PERFECT CUBE ROOTS OF LARGER NUMBERS BY USING VEDIC MATHEMATICS

A Thesis Submitted in partial fulfillment of the requirements for the award of degree of

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in

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Submitted by

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CERTIFICATE

I hereby certify that the dissertation entitled, **"Perfect Cube Roots of Larger Numbers by using Vedic Mathematics"**, which is being submitted by **Miss Harmanpreet Kaur** (Roll No. 301403005), in the partial fulfillment of the requirement for the award of the degree of **Master of Science** in the **School of Mathematics, Thapar University, Patiala**, comprises of candidate's own research work carried out under the supervision and guidance of **Dr. Sanjeev Bakshi** during the period from January 2016 to June 2016.

The part of the work presented in this dissertation has not been submitted either in part or in full to this or any other University/ Institute for the award of any degree.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ABSTRACT

We have witnessed phenomenal technological changes in the recent years leading better and faster applications in life. This phenomenal growth becomes more faster by using Vedic mathematics. Vedic sūtras and sub-sūtras, used in the computer field, reduces the processing time of the machines and gives the result in very less time than the time usually a machine takes. Secondly, in the present cut-throat competitive era, where time is the only constraint, we need to train our brain in such a way that it is able to do fast calculations in a fraction of a minute, and that too without pen and pencil. Vedic mathematics is an emerging tool for students appearing in various competitive examinations where speed and accuracy play a vital role. The techniques of Vedic mathematics create a paradigm shift from hard work to smart work. The methods mentioned in it helps to carry out the calculations mentally involving minimal paper use and saves almost one-tenth of the time taken by the traditional methods. One such method to find three-digit(or less) cube roots of exact cubes which serves as a base to this thesis, has been gracefully explained by Jagadguruji [3].

The present thesis is organized into three chapters which are briefly summarized as follows :

In **Chapter 1**, a brief introduction to Vedic mathematics, Vedic Sūtras and Jagadguruji is given. Also, the basic concepts of mathematics, multiplication and division are explained by using Sūtras with enough step-by-step solved examples.

In **Chapter 2**, a technique is given to check whether any given number has a perfect cube root or not. It also presents Jagadguruji's [3] work to find the three-digit(or less) cube roots of exact cubes and step-by-step approach to solve the examples. We proposed a method to find n-digit cube roots of exact cubes by generalizing and extending the Jagadguruji's [3] method with the help of the principle of mathematical induction.

In **Chapter 3**, drawback and future scope of the proposed method is included.

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The real spirit of achieving the goal is through the way of excellence and austerious discipline. First of all, I render my gratitude to the ALMIGHTY who bestowed self-confidence, ability and strength in me to complete this work.

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Most importantly, I express my very profound gratitude to my parents for providing me with unfailing support, continuous encouragement and blessings throughout my years of study and through the process of researching and writing the thesis.

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Harmanpreet Kaur

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Dedicated to my beloved parents

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Brief Introduction and Some Basic Concepts

Vedic Mathematics [1-8] deals mainly with various Vedic mathematical formulae and their applications for carrying out tedious and cumbersome arithmetical operations, and to a very large extent, executing them mentally. Some people may find it difficult, at first reading, to understand the arithmetical operations although they have been explained very lucidly by Jagadguruji [3]. It is not because explanations are lacking in any manner but because the methods are totally unconventional. Some people are so deeply rooted in the conventional methods that they probably, subconsciously reject to see the logic in unconventional methods.

The word 'Veda' has this derivational meaning, i.e. the fountainhead and illimitable store-house of all knowledge. This derivation, in effect, means, connotes and implies that the Vedas should contain within themselves all the knowledge needed by mankind relating not only to the so-called 'spiritual' matters but also to those usually described as purely 'secular', 'temporal' or 'worldly' and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions and that there can be no adjectival or restrictive epithet calculated to limit that knowledge down in any sphere, any direction or any respect whatsoever.

The Vedas are well-known four in number Ṛgveda, Sām-aveda, Yajurveda, Atharvaveda, but they have also the four Upavedas and the six Vedāṅgas all of which form an indivisible corpus of divine knowledge. It is the Pariśiṣṭa from Atharvaveda on which the vedic sutras are based. Vedic Mathematics is the collection of sixteen beautiful formulae from the Vedas, discovered by His Holiness, Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja. It is a gift to the world by Swamiji, who himself was a great scholar. All the sixteen sūtras and thirteen sub-sūtras were discovered by Swamiji between 1911 and 1918. Though they don't appear directly in Vedas, it is the ultimate discovery of Swamiji, after extensive research of Vedas and Upaveda.

Vedic mathematics is the name of the wind that has created revolutionary changes in fast calculations. It is the super-fast way of making all mathematical calculations easier and faster than the traditional one. Nowadays, it has become a must learn tool for students who want to perform faster and flawless calculations in few seconds.

All the sixteen formulae deal with the different branches of mathematics. The sūtras being one-line phrases, are easy to understand and remember. They speak for the subject's coherence and simplicity in handling mathematical problems. These sixteen formulae can be used to solve problems ranging from arithmetic to algebra and geometry to calculus and trigonometry. The importance of Vedic mathematics can be understood by the fact that complex mathematical questions which otherwise take numerous steps to solve can be solved through Vedic mathematics mentally or in a few seconds. These sūtras are so beautifully interrelated that a single formula can be used to perform different arithmetical operations. All the fundamental operations can be performed by various methods and the learner has a choice to use the method he/she feels comforts with. This process boosts confidence in one's ability and keeps fear away.

SNo.	Sūtra	Meaning
1.	Ekādhikena Pūrveṇa (also a corollary)	By one more than the previous one
2.	Nikhilaṃ Navataścaramaṃ Daśataḥ	All from nine and last from ten
3.	Ūrdhva-tiryagbhyām	Vertically and Crosswise
4.	Parāvartya Yojayet	Transpose and Apply
5.	Śūnyaṃ Sāmyasamuccaye	The summation is equal to zero
6.	(Ānurūpye) Śūnyamanyat	If one is in ratio, other one is zero
7.	Saṅkalana-vyavakalanābhyām	By addition and subtraction
8.	Pūraṇāpūraṇābhyām	By completion and non-completion
9.	Calana-kalanābhyām	Sequential motion
10.	Yāvadūnam	The deficiency
11.	Vyaṣṭisamaṣṭiḥ	Whole as one and one as whole
12.	Śeṣāṇyaṅkena Carameṇa	Remainder by last digit
13.	Sopāntyadvayamantyam	Ultimate and twice the penultimate
14.	Ekanyūnena Pūrveṇa	By one less than the previous one
15.	Guṇitasamuccayaḥ	The whole product is same
16.	Guṇakasamuccayaḥ	Collectivity of multipliers

Table 1.1: Sūtras and their Meanings

SNo.	Sub - Sūtra	Meaning
1.	Ānurūpyeṇa	Proportionality
2.	Śiṣyate Śeṣasaṃjñaḥ	Knowing remainder from remainder
3.	Ādyamādyenāntyamantyena	First by first and last by last
4.	Kevalaiḥ Saptakaṃ Guṇyāt	Only multiple of seven
5.	Veṣṭanaṃ	Osculation
6.	Yāvadūnaṃ Tāvadūnaṃ	Whatever be the deficiency, lessen it further
7.	Yāvadūnaṃ Tāvadūnikṛtya Vargañca Yojayet	Whatever the extent of its deficiency, lessen it further to that extent and set up the square of deficiency
8.	Antyayordaśake'pi	When the sum of last digits is ten
9.	Antyayoreva	Only the last term
10.	Samuccayaguṇitaḥ	Sum of coefficients in the product
11.	Lopanasthāpanābhyām	By Elimination and Retention
12.	Vilokanam	The product of the sum of coefficient
13.	Gunitasamuccayaḥ Samuccayaguṇitaḥ	The product of the sum of coefficients in the factor is equal to the sum of coefficients in the product

Table 1.2: Sub-sūtras and their Meanings

Śrī Bhāratī Kṛṣṇa Tīrthajī was the magnificient and divine personality that gracefully adorned the famous Govardhan Math, Puri. Very few persons can there be amongst the cultured people of India who know about his vast and versatile learning, his spiritual and educational attainments, his wonderful research achievements in the field of Vedic Mathematics and his consecration of all these qualifications to the service of humanity.

Jagadguruji was born of highly learned and pious parents in March, 1884. His father, late Sri P.Narsimha Shastri was then in service as a Tahsildar at Tinnivelly (Madras Presidency) who later retired as a Deputy Collector. He was better known among his disciples by the beloved name 'Jagadhguruji' or 'Gurudeva'. Jagadhguruji was named as 'Venkatraman' in his early days. He was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his career. He passed his matriculation examination from the Madras University in January, 1899, topping the list. He was awarded title of 'Saraswati' by Madras Sanskrit Association for being proficient in Sanskrit. He had profound impression of his Sanskrit Guru Sri Vedam Venkatrai Shastri whom Jagadguruji always remembered with deepest love, reverence, with tears in his eyes.

After winning the highest place in B.A. Examination, Sri Venkatraman Saraswati appeared at the M.A. Examination of the American College of Sciences, Rochester, New York, from Bombay Center in 1903; and in 1904, at the age of just twenty he passed his M.A. Examination in six subjects simultaneously securing the highest honours in all, which is perhaps the all time world-record of academic brilliance. His subjects included Sanskrit, Philosophy, English, Mathematics, History, Science.

From his very early days Jagadguruji was aware of the need of the right interpretation of "*Dharma*" which he defined as "the sum total of all the means necessary for speedily making and permanently keeping all the people, individually as well as collectively superlatively comfortable, prosperous, happy and joyous in all respects (including the physical, mental, intellectual, educational, economic, social, political, psychic, spiritual etc.)". He, therefore, always laid great emphasis on the necessity of harmonising the 'spiritual' and the 'material' spheres of daily life. He stood for the omnilateral and all - round progress simultaneously of both the individual and society towards the speedy realization of India's spiritual and cultural ideal, the lofty Vedāntic ideal of 'Pūrnatva' (perfection and harmony all - round).

He was mighty in his learning and voracious in his reading. A sharp intellect, a retentive memory and a keen zest went to mark him as the most distinguished scholar of his day. His leisure moments he would never spend in vain. He was always reading or eating something. There was no branch of knowledge which he did not know and that also '*śāstrically*'. He was equally learned in Chandaḥśāstra, Āyurveda and Jyotish Śāstra. He was a poet of uncommon merit and wrote a number of poems in Sanskrit in the praise of his Guru,Gods and Goddesses with charming flow of *Bhakti* so conspicuous in all his writings.

He was always perfectly impartial. Every one was equal in his eyes. He cared not for riches and position. Nothing but *Bhakti* could attract people to him, rich or poor, high or low, everybody had to go through the portals of *Bhakti* to approach his august presence. Everyone who had even two minute's conversation with him went out with the full conviction that he was the object of some special love of His Holiness.

Revered Guruji used to say that he had reconstructed the sixteen mathematical formulae from the Atharvaveda after assiduous research and 'Tapas' for about eight years in the forests surrounding Sringeri. These formulas are not to be found in the present recensions of Atharvaveda; they were actually reconstructed, on the basis of intuitive revelation, from materials scattered here and there in the Atharvaveda. Revered Gurudeva used to say that he had written sixteen volumes on these *Sūtras*, one for each *Sūtra* and that the manuscripts of the said volumes were deposited at the house of one of his disciples. Unfortunately, the said manuscripts were lost irretrievably from the place of their deposit and this colossal loss was finally confirmed in 1956. Revered Gurudeva was not much perturbed over this irretrievable loss and used to say that everything was there un his memory and that he could re-write the sixteen volumes.

In this field of mental arithmetic operations, the contributions of the famous mathematicians Trachtenberg [9] and Lester Meyers are very elementary as compared to that of Jagadguruji [3]. Recently, many researchers [1, 2, 4-8] have contributed a lot in this field.

1.2 Multiplication Using Vedic

Multiplication by using the following four Sūtras :-

(i) *Nikhilaṃ Navataścaramaṃ Daśataḥ*

(ii) *Ekādhikena Pūrveṇa*

(iii) *Ekanyūnena Pūrveṇa*

(iv) *Ūrdhva-tiryagbhyām*

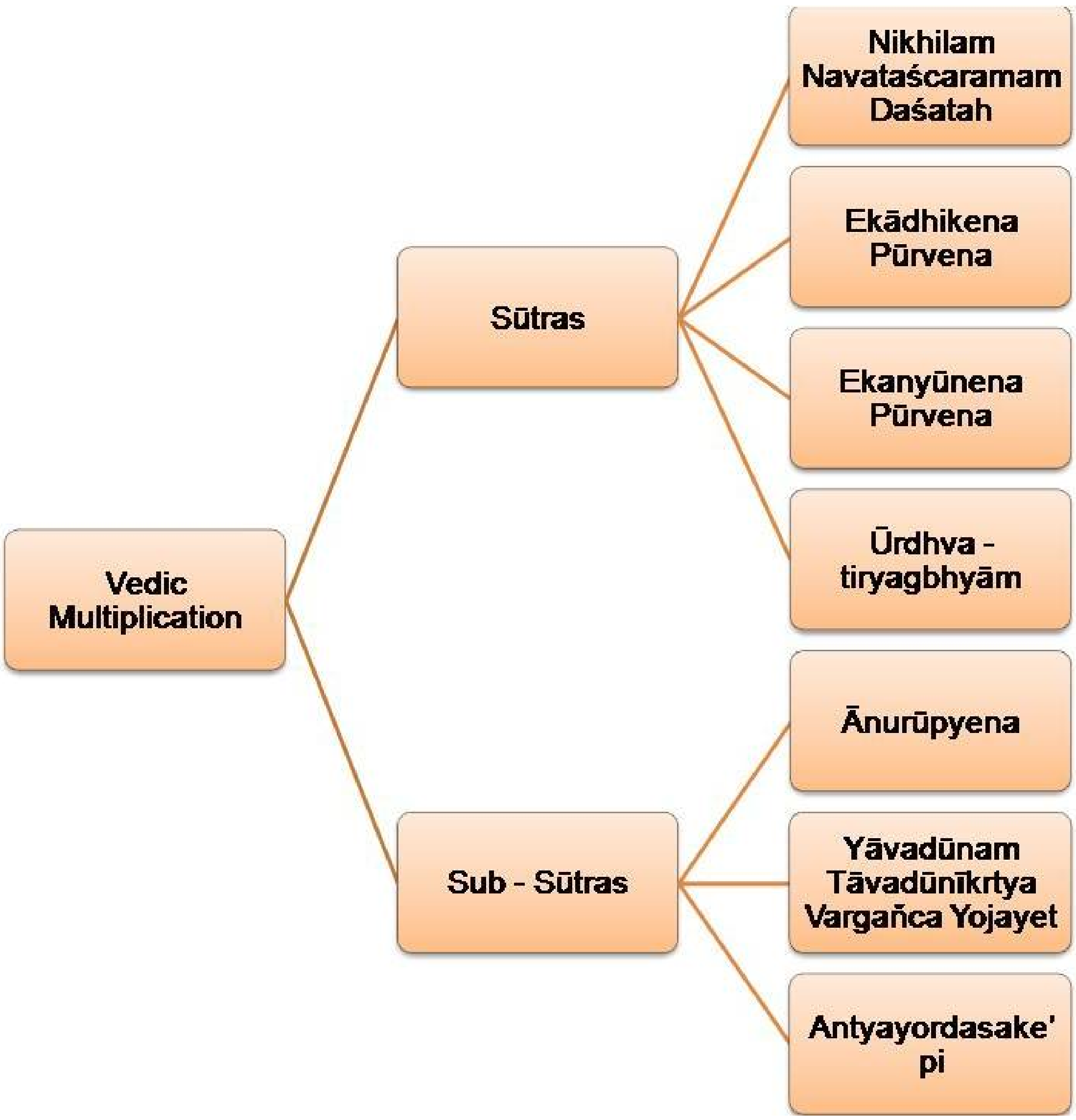

Figure 1.1: Sūtras and Sub-sūtras of Multiplication

(i) ***Multiplication by Nikhilaṃ Navataścaramaṃ Daśataḥ:-***

The meaning of this sūtra is "all from 9 and the last from 10". Basically this sūtra is used when both the multiplier and multiplicand are near to the base. The base should be in form of 10^n, where n is a natural number.

RULE:-

- Write the two numbers to be multiplied above and below.
- Write the deviation of multiplicand and multiplier from the base and place them next to the digit to be multiplied.
- The final result will have two parts.
  (a) The left hand part will be obtained by cross operation of two numbers written diagonally.
  (b) The right part will be obtained by multiplying the deviations.
- The number of digits in the right hand part will be in accordance to the number of zeros in the base.
  In simple words, if the base is 10, then the right hand part will have one digit. If the base is 100, then the right hand part will have two digits. If the base is 1000, then the right hand part will have three digits and so on.
- In case there is lesser number of digits in the right hand side, accommodate as many zeros before the right hand part so that the total number of digits in that part is equal to the number of zeroes in the base.

CASE I : When both the numbers are below the base.

Example 1.1 Multiply 95 by 91

Solution: Doing the following steps to solve,

a) Put the multiplicand and multiplier as shown

$$\begin{array}{r} 95 \\ \times \quad 91 \\ \hline \end{array}$$

b) Both the numbers are closer to the base 100, so take

Base = 100

Deviation of 95 = 95 - 100 = - 5

Deviation of 91 = 91 - 100 = - 9

c) Put the deviation at the right hand side along with the number to be multiplied.

$$\begin{array}{rrr} & 95 - & 5 \\ \times & 91 - & 9 \\ \hline \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals. Here 95 - 9 = 86 or 91 - 5 = 86 is written in the left hand part.

$$\begin{array}{rrr} & 95 - & 5 \\ \times & 91 - & 9 \\ \hline & 86 \diagup & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{rrr} & 95 - & 5 \\ \times & 91 - & 9 \\ \hline & 86 \diagup & 45 \end{array}$$

Therefore, 95 × 91 = 8645

CASE II : When both the numbers are above the base.

Example 1.2 Multiply 105 by 104

Solution: Doing the following steps to solve,

a) Put the multiplicand and multiplier as shown

$$\begin{array}{rr} & 105 \\ \times & 104 \\ \hline \end{array}$$

b) Both the numbers are closer to the base 100, so take

Base = 100

Deviation of 105 = 105 - 100 = 5

Deviation of 104 = 104 - 100 = 4

c) Put the deviation at the right hand side along with the number to be multiplied.

$$\begin{array}{rrr} & 105 + & 5 \\ \times & 104 + & 4 \\ \hline \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{rrr} & 105 + & 5 \\ \times & 104 + & 4 \\ \hline & 109 \diagup & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{rr} & 105 + \ 5 \\ \times & 104 + \ 4 \\ \hline & 109 \ / \ 20 \end{array}$$

Therefore, $105 \times 104 = 10920$

CASE III : When one number is less than the base and another is greater than the base.

Example 1.3 Multiply 122 by 98

Solution: Doing the following steps to solve,

a) Put the multiplicand and multiplier as shown

$$\begin{array}{rl} & 122 \\ \times & 98 \\ \hline \end{array}$$

b) Both the numbers are closer to the base 100, so take

Base = 100

Deviation of 122 = 122 - 100 = 22

Deviation of 98 = 98 - 100 = - 2

c) Put the deviation at the right hand side along with the number to be multiplied.

$$\begin{array}{rll} & 122 & + \ 22 \\ \times & 98 & - \ 2 \\ \hline \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{rll} & 122 & + \ 22 \\ \times & 98 & - \ 2 \\ \hline & 120 \ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{rll} & 122 & + \ 22 \\ \times & 98 & - \ 2 \\ \hline & 120 \ / & -44 \end{array}$$

f) When there is a minus($-$) sign at the right hand product, use the Nikhilam formulae which states "All from 9 and the last from 10". Hence subtract the right hand digit -44 from 100 and left hand part 120 will get diminished by 1.(i.e. $120 - 1 = 119$)

$$\begin{array}{rl} & 122 + 22 \\ \times & 98 \;-\; 2 \\ \hline & 119 \,/\, 100-44 \\ & 119 \,/\, 56 \end{array}$$

Therefore, 122 × 98 = 11956

(ii) ***Multiplication by Ekādhikena Pūrveṇa:-***

The literal meaning of this sutra is "one more than the previous one". This sutra is basically used for the squaring of numbers ending with 5.

Example 1.4 Multiply 25 by 25

Solution: Doing the following steps to solve,

a) Write the multiplier and multiplicand as shown below

$$\begin{array}{rcc} & 2 & 5 \\ \times & 2 & 5 \\ \hline & & / \end{array}$$

b) The answer will have two parts. R.H.S. will be 5×5 = 25.

$$\begin{array}{rcc} & 2 & 5 \\ \times & 2 & 5 \\ \hline & & /\ 25 \end{array}$$

c) The L.H.S. will be 2×3 (as the sutra says one more than the previous one). So, one more than 2 is 3. It becomes 2×3 = 6.

$$\begin{array}{rcc} & 2 & 5 \\ \times & 2 & 5 \\ \hline & 6 & /\ 25 \end{array}$$

Therefore, 25 × 25 = 625

Example 1.5 Multiply 95 by 95

Solution: Doing the following steps to solve,

a) Write the multiplier and multiplicand as shown below

$$\begin{array}{rcc} & 9 & 5 \\ \times & 9 & 5 \\ \hline & & / \end{array}$$

b) The answer will have two parts. R.H.S. will be 5×5 = 25.

$$\begin{array}{rcc} & 9 & 5 \\ \times & 9 & 5 \\ \hline & & /\ 25 \end{array}$$

c) The L.H.S. will be 9×10 (as the sutra says one more than the previous one). So, one more than 9 is 10. It becomes 9×10 = 90.

$$\begin{array}{rcl} & 9 & 5 \\ \times & 9 & 5 \\ \hline & 90 & / \ 25 \end{array}$$

Therefore, 95 × 95 = 9025

Example 1.6 Multiply 595 by 595

Solution: Doing the following steps to solve,

a) Write the multiplier and multiplicand as shown below

$$\begin{array}{rcl} & 59 & 5 \\ \times & 59 & 5 \\ \hline & & / \end{array}$$

b) The answer will have two parts. R.H.S. will be 5×5 = 25.

$$\begin{array}{rcl} & 59 & 5 \\ \times & 59 & 5 \\ \hline & & / \ 25 \end{array}$$

c) The L.H.S. will be 59×60 (as the sutra says one more than the previous one). So, one more than 59 is 60. It becomes 59×60 = 3540.

$$\begin{array}{rcl} & 59 & 5 \\ \times & 59 & 5 \\ \hline & 3540 & / \ 25 \end{array}$$

Therefore, 595 × 595 = 354025

(iii) ***Multiplication by Ekanyūnena Pūrveṇa:-***

The literal meaning of this sutra is "one less than the previous one". This one liner multiplication is the beauty of the Vedic Mathematics. This sutra is basically used when the multiplier digits consists entirely of nines. This sutra works under three conditions :-

- When the number of 9s in the multiplier are same as the number of digits in the multiplicand.
- When the number of 9s in the multiplier are more than the number of digits in the multiplicand.
- When the number of 9s in the multiplier are less than the number of digits in the multiplicand.

CASE I : When the number of 9s in the multiplier are same as the number of digits in the multiplicand.

RULE:-

- Subtract 1 form the multiplicand and write the result in L.H.S.
- Subtract the multiplicand by applying Nikhilaṃ Navataścaramamaṃ Daśataḥ Sūtra and write result in R.H.S.

Example 1.7 Multiply 6543 by 9999

Solution:

L.H.S. = Multiplicand - 1 = 6543 - 1 = 6542

R.H.S. = Apply the Nikhilaṃ method of subtraction and subtract the unit digit from 10 and the rest of the digits from 9. We get

9 - 6 = 3

9 - 5 = 4

9 - 4 = 5

10 - 3 = 7

$\therefore$ R.H.S = 3457

$$\begin{array}{rl} & 6543 \\ \times & 9999 \\ \hline & 6542 \;/\; 3457 \end{array}$$

Therefore, 6543 × 9999 = 65423457

CASE II : When the number of 9s in the multiplier are more than the number of digits in the multiplicand.

RULE:-

- Subtract 1 form the multiplicand and write the result in L.H.S.
- Subtract the multiplicand by applying Nikhilaṃ Navataścaramamaṃ Daśataḥ Sūtra and write result in R.H.S. But here the only difference is that we have to equal the number of digits in multiplicand by taking zero(s) before it. So, when we subtract all the digits from 9 and the last from 10 then we have to subtract zero(s) from the 9 also.

Example 1.8 Multiply 456 by 9999

Solution:

L.H.S. = Multiplicand - 1 = 456 - 1 = 455

R.H.S. = Apply the Nikhilaṃ method of subtraction and subtract the unit digit from 10 and the rest of the digits from 9. We get

9 - 0 = 9

9 - 4 = 5

9 - 5 = 4

10 - 6 = 4

$\therefore$ R.H.S = 9544

$$\begin{array}{rl} & 456 \\ \times & 9999 \\ \hline & 455\ /\ 9544 \end{array}$$

Therefore, 456 × 9999 = 4559544

CASE III : When the number of 9s in the multiplier are less than the number of digits in the multiplicand.

RULE:- This case is little bit different from the last two cases.

- Add as many zeros as the number of 9s to the multiplicand.
- Subtract the original multiplicand from the figure obtained in the above step.

Example 1.9 Multiply 1564 by 99

Solution: The multiplicand 1564 has 4 digits, whereas there are two 9s in the multiplier.

$$\begin{array}{rl} & 1564 \\ \times & 99 \\ \hline \end{array}$$

a) Since there are two 9s, put two zeros at the end of multiplicand 1564, making it 156400.

b) Subtract 1564 (the original multiplicand) from 156400.

$$\begin{array}{rr} & 156400 \\ - & 1564 \\ \hline & 154836 \end{array}$$

Therefore, 1564 × 99 = 154836

(iv) ***Multiplication by Ūrdhva-tiryagbhyām:-***

The literal meaning of this sutra is "vertically and cross-wise". This sutra is applicable to all the cases of multiplication. As the name suggests, the method consists of steps including vertical multiplication and crosswise multiplication giving the answer in one straight line.

Conventional Method

				3	2	1	4	2
		×		1	2	3	7	4
			1	2	8	5	6	8
		2	2	4	9	9	4	
		9	6	4	2	6		
	6	4	2	8	4			
3	2	1	4	2				
3	9	7	7	2	5	1	0	8

Vedic one-line Method

$$\begin{array}{r} 32142 \\ \times \quad 12374 \\ \hline 397725108 \end{array}$$

Here we have different patterns to multiply according to the number of digits in multiplicand and the multiplier.

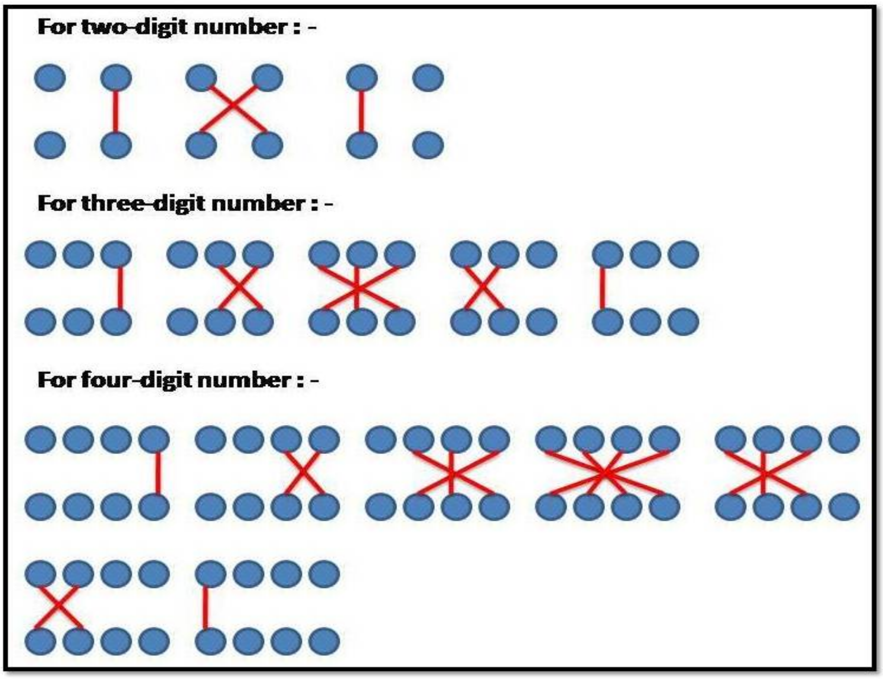

Figure 1.2: Different Patterns of Multiplication by Ūrdhva-tiryagbhyām Sūtra

CASE I : Two-digit multiply by Two-digit

RULE:-

- We vertically multiply the 2 digits in the right hand column.
- We cross multiply the digits in both the columns and add the products.
- Then we vertically multiply the digits in the left hand column.

Example 1.10 Multiply 12 by 13

Solution: Doing the following steps to solve,

a) Multiply vertically the right hand digit and write it under the unit's place.

$2 \times 3 = \mathbf{6}$

$$\begin{array}{rr} & 12 \\ \times & 13 \\ \hline & \mathbf{6} \end{array}$$

b) Multiply crosswise and add the results for the ten's place.

$(1\times3) + (1\times2) = \mathbf{5}$

$$\begin{array}{rr} & 12 \\ \times & 13 \\ \hline & \mathbf{56} \end{array}$$

c) Multiply vertically in the left hand side.

$1 \times 1 = \mathbf{1}$

$$\begin{array}{rr} & 12 \\ \times & 13 \\ \hline & \mathbf{156} \end{array}$$

Therefore, $12 \times 13 = 156$

CASE II : Three-digit multiply by Three-digit

Example 1.11 Multiply 413 by 321

Solution: Doing the following steps to solve,

a) Multiply vertically the right hand digit and write it under the unit's place.

$3 \times 1 = \mathbf{3}$

$$\begin{array}{rr} & 413 \\ \times & 321 \\ \hline & \mathbf{3} \end{array}$$

b) Multiply crosswise the digits in the middle and right hand column and add the results.

$(1\times1) + (3\times2) = \mathbf{7}$

$$\begin{array}{rr} & 413 \\ \times & 321 \\ \hline & \mathbf{73} \end{array}$$

c) Multiply crosswise the digits in the right and left hand columns and vertically the middle digits. Then add all the three

$(3\times3) + (4\times1) + (1\times2) = {}_1\mathbf{5}$

$$\begin{array}{rr} & 413 \\ \times & 321 \\ \hline & {}_1\mathbf{573} \end{array}$$

d) Multiply crosswise the digits in the middle and left column and add the results

$(4\times2) + (3\times1) = 11$

Adding the carry, $11 + 1 = {}_{1}\mathbf{2}$

$$\begin{array}{r} 413 \\ \times \quad 321 \\ \hline {}_{1}\mathbf{2573} \end{array}$$

e) Multiply vertically in the left column digits.

$4 \times 3 = 12$

Adding the carry, $12 + 1 = \mathbf{13}$

$$\begin{array}{r} 413 \\ \times \quad 321 \\ \hline \mathbf{132573} \end{array}$$

Therefore, $413 \times 321 = 132573$

In this way, we can extend the pattern to any number of digits.

1.3 Division Using Vedic

Division by using the following three Sūtras :-

(i) *Nikhilaṃ Navataścaramaṃ Daśataḥ*

(ii) *Ūrdhva-tiryagbhyām*

(iii) *Ekādhikena Pūrveṇa*

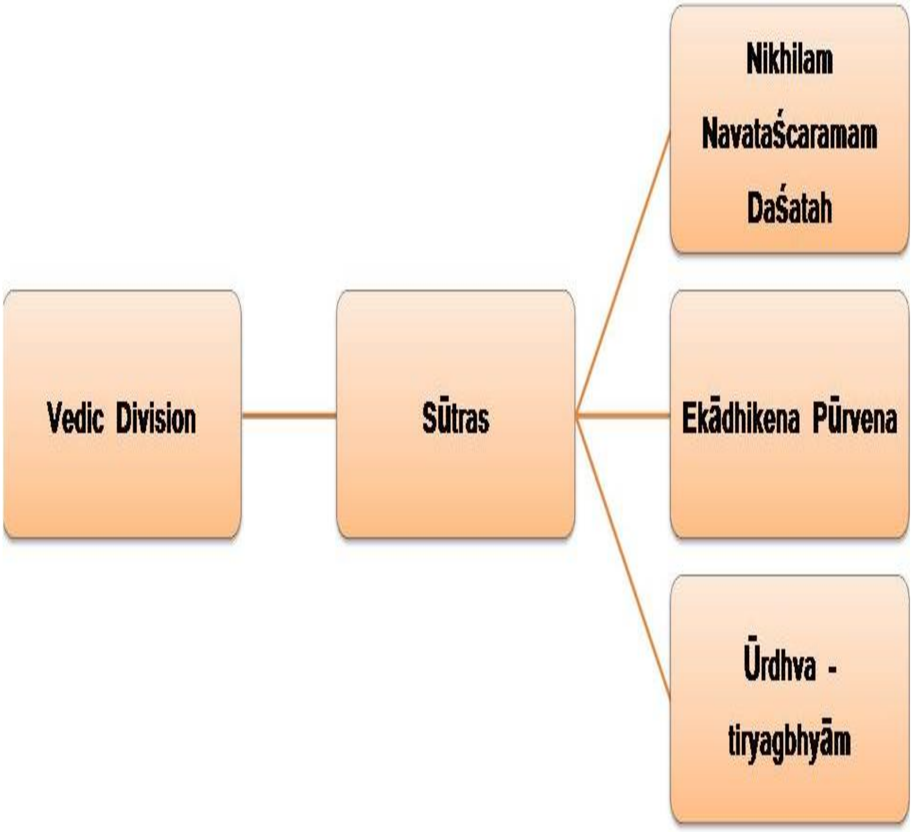

Figure 1.3: Sūtras of Division

(i) ***Division by Nikhilaṃ Navataścaramaṃ Daśataḥ:-***
The meaning of Nikhilam Sutra is "all from 9 and the last from 10". This sutra is useful when every digit of divisor is greater than 5.

RULE:-

- Take a base (in the power of 10) nearest to the divisor and write it's complement below the original divisor, in the divisor column.
  Complement = Base - Divisor
- Separate the extreme right of the dividend by drawing a slash line equal to the number of digits in the divisor. This block is known as the "Remainder Block" and the left block is known as the "Quotient Block".
- The number of digits to be placed in the remainder column should be equal to the number of zeros in the base.
- Carry down the first digit of divisor in the first column. This gives you the first digit of the quotient. Multiply the quotient digit by the complement and place it in the dividend column; next to the first digit of the dividend.
- Write mechanically the sum of the digits of the second column to get the second digit of the quotient.
- Repeat the process until you get a number in the remainder column. If the remainder is digit in the remainder column is less than that of the original divisor.

CASE I : When the remainder is less than the divisor

Example 1.12 Divide

(i) 12 by 9

(ii) 111 by 89

(iii) 12345 by 7999

(iv) 1010101 by 899997

Solution:

9)	1	/	2		89)	1	/	11
1		/	1		11		/	11
	1	/	3			1	/	22

Quotient = 1

Remainder = 3

Quotient = 1

Remainder = 22

7999)	1	/	2345
2001		/	2001
	1	/	4346

Quotient = 1

Remainder = 4346

899997)	1	/	010101
100003		/	100003
	1	/	110104

Quotient = 1

Remainder = 110104

CASE II : When the remainder is greater than the divisor

Example 1.13 Divide 198 by 88

Solution:

88)	1	/	98
12		/	12
	1	/	110

Here remainder 110 is greater than divisor. So, we will again repeat the process for 110.

88)	1	/	10
12		/	12
	1	/	22

Now, we will add the newly obtained quotient to the previous quotient and new remainder will be the final remainder.

Quotient = 1 + 1 = 2

Remainder = 22

Importance of Zero in Complement :-

While taking the complement from the base, we need to be very careful. If the base is 100, the complement should have two digits and if the base is 1000, the complement should have three digits. Eg.

If the divisor is 9, then complement from the base 10 will be 1.

If the divisor is 99, then complement from the base 100 will be 01.

If the divisor is 999, then complement from the base 1000 will be 001.

Example 1.14 Divide 11199171 by 99979

Solution: Base = 100000

Complement = 00021

No. of digits to be placed in Remainder column = 5

99979)	1	1	1	/	9	9	1	7	1
00021		0	0	/	0	2	1		
			0	/	0	0	2	1	
				/	0	0	0	2	1
	1	1	1	/	9	$^{1}1$	4	$^{1}0$	2
	1	1	1	/	$^{1}0$	1	5	0	2

Here remainder 101502 is greater than divisor. So, we will again repeat the process for 101502.

99979)	1	/	0	1	5	0	2
00021		/	0	0	0	2	1
	1	/	0	1	5	2	3

Now, we will add the newly obtained quotient to the previous quotient and new remainder will be the final remainder.

Quotient = 111 + 1 = 112

Remainder = 01523

(ii) ***Division by Ūrdhva-tiryagbhyām*:-**

The meaning of this sutra is "vertically and crosswise". This sutra is used when we have high degree of "x". If we have to divide the polynomials where the difference between highest power of x in dividend and divisor is greater than 5, Ūrdhva Sūtra is used. This gives the answer in just one line.

Example 1.15 Divide $12x^2 - 8x - 32$ by $x - 2$.

Solution:

$$x - 2)\ 12x^2 - 8x - 32$$

$$12x + 16 \ / \ 0$$

Explanation:-

i) $12x^2$ divided by x gives us $12x$.Therefore the first term of the quotient is $12x$.

ii) $12x$ multiplied by -2 (of the divisor) gives us $-24x$.

iii) But we want $-8x$ in the dividend.

iv) Therefore the second term of the quotient should be $+16$, because 16 multiplied by x gives $16x$ and we already have $-24x$ from the first term of the quotient and $-24x + 16x = -8x$.

v) Therefore the quotient now is $12x + 16$.

vi) 16 multiplied by -2, of the divisor, gives -32 which is the last term of the dividend. Therefore reminder is 0.

Quotient $= 12x + 16$

Remainder $= 0$

Example 1.16 Divide $x^3 + 7x^2 + 6x + 5$ by $x - 2$.

Solution:

$$x - 2)\ x^3 + 7x^2 + 6x + 5$$
$$x^2 + 9x\ \ + 24\ /\ 53$$

Explanation:-

i) x^3 divided by x gives x^2. Therefore the first term of the quotient is x^2.

ii) x^2 multiplied by -2 (of the divisor) is $-2x^2$.

iii) But we have $7x^2$ in the dividend. Therefore the second term of the quotient should be $+9x$ $[(9x \times x) - 2x^2 = 7x^2)]$.

iv) $9x$ multiplied by -2 is $-18x$. But we have $6x$ in the divisor.

v) Therefore the third term of the quotient should be $+24(24x - 18x = 6x)$.

vi) The quotient till now is $x^2 + 9x + 24$.

vii) 24 multiplied by -2 give -48. The last term of the divisor is 5.

viii) Therefore the reminder has to be $+53(53 - 48 = 5)$.

Quotient $= x^2 + 9x + 24$

Remainder $= 53$

(iii) ***Division by Ekādhikena Pūrveṇa:-***

Vedic Mathematics provides elegant technique for the division of fraction numbers (denominator ending with '9') by using the "Ekādhikena Pūrveṇa". The meaning of this sutra is "one more than the previous one" where:-

- The dividend is less than the divisor.
- The divisor is small i.e. of 2 or 3 digits.
- The last digit of the divisor ends with 9. Eg. 19, 29, 39, 49, 59 etc.

RULE:-

- Add 1 to the denominator and use that as the divisor.
- Remove the zero from the divisor and place a decimal point in the numerator at the appropriate position.
- Carry out a step by step division by using the new dividend.
- Every division should return a quotient and a remainder and should not be a decimal division.
- Every quotient digit will be used without any change to compute the next number to be taken as the dividend.

Example 1.17 Divide 5 by 29

Solution: Doing the following steps to solve,

a) Add 1 to the divisor to get 30

b) The modified division is now 5 / 30

c) Remove the zero from the denominator by placing a decimal point in the numerator giving the modified division as .5 / 3

d) We will not write the final answer as 0.16666

e) We will carry out a step by step division explained below:-

   i) Divide 5 by 3, to get a quotient(q) = 1 and remainder(r) = 2.

   ii) Write down the quotient in the answer line and remainder just below it to it's left as shown.

   0 . 5 / 3 = 0 . 1

   2

   iii) Now, the next number for division is 21 which on division by 3 gives q = 7 and r=0. The result now looks as

0 . 5 / 3 = 0 . 1 7

2 0

iv) Repeat the division at each digit to get the final answer to any desired level of accuracy. The next few digits are shown below.

On dividing 7 by 3, we get q = 2 and r = 1.

The result now looks as

0 . 5 / 3 = 0 . 1 7 2

2 0 1

v) On dividing 12 by 3, we get q = 4 and r = 0. The result now looks as

0 . 5 / 3 = 0 . 1 7 2 4

2 0 1 0

vi) The final result upto 8 digits of accuracy is

0 . 5 / 3 = 0 . 1 7 2 4 1 3 7 9

2 0 1 0 1 2 2 0

Hence 5 / 29 = 0.17241379

Chapter 2

Perfect Cube Roots of Larger Numbers

Before finding the cube root of any given number, firstly we will check whether the given number has a perfect cube root or not.

2.1 How will you determine whether the given number has a perfect cube root or not ?

The Vedic method has answer to this question. Find the digit sum of that number by using casting out nines method. If the digit sum of that number is found to be 0, 1 or 8, then the given number is a perfect cube and hence has a perfect cube root otherwise doesn't have a perfect cube root. This is because the Beejank of cube of numbers from 1 to 9 comes out to be 0, 1 or 8. This can be seen from the following table.

Number	Cube	Beejank of Number
1	1	1
2	8	8
3	27	0(9)
4	64	1
5	125	8
6	216	0(9)
7	343	1
8	512	8
9	729	0(9)

Table 2.1: Numbers, Cubes and their Beejank

Example 2.1 Is 1729 a perfect cube ?

Solution: The digit sum of 1729 is

$1 + 7 + 2 + 9 = 1$

Therefore, 1729 is a perfect cube.

Example 2.2 Is 9528127 a perfect cube ?

Solution: The digit sum of 9528127 is

$9 + 5 + 2 + 8 + 1 + 2 + 7 = 7$

Therefore, 9528127 is not a perfect cube.

2.1.1 How to find a perfect two-digit cube root of a cube number ?

Let us consider a two-digit number x_2x_1

where x_1 is unit place digit

x_2 is ten's place digit

$$(x_2x_1) = (10 \times x_2) + x_1$$
$$= 10x_2 + x_1$$
$$= x_2^* + x_1 \qquad \qquad \textit{where} \;\; x_2^* = 10x_2$$

Eg. $75 = (10 \times 7) + 5 = 70 + 5 = 75$

$$(x_2x_1)^3 = (x_2 + x_1)^3$$
$$= {x_2}^3 + 3x_1{x_2}^2 + \underbrace{3x_1^2x_2} + \underbrace{x_1^3} \tag{2.1}$$

where x_1^3 is the unit place

$3x_1^2x_2$ is the ten's place

$3x_1{x_2}^2$ is the hundred place

${x_2}^3$ is the thousand place

We need only last two terms (x_1^3 and $3x_1^2x_2$) from R.H.S. of expression (2.1) to find a two-digit cube root.

Working Rule :-

(i) Form the groups of three digits from extreme right to left side.

Number	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729

Table 2.2: Numbers and their Cubes

(ii) To determine Unit Place Digit :- Observe the above table to find the unit digit of cube root. Since it can be seen that cube of 0, 1, 4, 5, 6, 9 ends with 0, 1, 4, 5, 6, 9 respectively. But the cube of 2 ends with 8, cube of 3 ends with 7, cube of 7 ends with 3, cube of 8 ends with 2.

This means that if unit place digit of a cube is 0, 1, 4, 5, 6, 9 then unit place of cube root is also 0, 1, 4, 5, 6, 9 respectively. But if unit place digit of a cube is 2 then unit place digit of it's cube root is 8 and if unit place digit of a cube is 8 then unit place digit of it's cube root is 2. Similarly, if unit place digit of a cube is 3 then unit place digit of it's cube root is 7 and if unit place digit of a cube is 7 then unit place digit of it's cube root is 3.

(iii) To determine Ten's Place Digit :-

(a) Firstly, subtract the cube of unit place digit of cube root from the given number. Then ignore the unit digit and note the ten's place digit of the resultant number because we are finding the ten's place digit.

(b) Now, use $3x_1^2x_2 =$ ten's place digit of resultant number.

We have already calculated the value of x_1. Now, we have to find the value of x_2 in such a manner that when L.H.S. is totally solved, it's unit place digit matches the digit on the R.H.S.

Example 2.3 Find the cube root of $(250047)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$2 + 5 + 0 + 0 + 4 + 7 = 0$

It is a perfect cube.

Let cube root of $(250047)^{\frac{1}{3}} = (x_2x_1)$

i) Form the group of three digits from extreme right to left side.

   250,047

ii) Since unit digit of the cube is 7. Therefore, unit digit of cube root will be 3.

   $\therefore x_1 = 3$

iii) Now we have to subtract x_1^3 from the given number 250047.

   $x_1^3 = 3^3 = 27$

   250047 - 27 = 250020

   Ignore the unit place digit and note the ten's place digit and the remaining number.

   Ten's place digit is 2 and the remaining number is 25002.

iv) Use the remaining number 25002 to find the ten's place of cube root.

   Set $3x_1^2x_2 =$ a number whose unit digit is 2

   $3(3)^2x_2 =$ a number whose unit digit is 2

   $3(9)x_2 =$ a number whose unit digit is 2

   $27x_2 =$ a number whose unit digit is 2

   Now, set the value of x_2 in such a way that the number when multiplied by 27 gives the unit digit of resultant number as 2.

   By putting $x_2 = 6$, we get unit digit as 2 which matches the unit digit of R.H.S.

   $3(3)^2 \times 6 =$ a number whose unit digit is 2

   $3(9) \times 6 =$ a number whose unit digit is 2

   $27 \times 6 =$ a number whose unit digit is 2

   $162 =$ a number whose unit digit is 2

   $\therefore$ Ten's Place Digit $= 6$

Hence $(250047)^{\frac{1}{3}} = 63$

Example 2.4 Find the cube root of $(753571)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$7 + 5 + 3 + 5 + 7 + 1 = 1$

It is a perfect cube.

Let cube root of $(753571)^{\frac{1}{3}} = (x_2x_1)$

i) Form the group of three digits from extreme right to left side.

753,571

ii) Since unit digit of the cube is 1. Therefore, unit digit of cube root will be 1.

$\therefore x_1 = 1$

iii) Now we have to subtract x_1^3 from the given number 753571.

$x_1^3 = 1^3 = 1$

753571 - 1 = 753570

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 7 and the remaining number is 75357.

iv) Use the remaining number 75357 to find the ten's place of cube root.

Set $3x_1^2x_2$ = a number whose unit digit is 7

$3(1)^2x_2$ = a number whose unit digit is 7

$3(1)x_2$ = a number whose unit digit is 7

$3x_2$ = a number whose unit digit is 7

Now, set the value of x_2 in such a way that the number when multiplied by 3 gives the unit digit of resultant number as 7.

By putting $x_2 = 9$, we get unit digit as 7 which matches the unit digit of R.H.S.

$3(1)^2 \times 9$ = a number whose unit digit is 7

$3(1) \times 9$ = a number whose unit digit is 7

3×9 = a number whose unit digit is 7

$27 =$ a number whose unit digit is 7

$\therefore$ Ten's Place Digit $= 9$

Hence $(753571)^{\frac{1}{3}} = 91$

2.1.2 How to find a perfect three-digit cube root of a cube number ?

Let us consider a three-digit number $x_3x_2x_1$

where x_1 is unit place digit

x_2 is ten's place digit

x_3 is hundred place digit

$$\begin{aligned}(x_3x_2x_1) &= (100 \times x_3) + (10 \times x_2) + x_1 \\ &= 100x_3 + 10x_2 + x_1 \\ &= x_3^* + x_2^* + x_1 \qquad \textit{where} \;\; x_3^* = 100x_3 \;,\; x_2^* = 10x_2\end{aligned}$$

Eg. $375 = (100 \times 3) + (10 \times 7) + 5 = 300 + 70 + 5 = 375$

$$\begin{aligned}(x_3x_2x_1)^3 &= (x_3 + x_2 + x_1)^3 \\ &= (x_2 + x_1)^3 + {x_3}^3 + 3x_3(x_2 + x_1)(x_3 + x_2 + x_1) \\ &= {x_3}^3 + 3{x_3}^2x_2 + 3x_1{x_3}^2 + 3{x_2}^2x_3 + 6x_1x_2x_3 + {x_2}^3 + \underbrace{3x_1{x_2}^2 + 3x_1^2x_3} + \\ &\quad \underbrace{3x_1^2x_2} + \underbrace{x_1^3}\end{aligned} \tag{2.2}$$

where x_1^3 is the unit place

$3x_1^2x_2$ is the ten's place

$3x_1{x_2}^2 + 3x_1^2x_3$ is the hundred place

We need only last three terms (x_1^3, $3x_1^2x_2$ and $3x_1{x_2}^2 + 3x_1^2x_3$) from R.H.S. of expression (2.2) to find a three-digit cube root.

Working Rule :- The working rule for finding the unit and ten's place digit of a three-digit cube root is exactly same as we do to find a two-digit cube root. We have to do one more step to find the hundred place digit which is as follows :-

(i) After determining the value of ten's place (i.e. the value of x_2), subtract the value of $3x_1^2x_2$ from the remaining number.

(ii) Then again ignore the unit place digit and note the ten's place digit (say m).
Set $3x_1{x_2}^2 + 3x_1^2x_3 = \text{m}$
Now, put the values of x_1 and x_2. Then find the value of x_3 in such a way that when the L.H.S. is totally solved, it's unit digit should match the value of m.

CASE 1: When the number of digits in the cube of the number is greater than or equal to 7 and unit digit is an even number.

In such cases, there arise a problem that we can't find the value of x_2 as we can get two or more values for x_2. So, it becomes difficult for us to choose which value. For solutions to these kind of situations, we have to divide the number (whose cube root has to be extracted) by 8, until an odd cube emanates. Then apply the same above procedure on that new number to get the cube root.

CASE 2: When the number of digits in the cube of the number is greater than or equal to 7 and unit digit is 5.

In such cases, there arise a problem that we can't find the value of x_2 as we can get two or more values for x_2. So, it becomes difficult for us to choose which value. For solutions to these kind of situations, we have to divide the number (whose cube root has to be extracted) by 125, until an odd cube emanates. Then apply the same above procedure on that new number to get the cube root.

Example 2.5 Find the cube root of $(143055667)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$1 + 4 + 3 + 0 + 5 + 5 + 6 + 6 + 7 = 1$

It is a perfect cube.

Let cube root of $(143055667)^{\frac{1}{3}} = (x_3x_2x_1)$

i) Form the group of three digits from extreme right to left side.
143,055,667

ii) Since unit digit of the cube is 7. Therefore, unit digit of cube root will be 3.
$\therefore x_1 = 3$

iii) Now we have to subtract x_1^3 from the given number 143055667.

$x_1^3 = 3^3 = 27$

143055667 - 27 = 143055640

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 4 and the remaining number is 14305564.

iv) Use the remaining number 14305564 to find the ten's place of cube root.

Set $3x_1^2x_2 =$ a number whose unit digit is 4

$3(3)^2x_2 =$ a number whose unit digit is 4

$3(9)x_2 =$ a number whose unit digit is 4

$27x_2 =$ a number whose unit digit is 4

Now, set the value of x_2 in such a way that the number when multiplied by 27 gives the unit digit of resultant number as 4.

By putting $x_2 = 2$, we get unit digit as 4 which matches the unit digit of R.H.S.

$27x_2 =$ a number whose unit digit is 4

$27 \times 2 =$ a number whose unit digit is 4

$54 =$ a number whose unit digit is 4

$\therefore$ Ten's Place Digit $= 2$

v) Now we have to subtract $3x_1^2x_2$ from the number 14305564 obtained in step iii).

$3x_1^2x_2 = 3 \times 3^2 \times 2 = 3 \times 9 \times 2 = 54$

14305564 - 54 = 14305510

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 1 and the remaining number is 1430551.

vi) Use the remaining number 1430551 to find the hundred place of cube root.

Set $3x_1x_2^2 + 3x_1^2x_3 =$ a number whose unit digit is 1

$3x_1x_2^2 + 3x_1^2 \times x_3 =$ a number whose unit digit is 1

$3 \times 3 \times 2^2 + 3 \times 3^2 \times x_3 =$ a number whose unit digit is 1

$9 \times 4 + 3 \times 9 \times x_3 =$ a number whose unit digit is 1

$36 + 27x_3 =$ a number whose unit digit is 1

Now, set the value of x_3 in such a way that when it is multiplied by 27 and will be added to 36, unit digit of resultant number comes out to be 1.

By putting $x_3 = 5$, we get unit digit as 1 which matches the unit digit of R.H.S.

$36 + 27x_3 =$ a number whose unit digit is 1

$36 + 27 \times 5 =$ a number whose unit digit is 1

$171 =$ a number whose unit digit is 1

$\therefore$ Hundred Place Digit $= 5$

Hence $(143055667)^{\frac{1}{3}} = 523$

Example 2.6 Find the cube root of $(491169069)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$4 + 9 + 1 + 1 + 6 + 9 + 0 + 6 + 9 = 0(9)$

It is a perfect cube.

Let cube root of $(491169069)^{\frac{1}{3}} = (x_3x_2x_1)$

i) Form the group of three digits from extreme right to left side.

491,169,069

ii) Since unit digit of the cube is 9. Therefore, unit digit of cube root will be 9.

$\therefore x_1 = 9$

iii) Now we have to subtract x_1^3 from the given number 491169069.

$x_1^3 = 9^3 = 729$

491169069 - 729 = 491168340

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 4 and the remaining number is 49116834.

iv) Use the remaining number 49116834 to find the ten's place of cube root.

Set $3x_1^2x_2$ = a number whose unit digit is 4

$3(9)^2x_2$ = a number whose unit digit is 4

$3(81)x_2$ = a number whose unit digit is 4

$243x_2$ = a number whose unit digit is 4

Now, set the value of x_2 in such a way that the number when multiplied by 243 gives the unit digit of resultant number as 4.

By putting $x_2 = 8$, we get unit digit as 4 which matches the unit digit of R.H.S.

$243x_2$ = a number whose unit digit is 4

243×8 = a number whose unit digit is 4

1944 = a number whose unit digit is 4

$\therefore$ Ten's Place Digit = 8

v) Now we have to subtract $3x_1^2x_2$ from the number 49116834 obtained in step iii).

$3x_1^2x_2 = 3 \times 9^2 \times 8 = 3 \times 81 \times 8 = 1944$

14305564 - 1944 = 49114890

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 9 and the remaining number is 4911489.

vi) Use the remaining number 4911489 to find the hundred place of cube root.

Set $3x_1x_2^2 + 3x_1^2x_3$ = a number whose unit digit is 9

$3x_1x_2^2 + 3x_1^2x_3$ = a number whose unit digit is 9

$3 \times 9 \times 8^2 + 3 \times 9^2 \times x_3$ = a number whose unit digit is 9

$27 \times 64 + 3 \times 81 \times x_3$ = a number whose unit digit is 9

$1728 + 243x_3$ = a number whose unit digit is 9

Now, set the value of x_3 in such a way that when it is multiplied by 243 and will be added to 1728, unit digit of resultant number comes out to be 9.

By putting $x_3 = 7$, we get unit digit as 9 which matches the unit digit of R.H.S.

$1728 + 243x_3 =$ a number whose unit digit is 9

$1728 + 243 \times 7 =$ a number whose unit digit is 9

$3429 =$ a number whose unit digit is 9

$\therefore$ Hundred Place Digit $= 7$

Hence $(491169069)^{\frac{1}{3}} = 789$

Example 2.7 Find the cube root of $(890277128)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$8 + 9 + 0 + 2 + 7 + 7 + 1 + 2 + 8 = 8$

It is a perfect cube.

Let cube root of $(890277128)^{\frac{1}{3}} = (z_3 z_2 z_1)$

Here, the given cube number has 9 digits and also the unit digit is even.

So we will multiply and divide the given cube number by 8 until an odd cube emanates.

$$\begin{aligned}
(z_3 z_2 z_1) &= (890277128)^{\frac{1}{3}} \\
&= \left(8 \times \frac{890277128}{8}\right)^{\frac{1}{3}} \\
&= (8)^{\frac{1}{3}} \times \left(\frac{890277128}{8}\right)^{\frac{1}{3}} \\
&= (8)^{\frac{1}{3}} \times (111284641)^{\frac{1}{3}} \\
&= (2)^{3 \times \frac{1}{3}} \times (111284641)^{\frac{1}{3}} \\
&= 2 \times (111284641)^{\frac{1}{3}}
\end{aligned} \tag{2.3}$$

Now, we have to find the cube root of 111284641.

Let cube root of $(111284641)^{\frac{1}{3}} = (x_3 x_2 x_1)$

i) Form the group of three digits from extreme right to left side.

111,284,641

ii) Since unit digit of the cube is 1. Therefore, unit digit of cube root will be 1.

$\therefore x_1 = 1$

iii) Now we have to subtract x_1^3 from the given number 111284641.

$x_1^3 = 1^3 = 1$

111284641 - 1 = 111284640

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 4 and the remaining number is 11128464.

iv) Use the remaining number 11128464 to find the ten's place of cube root.

Set $3x_1^2x_2$ = a number whose unit digit is 4

$3(1)^2x_2$ = a number whose unit digit is 4

$3(1)x_2$ = a number whose unit digit is 4

$3x_2$ = a number whose unit digit is 4

Now, set the value of x_2 in such a way that the number when multiplied by 3 gives the unit digit of resultant number as 4.

By putting $x_2 = 8$, we get unit digit as 4 which matches the unit digit of R.H.S.

$3x_2$ = a number whose unit digit is 4

3×8 = a number whose unit digit is 4

24 = a number whose unit digit is 4

$\therefore$ Ten's Place Digit = 8

v) Now we have to subtract $3x_1^2x_2$ from the number 11128464 obtained in step iii).

$3x_1^2x_2 = 3 \times 1^2 \times 8 = 3 \times 1 \times 8 = 24$

11128464 - 24 = 11128440

Ignore the unit place digit and note the ten's place digit and the remaining number.

Ten's place digit is 9 and the remaining number is 1112844.

vi) Use the remaining number 1112844 to find the hundred place of cube root.

Set $3x_1x_2^2 + 3x_1^2x_3$ = a number whose unit digit is 4

$3x_1x_2^2 + 3x_1^2x_3$ = a number whose unit digit is 4

$3 \times 1 \times 8^2 + 3 \times 1^2 \times x_3$ = a number whose unit digit is 4

$3 \times 64 + 3 \times 1 \times x_3$ = a number whose unit digit is 4

$192 + 3x_3$ = a number whose unit digit is 4

Now, set the value of x_3 in such a way that when it is multiplied by 3 and will be added to 192, unit digit of resultant number comes out to be 4.

By putting $x_3 = 4$, we get unit digit as 4 which matches the unit digit of R.H.S.

$192 + 3x_3$ = a number whose unit digit is 4

$192 + 3 \times 4$ = a number whose unit digit is 4

204 = a number whose unit digit is 4

$\therefore$ Hundred Place Digit $= 4$

$$(111284641)^{\frac{1}{3}} = (x_3x_2x_1) = 481$$

Putting this value in expression (2.3).

$$\begin{aligned}(z_3z_2z_1) &= 2 \times (111284641)^{\frac{1}{3}} \\ &= 2 \times 481 \\ &= 962\end{aligned}$$

Hence $(890277128)^{\frac{1}{3}} = 962$

Example 2.8 Find the cube root of $(14706125)^{\frac{1}{3}}$.

Solution: Firstly we will check whether it is a perfect cube or not.

It's digit sum is

$1 + 4 + 7 + 0 + 6 + 1 + 2 + 5 = 8$

It is a perfect cube.

Let cube root of $(14706125)^{\frac{1}{3}} = (z_3z_2z_1)$

Here, the given cube number has 8 digits and also the unit digit is 5.

So we will multiply and divide the given cube number by 125 until an odd cube emanates.

$$\begin{aligned}
(z_3 z_2 z_1) &= (14706125)^{\frac{1}{3}} \\
&= \left(125 \times \frac{14706125}{125}\right)^{\frac{1}{3}} \\
&= (125)^{\frac{1}{3}} \times \left(\frac{14706125}{125}\right)^{\frac{1}{3}} \\
&= (125)^{\frac{1}{3}} \times \left(\frac{2941225}{25}\right)^{\frac{1}{3}} \\
&= (125)^{\frac{1}{3}} \times \left(\frac{588245}{5}\right)^{\frac{1}{3}} \\
&= (125)^{\frac{1}{3}} \times (117649)^{\frac{1}{3}} \\
&= (5)^{3\times\frac{1}{3}} \times (117649)^{\frac{1}{3}} \\
&= \ 5 \times (117649)^{\frac{1}{3}}
\end{aligned} \tag{2.4}$$

Now, we have to find the cube root of 117649.

Let cube root of $(117649)^{\frac{1}{3}} = (x_2 x_1)$

i) Form the group of three digits from extreme right to left side.

   117,649

ii) Since unit digit of the cube is 9. Therefore, unit digit of cube root will be 9.

   $\therefore x_1 = 9$

iii) Now we have to subtract x_1^3 from the given number 117649.

   $x_1^3 = 9^3 = 729$

   117649 - 729 = 116920

   Ignore the unit place digit and note the ten's place digit and the remaining number.

   Ten's place digit is 2 and the remaining number is 11692.

iv) Use the remaining number 11692 to find the ten's place of cube root.

   Set $3x_1^2 x_2$ = a number whose unit digit is 2

   $3(9)^2 x_2$ = a number whose unit digit is 2

$3(81)x_2 =$ a number whose unit digit is 2

$243x_2 =$ a number whose unit digit is 2

Now, set the value of x_2 in such a way that the number when multiplied by 243 gives the unit digit of resultant number as 2.
By putting $x_2 = 4$, we get unit digit as 2 which matches the unit digit of R.H.S.

$3(9)^2 \times 4 =$ a number whose unit digit is 2

$3(81) \times 4 =$ a number whose unit digit is 2

$243 \times 4 =$ a number whose unit digit is 2

$972 =$ a number whose unit digit is 2

$\therefore$ Ten's Place Digit $= 4$

$(117649)^{\frac{1}{3}} = 49$

Putting this value in expression (2.4).

$$\begin{aligned}(z_3 z_2 z_1) &= 5 \times (117649)^{\frac{1}{3}} \\ &= 5 \times 49 \\ &= 245\end{aligned}$$

Hence $(14706125)^{\frac{1}{3}} = 245$

For detailed study, one can refer page 323, 324 of Vedic Mathematics [3].

On the similar lines, we can also find perfect cube roots of larger numbers.Eg.

(i) $(653972032)^{\frac{1}{3}} = 868$

(ii) $(16836267547)^{\frac{1}{3}} = 2563$

(iii) $(864052711946936)^{\frac{1}{3}} = 95246$

Motivated from these calculations, we observe that we can give generalization of n-digits perfect cube roots of larger numbers.

2.2 Generalization

We proposed the following generalization

$$
\begin{aligned}
(x_2x_1)^3 &= (x_1)^3 + x_2^3 + 3x_2(x_1)(x_2+x_1)\\
(x_3x_2x_1)^3 &= (x_2+x_1)^3 + x_3^3 + 3x_3(x_2+x_1)(x_3+x_2+x_1)\\
(x_4x_3x_2x_1)^3 &= (x_3+x_2+x_1)^3 + x_4^3 + 3x_4(x_3+x_2+x_1)(x_4+x_3+x_2+x_1)\\
\vdots \qquad & \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots\\
(x_nx_{n-1}\cdots x_4x_3x_2x_1)^3 &= (x_{n-1}+x_{n-2}+\cdots+x_3+x_2+x_1)^3 + x_n^3 +\\
&\qquad 3x_n(x_{n-1}+x_{n-2}+\cdots+x_2+x_1)(x_n+x_{n-1}+\cdots+x_2+x_1)
\end{aligned}
$$

$$
\left(\prod_{k=1}^{n} x_k\right)^3 = \left(\sum_{k=1}^{n} x_k\right)^3 = \left(\sum_{k=1}^{n-1} x_k\right)^3 + x_n^3 + 3x_n\left(\sum_{k=1}^{n-1} x_k\right)\left(\sum_{k=1}^{n} x_k\right) \quad, \forall n \geq 2
$$

where x_1 is the unit place

x_2 is the ten's place

x_3 is the hundred place

x_4 is the thousand place

$\vdots \qquad \vdots$

x_{n-1} is the $10^{n-2}th$ place

x_n is the $10^{n-1}th$ place

Now, we shall prove our proposed generalization by the principle of mathematical induction.

2.3 Proof by the Principle of Mathematical Induction

Statement :

$$\left(\prod_{k=1}^{n} x_k\right)^3 = \left(\sum_{k=1}^{n} x_k\right)^3 = \left(\sum_{k=1}^{n-1} x_k\right)^3 + x_n^3 + 3x_n \left(\sum_{k=1}^{n-1} x_k\right)\left(\sum_{k=1}^{n} x_k\right) \quad , \forall n \geq 2 \quad (2.5)$$

Proof : Firstly we will prove that the result (2.5) is true for $n = 2$.

Putting $n = 2$,

$$\left(\prod_{k=1}^{2} x_k\right)^3 = \left(\sum_{k=1}^{2} x_k\right)^3 = \left(\sum_{k=1}^{1} x_k\right)^3 + x_2^3 + 3x_2 \left(\sum_{k=1}^{1} x_k\right)\left(\sum_{k=1}^{2} x_k\right).$$

$$(x_2 x_1)^3 = (x_2 + x_1)^3 = x_2^3 + 3x_2^2 x_1 + 3x_1^2 x_2 + x_1^3.$$

$\Rightarrow$ The result (2.5) is true for $n = 2$.

Let us suppose that the result is true for $n = m$.

$$\left(\prod_{k=1}^{m} x_k\right)^3 = \left(\sum_{k=1}^{m} x_k\right)^3 = \left(\sum_{k=1}^{m-1} x_k\right)^3 + x_m^3 + 3x_m \left(\sum_{k=1}^{m-1} x_k\right)\left(\sum_{k=1}^{m} x_k\right). \quad (2.6)$$

Now, we will prove that the result is true for $n = m + 1$.

i.e. we have to show that

$$\left(\prod_{k=1}^{m+1} x_k\right)^3 = \left(\sum_{k=1}^{m+1} x_k\right)^3 = \left(\sum_{k=1}^{m} x_k\right)^3 + x_{m+1}^3 + 3x_{m+1} \left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right).$$

Taking

$$\left(\sum_{k=1}^{m} x_k\right)^3 + x_{m+1}^3 + 3x_{m+1} \left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right)$$

$$= \underbrace{\left(\sum_{k=1}^{m-1} x_k\right)^3 + x_m^3 + 3x_m \left(\sum_{k=1}^{m-1} x_k\right)\left(\sum_{k=1}^{m} x_k\right)}_{\text{Using (2.6)}} + x_{m+1}^3 + 3x_{m+1} \left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right),$$

$$= \left(\sum_{k=1}^{m-1} x_k\right)^3 + x_m^3 + 3x_m \left(\sum_{k=1}^{m-1} x_k\right)\left(\sum_{k=1}^{m-1} x_k + x_m\right) + x_{m+1}^3 + 3x_{m+1} \left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right),$$

$$= \underbrace{\left(\sum_{k=1}^{m-1} x_k\right)^3 + x_m^3 + 3x_m\left(\sum_{k=1}^{m-1} x_k\right)^2 + 3x_m^2\left(\sum_{k=1}^{m-1} x_k\right)}_{\text{Combining these}} + x_{m+1}^3$$

$$+ 3x_{m+1}\left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right),$$

$$= \left(\sum_{k=1}^{m-1} x_k + x_m\right)^3 + x_{m+1}^3 + 3x_{m+1}\left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right),$$

$$= \left(\sum_{k=1}^{m} x_k\right)^3 + x_{m+1}^3 + 3x_{m+1}\left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m+1} x_k\right),$$

$$= \left(\sum_{k=1}^{m} x_k\right)^3 + x_{m+1}^3 + 3x_{m+1}\left(\sum_{k=1}^{m} x_k\right)\left(\sum_{k=1}^{m} x_k + x_{m+1}\right),$$

$$= \left(\sum_{k=1}^{m} x_k\right)^3 + x_{m+1}^3 + 3x_{m+1}\left(\sum_{k=1}^{m} x_k\right)^2 + 3x_{m+1}^2\left(\sum_{k=1}^{m} x_k\right),$$

$$= \left(\sum_{k=1}^{m} x_k + x_{m+1}\right)^3,$$

$$= \left(\sum_{k=1}^{m+1} x_k\right)^3.$$

$\Rightarrow$ The result (2.5) is true for $n = m + 1$.

Hence, by principle of mathematical induction result (2.5) is true $\forall n \geq 2$.

Using the proposed generalization, we can find the cube roots of larger numbers like

(i) $(428578861841456899832)^{\frac{1}{3}} = 7539518$

(ii) $(287276705480637429716240523)^{\frac{1}{3}} = 659832147$

2.4 Conclusions and Discussions

In this chapter, we have proposed the generalization of the work of Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Maharaja[3] on cube roots of exact cubes. We have extended his work to n-digits. The proof of this study is given by well known principle of mathematical induction. By using this generalization, it is quite easy to find the perfect cube root of any larger number with the help of Vedic mathematics, provided that it's perfect cube root exists. This act of innovation is only a small step in the field of Vedic mathematics and a large calculation time of any machine can be saved if we can write the program scripts by using these types of logics.

Chapter 3

Future Scope

After studying the Chapter 2, it may be easily concluded that the proposed method works only on the numbers those have perfect cube roots. For the future scope, this drawback can be removed by proposing such a generalization in which one can find any cube root (perfect as well as imperfect) of large numbers by using Vedic mathematic's logic.

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