Efficient Geometric Algorithms for Resource Location Models

A Thesis

Submitted in the fulfilment of the requirement

for the award of the degree of

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in

Computer Science and Engineering

Submitted by

Monika
(Regd. No. 951003006)

Dr. Deepak Garg, Supervisor
Professor & Head, Computer Science Engineering, Bennett University
&
Dr. Maninder Singh, Administrative Guide
Professor & Head, CSED, Thapar Institute of Engineering & Technology

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY

Computer Science and Engineering Department

Thapar Institute of Engineering & Technology

Patiala – 147001 (Punjab), India.

Table of Contents

CERTIFICATE

I hereby certify that the work which is being presented in this thesis entitled EFFICIENT GEOMETRIC ALGORITHMS FOR RESOURCE LOCATION MODELS, in fulfilment of the requirements for the award of degree of DOCTOR OF PHILOSOPHY submitted in Computer Science and Engineering Department (CSED), Thapar Institute of Engineering & Technology, Patiala, Punjab, is an authentic record of my own work carried out under the supervision of Dr. Deepak Garg. This work refers to the work of other researchers, which are duly listed in the reference section.

The matter presented in this thesis has not been submitted for the award of any other degree to this or any other university.

(Monika)

Registration No. 951003006

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge and belief.

Deepak Garg
Professor & Head,
Department of Computer Science & Engg.
Bennett University
Greater Noida, India.

Supervisor

Maninder Singh
Professor & Head,
CSED
Thapar Institute of Engg. & Tech.
Patiala, India.

Supervisor (Administrative)

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY

Computer Science and Engineering Department

Thapar Institute of Engineering &Technology, Patiala

CANDIDATE'S DECLARATION

I hereby certify that the work which presented in this thesis entitled "**Efficient Geometric Algorithms for Resource Location Models**" is being submitted to the Computer Science and Engineering Department in fulfilment of the requirement for the award of the degree of **Doctor of Philosophy.** The work submitted in this thesis is my own work done under the supervision of **Dr. Deepak Garg, Professor & Head, Computer Science Engineering, Bennett University, Noida.**

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute/University.

Monika
Reg. No. 951003006

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Date:
Place: Thapar Institute of Engg. & Tech., Patiala (Monika)

List of Figures

List of Tables

List of Abbreviations

ALA	Alternate Location-Allocation
ATL	Alternate Transportation–Location
CFLP	Capacitated Facility Location Problem
CMSWP	Capacitated Multiple-source Weber Problem
DT	Delaunay Triangulation
FLP	Facility Location Problem
FWP	Fermat Weber Problem
GRASP	Greedy Randomized Adaptive Search Procedure
IRCI	Iterative Route Construction and Improvement
IRP	Inventory Routing Problem
MDVRP	Multi-Depot VRP
MFLP	Multi-Facility Location Problem
MSCFLP	Multi-Source Capacitated Facility Location Problem
PDP	Pickup and Delivery Problem
SSCFLP	Single Source Capacitated Facility Location Problem
TDVRP	Time-dependent Vehicle Routing Planning
TSP	Traveling Salesman Problem
UFLP	Uncapacitated Facility Location problem
VD	Voronoi Diagram
VG	Visibility Graph
VR	Voronoi Region
VNS	Variable Neighborhood Search
VRP	Vehicle Routing Planning

List of Publications

Papers Published

- Monika Mangla, Deepak Garg "**Rapidly Converging Geometric p-center approach for non-convex polygon**" accepted in Turkish Journal for Electrical Engineering and Computer Science (SCI indexed).
- Monika Mangla, Deepak Garg **"An Automated Parking Guidance System for Megacities"** published in International Journal of Information Technology and computer science 2018 vol. 1.
- Monika Mangla, Deepak Garg "**Computational Geometry for Continuous Resource Location Model in a Competitive Environment**" published in International Journal of Computer Applications (IJCA), Vol 27 No. 5 pp. 1-4 August 2011, FCS, USA

Papers under Review in SCI Journals

- A paper entitled "**A Capacitated Facility Allocation Approach based on Residue for Constrained regions"** in Turkish Journal of Electrical Engineering and Computer Science.
- A paper entitled "**Optimal Layout Planning to maximize visitors for participants in an Industrial Trade Fair"** in International Journal of Industrial Engineering Theory and Practice.
- A paper entitled **"Optimal Layout Planning for Maximizing Revenue of a retails store"** communicated to Journal of Universal Computer Science.

ABSTRACT

Facility Location has been the most studied problem in the field of operation research and optimization during the past few decades. Given a set of candidate locations, the facility location problem focuses on finding an optimal set of locations to open facilities. The location of facilities is selected with an intention to minimize the total cost. This cost includes the facility opening cost of facilities along with connection cost (assignment cost) of clients (demand nodes) to facilities. This connection cost is often modeled as the weighted sum of metric distances among clients and allocated facilities. Thus Facility Location Problem aims to optimize the distance in order to minimize the assignment cost.

This basic Facility Location Problem has evolved in order to address realistic issues over time. This evolution helps in implementation of FLP to real-life applications. For instance, facilities may have some limited capacity in real life, thus limiting the number of demand nodes it can serve to. Few other examples of evolved FLP model are constrained FLP, Multifacility Location problem etc. FLP can be classified based on the objective functions mainly into *median problems* and *center problem*.

k-median problem, the most researched variant of FLP minimizes the assignment cost allowing at most *k* facilities. *k-median* problem is generally used for transport applications and thus has widespread application ranging from network design to data warehousing etc. *k-center* problem is mainly used for location of emergency services like ambulance station, fire brigade service etc. In *k-center* model, the aim is to minimize the distance of each facility to its farthest demand node. This aim ensures that even the farthest demand site will receive service within some stipulated time.

These location problems are NP-hard and thus have been widely studied by various researchers. Various popular approaches for FLP include metaheuristics, approximation

algorithms, and Computational geometry etc.

In this thesis, we particularly employ structures in computational geometry for Facility Location Problem. We use the spatial properties of the demand plane and facilities to solve location problems. Main contributions of this thesis in the field of facility location literature are as follows:

- We consider the *p-center* problem in a non-convex polygonal region and present an algorithm for locating p facilities in the demand plane such that the objective function of p-center is accomplished. We prove that the proposed algorithm rapidly converges to the optimal solution in comparison to the traditional approach.
- We undertake an allocation problem for capacitated resources and present an algorithm that utilizes various geometric structures for the same. We also incorporate the usage of residue ratio of the resources in the proposed algorithm for allocation. Finally, we observe that the proposed algorithm optimizes the average distance(f_{avg}). This improvement in f_{avg} becomes apparent as the total residue ratio of all resources lowers.
- Subsequently, we propose layout planning using well-known structures in computational geometry. For the layout planning, we consider layout planning on a departmental store and exhibition hall. The proposed approach is independent of the travel path undertaken by the customers. Using t-test, it has been validated that the proposed algorithm outperforms the traditional approach.
- Considering the widespread requirement of parking management with limited parking, we also propose an approach for an automated parking management system. The suggested approach takes information from sensors and suggests parking to the requesting vehicle. Various performance metrics have been discussed to validate parking management. The suggested approach is validated to achieve better performance metrics.

Chapter 1

1. Introduction

Whenever an organization wants to open a new outlet or to shift an existing unit (office, factory or warehouse), the most important question to be addressed is the location. Similarly, when an individual wants to purchase a site for residential purpose, the important factors to be examined are the availability of facilities like school, market, and hospital in its vicinity. It should also be examined that the chosen site for residential purpose is not close to the railway station, airports, nuclear plant, etc. Similarly, In commercial organization, location decision is related to various factors like transportation cost, availability of raw material, social and environmental impact, etc. Thus there exist several factors that influence location decision, and thus should be considered comprehensively. Thus, considering such widespread influence of location decision, it has been accepted as a prime factor for decision making.

Location decision, if taken carefully considering all major issues, results in escalation and growth of the business.

Additionally, location decision also plays an important role for government bodies during the location of public emergency services like a police station, ambulance service station, fire brigade service, etc. For these services, government bodies want to locate them so that each demand node could be provided service within the stipulated time. The purpose of stipulated time is to ensure that each demand node receives emergency service within this predetermined time for minimizing the loss of life and/or property. A poorly located emergency service fails to serve demand nodes within the stipulated time and eventually it fails to serve its purpose. Therefore, it is evident that location decision plays an influential role and thus requires devising an efficient mathematical model for location modelling.

Mathematical location model addresses various questions including [1]:

- How many facilities need to be sited?
- Where these facilities should be sited?
- What should be the size of each facility?
- Allocation of demand nodes to located facilities.

These questions are addressed by the objectives of the location model. As discussed earlier the objective function is inherently based on the resource under consideration. Emergency services consider minimization of the distance of farthest demand site to the facility. On the contrary, obnoxious facilities [2] (like garbage dump ground, nuclear plant, etc.) optimizes to maximize the distance of nearest demand site to the facility.

This thesis considers developing efficient location models that address all the above questions making use of various geometric structures. In the next section, we give an overview of the classification of the location model and discuss significant findings. Throughout the thesis, terms resource and facility have the same meaning.

1.1 Facility Location Model and Classification

Facility location handles the location of p facilities in a demand plane (open or closed) containing n demand sites while optimizing an objective function. The facility location models may be categorized based on various parameters: nature of demand plane, type of facility, number of facilities, nature of facilities, etc. Primarily, the location model can be broadly classified into four categories as shown in Figure 1.1

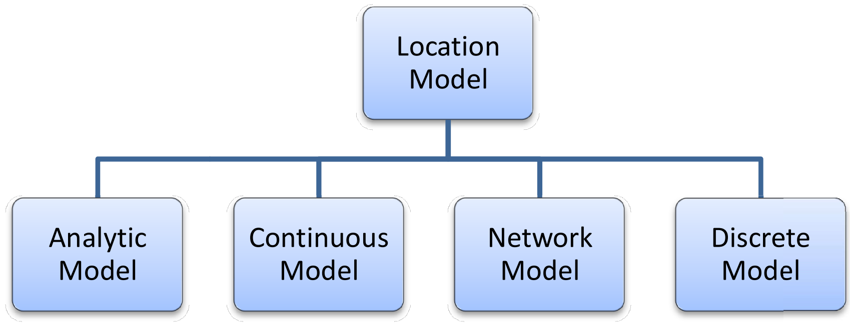

Figure 1.1: Classification of Location Model

The analytic model considers that the demand sites are considered to be distributed over the service area. Facilities are permitted to be located anywhere throughout the demand plane in the analytic model. This assumption of the distribution of demand sites in the service area limits its application [1]. The resource can be located anywhere throughout the demand plane with respect to assumed distribution of demand nodes in the plane.

Unlike analytic model, in continuous model, the location of demand nodes is well-known in advance. This information regarding location of demand nodes aids in determining location of facility. The continuous model also allows locating facilities at any location in the demand plane. For each model, determining optimal location for a facility is associated with its objective function. In order to optimally locate a facility, consideration of infinite possibilities is required which is computationally expensive and thus necessitates devising a specialized technique for analytic and continuous resource location model.

In the network location model, it is assumed that demands arise only within the network. Another restriction for network model is that facility can be placed on the nodes of the network only. Here, transportation and communication is considered to take place only along edges in the network. Network model can be applied for locating a server or a firewall in a network of an organization. Literature of network location model mainly focuses on devising polynomial time algorithms. A polynomial time algorithm for locating p facilities on a network is available. Linear time algorithms are also available for non-weighted network location model for one or two facilities.

Another major classification of the location model is a discrete location model. In discrete location model, the demand nodes are present throughout the demand plane. However, the resources are restricted to be located on finite set of candidate locations only. Unlike network model, the travel is not restricted along edges only.

As discussed earlier, the objective function depends on the facilities under consideration which generates another major classification of location model. The location model is mainly classified into covering based location model, median-based and other miscellaneous models, etc. based on objective function as illustrated Figure 1.2.

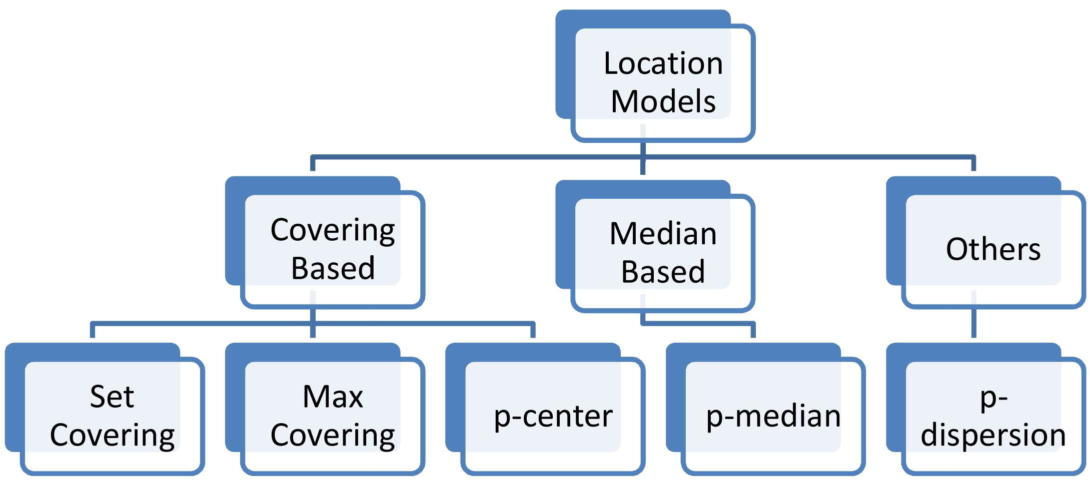

Figure 1.2: Categories based on the objective function

Covering model is used for a service that necessitates some critical coverage standard (in terms of distance and/or time). Such covering based model may be employed for locating emergency services like ambulance service, fire station service, etc. In such model, a demand node is considered to be covered if it receives the service within service standard. For example, a demand node is covered if it receives the service within t time otherwise remains uncovered. Further, covering model is basically implemented by i) set covering model ii) maximal covering model and iii) *p-center* model.

As discussed above, a demand node is assumed to be covered if it has at least one facility within coverage standard i.e. if the demand node receives the service with stipulated time t by minimum one resource. The objective of a set covering model is to cover all demand sites. This set covering model is used for high priority emergency services e.g. fire brigade and ambulance service. Set covering model was introduced in [3] and was later addressed by Toregas et al. [4]. In *Maximal covering problem,* emphasis is given to cover maximum demand nodes [5].

p-center location model is also a well-known variant of covering model. Here, the objective is to locate p resource in the demand plane so that the distance between each demand point and its nearest resource is minimized. Alternatively it can also be understood that the distance between each resource and its farthest demand node is minimized. The objective function of *p-center* is expressed as:

$$Z(X) = \min \left\{ \max\ d\,(d_i, X) \middle| \ 1 \leq i \ \leq n \right\}$$

Thus it determines location X for locating facility so that the distance of X from its farthest demand site $d_i |\ 1 \ \leq i \leq n$ is minimized.

In *median based* models, the objective is to optimize the total weighted distance between each demand site to its nearest resource, termed as *minisum*. Median based model is applicable for transportation and distribution context where travelled distance influences the total cost in a significant manner. For example, the *p-median* location model can be used for location of p warehouses of an organization in the service area to minimize the total transportation cost among p warehouses [6]. It can be mathematically formulated as [7]:

$$\min Z(X) = \sum_{i=1}^{n} h_i d(X, d_i)$$

Here d represents feasible distance among the facility and the demand sites.

Classical Fermat Weber Problem (FWP) is a popular example of median based location model. Weber problem considers n demand nodes each having a quantum of demand to be h_i. FWP locates a single resource at location (X, Y) so that it minimizes the total weighted distance between facility and demand nodes, represented by following equation.

$$\min_{X,Y} \sum_{i=1}^{n} h_i \sqrt{(x_i - X)^2 + (y_i - Y)^2}$$

As FWP locates single resource with an objective to minimize the total distance, it may also be termed as 1-median problem. Median based problems employ an iterative procedures e.g Weiszfeld algorithm. Drezner et al. have further enhanced the Weiszfeld algorithm in order to expedite its convergence towards an optimal solution [8].

Unlike covering and median-based location model, *p-dispersion* model is used for location of obnoxious facilities like a garbage dump, nuclear power plant, crematorium etc. In *p-dispersion* location problems, the emphasis is given to maximizing the gap for each resource (obnoxious) to its nearest demand site in the service area. This objective function is known as *maxmin* and is expressed as follows:

$$Z(X) = \max\{\min d(\, d_i, X) |\, 1 \leq i \leq n\}$$

As already discussed, *p-dispersion* is mainly used for obnoxious facilities but it can also be implemented to locate outlets of an organization in a competitive scenario. Furthermore, it can also be aptly used for locating additional stores of an industry when few stores are already present in the service area. In such scenario, the objective of the location model is to maximize the distance of new resource to existing resources in order to have the largest share of market.

Location model can further be categorized based on the nature of input: *static location model* and *dynamic location model*. Location model where the input remains same along time is called *static location model* while dynamism in the input along time results in *dynamic location models* [9][10][11]. In each location model discussed above, facilities can have a capacity constraint. Imposing capacity constraint on resources generates *capacitated location model* and *uncapacitated location model*. In capacitated location model, capacity constraint of the resource limits the number of demand sites it can be allocated to. On the contrary, an uncapacitated model does not limit the number of demand nodes to be served [12].

Furthermore, location model is also classified based on the geometric properties of facilities under consideration in the literature. This representation of a facility in terms of a geometric object aims to attempt the location model in an efficient manner. The popular classes of facilities based on its geometric properties are: point facility, line facility, and polygonal facility. Point facility can be used for location of facilities like a warehouse, store outlet of industry, etc. Location of road network or any other communication network comes under the category of line facility location. Similarly location of amusement park consisting of multiple rides, job line is considered as polygon facility location.

All location models discussed above are NP-Hard if p is given as input [13]. There exist few variants of facility location problem which are NP-Complete thus encouraging numerous researchers. These researchers have been practicing various tools and techniques to develop efficient algorithms that evaluate solutions closest to the exact solutions. Few of these approaches include soft computing, approximation algorithm, various heuristics, etc. In addition to all these tools, during the past few decades, computational geometry has also emerged as an effective tool for location models [14]. The application of geometric structures of computational geometry with reference to FLP has been discussed in this thesis.

1.2 Computational Geometry and Facility Location

Computational Geometry is the branch of computer science that is competent for attempting problems which can be expressed in form of geometry. This competence of handling the geometric problem is achieved as a result of the inclusion of efficient algorithmic tools for solving spatial problems. As a result of its efficiency in handling geometric problems, computational geometry can be utilized as an effective tool for location models. Consequently, during the past few years, location problem has gained intensive research by researchers in the computational geometry community.

As discussed, Computational Geometry is capable of efficiently processing spatial data by making use of geometric optimization. It has various algorithms, techniques and data structures that can collectively solve location problem effectively. Few popularly used geometric structures for location model are Voronoi diagram, Delaunay triangulation etc. These geometric structures can efficiently represent proximity and relationship of the objects, which acts like an elementary basis for decision making in the location model. Therefore computational geometry emerges as a promising choice for location model in robotics, Integrated Circuit design, Geographic Information System (GIS) etc. The popular geometric structures of Computational Geometry are as follows:

Convex Hull for Q, that contains numerous points, is the smallest enclosing convex polygon that encloses Q. In Convex Hull $CH(Q)$, the line segment pq for each pair (p, q) lies entirely in the $CH(Q)$ [7]. As $CH(Q)$ is the smallest polygon, no smaller polygon may contain Q entirely. Convex Hull is computed using Graham scan's algorithm in $\mathrm{O}(\mathrm{n}\ \log \mathrm{n})$ time. Authors in [15] [16] [17] have incorporated Convex Hull in location problems.

Voronoi Diagram (VD) is the most prevalent geometric structure for location model. This geometric structure divides the demand plane based on facilities into various regions (or cells). Let P contains n facilities in the service plane. VD divides the demand plane into n cells also known as regions. Voronoi region $\mathrm{VR_i}$ corresponding to a site $\mathrm{p_i} \in \mathrm{P}$ contains point q if $\mathrm{dist}(\mathrm{q}, \mathrm{p_i}) < \mathrm{dist}(\mathrm{q}, \mathrm{p_j}),\ \forall\ \mathrm{p_j}\ \in \mathrm{P}, \mathrm{i}\ \neq \mathrm{j}$. The VD for P, represented as (VD(P)) may also be understood as the union of $\mathrm{VR_i} \mid \forall\, \mathrm{i}$ i.e

$$VD(P) = \cup VR_i \mid P_i \in P$$

The region corresponding to a facility or site R_i is called Voronoi Region of R_i (VR_i).

From perspective of location modelling, all demand nodes in VR_i have resource R_i as its nearest resource in comparison to any other resource R_j. Therefore, a Voronoi diagram can be used in location modelling for Nearest Neighbour and Reverse Nearest Neighbour (RNN) queries.

Delaunay Triangulation for P ($DT(P)$), is another important geometric structure used to represent neighborhood relationship among P. In $DT(P)$, two sites p_i and p_j are connected by an arc i.e $(p_i, p_j) \in E(DT(P))$ if $VR(p_i)$ and $VR(p_j)$ share an edge in $VD(P)$. Therefore Delaunay Triangulation is also referred to as dual of Voronoi diagram. Delaunay Triangulation also serves as an effective tool for location modeling. Primarily, it can be used to optimize the initial and current solution during iterations of an algorithm.

Visibility Graph is a graph for a set of nodes (representing demand nodes and facilities) and obstacles in the Euclidean plane. There exists an edge among nodes if the path among these nodes does not pass through any obstruction and thus nodes are directly visible. Major application of visibility graph emerges in the situation when the demand plane contains constraints. Few other areas of its application include robotics and their motion planning etc.

This section has discussed the major classification of location model and application in the relevant area. We have also elaborated various geometric structures used to address location modelling. In practice, sometimes the location model can be convoluted by the presence of constraints in the service area and thus necessitates devising a specialized approach to handle it efficiently. The location model where demand plane contains constraints is termed constrained FLP and has been discussed in following subsection.

1.3 Constrained Facility Location Problem

Resource location handles the location of resources in the service region consisting number of existing resources. However, resource location models may be obscured by presence of some constraints (restrictions). These constraints may be in terms of placement of resource and size of resources (point, line or polygon) under consideration. The location model where demand plane contains constraints is called constrained location model. This constrained location problem is capable of simulating several real-world applications; thus it needs to be handled efficiently and distinctively.

In constrained location model, constraints may be present in form of *forbidden region, congested region* or *barriers* [18] [13]. The inclusion of Constraints (forbidden region, congested region or barrier) in the demand plane necessitates handling the same in a specialized manner. The rationale behind requiring a specialized approach is that the Euclidean distance (minimum possible distance) may not be possible due to presence of constraints in the service area. This requires devising a new distance metric that does not pass through any constraint for the constrained location model.

The *forbidden region* represents the area where the location of resources is prohibited but passing through this region is permitted. Example of a forbidden region can be a river where the location of the resource is prohibited but one can travel through the river using a boat. If the forbidden region is denoted by $F = \cup F_i \; \forall \, i$, then classical objective function for constrained p-center modifies as follows in order to implement the constrained location model:

$$Z(X) = \min\left\{\max\ d\,(d_i, X) |\ 1 \le i \ \le n\right\} |X\ does\ not\ belong\ to\ F$$

As discussed above, *Congested region* represents the area which can be passed through but at an additional cost (penalty) whereas the location of the resource is not possible. Few examples of the congested region include assembly areas where the location of resource is not permitted but travel through this area is permitted at additional cost due to congestion.

Barrier represents a region that can neither be passed through nor can any resource be located [19]f. In this area, thus barriers demonstrate the physical regions like lake and mountains etc. In order to have better understanding of constrained location model with barriers, related terms have been described as follows:

Let $\{\beta_1, \beta_2, \dots \dots \beta_n\}$ represent n polygonal barrier region and let $\beta = \cup_{i=1}^{n} \beta_i$ represent the entire barrier region in the demand plane. Now, from the definition of barriers it is evident that resource can be located only within feasible area $F = R^2 \backslash int(\beta)$. Here, $d_\beta(X, Y)$ is used to represent the shortest barrier distance which does not pass through β and is expressed as [20]:

$$d_\beta(X, Y) = \min\{\text{path}(X, Y): \quad \text{path}(X, Y) \in F\}$$

$d_\beta(X, Y)$ is evaluated using an important property, ***Barrier Touching Property*** (BTP) [21][22] which is illustrated and explained using Figure 1.3.

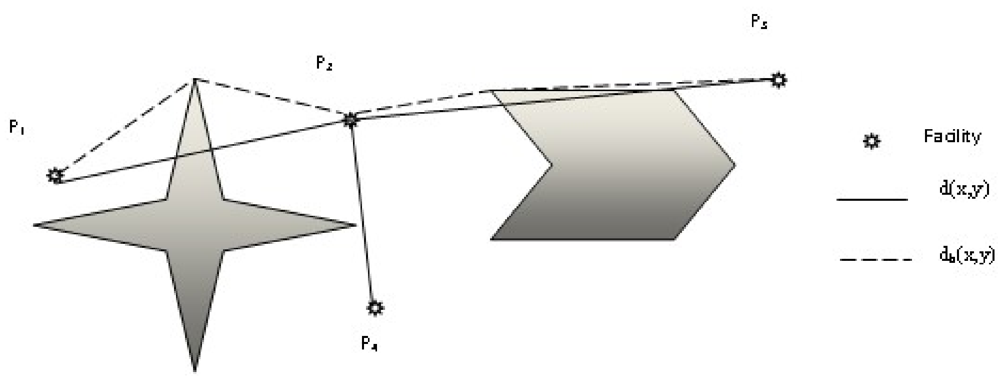

Figure 1.3: Illustration of Barrier Touching Property

According to BTP, for $X, Y \in F \mid X, Y$ are not directly visible, a shortest feasible path P among X and Y exists consisting of line segments with breaking points only at vertices of barrier polygons as shown in Figure 1.3.

In Figure 1.3, barriers are represented by shaded polygons. Therefore permissible path does not pass through the interior of a polygon. Here, solid and dashed lines between p_1 and p_2

represent the Euclidean distance and barrier distance respectively. It is clear that barrier distance d_β has a breaking point only at the vertex of the barrier.

Consideration of barriers in location model was introduced by Katz and Cooper [23] by considering one circular barrier. This was later extended to include polyhedral barriers thus expediting representation of physical barriers like a vehicle, mountain, etc.[21] [24]. Constrained facility location has many practical applications and thus needs to be studied carefully while making location decisions. In this thesis, we have considered p-center problem in continuous location modelling. The resources are considered to be capacitated thus limiting the number of demand nodes it can serve. Subsequently, barriers have also been considered for some cases and appropriate solution methodology has been presented.

1.4 Motivation and Outline of the Thesis

This thesis focuses on resource location models using geometric tools in an efficient manner. The introduction part describes the basics of location model and its classification. In chapter 1, we introduced the various location models and corresponding mathematical models for objective functions. This chapter exhibits that computational geometry is an efficient choice for handling location problems as it is capable of representing spatial data in an efficient manner. Constrained location model is also represented in this chapter that necessitates a specialized approach for solving location problems.

Chapter 2 is dedicated to the literature survey in the concerned area of research. We review the available literature focusing on the usage of geometric structures for location models. In particular, we concentrate on the p-center problem and its variants. This chapter also reviews the literature for p-median and obnoxious location models by using structures in computational geometry.

An algorithm for the p-center location model using geometric tools is given in chapter 3. We also present a heuristic based approach to solve p-center for non-convex regions using

Voronoi Diagram and Delaunay Triangulation.

Chapter 4 consider the allocation of demand nodes to capacitated resources while minimizing the service distance. For the same, a residue based approach is presented in chapter 4 which ensures that a demand node is allocated to nearest resource in the best possible manner.

Chapter 5 presents a layout planning approach for the discrete location model. We have proposed a layout planning algorithm. In this chapter, we have considered two instances for the layout planning viz. layout planning for the departmental store and exhibition planning. For both considerations, we have presented the illustration. The presented illustration observes that the suggested approach outperforms in comparison to the existing approach.

Finally in chapter 6, concluding remarks and future directions have been given.

Chapter 2

2. Literature Review

Location modeling has proved its application in the various real-time applications and thus inviting various researchers to work in this area. Many variants of the location problems have been identified and various researchers have used different approaches (heuristics, approximation algorithm, etc.) to handle these problems. Apart from various techniques to handle location problems, computational geometry has been identified to be an efficient technique for handling location problem. Computational Geometry is competent in addressing location modelling as it is capable to efficiently process spatial problems with geometric optimization [14]. This property of geometric structures has attracted researchers in the field of location modeling during the past few decades [25]. This chapter presents the findings by various researchers in location problems using tools and structures from Computational Geometry [26] and [27].

Location problems have a diverse range of applications in real life. Many a time, the objective is to place a resource as near as possible to the site of demand. Such problems are effectively implemented using *minmax* (*p-center*) function. In contrast, in few cases, the objective is to locate resource at the farthest possible location (in case of obnoxious facilities). Such an objective function is called *maxmin* objective function. Additionally, there are some instances when the objective is to minimize the total cost of transportation. Such Location problem is generalized by the *Minsum* objective (*p-median*) function [28][29] and is generally used for transport application in real life like Vehicle routing problem (VRP). This section focuses on the different solution approaches for location modelling. More specifically, emphasis is given to the usage of various geometric structures and tools to handle location modelling.

2.1 p-center

The *p-center* problem has always been an interesting area for researchers because of its applications in real life. In *p-center,* the objective is to find p circles covering the entire service space while minimizing the radius of the largest circle. Thus it minimizes the distance of farthest demand to the center of the circle. *p-center* is mainly used for the location of emergency services (police office, hospitals etc.) [13] as a result of its objective function

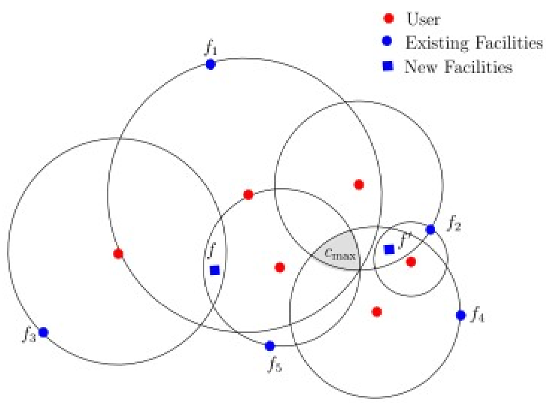

Figure 2.1: Demonstration of maxCov in l2 metric

The authors in [13] have focused their research on p-center location modeling [30][31][32]. The authors in [33] have shown *p-center* location problem is NP-complete and takes p as a part of the input. *p-center* is further categorized into continuous or discrete location modeling. According to the literature in the area of operations research, the location problem is termed as a continuous problem if there exists an infinite set of demand nodes. On the contrary, according to computational geometry, if candidate locations for locating facilities are infinite, it is termed as a continuous location model [34][35]. The authors in [36] have studied many versions of the p-center in correspondence to spatial properties of demand nodes. For *p-center,* numerous heuristic and approximation algorithms are also present in addition to exact algorithms [37][38][39]. The authors in [30][31] have implemented the p-center for graphs and trees. This work helps to implement p-center for many real-life problems like the location of a server in a network etc.

Various researchers have proposed the use of different approaches for solving p-center problems [13]. Numerous approaches have been devised by researchers for p-center location modeling. Among these tools, the Voronoi Diagram (VD) has proved to be a primary geometric object that has been successfully applied in location problems. The authors in [40] have presented an algorithm for discrete demand nodes where VD is used to further optimize the computational time. On the other hand, VD acts as a primary tool for algorithms in continuous demand set cases. Authors in [41][16] have also proposed a heuristic VD method for the continuous *p-center* problem. This work was later extended by [42] and [43]to solve the constrained space that will be discussed subsequently in the chapter.

Tamir et. al. [44] extended this research for p-center by considering the p-center problem for network model. In a network model, the choice for the location of a resource is limited to a subset of D. p-center for network model has also been reported in [45] and some heuristics have been presented in [46]. A well-researched application of p-center for network model is collection depot [47]. Tamir and Halman [44] proposed an $O\left(p^2 n^2 log^3(pn)\right)$ algorithm that uses parametric search technique [47] for solving the problem of collection depots. Authors in [44] studied the case where round trip's involving service center is omitted. The authors in [48] have addressed p-center problems on a network extensively.

2.2 p-median

If p locations are to be chosen for locating facilities, where the emphasis is laid on minimizing the total distance from each demand point to its closest resource, the problem is called *p-median* (also known as *minisum*) location problem [49], a well-known NP-complete problem [50]. The *p-median* is also known as Fermat Weber Problem (FWP) in the literature. Multiple resources can also be considered at the same time, known as the Multi-Facility Weber Problem (MFWP). MFWP locates multiple facilities in a planar space at the same time. It also addresses the allocation of a given set of customers to facilities. The objective of allocation in MFWP is to minimize the overall cost between facilities and associated customers [51].

p-median problems are difficult to compute even for moderately sized instances and thus heuristic-based methods need to be devised. The collection depot problem, a well-known median problem was introduced by [52]. The authors in [52] attempted *minsum* using an iterative procedure which eventually converges to local optima. Here *minsum* problem was examined for Euclidean and rectilinear distance on the plane. Authors in [53] examined the properties of solutions for MinMax and MinSum versions of the location problem. Authors in [54][55] and [56] have also discussed the *p-median* problem in their work. Later [57] used heuristics based methods to find an optimum solution. The authors in [57] considered 25 facilities. Thereafter Weiszfeld [58] modified the well-known Weber problem and proposed a planar 1-median model for placing single facility.

This research was escalated further by Eyster et al. [59], who developed a hyperboloid approximation procedure (HAP). *2-median* problem with 100 demand points was attempted by [60]. Authors in Captivo [61], Dai and Cheung[62], Hansen et al. [63], Hribar and Daskin [64], and Rolland et al. [65] have discussed various efficient heuristics for the p-median problem in their work. The inherent difficulty in finding the exact solution necessitates devising some bounding procedures to estimate the optimal value of objective function. Love and Yeong [66] have developed a bounding procedure for the 1-median problem, which was later extended to incorporate 2 or more facilities by Juel [67]. Drezner [60] also proposed a bounding procedure for *1-median*. This bounding procedure was extended further to handle multi-facility location by Dowling and Love [68]. Love and Dowling [69] developed a bounding method for *1-median* for the non-planar region. Thereafter Hansen et al. [63] handled MFWP by solving *p-median*. In [63] all fixed points are considered as facilities and then ALA algorithm is utilized to locate facilities. This was further extended by Gamal and

Salhi in [70] to generate efficient initial solutions.

Cooper [55] has proposed an Alternate Location-Allocation (ALA) algorithm, an iterative heuristic method for MFWP. This ALA algorithm is efficient from the perspective of solution quality and its time complexity. Gamal and Salhi [70] also explored a cellular heuristic, a 2-phase heuristic method. Brimberg et al. [71] combined VNS and ALA algorithm in their work to suggest a solution approach. Salhi and Gamal [72] has proposed a genetic algorithm (GA) for median based location problem. Furthermore, authors in Taillard [73] presented a decomposition heuristic where the problem is partitioned into smaller sub-problems which are subsequently solved using candidate list search.

2.3 Capacitated Resource Location Model

In reality, the majority of the resources have a capacity constraint that limits the number of demand nodes it can serve. This capacity constraint, therefore, restricts the number of demand nodes that can be allocated to resources, creating another well-known version of the location modeling called Capacitated Resource Location Model [74]. Capacity constraint may be imposed in any location model like p-center or p-median etc. The p-median (FWP), if contains capacitated resources, is called Capacitated Fermat Weber Problem (CFWP). CFWP can also be employed for multiple resources, generating Capacitated Multi-facility Weber Problem (CMFWP). Capacitated Multi-facility Weber Problem (CMFWP) is capable of implementing various real-life applications and thus has been extensively researched by the researchers.

Capacitated location-allocation problem is closely examined by various researchers [75][26]. All these papers have focused on single source Weber Problem. Contrary to this there exists a shortage of papers on capacitated Multiple Source Weber Problem (CMSWP). However, the author in [55] has proposed an exact and heuristic method for CMSWP. It starts with the generation of a basic feasible solution. It is followed by the construction of a connected graph of basic feasible solutions for location problem. Finally, solution minimizing the total cost of the model is believed to be an optimized solution.

The author in [76] proposed a branch and bound algorithm which is followed by the column generation approach to attempt the problem. Authors in [77] introduced an exact method for CMFWP considering rectangular distances. An exact method for problems considering Euclidean distances was proposed by Al-Loughani [78]. Authors in [78] present a branch-and-bound algorithm to enumerate vertices of feasible region in an implicit manner. Cooper [79] developed a heuristic method Alternate Transportation–Location (ATL) for CMFWP. Authors in [80] proposed a linear approximation of the problem and developed 3 heuristics for problems involving Euclidean, squared Euclidean and l_p distance. Zainuddin and Salhi [81] also introduced a perturbation-based heuristic method. This heuristic based algorithm is observed to outperform the classical ATL while tested for $n = 50$ given in [71]. Recently authors in [82] combined the ATL algorithm [79] with GRASP to obtain robust solutions.

In the beginning, authors in [83] used MSWP to find location of steam generators. Thereafter authors in [63] attempted MSWP using p-median. Authors in [71] also performed a comparative analysis of heuristics based on variable neighborhood search and genetic algorithms. Furthermore, a decomposition heuristic was proposed in [73] that partitions the given problem into smaller sub-problems. These sub-problems are later solved using a candidate list search for a number of centers. The authors in [84] handled the MSWP with help of vector quantization and self-organizing maps. Authors in [84] also considered Euclidean distance as well as rectilinear distance.

The authors in [85] extended this work and introduced a constructive and adaptive heuristics for CMFWP. The proposed technique restricts the placement of new resource in close proximity of previously located resource. This restriction is achieved using circles of some fixed radius. This radius is based on the sparsity of customers and facilities under consideration. This radius of the circle is gradually adjusted as each resource is located. The authors in [85] showed that this region-rejection method using circle provided encouraging results in terms of solution quality and time (computational). For the same, Authors in [70] proposed a constructive heuristic to find the initial location. Here authors introduced the concept of forbidden region to restrict the placement of resources near to each other. The author further extended the research in [86] and presented a cellular heuristic whereas authors in [72] used a genetic algorithm.

2.4 Constrained Weber Problem

Constrained Weber problem is a variant of MSWP. In constrained Weber problem, constraints may be available in form of a congested region, forbidden region or barriers [87]. These constraints impose some restrictions for the location of resources in the demand plane [13] [88] [89]. Such a location model is also known as the restricted location model. Constrained Weber problem is capable of implementing the real world problems, thus widening its application against the *p-center* problem.

Presence of these constraints limits the location of resources. In such model, resource can be located in feasible region only. The feasible region excludes the restricted region (if any exists) from the service area. If there is no restricted region for placement of resource, the feasible region coincides with the service area. Among all three types of constraints, the forbidden region has been extensively researched and well-solved. However, there still exist some challenges in the location problem with barriers and congested region.

These restricted location problems have been closely researched in the literature and have been mainly classified into three classes as follows:

i. **Barriers** prohibit location as well as and travel through.

ii. **Forbidden regions** prohibit location but can be travelled through.

iii. **Generalized Congested regions** prohibit the location of resource although travel is permitted at an additional cost.

A general example of constrained Weber problem is illustrated in Figure 2.2 [88].

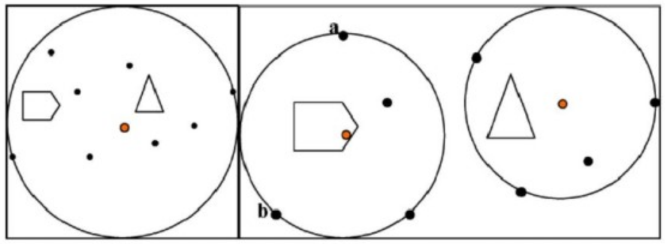

Figure 2.2: constrained p-center for p=1 and p=2

Authors in [90] [91][92][93] and [94] have focused their research in constrained location model. Other well-known research in restricted location modelling is given by [90] [95].

The inclusion of impenetrable barriers in the location model necessitates evaluation of shortest path while considering barriers. This was initially studied by Lozano-Perez and Wesley [96] and was followed by Larson and Li [97], Alt and Welzl [98]. In constrained location modeling, it is generally assumed that the size of a resource under consideration is infinitesimal, which was considered further by Savas et al.[94]

Authors in Francis et al.[99] have presented a scenario for location modeling to locate supply centers that supply parts to numerous demand centers. In this model, it is assumed that demand nodes and facilities are infinitesimal and thus pose no barrier for travelling. Although such an assumption is valid only if the size of the facility is negligible with respect to the size of service plane, otherwise this assumption does not hold true. For example, in layout problems of workstation, factory, etc., the objects (that may act as demand centers or resources) occupy considerable space and thus results in a barrier to travel. This necessitated the research for locating facilities in the presence of the forbidden region. The authors Katz and Cooper [23], Batta et al. [95], Larson and Sadiq [90] and, more recently, Butt and Cavalier [92], Hamacher and Nickel [91], Brimberg and Wesolowsky [100] and Savas et al. [94] have handled restricted location problem particularly with barriers.

Constrained Location problem with barriers has also been extensively researched in [101]. Although there exists extensive research for the center problem without barriers in the literature [102][99]. On the contrary, center problem with barriers still lacks enough research. MSWP with barrier was initially investigated in [23] considering one circular forbidden region and median objective function.

In constrained location modeling (in continuous service area), physical barriers can be simulated with the help of non-ordinary VD [103]. It is an important property as it enables location modelling to implement problems with physical obstacles [104] [105].

Authors in [106] considered the location of a resource in demand space having a single line barrier. This may correspond to the random occurrence of any barrier along a horizontal line on the plane. The authors [106] also established the concept of probabilistic barriers in location theory. Authors in [107] have also handled MSWP with barriers and proposed an algorithm. This line of research for MSWP with barriers was extended further by the authors in [108][92][109][93]. Authors in [108] and [92] proposed heuristics for the 1-median problem under the l_p distance metric.

The authors in [97] considered presence of polygonal barrier and proposed an efficient algorithm to find the shortest feasible rectilinear path. The authors in Nandikonda et al. [110] have addressed 1-center using l_1 distance metric for demand space containing arbitrarily shaped barriers. Suggested approach divides this feasible area among cells based on work in [90]. It is followed by the classification of the cells based on corners. The proposed solution procedure is polynomially bounded. The authors in [24] have obtained the dominating results for the 1-center location problem in the presence of barriers. The barriers were represented by convex polygons and thus l_1 distance was used. Here authors in [24] adopted the algorithm of Mitchell [111]in order to find bisectors, which were further used to decompose the feasible region into cells. This work differed from work in Nandikonda et al. [110] in terms of the shape of barriers and solution approach.

The authors in [21] considered polyhedral barriers and used visibility graph for computation of the barrier distance efficiently. Several heuristic and iterative algorithms are available in the literature that uses the visibility graph for computation of distance [108] [92] [93]. They considered the Euclidean metric for finding the distance [21].

Furthermore, authors in [90] obtained discretization result for *p-median* in demand space containing arbitrarily shaped barriers. In [90], the authors proposed the construction of a grid that split the feasible region into multiple cells. This was followed by conversion of the original problem into *p-median* network problem. The author concluded that facilities can be located optimally based on the grid points. This line of research was taken ahead by Batta et al. [95] by considering forbidden regions.

Recently, the authors in [43] used a heuristic algorithm to attempt *p-center* in a constrained environment. The service plane under consideration in [43] is continuous space [112]. The proposed method is implemented for the location of sirens, sensors, wireless antenna and object tracking etc. [113]. The authors in [13] suggested a Heuristic VD algorithm for the constrained *p-center* problem.

Constrained location modelling in continuous service area can be simulated with help of non-ordinary Voronoi diagram [103]. It is an important property as it enables location modelling to implement problems with obstacles like highways, rivers, hills etc.

In all above-mentioned literature, the size of resource was infinitesimal although such an assumption may not be always valid. Savas et al. in [94] considered the location of finite size facility in the presence of an arbitrarily shaped barrier. In [94] authors considered rectangular distance metric to implement the median objective function. The authors were successful in identifying the optimal location of a new finite-size facility with a fixed orientation with a fixed server location. This work of Savas et al. [94] for finite size facility was researched further by [114] Kelachankuttu et al. [115] and Sarkar et al. [18]. The authors in Sarkar et al. [114] considered median objective function for rectangular shaped facilities using rectilinear distance metric.

The authors in [116] escalated the line of research by considering resource location in an existing layout. The motive behind this work was findings of Savas et al. [94] that considered Placement of Finite-Size Facility for rectilinear distance metric in constrained demand plane. The work of [116] differed from earlier work as it did not permit overlapping of the new facility with existing facilities.

2.5 Obnoxious Facility Location

The authors in [117] present a location model that considers obnoxious facilities. Due to the obnoxious nature of facilities, its objective function is to maximize the gap between a facility and nearest demand node. During the past few decades, researchers have developed their interest in location model involving obnoxious facilities [118].

The obnoxious facility location problem, introduced by Church and Garfinkel [119]is well researched Carrizosa and Plastria[120], Drezner and Wesolowsky [117], Plastria and Carrizosa [121], Kaiser and Morin [122], Munoz-Perez and Saameno-Rodriguez [123] Romero-Morales et al. [124], and Tamir [125]. Erkut and Neuman [126] and Plastria [127] have presented an elaborated the obnoxious FLP. In [128] authors have proposed a solution method for locating an obnoxious facility in a planar network minimizing nuisance.

The authors in Ben-Moshe et al. [129] have also considered obnoxious facilities. A constraint of each region compulsorily having at least one facility was also considered in the study. Thereafter Qin et al. [130] solved the location of obnoxious facilities amidst a set of demand sites by using Voronoi diagram. The authors in Qin et al. [130] employed Voronoi diagrams to solve the problem when the demand sites are either points or weighted convex polygons. Consequently, authors in [131] presented sub-quadratic algorithms for locating two obnoxious facilities in the demand plane. Two branch-and-bound algorithms were introduced by Welch et al. [132] for obnoxious facilities. In [133] authors have employed methods from computational geometry to solve obnoxious multi-facility problems when p (number of facilities) is not known beforehand. The authors in [134] addressed the obnoxious center problem on trees.

The authors in [135] have introduced a mixed integer bi-objective programming approach for obnoxious facilities. The proposed approach is fostered by an urban waste management system. Authors in [136] considered various aspects of the analysis e.g. cost of facility, travel cost etc. The authors in [137] have incorporated the greenhouse effect and energy recovery to the model. Finally, Tralhão et al. [138] introduced an approach for locating waste containers and considered objective function as investment cost (containers), travel distance among other objectives. This model obtains the appropriate location and capacity of each obnoxious facility. This is a complex model as it includes the environmental cost for individual who lives in its vicinity and travel cost for individual who lives too far from obnoxious facilities. Such a combination of this “push” and “pull” factor creates a new class of facilities known as semi-obnoxious facilities [139] [124] [151] [152].

The authors in [138][140] have proposed a multi-objective approach for the location of semi-obnoxious facilities. These objectives are the minimization of cost and average distance between customers and facilities. Other factors under consideration are push and pull factor. These factors are implemented by minimizing the number of demand nodes located too close (push) or too far (pull) from an obnoxious facility. A different model needs to be adopted to address the problem with homogenous demand stakeholders. The authors in [135] have addressed two objectives in their work i.e investment cost and acceptability by the residents. The authors in problems [141][142] [124] [143] [140][144] have adopted a bi-objective model for location of semi-obnoxious facilities.

2.6 Routing Problems

From the literature survey, it became quite evident that the Voronoi diagram has played a major role in various location models. The Voronoi diagram has been combined with approximation techniques in order to handle numerous real-life applications. It is observed during the literature survey that transport activity is the most influential activity in logistics as it presents an economic challenge to the organization [145]. As a result of its economic influence, large numbers of researchers have developed their interest in VRP, an NP-hard optimization problem [146]. The objective of VRP is to identify a plan to serve the number of customers by a number of vehicles so that the cost of allocating vehicles to customers is minimized [147].

Apart from having various approaches for handling VRP, the Voronoi diagram is also observed as a primary tool in the Vehicle Routing Problem (VRP). The authors in boots and south [148] have attempted retail trade (VRP) using multiplicatively weighted Voronoi diagram. On the other hand, a multiplicatively weighted diagram was used by Galvao et al. [149] to solve urban freight problem.

Routing problem mainly focuses on distribution network which is used to transport products from warehouses and to customers [150]. Generally, the focus of the routing problem is to minimize the financial impact of routes on the organization that carries the distribution of goods or services. This can be achieved by minimizing the total travelled distance by all the vehicles providing service to all demand points. Numerous research papers are presented to report development in VRP [151] [152] [153][154]. The focus on VRP is increasing by leaps and bounds as a result of its real-life applications. Another motive behind focused attention in VRP is its growing complexity subject to operational constraints. Various classes of VRP have been identified and each class has received equal attention of researchers. Some of these classes are VRP with Loading Constraints, VRP with a time window and Multi-echelon VRP etc.

The vehicle routing problem was initially addressed by [146]. The authors in Dantzig and Ramser [146] considered routing of gasoline delivery truck between terminal and service stations. The number of possible routes exponentially increases with an increase in the number of service stations. The exponential increase in the number of possible routes intricate the work of finding an optimal route. The authors in [155] proposed an algorithm to obtain a near optimal solution. The proposed algorithm is based on the linear formulation and also considers the capacity constraint of each vehicle. This capacitated VRP is further classified into symmetrical capacitated VRP (SCVRP) and Asymmetrical capacitated VRP (ACVRP) [156].

Traditionally VRP considers Euclidean distance as a measure of evaluation. On the other hand, Euclidean distance contradicts the real requirement of delivery services as vehicles move on a real road network and thus time to travel may not be solely dependent on Euclidean distance [157] [158]. This model largely neglects the time, which is unconvincing [156] as travel time (between depots and customers) is a major characteristic of time-dependent VRP (TDVRP). Time to travel between two points may vary significantly during various time durations accounting the conditions such as variation in speed of the vehicle (based on traffic density). Consequently, TDVRP is a useful model to implement real-life problems [159] [160]. The earliest result in TDVRP was given by Cooke and Halsey [161] that extended the classical shortest path problem. The model in [161] did not consider multiple vehicles. Later authors in [162] handled TDVRP by presenting a mixed integer linear programming mathematical model.

The time-dependent VRP was further extended to incorporate time windows, which has grasped the great interest of researchers [163] [164] due to its similarity with real-life applications. An iterative route construction and improvement algorithms (IRCI) is available in the literature that is capable of handling constant or time-dependent speed problems with hard or soft time windows. Other researchers who worked in time-dependent VRPTW includes [165] [166]. In [167] the authors make use of approximate approaches for the VRPTW in shorter computational time. In [168] authors used a diversification and intensification technique for VRPTW. In [169] authors solved this problem by implementing a tabu search and further improved the best-known solutions. More solution methodologies have been presented in [170] [171] and [172]. In [173] and [174] a simulated annealing model for VRPTW is designed and results proved that these two approaches have successfully solved a large-scale instance of VRPTW. Authors in [175][176] have used the evolutionary computation, population-based metaheuristic algorithms. Genetic algorithm and set partition formulation is combined by Alvarenga et. Al. in [177] recently.

The earlier problem dial-a-ride by [178] has been reformulated as Pickup and delivery problem. There exists enough research in the area of pickup and delivery problem (PDP), simultaneous PDP and VRP with backhauls etc. These variants share a similar structure but have a slight difference that creates different variant within PDP. The authors in [179] [180] have surveyed the literature for PDP and have given a reasonable classification of PDP.

The problem of Multi-Depot VRP (MDVRP) was introduced by [181]. MDVRP contains multiple depots and each customer is visited by a vehicle which is assigned to some depot. MDVRP considers physical distribution problem of various goods to numerous locations such as delivery of meals, industrial gases, specific services, etc. In MDVRP, it is considered that vehicle originates and returns at the same depot. In this concern, various optimization approaches have been used to substantially optimize economic savings in MDVRP [182]. In literature authors have discussed various extensions of MDVRP e.g MDVRP with Time Windows [183] [184][185], MDVRP with Backhauls [186][187], MDVRP with Pickup and Delivery [188], MDVRP with Mix Fleet [189] [190] and Multi-depot Location Routing

Problem [191] [192] etc. Another version of the MDVRP is inter-depot MDVRP [193] [194] where the intermediate depots can be used for warehousing or recycling facilities.

All the above traditional VRP are assumed to work in a deterministic environment where all parameters are known ahead of construction of an optimized route. Furthermore, the whole operational environment remains unchanged while carrying out the routing plan. However, the real-world situation may not be always deterministic and static because several uncertainties may arise during the planning horizon. Few examples of such uncertainties are the addition of new customers, breakdown of vehicles etc. DVRP handles a dynamic model where customers are capable of making an online request (during planning horizon) which should be incorporated in the route planning. The research in DVRP is motivated by its real-life application in distribution systems, mobile ambulance service, rescue service, and cab service etc. [195].

The authors in [196] considered the problem of distributing products and vehicle dispatching known as Inventory Routing Problem (IRP). IRP ensures that there exist no stock outs for any customer [197][198]. The authors in [199][200][201] considered IRP for a single vehicle. This consideration does not match the nature of VRP and is unable to represent the complex nature of the real-world problem. The first exact algorithm for IRP was proposed by [202]. Authors in [202] used a branch-and-cut algorithm to handle the problem where the number of customers is limited to 50. Thereafter continuing this line of research, authors in [203] considered Vendor-Managed Inventory (VMI) where products are shipped from supplier to customer or among customers. This was further extended by the author in [204] by taking the multi-vehicle case into consideration.

Another variant of VRP is generalized VRP where customers are partitioned into clusters and vehicles are permitted to visit only one customer from each cluster [205]. This model is an extension of the problem of orienteering introduced by [206] which was subsequently worked upon in [207]. There are numerous similar problems available in the literature e.g selective TSP [208], the TSP with profits [209], VRP with selective backhauls [210] [211]. The authors in [212] provide an extensive survey on generalized VRP and its widespread applications.

2.7 Network Location model

In network models, nodes of the network represent the existing resources and the location of resource is limited to vertex of the network only. The cost of communication among two nodes (representing existing facilities) is represented by the edges in the graph, thus the shortest path in the network is an indicator for network distance. Network models are mainly focused on the transportation network. However, in network location modeling, the inclusion of continuities and other modeling parameters lacks rigorous research. The authors in [213] have incorporated the addition of mega nodes that can be added or removed at some finite access points. Furthermore, the author in Erkut [214] has suggested the addition of a candidate set outside a given transportation network for locating new facilities. In contrast to the set of candidate locations, Blanquero et al. [215] extended the network model by using a convex feasible region of upcoming locations. Network model with some points of potential hazards has been considered in [28].

2.8 Geometry structures for various location models

After literature survey, it is evident that location modelling can have various objective functions such as minimizing the total transportation cost, maximizing the coverage of demand sites, capturing the largest market share, etc. [216][217]. In all these types of applications, customers (demand nodes) are assumed to be spatially distributed over a geographical area (service area). These customers originate demand for service, which must be served by some resource. Therefore optimization process must focus on locating the facilities considering the quantum of demand and any other restriction (in terms of capacity of resource or any other operational restriction etc.).

Such facility location models are related to the geometrical and combinatorial problem and thus various approaches have been implemented to handle location modelling [6]. As a result, Computational geometry has emerged as an effective tool for spatial problem in the past few decades. Few such applications of computational geometry are robotics, distribution system, motion planning, pattern recognition etc. In Tamir and Halman [44] authors have shown the utilization of Voronoi diagrams for handling location problems.

The geometric object that has been most successfully used in location problems is the VD. Drezner [60] has proposed an algorithm for discrete demand set and continuous demand set making usage of Voronoi diagram. A heuristic VD method has been presented for *p-center* in continuous space [41][16], which is further extended by Plastria[218].

It has been observed in the literature that interest of researchers has increased in Voronoi diagram as a result of its widespread applications. Most widely used approach for constructing Voronoi diagram are combinatorial methods, incremental techniques, divide-and-conquer methods, and approximation algorithms [219][41][220][221].

Chapter 3

3. Proposed solution for p-center in non-convex regions

The objective of the *p-center* location model is to minimize the distance between a facility and its farthest demand node. Therefore, it is used for the location of emergency services. As already discussed *p-center* is NP-complete and thus extensive research has taken place to handle this problem. In this chapter, *p-center* for a non-convex demand plane has been discussed. Based on the objective function of *p-center,* the objective here is to locate p resources so that the distance of a demand point to its nearest resource is minimized. p-center is broadly categorized into two categories namely *continuous p-center* and *discrete p-center* [35]. In *continuous p-center*, the location of resources has no restrictions. The resources are located with reference to the location of existing resources while optimizing the objective function of *p-center.* On the contrary, in a *discrete location model*, there exists a finite set of candidate locations. Location of p resources in a discrete location model is chosen among the given set of candidate locations while optimizing the objective function [222] [35].

According to the mathematical formulation by Daskin [223], If $D = \{p_1 , p_2, \dots . p_n\}$ represents demand points, the objective is to find p centers $C = \{c_1 , c_2 , \dots \dots c_p\}$ such that maximum distance for each demand point to its closest resource is minimized [13].

$$Z(c) = \min_{1\le j\le p} \left\{ \max_{1\le i\le n} d(p_i, c_j) \right\}$$

where $d(p_i, c_j)$ represents the minimum feasible distance between demand point p_i and facility c_j.

p-center is an important variant of location problem and thus has attracted many researchers [224] [225]. As discussed previously, there are numerous applications in a wide range of areas that employ *p-center*. For instance, *p-center* can also be used for location of telecommunication servers in computer networks. *p-center* guarantees that each node in the region is served by at least one facility within stipulated standards in terms of time or distance (covering model).

The requirement of stipulated service standards is the result of work by Drezner et. al [226] who presented the most imperative aspect of the *p-center* problem. According to [226], distance beyond the threshold Δ (stipulated service standard) is considered constant. For instance, in fire brigade service the service provided is advantageous if it is received within the stipulated time (threshold Δ). Any service that reaches the point of demand beyond Δ becomes ineffectual and therefore damage caused is considered to be constant beyond Δ. This can be demonstrated by locating p circular disks of radius Δ centred at obtained locations maintaining that all demand nodes are covered by at least one disk [227]. When p similar circular disks (of the same radius) are centred at obtained locations, the minimum radius which covers every demand point is called coverage distance. The objective of *p-center* is to minimize the coverage distance such that each demand node has at least one facility within a radius of Δ.

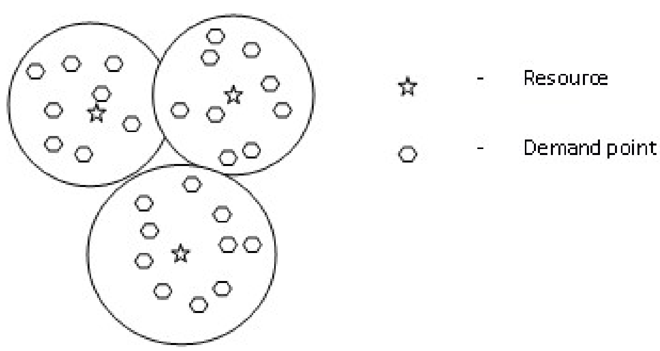

Figure 3.1: Illustrating 3-center problem

Figure 3.1 illustrates the 3-center problem. In this figure, asterisk and hexagons represent facilities and demand nodes respectively. The circles represent the area where each requesting node receives services by a facility located at its center.

As discussed earlier, as *p-center* is an NP-complete problem, various heuristics, techniques and approximation approaches are available in abundance. The best algorithm of the order of $O(n^{\sqrt{p}})$ is available for continuous *p-center* [228]. Authors in [228] [112] have discussed *1-center* and *2-center* problem which runs in $O(n^2 log^2 n)$ time.

It is also noticed in the literature that rectilinear *p-center* problem is optimally solvable in linear time and $O(n\ log\ n)$ for $p = 2$ and $p = 3$ respectively [112] [229]. Authors in [229] [230] escalated this general version of *p-center* by considering the weighted rectilinear *p-center* problem. *p-center* for graphs and trees is discussed in [231] [232] which can be used for handling client/server problem. Many other approaches are techniques are available in the literature to handle p-center.

In addition to various approaches for handling location modeling, computational geometry has also been also acknowledged as an efficient and effective tool for handling location problems. This acknowledgement is result of the efficiency of geometric structures in handling spatial problems. As a result, Computational geometry has been closely associated with the location problem since its inception. Over time, it has been established as an effective choice for handling all variants of spatial problems. In the subsequent section we discuss p-center in polygons and thereafter discuss the application of computational geometry to address this polygonal p-center problem.

3.1 p-center in Polygon

Earlier *p-center* problems considered the service plane to be continuous and infinite. However, if the service plane is bounded by a polygon, it is known as the polygonal *p-center* problem and necessitates some specialized approach. Polygonal p-center is applicable for locating resources in an enclosed region defined by a polygon. The requirement for a specialized approach is the result of discrepancy among Euclidean distance and realistic distance, particularly for non-convex polygons. This discrepancy between realistic distance and Euclidean distance is illustrated by following Figure 3.2.

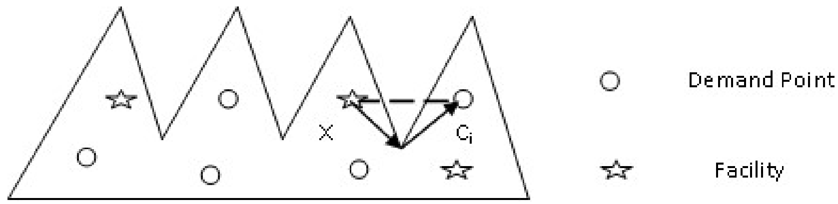

Figure 3.2: Illustration of actual distance and Euclidean distance

Figure 3.2 represents a comb-shaped non-convex polygon. In Figure 3.2, solid arrows represent the feasible distance between demand point c_i and Facility X whereas the dashed line represents the Euclidean distance. This inconsistency among actual and Euclidean distance is due to the existence of non-convex vertices in the service polygon. Now although service plane represented by convex polygons is similar to a basic *p-center* problem, non-convex service polygon necessitates a specialized approach. Here, we discuss usage of various structures in computational geometry to address this p-center in polygons.

3.2 Computational Geometry for p-center in polygon

As discussed in previous chapters, geometric structures have been used extensively in location modelling since its inception. Usage of computational geometric algorithms has grown manifold for solving numerous variations of location problems like p-center problem. A popular variant of p-center is polygonal p-center and requires a specialized approach unlike traditional p-center. In p-center problem for polygons, specialized approach is required as the feasible path may not be occasionally same as Euclidean path as represented in Figure 3.3. In Figure 3.3, the Euclidean path may go off the polygon and thus not advisable. In such scenario, the path among nodes needs to remain within polygon. Such a path is called feasible path and shown by dotted line in Figure 3.3. Distance between pair of farthest vertices is termed as geodesic or feasible diameter [233]. Mid-point of geodesic or feasible diameter is known as geodesic center (feasible radius) and is represented by diamond in Figure 3.3.

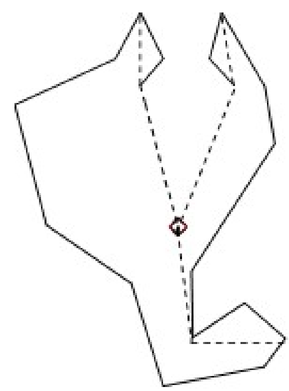

Figure 3.3: Illustration of Geodesic Center and Geodesic diameter

Furthermore, G_c within the convex region can be utilized to minimize the distance of farthest demand site from a facility. Moreover, according to the definition of G_c of the polygon, if the resource is located at G_c, the distance from resource to its farthest demand node is upper bounded by the radius of the polygon.

However, computation of feasible path or shortest path on a surface remains a challenging problem in computational geometry despite its widespread applications in various domains e.g computerized brain flattening [234] [235], texture mapping [236], surface partitioning [237][238], terrain navigation [239], and path planning. There exist some methods for computing the exact shortest path on non-parameterized surfaces [240] [241][242][243]. Several algorithms are available to find the feasible path between the pair of vertices [244][245]. These suggested algorithms are difficult to implement. Therefore, several algorithms for approximate shortest paths have been suggested by researchers [239][246] [244] [245] [247]. It can be used for applications where rapid solutions are desired and some loss of accuracy can be tolerated.

O'Rourke et al. [242] extended the algorithm in Sharir and Schorr [240] and Mount [241] and obtained the first algorithm to find the exact feasible path between two vertices in polynomial time. Mitchell et al. [248] also formulated continuous Dijkstra to find the shortest paths from a source point s. Authors in Chen and Han [249] devised an algorithm to compute feasible distance for a non-convex polyhedron that uses a tree structure. Kapoor [243] presented a sub-quadratic algorithm to find the shortest feasible path between two points. Kanai and Suzuki [250] also proposed an approximation algorithm to compute shortest feasible path which works in an iterative manner. Aleksandrov et al. [251] presented an approximation algorithm to compute the weighted feasible path with approximation ratio (1 + ϱ).

Authors in [233] propose a $O(n\log n)$ algorithm for G_c of the polygon. Now it becomes evident that the facility must be located at the G_c of the polygon for *1-center* thus bounding the maximum distance by feasible radius. In the following section, we propose an approach that implements various tools in computational geometry for *p-center* in polygon. The suggested approach is also competent in finding the optimal value of p such that each demand point has at least one resource within a threshold distance Δ.

3.3 Proposed Approach

According to the proposed approach, Geometric properties of demand location can be utilized to locate candidate facility sites in continuous space such that coverage is maximized [252]. It should be noted that Voronoi diagrams and Delaunay triangulations, for the past decades, have been playing a vital role in location modelling. In particular, they are increasingly applied for geometric modelling [253]. These tools of computational geometry help to understand geometric properties of demand sites and thus aids in location modelling. This subsection briefly discusses the Voronoi Diagram in connection to the *p-center problem*, followed by its dual Delaunay Triangulation.

3.3.1 Voronoi Diagram

Voronoi Diagram is a classification of demand points in the d-dimensional plane based on their distance to the nearest resource. Each such division is known as Voronoi region [254] [14]. A formal definition of the Voronoi diagram has already been given. Numerous algorithms for Voronoi diagram are available in the literature. The time complexity of Fortune's algorithm [255] to construct VD of n points is $\mathrm{O}(\mathrm{n} \log \mathrm{n})$. The VD can be employed to handle the nearest and reverse Nearest Neighbourhood (RNN) query in $\mathrm{O}(\log \mathrm{n})$.

Voronoi Diagram can also be used to address the allocation of demand points to nearest facilities. Quantum of demand for requesting nodes in Voronoi Region VR_i can help to determine the capacity of resources for Capacitated Facility Location. This determination of capacity ensures that facility P_i is capable to serve all requesting nodes in VR_i.

Suzuki and Okabe [256] have used the Voronoi Diagram and proposed a Voronoi Diagram based Heuristic (VDH) for continuous p-center. According to VDH, p centres are chosen as initial solution randomly. Thereafter these initial p solutions are used to construct Voronoi diagram. Construction of Voronoi diagram is followed by the computation of center of each Voronoi region (1-center problem). Thereafter, these initial solutions are refined during subsequent iterations until the termination condition holds.

Now demand nodes in the Voronoi Region, VR_i are served by facility p_i because it is the closest facility as per the definition of Voronoi diagram. According to the definition of Voronoi diagram, the farthest demand point from facility p_i lies at the boundary of VR_i. The distance between facility p_i and the farthest demand point in the Voronoi region VR_i is the radius of VR_i. If demand nods are assumed to be distributed in $\mathrm{VR_i}$, the objective function of p-center can be optimized by placing the facility at the center of the corresponding Voronoi region for convex regions. In contrast, p-center for the polygon is a non-convex optimization problem and thus it remains a challenge to find global optima [257][258].

Now a large number of vertices may be encountered during iterations of VDH as a result of non-convexity of Voronoi regions. The computation time for VDH in non-convex polygons significantly increases as a result of repeated execution of *1-center* [88]. This requires the efficient execution of a *1-center* algorithm for the non-convex region to further optimize the objective function. The proposed approach considers non-convex polygon and employs geometric properties of demand points in demand plane.

3.3.2 Delaunay Triangulation

In Delaunay Triangulation two Voronoi sites p_i and p_j are connected by an arc if $VR(p_i)$ and $VR(p_j)$ share a common edge. Lawson [259] initially proposed a flip algorithm to construct Plane Delaunay triangulations having the worst-case complexity to be $O(n^2)$. Delaunay triangulation is useful for convergence of initial solutions to the optimal solution during iterations of the proposed algorithm. Usage of Delaunay Triangulation results in fast convergence of initial solutions to the optimal solution. it aids in finding the direction of movement to refine the solution during iterations of the proposed algorithm. Moreover, employment of Delaunay triangulation results in rapid convergence of initial solution to optimized solution in the proposed algorithm discussed in the subsequent section.

3.3.3 Algorithm for polygonal p-center

The proposed algorithm handles p-center using Voronoi Diagram and Delaunay Triangulation of the demand points in a non-convex polygonal region. As already discussed, the proposed algorithm uses G_c in contrast to Euclidean distance. In the proposed algorithm, the feasible center Gc coincides with the *1-center*.

It is evident from literature in location modelling that each resource lies within Convex Hull of Demand Point $(CH(D))$[13]. Usage of Convex Hull limits the possibilities for the location of resources and thus reduces the complexity. Convex Hull of the demand points is the minimum enclosing polygon that encloses all demand points. Computing the convex hull for input is an important pre-processing step for location problem. To perform this step, we use the Quickhull algorithm. The proposed polygonal *p-center* approach is as follows:

Proposed Algorithm for polygonal *p-center* problem
1. *Construct convex hull* C *of the demand points in the region so that C does not go outside the demand plane represented using non-convex polygon.* 2. *Randomly select* p *demand points as the initial solution for the resources* $C = \{c_1, c_2 \ldots\ldots\ldots c_p\}$ 3. *Generate Voronoi diagram* $VD(C)$ *in the constrained region* 4. *Generate Delaunay triangulation* $T(C)$ 5. *for each facility* i *Assign all demand points in Voronoi region* $VR(c_i)$ *to set* S_i 6. *For each set* S_i $Dist_i = \max\{r_{min_i}\ for\ all\ members\ in\ the\ set\ S_i\}$. 7. $DIST = \max\{Dist_i\}\ for\ all\ facilities$ 8. *if* $DIST > threshold$ *then* $Diff\ =\ DIST - threshold$ 9. *if* $p = 1$ *or* 2 *Move resource* c_i *having* $Dist_i = DIST$ *towards* Gc *by* $Diff$ *provided it does not leave* C *else* *Move resource* C_i *having* $Dist_i = DIST$ *towards the longest edge in* $T(C)$ *by Diff provided does not leave* C 10. *If resource* C_i *has only one edge in* $T(C)$ *then* *move resource* C_i *towards farthest demand point by* $Diff$ *provided does not leave* C 11. *Reconstruct the Voronoi diagram* VD *and Delaunay Triangulation* T 12. *Repeat step 5 until termination conditions hold* 13 *If the termination condition does not hold for a specific number of iterations, increment the value of p and repeat step 2*

In the proposed approach step 1 constructs the Convex Hull C of the demand points. This is followed by a random selection of p locations as initial locations in step 2. Thereafter Voronoi Diagram and Delaunay Triangulations of these p locations are constructed in step 3 and step 4 respectively. Step 5 considers all demand points in the Voronoi Region for each resource that helps to find the radius for each resource in step 6. Further steps ensure that feasible radius of no resource exceeds the threshold Δ. Any resource having its feasible radius more than the threshold Δ is selected for relocation. The direction of relocation of the selected resource is determined by Delaunay Triangulation and magnitude of relocation is obtained using Δ according to step 8. The algorithm iterates until termination conditions are encountered. Furthermore, if the algorithm terminates with any demand point having no resource in the circle of radius threshold Δ, the value of p is then incremented and the process is repeated for the modified value of p. Thus proposed algorithm can also be used to find minimal p as it is a significant cost influencing factor. The initial value of p can be set to any arbitrary value, even to 1 that can be further increased if required thus finding an optimal number of resources.

In the proposed approach, step 1 constructs the Convex Hull of the demand points. The well-known methods for Convex Hull in $\boldsymbol{\Theta(n \log n)}$ are available in the literature. It is then followed by a random selection of $\boldsymbol{p}$ points as an initial solution, which is used for the construction of Voronoi Diagram. The Complexity of Voronoi Diagram for $\boldsymbol{p}$ points is $\boldsymbol{\Theta(p \log p)}$ where $\boldsymbol{p \ll n}$. As suggested in the approach; these initial solutions are improved using Delaunay Triangulations during iterations of the approach. Delaunay Triangulation used in step 4 of the algorithm is dual of Voronoi Diagram and thus needs no extra computation.

As demand points in the region are static, this static nature of demand points can be used to generate the Range tree. Generation of the Range tree helps in finding all demand points in $\boldsymbol{VR_i}$ for each resource $\boldsymbol{c_i}$ in $\boldsymbol{O(\log n)}$ time. According to the suggested approach, shifting of any resource necessitates regeneration of Voronoi Diagram. It does not require regenerating the complete Voronoi Diagram. The reason is that it only affects the Voronoi Region found in step 6 and its neighboring Voronoi Regions.

The number of iterations here is reasonably small due to a higher rate of convergence. This higher rate of convergence is achieved using $\boldsymbol{Diff}$ as the magnitude of movement for the selected resource. A Global selection of candidate resource for movement also helps in achieving a higher rate of convergence. Thus proposed approach is a polynomial time method for *p-center* in non-convex polygons.

3.3.4 Illustration of the proposed algorithm

This section illustrates the proposed algorithm for a non-convex region. Here demand points $(\boldsymbol{n})$ and resources $(\boldsymbol{p})$ is considered to be 50 and 5 respectively as shown in Figure 3.4. The outer polygon represents the non-convex polygon under consideration. Here, Convex Hull of demand points goes outside the given polygon region. It is evident that no resource should be located outside the polygon. Also, travel is permissible only within polygon. Thus, in order to eliminate such area from consideration, it is required to minimally reduce the Convex Hull so that Convex Hull is entirely located within the polygon region. Such reduced convex hull of given demand points is represented by inner polygon in Figure 3.4. The principle behind reducing (contracting) the convex hull is to eliminate the portion from consideration where facilities can't be located. Therefore, in the proposed approach, we employ minimized convex hull.

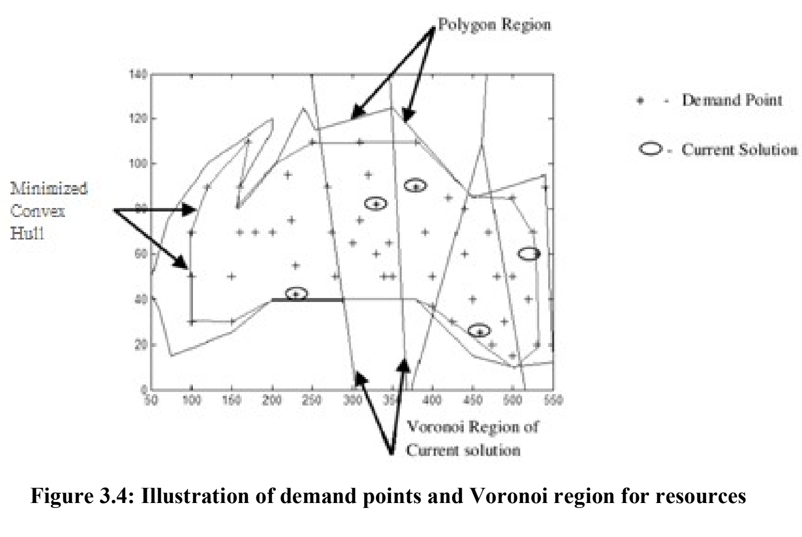

Figure 3.4: Illustration of demand points and Voronoi region for resources

As already discussed, no resource lies outside the Convex Hull. Demand points are shown by '+' symbol and '+' within ellipse represents the initial location of resources in Figure 3.4. As discussed above, chosen $\boldsymbol{p}$ locations are used to construct Voronoi Diagram $(\boldsymbol{VD})$. This Voronoi Diagram is represented by spatial decomposition in Figure 3.4. All demand points in $\boldsymbol{VR_i}$ will be allocated to resource $\boldsymbol{p_i}$ as it is their nearest facility.

Figure 3.5(a) represents the farthest demand point for $\boldsymbol{p_1}$ using a circle and the corresponding feasible radius. Now as stated in the algorithm, the resource having maximum feasible radius is selected for movement; thus resource $\mathbf{p_1}$ is selected during current iteration as shown in Figure 3.5 (a). Now the Delaunay Triangulation $\boldsymbol{T}$ is used to move selected resource. As per the spatial decomposition, selected resource $\mathbf{p_1}$ has only one neighboring Voronoi Region and therefore $\mathbf{p_1}$ has only one edge in corresponding $\boldsymbol{T}$. Hence it is moved towards the farthest demand point in $\boldsymbol{VR_1}$ by a magnitude equal to the difference of feasible radius and threshold Δ (VAL). This movement takes place along the feasible Path as shown in Figure 3.5 (b).

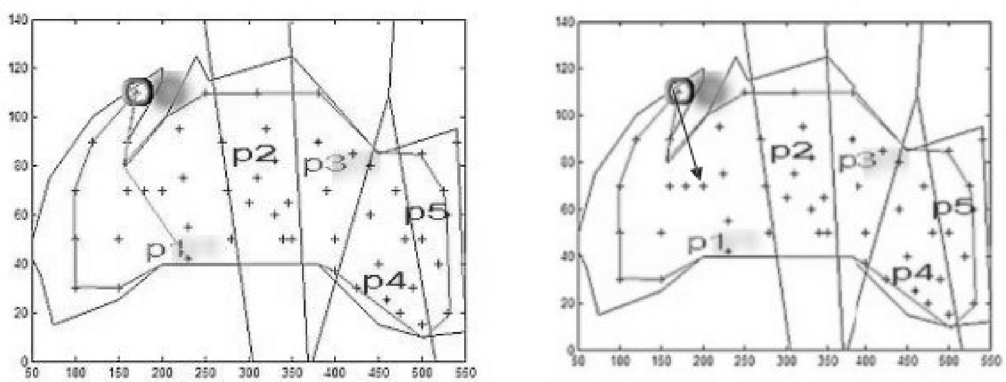

Figure 3.5: (a) selection of resource for relocation (b) Selecting direction for movement

The process continues until any resource has its feasible radius not exceeding the threshold Δ. This process will be executed until the termination conditions hold.

3.3.5 Simulation and Results

The proposed algorithm has been simulated to explain its working for $\boldsymbol{n} = \mathbf{80}$ demand points and $\boldsymbol{p} = \mathbf{5}$ resources. The threshold value Δ has been set to $\mathbf{170}$. A randomly generated initial solution is shown in Figure 3.4. The initial solution converges towards optimal locations during iterations of the algorithm.

Table 1: number of demand nodes allocated to each resource and its radius.

Iter.	Dist1	Dist2	Dist3	Dist4	Dist5	DIST	Diff
1	125.92(12)	108.78(15)	123.49(19)	197.04(11)	**231(23)**	231	61
					Resource p_5 needs to be moved		
2	125.92(17)	108.78(14)	140.13(20)	197.04(11)	**208.03(25)**	208.03	38.03
					Resource p_5 needs to be moved		
3	125.92(20)	107.32(15)	140.13(24)	**197.04(11)**	134.43(20)	197.04	27.04
					Resource p_4 needs to be moved		
4	125.92(20)	107.32(15)	140.13(23)	**170(12)**	134.43(20)	170	0
					No further movements are required		

During an iteration of the algorithm, the radius of each existing resource in $\mathbf{C}$ is calculated. This radius $\mathbf{dist_i}$ of resource $\mathbf{c_i}$ represents the distance from $\mathbf{c_i}$ to the farthest demand point in $\mathbf{VR_i}$. The radius $\mathbf{dist_i}$ for $\mathbf{i} = \{\mathbf{1}, \mathbf{2} \dots\dots.. \mathbf{p}\}$ has been shown in Table 1 during the iterations of the algorithm. Thus from Table 1, we see that during the first iteration, $\mathbf{dist_5}$ is largest that requires moving resource $\mathbf{p_5}$. Finding the direction of movement requires determining the longest edge in Delaunay Triangulation. The length of Delaunay edges has been given in Table 2. Thus using Table 1 and Table 2, it is determined that $\mathbf{p_5}$ should be moved towards $\mathbf{p_3}$ by 61 (Diff for $\mathbf{1^{st}}$ iteration).

The numbers marked in bold in Table 1 thus determines the resource that needs to be moved. While bold numbers in Table 2 determines the direction of movement for the selected resource. The value in pair of braces in Table 1 represents the number of demand points in $\boldsymbol{VR_i}$ of $\boldsymbol{p_i}$ during that iteration. It represents the number of demand points allocated to each resource. During the iteration of the algorithms, the resources continuously change their location and thus changing the number of demand points in $\boldsymbol{VR_i}$.

Table 2: Edge length in DT for direction of movement for resource obtained from Table 1.

Iter	What to move	P1	P2	P3	P4	P5
1	**P5**	179.06	151.82	**207.32**	X	X
		Resource p_5 moved in the direction of p3				
2	**P5**	**193.17**	105.06	110.44	X	X
		Resource p_5 moved in the direction of p1				
3	**P4**	X	X	56.32*	X	X
		Resource p_4 moved towards farthest demand point				

** represents only one edge in the Delaunay triangulation*

The same process is repeated during subsequent iterations of the algorithm. Using Table 1 and Table 2, the selected resource is moved towards a particular direction by $Diff$. It is observed from the illustration that $Diff$ significantly converges to 0, thus reducing the number of iterations. As shown, $Diff$ converges from 61 to 0 in four iterations only.

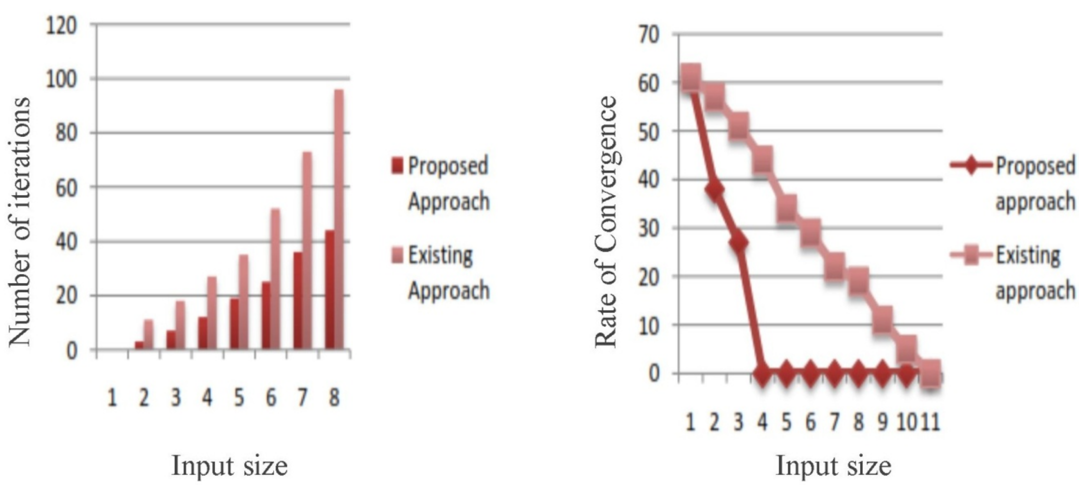

Figure 3.6: (a) number of iterations (b) rate of convergence

Thus we have successfully simulated the proposed algorithm. Contrary to the existing approach, it is seen that the initial solution rapidly converges towards an optimal solution in the proposed approach [260] [32]. During comparative analysis, it is observed that the proposed approach outperforms the existing approach [260]. The results of the comparison are shown in Figure 3.6. Figure 3.6 (a) and (b) illustrates the number of iterations and rate of convergence respectively.

The proposed approach is capable of implementing *p-center* for the non-convex demand region by utilizing the Voronoi Diagram and Delaunay Triangulation of the demand points. It is observed that the proposed algorithm outperforms the existing approach. It is also observed that using $\boldsymbol{Diff}$ as the magnitude of movement results in a higher convergence rate. The proposed algorithm is also capable of estimating an optimal value of $\boldsymbol{p}$ for *p-center* problems. The suggested algorithm can be further extended to reduce the number of iterations by estimating initial solutions and thus further reducing the execution time of the algorithm. The proposed approach can also be extended in the direction of solving polygonal *p-center* with constraints.

In this work, the focus is limited to the location of p resources in a non-convex demand plane that minimizes the total coverage distance. In our solution, we employ various Geometrical Structures such as Convex Hull, Voronoi diagrams and Delaunay triangulations for p-center in non-convex polygon for utilizing the geometric properties of demand sites. The proposed algorithm handles p-center using Voronoi Diagram and Delaunay Triangulation of the demand points in a non-convex polygonal region. Convex Hull is used to further reduce the complexity of the proposed approach. The results of simulation show that the initial solution rapidly converges towards an optimal solution in the proposed approach.

Here we handled locating p resources in a demand plane that is represented by non-convex region. The objective here is to minimize the maximum distance. In our solution, we employ the geometric properties of demand sites in addition to Geometrical tools such as convex hull, Voronoi diagrams and Delaunay triangulations for the solution. The proposed algorithm handles p-center in a non-convex polygonal region efficiently. In the suggested approach, the Voronoi diagram is used to reduce the run-time complexity of the proposed approach and thus initial solution rapidly converges to the optimal solution. The results of simulation show that the proposed approach outperforms the existing traditional approach by reducing the number of iterations. In the suggested approach, we have considered the non-convex demand region in absence of any constraint. The proposed approach can also be extended in the direction of solving polygonal p-center with constraints in form of holes.

Chapter 4

4.Proposed approach for Facility Allocation

Allocation of demand nodes to the facilities is an indispensable element of location modelling. Even the greatest location solution may lose its significance if the requesting sites are not allocated to the appropriate resource. Any inefficiency in allocation augments the transport cost despite the best location decisions. Allocation becomes more significant for emergency services as any inefficient allocation may result in the fatal loss. Despite recognition of allocation, it is observed that the allocation problem has been devoid of sufficient attention. Here, in this chapter, we consider the allocation of requesting nodes to the facilities, where facilities have a capacity constraint. Allocation of demand nodes is preceded by Location of facilities in the demand plane considering various parameters. After the allocation of facilities to the demand nodes, whenever there arises a demand, the allocated facility is responsible for serving this request. The objective of allocation is to minimize the service cost; therefore an appropriate approach needs to be devised to minimize the service cost. This cost includes connection and transportation cost between demand points to the allocated resources. In this chapter we consider allocation for capacitated facilities for a continuous demand plane with polyhedral barriers. Presence of barriers in the demand plane further obscures the problem. Here in this chapter, we propose a Residue Based Capacitated Facilities Allocation (RBCFA). The proposed approach considers a metric that determines how many demand nodes can be further allocated to the resource, known as residue ratio. This chapter proposes an algorithm for this residue based allocation and presents an illustration.

4.1 Background

The reason for rigorous research in the area of location modelling is that these problems are not only challenging and interesting but also widespread applications. Location modelling consists of the location of facilities in the demand plane, allocation of demand nodes to the resources and relocation of resources if required. During the literature survey, it is observed that although allocation is a key factor of the location model, still it is devoid of intensive research.

Location problem locates a facility $X \in R^2$ while existing facilities $EX = \{Ex_1, Ex_2 \ldots \ldots \ldots . . Ex_m\}$ are present in the demand plane with an objective to minimize the weighted distance from X to existing facilities. Let $d(X, Ex_i)$ represents the smallest distance from X to existing resources Ex_i and w_i represent associated weight, the problem is formalized as:

$$\min f(x) = \sum_{i=1}^{m} w_i d(X, Ex_i) \tag{4.1.1}$$

Here we consider two-dimensional continuous demand plane, therefore Euclidean distance is the measure of shortest distance.

Here in this chapter, we consider demand plane containing various constraints. These constraints may be present in the form of the forbidden region, congested region or barriers [93]. The forbidden region represents the area where the location of resources is prohibited while this region can be passed through. On the contrary, the congested region represents the area which can be passed through at an additional cost but the location of any resource is not permitted. Barriers represent the area that prohibits location as well as passage through this region, thus demonstrating the physical regions like lake, mountains etc. The inclusion of Constraints in the demand plane results into devising some specialized approach for location modelling. The requirement of a specialized approach arises as actual distance may not be similar to Euclidean distance.

For demand plane containing constraints, allocation focuses on assigning demand nodes to the resources so that objective function is optimized in the presence of constraints

[261][262]. Now in order to optimize the objective function, each requesting node should be allocated to the nearest facility. It may not be possible for each requesting node due to the capacity constraint of resources. Therefore capacity constraint for facilities necessitates devising specialized location modelling approach in order to optimize the objective function.

As discussed, this chapter focuses on the allocation of capacitated resources in a constrained demand region. During literature survey, it is noticed that numerous variants of location models are available e.g. Single source Capacitated FWP (SSCFWP) and Multiple source Capacitated FWP (MSCFWP) [263][264][18].

In SSCFWP each demand node is fully serviced by a single capacitated facility. Although in MSCFWP a demand node may obtain its demand from multiple capacitated facilities. In this chapter, we consider the presence of barriers in the demand plane. Thus we have limited our work to SSCFWP in the presence of barriers. Presence of barriers in demand planes further intricate the allocation problem and thus needs to be handled specifically. In the proposed approach, we have employed various geometric structures for handling SSCFWP with barriers e.g visibility graphs etc.

4.2 SSCFWP for constrained Demand plane

In order to formulate the mathematical model for SSCFWP, we define the following parameters:

n number of demand points

m number of facilities

req_i quantum of requirement for demand node i

C_j capacity of facility j

$dloc_i$ location of demand point i in 2d plane

$floc_j$ location of facility j in 2d plane

$$z_{ij} = \begin{cases} 1 & \text{if demand point i is connected to facility j} \\ 0 & \text{otherwise} \end{cases}$$

$$\min g = \sum_{i=1}^{n}\sum_{j=1}^{m} d(dloc_i, floc_j) z_{ij} \quad (4.2.1)$$

$$\sum_j z_{ij} = 1 \text{ for } i \in \{1,2,\dots.n\} \qquad Z_{ij} \in \{0,1\} \quad (4.2.2)$$

$$\sum_i req_i z_{ij} \leq C_j \text{ for } j \in \{1,2,\dots..m\} \quad (4.2.3)$$

$$z_{ij} \in \{0,1\} \quad \text{for } i \in \{1,2,\dots.n\} \text{ and } j \in \{1,2,\dots..m\} \quad (4.2.4)$$

The objective function stated earlier is formulated in Eq. 2.1. Eq. 2.2 ensures that the requesting node receives its service. Eq. 2.3 maintains that total demand served by a facility to all allocated demand nodes at all time is upper bounded by its capacity. According to SSCFWP, every demand point should be served by only one facility which is checked in Eq. 2.4. The focus of study in this chapter is limited to single source capacitated Fermat Weber Problem with constraints.

The inclusion of barrier in the location models was introduced by Katz and Cooper [23] [22] by considering one circular barrier which was later extended to incorporate polyhedral barriers. These barriers in the 2-dimensional plane are represented in form of polygons. Polygons representing barriers may be either convex or non-convex. Although in this chapter, we consider only convex polygonal barriers [265]. Exclusion of non-convex polygons is based on the work done by Klamroth [266] [267]. Another work by Butt [268] also supported the exclusion of non-convex polygons from consideration. Therefore any non-convex polygonal barrier can be easily substituted by its Convex Hull without affecting the solution of location model. Consequently, we consider only convex polygons based on the results of former researchers.

As discussed, here we consider demand plane containing barriers for SSCFWP. The inclusion of barriers in demand plane restricts consideration of Euclidean distance. Therefore new distance metric needs to be devised for constrained demand plane so we hereby define a new distance metric.

Let β_i represent a convex polygonal barrier region. Now $\beta = \cup_{i=1}^{n} \beta_i$ represents the entire barrier region in the demand plane. Now resource can be located only within feasible region $F = R^2 \setminus \mathrm{int}(\beta)$. The Euclidean distance between X and Y is represented by $d(X, Y)$. Let barrier distance $d_\beta(X, Y)$ represents the shortest distance which does not pass through barrier β [20]. Therefore

$$d_\beta(X, Y) = \min \{\mathrm{path}(X, Y): \quad \mathrm{path}(X, Y) \in F\} \tag{4.2.5}$$

Here Equation 4.2.5 maintains that barrier distance passes through feasible region F of the demand plane without passing through barrier region.

From the definition of $d(x, y)$ and $d_\beta(X, Y)$, Following conclusion can be drawn

$$d_\beta(X, Y) \geq d(X, Y) \tag{4.2.6}$$

$$d_\beta(X, Y) = d(X, Y) \qquad \text{if } d(X, Y) \in F \tag{4.2.7}$$

$$d_\beta(X, Y) > d(X, Y) \qquad \text{if } d(X, Y) \text{ intersects } \mathrm{int}(\beta) \tag{4.2.8}$$

As Euclidean distance is the smallest distance, barrier distance is lower bounded by Euclidean distance. This property has been given in Equation 4.2.6. Equation 4.2.7 represents the condition when Euclidean distance doesn't pass through the interior of a barrier. In such a case, barrier distance is same as Euclidean distance. Whenever Euclidean distance passes through β, barrier distance exceeds $d(X, Y)$ as given in Equation 4.2.8. In order to find $d_\beta(X, Y)$, we employ *Barrier Touching Property (BTP).*

Barrier Touching Property (BTP) proves to be an important property for $d_\beta(X, Y)$ in R^2 [266]. According to BTP, the shortest feasible path P between $X \;\&\; Y |\ X\ is\ not\ visible\ to\ Y$, has breaking point only at vertices of barrier polygons. Barrier Touching Property is demonstrated in Figure 4.1.

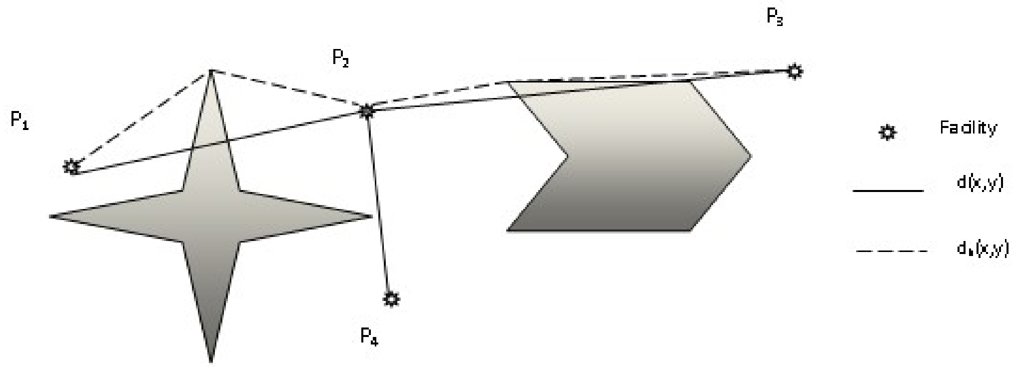

Figure 4.1: Barrier Touching Property

Here shaded polygons represent the Barrier region. Existing facilities $P = \{p_1, p_2, p_3, p_4\}$ in the demand plane are represented by stars. The Euclidean and barrier distance is represented by solid and dashed line respectively. Existing facility p_2 and p_4 are directly visible therefore Equation 4.2.7 satisfies in this case. On the other hand, for pairs (p_1, p_2) and (p_2, p_3) the Euclidean distance intersect barrier β. Therefore feasible distance is lengthened due to the presence of barrier and therefore inequality 3.4 becomes true.

According to Barrier Touching Property, barrier distance always has a bend at some vertex as represented in Figure 4.1. Consequently, visibility graphs can be utilized to find barrier distance in constrained SSCFWP.

4.3 Visibility Graph for constrained SSCWP

Barrier touching property leads to usage of visibility graphs for barrier distance in constrained location models. Let $CH(\beta)$represents Convex Hull of barrier region β and B is set vertices of $CH(\beta)$. Visibility graph is undirected graph $VG = < V, E > | V = Ex \cup B$. Here EX represents the set of existing facilities. Figure 4.2 represents the visibility graph for the structure of Figure 4.1. In Figure 4.2, Convex Hull of the barrier region is shown by the red solid line. Visibility arcs are represented by the dashed line. Visibility edge $arc(x, y) \in$

$E|x, y \in V, x$ is visible to y. Two vertices x and y are directly visible if $path(x, y) \in F$ i.e Euclidean path between X and Y doesn't pass through $int(\beta)$.

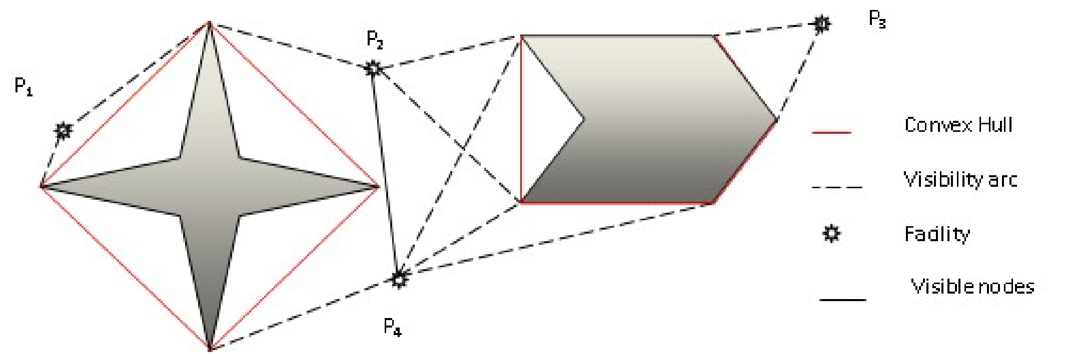

Figure 4.2: Visibility Graph

It is evident from Figure 4.2 that for a pair of vertices x and y, which are not directly visible i.e. $arc(x, y) \notin E(VG)$, the shortest path contains some vertices of B (BTP). In such a case, $path(x, y)$ consists of starting point x, a sequence of vertices from $CH(\beta)$ and finally the ending point y. As discussed, the first vertex in $d_b(x, y)$ remains x. Thereafter, The first vertex after x in $d_b(x, y)$ is called the projection point from x towards y [269]. If p_{xy} represent the projection point from starting vertex x for y, the feasible path length $l_b(x, y)$ is determined as:

$$l_\beta(x, y) = d(x, p_{xy}) + d_{\partial CH(B)}(p_{xy}, p_{yx}) + d(p_{yx}, y) \quad if\ arc(x, y) \notin E(VG) \qquad (4.3.1)$$

$$l_\beta(x, y) = d(x, y) \quad if\ arc(x, y) \in E(VG) \qquad (4.3.2)$$

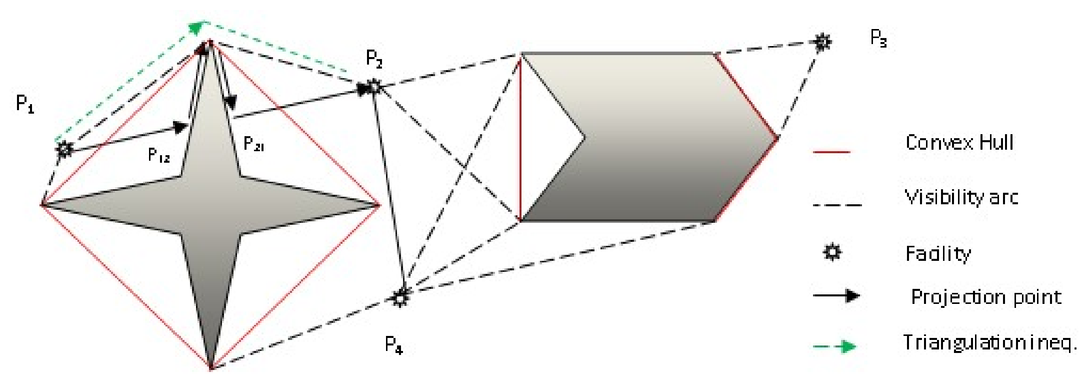

Figure 4.3: Illustration of projection point

Here equation 4.3.1 considers the case when two vertices x and y are directly not visible. In such a case the path length consists of Euclidean distance from x to p_{xy}, the shortest distance between p_{xy} and p_{yx} along the boundary of $CH(B)$ and finally Euclidean distance from p_{yx} to y. Figure 4.3 represents equality 3.4 with help of directed arrows. As observed p_1 and p_2 are directly not visible, the path followed is represented using directed arrows. Further, this path is optimized using triangular inequalities resulting in the path represented by dashed directed arrows which is same as visibility graph. Equation 4.3.2 represents the case of directly visible nodes and thus shortest path length is similar to Euclidean distance.

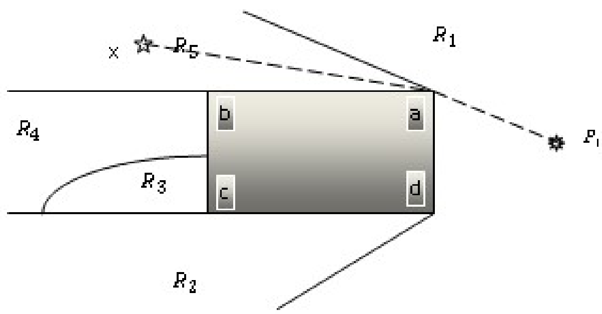

Figure 4.4: Partitioning of demand plane with respect to p_1

As discussed, the visibility graph can be used to find $path(x,y) \in F \mid x,y \in V(VG)$. Furthermore, the demand plane can be partitioned with respect to the projection point for a vertex of VG [267]. Five partitions of demand plane with respect to the projection point of p_1 are represented in Figure 4.4 by a solid line. Here all points from the same region have the same projection point for p_1. Thus any demand node $x \in R_2$ will always be approaching p_1 through projection point a i.e $p_{x,p_1} = a \; \forall \, x \in R_2$. Consequently, this planar division with respect to a point can be utilized to generalize the distance functions as follows:

$$d_{\beta}(x, p_1) = d(x, a) + d(a, p_1) \quad \forall \; x \in R_2 \tag{4.3.3}$$

Now as Figure 4.4 gives the partitions with respect to point p_1, similar partitioning is obtained for all the points. These partitions with respect to different points are then superimposed to obtain the partitions of the demand plane. This planar partitioning is used for the RBCFA proposed in the following section.

4.4 RBCFA: Proposed algorithm for allocation

Although location and allocation problems have been studied inseparably in the literature, the allocation has been lacking enough attention. Few approaches are available in the literature to handle multiple variants of the allocation. It is noticed in the literature that any demand point is always allocated to the nearest facility [13][88]. This allocation principle performs optimum for resources without capacity constraint. But here we consider SSCFWP for constrained demand plane.

In CFWP (capacitated FWP), each resource has capacity C_j which limits the total number of demand points it can be allocated to. Similarly, we consider one residue capacity of a resource res_j that represents the balance capacity of the resource. From the definition of C_j and res_j, it is clear that $res_j \leq C_j$. Initially, no demand point is allocated to resource j then $res_j = C_j$. Whenever a demand point is allocated to resource j, res_j decreases. Demand points can be allocated to resource j while $res_j > 0$.

Generally a demand point i is always allocated to its nearest resource j while $res_j > req_i$. Here in the proposed approach RBCFA, we consider a new metric *residue ratio*. Residue ratio of a resource j is the ratio of residue capacity to its total capacity. From the definition of *residue ratio*, it can be defined as:

$$R_j = res_j/C_j \mid j \in \{1,2 \ldots \ldots . . m\}$$

As res_j is upper bounded by C_j and $R_j \in [0,1]$. The motive behind the inclusion of residue ratio R_j is to accommodate future demands to their nearest resource. Although the gain of the proposed approach may not be appreciable in the beginning, it becomes significant as the difference between $\sum_{j=1}^{m} C_j$ and $\sum_{i=1}^{n} req_i$ diminishes.

In proposed RBCFA, the planar decomposition given in the previous section is further represented in form of undirected graph $G = \langle V|E \rangle$ with $V = P \cup R$. This graph contains all existing facilities and partitions as its vertices. Here $P = \{p_1, p_2 \ldots \ldots p_m\}$ and $R = \{r_1, r_2 \ldots \ldots r_n\}$ represents the existing facilities and partitions respectively. A partition r_i is an enclosed region and thus can be represented as $r_i = \{r_{i1}, r_{i2} \ldots \ldots \ldots r_{ik}\}$ by an ordered sequence of k vertices in a clockwise or counter-clockwise direction. As barrier is also represented as a convex polygon, it is also represented as an ordered pair of its vertices. Figure 4.5 represents the resulting partitions of demand plane having a barrier with respect to vertex p_1 as given by Butt and Cavalier [92].

As shown in Figure 4.5, the demand plane is partitioned as $\{r_1, r_2, \ldots \ldots r_5\}$. Each partition is shown with a different colour in Figure 4.5. As discussed, each partition represents a node of the corresponding graph in RBCFA. Furthermore, edges among these nodes represent the various types of relationships among the partitions. These various types of relations are:

i. partitions that share a single vertex

ii. partitions that share an edge

iii. partitions connected along ∂B (barrier)

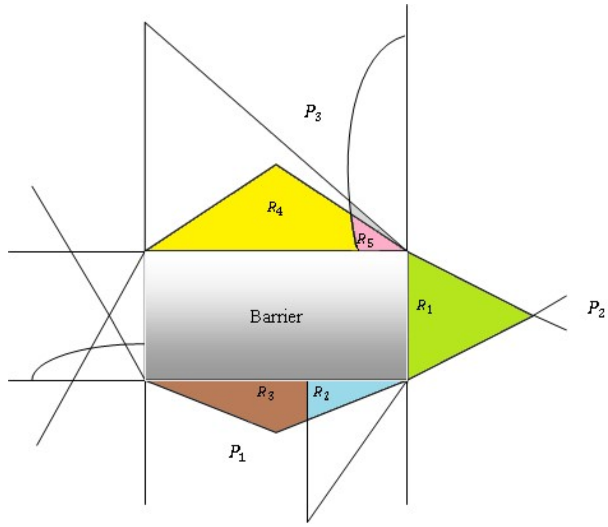

Figure 4.5: Illustration of planar division by Butt and Cavalier (1996)

Now, these different types of relations are represented as follow:

Whenever two regions are connected by vertex represented by cbv, equation 4.4.1 holds true

$$cbv(r_i, r_j) \in E(G)\ if\ \exists\ x, y\ s.t\ r_{ix} = r_{jy} \tag{4.4.1}$$

Similarly, if two regions are connected by an edge represented by cbe, equation 4.4.2 holds true

$$cbe(r_i, r_j) \in E(G)\ if\ \exists\ x, y\ s.t\ (r_{ix}, r_{ix+1}) = (r_{jy}, r_{jy+1}) \tag{4.4.2}$$

For two regions if 4.1 and 4.2 do not hold true, these are connected along the boundary of a barrier. The term to represent this type of relation is $cb\partial B$.

$$cb\partial B(r_i, r_j) \in E(G)\ if\ \exists\ x, y\ s.t\ r_{ix} \overset{\partial B}{\rightarrow} r_{jy} \tag{4.4.3}$$

Here in equation 4.4.3 $r_{ix} \overset{\partial B}{\rightarrow} r_{jy}$ represents that there exists a path along the boundary of barrier ∂B from r_{ix} to r_{jy}. It implies that r_{jy} is reachable from r_{ix} along ∂B.

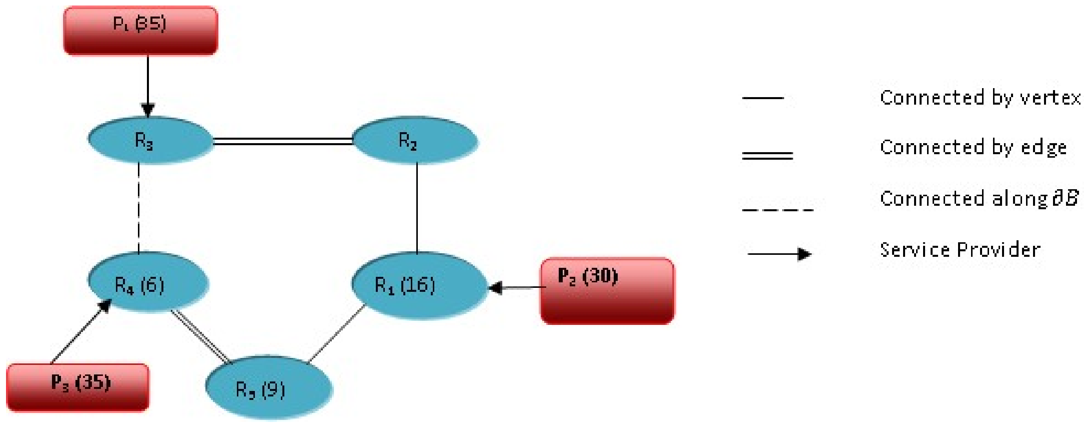

Figure 4.6: Graphical Representation for planar division

Furthermore an edge $(p_j, r_k) \in E(G)$ represents an edge among a facility p_j and a partition r_k. This edge indicates that each demand point in r_k has p_j as its nearest facility. Consequently, each demand point in r_k will be allocated to p_j unless its demand exceeds residue capacity res_j. We use the term Service Provider (SP) to represent this relationship. In such a case facility p_j also coincides with a vertex of r_k. According to the above description following equation is derived:

$$(p_j, r_k) \in E(G)\ if\ \exists\ x\ s.t\ r_{kx} = p_j \qquad (4.4.4)$$

This condition implies $SP(r_k) = p_j\ and\ p_j \in r_k$

Now as per RBCFA, we graphically represent the planar divisions from Figure 4.5 in following Figure 4.6.

Here in Figure 4.6, we represent two types of nodes. Blue ovals and red rectangles represent partitions and facilities respectively. Single solid line, double solid line and the dashed line represents that participating vertices are connected by a vertex, edge and connected along ∂B respectively. From Figure 4.6, it is also clear that facilities p_1, p_2 and p_3 are Service Provider for r_3, r_1 and r_4 respectively. Therefore each demand point from r_1 is ideally allocated to p_2 while demand doesn't exceed res_2.

According to RBCFA, the residue ratio $R_j \in [0,1]$ of a facility is also considered during the allocation of a demand point. Here we fix threshold θ for residue ratio $R_j | j \in \{1,2 \dots . m\}$ such that $0 \leq \theta \leq 1$. The threshold may be determined based on various parameters and is the same for all facilities. Therefore demand node i is allocated to facility j while $R_j > \theta$. Consequently, whenever a demand at node $i \in r_k$ arises it is allocated to the nearest resource p_j if $req_i \leq res_j$ and $R_j \geq \theta$. Thus demand points are allocated to nearest resource j while $R_j > \theta$ is maintained.

As given in Equation 4.2.1, the objective of the allocation is to minimize the travel cost by allocating demand point to its nearest resource. Capacity constraints restrict the allocation of entire demand points to their nearest resource. Therefore demand point i is always allocated to nearest resource j irrespective of req_i while $req_i \leq res_j$. It results in future demand nodes to be allocated to a farther resource as soon as $res_j < req_i$. This type of first come first served approach hinders in minimizing the objective function value in Equation 4.2.1. Now as per RBCFA, this first come first served approach is followed for a resource j only while $R_j \geq \theta$. The Inclusion of R_j maintains the allocation of demand nodes considering the future demand nodes for minimizing the travel cost. Average distance is a key parameter for comparing the performance of various allocation methods and is defined as:

$$f_{avg} = \frac{1}{n} \sum_{x} \sum_{y} d(x,y) c_{xy} \tag{4.4.5}$$

The assignment variable c_{xy} is 1 if x is allocated to resource y otherwise it remains 0. Now the objective function for allocation problems is minimizing the value f_{avg}.

The objective function f_{avg} in equation 4.4.5 is modified to following equation 4.4.6 for constrained allocation problem including barriers.

$$f_{avg} = \frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{m} d_{\beta}(x,y) c_{xy} \tag{4.4.6}$$

Here barrier distance $d_\beta(x,y) \mid x \in r_i, y \in r_j$ is determined using Equation 4.4.7. This equation considers all three types of relations of r_i and r_j as discussed in the previous section connected by vertex, connected by an edge or connected along ∂B.

$$d_\beta(x,y) = \begin{cases} d(x,\vartheta) + d(\vartheta,y) & for\ cbv(r_i,r_j) \in E(G) \\ d(x,y) & for\ cbe(r_i,r_j) \in E(G) \\ d(x,u) + d_{\partial B}(u,v) + d(v,y) \mid u \in r_i, v \in r_j & for\ cb\partial B(r_i,r_j) \in E(G) \end{cases} \quad (4.4.7)$$

In equation 4.4.7, $d_\beta(x,y) \mid x \in r_i, y \in r_j$ represents barrier distance whereas $d(x,y) \mid x \in r_i, y \in r_j$ represents Euclidean distance. Similarly, $d_{\partial B}(x,y) \mid x \in r_i, y \in r_j$ corresponds to distance along the boundary of Barrier region ∂B. Therefore Equation 4.4.7 is used to calculate the objective function of Equation 4.4.6.

Now as suggested in RBCFA, every demand node $x \in r_k$ is allocated to its nearest resource $p_j \mid SP(r_k) = p_j$ while $req_x \leq res_j$ and $R_j \geq \theta$. Now, whenever $req_x > res_j$ or $R_j < \theta$, it needs to be allocated to some other resource. This resource is selected based on the following Equation 4.4.8.

$$\min_i f(R_i, d_\beta(x,p_i)) = \alpha\, d_\beta(x,p_i) + \beta / R_i \quad \mid x \in r_y \ , p_i \in r_z \qquad (4.4.8)$$

Here α and β are the constants that may be decided based on service required. The reason for considering barrier distance in f is that demand plane is constrained by the presence of barriers. Equation 4.4.8 avoids allocation of future demand nodes to farther resources thus attempting to minimize the average travel distance that eventually minimizes the travel cost. We utilize f for allocation of demand nodes in the proposed approach RBCFA. The proposed approach is given as follows:

RBCFA: Algorithm for allocation a demand node x to an existing facility
1. Find containing Region $r_c \mid x \in r_c$ for demand node x.
2. Set $p_{sp} = SP(r_c)$
3. If $p_{sp} \neq Null$
If $res(p_{sp}) \geq req_x$ and $R(p_{sp}) \geq \theta$
Allocate x to p_{sp}

Update balance and residue ratio of p_{sp}

else

S = {y| y is Delaunay neighbor of p_{sp}}

$S' = \{z \mid z \in S \text{ and } res_z \geq req_x\}$

$X = \min_{j \in S'} f(R_j, d_\beta(SP(r_j), x)) \mid p_j \in r_t$

Allocate x to p_X

Update balance and residue ratio of p_X

Otherwise go to step 4

4. S = $\{ y | y \in P,\ y \text{ is reachable from } x \text{ through partitions } x \xrightarrow{R} y \}$

$S' = \{z \mid z \in S \text{ and } res_z \geq req_x\}$

$X = \min_{j \in S'} f(R_j, d_\beta(SP(r_j), x)) \mid p_j \in r_t$

Allocate x to p_X

Update balance and residue ratio of p_X

5. If $\frac{1}{m}\sum_{i=1}^{m} R_i < \delta$

Add a new resource to the plane

Now whenever a demand arises, containing region is obtained in step 1. It is followed by finding the Service Provider of the selected region in step 2. If the containing region has a service provider then a necessary condition for RBCFA is checked in step 3. Thereafter if the necessary condition satisfies, then the demand node is allocated to the service provider. If the condition is not satisfied then the equation 4.4.8 is evaluated for all Delaunay neighbours of the service provider. Delaunay neighbours of the service provider refer to the second nearest resource in all possible directions. Thus the resource that minimizes f among all Delaunay neighbors is selected for allocation. Whenever allocation is performed, balance capacity and residue ratio of the allocated resource is updated.

Now if containing region does not have a service provider, all reachable resources are considered into S. Furthermore $res_z \geq req_x \mid z \in S$ is evaluated and taken into S'. Thereafter, the resource which minimizes the average travel cost, is selected. Here we use Equation 4.4.7 to evaluate the barrier distance among the demand node and a region. Now Step 6 of RBCFA checks that average Residue Capacity should never go below a predetermined value (δ). Once it approaches below δ, it signals that the addition of a new resource is required in the demand plane. The following section simulates the proposed approach.

4.5 Simulation and Results

In order to simulate the proposed algorithm, a well-known instance by Butt and Cavalier [92] is considered. Here demand plane consists of a rectangular barrier and three resources. The demand plane is partitioned into five regions based on projection point as discussed previously in Figure 4.5. Graphical representation of given planar decomposition is represented in Figure 4.6. In order to simulate the proposed approach, the coordinates of the facilities and barrier are considered as follows:

Barrier B_1 (114,164)(272,164)(272,224)(114,224)(114,164)

Facility P_1 (191,107)

Facility P_2 (291,187)

Facility P_3 (155,263)

Furthermore, the coordinates of the partitions as shown in Figure 4.5 are as follows:

Partition R_1 (272,164)(291,187)(272,224)(272,164)

Partition R_2 (231,136)(272,164)(229,164)(231,136)

Partition R_3 (114,164)(191,107)(231,136)(229,164)(114,164)

Partition R_4 (114,224)(194,224)(191,250)(155,263)(114,224)

Partition R_5 (191,250)(194,224)(272,224)(191,250)

Now each resource j has capacity C_j as shown in Table 3 that limits the demand points it can be allocated to. Furthermore, res_j is initialized to C_j and is updated whenever a demand node patronizes or leaves the facility j. Following data represents the capacity of resources.

Table 3: Resources and corresponding capacity

Resource	1	2	3
Capacity	35	30	35

Similarly, each partition also has some quantum of demand that specifies the demand from this partition. Table 4 gives the quantum of demand for the partitions.

Table 4: Partitions and their quantum of demand

Partition	1	2	3	4	5
Quantum of	16	--	--	6	9

We have implemented RBCFA in MATLAB. Now as already discussed, this planar decomposition of demand plane is based on projection points. Now from Figure 4.5, it is clear that path from demand point $x|\ x \in r_i$ to resource p_j always passes through projection point ρ_{ij}. For example projection point from region r_5 for p_2 denoted by ρ_{52} will be always top right corner of the rectangular barrier. Thus equation 4.4.7 can be further optimized by finding the projection point for all regions with respect to each facility. Consequently, whenever a demand node $x \in r_i$ arises, ρ_{ij} can be used to calculate the distance between $x|\ x \in r_i$ and resource p_j as follows:

$$d_\beta(x, p_j) = d(x, \rho_{ij}) + d_\beta(\rho_{ij}, p_j)|\ x \in r_i \qquad (4.5.1)$$

Here d and d_β refer to the Euclidean and barrier distance respectively. Now here we are considering the static nature of barriers and facilities. It results in static boundaries of the partitions and subsequently projection point also. This static property of involved parameters leads to finding d_β component of Equation 4.5.1 in advance and using wherever required. Reusing d_β evade executing shortest path algorithm every time it is required and thus further optimizing the solution complexity of the proposed approach. The value of d_β based on considered coordinates is shown in the following Table 5.

Table 5: The values of d_β (ρ_{ij},p_j)

	p_1	p_2	p_3
r_1	80.10(272,164)	0	116.5(272,224)
r_2	0	19.17(272,164)	118.55(272,164) 71.20(114,164)
r_3	0	19.17(272,164)	118.55(272,164) 71.20(114,164)
r_4	82.09(272,224) 110.99(114,224)	19.43(272,224)	0
r_5	82.09(272,224) 110.99(114,224)	19.43(272,224)	0

In Table 5, double entry represents that either ρ_{ij} may be chosen depending upon the location of x. This prior calculation of the constant factor of $d_\beta(x, p_j)$ in Equation 4.5.1 further optimizes the approach.

Table 6: values of F_{avg} traditional approaches vs. RBCFA

demand pts	F_{min}	F_{max}	Traditional F_{avg}	RBCFA F_{avg}
5	15	65	44.8	44.8
10	15	65	35.8	35.8
20	15	65	30.4211	27.7368
30	13	65	22.7333	15.7667
40	13	65	18.6534	15.5750
50	13	65	13.1200	11.1400

We executed the RBCFA for the number of demand points ranging from 5 to 50. These demand points are randomly generated. Random generation of demand points may cause a slight variation in the value of objective function value during various executions. Thus we executed the algorithm five times for the same number of demand points and then averaged the obtained results. The proposed approach is compared with a traditional approach where demand point is allocated to the nearest resource while $req_i \geq res_j$.

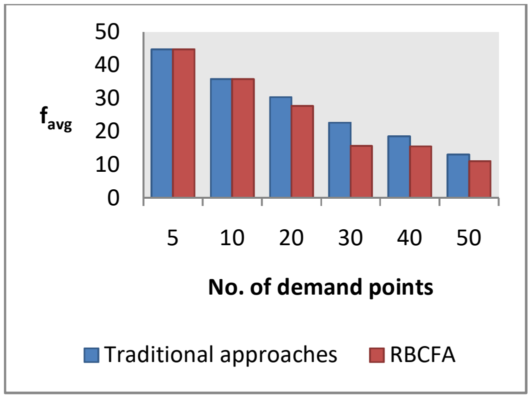

Figure 4.7: f_{avg} using traditional and RBCFA

From Table 6 and Figure 4.7 we observe that although the improvement of RBCFA over traditional approaches is not noticeable initially it becomes significant with an increase in the number of demand points. We have represented only average distance f_{avg} in Figure 4.7 as the minimum and maximum distance remains unaffected. Thus it becomes evident that the proposed approach performs better in comparison to the traditional approach.

Thus we proposed usage of barrier touching property to find projection point in the demand plane. Based on the projection point, the demand plane is partitioned into multiple regions. We use the proposed approach RBCFA for allocation of the demand points to capacitated facilities. We propose consideration of residue ratio of the capacitated resource during allocation of demand point to the resource. In this chapter, we have presented a simulation of the proposed approach and its comparison with traditional approaches. Consequently, it is validated that the inclusion of residue ratio results in further minimization of f_{avg}. This improvement in f_{avg} becomes apparent as the total residue ratio of all resources lowers. The research can be further enhanced in the direction of multi-source capacitated location problem with barriers for dynamic demand points.

Chapter 5

5. Suggested Layout planning using Geometric Structures

Nowadays, each industry is going through stringent competition. In such a competitive market, all industries explore all possible avenues to pull a maximum share of the market. Each industry aims to outshine using all possible alternatives to lure maximum customers. While marketing, advertising and discount offers had always been primary tools to maximize profit during the past few years. But recently, Layout Planning is also accepted to be another important factor that significantly affects the success of any professional body. Layout Planning has found its applications in various sectors like construction, robot path planning, VLSI design and hospitals [270] [271]. Thus it is seen that Layout Planning is an important aspect to be focused on.

Here we propose Layout Planning for various business outlets using various geometric structures in order to maximize the revenue generated. We consider two cases for the same: the first case considers layout planning for a retail store that contains numerous items. In the second case, we undertake the layout planning of an exhibition that houses various counters. In both cases, the proposed algorithm is independent of the travel path chosen by a customer to visit the business outlet (retail store or exhibition). We have also performed t-test in order to validate the proposed approach. It is proved with help of t-test that the proposed approach significantly improves existing layout in terms of generated revenues. Apart from layout planning, the chapter also discusses the parking issue to develop automated parking management system. The issue of parking management has been considered after realizing its influence on the health of humans and the environment.

5.1 Layout Planning for Departmental Store

From the literature survey, we observed that whenever a customer visits a retail store, he has a predetermined list of item to purchase but generally he ends up shopping many items which were not pre-listed. The listed items that the customer comes to purchase are called must-have items. On the other hand, the items which don't belong to the list but the customer may be tempted to purchase them are called impulse items. For example, items like grains, sugar, vegetables are the must-have items while decoration items, gift items (when not required) etc. may be considered as impulse items. In literature, it has been agreed from customer ehavior that purchase of must-have items is the result of a thoughtful and needful process while impulse items are purchased as a result of prompt stimuli [272][273]. Customer may be driven to purchase these impulse items by providing required stimulus in the form of physical proximity of impulse items to must-have items, attractive offers etc.[274].

The recent study shows that 30% to 50% of all purchase is classified as impulse buying by the customers themselves [275] [276][273][277]. Furthermore, must-have items are basic commodities and thus have lower marginal profit. On the other hand, impulse items make a huge share of the overall margin of the store. Another reason to focus on impulse items is that must-have items are daily needed items and therefore the customer is bound to buy these items. This strongly advocates the significance of impulse items in the overall profit of the retail store. Thus it necessitates strategic planning of retail store to maximize its revenue.

There exist many types of customers who visit a store, each having his choice, requirements, priorities and financial capability. For example, packaged food could be the must-have item for many while some may consider it as an impulse item. Thus must-have and impulse items may not be the same for all these customers. All these factors result in the classification of customers based on similarity in must-have and impulse items barring universal classification of items. Therefore, the first step is to classify must-have and impulse items for each class of customers. This classification of must-have and impulse items for each customers' class is then followed by devising a strategy that maximizes revenue [278][279]. The same strategic planning can be implemented for virtual stores also [280]. In the literature, Li [281] and Ozgormus [282] uses a classical model to optimally locate the products to increase the likelihood of their purchase. One very well accepted principle in the literature is that the purchase of impulse items can be maximized by exposing them to the maximum number of customers in the store [283].

A customer generally visits a store to purchase must-have items. However, he may be prompted to buy impulse items during this visit if he is made to pass through these impulse items while shopping must-have items. Consequently, rigorous research has been done to select the travel path of customer for a long time. Radio Frequency Identification (RFID) is also used to analyze shopping behaviour and therefore identify must-have and impulse items for all classes of customers [284]. Kholod et al. Proved the existence of a positive relationship between length of the travel path and shopping volume [285].

In connection with the existing literature, we hereby propose a Layout Planning to maximize the revenue of retail stores. We utilize the classification of must-have and impulse items. Based on the literature, we also assume that whenever a customer visits a store, he moves around the store to collect must-have items in the cart. During this movement, he may be prompted to purchase impulse items if layout planning has been performed in an efficient manner. Efficient layout planning ensures that each customer is exposed to maximum impulse items by locating them along their path in the store. The principle of Layout Planning is that the sale of impulse items is in direct proportion to their exposure to the customers. Therefore it necessitates performing Layout Planning in such a manner that distance travelled by a customer is maximized.

The path of a customer is determined by his must-have list and is therefore deterministically unknown. Furthermore, the path also depends upon the order in which the customer picks these must-have items. Generally, a customer is likely to choose the shortest path to another nearest must-have item from his current location and thus generating a huge range of the possible paths. Here we propose a Layout Planning that aims to maximize the path length of a customer in the store.

For any layout, revenue generated from must-have items is fixed and is independent of the layout. Layout Planning significantly affects revenue from impulse items, known as impulse value. Thus the layout having maximum impulse value is clearly considered as an optimal layout. Here we propose an approach that maximizes the impulse value of the layout. The proposed algorithm is presented in two stages. The first stage optimally locates the must-have items using p-dispersion model. The second stage then locates the impulse items following the location of must-have items during the previous stage. Usage of suggested two-step approach maintains that a customer needs to travel maximum distance. During this path, a customer is exposed to the largest number of impulse items. Exposure of these impulse items results in optimizing the impulse value of layout.

5.1.1 Problem Definition

This section discusses the mathematical model of the store layout problem. Here, we assume that the store has a single entry and exit point. It has $\boldsymbol{n}$ impenetrable racks of length $\boldsymbol{L}$ having blocks on both sides. $\boldsymbol{W}$ represents distance among two racks which a customer passes through. Items are to be placed in each block with an intention of maximizing the exposure of each item to maximal customers who visit the store. These $\boldsymbol{n}$ racks have been aligned in a parallel fashion as shown in Figure 5.1. Based on the current layout, the customer needs to travel rectilinear distance as the parallel layout of racks eliminates the possibility of any other distance from consideration. Customer may choose any path while shopping in the store. It is assumed that a customer will always choose the shortest path that covers all his must-have items.

Considering this layout, the number of blocks limits the number of items that can be placed in the store. Now the objective is to locate all items so as to maximize revenue generated by impulse items. The problem can be mathematically formulated as follows:

$$\text{Max}_{\,I \in S}\{\, \text{rev}(L_i)\}$$

Here, $\boldsymbol{L_i}$ represents a particular layout while $\boldsymbol{S}$ denotes the number of possible layouts. For a given layout, the value of revenue generated by impulse items is evaluated and the layout that generates maximum revenue by impulse items is considered to be the optimal layout. As discussed earlier, the motive behind excluding must-have items from consideration is that must-have items are the necessary items and thus its revenue is predetermined and nearly stable. On the other hand, revenue by impulse items is highly fluctuating and is dependent on the layout. Consequently, only impulse items are considered in the objection function.

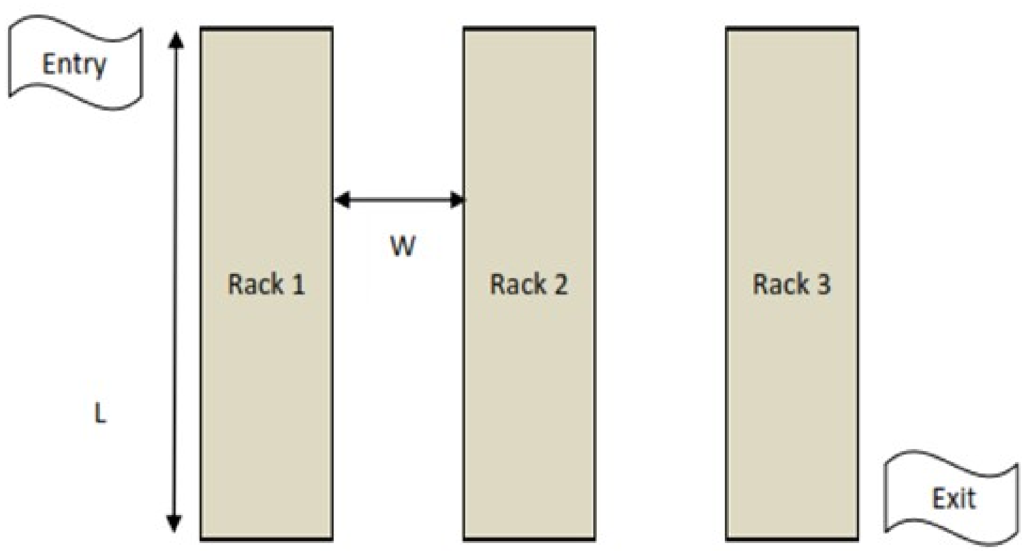

Figure 5.1: Illustration of Retail store layout

It is also proved that the distance travelled by a customer in the store also affects the items purchased in a positive manner. Thus the problem can be defined to find an optimal layout that maximizes the smallest distance that a customer travels while purchasing must-have items. Along with maximizing the smallest distance, the focus should also be given to make customer pass through the largest number of impulse items. In order to maximize the smallest distance travelled by a customer, it has been accepted in the literature that a well-known p-dispersion location model should be used to place all must-have items.

As discussed earlier, the objective of the p-dispersion location model is to locate p facilities in the demand plane such that the distance between demand node to its nearest facility is maximized. P-dispersion model is used for locating obnoxious facilities like dump yard, graveyard etc. In the proposed approach, p-dispersion is employed for locating must-have items. It maintains that must-have items are placed as far as possible and thus maximizing the distance travelled by a customer in the store. Location of must-have items (using p-dispersion location model) is followed by locating impulse items. The detailed algorithm for layout planning has been given in subsequent section.

5.1.2 Proposed Approach

Here Figure 5.1 illustrates the layout for a retail store with a single entry and exit point. Here, we consider four shelves aligned in parallel. Figure 5.2 represents the considered structure of the shelves. Each shelf has four blocks on both sides, thus having eight blocks for a rack. Homogeneity of blocks maintains that any item can be placed in any block thus providing a wide range of possible layouts. Value of a layout is determined to be the revenue generated by impulse items and therefore a layout that generates maximum revenue is considered to be the optimal layout.

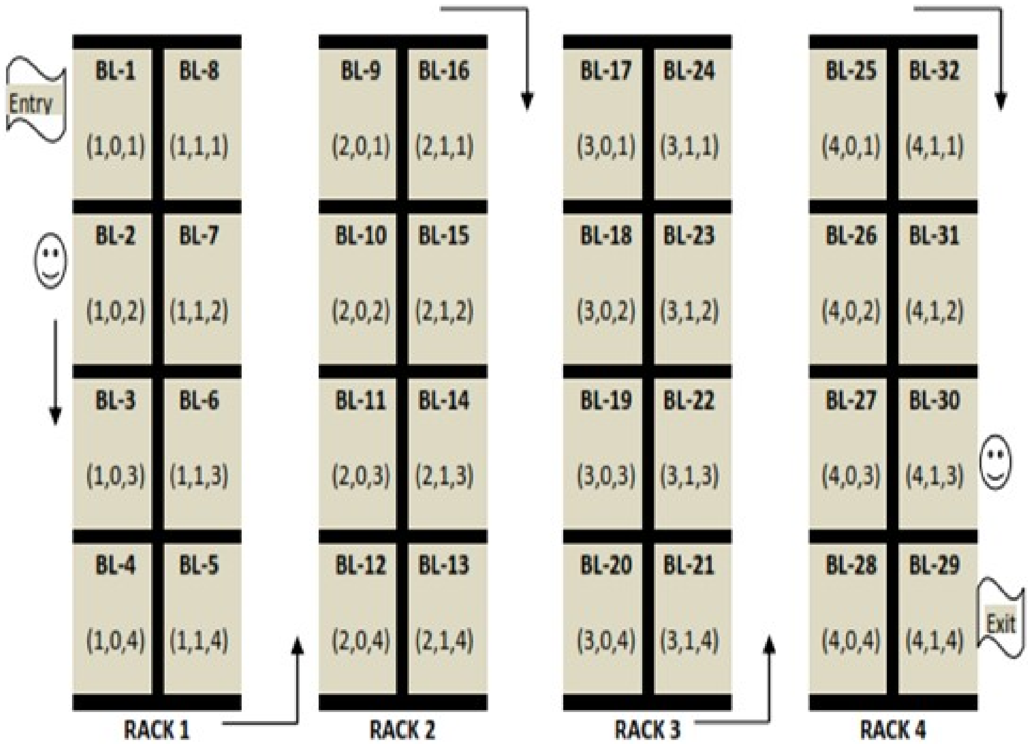

Figure 5.2: Illustration of Proposed Block Structure

Figure 5.2 represents four shelves where each shelf has four blocks on both sides. Now travel route for a customer depends upon his list of must-have items. The longest travel path where a customer passes through each block and thus is exposed to all items is represented by arrows.

Apart from block number, we refer a block by its position in the store like rack number, its side (left/right) and its location on that side. Now as observed in Figure 5.2, a block is represented by three values $\mathrm{i, j, k}$ representing shelf number, side of the shelf (left or right) and block number respectively. Here left and right side of the rack is represented by 0 and 1 respectively. For example, block 19 is represented as the third block on the left side of rack 3 or $B_{3,0,3}$. Thus it can be written as:

$$n: \; number\ of\ racks$$

$$m: \; number\ of\ blocks\ on\ one\ side\ of\ rack$$

$$Block\ B_{i,j,k} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m$$

From this definition, we have devised some relationships. A block is **adjacent** to another block b_y if they are on the same side of the rack and also share a boundary. Block b_x is **opposite** to another block b_y if these lie on different rack opposite to each other. Similarly, block b_x is **diagonally opposite** to another block b_y if both are lying diagonally opposite to each other on different racks. From these definitions it is observed that $block - 22$ and $block - 23$ are adjacent, $block - 13$ and $block - 20$ are opposite, $block - 19$ has two diagonally opposite blocks named $block - 13$ and $block - 15$. These relationships may be described as follows:

$$adj(b_{i,j,k})\ \{b_{i,j,k-1}, b_{i,j,k+1} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m\} \quad (5.1.1)$$

$$opp(b_{i,j,k}) = \begin{cases} b_{i-1,1,k} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m\ , \ if\ j = 0 \\ b_{i+1,0,k} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m, \ if\ j = 1 \end{cases} \quad (5.1.2)$$

$$diag(b_{i,j,k}) = \begin{cases} b_{i-1,1,k-1}, b_{i-1,1,k+1} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m\ , \ if\ j = 0 \\ b_{i+1,0,k-1}, b_{i+1,0,k+1} \mid 1 \leq i \leq n, 0 \leq j \leq 1, 1 \leq k \leq m\ , \ if\ j = 0 \end{cases} \quad (5.1.3)$$

Using equations 5.1.1 and 5.1.2, $diag(b_{i,j,k})$ may also be expressed as follows:

$$diag(b_{i,j,k}) = adj\left(opp(b_{i,j,k})\right) \quad (5.1.4)$$

With help of all these equations from 5.1.1 to 5.1.4, we see that we can find an adjacent, diagonal or opposite block for any block. Now considering the physical proximity of blocks, we have formulated distance metric among them using relations defined above. The distance metric $dist(b_x, b_y)$ represents the probability of coming across an item in a block b_y if a customer takes an item from b_x. From the definition of the distance metric, it is clear that adjacent blocks lie in close proximity in comparison to the opposite and diagonal blocks. Therefore we assign a value to distance metric as follows:

$$dist(b_x, adj(b_x)) = 1$$

$$dist(b_x, opp(b_x)) = 2$$

$$dist(b_x, diag(b_x)) = 3$$

Higher proximity of blocks has a higher probability of attracting customers to purchase contained items and thus is represented by the lower value of the distance metric.

As already discussed, we have considered k classification of customers. Each class has its set of must-have and impulse items. Furthermore, the profit earned by an impulse item is represented by its own marginal value which is considered during layout planning as it has significant impact on revenue of the retail store. Additionally, the proposed approach also considers all k classes of customers. A brief algorithm for the suggested approach is as follows:

Proposed Algorithm for layout planning

Input(racks_number, m, k, must-have[], I[])

1. Arrange the impulse items in non-increasing order of marginal value in List I.
2. For each blocks in B

   $Item(B_i) = null \mid 1 \leq i$
3. Set $n = k$, where k is number of customer classes.
4. Repeat following while $n > 0$

   i. Set $M_n = p$ must-have items for n^{th} class of customers

   ii. Locate p elements of M_n using p −dispersion location model,

   iii. For each block b_x where item $(b_x \in M_n)$

   If $(item(adj(b_x)) = null)$

   make $(item(adj(b_x)) = front(I)$ and $delete(front(I))$

   If $(item(opp(b_x)) = null)$

   make $(item(opp(b_x)) = front(I)$ and $delete(front(I))$

   If$(item(diag(b_x)) = null)$

   make $(item(diag(b_x)) = front(I)$ and $delete(front(I))$

   iv. $n = n - 1$

In the algorithm m and k represent the number of blocks on each side of rack and number of customers' classes. Here I contains the list of impulse items in non-decreasing order of marginal value. The algorithm initializes each block to null. During phase 1, algorithm starts locating must-have items using p-dispersion model that places these must-have items as dispersed as possible thus ensuring that each customer needs to travel longest shortest route in the store to get his list of must-have items. Subsequently, in phase 2, impulse item I for each class is chosen in order of margin value. Thus each impulse item is located considering the distance metric so as to maximize the impulse value of the layout.

As discussed earlier, a larger number of customers coming across impulse items induce a positive effect on impulse revenue. Therefore, whenever a must-have item is placed in a block x, all blocks at a distance 1 from b_x are checked if these are empty. Empty blocks at a distance of 1 from b_x are chosen to contain items from the front of list I. The process is repeated for all neighbouring blocks from b_x at distance of 2 and 3. Thus, surrounding M_k with impulse items ensures that all k classes of customers will come across these impulse items thus generates maximum revenue. Following section presents the illustration of the proposed algorithm and finally performs the comparison.

5.1.3 Result Analysis

We consider the test data given in [286]. The data set considers three racks where each rack has five blocks on one side thus making total 30 blocks. We consider 3 classes of customers for implementing the proposed approach. Table 7 describes the must-have and impulse item for each customer class. Table 8 describes the marginal value generated by all items in the store. These marginal values have been given per unit of an item.

Table 7: Classification of customers along with their must-have and impulse items

Category	Must-have items	Impulse items
I	I-2, I-11,I-12,I-13, I-23, I-25	I-7, I-28
II	I-2, I-11,I-12,I-20, I-22, I-30	I-15, I-16
III	I-2, I-3,I-11,I-12, I-20, I-22	I-24, I-29

Table 8: Items and Marginal value

Item	I-1	I-2	I-3	I-4	I-5	I-6	I-7	I-8	I-9	I-10	I-11	I-12	I-13	I-14	I-15
Margin	1.24	3.59	0.69	1.29	3.09	2.49	1.79	0.59	3.69	5.99	2.79	2.99	1.79	2.79	2.49
Item	**I-16**	**I-17**	**I-18**	**I-19**	**I-20**	**I-21**	**I-22**	**I-23**	**I-24**	**I-25**	**I-26**	**I-27**	**I-28**	**I-29**	**I-30**
Margin	2.59	2.29	1.85	2.99	6.99	4.39	2.29	3.25	3.79	4.49	1.69	2.59	0.35	3.33	2.59

Here, Figure 5.3 (a) demonstrates original layout while Figure 5.3 (b) demonstrates the layout planning obtained by implementing the proposed approach. Figure 5.3 represents the block numbers and its contained item number for the 30 items given in Table 8. Term $item(b_x)$ represents the item contained in block b_x e.g $item(b_{17}) = 17$ represents that block b_{17} contains $I - 17$ in Figure 5.3 (a) while in Figure 5.3 (b) $item(b_{17}) = 4$.

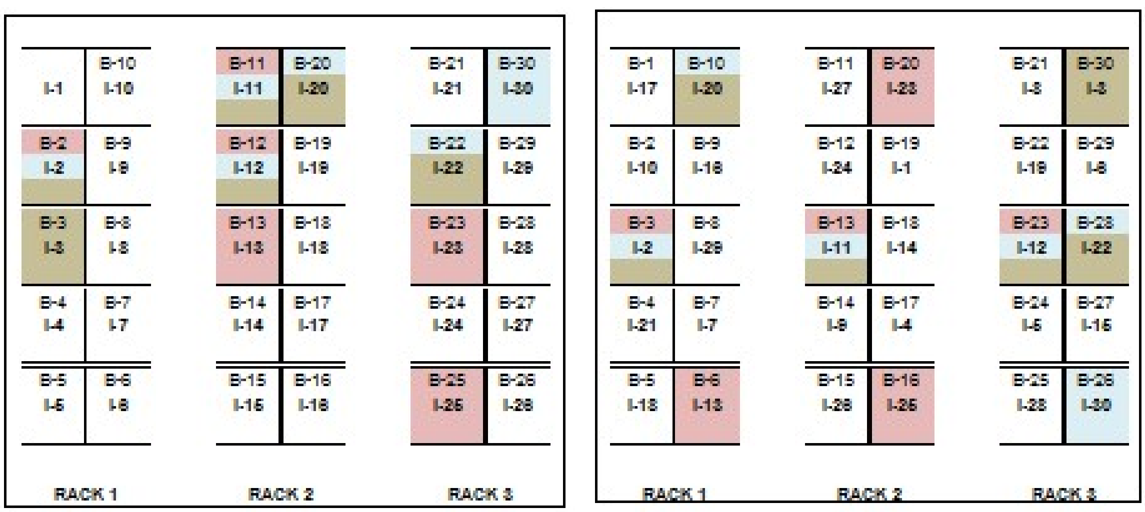

Figure 5.3: a) Original layout b) Proposed Layout

Now using Table 7 and Table 8, we implement the proposed algorithm. The layout planning of the items in the store has been demonstrated in the following Figure 5.3. Here Figure 5.3 gives the original layout planning while Figure 5.3 (b) illustrates the layout planning obtained after implementing the proposed algorithm. Here in Figure 5.3, coloured items represent must-have items. As discussed earlier, three classes of customers have been considered.

Therefore must-have items for class I, II and III have been shown using pink, blue and gray colour respectively. Thus it is clear from Figure 5.3 (b) that item 2, item 11 and item 12 are common must-have items for all three categories of customers and therefore have been placed as distant as possible. The objective of locating these common items most distant is that all categories of customers need to travel the longest distance to get their must items in the store and thus coming across the largest number of impulse items.

During result analysis, we have considered various cases. The first case of consideration is that a customer may be attracted by any impulse item which is placed in the vicinity of a must-have item at a distance which is not more than 1. The number of customers has been taken to be in the range of 10 to 100. The same assumption has been taken for all three classes of customers. Similarly, the case of considering impulse items that are placed at a $distance \leq 2$ and then $distance \leq 3$ are considered.

The obtained result has been demonstrated in Appendix $A1$ to $A6$. Appendix $A1 - A2, A3 - A4$ and $A5 - A6$ represent the impulse value when the distance of alluring impulse items is considered not more than $1, 2$ and 3 respectively. t-test has been performed on the obtained values considering the significance level of 5% and obtained results validated the proposed approach and thus can be implemented for layout planning of major retail stores.

5.2 Layout Planning for Exhibition

In this section, we undertake another instance of layout planning of exhibition that houses counters for various industries. In this case, we plan a layout that maximizes the number of visitors for all the counters that have participated in the exhibition or industrial trade fair. Here we employed Voronoi diagram that is an efficient geometric structure in handling spatial problems. The suggested algorithm considers discrete planning horizon, thus has a set of candidate facility locations. Here facilities refer to the counters of the participating industries in the exhibition. Furthermore, it is assumed that the planning horizon does not have any constraint for the location of stalls. The proposed layout maximizes the visitors for each participant irrespective of the path chosen by a person to move around the exhibition. The proposed approach can also be utilized for layout planning for a retail store, amusement park, art gallery etc.

In today's highly competitive world, each industry employs numerous tools to lure the maximum number of patrons. Each industry wants to promote its product to the largest size of the audience at the same time. Such methods include promotional advertisement through popular media channels. Although promotional advertisement is the most prominent tool to launch any product in the market at a large scale it has some limitation. One main limitation is that promotional advertisement has a short duration and is quite expensive. Moreover, customers don't get to physically examine and experience the product, which resists them to try this product. Such limitation can be overcome by some other methods like exhibitions where end users get to physically experience the product. In this section, the terms exhibition and trade fair have been used interchangeably.

Therefore, industrial exhibitions are the most promising platforms where industry can launch, showcase and promote its products to the largest size of the audience at a global level. In an industrial exhibition or trade fair, each participating industry sets up its counter where it demonstrates its products. Such exhibitions have various participating industries desirous of launching and promoting their products. A participating industry attains maximum benefit from an exhibition if the corresponding counter is visited by the maximum number of visitors failing which it loosens its purpose of luring the maximum customers. Thus it necessitates having a layout, which ensures that the maximum number of visitors visit each counter. Such visitors may eventually be upgraded to customers by offering lucrative schemes from corresponding industries. Therefore, an exhibition layout focuses on maximizing the number of visitors for each counter. In the literature, it is quite evident that location modeling plays a very important role for every business [287] [270][271]. Therefore, we present an approach that proposes a layout planning of the exhibition to maximize the visitors for each participating counter.

The motive behind focusing on maximizing visitors is based on the research finding in [285] [283]. Another motive for layout planning is research in the area of customer behavior. During the analysis of customer behavior, it is observed that purchase is a result of a thoughtful process. However, some items are purchased as a result of prompt stimuli [272] [275] without any prior planning in some cases[288][276][289][277]. Thus exposure of a product to customers is the prime stimuli that may eventually escalate its volume of sale. Therefore, it necessitates optimal layout planning for the exhibition so that each participating counter is visited by the maximum number of visitors.

According to the suggested approach, visitors to the exhibition may be categorized based on factors like age, gender and interest etc. similarly counters are also categorized into various classes. During layout planning of the exhibition, all categories of customers should be considered. Here we present an approach considering all these categories for layout planning, which aims to maximize visitors for each participating counter in the exhibition.

5.2.1 Problem Formulation

Here authors consider that the exhibition takes place on a two-dimensional continuous non-constrained plane having a single entry and exit for visitors to the exhibition. As discussed earlier, the visitors are categorized based on various factors like age, gender, choice etc. Similarly, counters are classified that helps in optimal layout planning of the exhibition. For example, young visitors to the exhibition are interested in automobiles and technology. This layout should ensure that every young visitor is made to pass through all counters of the corresponding category.

Here let $C = \{c_1, c_2, \dots \dots . c_m\}$ and $V = \{ v_1, v_2, \dots \dots . v_n\}$ represent m counters and n visitors in the exhibition. Here visitors have been classified into p category. Based on the classification of visitors, counters are also classified into p categories.

Following are some notations:

i represents visitors

j represents counters

k represents categories of visitors

$x_{ik} = 1$ if the visitor v_i is a member of visitor class k, otherwise $x_{ik} = 0$.

$y_{ij} = 1$ if the visitor v_i visits counter c_j, otherwise $y_{ij} = 0$.

$z_{jk} = 1$ if counter j represents an interesting counter for visitors of category k.

The objective of the layout planning for the exhibition is to maximize the number of visitors for each counter in the exhibition. It can be mathematically formulated as:

$$f = \max\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p} x_{ik}y_{ij}z_{jk} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p} \alpha\, x_{ik}y_{ij}z_{jl}\right) | l \neq k, \quad 0 \leq \alpha \leq 1 \qquad (5.2.1)$$

First part of objective function maintains that visitor v_i must visit the counter c_j (represented by $y_{ij} = 1$) if $x_{ik} = 1$ and $z_{jk} = 1$ for any possible value of k. It represents that each customer must visit every counter that belongs to its own category. For example, each young visitor must visit all counter for automobiles and technology. However, the second part maintains that each visitor visits the maximum number of counters apart from counters of his own category. Here constant α represents the probability of upgrading a visitor to a customer and thus lies in the range of 0 to 1. The value of constant α may be decided based on various factors involved. As per the research findings in [283][272][275][288], it has been observed that 30% to 50% of all purchase by a customer is the result of stimuli and is termed as impulse buying. Accordingly, the value of α may be taken as 0.4 based on these findings.

In order to maximize objective function value, it is proposed to classify the participating industries into *established industries* and *emerging industries*. An industry is *established* if the average turnover of the industry is more than Δ failing which it remains an *emerging industry*. The same can be expressed by the following inequality. Here $turnover_i$ represents the turnover of the industry for financial year i.

$$\frac{1}{n}\sum_{i=1}^{n} turnover_i > \Delta \qquad (5.2.2)$$

The Δ may be identified by financial experts and varies across industries. This further classification of industries into *established* and *emerging* assists to plan layout for exhibition maximizing objective function value. This multilevel classification of industries is represented in Figure 5.4.

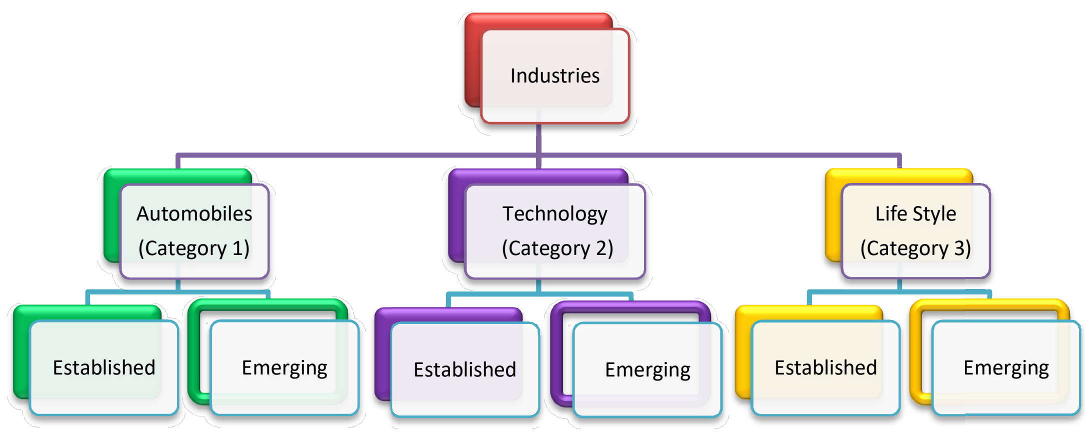

Figure 5.4: Multilevel classification of counters

The first level of classification is based on the various categories of the customer. Thereafter classification is based on inequality 2. As shown in Figure 5.4, the first level of classification represents various industries and the second level of classification represents corresponding established and emerging industries. Established and emerging industries are represented by a solid rectangle and curved unfilled rectangles. Different colours represent the category of different industries. Here authors propose a 2-phase algorithm for layout planning of an exhibition which aims to maximize the number of visitors for every stall of each industry.

5.2.2 Proposed Approach

The proposed approach for exhibition planning performs layout planning in two phases. The first phase locates established counters for all established counters. Location of established industries is followed by locating emerging industries during the second phase of the proposed approach. During location of established industries, p-dispersion method is employed, which maximizes the minimum distance among the pair of established counters. Usage of p-dispersion for locating established industries is based on the result obtained in the Kholod et al. [285]. Thereafter during the second phase, the location of established industries assists to determine the location of the emerging industries. Proposed approach utilizes the location of established industries in order to locate emerging industries.

The algorithm can be expressed as:

Proposed Algorithm for Layout Planning of an Exhibition Input: k customer classes
1. Find the *established* and *emerging* industries for all k classes
2. Phase 1: Location of *established* industries
For each class i
Let p be the number of *established* industries in class i
Locate p *established* industries of class i using p-dispersion model maximizing the minimum possible distance with *established* industries of all other classes as well.
3. Generate the *Voronoi Diagram* for these p *established* industries.
4. Phase 2: Location of *emerging* industries
For each class i
Locate the *emerging* industries of the class in the Voronoi Region of established industries along the Hamilton path covering these *established* industries of class$i$.

The underlying principle of the suggested approach is a heuristic that each customer of class k visits all counters of established industries for k class. This heuristic suggests that all emerging industries of k class should be located in the vicinity of established industries of k class. Location of established industries is followed by generation of Voronoi Diagram which is used to locate emerging industries. Emerging industries are then located in the Voronoi region of previously located established industries. The process is repeated for all possible classification of the customers.

Above algorithm represents a two-step approach for layout planning of an exhibition. The same is represented by an Illustration in Figure 5.5. Here the area for the exhibition is illustrated by a rectangle. There exist two classifications of customers and counters represented by green and blue colour. Here solid and non-solid rectangles represent the established and emerging industries respectively for both classes.

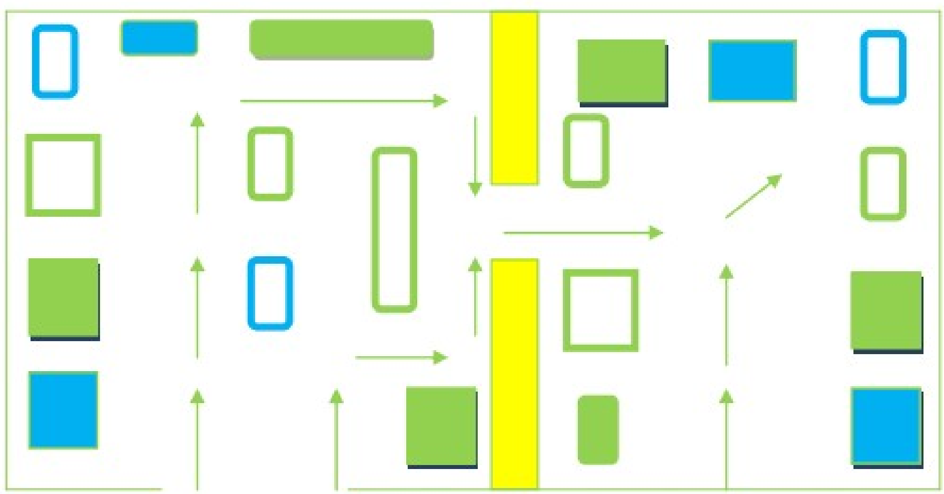

Figure 5.5: Illustration for Location of Established and Emerging Industries

As suggested in the above algorithm, emerging industries for a class should be placed in the vicinity of established industries of the corresponding class. Here Voronoi Region is used to determine the area of the vicinity for an established industry. This algorithm considers Hamilton path for each class of customers covering all established counters of the corresponding class. The motive behind considering Hamilton path is based on the fact that all customers for all classes visit all established industries of their class however the path may not be known. Hamilton path for a class gives the shortest possible path covering all established industries and thus acts as a measure of estimating the minimum distance travelled by the visitor. Therefore, any visitor of class i visits all established industries of class i in any order. The distance travelled by any visitor of class i in the exhibition who visits all established industries of class i is lower bounded by Hamilton path covering all established industries of class i. Consequently, the placement of emerging industries along the Hamilton path ensures that these emerging industries are visited by the maximum number of visitors of the corresponding class thus maximizing the value of objective function. We implemented the proposed approach for evaluating the effectiveness of the proposed approach. The following section presents the obtained results.

5.2.3 Implementation of Proposed Approach

The suggested approach has been implemented in MATLAB 2012 on i3 processor. The dimension of the plane under consideration is 30 by 30. We consider three classes of customers for the purpose of simplicity. For each class of customers, we consider 5 established and 20 emerging industries in the exhibition. During first phase of the proposed approach, we locate five established industries for all three classes using p-dispersion.

During the second phase of the proposed approach, the Voronoi diagram of the established industries for all $\boldsymbol{k}$ classes of customers is generated. Thereafter emerging industries of class $\boldsymbol{i}$ are located in Voronoi region established industry counter of the same class. We consider a varying number of customers, who visit the exhibition for each customer class. Table 9 represents the average value of the objective function for each class using a conventional layout and proposed layout approach. We assume the value of constant $\boldsymbol{\alpha}$ to be 0.4. Now for each customer, authors consider the Hamiltonian path (irrespective of class) that covers all established industries of his class. We also consider other counters along this Hamilton path to optimize the value of objective function. Table 9 represents the obtained average values as:

Table 9: value of average objective function for n customers

No. of visitors	Class 1		Class 2		Class 3	
	Random	Proposed	Random	Proposed	Random	Proposed
5	20.8	22.8	22.2	26.4	20.4	23.8
10	22.4	26.2	23.8	27.4	24.0	28
20	20.0	24.8	24.6	28.8	22.4	27.8
50	20.2	26.5	22	28.2	22.2	28.8
100	22.6	30.2	22.8	29.2	22.1	29.2

As we observe from the value obtained specified in Table 9, we observe that the proposed approach outperforms the random layout. The authors have performed a t-test comparison with a significance level of 5% to further validate and check the effectiveness of the proposed approach in comparison to random layout planning. t-test validates the proposed approach and thus concludes that the proposed approach improves the objective function. This improvement in the value of objective function becomes significant as the number of visitors increases.

Thus the authors have successfully implemented the layout planning for departmental store and exhibition using geometric structures. Moreover, the proposed approach is independent of the travel path chosen by a customer. This work can be further extended for planning retail stores, where the layout planning is not limited to the grid structure. The planning horizon consisting of constraints can also be considered for extending the current work.

5.3 Automated Parking Guidance System

Exponential growth in the number of vehicles with limited parking has become a serious issue, creating parking management as an obligatory area for research [290]. Moreover, with limited parking, the number of vehicles can be observed circling around looking for parking space resulting into increase in traffic and pollution. Thus over the past few years, researchers have been working intensively to provide an efficient parking management solution. In order to address this issue, cities are employing the Intelligent Transportation System (ITS) [291]. Currently, ITS is in the state of evolution to incorporate upcoming issues. ITS communicates among vehicles and thus obtains dynamic and real-time information for parking guidance.

Moreover, advancement in Information and Communication Technology (ICT) further escalates the guiding parking system by providing dynamic information to the central server [292] [293]. This information is used by the central server to suggest parking to the requesting vehicle by sending a parking suggestion message (PSM). PSM also contains relevant information like the tariff, distance and direction to the suggested parking. Thereafter, the receiver may choose to oblige or decline the suggestion based on his experience. The receiver may also choose to reserve the suggested parking until it reaches the suggested parking by bearing an additional cost (for reservation). Alternatively, if the driver declines the current suggestion, he may submit another request for parking guidance.

The architecture of the proposed automated parking guidance system consists of various components mainly sensors and communication devices [113]. The status of each parking is regularly observed using sensors and forwarded to the central server [294]. The server then uses this information for suggesting parking to the requesting vehicles. The server receives a request comprising of the current location of requesting vehicle and destination through personal navigation device [295]. Thereafter, parking is suggested based on received information. The updated position of the vehicle is also communicated to the central server at regular intervals so that the parking facility could estimate the expected time of arrival of a vehicle.

Initially, parking information was obtained using image processing [296] which has been completely replaced by usage of various types of sensors to collect the information [297] [298]over time. Authors have also chosen ZigBee protocol for its low installation cost [299] while authors in [300] suggested a vehicle to vehicle communication which can be used for route determination. Thus varieties of communication technologies are available in the literature for the parking management system.

5.3.1 Suggested Parking Management System

In the suggested parking management system, two metrics have been proposed: a*vailability metric* and *reservation metric*. *Availability metric* defines the probability of vacant parking spot when the driver reaches to the parking and is based on driving distance to the parking, walking distance from parking to the destination and number of vacant slots etc. Additionally, *reservation metric* considers the cost of reservation while driver reaches the suggested parking and is determined by parameters like parking cost, time to reach suggested parking etc. In order to present mathematical formulations for a suggested metric, the following terms need to be defined:

T_d : driving time to the suggested parking

W_d: walking distance from suggested parking to the destination.

P : parking cost.

n : number of vacant parking spots

n_a : number of cars arriving to the parking during T_d

n_e : number of cars leaving parking during T_d

P_a : degree of availability

From definition of n_a and n_e, it can be expressed as follows:

$$n_a = \frac{T_d}{ATBA} \qquad (5.3.1)$$

$$n_e = \frac{T_d}{ATBE} \qquad (5.3.2)$$

$$P_a = \frac{n_a}{n + n_e} \quad as \left(\frac{\frac{T_d}{ATBA}}{n + \frac{T_d}{ATBE}} \right) \qquad (5.3.3)$$

We also consider the situation when driver fails to get a vacant parking spot when he reaches the suggested parking p_s. In such a case, he needs to search again for another parking resulting in additional cost. Here C defines this additional cost and needs to be minimized:

$$C = \sum dist(p_s, p_i) * P_i, \quad for\ each\ i\ adj.to\ s \quad (5.3.4)$$

where p_i is parking adjacent to p_s and P_i represents its degree of availability. Now mathematical formulation for suggestion metric M_s, is defined as:

$$M_s = \alpha_1 T_d + \alpha_2 W_d + \alpha_3 P + \alpha_4 P_a + \alpha_5 C \quad (5.3.5)$$

Here αs are the predetermined constant for each factor. Similarly, the reservation metric M_r is defined as follows:

$$M_r = \beta_1 T_d + \beta_2 P + \beta_3 W_d \quad (5.3.6)$$

In the suggested approach, various performance metrics have been considered to evaluate its effectiveness. Following are some of these metrics.

i. Average Driving Distance
ii. Average Walking Distance
iii. Average level of congestion
iv. Average occupancy rate of parking spots
v. Average parking Failure rate
vi. Average requesting number

The suggested approach has the following steps for parking guidance

i. Request for parking guidance
ii. Determining the best parking for reservation
iii. Determining the best parking for suggestion
iv. Updating parking spot status

5.3.2 Simulation and Results

The proposed parking management system is implemented in MATLAB for the dataset given in [301]. It is assumed that the parking request is generated as a uniform random distribution between 0 and 1 has been considered. Different values of constants (α) and rectilinear distance have been considered. It has been implemented for $N = 500$. Table 10 represents values of different metrics which have been already discussed.

Table 10: performance metrics for Guided parking Management

N	Base preference	Avg. dis (driving) Km/car	Avg. dis (walking) Km/driver	Avg. occupancy Per parking (in %age)	Avg.parking failure
Case I-1	I	21.5605	0.9852	78.2173	0.01127
N= 500 (3803 parking Spots / 61845 cars)	II	21.4662	0.9417	77.6274	0.01234
	III	21.7290	0.9672	**79.2185**	0.01092
	IV	21.2202	0.9582	78.9123	**0.01063**
	V	**21.1185**	0.9761	79.0212	0.01147
	VI	21.2328	1.0012	78.5864	0.01124
	VII	21.1876	**0.9012**	78.2143	0.01232
	VIII	21.4562	1.0028	78.1462	0.01108
Optimized value Of the metric		**21.1185**	**0.9012**	**0.9012**	**0.01063**

It is observed that the suggested approach outperforms the existing approach. This improvement is the result of the inclusion of $ATBE$ of vehicles for parking suggestions. $ATBE$ helps to estimate the number of vacant parking spots, thus providing efficient suggestions. Following graphs in Figure 5.6 and Figure 5.7 represents the average driving and walking distance respectively for the existing and proposed approach.

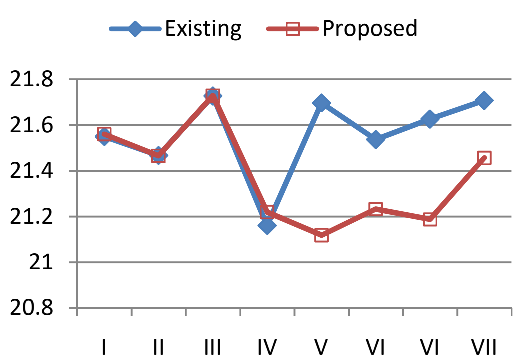

Figure 5.6: Average driving distance

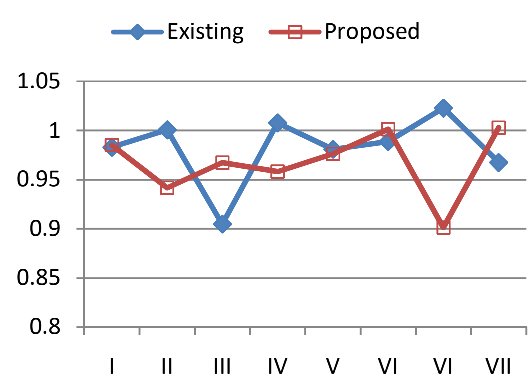

Figure 5.7: Average Walking distance

During simulation, the average occupancy rate of proposed approach becomes 78.49% in comparison to 57.33% of the existing approach. It becomes evident that the proposed approach improves the occupancy rate of parking by a remarkable margin. This improvement is obtained by considering the ATBE while suggesting parking. As a result, it further reduces the overall traffic/ congestion on the roads.

Chapter 6

6. Conclusion and Future Directions

This thesis is mainly focused on designing efficient resource location models using computational geometry tools. The presented study describes various optimization models that have been used to describe the spatial distribution of the facilities over a planar region. The work done considers three variants under location models: location, allocation and layout planning of the resources.

This thesis covers the location of facilities while optimizing an objective function. The objective function varies along the resources. Few examples of objective functions are *minsum*, *minmax* or *maxmin*. In the thesis, the location of facilities is followed by allocating demand nodes to the newly located resources. During the allocation of demand nodes, the objective is to allocate resources using their geometric properties while minimizing the expected service cost. Layout planning is also considered in this thesis.

In the thesis, we have developed and implemented some algorithms for location models, all of which are capable of performing efficiently and thus achieves best results. The primary structures used in designing and analyzing proposed algorithms are Voronoi diagrams, Delaunay triangulations, convex hull, visibility graphs and polygonal partitioning etc. Many factors are considered for location modelling. We exhibited the versatility of these techniques by applying them to a real-life scenario (such as layout planning).

Firstly, we focus on finding the optimal facility location for *p-center* in the non-convex region. Next, we consider the allocation of demand nodes to capacitated facilities in the presence of forbidden regions. Finally, we present an approach for layout planning. Two instances of layout planning are considered in the thesis: layout planning of an inventory store and an exhibition.

As discussed in chapter 1, Computational Geometry is a very powerful tool for spatial queries. Thus we have employed the structures in computational geometry for attempting *p-center* in the non-convex region. This chapter introduces the various location models and corresponding mathematical models for objective functions. This chapter exhibits that computational geometry is an efficient choice for handling location problems as it is capable of representing spatial data in an efficient manner. Constrained location model is represented in this chapter that necessitates a specialized approach. In the constrained service space, there are some restrictions for the location of facilities and therefore facilities cannot be situated at any arbitrary location. In the constrained location model, some constrained regions are given and the objective is locating minimal resources that cover all demand points.

Chapter 2 is dedicated to the review of available literature for usage of geometric structures in location modelling. In particular, we concentrate on the p-center problem also known as Fermat-Weber problem and its variants. As the location model is NP-hard, numerous heuristic based algorithms and approximations have been presented in addition to exact algorithms [37][38][39]. Algorithms developed by [41][16] employed Voronoi diagram for p-center in continuous demand plane. Such algorithms demonstrate how the Voronoi diagram can be used in the context of location modelling. This chapter also reviews the literature for p-median and obnoxious location models by using structures in computational geometry. Location model where facilities have capacity constraint also forms a section of this chapter, which discusses how this constraint is different from basic location model.

Chapter 3 presents a heuristic based approach to solve p-center for non-convex regions that utilizes the Voronoi diagram and Delaunay Triangulation. Here, we consider a threshold Δ for the distance i.e each demand node should have at least one resource within distance Δ. The proposed approach has a higher convergence rate and thus rapidly converges to the optimal solution. The proposed approach is also competent to estimate the optimal number of resources also (p). Capability to estimate the optimal number of resources helps in minimizing the cost of location modelling as the number of resources is a significant factor of cost. We have not considered the presence of barriers in the demand region in this chapter and work in this chapter can be further extended to consider non-convex region with barriers for p-center.

Chapter 4 of the thesis is dedicated to the allocation of demand nodes to the capacitated resources. The capacity constraint of the resources limits the number of demand nodes it can serve, thus requiring a specialized approach to handle allocation. In this chapter, we have proposed an approach for the allocation of demand nodes to capacitated resources while minimizing the service distance. In the proposed approach, we suggest the use of residue ratio for allocation. Residue ratio of the resource helps in minimizing the service cost. An illustration has been presented for the proposed approach and it has been observed that the suggested algorithm outperforms the conventional method of allocation for demand nodes. This work can be taken further to handle allocation of demand nodes to resources with lower boundary constraint.

In chapter 5, we have proposed a layout planning algorithm. In this chapter, we have considered two instances for the layout planning viz. layout planning for the departmental store and exhibition planning. In this chapter, we have employed the geometric structure of the demand plane and thus utilized the spatial properties of the problem. We have considered a grid layout for the departmental store. This consideration can be removed and thus the work can be extended further where the layout is not limited to the grid. Thereafter layout planning of an exhibition is presented. The chapter also presented the illustration of the proposed approach and also validated its effectiveness. The chapter also discussed automated parking management system to manage the issue of parking requirement. The proposed approach suggested parking to the requesting vehicle based on proposed metrics. The effectiveness of the proposed approach is validated with reference to various performance metrics for parking management.

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Appendix – A1

The following Table 11 contains the impulse value for the original and proposed layout for all three classes of customer for 10 customers. Here in Table 11, the range of distance is considered to be 1 only. The impulse values in bold represents the better impulse value.

Table 11: Impulse value for original and proposed layout (n = 10, Distance range = 1)

DISTANCE 1						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	4.03	**7.77**	2.99	**11.76**	5.99	**12.27**
2	4.03	**7.47**	2.99	**5.58**	**6.68**	**2.59**
3	2.79	**8.71**	0	**9.27**	0.69	**8.48**
4	2.79	**7.77**	0	**5.68**	5.99	**12.27**
5	1.24	**2.53**	0	**11.76**	5.99	**14.86**
6	1.24	**3.38**	0	**5.48**	6.68	**11.07**
7	1.24	**6.92**	0	**5.48**	0.69	**8.87**
8	0	**4.74**	2.99	**8.17**	0.68	**2.49**
9	4.03	**5.68**	0	**5.48**	0	**12.37**
10	0	**8.71**	2.99	**5.58**	5.99	**12.37**

Appendix – A2

The following Table 12 contains the impulse value for the original and proposed layout for all three classes of customer for 20 customers. Here in Table 12, the range of distance is considered to be 1 only. The impulse values in bold represents the better impulse value.

Table 12: Impulse value for original and proposed layout (n = 20, Distance range = 1)

DISTANCE 1						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	**2.79**	0.35	0	**5.48**	0	**8.87**
2	**4.03**	1.59	0	**11.76**	**5.99**	5.08
3	0	**3.38**	2.99	**9.27**	**6.68**	6.28
4	0	**3.38**	0	**5.48**	5.99	**6.28**
5	0	**7.77**	2.99	**9.17**	5.99	**14.86**
6	0	**7.77**	0	**2.99**	**5.99**	3.79
7	0	**2.14**	2.99	**11.86**	0.69	**8.87**
8	0	**5.98**	2.99	**2.99**	0.69	**5.99**
9	**2.79**	1.29	2.99	**9.27**	0.68	**8.48**
10	1.24	**7.47**	0	**6.78**	0	**12.37**
11	2.79	**6.53**	2.99	**11.76**	5.99	**12.27**
12	0	**2.53**	2.99	**11.76**	0.69	**2.59**
13	0	**5.68**	2.99	**8.17**	0.69	**11.07**
14	2.79	**8.71**	2.99	**11.76**	5.99	**8.87**
15	2.79	**3.38**	0	**9.17**	0	**3.79**
16	4.03	**7.47**	2.99	**5.58**	5.99	**12.27**
17	0	**3.08**	2.99	**6.68**	5.99	**11.07**
18	**4.03**	1.29	0	**9.37**	5.99	**11.07**
19	0	**0.35**	2.99	**11.76**	5.99	**8.87**
20	**4.03**	1.29	2.99	**11.86**	6.68	**11.07**

Appendix – A3

The following Table 13 contains the impulse value for the original and proposed layout for all three classes of customer for 10 customers. Here in Table 13, the range of distance is considered to be 2 only. The impulse values in bold represents the better impulse value.

Table 13: Impulse value for original and proposed layout (n = 10, distance range =2)

DISTANCE 2						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	**4.03**	0	2.99	**5.58**	5.59	**5.99**
2	4.03	**5.68**	2.99	**8.17**	0	**8.23**
3	1.29	**3.14**	**6.68**	3.68	0	**10.26**
4	1.24	**5.68**	6.68	**8.77**	0	**2.59**
5	0	**6.92**	2.99	**2.99**	0	**1.35**
6	0	**3.03**	**6.68**	3.09	0	**11.46**
7	**1.24**	0	**6.68**	3.09	**5.99**	3.33
8	**4.03**	3.14	6.68	**9.17**	5.99	**12.27**
9	0	**1.29**	2.99	**3.09**	0	**7.12**
10	0	**3.14**	2.99	**5.58**	0	**6.28**

Appendix – A4

The following Table 14 contains the impulse value for the original and proposed layout for all three classes of customer for 20 customers. Here in Table 14, the range of distance is considered to be 2 only. The impulse values in bold represents the better impulse value.

Table 14: Impulse value for original and proposed layout (n = 20, Distance range =2)

DISTANCE 2						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	4.08	**9.71**	0	**0**	0.69	**9.61**
2	0	**8.42**	0	**3.09**	**5.99**	1.85
3	1.29	**4.74**	**6.68**	5.08	5.99	**11.91**
4	**4.03**	2.79	2.99	**11.76**	0	**8.23**
5	**4.08**	0	2.99	**12.35**	0.69	**6.93**
6	0	**4.38**	**3.69**	3.68	5.99	**11.07**
7	**4.03**	3.14	2.99	**5.08**	6.68	**8.87**
8	**2.79**	0.35	0	**2.49**	0	**5.64**
9	4.03	**7.77**	2.99	**3.68**	0.69	**6.48**
10	4.03	**7.77**	2.99	**5.48**	0	**12.27**
11	4.02	**8.42**	2.99	**5.68**	5.99	**10.72**
12	0	**4.08**	6.68	**8.17**	0	**12.27**
13	1.24	**5.63**	2.99	**3.18**	0	**3.33**
14	1.24	**10.21**	0	**3.68**	0	**8.13**
15	4.08	**5.68**	0	**2.99**	0.69	**12.37**
16	1.24	**8.97**	**6.68**	3.58	**5.99**	3.13
17	**1.29**	0.35	0	**9.27**	0	**10.26**
18	0	**1.29**	2.99	**2.99**	0	**7.12**
19	4.08	**7.18**	0	**5.48**	0.69	**6.93**
20	**2.79**	1.24	2.99	**5.48**	0.69	**15.6**

Appendix – A5

The following Table 15 contains the impulse value for the original and proposed layout for all three classes of customer for 10 customers. Here in Table 15, the range of distance is considered to be 3. The impulse values in bold represents the better impulse value.

Table 15: Impulse value for original and proposed layout (n = 10, Distance range = 3)

DISTANCE 3						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	**3.03**	0	**3.58**	2.49	1.85	**2.59**
2	3.03	**5.08**	**6.68**	3.09	0.69	**8.87**
3	**4.08**	3.03	4.28	**6.78**	0.69	**3.33**
4	0	**5.43**	0	**3.09**	7.84	**11.46**
5	**2.79**	0	0	**0.59**	0	**3.79**
6	**3.03**	1.29	**6.78**	6.18	0	**2.59**
7	**1.24**	0	5.38	**6.27**	1.85	**11.46**
8	**2.79**	0	2.99	**5.68**	**7.84**	3.33
9	**5.32**	3.53	3.68	**6.27**	0	**9.78**
10	2.29	2.29	**4.78**	2.59	5.99	**8.13**

Appendix – A6

The following Table 16 contains the impulse value for the original and proposed layout for all three classes of customer for 20 customers. Here in Table 16, the range of distance is considered to be 3. The impulse values in bold represents the better impulse value.

Table 16: : Impulse value for original and proposed layout (n = 20, Distance range = 3)

DISTANCE 3						
	class I		class II		class III	
cust. #	i(original)	i(revised)	i(original)	i(revised)	i(original)	i(revised)
1	**1.79**	0	2.99	**8.17**	0	**6.28**
2	1.29	**2.29**	2.99	**6.18**	**5.99**	3.33
3	2.79	**4.74**	2.99	**11.86**	**7.84**	5.82
4	2.29	**5.98**	5.27	**7.37**	**5.99**	2.49
5	0	**5.43**	**8.36**	5.48	1.85	**10.72**
6	2.29	**4.43**	**6.78**	2.59	6.68	**12.27**
7	**3.53**	2.79	0	**2.59**	0	**2.49**
8	**1.24**	0	**3.09**	2.99	7.84	**13.11**
9	4.08	**4.39**	7.47	**8.57**	0	**5.08**
10	1.79	**7.53**	0	**6.28**	2.54	**6.38**
11	0	**3.53**	0	**6.87**	**8.53**	5.08
12	**6.37**	2.29	0.79	**2.99**	7.84	**9.78**
13	**3.53**	1.29	4.28	**6.78**	0	**8.13**
14	2.29	**4.39**	**3.58**	3.09	8.53	**11.07**
15	1.79	**4.93**	6.68	**11.16**	2.54	**10.26**
16	**1.29**	0	3.69	**8.17**	1.85	**3.79**
17	4.08	**5.63**	4.68	**8.17**	0	**7.12**
18	1.24	**1.59**	3.09	**3.09**	5.99	**8.97**
s19	**5.37**	3.53	1.69	**3.09**	**0.69**	0
20	**3.08**	0	3.58	**3.58**	0.68	**8.58**