AN INTEGRATED INVENTORY MODEL OF DETERIORATED ITEMS WITH EXPONENTIAL DEMAND AND LINEAR DETERIORATION RATE

A
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The Award Of The Degree Of

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July, 2018

CERTIFICATE

This is to certify that the dissertation titled "*An Integrated Inventory Model Of Deteriorated Items With Exponential Demand And Linear Deterioration Rate*" is a bonafide record of the work done in partial accomplishment of the requirements for the award of degree of Master of Science in Mathematics and Computing from Thapar Institute of Engineering and Technology, Patiala during the year 2016-2018 under the guidance of Dr. Mahesh Kumar Sharma.
I, certify that the matter embodied in this report is of candidate's own record and not submitted to any other university in any part or full form for the award of such a degree.

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This is to certify that the above statement made by candidate is correct and true to the best of my knowledge.

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ABSTRACT

In a system of inventory production manufacturer yields the item at a rate, discharges the quantities ordered to the customers in certain intervals and for successive deliveries stores the surplus inventory. In frequent real situations, manager for inventory holds the thousand of items in an inventory. Yet, these inventory model for single item are expanded for demand i.e., both for stationary and non-stationary. As in a competitive market the overview of the vendor and the buyer is to maximize their profit and an integrated policy is required by the vendor in general. Numerous models have been evolved in literature for deteriorated item with constant demand. Later on it has been considered that in real life demand of any item either depends on time or selling price.

The present thesis emphasis on an inventory model in which demand varies at an exponential rate and linear deterioration rate. The work is further divided into two chapters.

Chapter I is introductory in nature in which some basic inventory models have been discussed. An integrated inventory model of economic ordering policy of deteriorated items for vendor and buyer is also discussed. The literature related to this work has also been given in this chapter.

In chapter II, the economic ordering policy of deteriorated items for vendor and buyer (Yang and Wee(2000)) has been considered in which constant demand rate and deterioration rate is replaced by exponential and linear rate respectively.

Contents

CHAPTER I

1 Introduction

Inventory essentially deals with materials and material management is how we store the material for subsequent use. In every manufacturing organization we buy raw material, sometimes these raw material undergo manufacturing and become finished or semi finished products. In order to make these products we need to buy raw material. If demand of various products are known, these demands can be suitably multiplied to find out the requirement of each of these items that goes into final product.
Inventory may also be defined as stock of materials that a company or business holds to have a purpose of resale eventually or stock of goods which are kept for efficient working of a business or a company. A company may have small or large inventory or we can say companies have variation in inventories, comprising of small items such as food ingredients, stationary equipments etc and some big items such as vehicles or machines. An organization generally retains an equitable stock of goods to certify easy operations.

1.1 General Inventory Models

Inventory model is a model that helps in discovering the optimum level of inventories that must be preserved in a production process, organizing frequency of ordering, determining on quantity of goods or raw materials to be stored to provide successive service to customers without any delay in delivery. Probabilistic or deterministic nature of demand is a key factor that affects the output.

Inventory problems consist of receiving orders and also placing orders of given sizes at some intervals. If there is an item or material that is to be bought then there are two very important decisions that are to be made associated with purchase of the material or we can say inventory policy answers two questions given below:

1. How much to order ?
2. When to order ?

The answer to the first question is determined by economic order quantity by minimizing the cost model as given below:

Total inventory cost = Purchasing cost (it depends on price per unit of goods) + Setup cost (it tells about the expenses incurred during the placement of an order) + Holding cost (it tells about cost of maintaining goods in the stock) + Shortage cost (the penalties faced when we run out of stock)

Now further to answer the another question dependence is completely upon the system of inventory one is dealing with. The system might require periodic review (when the time for receiving the new order coincides with start of each period) or continuous review (when inventory is dropped to a point called reorder point then new orders are placed).

1.1.1 Classic Economic Order Quantity (EOQ) Model

Classic EOQ model is the basic model of many inventory models. It concerns with single item only. The main aim is to decide economic order quantity y which minimizes the total cost of an inventory system when there is constant demand and instantaneous order replenishment with no shortages allowed. The model is developed under following assumptions:

1. It deals with single item only.
2. The demand rate is constant and it is known.
3. The ordering cost is also constant.
4. Quantity discounts are not accessible.
5. No shortages are allowed.
6. Lead time constant and is known.

Let us assume:

- y = order quantity (number of units)
- D = demand rate (units per unit time)
- t_0 = ordering cycle (time units)

According to above assumptions, the inventory level follows the pattern as shown above in figure 1.1
When level of inventory is dropped to zero, an order is placed and received immediately of size y. Then at constant demand rate D stock is depleted uniformly. For this pattern, ordering cycle is:

$$t_0 = \frac{y}{D} \text{ time units}$$

The resulting average inventory level is given as:

$$\text{Average inventory level} = \frac{y}{2}$$

Two parameters of cost are required by this model:
Let the setup cost be denoted by K and holding cost be denoted by H. The total

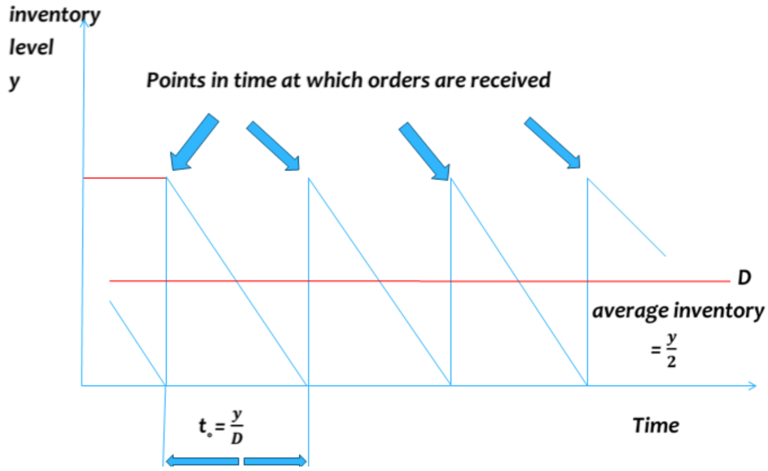

figure 1.1

cost per unit time can be calculated as:
$TCU(y)$ = set up cost per unit time + holding cost per unit time

$$= \frac{K + h(\frac{y}{2})t_0}{t_0}$$

$$= \frac{K}{\frac{y}{D}} + h(\frac{y}{2})$$

The most appropriate value for order quantity is decided by minimizing $TCU(y)$. Assuming y is continuous, we have a necessary condition given below to find value of y which must be optimal.

$$\frac{dTCU(y)}{dy} = \frac{KD}{y^2} + \frac{h}{2} = 0$$

As $TCU(y)$ is convex, the above condition is also sufficient. So from the solution of equation we have EOQ (y^*) :

$$y^* = \sqrt{\frac{2KD}{h}}$$

The optimum inventory is given as:

$$\text{Order } y^* = \sqrt{\frac{2KD}{h}} \text{units every } t_0^* = \frac{y^*}{D} \text{ time units}$$

In real life situations new orders cannot be received instantly so for such situations lead time might occur between the placement of an order and receipt of an order. In cases where lead time occurs and when the level of inventory gets dropped to to LD units then a point is achieved known as reorder point.

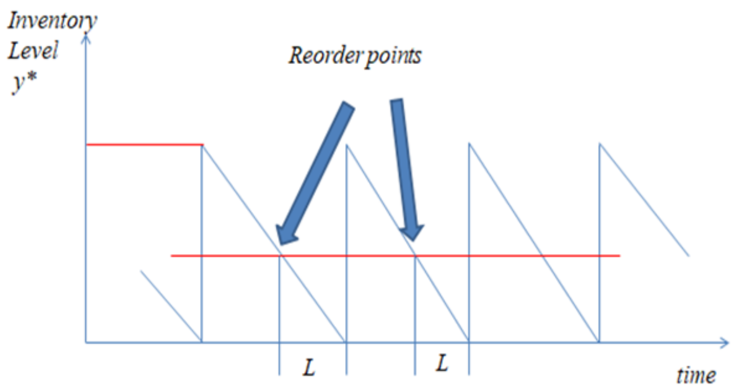

figure 1.2

Now in figure 1.2 , L denotes the lead time. It is also assumed that ordering cycle length t_0^* will be greater than lead time L (not generally). So for such situations where we get lead time greater that ordering cycle length, effective lead time is defined as:

$$L_e = L \quad nt_0^*$$

where n is the largest integer. Hence reorder point will occur at L_eD units and so we can order quantity y^* every time inventory drops to L_eD units.

1.1.2 Price Breaks Applied On Economic Order Quantity

In this model which is EOQ with price breaks the items of inventories might be bought at a discount whenever the limit q is exceeded by an order(of size y). The unit purchasing price c is given as:

$$c = \begin{cases} c_1, \ if \ y \leq q \\ c_2, \ if \ y > q \end{cases}$$

where $c_1 > c_2$
Hence,

$$\text{purchasing cost per unit time} = \begin{cases} \frac{c_1 y}{t_0} = \frac{c_1 y}{\frac{y}{D}} = Dc_1 \ , \ y \leq q \\ \frac{c_2 y}{t_0} = \frac{c_2 y}{\frac{y}{D}} = Dc_2 \ , \ y > q \end{cases}$$

The total cost per unit time is:

$$TCU(y) = \begin{cases} TCU_1(y) = Dc_1 + \frac{KD}{y} + \frac{h}{2}(y), y \leq q \\ TCU_2(y) = Dc_2 + \frac{KD}{y} + \frac{h}{2}(y), y > q \end{cases}$$

The functions TCU_1 and TCU_2 are shown in the figure 1.3, their minima occurs at a point y_m where,

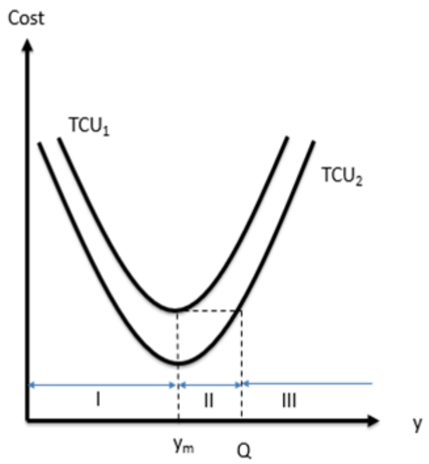

figure 1.3

$$y_m = \sqrt{\frac{2KD}{h}}$$

The cost function $TCU(y)$ gets started on the left along TCU_1 and falls to TCU_2 at the price break point q which might lie in zones I , II or III given in figure as $(0, y_m)$, (y_m, Q) , (Q, ∞) respectively. The value of Q is obtained from:

$$TCU_2(Q) = TCU_1(y_m)$$

From figure 1.4(a),(b),(c) , the optimum quantity y^* is:

$$y^* = \begin{cases} y_m, if\ q\ is\ in\ zones\ I\ or\ III \\ q, if\ q\ is\ in\ zones\ II \end{cases}$$

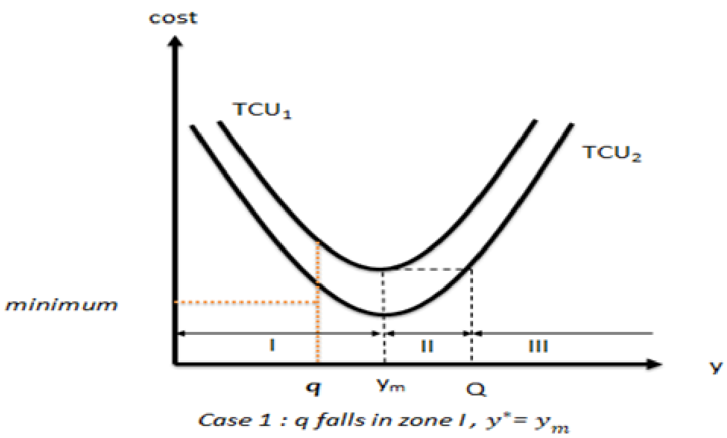

figure 1.4(a)

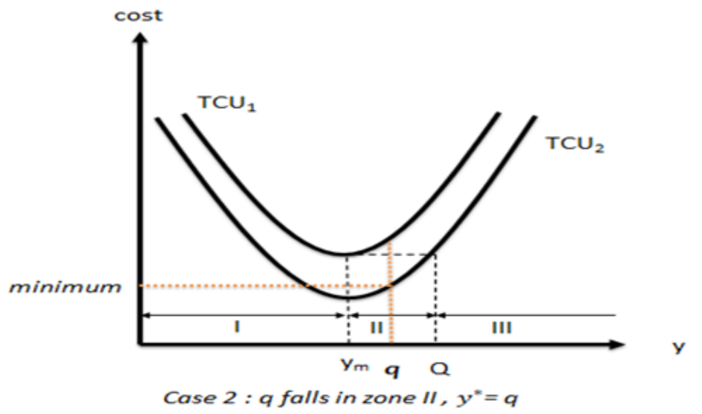

figure 1.4(b)

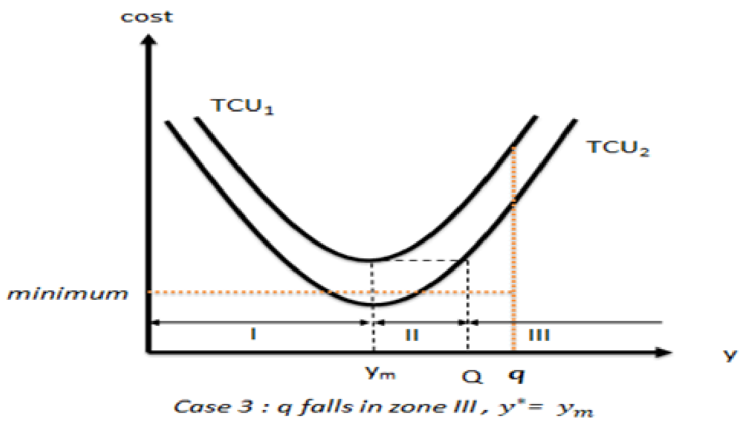

figure 1.4(c)

The following are steps given for determining y^*
Step 1. $y_m = \sqrt{\frac{2KD}{h}}$, if q is in zone I then $y^* = y_m$, else move further to step 2
Step 2. Determine the value of Q, if q is in zone II, $y^* = q$ otherwise q will be in zone III and $y^* = y_m$

1.1.3 Multi -Item EOQ With Storage Limitations

The model considers more than one item, fluctuations follow the pattern for inventory same as that in classic EOQ model(shortages are not allowed). The main difference from the classic EOQ model is that the items in this model are competing for a restricted storage space. Here,

D_i = rate of demand
K_i = cost of setup
h_i = Unit holding cost per unit time
y_i = Order quantity
a_i = Storage area requirement per inventory unit
A = Maximum available storage area
(for i = 1,2,...,n)

Under the assumption of no shortage, we have the following given expression:

$$\text{Minimize } TCU(y_1, y_2...y_n) = \sum_{i=1}^{n}(\frac{K_iD_i}{y_i} + \frac{h_iy_i}{2})$$

$$\text{subject to conditions : } \sum_{i=1}^{n} a_iy_i \leq A$$

$$y_i > 0, i = 1, 2, ..., n$$

The steps for the solving the model are :

step 1: Unconstrained optimal values of the order quantities are computed as :

$$y_i^* = \sqrt{\frac{2K_iD_i}{h_i}}, i = 1, 2, ..., n$$

step 2: Check, if unconstrained optimal values of y_i^* satisfies the storage constraint and if it does then values of y_i^* , i = 1,2,...,n are optimal else move further to next step.
step 3: Now make use of Lagrange multipliers method to find out constrained optimal values.

The Lagrangian function here is formulated as:

$$L(\lambda, y_1, y_2...y_n) = TCU(y_1, y_2...y_n) \quad \lambda(\sum_{i=1}^{n} a_iy_i \quad A)$$

$$= \sum_{i=1}^{n}(\frac{K_iD_i}{y_i} + \frac{h_iy_i}{2}) \quad \lambda(\sum_{i=1}^{n} a_iy_i \quad A), where \lambda < 0$$

The optimal values of y_i and λ are known from following given necessary conditions (as Lagrangian function is convex):

$$\frac{K_iD_i}{y_i^2} + \frac{h_i}{2} \quad \lambda a_i = 0$$

$$\sum_{i=1}^{n} a_iy_i + A = 0$$

We get:

$$y_i^* = \sqrt[2K_iD_i]{h_i \quad 2\lambda^*a_i}$$

Hence y_i^* is dependent on λ^*, now if $\lambda^* = 0$, y_i^* will give unconstrained solution.

1.2 INTEGRATED INVENTORY MODEL WITH DETERIORATING ITEMS FOR VENDOR AND BUYER (Yang and Wee(2000))

In this section an integrated inventory model with deteriorating items for single vendor and single buyer given by Yang and Wee (2000) has been discussed.
A supply chain is a network that generally consists of distributors, manufacturers and retailers. In a competitive world, buyer is the one who has the power to determine about the number of deliveries whenever an order is placed. The number of optimal deliveries opted by the buyer may not economical for the vendor. When the number of deliveries are considered in cooperation to the vendor, then the entire cost (which is integrated) can be minimized.
The supply chain coordination is the most important issue in supply chain management research. To get an outcome for such situations several researches have been done in view of integrated approach. Here an economic ordering policy of deteriorated items with constant rate of demand and production along with few required assumptions is developed. It's proven that integrated approach is a way better than the independent approach.

1.2.1 Assumptions

1. The rate of production is constant .
2. The rate of demand is constant.
3. Shortages are not allowed.
4. An item (single) with a constant rate of deterioration is considered.
5. Deterioration of the units is examined only after they have been received into inventory.
6. There is no replacement or rectification of deteriorated units.
7. Carrying cost is concerned to good units only.
8. Single producer and single distributor are considered.
9. There are several deliveries per order.
10. There is one production cycle only.

1.2.2 Notations

p = Production rate.
d = consumer's demand rate.
I_{t_1} = Inventory level that changes with time t_1 during production period.
I_{t_2} = Inventory level that changes with time during non-production period.
T_1 = The production period in each cycle.
T_2 = The non-production period in each cycle.
T = Time length of cycle.
θ = Rate of deterioration.
n = Number of deliveries per order.
$I_{pc}(t)$ = Vendor's inventory level.
$I_b(t)$ = Buyer's inventory level.
C_{ob} = Buyer's ordering cost per order.

C_{sv} = Vendor's setup cost,per production cycle.
C_{cb} = Buyer's inventory carrying cost,per time and per unit.
C_{cv} = Vendor's inventory carrying cost,per time per unit.
C_b = buyer's deterioration cost.
C_v = vendor's deterioration cost.
K_{0b} = Buyer's incoming control cost per delivery.
TC_b = Buyer's total cost function.
TC_v = Vendor's total cost function.
TC = integrated total cost function which includes TC_b and TC_v.

1.2.3 Formulation Of The Model

The main motive of this model is to discover the optimum profit for items having constant rate of deterioration and constant demand. The differential equations $I_{pc}(t)$ which describes the inventory level for a vendor with consumer demand are as follows:

$$\frac{dI_1(t_1)}{dt_1} = (p \quad d) \quad \theta I_1(t_1);\ 0 \le t_1 \le T_1 \tag{1.1}$$

$$\frac{dI_2(t_2)}{dt_2} = \quad d \quad \theta I_2(t_2);\ 0 \le t_2 \le T_2 \tag{1.2}$$

using Spiegal (1960) , the boundary conditions obtained are as $I_1(0) = I_2(T_2) = 0$ and solution obtained are as follows:

$$I_1(t_1) \exp \theta(t_1) = (p \quad d) \int \exp \theta(t_1) dt_1 + C_1$$

$$I_2(t_2) \exp \theta(t_2) = \int \quad \exp \theta(t_2) d + C_2$$

Applying these boundary conditions : $I_1(t_1) = 0$ and $I_2(T_2) = 0$

we get:

$$C_1 = \quad \frac{p \quad d}{\theta} \; and \; C_2 = \frac{d}{\theta} \exp \theta T_2$$

so the final equations are:

$$I_1(t_1) = \frac{p \quad d}{\theta}\Big(1 \quad \exp \quad (\quad \theta t_1)\Big); 0 \ \le t_1 \le T_1 \tag{1.3}$$

$$I_2(t_2) = \frac{d}{\theta}\Big(1 \quad \exp\Big(\theta(T_2 \quad t_2)\Big)\Big); 0 \ \le t_2 \le T_2 \tag{1.4}$$

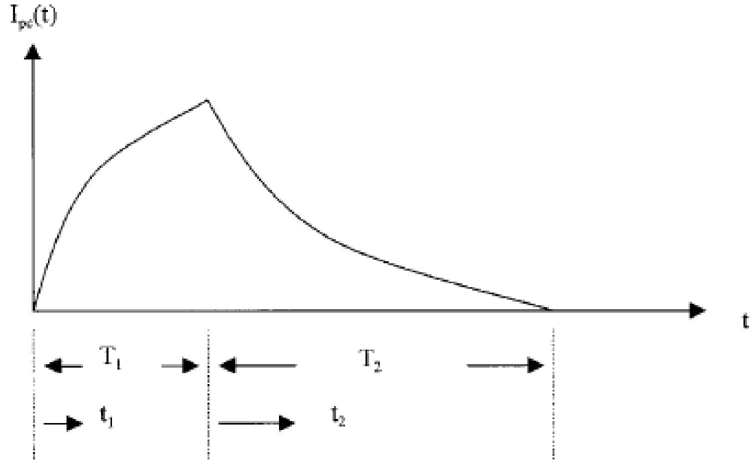

figure 1.5

The inventory level depicted in figure 1.5 with boundary conditions $I_1(T_1) = I_2(0)$

We get:

$$(p \quad d)(1 \quad \exp(\quad \theta T_1)) = \quad d(1 \quad \exp(\theta T_2)) \tag{1.5}$$

Value of θ is assumed to be very small $\theta \ll 1$, because deterioration rate can never be zero so by using taylor's expansion, following expression is obtained:

$$(p \quad d)T_1(1 \quad (1 \quad \theta T_1 \quad \frac{\theta T_1^2}{2} \quad \frac{\theta T_1^3}{6})) = \quad d(1 \quad (1 + \theta T_2 + \frac{\theta T_1^2}{2} + \frac{\theta T_1^3}{6}))$$

By solving it and neglecting the higher terms we get:

$$(p \quad d)T_1(1 \quad \frac{1}{2}\theta T_1) = dT_2(1 + \frac{1}{2}\theta T_2) \tag{1.6}$$

From Misra (1975) and from equation(1.6) , production period of each cycle is obtained as:

$$T_1 \approx \frac{d}{p \quad d}T_2(1 + \frac{t}{2}\theta T_2) \tag{1.7}$$

Total length obtained of the time cycle is given as:

$$T = T_1 + T_1 \tag{1.8}$$

$$T = \frac{d}{p \quad d} T_2 (1 + \frac{1}{2}\theta T_2) + T_2 \tag{1.9}$$

The final expression for total length of time cycle by solving equations is obtained as follows:

$$T \approx \frac{T_2}{p \quad d}(p + \frac{1}{2}\theta T_2) \tag{1.10}$$

Inventory function of n deliveries for a buyer per order is given as follows:

$$\frac{d}{dt} I_b(t) = \quad d \quad \theta I_b(t) \tag{1.11}$$

We get inventory level of the buyer as follows:

$$I_b(t) = \frac{d}{\theta}\left(\exp\left(\theta\left(\frac{T}{n} \quad t\right)\right) \quad 1\right); 0 \leq t \leq \frac{T}{n} \tag{1.12}$$

At $t = 0$, we get maximum inventory $I_b(t)$ as:

$$I_b(t = 0) = \frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) \quad 1\right) \tag{1.13}$$

Figure 1.6 shows the inventory level of buyer with respect to time. By the assumption of θ « 1 and by the taylor's expansion we applied earlier, the total cost function for the buyer TC_b is obtained as follows:

$$TC_b(t) = \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}} I_b(t)dt + C_b\frac{n}{T}\left(I_b(0) \quad \frac{T}{n}(d)\right)$$

Now substituting the values from equations (1.12) and (1.13) in above expression, we get:

$$TC_b(t) = \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\int \frac{d}{\theta}\left(\exp\left(\theta\left(\frac{T}{n} \quad t\right)\right) \quad 1\right)dt$$

$$+C_b\frac{n}{T}\left(\frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) \quad 1\right) \quad \frac{T}{n}d\right)$$

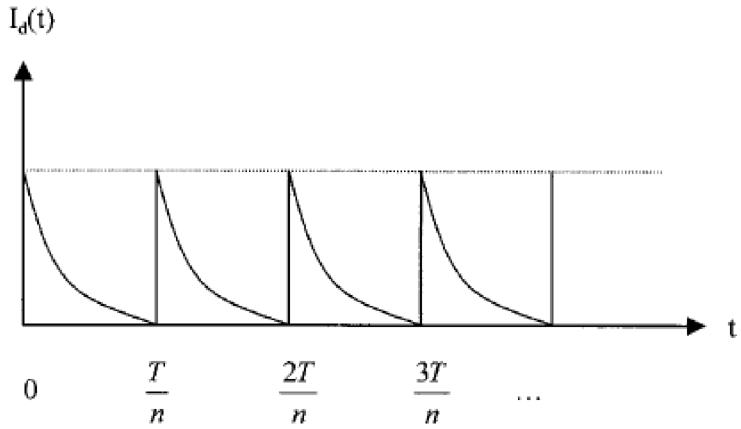

figure 1.6

$$= \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\frac{d}{\theta}\frac{1}{\theta}\left(\left(\quad 1 + \exp\frac{\theta T}{n}\right) \quad \frac{T}{n}\right)$$

$$+C_b\frac{n}{T}\left(\frac{d}{\theta}\exp\frac{\theta T}{n} \quad 1\right) \quad \frac{T}{n}d\Bigg)$$

$$= \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\frac{d}{\theta}\left(\frac{1}{\theta}\left(\quad 1 + 1 + \frac{\theta T}{n} + \frac{\theta^2 T^2}{2n^2} + \frac{\theta^3 T^3}{6n}...\right)\right)$$

$$+C_b\frac{n}{T} + C_b\frac{n}{T}\frac{d}{\theta}\left(\left(\left(1 + \frac{\theta T}{n} + \frac{\theta^2 T^2}{2n^2} + \frac{\theta^3 T^3}{6n}...\right) \quad 1\right) \quad \frac{Td}{n}\right)$$

$$= \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\frac{dT^2}{2n^2}\left(1 + \frac{\theta T}{3n} + \frac{\theta^2 T^2}{12n^2} + ...\right) + C_b\frac{n}{T}$$

$$+C_b\frac{n}{T}\frac{d}{\theta}\left(\frac{\theta^2 T^2}{2n^2} + \frac{\theta^3 T^3}{6n^3} + ...\right)$$

$$TC_b(t) \approx \frac{1}{T}(C_{ob} + nK_{0b}) + \frac{C_{ob}Td}{2n}\left(1 + \frac{\theta T}{3n}\right) + \frac{C_b T d\theta}{2n} \qquad (1.14)$$

The total cost function for the vendor is as follows:

$$TC_v = \frac{1}{T}(C_{sv} + nK_{0v}) + C_{cv}\frac{1}{T}\left(\int_0^{T_1} I_1(t_1)dt_1 \int_0^{T_2} I_2(t_2)dt_2 \quad n\int_0^{\frac{T}{n}} I_b(t)dt\right)$$

$$+C_v\frac{1}{T}\left(pT_1 \quad dT \quad n\left(I_b(0) \quad \left(\frac{T}{n}\right)d\right)\right)$$

Now from equations (1.3),(1.4),(1.12) and (1.13) the expression obtained is as follows for total cost of the vendor:

$$\frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\int_0^{T_1}\left(\frac{p\quad d}{\theta}(1\quad \exp(\quad \theta t_1))\right)dt_1$$

$$+\int_0^{T_2}\frac{d}{\theta}\Big(1\quad \exp(\theta(T_2\quad t_2))\Big)dt_2$$

$$n\int_0^{\frac{T}{n}}\left(\frac{d}{\theta}\left(\exp\left(\theta\left(\frac{T}{n}\quad t\right)\right)\quad 1\right)\right)dt$$

$$+C_v\frac{1}{T}\left(pT_1\quad dt\quad n\left(\frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right)\quad 1\right)\quad \left(\frac{T}{n}\right)d\right)\right)$$

Now again by the same assumption applied on θ

$$\begin{aligned}TC_v \approx \frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\frac{(p\quad d)T_1^2}{2}\left(1\quad \frac{\theta T_1}{3}\right)+\\ \frac{dT_2^2}{2}\left(1+\frac{\theta T_2}{3}\right)\quad \frac{dT^2}{2n}\left(1\quad \frac{\theta T}{3n}\right)\Bigg)+\\ \frac{C_v}{T}\left(pT_1\quad Td\quad \frac{d\theta T^2}{2n}\right)\end{aligned} \tag{1.15}$$

The function of total cost TC is the sum of buyer's cost TC_b and vendor's cost TC_v i.e.,

$$TC = TC_b + TC_v$$

Now by substituting the values of buyer's cost TC_b and vendor's cost TC_v from equations (1.14) and (1.15) in above equation for total cost(TC), the following expression is obtained:

$$\begin{aligned}TC = \frac{1}{T}(C_{ob}+nK_{0b})+\frac{C_{ob}Td}{2n}\left(1+\frac{\theta T}{3n}\right)+\frac{C_bTd\theta}{2n}\\ +\frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\frac{(p\quad d)T_1^2}{2}\\ \left(1\quad \frac{\theta T_1}{3}\right)+\frac{dT_2^2}{2}\left(1+\frac{\theta T_2}{3}\right)\\ \frac{dT^2}{2n}\left(1\quad \frac{\theta T}{3n}\right)\Bigg)+\frac{C_v}{T}\left(pT_1\quad Td\quad \frac{d\theta T^2}{2n}\right)\end{aligned} \tag{1.16}$$

1.3 Literature Review

Inventory is a term that symbolizes materials or commodities which a company uses for the objective of the sale and production. The basic apprehension is that it requires the theory of inventory to attain the optimum level of investment and to claim the control system which is effective in order to minimize the total cost of inventory. Management of inventory plays a vital role which indicates certain issues :

Firstly helps in smooth production and secondly helps in enhancement of profit by minimizing the investment.

In (1915) Harris carried forward his study on inventory models. Later on, the model purposed by Harris was generalized by Wilson in (1934) and gave a formula for Economic Order Quantity. These models were formulated based on certain assumptions for single and multiple items. Ghare and Schrader (1963) first analyzed the decay problem and considered a constant rate of decay. Then Covert and Philip (1973) extended this model for variable rate by two parameters Weibull distribution as assumption. Misra(1975) considered both constant and variable demand and lot size model of production with no back logging and finds optimal inventory management under inflation.

In (1981) Dave and Patel developed another model for deteriorating items with time proportional to demand. In this process EOQ model with linear demand rate changing with time and constant fraction of the deterioration was assumed. An approach for total inventory calculations used by Joglenkar(1990) by remodeling the complex problem and provides analytical result for cost expression. Furthermore, Wee(1993) formulated a model with partial back ordering as a assumption and some numericals are used to illustrate this notion and the policy of model leading to minimize cost has been shown. Then Wee in (1995) modified this model by considering the replenishment policy where the demand declines exponentially over a fixed item horizon and complete back ordering of demand is assumed. Continuing this study Wee in (1997) developed a model on the policy of replenishment with price dependent demand and rate of deterioration varying. Then Ganeshan (1999) again developed a model by keeping unit price as constant, retailers as identical and many suppliers along with single distribution center.

Various approaches are used to evaluate the total cost firstly when number of buyers are less then two and secondly when it is more than two buyers sensitivity analysis is conducted to formulate the changes in the parameters of total cost in joint total cost reduction for buyer and vendor both by using integrated approach.Assuming constant demand and production rate Ghiami and Williams (2015) evolved two echelon production inventory model for did your rating items. They commenced physical inventory and excellence talk of vendor in the above model during production and non-production time.In a lot of cases, demand of item is considered as a constant but will change according to time in real situations. Therefore, to evaluate the effect of time on demand Ouyang et al. (2005) developed a model for deteriorating with demand declining exponentially with partial backlogs and allowed shortages. In realistic inventory system, para meters are breakable and probabilistic. Vats(2014) Considered a model with demand as quadratic function is dependent with back locking depending on length of replenishment and shortages or not allowed analytical solutions has been attain for minimizing the cost, which is useful for inventory is where demand rate depends upon time and holding cost is constant. As Yang and Wee (2002) used heuristic approach to solve the model considered results were compared with yang and Wee (2000) to higher production rate than demand rate and drop in production time due to huge surplus is observed by Yang and Wee (2002) when there is the relaxation of huge surplus the solution obtained by Ghiami and Williams (2015) attains optimality .Rau et al. (2003) assumed that demand is smaller then production rate which drops a part of manufacturers production period in the developed model for single supplier, single manufacturer and single buyer.

CHAPTER II

2 An Integrated Inventory Model For Vendor And Buyer With Exponential Demand And Linear Rate Of Deterioration

2.1 Introduction

A constant demand rate and constant rate of deterioration for economic ordering policy of vendor and buyer has been considered by Yang and Wee (2000). In real situations, demand and deterioration rate of any product is not always constant. Keeping this in view, we developed an inventory model for deteriorating items in which both demand and deterioration rate have been replaced by exponential demand and linear deterioration rate respectively.

2.2 Assumptions and Notations

We have considered the same assumptions and notations proposed by Yang and Wee (2000) and also discussed in chapter I. However the constant demand and deterioration rate have been replaced by exponential demand and linear deterioration rate respectively and are given by:

$$d(t) = K\ exp(h \quad \beta t)$$

$$\theta = a + bt$$

2.3 Formulation Of The Model

The main motive of this model is to formulate a model for items with exponential demand and linear rate of deterioration items with exponential demand and linear rate of deterioration. Following Yang and Wee (2000) the differential equations governing the new inventory model are

$$\frac{dI_1(t_1)}{dt_1} = (p \quad Ke^{h \quad \beta t_1}) \quad (a + bt_1)I_1(t_1)\ ;\ 0 \leq t_1 \leq T_1 \tag{2.1}$$

$$\frac{dI_2(t_2)}{dt_2} = \quad Ke^{h \quad \beta t_1} \quad (a + bt_2)I_2(t_2)\ ;\ 0 \leq t_2 \leq T_2 \tag{2.2}$$

Using speigal (1960), the boundary conditions obtained are as
$I_1(0) = I_2(T_2) = 0$

And on solving equation (2.1) the inventory level in the first interval is given by:

$$I_1(t_1)e^{(at_1+b\frac{t_1^2}{2})} = \int (p \quad Ke^{h \quad \beta t_1})e^{at_1+b\frac{t_1^2}{2}}dt_1 + C_1$$

$$I_1(t_1)e^{(at_1+b\frac{t_1^2}{2})} = \frac{pe^{(at_1+b\frac{t_1^2}{2})}}{a+bt_1} \quad K\frac{e^{h \quad \beta t_1+at_1+b\frac{t_1^2}{2}}}{\beta+a+bt_1}$$

$$I_1(t_1) = \frac{p}{a+bt_1} \quad K\frac{e^{h \quad \beta t}}{\beta+a+bt_1} + C_1 e^{\quad at_1 \quad b\frac{t_1^2}{2}}$$

By boundary condition $I_1(t_1)$=0,

we have,

$$C_1 = \frac{K}{a \quad \beta}e^{h} \quad \frac{p}{a}$$

The final expression for $I_1(t_1)$:

$$I_1(t_1) = \frac{p}{a+bt_1} \quad K\frac{e^{h \quad \beta t_1}}{\beta+a+bt_1} + (\frac{K}{a \quad \beta}e^{h} \quad \frac{p}{a})e^{\quad at_1 \quad b\frac{t_1^2}{2}} \tag{2.3}$$

And on solving the differential equation (2.2) , the inventory level in the interval [0,T] is given by:

$$I_2(t_2)e^{(at_2+b\frac{t_2^2}{2})} = \quad K\int e^{h \quad \beta t_2+at_2+b\frac{t_2^2}{2}} + C_2$$

$$I_2(t_2)e^{(at_2+b\frac{t_2^2}{2})} = \frac{Ke^{h \quad \beta t_2+at_2+\frac{t_2^2}{2}}}{\beta+a+bt_2} + C_2$$

$$I_2(t_2) = \frac{Ke^{h \quad \beta t_2}}{\beta+a+bt_2} + C_2 e^{\quad at_2 \quad b\frac{t_2^2}{2}}$$

By boundary condition $I_2(t_2)$=0,

We have,

$$C_2 = K\frac{e^{h \quad \beta T_2}}{\beta+a+bT_2}e^{aT_2+b\frac{T_2^2}{2}}$$

The final expression for $I_2(t_2)$:

$$I_2(t_2) = \frac{Ke^{h \quad \beta t_2}}{\beta + a + bt_2} + K\frac{e^{h \quad \beta T_2 + aT_2 + b\frac{T_2^2}{2} \quad at_2 \quad b\frac{t_2^2}{2}}}{\beta + a + bT_2} \tag{2.4}$$

Further by using the boundary conditions:

$$I_1(T_1) = I_2(0)$$

we get the following result:

$$\frac{p}{a + T_1} \qquad K\frac{e^{h \quad \beta T_1}}{\beta + a + bT_1} + \frac{K}{a \quad \beta}e^{h \quad aT_1 \quad b\frac{T_1^2}{2}} \qquad \frac{p}{a}e^{\quad aT_1 \quad b\frac{T_1^2}{2}} \tag{2.5}$$

$$= \quad K\frac{e^{h}}{\beta + a} + K\frac{e^{h \quad \beta T_2 + aT_2 + b\frac{T_2^2}{2}}}{\beta + a + bT_2}$$

Now by applying binomial expansion and ignoring the higher order terms we get:

$$\frac{pbT_1}{a^2} + \frac{bkT_1}{(a \quad \beta)^2} + \frac{hbKT_1}{(a \quad \beta)^2} \qquad \frac{\beta KT_1}{\beta \quad a} \qquad \frac{aT_1K}{a \quad \beta} + pT_1$$
$$= \frac{KbT_2}{(a \quad \beta)^2} \qquad \frac{hbKT_2}{(a \quad \beta)^2} \qquad \frac{\beta KT_2}{a \quad \beta} + \frac{aKT_2}{a \quad \beta} \tag{2.6}$$

$$T_1[\frac{pb}{a^2} + \frac{bk}{(a \quad \beta)^2} + \frac{hbK}{\beta \quad a} \qquad \frac{\beta K}{\beta \quad a} \qquad \frac{aK}{a \quad \beta} + p]$$

$$= T_2[\frac{Kb}{(a \quad \beta)^2} \qquad \frac{hbK}{(a \quad \beta)^2} \qquad \frac{\beta K}{a \quad \beta} + \frac{aK}{a \quad \beta}$$

$$T_1 = T_2\frac{\frac{Kb}{(a \quad \beta)^2} \qquad \frac{hbK}{(a \quad \beta)^2} \qquad \frac{\beta K}{a \quad \beta} + \frac{aK}{a \quad \beta}}{\frac{pb}{a^2} + \frac{bk}{(a \quad \beta)^2} + \frac{hbK}{\beta \quad a} \qquad \frac{\beta K}{\beta \quad a} \qquad \frac{aK}{a \quad \beta} + p} \tag{2.7}$$

The total length is determined as follows:

$$T = T_1 + T_2 \tag{2.8}$$

Now by substituting the value of T_1, the total length becomes:

$$T = T_2\frac{\frac{Kb}{(a \quad \beta)^2} \qquad \frac{hbK}{(a \quad \beta)^2} \qquad \frac{\beta K}{a \quad \beta} + \frac{aK}{a \quad \beta}}{\frac{pb}{a^2} + \frac{bk}{(a \quad \beta)^2} + \frac{hbK}{(a \quad \beta)^2} \qquad \frac{\beta K}{\beta \quad a} \qquad \frac{aK}{a \quad \beta} + p} + T_2$$

$$T=\frac{\frac{KbT_2}{(a\quad\beta)^2}\quad\frac{hbKT_2}{(a\quad\beta)^2}\quad\frac{\beta KT_2}{a\quad\beta}+\frac{aKT_2}{a\quad\beta}+\frac{pbT_2}{a^2}}{\frac{pb}{a^2}+\frac{bk}{(a\quad\beta)^2}+\frac{hbK}{(a\quad\beta)^2}\quad\frac{\beta K}{\beta\quad a}\quad\frac{aK}{a\quad\beta}+p}$$

$$+\frac{\frac{bkT_2}{(a\quad\beta)^2}+\frac{hbKT_2}{(a\quad\beta)^2}\quad\frac{\beta KT_2}{\beta\quad a}\quad\frac{aKT_2}{a\quad\beta}+pT_2}{\frac{pb}{a^2}+\frac{bk}{(a\quad\beta)^2}+\frac{hbK}{(a\quad\beta)^2}\quad\frac{\beta K}{\beta\quad a}\quad\frac{aK}{a\quad\beta}+p}$$

$$T=\frac{\frac{pbT_2}{a^2}+pT_2}{\frac{pb}{a^2}+\frac{bk}{(a\quad\beta)^2}+\frac{hbK}{(a\quad\beta)^2}\quad\frac{\beta K}{\beta\quad a}\quad\frac{aK}{a\quad\beta}+p}\tag{2.9}$$

The differential equation for n deliveries per order (for buyer) is given as follows:

$$\frac{d}{dt}I_b(t)=\quad Ke^{h\quad\beta t}\quad(a+bt)I_b(t)\tag{2.10}$$

By solving the above equation, we get the the inventory function for buyer as given below:

$$I_b(t)=\frac{Ke^{h\quad\frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}\quad at\quad\frac{bt^2}{2}}}{\beta+a+\frac{bT}{n}}\quad\frac{Ke^{h\quad\beta t}}{\beta+a+bt};\ 0\le t\le\frac{T}{n}\tag{2.11}$$

Buyer's maximum inventory $I_b(t)$ at t=0 is:

$$I_b(t=0)=\frac{Ke^{h\quad\frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta+a+\frac{bT}{n}}\quad\frac{Ke^{h}}{\beta+a}\tag{2.12}$$

The total cost function TC_b for the buyer is expressed as:

$$TC_b(t)=\frac{1}{T}(C_{ob}+nK_{0b})+C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}}I_b(t)dt+C_b\frac{n}{T}\left(I_b(0)\quad\frac{T}{n}(e^{h\quad\beta t})\right)$$

By substituting the values from equation (2.11) and (2.12) in the above equation for buyer's total cost TC_b, the following expression is obtained:

$$
\begin{aligned}
TC_b(t) = \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}} \Bigg(& \frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2} \quad at \quad \frac{bt^2}{2}}}{\beta + a + \frac{bT}{n}} dt \\
& \frac{Ke^{h \quad \beta t}}{\beta + a + bt} dt \Bigg) \qquad (2.13) \\
+C_b\frac{n}{T}\Bigg(\Bigg(& \frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta + a + \frac{bT}{n}} \quad \frac{Ke^h}{\beta + a}\Bigg) \quad \frac{T}{n}(Ke^{h \quad \beta t})\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\Bigg(\frac{Ke^{h \quad \beta \frac{T}{n}}}{\left(\quad \beta + a + \frac{bT}{n}\right)\left(\quad a \quad \frac{bT}{n}\right)} \\
&+ \frac{Ke^{h \quad \beta \frac{T}{n} + \frac{aT}{n} + \frac{bT^2}{2n^2}}}{a\left(\quad \beta + a + \frac{bT}{n}\right)} + \left(\frac{K}{a \quad \beta} \quad \frac{hk}{a \quad \beta}\right)\frac{T}{n} \qquad (2.14) \\
&+\left(\frac{bK}{2(a \quad \beta)^2} + \frac{hbK}{2(a \quad \beta)^2} + \frac{K\beta}{2(a \quad \beta)}\right)\frac{T^2}{n^2} \quad \left(\frac{Kb\beta}{3(a \quad \beta)^2}\right)\frac{T^3}{n^3}\Bigg) \\
&+C_b\frac{n}{T}\Bigg(\Bigg(\frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta + a + \frac{bT}{n}} \quad \frac{Ke^h}{\beta + a}\Bigg) \quad \frac{T}{n}(Ke^{h \quad \beta t})\Bigg)
\end{aligned}
$$

The total cost function for the vendor is as follows:

$$
TC_v = \frac{1}{T}(C_{sv} + nK_{0v}) + C_{cv}\frac{1}{T}\left(\int_0^{T_1} I_1(t_1)dt_1 + \int_0^{T_2} I_2(t_2)dt_2 \quad n\int_0^{\frac{T}{n}} I_b(t)dt\right)
$$

$$
+C_v\frac{1}{T}\left(pT_1 \quad (Ke^{h \quad \beta t})T \quad n\left(I_b(0) \quad \left(\frac{T}{n}\right)(Ke^{h \quad \beta t})\right)\right)
$$

Now by substituting the values from equations (2.3), (2.4), (2.11) and (2.12) in the above expression for vendor's total cost TC_v, following result is obtained:

$$
\begin{aligned}
& TC_v = \tfrac{1}{T}(C_{sv} + nK_{0v}) + C_{cv}\tfrac{1}{T}\Bigg(\Bigg(\int_0^{T_1} \tfrac{p}{a+bt_1} \\
& K\tfrac{e^{h \quad \beta t_1}}{\beta+a+bt_1} + \left(\tfrac{K}{a \quad \beta}e^h \quad \tfrac{p}{a}\right)e^{\quad at_1 \quad b\frac{t_1^2}{2}} dt_1\Bigg) \\
& + \Bigg(\int_0^{T_2} \tfrac{Ke^{h \quad \beta t_2}}{\beta+a+bt_2} + K\tfrac{e^{h \quad \beta T_2 + aT_2 + \frac{bT_2^2}{2} \quad at_2 \quad \frac{bt_2^2}{2}}}{\beta+a+bT_2} dt_2\Bigg) \\
& n\Bigg(\int_0^{\frac{T}{n}} \tfrac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2} \quad at \quad \frac{bt^2}{2}}}{\beta+a+\frac{bT}{n}} \quad \tfrac{Ke^{h \quad \beta t}}{\beta+a+bt}dt\Bigg)\Bigg) \\
& +C_v\frac{1}{T}\left(pT_1 \quad (Ke^{h \quad \beta t})T \quad n\left(\frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta + a + \frac{bT}{n}} \quad \frac{Ke^h}{\beta + a} \quad \left(\frac{T}{n}\right)(Ke^{h \quad \beta t})\right)\right) \qquad (2.15)
\end{aligned}
$$

$$= \tfrac{1}{T}(C_{sv} + nK_{ov}) + C_{cv}\tfrac{1}{T}\Bigg(\Bigg(\Bigg(\Bigg(\tfrac{p}{a} \quad \tfrac{K}{a \quad \beta} \quad \tfrac{hK}{a \quad \beta}\Bigg)T_1$$

$$+ \Bigg(\tfrac{pb}{2a} + \tfrac{bK}{2(a \quad \beta)^2} + \tfrac{hbK}{2(a \quad \beta)^2} + \tfrac{K\beta}{2(a \quad \beta)}\Bigg)T_1^2 + \Bigg(\tfrac{Kb\beta}{3(a \quad \beta)^2}\Bigg)T_1^3$$

$$+ \tfrac{K}{a \quad \beta}\Bigg(\tfrac{e^{h \quad aT_1 \quad \frac{bT_1^2}{2}}}{a \quad bT_1}\Bigg) \quad \tfrac{p}{a}\Bigg(\tfrac{e^{\quad aT_1 \quad \frac{bT_1^2}{2}}}{a \quad bT_1}\Bigg) + \tfrac{Ke^h}{(a \quad \beta)a} \quad \tfrac{p}{a^2}$$

$$+ \Bigg(\Bigg(\tfrac{Ke^{h \quad \beta T_2}}{(\quad beta+a+bT_2)(\quad a \quad bT_2)}\Bigg) + \Bigg(\tfrac{K}{a \quad \beta} \quad \tfrac{hK}{2(a \quad \beta)}\Bigg)T_2$$

$$+ \Bigg(\tfrac{bK}{2(a \quad \beta)} + \tfrac{bhK}{2(a \quad \beta)^2} + \tfrac{K\beta}{2(a \quad \beta)}\Bigg)T_2^2 + \Bigg(\tfrac{Kb\beta}{2(a \quad \beta)^2}\Bigg)T_2^3 + \tfrac{Ke^{h \quad \beta T_2 + aT_2 + \frac{bT_2^2}{2}}}{a(\quad \beta+a+bT_2)}\Bigg)$$

$$n\Bigg(\frac{Ke^{h \quad \beta\frac{T}{n}}}{\Big(\quad \beta+a+\frac{bT}{n}\Big)\Big(\quad a \quad \frac{bT}{n}\Big)} + \frac{Ke^{h \quad \beta\frac{T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{a\Big(\quad \beta+a+\frac{bT}{n}\Big)} + \Bigg(\tfrac{K}{a \quad \beta} \quad \tfrac{hk}{a \quad \beta}\Bigg)\tfrac{T}{n}$$

$$+ \Bigg(\tfrac{bK}{2(a \quad \beta)^2} + \tfrac{hbK}{2(a \quad \beta)^2} + \tfrac{K\beta}{2(a \quad \beta)}\Bigg)\tfrac{T^2}{n^2} \quad \Bigg(\tfrac{Kb\beta}{3(a \quad \beta)^2}\Bigg)\tfrac{T^3}{n^3}\Bigg)$$

$$+ \frac{C_v}{T}\Bigg(pT_1 \quad (Ke^{h \quad \beta t})T \quad n\Bigg(\frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta + a + \frac{bT}{n}} \quad \frac{Ke^h}{\beta + a} \quad \frac{T}{n}(Ke^{h \quad \beta t})\Bigg) \tag{2.16}$$

The total cost function is obtained from sum of cost function of both vendor and buyer:

$$TC = TC_b + TC_v \tag{2.17}$$

Now by substituting the values from equations (2.15) and (2.17), we get the following result:

$$TC = \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\Bigg(\frac{Ke^{h \quad \beta\frac{T}{n}}}{\Big(\quad \beta + a + \frac{bT}{n}\Big)\Big(\quad a \quad \frac{bT}{n}\Big)} + \frac{Ke^{h \quad \beta\frac{T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{a\Big(\quad \beta + a + \frac{bT}{n}\Big)}$$

$$+\Bigg(\frac{K}{a \quad \beta} \quad \frac{hk}{a \quad \beta}\Bigg)\frac{T}{n} + \Bigg(\frac{bK}{2(a \quad \beta)^2} + \frac{hbK}{2(a \quad \beta)^2} + \frac{K\beta}{2(a \quad \beta)}\Bigg)\frac{T^2}{n^2} \quad \Bigg(\frac{Kb\beta}{3(a \quad \beta)^2}\Bigg)\frac{T^3}{n^3}\Bigg)$$

$$+C_b\frac{n}{T}\Bigg(\Bigg(\frac{Ke^{h \quad \frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta + a + \frac{bT}{n}} \quad \frac{Ke^h}{\beta + a}\Bigg) \quad \frac{T}{n}(Ke^{h \quad \beta t})\Bigg)$$

$$+\frac{1}{T}(C_{sv} + nK_{ov}) + C_{cv}\frac{1}{T}\Bigg(\Bigg(\Bigg(\Bigg(\frac{p}{a} \quad \frac{K}{a \quad \beta} \quad \frac{hK}{a \quad \beta}\Bigg)T_1$$

$$+\Bigg(\frac{pb}{2a} + \frac{bK}{2(a \quad \beta)^2} + \frac{hbK}{2(a \quad \beta)^2} + \frac{K\beta}{2(a \quad \beta)}\Bigg)T_1^2 + \Bigg(\frac{Kb\beta}{3(a \quad \beta)^2}\Bigg)T_1^3$$

$$+\frac{K}{a \quad \beta}\Bigg(\frac{e^{h \quad aT_1 \quad \frac{bT_1^2}{2}}}{a \quad bT_1}\Bigg) \quad \frac{p}{a}\Bigg(\frac{e^{\quad aT_1 \quad \frac{bT_1^2}{2}}}{a \quad bT_1}\Bigg) + \frac{Ke^h}{(a \quad \beta)a}$$

$$\frac{p}{a^2} + \Bigg(\Bigg(\frac{Ke^{h \quad \beta T_2}}{(\quad beta + a + bT_2)(\quad a \quad bT_2)}\Bigg) + \Bigg(\frac{K}{a \quad \beta} \quad \frac{hK}{2(a \quad \beta)}\Bigg)T_2$$

$$+\left(\frac{bK}{2(a\;\;\beta)}+\frac{bhK}{2(a\;\;\beta)^2}+\frac{K\beta}{2(a\;\;\beta)}\right)T_2^2+\left(\frac{Kb\beta}{2(a\;\;\beta)^2}\right)T_2^3+\frac{Ke^{h\;\;\beta T_2+aT_2+\frac{bT_2^2}{2}}}{a(\;\;\beta+a+bT_2)}\right)$$

$$n\left(\frac{Ke^{h\;\;\beta\frac{T}{n}}}{\left(\;\;\beta+a+\frac{bT}{n}\right)\left(a\;\;\frac{bT}{n}\right)}+\frac{Ke^{h\;\;\beta\frac{T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{a\left(\;\;\beta+a+\frac{bT}{n}\right)}+\left(\frac{K}{a\;\;\beta}\;\;\frac{hk}{a\;\;\beta}\right)\frac{T}{n}+\left(\frac{bK}{2(a\;\;\beta)^2}+\frac{hbK}{2(a\;\;\beta)^2}+\right.\right.$$

$$\left.\frac{K\beta}{2(a\;\;\beta)}\right)\frac{T^2}{n^2}\;\;\left(\frac{Kb\beta}{3(a\;\;\beta)^2}\right)\frac{T^3}{n^3}\Bigg)$$

$$+\frac{C_v}{T}\left(pT_1\;\;(Ke^{h\;\;\beta t)}T\;\;n\left(\frac{Ke^{h\;\;\frac{\beta T}{n}+\frac{aT}{n}+\frac{bT^2}{2n^2}}}{\beta+a+\frac{bT}{n}}\;\;\frac{Ke^{h}}{\beta+a}\;\;\frac{T}{n}(Ke^{h\;\;\beta t)}\right)\right) \qquad (2.18)$$

2.4 Validation Of Mathematical Expressions

On substituting $h=0,\ \beta=0,\ b=0,\ a=\theta$ and $K=d$, the demand and deterioration rate becomes constant and the expressions for the cost of buyer , vendor and the total cost becomes:

$$TC_b(t)\approx\frac{1}{T}(C_{ob}+nK_{0b})+\frac{C_{ob}Td}{2n}\left(1+\frac{\theta T}{3n}\right)+\frac{C_bTd\theta}{2n} \qquad (2.19)$$

$$\begin{aligned}TC_v\approx\frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\frac{(p\;\;d)T_1^2}{2}\left(1\;\;\frac{\theta T_1}{3}\right)+\\ \frac{dT_2^2}{2}\left(1+\frac{\theta T_2}{3}\right)\;\;\frac{dT^2}{2n}\left(1\;\;\frac{\theta T}{3n}\right)\Bigg)+\\ \frac{C_v}{T}\left(pT_1\;\;Td\;\;\frac{d\theta T^2}{2n}\right)\end{aligned} \qquad (2.20)$$

$$\begin{aligned}TC=\frac{1}{T}(C_{ob}+nK_{0b})+\frac{C_{ob}Td}{2n}\left(1+\frac{\theta T}{3n}\right)+\frac{C_bTd\theta}{2n}\\ +\frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\frac{(p\;\;d)T_1^2}{2}\\ \left(1\;\;\frac{\theta T_1}{3}\right)+\frac{dT_2^2}{2}\left(1+\frac{\theta T_2}{3}\right)\\ \frac{dT^2}{2n}\left(1\;\;\frac{\theta T}{3n}\right)\Bigg)+\frac{C_v}{T}\left(pT_1\;\;Td\;\;\frac{d\theta T^2}{2n}\right)\end{aligned} \qquad (2.21)$$

which is same as given by Yang and Wee (2000) given by equations (1.14), (1.15), (1.16) in chapter I.

2.5 Conclusion

In this chapter an inventory model for deteriorating items with exponential demand and linear rate of deterioration is formulated.

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