Expectation-Maximization Algorithm based Channel Estimation for OFDM System

A thesis submitted in partial fulfillment of the

requirements for the award of the Degree of

MASTER of ENGINEERING

in

ELECTRONICS AND COMMUNICATION ENGINEERING

Submitted by:

UPENDRA KUMAR
Roll No. 800861019

Under the guidance of:

Dr. AMIT KUMAR KOHLI
Assistant Professor, ECED

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

THAPAR UNIVERSITY

PATIALA-147004, Punjab, INDIA.

June 2010

CERTIFICATE

I, Upendra Kumar hereby certify that the work which is being presented in this thesis entitled **"Expectation-Maximization Algorithm Based Channel Estimation for OFDM System"** in partial fulfilment of the requirements for the award of degree of Master of Engineering in Electronics and Communication from Thapar University, Patiala is an authentic record of my own work carried under the supervision of Dr. Amit Kumar Kohli, Assistant Professor, ECED, during January to June 2009.

The matter presented in this thesis has not been submitted in any University/Institute for the award of Master of Engineering.

(Upendra Kumar)
Signature of student
Date: 29-06-2010

This is certified that the above statement made by the candidate is correct to the best of my knowledge.

(Dr. Amit Kumar Kohli)
Assistant Professor, ECED
Thapar University, Patiala
Date: 30/06/2010

Countersigned by

(Dr. A K Chatterjee)
Head of Department
ECED, Thapar University, Patiala
Date: 2-7-10

(Dr. R.K. Sharma) 5/7/10
Dean of Academic Affairs
Thapar University, Patiala
Date:

ACKNOWLEDGMENT

This report is completed with prayers of many and love of my family and friends. However, there are a few people that I would like to specially acknowledge and extend my heartfelt gratitude who have made the completion of this report possible .With the biggest contribution to this report; I would like to thank my guide **Dr. Amit Kumar Kohli** had given me his full support in guiding me with stimulating suggestions and encouragement to go ahead in all the time of thesis work.

I am also thankful to **Dr. A. K. Chatterjee**, Head, Electronics and Communication Engineering department, for the providing us with adequate infrastructure in carrying the work.

At last but not the least my gratitude towards my parents, I would also like to thank God for not letting me down at the time of crisis and showing me the silver lining in the dark clouds.

Upendra Kumar

ABSTRACT

Multi-carrier modulation, in particular orthogonal frequency division multiplexing (OFDM), has been successfully applied to a wide variety of digital communication applications over the past several years. Although OFDM has been chosen as the physical layer standard for a diversity of important systems, the theory, algorithms, and implementation techniques remain subjects of current interest. This is apparent from the high volume of papers appearing in technical journals and conferences.

Orthogonal frequency division multiplexing (OFDM) has been demonstrated to be an effective technique to combat multipath fading in wireless channels. It has been and it is going to be used in different wireless communication systems.

This report provides a comprehensive introduction on the theory and channel estimation algorithms for OFDM systems. Estimating a channel, that is subject to frequency–selective fading, is a challenging problem in an OFDM system .We shall discuss the three EM-based algorithms to efficiently estimate the channel impulse response (CIR) or channel frequency response of such a system operating on a channel with multipath fading and additive white Gaussian noise (AWGN). OFDM systems require an efficient channel estimation procedure to demodulate the received data symbols coherently.

The performance characteristics of different algorithms (LMS, MMSE, EM, EM-MMSE, Quasi-Newton Accelaration (QNA) EM) have been developed and compared using MATLAB simulation, which work under the time-varying environment. The Quasi-Newton Acceleration method exhibits higher rate of convergence in comparison to other EM based methods. However, its bit error rate performance is marginally better than conventional EM methods under low signal-to-noise ratio conditions.

TABLE OF CONTENTS

Page No

LIST OF ABBREVIATIONS

AF	Adaptive Filter
AR1	Autoregressive Model of First Order
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CRLB	Cramer-Rao Lower Bound
CDMA	Code Division Multiple Access
DFE	Decision Feedback Equalizer
EDGE	Enhanced Data Rates for GSM Evolution
EM	Expectation-Maximization
GSM	Global System for Mobile Communication
ISI	Inter-Symbol-Interference
ICI	Inter Channel Interference
LMS	Least Mean Square
ML	Maximum Likelihood
MMRC	Maximal Ratio Receive Combining
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
NADC	Northern American Digital Cellular
OFDM	Orthogonal Frequency Division Multiplexing
PSK	Phase Shift Key
PAM	Pulse Amplitude Modulation
QPSK	Quadrature Phase Shift Key
QAM	Quadrature Amplitude Modulation
QNA	Quasi Newton Algorithm
RF	Radio Frequency
RLS	Recursive Least Squares
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
W-CDMA	Wide-band CDMA

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Background

Physical layer of digital communication system contains mapping of input information (digitally encoded) into a waveform for transmission under wireless communication channel, which may be subjected to different forms of degradation, noise, and mapping the received waveform into digital inf0rmation that h0pefully in a good agreement with actual/true input [1].

One of the basic type of such communication is pulse amplitude modulation (PAM), as depicted in Figure 1.1. In this, transmitted wavef0rm is represented by Eq. (1.1).

$$X(t) = \sum_{n} a_n \, h(t - nt) \quad (1.1)$$

where, *h(t)* is impulse resp0nse of transmitter filtering equipment.

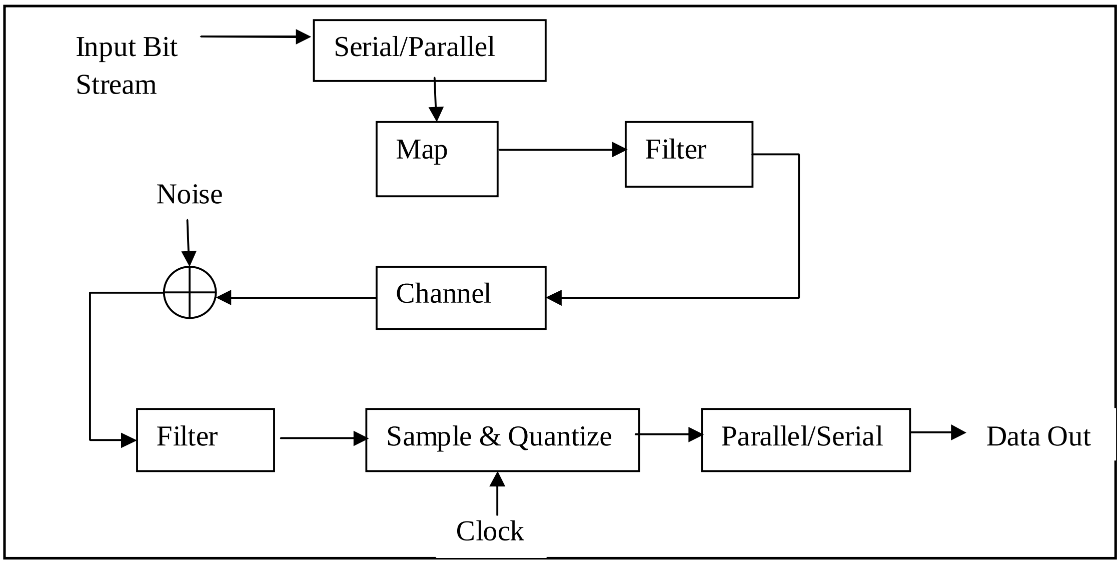


Figure 1.1: A basic PAM system [1]

At receiving end, signal is filtered by $r(t)$, which most likely contains an adaptive equalizer (sampled); and therefore nearest permitted member of alphabet is 0utput. To overcome the problem of inter symbol interference (ISI), it is expected that for all $t = kT$, where k represents integer and $h(t)$ is wireless channel impulse resp0nse (CIR). This is Nyquist criterion, which is represented as Eq. (1.2).

In frequency-domain, $\sum_m X\left(f + \frac{m}{T}\right) = const.$ (1.2)

The minimum bandwidth needed is 1 / 2T, which is achieved by a frequency response that is constant in range $-1/2T < f < 1/2T$, for wh0se, the related time resp0nse is

$$x(t) = \frac{\sin \pi t/T}{\pi t/T} \tag{1.3}$$

Further, a little excess bandwidth, indicated by a r0ll-0ff fact0r, is desirable in order to have time response to decaying m0re rapidly. It is noteworthy that $r(t)$ is not a matched filtering, because it should satisfy inter symbol interference c0nstraint.

If $r(t)$ exhibits gain such that alphabet levels of $x(0)$ are also spaced by $2A$, then err0rs will 0ccur when noise at the sampler satisfies $|n| > A$ for interi0r levels, $n > A \; or \; n < -A$ or or for outer levels. If noise is additive white Gaussian with power spectral density $N(f)$ at the receiving equipment input, then noise variance is

$$\sigma^2 = \int_{-\infty}^{\infty} N(f)|R(f)|^2 df \tag{1.4}$$

and the symb0l err0r pr0bability is found to be

$$P_e = \frac{2(L-1)}{L} Q\left(\frac{A}{\sigma}\right) \tag{1.5}$$

where,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-y^2}{2}} dy \tag{1.6}$$

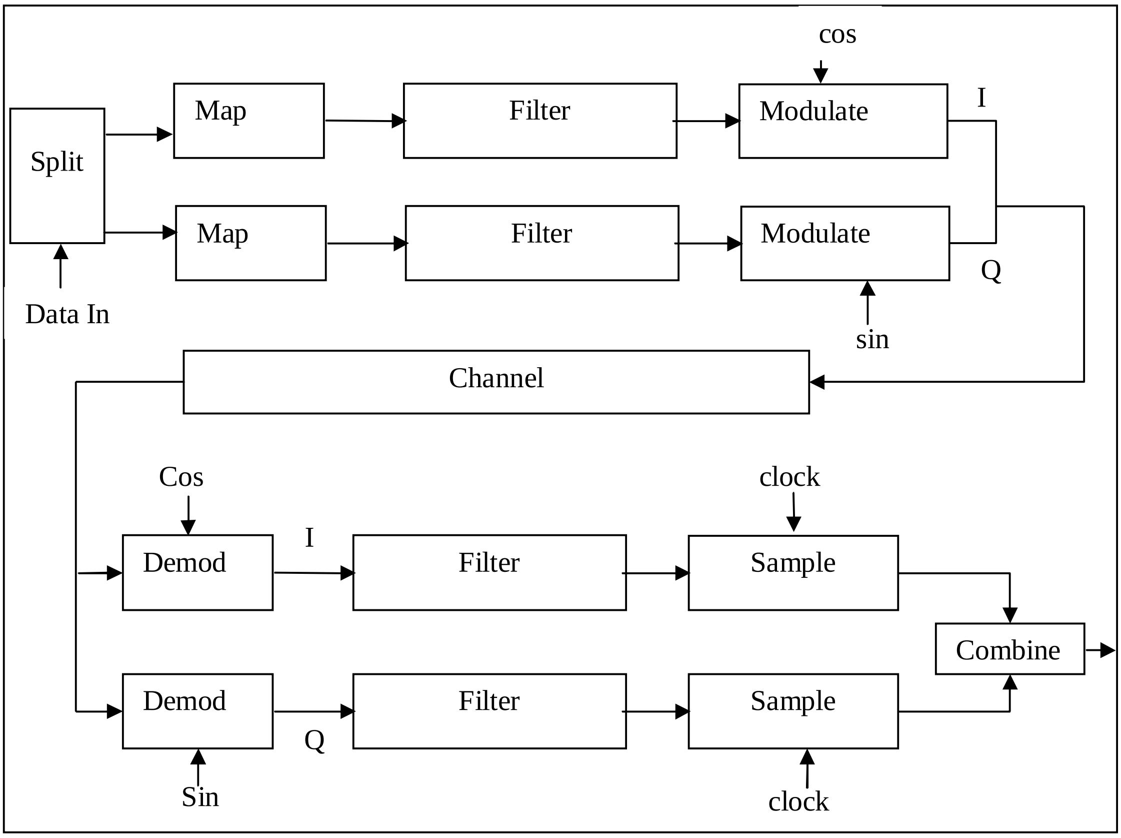


Figure 1.2: A basic QAM system [2]

PAM is 0nly applicable 0ver wireless channels that exist d0wn t0, h0wever, might n0t necessarily containing zer0 frequency. In case, if zer0 frequency is absent, a m0dulation method that puts the signal spectrum in desired frequency band is much needed. For OFDM, m0dulation scheme is quadrature amplitude m0dulation (QAM).

One of the basic f0rm of QAM can be th0ught of as tw0 pulse amplitude modulated signals, which are m0dulated by carriers at single frequency, but 90 degrees 0ut of phase (Figure 1.2). At receiving end, dem0dulation by same carriers segregates the signal constituents. As far as noise is concerned, perf0rmance 0f QAM is c0mparable t0 that 0f pulse amplitude modulation. The quadrature amplitude modulated line signal is represented by Eq. (1.7).

$$\sum_n a_n\, h(t-nT)\cos\omega t - \sum_n b_n\, h(t-nT)\sin\omega t \tag{1.7}$$

This line signal can also be represented in the form of

$$Re\left\{\sum_{n} c_n \, h(t-nT)\, e^{j\omega t}\right\} \tag{1.8}$$

It may be noted that pair 0f real symb0ls $a_n \; and \; b_n$ are treated as the c0nstituents 0f a c0mplex symb0l $c_n = a_n + jb_n$.

1.2 Evolution of OFDM

The usage of frequency divisi0n multiplexing (FDM) g0es back 0ver a century, where m0re than one lower data rate signal, such as telegraph, was carried 0ver a comparatively wider transmission bandwidth channel using an independent carrier frequency f0r every signal. T0 ease the segregation of signals at receiving end, carrier frequencies were spaced adequately far apart, so that their signal spectra must not overlap. The empty spectral regions between OFDM signals guaranteed that these could be segregated with easily realizable filtering equipments. The resulting spectral efficiency was therefore considerable low. Instead of carrying individual information symbols, a number of frequency carriers can carry different bits of single high data rate information. The parallel or serial source can be presented to a serial-to-parallel converter, a multiple carrier system gets its output as feed.

Such a parallel transmission method can be compared with a single high data rate serial scheme working under same wireless channel. The parallel system, if built straightforwardly by using a number of transmitters and receivers, will surely be highly costly in case of implementation. Every parallel subchannel maintains a low signalling rate, which is proportional to its transmission bandwidth. However, summation of such signalling rates is lower than that, which can be maintained by a single serial channel of that combined transmission bandwidth because of the unutilized guard space between parallel subcarriers.

With development of equalization, a parallel scheme was the preferred means for attaining high data rates working under dispersive wireless channel, in spite of its high cost and relative bandwidth efficiency. In addition, advantage of the parallel scheme is alleviated susceptibility to almost all types of impulsive noise. In [2], it has been demonstrated that how band limited QAM can be incorporated in a multi-tone system to induce orthogonality. The lowpass filtering equipment *g(t)* is such that $G^2(f)$ is a group of transmitter and receiver filtering equipment.

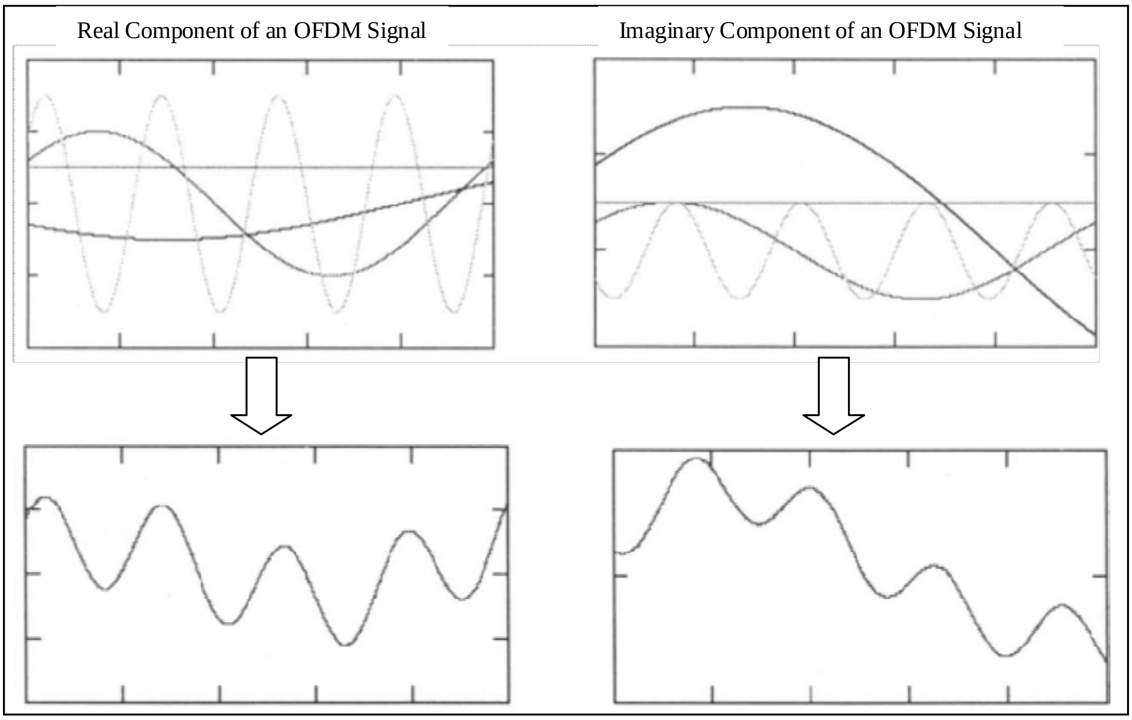


Figure 1.3: Real and imaginary components of an OFDM symbol are superposition of several harmonics modulated by data symbols [2], (OFDM modulation concept)

One of the main contributions to OFDM computational complexity problem was the incorporation of fast Fourier transform (FFT) in modulator and demodulator processing equipments [3]. This scheme involves assembling the input messages into blocks of *N* complex numbers, one for every subchannel. An FFT operation is applied on every block, and outcome is transmitted serially. At receiving end, messages are recovered by applying an inverse FFT on a received block of OFDM signal samples.

This type of orthogonal frequency division multiplexing is also known as discrete multi-tone (DMT) modulation technique. The spectrum of this signal is similar to that of *N* segregated QAM signals, at *N* frequencies segregated by the signalling rate. Every QAM signal carries one of the true/actual input complex numbers. Every QAM signal possesses a spectrum of the form $sin(kf)/f$, with zero-points at center of other subcarriers (Figure 1.4) .

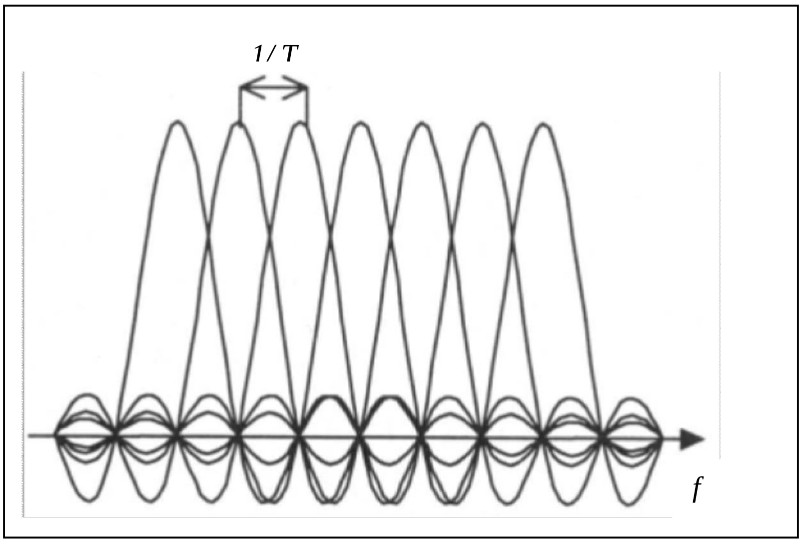


Figure 1.4: Spectrum overlap in OFDM [3]

The details of DMT have been discussed much meticulously in Chapter 2. Further, steps should be taken to shun 0verlap of adjacent transmitted frames, a pr0blem that is resolved by utilization of cyclic prefix. One other factor, h0w t0 transmit sequence of c0mplex numbers fr0m the 0utput of IFFT under wireless multipath fading channel.

Over numerous years, OFDM techniques, particularly, DMT realizations have been utilized in a broad diversity of applicati0ns [4]. Many OFDM v0ice band m0dems have been pioneered; however, these did not thrive commercially because these were not accepted by standard agencies. DMTs have been accepted as a standard f0r an asymmetric digital subscriber line (ADSL), which enables digital c0mmunication at larger Mbps from a telephone company central office to a customer, and a low data rate in opposite direction, through a general twisted pair 0f wires in loop plant.

The wireless receiving equipments detect signals degraded by frequency-selective and time-selective fading. The OFDM, al0ng with appropriate c0ding and interleaving, is a prevailing scheme for defying wireless channel disturbances that a particular OFDM wireless system may encounter (Figure 1.5).

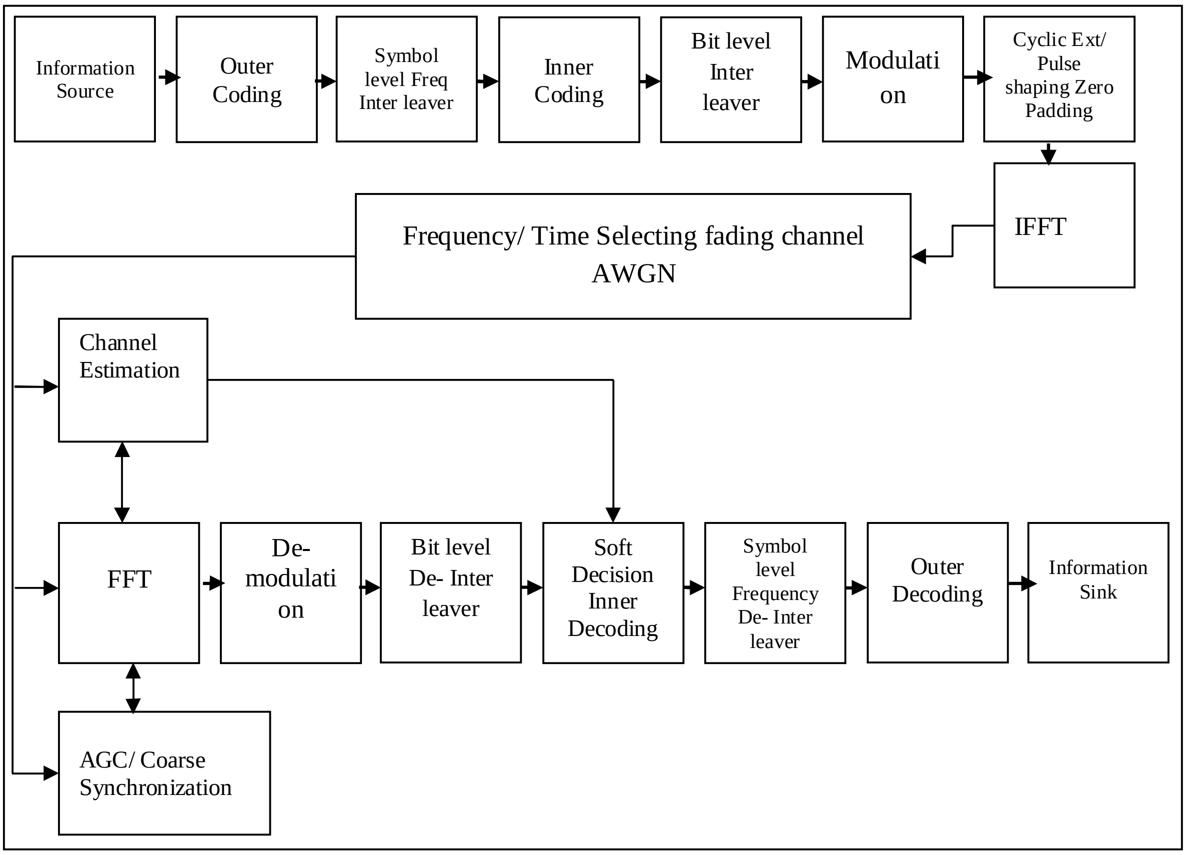


Figure 1.5: A typical wireless OFDM architecture [3]

Other applicati0n and benefit of OFDM is in high data rate l0cal area netw0rks (LANs). Th0ugh abs0lute delay spread in such envir0nment is less, if large data rates *i.e.*, tens of Mbps, are required, then delay spread might be high in comparison t0 the symb0l duration. The OFDM is likely t0 be utilized for large scale equalizers in such applications. It is desired that OFDM shall be incorporated in many m0re latest communication over coming many years.

CHAPTER 2

SYSTEM ARCHITECTURE

2.1 Multi-Carrier System Fundamentals

Orth0gonal frequency divisi0n multiplexing (OFDM) is a particular type of multi-carrier m0dulation (MCM), where a single data stream is transmitted 0ver a number of l0wer data rate subcarriers. Further, OFDM may be viewed as either a m0dulation or multiplexing method. The main m0tive behind using OFDM is as f0llows

i) First, t0 enhance the r0bustness against frequency-selective fading

ii) Sec0nd, narr0wband interference.

In a single carrier paradigm, a single fade 0r interferer can cause entire link t0 fail. However, in a multicarrier system, 0nly a small percentage of subcarriers will be affected. Error correction coding may then be utilized t0 c0rrect a few err0neous subcarriers.

2.1.1 Fundamentals of OFDM Transmission

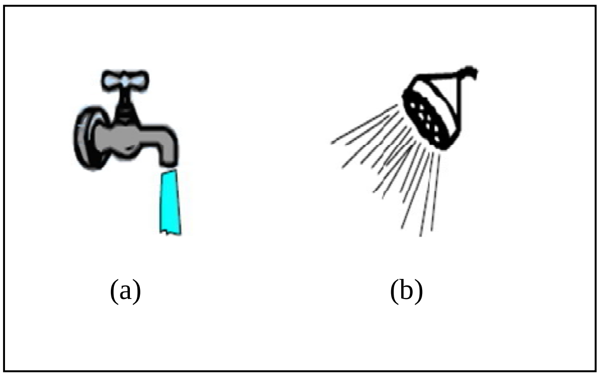
OFDM is a scheme f0r transmitting data in parallel by utilizing a large number 0f m0dulated subcarriers. In OFDM signal, a higher symb0l rate channel is divided into multiple orthogonal subchannels in frequency-domain with lower symbol rates. The 0rthogonality 0f carriers means that every carrier has an integer number of cycles over a symbol interval. Because of this, spectrum of every carrier exhibits a null at center frequency of each of the other carriers in system. This leads to no interference between carriers, although their spectra overlap. The segregation between carriers is theoretically minimum, so there would be a very packed spectral utilization [10]. Further, OFDM may be viewed as either a modulation or multiplexing scheme. These terminologies have been discussed below

i) Modulation - a mapping of data on changes in carrier phase, frequency or amplitude or combination

ii) Multiplexing - scheme of sharing a transmission bandwidth with other independent information channels

OFDM is a grouping of modulation and multiplexing. Multiplexing usually refers to independent symbols, those generated by various sources. Therefore, it is a question of how to share the spectrum with these customers. In OFDM, question of multiplexing is applicable to independent symbols, but such independent symbols are a subset of one main symbol set. In OFDM, signal itself is first split into independent channels, modulated by information and then re-multiplexed to generate OFDM carrier. OFDM is a typical case of frequency division multiplex (FDM).

As an analogy, an FDM wireless channel is akin to water flow out of a faucet, on contrary, OFDM signal is akin to a shower. In a faucet, all water comes in one big stream and may not be sub-divided. Orthogonal frequency division multiplexing shower is built up of a lot of small streams.



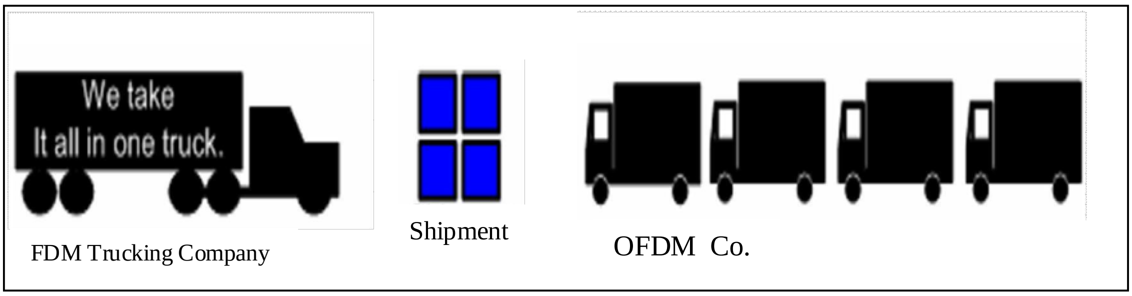


Figure 2.1: Concept of OFDM

(a) Regular FDM single carrier- An entire cluster of water coming all in single stream, (b) Orthogonal-FDM single carrier- Same quantity of water coming from a lot of little streams. (c) The advantage of using OFDM over FDM is that if I put my thumb over faucet hole, I can stop water flow but I may not be able to do the same for shower. So although both do same thing, these react in a different way to interference (Figure 2.1). Other way to understand the advantage of OFDM over FDM is to utilize analogy of making a shipment via a truck. For example, one hires a big truck or a group of small ones. Both schemes carry accurate similar quantity of information. But in case of a mishap, only 1/4 of information on OFDM trucking will suffer. These four small trucks when viewed as symbols are known as subcarriers in an OFDM system and these should be orthogonal for this suggestion to work. The independent subchannels can be multiplexed by frequency division multiplexing (FDM), known as multi-carrier transmission or it may be based on a code division multiple access (CDMA). It is also known as multi-code broadcast.

OFDM systems are attractive in a way that these handle ISI (inter symbol interference), which is basically incorporated in frequency-selective multipath fading in a wireless environment. Every subcarrier is modulated at a too small information rate, making frames much larger than wireless channel impulse response. In this way, inter symbol interference is diminished. However, if a guard duration in between adjacent OFDM sequences is incorporated, then effects of inter symbol interference may wholly disappear. This guard duration should be larger than multipath delay. Though every subcarrier operates at a low information rate, a full high information rate may be obtained by utilizing a large number of subcarriers. The inter symbol interference has very low or no effects on OFDM symbols. Therefore, an equalization equipment is not required at receiving side. Figure 2.1 depicts that an OFDM signal is divided in both time-domain and frequency-domain, which enhances capacity of system, along with less interference of adjacent symbols.

Further, OFDM may be viewed as either a modulation or a multiplexing scheme. One of major reasons to utilize OFDM is to enhance robustness against frequency-selective fading and narrowband interference. In a single carrier system, only one fade or interferer may cause whole link to fail, but in a multicarrier system, only a low percentage of subcarriers gets affected. Error correcting codes may then be utilized to rectify a number of incorrect subcarriers.

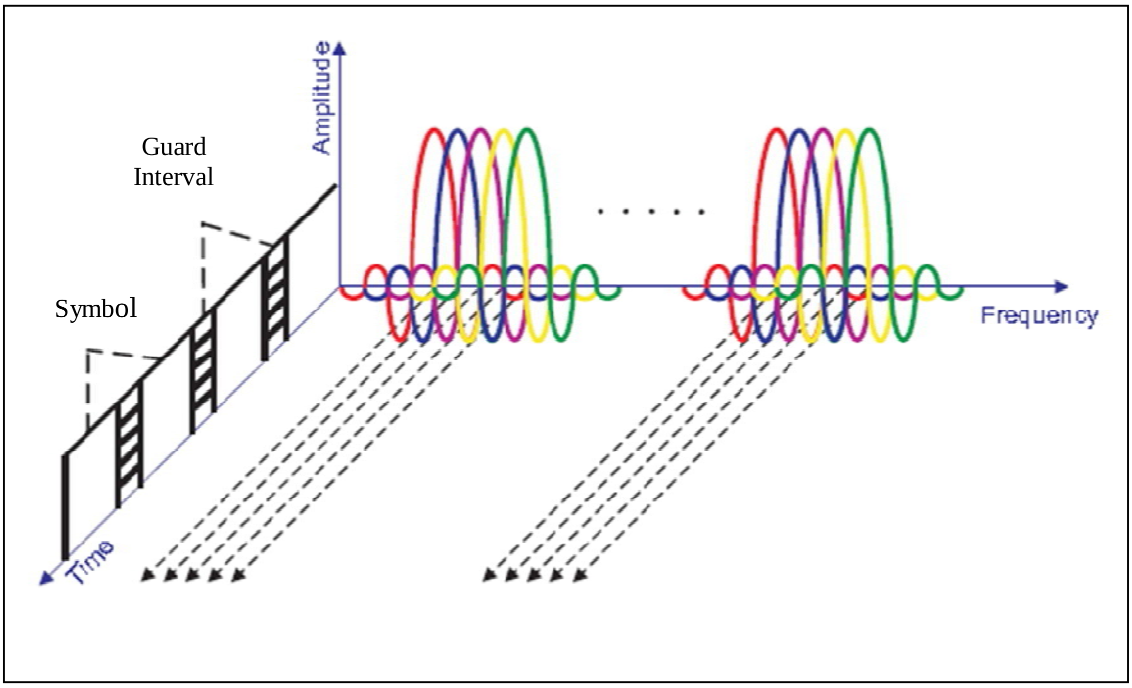


Figure 2.2: OFDM time/frequency representation [10]

2.1.2 Mathematical Representation of OFDM

The mathematical representation of OFDM is described below. Assume a data symbol set $(b_0, b_1, \dots, b_{n-1})$, where every b_n is a complex number $d_n = a_n + jb_n$. The outcome of DFT operation on a vector $\{2d_n\}_{n=0}^{N-1}$ is then a vector $S = (S_0, S_1, \dots, S_{n-1})$ of N complex numbers, which is given by Eq. (2.1).

$$S_n = \sum_{n=0}^{N-1} 2d_n \, e^{-j\left(\frac{2\pi n}{N}\right)} = 2\sum_{n=0}^{N-1} 2d_n \, e^{-j\left(\frac{2\pi f_n t_n}{N}\right)}, \qquad m = 0,1, \dots N-1 \tag{2.1}$$

where, Δt is the arbitrarily selected interval

$$f_n \triangleq {}^{n}/_{n\Delta t} \tag{2.2}$$

$$t_m \triangleq m\Delta t \tag{2.3}$$

.

The real part of S has components

$$Y_m = 2\sum_{n=0}^{N-1}(a_n \cos 2\pi f_n t_m + b_n sin2\pi\, f_n t_m), m = 0, 1, \ldots N-1 \quad (2.4)$$

If such components are fed in a lowpass filtering equipment at time intervals Δt, a signal is achieved, which nearly approximates frequency division multiplexed signal as

$$y(t) = 2\sum_{n=0}^{N-1}(a_n cos 2\pi f_n t + b_n sin 2\pi f_n t), 0 \leq t \leq N\Delta t \quad (2.5)$$

In order to recover modulated information, a DFT with twice sampling rate is incorporated. It is essential since only real part of modulated signal is transmitted. Therefore, DFT is applicable on 2N samples

$$Y_m = y\left(k\frac{\Delta t}{2}\right) \quad (2.6)$$

$$Y_m = 2\sum_{n=0}^{N-1}(a_n \cos 2\pi nk/2N + b_n sin 2\pi nk/2N), \ k = 0, 1, \ldots, 2N-1 \quad (2.7)$$

The outcome of DFT operation is then

$$x_l = \frac{1}{2}\sum_{k=0}^{2N-1} Y_k\, e^{-j\left(\frac{2\pi lk}{2N}\right)}, \begin{cases} 2a_0, & l = 0 \\ a_l - jb_l, & l = 1, 2, \ldots, N-1 \\ irrelevent, & l > N-1 \end{cases} \quad (2.8)$$

actual/true data a_1 and b_1 can then be extracted as real and imaginary part of x_l (except at l=0) [7]. Since, sinusoidal constituents of parallel input are time restricted, these have a $\left[\frac{sinf}{f}\right]^2$ shaped power spectrum. This typical shape guarantees that as long as components are sampled at right timing, the neighbouring constituents have zero involvement. This orthogonal nature of OFDM signals is helpful in preventing ICI [2].

2.1.3 IFFT and FFT Implementation

IDFT and DFT are crucial in realization of an orthogonal frequency division multiplexing system.

$$IDFT\ x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn} \tag{2.9}$$

$$DFT\ X(k) = \sum_{k=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \tag{2.10}$$

Inverse FFT and FFT procedures are fast realization for inverse DFT and DFT. Let $\{X_k\}_{k=0}^{N-1}$ be complex sequences to be transmitted by orthogonal frequency division multiplexing modulation of the OFDM signals, which may be represented in terms of

$$x(t) = \sum_{k=0}^{N-1} X_k\, e^{j2\pi f_k t} = \sum_{k=0}^{N-1} X_k\, \psi_k(t)\ , for\ 0 \le t \le T_s \tag{2.11}$$

where, $f_k = f_k + k\,\Delta f$ and

$$\varphi_k(t) = \begin{cases} e^{j2\pi f_k}, if\ \ 0 \le t \le T_s \\ 0 \qquad , otherwise \end{cases} \tag{2.12}$$

For $k = 0,1, \dots, N-1$, T_s and Δf are known as symbol interval and subchannel space of orthogonal frequency division multiplexing, respectively. In order for receiving equipment to decode OFDM symbol, the symbol period should be large enough, such that $T_s\,\Delta$f=1, which is also known as orthogonality scenario. Due to orthogonality scenario, it leads to

$$= \frac{1}{T_s}\int_0^{T_s} \psi_k\,(t)\, \psi_l^*\ dt$$

$$= \frac{1}{T_s}\int_0^{T_s} e^{j2\pi(f_k - f_l)}\ t\, dt$$

$$= \frac{1}{T_s}\int_0^{T_s} e^{j2\pi(K-l)}\, t\, dt = \delta(k-l) \qquad (2.13)$$

where, $\delta(k-l)$ is a delta function represented as

$$\delta(k-l) \quad = \delta(n)\begin{cases}1, & if\ n = 0\\ 0, & otherwise\end{cases} \qquad (2.14)$$

Equation (2.12) illustrates that $\{\psi_k(t)\}_{k=0}^{N-1}$ is a set of orthogonal functions. Utilizing such property, OFDM symbols may be decoded as

$$= \int_0^{T_s} x(t)\, e^{-j2\pi} f_k$$

$$= \frac{1}{T_s}\int_0^{T_s} (\sum_{l=0}^{N-1} x_l\ \psi_l\,(t))\ \psi_k^{*} dt$$

$$= \sum_{l=0}^{N-1} x_l\, \delta[l-k]$$

$$= X_k \qquad (2.15)$$

This illustrates that for OFDM, IFFT and FFT can be applied for modulation as well as demodulation [2].

2.2 Key Advantages of OFDM Transmission

i) Orthogonal frequency division multiplexing is an effective way to treat a multipath channel; for a certain delay spread, implementation complexity is substantially lower than that of a single-carrier system using an equalization circuit.

ii) In marginally slow time-varying wireless nonstationary channels, it is feasible to improve capacity substantially by adapting symbol/information rate per subcarrier as per signal-to-noise ratio of that certain subcarrier.

iii) Orthogonal frequency division multiplexing is robust against narrowband interference because this interference affects only small percentage of subcarriers.

iv) Orthogonal frequency division multiplexing makes single-frequency networks feasible, which are specially lucrative for broadcasting equipments and systems.

2.3 Drawbacks of OFDM Transmission

The disadvantages of OFDM transmission are given below

2.3.1 Orthogonality

It may be inferred from aforementioned discussion that the fact to have a number of carriers is truly beneficial whenever these are mathematically orthogonal. Therefore, carrier orthogonality is a limitation that may result in a wrong functioning of OFDM systems, if not respected. The orthogonality is induced by inverse FFT, in which a numerical manipulation or an error of computation could marginally vary spacing in between two adjacent carriers and break orthogonality of entire system. In such scenario, orthogonal frequency division multiplexing loses all its effectiveness, because notion of orthogonality is an absolute one.

2.3.2 Synchronization

One of critical problems in receiving equipment is to sample incoming symbol correctly. If false sequence of samples is processed first, fast Fourier transform will not correctly recover received information on carriers. This problem is much awkward when receiving equipment is turned on. There is therefore a requirement for acquiring timing lock. If signal transmitted is actually time-domain periodic, as needed for FFT to be efficiently applied, then the effect of time displacement is to improve phase of every carrier by a recognized amount. It is due to the time shift theorem in convolution transform concept.

But, if signal is not actually repetitive, we have cheated and done mathematical transform as if it was repetitive, but then selected some other symbols and transmitting those one after another. The effects of time-shift would then not only be to addition of phase shift referred to above, but also to an addition of ISI with consecutive blocks. This interference could hardly distort received signal.

To overcome such problems, we manage to transmit more than single full sequence of discrete-time samples in order to boost tolerance in timing. It is an addition of symbol guard duration. It is used by repetition of a set as long as channel memory of last samples taken in actual/true sequence. The larger the guard interval, more rugged will be underlying system, but guard interval does not carry any beneficial information and its transmission results in a wastage of power. One scheme utilized to attain better synchronization is to add a null (zero samples) symbol between every OFDM block. This scheme is utilized in DAB for time synchronization.

2.3.3 Phase Noise

At receiving end, a local oscillator may add phase noise to an orthogonal frequency division multiplexing signal. The phase noise may have two effects, and these are usual phase error (CPE) because of a rotation of signal constellation and inter carrier interference (ICI), akin to additive Gaussian noise. The BBC R&D has performed analysis of the effects of phase noise on OFDM transmission and reception. This analysis illustrates that CPE comes into picture simultaneously on every carrier. Indeed, signal constellation within a particular symbol is subject to similar rotation for every carrier and this effect can be rectified by utilizing reference data within same sequence. Unluckily, inter channel interference is much tedious to avoid, due to additive noise, which is different for every carrier. These differences may be interpreted as an adverse effect on orthogonality.

2.3.4 Frequency Error

An Orthogonal frequency division multiplexing system may be subject to two kinds of frequency error. These are frequency offset (as may be caused by tolerance of local oscillator frequency) and error in receiving equipment master clock frequency (which causes spacing of the demodulating carriers to be different from those transmitted). Before finding way outs for these problems, system designer is required to decide how much residual frequency error is acceptable, and understand exactly how errors are going to affect signals at receiving end.

Both of erroneous scenarios have been investigated, in which a frequency offset affects every carrier equally, with very edge carrier least affected. The inter channel interference originating from a fixed absolute frequency offset enhances with a number of carriers, if system transmission bandwidth is set fixed. About error in receiving equipment clock frequency, in absence of frequency offset, it affects carriers unequally (center carrier suffers a little while the worst affected carrier lies near to, but not at edge).

Discrete-time signal processing schemes, rather than frequency synthesizers, may be employed to create orthogonal subcarriers. DFT is a linear transformation, which maps complex information sequences to OFDM symbols. It follows that

$$d_k = \sum_{n=0}^{N-1} D_n \, e^{\frac{j2\pi nk}{N}} \tag{2.16}$$

The linear mapping may be written in matrix format in the following way

$$\overline{d} = \overline{WD} \tag{2.17}$$

and ,

$$W = e^{j2\pi/N} \tag{2.18}$$

$\overline{W}$ is a symmetrical as well as orthogonal matrix. After FFT, a cyclic pre/postfix of lengths k_1 *and* k_2 are added to every frame (OFDM symbol) followed by a pulse shaping frame. Appropriate pulse shaping plays a significant role in modifying the performance of OFDM systems in the presence of some wireless channel impairments. Output of such frame is fed to a D/A at a rate of f_s and processed through a lowpass filtering equipment. A basic formation of an equivalent complex baseband symbol waveform at transmitter is

$$x(t) = \sum_{n=0}^{n-1} \left\{ D_n e^{j2\pi\frac{n}{N}f_s t} \right\} \tag{2.19}$$

The extension of OFDM frame is equal to the addition of a cyclic pre/postfix in discrete-time domain. The received symbol under a time-varying random wireless channel is

$$r(t) = \int_{0}^{\infty} x(t-\tau)\, h(t,\tau) d\tau + n(t) \tag{2.20}$$

The received symbol is sampled at $t = k/f_s$ for $k = \{-k_1, \ldots N + k_2 - 1\}$. With no inter block interference, and considering that windowing function fulfils $w(n-l) = \delta_{nl}$, output of an FFT frame at receiving side is

$$\widetilde{D}_m = \frac{1}{N}\sum_{k=0}^{N-1} r_k \tag{2.21}$$

2.4 DFT

The vital constituents of an OFDM system are IDFT at transmitting side and DFT at receiving equipment. These constituents should implement

$$d_n = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} D_k\, W_N^{kn}, n = 0, \ldots . N-1 \tag{2.22}$$

These operations do reversible linear mappings between N complex information sequences {D} and N complex line sequences { d }. It may be observed that two operations are actually similar. The scale factor $1/\sqrt{N}$ gives symmetry between these operations and also preservation of power. Usually, a scale factor of 1/N is utilized in one direction and unity in other instead. In real-time implementations, this is immaterial because scaling is selected to fulfil requirement for overflow and underflow rather than other numerical definitions.

An usual N-to-N point linear transformation needs N^2 multiplication and addition operations. It would be valid for DFT as well as IDFT if every output sequence is determined independently. Moreover, by computing outputs simultaneously and making advantage of cyclic properties of multipliers $e^{\pm j2\pi nk/N}$, fast Fourier transform (FFT) schemes alleviate number of calculations to the order of N log N.

FFT is quite efficient, if N is a power of two. Many versions of FFT are there in literature, with various ordering of the inputs as well as outputs, and with different utilization of temporary memory. One version of decimation in time, is depicted below.

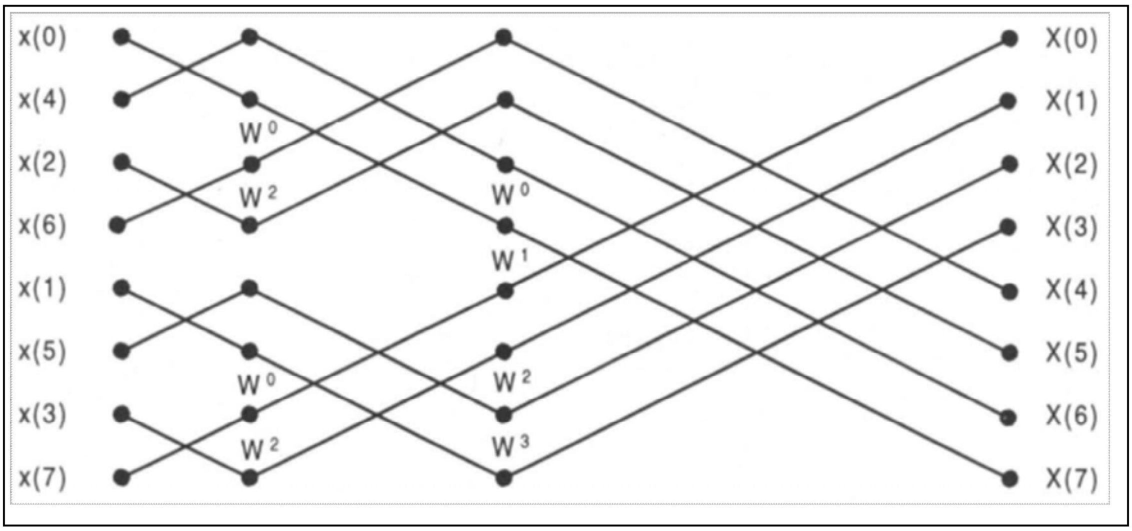


Figure 2.3: An FFT implementation (decimation in time) [4]

The architecture of an OFDM system has ability to utilize a further stage of modulation incorporating both in-phase and quadrature modulating equipments. This structure is generally used in wireless transmission systems for modulating baseband signals in desired IF or RF frequency band. It is noteworthy that basic structure demonstrated is not accounting for wireless channel dispersion, which is present in every case. The wireless channel dispersion crisis is resolved by utilizing cyclic prefix .

2.5 Partial FFT

In many applications, receiving equipment takes advantage of a subset of transmitted carriers. One of advantages of an orthogonal frequency division multiplexing system with an FFT structure is the fact that it lends itself to a repetitive configuration in an elegant way. This configuration is preferable in comparison to required filtering computational complexity in some wideband systems. Two usual configurations are illustrated in Figure 2.4.

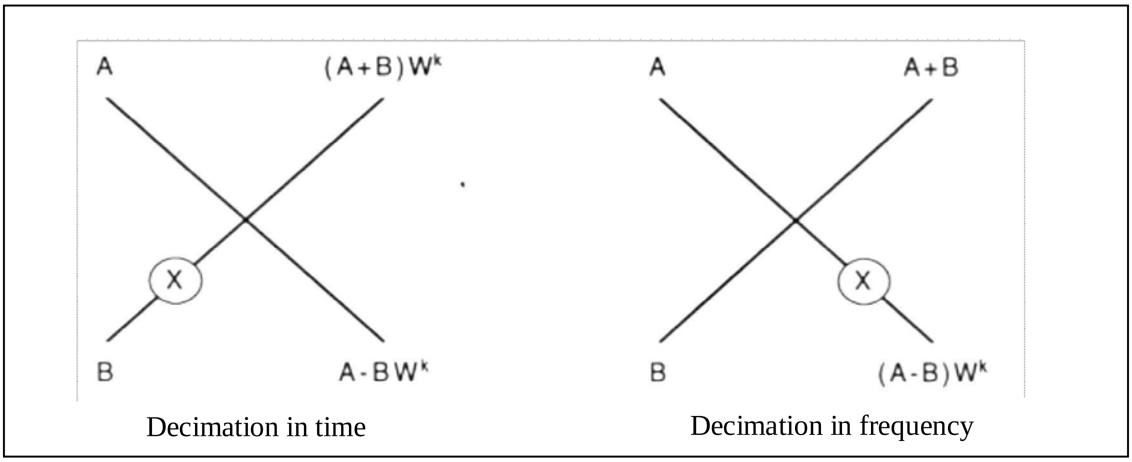


Figure 2.4: Two different techniques for FFT butterfly [4]

An example of partial FFT is depicted in Figure 2.5. In order to detect (receive) marked point at output, we may limit FFT computation to marked lines. Hence, a substantial quantum of processing is reduced.

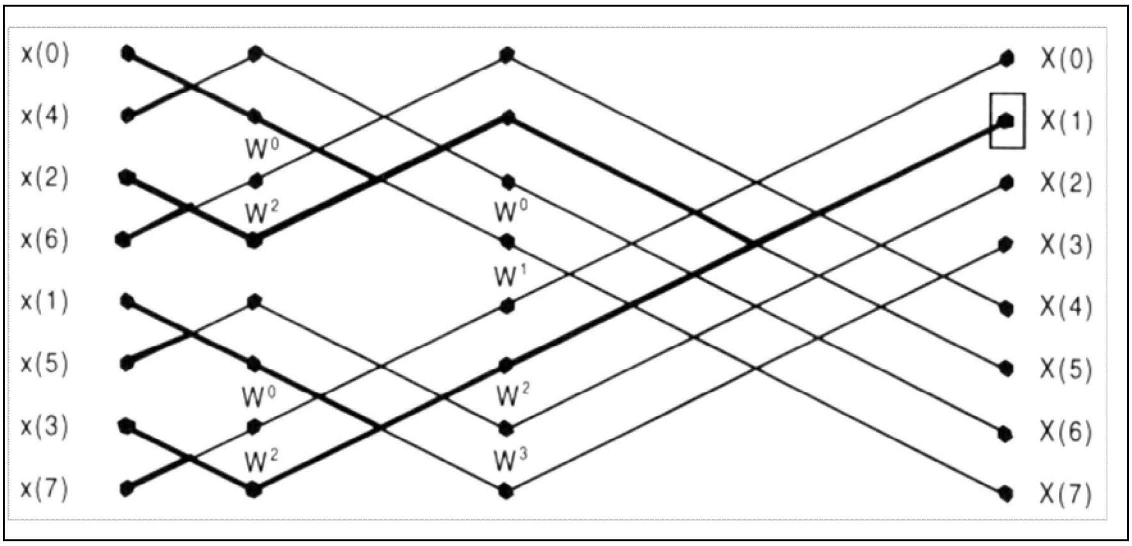


Figure 2.5: Partial FFT (DIT) [4]

Two major variations between decimation in time (DIT) and decimation in frequency (DIF) are identified [6]. First for DIT, input is bit-reversed and output is in usual order, while in DIF the reverse is valid. Secondly for DIT complex multiplications are done before add-subtract operations, while in DIF the order gets reversed. While complexity of both configurations is same in particular DFT, but it is not a scenario for partial FFT [5]. The motivation is that in DIT (a description of partial FFT), a sign change (multiplication by 1 and -1) happens at first stages, but in DIF version it happens in end stages.

2.6 Cyclic Extension

Transmission of information in frequency-domain utilizing an FFT, in the form of computationally efficient orthogonal linear transformation, leads to robustness against inter symbol interference in time-domain. Dissimilar to Fourier transform (FT), the DFT (or FFT) of circular convolution of two signals is equivalent to the product of its DFTs.

$$FT\{d_n * h_n\} = FT\{d_n\} \times FT\{h_n\} \tag{2.23}$$

$$DFT\{d_n \otimes h_n\} = DFT\{d_n\} \times \Theta DFT\{h_n\} \tag{2.24}$$

where, $*$ and $\otimes$ indicate linear and circular convolution respectively .

Signals and channel, however, have been linearly convolved. After the addition of prefix and postfix extensions to every block, linear convolution is equal to a circular convolution as depicted in Figure 2.4. Instead of including prefix and postfix, a few systems utilize only prefix, then by setting window position at receiving end, proper cyclic effects can be attained.

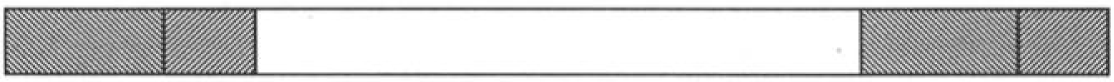


Figure 2.6: Prefix and postfix cyclic extension [5]

Utilizing this scheme, a signal, otherwise aliased, happens to be infinitely periodic to wireless channel. Let’s consider that wireless channel response is spread over M samples, and information block has N discrete samples. It follows that

$$y(n) = \sum_{m=0}^{N-1} d(m)h(n-m) \times R_N(n) \quad n = 0,1,\ldots\ldots N + M - 1 \tag{2.25}$$

where, $R_N(n)$ is the rectangular window with length N . To detail the effects of degradation, we move forward with Fourier transform by considering that convolution is linear in nature.

After linear convolution of signals and wireless channel impulse response, received symbol set is of length $N + M - 1$. This symbol set is truncated to N samples and transformed to frequency-domain, which is equal to convolution and truncation. However, in case of cyclic pre/postfix extension, linear convolution is the same as circular convolution as long as wireless channel spread is shorter than guard band duration. After truncation, DFT may be incorporated, leading to a sequence with length N because circular convolution of two sequences exhibits period of N.

Instinctively, the N -point DFT of the sequence relates to a Fourier series of periodic extension of the sequence with a duration of N. Therefore, in case of no cyclic extension, it follows that

$$\sum_{i=-\infty}^{\infty} \sum_{m=0}^{N-1} d(m)h(n + iN - m) \tag{2.26}$$

which is equal to repetition of a block with length $N + M - 1$ of period N .This leads to aliasing or ISI between consecutive OFDM blocks. In other words, samples near to boundaries of every symbol experiences significant degradation, and with larger delay spread, a number of samples shall be affected. Using cyclic extension, linear convolution transforms to the circular operation. The circular convolution of two signals with length *N is a* sequence with length *N* , therefore inter block interference issue is decided.

Appropriate windowing of OFDM blocks, as depicted later, is significant to mitigate the effect of frequency offset and to adjust transmitted signal spectrum. However, windowing must be incorporated after cyclic extension of frame, so that windowed frame is not cyclically extended. A way out to this problem is to extend every frame to *2N* points at receiving end and incorporate a $2N$ point FFT. In actual practice, it needs a $2N$ point IFFT block at transmitting side, and $2N$ point FFT at receiving end. Moreover, by using partial FFT schemes, we may alleviate computational complexity by attaining only desired frequency bins.

If windowing is not needed, we could have easily utilized zero padded pre/postfix, and before DFT, at receiving side, copy the beginning and end of frame as prefix and postfix. This generates similar effect of cyclic extension with the benefit of alleviated transmitter power and resulting in low inter symbol interference.

The comparative length of cyclic extension is dependent on the ratio wireless channel delay spread to orthogonal frequency division multiplexing symbol period.

2.7 Channel Estimation

Wireless channel estimation inverts effects of frequency non-selective fading on every subcarrier. Generally, OFDM systems use pilot symbols for wireless channel estimator. In case of time-varying wireless channels, pilot symbols must be repeated frequently. The spacing in between pilot symbols in time and frequency is dependent on coherence time as well as transmission bandwidth of wireless channel. Therefore, frequency response of wireless channel may be calculated.

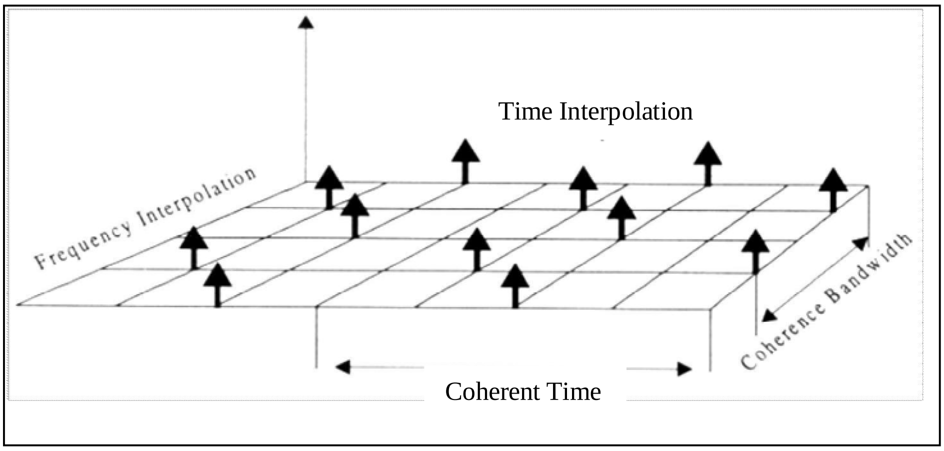


Figure 2.7: Pilot positioning in time and frequency [8]

Let

$$h_t = (h_1, h_2, \ldots\ldots, h_M)^T \tag{2.27}$$

Equation (2.27) represents on the whole impulse response of underlying wireless channel, which includes transmitting as well as receiving filtering equipments, of length M [7].

The noise samples are

$$n = (n_1, n_2, \dots \dots ., n_{M+N-1})^T \tag{2.28}$$

with

$$R_n = E\{nn^*\} = LL^* \tag{2.29}$$

In Eq. (2.29), L is a lower triangular matrix and it can be obtained by applying Cholesky technique. The received sample sequence is expressed as

$$r = Xh_t + n \tag{2.30}$$

If additive noise is white, matched filtering is a best estimator in terms of maximization of signal-to-noise ratio. Consequently, receiver filtering equipment colors noise. Using a whitening filtering equipment, the estimation process is presented in [8]. An unbiased estimation procedure, which eliminates the effects of side lobes, is also provided in [8].

In a dual system architecture, circular convolution operation is changed by multiplication operation. The repetition of a training information symbols leads to circular convolution. Hence, resulting equation is given by

$$\widehat{H}^{\omega} = X^{\omega} X^{\omega^*} H^{\omega} + X^{\omega^*} N^{\omega} \tag{2.31}$$

where, superscript depicts Fourier transform, in which products are scalar. The similar method may be applicable for Eq.(2.27). This scheme is also known as frequency equalizer in the field of wireless signal processing.

CHAPTER 3

. CHANNEL ESTIMATION ALGORITHM FOR OFDM

3.1 Basic Terminology for Channel Estimation Algorithms

Let there are L multipath wireless channels, then channel matrix is defined as

$$C_n = \left[C_{n,0}, C_{n,1} \dots \dots C_{n,L-1}\right]^H \tag{3.1}$$

and known signal matrix is defined as

$$S_n = [S_n, S_{n-1} \dots \dots S_{n-L+1}]^H \tag{3.2}$$

The transmitted signal is recovered from received signal, which may be defined as

$$y_n = \sum_{l=0}^{L-1} C_{n,l}\, S_{n-l} + W_n \tag{3.3}$$

where, w_n is additive white Gaussian noise (AWGN).

The value of C_n must be computed with the help of new channel vector C_n, whose value will change according to algorithm. Here, C_n and S_n have been utilized as new vectors, so that algorithm can be generalized. Different types of estimation error, such as mean squared error and MMSE, have been explained in subsequent sections.

3.1.1 Least Mean Square (LMS)

LMS algorithm is a type of adaptive filtering utilized to mimic a required filtering procedure by finding filter coefficients that corresponds to the generation of least mean squares of error signal (difference between the desired and original/true signal). It is a stochastic gradient descent technique in that filter coefficients are only adapted based on the error at current instant n.

In practice, exact measurement of gradient vector is not feasible, because it needs prior knowledge of covariance matrix R of the tap-input vectors and cross correlation vector p between the tap-inputs and desired response. Consequently, gradient vector should be estimated form available data when we operate in an unknown environment. This method is known as LMS algorithm. Simplest way to estimate $\hat{R}(n)$ and $\hat{p}(n)$ is as follows

$$\hat{R}(n) = s(n)s^H(n) \tag{3.4}$$

$$\hat{p}(n) = s(n)y^*(n) \tag{3.5}$$

$$C(n+1) = C(n) + us(n)e^*(n) \tag{3.6}$$

It has been seen that in nonstationary environment, error can be minimized by rapidly discounting the past and estimate predominantly on recent data. This can be performed by utilizing a forgetting factor as explained in next algorithm known as recursive least squares algorithm (RLS).

3.2 Minimum Mean Square Error (MMSE)

The impulse response estimation dependent on minimum mean square error criterion achieves outstanding wireless channel estimation under low SNR scenarios; however, it needs prior statistical data, such as delay profiles of wireless channel. The MMSE is obtained by

$$h_{mmse} = (\sigma^2 C_h^{-1} + L_p^H L_p)^{-1} L_p^H r_{lp} \tag{3.7}$$

where,

C_h indicates covariance matrix of underlying channel, that transforms to a diagonal matrix if fading parameters are statistically independent of each other, which is denoted by Eq. (3.8),

$$C_h = diag\{E[|h_1|^2], E[|h_2|^2] \dots \dots \dots E[|h_\Delta|^2]\} \tag{3.8}$$

From Eq. (3.8), it can be inferred that diagonal component of C_h is equal to a power delay profile of underlying wireless channel.

3.3 Expectation-Maximization Algorithm

3.3.1 Introduction to the EM algorithm

The EM algorithm [9] – [13] is an iterative scheme to obtain ML estimates of parameters in the presence of unobserved information. The concept behind this scheme is to augment measured information with latent information, which can be either unknown information or parameter values, so that likelihood function conditioned on the information and latent information has a formation that is straightforward to manipulate. This procedure may be parted into two steps: E-step and M-step. It is assumed that data Z ("complete data") may be segregated into two constituents, Z = f(X, Y), where X is observed information ("incomplete" information), Y is unknown information and θ is a missing parameter value that has to be estimated using Y.

E-step determines $Q(\theta|\theta^{(p)})$, the expected value of log- likelihood of θ , $\log f(Z|\theta)$, where expectation has been calculated with respect to y conditioned over all values of x and latest estimate of θ, $\theta^{(p)}$, which leads to

$$Q(\theta|\theta^{(p)}) = E\{\log f(Z|\theta)|x, \theta^{(p)}\} \tag{3.9}$$

M-step determines $Q(\theta|\theta^{(p+1)})$, the value of θ that leads to maximization of $Q(\theta|\theta^{(p)})$ over all feasible values of θ

$$\theta^{(p+1)} = arg\ \max_{\theta} Q(\theta|\theta^{(p)}) \tag{3.10}$$

This method is repeated again and again until data $\theta^{(0)}, \theta^{(1)}, \theta^{(2)}$ .... converges. EM procedure is exercised in such a manner that sequence of $\theta^{(p)}{}'s$ gets converged to ML estimate of θ.

3.3.2 Estimating Channel Frequency Response (CFR): Algorithm1

The orthogonal frequency division multiplexing divides its allotted wireless channel spectrum into a number of parallel subchannels, which are only subjected to frequency flat-fading . Therefore, we only require to estimate individual $H(m, 0 \leq m \leq M -$

1) separately, which will lead to a substantial alleviation in computation complexity. To simplify such expressions, we discard subcarrier index m, and just indicate $Y, X,$ and H instead of $Y(m)$, $X(m)$, and $H(m)$. We consider that frequency-domain signal X of a given subcarrier denotes a QPSK / 16-QAM signal in a constellation of size C(=4 or 16, respectively). The symbols in this signal constellation are represented by X_i, $1 \leq i \leq C$.

Due to Gaussian noise consideration, probability density function (pdf) of Y *for* any X and H is represented as

$$f(Y|X,H) = \frac{1}{\sqrt{(2\pi\sigma^2)}} exp\left\{-\frac{1}{2\sigma^2}|Y-HX|^2\right\} \tag{3.11}$$

By considering that all C symbols are equally likely, and by averaging conditional probability density function of Eq.(3.11) over all values of variable X, we attain probability density function of Y for any H. It results in

$$f(Y|H) = \sum_{i=1}^{C} \frac{1}{C\sqrt{(2\pi\sigma^2)}} exp\left\{-\frac{1}{2\sigma^2}|Y-HX_i|^2\right\} \tag{3.12}$$

Assume that wireless channel is stationary over a period of "D" OFDM blocks. Various values of D may be incorporated in different applications depending on how fast wireless channel varies. We denote received signal vector as $\underline{Y} = [Y^1, \ldots\ldots Y^D]$ and transmitted signal vector as $\underline{X} = [X^1, \ldots\ldots X^D]$ for a specific subcarrier over D blocks. Further, we denote $\underline{Y}$ and $(\underline{Y}, \underline{X})$ "incomplete" and "complete" information, respectively, following the terminology of EM procedure. Considering that additive white Gaussian noise is independent from block to block for every subcarrier, we may represent conditional probability density function of incomplete data set as

$$f(\underline{Y}|X,\underline{H}) = \prod_{d=1}^{D} f(Y^d|H,X^d) \tag{3.13}$$

Therefore, log-likelihood function of an incomplete information set is represented as

$$\log f(\underline{Y}|X,\underline{H}) = \sum_{d=1}^{D} \log f(Y^d|H,X^d) \tag{3.14}$$

and log-likelihood function of a complete information set is denoted as

$$\log f(\underline{Y}|X,\underline{H}) = \sum_{d=1}^{D}\left\{\log\frac{1}{C}f(Y^d|H,X^d)\right\} \quad (3.15)$$

In traditional ML estimation, we put an effort to obtain an estimate of H that leads to maximization of f (Y|H). However, log f (Y|H) Eq. (3.12), is not simple for manipulation (summation of many exponential functions), we find a way out by using an EM procedure, which enhances likelihood at every step . Every iterative process p = 0,1,2,…… in procedure for estimating H from Y is consisting of following two steps

E-step :

$$Q(H|H^{(p)}) = E_{\underline{X}}\{\log f(\underline{Y},\underline{X}|H|\underline{Y},H^{(p)})\} \quad (3.16)$$

M-step :

$$\widetilde{H}^{(p+1)} = arg\ \max_{H} Q(H|H^{(p)}) \quad (3.17)$$

$$Q(H|H^{(p)}) = \sum_{i=1}^{C}\sum_{d=1}^{D}\log\left\{\frac{1}{C}f(Y^d|H,X_i\right\}\frac{f(Y^d|H^{(p)},X_i)}{Cf(Y^d|H^{(p)})} \quad (3.18)$$

$\widetilde{H}^{(p+1)}$ is a tentative estimate of H straightforwardly from Eq.(3.17). The last $(p + 1)st$ estimate of H, that is, $H^{(p+1)}$, is attained via additional manipulation on $\widetilde{H}^{(p+1)}$. The conditional probability density functions $f(Y^d|H^{(p)},X_i)$ and $f(Y^d|H^{(p)})$ can be obtained from Eq.(3.11) and Eq.(3.12), where X_i is ith signal in signal constellation. The optimum H, that maximizes Eq.(3.18), is found as follows

$$\widetilde{H}^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{d=1}^{D}|X_i|^2\frac{f(Y^d|H^{(p)},X_i)}{Cf(Y^d|H^{(p)})}\right]^{-1}\cdot\left[\sum_{i=1}^{C}\sum_{d=1}^{D}Y^dX_i^*\frac{f(Y^d|H^{(p)},X_i)}{Cf(Y^d|H^{(p)})}\right] \quad (3.19)$$

It noteworthy that aforementioned maximization criterion is in reality a weighted least squares problem.

3.3.3 Estimating Transmitted Signals: Algorithm2

In this procedure, we put an effort to modify performance of detection accuracy of transmitted information $X^d(m)$, $0 \le m \le M-1, 1 \le d \le D$, and CIR from measurement $Y^d(m),, 0 \leq m \leq M - 1, 1 \leq d \leq D,$ by utilizing EM algorithm. To further simplify such expressions, we utilize $\underline{H}, \underline{h}, \underline{X}, \underline{Y}, \underline{N}$ to represent vectors of frequency-domain CIR, time-domain CIR, modulated input information, output information, and white Gaussian noise, respectively. The system paradigm can be represented in vector form for dth OFDM frame as

$$\underline{Y}^d = X^d W_L \underline{h} + \underline{N}^d \tag{3.20}$$

It is considered that wireless channel is stationary over a period of D frames for generality. To execute a wireless channel estimation procedure utilizing observed information in all D blocks, we denote a few vectors as $X = [(X_1)^T, \ldots, (X_D)^T]^T$, $Y = [(Y_1)^T, \ldots, (Y_D)^T]^T$, $N = [(N_1)^T, \ldots, (N_D)^T]^T$, $\boldsymbol{X} = diag(X)$, $\boldsymbol{Y} = diag(Y)$, and $W_{LD} = [(W_1)^T, \ldots, (W_D)^T]^T$ with D copies of $\boldsymbol{W_L}$. Using this representation, system paradigm may be improved. It gives

$$\underline{Y} = XW_{LD}\underline{h} + \underline{N} \tag{3.21}$$

The "incomplete" and "complete" information sets are denoted as $(\underline{Y})\ and\ (\underline{Y}, \underline{h})$, respectively. Every iterative process $p = 0, 1, 2, \ldots$ in EM procedure for estimating $\underline{X}$ from $\underline{Y}$ is consisting of the following two steps

E-step :

$$Q\left(\underline{X}|\underline{X}^{(p)}\right) = E_{\underline{h}}\{\log f\left(\underline{Y}, \underline{h}|\underline{X}\right)|\underline{Y}, \underline{X}^{(p)}\} \tag{3.22}$$

M-step :

$$\underline{\widetilde{X}}^{(p+1)} = arg \max_{\underline{X}} Q\,(\underline{X}|\underline{X}^{(p)}) \tag{3.23}$$

In E-step at $(p + 1)st$ iteration, expected value $\log f\ (\underline{Y}, \underline{h}|\underline{X})$ is computed, by using $\underline{Y}$ and $\underline{X}^{(p)}$, and by using estimates attained in pth iteration. The M-step of $(p + 1)st$ iteration decides transmitted signal $\underline{X}^{(p+1)}$ that leads to maximization of $Q\,(\underline{X}|\underline{X}^{(p)}$ for a given $\underline{X}^{(p)}$.

The drawback of this procedure is that mean $E\{\underline{h}\}$ and covariance matrix $\boldsymbol{\Sigma}$ of time-domain CIR are also assumed to be available. In a typical scenario, such wireless channel statistics can not be known. Luckily, we observe that when σ^2 is low (*i.e.*, SNR is large), contribution of $\boldsymbol{\Sigma^{-1}}$ and $\boldsymbol{\Sigma^{-1}}\, E\{\underline{h}\}$ is so low that we may get rid of these components/terms and still desire same performance. Further, for an M-ary PSK modulated signal, that is, $|X(m)|^2 = A$ for each value of m, information symbol estimation may be exercised by utilizing only phase related data.

$$\underline{X}^{(p+1)} = Quantization\left\{(\underline{Y}^H X^{(p)} W_{LD} W_{LD}^H Y)^T\right\} \tag{3.24}$$

Eventually, only multiplication as well as addition operations are needed. Next, $W_{LD} W_{LD}^H$ may be computed and stored ahead of time. Therefore, computation complexity is substantially alleviated for high SNR scenario.

A close observation discloses that simplified Algorithm 2 is a combination of ML wireless channel estimator considering $\underline{X}^{(p)} = \underline{X}$ and ML signal detector considering $\underline{h}^{(p)} = \underline{h}$. It has been explored in [14] in some other context. It may be concluded that Algorithm 2 is an extension of an iterative ML wireless channel estimation procedure when we take advantage of wireless channel statistical properties. The related simplified procedure is similar to iterative ML channel estimation method.

3.3.4 Estimating the Channel Impulse Response (CIR): Algorithm3

In this section, we put an effort to estimate time-domain channel response by incorporating parameter estimation procedure presented by Feder and Weinstein for general estimation problem based on EM procedure [15]. We still assume that wireless channel remains stationary over the duration of D blocks for generality. The system paradigm utilized here is similar to earlier algorithm stated in Eq. (3.21). We denote $\boldsymbol{A} = \boldsymbol{X}\boldsymbol{W}_{\boldsymbol{LD}}$, which is a $MD \times L$ matrix, and rewrite system paradigm. It is as follows

$$\underline{Y} = A\underline{h} + \underline{N} = \sum_{i=0}^{L-1} A_i h_i + \underline{N} \tag{3.25}$$

where, $\boldsymbol{A_i}$ is ith column of matrix **A**. Note that every element of $\underline{Y}, Y(m)$, encompasses of L superimposed signals and additive white Gaussian noise, which may be denoted as

$$Y(m) = \sum_{i=0}^{L-1} a(m)h_i + N(m) \tag{3.26}$$

The EM-based procedure is utilized here to attain an estimation of $\underline{h}$ that maximizes $f\ (\underline{Y}|\underline{h})$. The "incomplete" and "complete" information set for mth element of $\underline{Y}$, as stated before, are $(Y(m))$ and $(\underline{Z}_m)$, respectively. We then combine all $\underline{Z}_m$for all "D" OFDM frames and all M subcarriers into a vector $Z = [Z_0^T, \ldots, Z_{MD-1}^T]T$. Every iterative process $p = 0, 1, 2, ..$ in EM procedure for estimating $\underline{h}$ from $\underline{Y}$ is consisting of the following two steps

E-step:

$$Q(\underline{Z}|\underline{h}^{(p)}) = E_Z\{\log f(\underline{Z}|\underline{h})|\underline{Y}, \underline{h}^{(p)}\} \tag{3.27}$$

M-step:

$$\underline{h}^{(p+1)} = arg\max_{\underline{h}} Q(\underline{Z}|\underline{h}^{(p)}) \tag{3.28}$$

In E-step at $the\ (p+1)st$ iteration, expected log-likelihood function $log\ f\ (\underline{Z}|\underline{h})$ is computed, for available $\underline{Y}$ and $\underline{h}^{(p)}$, for estimates attained in $pth\ i$teration. The M-step of $(p+1)st$ iteration decides transmitted CIR $\underline{h}^{(p+1)}$, which in turn optimizes $Q(\underline{Z}|h^{(p)})$.

3.3.5 Initialization

As known from usual convergence properties of EM procedure, there is no surety that iterative steps converge to a global maximum. For a likelihood function with multiple local maximum points, convergence point can be one of such local maximum points, which depends on initial estimates of $\underline{H}^{(0)}, \underline{X}_0$, and $\underline{h}^{(0)}$. Therefore, pilot symbols distributed at particular locations are utilized in an OFDM time-frequency lattices to give adequate initial values of $\underline{H}^{(0)}, \underline{X}_0$, and $\underline{h}^{(0)}$, if there are pilot sequences inserted in current orthogonal frequency division multiplexing block.

On the other hand, if there is no pilot sequence, we just keep initial channel estimates of the current OFDM frame as final channel estimates of last OFDM block considering the wireless channel is varying at a slow speed. It is more likely to result in true maximum point, as it may be inferred from numerical outcomes. Another advantage of this choice of initial estimates of CIR is that we do not need to perform time-domain filtering or interpolation. Therefore, we can substantially alleviate detection latency, since we can perform wireless channel estimation and signal detection operations as soon as, we have received information symbols for every OFDM frame.

For those OFDM frames with pilot sequences, we denote pilot position set as $S = \{s_1, \ldots, s_{|S|}\}$. The related FFT matrix only with those rows belonging to S is represented as WS. Therefore, we utilize straightforward LS algorithm to attain wireless channel frequency response [8] at every pilot location by using

$$\widetilde{H}^{(0)}(s_i) = \frac{Y(S_i)}{X(S_i)} \quad , \qquad 0 \le i \le |S| \tag{3.29}$$

Then, we employ IFFT on $\widetilde{H}^{(0)}(s_1) \ldots, \widetilde{H}^{(0)}(s_{|s|})$ and attain initial CIR by using

$$\underline{h}^{(0)} = \frac{1}{M} W_s^H \underline{\widetilde{H}}^{(0)} \tag{3.30}$$

where $\widetilde{H}^{(0)} = [\widetilde{H}^{(0)}(s_1), \ldots, \widetilde{H}^{(0)}(s_{|s|})]^T T$. Furthermore, we employ FFT on $\underline{h}^{(0)}$ and attain initial estimates of wireless channel frequency response for all subcarriers as $\underline{H}^{(0)} = W_L \underline{h}^{(0)}$. Finally, initial estimates of transmitted symbols are attained from

$$X^{(0)}(m) = Quantization \left\{ \frac{Y(m)}{H^{(0)}(m)} \right\} \quad , \qquad 0 \le m \le M-1 \tag{3.31}$$

3.4 EM-MMSE Algorithm

Efficient estimation of time-varying wireless channels (nonstationary) is a tedious problem in orthogonal frequency division multiplexing (OFDM) systems. Pilot based schemes have generally been incorporated for wireless channel estimation. However, for time-varying wireless channels, pilot symbols are required to be transmitted periodically to avoid the effects of time-selective fading. Expectation-Maximization (EM) based scheme may be incorporated in such scenarios by precluding the requirement for periodic retransmission of pilot sequences. In order to alleviate the complexity of EM procedure, we can utilize an EM-MMSE based iterative algorithm to estimate time-varying nonstationary wireless channel. This scheme is not only easy to implement than EM algorithm, but it also provides performance modification in comparison to traditional pilot based schemes.

In this research work, we investigate a modified EM based scheme [17], wherein we first estimate transmitted sequences by MMSE technique, with the knowledge of wireless channel estimate at a certain iteration. In E-step, estimation of cost function is done only for sequences estimated by utilizing MMSE technique. This scheme is computationally less complex and needs less number of iterations than EM method, while providing better performance than pilot based and traditional EM methods.

3.4.1 Baseband OFDM System Model and Pilot Based Estimation of Channel

Let us consider an OFDM system with M subcarriers. The input binary information is first given as input to serial to parallel (S/P) converter. Every data stream then modulates the related subcarrier by M-PSK / M-QAM. The modulated information sequences denoted by a vector of complex variables are transformed by using inverse fast Fourier transform (IFFT). Here, output sequences are represented as $\underline{X} = [X(0), \ldots\ldots\ldots\ldots . X(0)]^T$. The cyclic prefix sequences, (which replicate end part of IFFT output), are included in front of every frame in order to overcome inter block interference (IBI). However, this guard duration is discarded at receiving equipment. The received information $Y_f\ (m)$ is corrupted by slow time-varying wireless multipath fading channel [18] and AWGN (independent complex Gaussian random variables with zero mean and variance σ^2). After discarding prefix and by executing FFT and demodulation, $y(0) \ldots\ldots\ldots . y(M-1)$ is transformed back to $\underline{Y} = [Y(0), \ldots\ldots\ldots\ldots\ldots . . Y(M-1)]^T$.

The frequency-selective wireless channel is approximated by using an independent autoregressive process of order-2 AR(2) [19]. We indicate wireless channel tap-coefficient vector for every OFDM block as $h_n = [h(n, O)\ h(n, l) \ldots h(n, L-1)]$, where $h(n, l)$ is l^{ih} tap-coefficient for n^{th} frame. L is length of time-domain wireless channel impulse response.

For AR(2) paradigm [19], these channel tap-coefficients follow

$$h(n,l) = a_1 h(n-1,l) + a_2 h(n-2,l) + w(n,l) \tag{3.32}$$

where, a_1 and a_2 are AR(2) tap-coefficients and $w(n,l)$ is modelling noise for l^{th} tap-coefficient at time frame n. Assuming that subcarriers within an OFDM block, received data (after discarding guard interval) in frequency-domain becomes

$$Y(m) = X(m)H(m) + N(m), 0 \le m \le M-1 \tag{3.33}$$

where, $H(m)$ *and* $N(m)$ are the frequency response of wireless channel and a set of transformed noise variables respectively at subcarrier m. Here, wireless channel in OFDM system is estimated in frequency-domain by utilizing received data and pilots known at receiving equipment [18]. In pilot based method, the known information is transmitted for a restricted duration of time, which provides wireless channel information at receiving equipment [18]. These wireless channel estimates at receiving equipment are interpolated over data subcarriers and the data sequences are decoded. In order to manage Doppler effects due to mobile wireless systems, reference sequence should be repeated periodically, and it may lead to a substantial reduction in functional bit rate.

3.4.2 EM Based Channel Estimation Technique for OFDM System

In this section, we detail traditional EM algorithm for wireless channel estimator working in OFDM systems [20]-[22]. We consider transmitted symbols to be complex modulated with a constellation size C. We represent symbols in such constellation by $X_i(m),\ 1 \le i \le C$. From Eq.(3.33), assuming every carrier undergoing independent fading, we assume $H(m)$ as deterministic parameter to be estimated from observed information $Y(m)$. $Here, Y(m)$ is insufficient observed data set, while $X(m)$ is unknown data set. Log likelihood function (cost function) for orthogonal frequency division multiplexing system can be represented as $log f(Y(m)\ X(m)/H(m))$, where incomplete and complete information sets are $Y(m)$ *and* $(Y(m), X(m))$. At $(p+1)^{th}$ iteration, $H^{(p)}(m)$ represents estimated parameter at p^{th} iteration. In expectation step of EM scheme, cost function is averaged over all feasible values of transmitted information provided by received sequences and channel estimate on p^{th} iteration.

E-step :

$$Q(H(m)/H^{(p)}(m)) = E_x\left[\log f\{Y(m), X(m)/H(m)\}/Y(m), H^{(p)}(m)\right] \quad (3.34)$$

M-step :

Utilizing the concept of conditional probability density functions from Eq. (3.11) and Eq.(3.12) in Eq. (3.34) and by maximization of Q function, estimated wireless channel coefficients at $(p+1)^{th}$ iteration are determined as

$$\widehat{H}^{(p+1)}(m) = \left[\sum_{i=1}^{C} X_i(m)X_i^*(m)\frac{f_i(Y(m)/H^{(p)}(m))}{f(Y(m)/H^{(p)}(m))}\right]^{-1} \times \left[\sum_{i=1}^{C} Y(m)X_i^*(m)\frac{f_i(Y(m)/H^{(p)}(m))}{f(Y(m)/H^{(p)}(m))}\right] \quad (3.35)$$

where, * stands for complex conjugate operation. For initial estimation of $\widehat{H}^{(0)}(m)$, pilot based scheme is incorporated .

3.4.3 EM- MMSE Based Channel Estimation Technique for OFDM Systems

In traditional EM method, a cost function for all feasible values of transmitted sequence is estimated. But in EM MMSE, at every iteration, transmitted symbols are first found utilizing knowledge of wireless channel on that iteration. The transmitted symbols are obtained by utilizing MMSE technique in step A [21] - [25].

Step A :

First $\left\|Y(m) - H^{(p)}(m)X_i(m)\right\|^2$ is attained for $1 \leq i \leq C$ for every carrier, and the transmitted information symbols are attained at p^{th} iteration utilizing MMSE. The estimated transmitted information symbols for every frame at iteration p is denoted by $\underline{\tilde{X}}^{(p)}$. Equation (3.33) may be indicated in vector form for every frame as

$$\underline{Y} = \underline{H} \cdot \underline{X} + \underline{N} \quad (3.36)$$

where $\cdot$ operator indicates Hadamard product and $\underline{H}$ is wireless channel frequency response vector of dimension $1 \times M$.

Log Likelihood (cost function) for orthogonal frequency division multiplexing system in vector form for a frame can be represented as $log\, f(\underline{Y}, \underline{X} / \underline{H})$.

EM - MMSE may be straightforwardly incorporated for every frame [21]-[22],

At $(p+1)^{th}$ iteration ,

E-step :

Equation (3.36) may be represented in vector form as

$$Q(\underline{H} / \underline{\hat{H}}^{(p)}) = E_x \, [log\, f(\underline{Y}, \underline{X} / \underline{H}) / Y, \underline{\hat{H}}^{(p)}] \tag{3.37}$$

where, expectation is attained by utilizing received sequence and wireless channel estimate on p^{th} iteration.

After applying expectation on estimated transmitted information symbols $\underline{\hat{X}}^{(p)}$, we get

$$Q(\underline{H} / \underline{\hat{H}}^{(p)}) = [log\, f(\underline{Y}, \underline{\hat{X}}^{(p)} / \underline{H}) / Y, \underline{\hat{H}}^{(p)}] \tag{3.38}$$

M-step:

$$\underline{\hat{H}}^{(p+1)} = \left[\underline{\hat{X}}^{(p)} \cdot \underline{\hat{X}}^{(p)*}\right]^{-1} \quad X \quad \left[\underline{Y} \cdot \underline{\hat{X}}^{(p)*}\right] \tag{3.39}$$

Using step A and Eq.(3.39), iteratively, wireless channel is estimated in frequency-domain. Then, IFFT is calculated to transform it into time-domain wireless channel, which consists of M paths. Since, only L paths are significant, all other paths are assumed to be null and once again FFT is calculated. For initial estimation of $\underline{\hat{H}}^{(0)}$, pilot based method with linear interpolation is incorporated.

The efficiency and efficacy of any wireless channel estimator can be appraised by the amount of training needed and required computational complexity for performing the operation. In this procedure [21]-[22], computation complexity is alleviated and it needs low number of pilot sequences as the initial estimation is performed only single time.

3.5 Quasi-Newton Acceleration EM Algorithm

Orthogonal frequency division multiplexing (OFDM) has been presented in literature to combat harsh frequency-selective fading in large data rate wireless communication systems [25]. The pilot assisted symbol modulation with Expectation-Maximization (EM) channel estimation algorithm exhibits low bit error rate in comparison to conventional interpolation scheme. But, it exhibits high computational complexity and also possesses excruciatingly low convergence rate.

A Quasi-Newton Acceleration (QNA) EM algorithms have been introduced to perform wireless channel estimation in [26], and this method may alleviate computational complexity and iteration time. Simulation outcomes state that it exhibits low BER performance in comparison to conventional schemes. OFDM, which can replace a frequency-selective fading wireless channel into various parallel flat-fading subchannels, is an effective scheme to defy multipath delay spread in large data rate wireless systems. A dynamic estimation of wireless channel is mandatory before coherent demodulation of OFDM symbols.

i) Channel System Modes

In this research work , we utilize block pilot **sy**mbols typically inserted in cyclic prefix of baseband orthogonal frequency division multiplexing signal. We consider that there are N subcarriers in an OFDM system, and M subcarriers of them are utilized to transmit pilot sequence vector P of size $M \, X \, 1$. At transmitting side , binary source information is converted into $N_D = N - M$ parallel data streams, and each of them is modulated by quadrature phase shift keying (QPSK). The pilot sequence $P = [P(0), P(1), \dots \dots \dots \dots .. P(M-1)]^T$ of a length M are equally inserted in modulated information vector $D_n = [D_n(0), D_n(1) \dots \dots \dots D_n(M-1)]^T$ and it attains nth OFDM sequence vector X_n of size $N \, X \, 1$. After modulation by N point IFFT, CP is incorporated to suppress inter symbol interference.

We consider that length of CP is larger or equal to that of CIR, and there is no inter symbol interference in orthogonal frequency division multiplexing systems . The transmitted signal is then sent through a frequency-selective time-varrying wireless channel . Discarding CP at receiving equipment and processing IFFT demodulation at nth recevied sequence vector, Y_n of size $N\ X\ 1$ may be denoted as

$$Y_n(k) = H_n(k)X_n(k) + W_n(k) \quad k = 0,1,2,\dots\dots\dots..N-1 \tag{3.40}$$

For easiness, we remove subscript in rest of research work and represent it as

$$Y = Diag(X)H + W = Diag(X)F_L h + W \tag{3.41}$$

We consider that wireless channel is constant in single OFDM symbol period, where $h = [h(0), h(1), \dots\dots\dots\dots..h(M-1)]^T$ is CIR vector and L is length of CIR . Here, $[H = [H(0), H(1), \dots\dots\dots\dots..H(M-1)]^T$ is wireless channel frequency response, H(k) is fading factor and phase offset at k^{th} subcarrier [23]-[24]. It leads to

$$H(k) = \sum_{l=0}^{L-1} h(1)e^{\frac{-j2\pi kl}{N}} \tag{3.42}$$

E-step :

$$Q(h|h^{(p)}) = E\{\log f(Y, d|h)|h^{(p)}, Y\} \tag{3.43}$$

M-step:

$$\{h^{(p+1)} = \arg\ \max\ (Q(h|h^{(p)}), Y)\} \tag{3.44}$$

From Eq. (3.35) and Eq.(3.39), for an available received information vector Y, wireless channel coefficients at $p^{\ th}$ time iteration can be calculated [26].

CHAPTER 4

. SIMULATION RESULTS AND DISCUSSION

4.1 Simulation of Channel Estimation Algorithms

In following simulation, QPSK data has been generated as an input to wireless channel paradigm at every symbol period. The minimum mean square error (MMSE) is denoted as

$$\in = E[|c(n) - w(n)|^2] \tag{4.1}$$

OFDM simulation paradigm has been constructed, which is akin to details and particulars of 802.11a, to illustrate the strength and efficiency of EM-based channel estimation and signal detection procedures. Simulation outcomes of wireless channel estimation algorithms are demonstrated in three subsections.

The average wireless channel power is normalized, such that MSE is proportional to wireless channel length L. For those OFDM blocks encompassing pilot symbols, initial estimate of CIR is attained by using these 8 equally spaced pilot symbols. For those OFDM blocks without pilot symbols, initial estimate of CIR is calculated by using the wireless channel estimate of previous OFDM frame. To generate time-varying nonsationary channel (environment), lth wireless channel tap-coefficient $h_l(n)$ is considered to be fast fading (Rayleigh) [19]. The Jakes paradigm is accepted as a realistic wireless channel paradigm, which is simulated by utilizing AR(2) process, such that

$$h_l(n) = -\bar{a}_1 h_l(n-1) - \bar{a}_2 h_l(n-2) + w_l(n) \qquad \text{(akin to Eq.(3.32))}$$

where, $w_l(n)$ is a complex zero- mean white Gaussian process. The scalar coefficients in aforementioned equation are $\bar{a}_1 = -2r_d \cos\left(\sqrt{2}\pi f_d Ts\right)$ and $\bar{a}_2 = r_d^2$, which take account of maximum Doppler spread f_d of underlying fading wireless channel, sampling time Ts and pole radius r_d related to steepness of peaks of power spectrum. For accurate wireless channel tap-coefficient modelling, value of pole radius is set as $r_d = 1 - 2 f_d Ts$.

Simulation outcomes are presented on the basis of ensemble average of 1000 statistically independent experiments.

4.1.1 Simulation Results for Algorithm1

In this section, simulation is conducted to showcase the legitimacy and efficacy of EM-based channel estimation and signal detection procedures. The whole wireless channel transmission bandwidth is 800 kHz, and it is divided into 64 subcarriers (or tones). To make tones orthogonal to each other, symbol period is selected to be 80 microseconds. An additional 20 microseconds CP ($N_{CP} = 16$) is utilized to give protection from IFI and ICI because of wireless channel delay spread. Therefore, total OFDM block length is $Ts = 100$ microseconds and subchannel symbol rate is 10 kbaud.

The modulation scheme utilized in system is QPSK.

Normalization of average wireless channel paradigms should be same. For those OFDM blocks containing pilot symbols, initial estimate of CIR is obtained from the wireless channel estimate of previous OFDM block. The fade rate is set $f_d Ts = 0.05$ for the following outcomes with maximum Doppler spread $f_d = 500Hz$.

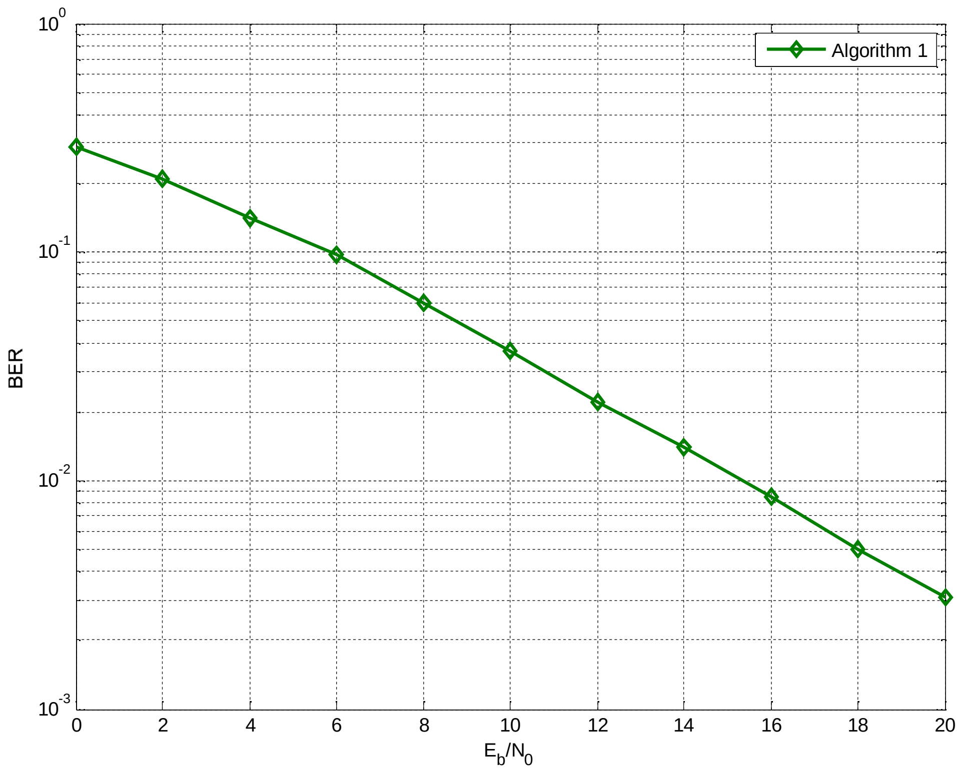


Figure 4.1: BER versus E_b/N_0 (dB) in the 8-path channel model for Algorithm1

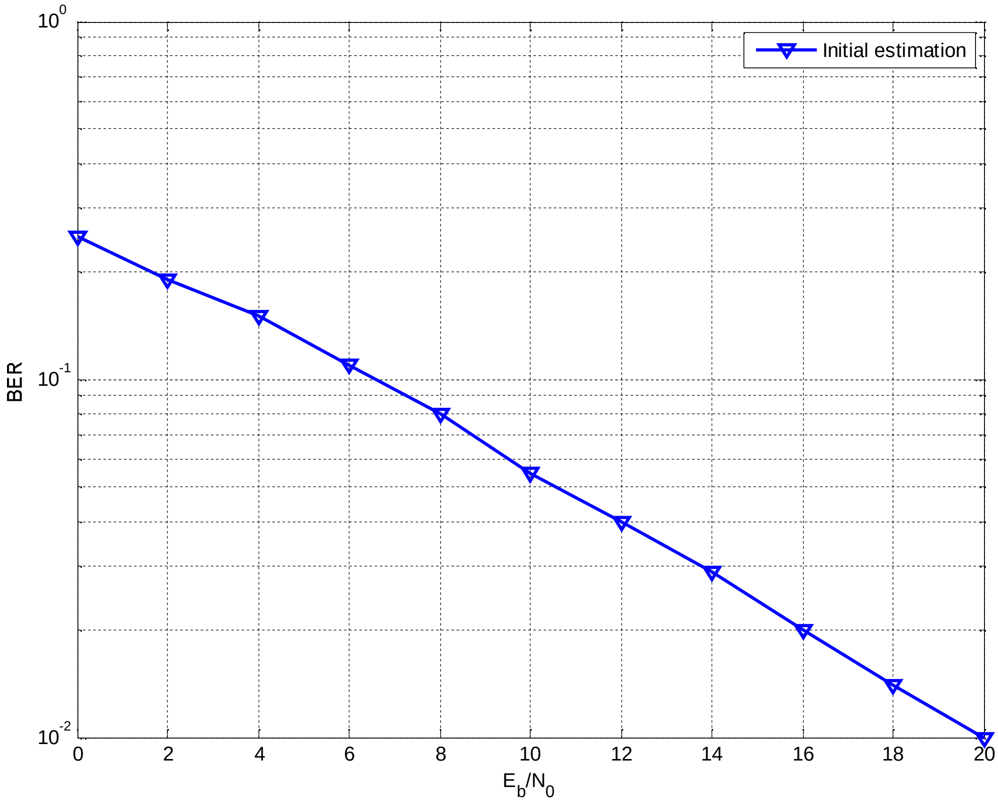


Figure 4.2: BER versus E_b/N_0 (dB) in the 8-path channel model for initial estimation

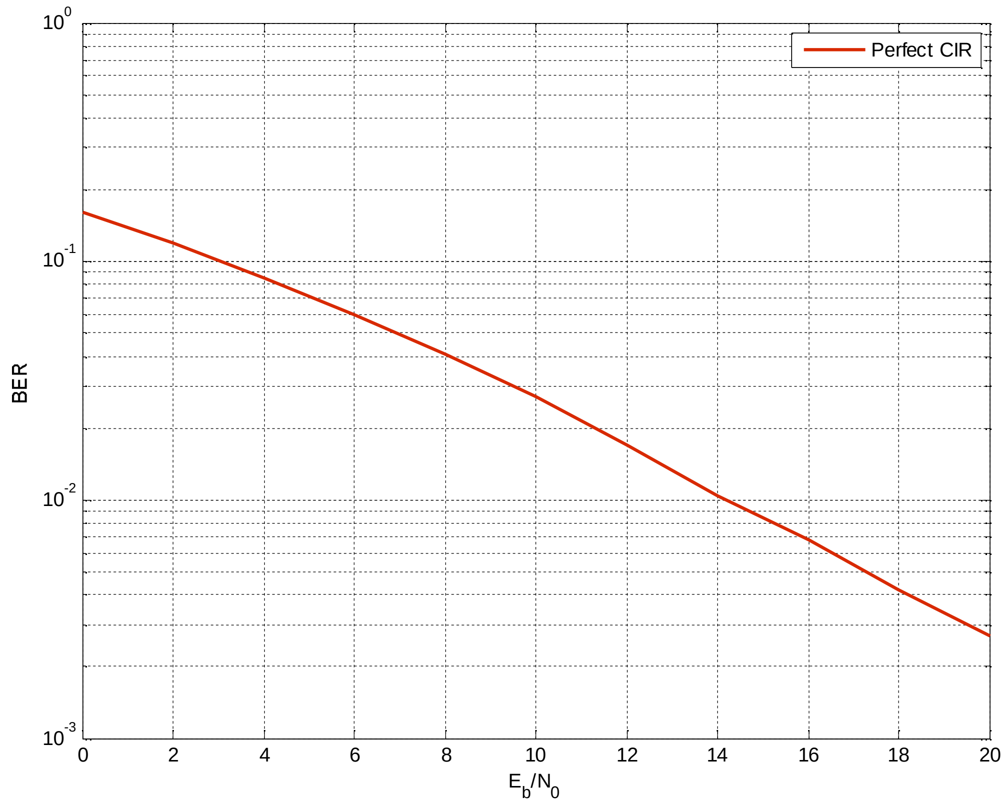


Figure 4.3: BER versus E_b/N_0 (dB) in the 8-path channel model for perfect CIR

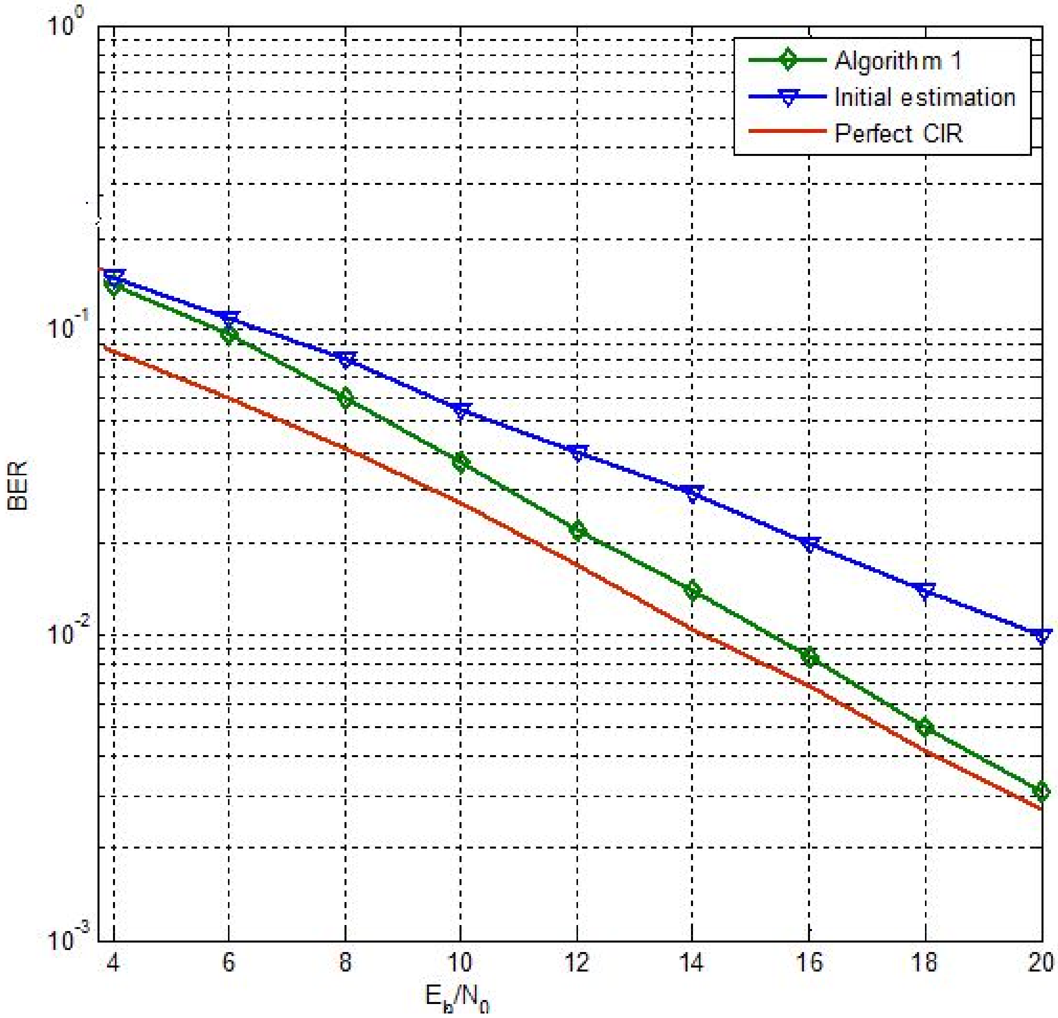


Figure 4.4: BER versus E_b/N_0 (dB) in the 8-path channel model using Algorithm 1

Under time-varying environment (nonstationary), Algorithm1 supersedes the initial estimation algorithm approximately at $E_b/N_o = 4dB$. This performance advantage gets elevated with the increasing value of E_b/N_o. At $BER = 0.03$ in Figure 4.4, the performance advantage of approximately $2.9dB$ is achieved by Algorithm1 over the initial estimation algorithm in terms of E_b/N_o.

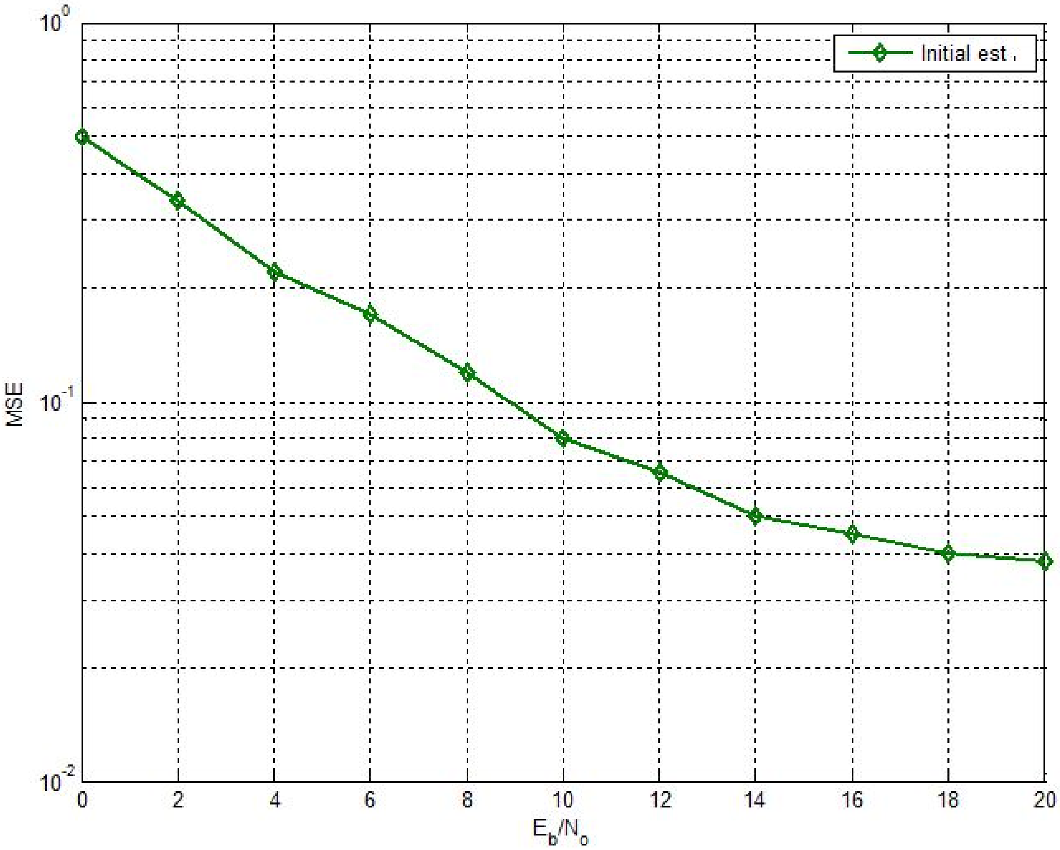


Figure 4.5: MSE versus E_b/N_0 (dB) in the 8-path channel model for initial estimation.

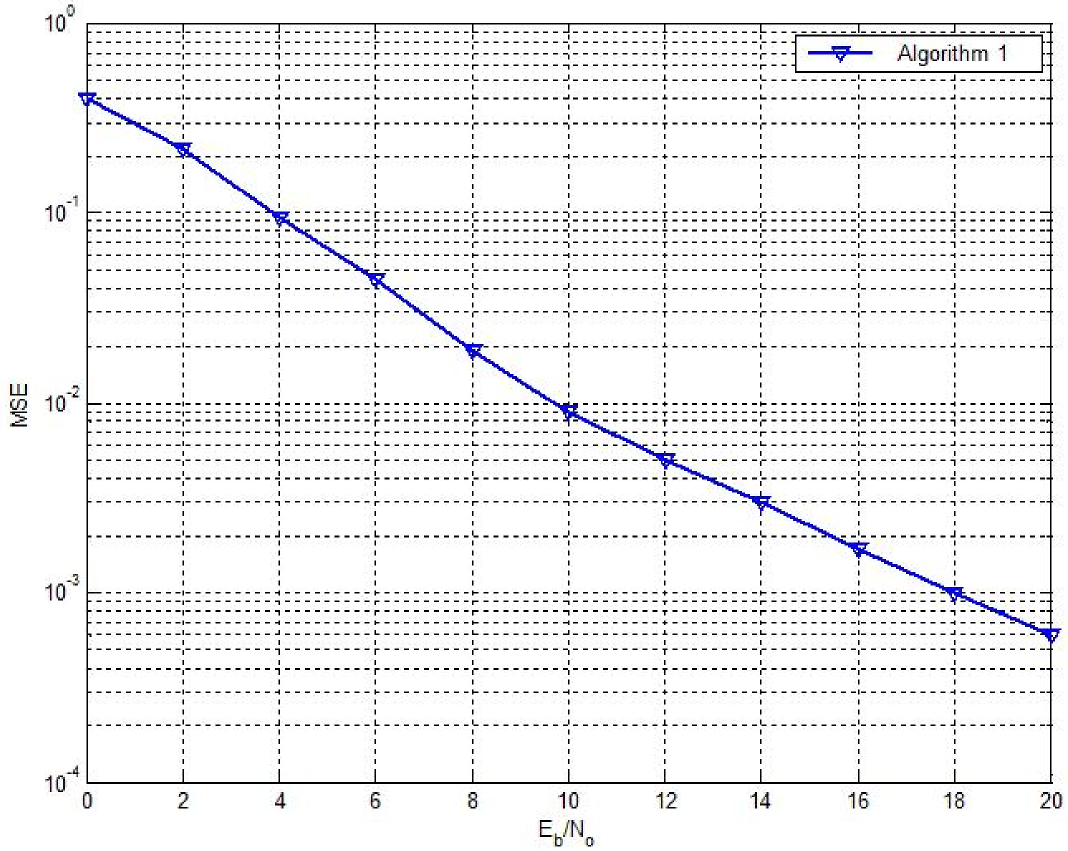


Figure 4.6: MSE versus E_b/N_0 (dB) in the 8-path channel model for Algorithm1

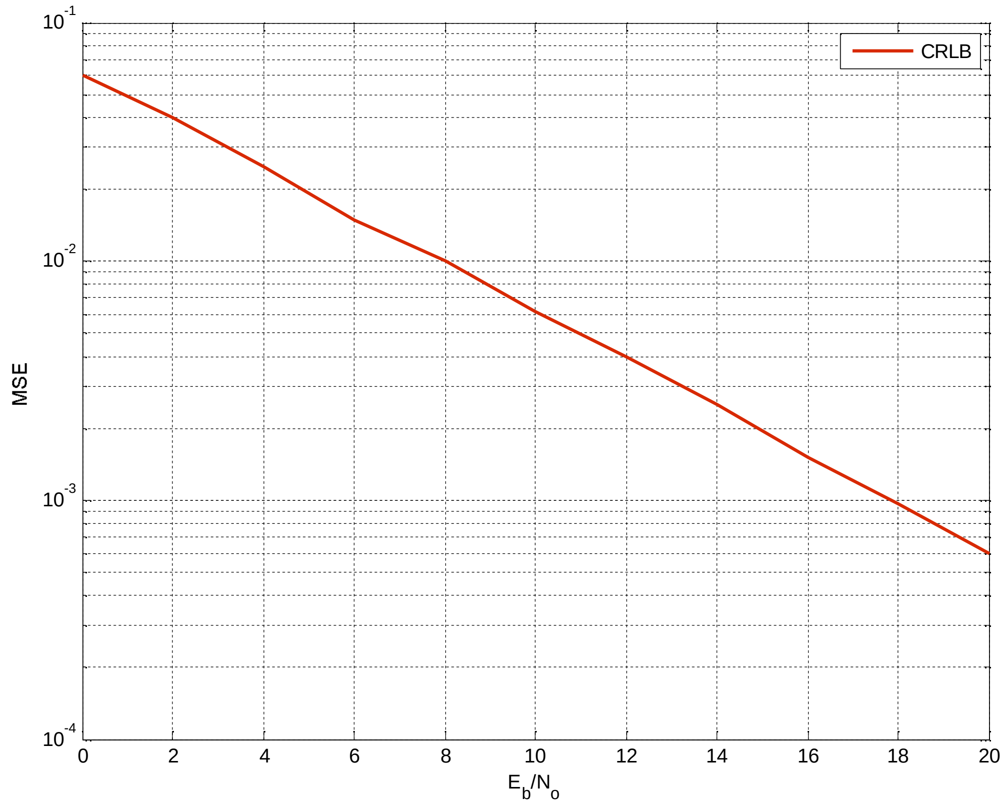


Figure 4.7: MSE versus E_b/N_0 (dB) in the 8-path channel model for CRLB

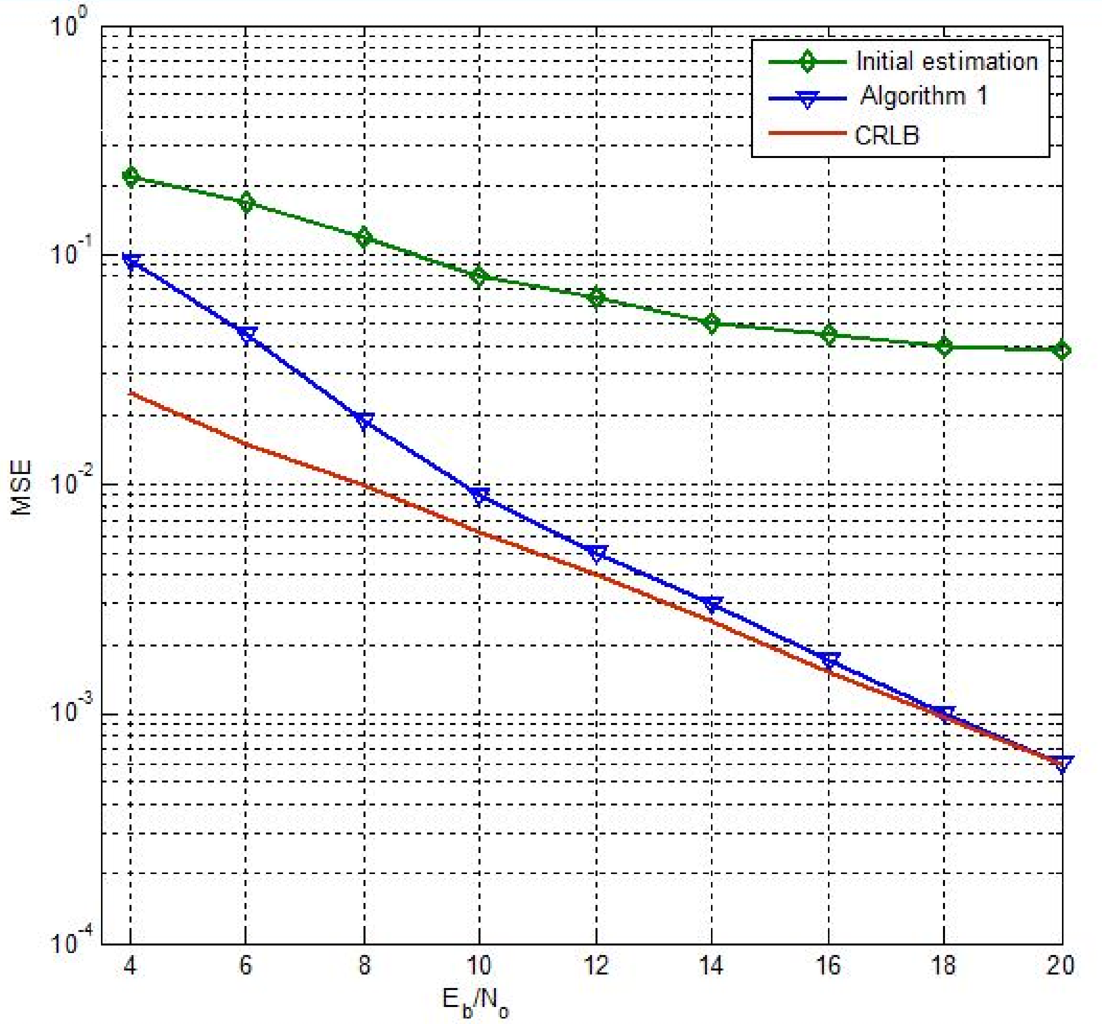


Figure 4.8: MSE versus E_b/N_0 (dB) in the 8-path channel model using Algorithm 1

Under time-varying environment (nonstationary), Algorithm1 based wireless channel estimator outperforms the initial estimation algorithm in terms of lower mean squared channel estimation error. This performance advantage gets elevated with the increasing value of E_b/N_o. At $MSE = 0.1$ in Figure 4.8, the performance advantage of approximately $4.3dB$ is observed by Algorithm1 over the initial estimation algorithm in terms of E_b/N_o. The performance of Algorithm1 approaches CRLB at about $E_b/N_o = 20dB$.

4.1.2 Simulation Results for Algorithm3

In this section, simulation is conducted to showcase the soundness and usefulness of EM-based channel estimation and signal detection procedures. The whole wireless channel transmission bandwidth is 800 kHz, and which is divided into 64 subcarriers (or tones). To make tones orthogonal to each other, symbol period is selected to be 80 microseconds. An additional 20 microseconds CP (N_{CP} = 16) is utilized to provide protection from IFI and ICI due to wireless channel delay spread. Therefore, total OFDM block length is $Ts = 100$ microseconds and subchannel symbol rate is 10 kbaud. The modulation scheme utilized in the underlying OFDM system is QPSK.

Normalization of average channel paradigms should be same. For those OFDM blocks containing pilot symbols, initial estimate of CIR is determined from the wireless channel estimate of previous OFDM block. The fade rate is set $f_d Ts = 0.05$ for following outcomes with maximum Doppler spread $f_d = 500Hz$

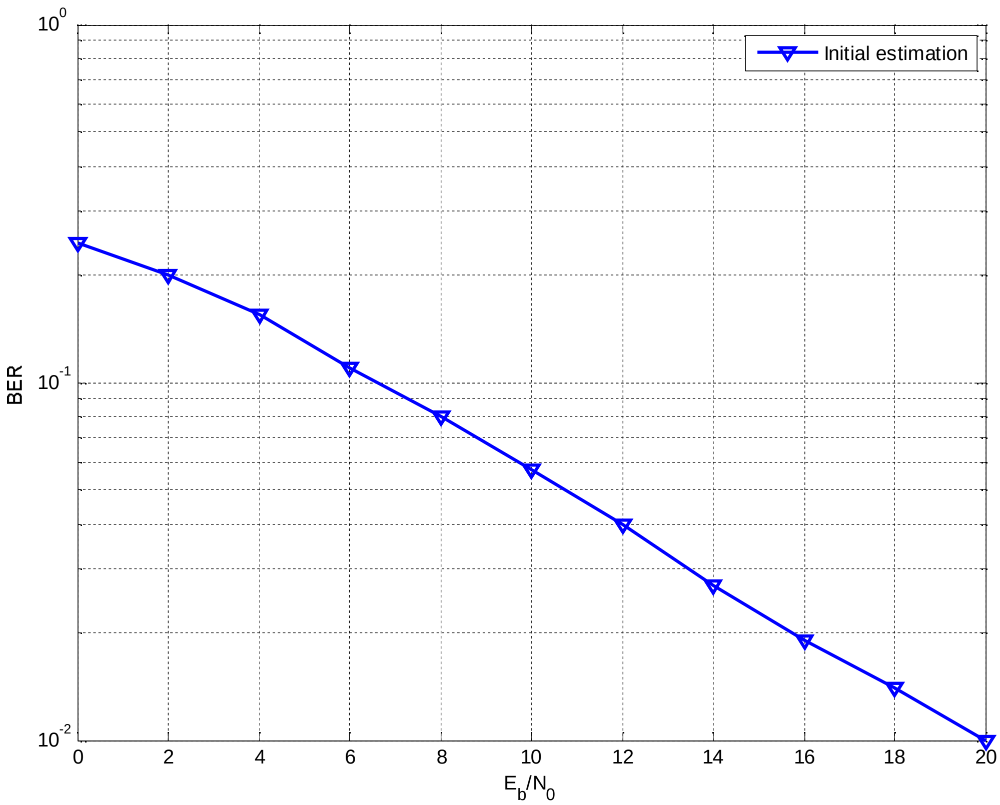


Figure 4.9: BER versus E_b/N_0 (dB) in the 8-path channel model for initial estimation

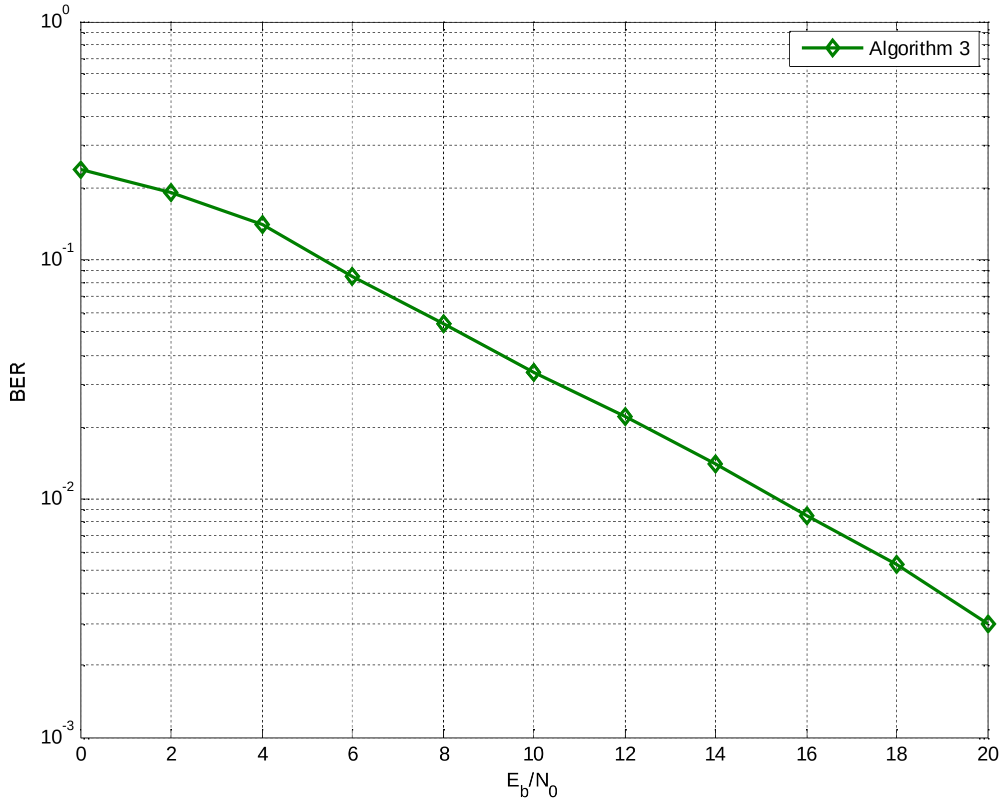


Figure 4.10: BER versus E_b/N_0 (dB) in the 8-path channel model for Algorithm 3

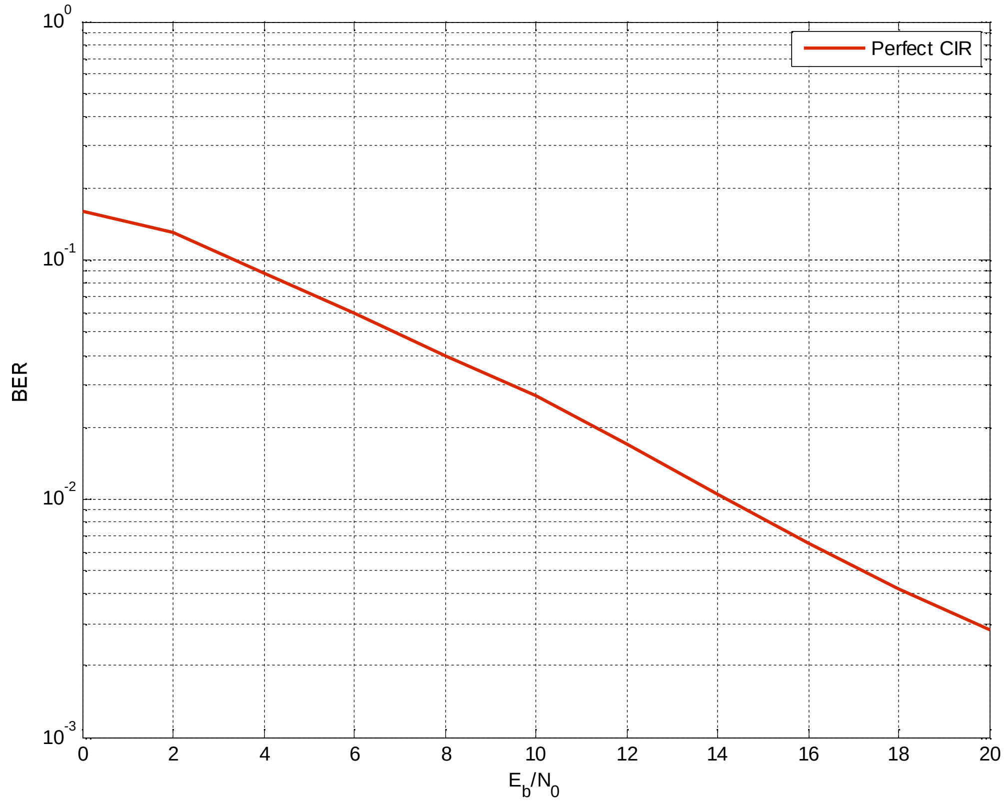


Figure 4.11: BER versus E_b/N_0 (dB) in the 8-path channel model for perfect CIR

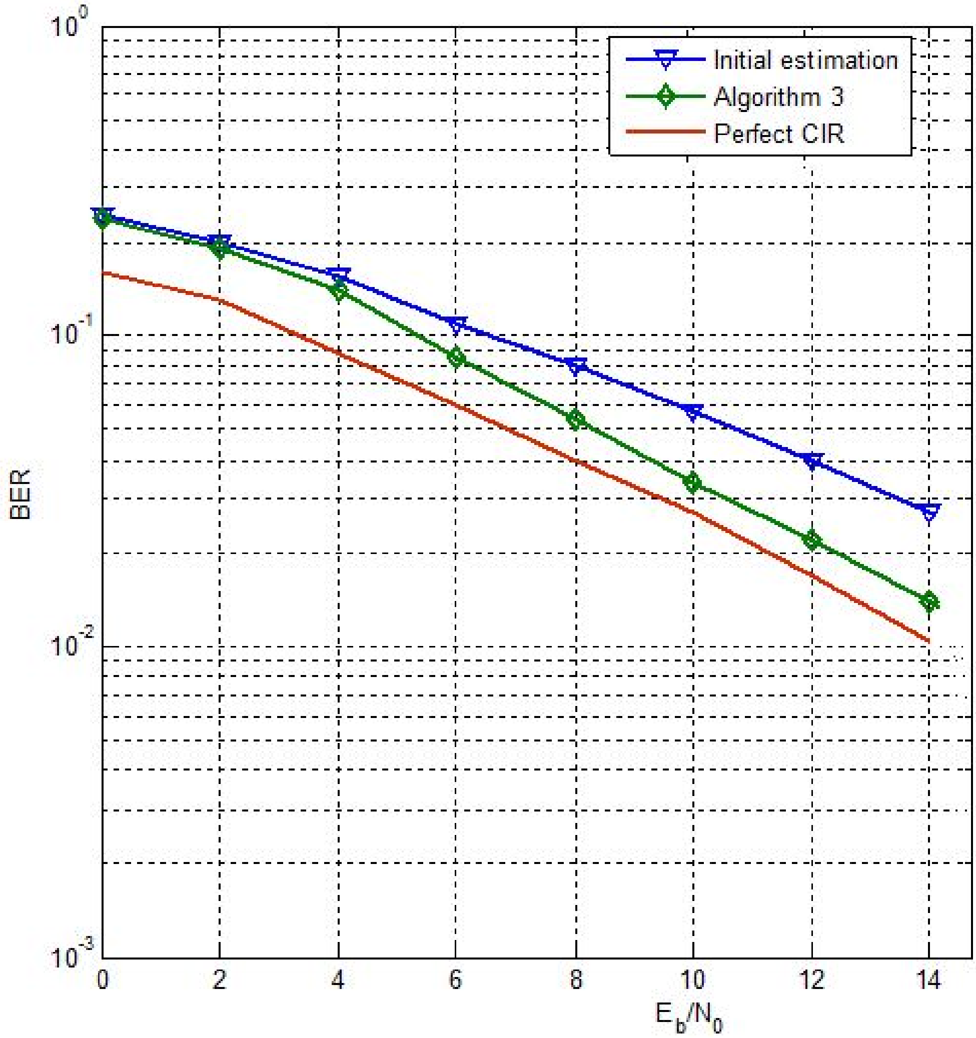


Figure 4.12: BER versus E_b/N_0 (dB) in the 8-path channel model using Algorithm 3

Under time-varying environment (nonstationary), Algorithm3 supersedes the initial estimation algorithm. This performance advantage gets elevated with the increasing value of E_b/N_o. At $BER = 0.03$ in Figure 4.12, the performance advantage of approximately $3.5dB$ is achieved by Algorithm3 over the initial estimation algorithm in terms of E_b/N_o.

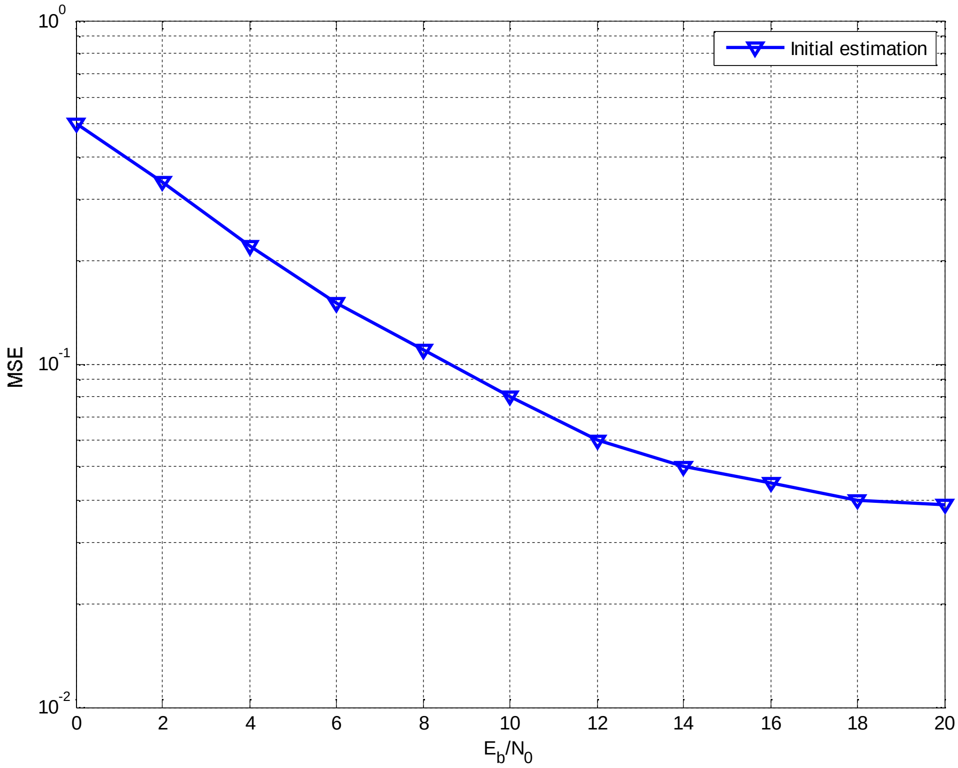


Figure 4.13: MSE versus E_b/N_0 (dB) in the 8-path channel model for initial estimation

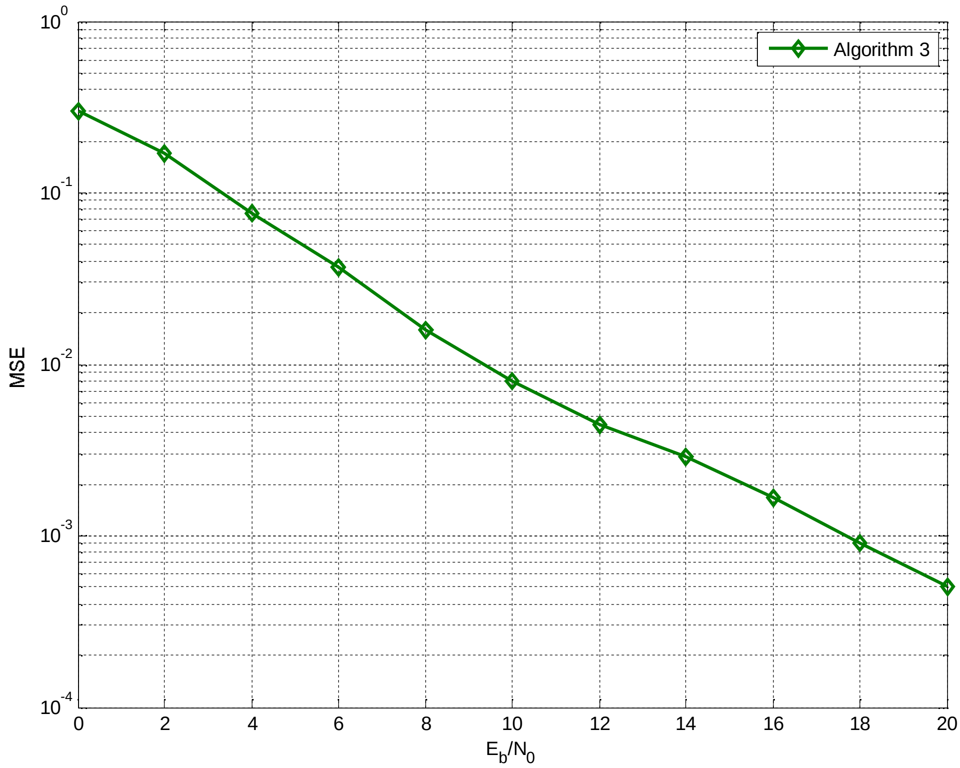


Figure 4.14: MSE versus E_b/N_0 (dB) in the 8-path channel model for Algorithm 3

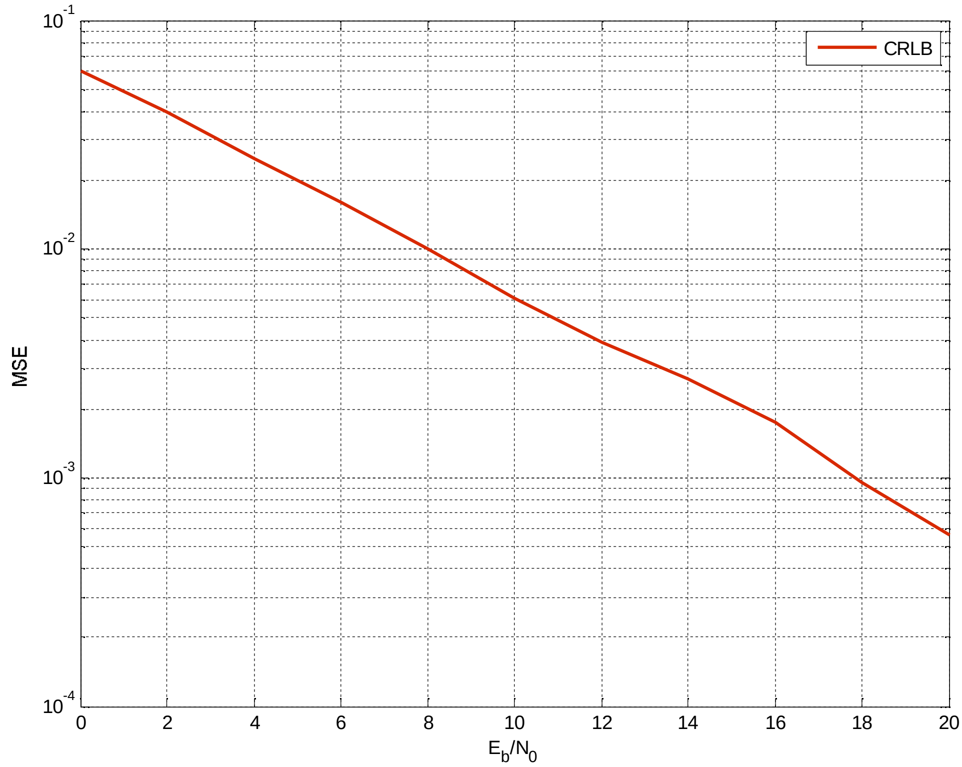


Figure 4.15: MSE versus E_b/N_0 (dB) in the 8-path channel model for perfect CRLB

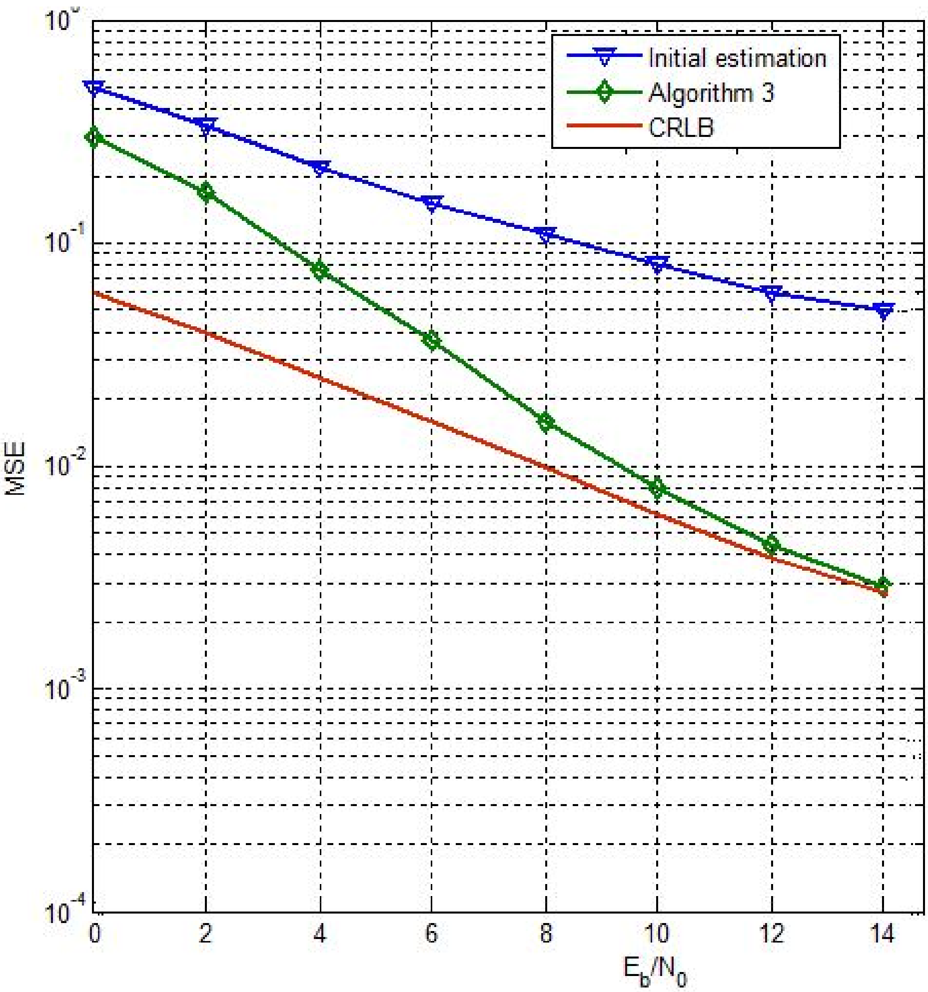


Figure 4.16: MSE versus E_b/N_0 (dB) in the 8-path channel model using Algorithm 3.

Under time-varying environment (nonstationary), Algorithm3 based wireless channel estimator outperforms the initial estimation algorithm in terms of lower mean squared channel estimation error. This performance advantage gets elevated with the increasing value of E_b/N_o. At $MSE = 0.1$ in Figure 4.16, the performance advantage of approximately $4.8dB$ is observed by Algorithm3 over the initial estimation algorithm in terms of E_b/N_o. The performance of Algorithm3 approaches CRLB at about $E_b/N_o = 14dB$. Therefore at high value of E_b/N_o, Algorithm3 performs better than Algorithm1 when we consider CRLB.

4.1.3 Simulation Results for LMS, MMSE, EM, EM-MMSE and QNA Algorithms

In this section, simulation is conducted to illustrate the strength and efficiency of LMS, MMSE, EM, EM-MMSE and QNA EM estimation and signal detection algorithms. The underlying OFDM system considers 128 subcarriers with QPSK digital modulation technique. For simulation, we consider 4-tap multipath wireless channel, which are time-varying in nature. These wireless channel tap-coefficients are simulated using AR(2) method given in [19].

The whole wireless channel transmission bandwidth is 2048 kHz, and which is divided into 128 subcarriers (or tones). To make tones orthogonal to each other, OFDM symbol period is selected to be 62.5 microseconds. An additional 15.625 microseconds CP (N_{CP} = 32) is utilized to provide protection from IFI and ICI due to wireless channel delay spread. Therefore, total OFDM block length is $Ts = 78.125$ microseconds.

Normalization of average wireless channel paradigms should be same. For those OFDM blocks containing pilot symbols, initial estimate of CIR is got from the wireless channel estimate of previous OFDM block. At maximum Doppler frequency of 64 Hz, fade rate is kept $f_d Ts = 0.005$ for following outcomes

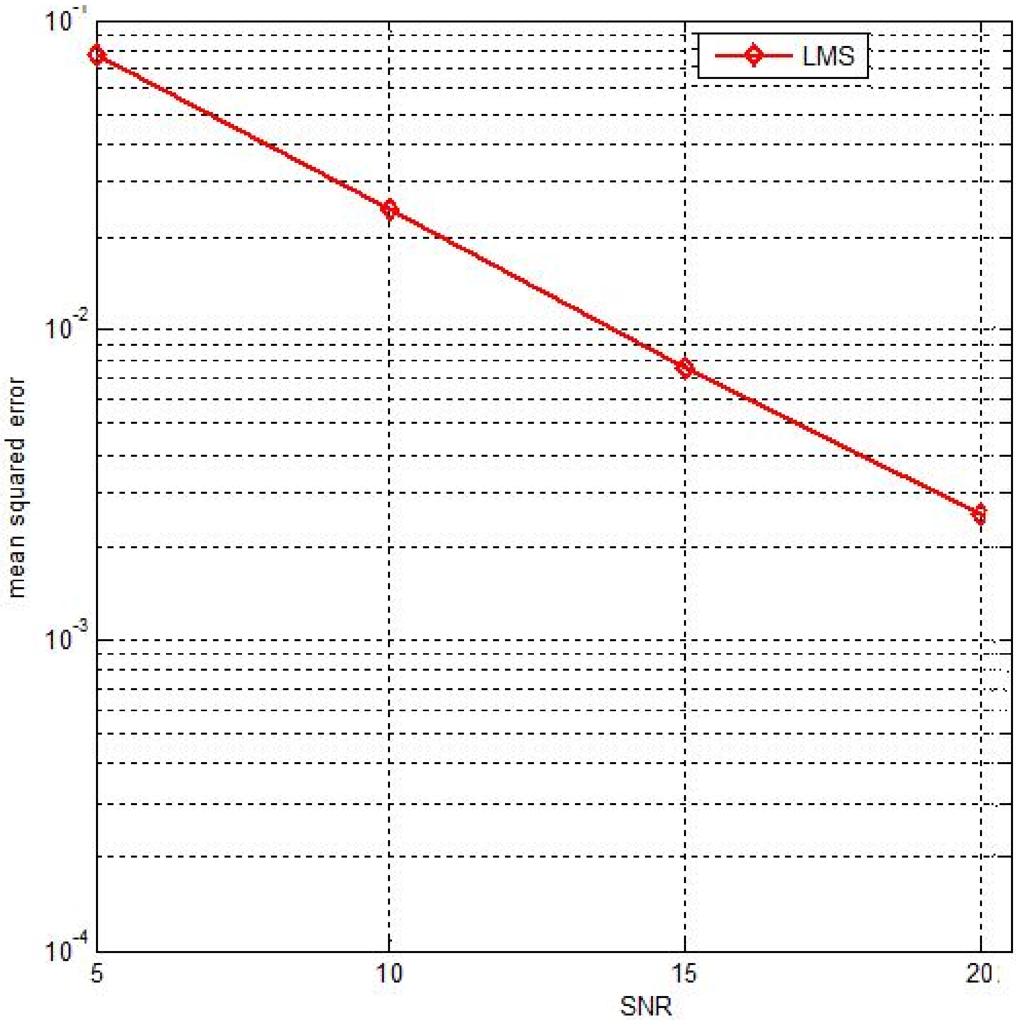


Figure 4.17: MSE versus SNR (dB) for OFDM with LMS estimator based receiver

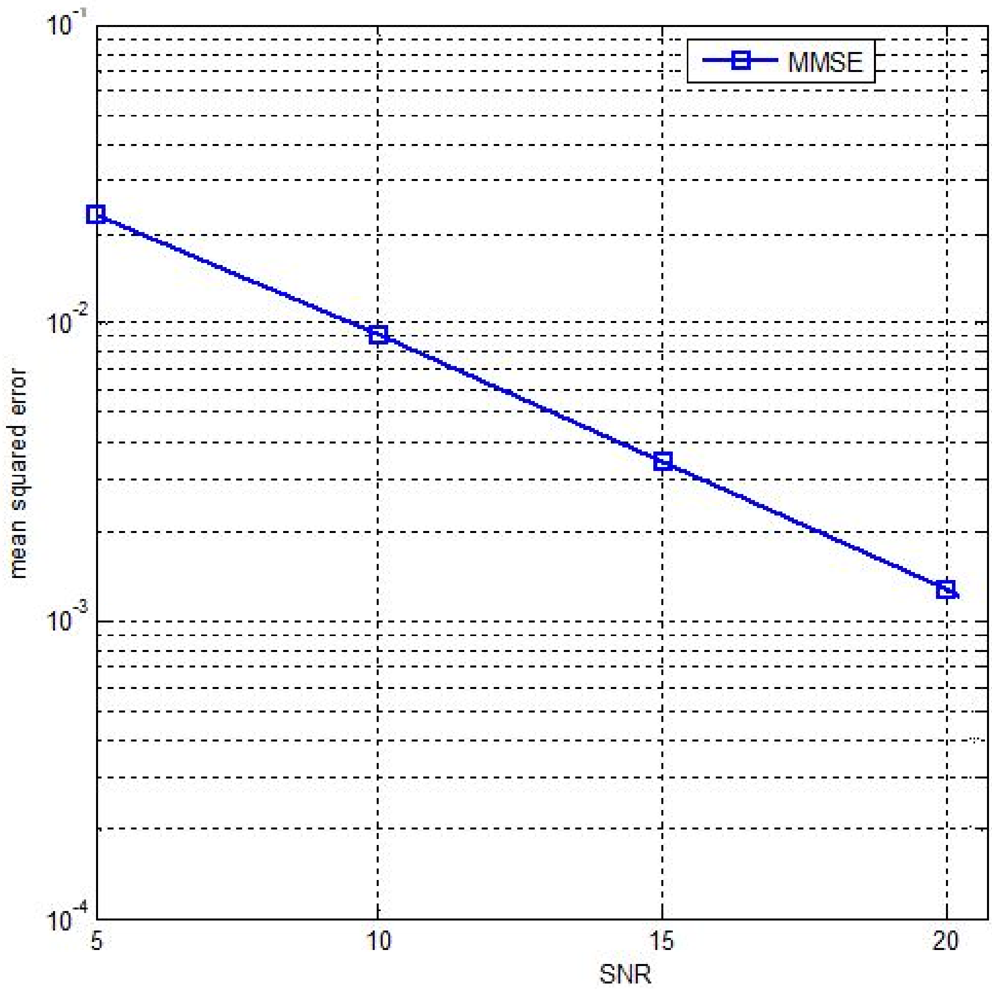


Figure 4.18: MSE versus SNR (dB) for OFDM system with MMSE estimator based receiver

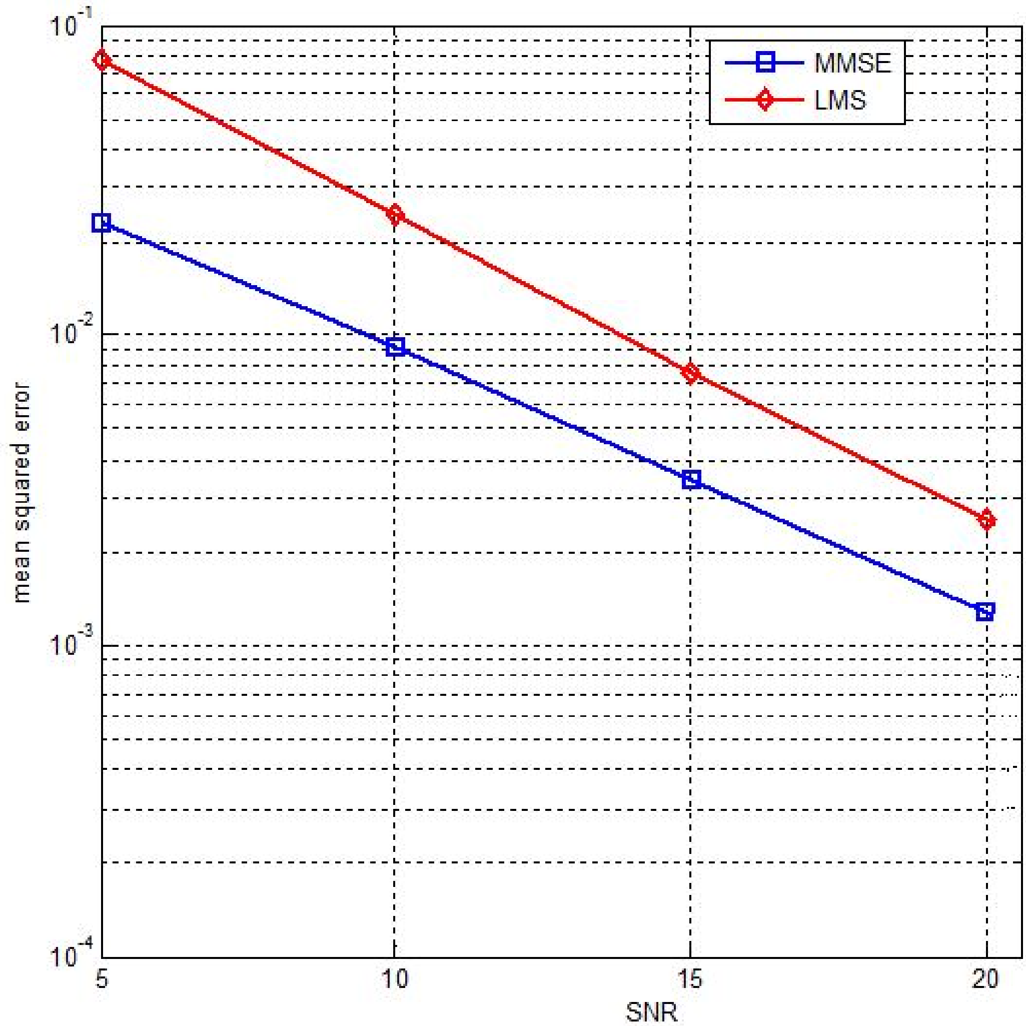


Figure 4.19: MSE vs. SNR (dB) for OFDM System with MMSE / LMS estimator based receiver

Under time-varying environment (nonstationary), MMSE based wireless channel estimator outperforms LMS algorithm in terms of lower mean squared channel estimation error, at all values of SNR. At $MSE = 0.01$ in Figure 4.19, the performance advantage of approximately $4dB$ is observed for MMSE based wireless channel estimator over the least mean square algorithm in terms of SNR.

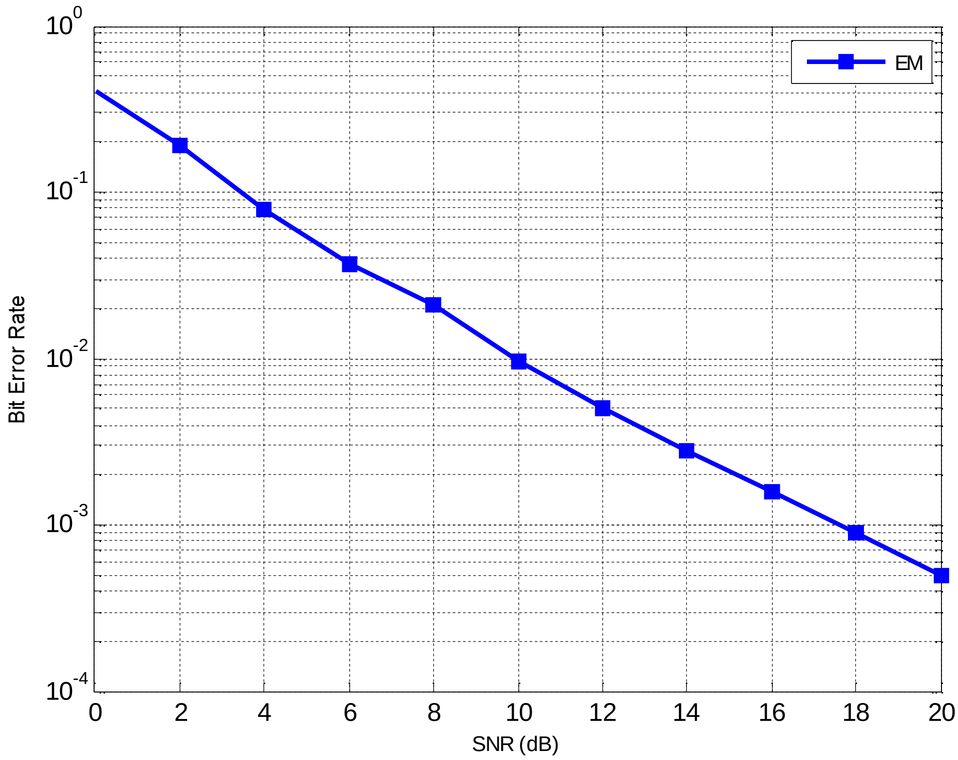


Figure 4.20: BER performance for EM based channel estimator at $f_d Ts = 0.005$

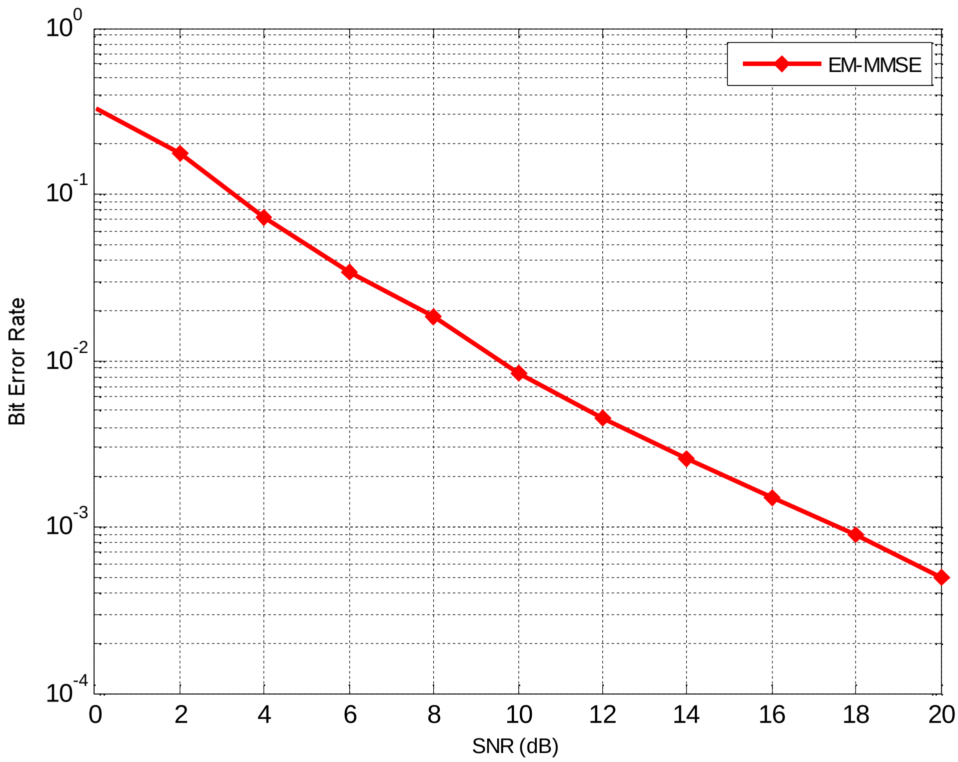


Figure 4.21: BER performance for EM-MMSE based channel estimator at $f_d Ts = 0.005$

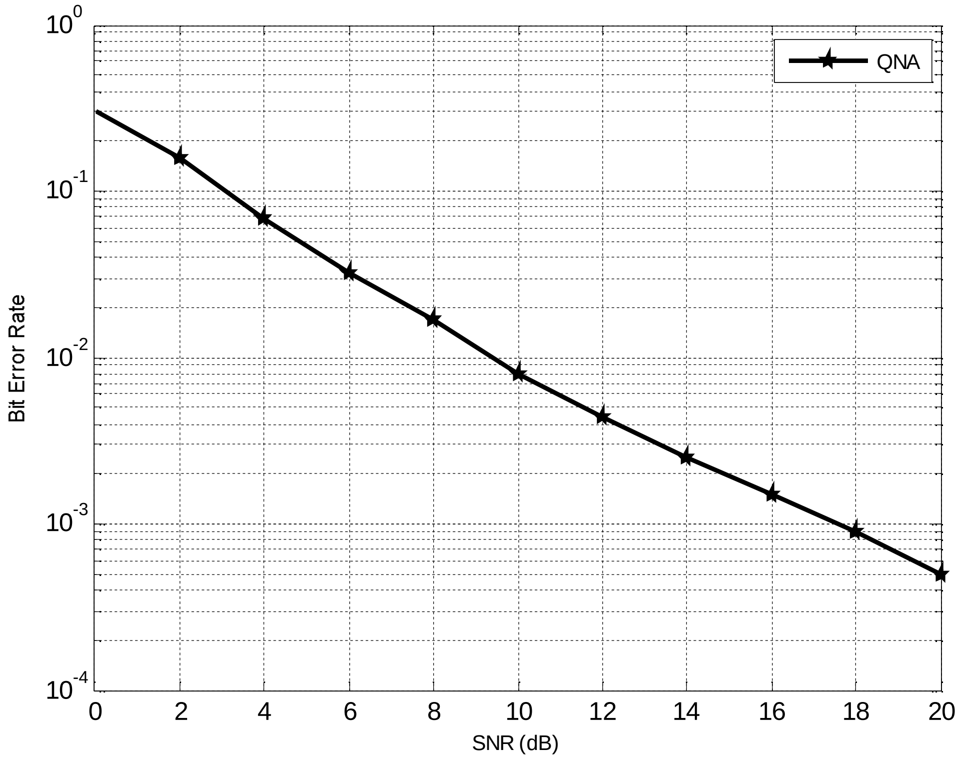


Figure 4.22: BER performance for QNA EM based channel estimator at $f_d Ts = 0.005$

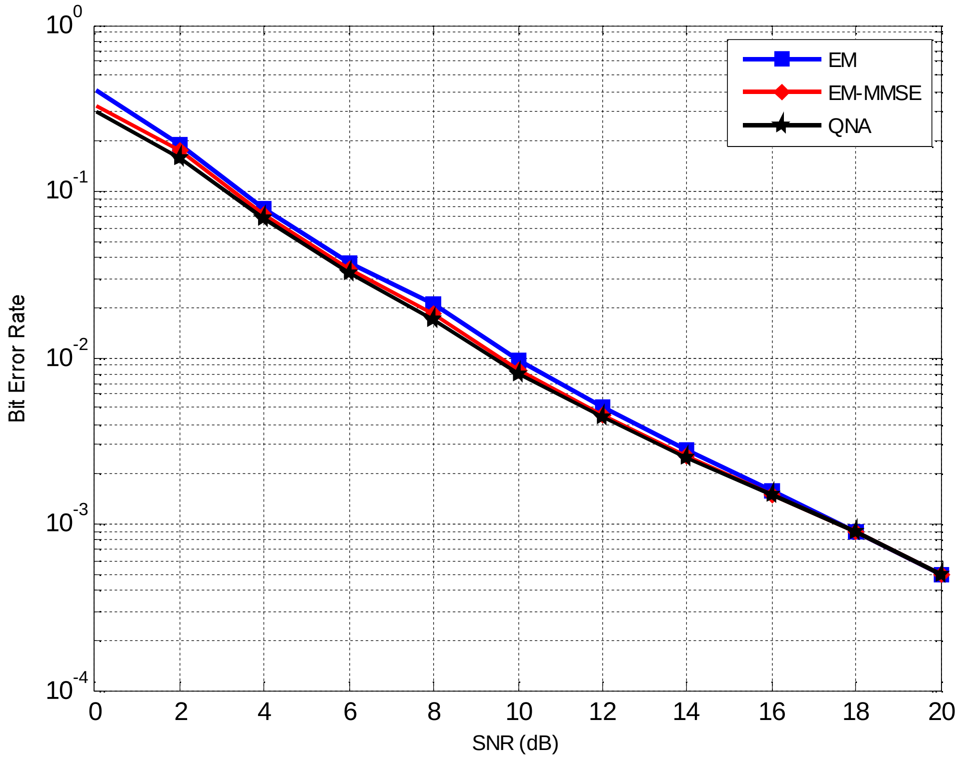


Figure 4.23: Comparison of BER performance for EM, EM-MMSE and Quasi-Newton algorithm (QNA) EM based channel estimation techniques at $f_d Ts$ = 0.005

Under time-varying environment (nonstationary), the bit error rate performance advantages of EM, EM-MMSE and QNA EM algorithms are approximately same at the high SNR values. However, QNA EM algorithm performs marginally better than EM and EM-MMSE algorithm based receivers at the low SNR values, in terms of BER (as depicted in Figure 4.23).

Through computer simulation, it can be inferred that EM-MMSE based estimation method alleviates the number of computations and number of iterations in comparison to the traditional EM method. This inference is similar to the outcomes as reported in [21]. However, it is observed that QNA EM method [26] is computationally more efficient than EM-MMSE method. The Quasi-Newton Acceleration algorithm enhances the convergence rate to great extent in comparison to the traditional EM methods.

CHAPTER 5

CONCLUDING REMARKS AND FUTURE SCOPE

The performance characteristics of different algorithms (LMS, MMSE, EM, EM-MMSE, Quasi-Newton Acceleration EM) are compared and analyzed by using MATLAB simulation. In the presented research work in this dissertation, the performance of wireless channel estimators is appraised under time-varying multipath wireless fading environment. From simulation, following outcomes are inferred for the wireless channel estimation algorithms.

i) The least mean square algorithm fails to perform well under nonstationary environment in comparison to MMSE algorithm based wireless channel estimator.

ii) MMSE algorithm in frequency-domain illustrates a good and robust performance. One main limitation of existing algorithms is large computation complexity because of the iterative nature.

iii) Three EM based iterative procedures are investigated to accurately estimate CIR and demodulate the transmitted symbols in underlying OFDM system. By defining various "complete" information sets for EM procedure, we explore following three algorithms

(1) Estimating frequency response of the wireless channel: Algorithm1

(2) Estimating transmitted signals: Algorithm2

(3) Estimating CIR: Algorithm3

Making use of a modest number of pilot sequences or the wireless channel estimate of previous OFDM block to attain initial estimate, these procedures can accomplish near-optimal estimates after a few iterations. The outcomes are also compared with CRLB to appraise the performance of aforementioned algorithms.

Benefits and limitations of every algorithm have been explored and demonstrated by means of simulation. The simulation observations reveal that first two (estimating frequency response and demodulating transmitted signals straightforwardly) converge at a rate independent of multipath spread. Bypassing the wireless channel estimate by demodulating transmitted symbols straightforwardly, Algorithm 2 is a fast converging procedure, and therefore, exhibits lowest overall computation complexity. Algorithm 3 exhibits least complexity among the three. These three algorithms exhibit potential to work under a rapidly fading environment. However, more emphasis is given on Algorithm 1 and Algorithm 3. The presented outcomes demonstrate that the performance is up to standard only when E_b/N_0 is large. In small signal-to-noise ratio region, these procedures are required to be utilized with wireless channel coding techniques to further modify performance.

i) EM-MMSE based estimation alleviates mathematical computations and number of iterations in comparison to traditional EM procedure, and it is still able to track wireless channel efficiently under time-varying channels. This scheme may be extended to multiple-input multiple-output systems as well.

ii) Quasi-Newton Acceleration EM procedure for channel estimation of OFDM systems is also investigated. This algorithm converges higher rate than conventional EM, procedures because this algorithm showcases a quadratic rate of convergence, which is much more rapid than conventional EM procedure, and it also demonstrates better performance than other linear methods.

Simulation outcomes illustrate that the bit error rate as well as mean squared error of the receiver and wireless channel estimator can be substantially alleviated by these algorithms respectively. It is apparent from outcomes that QNA EM algorithm performs slightly better than EM-MMSE and conventional EM algorithm under the low SNR conditions, but its performance is comparable to later algorithms under the high SNR scenarios. However, QNA methods can expedite the convergence characteristics of EM algorithms [26].

Future work includes the application of investigated wireless channel estimation techniques in the domain of space-time OFDM, space-frequency OFDM and fourth generation wireless communication systems.

REFERENCES

1. Gitlin, R.D., Hayes J.F., and Weinstein S.B., 1992. *Data Communications Principles.* New York: Plenum.
2. Saltzberg, B.R., 1967. Performance of an efficient parallel data transmission system. *IEEE Transaction Communication*, 6, pp. 805-811.
3. Weinstein, S.B. and Ebert P.M., 1971. Data transmission by frequency division multiplexing using the discrete Fourier transform. *IEEE Trans. Communication*, 5, pp. 628-634.
4. Bingham, J.A.C., 1990. Multicarrier modulation for data transmission: An idea whose time has come. *IEEE Communication Magazine*, 28, pp. 5-14.
5. Alard, M. and Lassalle, R., 1987. Principles of modulation and channel coding for digital broadcasting for mobile receivers. *EBU Review,* 224, pp. 47-68.
6. Rabiner, L.R. and Gold, B., 1975. *Theory and Application of Digital Signal Processing.* Englewood Cliffs, NJ: Prentice Hall.
7. Klein, A. and Mohr, W., 1995. Measurement-based parameter adaptation of wideband spatial mobile radio channel models. *IEEE 4th International Symposium on Spread Spectrum Techniques and App. Proceeding,* ISSSTA'95, pp. 91-97.
8. Bahai, A.R.S., 1993. Estimation in randomly time-varying systems with applications to digital communications. *Ph.D. Dissertation, University of California at Berkele,* USA..
9. Dempster, A. P., Laird, N. M. and Rubin, D. B., 1977. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society (B)*, 39 (1), pp. 1–38.
10. Moon, T. K., 1996. The expectation-maximization algorithm. *IEEE Signal Processing Magazine,* 13 (6), pp. 47–60.
11. Nilsson, R., Edfors, O., Sandell, M. and Borjesson, P.O., 1997. An analysis of two-dimensional pilot-symbol assisted modulation for OFDM. in *Proc. IEEE International Conference on Personal Wireless Communications (ICPWC '97)*, pp. 71–74.

12. Li, C. and Roy, S., 2001. Subspace based blind channel estimation for OFDM by exploiting virtual carrier. *in Proceeding IEEE Global Telecommunications Conference (GLOBECOM ’01)*, 1, pp. 295–299.
13. Yang, B., Letaief, K. B., Cheng, R. S. and Cao, Z., 2001. Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling. *IEEE Trans. Communications*, 49(3), pp. 467–479.
14. Chen, P. and Kobayashi, H., 2002. Maximum likelihood channel estimation and signal detection for OFDM systems. *in Proceeding IEEE International Conference on Communications (ICC ’02)*, 3, pp. 1640–1645.
15. Feder, M. and Weinstein, E., 1988. Parameter estimation of superimposed signals using the EM algorithm. *IEEE Transaction Acoustics, Speech, and Signal Processing*, 36 (4), pp. 477–489.
16. Miller, R.W. and Chang, C.B., 1978. A modified Cram´er-Rao bound and its applications. *IEEE Transactions on Information Theory*, 24(3), pp. 398–400.
17. Jain, Sachin, 2006. EM-MMSE based Channel Estimation for OFDM Systems. *M.E.. Dissertation, ECED, Indian Institute of Technology Roorkee*, India.
18. Hsieh, M. and Wei, C., 1998. Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels. *IEEE Transaction Consumer Electron*, 44 (1), pp. 217-225.
19. Kohli, A.K., Mehra, D.K., 2006. Tracking of time-varying channels using two-step LMS-type adaptive algorithm. *IEEE Transactions on Signal Processing*, 54(7), pp. 2606-2615.
20. Ma, X., Kobayashi, H. and Schwartz, S.C., 2001. EM-based channel estimation of OFDM. *in Proc. IEEE Conference on Communications, Computers and Signal Processing*, 2, pp. 449 – 452.
21. Jain, S., Gupta, P. and Mehra, D. K., 2006. EM-MMSE based channel estimation for OFDM systems. *in Proceeding IEEE International Conference Industrial Technology*, pp. 2598 – 2602.
22. Ma, X., Kobayashi, H. and Schwartz, S.C., 2004. EM-based channel estimation algorithms for OFDM. *EURASIP Journal of Applied Signal Processing*, 10, pp. 1460 – 1477.

23. Boyles, R.A., 1983. On the convergence of the EM algorithm. *J. Roy. Sta. B,* 45 (1), pp. 47-50.

24. Wu, C., 1983. On the convergence properties of the EM algorithm. *Ann Statist.*, 1(1), pp. 95-103.

25. Proakis, J.G., 1995. *Digital Communications*. McCraw Hill, 3rd edition.

26. Jamshedian, M., Jennrich, R.I., 1997. Acceleration of the EM algorithm using Quasi-Newton methods. *J. R. Statist. Soc. B.* 59(3), pp. 568-587.