CIRCUIT REALIZATION OF FRACTIONAL ORDER PROPORTIONAL INTEGRAL CONTROLLER FOR DC MOTOR

A Dissertation submitted in fulfillment of the requirements for the Degree
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Submitted by

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DECLARATION

I hereby certify that the work which is presented in dissertation entitled, "Circuit realization of Fractional Order Proportional Integral controller for DC motor", in partial fulfillment of the requirements for the award of the degree of Master of Engineering in Electronic Instrumentation and Control, submitted to Electrical & Instrumentation Engineering Department of Thapar Institute of Engineering & Technology (Deemed to be University) is as authentic record of my own work carried under the supervision of Dr. Swati Sondhi. It refers others researcher's work which are duly listed in the reference section. The matter contained in this dissertation has not been submitted, neither in part nor in full to any other degree to any other university or institute except as reported in text and references.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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ABSTRACT

Fractional order controllers are more flexible and perform in a better way than the integer order controllers i.e. simple Proportional Integral or Proportional Integral Derivative controller. The fractional order controller has the properties such as eliminating the steady state error, robustness towards the plant gain variations and also good disturbance rejection. In this work fractional order controller is designed for DC motor to control its speed and analogue realization of the controller is being performed. Mathematical model of DC motor is calculated. Stabilization of the system is done by calculating the parameters of the PI^{λ} controller. Then the values of the parameters that are obtained are been plotted in the (k_p, k_i) plane and the stability region for the PI^{λ} controller is observed. The resulted fractional order controller is designed with the help of Fractional Power Pole method. Circuit realization of fractional order Proportional Integral controller is done by approximating the rational function and it known as approximation method. Circuit testing of the controller is done through simulations. The proposed controller gives the desired results as the output tracks the set reference point.

Keywords: Fractional Order Proportional Integral (PI^{λ}), Proportional gain (k_p), Integral gain (k_i)

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION TO DC MOTOR

DC motors consists of armature winding, stator, rotor and commutator. Whenever voltage is being applied to the coils, motion is generated by the torque produced in the armature [1].

Stator- It is the part of the motor that remain motionless. It consists of pieces of permanent magnet pole. An electromagnet is used to create magnetic field. Stator is formed by wounding coil on a magnetic component.

Rotor- It is the part which rotates and carries the mechanical rotations. It consists of windings that are connected with the help of commutators to the external circuit. Stator and rotor are separated with the help of air-gap.

Commutator- It is made from piling mica and copper segments together for insulation between them. It is a moving part and is used to opposite the direction of current which is between the rotor and external circuit [2]. The winding is positioned in the rotor and supply current is brought to it with the help of commutator from mains with brushes in DC motor.

1.1.1 Principle of DC motor

The principle states that as soon as a conductor having current (I) is located in a magnetic field (B) and conductor gets force (F). It is related to electromagnetism [1]. The force is shown as:

$$Force\ (F) = B \times I \times L \tag{1}$$

Where

$F =$ Force applied

$B =$ Magnetic Field applied

$L =$ Length

$I =$ Current in conductor

It is also called Lorentz force. The motor runs when the rotor is rotated by the action of torque because of force.

1.1.2 Modelling of DC motor

Armature controlled dc motor is considered and by ignoring the effect of demagnetization [3]. Circuit around the magnetic field is linear and field voltage is constant i.e. i_f . Figure 1.1 shows DC motor which is being restricted with the help of an armature and its transfer function is calculated.

Let

$L =$ Self inductance

$R_a =$ Armature resistance

$i_a =$ Armature current

$i_f =$ Field current

$V =$ Applied voltage

$\theta =$ Displacement

$J =$ Moment of inertia

$T =$ Torque developed by the motor

$B =$ Coefficient of friction

$E =$ Induced emf in an armature

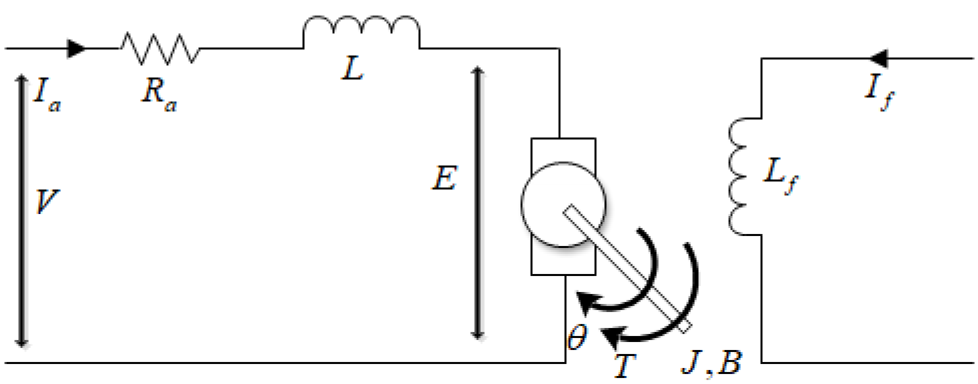

Figure1.1: Armature controlled DC motor [3]

Apply Kirchhoff's voltage law in the circuit as given in Figure 1.1

$$V = R_a i_a + L\frac{di_a}{dt} + E \tag{2}$$

Since field current i_f is constant the flux φ will be constant. When armature is rotating emf is induced across it [4].

$$E \propto \varphi \times \omega \tag{3}$$

$$E = K_b \omega \tag{4}$$

$$E = K_b \frac{d\theta}{dt} \tag{5}$$

$\omega =$ Angular Velocity

$K_b =$ Back emf constant

Now torque is a product of torque constant and armature current [5].

$$T \propto f \times i_a \tag{6}$$

$$T = K \times i_a \tag{7}$$

$K =$ Motor torque constant

The differential equation which is resulted from armature circuit is as shown [5]:

$$L \frac{di_a}{dt} + R_a i_a + E - V = 0 \tag{8}$$

The torque equation [5] is obtained as follow:

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \tag{9}$$

Take the Laplace transform of Equation (2)

$$V(s) - E(s) = \mathrm{I}_a(s)\ (R_a + Ls\ ) \tag{10}$$

Take the Laplace transform of Equation (5)

$$E(s) = K_b \times s\theta(s) \tag{11}$$

Take the Laplace transform of Equation (9)

$$T(s) = \ K\ I_a(s) = \left(Js^2 + Bs\right)\theta(s) \tag{12}$$

By using these equations block diagram is made with a feedback loop [6] represented in Figure 1.2 and then the block reduction method is performed to obtain its input output relationship in terms of transfer function and is shown in Figure 1.3.

Now determining the transfer function by block reduction method [5] is given as below:

$$\frac{\theta(s)}{V(s)} = \frac{K}{s\left(R_a\left(1+\frac{sL}{R_a}\right)B\left(1+s\frac{J}{B}\right)+KK_b\right)} \tag{13}$$

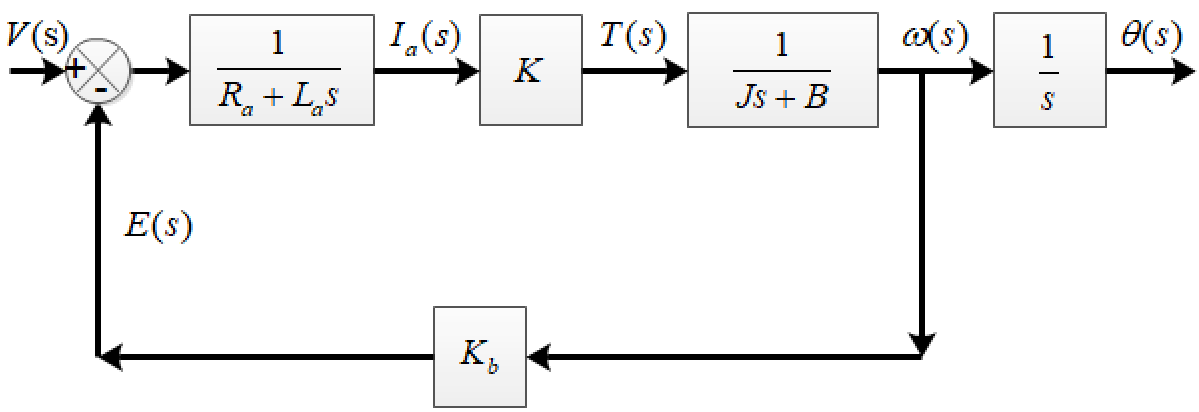

Figure1.2: Representation of DC motor restricted with armature [3]

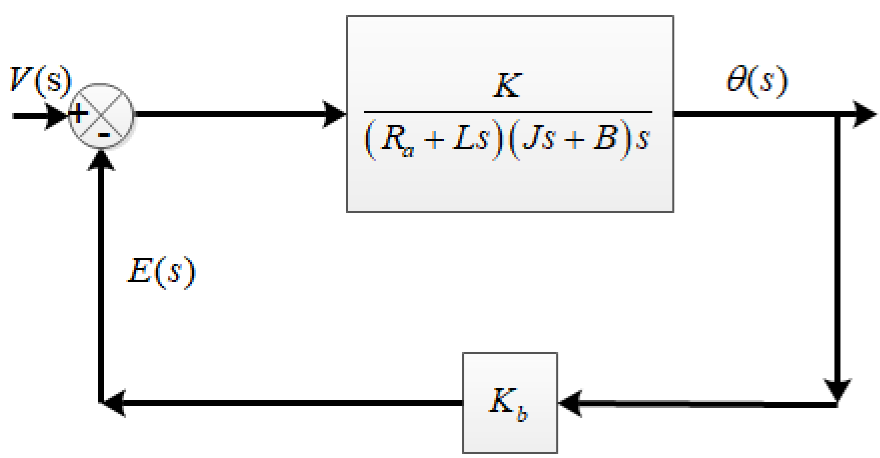

Figure1.3: Simplified representation of DC motor

The relationship between output and input for speed controlled DC motor is calculated from Figure 1.4 and represented by taking an angular velocity and the applied voltage as output and input respectively. The function obtained [3] is represented as:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K}{R_a\left(1+\frac{sL}{R_a}\right)\times B\left(1+s\frac{J}{B}\right)+KK_b} \tag{14}$$

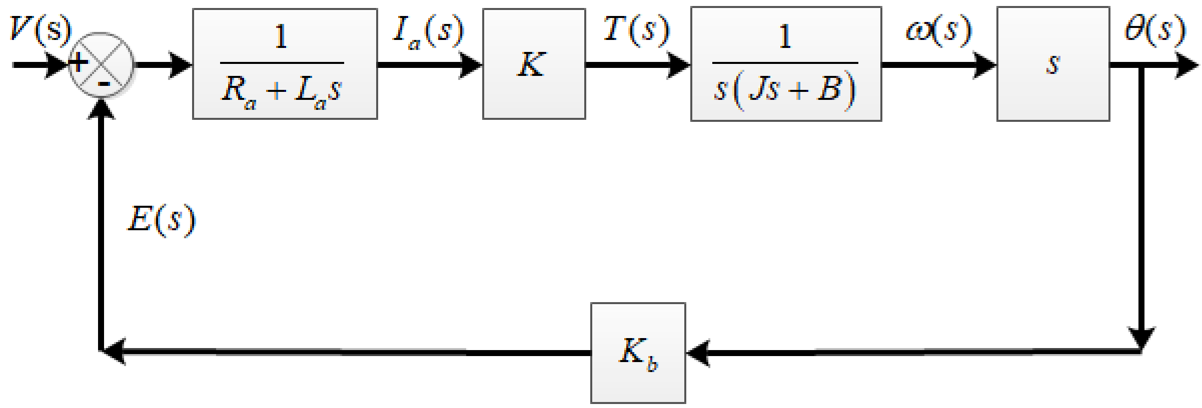

Figure1.4: Representation of speed control DC motor [3]

1.1.3 The Series RLC circuit

In series RLC circuit shown in Figure 1.5 similar current flows through all the components in one loop. The RLC circuit response will change with the frequency [7]. Source voltage value will be the combination of individual voltages of the components and calculated by Kirchhoff's voltage law (KVL).

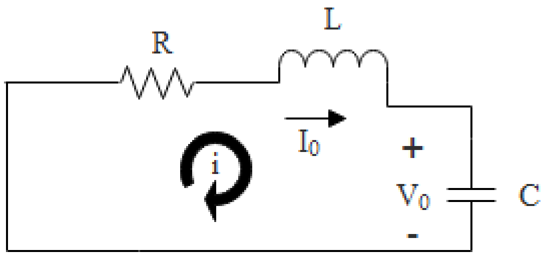

Figure1.5: Series RLC circuit [7]

$$i(0) = I_0 \tag{15}$$

$$v_c(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0 \tag{16}$$

Where v_c = Voltage across capacitor

Applying KVL to the circuit

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} tdt = 0 \tag{17}$$

Taking differentiation

$$\Rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{t}{LC} = 0 \tag{18}$$

To solve Equation (18), the value of $\frac{di(0)}{dt}$ should be known.

Equation (15), (16) and (17) gives

$$Ri(0) + L\frac{di(0)}{dt} + V_0 = 0 \tag{19}$$

$$\frac{di(0)}{dt} = -\frac{(RI_0 + V_0)}{L} \tag{20}$$

Let

$$i = Ae^{st} \tag{21}$$

A and s are assumed to be constants [7].

$$As^2 e^{st} + \frac{AR}{L} se^{st} + \frac{A}{LC} e^{st} + 0 \tag{22}$$

$$Ae^{st}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0 \tag{23}$$

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ is known as the characteristic equation.

Solving the characteristic equation two roots are obtained [7] which are as follow:

$$s_1 = -\frac{R}{2L} + \sqrt{\frac{R^2C - 4L}{4L^2C}} \tag{24}$$

$$s_2 = -\frac{R}{2L} - \sqrt{\frac{R^2C - 4L}{4L^2C}} \tag{25}$$

1.1.4 Equating DC motor transfer function to RLC circuit

To simulate the DC motor an electronic equivalent is obtained. The RLC circuit transfer function of 2nd order is given as follow:

$$Transfer\ Function\ =\ \frac{\left(\frac{1}{LC}\right)}{\left(s^2+\left(\frac{R}{L}\right)s+\frac{1}{LC}\right)} \tag{26}$$

Comparing the RLC circuit equation among DC motor relationship of input and output i.e. transfer function $K,\ R,\ L$ and C are calculated.

1.2 FRACTIONAL CALCULUS

Fractional calculus is used for real and complex number powers of derivatives and integrals. The main concept of fractional calculus is a fractional derivative.

Differintegral operator: It is a generalized form of differential and integral operator and denoted by aD_t^γ. Oldham and Spanier [8] named it as differintegral operator as have both derivative and integral. It is represented as follow:

$$aD_t^\gamma = \begin{cases} \frac{d^\gamma}{dt^\gamma}, & \Re(\gamma) > 0 \\ 1, & \Re(\gamma) = 0 \\ \int_a^t \left(d\tau\right)^{-\gamma}, & \Re(\gamma) < 0 \end{cases} \tag{27}$$

In above equation γ is used for the fractional order, a is unvarying value and $\Re(\gamma)$ is the fractional order real part.

Grunwald - Letnikov gave the most fundamental definition [9] of fractional integral and derivative having order γ.

$$aD_t^\gamma f(t) = \lim_{h\to\infty} \frac{1}{h^\gamma} \sum_{j=0}^{\frac{t}{h}-\frac{a}{h}} (-1)^j \binom{\gamma}{j} f(t-jh) \tag{28}$$

$$\binom{\gamma}{j} = \frac{\Gamma\left(\gamma+1\right)}{\Gamma\left(\gamma-j+1\right)\Gamma\left(j+1\right)} \tag{29}$$

$\Gamma(x)$ is Euler's gamma function, $f(t)$ is function applied, h is increment in time, $\lfloor . \rfloor$ is flooring operator. The above definition shows that finite and infinite number of series is required by integer-order derivatives and fractional-order derivatives respectively [9].
Laplace transform method is used for analysis and synthesis of control system. When differintegral operator is undergone Laplace transform [10].

Reimann-Liouville Fractional Derivative:
To evaluate the fractional derivative of this type ${}_aD_t^{\gamma} f(t)$ of power function [11] is used.
Assume that $n-1 \leq \gamma < n$

$${}_aD_t^{\gamma} f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int \left(\frac{f(\tau)d\tau}{(t-\tau)^{\gamma-n+1}} \right) \tag{30}$$

1.2.1 Fractional Calculus Properties

- When $f(t)$ becomes an analytic function then the differentiation of it is also analytic.
- When $\alpha = 0$ then ${}_aD_t^{0} f(t) = f(t)$.
- The differentiation and integration of the fractional operator is linear.
- Additive index law

$${}_aD_t^{\alpha}\, {}_aD_t^{\beta} = {}_aD_t^{\beta}\, {}_aD_t^{\alpha} = {}_aD_t^{\alpha+\beta} f(t) \tag{31}$$

1.3 FRACTIONAL ORDER SYSTEMS

Fractional order means dynamic model having non integer order. Non integer order is used to characterize the real dynamic systems [12] in a better way. Let us consider the transfer function of non integer order:

$$G(s) = \frac{N(s)}{D(s)} \tag{32}$$

$G(s) =$ Transfer function having non integer order

$$G(s) = \frac{b_n s^{\beta_n} + b_{n-1} s^{\beta_{n-1}} + \ldots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}} \tag{33}$$

$$G(s)=\frac{\sum_{i=0}^{n} b_i s^{\beta_i}}{\sum_{i=0}^{n} a_i s^{\alpha_i}} \tag{34}$$

Where $a_i, b_{i,} \beta_{n \text{ to } 0} \geq 0$ and $\alpha_{n\ to\ 0} \geq 0$ are arbitrary real numbers [12].

$$\sum_{i=0}^{n} a_i D^{\alpha_i} y(t) = \sum_{i=0}^{n} b_i D^{\beta_i} u(t) \tag{35}$$

$y(t)$ and $u(t)$ represents output and input of plant respectively.

1.3.1 Fractional order PI^{λ} and $PI^{\lambda}D^{\mu}$ control systems

A Single Input Single Output system with non integer system as represented in Figure 1.6. The Figure 1.6 y represents the outcome of the system, r represents the reference for the system, e represents the difference between output and input and u represents the signal from the controller. In PI^{λ} / $PI^{\lambda}D^{\mu}$ controller $G(s)$ and C(s) represent the system and controller transfer function [12]. The system may be chosen as simple PI/PID or fractional order PI/PID.

The controller transfer function is obtained by the output that is in form of control signal and the input that is in form of error [13]. The transfer function [12] of controller is given as:

$$C(s)=\frac{U(s)}{E(s)}=k_p+\frac{k_i}{s^{\lambda}}+k_d s^{\mu} \tag{36}$$

Where λ and μ lie between 0 and 2.

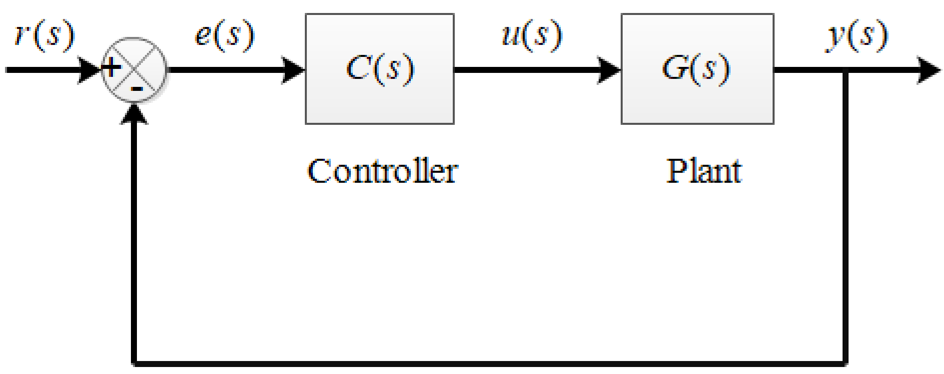

Figure1.6: Single Input Single Output Non Integer order control system [12]

Eliminating the differential element, the $PI^{\lambda}D^{\mu}$ controller is changed to PI^{λ} controller represented in the form [12]:

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s^{\lambda}} \tag{37}$$

When $\lambda = 1$ and $\mu = 1$ in Equation (36), gives a PID controller, $\lambda = 1$ and $\mu = 0$ gives a PI controller and $\lambda = 0$ and $\mu = 1$ gives a PD controller. Fractional order controller is robust [14] and the properties are adjusted in a suitable manner.

1.3.2 Linear Time Invariant (LTI) Non Integer order system stability

LTI is stable when roots value is negative for the characteristic polynomial and for complex conjugate it has negative real part and lie on left side of complex plane [15]. The non integer and integer order differ in the stability [16].

When the non integer order LTI system is stable roots lie in complex plane as given in Figure 1.7. The fractional order system is shown by:

$${}_aD_t^q x(t) = Ax(t) + Bu(t) \tag{38}$$

$$y(t) = Cx(t) \tag{39}$$

Where $x \in R^n, y \in R^p$ and $u \in R^T$ are given for state vectors, input vectors and output vectors of the system and q is for the fractional order [17].

System given in Equation (38) and Equation (39) is stable if [17]

$$|\arg(eig(A))| > q\frac{\pi}{2} \tag{40}$$

Where q lie between 0 and 1.

Matignon's stability theorem [17]: For the stability of non integer function $G(s) = \frac{N(s)}{D(s)}$

$$|\angle(\sigma)|\, q\frac{\pi}{2}, \forall_{\sigma} \in C, P(\sigma) = 0 \tag{41}$$

Equation (41) must lie in σ-plane.

In Equation (41) value of $\sigma = s^q$. When there is $\sigma = 0$ it is unstable. When $q = 1$ in the complex plane it has all poles in left half side.

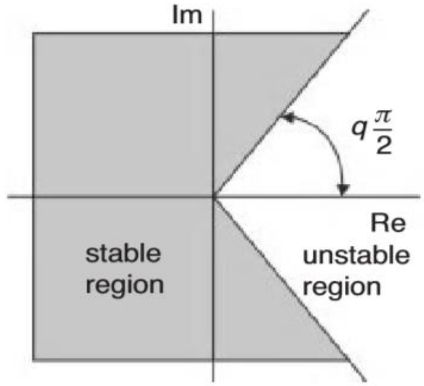

Figure1.7: Non Integer order stability region [17]

1.3.3 Stability with the help of Fractional Order PI^{λ} controller

The system considered with the unity feedback [12] shown in Figure 1.6. $G(s)$ is for the system to control and $C(s)$ is a PI^{λ} controller.

$$G(s) = \frac{N(s)}{D(s)} \tag{42}$$

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s^{\lambda}} \tag{43}$$

To calculate the parameters of the controller the output of system given as:

$$y(s) \;=\; \frac{C(s)G(s)}{1+C(s)G(s)} r(s) \tag{44}$$

$1+C(s)G(s)$ in above Equation (44) is known as fractional-order characteristic polynomial. The fractional order characteristic polynomial of control system is calculated with the help of Equations (42), (43), and (44) and is obtained as [18]:

$$P(s;k_p;k_i;\lambda) = \mathrm{s}^{\lambda}\mathrm{D(s)} + (\mathrm{k}_p\mathrm{s}^{\lambda} + \mathrm{k}_i)\,\mathrm{N(s)} \tag{45}$$

$$P(s;k_p;k_i;\lambda) = \sum_{i=0}^{n}\left[a_i\,\mathrm{s}^{(\alpha_i+\lambda)} + \mathrm{k}_p\mathrm{b}_i\mathrm{s}^{(\beta_i+\lambda)} + \mathrm{k}_i\mathrm{b}_i\mathrm{s}^{(\beta_i)}\right] \tag{46}$$

Fractional order PI controller having three parameters $\left(\lambda, k_p, k_i\right)$ is bounded input bounded output stable when the polynomial defined above lies in open left hand side [18].

In parameter space P the stability domain S is the region where $\left(\lambda, k_p, k_i\right) \in S$. To design the PI^{λ} controller stability domain S must be determined.

The characteristic polynomial [18] can be described as:

$$P(s) = p_k s^{q_k} + p_{k-1} s^{q_{k-1}} + \ldots + p_1 s^{q_1} + p_0 s^{q_0} = \sum_{i=0}^{k} p_i s^{q_i} \tag{47}$$

In the above equation p_i represent the coefficients and q_i represent the fractional orders of the Fractional Order Characteristic Polynomial (FOCP). p_i coefficients is function of both k_p and k_i depending on the order of numerator and denominator of polynomials.

In parameter space P there are two domains: stability and instability domains and the boundaries that lie between them are represented in 3 parts [12]:

i. Real Root Boundary (RRB):

   At $s=0$ the imaginary axis is being crossed by a real root. The P(s) given in Equation (47) is substituted by $s=0$ a real root boundary is obtained. The boundary can be represented as $p_0 = 0$ only when the least order of the FOCP i.e. $q_0 = 0$ has value equal to zero i.e. $s^{q_0} = 1$.

ii. Complex Root Boundary (CRB):

   At $s = j\omega$ the imaginary axis are being crossed by a complex roots pair. In Equation (47) the real and the imaginary parts become zero so equation is unstable.

iii. Infinite Root Boundary (IRB):

   At $s = \infty$ the imaginary axis are being crossed by a real root. In Equation (47) take $p_k = 0$ and boundary is indentified.

1.3.4 Advantages of Fractional PID

i. Reduce steady-state error.
ii. Robustness to the change of gain.
iii. Reject noise having high frequency.
iv. More parameters to be tuned and improved control.

1.4 STRUCTURE OF THE DISSERTATION

Chapter 2 – Literature Review:

In this chapter various papers are discussed related on fractional order controllers, DC motor and stability of the controllers.

Chapter 3 – Proposed Methodology:

In this chapter the problem to be solved is given and solutions are presented to solve it. The transfer function of DC motor and its stabilizing parameters are obtained. The values to design fractional order controller are calculated.

Chapter 4 – Results and Discussion:

In this chapter detailed information on how the simulation is obtained in a PSpice environment is given and the values of the controller are obtained.

Chapter 5 – Conclusion:

In this chapter conclusion is given.

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CHAPTER 2

LITERATURE REVIEW

Speed Control of DC Motor Using PID Controller Based on Artificial Intelligence Techniques [17]

In this paper by choosing appropriate parameters of a PID a DC motor for a speed controller is designed with the help of two techniques. First one is the Genetic Algorithm and the second one is Adaptive Neuro – Fuzzy Inference System. In this the DC motor is represented as a non linear model. Also the comparison is made between tuning methods that are being used to calculate the PID controller parameters. Different tuning methods are: Ziegler and Nicholas, Genetic Algorithm and Adaptive Neuro – Fuzzy Inference System is used. The DC motor modelling is done for third order system.

The PID which is designed with the help of Adaptive Neuro – Fuzzy Inference System related algo on genetics responses fast and show less error than the classical method. The methods that have been stated in this can also be used for higher order systems. When Genetic Algorithm is applied with the Adaptive Neuro – Fuzzy Inference System or without it the optimization is obtained.

Fractional order PID controller for load frequency control [19]

Load frequency control is used in systems where power is an essential element. The load frequency control should not change with the parameters and should reject disturbances entering in the systems. The fractional order controller removes the error, does not change with the changes occurred in the plant and reject the disturbances therefore it is better than already available options. In this paper fractional order PID with load frequency control is made for the turbines. The fractional order controller is made by combining all three individual controllers of fractional P, I and D. Two types of turbines are used steam and hydro turbines.

The model and fractional order controller is made for each of them. Integral error criterion is used for the controller to be optimized. To check the controller robustness 50% uncertainty is being added to the parameters of the system. The fractional order controller is used to check uncertainty. It is noticed that the controller works in an improved manner than other techniques.

Fractional order PID controller for perturbed load frequency control using Kharitonov's theorem [20]

In designing of load frequency control the main problem is related to the uncertainty and the disturbances. The fractional order controller is used because it reduces error and provides rejection to the disturbances. Interval model is used for the plant because it provides more robustness as a number of values is selected for the parameters. Here combining of the load frequency control with interval model is done. Limits are not permanent in this method and can vary accordingly. The designing of controller is done with the help of . In this theorem there are four polynomials and to check the stability the Routh Hurwitz method is used. If the polynomials present are stable in Routh Hurwitz method then the system is stable. The largest stability region is taken. Optimization is done with the help of integral error methods.

This lead to show good performance but sometimes changing the parameters in an abrupt manner can break the system.

The comparison is made with other techniques and it is noted that the controller provided in this paper shows an improved performance and it is stable for large time. The controller for non reheated turbine removes the disturbances entering to the system prior to the peak and is robust. The controller for reheated turbine also removes the disturbances. Various integral error criterions are used to compare different controllers. The system developed in this paper is much capable for robustness and reduce the chances for the system to get break. This method can be extended to the multi area load frequency control.

Simulation work on Fractional Order PI^{λ} Control Strategy for speed control of DC motor based on stability boundary locus method [21]

In this paper to control the speed of DC motor a Fractional Order Proportional Integral controller is designed. DC motor modelling is done and with the help of it a controller is made using stability boundary locus method. The global stability region is obtained by with the help of λ in (k_p, k_i) plane. In this DC motor has a second order system. The parameters that are used to stabilize the controller is calculated with the help of global stability region. With the help of different test points residing either inside or outside of a stability locus curve the global stability region is made. Point tracking of the non integer order controller is performed and being related to the Integer Order Relay Feedback Proportional Integral controller also the loading tracking is done. Stability for both the systems is analyzed.

Fractional order PID controller for power control in perturbed pressurized heavy water reactor [22]

In this paper fractional order controller for interval method is used to control the power. The system taken is non linear Pressurized Heavy Water Reactor. Method of stability boundary locus is used. Eight models of the reactor are considered using Edge theorem. The integral error methods are used for optimization. Different step back conditions are considered. When the perfect value of parameters are not available then a certain range of values are taken this type of method is called interval method.

Edge theorem states that if bounded polynomials are stable then all other polynomials are stable. Pressurized Heavy Water Reactor is related to rods (control, adjust) and zone control compartments. When it is used under different values of the load or any other condition where the power had to decrease. To implement this control rods are used. Different transfer function for the method of interval is taken from the reactor. The stability regions are drawn for each of them. Simulations are done on different transfer function with different rod positions. The set point tracking is better in the controller for all the different functions. Comparison of the performance is done with other controllers. The proposed controller show good performance and robustness. Safety for breakdown is also presented.

Speed Control of DC Motor Using PID and FOPID Controllers Based on Differential Evolution and PSO [23]

In this paper DC motor speed calculated using PID and Fractional order Proportional Integral Derivative. Differential Evolution and Particle Swarm Optimization are methods for tuning Fractional order Proportional Integral Derivative FOPID controller. The DC motor has a second order system. To eliminate the errors that is in the process variable which is measured and in a set point which is being desired. For the performance calculation the fitness function is taken as integral of time weighted absolute error.

With the help of soft computing techniques the better results are observed. The Differential Evolution improves the performance of FOPID in time domain rejects the noise and is robust. The FOPID-PSO gives better dynamic properties than PID-PSO. The Differential Evolution gives improved convergence and stability.

Fractional-Order PI Control of DC Servo System Using the Stability Boundary Locus Approach [24]

In this paper a servo system is stabilized by using Fractional Order PI controller with help of locus boundary method. Path to perform hardware implementation on system precision modular servo system is used. MATLAB and Simulink are used to get the performance of the controllers. Concept of gain phase testor is used to design the controller. The global stability region is observed in the intersection of both complex and real boundaries.

Simulation results show that the target value is reached in near about 2 sec. The tracking of the controller is smooth because it does not have an overshoot and error. For the hardware testing of the DC motor three cases are considered. In the first case no load is applied to the motor to see the performance of controller. The speed that is to be required is attained in 1 sec. In the second case the magnetic brakes for 3 sec act as a load. In this the disturbances that are external to the system are rejected by the controller in near about 2 sec. In the third case at 5 sec the load gets detached from the motor. In this speed to be required is attained in 2 sec and after detaching the load in 2 sec the disturbances are settled.

Stabilization using fractional-order PI and PID controllers [12]

In this paper the dynamic system having fractional orders are stabilized with the help of Fractional Order Proportional Integral and Fractional Order Proportional Integral Derivative controllers. The stability of region of FOPI and FOPID controller are plotted in $\left(k_p, k_i\right)$ plane and $\left(k_p, k_i, k_d\right)$ space respectively. The system is stable if the right half of plane which is closed does not have any root and known as stability of bounded input bounded output. To stabilize the system the controller parameters is calculated step by step. Electrical radiator is taken and its transfer function which is of fractional order is used for further calculations. With the help of $\left(k_p, k_i\right)$ values the complex root boundary curve and real root boundary curve is made. The intersection of both the curves gives global stability region. For Fractional Order Proportional Integral Derivative controller stability region real root boundary and infinity foot boundary line is considered. Depending on the order of derivative part different number of regions is obtained. To stabilize the fractional PID controller a temperature controlled system is taken and its parameters values are obtained. The order of derivative part will lead to number of stability regions in this case it comes three. When derivative order has value less than 1 it gives the smallest global stability region when evaluated with other two.

Fractal System as Represented by Singularity Function [25]

In this paper physical phenomenon when characterized on log-log Bode plot gives a fractional slope. To study the dynamical behavior of the system a discussion on the method of arranging pole zero in cascade manner is done. To solve a fractal system having a single value is given a singularity structure in the paper. An approximation method which is in a graph form is used to select the pole zero pairs or singularities. Poles and zeros are in a geometric progression. It shows that the slope obtained from the approximation of a transfer function give same slope as the original transfer function. Also it shows that the calculation of system's distribution spectrum can be easily done. The RC structure replaces these pole zero pairs. The error between the required line and the zig-zag lines is considered to be constant. Minimum the error larger the pole zero number. The single fractal system is extended to the multiple fractal system it has many fractional power poles. Result of multiple fractal system is shown by the combination of single fractal system. Examples of both the single and multiple fractal systems are given. The error shown by the fractional or multiple fractal system and the bandwidth will give total pole-zero terms for the structure. The phase spectra are also obtained.

Analogue realisation of fractional-order integrator, differentiator and fractional $PI^{\lambda}D^{\mu}$ controller [18]

In this paper a method to approximate with the help of rational function for the non integer order integrator and differentiator is presented. The fractional power pole and zero method are used to model the non integer order integrator and differentiator respectively in a specific frequency. Then the approximation method rationalize the fractional power pole which replaces the original line with the number of zig-zag lines which are alternating slopes of 0 and -20 db/dec. This method was further extended to fractional power zero. To calculate the parameters the graphical method is used. Then the rational function is being expanded by the partial fraction. The partial fraction of fractional order integrator is similar to the impedance of parallel RC cells connected in series. The analogue realization circuit is made from the RC structure. The rational function is used to approximate fractional order differentiator and its partial function is similar to the admittance of series RC cells connected parallel. Similarly for fractional PID controller the irrational transfer function is converted into rational one. Then its analogue realization is done by parallel combination of proportional, integration and derivative components and each component is realized separately. Then the realization of controller is made with Op-amps, integration, differential and proportional elements.

Optimal Parameter Selection of Fractional Order PI^{λ} Controller for Speed Control of DC Motor [26]

In this paper dynamic systems are described by using non integer order differentiator and integrator. The speed control, modelling of motor being discussed and also the comparison in methods for selecting the controller is done. For the control scheme two loops are considered inner loop for integer order proportional integral controller and outer loop for non integer order proportional integral controller. The two techniques Particle swarm optimization (PSO) and Bacterial foraging optimization is used. A new function is proposed which is non linear by using error, overshoot, settling and rising time. It is shown that Bacterial foraging optimization technique shows better result to minimize the settling time. It can be shown on system of real time. In the company of 50% torque load the tracking of speed in the controllers is observed.

Efficient Analog Implementations of Fractional-Order Controllers [27]

In this paper the non integer order controllers are realized. Feasibility of the network that comes out as a result and the performance of controller are checked. The fractional order process and controllers are done with the approximation of fractance component. To work with fractance network a toolset is being designed in MATLAB. An object plays an important role in FOMCON. Example of fractance network is given and is shown how a main structure is being broken down into substructures. The approximation is done to make hardware by generalizing different structures of the network and make it to come under one object.

CHAPTER 3

PROPOSED METHODOLOGY

3.1 OBJECTIVES

- Mathematical modelling of DC motor.
- Stabilization of Fractional Order Proportional Integral (FOPI) controller for DC motor.
- Analogue realization of Fractional Order Proportional Integral controller.
- Circuit testing through simulation.

3.2 PROPOSED WORK FLOW/ METHODOLOGY

System Description:

An armature controlled DC motor [3] is selected and its parameters are to be calculated for speed control. Modelling of DC motor represented in section 1.1.2 and DC motor is shown in Figure 3.1. $G(s)$ gives the output and input relationship and DC motor transfer function represented below:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K}{(Js+B)(Ls+R)+K^2} \tag{48}$$

In the above equation

$J =$ Moment of inertia

$R =$ Resistance of armature

$L =$ Self inductance

$B =$ Friction coefficient of motor

$K =$ Motor torque constant

$V(s) =$ Armature voltage considered the input

$G(s) =$ Transfer function of motor

$\omega(s) =$ Angular speed of motor shaft

Specification of the motor:

Selected DC motor as shown in Figure 3.1 with following specifications and all the calculations are done for getting its transfer function:

Figure3.1: DC motor of 1000rpm

Table 3.1: DC Motor Parameters

Moment of Inertia	0.03 Kgm^2
Maximum Speed of motor	1000 rpm
Coefficient of friction	0.019 Nms
Back emf constant	0.1331
Motor torque constant	0.1331
Armature Resistance	6Ω
Armature Inductance	$4.5mH$

Transfer function of DC motor:

On substituting the parameters values those are specified above in Equation (48). The mathematical model is obtained as:

$$G(s) = \frac{1.01}{0.001025s^2 + 1.367s + 1} \tag{49}$$

3.2.1 Stability of Non Integer Order PI and PID controllers

Taking fractional-order control system [12] where $G(s)$ is the plant or system taken and $C(s)$ is PI^{λ} controller which has form of

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s^{\lambda}} \tag{50}$$

To stabilize the system the calculation of parameters of the Fractional Order PI^{λ} controller is performed. The output is represented [12] as follow:

$$y(s) = \frac{C(s)G(s)}{1+C(s)G(s)} r(s) \tag{51}$$

Put $1+C(s)G(s)=0$

$$s^{\lambda}(0.001025s^2 + 1.367s + 1) + (k_p s^{\lambda} + k_i)(1.01) = 0 \tag{52}$$

Put $s = j\omega$

$$0.001025(j\omega)^{\lambda+2} + 1.367(j\omega)^{\lambda+1} + (j\omega)^{\lambda} + (k_p(j\omega)^{\lambda} + k_i)(1.01) = 0 \tag{53}$$

It is known [12] that

$$j^{\lambda} = \cos(\lambda\frac{\pi}{2}) + j\sin(\lambda\frac{\pi}{2}) \tag{54}$$

Put this value in the Equation (49)

$$\begin{aligned}&0.001025\left[\left(\cos(\lambda+2)\frac{\pi}{2}\right)+j\ \sin\left((\lambda+2)\frac{\pi}{2}\right)\right]\omega^{\lambda+2}+1.367\left[\left(\cos(\lambda+1)\frac{\pi}{2}\right)+j\ \sin\left((\lambda+1)\frac{\pi}{2}\right)\right]\omega^{\lambda+1}\\&+\left[\left(\cos(\lambda)\frac{\pi}{2}\right)+j\ \sin\left((\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}+1.01k_p\left[\left(\cos(\lambda)\frac{\pi}{2}\right)+j\ \sin\left((\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}+1.01k_i=0\end{aligned} \tag{55}$$

Separate real and imaginary term and then equate both the terms equal to zero

$$\begin{aligned}&0.001025\left[\left(\cos(\lambda+2)\frac{\pi}{2}\right)\right]\omega^{\lambda+2}+1.367\left[\left(\cos(\lambda+1)\frac{\pi}{2}\right)\right]\omega^{\lambda+1}+\left[\left(\cos(\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}\\&+1.01k_p\left[\left(\cos(\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}+1.01k_i=0\end{aligned} \tag{56}$$

$$0.001025\left[\sin\left((\lambda+2)\frac{\pi}{2}\right)\right]\omega^{\lambda+2}+1.367\left[\sin\left((\lambda+1)\frac{\pi}{2}\right)\right]\omega^{\lambda+1}$$
$$+\left[\sin\left((\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}+1.01k_p\left[\sin\left((\lambda)\frac{\pi}{2}\right)\right]\omega^{\lambda}=0 \tag{57}$$

By solving both the equations we get the values corresponding to k_p and k_i

$$k_p=\frac{-\left[0.001025\left[\sin\left((\lambda+2)\frac{\pi}{2}\right)\right]\omega^{2}+1.367\left[\sin\left((\lambda+1)\frac{\pi}{2}\right)\right]\omega^{1}+\left[\sin\left((\lambda)\frac{\pi}{2}\right)\right]\right]}{1.01\left(\sin\left(\frac{\lambda\pi}{2}\right)\right)} \tag{58}$$

$$k_i=\frac{\left[1.3534\omega^{\lambda+1}\right]}{\sin\left(\frac{\lambda\pi}{2}\right)} \tag{59}$$

Get value of ω from them. Set of global stability regions are found out with the help of different values of λ lie between (0, 2) for the PI^{λ} controller. Plot graph of (k_p, k_i) with the help of λ and ω in MATLAB. Figure 3.2 and Figure 3.3 represent the region for specific λ.

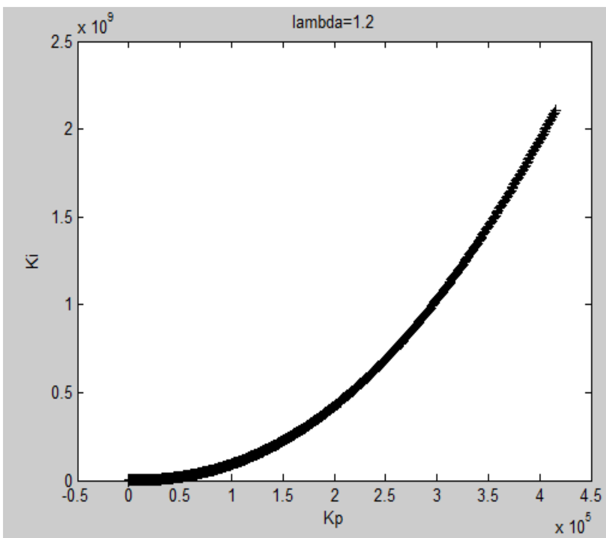

Figure3.2: Graph for λ= 1.2

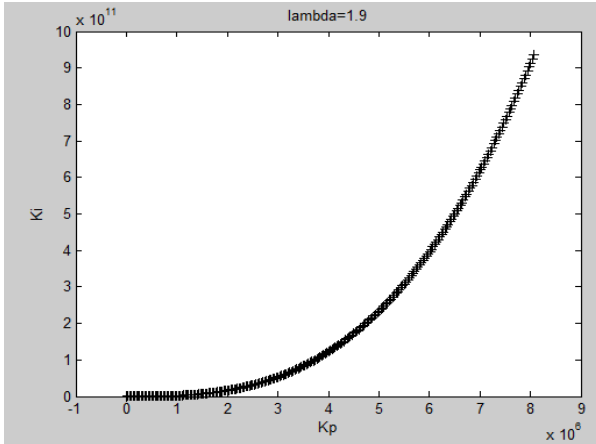

Figure3.3: Graph for λ= 1.9

$\lambda = 1.2$ provides the best global stability region.

Placing the values of the parameters in Equation (50) the controller *C(s)* is represented in form:

$$C(s) = 2.5732 + \frac{1.45204}{s^{1.2}} \tag{60}$$

3.2.2 Approximation Method

The log-log Bode plot has fractional slope on it and it helps to characterize the fractal system [14]. The dynamic behavior of the system is studied with the help of singularity function based method which consists of branches that are been cascaded of numbers of poles-zero pairs [25]. At very low frequency it gives finite magnitude therefore fractional power pole is most precise representation of frequency behaviour. In the frequency domain a single fractal system modelled by transfer function of single FPP [25] represented in form of:

$$\mathrm{H(s)} = \frac{1}{\left(1 + \frac{\mathrm{s}}{\mathrm{p_T}}\right)^{\mathrm{m}}} \tag{61}$$

$1/p_T$ is constant of time and m lie between 0 and 1

Approximation method used here approximate $-20m\ db/decade$ slope line with large numbers zig-zag lines that are joined with each other having the varying slopes of $0\ dB/decade$ and $-20\ dB/decade$ [25] it is represented in Figure 3.4. Approximate of the line of $-20m\ dB/decade$

with zeros and poles shown in Figure 3.4. The black line (thick) is for $-20m\ dB/decade$ slope and red line (zigzag) is for poles and zeros of fractional power pole that are approximated.

- In Figure 3.4. from $\log p_T$ point shift towards the X-axis till a point where the line having slope of $-20m\ dB/decade$ is ydb far away that mean BC equals to y. p_0 is the first pole at B. Taking assumption of error y equal to 1 db.
- Starting again at B make a line of $-20\ dB/decade$ to E where BE line and the original line of $-20m\ dB/decade$ line of slope remains y distance away from each other therefore EL equal to y. Extend the EL line upwards towards the X axis till D that gives zero z_0 .
- Repeating the steps to calculate all zeros and poles.

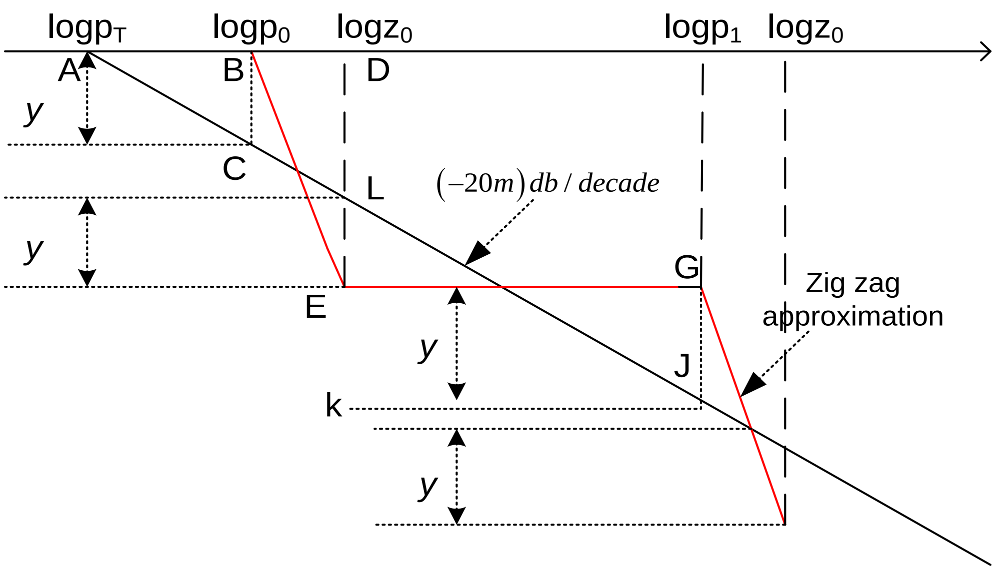

Figure3.4: Approximation Method [25]

By taking triangle ABC

$$\frac{y}{\log p_0 - \log p_T} = 20m, \log p_0 = \log p_T + 20m \tag{62}$$

Therefore the first pole [25]:

$$p_0 = p_T 10^{\left[\frac{y}{20m}\right]} \tag{63}$$

The relationship is represented as:

$$\frac{2y}{\log z_0 - \log p_0} + 20m = 20 \tag{64}$$

Therefore the first zero:

$$z_0 = p_0 10^{\left[\frac{y}{10(1-m)}\right]} \tag{65}$$

The second pole and second zero is:

$$p_1 = z_0\ 10^{\left[\frac{y}{10m}\right]},\ z_1 = p_1\ 10^{\left[\frac{y}{10(1-m)}\right]} \tag{66}$$

Where p_T is the corner frequency

Let

$$a = 10^{\left[y/10(1-m)\right]} \text{ and } b = 10^{\left[y/10m\right]} \tag{67}$$

Therefore

$$p_i = (ab)^i\ p_0 \text{ and } z_i = (ab)^i\ ap_0 \tag{68}$$

Therefore the approximated transfer function of fractional power pole [25] is:

$$H(s) = \frac{1}{\left(1 + \frac{s}{p_T}\right)^m} = \frac{\prod_{i=0}^{N-1}\left(1 + \frac{s}{(ab)^i\ ap_0}\right)}{\prod_{i=0}^{N}\left(1 + \frac{s}{(ab)^i\ p_0}\right)} \tag{69}$$

Maximum frequency $\omega_{\max}$ the rational approximation is done by N pairs of zeros and poles. The spread of frequency within log scale is $\log \omega_{\max} - \log p_0$. The ratio between the 2 poles is:

$$\frac{p_1}{p_0} = \ldots = \frac{p_N}{p_{N-1}} = ab \tag{70}$$

The distance between 2 poles in log scale [25] is:

$$(\log p_1 - \log p_0) = (\log p_2 - \log p_1) = ... = (\log p_N - \log p_{N-1}) = \log(ab) \tag{71}$$

$$N - 1 = \frac{\log\left(\omega_{\max}/p_0\right)}{\log\left(ab\right)} \tag{72}$$

Integer approximation [25] of above equation is represented as:

$$N = \text{integer}\left[\frac{\log\left(\omega_{\max}/p_0\right)}{\log\left(ab\right)}\right] + 1 \tag{73}$$

3.3 DESIGNING OF THE CONTROLLER

The equation to design a controller for a non integer order and is in a form of FOPI controller. $C(s)$ is the function of controller represented in form of:

$$C\left(s\right) = 2.5732 + \frac{1.45204}{s^{1.2}} \tag{74}$$

The challenge is to make the hardware implementation of the controller.

3.3.1 Fractional Order Integrator

Fractional order integrator in frequency domain [18] is given by following:

$$G\left(s\right) = \frac{1}{s^m} \tag{75}$$

Where $s = j\omega$ and m is positive number.

For frequency band between $\left(\omega_L, \omega_H\right)$ operators of non integer order can be model with the help of fractional power pole [18] and it is represented in form of:

$$G\left(s\right) = \frac{K_I}{\left(1 + s/\omega_c\right)^m} \tag{76}$$

Assuming that ω belongs to (ω_L, ω_H)

$$G\left(s\right) = \frac{K_I}{\left(s/\omega_c\right)^m} = \frac{K_I {\omega_c}^m}{s^m} = \frac{1}{s^m} = G_I\left(s\right) \tag{77}$$

With $K_I = \left(\frac{1}{\omega_c^{\,m}}\right)$ and ω_c is corner frequency equal to $-3\,\mathrm{m}\,\mathrm{dB}$ and the value [18] is taken with the help of ω_{L}:

$$\omega_c = \omega_{\mathrm{L}} \sqrt{\left(10^{\left(\varepsilon/10\mathrm{m}\right)} - 1\right)} \tag{78}$$

ε is maximum permitted error.

To rationalize the equation by approximation method [18]:

$$G_I(s) = \frac{K_I}{\left(1+\left(s/\omega_c\right)\right)^m} \cong K_I \frac{\prod_{i=0}^{N-1}\left(1+\left(s/z_i\right)\right)}{\prod_{i=0}^{N}\left(1+\left(s/p_i\right)\right)} \tag{79}$$

The various parameters can be explained with the help of graph [18]. Parameters $a,\ b,\ p_o,\ z_o$ *and* N solved with the help of Equation (66), (67), (68) and (73) with y (specified error) and $\omega_{\max}$ (frequency band).

Equations (63) – (73) fractional order integrator is mostly approximated with the help of a rational function [18] given in the form of:

$$G(s) = \frac{1}{s^m} = \frac{K_I}{\left(1+\left(s/\omega_c\right)\right)^m} = K_I \frac{\prod_{i=0}^{N-1}\left[1+\left(s/z_0\,(ab)^i\right)\right]}{\prod_{i=0}^{N}\left[1+\left(s/p_0\,(ab)^i\right)\right]} \tag{80}$$

In frequency form the integrator [18] is given in form of:

$$G_I(s) = \sum_{i=0}^{N} \frac{k_i}{\left[1+\left(s/(ab)^i\, p_0\right)\right]} \tag{81}$$

k_i are the residues of poles [18] and is solved in form of:

$$k_i = K_I \frac{\prod_{j=0}^{N-1}\left[1-\left(\frac{(ab)^{i-j}}{a}\right)\right]}{\prod_{j=0,i\neq j}^{N}\left[1-(ab)^{i-j}\right]} \quad \text{for } i = 0,1,\ldots,N \tag{82}$$

The integrator of fractional order [28] is similar to the impedance Z(s) of the N+1 parallel RC cells which are joined in series [18] as shown in Figure 3.5 and given as

$$Z(s) = \sum_{i=0}^{N} \left(\frac{R_i}{1 + sR_iC_i} \right) \tag{83}$$

Hence on comparing Equations (80) and (82) for $i = 0,1,\ldots,N$, values of the capacitors and the resistors for the analogue circuit [29] that is to be used in the fractional-order integrator represented [18] in the form of:

$$R_i = k_i \tag{84}$$

$$C_i = \frac{1}{(ab)^i \times p_0 \times k_i} \tag{85}$$

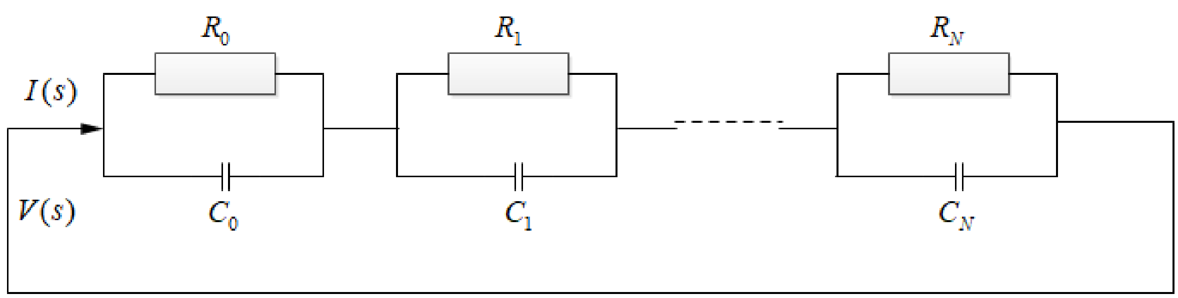

Figure3.5: Fractional Order Integrator converted in RC circuit [18]

3.3.2 Fractional Order Proportional Integral controller

Combining the proportional, integration element having fractional or non integer order to form a PI^{λ} controller. A parallel connection is made between both the elements as shown in Figure 3.6.

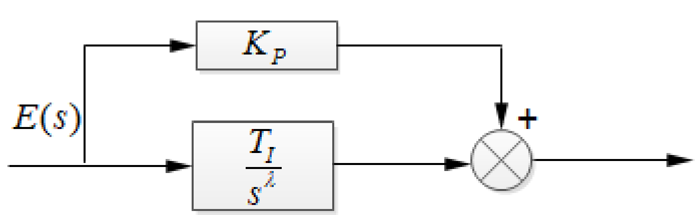

Figure3.6: Structure of fractional combination of proportional and integral element [18]

Irrational transfer function [18] of the controller is represented in Equation (50) and it can be shown in the form of:

$$C(s) = K_P + \left[\frac{T_I}{s}\right]\left(\frac{1}{s^{m_I}}\right) \tag{86}$$

K_P = Proportional element of the fractional order controller

T_I = Integration constant

λ = constant

$\frac{T_I}{s}$ = First order integrator

$\frac{1}{s^{m_I}}$ = Fractional order integrator

m_I should lie between 0 and 1.

Irrational transfer function of non integer order [30] being converted to rational function by an approximation method.

The integrator element is modelled with the help of Fractional Power Pole [18]. So the Equation (86) is represented [18] as follow:

$$C(s) = K_P + \left[\frac{T_I}{s}\right]\left(K_I \frac{\prod_{i=0}^{N-1}\left(1+\left(s/z_i\right)\right)}{\prod_{i=0}^{N}\left(1+\left(s/p_i\right)\right)}\right) \tag{87}$$

From Figure 3.6 the controller equation can also be written as follow:

$$C(s) = \frac{U(s)}{E(s)} = \frac{U_P}{E(s)} = \frac{U_I}{E(s)} \tag{88}$$

The proportional element K_P is realized by an Op-amp as shown in Figure 3.7. Therefore

$$\frac{U_P}{E(s)} = K_P = \frac{R_{P_2}}{R_{P_1}} \tag{89}$$

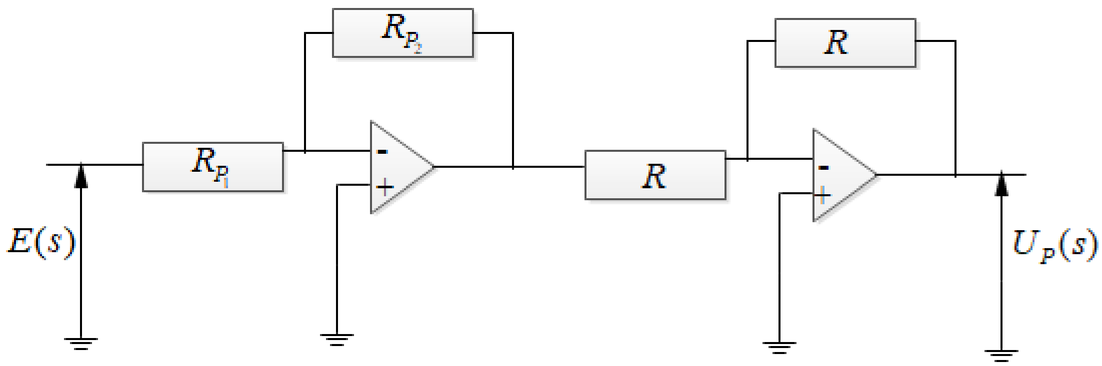

Figure3.7: Analogue circuit realization for proportional element [18]

The integration element [18] is shown as:

$$\frac{T_I}{s^{\lambda}} = \left[\frac{T_I}{s}\right]\left(K_I \frac{\prod_{i=0}^{N-1}\left(1+\left(s/z_i\right)\right)}{\prod_{i=0}^{N}\left(1+\left(s/p_i\right)\right)}\right) \tag{90}$$

Decomposing the Equation (90) replacing with simple function [18] it is given as:

$$\frac{T_I}{s^{\lambda}} = \left[\frac{T_I}{s}\right]\left(\sum \frac{k_{I_i}}{1+\left(s/p_{I_i}\right)}\right) \tag{91}$$

Values of the residues k_{I_i} are calculated with the help of Equation (82). The analogue realization circuit for integration element [18] is shown in Figure 3.8. Therefore

$$\frac{U_I}{E(s)} = \frac{T_I}{s^{\lambda}} = \frac{Z_{I_2}}{Z_{I_1}} \tag{92}$$

$$Z_{I_1} = \left(\frac{1}{T_I}\right)s = L_I s \tag{93}$$

$$Z_{I_2}(s) = \sum_{i=0}^{N_I} \left(\frac{R_{I_i}}{1 + sR_{I_i} C_{I_i}} \right) \tag{94}$$

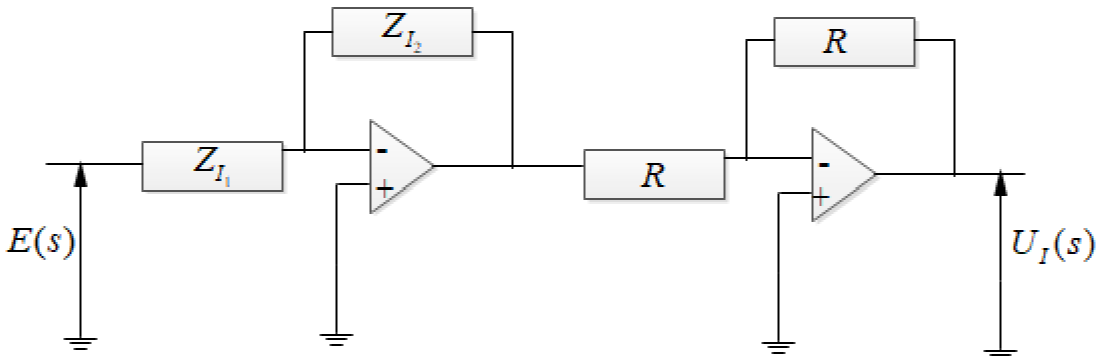

Figure3.8: Analogue circuit realization for integration element [18]

3.4 DESIGN PROCEDURE

To simulate the DC motor an electronic equivalent is obtained. The RLC circuit transfer function of 2^{nd} order is given in Equation (95).

$$T.F. = \frac{\left(\frac{1}{LC} \right)}{\left(s^2 + \left(\frac{R}{L} \right) s + \frac{1}{LC} \right)} \tag{95}$$

Comparing Equations (95) and (49)

$$Gain\, K = 1.01 \tag{96}$$

$$\frac{1}{LC} = 1 \quad \text{and} \quad \frac{R}{L} = 1.367 \tag{97}$$

Assuming

$$L = 10\ mH \tag{98}$$

$$R = 0.01367\ K\Omega \tag{99}$$

$$C = 100\ \mu F \tag{100}$$

Where R (Resistance), L (Inductance) and C (Capacitance) of the RLC circuit used to design and modelling of the DC motor.

Fractional order integrator is considered which is given in Equation (78).

Here value of m is taken to be equal to 0.2

The FPP technique is used to form model of fractional order integrator as given in above section. It is represented [25] by the following way:

$$G(s)=\frac{K_I}{\left(1+\left(\frac{s}{\omega_c}\right)\right)^{0.2}}=\frac{0.78321}{\left(1+\left(\frac{s}{3.3930}\right)\right)^{0.2}} \tag{101}$$

Assuming,

ω >> ω_c where ω_c is the -3mdb

$$K_I=\frac{1}{\omega_c^m} \tag{102}$$

$$\omega_c=\omega_l\sqrt{10^{\left(\frac{\varepsilon}{10m}\right)}\text{-}1}=3.3930\ \text{rad}/\text{s} \tag{103}$$

Therefore

$$K_I=\frac{1}{\omega_c^m}=0.78321 \tag{104}$$

$$G_c=\frac{1}{1.2767+s^{0.2}} \tag{105}$$

Using the graphical method parameters are found in relation to the FPP technique [25].

Value of error $y=1\ db$,

$$a=10^{\left[\frac{y}{10(1\text{-}m)}\right]}=1.3335 \tag{106}$$

$$b=10^{\left[\frac{y}{10(m)}\right]}=3.1622 \tag{107}$$

$$p_o=\omega_c\times10^{\left(\frac{y}{20m}\right)}=6.0337 \tag{108}$$

$$z_o=ap_o=8.0459 \tag{109}$$

$$N = Integer\left[\frac{\log\left(10^7 / 6.0337\right)}{\log\left(1.335\times 3.1622\right)}\right]+1=10 \tag{110}$$

From Equation (82) and (95),

$$R_i = k_i \tag{111}$$

$$k_i = K_I \frac{\prod_{J=0}^{N-1}\left[1-\left(\left(ab\right)^{(i-j)} / a\right)\right]}{\prod_{J=0,i\neq j}^{N}\left[1-\left(ab\right)^{(i-j)}\right]} \tag{112}$$

Put values resulted in Equation (112) to Equation (111) the resistances are calculated.

$$R_o=k_o=0.2151985344 \tag{113}$$

$$R_1=k_1=0.1446480193 \tag{114}$$

$$R_2=k_2=0.1063173002 \tag{115}$$

$$R_3=k_3=0.07937031532 \tag{116}$$

$$R_4=k_4=0.05946273934 \tag{117}$$

$$R_5=k_5=0.04459747298 \tag{118}$$

$$R_6=k_6=0.03349630344 \tag{119}$$

$$R_7=k_7=0.2531929962 \tag{120}$$

$$R_8=k_8=0.01953777549 \tag{121}$$

$$R_9=k_9=0.05715179544 \tag{122}$$

$$R_{10}=k_{10}=0.03847082174 \tag{123}$$

The value of capacitor can be calculated by putting the values obtained in Equations (113) to (123) in Equation (85)

$$C_o=0.7701529403 \tag{124}$$

$$C_1=0.2717198972 \tag{125}$$

$$C_2=0.08766932013 \tag{126}$$

$$C_3=0.02784909703 \tag{127}$$

$$C_4 = 8.815398658 \times 10^{-3} \quad (128)$$

$$C_5 = 2.787367707 \times 10^{-3} \quad (129)$$

$$C_6 = 8.80086337 \times 10^{-4} \quad (130)$$

$$C_7 = 2.761138035 \times 10^{-5} \quad (131)$$

$$C_8 = 8.485595732 \times 10^{-5} \quad (132)$$

$$C_9 = 6.87931537 \times 10^{-6} \quad (133)$$

$$C_{10} = 2.423601879 \times 10^{-6} \quad (134)$$

From Equation (74) calculate further to get analogue realization circuit values. Therefore

$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{T_I}{s^{\lambda}} = 2.5732 + \frac{1.45204}{s^{1.2}} \quad (134)$$

$$K_P = \frac{R_{P_2}}{R_{P_1}} = 2.5732 \quad (135)$$

$$R_{P_1} = 0.388662 \text{ k}\Omega \quad (136)$$

$$R_{P_2} = 1 \text{ k}\Omega \quad (137)$$

$$Z_{I_1} = 0.6886 \text{ k}\Omega \quad (138)$$

From Equation (93) it is known that Z_{I_2} is the RC ladder structure

Combining both the realization of proportional and integration element the fractional order PI^{λ} controller is represented in Figure 3.9.

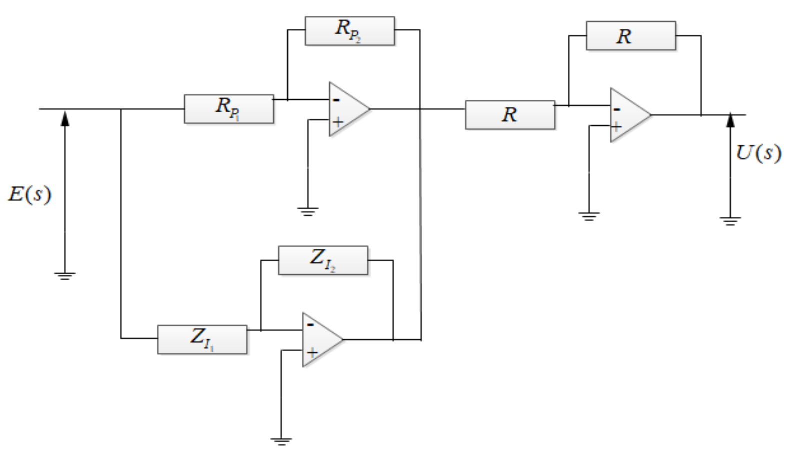

Figure3.9: Analogue circuit realization for Fractional Order PI controller [18]

CHAPTER 4

RESULTS AND DISCUSSION

4.1 SIMULATION

SPICE (Simulation Program for Integrated Circuits Emphasis) version of PC called PSpice (Personal Simulation Program with Integrated Circuit Emphasis). It has different types of libraries like analog and digital libraries with different components [31]. Simulation of the circuit is being done in it. Cadence OrCAD PCB Designer in addition with PSpice has three main applications:

Capture:

It is mainly used to draw a circuit on the screen. It is known as schematic capture. It is more flexible than paper and pencil drawing. The errors are corrected quickly and resolved easily.

PSpice:

The circuit made in capture is simulated. The result is shown in a graph and can evaluate its behavior and performance.

PCB Editor:

It is mainly used to design printed circuit boards. Output is in terms of files and is sent to the manufacturer.

4.1.1 Procedure to obtain simulations

1. Run the Capture.
2. Choose File then click on New and then on Project.
3. A dialogue box showing New Project appears.
4. Select the button showing Analog or Mixed A/D button. Write a name to the project and create a project.
5. A dialogue box showing Create PSpice Project appears and select the button showing create a blank project and press OK.
6. Schematics window opens up and circuit is made in it.
7. Select Place and then Part from the menu bar.

8. A dialogue box appears having list of parts. Click on ANALOG in the libraries.
9. Scroll the part list and choose the component which is required. Click OK.
10. With the help of mouse place the component where it is required and then right click the mouse to select End Mode.
11. With the help of SOURCE library battery can be added. Click OK and place the source.
12. Select place and then wire or use wiring tool to wire the components.
13. Place the cursor from the starting point to the terminal where to finish it and interconnect all the components.
14. Right click and select End Wire.
15. To place a ground in the circuit choose Place then Ground from the menu bar. Choose 0/CAPSYM from the list and click OK. Place the ground.
16. The values of the components can be changed if required. Double click on the component to change its value.
17. After building the circuit it is to be simulated.
18. Choose PSpice and then to New Simulation Profile. Write the name on it and click CREATE.
19. A dialogue box with Simulation Settings appears. The type of analysis can be selected from the options available. Click Apply and then OK.
20. To run the simulation of the drawn circuit select PSpice and then Run or the arrow option that is available in toolbar.
21. Graph showing the result will appear.
22. Process can be repeated by changing the settling time and a new simulation result can be obtained.

4.1.2 Capture and Simulation of a circuit

Draw the circuit with the help of the values obtained from the calculations of the transfer function and fractional controller [32]. Run the circuit and obtain its simulation to know about its behavior and the performance. Following steps will be done:

1. Placing the components.
2. Setting the values according to the requirement.
3. Wiring the components.

4. Simulations to be performed.

The circuit of FOPI controller with motor is drawn in OrCAD Capture as shown in Figure 4.1. First the integral element of the fractional controller is made. Z_{I_2} from Figure 3.9 is replaced with the RC structure. The values of resistances are put from Equation (113) to Equation (123) and the values of capacitances are put from Equation (124) to Equation (134). The resistances and capacitances are placed from the place part option.

$R_1,\ R_2,\ R_3,\ R_4,\ R_5,\ R_6,\ R_7,\ R_8,\ R_9,\ R_{10}\ \textit{and}\ R_{11}$ are placed and values are changes according to the values obtained. Parallel RC cells are connected in series to form the structure therefore $C_1,\ C_2,\ C_3,\ C_4,\ C_5,\ C_6,\ C_7,\ C_8,\ C_9,\ C_{10}\ \textit{and}\ C_{11}$ are placed and the values are changed according to the values obtained. The Z_{I_1} value is obtained in Equation (139) therefore the value of resistance R_{12} is put equal to 0.6886 $\mathrm{k\Omega}$. Then Z_{I_2} in form of RC structure and Z_{I_1} is connected to an Op-amp U1.

For the proportional element as given in Figure 3.9. The values of R_{P_1} and R_{P_2} are calculated in Equation (137) and Equation (138). $R_{13},\ R_{15}$ are the resistances of the proportional element and equal to R_{P_1} and R_{P_2} respectively. The resistances are connected to Op-amp U2.

Then all the components are wired together to form the FOPI controller and further connected with equivalent of DC motor which is obtained by converting the second order transfer function. The gain in DC motor obtained in Equation (96) is put in R_{17} and connected to Op-amp U3.

The values of equivalent resistance, capacitance and inductance of the DC motor are obtained from Equation (98) to Equation (100).

The values are put in $R_{18},\ L_1,\ C_{12}$ and are wired together. Then a source is added to give the supply to the circuit and the upper and lower limit of voltage is set accordingly and the delay, rise and fall time is given.

Ground is placed at the bottom of the circuit. Voltage knobs markers are used at input i.e. before the supply and output i.e. before capacitor to check the simulation results corresponding to the circuit.

After completing the circuit click on the PSpice option and then to New Simulation Profile to obtain the simulation of the circuit. Give a name and then click Create. After clicking on Create the dialog box of Simulation Settings appear. The time for which the simulation on the circuit needs to be performed is written in Run To Time [34]. After putting the values click on Apply option and the graph with the result will be obtained.

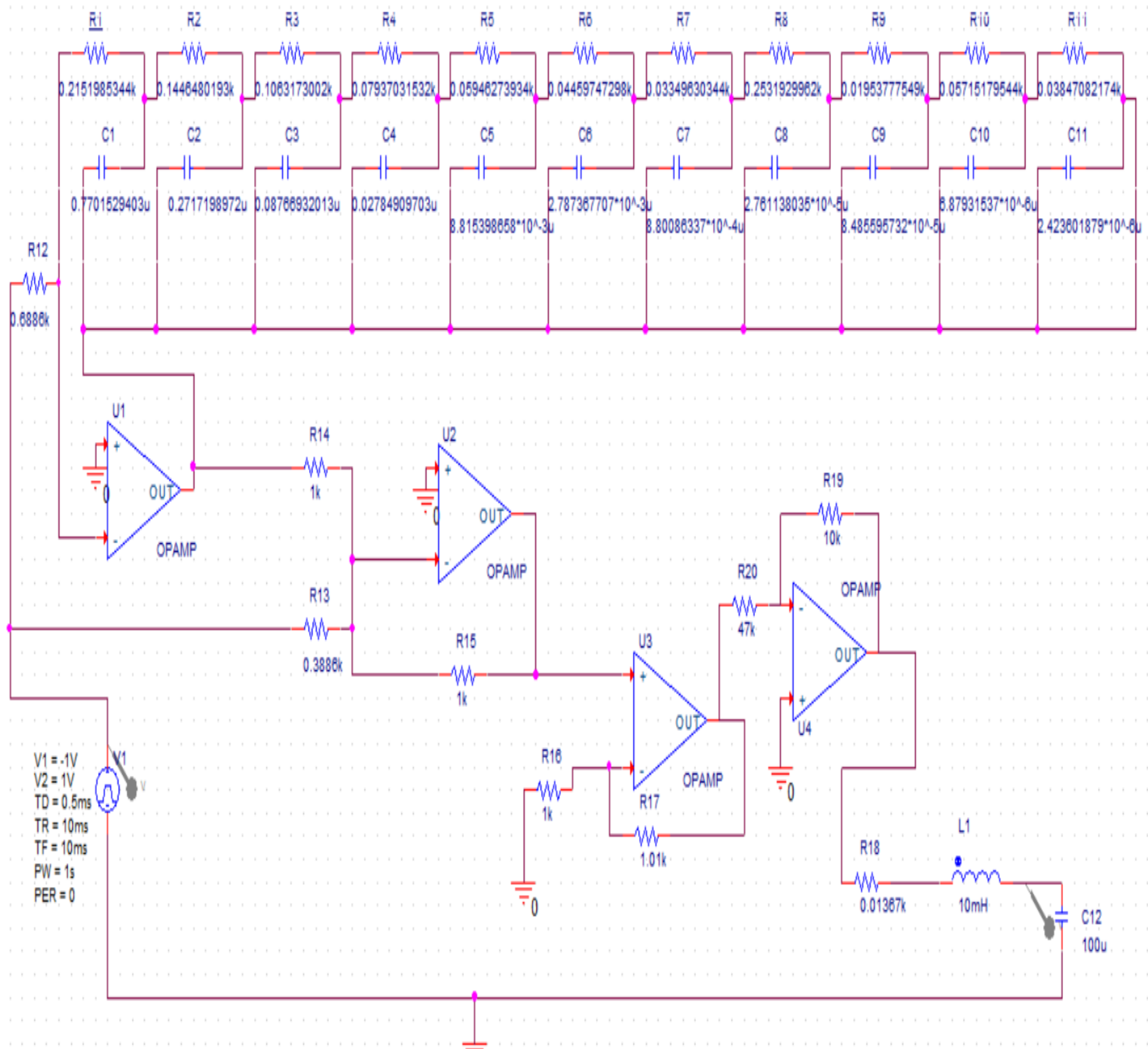

Figure4.1: Circuit of FOPI controller in OrCAD

4.2 HARDWARE IMPLEMENTATION

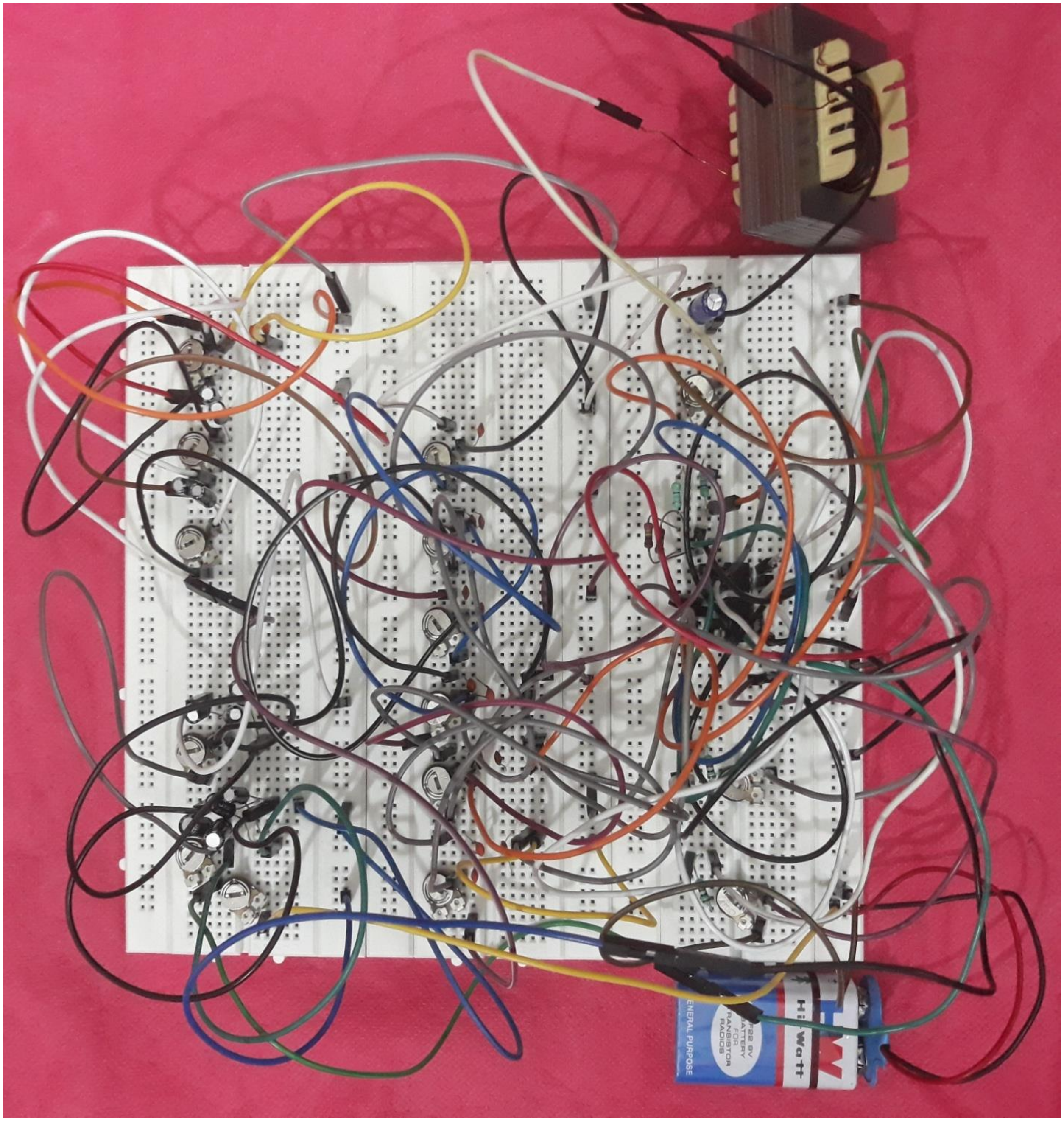

Figure4.2: Hardware Implementation

4.3 SIMULATION RESULTS

The results obtained from the circuit when it is simulated in PSpice environment. To get the response pulse wave is used as an input to the circuit. In source $V1$ is the upper limit which is set to be -1V and $V2$ is set to be 1V. The time delay is given as 0.5 ms, the rise time is given as 10ms and the fall time is given as 10 ms. The width of the pulse is set to be 1 s.

Now putting these values and creating a simulation profile it is observed that the output comes equivalent to the target value which is been set as 1V as shown in Figure 4.3. The response of the system has been shown.

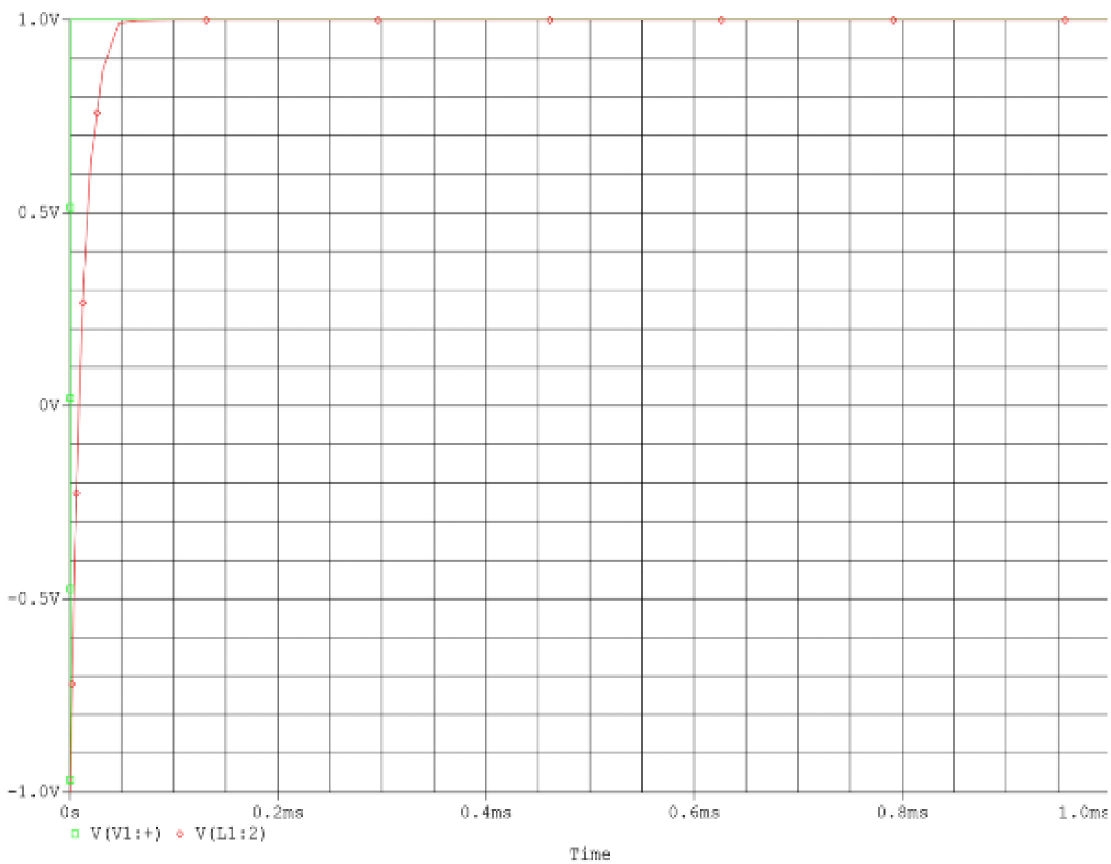

Figure4.3: Response of Fractional Order Proportional Integral controller

Table 4.1: Result obtained from simulation

	Rise Time	**Settling Time**	**Overshoot**
FOPI	**0.025 ms**	**0.05 ms**	**0**

Reference set point = 1V

The rise time obtained from the simulation of the proposed controlled is equal to 0.025 ms. The time taken to settle down the signal is equal to 0.05 ms and no overshoot is occurred.

CHAPTER 5

CONCLUSION

5.1 CONCLUSION

Fractional order means dynamic model having non integer order. Fractional order controllers are more flexible because they provide more parameters and perform in a better way than the integer order controllers. In this work an armature controlled dc motor is considered. The transfer function of DC motor is being calculated for controlling its speed. The series RLC circuit is obtained to get the equivalent of DC motor transfer function. Equating the DC motor transfer function to RLC circuit the values of R, L and C are calculated. Single Input Single Output system with non integer system is taken. PI controller has three parameters $\left(\lambda, k_p, k_i\right)$ and its values are being calculated. Fractional order PI controller is realized and stabilized with the help of an approximation method. The simulation is been performed on the circuit designed in PSpice environment. It is noted that the simulation result of the controller obtained is being equal to the set reference point.

LIST OF PUBLICATIONS

Manpreet Kaur, Dr. Swati Sondhi, "Circuit Realization of Fractional Order Proportional Integral Controller for DC Motor," National Conference on Recent Trends in Electrical, Electronics & Communication Engineering (RTEECE-2K19) Presented at BPIT, New Delhi on 24 January 2019.

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