Study of Baryon Decuplet Particles

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Submitted by
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CERTIFICATE

This is to certify that the thesis entitled **"Study of Baryon Decuplet Particles"** being submitted by **Vikas (Roll No.301204013)** of M.Sc. (physics), Thapar University, Patiala. He has carried out this thesis by himself under my supervision. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

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I hereby declare that the Dissertation **"Study of Baryon Decuplet particles"** is the work carried out by me under the supervision of **Dr. Alka Upadhyay**. I have not submitted this work anywhere else for the award of any degree.

Vikas

Acknowledgement

My third semester of M.Sc. (Physics) had started, then I saw the courses in this semester. There was a course named Particle Physics (High Energy Physics), which is very surprising for me, because I had no knowledge about this subject before this semester. Next day when ALKA Madam came, and delivered the lecture, then I came to know about the science, which is beyond the proton and neutron or simply the nuclear science. That lecture gave me the knowledge of fundamental particles. ALKA madam had taught this subject in a very interesting manner. My friend Ashish also took interest in this subject. This subject undoubtedly very interested but not so easy for me. With the help of my teacher I could understand many basic concepts in this field.

So, In fourth semester I decided to take my project work with Dr.Alka Upadhyay. She solved my many problems in this field. She has done her Ph.D. from Vadodra University and PDF from IISc.(Bangalore).She is a brilliant teacher and also a very nice person. Her nature is very polite. Also a great role to complete my thesis is of Meenakshi Madam. She has given me her precious time to solve calculations and about Mathematician. Ashish has also helped me a lot.

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ABSTRACT

We suggest a general formalism to treat a baryon as a composite system of three quarks and a “sea”. In this formalism, the sea is a cluster which can consist of gluons and quark- antiquark pairs. Historically, the static SU (6) quark model provide a good description of hadrons .baryons (mesons) are color singlet combination of three quarks (quarks – antiquark pairs) in the appropriate flavour and spin combination. The space time part of a hadrons wave function can be determined by using a specific model of confinement, e.g., the bag model, simple harmonic oscillator model, or other phenomenological models. The existence of a quark gluon interaction implies that quark antiquark pairs can be created by the virtual gluons emitted from valence quarks. These quark antiquark pairs are so called sea quarks. Usually the, “sea” means a contribution of the virtual gluons and sea quark – antiquark pairs. Although deep inelastic muon nucleon scattering shows that the sea components (quark –antiquark pairs and gluons indeed exist and play a very important role (e.g. gluons carry about one- half of the nucleon momentum and sea dominates small-x behavior of structure functions), it is commonly believed that in the low energy regime, static properties of hadrons are dominated by their valence components. However, it has been shown that the sea contributions may change the structure of hadrons and modify their low energy properties. Using the QCD interaction Hamiltonian and MIT bag model, Donghue and Golowich (DG) calculated the probabilities of different sea quark components in hadrons. In these models a mixing of q^3 and q^3+ gluon, in which a color 8_c gluon coupled to an 8_c q^3 state to form a color singlet.

However, the “sea” could be a gluon or a quark antiquark pair, or even more complicated, for instance a multi gluon state, multi quark antiquark pairs or gluon(s) plus quark antiquark pair(s) .Since the baryons should be colorless and a q^3 state can be in color states $1_{c,}$ 8_c and $10_{c,}$ the “sea ‘should also be in corresponding color states to form a color singlet baryon. In this work I have constructed the Wave Function for Baryon Decuplet Particles under the constraint of color singlet and S Wave.

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2. Theory of Statistical Model (Sea Model)

CHAPTER: 1 INTRODUCTION

The universe is filled with matter that exists in different states, namely, the solid, liquid, gas and plasma. The question now arises – what the matter is made of?

The answer of this question lies primarily in the realm of a particle physicists, who keep on trying to know the ultimate constituent (constituents) of matter. This ultimate or the fundamental constituent which may also be called constituent of matter at its smallest scale of size is also known as the elementary particle or the fundamental particle. We mean by a particle an object that behaves like a point, which does not divided further. This point is supposed to carry a well defined mass and a charge.

What must be considered an elementary particle depends on our knowledge on the subject in a particular period of time. If we go back to the period when only meta physical thoughts without much more scientific analysis dominated , we would say that air, water, fire, sky and earth were the fundamental constituent of every matter. Time to time according to our knowledge, the fundamental particles goes on changing.

1.1 Journey of fundamental particle in high energy physics.

- 1800: William Hershel discovers "heat rays"
- 1801: Johann Wilhelm Ritter [1] made the hallmark observation that invisible rays just beyond the violet end of the visible spectrum were especially effective at lightening soaked paper. He called them "oxidizing rays" to emphasize chemical reactivity and to distinguish them from "heat rays" at the other end of the invisible spectrum (both of which were later determined to be photons). The more general term "chemical rays" was adopted shortly thereafter to describe the oxidizing rays, and it remained popular throughout the 19th century. The terms chemical and heat rays were eventually dropped in favor of ultra violet and infra red radiation, respectively.

- 1895: Discovery of the ultraviolet radiation below 200 nm, named vacuum ultraviolet (later identified as photons) because it is strongly absorbed by air, by the German physicist victor Schumann [2].

- 1895: X Rays produced by Wilhelm Rontgen [3] (later identified as photons)
- 1897: Electron discovered by J.J. Thomson [4].
- 1899: Alpha Particle discovered by Ernest Rutherford in Uranium Radiation [5].
- 1900: Gamma Ray (a high-energy photon) discovered by Paul Villard [6] in uranium decay
- 1911: Atomic nucleus identified by Ernest Rutherford based on scattering observed by Hans Geiger and Ernest Marsden [7].
- 1919: Proton discovered by Ernest Rutherford [8].

- Neuron discovered by James Chadwick [9] (predicted by Ernest Rutherford) [10]
- 1932: Anti Electron (or positron) the first antiparticle, discovered by Carl D. Anderson [11] (proposed by Paul Dirac in 1927 and by Ettore Majorana 1928)
- 1937: Muon (or mu lepton) discovered by Seth Neddermeyer, Carl D. Anderson, J.C. Street, and E.C. Stevenson, using cloud chamber measurements of cosmic rays [12] (It was mistaken for the pion until 1947)[13].
- 1947: Pion (or pi meson) discovered by C.F. Powell's group (predicted by Hideki Yukawa in 1935) [14].
- 1947: Kaon (or K meson), the first strange particle discovered by George Dixon Rochester and Clifford Charles Butler [15].
- 1947: Λ^0 discovered during a study of cosmic ray interactions.
- 1955: Antiproton discovered by Owen Chamberlain, Emilio Segre, Clyde Wieg and Thomas Ypsilantis.[16]
- 1956: Electron – Antineutrino detected by Frederick Reines and Clyde Cowan (proposed by Wolfgang Pauli in 1930 to explain the apparent violation of energy conservation in beta decay)[17]. At the time it was simply referred to as *neutrino* since there was only one known neutrino.
- 1962: Muon Neutrino (or mu neutrino) shown to be distinct from the electron neutrino by a group headed by Leon Lederman. [18]
- 1964: Xi baryon [19] discovery at Brookhaven National Laboratory
- 1969: Partons internal constituents of hadrons) observed in deep inelastic scattering experiments between protons and electrons at SLAC [20] [21] this was eventually associated with the quark model (predicted by Murray Gell Mann and George Zweig in 1964) and thus constitutes the discovery of the up quark, down quark and strange quark.
- 1974: J/Ψ meson discovered by groups headed by Burton Richter and Samuel Ting, demonstrating the existence of the Charm quark [22] [23] (proposed by James Bjorken and Sheldon Lee Glashow in 1964 [24])
- 1975: Tau discovered by a group headed by Martin Perl [25].
- 1977: Upsilon Meson discovered at Fermi Lab, demonstrating the existence of the Bottom quark [26](proposed by Kobayashi and Maskawa in 1973)
- 1979: Gluon observed indirectly in three jet events at DESY [27].
- 1983: W and Z boson discovered by Carlo Rubbia, Simon Van der Meer and the CERN UA1 collaboration [28] [29] (predicted in detail by Sheldon Glashow Abdus Salam and Steven Weinberg)
- 1995: Top quark discovered at Fermi Lab[30,31].
- 1995: Ant hydrogen produced and measured by the LEAR experiment at CERN [32].
- 2000: Tau neutrino first observed directly at Fermi lab [33].
- 2011: Antihelium-4 produced and measured by the STAR detector the first particle to be discovered by the experiment

- 2012: A particle exhibiting most of the predicted characteristics of the Higgs boson discovered by researchers conducting the Compact Muon Solenoid and ATLAS large compared experiments at CERN's Large Hadron Collider [34].

Therefore to investigate this point object (particles), our studies should be at very small distances. Interactions of the particles must be described by quantum mechanics and the present state of experimentation has allowed the study of physics at distances scales down to about 10^{-16} cm. These studies typically require high energy, in fact the energies in such particle possesses are frequently large compared to the masses of the particle involved, implying relativistic motion. As a consequence, the production of new particles through the interaction is typical, since the large kinetic energy carried by the interacting particles can provide the energy for the creation of additional particles.

To incorporate relativity into quantum mechanics, will require some new ideas. In particular, the requirement of relativistic invariance is a strong constraint on how we can picture the interactions between the particles. Basic interactions must be local in character. There are no instantaneous interactions at a distance. We thus picture a particle process or quantum event as an interaction at a point in space – time of objects of very small size (nearly fundamental).

In particle physics all the particle obey the same quantum rules, which replace the classical picture of a particle. In particle physics we talk about the field of particle rather a single particle.

The basic quantum event allows for the creation and absorption of particles, as first seen most clearly in the photo electric effect. The particles involved carry energy and momentum, as well as other attributes such as spin and charge. These are absorbed or created fully in the event and the possible values are governed by the allowed quantum states for each particle type. Although the interactions are local, the quantum particles are not fully localized, but rather their “presence” is spread over space time (e.g. described by a wave function). Thus calculating the effect of the local interactions in real situations always involves adding contributions spread over space time.

1.2 What do we measure:-

The study of the forms and interactions of matter is based on observations of natural systems and on experiments arranged to create controlled interactions. The latter, using large accelerators have become the primary research tool for looking at matter at very short distance scales. The present state of experimentation has probed energy scales up to approximately 200GeV. This corresponds to distances on the order of 10^{-16}cm. For some type of measurements we understand the physics down to 10^{-17}cm. The types of observations typically made in experiments are characterized below [35].

1.21 The particle spectrum:-

This involves the determination of the base states consisting of a single stable isolated particle, along with any quantum numbers needed for a complete description of these energy eigen- states. Such a particle may be a composite object. These states are eigen states of the Hamiltonian operator that determine the time evolution of the system; they provide information on the objects and interactions the Hamiltonian describes.

1.22 Scattering of particles:-

In addition to seeking the spectrum of individual particles, we can create collisions between particles. This leads to measuring the results of a scattering experiment. Here beams of particles are directed onto each other or onto stationary targets. The resulting collisions allow us to discover new final state particles, as well as the characteristics of the interactions of the initial particles.

A beam of scattering experiments typically originates with either low energy electrons stripped from materials, or protons from ionized hydrogen. These particles are accelerated and can be used directly in experiments, or they can be used to make beams of secondary particles created in collisions with intermediate targets. These secondary beams can then be used in scattering experiments.

The final collision of interest involves two particles at a time, one from the beam and one from the final target. Prior to collision, these were two widely separated, non interacting particles. Following the collision, we measure the final particles at a distance far removed from the collision region. Here the state can again be isolated particles. To achieve maximum total energy, the target is often chosen to be a beam of high momentum particles.

The states used to describe the motion of the initial and final particles are generally be momentum eigen states. Generally we determine the momentum to find out the final state.

To complete the space time description of a momentum eigen state requires specification of the spin projection along the direction motion, called the helicity. Final states of different helicity are distinguishable and do not interfere. For the initial colliding particles the states can be specially prepared to have unequal populations of the various helicities providing a polarized initial state. In the common case, where nothing is done to preferentially populate the spin states, the initial particles contain an incoherent statistical mixture of all helicities, often reflecting the state from the initial particle source.

Since the evolution of the system is determined by the quantum principles, the description of the scattering process is contained in amplitude that is a function of the initial and final momenta and helicities. i.e.

$$A\,(\vec{P}_{1,}\lambda^{i}_{1}, \vec{P}_{2}\,,\lambda^{i}_{2};\ \vec{k}_{1}\,,\lambda^{f}_{1};\_\,\_\,\_\,\_\,\_\,;\vec{k}_{n}\,,\lambda^{f}_{n}\,)$$

Where P_{1}, λ^{i}_{1}, P_2, λ^{i}_{2} are the initial momenta and helicities and k_1, λ^{f}_{1};_ _ _ _ ;k_n ,λ^{f}_{n} are the analogous quantities for the final particles, for the case of n final particles. Rates are determined by the square of the amplitude yielding a density of events in the momentum space describing the n final particles. The calculation must yield an answer that is Lorentz covariant, which will provide a constraint on how the momenta appear in the amplitude. Finally the amplitude must be linear in the spin degrees of freedom by the linearity requirement. In the case of identical particles in the final state, the amplitude must reflect the spin – statistics relation obeyed by all systems, including identical complex composite systems emerging as separate isolated particles. If particles I and j are identical, the amplitude must satisfy

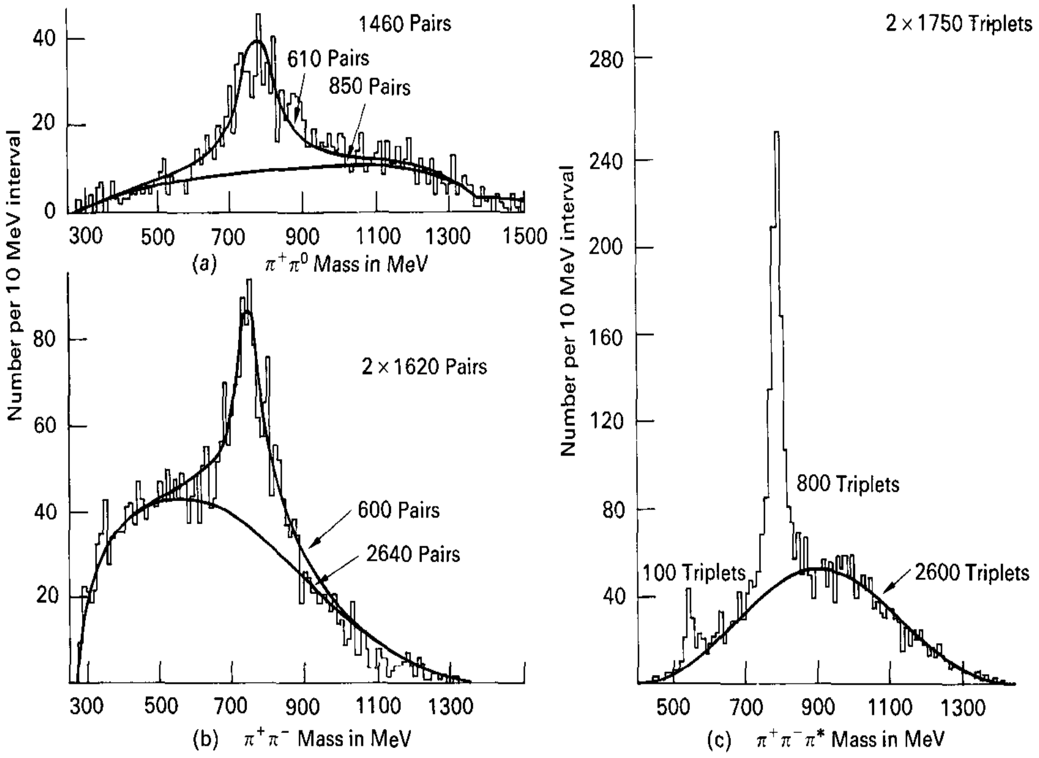
$$A(\_\_\_ \,;\, \vec{K}_i , \lambda^{f}_{i} \_\_\_ \,;\, \vec{K}_j , \lambda^{f}_{j} \_\_) = +- A(\_\_\_ \,;\vec{K}_j , \lambda^{f}_{j};\_\_\_;\, \vec{K}_i, \lambda^{f}_{i} \_)$$

With the positive sign for particles of integral spin (called bosons) and the negative sign for particles of half integral spin (called fermions). Care must also be taken that physically indistinguishable configurations are not counted more than once when calculating rates by integrating over the momentum space.

1.23 Production of resonances:-

In a hadron - hadron reaction, the probability that the two participating hadrons will really interact depends on the energy involved in the interaction process. The energy involved means the rest energies plus kinetic energies of hadrons participating in the reactions. If the amount of energy involved changes, the probability of interaction of two hadrons also changes; it may increase or decrease with increasing energy.

However at certain values of energy, the reaction (interaction) probability is observed to increase sharply and the reaction is much more likely to occur at these values than at other energies. Let us assume that in a hadron –hadron interaction the energy involved was such that the interaction did take place. If the interaction process is scattering the incoming and outgoing hadrons would be same and in the case of other reactions outgoing particles are different from the incoming. The mechanism is that when incoming hadrons have interacted they first join together to form a new hadron, which lives for a very short time ($\approx 10^{-23}$s) and then decays. The decaying of this intermediate hadron may again produce the incoming pair of hadrons or new particles may come out. These short lived hadron states are called resonances. They are called resonances because the probability of their formation is maximum at certain values of energy.

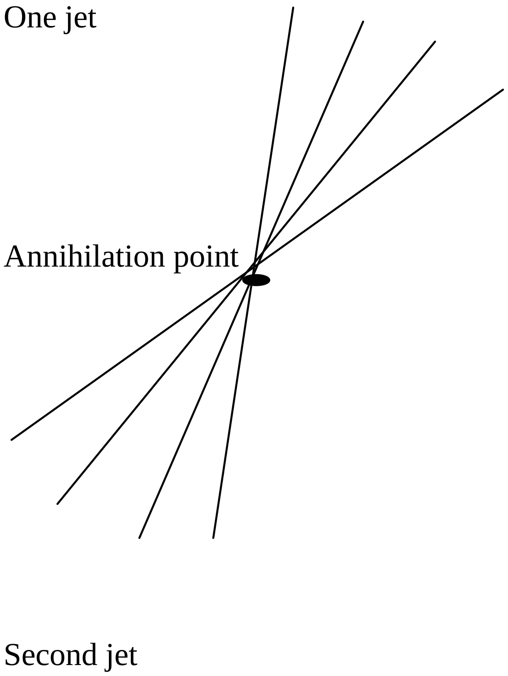


Mass distributions of (a) $\pi^+\pi^0$ pairs from reaction (b) $\pi^+\pi^-$ pairs from reaction. The smooth curves indicate the distributions expected from the phase space (c) The π^+, π^-, π^0 invariant mass spectrum from reaction. The narrow peak at 785 MeV corresponds to the $\omega \rightarrow 3\pi$ (From Alff et al. ,1962)

1.24 Jets at high energies:-

In very high energy scattering processes, producing strongly interacting particles, the number of particles produced can be very large. Focusing on the distribution function that keeps track of all of the many particles can obscure the global characteristics of the final state. We find that the final state particles can often be grouped together into individual "jets" that reflect the underlying physics more clearly. Each jet contains a number of particles close together in momentum space and separated from the other particles in the scattering event. Unlike resonances, which escape the collision volume and decay through an independent of the momentum of the resonance or how it originated, the jets are not independent of each other in an event. Thus there is no well-defined average number of particles characterizing the decay of a single jet.

The simplest process illustrating the production of jets is the annihilation of a positron with an electron into strongly interacting particles emerging in two jets. An example of how such a two jet event might look is shown. The average number of particles in the final state is determined by the pair of jets and grows with the invariant masses(or total center of mass energy)of the jet pair. As this energy grows, the final particles are more and more collimated into the two Jet structures, making it progressively easier to define the directions and energies.



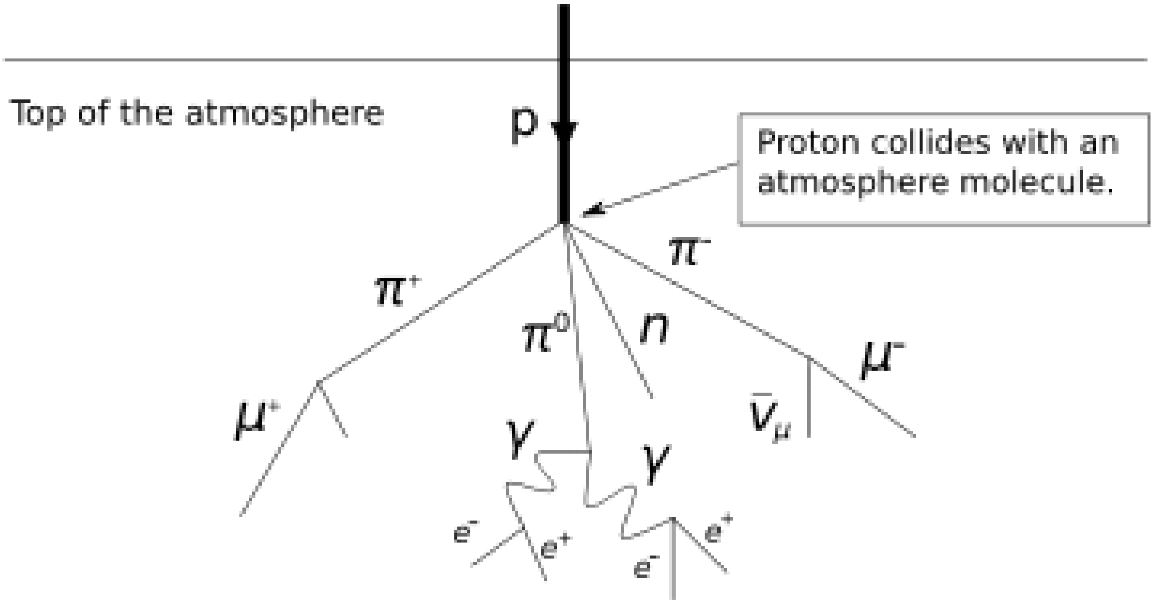
1.3 How do we produce elementary particles:-

Electrons and protons are no problem; these are the stable constituents of ordinary matter. To produce electrons one simply heats up a piece of metal, and they come boiling off. And to produce a beam of electrons, one then set up a positively charged plate nearby, to attract them over, and cuts a small hole in it; the electrons that make it through the hole constitute the beam, such an electron gun is the starting element in a television tube or an oscilloscope or an electron accelerator.

To obtain protons, we should ionize hydrogen. If we are using the protons as a target, then there is no need to bother about the electrons because electrons are very light, when an energetic particle comes, it knocks out the electrons. Thus a hydrogen atom is basically considered as a proton. For more exotic particles there are three main sources; cosmic rays, nuclear reactors and particle accelerators.

1.31 Cosmic Rays:-

The earth is constantly bombarded with high energy particles (principally protons) coming from outer space. What the source of these particles might be remains something of a mystery; at any rate, when they hit atoms in the upper atmosphere they produce showers of secondary particles (mostly muons, by the time, they reach ground level) , which rain down on us all the time. As a source of elementary particles, cosmic rays have two virtues, they are free and their energies can be enormous – far greater than we could possibly produce in the laboratory. But they have two major disadvantages. The rate at which they strike any detector of reasonable size is very low and they are completely uncontrollable. So cosmic ray experiments call for patience and luck.



Primary cosmic particle collides with a molecule of atmosphere.

1.32 Nuclear Reactor:-

When a radioactive nucleus disintegrates, it may emit a variety of particles – neutrons, neutrinos and what used to be called alpha rays (actually, alpha particles, which are bound states of two neutrons plus two protons), beta rays (actually, electrons or positrons) and gamma rays (actually photons)

1.33 Particle Accelerators:-

You start with electrons and protons accelerate them to high energy, and smash them into target. By skillful arrangements of absorbers and managements, you can separate out of resulting particle species you wish to study. Nowadays it is possible in this way to generate intense secondary beams of positrons, muons, pions, kaons and anti protons, which in turn can be fired at another target. The stable particles – electrons, protons, positrons and anti –protons - can even by fed into giant storage rings in which, guided by powerful magnets, they circulates at high speed for hours at a time, to be extracted and used at the required moment.

In general, the heavier the particle, which we want to produce, the higher must be the energy of the collision. That's why, light weight particles tends to be discovered first, and as time goes on, accelerators become more powerful, heavier and heavier particles are found. At present the heaviest known particle is the Z^0, with nearly 100 time's mass of the proton. It turns out that the particle gains enormously in energy if you collide two high speed particles head on, as opposed to firing one particle at a stationary target. Therefore most contemporary experiments involve colliding beams from intersecting storage rings; if the particles miss on the first pass, they can try again the next time around. Indeed, with electrons, and positrons (or protons and anti-protons) the same ring can be used, with the plus charges circulating in one direction and the minus charges in the other.

There is another reason why particle physicists are always pushing for higher energies; In general, The higher the energy of the collision, the closer the two particles come to one another. So if you want to study the interaction at very short range, you need very energetic particles. In quantum mechanical terms, a particle of momentum þ has an associated wave-length (λ), given by the De Broglie formula,

$\lambda = h/p$

Where h is Planck̕s constant. At large wave lengths (low momenta) you can only hope to resolve relatively large structures; in order to examine something extremely small, you need comparably short wave – lengths, and hence high energy or momenta.

1.4 How do we detect elementary particles:-

There are many kinds of particle detectors – Geiger counters, cloud chambers, bubble chambers, spark chambers, photographic emulsions, Cerenkov counters, scintillators, and photomultipliers and so on. Actually a typical modern detector has whole arrays of these devices, wired up to a computer that tracks the particles and displays their trajectories on a television screen. The details do not concern us But there is one thing to be aware of: most detection mechanisms rely on the fact that when high energy charged particles pass through matter they ionize atoms along their path The ions then act as "seeds" in the formation of droplets (cloud chamber) or bubbles (bubble chamber) or sparks (spark chamber), as leave no tracks. Evidently the bubble chamber was placed between the poles of a giant magnet. In a magnetic field B, a particle of charge q and momentum p will move in a circle of radius R given by the famous cyclotron formula:

$R = \frac{pc}{qB}$, where c is the speed of light. The curvature of the track in a known magnetic field thus affords a very simple measure of the principle's momentum. Moreover we can immediately tell the sign of the charge from the direction of the curve.

1.5 Types of interaction:-

1.51 Electromagnetic interactions:-

Each type of interaction is typified by an inherent strength or coupling constant. This coupling constant enters in the matrix element for the process under consideration, which when squared gives the decay probability or cross-section. In electromagnetic phenomena involving the interactions between charged particles and photons, the characteristic coupling constant is,

$$\frac{e^2}{\hbar c} = \alpha \approx \frac{1}{137}$$

The quantity α is called the fine structure constant, because it determine the magnitude of the spin – orbit splitting in atomic spectra. As an example, the cross-section for Compton scattering of a photon by an electron is of order

$$\left(\frac{\hbar}{mc}\right)^2 \left(\frac{e^2}{\hbar c}\right)^2$$

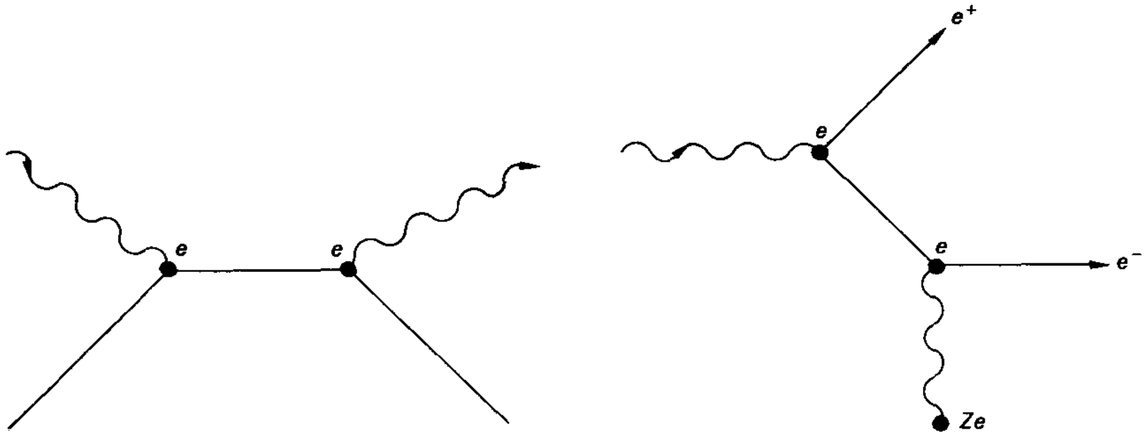
Here the magnitude of length involved is the electron Compton wavelength ħ/mc.We can visualize the process as the absorption of a photon by an electron followed by its reemission as in figure. At each vertex we put a factor e so that when the amplitude is squared we obtain a factor e^4 or α^2. For Compton scattering by a proton, the cross-section would be $(\hbar/mc)^2\alpha^2$. For pair production in the field of a nucleus Z as in figure, photons are emitted or absorbed at three vertices, so that the cross-section is of the order $(\hbar/mc)^2\alpha^3$. Relatively few particle states decay principally via the electromagnetic interaction ; it occurs where decay by strong interactions is forbidden by the conservation laws. Examples are :-

$$\Pi^0 \rightarrow 2\gamma$$
$$\Sigma^0 \rightarrow \gamma + \Lambda$$

In the π^0 decay one of the γ's may undergo "internal conversion"to give a so called Daltiz pair,

$$\Pi^0 \rightarrow e^+ e^- \gamma,$$

the branching ratio for this process is of order α. The life times involved in these processes are of the order of 10^{-16}sec, i.e. long compared with the characteristic nuclear time, 10^{-23}sec. Electromagnetic decays do not



Electron

Diagrammatic representation of electromagnetic processes (a) Compton scattering (b) Pair production in the field of a nucleus Z.

Necessarily have to include real photons as end products. For example, the decay of the eta –meson,

$\eta \rightarrow \pi^+ + \pi^- + \pi^0$ is electromagnetic since it is forbidden by the selection rules for strong interactions.[36]

1.52 Strong interactions:-

Although the strong interaction between the hadrons cannot be characterized by a unique coupling constant, it is clear that the strong, as judged from observed cross-sections, is much larger than the electromagnetic coupling. For example, at about 1Gev incident energy, the total pion nucleon cross-section is about 10^{-26}cm^2, compared with 10^{-29}cm^2 for the electromagnetic process of pion photo-production,

$$\gamma + p \rightarrow \pi^0 + p$$

It is possible to analyze the low energy pion-nucleon elastic scattering in terms of a dimensionless coupling constant. If this cross-section is measured in terms of the nucleon Compton wavelength, one obtains for this particular strong coupling,

$$\frac{g^2}{\hbar c} \approx 15$$

The life times of states decaying by strong interactions are of the order of the "characteristic nucleon time". This is equal to the range R_0 of the strong forces, of order $\frac{\hbar}{m_\pi c} = 1.4 \times 10^{-13}$cm, divided by c, and is thus $\approx 10^{-23}$sec. Because of the very short lifetime τ, such states are generally rather broad, the width being given by

$$\Gamma = \hbar/\tau$$

i.e. of order 10 to 100 MeV. Such state are called resonances, since they can be formed by collision of the particles into which they decay, and are then signified by a bump or resonance in the collision cross-section at the appropriate centre of mass energy(equal to the mass of the resonance). An example is the resonance peak appearing in the π^{+}ƥ elastic scattering cross-section, for a pion-proton invariant mass of 1236MeV, and called the $N^{*}(1236)$ state.

$$\pi^{+} + p \rightarrow N^{*}\,(1236) \rightarrow \pi^{+} + p$$

The width of the resonance in this case is 120 MeV. Although it does not have a unique mass and has an extremely transient existence, travelling only 10^{-13}cm before decay, it has definite quantum numbers (spin-parity $3/2^{+}$, isospin 3/2), and is thus a well-define state. Its lifetime cannot, of course, be measured, but is inferred from the observed width.

The conventional notation is that states decaying by strong interactions are called resonances, and those decaying by weak or electromagnetic interactions are called particles. Generally, one can measure the life time, but not the width, for particles. Thus a pion decays weakly, with a measured life time; $\tau = 2.5 \times 10^{-8}$ sec; the characteristic decay length, $c\tau$ is therefore quite large – 7.5 meters. Its width, according to the above formula is, 2×10^{-8} eV and thus unmeasurable.

1.53 Weak interactions:-

The weak interactions are described in terms of the Fermi to G^{2} and to kinematic factors (phase space). A dimensionless measure of the weak interaction can only be obtained if one defines a length, for example, the pion Compton intermediate vector boson W- analogous to the photon of the electromagnetic field –has not yet been observed, and its mass is unknown. So, the weak interaction could be inherently weak, or rather strong but of extremely short range. If we take the pion Compton wavelength as the dimension of length we obtain

$$G = 1.4 \times 10^{-49}\text{erg cm}^{3} = 2.3 \times 10^{-7}\hbar c\,(\hbar/m_{\pi}c)^{2} = 10^{-5}\,\hbar c\,(\hbar/m_{p}c)^{2}$$

Or $$(\sqrt{G}\,)^{2}/\hbar c \approx 10^{-7}(\hbar/m_{\pi}c)^{2}\,,$$

where $\sqrt{G}$ can be thought of as the “weak charge” carried by a particle analogous to the electromagnetic charge, e .It will be noted that this value is very small compared with $e^{2}/\hbar c \approx 10^{-2}$ or $g^{2}/\hbar c \approx 10$ for the other interactions. As one might expect, the cross-sections for weak reactions are extremely feeble. For example, at 1 GeV bombarding energy, the cross-section for weak pion production by neutrinos,

$$\nu + p \rightarrow \pi^{+} + p + \mu^{-}$$

Is only about 10^{-38}cm^{2}, i.e. some 10^{-12} times that for typical hadron cross- sections. The strong interaction cross-sections correspond to mean free paths in solid materials measured in terms of centimeters, whilst the neutrino mean free path is of the order of the earth-sun distance. The life time of decay via the weak interactions is correspondingly long. Nuclear β-decay lifetimes range from 10^{-6}sec to thousands of years. For the decay of pions, kaons, and hyperons, the lifetimes are in the range 10^{-10} to 10^{-8} sec. i.e. of order

10^{14} times longer than the characteristic nuclear time, and thus typical of the weak interactions.

1.54 Gravitational interaction:-

Every particle feels the force of gravity; the gravitational interaction therefore is universal. The classical theory of gravity is the well-known Newton's law of gravitation, which says that the force of attraction between two particles of masses $m_1$and m_2 at a distance r apart is given by

$$F = G\, m_1\, m_2/r^2$$

Where G is 6.69×10^{-11} N $m^2 kg^{-2}$ is the gravitational constant.

Special theory of relativity assumes that the space time is flat. As an example we can say that, the surface of this page is flat whereas the surface of a ball is curved. When special theory of relativity could not bring about a generalization of Newton's law of gravitation, Einstein made a revolutionary suggestion that gravity, is not a force like other forces, but it is a consequence of the fact that the space-time is curved, not flat. A theory of relativity, which is based on the curved space-time, is called the general theory of relativity and hence Einstein's general theory of relativity is the relativistic generalization of Newton's law of gravitation.

For the quantum mechanical theory of gravity, we have to think of the gravitational field. In this description, the force between two matter particles is carried by the quantum of gravitational field, which is called graviton, an uncharged mass less particle having spin 2. The gravitational force between say earth and moon is described as the exchange of gravitons between the particles that make earth and moon. Through the exchanged particles (gravitons) are virtual, they produce a measurable effect that moon orbits the earth. The real gravitons make up what we call gravitational waves, which however, have not been observed. Because of mass less quantum of gravitational field, its range is infinite. A completely satisfactory quantum theory of gravity has not yet been developed and it is assumed that at the elementary particle level, the gravitational interaction is too weak to play a role, hence just ignored.

A summary of the couplings in the various interactions is given below in the table.

Interaction	Typical coupling constant	Typical cross-section	Typical life-time	Associated particle	Range
Strong	$g^2/\hbar c \approx 15$	$10^{-26} cm^2 = 10^4 \mu b$	10^{-26}sec	Gluon	10^{-15}m
Electromagnetic	$e^2/\hbar c \approx 1/137$	$10^{-29} cm^2 = 10 \mu b$	10^{-16}sec	Photon	Infinite
Weak	$\approx 10^{-7}$	$10^{-38} cm^2 = 10^{-8} \mu b$	10^{-8}sec	W^{+-} , Z^0	10^{-17}m
Gravitational	$\approx 10^{-45}$	-------------	10^{16}sec	Graviton	Infinite

1.6 Conservation rules:-

From the above study it is clear that the numerical strengths of the various interactions are not well defined. However, they are also quite clearly differentiable in terms of the type of conservation rules which they are found to obey.

Just like charge and energy, momentum and angular momentum, baryon and lepton number are absolutely conserved in all interactions, other quantities, conserved in strong interactions, are violated in weak and electromagnetic interactions. Thus weak interactions appear to obey the rule $\Delta I = 1$ for non-strange decays, and $\Delta I = ½$ for $\Delta S = 1$ decays, while $\Delta S = 2$ transitions are forbidden.

Conserved quantity	Strong interaction	Electromagnetic interaction	Weak interaction
Baryon number(B)	Yes	Yes	Yes
Lepton Number(L)	Yes	Yes	Yes
Iso-spin(I)	Yes	No	No ($\Delta I = 1$ or ½)
G-Parity (G)	Yes	No	No
Strangeness (S)	Yes	Yes	No ($\Delta S = 1$)
Parity (P)	Yes	Yes	No
Charge conjugation (C)	Yes	Yes	No
CP	Yes	Yes	Yes, (10^{-3}violation in K^0decay)

1.7 Symmetries and Lagrangian:-

Nature is symmetric and the laws of symmetry are applicable everywhere .Thus symmetry is non-distinguishability of an object or structure across a dividing line or a point. This implies that by doing something, we wish to observe the change in the object. If the operation (doing something) does not bring about any change in the object, the object is said to be symmetric under that particular operation. i.e. if a system remains unchanged (invariant) under the operations of translations in time, translations in space and rotation about a point, the system is said to be symmetrical with respect to these operations and follow from these three symmetries, the conservation law of energy, linear momentum and angular momentum respectively. From the following table it will be clear that which transformation in the physics corresponds to which, conservation law:-

Symmetric operation	Conservation laws
Rotation about a point	Conservation of angular momentum
Translation in time	Conservation of energy
Translation in space	Conservation of linear momentum
Mirror reflection	Parity
Change in sign of charge	Charge conjugation

1.8 Lagrangian density:-

A technique for arriving at the equation of motion in classical mechanics is the use of a Lagrangian and a variational principle. It is possible to derive the field equations using an analogous approach based on a Lagrangian. Since the field is defined over all space, the Lagrangian is specified by a density, also called the Lagrangian for simplicity, which is a Lorentz invariant space-time function of all of the fields and their first derivatives. For the case of the free scalar field, the function is L (φ, $\frac{\partial\varphi}{\partial x\mu}$). We define the action, S as;

$$S = \int_{t1}^{t2} dt \int d^3x L$$

Requiring the action to be stationary, that is δS = 0, for φ varying in space-time arbitrarily, generates the field equation from L. The variation φ is, however taken to vanish at t_1, t_2 and the extremities of the space integral. For L depending on more fields, this procedure applied to each field will generate equations for each of the independent fields. For the scalar field,

$$\delta S = \int_{t1}^{t2} dt \int d^3x [\frac{\partial L}{\partial\varphi} \delta\varphi + \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})} \delta(\frac{\partial\varphi}{\partial x\mu})]$$

Integrating the second term by parts, with the total derivative yielding a term that vanishes at the extremities of the integration region, gives

$$\delta S = \int_{t1}^{t2} dt \int d^3x [\frac{\partial L}{\partial\varphi} - \frac{\partial}{\partial x\mu} \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}] \delta\varphi$$

Requiring this term to vanish for $\delta\varphi$ an arbitrary function of $\varkappa$ and t yields the Euler-Lagrange equation:-

$$\frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}) - \frac{\partial L}{\partial\varphi} =$$

To derive the free field Klein-Gordon equation using the Euler-Lagrange equation, a choice for L_{free} is, $L_{free} = \frac{1}{2}[\frac{\partial\varphi}{\partial x\mu}\frac{\partial\varphi}{\partial x\mu} - m^2\varphi^2]$

$$L_{Int} = -\varphi\, \rho \,, \qquad \text{Where} \quad [\nabla^2 - \frac{\partial^2}{\partial t^2}]\, \varphi - m^2\varphi = \rho$$

And φ is a Lorentz scalar. Which then yields the correctly modified field equation, Since we are particularly interested in the interactions, the usual focus of attention is the form and consequence of L_{int}, which must be generalized to apply to the correct set of particles and interactions present. We can define a Hamiltonian density, by analogy with the mechanical case, as

$H = \frac{\partial\varphi}{\partial t}\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial t})} - L$ with the Hamiltonian, $H = \int H d^3x$. We will make the connection to quantum mechanics, through the Hamiltonian. H must be Hamiltonian, which implies that L is as well. The contribution of the interaction term in L provides an interaction term in H :

$H_{Int} = \frac{\partial\varphi}{\partial t}\frac{\partial LInt}{\partial(\frac{\partial\varphi}{\partial t})} - L_{Int}$. For the scalar field just discussed, $H_{Int} = - L_{Int} = \rho\varphi$.

As an example, for a static source at $\varkappa_2$ in the field of another static source at $\varkappa_1$. We can calculate the interaction energy by integrating $\rho\varphi$ over all space. According to Yukawa potential, we have $\varphi(r) = -\frac{g}{4\pi r}e^{-mr}$. Therefore we have

$$\int \varphi(x) g \delta^3(x - x^2) d^3x = -\frac{g^2}{4\pi}\, e^{-m|x_1-x_2|}\frac{}{|x_1-x_2|}$$

The interaction potential energy is attractive, with a range determined by 1/m. This is the scalar theory analog of the coulomb interaction energy in electrostatics.

1.9 Lagrangian:-

If the Lagrangian is invariant under a continuous group of transformations, then there exist locally conserved quantities constructed from the fields and their derivatives. These quantities like the electric charge, flow in space-time and are described in terms of currents. This is called no ether's theorem. The conserved quantities in nature tell us which symmetries to build into the Lagrangian. In addition an internal symmetry will result in a spectrum of quantum states that are related by symmetry transformations. The mechanism for evasion of this expectation, that is, a symmetry of the Lagrangian not seen in the spectrum of states, is called spontaneous symmetry breaking. This intriguing phenomenon, believed to be responsible for mass generation. A symmetry familiar from mechanics is the invariance of the equations of the motion under translation of the origin in space –time. For the fields this symmetry corresponds to the Lagrangian having the same form if the $\varkappa_\mu$ are used as coordinates, or if $\varkappa'_\mu = \varkappa_\mu + \epsilon_\mu$ are used. This requires that $\varkappa_\mu$does not appear explicitly, only the fields and their derivatives can appear. We calculate the change in l due to an infinitesimal displacement;

$$\delta L = L' - L = \epsilon_\mu \frac{\partial L}{\partial x\mu},$$

We can however write this in terms of changes of φ and $\frac{\partial\varphi}{\partial x\mu}$, Since L depends only on these; therefore using the chain rule for differentiation,

$$\epsilon_\mu \frac{\partial L}{\partial x\mu} = \frac{\partial L}{\partial\varphi}\epsilon_\nu \frac{\partial\varphi}{\partial x\nu} + \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\epsilon_\nu \frac{\partial}{\partial x\nu}\frac{\partial\varphi}{\partial x\mu}$$

Using the Euler-Lagrange equation to replace $\frac{\partial L}{\partial\varphi}$ by $\frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})})$ gives

$$\epsilon_\mu \frac{\partial L}{\partial x\mu} = \epsilon_\nu \frac{\partial}{\partial x\mu}\left[\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\ (\frac{\partial\varphi}{\partial x_\nu})\right]$$, we can write this as ,

$$= \epsilon_\nu \frac{\partial}{\partial x\mu}\left[\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\ (\frac{\partial\varphi}{\partial x_\nu})] - g_{\mu\nu}L\right] = 0$$

Since the ϵ_ν are arbitrary, the tensor

$$T_{\mu\nu} = \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\ (\frac{\partial\varphi}{\partial x_\nu})] - g_{\mu\nu}L\ , \qquad \text{satisfies} \qquad \frac{\partial T\mu\nu}{\partial x\mu} = 0$$

Using the divergence theorem, it is then easy to show that the quantities $P_\nu = \int d^3x T_{0\nu}$ are conserved over time, that is $dP_\nu/dt = 0$. T_{00} is the Hamiltonian density H, and the corresponding conserved quantity H is the total energy. Its conservation follows from time translation invariance.

For the non-relativistic case, we know that the absolute phase of the wave function is not measurable. We look at the analog of this for the scalar field ,stated as a symmetry, we want the replacement of φ by $\varphi' = e^{i\theta}\varphi$to yield the same form of the Lagrangian. To achieve this

The Lagrangian, $L = \frac{\partial\varphi*}{\partial x\mu}\frac{\partial\varphi}{\partial x\mu} - m^2\varphi^*\varphi$,

is clearly invariant with regard to the overall space is called a global gauge transformation.

By varying φ and φ^*separately, it is seen that both fields satisfy the free field Klein-Gordon equation with the same mass. We next show that the symmetry leads to a conserved current J_μ. The associated time independent quantity $Q = \int J_\circ d^3x$ is called a conserved charge. Under an infinitesimal phase change;

$$\varphi \rightarrow (1+i\theta)\varphi\ , \qquad \varphi^* \rightarrow (1-i\theta)\varphi^*\ , \text{ and } \quad L \rightarrow L$$

Thus $$0 = \delta L = \frac{\partial L}{\partial \varphi}\delta\varphi + \frac{\partial L}{\partial \varphi *}\delta\varphi * + \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\delta\frac{\partial\varphi}{\partial x\mu} + \frac{\partial L}{\partial(\frac{\partial\varphi *}{\partial x\mu})}\delta\frac{\partial\varphi *}{\partial x\mu}$$

Here $$\delta\varphi = i\theta\varphi , \qquad \delta(\frac{\partial\varphi}{\partial x\mu}) = i\theta\,\frac{\partial\varphi}{\partial x\mu}$$

Replacing, $\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}\frac{\partial\varphi}{\partial x\mu}$ by $\frac{\partial}{\partial x\mu}[\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}.\varphi] - \frac{\partial}{\partial x\mu}[\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}.]\,\varphi$ gives:

$$0 = i\theta\,[\frac{\partial L}{\partial\varphi} - \frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})}] - i\theta\,[\frac{\partial L}{\partial\varphi *} - \frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi *}{\partial x\mu})}] + i\theta\,[\frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi}{\partial x\mu})})\,\varphi - \frac{\partial}{\partial x\mu}(\frac{\partial L}{\partial(\frac{\partial\varphi *}{\partial x\mu})})\,\varphi *]$$

The first two terms vanish by the Euler- Lagrange equations. Since θ is arbitrary, the last term, using the explicit formula for L above, gives

$$\frac{\partial}{\partial x\mu}J_\mu = 0, \qquad J_\mu = i\,(\varphi *\frac{\partial\varphi}{\partial x\mu} - \varphi\,\frac{\partial\varphi *}{\partial x\mu})$$

To arrive at a conserved current we had to double explicitly the number of fields being considered. This can be seen in another way. Taking

$$\varphi = \frac{\varphi_1 + i\,\varphi_2}{\sqrt{2}}, \quad \varphi^* = \frac{\varphi_1 - i\,\varphi_2}{\sqrt{2}},$$

We can write L in terms of the real fieldsφ_1 and φ_2. The result is $L = L(\varphi_1) + L(\varphi_2)$ that is two independent scalar field Lagrangian, but with both fields having the same mass. The phase transformation for φ and φ^* is equivalent to an orthogonal transformation (a real rotation) in the φ_1 and φ_2 space. From the point of view of φ_1 and φ_2, the symmetry arises because the φ_1 and φ_2 particles are degenerate in mass. The doubling of the degrees of freedom for a field describing a particle that carries charge will be seen later in this chapter to correspond to the total charge Q can be either positive or negative, depending on how many particles or antiparticles are present.

Maintaining charge conservation even in the presence of interactions constrains the choices available for L_{Int}; L_{Int} must maintain the global gauge symmetry. For e.g. a term of the type $-\lambda(\varphi^*\varphi)^2$ maintains the symmetry. This represents an interaction between the scalar particles themselves, which conserves the charge and leaves the expression for J_μ unchanged.

Since the Lagrangian formalism is the same for the classical and quantum case, we can use the familiar classical field equations for electromagnetism directly. When we come to the more complex equations and symmetries of the strong and weak interactions, the Lagrangian will provide the most direct approach for formulating the complete theory.

In addition, L provides the Hamiltonian that we will use for construction of the quantum theory. Except for the bound states of spin ½ particles, no spin 0 particle has yet been seen, so much of our subsequent discussion will focus on particles of spin ½ and spin 1.[36]

1.10 Spin 1 particle:-

We turn next to the case of a free massive particle with spin 1, called a vector field. The spin dependence $\chi(P_\mu)$ is specified by a 4- vector e_μ called the polarization vector. In the particle rest frame, e_μ is a space vector, which transforms as spin 1 under spatial rotations. There are thus three independent spin choices in the rest frame, with space components $\hat{e}_x$,$\hat{e}_y$, $\hat{e}_z$ providing the simplest set. These satisfy the Lorentz covariant conditions e.p = 0, e.e = 1, which then hold in any frame. We write for the free particle vector field in a general frame: $A_\mu = e_\mu e^{-ip.x}$

Choosing $\vec{P}$ in the z-direction, we can Lorentz transform the individual spin vectors given above to get three choices for e_μ: (0,1,0,0), (0,0,1,0), or 1/m (p,0,0,E), where the first component in parentheses is the time-component and $p \equiv |\vec{p}|$. the first two choices are called transverse polarization, the third is called longitudinal.

A convenient, related basis is given by the helicity states, which are the states with fixed angular momentum component along $\vec{p}$ To calculate these we can take states of given angular momentum in the rest frame along the axis given by $\vec{p}$ and then boost these, so the momentum is $\vec{p}$ .Taking as an example $\vec{p}$ along z, the states in the rest frame have spatial components.

$$J_z = +-1, \quad +- \frac{(\hat{e}x +- i\hat{e}y)}{\sqrt{2}}$$

$$J_z = 0, \quad \hat{e}_z$$

Under a rotation of the vectors by an angle θ about the z-axis, these change by a factor $e^{-iJz\theta}$, indicating the correct J_z. Boosting, the value of J_z becomes the helicity, which we denote by λ. Thus (keeping the unit vector notation for the spatial part of the transverse spin states):

$$\lambda = + - 1, \qquad e_\mu(\lambda) = + - (0,\frac{(\hat{e}x +- i\hat{e}y)}{\sqrt{2}}$$

$$\lambda = 0, \qquad e_\mu(\lambda) = 1/m\ (p, 0, 0, E).$$

To specify fully a given free particle state, we have to specify the momentum p and the helicity λ.

We can generalize the procedure above to arrive at fields for massive particles with other integral values of spin. For example, for spin 2 there are 2J+1 = 5 base states. These are specified in the rest fame by five choices for J_z. Based on the representations of the rotation group, the spin states are given by traceless symmetric tensors. The five linearly independent such tensors correspond to five choices we use to construct the field amplitudes.

1.11 Spin ½ particles:-

The electron is a spin ½ particle, which implies that each momentum state has two possible helicities, $\lambda = +1/2$ or $\lambda = -1/2$. The states in the particle rest frame can be determined by looking at the spin ½ representation of the rotation group. We can describe two spin choices in terms of base states:

$$X^{+} = \binom{1}{0} \quad \text{and} \quad \chi^{-} = \binom{0}{1}$$

These states called spinors correspond to spin +1/2 and -1/2 along a chosen axis, which we take to the z-axis.

The spin operator in the fermion rest frame is $\vec{S}$, which is given in the basis above by $\vec{S} = \frac{\vec{\sigma}}{2}$,with

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

the Pauli spin matrices. A question comes, why can $\vec{S}$ be a legitimate spin operator? The answer is that it satisfies the angular momentum commutation relation: $[S_i , S_j] = i\epsilon_{ijk}S_k$,

Where ϵ_{ijk} is the fully antisymmetric symbol. As a result, the three numbers $\chi^{\dagger}\vec{S}\chi$, for a general state χ , transform as a vector and thus are appropriate for coupling to other vectors, for example the magnetic field, to make an operator that transforms correctly as a scalar under rotations. The spin state is measurable; for example, in a magnetic field the two spin states of an electron split in energy.

For integral spin, our knowledge of how to Lorentz transform vectors allow us to take the rest-frame solutions and boost them, generating the general free particle solution. For the spinor case this procedure is not so obvious.

1.12 GROUP THEORY and ELEMENTARY PARTICLE SYMMETRY

Group theory is the simplest and purest form of the algebra. Group theory is the part of the mathematics but it has a great role in the particle physics. Group theory solves many problem in particle physics.

Now consider that we have a set of symmetry transformations in which each transformation can be seen as an element of the set. Obviously, each transformation obeys the property of a group and we call such groups as symmetry groups. These groups are very important in physics and chemistry. In the following we will briefly describe the properties of various symmetry groups and also their representations.

1.121 Groups and their properties:-

We take the general transformations and consider a set of transformations in which each transformation constitutes an element of the set. If the set possesses the properties of closure, identity, inverse and associativity, it is called a group. These properties are explained below; Let us denote the ith transformation by the element M_i.

1.122 Abelian /Non-Abelian groups:-

If the elements of a group commute, i.e. if $M_iM_j = M_jM_i$, the group is called Abelian. If $M_iM_j \neq M_jM_i$, the group is called non- Abelian.

1.123 Finite /Infinite groups:-

If the number of elements of a group is finite, the group is called finite group and a group with infinite number of elements is called an infinite group.

1.124 Discrete / Continuous group:-

If the group is infinite, it may be discrete or continuous. When the infinite number of elements in a group is denumerably infinite, the group is called discrete. If the elements are non-denumerably infinite, the group is called continuous. By denumeragble, it is meant that the number of elements can be counted by correspondence with infinite set of integers. _ _ _-4,-3,-2,-1,0,1,2,3,4,_ _ _is the infinite set of integers.The group of all integers , in which addition r3eplaces the multiplication to define the closure property,is a discrete group and the group of all real numbers or a group of all rotations in a plane is continuous. Obviously, all finite all, finite groups are discrete.

A group is said to be continuous if the elements of the group observe the definition of nearness or continuity. The elements of a continuous group can be characterized by or labeled by a set of real parameters $a_1, a_2, a_3, \_ \_ \_ a_n$. In which at least one parameter varies continuously over a certain interval. For example, the set of transformations,

$$x' ax + b, \quad a \neq 0 .$$

In which a and b are two parameters, forms a group. The parameters a and b vary continuously from $-\infty$ to $+\infty$ and we say that the group of transformations is a two parameter continuous group. The set of real parameters must be such that all the elements of the group can be characterized by it. If the number of continuous parameter is r where

$1 \leq r \leq n$, we call that r is the order of continuous group. It may be recalled that if the infinitesimal change in one of the factors of the product produces only an infinitesimal change in the product, the definition of continuity is said to have been observed. Under the definition of continuity if the group manifold forms a topological space, the group is said to be a topological group. The simplest topological groups are those in which the elements can be put in one to one correspondence with the points of a subset of a r-dimensional real inner product space. This subset is called parameter space.

1.13 Simple and semi-simple groups:-

The notions of simple and semi-simple groups play an important role in group theory. To define them, we have to understand the invariant subgroups. It is easy to understand the meaning of a subgroup. If we select from the elements of a group G a subset H, i.e. H is contained in G and set H itself forms a group under the same law of combination that was used in G, then H is said to be a subgroup of G.

Starting from a subgroup H of G we may denote any element of H by h and write g for any element of G. Spanning over all the elements of H we can construct a set of elements $\{g\ h\ g^{-1}\} = g\ H\ g^{-1}$. This set of elements $g\ H\ g^{-1}$ is again a subgroup of G. Different elements of G, i.e. different g, would form different conjugate subgroups. It may happen that, $g\ H\ g^{-1} = H$ implying that all the conjugate subgroups of H in G are identical with H. When this happens, we say that the subgroup H is an invariant subgroup or self conjugate subgroup in G.

A group G is said to be simple, if it has no invariant subgroups. A subgroup is said to be semi-simple, if it has no Abelian invariant subgroups. Invariant groups of elementary particle physics like the unitary symmetry group and the Lorentz group belong to these categories.

1.14 Lie groups:-

Lie groups are those continuous groups, whose parameter space is locally Euclidean. By locally Euclidean, we mean that the parameter space within the infinitesimal vicinity of any element of the continuous group is taken as finite dimensional Euclidean space. E may recall that the elements of an n-parameter continuous group are labeled by n-continuous real parameters. That is any element R(a) of the group is $R(a) = R(a_1 ,a_2 ,\_ \_ \_ a_n)$ and the elements of the continuous groups are in one to one correspondence with the points of a subset of n dimensional real inner product(scalar product) space, which is called the parameter space.

In other words, we may say that a continuous group G is called an n-parameter Lie group if a certain nearness of its elements can be mutually uniquely and mutually continuously mapped into a certain region of n-dimensional real Euclidean space. The n-parameter Lie group is also called a Lie group of dimension n. Practically all the continuous groups appearing in particle physics are Lie groups and generally, the most important ones are the U(n), SU(n),O(n) and SO(n) groups

3.7 Unitary group in n-dimensions U(n):-

A set of all n × n unitary matrices, obeying the defining properties of a group, forms the U(n) group. The unitary matrices, for which the inverse is equal to transpose conjugate: $U^{-1} = \overline{U}^{*}$, constitute the elements of the U (n) group. For n = 1, the group is U (1), which is Abelian. For n >1, U (n) group is non-Abelian. Te number of essential real parameters for this group is n^2, which vary over a finite range and hence the Lie group U(n) is closed. The U (1) group consists of 1 × 1 unitary matrices, i.e. they are phase transformations $e^{i\theta}$. The U (1) group appears as the Abelian group of gauge(phase) transformations connected with the conservation of additive charges. The groups SU (n) and O (n) are the subgroups of U (n).

3.8 Special unitary group in n dimensions SU(n):-

If the unitary matrices of U(n) group have their determinant equal to one, they are called special (det U =1) matrices and the group whose elements are these special (s) matrices is called the SU(n) group. In the SU (n) family, we frequently come across with SU (2), SU (4), SU (5) and SU (6) groups which are the special unitary groups respectively in two, three, four, five and six dimensions.

SU (2) group is important as it can describe both spin and iso-spin. We may therefore have SU (2) spin and SU (2) iso-spin groups. SU (3) group is the generalization of SU(2) iso-spin group where we include hypercharge. There is SU(3) of flavor (u, d, s) and SU(3) of color(R, G, B), When spin is included in the consideration of SU (2) iso-spin group. We have the SU (4) group of iso-spin and spin. The unification of strong, weak and electromagnetic interactions requires the SU(5) group. When SU(3) of flavor and SU(2) of spin are combined (direct product) , we get the SU(6) group, which forms the basis of quark model of hadrons.

3.9 SU (2) Symmetry:-

We know that proton and neutron are identical as far as the nuclear force is concerned, but differ in their electromagnetic interactions. Thus, it is possible to imagine a group of symmetry operators which could transform a neutron into a proton (or proton into neutron)in the absence of an electromagnetic field. The proton and neutron would then form the fundamental representations of the group. The existence of such symmetry implies that something remains constant under the strong interaction. This is known as iso-spin and is ½ for proton as well as for neutron. The component of the iso-spin, T_3 is +1/2 for the proton and -1/2 for the neutron. The operators of the symmetry group thus change the co-ordinates of iso-spin in such a way as to reverse the sign of T_3. It can also be expressed as: the strong interactions are assumed to be invariant under rotations in the isotopic spin space.

The particular symmetry group applicable to iso-spin conservation is a form of unitary symmetry known as U (2), which can be expressed by a set of 2 × 2 matrices. This group may be reduced to a special unitary group SU (2). It is special because a restriction reduces by unity the number of operators in the group. The two dimensions refer to the two basic states which make up the fundamental representation in this case. The restriction of special reduces the number of operators 2 × 2 = 4 to three. The group is then said to have three generators.

By the use of the algebra of the SU (2) group it can be shown that all irreducible representations of the symmetry group consists of a multiplet of 2T+1 states. All the members of the multiplet have the same iso-spin T and are essentially identical except for charge. If the symmetry was exact, i.e. iso-spin is strictly conserved; the components of a multiplet would differ in charge and T_3. The SU (2) symmetry is violated by the electromagnetic interaction for which conservation of iso-spin is not applicable.

The nucleon states |p> , |n> have anti nucleon states $|\bar{p}>, |\bar{n}>$. Omitting |> brackets for clarity and separating the trace from traceless part, the combination of nucleon with an anti-nucleon may be represented as

$$\begin{pmatrix} p \\ n \end{pmatrix} \times (\bar{p}\bar{n}) \rightarrow \frac{p\bar{p}+n\bar{n}}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1/2(p\bar{p}-n\bar{n}) & p\bar{n} \\ n\bar{p} & 1/2(p\bar{p}-n\bar{n}) \end{pmatrix}$$

The first term of right hand side represents singlet (η meson=0, J= 0^-) and the second term represents the triplet array (pions T=1, J= 0^-). The second term can be written in array from with normalizing factors

$$\begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & \pi^0/\sqrt{2} \end{pmatrix}$$

Under the SU (2) group operations, the particle states transform into each other with in these multiplet

1.18 Eightfold way [SU (3) symmetry]:-

Since SU (2) group cannot accommodate the hyper charge quantum number, the more general theory SU (3) has been used which also includes SU (2). SU (3) stands for special unitary group in three dimensions. The term, three dimensions refer to the three basic states which make up the fundamental representation in this case. In a three dimension unitary group there are, in general 3 × 3 = 9 operators, but the restriction of "special" reduces the number to eight. The group is then said to have eight generators. Gell Mann has referred to the resulting group of symmetry operators as the eightfold way, named for Buddha's Eightfold Path to Nirvana, comprising eight right actions. Three of the generators apply to three components of isospin, as in SU (2) and a fourth is associated with hypercharge. The remaining four also involve hypercharge in a different way.

Application of the group algebra showed that the SU (3) symmetry should give rise to six supermultiplets, containing 1, 8,8,10,$\overline{10}$ and 27 members. The $\overline{10}$ multiplet is equivalent to the 10 but with hypercharges of opposite signs. In each of these multiplets the parity and intrinsic spin of members are the same, while the hypercharge and the isotopic spin are not same. Among above mentioned groups, 8 and 10 member groups are of particular interest.

In the case for B=0 we may form particle anti-particle states to fill a 3×3 array.

$$\begin{bmatrix} p \\ n \\ \Lambda \end{bmatrix} \times (\bar{p}\bar{n}\bar{\Lambda}) \rightarrow$$

$$\left\{\begin{matrix} 1/3(2p\bar{p} - n\bar{n} - \Lambda\bar{\Lambda}) & p\bar{n} & p\bar{\Lambda} \\ n\bar{p} & 1/3(-p\bar{p} + 2n\bar{n} + \Lambda\bar{\Lambda}) & n\bar{\Lambda} \\ \Lambda\bar{p} & \Lambda\bar{n} & 1/3(-p\bar{p} - n\bar{n} + 2\Lambda\bar{\Lambda} \\ & & \end{matrix}\right\}$$

It can be identified with known spin zero mesons

$$\left\{\begin{matrix} \frac{\pi^0}{\sqrt{2}} + (\frac{\eta}{\sqrt{6}}) & \pi^+ & k^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + (\frac{\eta}{\sqrt{6}}) & k^0 \\ k^- & k^0 & -2\eta/\sqrt{6} \\ & & \end{matrix}\right\}$$

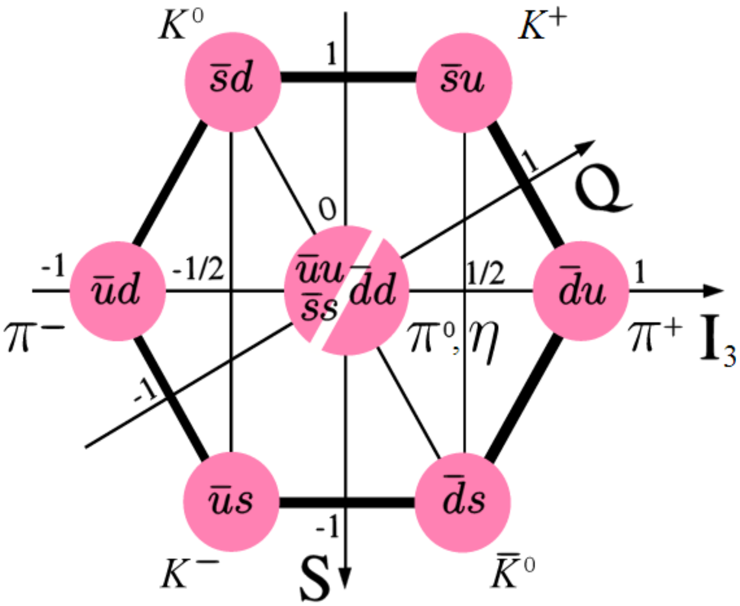
The neutral π and η mesons are now written as

$$\Pi^0 = (\mathrm{p}\,\bar{p} - \mathrm{n}\,\bar{n})/\sqrt{2}\ ; \qquad \eta = (\mathrm{p}\,\bar{p} + \mathrm{n}\,\bar{n} + 2\Lambda\,\bar{\Lambda})/\sqrt{6}\ .$$

There is in addition the symmetrical neutral combination or singlet

$$\eta' = (\mathrm{p}\,\bar{p} + \mathrm{n}\,\bar{n} + \Lambda\bar{\Lambda}\,)/\sqrt{3}\ .$$

The matrix array contains a total of eight states and is known as an octet. Since the mesons are formed from the fermions particle-antiparticle pairs, hence have odd parity. These eight particles with B=0, and $J^P = 0^-$ should be arranged as:



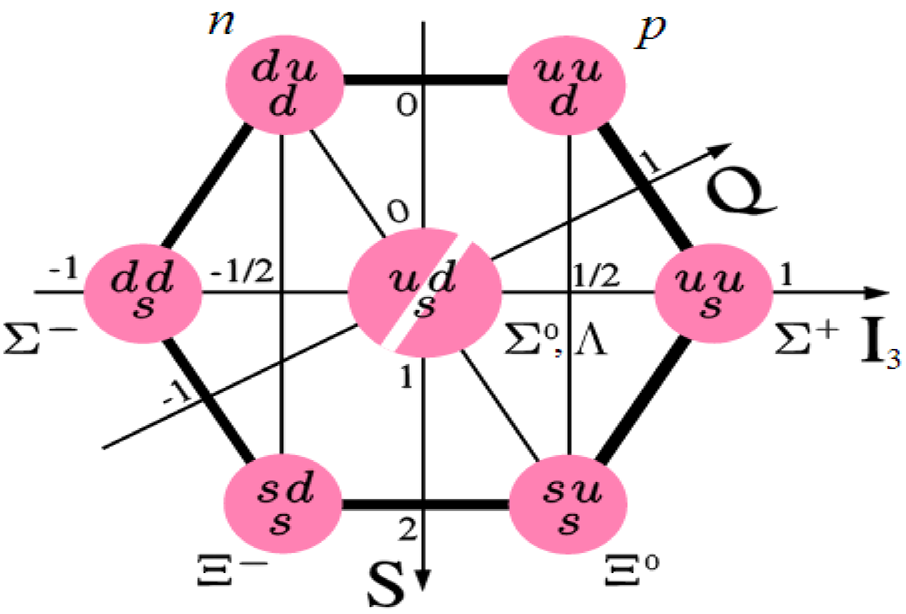
One triplet with	Y=0,	T= 1	8 members	π^- , π^0, π^+
One doublet with	Y=1,	T=1/2		k^0, k^+
One doublet with	Y= -1,	T=1/2		$\bar{k}^0$, k^-
One singlet with	Y=0,	T=0		η^0

When this was first postulated, there were seven ground state mesons. The missing meson was expected to have Q = 0, Y=0, T=0 and was predicted both by Gell-Mann and by Ne'eman. This eighth meson was named η^0 meson. An octet of metastable mesons (zero intrinsic spin) is shown.

In 1962, Gell-mann and in 1961 Ne'eman pointed out that the baryon also formed an octet array which corresponded with the octet representation of SU (3). The octet array is as :

$$\left\{ \begin{array}{ccc} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & (\frac{-\Sigma}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}}) & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{array} \right\}$$

These are eight in numbers (p, n, Λ^0, $\boldsymbol{\Sigma}^+$, $\boldsymbol{\Sigma}^-$, $\boldsymbol{\Sigma}^0$, Ξ^- , Ξ^0). For these $J^P = ½^+$, B= 1. The eightfold way arrangement of this baryon octet is shown



Another interesting case is that of the boson octet with $J^P= 1^-$ .The particles are :

K^{*0}(T_3 = -1/2, Y = 1), K^{*+} (T_3 = ½, Y = 1),

ρ^-(T_3 = -1, Y = 0), φ^0& ρ^0 (T_3= 0, Y= 0), ρ^+(T_3= 1,Y = 0),

K^{*-} (T_3= -1/2, Y = -1), $\overline{K}^{*\,0}$(T_3 = ½, Y = -1)

For $J^P = 1^-$, a boson singlet ω^0 (T = 0, Y = 0) has also been suggested. Neither the mass of φ^0 norω^0 fits with the expected value of M_0 in above relation. The mass of ω^0 and φ^0 are found approximately equally distant from the expected value (M_0 = 930 MeV/c^2) of mass. Gell-Mann and later Sakurai explained this by assuming ω^0 and φ^0 approximately the following mixtures of the octet and singlet states.

$$\text{Real } \omega = \sqrt{1/3}\,\varphi - \sqrt{2/3}\,\omega, \qquad \text{Real } \varphi = \sqrt{2/3}\,\varphi + \sqrt{1/3}\,\omega,$$

Without really knowing which particle belongs in the octet state and which in the singlet, the φ^0 has been arbitrarily placed in the octet.

Gell-Mann derived a relationship among the average masses of the components of the four baryon multiplets in the intrinsic spin half octet, given as; $2(M_N+M_\Xi) = 3M_\Lambda+M_\Sigma$, where M_N is the average nucleon mass and the average masses f the hyperons are indicated by the respective subscripts.

Since the SU(3) couplings are the same for any octet one might expect the results for the baryon octet to apply without change to the meson octets. Thus we predict

$$2(M_K{}^0+M\,\bar{k}^0) = 3M_\pi{}^0+3M_\eta{}^0$$

Since the masses of the K^0 and $\bar{K}^0$ are nearly equal, thus one may expect, $3M_\eta{}^0 = 4M_K{}^0 - M_\pi{}^0$.

The substitution of experimental masses shows a discrepancy of about 12%. Gell-Mann-Okubo suggested that the mass formula could be better if the square of the mass was used in place of the mass, i.e.

$$M^2{}_{\eta 0} = 1/3(4M^2{}_K{}^0 - M^2{}_\pi{}^0).$$

The above relations are special cases of the general formula, given by M.Gell-Mann and S. Okubo,

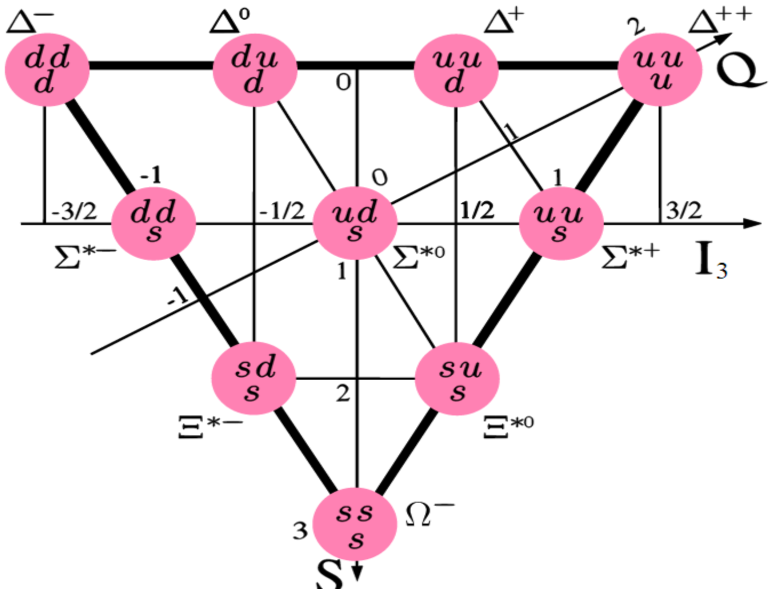
$$M(T,Y) = M_0[1+aY+b\{T(T+1)-1/4Y^2\}],$$

Where a and b are constants for a particular multiplet.

The baryon octet for intrinsic spin $3/2^-$ consists of one doublet (Y= +1; resonance states N^{*0} with T_3= -1/2 and N^{*+} with T_3 =1/2); one doublet (Y = -1; resonance states $\Xi^{*\,-}$ with T_3 = -1/2; and $\Xi^{*\,0}$ with T_3 = ½); one triplet (Y = 0; resonance states Y^{*0} with T_3 = -1, Y^{*0} with T_3 =0 and Y^{*+} with T_3 = +1)and one singlet (Y = 0, T_3= 0 and member is $Y^{*(0)}$).

A more remarkable prediction was the case of the ten-fold baryon group with intrinsic spin $3/2^+$. Theory predicted that there should be a group of ten members: Y=1, T= 3/2, quartet (nucleon resonance denoted by $N_{3/2}$ and exists in four charge states), Y=0, T=1, triplet (hyperon resonance $Y_1{}^*$ which is containing excited **Σ** particles), Y=-1, T= 1/2 , doublet (hyperon resonance which is equivalent to excited cascade particles Ξ),Y= -2, t=0, singlet (a particle **Ω** with charge -1).

In 1962, when the proposal was made to incorporate the N, Y_1, and Ξ resonance in a decuplet, the tenth particle was unknown. The existence of such a particle was predicted by Gell-Mann and was named omega (Ω). The baryon decuplet for intrinsic spin $3/2^+$ is shown in figure.



Before the Ω^- had been discovered, the values of a and b could be found in order to fit the Y= 1, 0 and -1. Thus the mass of the Y = -2 member was predicted as 1676 MeV, while the measured value is 1675 MeV.

Although all the other members of the baryon decuplet for intrinsic spin 3/2 are resonant states, decaying by the strong interaction, Gell-Mann noted that the Ω^- particle should decay by the weak interaction. Possible decay modes are

$$\Omega^- \rightarrow \Xi^- + \pi^0$$

$$\Omega^- \rightarrow \Xi^0 + \pi^-$$

$$\Omega^- \rightarrow K^- + \Lambda$$

It is clear that in these modes baryon number is conserved while iso-spin, hypercharge and strangeness are not conserved. These conservations only hold for strong interaction, hence the decay of Ω^- is by weak interaction.[37]

1.19Young Tableaux:

By working through the example of SU(2) in detail and then generalizing to S U (3) we have explicitly seen how to combine the fundamental representations and have constructed the SU(3) 1,8 and 10 representations for the qqq system. Now that a fourth flavour of quark is believed to exist one might consider a fundamental quartet (c, u, d, s) and by proceeding through the analogous steps to those just discussed one could obtain the S U (4) representations of the qqq system. Alternatively one could construct the SU(6)representations of qqq that arise when the uds quarks with spin! form the fundamental representation u↑, d↑, s↑, u↓, d↓, s↓ of SU (6). By explicit constructions you would find that [38]

$$6 \otimes 6 = 1 \oplus 35$$

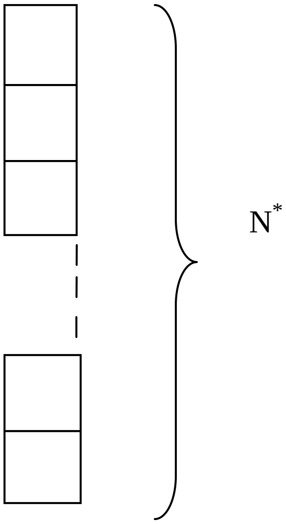
$$6 \otimes 6 \otimes 6 = 56 \oplus 70 \oplus 70 \oplus 20$$

In turn you could calculate the representations of an SU (8) group generated by the fundamental representation c↑, u↑, d↑, s↑,c↓, d↓, u↓,s↓ .

Instead of proceeding in this tedious fashion afresh for each S U (N) it would be much more elegant if there were some general techniques applicable for arbitrary S U (N) which would enable us to easily deduce the dimensions of the irreducible representation arising from products of other representation of the groups. . Elegant and extremely rapid for calculation are the techniques of Young tableaux. They also have the merit of being fun to play with. Suppose that we are interested in SU (N). The fundamental representation is denoted by a box.

$$\square = \text{dimension } N$$

while a column of N-1 boxes denotes the conjugate representation N^*



Hence in SU(2) $\square$ = 2 or 2^* but in SU(3) we find

$$\square = 3 \text{ or } \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} = 3^*$$

and so the fundamental and conjugate representations are the same in SU(2) but differ in SU(3) and higher groups SU(N≥3).

Young tableaux with only one row are associated with an anti-symmetric representation.

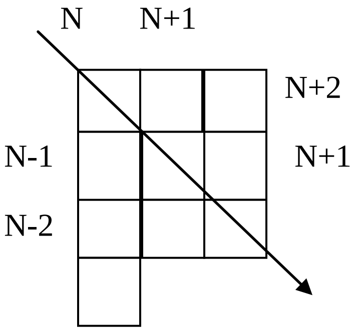
Now imagine that we wish to obtain the product of two such representations. We can add the second box to the first either to make a line or column of two boxes (the general set of rules is given in Hammermesh, 1963). The dimension of the representations corresponding to the line or column is given underneath both for SU(2) and SU(3). We have already seen how to calculate these dimensions explicitly

SU(2) : $2 \otimes 2 = 1 \oplus 3$

SU(3) : $3 \otimes 3 = 6 \oplus 3^{*}$

In general the product of two fundamental representations has the above row and column structure. Now the question arises, How do we calculate the resulting dimensions for arbitrary SU(N)? To calculate the dimension of any array of boxes there is a recipe which involves forming the ratio of two numbers. We assert that the way to calculate the numerator and denominator is as follows.

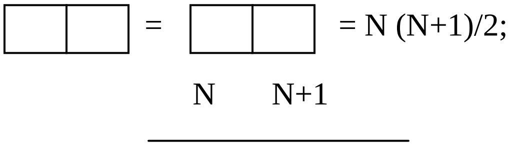
Calculation of the numerator:- For any given diagram representing a product of representations of SU(N) insert N in each of the diagonal boxes starting from the top left-hand corner, along the diagonals immediately above and below insert N + 1 and N -1 respectively. In the next diagonals insert N + 2 and so forth.

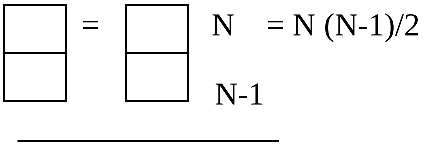


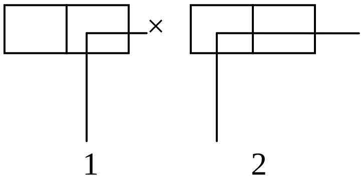
The numerator of our expression will be the product of all these numbers.

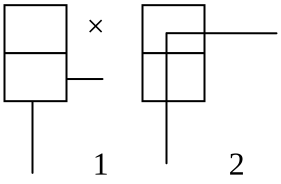
The- calculation of the denominator: This will be the product of the "hooks". Each box has a value of the hook associated with it and to find this value do the following. Draw a line entering the right-hand end of the row in which the box lies. On entering the box, this line turns downwards through 90° and then proceeds down the column until it leaves the diagram. The total number of boxes that the line has passed through, including the box in question, is the value of the hook associated with that box. The product of all the hooks is the denominator.

We will apply these rules to calculate the dimensions of our particular example met previously. It is trivial to verify that $\square$ indeed has dimension N (as it should!) and that

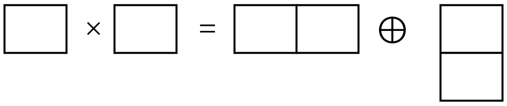








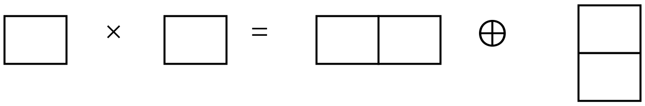
So that finally



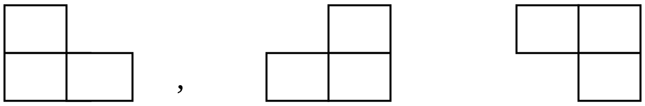
= N(N+1)/2 ⊕ N(N-1)/2

Hence for N = 2 we have that 2 ⊗ 2 = 3 ⊕ 1. For S U (3) we see that 3 ⊗ 3 = 6 ⊕ 3* . In SU(3) the column of two boxes has dimension three by the hook rule and we recognize it as

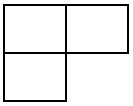
being the conjugate representation (a column of N -1 boxes in SU(N)) since it is a two-box column in SU(3). As an exercise verify that a column of N boxes in S U (N) is always the singlet srepresentation. Now let's combine three objects in SU(N). We have already seen that



and so we will now add another box to the two-box row and column. There is a detailed list of rules for forming allowed diagrams which are set out in Hammermesh (1963). For our present purpose we need only note that diagrams must not be concave upwards nor concave towards the lower left .Hence,

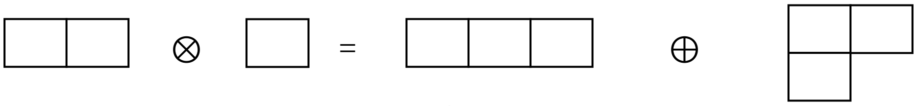


are all forbidden

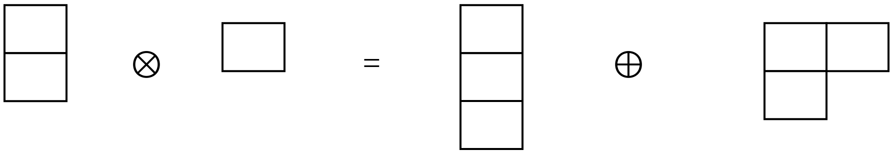


is allowed.

One combines the third box to the original two in all possible ways subject to this constraint. This yields the following topologies.



$= N (N+1) (N+2)/6 \quad \oplus \quad (N-1)N(N+)/3$, while



$= N (N-1) (N-2)/6 \quad \oplus \quad (N-1) N (N+1)/3$

and we have written the dimensions of these diagrams alongside. As an exercise use the hook rule already described and verifies these dimensions. To save effort in constructing diagrams note that in SU(N) no diagram need be drawn which has more than N boxes in any column since such a diagram trivially has dimension of zero. Hence for N = 2 the dimensionality of the tableaux are

$$(2 \otimes 2) \otimes 2 = (4 \oplus 2) \oplus 2$$

and the column is excluded as it has three boxes and we are discussing N = 2. For N = 3 we obtain $(3 \otimes 3) \otimes 3 = (10 \oplus 8) \oplus (8 \oplus 1)$

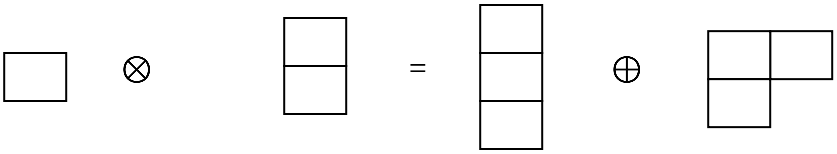
Other examples that are of physical interest include SU(4) and SU(6)

Where, $(4 \otimes 4) \otimes 4 = (20 \oplus 20) \oplus (20 \oplus 4)$

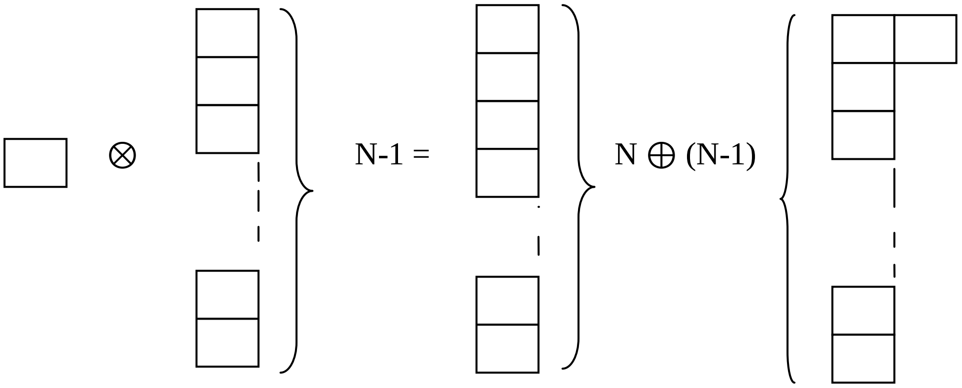
$$(6 \otimes 6)\otimes 6 = (56 \oplus 70) \oplus (70 \oplus 20)$$

respectively. If your appetite is now whetted so that you are stimulated to go and combine more complicated diagrams, e.g. $8 \otimes 8$ in SU(3), then you will need to know the set of rules that are to be obeyed in order to form legal diagrams. Once the legal diagrams have been formed, the dimensions can be calculated by the standard procedure involving the hook rule. The rules for legal diagrams can be found in Hammermesh (1963).

As a final illustration we can combine 3 and 3* in SU(3) to obtain the representations of the q$\bar{q}$mesons. We find



since all other topologies of connected boxes that contain the original single and two box columns will be concave upwards or towards the lower left and hence are forbidden. The dimensions are immediately found to be 1 and 8. In general one can see that in SU(N) the product of N and N* yields the topologies



And so $N \otimes N^{*} = 1 \oplus (N^{2} - 1)$

CHAPTER: 2 THEORY OF STATISTICAL MODEL (SEA MODEL)

We suggest a general formalism to treat a baryon as a composite system of three quarks and a "sea". In this formalism, the sea is a cluster which can consist of gluons and quark- antiquark pairs. Historically, the static SU (6) quark model provide a good description of hadrons .baryons (mesons) are color singlet combination of three quarks (quarks – antiquark pairs) in the appropriate flavour and spin combination. The space time part of a hadrons wave function can be determined by using a specific model of confinement, e.g., the bag model, simple harmonic oscillator model, or other phenomenological models. The existence of a quark gluon interaction implies that quark antiquark pairs can be created by the virtual gluons emitted from valence quarks. These quark antiquark pairs are so called sea quarks. Usually the, "sea" means a contribution of the virtual gluons and sea quark – antiquark pairs. Although deep inelastic muon nucleon scattering shows that the sea components (quark –antiquark pairs and gluons indeed exist and play a very important role (e.g. gluons carry about one- half of the nucleon momentum and sea dominates small-x behavior of structure functions), it is commonly believed that in the low energy regime, static properties of hadrons are dominated by their valence components. However, it has been shown that the sea contributions may change the structure of hadrons and modify their low energy properties. Using the QCD interaction Hamiltonian and MIT bag model, Donghue and Golowich (DG) calculated the probabilities of different sea quark components in hadrons. In these models a mixing of q^3 and q^3+ gluon, in which a color 8_c gluon coupled to an 8_c q^3 state to form a color singlet.

However, the "sea" could be a gluon or a quark antiquark pair, or even more complicated, for instance a multi gluon state, multi quark antiquark pairs or gluon(s) plus quark antiquark pair(s) .Since the baryons should be colorless and a q^3 state can be in color states $1_{c,}$ 8_c and $10_{c,}$ the "sea 'should also be in corresponding color states to form a color singlet baryon. In addition, the "sea" spin is not required to be one (as in the single gluon case). Further-more if the 'sea" is in an S-wave state relative to q^3 system ,conservation of the angular momentum restricts that sea spin can only be 0,1,or 2 to give a spin ½ baryon . If the sea is in a P-wave state ,then its spin could be 0,1,2 or 3 . We can construct the wave functions with the help of shift operators

and SU(3) algebra. Proceeding as follows:-

2.1 The sub algebra of the SU (3) Lie Algebra and the shift operator:-

The operators ($\hat{T}_1$,$\hat{T}_2$,$\hat{T}_3$) full fill the lie algebra of SU(2) $[\hat{T}^{+},\hat{T}_{-}]_- = 2\hat{T}_3$, $[\hat{T}_3 \hat{T}_{+-}]_- = \pm\hat{T}_{+-}$. The operator $\hat{T}$i (i=1,2,3) form a closed sub algebra .The same holds for the operators $\hat{U}_+$,$\hat{U}_-$,$\hat{U}_3$, for which we have the relations $[\hat{U}_+\hat{U}_-]_-$ = $2\hat{U}_3$, $[\hat{U}_3\hat{U}_{+-}]$ = $\pm\hat{U}_{+-}$. [39]

The last of these identities follows from the commutators,

$$[\hat{T}_3\hat{U}_+]_- = \pm ½\, \hat{U}_{+-} \;\Big|\; \times\ -½. \qquad [\hat{Y},\hat{U}_{+-}] = \pm\, \hat{U}_{+-}½ \;\Big|\; \times\ ¾,$$

$$\left[\hat{U}_3\hat{U}_{+-}\right] = \pm\tfrac{3}{4}\ \hat{U}_{+-} \pm \tfrac{1}{4}\,\hat{U}_{+-} = \pm\,\hat{U}_{+-}\ .$$

Similarly one can obtains $\left[\hat{V}_+\hat{V}_-\right] = 2\hat{V}_3$, $[\ \hat{V}_3\ , \hat{V}_{+-}\ ] = \hat{V}_{+-}$. The T span algebra (operator $\hat{T}i$) as well as the U span algebra (operator $\hat{U}i$) and the V span algebra (operator $\hat{V}i$) are closed. All three of them are sub algebras of SU(3) and each individually matches the algebra of the angular momentum operators [Lie algebra SU(2)].The operators $\hat{T}_{+-}$, $\hat{U}_{+-}$, $\hat{V}_{+-}$ are also shift operators , i.e. raising ($\hat{T}_+$, $\hat{U}_+$, $\hat{V}_+$) and lowering ($\hat{T}_-$, $\hat{U}_-$, $\hat{V}_-$) operators . The question arises as to which quantum number gets raised and lowered, respectively. To answer this we consider the final equation $\left[\hat{Y}, \widehat{\ \hat{T}_3}\right]_-$ = 0. Accordingly the operator $\hat{Y}$ and $\hat{T}_3$ may be simultaneously diagonalized. We denote the common eigen state by $T_3Y\rangle$

and have $T_3\,|Y\rangle$ = T₃ $|\,T_3Y\rangle$---------(1) $|\hat{Y}\ T_3Y>$ = Y $\Big|T_3Y\rangle$,

from $\left[\hat{T}_3\hat{V}_{+-}\right]_-$ = ± ½ $\hat{V}_{+-}$. , $(\,\hat{T}_3\hat{V}_{+-} - \hat{V}_{+-}\,)\Big|\ T_3Y\rangle$ = ± ½ $\hat{V}_{\pm}\,\Big|T_3Y\rangle$,

From above equation (1) we have $\hat{T}_3(\hat{V}_{\pm}\ \Big|T_3Y\rangle - \mathrm{T}_3\ \hat{V}_{+-}\Big|\ T_3Y\rangle = \pm\,\hat{V}_{+-}\Big|T_3Y\rangle$

Follows that $\hat{T}_3\ (\ \hat{V}_{+-}\ \ T_3Y\rangle) = (\ T_3 \pm \tfrac{1}{2})\ (\hat{V}_{\pm}\ \ T_3Y\rangle)$

Implies $\hat{V}_{+-}\ \ T_3Y\rangle = \sum_{Y'} N$ (T₃, Y, Y') T₃ ± ½, Y' >

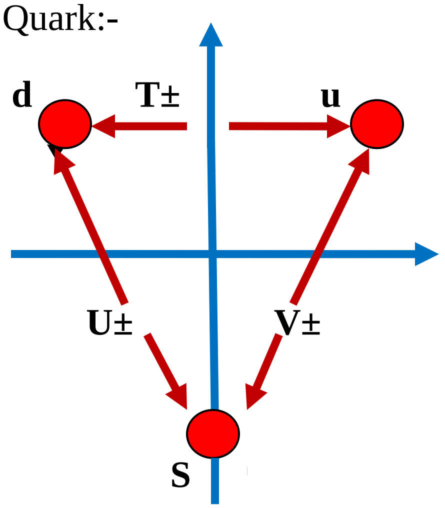
The normalization factors N (T₃, Y, Y') occurring in the above equation may depend on the quantum numbers T₃ and Y. Thus the operators $\hat{V}_{+-}$ transform a state with quantum number T₃ into a state with quantum number T₃ ± ½ and yet unknown hypercharge Y'.$\hat{V}_{+-}$ raises and lowers, respectively the quantum number T₃ by ½. By the same arguments we deduce, from $\left[\hat{T}_3\hat{U}_{+-}\right]_-$ = ±½ $\hat{U}_{+-}$, the relation $\hat{T}_3$($\hat{U}_{+-}|$ T₃ Y>) = (T₃ ± ½) ($\hat{U}_{+-}|T_3Y\rangle$).

Hence $\hat{U}_{+-}$ lowers and raises, respectively, the quantum numbers T₃ by ½ .From the relation $\left[\hat{Y}, \hat{V}_{+-}\right]_-$ = ± $\hat{V}_{+-}$, follows analogously,

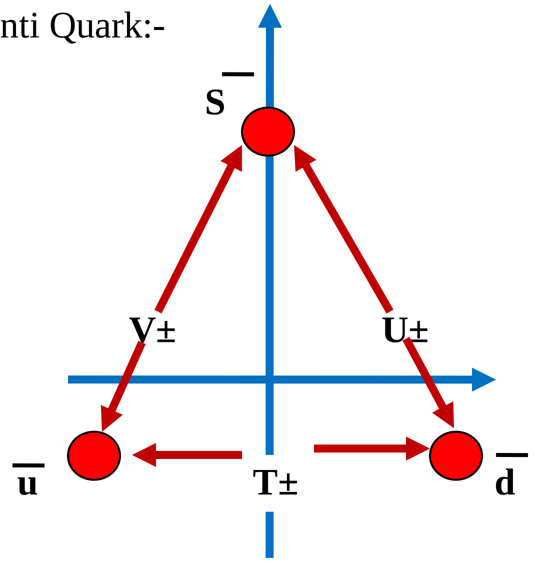
$$\hat{Y}\,(\hat{V}_{+-}\,\Big|T_3, Y\rangle) = (\mathrm{Y} \pm 1)\ (\hat{V}_{+-}\Big|T_3Y\rangle).$$

Hence $\hat{V}_{+-}$ raises and lowers the quantum number Y by 1. Finally the commutators $\left[\hat{Y}, \hat{U}_{+-}\right]_-$ = ± $\hat{U}_{+-}$ yields the equation for the eigen values, $\hat{Y}\ (\ \hat{U}_{+-}\Big|T_3, Y\rangle) = (\mathrm{Y} \pm 1)\ (\hat{U}_{+-}T_3Y\rangle)$. Hence $\hat{U}_{+-}$ raises and lowers the quantum number by 1, because of $\left[\hat{Y}, \hat{T}_{+-}\right] = 0$, the operators $\hat{T}_{+-}$ do not change the quantum number Y. in view of the algebra of angular momentum we already knew that the quantum number T₃ may be integer or half integer valued. This is evident because $\hat{V}_{+-}$ and $\hat{U}_{+-}$ shifts T₃ by half integer units and $\hat{T}_{+-}$change it by integer unit. We leave open the question of the units in which V has to be measure

Quark:-



Anti Quark:-

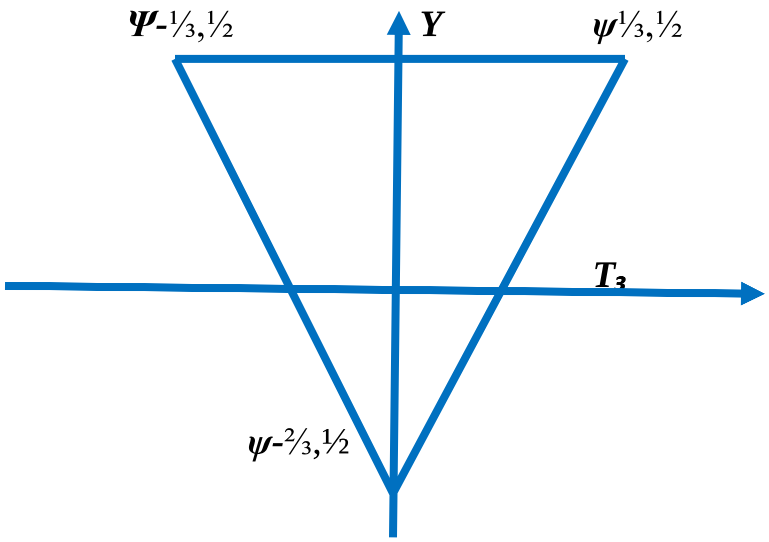


$T_+ d = u$; $T_- u = d$; $T_+ \bar{u} = - \bar{d}$; $T_- \bar{d} = - \bar{u}$

$V_+ s = u$; $V_- u = s$; $V_+ \bar{u} = - \bar{s}$; $V_- \bar{s} = - \bar{u}$

$U_+ s = d$; $U_- d = s$; $U_+ \bar{d} = - \bar{s}$; $U_- \bar{s} = - \bar{d}$

Operators	T_3	T_3	Y	Y
	Raises T_3 by	Lowers T_3 by	Raises Y by	Lowers Y by
T_+	1 unit		0	0
U_+		½ unit	1 unit	
V_+	½ unit		1 unit	
T_-		1 unit	0	0
U_-	½ unit			1 unit
V_-		½ unit		1 unit



$$\hat{T}_- \boldsymbol{\psi}^3_{1/3,1/2} = \boldsymbol{\psi}^3_{1/3,-1/2}$$

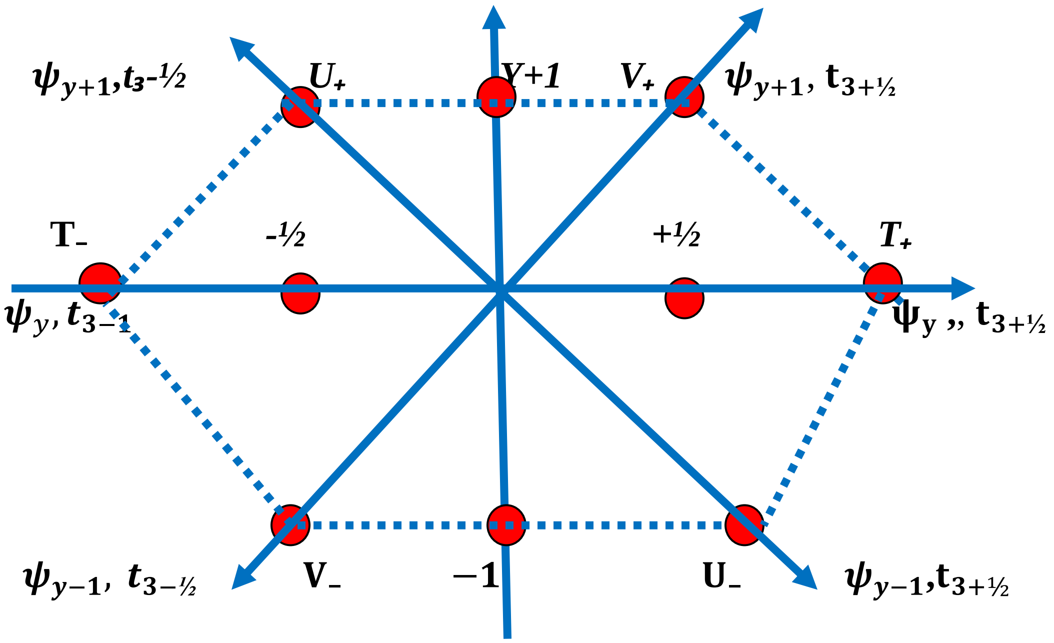
$$\hat{V}_- \boldsymbol{\psi}^3_{1/3,1/2} = \boldsymbol{\psi}^3_{-2/3,0}$$

$$\hat{T}_+ \boldsymbol{\psi}^3_{1/3-,1/2} = \boldsymbol{\psi}^3_{1/3,1/2}$$

$$\hat{U}_- \boldsymbol{\psi}^3_{1/3-,1/2} = \boldsymbol{\psi}^3_{-2/3,0}$$

$$\hat{V}_+ \boldsymbol{\psi}^3_{-2/3,0} = \boldsymbol{\psi}^3_{1/3,1/2}$$

$$\hat{U}_+ \boldsymbol{\psi}^3_{-2/3,0} = \boldsymbol{\psi}^3_{1/3,-1/2}$$



1. Applying $\hat{T}_{-}$ operator on Δ^{++} because it changes the t_3 by -1 and we get the Δ^{+}.
   $\therefore\ N\hat{T}_{-}|\Delta^{++}\rangle = \Delta^{+}$, where N is a normalization constant.

$N(\hat{T}_{-}(1)+\hat{T}_{-}(2)+\hat{T}_{-}(3))[\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)]\chi$, where χ is for spin [∵ χ for the triple elementary do not change.]

$= N.\chi[\hat{T}_{-}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)] + [\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\hat{T}_{-}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)] + [\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\hat{T}_{-}(3)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)]$

$= N\chi\{[\psi_{\frac{1}{3},-\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)]+[\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},-\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)]$
$+[\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},-\frac{1}{2}}(3)]\}$ -------------------- (a)

Now the factor N (normalization constant) can be calculated. As we know
that $\int_{-\infty}^{\infty}\boldsymbol{\psi\psi}^{*}\,d\psi = 1 \implies \langle\Delta^{+}|\Delta^{+}\rangle = 1$
From equation (a)

$\int\{N^{*}\overline{\chi}\,[\overline{\psi}_{\frac{1}{3},-\frac{1}{2}}(1)\,\overline{\psi}_{\frac{1}{3},\frac{1}{2}}(2)\,\overline{\psi}_{\frac{1}{3},\frac{1}{2}}(3)]+[\overline{\psi}_{\frac{1}{3},-\frac{1}{2}}(1)\,\overline{\psi}_{\frac{1}{3},-\frac{1}{2}}(2)\,\overline{\psi}_{\frac{1}{3},\frac{1}{2}}(3)] + \{[\overline{\psi}_{\frac{1}{3},\frac{1}{2}}(1)\,\overline{\psi}_{\frac{1}{3},\frac{1}{2}}(2)\,\overline{\psi}_{\frac{1}{3},-\frac{1}{2}}(3)]\ N*\chi\,[\psi_{\frac{1}{3},-\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)]+[\psi_{\frac{1}{3},-\frac{1}{2}}(1)\,\psi_{\frac{1}{3},-\frac{1}{2}}(2)\,\psi_{\frac{1}{3},\frac{1}{2}}(3)] +[\{[\psi_{\frac{1}{3},\frac{1}{2}}(1)\,\psi_{\frac{1}{3},\frac{1}{2}}(2)\,\psi_{\frac{1}{3},-\frac{1}{2}}(3)]$

$$\implies \int NN^{*}\,|\chi|^{2}\,[\psi_{I}\overline{\Psi_{I}} + \psi_{I}\overline{\Psi_{II}} + \psi_{I}\overline{\psi_{III}} + \Psi_{II}\overline{\Psi_{I}} + \Psi_{II}\overline{\Psi_{II}} + \Psi_{II}\overline{\Psi_{III}} + \Psi_{III}\overline{\psi_{I}} + \Psi_{III}\overline{\Psi_{II}} + \overline{\Psi_{III}}\Psi_{III}]\,d^{3}x_{1}d^{3}x_{2}d^{3}x_{3}$$

As we know that $\Psi\Psi^{*} = |\Psi|^{2}$

$\therefore \int |N|^2 |\chi|^2 \, [\Psi_I{}^2 + \overline{\Psi}_I \Psi_{II} + \overline{\Psi}_I \Psi_{III} + \overline{\Psi}_{II} \Psi_I + \Psi_{II}{}^2 + \overline{\Psi}_{II} \Psi_{III} + \overline{\Psi}_{III} \Psi_I + \overline{\Psi}_{III} \Psi_{II} + \Psi_{III}{}^2 \;]$ $d^3x_1 d^3x_2 d^3x_3$ ---------------- (b)

Ψ_x is orthonormal. i.e. $\int_{-\infty}^{\infty} \Psi_x^* \Psi_x \mathrm{dx} = 1$; if m = n, 0 if m ≠ n

Therefore equation (b) becomes

1 = $|N|^2$ [1+0+0+0+1+0+0+0+1]; N = $1/\sqrt{3}$

Therefore | $\Lambda^+ \rangle$ = $1/\sqrt{3}$ [ψ ⅓,-½ (1) ψ ⅓,½ (2) ψ ⅓,½ (3)] + [ψ ⅓,½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)] + [ψ ⅓,½ (1) ψ ⅓,½ (2) ψ ⅓,-½ (3)] $\chi_{½,½}$ (1) $\chi_{½,½}$ (2) $\chi_{½,½}$(3)

2. Applying $\hat{T}_-$ operator on Δ^+ because it changes the t $_3$ by -1 and we get the Δ^0
   $\therefore$ N$\hat{T}_-$ $|\Delta^+\rangle$ = Δ^0 , where N is a normalization constant

N ($\hat{T}_-$ (1) + $\hat{T}_-$ (2) + $\hat{T}_-$ (3)) [$1/\sqrt{3}$ [ψ ⅓,-½ (1) ψ ⅓,½ (2) ψ ⅓,½ (3)]+[ψ ⅓,½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)]+[ψ ⅓,½ (1) ψ ⅓,½ (2) ψ ⅓,-½ (3)] $\chi_{½,½}$ (1) $\chi_{½,½}$ (2) $\chi_{½,½}$(3)

= N/$\sqrt{3}$ { [$\hat{T}_-$ (1) (ψ ⅓,-½ (1) ψ ⅓,½ (2) ψ ⅓,½ (3)) + [$\hat{T}_-$ (1) ψ ⅓,½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)] + [$\hat{T}_-$ (1) ψ ⅓,½ (1) ψ ⅓,½ (2) ψ ⅓,-½ (3)] } $\chi_{½,½}$ (1) $\chi_{½,½}$ (2) $\chi_{½,½}$(3) + { [ψ ⅓,-½ (1) $\hat{T}_-$ (2) ψ ⅓,½ (2) ψ ⅓,½ (3)]+ [ψ ⅓,½ (1) $\hat{T}_-$ (2) ψ ⅓,-½ (2) ψ ⅓,½ (3)] + [ψ ⅓,½ (1) $\hat{T}_-$ (2) ψ ⅓,½ (2) ψ ⅓,-½ (3)] } + { [(ψ ⅓,-½ (1)ψ ⅓,½ (2) $\hat{T}_-$ (3)ψ ⅓,½(3))]+ [ψ ⅓,½ (1) ψ ⅓,-½ (2) $\hat{T}_-$ (3) ψ ⅓,½ (3)] + [ψ ⅓,½ (1) ψ ⅓,½ (2) $\hat{T}_-$ (3) ψ ⅓,-½ (3)] } $\chi_{½,½}$ (1) $\chi_{½,½}$ (2) $\chi_{½,½}$(3)

= N/$\sqrt{3}$ { ψ ⅓,-½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)] +[ψ ⅓,-½ (1) ψ ⅓,½ (2) ψ ⅓,-½ (3)] + [ψ ⅓,-½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)] +[ψ ⅓,½ (1) ψ ⅓,-½ (2) ψ ⅓,-½ (3)] +[ψ ⅓,-½ (1) ψ ⅓,½ (2)

ψ ⅓,-½ (3)]+ [ψ ⅓,½ (1) ψ ⅓,-½ (2) ψ ⅓,-½ (3)]

{ ∵ [$\psi_{1/3,-3/2}$ (1) ψ ⅓,½ (2) ψ ⅓,½ (3)] = [ψ ⅓,½ (1) $\psi_{1/3,-3/2}$ (2) ψ ⅓,½ (3)]

= [ψ ⅓,½ (1) ψ ⅓,½ (2) $\psi_{1/3,-3/2}$ (3)] = 0 }

= 2 N/$\sqrt{3}$[ψ ⅓,-½ (1) ψ ⅓,-½ (2) ψ ⅓,½ (3)] + [ψ ⅓,-½ (1) ψ ⅓,½ (2) ψ ⅓,-½ (3)]

+ [ψ ⅓, ½ (1) ψ ⅓,-½ (2) ψ ⅓,-½ (3)] ------------------- (a)

Now the factor N (normalization constant) can be calculated. As we know, That $\int_{-\infty}^{\infty} \Psi\Psi^{*}\, d\psi = 1 \quad \Rightarrow \quad \langle \Delta^0 | \Delta^0 \rangle = 1$

$1 = 4/3 \int NN^{*} \{ [\overline{\psi}\, ⅓,-½\,(1)\, \overline{\psi}\, ⅓, ½\,(2)\, \overline{\psi}\, ⅓,-½\,(3)] + [\overline{\psi}\, ⅓,-½\,(1)\, \overline{\psi}\, ⅓,-½\,(2)$ $\overline{\psi}\, ⅓,½(3)] + [\overline{\psi}\, ⅓,½\,(1)\, \overline{\psi}\, ⅓,-½(2)\, \overline{\psi}\, ⅓,-½\,(3)] \} \{ [\psi\, ⅓,-½\,(1)\, \psi\, ⅓,-½\,(2)$ $\psi\, ⅓,½\,(3)] + [\psi\, ⅓,-½\,(1)\, \psi\, ⅓,½\,(2)\, \psi\, ⅓,-½\,(3)] + [\psi\, ⅓,½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3)] \}$

$\Rightarrow \quad 1 = 4/3\, N^2 [\int . [\overline{\Psi_I}\Psi_I + \overline{\Psi_I}\Psi_{II} + \overline{\Psi_I}\Psi_{III} + \overline{\Psi_{II}}\Psi_I + \overline{\Psi_{II}}\Psi_{II} + \overline{\Psi_{II}}\Psi_{III} +$ $\overline{\Psi_{III}}\Psi_I + \overline{\Psi_{III}}\Psi_{II} + \overline{\Psi_{III}}\Psi_{III}]\, \chi(1)\, \chi(2)\, \chi(3)\, d^3x_1 d^3x\, d^3x_3$

$1 = 4/3\, N^2 [\int (|\Psi_I|^2 + |\Psi_{II}|^2 + |\Psi_{III}|^2)\, d^3x_1 d^3x_2 d^3x_3$

$1 = 4/3\, N^2 [1+1+1]$; $N = ½$. From equation (a) we get,

$1/\sqrt{3}\ \{[\psi\, ⅓, -½\,(1)\, \psi\, ⅓,-½(2)\, \psi\, ⅓, ½\,(3)] + [\psi\, ⅓,-½\,(1)\, \psi\, ⅓, ½(2)\, \psi\, ⅓,-½\,(3)] +$ $[\psi\, ⅓, ½\,(1)\, \psi\, ⅓,-½(2)\, \psi\, ⅓,-½\,(3)]\}$

3. Applying $\hat{T}_{-}$ operator on Δ^0, because it changes the t_3 by -1 and we get the Δ^-.
∴ $N\, |\hat{T}_{-}\Delta^0\rangle = \Delta^-$. , where N is a normalization constant.

$N(\hat{T}_{-}(1) + \hat{T}_{-}(2) + \hat{T}_{-}(3))\, [1/\sqrt{3}\ \{[\psi\, ⅓, -½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,½\,(3)] + [\psi\, ⅓,-½\,(1)\, \psi\, ⅓,½(2)\, \psi\, ⅓,-½\,(3)] + [\psi\, ⅓, ½\,(1)\, \psi\, ⅓,-½(2)\, \psi\, ⅓,-½\,(3)] \}\chi_{\frac{1}{2},\frac{1}{2}}(1)\, \chi_{\frac{1}{2},\frac{1}{2}}(2)\, \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$= N/\sqrt{3}\ [\{ [\hat{T}_{-}(1)\, (\psi\, ⅓,-½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,½\,(3)) + [\hat{T}_{-}(1)\, \psi\, ⅓,-½\,(1)\, \psi\, ⅓,½\,(2)$ $\psi\, ⅓,-½\,(3)] + [\hat{T}_{-}(1)\, \psi\, ⅓,½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3)] \}\chi_{\frac{1}{2},\frac{1}{2}}(1)\, \chi_{\frac{1}{2},\frac{1}{2}}(2)\, \chi_{\frac{1}{2},\frac{1}{2}}(3) +$

$\{ [\psi\, ⅓,-½\,(1)\, \hat{T}_{-}(2)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,½\,(3)] + [\psi\, ⅓,-½\,(1)\, \hat{T}_{-}(2)\, \psi\, ⅓,½\,(2)\, \psi\, ⅓,-½\,(3)] +$ $[\psi\, ⅓,½\,(1)\, \hat{T}_{-}(2)\psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3)] \} +$

$\{ [(\psi\, ⅓,-½\,(1)\psi\, ⅓,-½(2)\, \hat{T}_{-}(3)\psi\, ⅓,½(3))] + [\psi\, ⅓,-½\,(1)\, \psi\, ⅓,½\,(2)\, \hat{T}_{-}(3)\, \psi\, ⅓,-½\,(3)] +$ $[\psi\, ⅓,½\,(1)\, \psi\, ⅓,-½\,(2)\, \hat{T}_{-}(3)\, \psi\, ⅓,-½\,(3)] \}\, \chi_{\frac{1}{2},\frac{1}{2}}(1)\chi_{\frac{1}{2},\frac{1}{2}}(2)\, \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$= N/\sqrt{3}\ [0+0+ \psi\, ⅓,-½\,(1)\psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3) + 0 + \psi\, ⅓,-½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½(3) +$ $0 + \psi\, ⅓,-½\,(1)\, \psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3) + 0 + 0]\ \chi_{\frac{1}{2},\frac{1}{2}}(1)\, \chi_{\frac{1}{2},\frac{1}{2}}(2)\, \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$= 3\, N/\sqrt{3}\ [\psi\, ⅓,-½\,(1)\psi\, ⅓,-½\,(2)\, \psi\, ⅓,-½\,(3)]\, \chi_{\frac{1}{2},\frac{1}{2}}(1)\, \chi_{\frac{1}{2},\frac{1}{2}}(2)\, \chi_{\frac{1}{2},\frac{1}{2}}(3)$ --------(a)

Now the factor N (normalization constant) can be calculated. As we know, That $\int_{-\infty}^{\infty} \Psi\Psi^* \, d\psi=1 \Longrightarrow \langle \Delta^- | \Delta^- \rangle = 1$

$1 = \sqrt{3} \int NN^* \{ \ [\overline{\psi}\ \tfrac13,-\tfrac12\ (1)\ \overline{\psi}\ \tfrac13,-\tfrac12\ (2)\ \overline{\psi}\ \tfrac13,-\tfrac12\ (3)]\ [\ \psi\ \tfrac13,-\tfrac12\ (1)\ \ \psi\ \tfrac13,-\tfrac12\ (2)$

$\Psi\ \tfrac13,-\tfrac12\ (3)] d^3x_1 d^3x_2 d^3x_3$

$1 = 3\ |N|^2$; $N = 1/\sqrt{3}$, therefore from equation (a) we will get ,

$|\Delta^- \rangle = [\psi\ \tfrac13,-\tfrac12\ (1)\ \psi\ \tfrac13,-\tfrac12\ (2)\ \Psi\ \tfrac13,-\tfrac12\ (3)]\ \chi$

4. Now to calculated the wave function of Σ^{*+}we have ,

$N\ (\hat{V}_-(1) + \widehat{V_-}(2) + \hat{V}_-\ (3))\ [\psi\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ \tfrac13, \tfrac12\ (3)]\ \chi_{\frac12,\frac12}\ (1)\ \chi_{\frac12,\frac12}\ (2)\ \chi_{\frac12,\frac12}(3)$

$|\Sigma^{*+}> = N\ \{\ \hat{V}_-(1)\ [\psi\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ \tfrac13, \tfrac12\ (3\ )\ ] + [\psi\ \tfrac13, \tfrac12\ (1)\ \hat{V}_-\ (2)\ \ \psi\ \tfrac13, \tfrac12\ (2)$

$\psi\ \tfrac13, \tfrac12\ (3)] + [\ \psi\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \hat{V}_-\ (3))\ \psi\ \tfrac13, \tfrac12\ (3)\ ]\ \chi_{\frac12,\frac12}\ (1)\ \chi_{\frac12,\frac12}\ (2)\ \chi_{\frac12,\frac12}\ (3)$

$= N\ [[\psi\ -\tfrac23, o\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ \tfrac13, \tfrac12\ (3)] + [\psi\ \tfrac13, \tfrac12\ (1)\psi\ -\tfrac23, o\ (2)\ \psi\ \tfrac13, \tfrac12\ (3)] +$
$[\psi\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ -\tfrac23, o\ (3)\chi_{\frac12,\frac12}(1)\chi_{\frac12,\frac12}(2)\ \chi_{\frac12,\frac12}(3)\ ]$ -------------- (a)

Now the factor N (normalization constant) can be calculated. As we know,

That $\int \Psi\Psi^*\ d\psi=1 \Longrightarrow \langle \Sigma^{*+} | \Sigma^{*+} \rangle = 1$

$1 = \int NN^*\ \{\ [\overline{\psi}\ -\tfrac23, o\ (1)\ \overline{\psi}\ \tfrac13, \tfrac12\ (2)\ \overline{\psi}\ \tfrac13,\tfrac12\ (3)\ ]\ [\ \ \psi\ -\tfrac23,o\ (1)\ \ \psi\ \tfrac13,\tfrac12\ (2)$

$\Psi\ \tfrac13, \tfrac12\ (3)] + [\overline{\psi}\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (1)\ \overline{\psi}\ -\tfrac23, o\ (2)\ \psi\ -\tfrac23, o\ (2)\ \overline{\psi}\ \tfrac13, \tfrac12\ (3)\psi\ \tfrac13, \tfrac12\ (3)] + [\overline{\psi}\ \tfrac13, \tfrac12\ (1)$

$\psi\ \tfrac13, \tfrac12\ (1)\ \overline{\psi}\ \tfrac13, \tfrac12\ (2)\ \psi\ \tfrac13, \tfrac12\ (2)\ \ \overline{\psi}\ -\tfrac23, o\ (3)\ \psi\ -\tfrac23, o\ (3)\ ]\ d^3x_1 d^3x_2 d^3x_3\ ]$

$1 = |N|^2\ [3]$; $N = 1/\sqrt{3}$, from equation (a), we have

$|\Sigma^{*+}> = 1/\sqrt{3}\ [\psi\ -\tfrac23, o\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ \tfrac13, \tfrac12\ (3)\ ] + [\psi\ \tfrac13, \tfrac12\ (1)\psi\ -\tfrac23, o\ (2)\ \psi\ \tfrac13, \tfrac12\ (3)] +$

$[\psi\ \tfrac13, \tfrac12\ (1)\ \psi\ \tfrac13, \tfrac12\ (2)\ \psi\ -\tfrac23, o\ (3)\ ]\chi_{\frac12,\frac12}(1)\ \chi_{\frac12,\frac12}\ (2)\ \chi_{\frac12,\frac12}(3)$

Which is the required wave function for Σ^{*+}

5. Applying) $\hat{T}_-$ operator on Σ^{*+} because it changes the t_3 by -1 and we get theΣ^{*-}

$N\ \hat{T}_-|\ \Sigma^{*-} \rangle\ = \Sigma^{*0}$, where N is a normalization constant

$= N\{\,[1/\sqrt{3}\ [\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] +$

$[\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}\,(3)]\}\ \chi_{\frac{1}{2},\frac{1}{2}}(1)\ \chi_{\frac{1}{2},\frac{1}{2}}(2)\ \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$N.1/\sqrt{3}\ [(\hat{T}_-(1) + \hat{T}_-(2) + \hat{T}_-(3))]\ \{\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] +$

$[\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}(3)]\}\chi_{\frac{1}{2},\frac{1}{2}}(1)\chi_{\frac{1}{2},\frac{1}{2}}(2)\ \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$= N.1/\sqrt{3}\ [(\hat{T}_-(1)\ \psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\,(\hat{T}_-(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)$

$\Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\,\hat{T}_-(1)\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}(3)] + [\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \hat{T}_-(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)$

$\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \hat{T}_-(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \hat{T}_-(2)\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)$

$\psi\,-\tfrac{2}{3},\mathrm{o}(3)\,] + [\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \hat{T}_-\ (3)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \hat{T}_-(3)$

$\Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\tfrac{1}{3},\tfrac{1}{2}(1)\psi\tfrac{1}{3},\tfrac{1}{2}(2)\ \hat{T}_-(3)\ \psi\tfrac{2}{3},_0(3)]\}\ \chi_{\frac{1}{2},\frac{1}{2}}(1)\ \chi_{\frac{1}{2},\frac{1}{2}}(2)\ \chi_{\frac{1}{2},\frac{1}{2}}(3)$

$= N/\sqrt{3}\ [\,\psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},_0\,(2)\ \Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\psi\,-\tfrac{2}{3},\mathrm{o}(3)] +$

$[[\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}\,(3)\,] +$

$[\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(3)] + [\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)$

$\Psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(3)]\,\}\ \chi_{\frac{1}{2},\frac{1}{2}}(1)\ \chi_{\frac{1}{2},\frac{1}{2}}(2)\ \chi_{\frac{1}{2},\frac{1}{2}}(3)$--------------(a)

Now the factor N (normalization constant) can be calculated. As we know,

that $\int_{-\infty}^{\infty}\Psi\Psi^{*}\,d\psi = 1 \quad \Longrightarrow \quad \langle \Sigma^{*\,0} | \Sigma^{*0} \rangle = 1$

$1 = 1/3\int NN^{*}\{\,[\overline{\psi}\,\tfrac{1}{3},-\tfrac{1}{2}(1)\ \overline{\psi}\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \overline{\psi}\,\tfrac{1}{3},\tfrac{1}{2}\,(3)\,]\ [\,\psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(1)\ \psi-\tfrac{2}{3},\mathrm{o}\,(2)\Psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)] +$

$[\psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(1)\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}\,(3)]\ [\,\overline{\psi}\,\tfrac{1}{3},-\tfrac{1}{2}\,(1)\ \psi\,\overline{\tfrac{1}{3}},\tfrac{1}{2}\,(2)\ \psi\,\overline{-\tfrac{2}{3}},\mathrm{o}\,(3)] +$

$[\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\ \psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},\tfrac{1}{2}\,(3)]\ [\,\overline{\psi\,-\tfrac{2}{3}},\mathrm{o}\,(1)\ \psi\,\overline{\tfrac{1}{3}},-\tfrac{1}{2}\,(2)\ \psi\,\overline{\tfrac{1}{3}},\tfrac{1}{2}\,(3)\,] +$

$[\,\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(2)\ \psi\,-\tfrac{2}{3},\mathrm{o}(3)] + [\psi\,\overline{\tfrac{1}{3}},\tfrac{1}{2}\,(1)\ \psi\,\overline{\tfrac{1}{3}},-\tfrac{1}{2}\,(2)\ \psi\,\overline{-\tfrac{2}{3}},\mathrm{o}\,(3)] +$

$[\psi\,-\tfrac{2}{3},\mathrm{o}\,(1)\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(2)\ \psi\,\tfrac{1}{3},-\tfrac{1}{2}\,(3)\,]\ [\psi\,\overline{-\tfrac{2}{3}},\mathrm{o}\,(1)\ \psi\,\overline{\tfrac{1}{3}},\tfrac{1}{2}\,(2)\ \psi\,\overline{\tfrac{1}{3},-}\,\tfrac{1}{2}\,(3)] +$

$[\psi\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\psi\,-\tfrac{2}{3},\mathrm{o}\,(2)\ \Psi\tfrac{1}{3},-\tfrac{1}{2}\,(3)]\ [\overline{\Psi}\,\tfrac{1}{3},\tfrac{1}{2}\,(1)\ \overline{\psi\,-}\tfrac{2}{3},\mathrm{o}\,(2)\,\overline{\psi}\,\tfrac{1}{3},-\tfrac{1}{2}\,(3)]\ |\chi|^{2}$

$1 = 6\,|N|^{2};\quad N = 1/\sqrt{2}$

Therefore from equation (a), we will get

$|$

$\Sigma^{*\,0}\rangle$ = 1/$\sqrt{6}$ [ψ ⅓, -½ (1)ψ -⅔, o (2) Ψ ⅓, ½ (3)] + [ψ ⅓, - ½ (1) ψ ⅓, ½ (2) ψ -⅔, o (3)] + [[ψ -⅔, o (1) ψ ⅓, - ½ (2) ψ ⅓, ½ (3)] + [ψ ⅓, ½ (1)ψ ⅓, - ½ (2) ψ -⅔, o (3)] + [ψ -⅔, o (1) ψ ⅓, ½ (2) ψ ⅓,- ½ (3)] + [ψ ⅓, ½ (1)ψ -⅔, o (2) Ψ ⅓, -½ (3)] } $\chi_{1/2,1/2}(1)\chi_{1/2,1/2}(2)\,\chi_{1/2,1/2}(3)$

Which is the required wave function for $\Sigma^{*\,0}$

6. To calculate the wave function forΞ^{*0}, we use $\hat{V}$ operator.

$\hat{V}_{-}(1)+\hat{V}_{-}(2)+\hat{V}_{-}(3)\,|\Sigma^{*\,+}\rangle = \Xi^{*0}$,

$\hat{V}_{-}(1)+\hat{V}_{-}(2)+\hat{V}_{-}(3)|\mathrm{N}/\sqrt{3}$ { [ψ -⅔, o (1) ψ ⅓, ½ (2) ψ ⅓, ½ (3)] + [ψ ⅓, ½ (1)ψ -⅔, o (2) ψ ⅓, ½ (3)] + [ψ ⅓, ½ (1) ψ ⅓, ½ (2) ψ -⅔, o (3)] $\chi_{1/2,1/2}$ (1) $\chi_{1/2,1/2}(2)$ $\chi_{1/2,1/2}(3)$}

= N/$\sqrt{3}$ { [$\hat{V}_{-}$ (*1*) [ψ -⅔, o (1) ψ ⅓, ½ (2) ψ ⅓, ½ (3)] + [$\hat{V}_{-}(1)$ ψ ⅓, ½ (1)ψ -⅔, o(2) ψ ⅓, ½ (3)] + [$\hat{V}_{-}(1)$ ψ ⅓, ½ (1) ψ ⅓, ½ (2) ψ -⅔, o (3)] + [ψ -⅔, o (1) $\hat{V}_{-}(2)$ ψ ⅓, ½(2) ψ ⅓, ½ (3)] + [ψ ⅓, ½ (1) $\hat{V}_{-}$(2) (2) ψ -⅔, o (2) ψ ⅓, ½ (3)] +[ψ ⅓, ½ (1) $\hat{V}_{-}(2)$ ψ ⅓, ½ (2) ψ -⅔, o (3)] + [ψ -⅔, o (1) ψ ⅓, ½ (2) $\hat{V}_{-}(3)$ ψ ⅓, ½ (3)] + [ψ ⅓, ½ (1)ψ -⅔, o (2) $\hat{V}_{-}(3)$ ψ ⅓, ½ (3)] +[ψ ⅓, ½ (1) ψ ⅓, ½ (2) $\hat{V}_{-}$ (3)ψ -⅔, o (3)] } $\chi_{1/2,1/2}(1)\chi_{1/2,1/2}(2)\,\chi_{1/2,1/2}(3)$

= N/$\sqrt{3}$ { [0 + ψ -⅔,o (1)ψ -⅔, o(2) ψ ⅓, ½ (3)] + [ψ-⅔, o (1) ψ ⅓, ½ (2) ψ -⅔, o (3)] + [ψ -⅔, o (1) ψ -⅔, o (2) ψ ⅓, ½ (3)] + 0 + [ψ ⅓, ½ (1) ψ -⅔, o (2) ψ -⅔, o (3)] + [ψ -⅔, o (1)ψ ⅓, ½ (2)ψ -⅓, o(3)] + [ψ ⅓, ½ (1) ψ -⅔, o (2) ψ -⅔, o (3)] + 0] }

= N/$\sqrt{3}$ ×2 {[ψ -⅔,o (1)ψ -⅔, o(2) ψ ⅓, ½ (3)] + [ψ-⅔, o (1) ψ ⅓, ½ (2) ψ -⅔, o (3)] + [ψ -⅔, o (1) ψ -⅔, o (2) ψ ⅓, ½ (3)]} ---------------- (a)

Now the factor N (normalization constant) can be calculated. As we know

that $\int_{-\infty}^{\infty} \Psi\Psi^{*}\, d\psi = 1 \;\Longrightarrow\; \langle \Xi^{*0}|\Xi^{*\,0}\rangle = 1$

$|N|^2 \times 4/3\,[1+1+1] = 1$; N = ½

Put this value of N in equation (a)

$|\Xi^{*0}\rangle = 1/\sqrt{3}$ [ψ -⅔, o (1)ψ -⅔, o (2) ψ ⅓, ½ (3)] + [ψ-⅔, o (1) ψ ⅓, ½ (2)ψ -⅔, o (3)] +

[ψ -⅔, o (1) ψ -⅔, o (2) ψ ⅓, ½ (3)]. This is the required wave function for, Ξ^{*0}

2.2 Construction of wave function in statistical model :-

The three (valence) quark wave function of the baryon can be written as [40]

$$\boldsymbol{\psi} = \boldsymbol{\phi}\,(\;|\Phi\rangle|\chi\rangle\;\psi|\rangle)(\;\xi\;)\rangle$$

Where Φ, , ψ and ξ denote flavor, color, and space time q^3 wave function. For the lowest- lying hadrons, quarks appear to be in S- wave states and the space – time q^3 wave function ξ is total symmetric under permutations of two quarks. Hence the flavor-spin –color part Φ should be total anti-symmetric under $q_i \leftrightarrow q_j$. In the conventional quark model , the color wave function ψ is taken to be total anti-symmetric, i.e. , a color singlet .

Total spin = 3/2 for decuplet particles and parity remains positive. We have to add $q\bar{q}$ and gg pair ⊗in a valence (qqq) state For this we keep in mind the spin and flavor of the baryon state. Where in case of color we consider it is color singlet state for simplicity.

Case 1:- For quark anti-quark pairs

(spin):- $q\,\bar{q} = 1/2 \otimes 1/2$ [spin has multiplicative property]

$= 0, 1$ _ _ _ _ _ _(1)

(Color):- Every quark can take three colors i.e. red, blue, and green. Therefore we have

$3 \otimes 3 = 1 + 8$ _ _ _ _ _ (2)

Where 1 stands for color singlet.

Case 2:- For a gluon pair.

Spin:- $1 \otimes 1 = 0, 1, 2.$ _ _ _ _ _ _ (3)

Color: - A gluon has 8 colors, therefore we have

$8 \otimes 8 = 1+8+10+\overline{10}+27$ _ _ _ _ _(4)

From (2) and (4) we have

$1 \otimes 1,\ 1 \otimes 8,\ 1 \otimes 10,\ 1 \otimes \overline{10},\ 1 \otimes 27,\ 8 \otimes 1,\ 8 \otimes 8,\ 8 \otimes 10,\ 8 \otimes \overline{10},\ 8 \otimes 27.$ _ _ _ _ _(5)

From these we take the simple case i.e. color singlet.

From (1) and (3)

$0 \otimes 0,\ 0 \otimes 1,\ 0 \otimes 2,\ 1 \otimes 0,\ 1 \otimes 1,\ 1 \otimes 2 = 0,1,2,3.$ _ _ _ _ _ _ (6)

Spin = 0, 1, 2 - corresponds to S wave. And

Spin = 3 – corresponds to P wave which can be neglected for simplicity.Therefore sea can be represented with valence as follows:-

1. $\boldsymbol{\varphi}_{10}^{3/2}H_0\,G_1$:- Symmetric
2. $\boldsymbol{\varphi}_{10}^{3/2}H_0\,G_8$:- Symmetric
3. $\boldsymbol{\varphi}_{10}^{3/2}H_0\,G_{10}$:- Anti-Symmetric
4. $\boldsymbol{\varphi}_{10}^{3/2}H_0\,G_{10}$– :- Anti-Symmetric
5. $\boldsymbol{\varphi}_{10}^{3/2}H_1\,G_1$:- Anti-Symmetric
6. $\boldsymbol{\varphi}_{10}^{3/2}H_1\,G_8$:- Anti-Symmetric
7. $\boldsymbol{\varphi}_{10}^{3/2}H_1\,G_{10}$– :- Symmetric
8. $\boldsymbol{\varphi}_{10}^{3/2}H_2\,G_1$:- Symmetric
9. $\boldsymbol{\varphi}_{10}^{3/2}H_2\,G_8$:- Symmetric
10. $\boldsymbol{\varphi}_{10}^{3/2}H_2\,G_{10}$– :- Anti-Symmetric [41]

From these we take, $\boldsymbol{\varphi}_{10}^{3/2}H_0\,G_1$ (Symmetric), $\boldsymbol{\varphi}_{10}^{3/2}H_1\,G_1$ (Anti-Symmetric), $\boldsymbol{\varphi}_{10}^{3/2}H_2\,G_1$ (Symmetric)

Therefore we have a sea component with valence part $\boldsymbol{\varphi}$ (10, 3/2, 1) in a baryon decuplet wave function as below.

$$\frac{1}{N}\Big[a_1\boldsymbol{\varphi}(10,3/2,1)\,H_0\,G_1 + a_2\Big[\sqrt{\frac{3}{5}}\,\boldsymbol{\varphi}(10,3/2,1)H_{1,0} - \sqrt{\frac{2}{5}}\boldsymbol{\varphi}(10,1/2,1)\,H_1, G_1\Big] \otimes G_1 +$$

$$a_3\Big[\sqrt{\frac{2}{5}}\,\boldsymbol{\varphi}(10,3/2,1)\!\downarrow H_{2,2} - \sqrt{\frac{2}{5}}\,\boldsymbol{\varphi}(10,3/2,1)\!\uparrow H_{2,1} + \sqrt{\frac{1}{5}}\,\boldsymbol{\varphi}(10,3/2,1)\,H_{2,0}\Big] \otimes G_1\Big]$$

Now $\boldsymbol{\varphi}(10, 3/2, 1)\,H_0\,G_1 = \boldsymbol{\varphi}_{10}^{3/2}\!\uparrow \otimes H_1 = \boldsymbol{\varphi}_{a2}^{3/2}\!\uparrow \psi_1^{A}$

$$\varphi 3/2 \uparrow\rangle = \frac{1}{N}\big[\boldsymbol{\varphi}_{10}^{3/2}\!\uparrow H_0\,G_1\,a_1 + a_2\,(\boldsymbol{\varphi}_{10}^{3/2}\!\uparrow \otimes H_1)\,G_1 + a_3(\boldsymbol{\varphi}_{10}^{3/2}\!\uparrow \otimes H_2)\,G_1\big]$$

$$= \frac{1}{N}\big[\boldsymbol{\varphi}_{10}^{3/2}\!\uparrow H_0\,G_1\,a_1 + a_2\,(\boldsymbol{\varphi}_{a2}^{3/2}\!\uparrow \otimes \psi_1^{A})\,G_1 + a_3(\,{}_{10}^{3/2}\!\uparrow \otimes \psi_1^{A})\,G_1\big]$$

For any operator $\hat{O}$ which only depends on quark flavour and spin does not depend on the color and space time, we have

$$\left\langle \frac{\Phi 3}{2}\uparrow \middle|\hat{O}\middle| \frac{\varphi 3}{2}\uparrow\right\rangle = \frac{1}{N^2}\Big[a_1^2\left\langle (\,\Phi_{10}^{\frac{3}{2}\uparrow}\middle|\hat{O}\middle|\Phi_{10}^{\frac{3}{2}\uparrow}\right\rangle + a_2^2\left\langle (\,\Phi_{10}^{\frac{3}{2}\uparrow}\middle|\hat{O}\middle|\Phi_{10}^{\frac{3}{2}\uparrow}\right\rangle +$$

$$a_3^2\left\langle (\,\Phi_{10}^{\frac{3}{2}\uparrow}\middle|\hat{O}\middle|\,\Phi_{10}^{\frac{3}{2}\uparrow}\right\rangle$$

Where $N^2 = a_1^2 + a_2^2 + a_3^2$

These coefficients can be calculated by fitting process in mathematician. In this we use the experimental data of magnetic moment. Hence we calculated

$a_1 = 1.41$; $a_2 = 1.51$; $a_3 = 0.6$;

Therefore $N^2 = 4.7038$

The first term is the conventional quark model result. In this a_2 is from vector sea and a_3 is from tensor sea.

If operator $\hat{O}$ has a form such as $\hat{O}=\sum_i \hat{O}_{f\sigma_z^i}$, where $\hat{O}_f$ depends only on the flavor of the quark and σ_z^i is the spin projection (z direction) operator of i^{th} quark.

CHAPTER: 3 MAGNETIC MOMENTS OF BARYON DECUPLET PARTICLES IN NON – RELATIVISTIC QUARK MODEL

3.1 Magnetic Moments in simple quark model:-

1. Δ^{++}

   The wave function is u u u

   $$\therefore \left\langle u\;u\;u \middle| \frac{e}{2m} \middle| u\;u\;u \right\rangle = \frac{2}{3}\times\frac{1}{2m_u} + \frac{2}{3}\times\frac{1}{2m_u} + \frac{1}{2m_u}$$

   $$= \mu_u + \mu_u + \mu_u = 3\mu_u \qquad [\because \frac{e}{2m_u} = \mu_u]$$

   $$= 3\times 1.86\,\mu_N = 5.58\,\mu_N \qquad [\because \mu_u = 1.86\,\mu_N]$$

2. Δ^{+}

   The wave function is, 1/$\sqrt{3}$ (uud + udu + duu)

   $$\left\langle \frac{1}{\sqrt{3}}(u\,u\,d + u\,d\,u + d\,u\,u) \middle| \frac{e}{2m} \middle| \frac{1}{\sqrt{3}}(u\,u\,d + u\,d\,u + d\,u\,u) \right\rangle$$

   $$= 1/3\,[\,\mu_u + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_u\,]$$

   $$= 1/3\,[6\,\mu_u + 3\mu_d\,] = 2\times 1.86 - 0.93 = 2.79\,\mu_N \qquad [\because \mu_d = -\,0.93\,\mu_N\,]$$

3. Δ^{0}

   The wave function is given by, 1/$\sqrt{3}$ (udd + dud + ddu)

   $$\therefore \left\langle \frac{1}{\sqrt{3}}(u\,d\,d + d\,u\,d + d\,d\,u) \middle| \frac{e}{2m} \middle| u\,d\,d + d\,u\,d + d\,d\,u\;\frac{1}{\sqrt{3}} \right\rangle$$

   $$= 1/3\,[\mu_u + \mu_d + \mu_d + \mu_d + \mu_u + \mu_d + \mu_d + \mu_d + \mu_u\,]$$

   $$= \mu_u + 2\,\mu_d$$

   $$= [1.86 + 2\times -0.93]\,\mu_N = 0\,\mu_N$$

4. Δ^{-}

   The wave function is ddd

   $$\therefore \left\langle d\;d\;d \middle| \frac{e}{2m} \middle| d\;d\;d \right\rangle = \mu_d + \mu_d + \mu_d = 3\,\mu_d = [3\times -0.93]\,\mu_N = -2.79\mu_N$$

5. Σ^{*+}

   The wave function is, 1/$\sqrt{3}$ (uus + usu + suu)

   $$= \left\langle \frac{(\text{uus} + \text{usu} + \text{suu})}{\sqrt{3}} \middle| \frac{e}{2m} \middle| \frac{(\text{uus} + \text{usu} + \text{suu})}{\sqrt{3}} \right\rangle$$

   $$= 1/3\,[\,\mu_u + \mu_u + \mu_s + \mu_u + \mu_s + \mu_u + \mu_s + \mu_u + \mu_u\,]$$

   $= 2\mu_u + \mu_s$ [∵ μ_s = -o.58 μ_N]

   $$= [2\times 1.86 - 0.58]\,\mu_N = 3.14$$

6. Σ^{*0}

   The wave function is, 1/$\sqrt{6}$ (uds + usd + dus + dsu + sud + sdu)

$$\left\langle \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu) \middle| \frac{e}{2m} \middle| \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu\,) \right\rangle$$

$= 1/6\ [\ \mu_u + \mu_d + \mu_s + \mu_u + \mu_s + \mu_d + \mu_d + \mu_u + \mu_s + \mu_d + \mu_s + \mu_u + \mu_s + \mu_u + \mu_d + \mu_s + \mu_d + \mu_u]$

$= \mu_u + \mu_d + \mu_s$

= [1.86 – 0.93-0.58] μ_N = 0.29 μ_N

7. Σ^{*-} The wave function is given by, $1/\sqrt{3}$ (dds + dsd + sdd)

$=\left\langle \frac{(\text{dds} + \text{dsd} + \text{sdd})}{\sqrt{3}} \middle| \frac{e}{2m} \middle| \frac{(d\,d\,s + d\,s\,d + s\,d\,d)}{\sqrt{3}} \right\rangle$

$= 1/3\ [\ \mu_d + \mu_d + \mu_s + \mu_d + \mu_s + \mu_d + \mu_s + \mu_d + \mu_d\ ]$
$= 2\,\mu_d + \mu_s$
= [2×-0.93 – 0.58] μ_N = -2.44 μ_N

8. Ξ^{*0} The wave function is given by, $1/\sqrt{3}$ (uss + sus + ssu)

$=\langle 1/\sqrt{3}\,(u\,s\,s + s\,u\,s + s\,s\,u\,) | e/2m | 1/\sqrt{3}\,(u\,s\,s + s\,u\,s + s\,s\,u\,)\rangle$

$= 1/3\ [\ \mu_u + \mu_s + \mu_s + \mu_s + \mu_u + \mu_s + \mu_s + \mu_s + \mu_u\ ]$

$= \mu_u + 2\ \mu_s$

= [1.86 – 1.16] μ_N = 0.70 μ_N

9. Ξ^{*-} The wave function is given by, $1/\sqrt{3}$ (dss + sds + ssd)

$=\langle 1/\sqrt{3}\,(d\,d\,s + s\,d\,s + s\,s\,d\,) | e/2m | 1/\sqrt{3}\,(d\,d\,s + s\,d\,s + s\,s\,d\,)\rangle$

$= 1/3\ [\ \mu_d + \mu_d + \mu_s + \mu_s + \mu_d + \mu_s + \mu_s + \mu_s + \mu_d]$

$= \mu_d + 2\ \mu_s$

= [– 0.93+2×-0.58] μ_N = -1.09 μ_N

10. Ω^-

The wave function is sss ∴ $\left\langle s\,s\,s \middle| \frac{e}{2m} \middle| s\,s\,s \right\rangle = \mu_s + \mu_s + \mu_s = 3\ \mu_s$ $[\because \frac{e}{2m_s} = \mu_s\ ]$
= 3× -0.58 μ_N = -1.74 μ_N

3.2 MAGNTIC MOMENTS OF DELTA PARTICLES FROM SEA WAVE FUNCTION

a) Δ^{++}

$$\left\langle \varphi 3/2 \uparrow \middle| \hat{O} \middle| \varphi 3/2 \uparrow \right\rangle = \frac{1}{N^2} [\, a_1^2 \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle + a_2^2 \; [\; \frac{3}{5} \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle] +$$

$$\frac{2}{5} \left\langle \Phi_{10}^{1/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{1/2\uparrow} \right\rangle] + a_3^2 [\, \frac{2}{5} \left\langle \Phi_{10}^{-1/2\downarrow} \middle| \hat{O} \middle| \Phi_{10}^{-1/2\downarrow} \right\rangle + \frac{2}{5} \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle +$$

$$\frac{1}{5} \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle]$$

In these cases color remains singlet.

$$= a_1^2 [\langle u\uparrow u\uparrow u\uparrow | O_f^i \sigma_3^i | u\uparrow u\uparrow u\uparrow \rangle] + a_2^2 [\, \frac{3}{5} \langle u\uparrow u\uparrow u\uparrow | O_f^i \sigma_3^i | u\uparrow u\uparrow u\uparrow \rangle + \frac{2}{5} \frac{1}{3}$$

$$\langle u\uparrow u\downarrow u\uparrow + u\downarrow u\uparrow u\uparrow + u\uparrow u\uparrow u\downarrow | O_f^i \sigma_3^i | u\uparrow u\downarrow u\uparrow + u\downarrow u\uparrow u\uparrow + u\uparrow u\uparrow u\downarrow \rangle$$

$$+ a_3^2 [(\frac{2}{5} \frac{1}{3} \langle u\uparrow u\downarrow u\downarrow | O_f^i \sigma_3^i | u\uparrow u\downarrow u\downarrow \rangle) + \frac{2}{5} \langle u\uparrow u\uparrow u\uparrow | O_f^i \sigma_3^i | u\uparrow u\uparrow u\uparrow \rangle +$$

$$\frac{1}{5} \langle u\uparrow u\uparrow u\uparrow | O_f^i \sigma_3^i | u\uparrow u\uparrow u\uparrow \rangle]$$

$$= a_1^2 [3\mu_u] + a_2^2 [\, \frac{9}{5} \mu_u + \frac{2}{5} \frac{1}{3} 3 \mu_u] + a_3^2 [\, \frac{2}{5} \frac{1}{3} \; (-3 \mu_u + \frac{2}{5} (3\mu_u) + \frac{1}{5} (3\mu_u)]$$
$$= a_1^2 [3\mu_u] + a_2^2 [\, \frac{11}{5} \mu_u] + a_3^2 [\, \frac{2}{5} \frac{1}{3} \; (-3 \mu_u) + \frac{2}{5} (3\mu_u) + \frac{1}{5} (3\mu_u)$$
$$= a_1^2 [3\mu_u] + a_2^2 [\, \frac{11}{5} \mu_u] + a_3^2 [\, \frac{7}{5} \mu_u]$$

b) Δ^{+}

The total symmetric flavor wave function is $1/\sqrt{3}$ [u u d + u d u + d u u] We have

$$\left\langle \varphi 3/2 \uparrow \middle| \hat{O} \middle| \varphi 3/2 \uparrow \right\rangle = \frac{1}{N^2} [a_1^2 \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle + a_2^2 \; [\; \frac{3}{5} \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle] +$$

$$\frac{2}{5} \left\langle \Phi_{10}^{1/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{1/2\uparrow} \right\rangle] + a_3^2 [\; \frac{2}{5} \left\langle \Phi_{10}^{-1/2\downarrow} \middle| \hat{O} \middle| \Phi_{10}^{-1/2\downarrow} \right\rangle + \frac{2}{5} \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle +$$

$\frac{1}{5}\langle(\Phi_{10}^{3/2\uparrow}|\hat{O}|\Phi_{10}^{3/2\uparrow}\rangle]$

$\frac{a_1^2}{3}\langle u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ |O_f^i\sigma_3^i| u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\rangle + \frac{3}{5}$

$\frac{a_2^2}{3}[\ \langle u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ |O_f^i\sigma_3^i| u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\rangle +$

$\frac{2}{5}\ \frac{1}{3}\langle u\uparrow u\downarrow d\uparrow + u\downarrow d\uparrow u\uparrow + d\uparrow u\uparrow u\downarrow\ |O_f^i\sigma_3^i| u\uparrow u\downarrow d\uparrow + u\downarrow d\uparrow u\uparrow + d\uparrow u\uparrow u\downarrow\ \rangle] +$

$\frac{a_3^2}{3}[\ \frac{2}{5}\langle u\uparrow u\downarrow d\downarrow + u\downarrow d\uparrow u\downarrow + d\downarrow u\downarrow u\uparrow\ |O_f^i\sigma_3^i| u\uparrow u\downarrow d\downarrow + u\downarrow d\uparrow u\downarrow + d\downarrow u\downarrow u\uparrow\ \rangle$

$+\frac{2}{5}\frac{1}{3}\langle u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ |O_f^i\sigma_3^i|\ u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ \rangle$

$+\frac{1}{5}\frac{1}{3}\langle u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ |O_f^i\sigma_3^i|\ u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow\ + d\uparrow u\uparrow u\uparrow\ \rangle]$

$= a_1^2/3\ [\ \mu_u + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_u\ ]$

$+ a_2^2\ [\ \frac{3}{5}\ \frac{1}{3}(\ \mu_u + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_u)\ +\ \frac{2}{5}\ \frac{1}{3}(\ \mu_u - \mu_u + \mu_d - \mu_u + \mu_d + \mu_u$
$+ \mu_d + \mu_u - \mu_u\ )]$

$+ a_3^2\ [\frac{2}{5}\ \frac{1}{3}(\mu_u - \mu_u - \mu_d - \mu_u + \mu_d - \mu_u - \mu_d - \mu_u + \mu_u\ ) + \frac{2}{5}\ \frac{1}{3}(\ \mu_u + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u +$

$\mu_d + \mu_u + \mu_u\ ) +\ \frac{1}{5}\ \frac{1}{3}(\mu_u + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_d + \mu_u + \mu_u\ )\ ]$

$= a_1^2/3\ [\ 6\mu_u + 3\ \mu_d\ ] + a_2^2[\ \frac{3}{5}\ \frac{1}{3}(6\mu_u + 3\ \mu_d) +\ \frac{2}{5}\ \frac{1}{3}(\ 3\ \mu_d\ )\ ] + a_3^2\ [\ \frac{2}{5}\ \frac{1}{3}(\ -2\ \mu_u -\ \mu_d\ ) +$

$\frac{2}{5}\ \frac{1}{3}(6\mu_u + 3\ \mu_d\ ) + \frac{1}{5}\ \frac{1}{3}(6\mu_u + 3\ \mu_d\ )\ ]$

$= a_1^2\,(2\mu_u + \mu_d) + a_2^2[\ \frac{3}{5}(2\mu_u +\ \mu_d\ ) +\ \frac{2}{5}\ \ \mu_d\ ] + \frac{a_3^2}{15}[16\ \mu_u\ + 7\ \mu_d\ ]$

c) Δ^0

The total symmetric flavor wave function is $^1/_{\sqrt{3}}$ [u d d + d u d + d d u].

We have the sea wave function as follow:-

$\langle \varphi 3/2\uparrow |\hat{O}| \varphi 3/2\uparrow\rangle\ =\ \frac{1}{N^2}\ [a_1^2\ \langle(\Phi_{10}^{3/2\uparrow}|\hat{O}|\Phi_{10}^{3/2\uparrow}\rangle\ + a_2^2\ [\ \frac{3}{5}\ \langle(\Phi_{10}^{3/2\uparrow}|\hat{O}|\Phi_{10}^{3/2\uparrow}\rangle\ ] +$
$\frac{2}{5}\ \langle \Phi_{10}^{1/2\uparrow}\ |\hat{O}|\Phi_{10}^{1/2\uparrow}\rangle\ ] + a_3^2\ [\ \frac{2}{5}\langle\Phi_{10}^{-1/2\downarrow}\ |\hat{O}|\Phi_{10}^{-1/2\downarrow}\rangle + \frac{2}{5}\langle(\Phi_{10}^{3/2\uparrow}|\hat{O}|\Phi_{10}^{3/2\uparrow}\rangle\ +$

$\frac{1}{5}\left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle]$

$\frac{a_1^2}{3}\langle u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow \mid O_f^i \sigma_3^i | u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow \rangle$

$+\frac{a_2^2}{3}[\ \langle u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow + d\uparrow d\uparrow u\uparrow | O_f^i \sigma_3^i | u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow \rangle +$

$\frac{2}{5}\ \frac{1}{3}\langle u\uparrow d\downarrow d\uparrow + d\downarrow u\uparrow d\uparrow + d\uparrow d\uparrow u\downarrow | O_f^i \sigma_3^i | u\uparrow d\downarrow d\uparrow + d\downarrow u\uparrow d\uparrow + d\uparrow d\uparrow u\downarrow \rangle] +$

$\frac{a_3^2}{3}\ [\ \frac{2}{5}\langle u\uparrow d\downarrow d\downarrow + d\downarrow u\uparrow d\downarrow + d\downarrow d\downarrow u\uparrow | O_f^i \sigma_3^i | u\uparrow d\downarrow d\downarrow + d\downarrow u\uparrow d\downarrow + d\downarrow d\downarrow u\uparrow \rangle$

$+\frac{2}{5}\frac{1}{3}\langle u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow | O_f^i \sigma_3^i |\ u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow \rangle$

$+\frac{1}{5}\frac{1}{3}\langle u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow | O_f^i \sigma_3^i |\ u\uparrow d\uparrow d\uparrow + d\uparrow u\uparrow d\uparrow \; + d\uparrow d\uparrow u\uparrow \rangle$

$= a_1^2/3\ [\ \mu_u + \mu_d + \mu_d + \mu_d + \mu_u + \mu_d + \mu_d + \mu_d + \mu_u\ ]$

$+ a_2^2\ [\ \frac{3}{5}\ \frac{1}{3}(\mu_u + \mu_d + \mu_d + \mu_d + \mu_u + \mu_d + \mu_d + \mu_d + \mu_u\ )\ \ +\ \frac{2}{5}\ \frac{1}{3}(\ \mu_u - \mu_d + \mu_d - \mu_d + \mu_u +$

$\mu_d + \mu_d + \mu_d - \mu_u\ )]$

$+ a_3^2\ [\ \frac{2}{5}\ \frac{1}{3}(\mu_u - \mu_d - \mu_d - \mu_d + \mu_u - \mu_d - \mu_d - \mu_d + \mu_u\ ) + \frac{2}{5}\ \frac{1}{3}(\ \mu_u + \mu_d + \ \mu_d + \mu_d + \mu_u + \mu_d$

$+ \mu_d + \mu_d + \mu_u\ ) +\ \frac{1}{5}\ \frac{1}{3}(\mu_u + \mu_d\ + \mu_d + \mu_d + \mu_u + \mu_d\ + \mu_d + \mu_d + \mu_u\ )\ ]$

$= a_1^2\ [\ (\mu_u\ +\ 2\ \mu_d)\ ] + a_2^2\ [\ \frac{3}{5}(\mu_u + 2\ \mu_d\ ) +\ \frac{2}{15}(\mu_u + 2\ \mu_d\ )\ ] + a_3^2\ [\mu_u\ + \frac{2}{5}\mu_d\ ]$

d) Δ^-

The total symmetric flavor wave function is d d d.

We have the sea wave function as follows:-

$\langle \varphi 3/2 \uparrow | \hat{O} | \varphi 3/2 \uparrow \rangle\ =\ \ \frac{1}{N^2}[a_1^2 \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle\ + a_2^2\ \ [\ \frac{3}{5}\ \left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle] +$

$\frac{2}{5}\ \left\langle \Phi_{10}^{1/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{1/2\uparrow} \right\rangle] + a_3^2\ [\ \frac{2}{5}\left\langle \Phi_{10}^{-1/2\downarrow} \middle| \hat{O} \middle| \Phi_{10}^{-1/2\downarrow} \right\rangle + \frac{2}{5}\left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle +$

$\frac{1}{5}\left\langle (\Phi_{10}^{3/2\uparrow} \middle| \hat{O} \middle| \Phi_{10}^{3/2\uparrow} \right\rangle]$

$= a_1^2\ [\langle d\uparrow d\uparrow d\uparrow | O_f^i \sigma_3^i | d\uparrow d\uparrow d\uparrow \rangle] + a_2^2\ [\ \frac{3}{5}\ \langle d\uparrow d\uparrow d\uparrow | O_f^i \sigma_3^i | d\uparrow d\uparrow d\uparrow \rangle +\ \frac{2}{5}\ \frac{1}{3}$

$$\left\langle d\uparrow d\downarrow d\uparrow + d\downarrow d\uparrow d\uparrow + d\uparrow d\uparrow d\downarrow \left| O_f^i \sigma_3^i \right| d\uparrow d\downarrow d\uparrow + d\downarrow d\uparrow d\uparrow + d\uparrow d\uparrow d\downarrow \right\rangle$$

$$+\mathrm{a}_3^{\,2}\,[(\tfrac{2}{5}\tfrac{1}{3}\left\langle d\uparrow d\downarrow d\downarrow \left| O_f^i \sigma_3^i \right| d\uparrow d\downarrow d\downarrow \right\rangle) + \tfrac{2}{5}\left\langle d\uparrow d\uparrow d\uparrow \left| O_f^i \sigma_3^i \right| d\uparrow d\uparrow d\uparrow \right\rangle +$$

$$\tfrac{1}{5}\left\langle d\uparrow d\uparrow d\uparrow \left| O_f^i \sigma_3^i \right| d\uparrow d\uparrow d\uparrow \right\rangle\,]$$

$$= \mathrm{a}_1^{\,2}\,[3\mu_{\mathrm{d}}] + \mathrm{a}_2^{\,2}\,[\tfrac{9}{5}\mu_{\mathrm{d}} + \tfrac{2}{5}\,\tfrac{1}{3}\,3\,\mu_{\mathrm{d}}] + \mathrm{a}_3^{\,2}\,[\tfrac{2}{5}\tfrac{1}{3}\,(-3\,\mu_{\mathrm{d}} + \tfrac{2}{5}(3\mu_{\mathrm{d}}) + \tfrac{1}{5}(3\mu_{\mathrm{d}})\,]$$

$$= \mathrm{a}_1^{\,2}\,[3\mu_{\mathrm{d}}] + \mathrm{a}_2^{\,2}\,[\tfrac{11}{5}\mu_{\mathrm{d}}] + \mathrm{a}_3^{\,2}\,[\tfrac{2}{5}\tfrac{1}{3}\,(-3\,\mu_{\mathrm{d}}) + \tfrac{2}{5}(3\mu_{\mathrm{d}}) + \tfrac{1}{5}(3\mu_{\mathrm{d}})\,]$$

$$= \mathrm{a}_1^{\,2}\,[3\mu_{\mathrm{d}}] + \mathrm{a}_2^{\,2}\,[\tfrac{11}{5}\mu_{\mathrm{d}}] + \mathrm{a}_3^{\,2}\,[\tfrac{7}{5}\mu_{\mathrm{d}}]$$

CHAPTR: 4 RESULTS AND CONCLUSION

Magnetic moments for Decuplet Particles (Delta Particles) are as follow:-

1. $\Delta^{++} = a_1^2 [3\mu_u] + a_2^2 [\frac{11}{5}\mu_u] + a_3^2 [\frac{7}{5}\mu_u]$
2. $\Delta^{+} = a_1^2 (2\mu_u + \mu_d) + a_2^2 [\frac{3}{5}(2\mu_u + \mu_d) + \frac{2}{5}\mu_d] + \frac{a_3^2}{15}[16\mu_u + 7\mu_d]$
3. $\Delta^{0} = a_1^2 [(\mu_u + 2\mu_d)] + a_2^2 [\frac{3}{5}(\mu_u + 2\mu_d) + \frac{2}{15}(\mu_u + 2\mu_d)] + a_3^2 [\mu_u + \frac{2}{5}\mu_d]$
4. $\Delta^{-} = a_1^2 [3\mu_d] + a_2^2 [\frac{11}{5}\mu_d] + a_3^2 [\frac{7}{5}\mu_d]$

Numeric values of magnetic moments:-

Sr.No.	Particles	Calculated Values	Experimental Values
1.	Δ^{++}	5.00 μ_N	5.58 μ_N
2.	Δ^{+}	2.05 μ_N	2.79 μ_N
3.	Δ^{0}	0 μ_N	0 μ_N
4.	Δ^{-}	-2.17 μ_N	-2.79 μ_N

Conclusion:-

In this we have construct the baryon wave function in statistical model for decuplet particles. There is a contribution of sea with valence part in a baryon state, actually sea is the fock states having gg or $q\bar{q}$ pairs. In our study we have taken the sea as gg pair, $q\bar{q}g$ or $\bar{q}q\bar{q}q$. The spin of sea is 0,1,2,3. In our study we constraint the sea with spin 0,1,2 and color singlet for simplicity. There is definitely a contribution of sea in baryon properties .We have find out the magnetic moments for delta particles, after modification in valence quark wave function with sea component .It should be noted that our results and conclusions are subjected to the following poits.

(1) The sea and the three quarks are considered in S-Wave state (spin = 0,1,2). Higher magnetic moments are neglected.
(2) The sea is assumed to be flavorless and has been specified with total quantum number.
(3) Relativistic corrections have been neglected, although the inertial motion of light quarks in a baryon is expected to have relativistic motion.
(4) All the calculations are performed in baryon rest frame.
(5) The sea is assumed to be color singlet to make whole the wave function antisymmetric.
(6) We have lack of experimental data for decuplet particles. We have not the experimental value of magnetic moment except Δ^{++}. So, we have used the same data of coefficients

(a_1,a_2,a_3) for Δ^+, Δ^0, Δ^- . In future we can work for strange particles with same concept. Also we can find out other properties of decuplet particles after removing the constraints of spin and color. After removing the constraints we can construct the wave function in statistical model which satisfied all the properties correctly.

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