

School Bus Routing and Scheduling Based on Branch and Bound Approach

Thesis submitted in partial fulfillment of the requirements for the award of degree of

Master of Technology
in
Software Engineering

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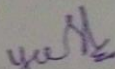


COMPUTER SCIENCE AND ENGINEERING DEPARTMENT
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July 2015

CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled, "*School Bus Scheduling Based on Branch and Bound Approach*" in partial fulfillment of the requirements for the award of degree of Master of Engineering in *Software Engineering* submitted in Computer Science and Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of *Dr. Sushma Jain* and refers other researcher's work which are duly listed in the reference section.

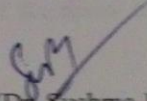
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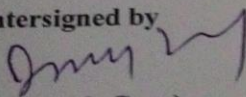
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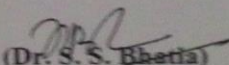
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ABSTRACT

The congestion in the traffic is the most basic problem that most of the cities facing now a days because more and more people are moving towards urban areas. As more and more people are moving toward urban areas so the number of schools are increasing and thus number of students travelling through school buses it is required to route and schedule school buses in an optimized manner. This problem is arising in most of the schools in urban cities, most of the parents are complaining about the travelling time and distance student has to travel because of no proper routing and scheduling. Hence, there is a need to find a way of minimizing the students travelling time. If the school buses are not routed and scheduled in an optimized manner it will result in loss of time, resources and money. So from there comes the concept of School Bus Routing and Scheduling Problem (SBRSP). It is a special case of Vehicle Routing Problem and is an NP-Complete problem. So, it is unlikely to get result in polynomial time. So for smaller problems we have given this approach. In this proposed work we have proposed Branch and Bound approach with combination of Hungarian algorithm on Homogenous Bus Fleet. The Branch and bound algorithm provide optimal solution for the smaller problems in polynomial time. In this proposed work we have taken care of several constraints like time window of schools, capacity of buses etc. For a group of schools it will provide an optimal solution that will help those schools to optimize the bus routes, number of buses used and thus optimizing cost. This will also help in reducing pollution as the distance travelled by buses is optimized. The proposed approach has been applied on the input set of data obtained from simulated using our proposed algorithm.

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1. Introduction

School Bus Routing and Scheduling is a Problem that is faced by schools in metro cities. The Bus service that schools provide to students is committed to give them a convenient, most optimal and less time consuming route. This involves many aspects like:

1. Allocate Bus stops to students
2. Allocate stops to buses
3. Allocate students to Buses
4. Finding out Routes
5. Scheduling the Buses

The problem is that there is no scientific way to assign and schedule buses to most suitable route and it is mostly seen that the school buses are routed spontaneously in real life and hence there is problem of optimization of time and thus cost. Moreover there are such cases where students has to travel much more than they are required if the routes are planned well. So it is required to schedule and route buses and where to pick up students and where to drop them. In this the constraints of time window and maximum capacity of buses should be maintained. This problem has some constraints that are given below:

1. Time window of the schools
2. Maximum capacity of the buses
3. Student pick up and drop point should be same
4. Assigning each student to a single bus
5. Assigning nonetheless one bus to each bus stop

School Bus Scheduling and Routing Problem like other problems seeks to give an efficient way to schedule and route school buses. The way out will help in many ways as it is going to reduce overall travel time of the students who otherwise have to travel much more because of no proper routing and scheduling of buses, It will help in environmental grounds also as the buses have to travel less distance and thus contamination will be less. The problem of School Bus Routing and Scheduling problem is a subpart of a larger problem known as vehicle routing problem.

1.1 Vehicle Routing Problem

Vehicle Routing Problem denotes the problem of the efficient use of vehicles that has to make a number of pickups and delivery within a certain time limit. The problem consists of several aspects like which customer should be served by which vehicle and in which order so that overall distance travelled by the vehicles should be minimum while considering all the constraints like time window of the delivery and vehicle capacity. So, the main purpose of this is to minimize the overall cost while maximizing the customer satisfaction.

Vehicle routing problem is the problem in which vehicles are located at a fundamental position has to serve the customers located at different positions in the city [1]. This problem has a great importance in many fields. The fig 1 give below illustrate this problem:

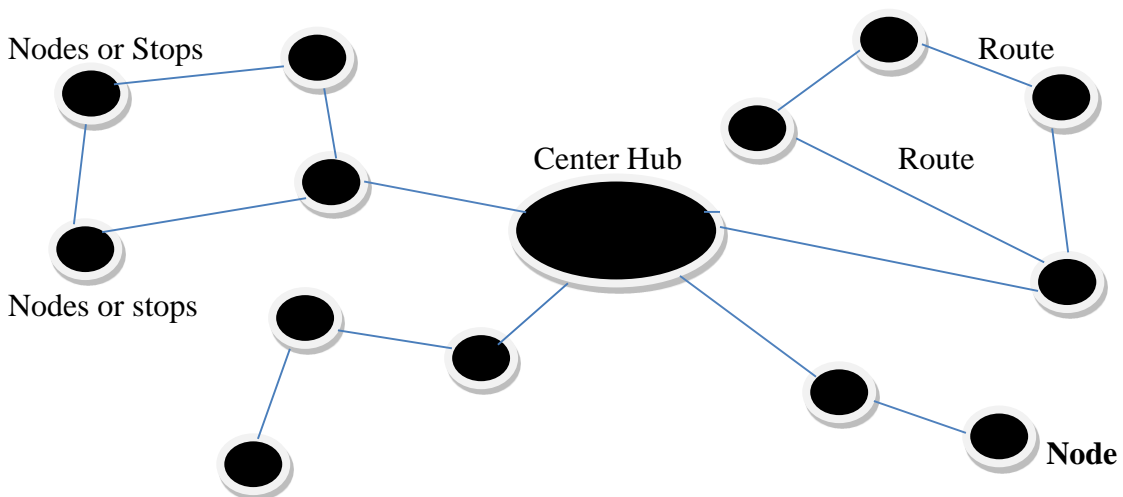


Fig 1: Vehicle Routing Problem

1. **Centre Hub:** It is the location from where all the vehicle start their journey and end their journey.
2. **Nodes (Stop):** These are the location where vehicle have to pick up or deliver something (pick up and drop points).
3. **Route:** These are the path on which vehicle has to travel that will contain number of pickup and delivery nodes.

It has led us to many problems like number of accidents are also increasing day by day. Given below is the data of Hyderabad city.

- a. Fig 2 shows a relation between year from 2000 to 2004 and number of collisions happened during those years and we can clearly see that number of collisions have increased rapidly.
- b. Fig 3 shows a relation between year from 2000 to 2004 and number of people injured during those years and it explains that number of injured people has also increased rapidly.

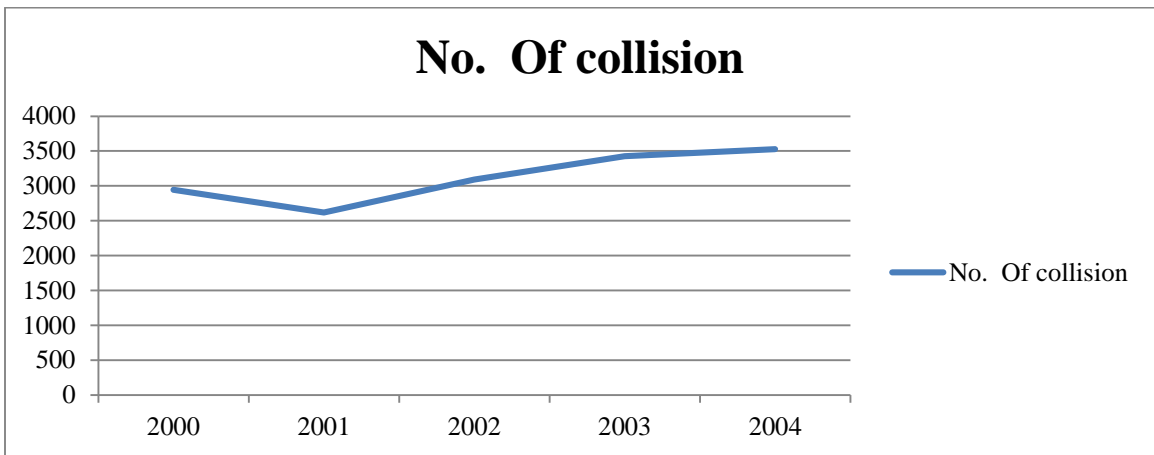


Fig 2: Years and number of Collision

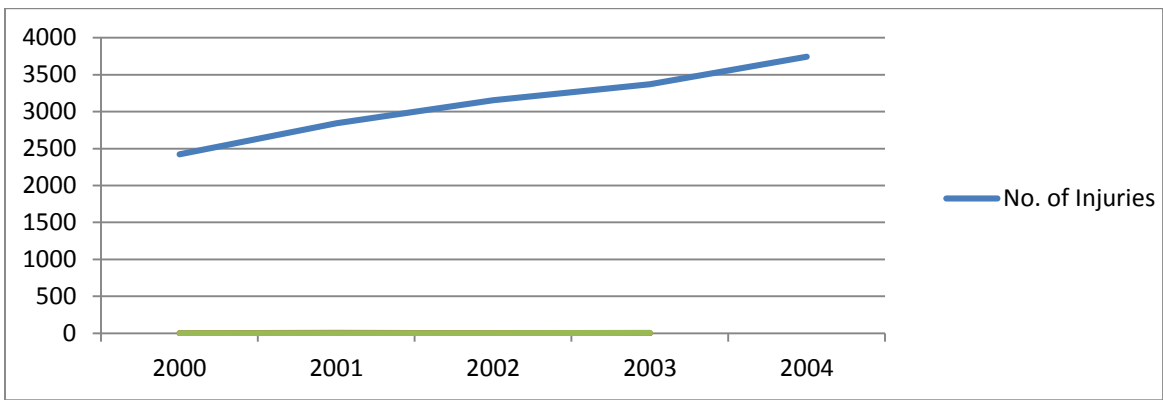


Fig 3: Year and Number of Injuries

1.2 Variations of Vehicle Routing Problem

Vehicle Routing Problem has several variations and each of them has their different importance in real life environment [33]. The following diagram shows the variations of Vehicle Routing Problem:

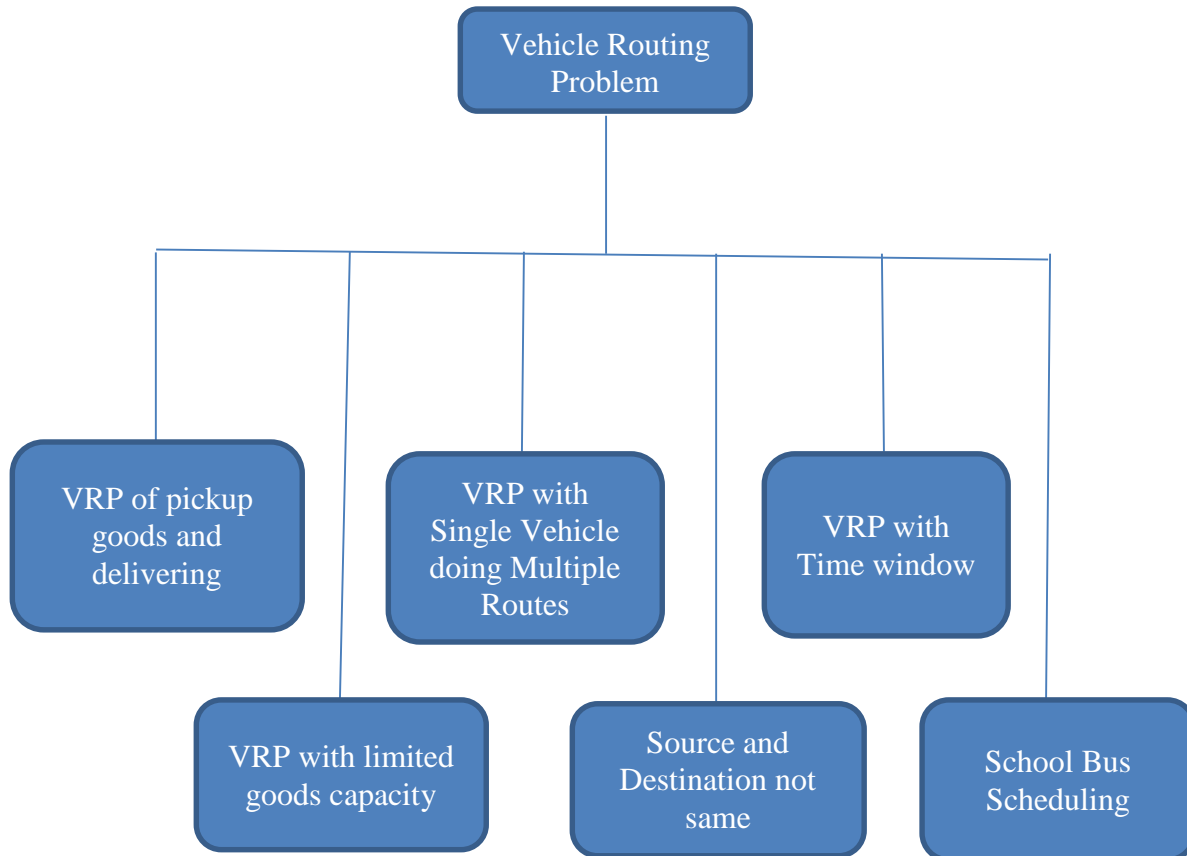


Fig 4: Variations of Vehicle Routing Problem

1. **Vehicle routing problem of pickup goods and delivering them:** In this goods has to be picked up from one location and has to be delivered to some other location and we have to find optimal path for picking up the goods and delivering them to the desired location [34].
2. **Vehicle routing problem with limited goods capacity:** This problem involves fleet of vehicles that has to serve passengers or carry goods to geographically

dispersed locations from a central depot [35]. Here each customer has a certain demand and each vehicle has a certain capacity, the purpose is to maximize the customer satisfaction and minimising the overall cost.

3. **Vehicle doing more than one trip:** Vehicle routing problem in which vehicle can do more than one trips that is a single vehicle can travel on more than one route [36]. Here each route has customers with their demand and the demand should be satisfied with minimal cost possible.
4. **Vehicle starting point and come back point may not be same:** Vehicle routing problem with an approach where vehicle is not required to come back to the depot. So in this a vehicle may not come back to the starting point and may start from that location next day. Here the use of multi depot is done.
5. **Vehicle routing problem with specific time window:** The delivery of the material that has to be delivered should be in given time, so this problem is related to time window [37]. In this problem there are three types of decisions that are taken:
 1. **Assigning:** In this approach decision have to be taken which vehicle has to be assigned to which route. In case of public transport decision has to be taken which transport vehicle has to be assigned to which bus stop.
 2. **Routing:** This approach decides which of the routes will be optimum for the vehicle. This decision helps to reduce the distance travelled and thus reduce the travel time.
 3. **Scheduling:** This approach decides which vehicle has to be scheduled which time and at which route.
6. **School bus routing problem:** School Bus Routing and Scheduling is a scheduling Problem that is faced by schools. The problem deals with two main aspects one is routing and another scheduling of buses. It is NP-Complete problem. The problem Consists of various schools in which each of them has their fix time window, they have their fix stops and number of children at each stop are fixed and has to complete the trip [2-3]. In the proposed work we are going to optimize the overall trip.

Travelling Salesman Problem

It is a real life problem. According to this problem there is a salesman that has to travel m cities and come back to the starting point covering the smallest possible distance.

The traveling salesman problem can be denoted as follows:

TSP = $\{(G, f, t): G = (V, E) \text{ a complete graph}$

And f is a function $V \times V \rightarrow Z$

And $t \in Z$, G is a graph that contains a traveling salesman tour with cost that does not exceed t . The solution of the Travelling salesman problem is Hamilton Circuit. It is a kind of NP-Hard problem. The brute force algorithm can be used to solve the problem, it is guarantee that it will find the solution but it will take long time.

1.3 School Bus Routing and Scheduling

School bus routing and scheduling problem is a variation of Vehicle routing problem. This problem deals with the routing and scheduling of buses in an optimal manner. The problem faced is that various schools has their fix time window, they have their fix stops and number of children at each stop are fixed and has to complete the trip [2-3]. Here we are going to optimize the overall trip.

There are three main factors that make school bus scheduling unique:

- a. The overall cost of running school bus should be optimized.
- b. The overall demand of the service should be fulfilled.
- c. The school bus scheduling should be equally distributed and reliable for all students of all schools.

It is a NP-Complete problem.

P Class of problems: These are those problems that can be solved in polynomial time with deterministic algorithm [29].

NP Problem: consists of all those problems that can be solved in polynomial time by a non-deterministic algorithm [30].

NP-Complete Problem: A problem β is NP-Complete if and only if $\beta \in NP$ and all other problems in NP polynomial transforms to β . However, so far there is no polynomial time algorithm that has been discovered for any NP-Complete problem [4].

NP-Hard Problems: A problem is said to be an NP-hard if there exist NP-complete problem which can be reduced to it in a polynomial time [30].

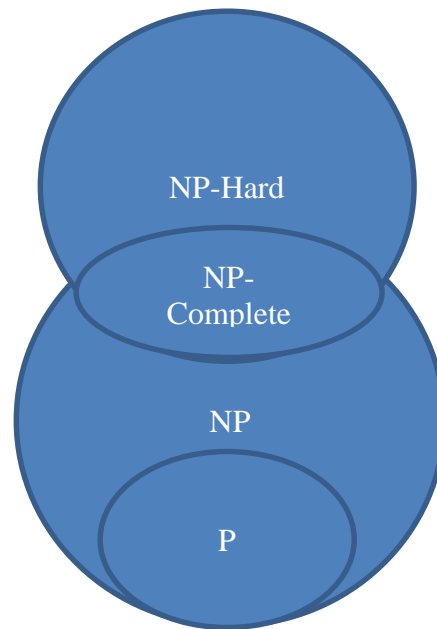


Fig 5: NP-Complete Problem

Let us take an example to see what the problem exactly is and what could be a possible solution for the given problem that we are going to find out. Now given below example shows school bus scheduling and what could be a possible solution for the problem [8].

Here let us consider there are three schools and they are having four, three and four number of trips respectively [25]. There is already defined time window for each of these schools. Student should reach school within that time window. When one bus can do two trips without violating the bus capacity constraints and without violating the school time window of the designated schools then that bus could be scheduled to do both the trips. Let us consider there are three schools and there time windows are T_1 , T_2 and T_3 . Now let $T_1 \leq T_2 \leq T_3$. There are four bus routes (r_1, r_2, r_3, r_4) of school 1, three bus routes ($r_5, r_6,$

r7) of school 2 and four bus routes (r8, r9, r10, r11) of school 3. Fig. 6 shows the bus routes of school and this can be considered as our sample problem.

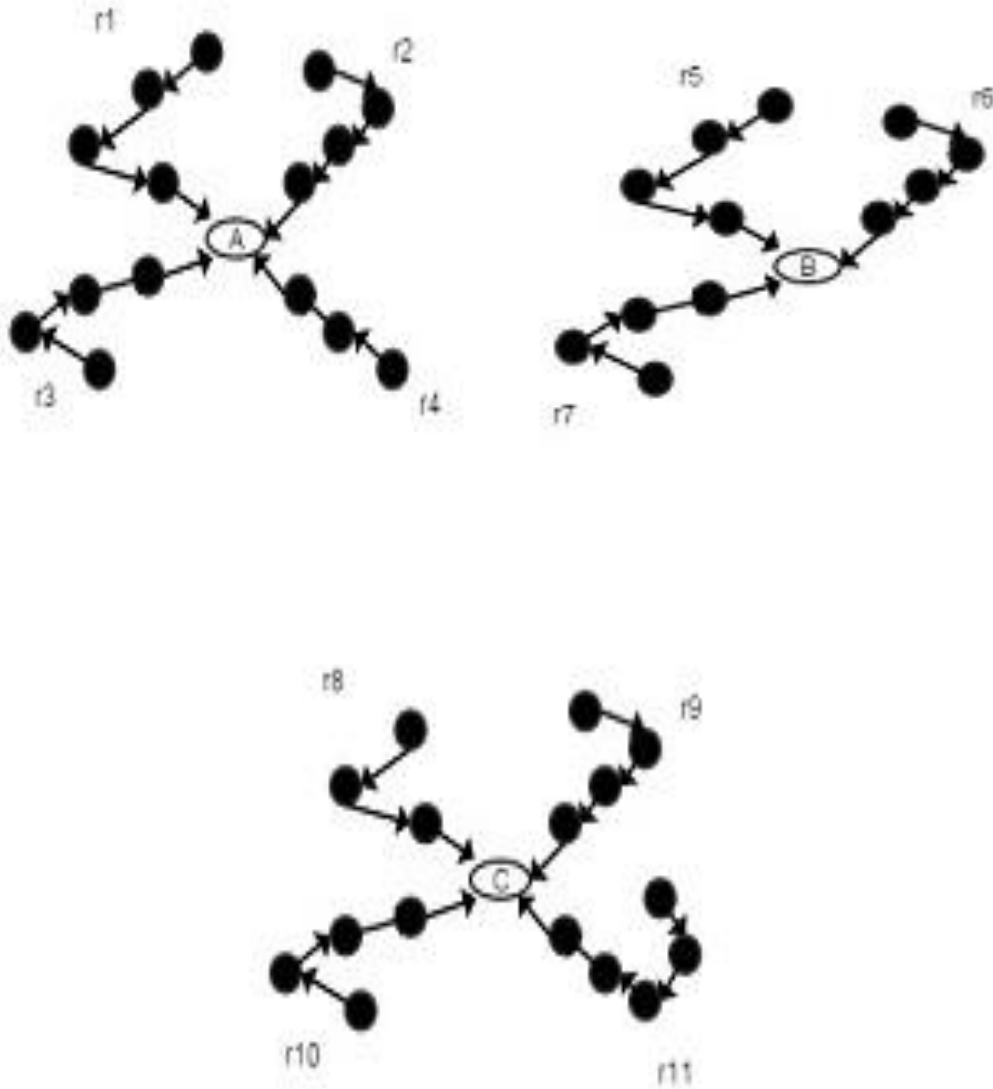


Fig 6: Sample Problem

Now the solution of this sample problem should be a solution that can optimize the overall running time of the buses and that can help us to minimize the overall cost. The solution will do it by using

lesser number of buses in an optimized manner. Fig. 7 shows the sample solution of the above given sample problem in fig. 6.

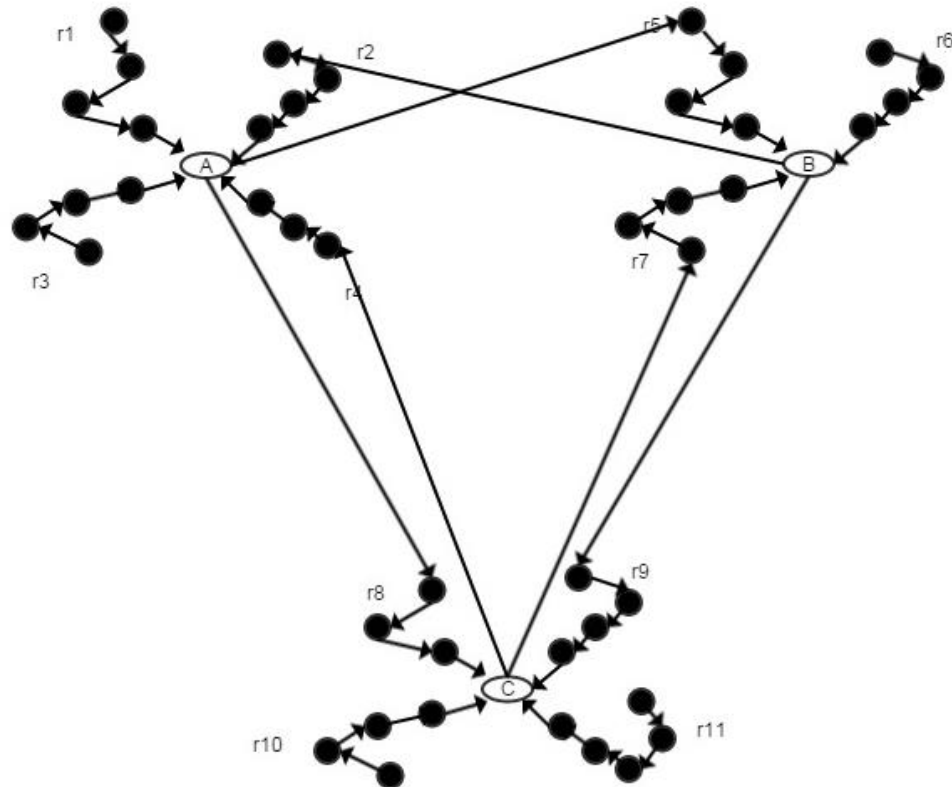


Fig 7: Sample Solution

The school bus problem can be further divided into following sub problems:

1. **Selection From available Bus Fleet:** First thing is to select from available bus fleet (homogenous bus fleet and heterogeneous bus fleet).
2. **Bus stop Selection:** Selection of bus stops from the given set of potential bus stops is required.

3. **Assigning students to a bus stop:** Then we have to assign the students to the particular bus stops from where the school bus is going to pick them.
4. **Assigning bus stops to buses:** We have to assign bus stops to the particular buses.
5. **Routing:** Then, we have to determine the bus route.
6. **Scheduling:** Then, we have to determine the bus schedule.

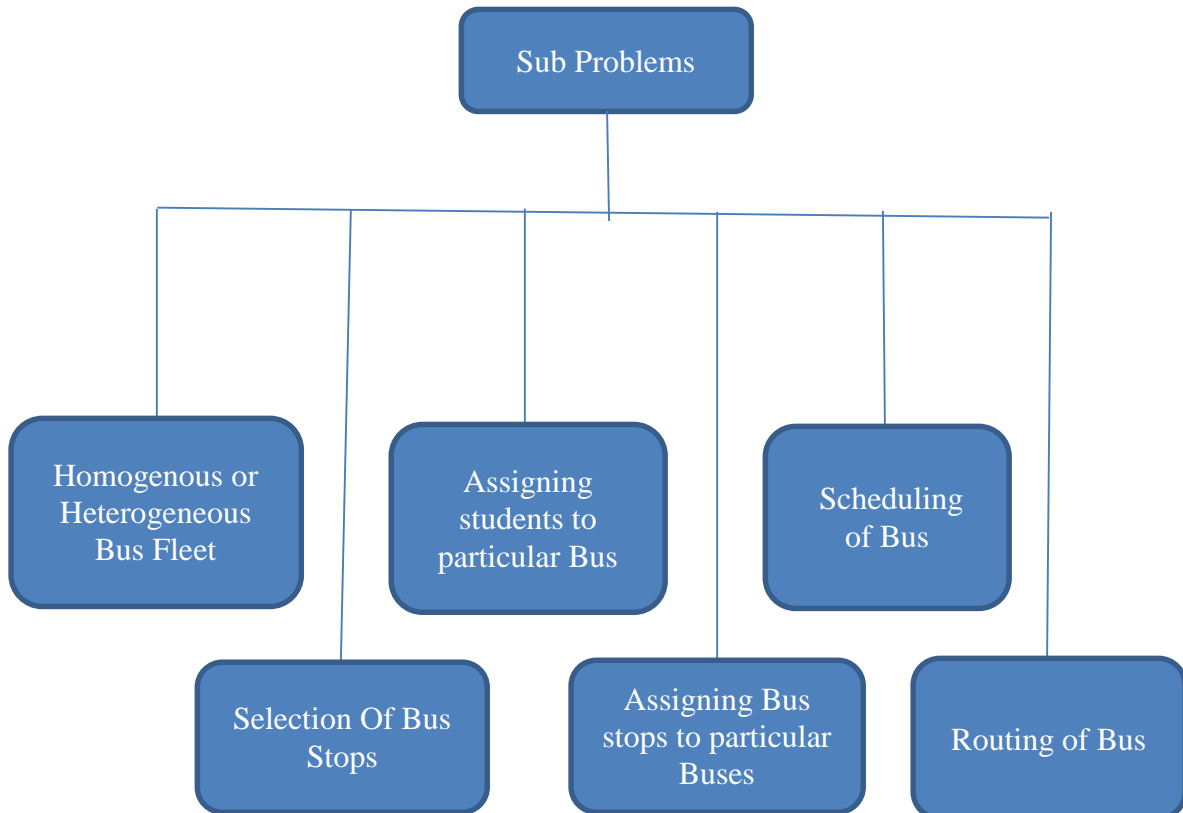


Fig 8: Sub Problems of School Bus Scheduling

1.4 Advantages of School Bus Routing and Scheduling:

1. **Environmental Welfares:** As on optimization the overall distance travelled by the buses decreases, the greenhouse gas omission done by them also decreases which in turn helps to decrease pollution.

2. **Time Saving:** Because of school bus routing the routes are optimized and thus the time to travel of most of the students decreases.
3. **Cost Saving:** As the distance travelled and number of buses required by the schools decreases the overall cost decreases.
4. **Congestion in Traffic Decreases:** As the number of buses travelling decreases the overall congestion in traffic also decreases.

1.6 Structure of Thesis

The summary of each of the chapter described in the thesis is given below:-

Chapter 2 will provide state of art how this problem came into existence and how it got evolved and what are the work done on it.

Chapter 3 illustrates the problem statement and the goals of the proposed work.

Chapter 4 provides proposed work that is branch and bound approach and thus scheduling and route school buses.

Chapter 5 discusses the evaluation parameters and the results obtained in proposed work and its comparison with other approaches.

Chapter 6 renders the conclusion, summary and future scope of the proposed work.

2.1 Evolution of School Bus Routing

Newton *et al.* [4] have proposed their approach and they considered the constraint that a student will take only one school bus and a school bus will service only one school. However, their assumptions were overly simplified. Gavish *et al.* [5] have proposed a bus scheduling problem in which time window of trip was given and this was called as Transportation problem. Carraresi and Gallo [6] have proposed the same problem as an assignment problem. An assignment problem is the problem in which the number of resources is equal to the number of sources. Braca *et al.* [7] have proposed the School Bus Routing of New York City School. Most of the Literature in the past deal with one bus at a time but this approach deal with whole of the collection of the buses with randomized insertion-based heuristic. Löbel [8] have proposed the multiple stations vehicle scheduling problem (MDVSP) in public transport, it is very similar to school bus routing problem with heterogeneous bus fleet, and gave a conclusion that it is a NP-complete problem and gave a solution that has a column generated approach. In this there were several depot but there was a condition that the bus should start and end at the same depot. Spada *et al.* [9] have proposed Heuristic approach on multiple school bus routing problem. Different schools were taken care on the basic of their increasing order of their timing, and the routes of each and every school is calculated with the help of greedy approach. And if possible the routes of the buses were merged to get better results. After merging the routes he applied Tabu search on it to optimize it. Fügenschuh [10] have proposed school bus scheduling but also took care of transshipment of student between buses. He then evaluated the problem based on branch and cut approach. In this homogenous school bus was considered and due to the complexity of this method it was not an optimal solution. And moreover the school start time and the start time of school buses at every point was considered fixed. So it is problem in real time world.

2.2 Exact Methods

2.2.1 Hungarian Algorithm on School Bus Routing

Kim *et al.* [26] have proposed Hungarian algorithm using homogenous bus service and having fixed start time. They handled a special case of problem here. In their approach they consider some constraints like the start service time and end service time of stops are already fixed and all of the trips can be served by any of the vehicles [11]. So it can be considered as assignment problem. In their proposed work they considered that $cost_{ij}$ is the cost between nodes i and j , when stop i can be followed by stop j without violating the given start service time and end service time.

$$MinZ = \sum_{i=1}^n \sum_{j=1}^n cost_{ij} y_{ij} \quad (1)$$

And the constraints are:

$$1. \sum_{i=1}^n y_{ij} = 1, \quad \forall j \quad (2)$$

$$2. \sum_{j=1}^n y_{ij} = 1, \quad \forall i \quad (3)$$

Kim have proposed a six step algorithm to solve this problem and that algorithm is given below:

Step 1: Find the smallest value from each row and subtract it from all the elements of the same row.

Step 2: Find the smallest value from each column and subtract that smallest value from all the element of that column.

Step 3: Now lines are drawn from appropriate column and rows where the element value is zero but minimum number of lines are used.

Step 4: If it is find out that the number of the covering lines is equal to n than an optimal assignment of those zeros possible and our algorithm ends here.

Step 5: If it is find out that the number of the covering lines is not equal to n than an optimal assignment of those zeros is not possible and we have to proceed further.

Step 6: Now again find out the smallest value that is still not covered by any of the line. Now we are going to subtract this value from each of the uncovered row and are going to add it to covered row and return to step 3.

2.2.2 Dynamic Programming

Dynamic programming is an optimization technique that is used for solving the complex problems by dividing that complex problem into simpler sub problems. It was proposed by Richard Bellman [12] and it was first used to solve sequential problem and it later extended to solve combinational problems by dividing those bigger problems into sub problems. Then the solution of these sub problems is combined to make the complete solution.

Various researches have proposed a solution for School Bus scheduling and Routing Problem using Dynamic Programming. Bektas, and Elmastas [13] have proposed an integer programming model and they pronounced the exact method solution approach for School Bus Routing Problem. Schittekat *et al* [14] developed an integer programming model for SBRP and they gave an exact method for the solution of sub problems. Fugenschuh A. [32] has also proposed the solution for school bus scheduling using integer programming.

2.3 Approximation Methods

These methods are used to find out the solution for the optimization problem but these solutions are approximate solutions. They get the results which may or may not be optimal but the results are close to the optimal solution. Approximation algorithms are being used in those problems where the exact polynomial time algorithm exists but there time complexity is really high which not feasible.

Various researches have proposed a solution for School Bus scheduling and Routing Problem using Approximation Methods, Hlaing and khine [27] have proposed travelling salesman problem as one of the approximation methods used to solve these problems. It is the problem of finding shortest tour between the n cities and thus minimizing the overall cost and after tour the salesman has to come to the starting city. This approximation method uses the concept of minimum spanning tree. Barr, *et al.* [24] have proposed this approach. Minimum spanning tree is a spanning tree whose weight is less than any other spanning tree. Two algorithm are used for it:

1. Prims Algorithm
2. Kruskal Algorithm

The following steps are taken in this method:

1. Compute minimum spanning tree by using any of the algorithm.
 2. Double the edges of the spanning tree (to get a graph in which each node has even degree).
 3. Then construct an Eulerian Tour.
 4. Construct a valid tour that visits each node exactly once using short-cuts.
- The tour contains two copies of each edge in MST, $\text{cost}(\text{tour}) = 2 \cdot \text{cost}(\text{MST})$.

2.4 Heuristics Approach

Zeng and Mouskos [15] have proposed Heuristics approach that finds the solution from all of the feasible solution and it does not guarantee to find out the optimal solution and this approach is used when we know that the problem we are dealing with is NP-Hard problem. They produce quickly solutions that are good enough for solving problem at hand but the solutions may not be the best of all. Below are the some of the heuristics approaches:

2.4.1 Ant Colony Optimization Algorithm

This approach is a probabilistic approach and it uses Meta heuristic technique for solving the complex problems and is aimed at finding the optimal route in a graph based on behavior of ants seeking a route between their ant colony and the food [16]. This is a technique used by real ants in real life to find the shortest path from their colony and finally all the ants follow that same shortest path. The algorithm basically makes two local decision policies:

1. Trail
2. Attractiveness

It goes through following steps:

1. Initialization
2. Construction
3. Trail Update
4. Termination Condition

This technique has been used in many complex optimization problem solution. Hlang and Khine [17] have proposed this technique to solve the travelling salesman problem (TSP) since it is one of the high performance computing method for TSP.

2.4.2 Scatter Search Algorithm

It is a Meta heuristic approach which uses evolutionary approach for solving had optimization problems [18]. This approach operates on a set of Reference Sets and combines them to make a new solution. Cavique *et al* [31] have proposed two phase procedure:

1. **Initial Phase:** In this a set of solution is generated and the best solution is chosen to be a part of reference set.
2. **Evolutionary Phase:** This step repeats until we find the solution.

Glovev [28] have proposed an approach that had integer programming problems and one of the study on it was based on Germany and denoted the school bus scheduling in Germany. In Germany every second student goes to school by public bus transport, so we want to integrate it with public bus transport. So Germany government wanted to make an optimised solution for optimising cost and routing and scheduling. It was found out that number of buses can be optimised and thus cost can be optimised together with the starting time problem, i.e. in this time window has to be considered.

This approach deals with real time situations. In this approach changing of bus is not allowed. Proposed work integrated school start times with public transport. In this approach the real time situations are considered and the integration is done so that all the requirement of the rural students can be considered. By this approach we can optimise the cost and schedule. In this approach different cut planes are considered and heuristic approach is done. The problem with this is that it takes too much time to optimise the solution

2.4.3 Simulated Annealing

Kirkpatric *et al.* [19] have proposed a probabilistic iterative method and he used tmethod to solve complex optimization problems. The algorithm starts from choosing an initial solution and generate a new solution that is neighborhood of chosen initial solution. Then the change of objective function σ is calculated. And then the solution reaches near to its

optimal value as the objective function goes on decreasing. The process continues until the given stopping criteria is met.

Various Researchers has provided solution to School Bus Routing and Scheduling using Simulated Annealing. Banos, and Raul [41] have proposed a solution of vehicle routing problem in which he took care the constraint of time window. Wang [42] has proposed solution for vehicle routing problem which took care of constraint of time window and simultaneous pickup and delivery.

2.4.4 Tabu Search Algorithm

Tabu search algorithm is a global heuristic approach which tries to avoid falling into local optima by restricting some of the solutions from selection for a given period of time. Gendreau *et al* [20] used tabu search heuristic to solve VRP with capacity and route length restrictions. The school bus routing problem (SBRP) discussed in this work is an NP-hard problem. Since heuristic algorithms are suitable for NP-hard " problems, this work proposed to be used to solve SBRP.

Various Researchers has provided solution to School Bus Routing and Scheduling using Tabu Search. Escobar [38] has proposed tabu search algorithm for multi- depot vehicle routing problem. In his approach he provided solution where bus start point can be from different depot. Brandão [39] has proposed a solution for vehicle routing problem where he used heterogeneous bus fleet. Nguyen [40] has proposed which took care various constraints like multi-depot, time window etc.

2.5 Multi-depot vehicle scheduling problems with time windows and Waiting cost

Desaulniers, *et al.* [21] have proposed multi-depot vehicle scheduling problem with time windows (MDVSPTW) will be considered. In this problem number of buses are given in a depot and their time windows are also given, the purpose of this is to minimise or optimize the cost by scheduling the buses according to time window. In this proposed work he considered constraint the buses should start and end at the same point. Each scheduling thus contains a number of things like loading, unloading and stoppage time and the start and end point of the buses and the time window should start at the start of the service. The

cost of each bus depends on its idle time, the distance travelled by the bus and the occupancy in the bus. The cost also depends upon the repairs done on the buses. In the real time environment the waiting time for the buses may also be taken in consideration. He proposed following solution.

- a. It produces an optimized solution.
- b. It provides optimized solution on the basis of heuristic approach.
- c. It outdone some of the specialized algorithm that of Bianco *et al.*
- d. Waiting cost should be considered.

Problem with this that it considers approximate value for that waiting time cost and this may be misleading. And this does not even optimized the routing of the buses which can further decrease the overall cost.

2.6 A metaheuristic for the school bus routing problem with bus stop selection

Schittekat *et al.* [22] have proposed work which considered the real life conditions and constraints in the approaches. But this proposed work considered the selection of the route as well as the selection of the stations to go to the route. These are two important sub problems that is present and a solution approach of this is done.

This approach considers the following three things:

- a. To find set of stops to visit.
- b. To find for each student to which stop he should go that it is optimized for him.
- c. To minimize the total distance travelled by the student

3. Problem Statement

3.1 Barriers in the Previous work

School Bus Routing and Scheduling Problem is the problem that has to deal with various issues. The major challenges is to decrease student travel time and the overall distance travelled by the buses. Along with this, the solution has to deal with various constraints like the capacity of the buses and pick up and drop points for the students. Routing should be done in such a way that student travelling time should be minimized. Another important aspect is to take care of the time window. Time window means that the student should reach school with in specified time of the school.

Till now, most of the proposed works are focus on lower bound of time window like Hungarian Algorithm. This lower bound approach was also used in integer programming model. The disadvantage of this approach is that it will break the constraint of time window. Heuristic methods are used to solve this problem. Some researchers have worked on approximation method to solve this problem. The problem with that solution is that it does not guarantee optimal solution. Some Researchers proposed the solution approach using heuristic approaches that even considered the waiting time that vehicle may have to do in case of a delivery to a customer. The previous approaches has following barriers:

- Upper bound of time window is not took care in most of the previously proposed work.
- Heterogeneous bus fleet service has not been used in most of the previously proposed work.
- Heuristic approach used does not provide guarantee that we are going to get an optimal solution.

3.2 Problem Statement

School Bus Routing and Scheduling is a Problem that is faced by schools in metro cities. The problem deals with two main aspects one is routing and another scheduling of buses. The problem is that there are various schools has their fix time window, they have their fix stops and number of children at each stop are fixed and has to complete the trip with

maximizing the student satisfaction and minimizing the overall distance and hence cost. This study that we are doing aims to develop a mathematical model and a branch and bound algorithm that would minimize the total time spent by students in travelling and to reduce the overall distance travelled. This algorithm is proposed to take care of upper bound of time window and thus reduce the overall travel time and distance travelled by the students.

3.3 Objective of the Proposed Work

Objectives of proposed methodology are as follows:

- Make a mathematical model to schedule and route school buses in a manner which takes care of upper bound of the time window of the schools.
- Implementation of Branch and Bound approach for the solution of our problem.
- Analysis of sample data using the developed mathematical model and algorithm.

4.1 Mathematical Model

In our proposed work we are using Homogenous Bus Fleet. Homogenous buses are the buses which are consistent that is they are of same capacity and same priority [4].

Following notations are used in this mathematical model:

- $I, j =$ stop index
- $S =$ set of stops
- $S_0 = S \cup \text{depot}(0)$
- $S_{n+1} = S \cup \text{depot}(n + 1)$
- $y_{ij} =$
1, if stop j has been served just after we serve stop i , otherwise 0 (distance)
- $t_{ij} = 1$, if trip j is followed by trip i ; 0 otherwise
- $x_i =$ service start time at stop i
- $\omega^+(i) =$ set of all the steps That can be directly connected without the violation of the time window + the depot index $(n+1)$ to stop myself.
- $\omega^-(j) =$ set of all the stops That can be directly preceded without the violation of the time window + the depot index (0) to stop j .
- $F_0 =$ A large number
- $F = \text{big } F (F \gg F_0)$
- $l_i =$ it is the service time for the trip i
- $[a_i, b_i] =$ time window for the stop i

Our model for the homogenous fleet problem is as follows

$$\text{Min } (\sum_{i \in S_0} \sum_{j \in S} t_{ij} y_{ij} + F_0 \sum_{j \in S} y_{0j}) \quad (1)$$

(Here we are minimizing the overall distance travelled)

Constraints are as follows:

$$1. \sum_{i \in \omega^-(j)} y_{ij} = 1, \quad \forall j \in S \quad (2)$$

(All buses should be assigned to each of the stops)

$$2. \sum_{i \in \omega^-(j)} y_{ij} = \sum_{i \in \omega^-(j)} y_{ji}, \quad \forall j \in S \quad (3)$$

(Number of buses arriving at a stop should be equal to the number of buses departing from that stop)

$$3. \quad x_i + l_i + t_{ij} - x_j \leq (1 - x_{ij})F, \quad \forall j \in S, \forall i \in S \quad (4)$$

(It states that when we serve stop j after stop I than service start of stop j should be equal to or greater than service start time of stop I plus the time taken between the two stops)

$$4. \quad a_i \leq x_i \leq b_i, \quad \forall i \in S \quad (5)$$

(It shows the constraint of the time window)

4.2 Hungarian Algorithm

Hungarian algorithm uses homogenous bus service and having fixed start time. In their approach they consider some constraints like the start service time and end service time of stops are already fixed and all of the trips can be served by any of the vehicles. So it can be considered as assignment problem. In their proposed work they considered that $cost_{ij}$ is the cost between nodes i and j, when stop i can be followed by stop j without violating the given start service time and end service time.

Mathematical Model:

$$MinZ = \sum_{i=1}^n \sum_{j=1}^n cost_{ij} y_{ij} \quad (6)$$

(Distance travelled multiplied by cost should be minimum)

And the constraints are:

$$1. \quad \sum_{i=1}^n y_{ij} = 1, \quad \forall j \quad (7)$$

$$2. \quad \sum_{j=1}^n y_{ij} = 1, \quad \forall i \quad (8)$$

Kim have proposed a six step algorithm to solve this problem and that algorithm is given below:

Step 1: Find the smallest value from each row and subtract it from all the elements of the same row.

Step 2: Find the smallest value from each column and subtract that smallest value from all the element of that column.

Step 3: Now lines are drawn from appropriate column and rows where the element value is zero but minimum number of lines are used.

Step 4: If it is find out that the number of the covering lines is equal to n than an optimal assignment of those zeros possible and our algorithm ends here.

Step 5: If it is find out that the number of the covering lines is not equal to n than an optimal assignment of those zeros is not possible and we have to proceed further.

Step 6: Now again find out the smallest value that is still not covered by any of the line. Now we are going to subtract this value from each of the uncovered row and are going to add it to covered row and return to step 3.

4.3 Branch and Bound Algorithm

In our proposed work we have used Branch and Bound approach on homogenous bus fleet. In our approach we have considered many constraints like time window, capacity of buses etc. Branch and Bound approach has following steps:

Step 1: First step is initialization step. In this step we are going to select the problem that we have to solve using this approach.

Step 2: Second step is known as branching step. In this step we are going to divide our problem into sub problems.

Step 3: This is step on which evaluation of sub problem takes place.

Step 4: It is called decision step. In this step we are going to see if our solution is infeasible or not.

Step 5: If our problem is infeasible then we are going to delete that sub problem.

Step 6: If our problem is feasible than we are going to update our bound and add that sub problem to queue and update it.

Step 7: This is the final step known as stopping step. If our solution is found then wer are going to stop.

Branch and Bound algorithm is very important in finding solution of complex problems by diving complex problems into small sub problems.

4.3 Solution Approach

Hungarian algorithm has some problem in case of time window it was not able to handle the case of upper bound. In earlier solution we saw that when start time of the stops are fixed and the link between S1, S2 and S2, S3 are feasible, then the link from S1 to S2 to S3 should be feasible but this may not hold when a time window is given. Let us see this from given below example:

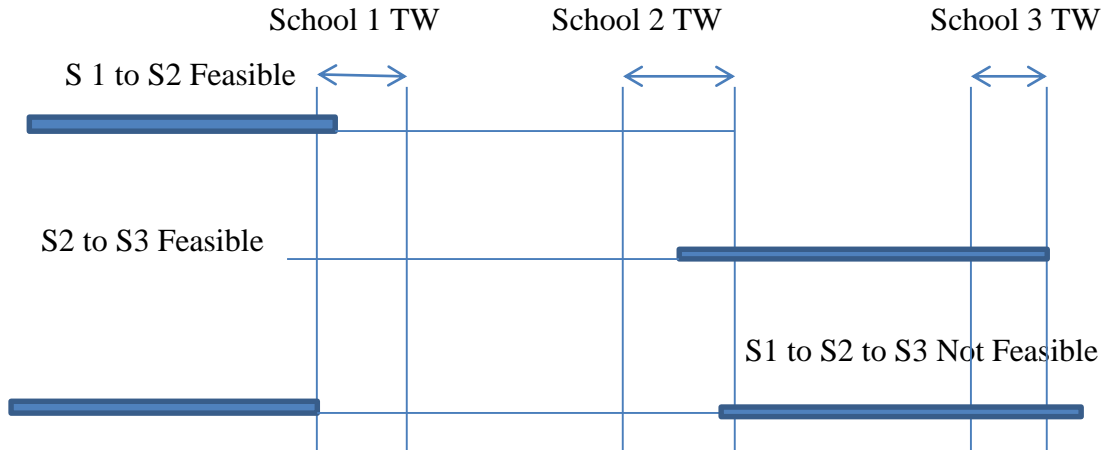


Fig 8: Problem when time window is given

Here vertical lines show the time window of the given schools. Here we can see that although trip from S1 to S2 and from S2 to S3 can be done by a single bus alone but trip from S1 to S2 to S3 cannot be done by that bus alone due to the violation of the time window of one of the school.

We can get an upper bound solution if we set the service start time to the latest possible time and use the assignment problem based approach. One possible solution for this is that the each trip is served to the latest time window. And in this if link from S1 to S2 is feasible and the link from S2 to S3 is feasible then the link from S1 to S2 to S3 is also guaranteed to be feasible. These assumptions that are made here are kind of relaxation [5]. Here vertical lines show the time window of the given schools. In this also the service start time of each of the trip is fixed. In the given below diagram we can see that trip from S1 to S2

and from S2 to S3 can be done by a single bus alone but trip from S1 to S2 to S3 can be done by that same bus. Here is the diagram showing that:

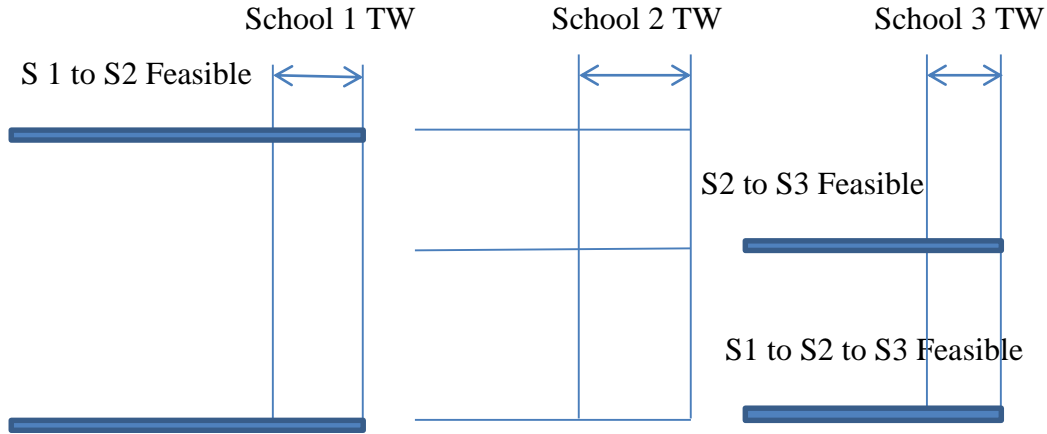


Fig 9: Upper bound case

Based on this we have given Branch and Bound algorithm. Here let us consider B_u and B_l Are the upper bound and lower bound respectively. Initially, both of these are calculated and saved. If we find out the solution of the upper bound is not optimal that is the value that comes out of the upper bound is not same as lower bound than the branching takes place. When the link from 1 to 2 is infeasible because of the time window than current node will be branched. In one of them the link between is left and in another branch the link is added. If the link is added than its travel time and stop time at stop 2 is also added. In this we are also going to adjust the time window. The process is continued until an optimal solution is found or the maximum computational time has reached. Detailed steps of this algorithm are as follows:

1. It is the initialization step:

First set the initial values of the lower bound and the upper bound. Then we are going to solve it using Hungarian Algorithm. If in this we find out that lower bound solution is feasible than we are going to stop the procedure because it is going to be the optimal solution.

Else set upper bound and lower bound to A_u A_l . $A_u = f(\mathbf{B}_u)$, $A_l = f(\mathbf{B}_l)$.

2. Stopping step:
 If our problem list is empty or the computation time has reached than A_u will be the best objective value.
 Else retrieve the B_l from the list and continue to step 3.
3. Branching:
 Here we are going to find an infeasible link from 1 to 2 from the lower bound assignment problem. Then we are going to branch it into two parts:
 First, that does not contain the link (CL).
 Second, that does not contain the link.
4. Bounding 1:
 Set prohibited assignment problems, lower and upper bound and solve them using the Hungarian algorithm. If the lower bound of this prohibited assignment problem is feasible to the original problem then
 If $B_u > f(\text{prohibited assignment problem lower bound})$, set
 $B_u = f(\text{prohibited assignment problem lower bound})$ Proceed to step 5
 Else
 If $B_u < f(\text{prohibited assignment problem lower bound})$, go to step 5.
 Else add that lower bound value to partial list and go to step 5.
5. Bounding 2:
 Set the values of Cl_u and Cl_l and then we are going to solve it using the Hungarian algorithm. If the lower bound of this combine assignment problem is feasible to the original problem then
 If $B_u > f(\text{combine assignment problem lower bound})$, set
 $B_u = f(\text{combine assignment problem lower bound})$ Proceed to step 6
 Else
 If $B_u < f(\text{combine assignment problem lower bound})$, go to step 6.
 Else add that lower bound value to partial list and go to step 6.
6. Bounding 3:
 Delete all of those nodes whose value of the lower bound is greater than that of the upper bound from the partial problem list. Now go to step 2.

5. Results and Discussion

In our proposed work we are using data taken from different schools and these school take students from different areas of a particular city. The result of the approach is found out by using the dataset. The dataset has the following parameters:

- Number of Schools
- Source of Bus
- Destination of Bus
- Distance Travelled
- Number of Stops
- Number of students

5.1 Evaluation Parameters

- Number of Buses Required:** The first parameter that we are going to find out is the total number of buses required in the overall journey. Our purpose is to find out a solution in which number of buses required are minimized, but without breaking the time window of any of the schools.
- Total Distance Covered:** The Second parameter to be evaluated is the total distance travelled. Our purpose is to find out a solution in which distance travelled is minimized without breaking the time window of any of the schools because as the travelled distance decreases cost of operating it also decreases.
- Time for evaluating Results:** The third parameter is to be evaluated is the time taken to evaluate the results. The time taken should not be more than an hour as it would not be optimal.

Input Data:

Out of these Schools School one has data given in Table 1. It shows number of students, number of buses required for school, distance travelled by those buses every day and number of trips required to serve these students.

Table 1: School one Data

Name of School	Number of trips	Number of students	No. of Buses Used	Distance Covered by Buses
School1	4	150	4	52

The given fig. 13 shows the details of the school 1 routes and distance of each route.

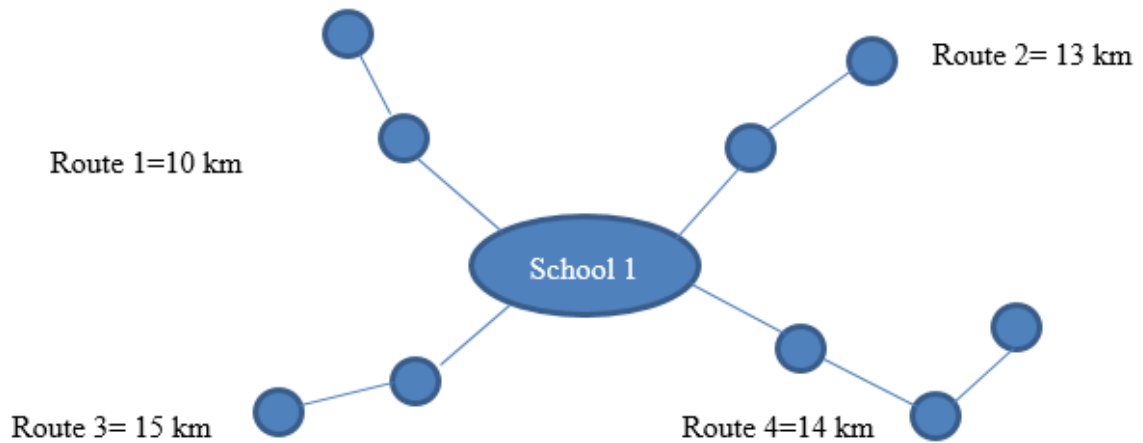


Fig 13: School 1 Route details

The input data for school number two is given in Table 2. It shows number of students, number of buses required by school, distance travelled by those buses every day and number of bus trips required to serve these students.

Table 2: School number two Data

Name of School	Number of trips	Number of students	No. of Buses Used	Distance Covered by Buses
School2	11	470	11	120

The given fig. 14 shows the details of the school 2 routes and distance of each route.

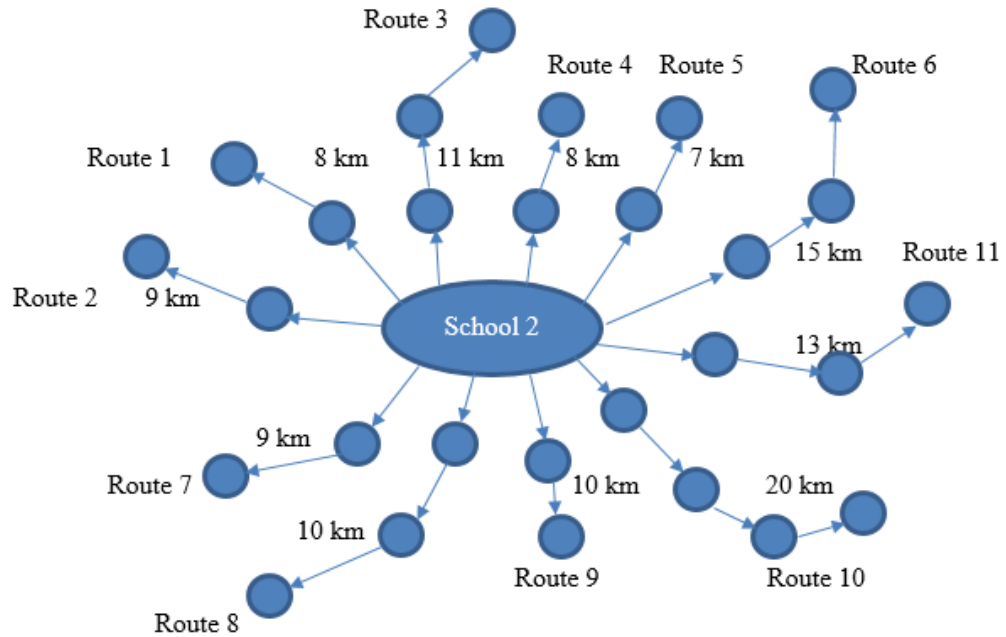


Fig 14: School 2 Route details

And in similar way data from different schools are taken and there combined result is seen before our algorithm implementation and after our algorithm implementation. Given below table 3 shows the input data set that is used in our approach.

Table 3: Experimental data for problem

Number of Schools	Number of trips	Number of students	Distance Travelled
1	4	150	52
2	15	620	172
3	20	1000	264
4	25	1200	336
10	65	2800	736

Fig 15 shows the result of our approach for school 1 and school 2 combined when we applied our approach on their data. From this we can see this that the number of route has been decreased from 15 to 11 and distance travelled also decreased from 172 to 133 km.

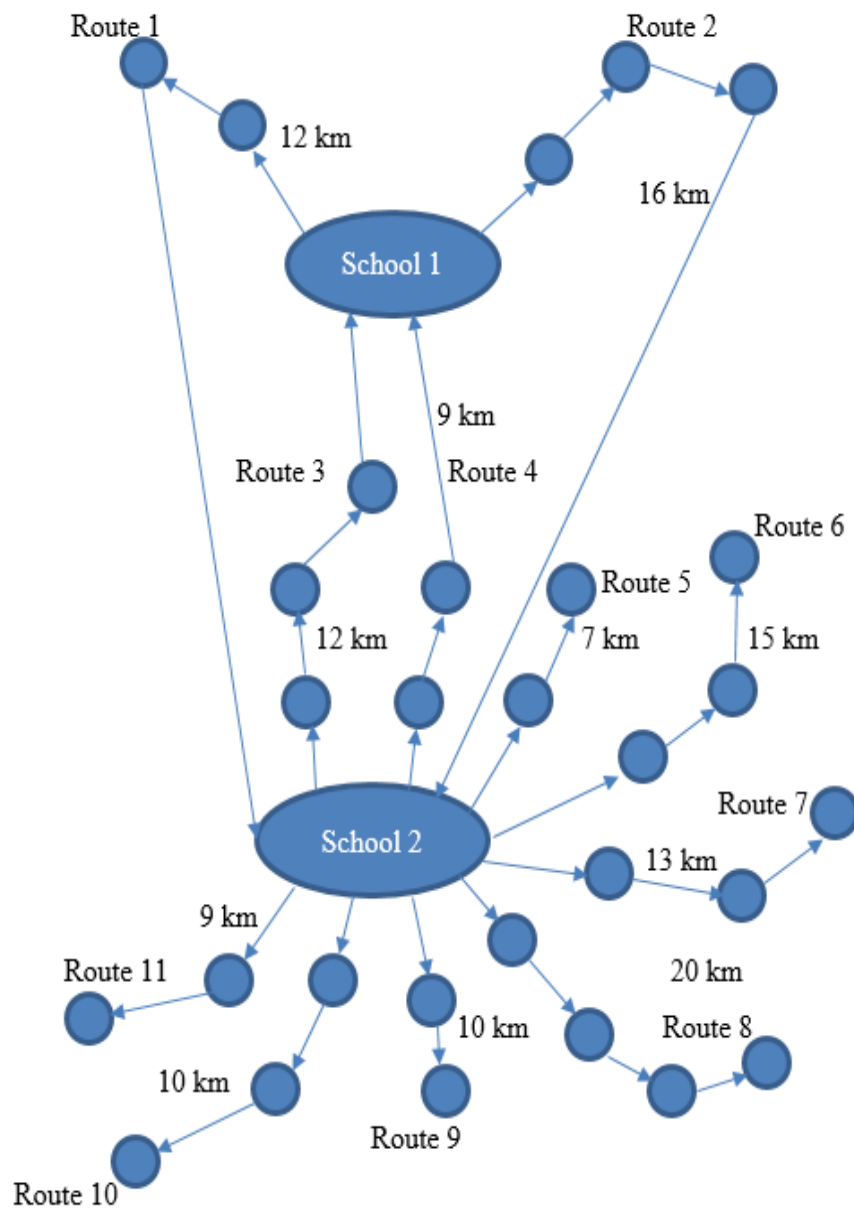


Fig 15: Solution using Branch and Bound Approach

5.2 Implementation of the Algorithm

The Branch and Bound approach was used to solve this problem. We have implemented C# to implement our algorithm and SQL is used as database for it. Our model was subjected to several constraints like time window and these constraints should not be violated. The model was computed on computer having 16 GB RAM and Windows 7 as operating system. This approach finds solution that is better than previous results used by the schools.

5.3 Simulation Result

The data given in table 5 is given as input in our model and we have applied our approach on it and got the results given in table 6.

Table 4: Result of Branch and Bound Approach

Assignment Problem using the Branch and Bound				
No. of Schools	No. of Bus	Distance Covered	Time in (ms)	Number of students
1	4	52	<1	150
2	11	133	<1	620
3	14	180	<1	1000
4	17	222	<1	1200
10	40	450	1 hour	5100

5.6 Result Analysis

We have applied our approach on the given set of data and we have got the simulation results. Here in this section we are going to analyze those results.

The table given below shows the percentage of saving in terms of the total distance travelled by the buses and we can see from this that the percentage of saving goes on increasing as number of schools goes on increasing

Table 5: Comparison Based on Distance Travelled

Number Of Schools	Distance Travelled(Before)	Distance Travelled (After implementation)	% of Saving
1	52	52	0
2	172	133	28.76
3	264	180	32.82
4	336	222	33.93
10	736	450	38.86

Figure 15 shows the Comparison of distance travelled before and after implementation of our algorithm with respect to number of schools.

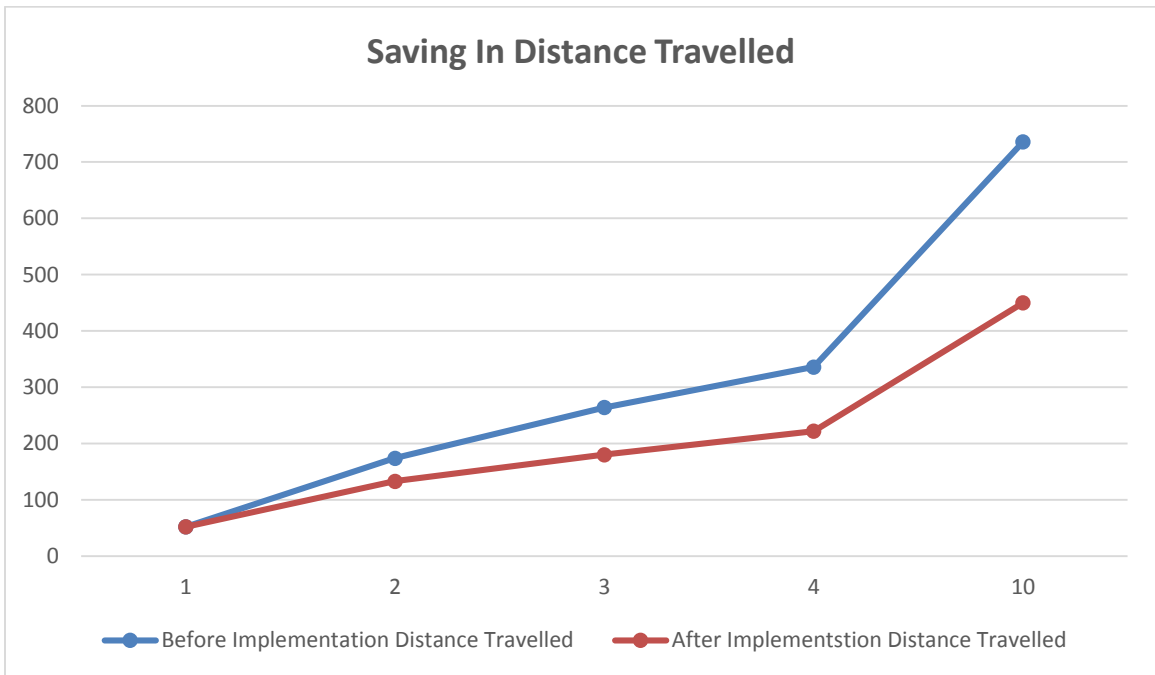


Fig 16: Number of Schools and Distance Travelled

The table 6 given below shows the percentage of saving in terms of the number of buses used by the schools and we can see from this that the percentage of saving goes on increasing as number of schools goes on increasing.

Table 6: Comparison Based on Number of Buses

Number Of Schools	Number of Buses (Before)	Number of Buses (After implementation)	% of Saving
1	4	4	0
2	15	11	33.33
3	20	14	35.00
4	25	17	40.00
10	65	40	44.67

Fig 16 shows the Comparison of distance travelled before and after implementation of our algorithm with respect to number of schools.

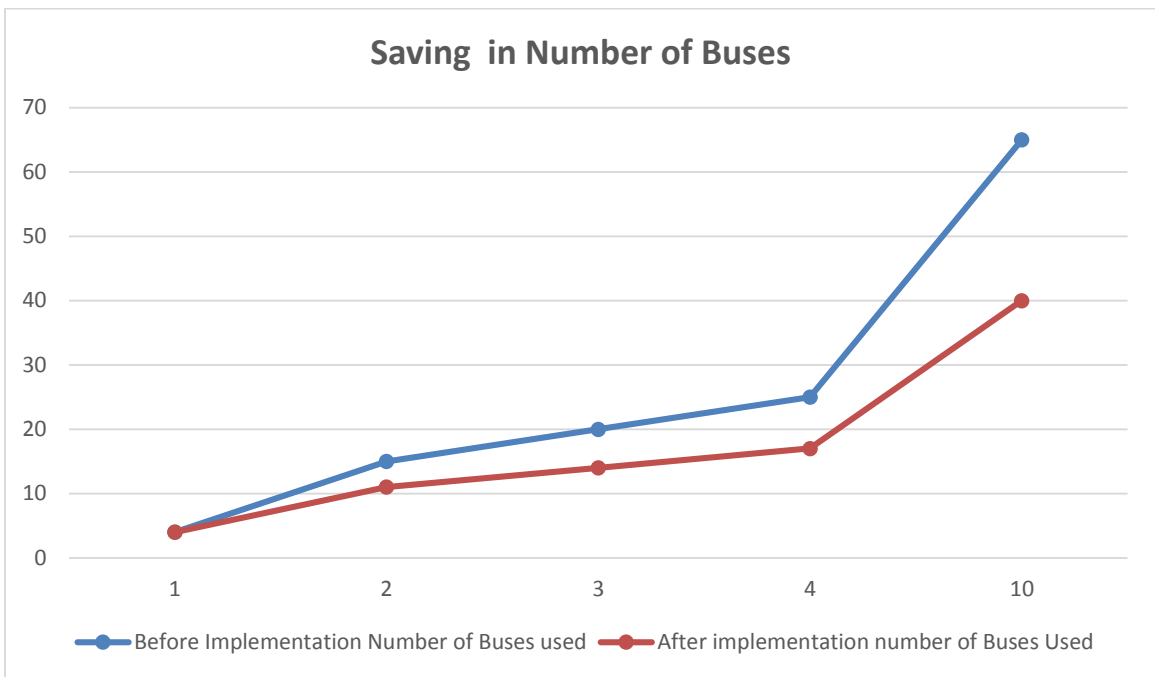


Fig 17: Number of Schools and Number of Buses

6. Conclusions and Future Scope

6.1 Conclusions

In this study the model for optimizing bus scheduling of real life school bus routing problem has been developed. With the help of Branch and Bound Approach we are able to find out the solution of our problem. We are able to find out the route and schedule the buses of schools. With this solution we can decrease the total number of bus required to service the schools. The solution shows that the total distance travelled is also decreased and thus the overall cost of operating the school buses is also decreased. This algorithm can find solution faster for smaller problems. Algorithm also help us to finalize route in such a way that overall travel time of each and every student is decreased significantly.

So overall it will help everyone from school management to school students. The congestion in traffic can also be decreased and thus the environmental pollution is also decreased. The result shown here are much better than the previous approaches like Hungarian Algorithm. The limitation of this approach is that above a certain amount of data the approach will take a long time which is not feasible.

6.2 Future Scope

1. In our approach we are focusing on Homogenous Bus Fleet and in future we can focus on Heterogeneous Bus fleet.
2. The result found by this approach a certain amount of data are not feasible so we can apply heuristic approach to this solution to get the feasible result even for the large amount of data.
3. Here we are considering that all the buses have same capacity and all the buses are of same time so in future we can work on the approach which is applied on the buses having different capacity.

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List of Publications

Yash Kumar and Dr. Sushma Jain, "School Bus Routing Based on Branch and Bound Approach", IEEE International Conference on Computer, Communication and Control, MGI Indore, INDIA.2015, [Accepted]

Video Presentation

Video Presentation Link-----><https://www.youtube.com/watch?v=0tddTNaLSao>.