FORMATION OF CAVITY SOLITON USING STANDING WAVE METHOD

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I affectionately dedicate this thesis to my loving family

Certificate

I hereby certify that the work which has been presented in this thesis entitled,"Formation of Cavity Soliton using Standing Wave Method",submitted in the partial fulfillment of the requirements for the award of degree of Masters of Science in Physics at Thapar University,Patiala,is an authentic record of my own work carried out under the supervision of Dr.Soumendu Jana,Associate Professor,School of Physics and Material Science and refers other researcher's work which are duly listed in the refrence section.The intellectual content of this thesis is the product of my own work,and contains no material which to a substantial extent has been accepted for the award of any other degree at this or other Educational Institution,except where due acknowledgement is made in thesis.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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Contents

Abstract

We generated localized patterns in dissipative cavity by standing wave approach. Two dissipative systems are considered, namely, Vertical Cavity Surface Emitting Laser (VCSEL) and VCSEL coupled with Frequency Selective Feedback (FSF) including perturbation gradient. The traveling wave (TW) solutions found by using He's Variational Method are considered. The TW solutions are superimposed to form standing wave (SW) patterns inside a cavity for gradient as well as non-gradient systems. Such SW pattern is as steady as a cavity soliton. These standing waves can be referred as cavity solitons if they satisfy three necessary conditions for the formation of soliton, namely, exponential localization, bi-stability and freedom of localization. These standing wave patterns/cavity solitons has many wide ranging applications in all-optical processing, memory devices, etc.

Chapter 1

INTRODUCTION

1.1Introduction to Cavity Solitons:

A soliton is a localized structure of waves which has the ability to preserve its shape even after collision with another soliton. It propagates with self similar shape. Soliton is called so because just like protons, electrons, etc. its behaviour is similar to a particle during interaction. The first ever soliton was observed in water waves of a union canal by a Scottish engineer, John Scott Russell in 1834.A heap of water formed in front of the boat and it continued to travel with unchanged shape even after the halt of boat [1]. In conservative systems, the formation of soliton requires the counterbalance between diffraction/dispersion and nonlinearity while in non-conservative system; a continuous energy flow is required in addition to aforesaid balance to keep the soliton alive. The solitons obtained in non-conservative systems are called dissipative solitons.

Cavity solitons (CS) are dissipative solitons inside a laser cavity that appear on a plane transverse to the cavity axis in the presence of nonlinear medium. These are dark spots on a light background or bright spots of light on dark background. A soliton is called cavity soliton if it has: (i) exponential localization (ii) bi-stability (iii) freedom of localization [2]. The concept of cavity solitons originated from theoretical paper of Moloney and coworkers. According to their model, a gaussian beam is passed through ring cavity containing self-focusing kerr medium. VCSEL (Vertical Cavity Surface Emitting Laser) is an example of a system in which cavity solitons can be obtained. VCSELS are ideal candidate to observe the soliton structures because the effective length of the cavity is of the order of 10 micrometers, have almost plane mirrors and the active medium lengths of the order of quarter wavelength. Moreover, there is a freedom of having more than 150 micrometer in diameter. Thus, VCSELS can be used to study spatiotemporal dynamics [3]. The main property of the cavity solitons is that its correlation length should be smaller than the size of the system. This condition will allow the cavity solitons to behave independently of each other. Localized structures having single intensity peaks can be referred to as Cavity Solitons in optics. CS can be characterized experimentally by the following properties. (i)They are self localized but do not depend upon boundary conditions (ii) CS can be independently manipulated and can exist in many transverse locations of the cavity (iii) CS can be set into motion. CS is formed under the coexistence of patterned and homogeneous stationary states. Depending upon the initial conditions, the solution may approach one or another state for same control parameters. Semiconductor systems are preferred over optical systems for observing CS because (i) CS observed in the semiconductor materials lie in small spatial scales and fast time scales. (ii)In semiconductor materials, the timescale for the formation of CS is of the order of carrier recombination time. This recombination time is of the range of nanoseconds and much smaller than other macroscopic systems. The size of the CS formed depends upon diffraction length 'a' and a = $\alpha \sqrt{L\lambda F}$.Here, L is the length of the cavity, λ is the wavelength of light and F is the Resonator Finesse. CS requires large and uniform aspect ratio systems. This means that the transverse extent of the cavity should be larger than the longitudinal extent so that there can be many transverse modes and there is no correlation among different locations of the cavity. Optical pumping is very useful for creating CS. The pump beam can be easily shaped up using optical pumping. The number of defects can be reduced by using optical pumping. The pump field is shaped up in the transverse as well as in the propagation direction. However, thermal management is a matter of concern while using optical pumping.

A homogeneous optical beam which covers the transverse section drives the nonlinear medium in an optical cavity. The system is set in such a manner that the saturable medium absorbs whole of the incoming beam so that the output intensity obtained is zero. A narrow, short optical pulse can ignite small region of device if the condition for bi-stability is obtained. This pulse is then called a writing beam. A holding beam is used to hold and sustain a CS. If the holding beam and writing beam are in phase with each other, then we get a CS. Similarly, many cavity solitons can be obtained at different locations. It should be noted that the small cavity length as compared to the transverse size is necessary for creating CS. By using writing pulse with opposite phase to the holding beam at the same place at which it was formed, CS can be erased too. CS can be manipulated in different kinds of gradients, for example, phase gradient, intensity gradient, etc. Gradients can arise due to several reasons. The gradient may arise during the growth of device and its fabrication. In electrically injected devices, the uneven distribution of carrier injection can also cause gradients. The misalignment of optical axis of VCSEL and the holding beam can also be one of the reasons [4].Uneven distribution of intensity in the holding beam and the defects in the structure of VCSELS can also cause gradients. All these factors greatly influence the creation as well as stability of CS. They are set into motion under the influence of gradients. This thesis will cover CS in two kinds of systems: (i) VCSEL and (ii) VCSEL coupled with frequency selective feedback (FSF) including perturbation gradient.

There are many methods to form CS like separation method, variational method [5], etc. We are following an alternative approach to form CS by using standing wave method. We started with the complex Ginzburg Landau equation (CGLE).Then its traveling wave solutions were considered for Sech and Cosh case. These solutions are then allowed to superimpose and form standing waves. These standing waves will then be called as CS if they satisfy the following three conditions [2]: (i) exponential localization (ii) present and absent under same conditions (iii) freedom of location. Different solutions can be obtained by using different profile functions. These solutions can be used in pattern formation and switching. They can be used to find complex localized patterns in VCSEL.

CS has very wide applications. They are used in parallel information processing. Since they can be erased or excited by perturbation at transverse location in nonlinear medium so they play the role of spatial logical bits or pixels. The appearance or disappearance of CS can be controlled because they can be switched ON and OFF independently. By adjusting the gradient of phase or intensity, the position and velocity of CS can be controlled [4]**.** Arrays of CS are used in parallel information processing. The application of CS in digital image processing is also expected [5]**.** The periodic motion of a CS can be revealed by putting a detector at one point of the trajectory. In this way, periodic train of pulses can be realized. CS can be trapped in the vicinity of the pulse. They can be stopped for a while and then released along desired direction [6]. An optical control pulse can switch the CS on and off. Thus, they are called as optical bits. CS is used in all optical networking. Holding beams are used to control the position of CS. Some of the spatial variations arise due to the holding beam. Spatial filtering can be used to minimize short scale noise. Gaussian intensity profile drives the SDS to the centre of the beam. The CS moves under the influence of amplitude and phase gradients. Mobility is the most promising property of CS in terms of applications. The velocity of CS is affected by the gradients. CS is also useful in all optical delay line. All optical routers will be required in future photonic networks in order to have high speed switching. If new data packets arrive when the router is busy, they need to be buffered in an optical delay line. Thus delay lines are said to be the key elements of these kinds of networks. The performance of delay line is governed by two factors :(i) the delay bandwidth product M (ii) the maximum operating speed. The maximum number of bits that can be stored in a delay line is represented by M. The CS must be distant enough in order to avoid the patterns that may arise due to interaction between different CS. The distance of about ten carrier lifetimes is considered as safe [8]. CS can be used in the determination of material structure (soliton force microscopy). By injecting a small beam, gradients can be measured. A linear phase gradient can be generated by tilting the HB with respect to the axis that is normal to VCSEL. Angle of incidence of HB is used to control the amount of gradient. The strength and position of defects in semiconductor devices can be determined by the motion of CS. This is known as soliton force microscopy (SFM).Self-localized states of light in time as well as space domain are called cavity light bullets (CLBs). Self-focusing is balanced by dispersion and group velocity dispersion (GVD) is balanced by Self Phase Modulation (SPM) simultaneously. 3D self localized states of light can lead to new applications. CLBs are not freely propagating as they are formed inside a cavity. They are independently controllable and addressable. The necessary condition for the formation of CLBs is that the carrier recombination times should be fast. If this is not so, the carrier dynamics may lag behind photon dynamics and the CLB may not be formed [4].

1.2 Literature Review:

Soliton is a topic which can be used to describe many phenomena, ranging from waves on a water surface to ultra short pulses in optical fibers. The fundamental property of solitons is that they can propagate long distance without changing their form. Soliton is localized solution of partial differential equations that describes the evolution of nonlinear system with infinite degrees of freedom. There are two kinds of system, namely, hamiltonian systems and dissipative systems. These systems are quite different from each other. The soliton is formed as a result of counterbalance between dispersion (diffraction) and nonlinearity in hamiltonian systems. The beam spreads due to diffraction but it gets focused due to nonlinearity. Thus, the beam becomes narrow. In dissipative systems, loss and gain should also be balanced in addition to the counterbalance between nonlinearity and diffraction/dispersion. Both the systems differ qualitatively from each other. Solitons in Hamiltonian systems are one or two parameter families while dissipative solitons are fixed solutions. The single complex Ginzburg Landau equation (CGLE) gives one family of solitons in case of Hamiltonian systems. If the number of coupled equations are increased, the number of soliton families increase. In case of dissipative systems, we get multiple solutions for single equation i.e. CGLE. Only the fundamental mode (lowest branch) is stable in case of hamiltonian systems whereas several solitons can be made stable simultaneously in dissipative systems [9].

The soliton structures that are trapped between reflecting surfaces particularly in a cavity are known as cavity solitons. This leads to the introduction of new properties. This widens the class of materials where solitons can be discovered. Some of the experiments have also shown the collision between cavity solitons. Solitons are formed when nonlinearity counter balances diffraction (or dispersion). Solitons are robust structures and propagate without changing their form. Cavity Solitons are dynamically stable and non-diffracting. CS will move under the influence of gradients and it will stop moving when it reaches a location that is free of any gradients [5].

Cavity solitons are formed in a dissipative environment. If the value of system parameters is fixed, the properties of CS are automatically fixed. CS are said to be rigid. By using external control beams, CS can be written and erased. By the introduction of amplitude or phase gradients, it is possible to control the velocity and location of CS. The light spots (CSs) can be written or erased in the transverse plane on any desired location. The transverse plane is orthogonal to the propagation direction of the beam. Optical resonators which contain nonlinear materials are usually used to produce CS. A stationary holding beam (HB) is injected to the cavity in order to provide energy to the system. An electric current is used to provide energy in case of semiconductor amplifiers. By switching 'ON' the holding beam, the CS can persist after the pulse also. Thus, many CS can be written using several pulses [10]. CS can be erased by using a HB that is out of phase with respect to holding beam. If the distance between two CS is larger than the certain minimum distance, then the two CS do not interact. Below this minimum distance, there is a possibility for two CS to reach an equilibrium distance. The locking of tails of CS can help in determination of equilibrium distance. The formation of clusters is also possible. The two CS fuse to form a single CS below a certain minimal distance. CS can be set into motion in presence of noise. The presence of phase or amplitude gradients in the presence of holding beam can also cause the motion of CS. Velocity of CS is proportional to the gradient. Phase gradients are very beneficial for many applications. The negative effects that arise due to noise and amplitude gradients can be neutralized by phase gradients. The most important practical application of CS is in semiconductor micro-resonators. The compact size and fast response of the material are two of the most important advantages.

When two level atomic medium is damped by squeezed vacuum field, a rich laser pattern is formed. Depending upon the ratio between cavity detuning (Δ) and squeezing parameter (M) of the vacuum field, two regimes were found. If $M>\Delta$, the pattern formed is different as compared to the usual case of lasers. Phase solitons and phase domains were obtained in this case. This was the first prediction of such type of patterns inside an amplifying cavity. If $M<\Delta$, the behaviour of the system is similar to the usual case of lasers. A new kind of pattern is formed by very weak and strong propagating traveling waves instead of tilted waves [11]. Furthermore, in the transition between these two regimes, a different type of pattern is formed. This pattern represents the features of both regimes. A stability analysis was carried out to confirm the results.

CSs are dark (bright) intensity peaks over a bright (or dark) homogeneous background. CS can be switched 'ON' or 'OFF' by using coherent pulses. The CSs are sensitive to intensity or phase gradients of intra-cavity field. This makes them to drift freely across the transverse section of the device. It is possible to obtain an optimal condition in a laser with saturable absorber. When the patterned state coexists with homogeneous background, the condition is said to be an optimal condition [12]. By using a VCSEL with saturable absorber, the system becomes more compact, robust and simple because of the elimination of HB. The motion of patterns in nonlinear media was demonstrated. Pattern formation takes place due to the presence of nonlinear medium. Control of geometry and size of optical patterns is also possible. The optical patterns may break into regular or irregular shapes. The velocity of patterns has constant magnitude and direction. The motion of 2D patterns was named as scrolling. The velocity of the structures is independent of the size of the system. The universal feature of pattern forming systems is the spontaneous scrolling motion of 2D disordered structures. The scrolling motion in 2D is because of the presence of boundary conditions and asymmetry of disordered structures [13].

The VCSEL with frequency selective feedback supports traveling wave solutions. In this model, it was shown that a single transverse traveling wave coexists with homogeneous solutions. This coexistence implies localized solutions. The standing waves and self localized traveling waves satisfy an important criterion of CS [4]. The CS requires bistability, freedom of location and potential localization. CSs possess particle like properties. The interaction of localized standing waves and localized traveling waves with external forces as well as with each other each other is an important research topic for future investigation.

The property of laser cavity solitons (LCS) in VCSEL with feedback from a volume bragg Grating (VBG) has been studied experimentally. Different trapping states can be achieved by tilting the VBG [14]. On increasing the current, the CS split from a single peak to 2 peaks. On increasing the current further, more complex arrangements can be achieved. CS shows polarization effects and CS splitting.

By introducing periodic modulations to the phase or linear gradients, the velocity, position and merging of CS with optimal injections can be easily controlled in VCSELS. By varying the amplitude and gradient, the velocity of CS and delay in all optical delay lines can be easily controlled. The drift in the cavity detuning parameter can be compensated by introducing variation in the input frequency. So, in this manner, the velocity of CS up to five times greater can be reached [15]. The merging of CS can play a vital role in correction of errors as well as in read out of delay line. The instabilities that affect the steady homogeneous state play an important role for determination of favorable and necessary condition for CS existence. If the ratio of carrier lifetimes in absorber and amplifier take approximate values, then the solitons can spontaneously move in Cavity Soliton Laser (CSL) based on VCSEL with saturable absorber [6]. A soliton can move on circle-shaped trajectories along the boundary in devices with finite circular cross-sections. The velocity of CS is of the order of few µs/ns. CS moves along the boundary if the pump has circular profile.

Pattern formation takes place due to nonlinearity in spatially extended systems. CS in driven systems are said to be 'slaved' because their frequency, phase and polarization is controlled by the holding beam. In a cavity soliton laser (CSL), CS are self sustained and do not require any holding beam [7]. The interaction properties and dynamics of CS will be affected by relative phase between them. CSs were demonstrated experimentally in laser with saturable absorber, with an injected signal and in laser with frequency selective feedback.

Nonlinear optical cavities driven by lasers can support different types of optical patterns. The intensity peaks that appear in such kind of optical patterns are known as CSs. Cavity Solitons are addressable individually. It is possible to locate CSs at discrete points by using Lugaito Lefever equation in order to model the perfect homogeneous cavity. The information capacity of the dynamical system can be evaluated by interpreting such patterns. For telecommunication purposes, CS can be considered as carriers of optical information. CS is not affected by boundary conditions but in case of finite size optical cavity, this may not be the case. CSs are strongly influenced by boundary conditions in this case. Even in absence of an imperfection, CSs can be drived to discrete locations by boundary conditions. These locations are different from the locations where CSs are generated [16]. Thus, a fundamental limit is set on the number of binary digits that can be encoded with CSs. This gives information capacity of the system. On one hand, the localized patterns which overlap with boundary were identified as edge states. On the other hand, contrary to the previous intuition, localized states were attracted to discrete locations.

1.3 Motivation:

The method of using standing waves to create CS is known but not explored much. The generation of CS using standing wave method has a very wide potential to show CS. A standing wave or a stationary wave is formed as a result of interference of two waves which are moving in opposite directions. When the interfering waves are non-tilted, superposition takes place at most of the points so we get many fringes but in case of tilted waves, the interference takes place at confined regions. There is pattern formation due to the superposition of waves (standing waves). The standing wave approach provides an alternate method for generation of CS.

1.4 Objective:

The main objective of our investigation is to show the formation of cavity soliton using standing wave method.

Chapter 2

NONLINEAR OPTICS, SOLITON AND PATTERN FORMATION

Cavity soliton generation is assisted by nonlinear phenomenon. In this chapter, we discuss about nonlinear optics, different optical nonlinearity, some relevant nonlinear (and linear) phenomena for better understanding of CS. Since we used an approach where traveling waves superimpose to create standing wave pattern, we discussed about traveling waves, standing waves and pattern formation. We also elaborated methodology relevant for this investigation.

2.1 Beginning of Nonlinear Optics:

The era of nonlinear optics started after first demonstration of laser by T. Maiman at Hughes Research Laboratories in 1960 [17]. In 1961, the invention of SHG (Second Harmonic Generation) marked the beginning of nonlinear optics. Nonlinear optics is the branch of optics that deals with the interaction of highly intense light with matter [3]. The nature of interaction between matter and light was considered to be linear before the invention of laser. In this case $\vec{P}$(Electric polarization of an optic material) is linearly proportional to $\vec{E}$ (Electric field of incident radiation). $\vec{P}$ and $\vec{E}$ are related by the relation: $\vec{P} = \chi\vec{E}$. Here, χ is linear susceptibility of an optical medium. Optical phenomenon observed using conventional light is referred to as "optics of weak light" or "linear optics". In case of an intense light (like laser), the electric polarization is not linearly dependent on $\vec{E}$. The nonlinear optics is called "optics of intense light". An electric field of about 1KV/cm is required in order to induce nonlinearity in an optical medium. The required beam intensity is about 2.5 KW/cm square. Evolution of refractive index (Intensity dependent) is one of the important consequences of light-matter interaction. The refractive index n susceptibility χ are related as: n=$\sqrt{1+\chi}$

Many phenomena like self phase modulation, spectral broadening, self trapping, self focusing/defocusing arise because of the intensity dependent n. The invention of solitons inside the optical fibers has enhanced the study of nonlinear optics.

Non-Resonant Interactions: Non-Resonant interaction is the interaction of electric dipoles under the effect of intense light. These electric dipoles are created because of positively charged atomic cores and electrons in matter.

Electron Cloud Distortion Model: Optical materials are made of negatively charged electrons and positively charged ions. The atoms do not show any net dipole moment in the absence of electromagnetic radiation. In the presence of electric field of electromagnetic radiation, the positive ions start moving along the direction of the field and negative electrons start moving in opposite direction. This results in separation of charges. Thus we get a net dipole moment.

Resonant Interaction: It is a light-matter interaction that leads to the absorption of incident radiation in case of coherent as well as incoherent light.

There are different types of Nonlinearities. They are:

1. Kerr Nonlinearity: It is the simplest nonlinearity. The refractive index n of a nonlinear medium is given by:

$$n(\omega) = n_0(\omega) + n_2(\omega)I$$

Here, '$n_\circ$' is linear refractive index of material. ' ω' is frequency of incident radiation. ' $n_2(\omega)I$ ' is nonlinear contribution of intense light. n_2 is related to nonlinear susceptibility of third order. Because of the susceptibility of third order, this nonlinearity is also known as "Cubic Nonlinearity". SiO_2 is the most common example of kerr nonlinear medium. Many materials, for example,diamond,TiO_2,GaAs,Al_2O_3,Si,ZnSe,CdS,CS_2,methanol,ethanol,CCl_4,etc. also show cubic nonlinearity. Soliton based optical communication system is one of the possible applications of kerr nonlinearity.

2. Cubic Quintic Nonlinearity: Competing Nonlinearity

General form of Cubic Quintic Nonlinearity is: $n(I) = n_p I^p + n_{2p}\, I^{2P}$

Here, p is positive constant. n_p and n_{2p} have opposite signs. This implies that n_p $n_{2p} < 0$.Then $n_p > 0$and $n_{2p} < 0$<represent self-focusing and self-defocusing respectively. Since the focusing and defocusing nonlinearity are competing with each other, the term that is used here is "competing nonlinearity". The effective coefficient is resultant of both the nonlinearities. If we take *p=1*, then it represents a special case of nonlinearity. It is expressed as:

$$n(I) = n_0 + n_2 I + n_4 I^2$$

n_4 is nonlinear coefficient of fifth order. The example of material that shows cubic quintic nonlinearity is PTS (Para Toulene Sulphonate).Some organic polymers also possess nonlinearity of this kind. It is important to note that quintic nonlinearity is weak as compared to cubic nonlinearity.

3. Saturating Nonlinearity: In some materials, optical nonlinearity saturates at high power. This is called Saturating Nonlinearity**.**

$$n = n_0 I \left({}^{I}/_{I_s} \right)$$

I_s represents saturation intensity. *I* represents intensity of incident beam. The examples of materials that possess saturating nonlinearity are: Rb vapour, CS_2, etc.

4. Transitive Nonlinearity: This nonlinearity transits from one form to another on increasing intensity of incident light. It is useful in switching and all optical logic devices.

There are also many other kinds of nonlinearity.

We are using cubic nonlinearity in our system.

2.2 Some Important Terms:

(i) Dispersion: Because of the variation of refractive index with frequency, the frequency components of pulse or an electromagnetic wave get separated. In optics, this phenomenon is called dispersion. Since different frequency components travel at different speeds in medium, it results in spreading of pulse in time domain. This phenomenon is known as Group Velocity Dispersion (GVD).Normal and Anomalous are the two kinds of dispersive medium. High frequency components travel fast as compared to slow frequency components in a normal dispersive medium. In an anomalous dispersive medium, slow components of frequency of travel faster as compared to high frequency components.

(ii) Self-focusing: The phenomenon of focusing of pulse or beam because of self-induced lens is known as self-focusing. The self-focusing can be either dynamic self-focusing or steady state self-focusing. Because of the slow variation (temporal) of the beam amplitude, the steady state self-focusing is not dependent on time. When the self-focusing becomes time dependent for pulse of short duration ($10^{-9} - 10^{-8}$ sec), then it is called dynamic self-focusing.

(iii) Self Phase Modulation (SPM): The interaction of intense light with matter modifies the phase profile as well as geometry of beam or pulse. Self Phase Modulation (SPM) is the phenomenon in which phase shift is induced in the pulse or beam because of intensity profile of its own. Note that the pulse shape does not change during SPM. The intensity profile for a moving pulse can be expressed as: $I = I_0 exp(-T^2/T_\circ^2)$. Here, I_0 is peak intensity. Time T is measured in moving frame of reference and $T_\circ$ is temporal scaling parameter. This is generally taken as half width of pulse at the point where intensity is $1/e$.

(iv) **Cross Phase Modulation(XPM):**In a system, when one beam experiences a phase shift because of intensity profile of another beam then it is known as Cross Phase Modulation(XPM). Kerr nonlinearity is simplest nonlinearity that gives rise to XPM. Moreover, XPM can be observed with saturating as well as cubic nonlinearity also. XPM is independent of time for continuous co-propagating waves [3]. XPM is time dependent for multi channel systems because it becomes important only when there is interaction between the pulse or beams that are traveling with different v_g (group velocity).XPM becomes insignificant when the faster pulse surpasses the slower pulse completely.

For CS generation, dispersion/self-diffraction should be balanced by nonlinearity induced SPM/self-focusing.

2.3 Soliton and its Background:

A soliton can be considered as localized state of waves that is capable of maintaining its shape. It maintains its shape even after undergoing collision with another soliton. It behaves particle-like during interaction. Soliton [3] can be found in light waves, plasma, water waves, etc. Soliton can exist in any of the four states of matter namely, liquid (example: organic materials like PTS), solid (example: crystals, polymers, etc.), gas (example: CS_2, Sodium vapour, etc.) and plasma. Soliton is an interesting research topic and has a wide scope in many fields like electronics, photonics, solid state physics, plasma physics, cosmology, oceanography, mechanical vibrations, superconductivity and fluid dynamics.

The first soliton was observed in 1834 by a Scottish Engineer, John Scott Russel. It was observed in the water waves of a union canal. A heap of water got collected in front of a moving boat and it continued to travel with same shape even after the halt of boat. He, later on, regenerated this water wave and named it as "wave of translation". He reported this wave of translation after 10 years. In 1965, Norman Zabusky and Martin Krusal [18] named this wave of translation as "soliton".

Optical solitons can be studied in various conditions and systems. This gives rise to variety of solitons. Depending upon their confinement in space and time, solitons can be classified as spatial and temporal solitons respectively. Spatial solitons remain confined in space domain while the temporal solitons remain confined in time domain. Spatiotemporal soliton is a third kind of soliton that remains confined in both time as well as space domain.

2.4 Basic Types of Soliton:

(1) Spatial Soliton: Self-diffraction causes the broadening of an optical beam with finite dimensions. The nonlinearity in the system can focus or defocus the pulse or beam. The beam propagates as self-trapped beam when self-diffraction and self-focusing perfectly balance each other. The soliton formed as a consequence of counterbalance between self-diffraction and self-focusing is known as spatial soliton. Since there is no change in spatial dimensions, so it is called as "Spatial Soliton". For example, let us consider a beam with gaussian intensity distribution propagating in a nonlinear medium. At the region of beam's peak intensity, the refractive index n is maximum and it decreases along both the edges of a gaussian pulse. The medium starts behaving like a convex lens as the curvature of phase front increases with propagation. Thus, this leads to the self-focusing of the beam.

Self trapping in (1+1) D case is stable in nonlinear medium. (2+1) D spatial solitons are also stable.P_c(Critical Power of self-focusing) is inversely proportional to r (beam radius).If the power of the beam is greater than critical power, i.e., P>P_c (then self-focusing of the beam takes place. More self-focusing implies higher P_c. As the beam radius decreases, P_c increases. If critical power is greater than power of the beam, i.e., the beam starts getting diffracted. Since the beam radius increases, so critical power decreases. Now when P_c again becomes less than P, the self-focusing again becomes dominant. This continues and thus an oscillating stability leads to self-trapping of pulse or beam. Spatial solitons are observed in many types of systems like optical cavity, bulk medium, etc. Spatial solitons have many potential applications. One of its most important application is controlling light by light (i.e., all-optical switching).Its other applications include all optical logic gates, beam steering and optical writing. They can be used in fabrication of quantum computers.

(2)Temporal Soliton: Both nonlinearity and dispersion induced SPM produces chirps when the pulse propagates in a dispersive nonlinear medium. These chirps can be of opposite nature under appropriate condition. The opposite chirps cancel each other under perfect counterbalance condition. As a result, the pulse propagates with constant temporal and spectral width. The localized pulse is then called as temporal soliton. This can also be called as longitudinal soliton sometimes. The leading edge of pulse is blue shifted and trailing edge is red shifted when the anamolous dispersion is dominant. This leads to the temporal spreading of pulse. When nonlinearity dominates, a chirp develops because of SPM, which is opposite to that due to GVD.SPM does not broaden the pulse in temporal domain but it spreads it in spectral domain. Both temporal as well as spectral widths remains constant when there is counterbalance between the chirps due to SPM and GVD. Temporal soliton is always (1+1) D.

(3) Spatiotemporal Soliton: Spatiotemporal solitons (STS) are those solitons that remain self-localized in space as well as time domain. They are also known as "super spikes" or "light bullets". STS originate because of the simultaneous effects of both spatial and temporal soliton. They are formed when GVD balances SPM and self-focusing balances self-diffraction simultaneously. The propagation of STS can be represented as (3+1) D NLSE. The solution of this equation will give a STS. They are stable in 1D but not in higher dimensions.STS have potential application in all optical processing devices. Each STS represents a bit of information.

Basic differences between temporal and spatial solitons :

(i)Medium sample length (of the order of kms) is required for the formation of temporal soliton. Small sample length (of the order of cms) is required or the formation of spatial soliton.

(ii)Temporal solitons are (1+1) D structures. Spatial solitons have higher dimensions because space has high dimensionality.

(iii)Temporal solitons can be found where transverse dimensions are negligible (For example: in optical fibers).Spatial solitons are formed in systems which have non-negligible transverse dimensions (For example: gas filled cell, etc.)

(iv)Temporal solitons have less degree of freedom. Spatial solitons have more degrees of freedom.

The soliton dynamics can be presented by a Nonlinear Schrodinger Wave Equation (NLSE) for a very ideal conservative case. However, most of the systems may be modeled as perturbed NLSE. In the proceeding sections, we discuss the basics of NLSE and perturbed NLSE.

2.5 Some Important Equations:

Nonlinear Schrodinger Equation (NLSE) is nonlinear dispersive PDE (Partial Differential Equation) and possesses infinite degrees of freedom. It is a Hamiltonian system. NLSE is used to represent variety of nonlinear systems [3]. This equation can be called as nonlinear perturbation of Schrodinger Wave Equation. The simplest NLSE can be written as:

$$i\,\frac{\partial A}{\partial z} + \frac{1}{2}\frac{\partial^2 A}{\partial y^2} + |A|^2 A = 0$$

Here, A represents slowly varying envelope of electromagnetic radiation. z represents propagating distance. y depends on whether the propagation is temporal or spatial.

In case of perturbed NLSE, the R.H.S of the above equation is non-zero.

2.6 Introduction to traveling waves, their superposition (Interference) and standing Waves:

Traveling Waves: A wave is a disturbance or perturbation that is caused by a moving object in a medium. The waves transport energy while traveling from one location to another in a medium. The waves propagate in a medium through particle-particle interaction. The particle gets displaced from its equilibrium position when it experiences a push or pull from its adjacent neighbour. In this way, a distinct wave pattern can be observed traveling through the medium. This wave continues to travel uninterrupted until it meets another wave or boundary of the medium [19]. The wave pattern traveling through the medium is known as traveling wave. An ocean wave is the most common example of traveling wave. Consider an elastic cord whose ends are, for example, two meters apart. The wave becomes confined in a small region if we introduce a wave into this elastic cord. This wave will immediately reach the end of cord. The wave will now reflect back and travel in opposite direction. The incident wave will interfere with the reflected portion of the wave and produce new pattern in the medium. The new pattern changes with time.

A wave can be observed when we drop a stone into quiet lake. The transfer of energy takes place when traveling waves move from one location to another. The energy is transported from the wave to the floating stick. The energy from the speaker system of a stereo gets transferred to our ears through sound waves. The energy is received from the sun in the form of electromagnetic waves (EM waves).No net transfer of matter is required for the transport of energy from a wave. Every wave is characterized by change of physical variable that propagates through space. The physical variable oscillating in sound waves is pressure. Because of vibratory motion of matter, sound waves are referred to as ’matter waves’. The electric and magnetic fields are physical variables that oscillate in an EM wave. Light is an EM wave. EM waves can propagate through vacuum but matter waves cannot. The waves can be either longitudinal or transverse. In longitudinal waves, the oscillations occur in a direction parallel to the direction of propagation. In transverse waves, the oscillations occur in a direction perpendicular to the propagation direction. Mathematically, a wave can be represented as: $y = A\ Sin\ (\omega t - kx + \theta)$

Here A represents amplitude of the wave, ω represents angular frequency (in radians per second), $t$represents time (in seconds), x is the distance from the source of oscillation, k is wave number and θ represents phase angle.

Superposition of waves: When two or more waves travel at the same time through the same medium, the principle of superposition is applied. The waves, without being disturbed, pass through each other. The sum of the individual wave displacements gives the net displacement of medium at any point in time or space. The principle of superposition is valid for waves of finite length.

Interference: Interference is a phenomenon in which the superposition of waves takes place in order to form a resultant wave of lower, same or greater amplitude. Interference occurs because of the interaction of coherent waves with each other. Interference can be observed with light waves, matter waves, etc. The interference can be either constructive or destructive. If the two (or more) interfering waves are in phase, i.e., ϕ=0, then the interference is said to be constructive. Here, ϕ represents phase angle. The amplitude of the resultant wave obtained is sum of amplitudes of individual waves. Contrary to this, if two (or more) interfering waves are out of phase, i.e., ϕ=180, then interference is said to be destructive. In this case, the interfering waves cancel out each other's effect.

Standing waves: A standing wave (SW) is a wave in which each point on the axis of wave has constant amplitude. Nodes are the locations in a wave where the amplitude is minimum. Antinodes are the locations in a wave where the amplitude is maximum. Michael Faraday noticed the standing waves on the surface of liquid in a vibrating container in 1831.The term 'standing wave' was coined by Franz Melde in around 1860 [20]. Standing waves can be formed in many different ways. SW can be formed if the medium and wave are moving in opposite direction with respect to each other. They can also be formed as a result of interference of two waves that are moving in opposite direction in a stationary medium. Resonance is the most common cause of standing waves. For example, it can be formed by the interference of sound waves, visible light, X-rays, etc. In general, the two identical traveling waves moving in opposite direction can form standing waves. A SW is a wave that does not appear to move. It can be formed when motion of wave is restricted to a finite region, for example, the string of a guitar can produce SW. Nodes are the stationary points in a SW. The frequency of a SW is related to the number of antinodes. The frequency is measured in Hertz (Hz).The formula is: $f = n\frac{v}{2L}$

Here, L represents length of the string, v represents speed of the waves, f represents frequency of SW and n gives the number of antinodes in a SW.

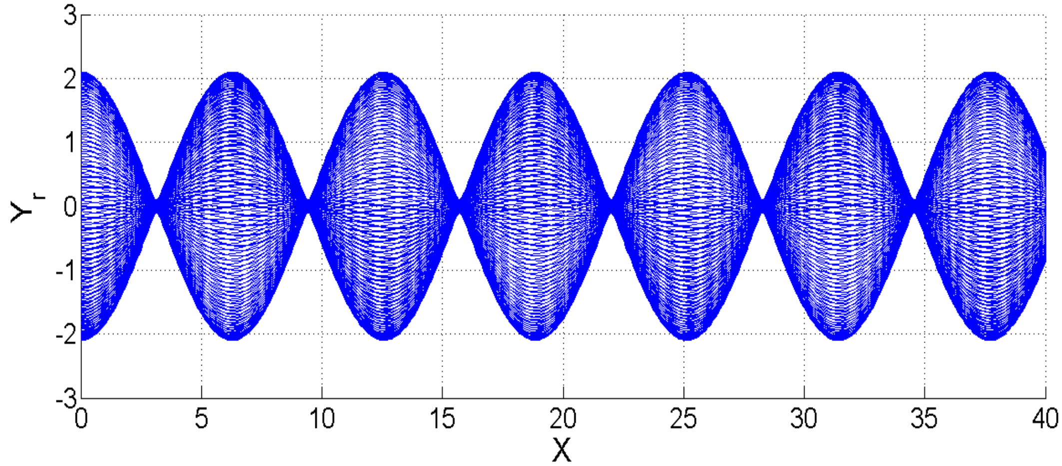


Fig.2.1. Standing Wave Pattern obtained by superposition of two traveling waves.

2.7 Pattern Formation:

Pattern formation is universal and it can be formed in different systems. The presence of at least two interacting fields leads to the formation of patterns. Out of the interacting fields, at least one of the fields must be propagating. Different patterns can be seen in ground, granular medium, etc. Pattern formation can take place in coherent as well as incoherent systems.

(i)Pattern formation for Coherent Systems: When plane wave of coherent light propagates in a self-focusing medium, the filamentation occurs during the propagation of beam in a medium. Despite of the fact that the propagation of a plane wave in self-focusing medium is the solution of nonlinear wave equation, the fragmentation occurs. It is invariant under propagation in z. The filamentation or fragmentation occurs because of the noise in the system. There will always be some kind of noise in the medium, no matter how uniform or homogeneous the medium is. The noise can be in the form of perturbation in refractive index, intensity, imperfections in medium, etc. Earlier, pattern formation and soliton were observed with coherent light only but later on, pattern formation was also observed with incoherent light. Patterns appear in the system in which the different places in space are not correlated [21]. Such kinds of systems are called as classical systems. In case of coherent light, there is no diffraction for a coherent periodic perturbation. This can be understood by using the concept of interference. The periodic perturbation can be viewed as interference of one plane wave with another plane wave that is arranged at symmetric angles with respect to propagation direction. Thus the periodic perturbation is invariant throughout the propagation. There is nothing to oppose self-focusing as there is no diffraction. Thus, even the slight nonlinearity is sufficient for pattern formation. The patterns will have large periodicity. There will always be filamentation in this case. No threshold value is required for pattern formation in this case.

(ii)Pattern formation for Incoherent Systems: In case of incoherent light, on the other hand, pattern formation takes place only if nonlinearity is above threshold. The threshold value depends upon degree of spatial correlation. There will be diffraction for an incoherent light for an incoherent periodic perturbation. The diffraction varies as the inverse of the correlation distance. Nothing will happen in the medium when we start from linear medium and then increase the nonlinearity because, at the beginning, the nonlinearity will not be strong enough in order to overcome the incoherent diffraction. Small perturbations of low intensity will fade away. On further increase in nonlinearity, we will reach the threshold value. Once the threshold value is attained, incoherent diffraction dominates over nonlinearity induced focusing. Thus, the perturbation will form a highly modulated pattern. When an incoherent beam of uniform intensity propagates in a self-focusing medium, patterns can arise from noise in an extended nonlinear medium.

(iii)Pattern formation in temporally and spatially incoherent cavity: A spatially incoherent cavity is defined as the cavity in which the transverse correlation distance of phase is smaller than the dimensions of the patterns that are involved in the cavity.

Pattern formation in nonlinear medium using feedback is an extensive topic of research in optics. The most common feedback can be obtained by designing a cavity such that it produces interference effects on being excited by the electromagnetic radiations. The light in these cavities is spatially and temporally coherent.

(iv)Pattern Formation in Cavity that is longer than the coherence length of light in it: Evolution of patterns in passive nonlinear cavity was studied. This cavity was taken to be longer than the coherence length of light that is circulating in it. On increasing the feedback, the patterns exhibit spatial line narrowing. This resembles the narrowing of lines in lasers. Patterns start from noise naturally. Some of the frequencies may experience higher gain and get stabilized through feedback in cavity, thus leading to the formation of pattern. Such kinds of patterns appear in laser cavities as well as passive cavities.

Patterns arising in cavities are different from the patterns that arise via MI (Modulational Instability) when the propagation takes place during the absence of feedback. A definite threshold is required for the formation of pattern inside the cavity. The resonators that were considered earlier for studying pattern formation were fully coherent. This means that the beam is coherent with the feedback beams and is spatially coherent and monochromatic. But in case of passive incoherent cavity, the feedback beams are mutually incoherent [22].This means that the relative phase among different coherent beams is random. It was observed that the relative phase between beams from different cycles and the resonant frequency of the cavity do not play any role in the formation of pattern. The beams from different cycles interact with each other. This interaction is similar to XPM (Cross Phase Modulation) but it leads to different phenomenon. It was found that on increasing the nonlinearity or feedback, the patterns inside the cavity exhibit spatial line narrowing.

2.8 Methodology:

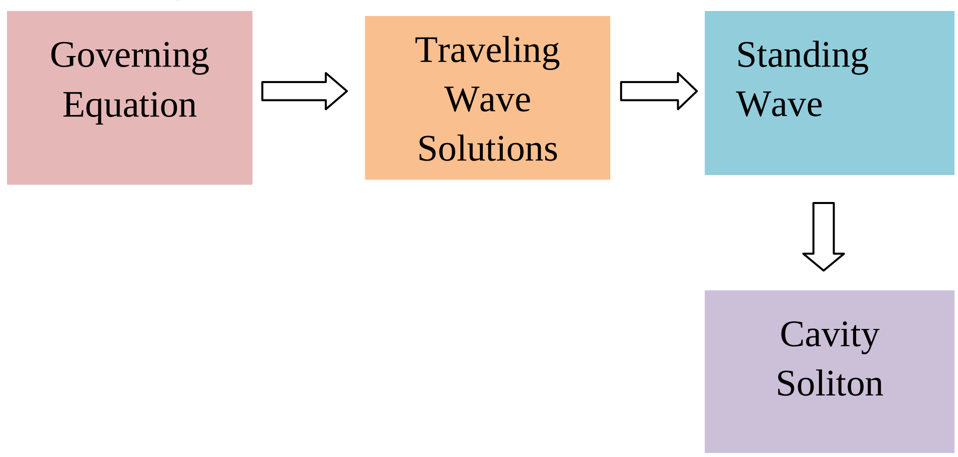


Fig.2.2. Blue Print of Patterns/Cavity Soliton inside a cavity.

We generated standing wave patterns for two dissipative systems, one is VCSEL and another is VCSEL with feedback including perturbation gradient. In first system, He's Variational Method gives traveling wave solutions of the CGLE with cubic nonlinearity (Governing Equation) by using different profile functions such as standard sech TW and non-standard cosh-gaussian TW inside a tilted cavity. The tilted traveling waves interact(superimpose) with one another and form standing waves. These standing wave patterns can be referred to as CS if they satisfy three basic conditions for CS, namely, bistability, freedom of location and exponential localization. Here, the superposition of the TW solutions for cosh-gaussian TW and sech TW case has been done and patterns (CSs) are formed for both the systems. For second system, CGLE with frequency selective feedback (FSF) and perturbation gradient is used as the Governing Equation. The approach used here is similar to that for first system in order to form different standing wave patterns after the superposition of TW solutions.

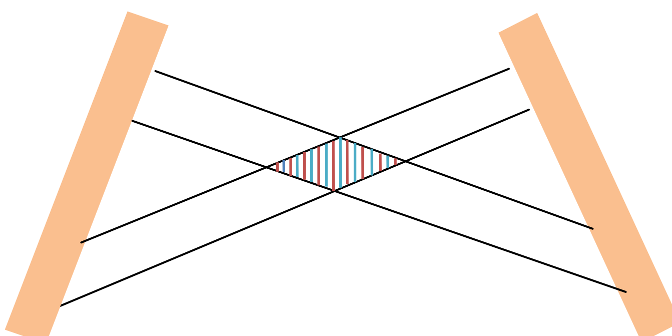


Fig.2.3. Cavity showing the superposition of tilted waves

Chapter 3

FORMATION OF CAVITY SOLITON

We investigated patterns/CSs through standing wave method for two different systems: (1) VCSEL (2) VCSEL with FSF including perturbation gradient.

3.1 Model 1: VCSEL

Complex Ginzburg Landau Equation (CGLE) is one of the most important nonlinear partial differential equation in the field of physics which describes the phenomenon of nonlinearity to superconductivity, super-fluidity, second order phase transitions, Bose Einstein Condensation, etc. The Ginzburg Landau theory (also known as Landau-Ginzburg theory) is a mathematical physical theory that is used to describe superconductivity of type 1 superconductors. This theory was named after Vitaly Lazervich Ginzburg and Lev Landau.

The Ginzburg Landau Equation is very useful in studying pattern formation. The Ginzburg Landau Equation has wide range of applications. It combines the fundamental features like nonlinearity, diffraction, dissipation and gain that underline the formation of patterns in nonlinear medium. CGLE is isotropic and translationally invariant. Cosh Gaussian traveling wave (TW) solution of a CGLE was derived. This solution describes dissipative semiconductor laser cavity. Sech TW solution was also obtained. Many distinct solutions were obtained in the case of Cosh Gaussian but in case of Sech, only one solution was obtained. Likewise, many solutions can be obtained using different profile functions. The Cosh Gaussian profile is generic and can be useful for further investigation on pulse dynamics, pattern formation, etc. Dissipative solitons (DSs) are self sustaining localized structures which are obtained from equilibrium in a dissipative nonlinear system. DSs occur in large variety of systems. An external energy is supplied in a dissipative system in order to keep the soliton alive. CSs are DSs. CSs are known to be propagating solitons that are bounded by mirrors of cavity. In VCSEL, CGLE (Complex Ginzburg Landau Equation) with cubic nonlinearity is an important model that is used for studying optical dissipative solitons and pattern formation [23]. The generalized CGLE used is:

$$\left(\frac{\partial}{\partial t} - \varepsilon\right)\psi = (1 + ic_1)\frac{\partial^2\psi}{\partial x^2} - (1 - ic_3)||\psi|^2\psi$$

Here, ψ represents slowly varying field envelope. x represents spatial coordinates that is transverse to the cavity axis. t represents cavity round trip time. The coefficients c_1, c_3 and ε are real value cavity parameters. The term on L.H.S of equation, i.e., $(\frac{\partial}{\partial t} - \varepsilon)$ describes basic cavity dynamics. Here, ε is cavity loss coefficient. The first term on R.H.S. of equation, i.e., $(1+ic_1)\frac{\partial^2\psi}{\partial x^2}$ represents spatial coupling. Here, real part is diffusive coupling and imaginary part is diffractive coupling. The last term on the R.H.S. of equation, i.e., $(1-ic_3)|\psi|^2\psi$ represents the nonlinearity induced gain or loss in the system. Here, real and imaginary part represents cubic nonlinear loss and gain respectively. If we take c_1=c_3=0 then the above equation becomes Ginzburg Landau Equation [24]. On eliminating 1s and setting ε=0, the CGLE becomes NLSE, i.e.

$$\frac{\partial\psi}{\partial t} = ic_1\frac{\partial^2\psi}{\partial x^2} + ic_3\,|\psi|^2\psi$$

CGLE is commonly used model for pattern forming systems as it supports many complex and coherent patterns. The tilted traveling waves (TW) can be excited in a VCSEL coupled with feedback. Here, we are considering a coherent cavity. By tilting the cavity, the traveling waves inside the cavity can be tilted. Solution of such system is considered as:

$$\psi(x,t) = g(\xi)e^{i(-kx+\omega t+\theta)}$$

Where $g(\xi)$ is the pulse shape, $\xi = x - vt$, v is the velocity of soliton , k is the soliton wave number , ω is the soliton frequency and θ is phase constant.

3.2 Model 2: (VCSEL + FSF) with perturbation gradient

In a VCSEL coupled with feedback, the pulse dynamics in a laser cavity is represented by the CGLE [25]:

$$\left(\frac{\partial}{\partial t}+1\right)E(x,t)-i\frac{\partial^2 E(x,t)}{\partial x^2}-\mu(1-i\alpha)(1-|E|^2)E(x,t)-\sigma(a-ib)E(x,t)$$

Where, $E(x,t)$ represents the slowly varying field envelope of the electric field of laser pulse, t represents cavity round trip time and x represents the coordinate transverse to the cavity axis. Here, a = $\Gamma_\circ / (\Gamma_\circ^2 + \Omega_\circ^2)$, b= $\Omega_\circ^2 / (\Gamma_\circ^2 + \Omega_\circ^2)$ while $\Omega_\circ$ is resonant frequency, while $\Gamma_\circ$ represents the line width enhancement parameter of feedback field, σ is the coupling constant or feedback strength. M is the scaled gain responsible for stabilization of off state in VCSEL. Line width enhancement parameter α attains large positive values for VCSEL usually. The first term in the above equation is the evolution of wave envelope $E(x,t)$ and linear loss inside cavity. The second term represents diffraction term .The third term consists of Kerr nonlinearity and nonlinear loss /gain depending upon sign of coefficients in the transverse direction of cavity. The last term in the equation is the contribution of frequency selective feedback (FSF). The above equation can be rewritten as perturbed nonlinear Schrodinger Equation as:

$$i\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}-(\sigma b+\alpha\mu)E+\alpha\mu|E|^2E=iR(E,E^*)$$

Where, $R(E,E^*)$ =$(\sigma a-1+\mu)E-\mu|E|^2E$, is the perturbation term, nullifying which an unperturbed NLSE can be retrieved. The following perturbation term is introduced in the present investigation:

$$R(E,E^*)=\frac{\partial}{\partial x}\left[(\sigma a-1+\mu)E-\mu|E|^2E\right]$$

This term represents slope of perturbation which has been used in anticipation to include the effect of modulation of perturbation in spatially distributed system. The solution of this system in

1D is considered as [26, 27]:

$$E(x,t)=\psi(s)\exp(i(-kx+\omega t+\theta))$$

Where $\psi(s)$ is the pulse shape, $s=x-vt$, v is the velocity of soliton , k is the soliton wave number , ω is the soliton frequency and θ is phase constant. The TW solution is:

$$E(x,t) = \left[\frac{2(-\sigma b - \omega + k^2 - \alpha\mu - \sigma a + 1 - \mu)}{\mu(k-\alpha)}\right]^{\frac{1}{2}} Sech\, [(-\sigma b - \omega + k^2 - \alpha\mu - \sigma a + 1 - \mu\ \ )^{\frac{1}{2}}]\, \exp(i(-kx + \omega t + \theta))$$

3.3 Cavity Soliton formation by standing wave method:

(I). For Model 1: VCSEL

We formed CS with standard sech profile and cosh-gaussian profile.

Case(i): Sech Standard TW

The profile function (traveling wave solution) in this case is:

$$g(\xi) = S\ Sech\ (T\xi)$$

Where S and T are taken from Ref. [28]:

$$S = \left[\frac{2\left(\omega + c_1 k^2 + \frac{2k(k^2-\varepsilon)}{v+2c_1 k}\right)}{\left(c_3 - \frac{2k}{v+2c_1 k}\right)}\right]^{\frac{1}{2}}$$

And

$$T = \left[-\frac{\left(\omega + c_1 k^2 + \frac{2k(k^2-\varepsilon)}{v+2c_1 k}\right)}{\left(c_1 + \frac{2k}{v+2c_1 k}\right)}\right]^{\frac{1}{2}}$$

The traveling wave solutions are now superimposed to create standing waves. We discuss standing wave pattern formation for different situations by taking: Φ= π/4, c_1=0.3, c_3=-0.8, I_0=ε, k=0.3, ε=1, ω=c_3:

(1)Variation with different tilt angle θ: On varying θ from 0˚to 25˚, we get 4 peaks. When θ is further increased up to 55, we get 3 peaks. From θ=56˚to 70˚, we get 2 peaks.1 peak is obtained on varying θ from 75˚ to 100˚ except at θ=80˚ and 100˚ where 2 peaks are obtained. **In general, the number of peaks decreases on increasing theta. Note that the number of peaks is the measure of the number of Cavity Solitons formed. Thus, if 4 peaks are formed, it means that 4 CSs are formed.**

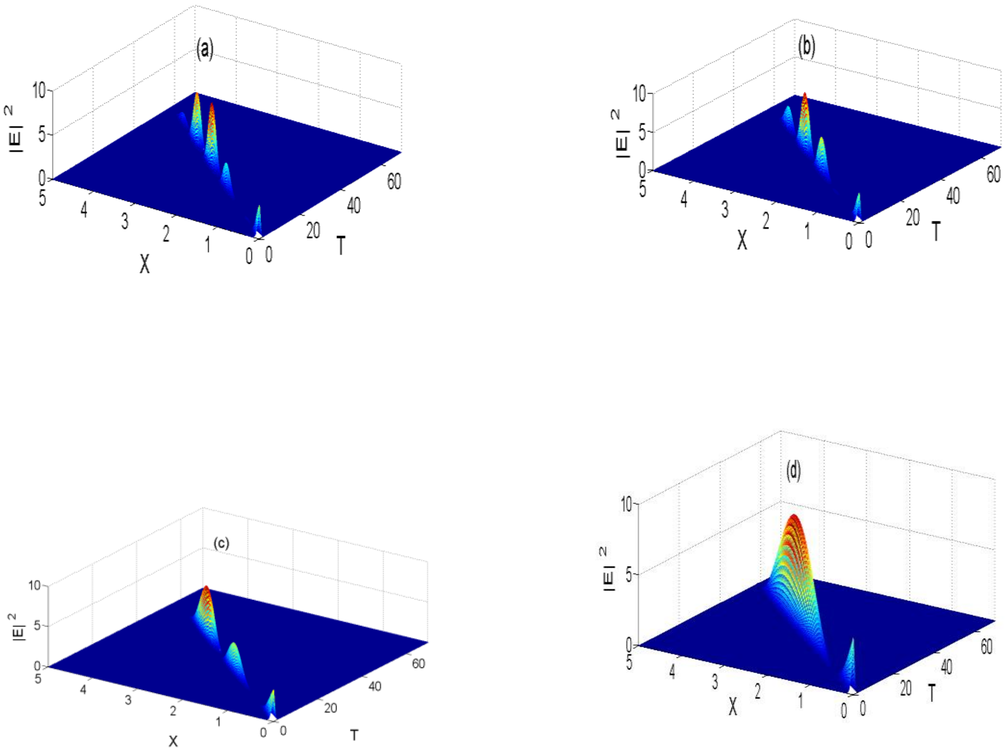


Fig.3.1. Generation of cavity soliton with different tilt angle θ. Parts (a), (b), (c), and (d) correspond to θ=0˚,30˚,60˚and 90˚ respectively.

(2)Variation of Peak Value (PV) with absorption coefficient (A): **On increasing the absorption coefficient (A), the Peak Value decreases.**

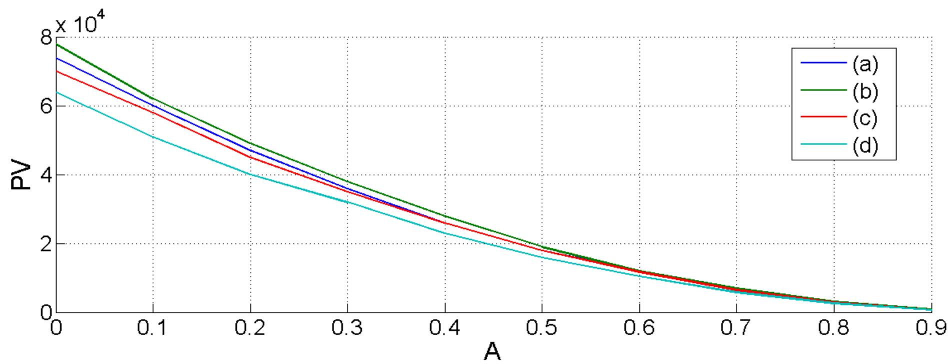


Fig.3.2. Variation of Peak Value(PV) with Absorption Coefficient (A) .The curves (a) ,(b), (c) and (d) correspond to the variation at θ= 30˚, 0˚,45˚and 60˚ respectively.

(3)Variation with cavity loss coefficient (ε): Variation with ε was checked by keeping θ fixed for different cases. By varying ε from 1 to 10:- 4 peaks were obtained for fixed θ=0˚, 3 peaks were obtained for fixed *θ*=30˚, 4 peaks were obtained for fixed θ=45˚ and 2 peaks were obtained for fixed θ=60˚.

(4)Variation with different reflection coefficient(*r*): By keeping θ=45˚ (fixed) ,the reflection coefficient was varied and and the number of peaks were observed.5 peaks were observed on varying *r* from 0.1 to 0.8 with maxima at 2.3 ,2.6 ,3.1 ,3.5 ,4 ,4.6 ,5.1 ,5.8 ,6.4 ,7 respectively. 3 peaks were observed for *r*=0.9 and *r*=1 with maxima at 6.4 and 7 respectively. By keeping theta=30˚(fixed), 6 peaks were obtained on varying *r* from 0.1 to 0.7 with maxima at 2.4, 2.7, 3.3, 3.8, 4.3, 4.9 and 5.6 respectively. For *r*=0.8 to 1, 3 peaks were obtained with maxima at 6.2, 6.9 and 7.8. **In general, the number of peaks decreases on increasing r**.

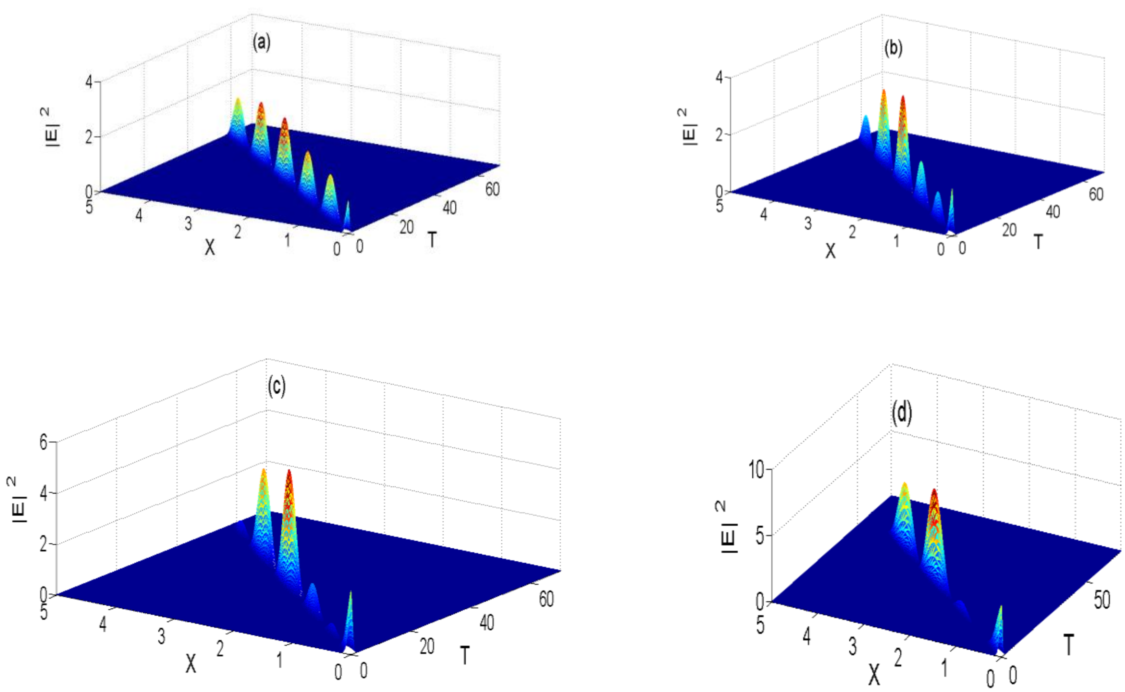


Fig.3.3. Generation of standing wave patterns/cavity solitons with different reflection coefficient(r). Parts (a), (b), (c) and (d) correspond to r = 0.1, 0.3, 0.6, 0.9 respectively.

Case(ii): Cosh Gaussian Non-Standard TW

The two (or more) Cosh TW are superimposed in order to form Standing Waves and these standing waves formed Cavity Soliton (CS).The profile function in this case is:

$$g(\xi) = P\,Cosh\,(Q\xi)\,\exp\left(-\frac{\xi^2}{R^2}\right)$$

where , P is pulse amplitude, Q represents cosh parameter and R represents pulse width

Since it is very difficult to calculate the values of P, Q and R numerically, so these values are determined graphically (ref. to fig.4 of [28]) .

The solution points are: A(P, Q) = (2.7994, 0), B(P, Q) = (1.9402, 4.7740), C(P, Q) = (0.1217, 13.0282), D(P, Q) = (0.0908, 13.5247), E(P, Q) = (0.0897, 13.5435), F(P, Q) = (0.0603, 14.1817), G(P, Q) = (0.0600, 14.1817), G(P, Q) = (0.0600, 14.1859) and H(P, Q) = (0.0301, 15.2294). Note that all the variations are shown for pt D here.

Taking: c1 = 0.3, c3 = -0.8, $I_{\circ}$=ε, ε=1, k=0.3, ω=c3, v=11.7279, R=0.3, the cavity solitons/standing wave patterns are generated under different conditions as shown:

(1)Variation with different tilt angle (θ): The variation of intensity($|E|^2$) with different angles was studied and the intensity profile does not follow a regular trend. The numbers of peaks are different with different θ. Each peak represents a cavity soliton. The number of peaks is the measure of the number of CSs formed. On increasing T, the fluctuations in intensity are observed for different snapshots of time. The dip in cosh traveling wave can be seen. It seems as if two similar patterns are observed behind each other. The results with different tilt angle are shown:

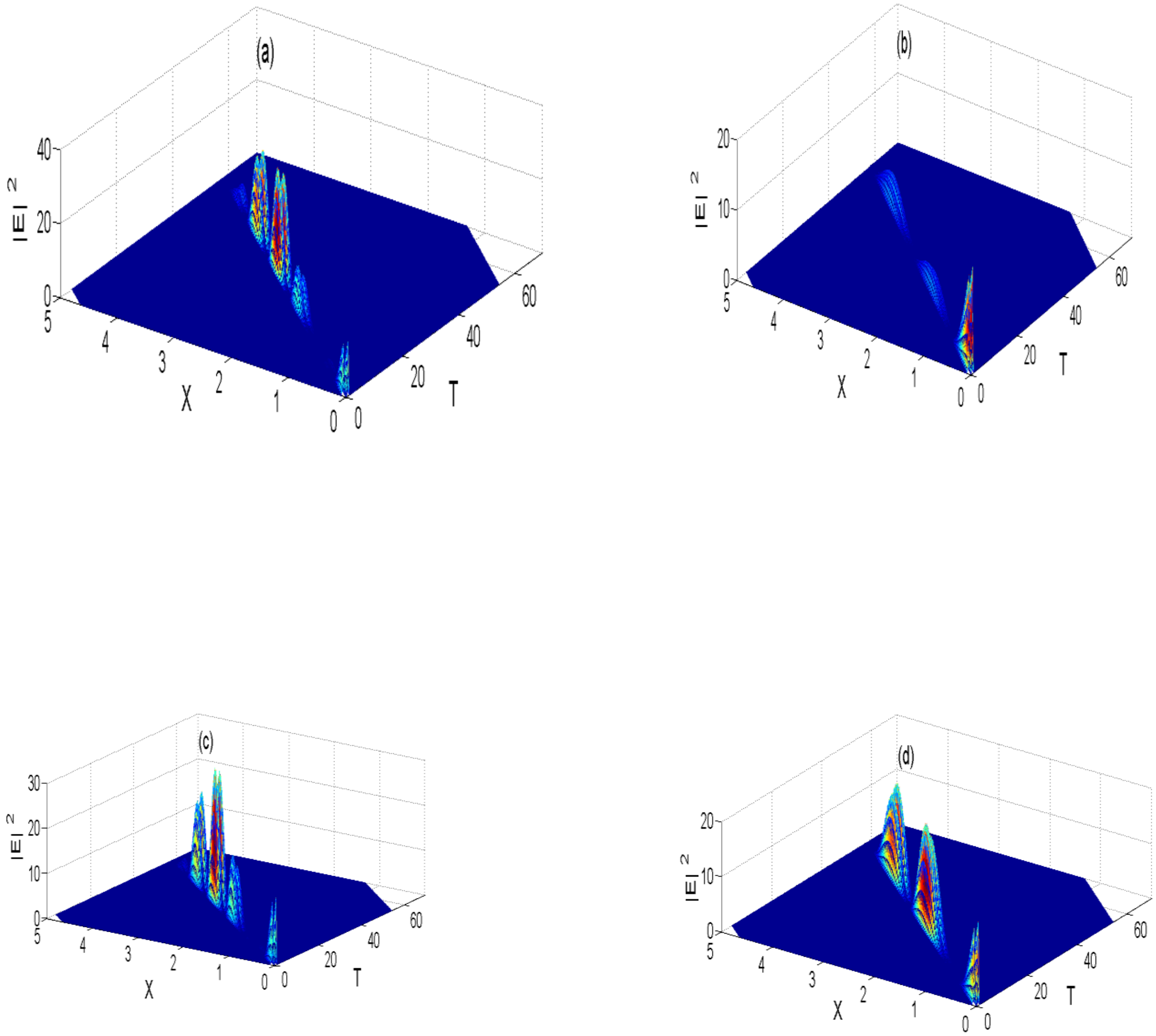


Fig.3.4. Standing wave patterns/cavity solitons with different tilt angle θ. Part(a),(b),(c) and (d) correspond to θ=0˚,30˚,60˚and 90˚respectively.

(2)Variation in Peak Value (PV) with Absorption Coefficient (A): **Peak Value (PV) decreases on increasing the absorption coefficient (A).**

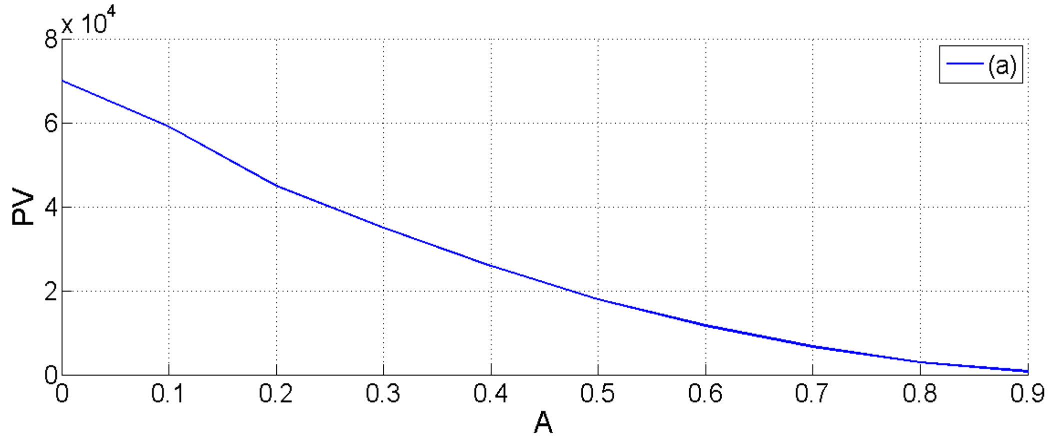
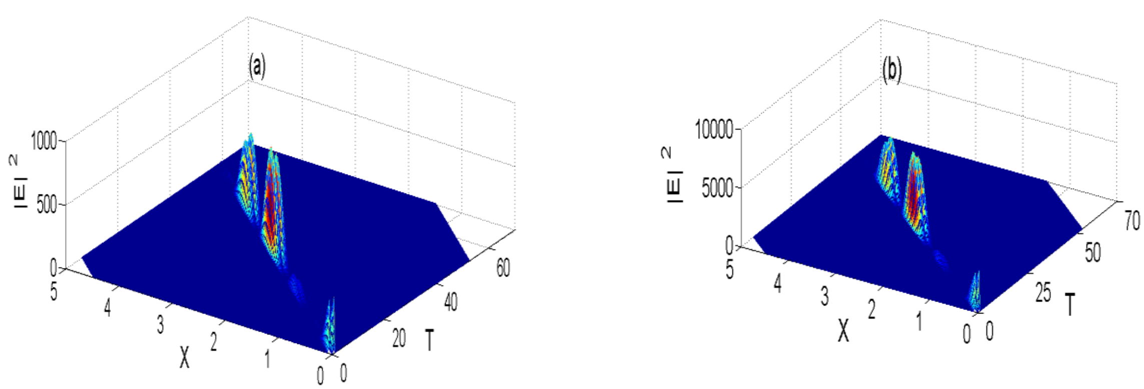


Fig.3.5. Variation in Peak Value (PV) with Absorption coefficient (A). Curve (a) corresponds to variation at θ=45˚.

(3)Variation with cavity loss coefficient(ε): On varying ε from 1 to 10, 4 peaks were observed for fixed θ= 0˚, 3 peaks were observed for fixed θ=30˚, 4 peaks were observed for θ=45˚ and 2 peaks were observed for θ=60˚. **This variation is same as that of Sech TW.**

(4)Variation with different reflection coefficient(*r*): 4 peaks were observed at *r*=0.1, on varying *r* from 0.2 to 1, 3 peaks were observed. **In general, the number of peaks decreases and maxima increases on increasing *r*.**



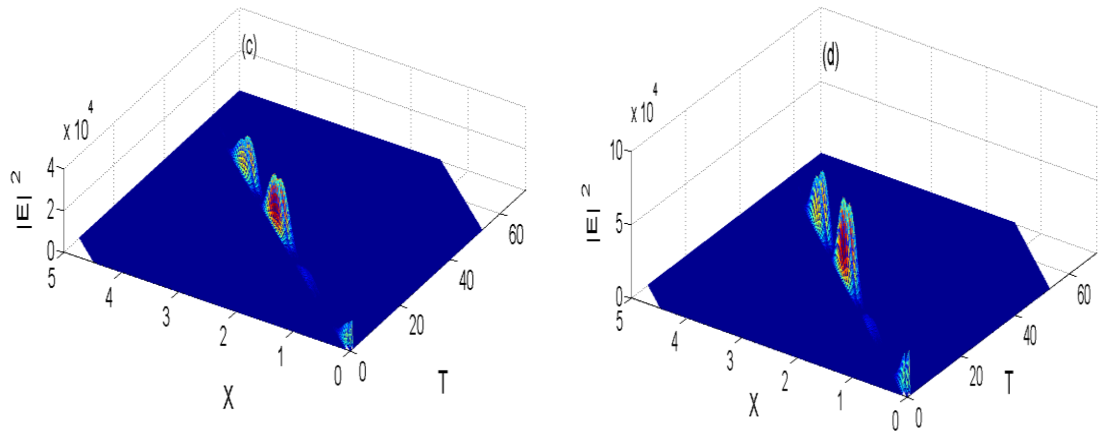


Fig.3.6. Generation of patterns for different reflection coefficients. Parts(a), (b), (c) and (d)correspond to r= 0.1, 0.3, 0.6 and 0.9 respectively.

(II). For Model 2: VCSEL with FSF and perturbation gradient

We formed CSs for standard sech, non-standard cosh-gaussian and Gaussian.

In a Vertical Cavity Surface Emitting Laser (VCSEL) with perturbation gradient and frequency selective feedback (FSF), the traveling wave solutions were obtained. In a VCSEL, high reflectivity Distributed Bragg Reflectors(mirrors) are used in order to make an optical cavity. A DBR mirror is a periodic structure of alternating dielectric layers having indices n_1 and n_2. At wavelength λ_B, there is maximum reflectivity. The reflection coefficient $r(\lambda_B)$ in a DBR is determined as:

$$r(\lambda_B) = \frac{\left(\frac{n_2}{n_1}\right)^{2m}\left[\left(\frac{n_0}{n_3}\right)-1\right]}{\left(\frac{n_2}{n_1}\right)^{2m}\left[\left(\frac{n_0}{n_3}\right)+1\right]}$$

The refractive index of the cavity is denoted by n_0 and the number of periods in a DBR is denoted by m. The value of reflection coefficient obtained from this formula is used here to study formation of patterns in non gradient systems. The reflectivity *(R)* is calculated by using formula:

$$R = |r(\lambda_B)|^2$$

On substituting n_2=3.49, $n_1 = 3.06$, $n_0 = 14.14, n_3 = 3.65$ and $m = 30$, the reflectivity comes out to be 0.9 .

The variation of reflectivity*(R)* with the number of periods m is studied .Initially, as m increases, *(R)* increases and for higher values of m, the graph saturates. Similar trend is observed when n_0/n_3 (or) n_2/n_1 is plotted with respect to *(R).* The graphs are shown below:

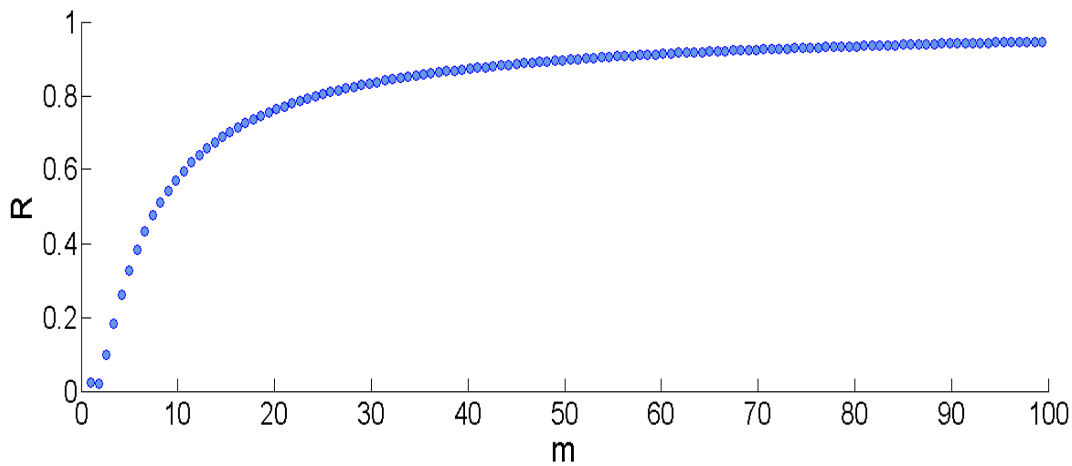


Fig 3.7 Variation of R with m.

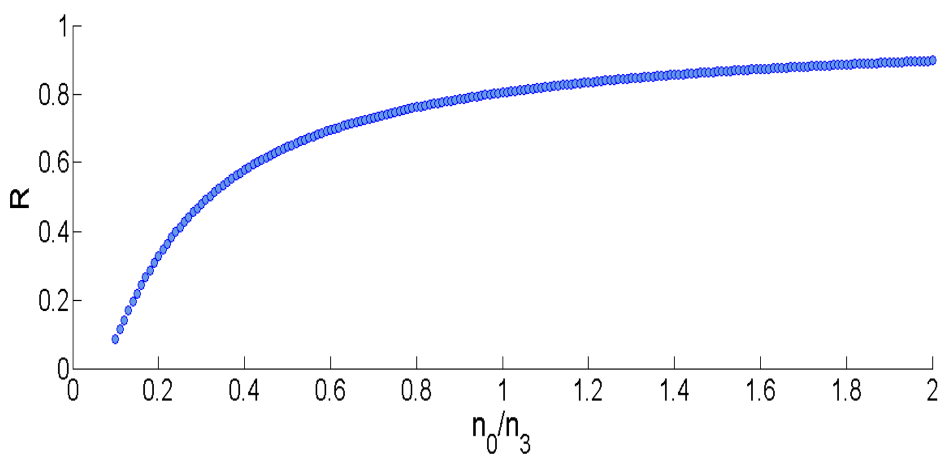


Fig.3.8 Variation of R with n_0/n_3.

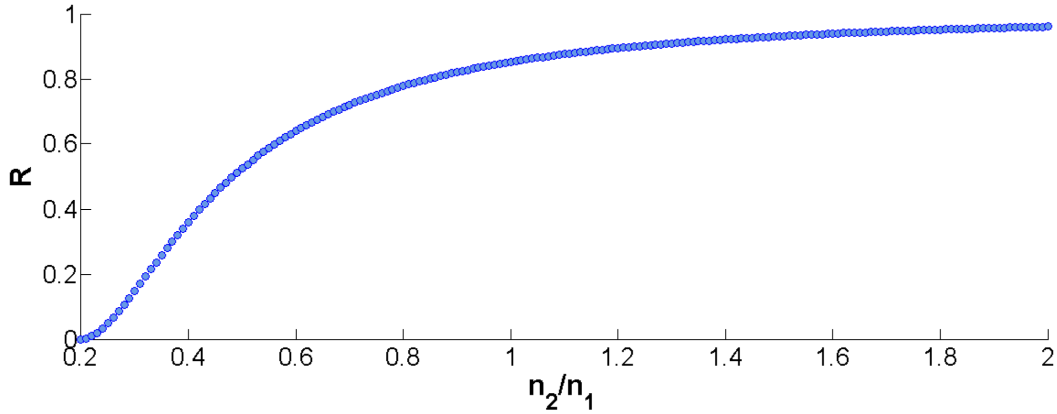


Fig.3.9. Variation of R with n_2/n_1.

The standard Sech type and the non-standard Cosh-Gaussian type solutions were derived. He's Variational Method was used to solve the Complex Ginzburg Landau Equation (CGLE) with perturbation gradient. This equation is the Governing Equation of the system. A parametric region was identified that corresponds to stable soliton solution. Cosh-Gaussian soliton solutions can be used to study CS and to realize all-optical tunable devices. Such solutions have wide applications in data processing and controlling units, delay lines, all-optical devices, delay lines, optically addressable displays and all-optical devices. Different results are obtained for Sech type, Cosh-Gaussian type and Gaussian type profiles.

Case(i): Standard Sech TW:

The Standard Sech profile function used is:

$$\psi(s) = PSech\ (Rs)$$

Where, P and R represent amplitude and inverse of width of soliton respectively.

Case(ii): Non-Standard Cosh Gaussian TW

Non-standard Cosh gaussian profile function used here is:

$$g(\xi) = A\ Cosh\ (Bs)\exp\left(-\frac{s^2}{T^2}\right)$$

Where, A is pulse amplitude, B represents cosh parameter and T represents pulse width .

Case(iii): Non-Standard Gaussian TW

Non-standard Cosh profile function used here is:

$$g(\xi) = A \exp\left(-\frac{s^2}{T^2}\right)$$

Where, A is pulse amplitude and T represents pulse width . Note that this profile function can be obtained from non-standard cosh-gaussian case by substituting B=0.

The traveling wave solutions are now superimposed to create standing waves. We discuss standing wave pattern formation for different situations by taking: $\Phi=\pi/4$, $c_1 = 0.3$, $c_3 = -0.8$, $I_0 = \varepsilon$, $\varepsilon = 1$, $k = 0.3$, $\omega = c_3$, $v = 11.7279$, $R = 0.3$, $A = 0.6236$, $B = 2.6205$ and $r = 0.9$.

The results are shown below:

(1) Circular Sech: The fluctuations in the intensity are observed for different snapshots of time.

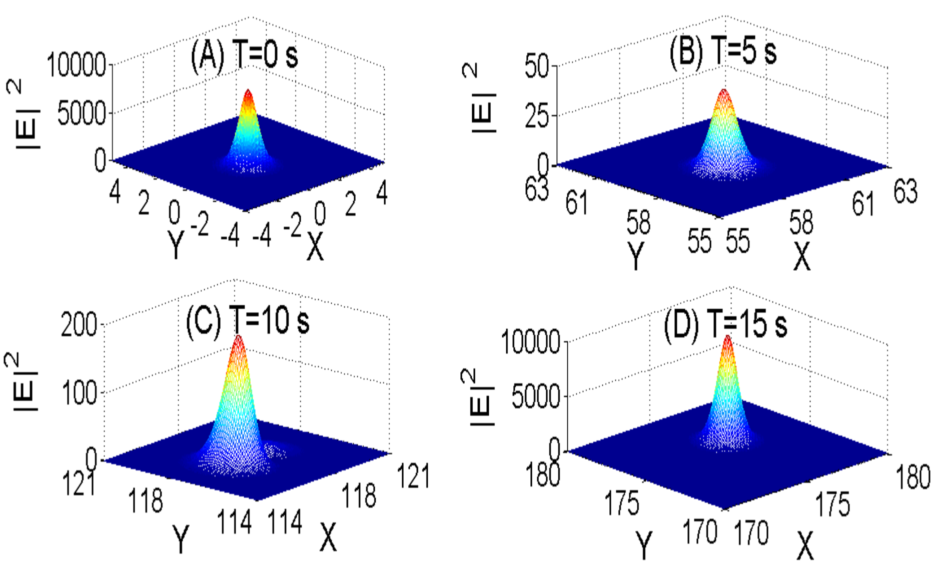


Fig.3.10. The 3D profile of Circular Sech .Part (A) ,(B) ,(C) and (D) correspond to variation at T= 0 ,5, 10 and 15 seconds respectively.

(2)Circular Cosh-Gaussian: At T=0 s, the intensity is almost same on the periphery of the annular pattern. As t is increased to T=5 s, uneven intensity is observed at the periphery. The intensity is more at one end and comparatively less at the other end of the periphery. On further increase in T, different patterns are observed as shown:

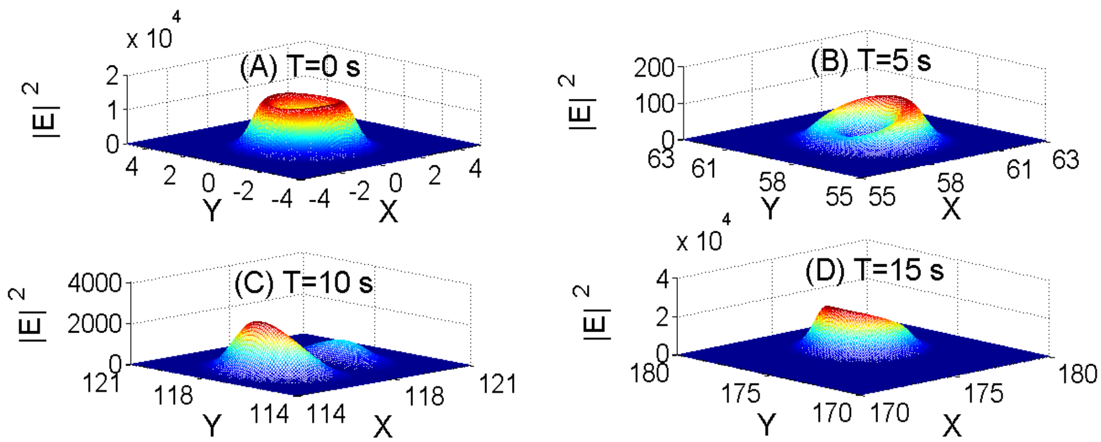


Fig.3.11. The 3D cavity soliton profile for Circular Cosh-Gaussian Circle .Part (A) ,(B), (C) and (D) correspond to variation at T= 0 ,5 ,10 and 15 seconds respectively.

(3)Circular Gaussian: The fluctuations in the intensity $(|E|^2)$ are observed for different values of time *(T)*.The intensity changes with increase in T but the actual shape of peak remains same except at T=10 s.

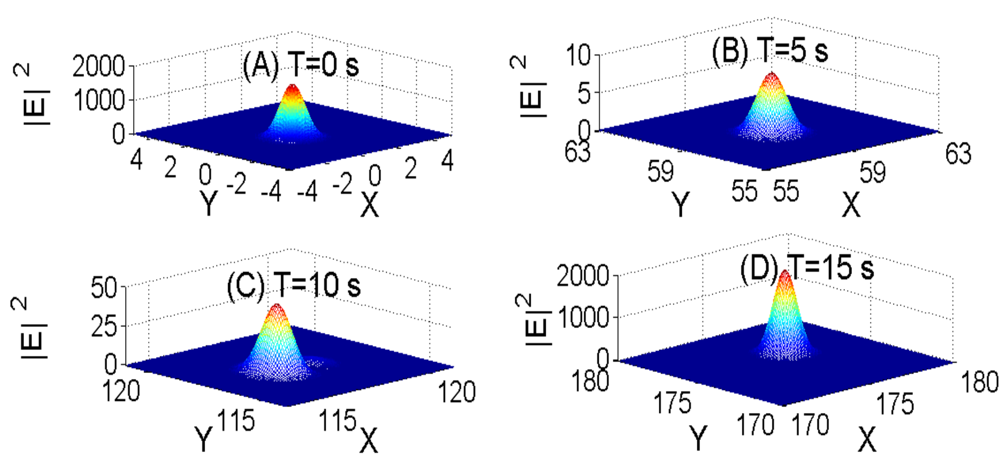


Fig.3.12. The 3D cavity soliton profile for Circular Gaussian .Part (A) ,(B) ,(C) and (D) correspond to variation at T= 0 ,5 ,10 and 15 seconds respectively.

(4)Assymetric Sech: Fluctuations in intensity ($|E|^2$)| are observed for different values of time *(T)* but the actual shape of the peak is preserved except at t=10 s.

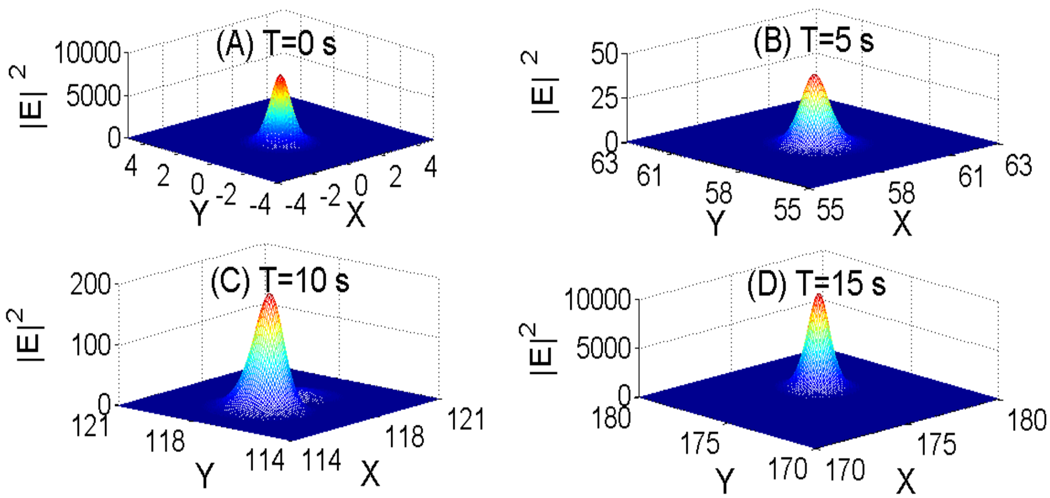


Fig.3.13. Generation of standing wave patterns/cavity solitons for assymetric sech .Part (A) ,(B), (C) and (D) correspond to variation at T= 0 ,5 ,10 and 15 seconds respectively.

(5)Assymetric Cosh-Gaussian Ellipse: At T=0 s, the intensity is almost same on the periphery of the annular pattern. As T is increased to T=5 s, uneven intensity is observed at the periphery. The intensity is more at one end and comparatively less at the other end of the periphery. On further increase in T, different patterns are observed.

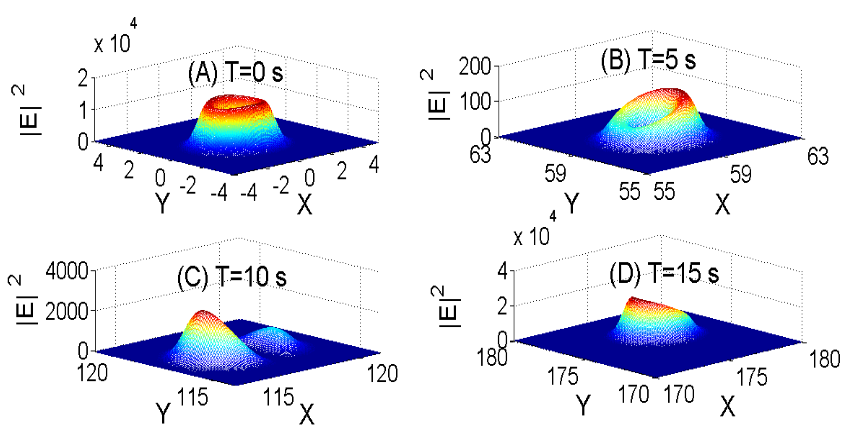


Fig.3.14. CS profiles for asymmetric cosh-gaussian .Part (A) ,(B) ,(C) and (D) correspond to variation at T= 0 ,5 ,10 and 15 seconds respectively.

(6) Assymetric Gaussian: The intensity $(|E|^2)$ decreases on increasing T from T=0s to 5s. On further increase in T, two peaks are observed at T=10 s and a single peak is observed for t=15 s.

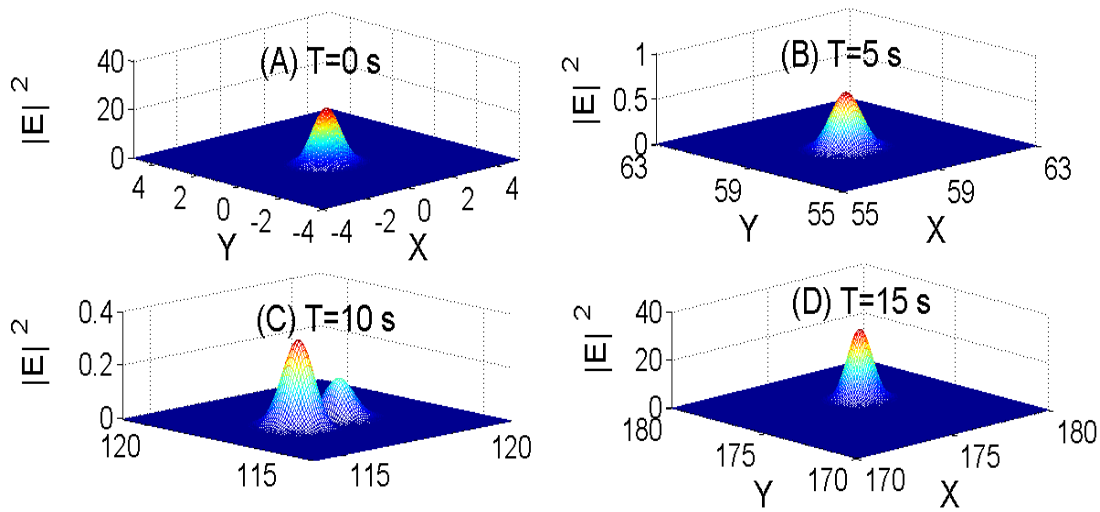


Fig.3.15. The 3D cavity solitons for assymetric gaussian. Part (A) ,(B) ,(C) and (D) correspond to variation at T= 0 ,5 ,10 and 15 seconds respectively.

(7)Variation with different ellipticity (Assymetric Sech): Different intensities $(|E|^2)$ are observed for different values of ellipticity*(e)* but a single peak is observed in all the cases.

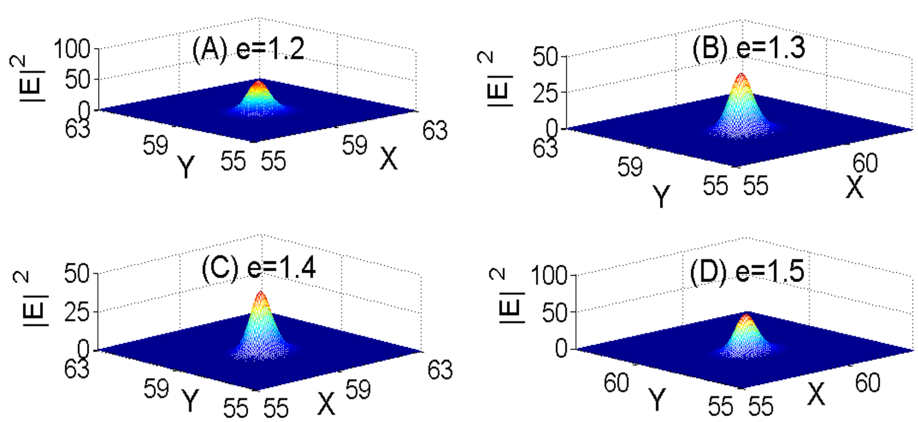


Fig.3.16. Variation of standing wave patterns with different ellipticity(e) for Sech . Part (A),(B), (C) and (D) correspond to variation at e= 1.2 ,1.3 ,1.4 and 1.5 seconds respectively.

(8)Variation with different ellipticity *(e)* for (Assymetric Cosh-Gaussian): Annular patterns are observed for Cosh-Gaussian case. The intensity is more at one end of the periphery and less at the other end. As *e* is increased, the intensity ($|E|^2$) changes at the periphery but the actual profile remains same.

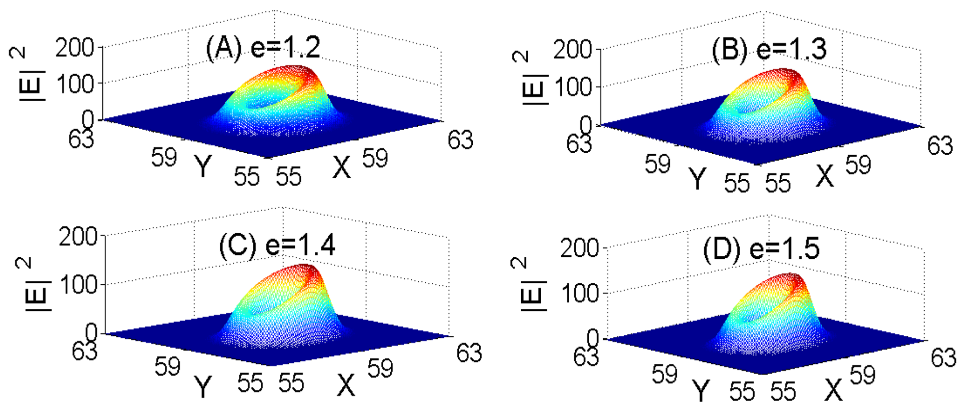


Fig.3.17. Cavity solitons/standing wave patterns with different ellipticity(e) for Cosh-Gaussian.

(9)Variation with different ellipticity*(e)* for Gaussian: Only slight changes are seen for different values of *e* .The actual intensity profile remains same for all the cases.

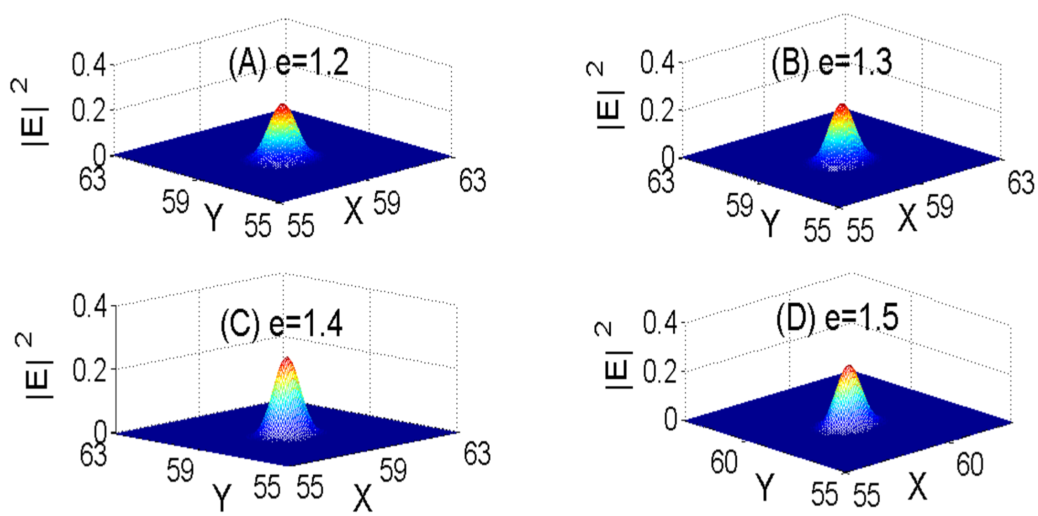


Fig.3.18. CS profiles with different ellipticity(e) for Gaussian.

(10)Variation with different A, B value (Cosh-Gaussian): **At fixed T=1.02, the following intensity profiles are observed for different values of A and B.**

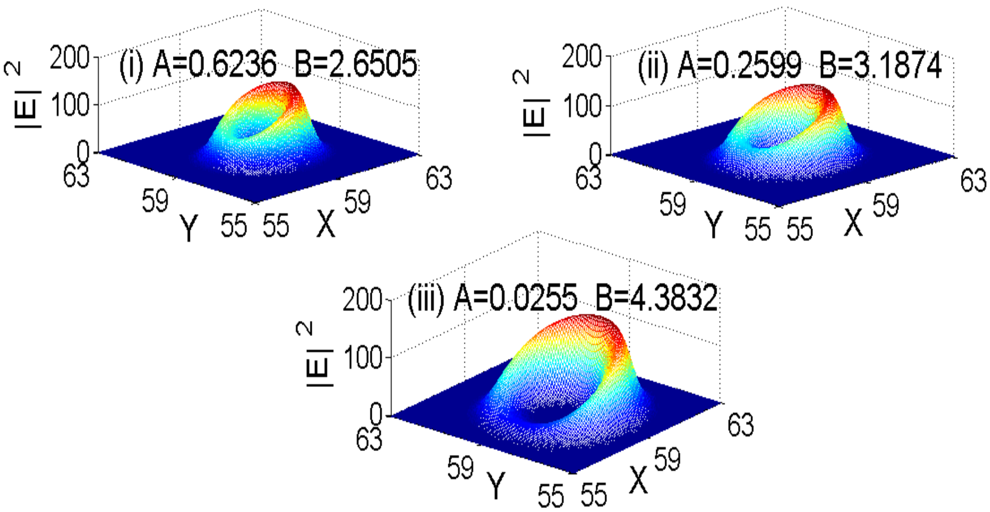


Fig.3.19. Standing wave patterns obtained for three set of values of A, B value for Cosh-Gaussian.

(11)Variation in intensity ($|E|^2$) with different tilt angle θ for Cosh-Gaussian: At θ=0˚, the pattern formed is not an annular one and the dip is very large. On increasing θ, annular patterns are formed. The variation in intensity with different angle θ is shown below:

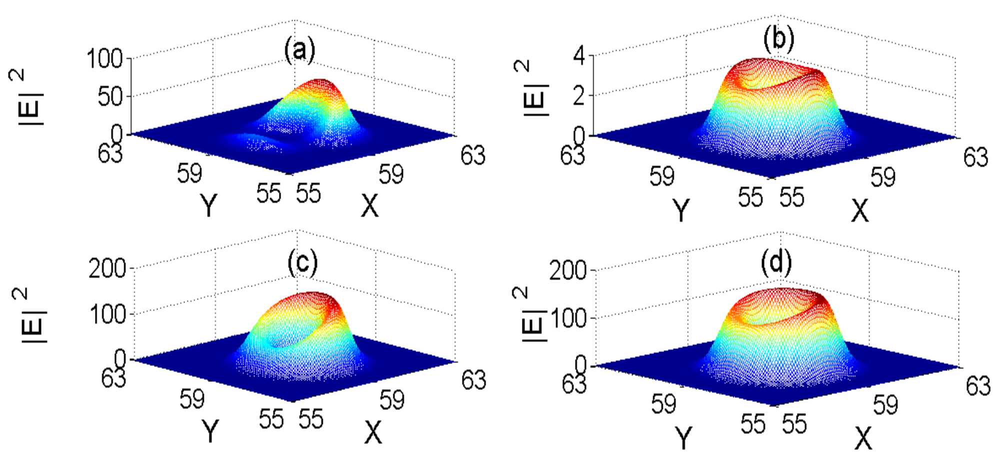


Fig.3.20. Patterns obtained with different tilt angle θ .Parts (a) ,(b) ,(c) and (d) correspond to θ=0˚, 15˚, 30˚and 45˚respectively.

(12)Variation with different reflection coefficients(r) for circular Cosh-Gaussian: On varying r, the following patterns are observed. The intensity changes on changing r, but the shape of the pattern remains annular in all the cases.

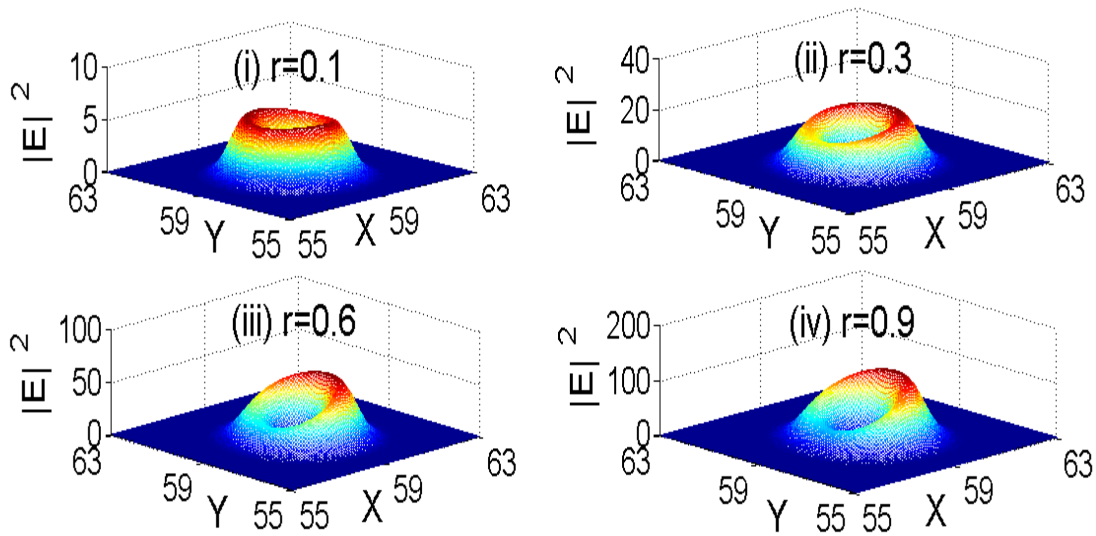


Fig.3.21. Variation in standing wave patterns with different reflection coefficient(r).

Till now, we used high reflection coefficient i.e , r=0.9. In a special situation, we illustrate generation of CS for r=0.1.

(1)For circular cosh-gaussian: The intensity ($|E|^2$) changes on changing T. Initially, the intensity is almost uniform at the periphery of the annular structure. On increasing, the intensity is found to be increasing at one end of the periphery and decreasing on another end of periphery.

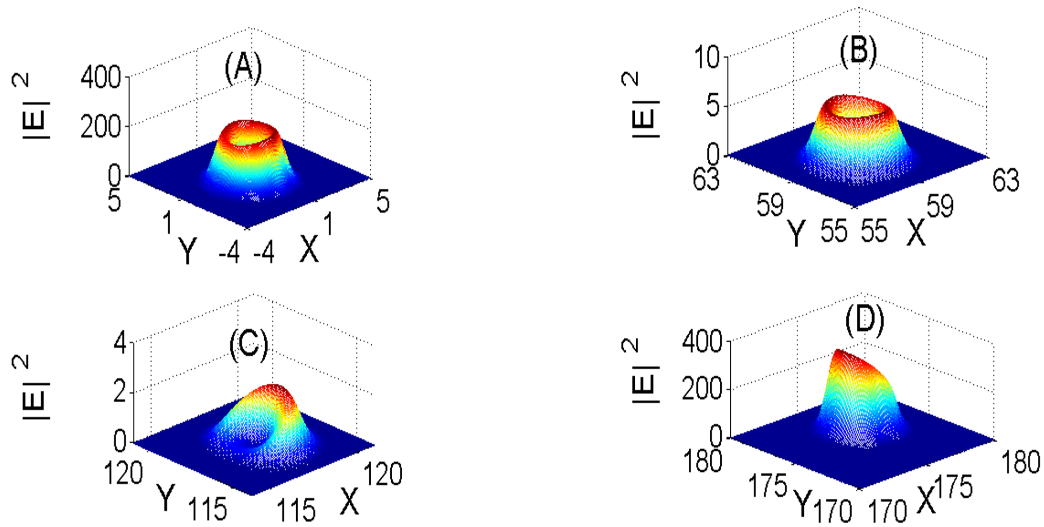


Fig.3.22. Part (A) ,(B) ,(C) and (D) correspond to the generation of patterns for circular Cosh-Gaussian Circle at T= 0 ,5 ,10 and 15 sec respectively.

(2)For circular Gaussian: On varying *T*, the intensity profile does not follow a regular trend and changes irregularly.

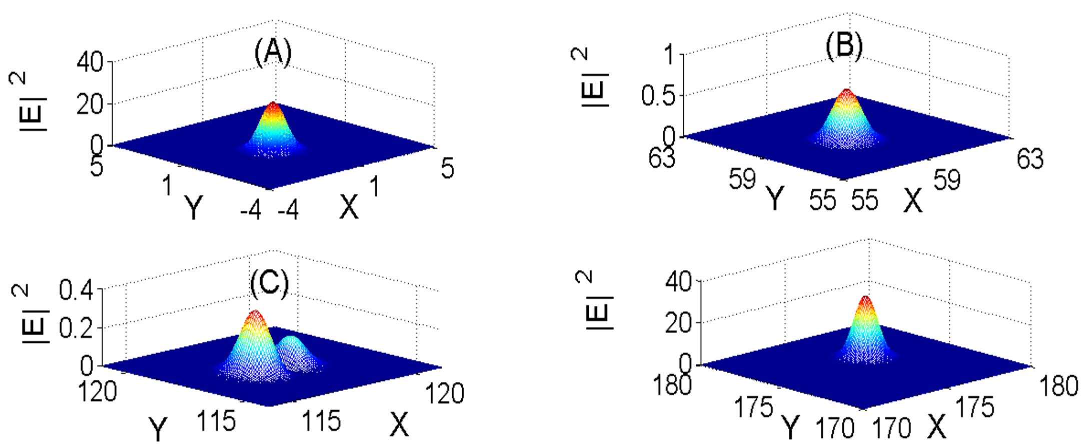


Fig.3.23. Part (A) ,(B) ,(C) and (D) correspond to CS profiles for circular gaussian at T= 0, 5, 10 and 15 seconds respectively.

(3)For asymmetric Gaussian : fluctuations in intensity profile are observed for different values of time.

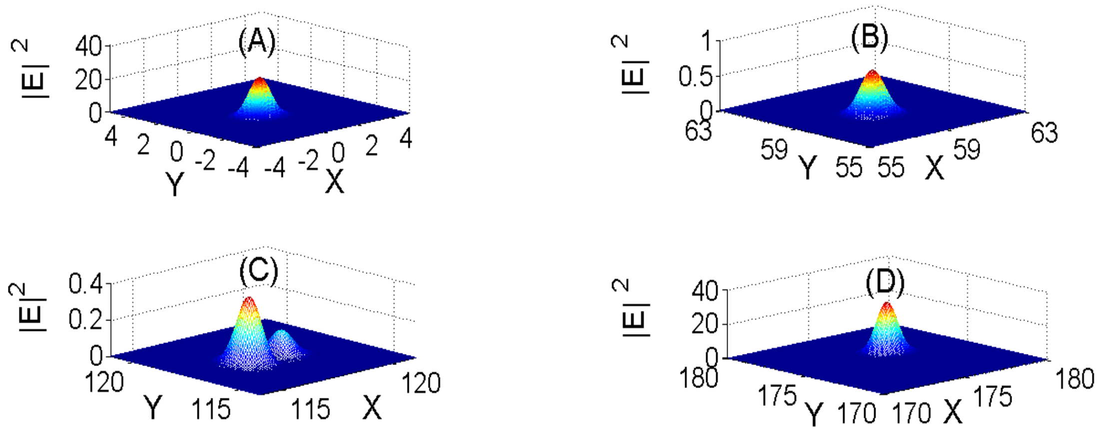


Fig.3.24. Part (A) ,(B) ,(C) and (D) correspond to variation in standing wave patterns at T=0, 5, 10 and 15 seconds respectively.

(4)For symmetric/circular Cosh - Gaussian (with different θ): The uneven distribution in intensity ($|E|^2$) is observed for different values of angle θ.

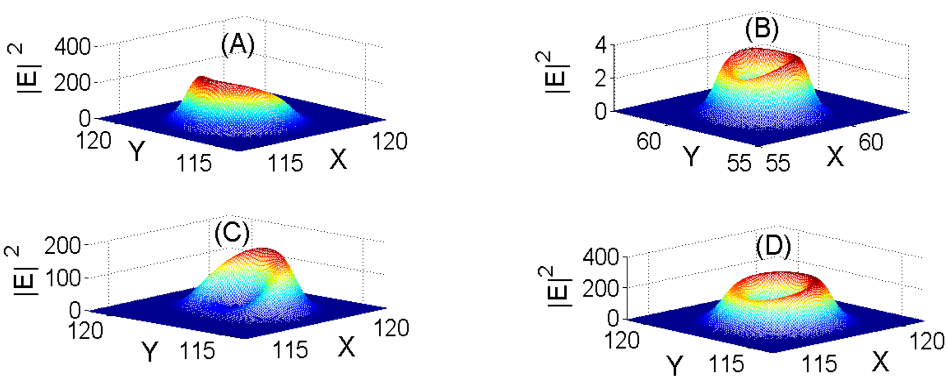


Fig.3.25. Parts (A) ,(B) ,(C) and (D) correspond to cavity soliton profiles at θ=0˚, 15˚, 30˚and 45˚respectively.

(5)For Gaussian Circle (with different θ): The following patterns are observed while studying the variation in intensity ($|E|^2$) for different θ and only one peak is observed in each case.

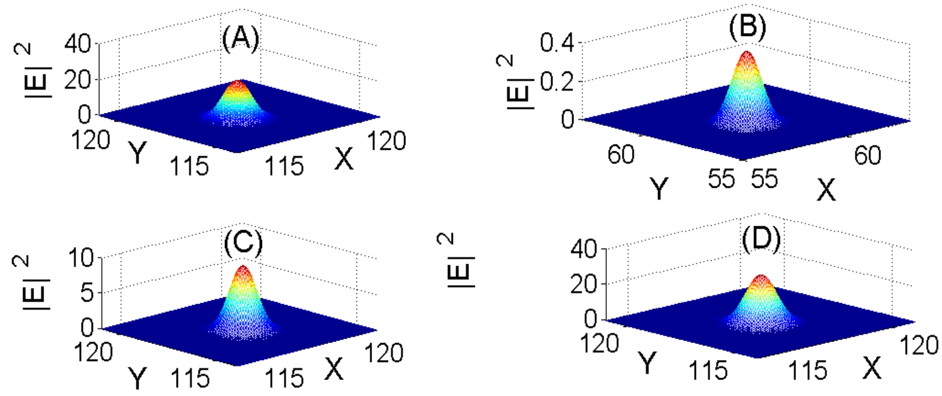


Fig.3.26. For Gaussian Circle , Parts (a) ,(b) ,(c) and (d) correspond to θ=0˚, 15˚, 30˚ and 45˚respectively.

(6)In case of variation in intensity ($|E|^2$) with different A, B value for asymmetric Cosh-Gaussian , different patterns are observed as shown :

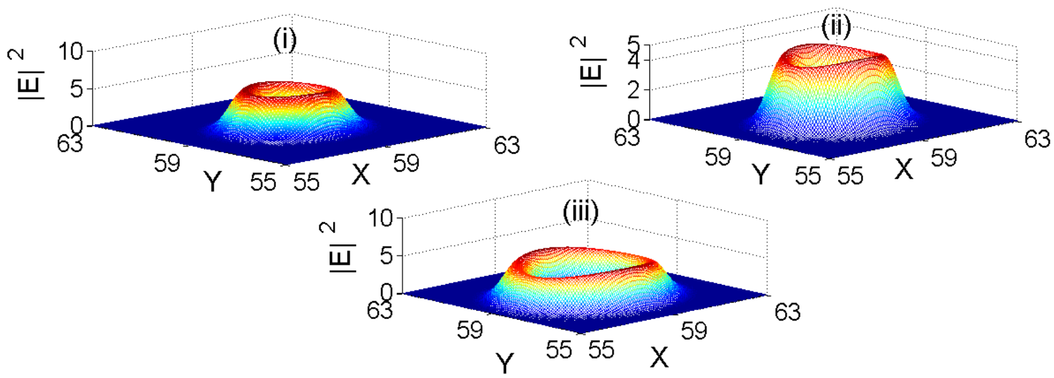


Fig. 3.27. Part (i) corresponds to A=0.6236 and B=2.6505, (ii) corresponds to A=0.2599 and B=3.1874, (iii) corresponds to A=0.0255 and B=4.3832 for asymmetric Cosh-Gaussian

In the proceeding curves, we show different generalized CS patterns for different times:

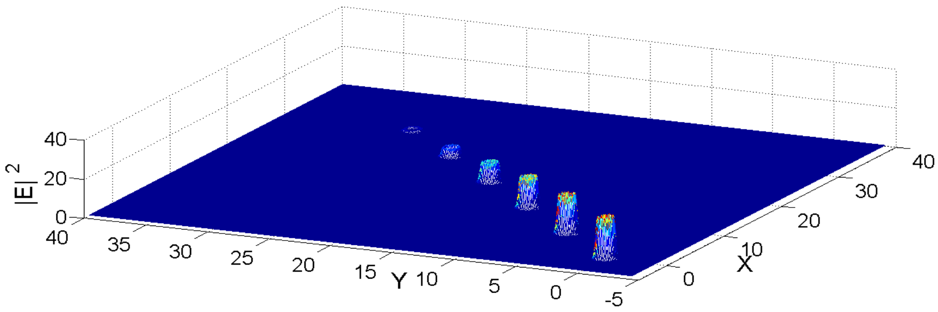


Fig.3.28. Generalized pattern for Symmetric Cosh

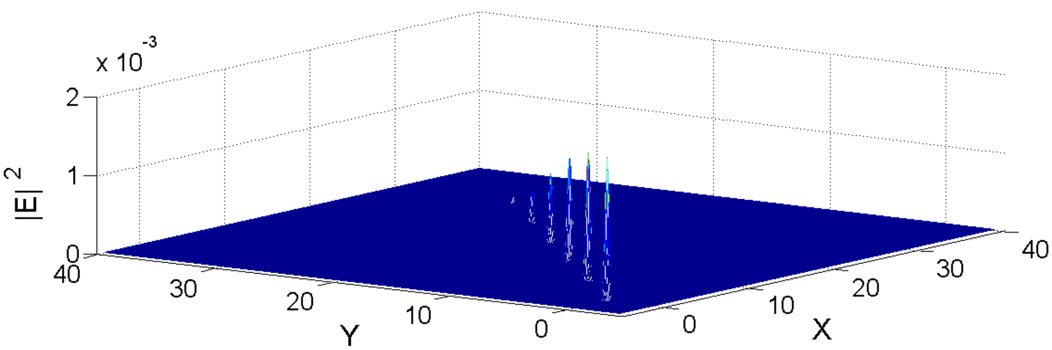


Fig.3.29. Generalized pattern for Symmetric Gaussian

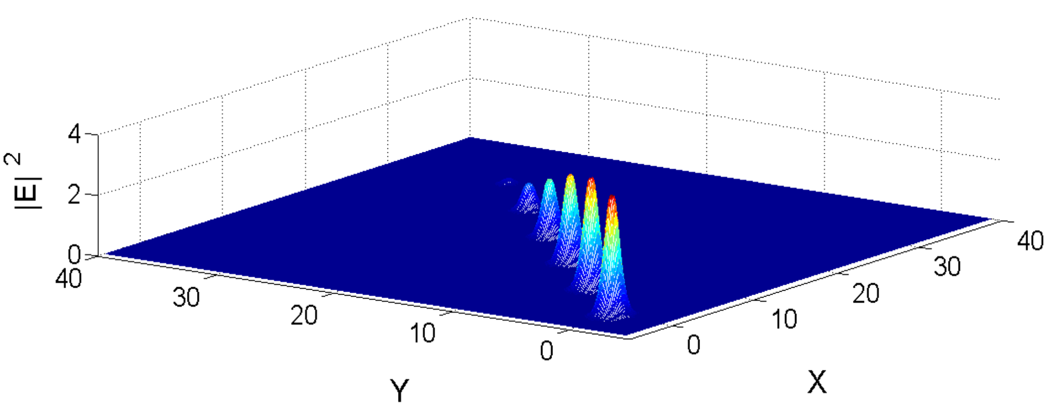


Fig.3.30. Generalized pattern for Sech Circle

3.4 Conclusion

The standing wave patterns (cavity solitons) inside cavity are observed after superposition of TW for VCSEL and VCSEL coupled with FSF including perturbation gradient. The variations in cavity soliton profile are observed with respect to different angles, reflection coefficients, etc. Different SW patterns are observed for different cases. For the system including FSF and perturbation gradient, annular patterns are observed in some cases. If one SW pattern indicates one memory, another SW pattern indicates another memory. The variation in different parameters for distributed bragg reflectors(DBR) is also studied. DBR are used to form an optical cavity in the system. Cavity solitons can be used for communication purposes, memory storage etc.

List of figures:

Fig.3.25. Parts (A) ,(B) ,(C) and (D) correspond to cavity soliton profiles at $\theta=0^{\circ}$, 15°, 30° *and* 45° *respectively.*

Fig.3.26. For Gaussian Circle , Parts (a) ,(b) ,(c) and (d) correspond to $\theta=0^{\circ}$, 15°, 30° *and* 45° *respectively.*

Fig.3.27. Part (i) corresponds to A=0.6236 and B=2.6505, (ii) corresponds to A=0.2599 and B=3.1874, (iii) corresponds to A=0.0255 and B=4.3832 for asymmetric Cosh-Gaussian.

Fig.3.28. Generalized pattern for Symmetric Cosh-Gaussian.

Fig.3.29. Generalized pattern for Symmetric Gaussian.

Fig.3.30. Generalized pattern for Sech Circle.

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