from Eq. (2) and (3) as

$$I_0(\omega) = |F(\omega)|^2 \qquad \ldots(4)$$

Adopting far-field approximation the propagating field can be expressed at a distance z as

$$E(x, y, z, \omega) = \frac{i}{\lambda z} \exp(-ikz)_i \int_{-a}^{a} \int_{-b}^{b} E(x_0, y_0, 0, \omega) \exp\left[\frac{ik}{z}(xx_0 + yy_0)\right] dx_0 dy_0 \qquad \ldots(5)$$

where, $k(= 2\pi/\lambda)$ is the wave number.

However we consider 2D case for our present investigation for field power spectrum.

$$I_z(x, y, z, \omega) = E(x, y, z, \omega)\, E^*(x, y, z, \omega) = |E(x, y, z, \omega)|^2 \qquad \ldots(6)$$

Observation shows that

$$I_z(\omega) = I_0(\omega) \times M \qquad \ldots(7)$$

where M is the modifier. This modifier and hence the I_z are utilized to study far field spectral behaviour and spectral switching.

However, as we propose aperture lattice, the modifier is expected to be much more complex as well as interesting in nature. We will use the aforesaid methodology for multiple aperture / aperture lattice. The effect of introducing multiple aperture/aperture lattice is discussed below [3].

We now consider a screen that contains a large number of specially oriented apertures.

Let o_1, o_2, ………… O_n the set of specially situated points, one in each aperture and let the coordinates of those points referred to a set of axes in the apertures be (x_1,y_1), (x_2,y_2),……………………(x_n,y_n). The light distribution in the Fraunhofer diffraction pattern is then given by

$$E(x,\ y) = c\sum_n \iint e^{-ik[(x_n+x_0)x+(y_n+y_0)y}]\,dx_0 dy_0$$

$$= c\sum_n e^{-ik[x x_n + y y_n]} \iint e^{-ik(x x_0 + y y_0)}\,dx_0\,dy_0 \quad \text{…(8)}$$

The integration expresses the effect of a single aperture, and the sum is represents the superposition of the coherent diffraction patterns. If 1^0 (p, q) is the intensity distribution arising from a single aperture, then total intensity is given by

$$I(x,\ y) = I_0\ (x,\ y)\left|\sum_n e^{-ik(x x_n + y y_n)}\right|^2$$

$$= I_0\ (x,\ y)\sum_n \sum_m e^{-ik[x(x_n - x_m)+y(y_n - y_m)]} \quad \text{…(9)}$$

However, neglected the dependence of I^0 on p, q and only study the effect of superposition. For *N=2*, eq.(9) reduces to

$$I = I_0\ \{2 + e^{-ik[x(x_1-x_2)+y(y_1-y_2)]} + e^{-ik[x(x_2-x_1)+y(y_2-y_1)]}$$

$$= 4 I_0 \cos^2 \frac{1}{2}\delta \quad \text{…(10)}$$

Where, $\delta = k[x(x_2 - x_1) + y(y_2 - y_1)]$

Let us now consider the effect of a large number of apertures. We shall see that quite different result is obtained, on whether the apertures are distributed regularly or irregularly over the screen.

When the aperture are consider irregularly distributed over the screen, with terms m and n in the double sum will fluctuate rapidly between +1, and-1 as m and n takes different values. The total intensity is N times the intensity of the light diffracted by a single aperture:

$$I(x,\ y) \approx NI_0(x,\ y) \qquad \ldots(11)$$

We, however, are interested in regular distribution of apertures, for which the scenario is quite different and interesting. Here we present some of the aperture design, which will be used to investigate spectral switching.

Figure 1(a) and 1(b) are the 1D array and1(c) in 2D array of apertures. Since, investigating 2D array is straightforward extension of 1D case; we will confine our investigation for 1D aperture array only. Thus we discuss single, double and four aperture case. This investigation will be a guideline for the other array of apertures.

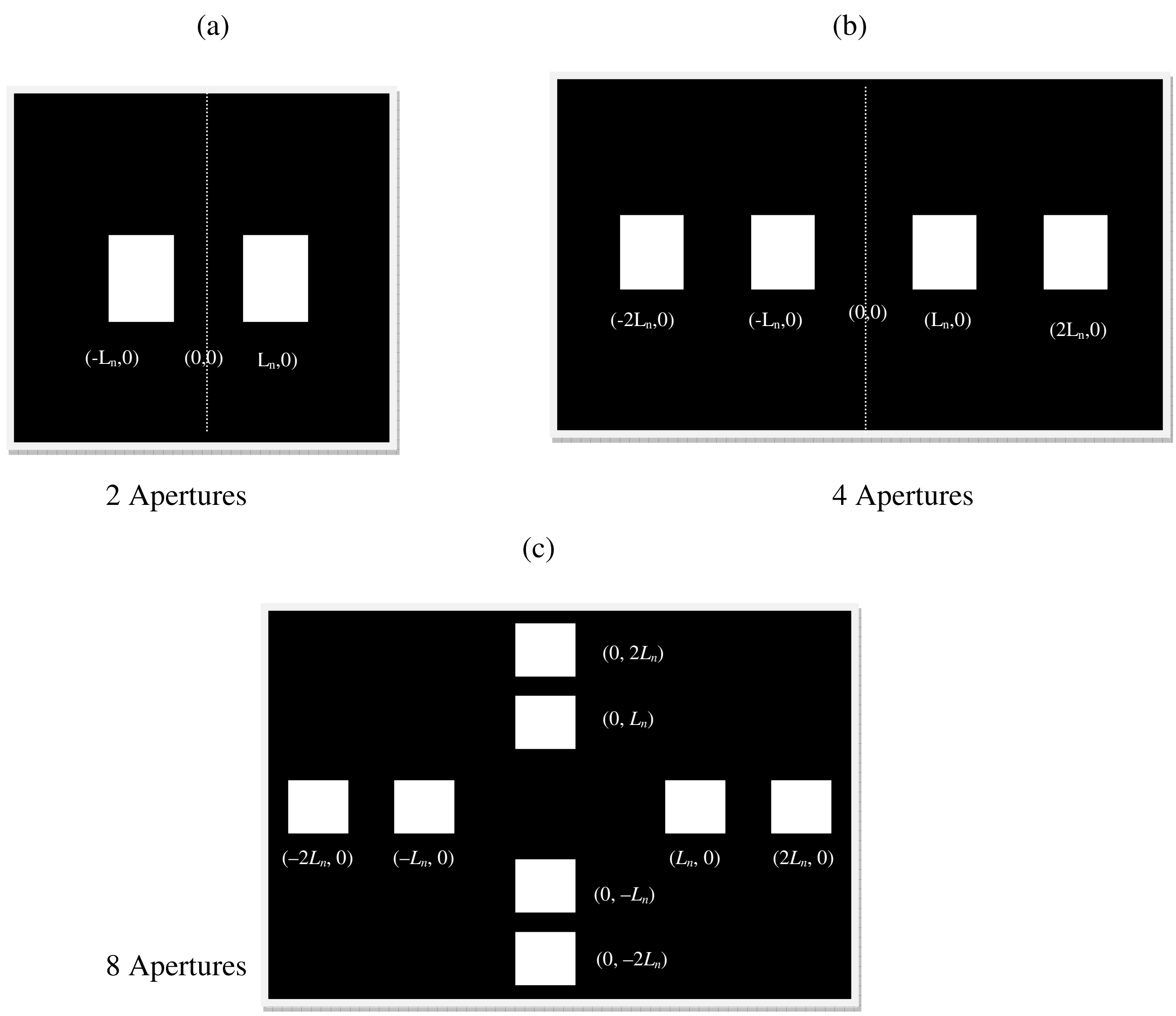


Fig.1 . (a, b)1D and (c) 2D array of apertures

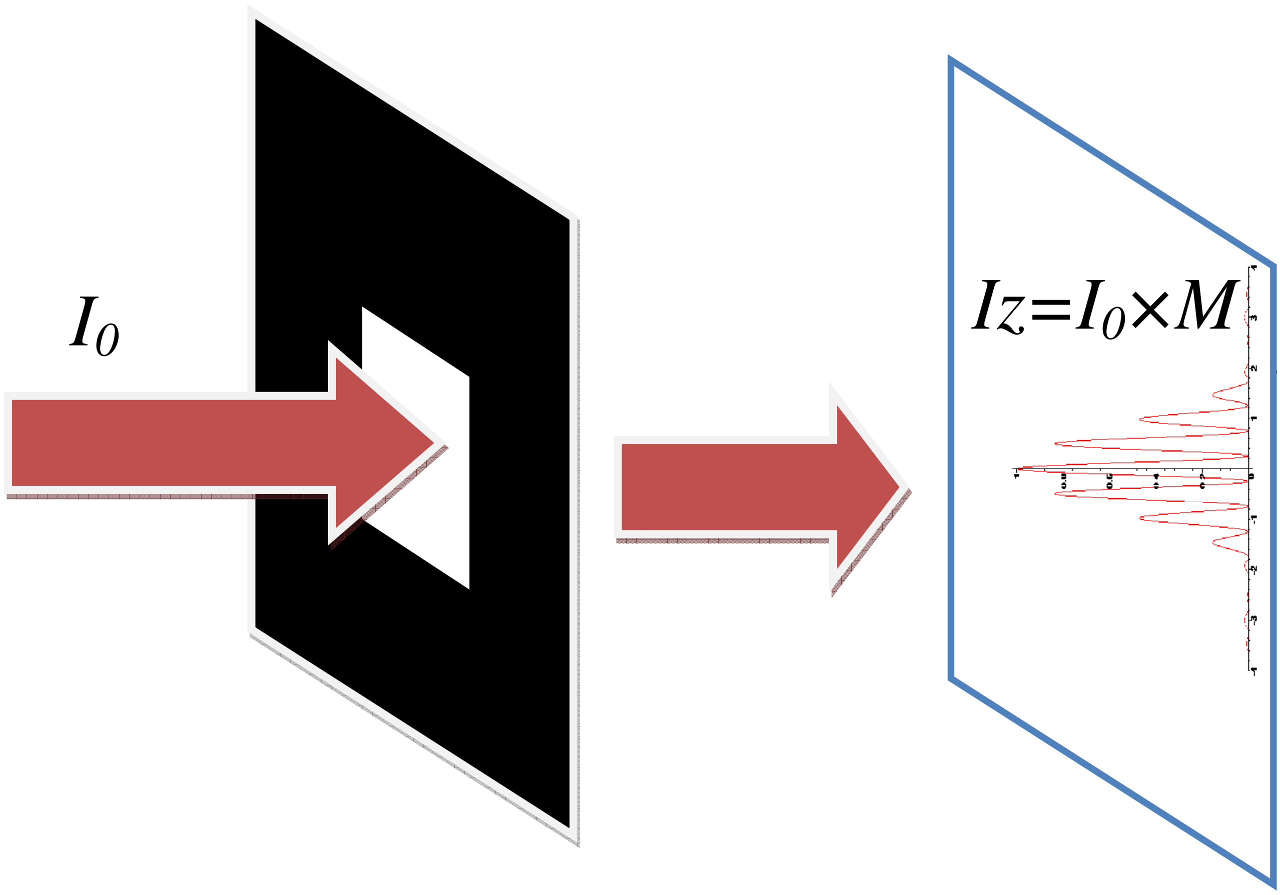


Fig. 2 Schematic diagram for the single aperture diffraction

2.1 Results And Discussion:

In our investigation we use Gaussian pulse profile as given by eq.(2). Fig.3 shows the unchirped and chirped Gaussian pulse profile.

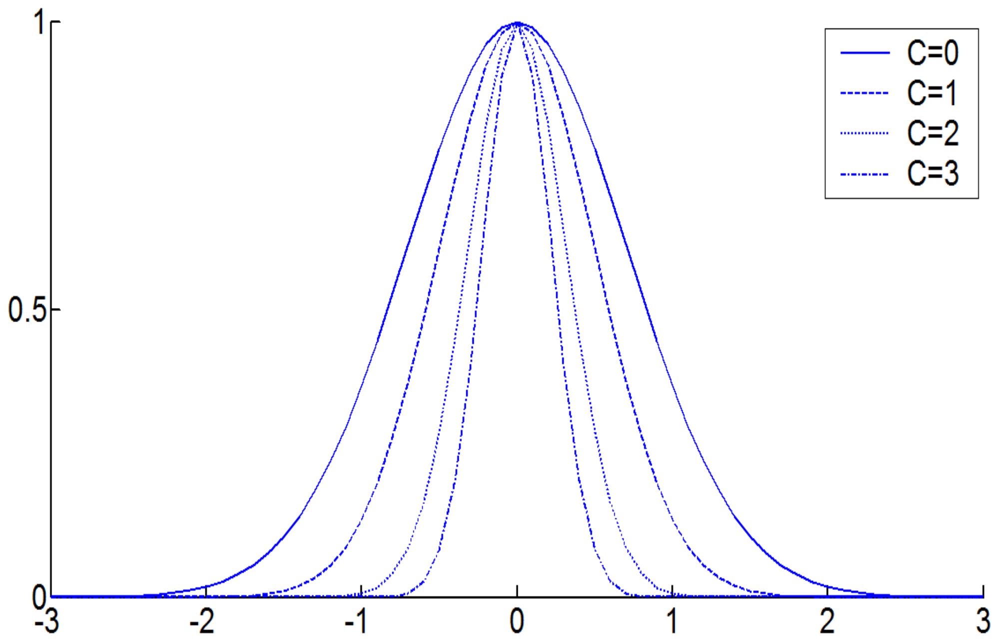


Fig.3 Chirped and unchirped Gaussian pulse profile

With this Gaussian pulse profile the initial electric field , we get

$$E_0 = \frac{T\sqrt{1-ic}}{1+c^2}\; e^{-x_0^2/\omega_0^2}\; e^{\frac{-T^2(1-ic)}{2(1+c^2)}(\omega-\omega_0)^2} \qquad \ldots(12)$$

The corresponding initial power spectrum (i.e., at $z = 0$ plane)

$$I_0 = \frac{T^2}{\sqrt{1+c^2}}\; e^{-\frac{T^2}{1+c^2}(\omega-\omega_0)^2} \qquad \ldots(13)$$

The field represented by eqn. (12) follows Huygens–Fresnel integration during its propagation along z. The form of electric field at $z = z$ plane will become

For single aperture:

$$E_1 = \frac{1}{2}\sqrt{i\frac{z_0}{z}\frac{\omega}{\omega_0}}\; e^{-ikz} \times \sqrt{\frac{T^2(1-ic)}{1+c^2}}\; e^{-\frac{T^2(1-ic)}{2(1+c^2)}(\omega-\omega_0)^2}\; e^{-\alpha^2\left(\frac{\omega}{\omega_0}\right)^2} \times \left[erf\left(\delta - i\,\alpha\frac{\omega}{\omega_0}\right) + erf\left(\delta + i\alpha\frac{\omega}{\omega_0}\right)\right]$$

…(14a)

For double apertures:

$$E_2 = \frac{1}{2}\sqrt{i\frac{z_0}{z}\frac{\omega}{\omega_0}}\, e^{-ikz}\left(e^{2\pi i\alpha L_n \frac{z}{z_0}\frac{\omega}{\omega_0}} + e^{-2\pi i\alpha l\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)\times\sqrt{\frac{T^2(1-ic)}{1+c^2}}\, e^{-\frac{T^2(1-ic)}{2(1+c^2)}(\omega-\omega_0)^2}\, e^{-\alpha^2\left(\frac{\omega}{\omega_0}\right)^2}$$

$$\times\left[erf\left(\delta - i\,\alpha\frac{\omega}{\omega_0}\right) + erf\left(\delta + i\alpha\frac{\omega}{\omega_0}\right)\right]$$

…(14b)

For four apertures:

$$E_4 = \frac{1}{2}\sqrt{i\frac{z_0}{z}\frac{\omega}{\omega_0}}\, e^{-ikz}\left(e^{2\pi i\alpha L_n \frac{z}{z_0}\frac{\omega}{\omega_0}} + e^{-2\pi i\alpha l\frac{z}{z_0}\frac{\omega}{\omega_0}} + e^{4\pi i\alpha L_n \frac{z}{z_0}\frac{\omega}{\omega_0}} + e^{-4\pi i\alpha L_n \frac{z}{z_0}\frac{\omega}{\omega_0}}\right)$$

$$\times\sqrt{\frac{T^2(1-ic)}{1+c^2}}\, e^{-\frac{T^2(1-ic)}{2(1+c^2)}(\omega-\omega_0)^2}\, e^{-\alpha^2\left(\frac{\omega}{\omega_0}\right)^2}\times\left[erf\left(\delta - i\,\alpha\frac{\omega}{\omega_0}\right) + erf\left(\delta + i\alpha\frac{\omega}{\omega_0}\right)\right]$$

…(14c)

The corresponding diffraction induced far-field modified power spectrum are as follows:,

For single aperture:

$$I_{z1} = \frac{T^2\sqrt{1+c^2}}{(1+c^2)}\, e^{-\frac{T^2(\omega-\omega_0)^2}{(1+c^2)}}\times\frac{1}{4}\frac{z_0}{z}\frac{\omega}{\omega_0}e^{-2\alpha^2\left(\frac{\omega}{\omega_0}\right)^2}\left[erf\left(\delta - i\alpha\frac{\omega}{\omega_0}\right) + erf\left(\delta + i\alpha\frac{\omega}{\omega_0}\right)\right]^2$$

…(15a)

For double apertures:

$$I_{z2} = \frac{T^2\sqrt{1+c^2}}{(1+c^2)}\, e^{-\frac{T^2(\omega-\omega_0)^2}{(1+c^2)}}\times\frac{1}{4}\frac{z_0}{z}\frac{\omega}{\omega_0}e^{-2\alpha^2\left(\frac{\omega}{\omega_0}\right)^2}\left[erf\left(\delta - i\alpha\frac{\omega}{\omega_0}\right) + erf\left(\delta + i\alpha\frac{\omega}{\omega_0}\right)\right]^2$$

$$\times\left(2 + e^{4\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}} + e^{-4\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)$$

…(15b)

For four apertures:

$$I_{z4}=\frac{T^2\sqrt{1+c^2}}{\left(1+c^2\right)}\,e^{-\frac{T^2(\omega-\omega_0)^2}{\left(1+c^2\right)}}\times\frac{1}{4}\frac{z_0}{z}\frac{\omega}{\omega_0}e^{-2\alpha^2\left(\frac{\omega}{\omega_0}\right)^2}\left[erf\left(\delta-i\alpha\frac{\omega}{\omega_0}\right)+erf\left(\delta+i\alpha\frac{\omega}{\omega_0}\right)\right]^2$$

$$\times\left[4+2\left(e^{2\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}+e^{-2\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)+\left(e^{4\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}+e^{-4\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)\right.$$

$$\left.+2\left(e^{6\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}+e^{-6\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)+\left(e^{8\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}+e^{-8\pi i\alpha L_n\frac{z}{z_0}\frac{\omega}{\omega_0}}\right)\right]$$

…(15c)

Here, λ_0 is the central wavelength, $z_0(=\pi w_0^2/\lambda_0)$ is the Rayleigh length, $\alpha\left(=x/(z\theta_0)\right)$ is the normalized angles of diffraction in x direction, $\theta_0(=\lambda_0/\pi w_0)$ is the far-field divergence angle, $\delta\left(=a/w_0\right)$ is the truncation parameter along x direction, and "*erf*(*x*)" stands for error function of the argument. *M* can be identified as spectral modifier that arises due to diffraction of the pulse through the finite aperture. The expression of M is the simplified one, which is valid only at far field (i.e., z_0 / z << 1). Eq.15(a-c) have been used subsequently to investigate the far-field spectral behaviour of the pulse.

2.1.1 Initial power spectrum:

The initial power spectrum of the Gaussian pulse is given by the eq.(13). Fig.4 shows the on-axis initial power spectrum of chirped and unchirped Gaussian pulses. From the figure it is clear that the initial spectrum is symmetrically distributed around the central frequency.

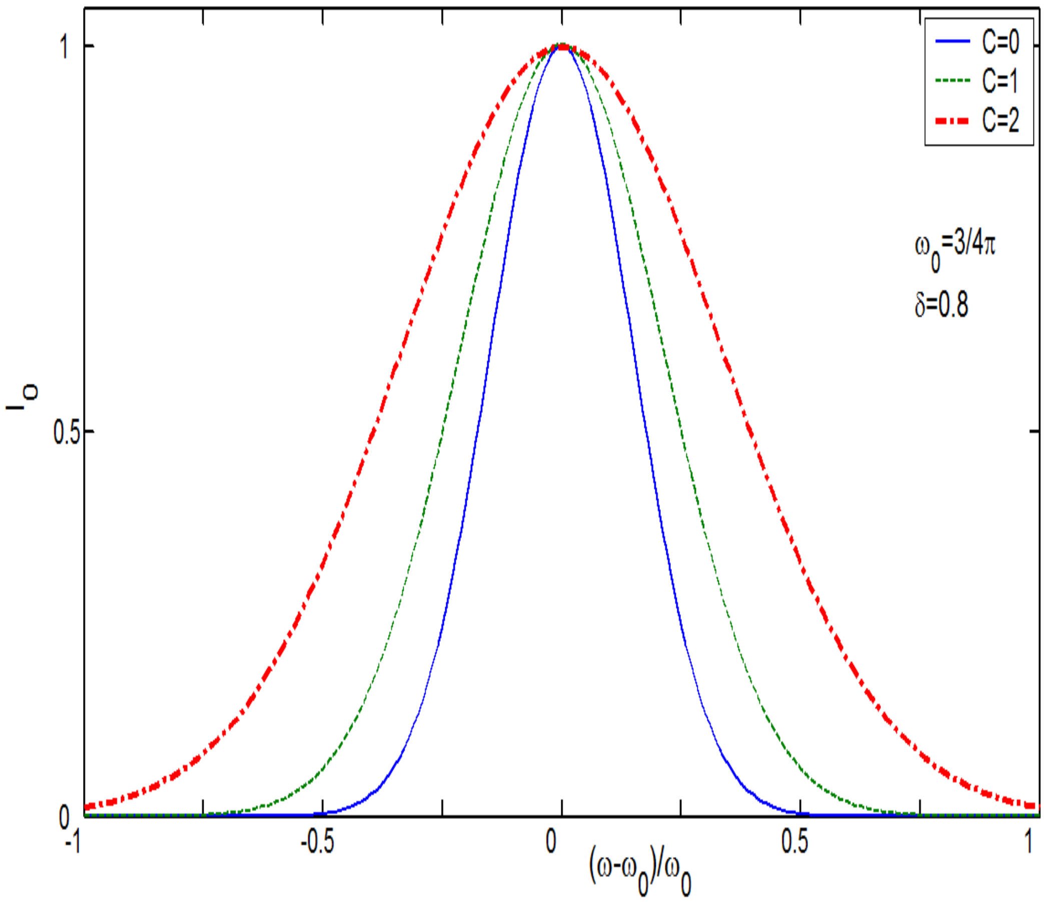


Fig.4: Normalized on-axis Initial power spectrum (i.e., at $z = 0$) of Gaussian pulse ***for single aperture****. Solid, dashed and dash dot lines correspond to C =0, 1 and 2 respectively.*

2.1.2 Modifier:

We now investigate the spectral modifier which solely arises due to the apertures. Figure (5) shows 3D spectral modifier for symmetrical and asymetrical case.

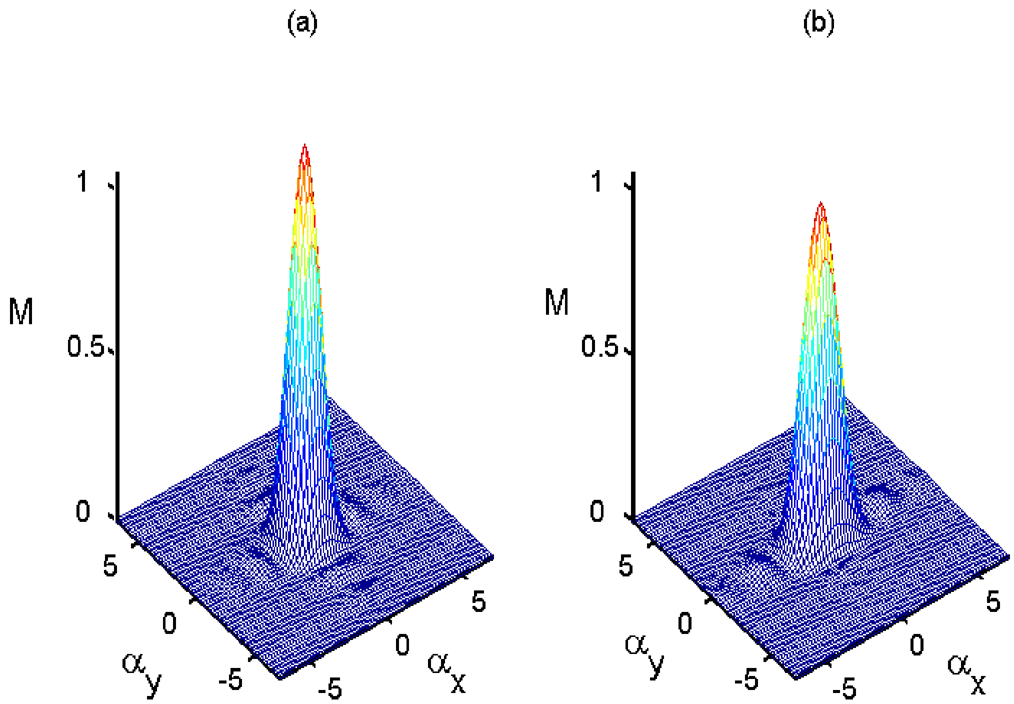


Fig.5: Typical 3D spectral modifier at the central frequency (ω_0) for (a) symmetrical aperture $(\delta_x = \delta_y = 0.8)$ *and (b) asymmetrical aperture* $(\delta_x = 1.0, \delta_y = 0.6)$.

In our investigation we are assuming the lower dimension for investigation as it can successfully describe the system and the results and correctly predicts the results for higher order case. Thus instead of taking 3D modifier we take the cross section of it, i.e., 2D modifier.

In our simplified to 2D case, we get the following modifier for different set of apertures. Figure 6 and 7 shows the modifier due to single, double and four apertures.

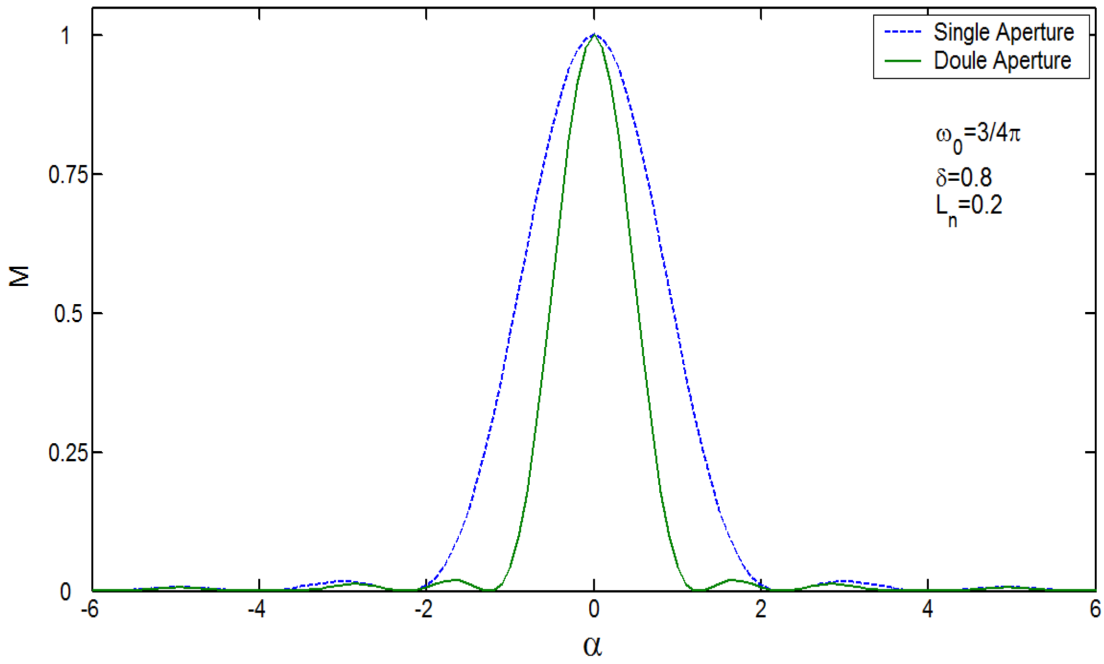


Fig.6: Spectral modifiers at the central frequency (ω_0) for single and double apertures.

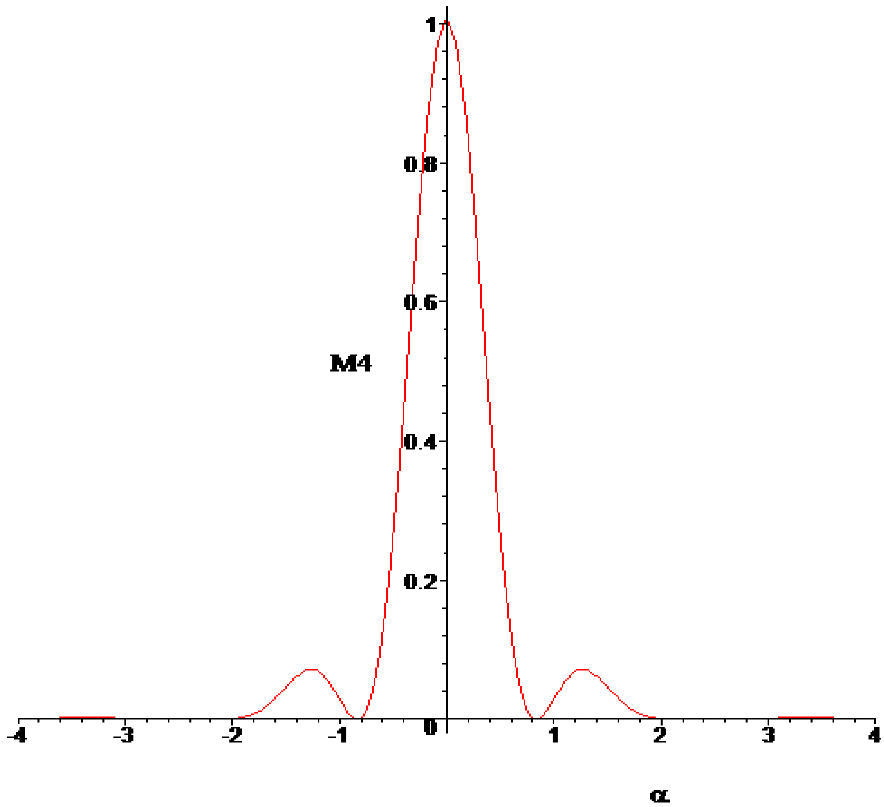


Fig.7: Spectral modifiers at the central frequency (ω_0) for four apertures.

From the figures it is clear that the phase singular points occur at different diffraction angle (α) in different cases. These phase singular points are the location of spectral anomaly and hence spectral switching. Therefore controlling the number of apertures we can control the spectral switching. This is one of the most important finding with such aperture lattice, we consider for switching.

TABLE-1 shows the critical angle for switching for different orders with different set of apertures. It is clear from the table-1 that the 3rd order switching can be more effective. Also switching can be done at lower angle of diffraction with large number of aperture.

TABLE-1

Values of critical angle of switching

	Single aperture	Double aperture	Four aperture
α_c for 1st order switching	----	1.308	0.8
α_c for 2nd order switching	2.368	2.256	2.0
α_c for 3rd order switching	~4 *	~4 *	~4 *

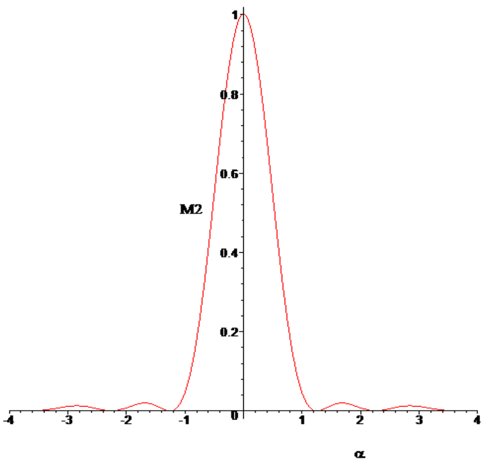
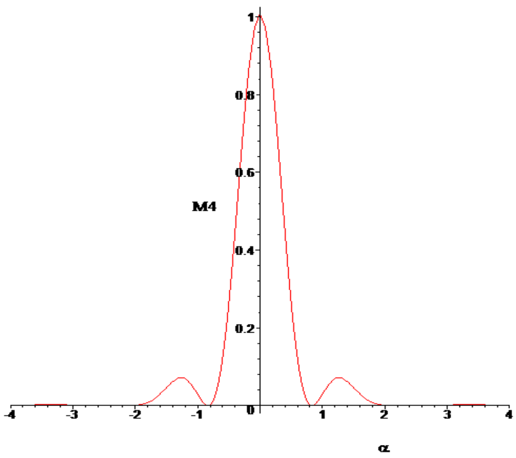
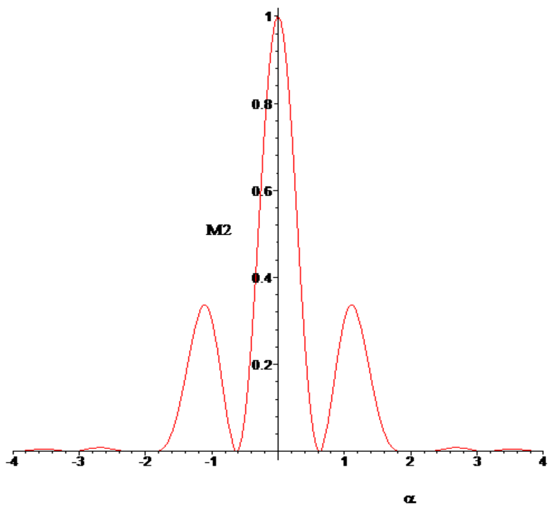
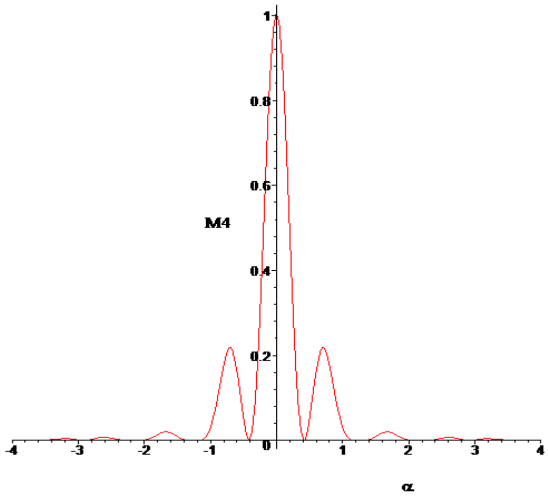
* (wide range, not suitable for switching)

Following are the comparative study of the modifiers for double and four-aperture case.

Another interesting feature of the modifier and hence the far field spectrum is dependence on L_n, the normalized separation between apertures (Refer to figure 1). Following table is the demonstration of the effect of aperture separation with switching.

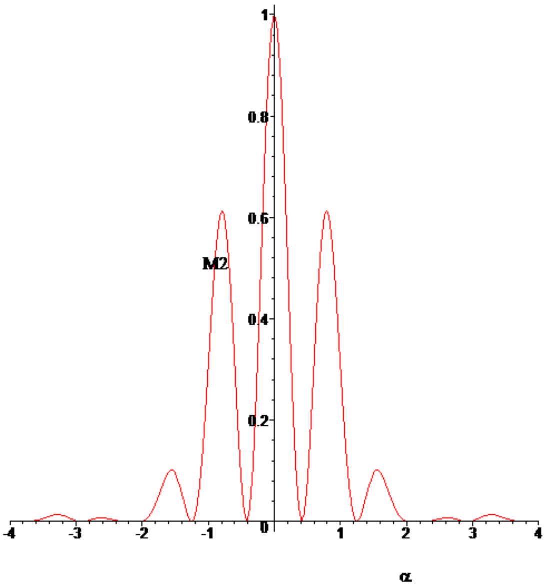
TABLE-2

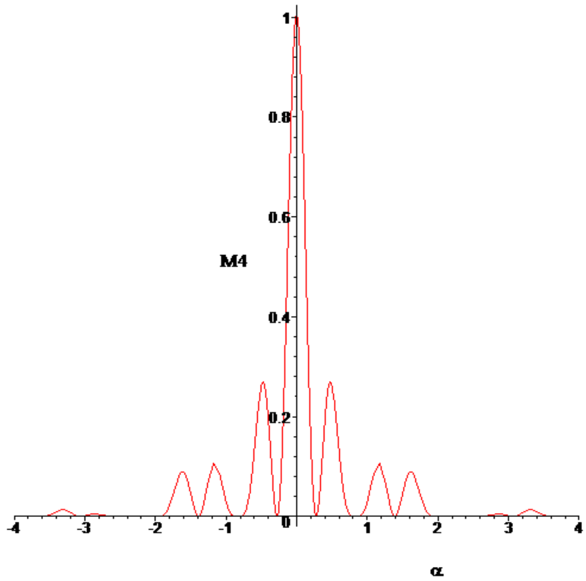
Modifier for double and four aperture case at different L_n

L_n	DOUBLE APERTURE	FOUR APERTURE
$L_{n}=$ *0.2*		
$L_{n}=$ *0.4*		

L_{n}=

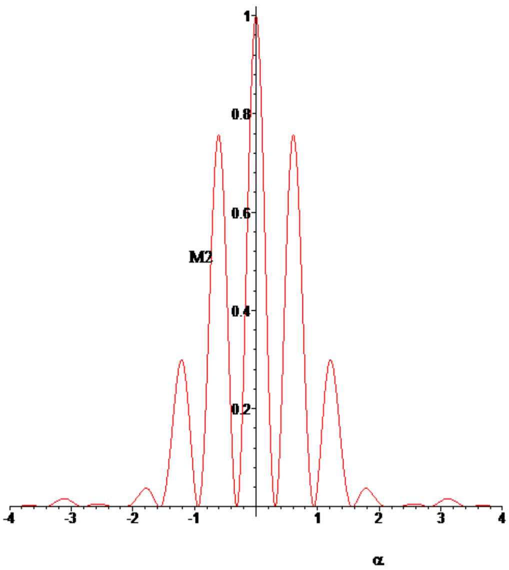
0.6

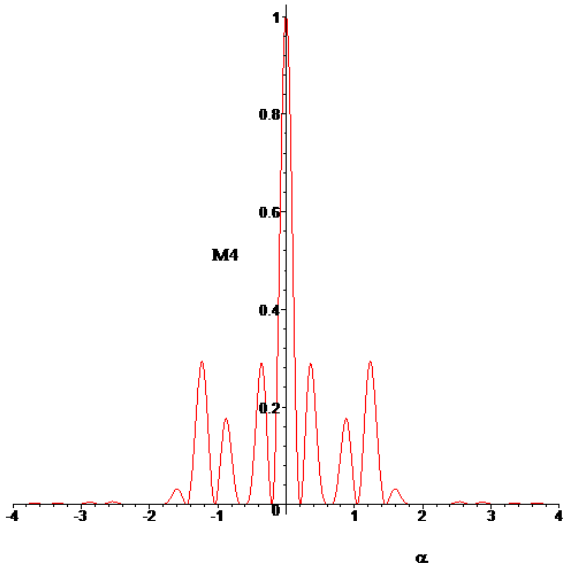


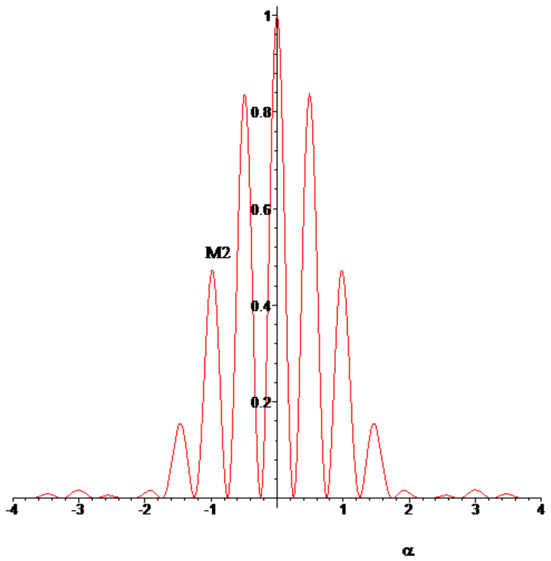
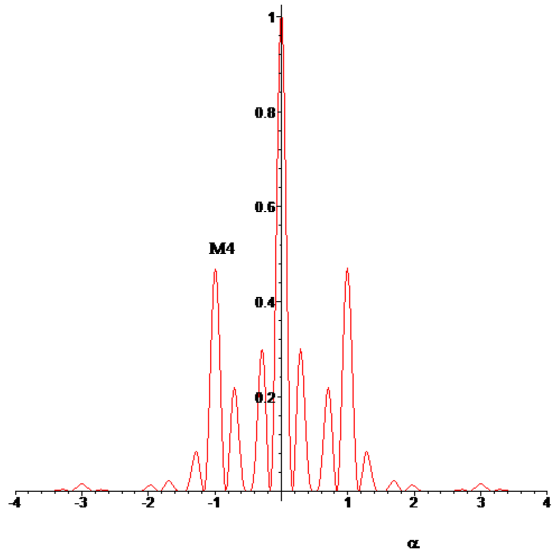
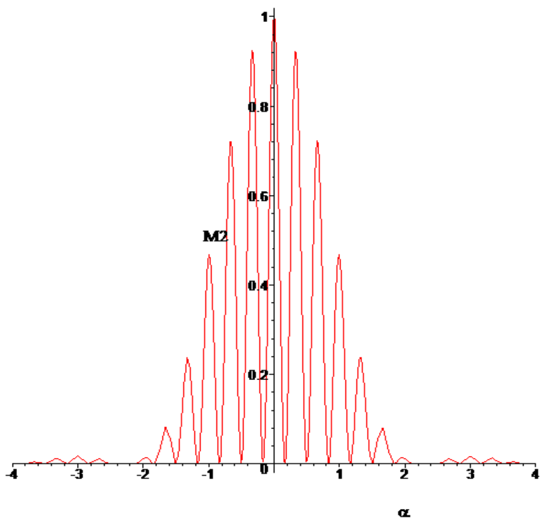
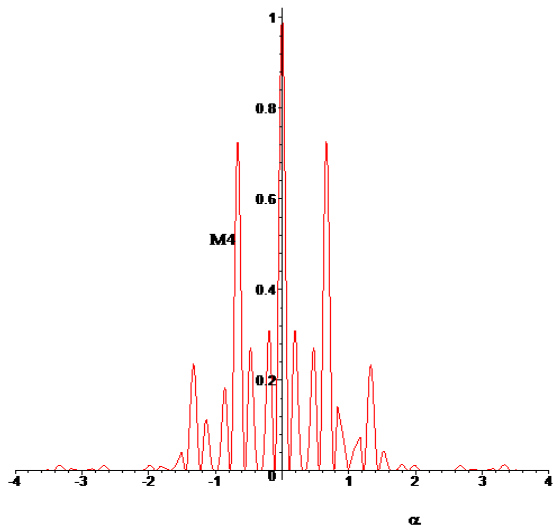


L_{n}=

0.8





L_n= 1.0		
L_n= 1.5		

Simple observation reveals that as L_n increases no. of singular points increases for a fixed diffraction cone. This indicates that at a certain angle of diffraction the higher order switching will occur with large value of L_n. Therefore for a fixed diffraction angle different order switching can be achieved by controlling L_n. In other words by controlling the value of L_n one can control spectral switching. i.e., spectral switching is tunable by controlling L_n. This is one of the important finding in our investigation. Also comparison between double aperture and four apertures shows that introducing more number of apertures one can enhance order number of switching for a particular diffraction angle. So a particular order switching can be control by the number of apertures this finding is very important because by choosing no. of open apertures one can get a desired signal at a particular location. This finding has immense potential application in optical computation all optical switching and data processing.

With the knowledge of above discussed modifier we now investigate far field power spectrum and spectral anomaly and hence spectral switching for different set of apertures. This investigation on for field can be subdivided into two lines one on axis far field power spectrum (i.e. $\alpha = 0$) and off axis $(\alpha \neq 0)$

2.1.3 On-axis far-field power spectrum:

The expression for on-axis far-field power spectrum or modified power spectrum can be derived by setting α=0 in Eq.15(a-c). The values of the parameters used in our analysis are follows: δ=0.8 (unless stated otherwise) and $T_0 \approx 10^{-15} s$. The central wavelength λ_0 (corresponding to ω_0) of the laser source is 800nm. Ti: Sapphire laser can generate shortest optical pulses (<5 fs) having spectrum centered at $\lambda_0 = 800nm.$

In our case the on-axis far-field power spectrum (i.e., at $z = 0$) are of following form:

For single aperture:

$$I_0(0,z,\omega) = \left[I_0 \frac{1}{4} \frac{z}{z_0} \frac{\omega}{\omega_0} (erf(\delta))^2 \right] \qquad \ldots(13a)$$

For double apertures:

$$I_0(0,z,\omega) = 4 \times \left[I_0 \frac{1}{4} \frac{z}{z_0} \frac{\omega}{\omega_0} (erf(\delta))^2 \right] \qquad \ldots(13b)$$

For four apertures:

$$I_0(0,z,\omega) = 12 \times \left[I_0 \frac{1}{4} \frac{z}{z_0} \frac{\omega}{\omega_0} (erf(\delta))^2 \right] \qquad \ldots(13c)$$

Fig. 8 & 9 depicts the on axis far field power spectrum for single and double aperture. Interestingly, they look very same however equation 13 support this. Both the figure 8 and 9 shows that the on axis far field power spectrum is blue shifted for both charped and uncharped pulse. The blue shift monotonically increases with increase in value of C. We expect similar on axis far field power spectrum for higher number of apertures.

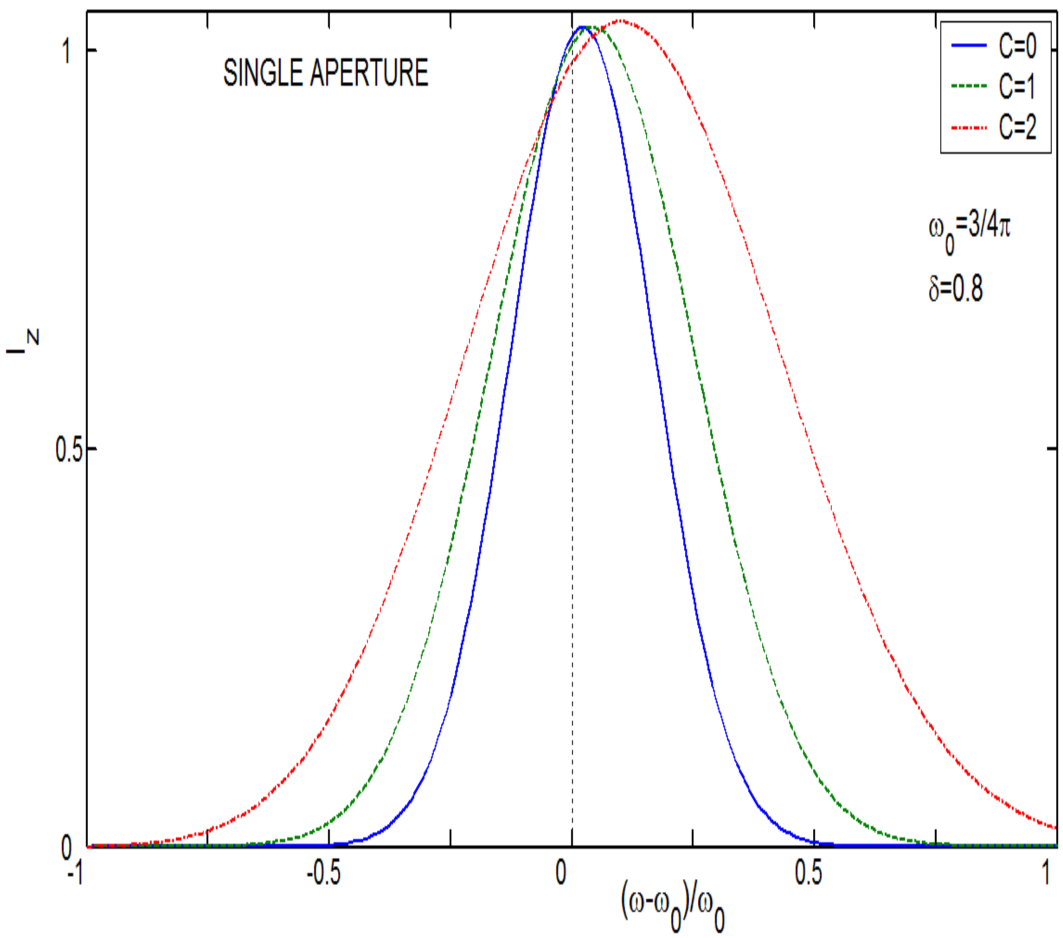


Fig.8: Normalized on-axis Far Field power spectrum (i.e., at $z=0$) of Gaussian pulse ***for single aperture****. Solid, dashed and dash dot lines correspond to C =0, 1 and 2 respectively.*

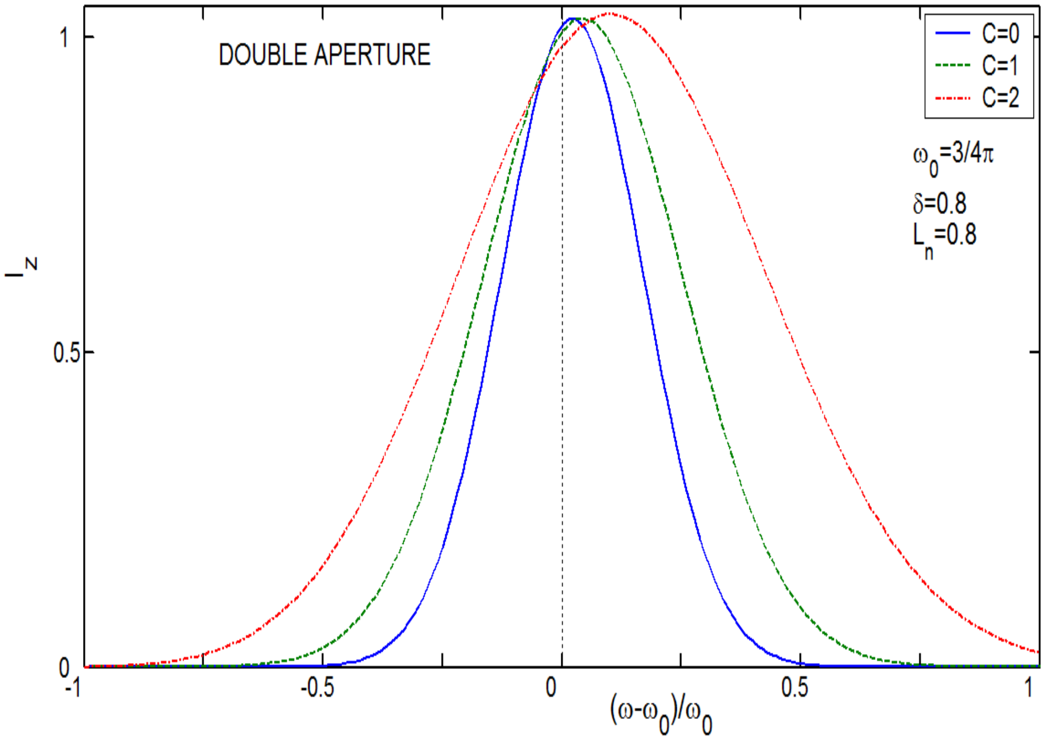


Fig.9: Normalized on-axis Far Field power spectrum (i.e., at $z = 0$) of Gaussian pulse ***for double aperture****. Solid, dashed and dash dot lines correspond to C =0, 1 and 2 respectively.*

2.1.4 Off-axis far-field power spectrum:

For the off-axis case $\alpha \neq 0$. We keep on increasing the value of α. At a certain value of α, the spectrum is divided into two peaks of equal height. This angle is known as critical angle α_c. At an angle smaller than α_c the spectrum shifts towards lower frequency region, i.e., the spectrum is red shifted and for larger angle the spectrum shifts towards higher frequency region, i.e., blue shifted. Therefore, by simply varying the angle of diffraction one can switch over from one colour to another colour. This phenomenon is known as spectral switching.

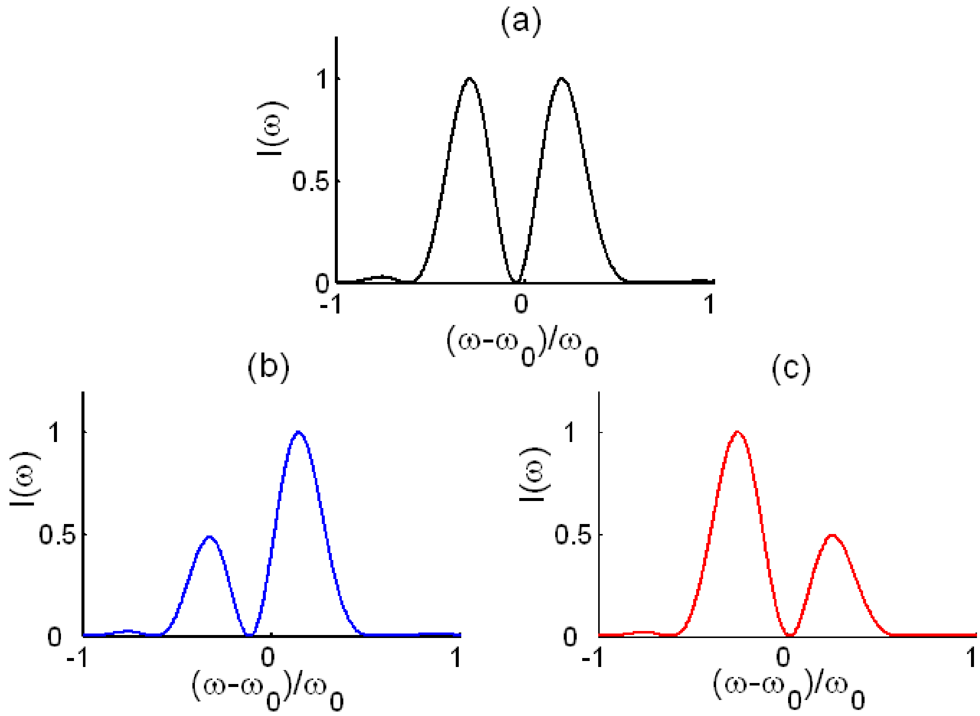


Fig.10: Demonstration of a typical spectral switching phenomenon.

(a) Spectrum at critical angle $(\alpha = \alpha_c = 2.368)$,

(b) blue shifted spectrum $(\alpha = 2.541 > \alpha_c)$,

(c) red shifted spectrum

Points of phase singularities correspond to critical angle α_c switching. While discussing modifier we already predicted the critical angle α_c for different orders and set of apertures. Here we search the critical angle for switching α_c by investigating the far-field power spectrum. Our simulation exactly matches with the data presented in TABLE-1.

We also showed in TABLE-2 that switching phenomena is dependent on the normalized distance of separation of the apertures,i.e., L_n.

The variation of the critical angle for switching with the normalized distance of separation of the apertures L_n is depicted in Fig.11. This clearly supports the possibility of tunable spectral switching.

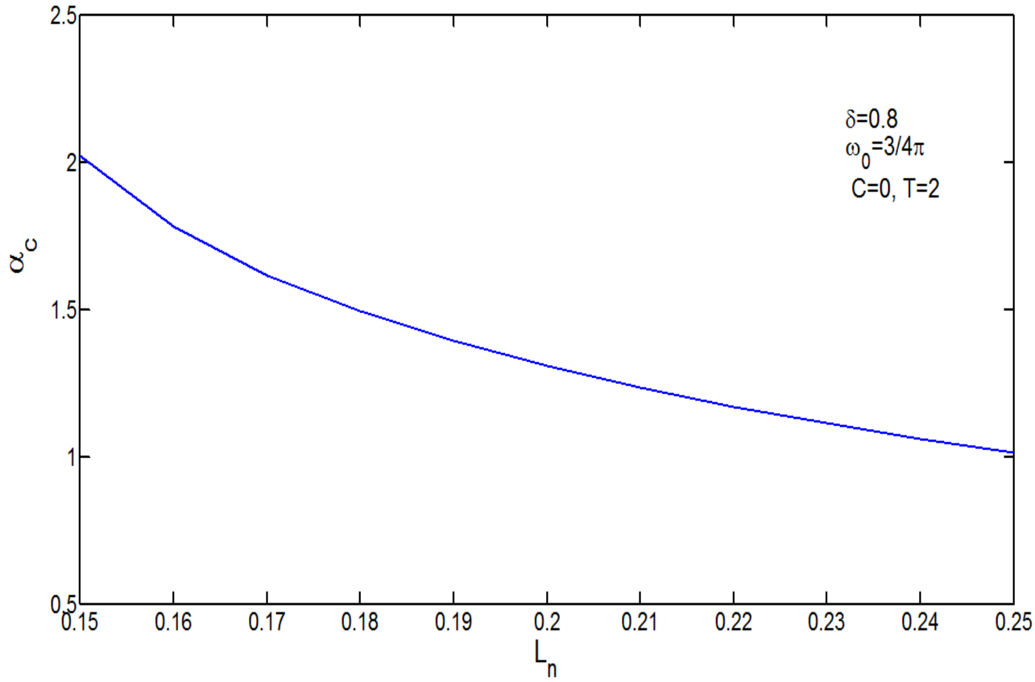


Fig.11: The variation of the critical angle for switching with the normalized distance of separation of the apertures (for 1st order switching).

Yet another interesting representation of our investigation is in Fig. 12. Here we portray the spectral shift $(\Delta\omega)$ with the angle of diffraction (α_x) for different aperture set. This shows that a particular order switching happens at lower angle of diffraction (α_x) for larger number of apertures. This is, however, prominent in lowers order switching as higher order switching occurs almost at same angle of diffraction (α_x). Moreover the transition from red to blue region is not appreciably sharp.

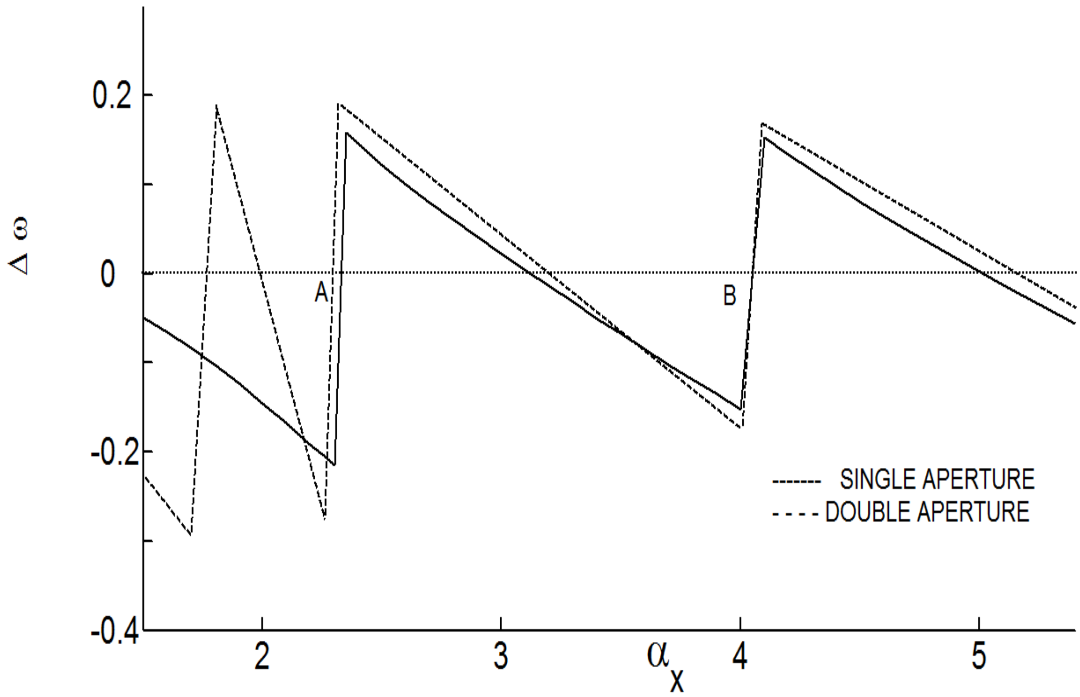


Fig.12: Variation of spectral shift $(\Delta\omega)$ *with the angle of diffraction* (α_x) *for different aperture set.* $T = 2$, $\delta = 0.8$. *Solid line is for single aperture, whereas dotted one indicates double aperture .*

2.2 Conclusion:

We invested spectral switching with multiple apertures with different orientation. We found the diffraction-induced anomalous spectral behaviour and hence spectral switching due to the aperture lattice. Our investigation shows that spectral switching can be tuned by controlling the separation between the apertures. It also be tuned by changing the number of apertures.

These findings are important as they can have significant potential applications in data processing, all-optical switching, space communication and optical computing. Also the results will provide an interesting guideline for further research with other orientation of apertures.
