STUDY OF VIBRATION CHARACTERISTICS OF CANTILEVER BEAMS OF DIFFERENT MATERIALS

A Dissertation

Submitted in Partial fulfillment of the requirement for the award of

Degree of
MASTER OF ENGINEERING
IN
CAD/CAM & ROBOTICS

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July 2012

CERTIFICATE

I hereby declare that the work which is being presented in the dissertation work entitled, **"STUDY OF VIBRATION CHARACTERISTICS OF CANTILEVER BEAMS OF DIFFERENT MATERIALS"** in the partial fulfillment of the requirements for the award of degree of **Master of Engineering in Mechanical Engineering in CAD/CAM & ROBOTICS** submitted my own work carried out under the supervision of **Dr. S.P. Nigam and Dr. Naveen Kwatra.**

The matter presented in this thesis has not been submitted for award of any other degree of this or any other university.

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ACKNOWLEDGEMENT

Words are ofter less to reveal ones deep regards, with an understanding that work like this can never be the outcome of a single person. I take this opportunity to express my profound sense of gratitude and respect to all those who directly or indirectly helped me through the duration of this work.

I take the opportunity to express my heartfelt adulation and gratitude to my supervisor **Dr. S.P. NIGAM** for his unreserved guidance, constructive suggestions, thought provoking discussions and unabashed inspiration in the nuturing work. It has been a benediction for me spend many present work is testimony to his activity, inspiration and ardent personal interest, taken by him during the course of his work in its present form. I am grateful to Dr. Ajay Batish, Professor and Head, Department of Mechanical Engineering for his encouragement & inspiration for execution of the thesis work .

I am highly thankful to **Dr. NAVEEN KWATRA** Associate professor, Department of Civil Engineering for his invaluable guidance & Support.

I am also very thankful to Mr. Sanjeev Kumar Assistant Professor, Department of Mechanical Engineering, B.B.S.B. Engineering College Fatehgarh Sahib for his intellectual support and cooperation during experimental investigation.

I take pride of myself being son of ideal parents for their everlasting desire, sacrifice, affectionate blessings, and help, without which it should not have been possible for me to complete my studies. I am also very thankful to Mr. Prabhjeet Singh co-editor, Science of healing magazine for his valuable suggestions.

I take this opportunity to thank all my friends especially Jaspreet Singh and Sunny for their help and moral support. I would like to thank all the members and employees of Mechanical Engineering Department, Thapar University, Patiala for their everlasting support. Above all, I express my indebetedness to the "ALMIGHTY" for all his blessing and kindness.

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ABSTRACT

Estimating damping in structure made of different materials (steel, brass, aluminum) and processes still remains as one of the biggest challengers. All materials posses certain amount of internal damping, which manifested as dissipation of energy from the system. This energy in a vibratory system is either dissipated into heat or radiated away from the system. Material damping or internal damping contributes to about 10-15% of total system damping. The main objective of this thesis is to estimate the damping ratio, natural frequency of aluminum, brass and steel by free vibration analysis experimentally & verify theoretically. Cantilever beams of required size & shape are prepared for experimental purpose & damping ratio is investigated. Damping ratio is determined by half-power bandwidth method. It is observed that damping ratio is higher for steel than brass than aluminum.

This thesis presents results of an experimental free vibration analysis of beams made with different materials such as Brass, aluminum and steel. The theoretical modal analysis was also done in ANSYS to compare the results. The beams were excited using wooden mallet and signals were catched with the help of accelerometer attached with VIB SCANNER instruments. Then FRF (Frequency response functions) were obtained using omnitrend software to identify fundamental natural frequency and damping ratios.

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NOMENCLATURE

SYMBOLS	DESCRIPTIONS
ω_n	Natural Frequency (rad/s)
ω_{nf}	Circular Natural Frequency (rad/s)
f_n	Natural Frequency (Hz)
ζ	Damping Ratio
C	Damping Coefficient
C_C	Critical Damping coefficient
E	Modulus of elasticity
I	Moment of inertia
ρ	Mass density
A	Cross- Sectional Area
ψ	Angle of Rotation

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium. Most vibrations are undesirable in machines and structures because they produce increased stresses, energy losses, cause added wear, increase bearing loads, induce fatigue, create passenger discomfort in vehicles, and absorb energy from the system. Rotating machine parts need careful balancing in order to prevent damage from vibrations.

Vibration occurs when a system is displaced from a position of stable equilibrium. The system tends to return to this equilibrium position under the action of restoring forces (such as the elastic forces, as for a mass attached to a spring, or gravitational forces, as for a simple pendulum). The system keeps moving back and forth across its position of equilibrium. A system is a combination of elements intended to act together to accomplish an objective. For example, an automobile is a system whose elements are the wheels, suspension, car body, and so forth. A static element is one whose output at any given time depends only on the input at that time while a dynamic element is one whose present output depends on past dynamic. In the same way we also speak of static and dynamic systems. A static system contains all elements while a dynamic system contains at least one dynamic element.

A physical system undergoing a time-varying interchange or dissipation of energy among or within its elementary storage or dissipative devices is said to be in a dynamic system. All of the elements in general are called passive, i.e., they are incapable of generating net energy. A dynamic system composed of a finite number of storage elements is said to be lumped & discrete, while a system containing elements, which are dense in physical space, is called continuous system. The analytical description of the dynamics of the discrete case is a set of ordinary differential equations, while for the continuous case it is a set of partial differential equations. The analytical formation of a dynamic system depends upon the kinematic or geometric constraints and the physical laws governing the behaviour of the system.

1.1.1 Elementary Parts of Vibrating System

In general, a vibrating system consists of a spring (a means for storing potential energy), a mass or inertia (a means for storing kinetic energy), and a damper (a means by which energy is gradually lost) as shown in Fig. 1.1. An undamped vibrating system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternatively. In a damped vibrating system, some energy is dissipated in each cycle of vibration and should be replaced by an external source if a steady state of vibration is to be maintained.

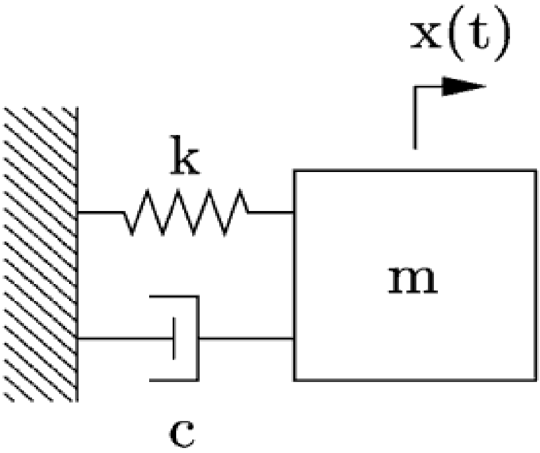

Fig.1.1 Elementary parts of vibrating system [33]

1.1.2 Vibration and Damping [23]

Damping is the resistance offered by a body to the motion of a vibratory system. The resistance may be applied by a liquid or solid internally or externally. The main advantage of providing damping in mechanical systems is just to control the amplitude of vibration so that the failure occurring because of resonance may be avoided.

Vibration damping plays an important role in machines and structures by improving performance and stability, reducing noise and increasing lifetime.
In mechanics, Damping may be realized using a dashpot. This device uses the viscous drag of a fluid, such as oil, to provide a resistance that is related linearly to velocity. The damping force F_C is expressed as follows:

$$F_C = -C\frac{dx}{dt} \tag{1.1}$$

where C is the viscous damping coefficient, given in units of Newton seconds per meter (N s/m).

Generally, damped harmonic oscillators satisfy the second-order differential equation:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n{}^2 x = 0 \tag{1.2}$$

where ω_n is the undamped angular frequency of the oscillator and ζ is a constant called the damping ratio.

The value of the damping ratio ζ determines the behavior of the system. A damped harmonic oscillator can be:

- Overdamped (ζ > 1): The system returns (exponentially decays) to equilibrium without oscillating. Larger values of the damping ratio ζ return to equilibrium more slowly.
- Critically damped (ζ = 1): The system returns to equilibrium as quickly as possible without oscillating. This is often desired for the damping of systems such as doors.
- Underdamped (0 < ζ < 1): The system oscillates (at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.
- Undamped (ζ = 0): The system oscillates at its natural resonant frequency (ω_n).

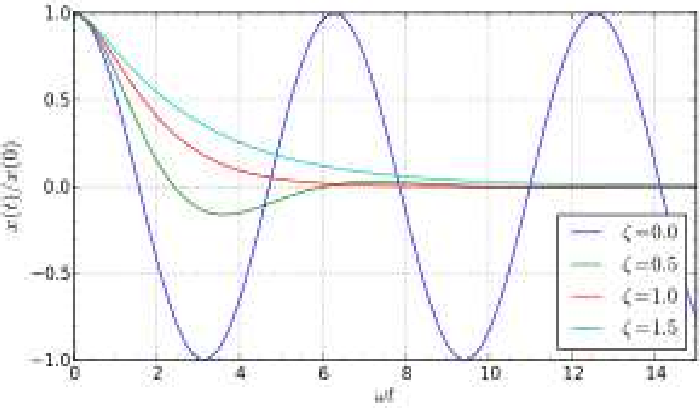

Fig1.2 Time dependence of the system behavior on the value of the damping ratio ζ, for undamped (blue), under-damped(green), critically damped (red), and over-damped (cyan)cases, for zero-velocity initial condition.[23]

1.1.3 Continuous Systems and Discrete Systems

Most of the mechanical and structural systems can be described using a finite number of degrees of freedom. However, there are some systems, especially those include continuous elastic members, have an infinite number of degree of freedom. Most mechanical and structural systems have elastic (deformable) elements or components as members and hence have an infinite number of degrees of freedom. Systems which have a finite number of degrees of freedom are known as discrete or lumped parameter systems, and those systems with an infinite number of degrees of freedom are called continuous or distributed systems.

1.2 FREE VIBRATION OF A SINGLE DEGREE OF FREEDOM SYSTEM [24]

The most basic mechanical system is the single degree of freedom system, which is characterized by the fact that its motion is described by a single variable or coordinates. Such a model is often used as an approximation for a generally more complex system. Excitations can be broadly divided into two types, initial excitations and externally applied forces. The behavior of a system characterized by the motion caused by these excitations is called as the system response. The motion is generally described by displacements.

1.2.1 Free Vibration of an Undamped Translation System

The simplest model of a vibrating mechanical system consists of a single mass element which is connected to a rigid support through a linearly elastic massless spring as shown in Fig. 1.3. The mass is constrained to move only in the vertical direction. The motion of the system is described by a single coordinate x(t) and hence it has one degree of freedom (DOF).

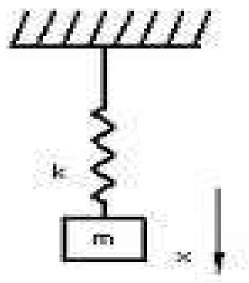

Fig 1.3 Spring mass system [33]

1.3 TYPES OF DAMPING [33]

There are mainly four types of damping used in mechanical systems:

1.3.1 Viscous damping

When the system is allowed to vibrate in a viscous medium, the damping is called as viscous. Viscosity is the property of a fluid by virtue of which it offers resistance to the motion of one layer over the adjacent one.

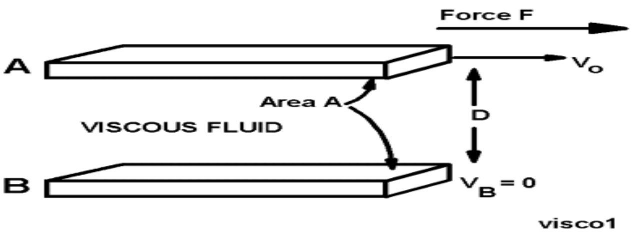

Fig. 1.4 Viscosity [23]

This can be explained by fig.1.4 where two plates are separated by fluid film by thickness t, the upper plate is allowed to move parallel to the fixed plate with velocity v_0. The net force required is:-

$$F = \frac{\mu A}{t} v_0 \qquad (1.3)$$

A is the area of plate , t is the thickness of the fluid film, μ is the coefficient of viscosity

The energy dissipation in viscous damping can be written as:-

$$\Delta E = \pi c \omega_n A^2 \qquad (1.4)$$

From the above equation it is clear that the energy dissipation per cycle is proportional to the square of the amplitude of motion.

1.3.2 Coulomb damping

When one body is allowed to slide over the other, the surface of one body offers some resistance to the movement of other body on it. This resisting force is called force of friction. Thus force of friction arises only because of relative movement between the two surfaces. Some amount of energy is wasted in overcoming this friction as the surfaces are dry.

So it is called dry friction. The general expression for coulomb damping is:-

$$F = \mu R_N \quad (1.5)$$

where is the coefficient of friction and is the normal reaction. Frictional force is proportional to the normal reaction on the mating surface. The system is shown in the fig.1.5

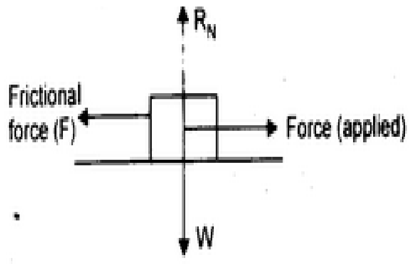

Fig.1.5 System showing force of friction and normal reaction [33]

1.3.3 Structural damping

It is the inherent characteristics of the material and the resistance is offered by the elastic properties from within the body. This type of damping arises because of intermolecular friction in the structure which opposes its movement. The magnitude of this damping is very small as compared to other types of damping. Elastic materials during loading and unloading form a loop known as hysteresis loop. The area of this loop is the amount of energy dissipated in one cycle during vibration.

$$E = \pi k \lambda A^2 \quad (1.6)$$

Where A represents the amplitude of vibration and λ is dimensionless damping factor

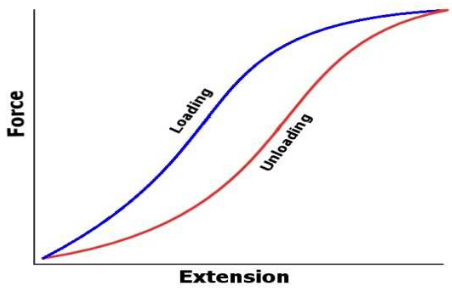

Fig.1.6 Hysteresis loop [30]

1.3.4 Non-Linear, Slip or Interfacial damping

The machine elements are connected through various types of joints. Microscopic slip occurs on the interfaces of machine elements which causes dissipation of vibrational energy when the interface of the machine elements or parts in contact are under fluctuating loads. The amount of damping depends on surface roughness of contacting parts, contact pressure and the amplitude of vibration. The energy dissipated per cycle depends upon the co-efficient of friction, the pressure at the contacting parts and amplitudes. There is an optimum value of pressure for which the energy dissipated is maximum. This value is different for different amplitudes. Larger the energy dissipation, larger is the effective damping in the system.

1.4 MEASURES OF DAMPING [24]

Several techniques are used to quantify the level of material damping in a structure:

1.4.1 Half-Power Bandwidth Method

For the SDOF, the structure will possess a classic compliance a response as shown in figure 1.7. The level of material damping can be subjectively determined by noting the sharpness of the resonant peak at ω_0 the more rounded shape, the more damping present in the structure. For a quantitative measure of damping, the half power bandwidth method can be employed. As defined in Equation 1.2, the damping of the structure can be determined from the ratio of $\Delta\omega$ to ω_0 with $\Delta\omega$ determined from the half power point down from the resonant peak value, A_{max} (equal to the inverse of the amplification factor Q). On a decibel scale, this corresponds to a -3 dB drop from the peak. For that reason, this damping measurement technique is also referred to as the 3 dB method.

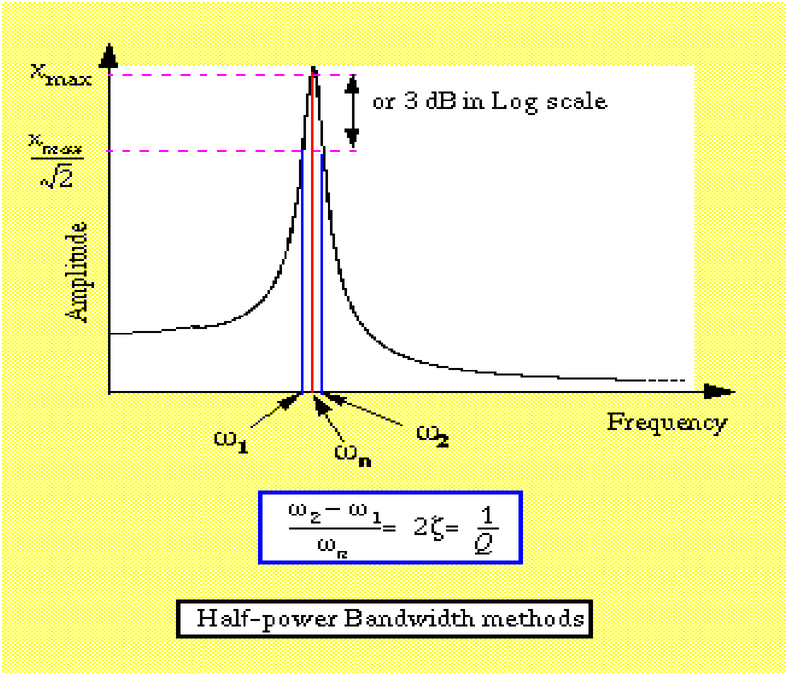

Fig.1.7 Frequency response of a SDOF system [24]

To estimate damping ratio from frequency domain, we may use half-power bandwidth method. In this method, FRF amplitude of the system is obtained first. Corresponding to each natural frequency, there is a peak in FRF amplitude. 3 dB down from the peak there are two points corresponding to half power point, as shown in the figure below. The more the damping, the more the frequencies range between these two points. Half-power bandwidth (BD) is defined as the ratio of the frequency range between the two half power points to the natural frequency at this mode.

1.4.2 Log Decrement Method

Logarithmic decrement method is used to measure damping in time domain. In this method, the free vibration displacement amplitude history of a system to an impulse is measured and recorded. A typical free decay curve is shown as below. Logarithmic decrement is the natural logarithmic value of the ratio of two adjacent peak values of displacement in free decay vibration.

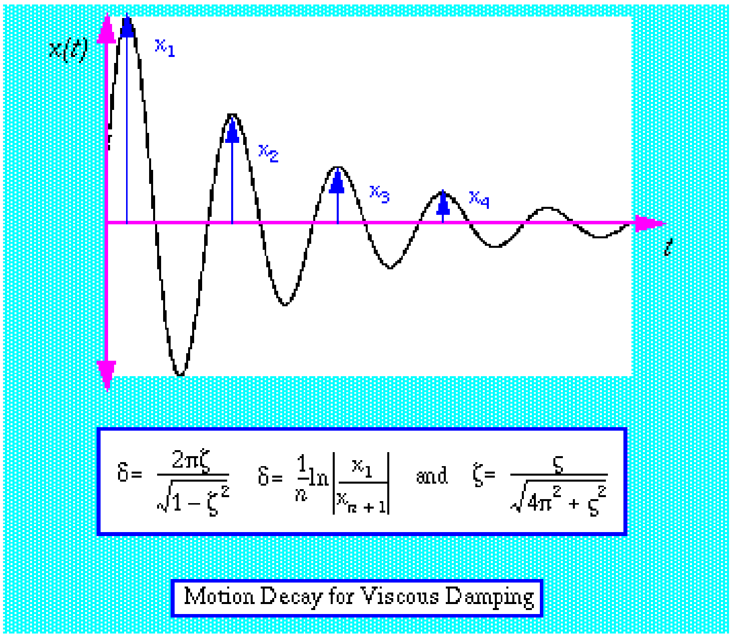

Fig.1.8 Motion Decay for viscous damping [24]

1.4.3 Hysteresis Loop Method

Another method to calculate damping can be achieved by calculating the energy loss per cycle of oscillation due to steady state harmonic loading. Again assume the complex spring element of is subjected to the cyclic stress (Force) resulting in a strain response (Extension) by plotting the instantaneous stress versus strain for a given cycle of motion, the hysteresis curve of fig.1.9 is generated. The area captured within the hysteresis loop D is equal to the dissipated energy per cycle of harmonic motion. For reasonable levels of damping, the loop area can be used to calculate damping, as shown in equation:-

$$\eta = \frac{D}{\pi \sigma_0 \omega_0}$$

(1.7)

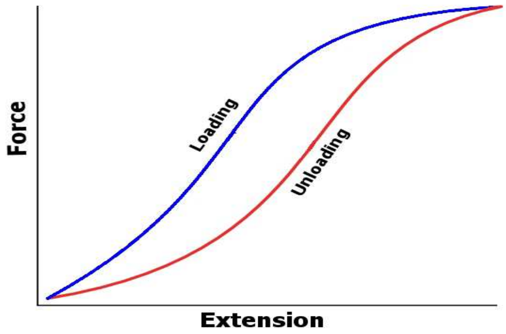

Fig.1.9 Hysteresis loop [24]

1.5 BEAM [25]

A beam is a horizontal or vertical structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending movement.

Beams are traditionally descriptions of building or civil engineering structural elements, but smaller structures such as truck or automobile frames, machine frames, and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion.

1.5.1 Types of Beams

Beams are characterized by their profile (the shape of their cross-section), their length, and their material. In contemporary constructions, beams are typically made of steel, reinforced concrete or wood. One of the most common types of steel beam is the I-beam or wide - flange beam (also known as a "universal beam" or, for stouter sections, a "universal column"). This is commonly used in steel-frame buildings and bridges. Other common beam profiles are the C- channels, the hollow structural section beam, the pipe, and the angle.

Beams are also described by how they are supported. Supports restrict lateral or rotational movements so as to satisfy stability conditions as well as to limit the deformations to a certain allowance. A simple beam is supported by a pin support at one end and a roller support at the other end. A beam with a laterally and rotationally fixed support at one end with no support at the other end is called a cantilever beam. A beam simply supported at two points and having one end or both ends extended beyond the supports is called an overhanging beam. Fig.1.10 showing the different types of beams.

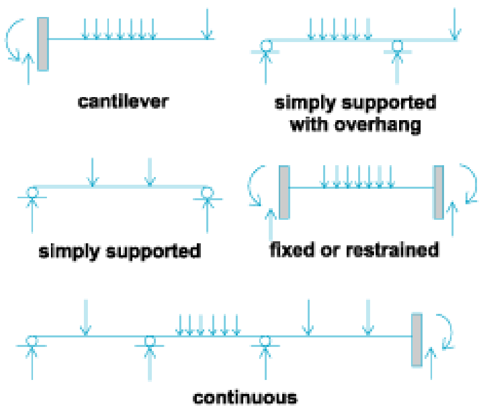

Fig.1.10 Different types of beams [23]

1.6 THEORY OF VIBRATION OF BEAMS

1.6.1 Timoshenko theory of Beams [32]

The Timoshenko beam theory was developed by Ukrainian-born scientist Stephen Timoshenko in the beginning of the 20th century. The model takes into account shear deformation and rotational inertia effects, making it suitable for describing the behaviour of short beams, sandwich composite beams or beams subject to high-frequency excitation when the wavelength approaches the thickness of the beam. The resulting equation is of 4th order, but unlike ordinary beam theory - i.e. Bernoulli-Euler theory - there is also a second order spatial derivative present. Physically, taking into account the added mechanisms of deformation effectively lowers the stiffness of the beam, while the result is a larger deflection under a static load and lower predicted eigen frequencies for a given set of boundary conditions. The latter effect is more noticeable for higher frequencies as the wavelength becomes shorter, and thus the distance between opposing shear forces decreases.

If the shear modulus of the beam material approaches infinity - and thus the beam becomes rigid in shear - and if rotational inertia effects are neglected, Timoshenko beam theory converges towards ordinary beam theory.

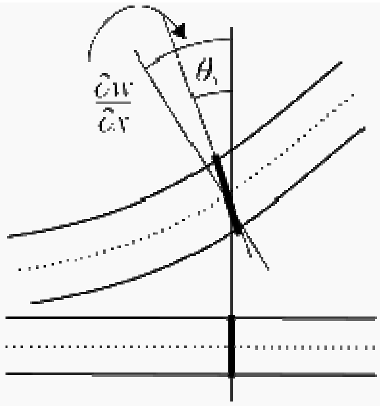

Fig . 1.11 Deformation of a Timoshenko beam. The normal rotates by an amount $\theta_x = \psi(x)$ which is not equal to $d\omega/dx$[32]

In static Timoshenko beam theory without axial effects, the displacements of the beam are assumed to be given by

$$u_x(x,y,z) = -z\psi(x)$$
$$u_y(x,y,z) = 0$$
$$u_z(x,y) = \omega(x) \quad (1.8)$$

where (x,y,z) are the coordinates of a point in the beam, u_x, u_y, u_z are the components of the displacement vector in the three coordinate directions, ψ is the angle of rotation of the normal to the mid-surface of the beam, and ω is the displacement of the mid-surface in the z-direction.

The governing equations are the following uncoupled system of ordinary differential equations:

$$\frac{d^2}{dx^2}(EI\frac{d\psi}{dx}) = q(x,t)$$
$$\frac{d\omega}{dx} = \psi - \frac{1}{KAG}\frac{d}{dx}(EI\frac{d\psi}{dx}) \tag{1.9}$$

The Timoshenko beam theory for the static case is equivalent to the Euler Bernoulli theory when the last term above is neglected, an approximation that is valid when

$$\frac{EI}{KL^2AG} << 1 \tag{1.10}$$

where L is the length of the beam.

Combining the two equations gives, for a homogeneous beam of constant cross-section,

$$EI\frac{d^4x}{dx^4} = q(x) - \frac{EI}{KAG}\frac{d^2q}{dx^2} \tag{1.11}$$

In Timoshenko beam theory without axial effects, the displacements of the beam are assumed to be given by:-

$$u_x(x,y,z,t) = -z\psi(x,t); u_y(x,y,z,t) = 0; u_z(x,y,z,t) = \omega(x,t) \tag{1.12}$$

where (x,y,z) are the coordinates of a point in the beam, u_x, u_y, u_z are the components of the displacement vector in the three coordinate directions, ψ is the angle of rotation of the normal to the mid-surface of the beam, and ω is the displacement of the mid-surface in the z-direction.

Starting from the above assumption, the Timoshenko beam theory, allowing for vibrations, may be described with the coupled linear partial differential equations

$$\rho A\frac{\partial^2\omega}{\partial t^2} - q(x,t) = \frac{\partial}{\partial x}[KAG(\frac{\partial\omega}{\partial x} - \psi)]$$
$$\rho I\frac{\partial^2\psi}{\partial t^2} = \frac{\partial}{\partial x}(EI\frac{\partial\psi}{\partial x}) + KAG(\frac{\partial\omega}{\partial x} - \psi) \tag{1.13}$$

where the dependent variables are $\omega(x,t)$, the translational displacement of the beam, and $\psi(x,t)$, the angular displacement. Note that unlike the Euler Bernoulli theory, the angular deflection is another variable and not approximated by the slope of the deflection. Also,

- ρ is the density of the beam material (but not the linear density).
- A is the cross section area.
- E is the elastic modulus
- G is the shear modulus.
- I is the second moment of area.
- K, called the Timoshenko shear coefficient, depends on the geometry. Normally, $K = \frac{5}{6}$ for a rectangular section.
- $q(x,t)$ is a distributed load (force per length).

These parameters are not necessarily constants.

For a linear elastic, isotropic, homogeneous beam of constant cross-section these two equations can be combined to give

$$EI\frac{\partial^4\omega}{\partial x^4} + m\frac{\partial^2\omega}{\partial t^2} - (\rho I + \frac{E\,\mathrm{Im}}{KAG})\frac{\partial^4\omega}{\partial x^2\partial t^2} + \frac{Jm}{KAG}\frac{\partial^4\omega}{\partial t^4} = q(x,t) + \frac{\rho I}{KAG}\frac{\partial^2 q}{\partial t^2} - \frac{EI}{KAG}\frac{\partial^2 q}{\partial x^2} \quad (1.14)$$

Further the above equation can be solved by applying the boundary conditions with the help of numerical methods.

1.6.2 Euler Bernoulli Beam Theory [34]

Euler Bernoulli's Beam Theory also known as engineer's beam theory or classical beam theory is a simplification of the linear theory of elasticity which provides a means of calculating the load-carrying and deflection characteristics of beams . It covers the case for small deflections of a beam which is subjected to lateral loads only. It is thus a special case of Timoshenko beam theory which accounts for shear deformation and is applicable for thick beams. It was first enunciated circa 1750, but was not applied on a large scale until the development of the Eiffel tower and the Ferris wheel in the late 19th century. Following these successful demonstrations, it quickly became a cornerstone of engineering and an enabler of the second industrial revolution.

Additional analysis tools have been developed such as plate theory and finite element analysis, but the simplicity of beam theory makes it an important tool in the sciences, especially structural and mechanical engineering.

Fig.1.12 This vibrating glass beam may be modeled as a cantilever beam with acceleration, variable linear density, variable section modulus, some kind of dissipation, springy end loading, and possibly a point mass at the free end. [34]

The Euler-Bernoulli equation describes the relationship between the beam's deflection and the applied load:

$$\frac{d^2}{dx^2}(EI\frac{d^2\omega}{dx^2}) = q \tag{1.15}$$

The curve $\omega(x)$ describes the deflection of the beam in the z direction at some position x (the beam is modeled as a one-dimensional object). q is a distributed load, in other words a force per unit length (analogous to pressure being a force per area); it may be a function of x, ω or other variables.

Note that E is the elastic modulus and that I is the second moment of area. I must be calculated with respect to the centroidal axis perpendicular to the applied loading. For an Euler-Bernoulli beam not under any axial loading this axis is called the Neutral axis.

Often, EI is a constant, so that:

$$EI\frac{d^4\omega}{dx^4} = q(x) \tag{1.16}$$

This equation, describing the deflection of a uniform, static beam, is used widely in engineering practice. For more complicated situations the deflection can be determined by solving the Euler-Bernoulli equation using techniques such as the "slope deflection method", "moment distribution method", "moment area method, "conjugate beam method", "the principle of virtual work", "direct integration", "Castigliano's method", "Macaulay's method" or the "direct stiffness method".

Sign conventions are defined here since different conventions can be found in the literature. In this article, a right handed coordinate system is used as shown in the figure, Bending of an Euler-Bernoulli beam. In this figure, the x and z direction of a right handed coordinate system are shown. Since $e_z * e_x = e_y$ where e_x, e_y and e_z are unit vectors in the direction of the x, y, and z axes respectively, the y axis direction is into the figure. Forces acting in the positive x and z directions are assumed positive. The sign of the bending moment is positive when the torque vector associated with the bending moment on the right hand side of the section is in the positive y direction (i.e. so that a positive value of M leads to a compressive stress at the bottom fibers). With this choice of bending moment sign convention, in order to have $dM = Qdx$, it is necessary that Q the s0hear force acting on the right side of the section be positive in the z direction so as to achieve static equilibrium of moments. To have force equilibrium with $dQ = qdx$, the loading intensity must be positive in the minus z direction. In addition to these sign conventions for scalar quantities, we also sometimes use vectors in which the directions of the vectors are made clear through the use of the unit vectors.

Successive derivatives of ω have important meanings where ω is the deflection in the z direction:

- ω is the deflection.
- $\frac{d\omega}{dx}$ is the slope of the beam.
- $M = -EI\frac{d^2\omega}{dx^2}$ is the bending moment in the beam.
- $-\frac{d}{dx}(EI\frac{d^2\omega}{dx^2}) = Q$ is the shear force in the beam.

The stresses in a beam can be calculated from the above expressions after the deflection due to a given load has been determined.

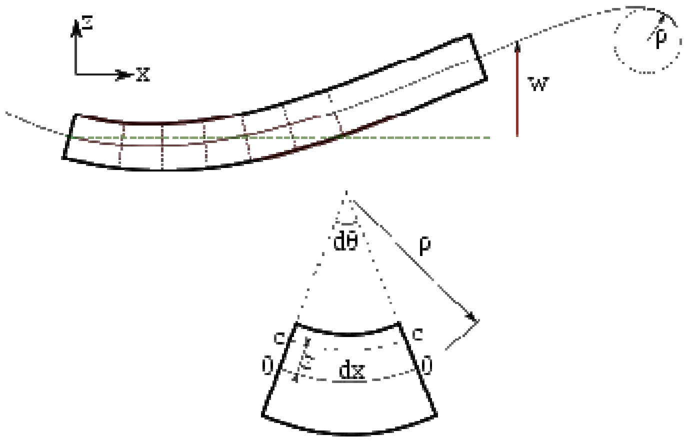

Fig.1.13 Bending of an Euler-Bernoulli beam. Each cross-section of the beam is at 90 degrees to the neutral axis. [34]

1.6.2.1 Boundary considerations

The beam equation contains a fourth-order derivative in x. To find a unique solution $\omega(x,t)$ we need four boundary conditions. The boundary conditions usually model supports, but they can also model point loads, distributed loads and moments. The supports or displacement boundary conditions are used to fix values of displacement (ω) and rotations $\frac{d\omega}{dx}$ on the boundary. Such boundary conditions are also called Dirichlet boundary conditions.

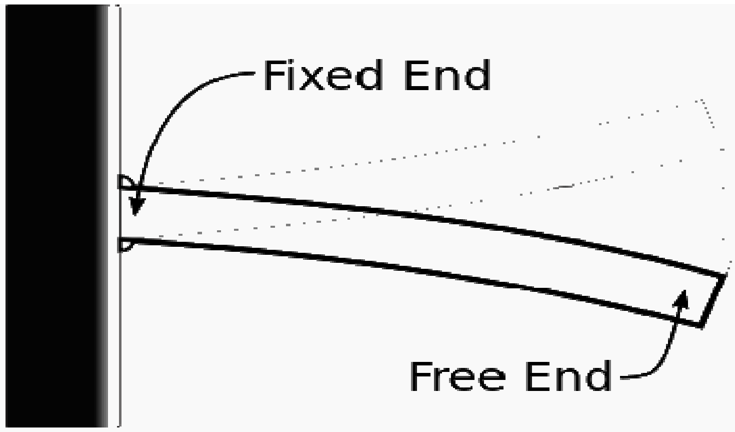

Fig.1.14 A cantilever beam [34]

As an example considers a cantilever beam that is built-in at one end and free at the other. At the built-in end of the beam there cannot be any displacement or rotation of the beam. This means that at the left end both deflection and slope are zero. Since no external bending moment is applied at the free end of the beam, the bending moment at that location is zero. In addition, if there is no external force applied to the beam, the shear force at the free end is also zero.

Taking the x coordinate of the left end as 0 and the right end as L (the length of the beam), these statements translates to the following set of boundary conditions (assume EI is a constant):

$$\omega|_{x=0} = 0; \frac{\partial \omega}{\partial x}|_{x=0} = 0 \qquad \text{(fixed end)} \qquad (1.17)$$

$$\frac{\partial^2 \omega}{\partial x^2}|_{x=L}; \frac{\partial^3 \omega}{\partial x^3}|_{x=L} = 0 \qquad \text{(free end)} \qquad (1.18)$$

A simple support (pin or roller) is equivalent to a point force on the beam which is adjusted in such a way as to fix the position of the beam at that point. A fixed support or clamp, is equivalent to the combination of a point force and a point torque which is adjusted in such a way as to fix both the position and slope of the beam at that point. Point forces and torques, whether from supports or directly applied, will divide a beam into a set of segments, between which the beam equation will yield a continuous solution, given four boundary conditions, two at each end of the segment. Assuming that the product EI is a constant, and defining $\lambda = F/\ EI$ where F is the magnitude of a point force, and $\tau = M/EI$ where M is the magnitude of a point torque, the boundary conditions appropriate for some common cases is given in the table below. The change in a particular derivative of w across the boundary as x increases is denoted by Δ followed by that derivative. For example, $\Delta\omega'' = \omega''(x+) - \omega''(x-)$ where $\omega''(x+)$ is the value of ω'' at the lower boundary of the upper segment, while $\omega''(x-)$ is the value of ω'' at the upper boundary of the lower segment. When the values of the particular derivative are not only continuous across the boundary, but fixed as well, the boundary condition is written e.g. $\Delta\omega'' = 0$ which actually constitutes two separate equations (e.g. $\omega''(x-) = \omega''(x+) =$ fixed). As $\frac{\partial\omega}{\partial x} = \omega'$, $\frac{\partial^2\omega}{\partial x^2} = \omega''$, $\frac{\partial^3\omega}{\partial x^3} = \omega'''$.

BOUNDARY	ω'''	ω''	ω'	ω
Clamp			$\Delta\omega' = 0$	$\Delta\omega = 0$
Simple support		$\Delta\omega'' = 0$	$\Delta\omega' = 0$	$\Delta\omega = 0$
Point force	$\Delta\omega''' = \lambda$	$\Delta\omega'' = 0$	$\Delta\omega' = 0$	$\Delta\omega = 0$
Point torque	$\Delta\omega''' = 0$	$\Delta\omega'' = \tau$	$\Delta\omega' = 0$	$\Delta\omega = 0$
Free end	$\omega''' = 0$	$\Delta\omega'' = 0$		
Clamp at end			ω' fixed	ω fixed
Simply supported end		$\omega'' = 0$		ω fixed
Point force at end	$\Delta\omega''' = \lambda$	$\omega'' = 0$		
Point torque at end	$\Delta\omega''' = 0$	$\omega'' = \tau$		

Table: 1.1 Point forces and torques are located between two segments, there are four boundary conditions, two for the lower segment, and two for the upper. [34]

1.7 CANTILEVER BEAMS [25]

Another important class of problems involves cantilever beams. A system is said to be a cantilever beam system if one end of the system is rigidly fixed to a support and the other end is free to move. Cantilevers are widely found in construction, in cantilever bridges and balconies . In cantilever bridges the cantilevers are usually built as pairs, with each cantilever used to support one end of a central section. Another use of the cantilever is in fixed-wing aircraft design, Cantilevered beams are the most ubiquitous structures in the field of microelectro mechanical systems (MEMS).A cantilever rack is a type of warehouse storage system.

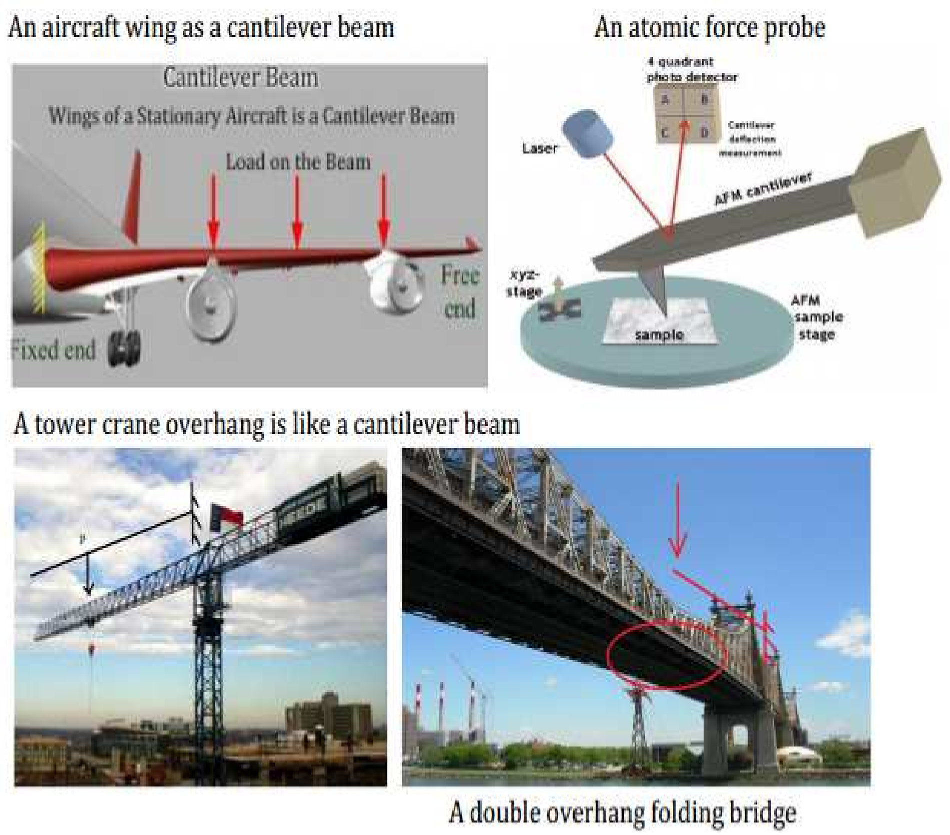

Fig. 1.15 Applications of cantilever beams [25]

Vibration analysis of a cantilever beam system is important as it can explain and help to analyse a number of real life systems there by helpful in making design changes accordingly for the most efficient systems.

The bending moments (M), shear forces (Q), and deflections (ω) for a cantilever beam subjected to a point load at the free end and a uniformly distributed load are given in the figure below:

Distribution	Max. value
Cantilever beam with end load	
$M(x) = P(x - L)$	$M_A = PL$
$Q(x) = P$	$Q_{\max} = P$
$w(x) = \frac{Px^2(3L-x)}{6EI}$	$w_C = \frac{PL^3}{3EI}$

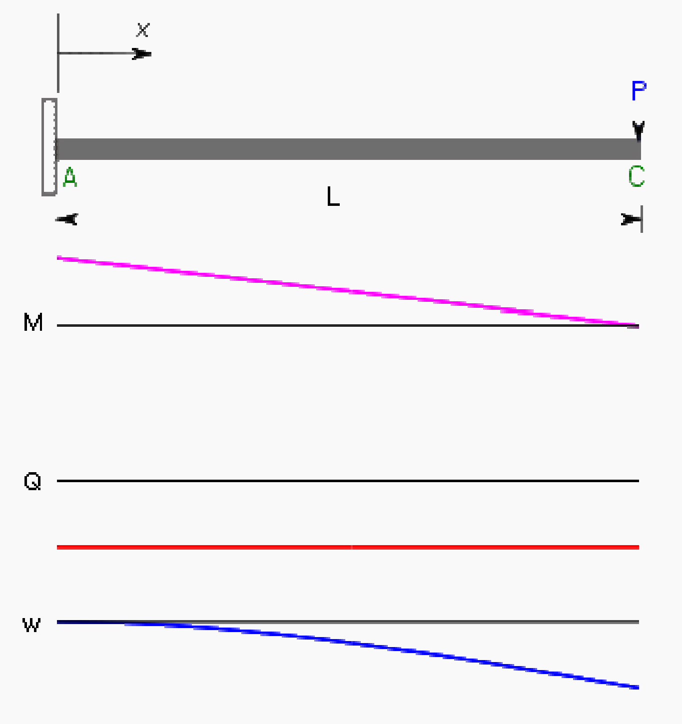

Fig. 1.16 The bending moments (M), shear forces (Q), and deflections (ω) for a cantilever beam subjected to a point load at the free end and a uniformly distributed load.[25]

1.7.1 Theory of Free Vibration of Cantilever Beams [25]

For a cantilever beam subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as**:-**

$$\frac{d^2}{dx^2}\{EI(x)\frac{d^2Y(x)}{dx^2}\} = \omega_n^{\,2} m(x)Y(x) \qquad (1.19)$$

Where, E is the modulus of rigidity of beam material, I is the moment of inertia of the beam cross-section, $Y(x)$ is displacement in y direction at distance x from fixed end, ω_n is the circular natural frequency, m is the mass per unit length, $m = \rho A(x)$, ρ is the material density, x is the distance measured from the fixed end.

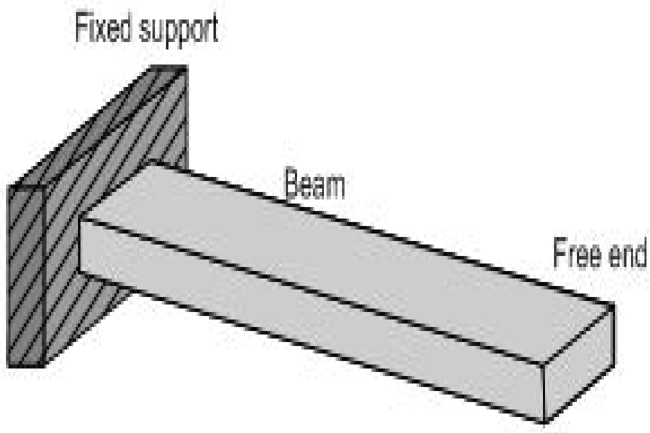

Fig.1.17 (a): A cantilever beam [30]

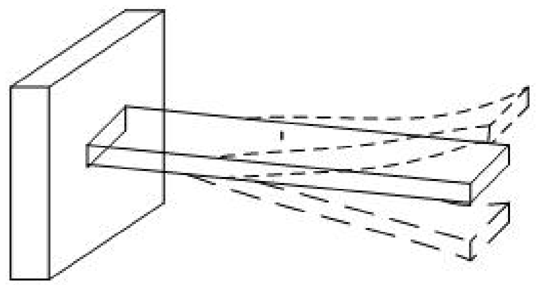

Fig.1.17 (b): The beam under free vibration [30]

Fig.1.17 (a) shows of a cantilever beam with rectangular cross section, which can be subjected to bending vibration by giving a small initial displacement at the free end; and Fig.1.17 (b) depicts of cantilever beam under the free vibration.

Following are the boundary conditions for a cantilever beam (Fig.1.17)

$$\text{at} \quad x = 0, Y(x) = 0, \frac{dY(x)}{dx} = 0 \tag{1.20}$$

$$\text{at} \quad x = l, \frac{d^2Y(x)}{dx^2} = 0, \frac{d^3Y(x)}{dx^3} = 0 \tag{1.21}$$

For a uniform beam under free vibration from equation (1.19), we get

$$\frac{d^4Y(x)}{dx^4} - \beta^4 Y(x) = 0 \tag{1.22}$$

with $$\beta^4 = \frac{\omega_n^2 m}{EI} \tag{1.23}$$

Using the boundary condition from Eq. (1.20) & Eq. (1.21), we obtain the frequency equation as

$$\cos\beta_n L + \cosh\beta_n L = -1 \qquad (1.24)$$

which must be solved numerically and it yields an infinite of solutions of β_n.

Corresponding to the eigen values of β_n, the mode shapes for a continuous cantilever beam is given as

$$f_n(x) = A_n\{(\sin\beta_n L - \sinh\beta_n L)(\sin\beta_n x - \sinh\beta_n x) + (\cos\beta_n L + \cosh\beta_n L)(\cos\beta_n x - \cosh\beta_n x)\} \qquad (1.25)$$

Where

$$n = 1,2,3.....\infty \text{ and } \beta_n L = \alpha_n$$

A closed form solution of the circular natural frequency ω_{nf}, from above equation of motion and boundary conditions can be written as,

$$\omega_{nf} = \alpha_n^{\ 2}\sqrt{\frac{EI}{mL^4}} \qquad (1.26)$$

The Eq.(1.26) is satisfied by a number of values of $\beta_n L$ corresponding to each normal mode of oscillation, which for first three modes are given as:-

α_n = 1.875, 4.694, 7.855

So,

First natural frequency

$$\omega_{nf} = 1.875^2\sqrt{\frac{EI}{\rho AL^4}} \qquad (1.27)$$

Second natural frequency

$$\omega_{nf} = 4.694^2 \sqrt{\frac{EI}{\rho AL^4}} \tag{1.28}$$

Third natural frequency

$$\omega_{nf} = 7.855^2 \sqrt{\frac{EI}{\rho AL^4}} \tag{1.29}$$

The natural frequency is related with the circular natural frequency as

$$f_{nf} = \frac{\omega_{nf}}{2\pi} Hz \tag{1.30}$$

where *I*, the moment of inertia of the beam cross-section, for a circular cross-section it is given as

$$I = \frac{\pi}{64} d^4$$

Where, *d* is the diameter of cross section and for a rectangular cross section

$$I = \frac{bd^3}{12} \tag{1.31}$$

Where *b* and *d* are the breadth and width of the beam cross-section as shown in the Fig. 1.18.

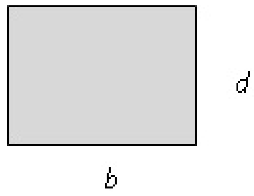

Fig.1.18 Cross-section of the cantilever beam

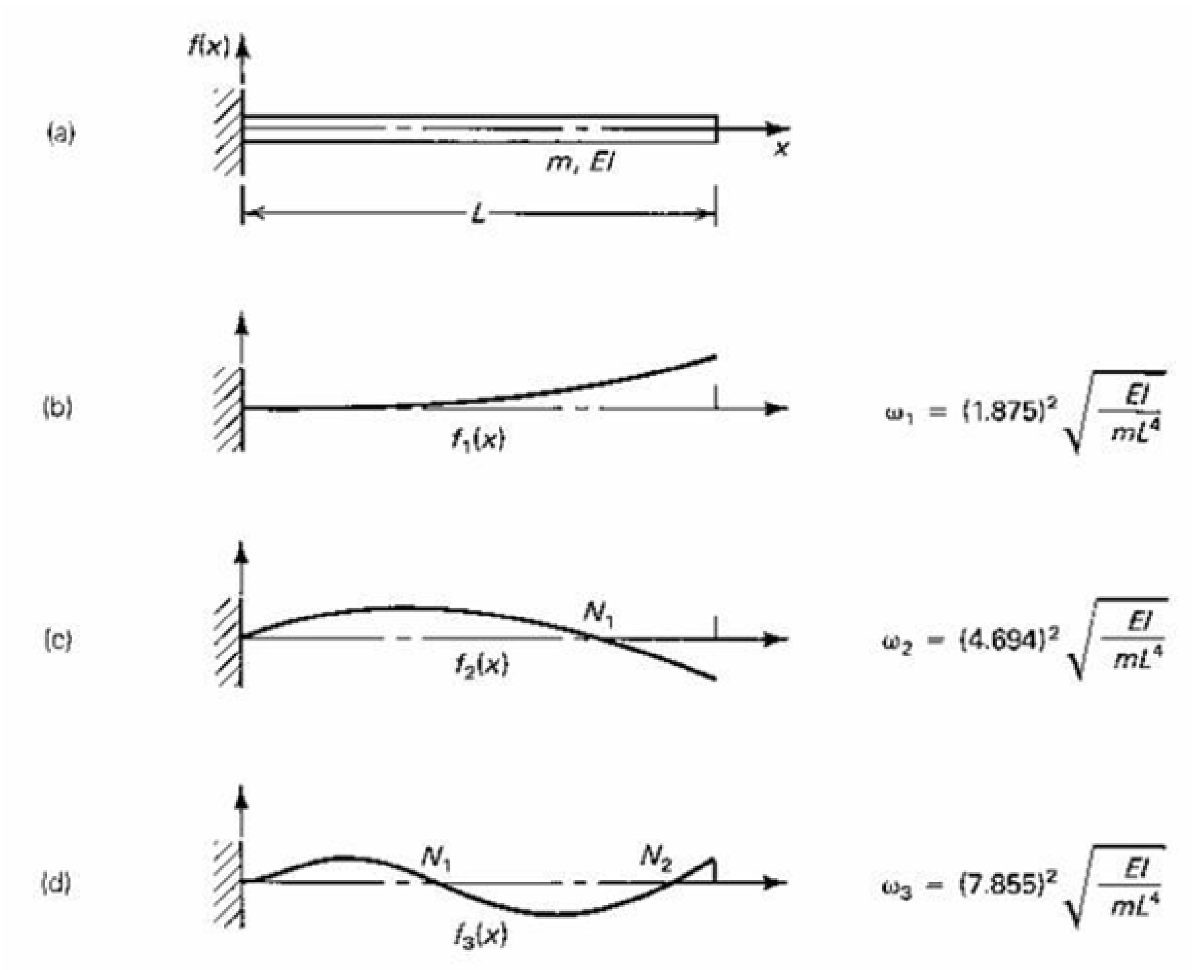

Fig. 1.19 The first three undamped natural frequencies and mode shape of cantilever beam [30]

1.8 FRF (Frequency Response Function)

The FRF (frequency response function) is the measure of any system output spectrum in response to an input signal. A linear system such as an SDOF or an MDOF, when subjected to sinusoidal excitation will respond sinusoidally at the same frequency and at specific amplitude that is characteristic to the frequency of excitation. The phase difference b/w the response and the excitation will vary with frequency.

The characteristics of a system that describe its response to excitation as the function of frequency is the frequency response function. It is defined as the ratio of the output to the input as a function of frequency. Accelerometer measures the vibration levels at several points on the structure and signal analyzer computes the FRF.

1.9 MODAL ANALYSIS [26]

Modal analysis is a worldwide used methodology that allows fast and reliable identification of system dynamics in complex structures. In the last decades several methods have been developed in quest to improve accuracy of modal models extracted from test data and to enlarge the applicability of modal analysis in industrial context.

Structures vibrate in special shapes called mode shapes when excited at their resonant frequencies. A mode shape is the characteristics deformation shape defined by relative amplitudes of the extreme positions of vibration of a system at a single natural frequency. The modal parameters are the natural frequencies, damping ratios and modal masses associated with each of the mode shapes. Under normal operating conditions, the structure will vibrate in a complex combination of all the mode shapes. Modal analysis refers to measuring and predicting the mode shapes and frequencies of a structure.

1.9.1 EXPERIMENTAL MODAL ANALYSIS

Experimental modal analysis is to identify the modal response of an existing structure to help to solve a vibration problem. Impact test or bump test is used in the experimental modal analysis. Impact testing is a fast and low cost of finding the modes of machines and structures. Experimental modal analysis consists of exciting the structure with an impact hammer, measuring the FRF between the excitation and many points on structure and then using software to visualize the mode shapes. Accelerometers measure the vibration levels at several points on the structure and a signal analyzer computes the FRF. The structures are divided into a grid pattern with enough points to cover the entire grid structure. The number of measurements points is determined by size and complexity of the structure and the highest resonant frequency of interest.

Each FRF identifies the resonant frequencies of the structure and modal amplitudes of the measuring grid point associated with FRF. The modal amplitude indicates the ratio of vibration acceleration divided by force input. The objective of modal analysis is to understand the mode shapes by visualizing the deformed geometry. The FRF is a fundamental measurement that isolates the inherent dynamic properties of a mechanical. The process of determining modal parameters from experimental data involves several phases including:

- Modal analysis theory
- Experimental Modal Analysis Methods
- Modal Data Acquisition
- Modal Parameter Estimation
- Modal Data Presentation/Validation

Experimental modal analysis is a supportive tool to the study of various vibration problems. In addition to verifying the theoretical results, it helps to study the actual dynamic behavior of a structure away from idealization. Modal analysis is “the representation of the dynamic properties of an elastic structure in terms of its modes of vibration”. The elastic structure tends to bend, twist and vibrate when subjected to an external dynamic force. The dynamic properties are also known as the modal parameters which are unique for the structure. The modal parameters are:

- Natural Frequency
- Damping Factor
- Mode Shape

The physical appearance of the three modal parameters is expressed by the "way" the structure behaves dynamically. When a structure vibrates at a certain frequency, it will possess a unique shape of motion known as the mode shape. At the same time, it dissipates the energy, absorbed from the external force, at a rate known as the damping factor. The frequency at which the structure may vibrate is the natural frequency.

The determination of the modal parameters is the key to solve the problem created such as noise, excessive vibration, failure of mechanical system due to fatigue etc. The modal parameters are determined by two methods: Sine test method and Frequency response function method as described earlier.

1.10 OBJECTIVE OF THE PRESENT WORK

The objective of the thesis is to study:

- free vibration of a cantilever beam using Euler's Bernoulli beam theory.
- the vibration characteristics of a cantilever beam made with different materials such as mild steel , brass and aluminium.
- the natural frequencies and mode shapes of a cantilever beam and effect of different parameters like thickness, length of the beams on these. Both theoretical and experimental techniques are to be used and to estimate modal damping for the fundamental mode.

1.11 SCOPE OF WORK

To achieve the above objectives, the scope of this work for the project generally involves the following:

a) To build up the experiment test rig for testing.
b) To catch the signals of vibration using VIB SCANNER to obtain the natural frequency of beams of different materials by varying the parameters like thickness ,length etc. and modal damping for the fundamental mode.
c) To compare the damping ratio of different materials under different parameters.
d) To do the harmonic and structure analysis of beams of different materials using ANSYS.

1.12 OUTLINE OF THESIS

CHAPTER 2 is a presentation of previous studies of free vibration techniques, material damping techniques to measure damping of materials. This includes theoretical and experimental investigations done by other investigators.

CHAPTER 3 is an experimental procedure and methodology.

CHAPTER 4 describes experimental investigation.

CHAPTER5 presents a description of finite element modeling in ANSYS. Harmonic analysis is also included in this chapter.

CHAPTER 6 The experimental results and analysis presented in details. The study of the results in terms of the parameters is also presented.

CHAPTER 2

LITERATURE REVIEW

A wealth of literature exists in the area of vibrations of beams but while going through the literature regarding material damping of cantilever beams it has been figured out that still a lot of work has to be done regarding it. Usually whenever study of various materials has been done the focus of researchers has been damping, mode shapes, resonant frequency, etc. but material damping have not been paid much attention. Some important literatures are given below:

1. **H H Yoo and S H Shin [4]** Vibration analysis of a rotating cantilever beam is an important and peculiar subject of study in mechanical engineering. There are many engineering examples which can be idealized as rotating cantilever beams such as turbine blades or turbo engine blades and helicopter blades .For the proper design of the structures their vibration characteristics which are natural frequencies and mode shapes should be well identified. Compared to the vibration characteristics of non rotating structures those of rotating structures often vary significantly. The variation results from the stretching induced by the centrifugal inertia force due to the rotational motion. The stretching causes the increment of the bending stiffness of the structure which naturally results in the variation of natural frequencies and mode shapes. The equations of motion of a rotating cantilever beam are derived based on a new dynamic modeling method. With the coupling effect ignored the analysis results are consistent with the results obtained by the conventional modelling method. A modal formulation method is also introduced in this study to calculate the tuned angular speed of a rotating beam at which resonance occurs.

2. **Mousa Rezaee and Reza Hassannejad [14]** derived a new analytical method for vibration analysis of a cracked simply supported beam is investigated. By considering a non linear model for the fatigue crack, the governing equation of motion of the cracked beam is solved using perturbation method. The solution of the governing equation reveals the super harmonics of the fundamental frequency due to the nonlinear effects in the dynamic response of the cracked beam. Furthermore, considering such a solution, an explicit expression is also derived for the system damping changes due to the changes in

the crack parameters, geometric dimensions and mechanical properties of the cracked beam. The results show that an increase in the crack severity and approaching the crack location to the middle of the beam increase the system damping.

In order to validate the results, changes in the fundamental frequency ratios against the fatigue crack severities are compared with those of experimental results available in the literature. Also, a comparison is made between the free response of the cracked beam with a given crack depth and location obtained by the proposed analytical solution and that of the numerical method. The results of the proposed method agree with the experimental and numerical results.

3. **Chih Ling Huang,Wen Yi Lin,Kuo Mo Hsio[7]**Rotating beams are often used as a simple model for propellers, turbine blades, and satellite booms. The free vibration frequencies of rotating beams have been extensively studied. Rotating beam differs from a non-rotating beam in having additional centrifugal force and Coriollis effects on its dynamics. The natural frequency of the flap wise bending vibration, and coupled lagwise bending and axial vibrations investigated for the rotating beam. A method based on the power series solution is proposed to solve the natural frequency of very slender rotating beam at high angular velocity. The rotating beam is subdivided into several equal segments. The governing equations of each segment are solved by a power series. Numerical examples are studied to demonstrate the accuracy and efficiency of the proposed method. The effect of Coriolis force, angular velocity, and slenderness ratio on the natural frequency of rotating beams is investigated. The Free vibration of the beam is measured from the position of the steady state axial deformation.

4. **H.Ding,G.C. Zhang,LQ Chen[16]**The axially moving beams has several applications, including robot arms, conveyor belts, high-speed magnetic tapes, and automobile engine belt. Understanding the vibrations of axially moving beams are important for the design of the devices. Recent developments in research on axially moving structures have been reviewed. Natural frequencies of nonlinear coupled planar vibration are investigated for axially moving beams in the supercritical transport speed ranges. The straight equilibrium configuration bifurcates in multiple equilibrium positions in the supercritical regime. The

finite difference scheme is developed to calculate the non-trivial static equilibrium. The equations are cast in the standard form of continuous gyroscopic systems via introducing a coordinate transform for non-trivial equilibrium configuration. Under fixed boundary conditions, time series are calculated via the finite difference method. Based on the time series, the natural frequencies of nonlinear planar vibration, which are determined via discrete Fourier transform (DFT), are compared with the results of the Galerkin method for the corresponding governing equations without nonlinear parts. The effects of material parameters and vibration amplitude on the natural frequencies are investigated through parametric studies.

5. **Liao-Liang Ke,Jie Yang, Sritawat Kitipornchai,Yang Xiang[13]**Free vibration and elastic buckling of beams made of functionally graded materials (FGMs) containing open edge cracks are studied in this paper based on Timoshenko beam theory. The crack is modeled by a massless elastic rotational spring. It is assumed that the material properties follow exponential distributions along beam thickness direction. Analytical solutions of natural frequencies and critical buckling load are obtained for cracked FGM beams with clamped-free, hinged-hinged, and clamped-clamped end supports. A detailed parametric study is conducted to study the influences of crack depth, crack location, total number of cracks, material properties, beam slenderness ratio, and end supports on the free vibration and buckling characteristics of cracked FGM beams.

6. **M.Shavezipur,S.M. Hashemi[10]** A set of differential equations governing triply coupled vibrations of centrifugally stiffened beams, a refined dynamic finite element (RDFE) method is developed. The application of the proposed method is demonstrated to obtain numerical results for several examples. Some of these results are compared with those obtained from classical FEM and published results. As it was confirmed by numerical results, the RDFE method is a reliable solution method with drastically higher convergence rates compared to other numerical methods. The RDFE can be advantageously used when multiple natural frequencies and/or higher modes of the beam structures are of interest. It is important to note that the method is not limited to the equations introduced in this paper and can be extended to more advanced models which

may include more geometric and material coupling terms. Many aerospace and terrestrial structures, such as aircraft wings, propeller blades, solar panels and satellite antenna, compressor, turbine and helicopter rotor blades, space structures, bridges, etc. can be modelled as a combination of beam elements with two or three coupled governing differential equations.

7. **Michael I Frisswell,John E Mottershead[19]** A Method is proposed for the replacement of unknown stiffness with rigid connections in two systems of equation from a finite element model and from measured response functions. The frequency response can be determined from standard modal tests and no special forcing arrangements or physical constraints are needed. The only use of constraints in mathematics where the physical behaviour of constrained system is inferred from the unconstrained measurements and predictions are obtained from constrained finite element equations since stiffness which are replaced by rigid connections can't experience any elastic strain they can have no effect on the inferred measurements from an elastic structure. The method can be used to determine erroneous connections in a finite element model or to locate discrete non-linearities. Simulated examples are used to illustrate the application of the technique.

8. **Hamid Zabihi Ferezqi,Masoud Tahani,Hamid Ekhteraei Touss[15]** This paper presents an analytical investigation of the free vibrations of a cracked Timoshenko beam made up of functionally graded materials (FGMs). It is assumed that the beam is constructed of FGM materials with a power law variation of metal-ceramic volume fraction. The perspective of wave method is adopted for the analysis. The method considers the nature of the propagation and reflection of the waves along the beam. Consequently, the propagation, transmission and reflection matrices for various discontinuities located on the beam are derived. Such discontinuities may include crack, boundaries or change in section. By combining these matrices a global frequency matrix is formed. In order to investigate the effect of the beam's structural synthesis, different natural frequencies are obtained and studied.
9. **R. Lassoued, M. Guenfoud[8]**An accurate procedure to determine free vibrations of beams and plates is presented. The natural frequencies are exact solutions of governing

vibration equations witch load to a nonlinear homogeny system. The bilinear and linear structures considered simulate a bridge. The dynamic behavior of this one is analyzed by using the theory of the orthotropic plate simply supported on two sides and free on the two others. The plate can be excited by a convoy of constant or harmonic loads. The determination of the dynamic response of the structures considered requires knowledge of the free frequencies and the shape modes of vibrations. The formulation is based on the determination of the solution of the differential equations of vibrations. The boundary conditions corresponding to the shape modes permit to lead to a homogeneous system.

10. **Metin O Kaya[9]**There has been a growing interest in the analysis of the free vibration characteristics of elastic structures that rotate with constant angular velocity. Numerous structural configurations such as turbine, compressor and helicopter blades, spinning spacecraft and satellite booms fall into this category. A simple equation (known as the Southwell equation), which is based on the Rayleigh energy theorem to estimate the natural frequencies of rotating cantilever beams. Earlier studies mainly focused on Euler Bernoulli beams. However, due to the inclusion of shear deformation and rotary inertia effects, Timoshenko beam theory is more accurate than Euler Bernoulli beam theory. Therefore, considerable research has been carried out on the free vibrations of rotating Timoshenko beams, recently. Recently, the Dynamic Stiffness Method for a rotating cantilever Timoshenko beam that is based on Fresenius series expansion and claims its superiority of finding more correct results. On the other hand, the advantage of the DTM is its simplicity and high accuracy.

11. **M.Shahidi,M.Bayat,I.Pakar,GR Abdollahzadeh[17]**In this paper, the nonlinear governing equation of tapered beams, attempt has been made to analyze the nonlinear behavior of tapered beams analytically. The nonlinear governing equation is solved by employing the variational approach method (VAM) and Improved Amplitude-Formulation (IAFF). Despite the increasing expenses of building structures to maintain their linear behavior, nonlinearity has been inevitable and therefore, nonlinear analysis has been of great importance to the scientists in the field. The major concern is to assess excellent approximations to the exact solutions for the whole range of the oscillation amplitude,

reducing the respective error of angular frequency in comparison with the VAM and IAFF. The effect of vibration amplitude on the nonlinear frequency is discussed. It is predicted that there can be wide application of VAM and IAFF in engineering problems, as indicated in this paper.

12. **Sabah Mohammed Jamel Ali, Ziad Shakeeb Al-Sarraf[21]** A numerical solution to the frequency equation for the transverse vibration of a beam (Simply Supported with symmetric overhang) is done. It is proposed two limiting cases of abeam with no overhang, and no span. This agrees with the cases in which the supports are at the nodal Points of a freely vibrating beam. Also the numerical results compared with the analytical solutions for this study are coincident. An approximation to the solution of the frequency equation for beams with small overhang is presented and compared with the numerical solution. This approximation is quite useful to determine a beam's flexural stiffness (EI), or modulus of elasticity (E), by free vibrating of a simply supported beam.

13. **W.L. LI[22]** A simple and unified approach is presented for the vibration analysis of a generally supported beam. The flexural displacement of the beam is sought as the linear combination of a Fourier series and an auxiliary polynomial function. The polynomial function is introduced to take all the relevant discontinuities with the original displacement and its derivatives at the boundaries and the Fourier series now simply represents a residual or conditioned displacement that has at least three continuous derivatives. As a result, not only is it always possible to expand the displacement in a Fourier series for beams with any boundary conditions, but also the solution converges at a much faster speed. The reliability and robustness of the proposed technique are demonstrated through numerical examples.

14. **Gurgoze,H.Erol[5]** The frequency response function is obtained through a formula, which was established for the reacceptance matrix of discrete systems subjected to linear constraint equations. The comparison of the numerical results obtained with those via a boundary value problem formulation justifies the approach used here. Frequency response is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system. It is a measure of

magnitude and phase of the output as a function of frequency, in comparison to the input. In simplest terms, if a sine wave is injected into a system at a given frequency, a linear system will respond at that same frequency with a certain magnitude and a certain phase angle relative to the input. Also for a linear system, doubling the amplitude of the input will double the amplitude of the output.

15. **JinsuoNie,Xing Wei[18]** This paper is aimed at determining how material dependent damping can be specified conveniently in ANSYS in a mode superposition transient dynamic analysis. A simple cantilever beam is analyzed using various damping options in ANSYS. The mode superposition method is often used for dynamic analysis of complex structures, such as the seismic Category I structures in nuclear power plants, in place of the less efficient full method, which uses the full system matrices for calculation of the transient responses. In such applications, specification of material-dependent damping is usually desirable because complex structures can consist of multiple types of materials that may have different energy dissipation capabilities. A recent review of the ANSYS manual for several releases found that the use of material-dependent damping is not clearly explained for performing a mode superposition transient dynamic analysis. This paper includes several mode superposition transient dynamic analyses using different ways to specify damping in ANSYS, in order to determine how material-dependent damping can be specified conveniently in a mode superposition transient dynamic analysis.

16. **Shibabrat Naik, Wrik Mallik[20]**Studied of substantial importance in complaint structures, now days ,are the dynamic parameters such as the modal frequencies and damping constant of their components. These parameters are the essential technical information required in engineering analysis and design. In addition this information is needed for numerical simulations and finite element modeling to predict the response of structures to a variety of dynamic loadings. In this work, experimental modal testing of a cantilever beam has been performed to obtain the mode shapes, modal frequencies and the damping parameters. A fast fourier transform analyzer, PULSE lab shop was used to obtain the frequency response functions and subsequent extraction of modal data was

performed using ME's scope. These modal parameters were then checked using finite element analysis software, ANSYS which were found to comply with the experimental results. The range of applications for modal data is vast and includes checking modal frequencies, forming qualitative descriptions of the mode shapes as an aid to understanding dynamic structural behaviour for trouble shooting, verifying and improving analytical models. It is with this objective that the experimental method was standardized and thus the mathematical model can be updated further.

The experimental methods include obtaining the FRF plots from a cantilever beam and then using the ME's scope to obtain the various parts of the FRF plots like the magnitude, phase, real, imaginary. Then the modal data was analyzed to obtain different parameters which were further compared with the model developed in ANSYS.

17. **D.Ravi Prasad[11]**Modal analysis is a process of describing a structure in terms of its natural characteristics which are the frequency, damping and mode shapes –its dynamic properties. The change of modal characteristics directly provides an indication of structural condition based on changes in frequencies and mode shapes of vibration. This paper presents results of an experimental modal analysis of beams with different materials such as steel, brass, copper and aluminum. The beams were excited using an impact hammer excitation technique over the frequency range of interest, 0-2000 Hz. Response functions were obtained using vibration analyzer. The FRFs were processed using NV solutions modal analysis package to identify natural frequencies, damping and the corresponding mode shapes of the beam.

CHAPTER 3

GENERAL PROCEDURE & EXPERIMENTAL SETUP

3.1 INSTRUMENT SETUP

The experiment test rig was build in the vibration lab at Baba Banda Singh Bahadur Engg. College, Fatehgarh Sahib under the supervision of Assistant Prof. Sanjiv Kumar. The instruments used in the testing are shown in Fig. 3.1 and Fig. 3.2. The experimental vibscanner system consists of three main components; (i) portable data collectors (ii) accelerometer (iii) data acquisition system. The vibscanner is used for the diagnosis and recording of conditions of test specimens. The accelerometer is used to convert the mechanical motion of the structure into an electrical signal. The data acquisition system is used to convert the analog signals into digital format. Software called OMNITREND is then used to execute signal processing and analysis.

3.2 VIBSCANNER DETAIL

VIBSCANNER is a measuring instrument for the diagnosis and recording of machine conditions. The transducers required for this are already integrated so that the time consuming task of changing to different transducers and cables is no longer necessary.

VIBSCANNER is also a data collector. You can use it to transfer the stored measurement results to your PC. The OMNITREND software processes the data and checks whether any limits have been exceeded. For the visualization, the data are displayed graphically in a diagram (trend, spectrum, time signal). A comprehensive report function supports the documentation and archiving of the machine data acquired.

Together with the VIBCODE transducer, the VIBSCANNER recognizes the measurement location number absolutely safely and reliably and performs the necessary measurement tasks virtually independently.

Fig 3.1 VIB SCANNER

Fig. 3.2 Accelerometer [35]

Key features

- Built-in transducers for

  - Vibration velocity / acceleration / displacement

  - Bearing condition (shock pulse)

  - RPM, temperature, cavitations

- Almost any type of transducer can be connected (current, voltage, displacement, ICP)
- Measurement / Input of process parameters
  (e.g. continuous flow rate)
- Evaluation of the measurement results relative to warning/alarm thresholds (ISO)
- Shockproof, waterproof enclosure (IP 65)
- Intrinsic safety is available as an option.

Channels	1x analog: - Vibration sensors (Current Line Drive - CLD, ICP) - Instrument outputs (AC & DC) 1x digital: - Trigger (TTL) Built-in sensors: - Vibration, Shock pulse & Pump cavitation - Temperature - RPM
Meas. Range	Displacement: 9,000 µm (p-p) Velocity: 9,000 mm/s (p-p) Acceleration: 6,000 m/s^2 (p-p) RPM: 60 .. 60,000 rpm Temp.: -50 .. +100°C Low signal voltage: ± 30V Low signal current: ± 20mA

Dynamic range	60 dB
F max	10 kHz
Max. FFT lines	6,400
Memory	512 MB
Prot. Class	IP 65
Display	LCD, pixel, backlit
Supply	Rechargeable battery

Table 3.1 Technical Specifications [35]

3.3 DESCRIPTION OF SPECIMENS (CANTILEVER BEAMS)

ALUMINIUM	
Flexural Member	Beam
Material	Aluminium
Length	690,500 mm
Width	25.4 mm
Depth	6,3 mm
Boundary Condition	Cantilever
Mass Density	2700 kg/m^3
Modulus of Elasticity	69-70 GPa

BRASS	
Flexural Member	Beam
Material	Brass
Length	690,500 mm
Width	25.4 mm
Depth	6,3 mm
Boundary Condition	Cantilever
Mass Density	7037 kg/m^3
Modulus of Elasticity	103.4GPa

MILD STEEL	
Flexural Member	Beam
Material	Mild Steel
Length	690,500 mm
Width	25.4 mm
Depth	10,6 mm
Boundary Condition	Cantilever
Mass Density	7850 kg/m^3
Modulus of Elasticity	200 GPa

Table 3.2 Geometric and material properties for the test specimens

3.4 EXPERIMENTAL PROCEDURE & TEST METHODOLOGY

Free vibration is conducted on the test specimens to obtain its dynamic characteristics including natural frequencies and damping ratios. The beam is clamped on the table with the help of clamping device arrangement. The impact is applied by striking the free end of the test specimen (horizontally mounted ,slender, uniform cross section, M.S., Brass, Aluminium(6063), cantilever beam) on the table with one end fixed has dimensions (690 mm x 25.4 mm x 6 mm),(690 mm x 25.4 mm x 3 mm),(500 mm x 25.4 mm x 6 mm),(500 mm x 25.4 mm x 3 mm) as shown in Figure(3.3) by using a mallet.

Fig 3.3 Experimental Set Up

During free vibrations, the dynamic responses of the beam are measured through the accelerometer as shown in fig. 3.2. For this test, the location of accelerometer at free end is carried out in order to extract the signals of vibration. The layout of the sensors on the test specimen is depicted in Fig. 3.3. The vertically mounted accelerometer at free end is used primarily for measuring the response in terms of acceleration. A data acquisition system i.e. vibscanner is used to store the record data and transfer measured data to the pc for post processing. Frequency response functions (FRFs) were obtained using vibscanner software and were processed using OMNITREND Solutions analysis to identify natural frequencies, damping and the corresponding mode shapes of the beams.

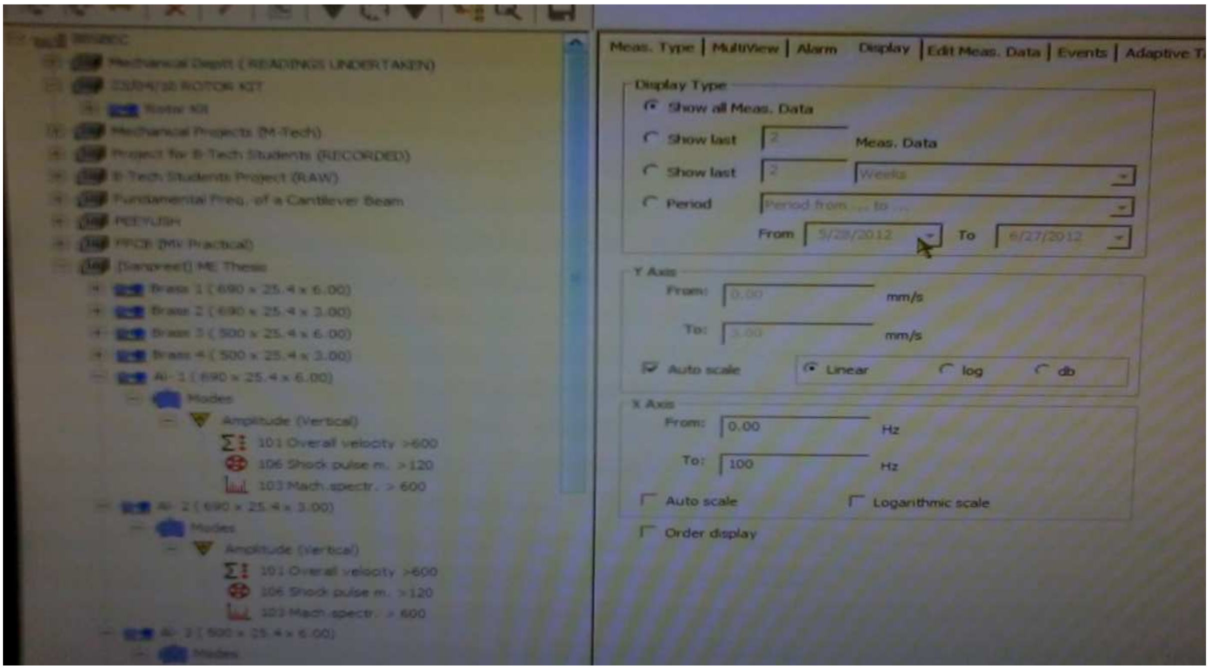

Fig. 3.4 Omnitrend Software used for generating spectrum of signals catched during vibration

3.4.1 Measurement procedure

1. A beam of a particular material (mild steel, brass, aluminium), dimensions(L , w , d) was used as a cantilever beam
2. The fixed end was made by fixing the beam with the help of clamp (bolt is attached)

fixed on the table.

3. The connections of the vibscanner, accelerometer were properly made.
4. Accelerometer was placed at the free end of the cantilever beam, to measure the vibration response.
5. The free end of a cantilever beam was struck with a wooden mallet and beam starts vibrating.
6. All the data was recorded obtained from the vibrating beam with the help of vibscanner as accelerometer is attached to it.
7. The experiments were repeated to check the repeatability of the experimentation (i.e. vibration data).
8. Repeat the whole experiment for different material by changing the parameters i.e. length & thickness.
9. The whole set of data was recorded and then the data was imported into the PC , further processing and analysis was done using OMNITREND software.

CHAPTER 4

EXPERIMENTAL INVESTIGATION

The experimental set up was build in the vibration lab at B.B.S.B. Engineering College. Free vibration is conducted on the test specimens to obtain its dynamic characteristics i.e. natural frequencies and damping ratios. During free vibrations, the dynamic responses of the beam are measured through the accelerometer. The accelerometer is placed at the free end to extract the signals of vibrations and import the data into PC. FRF (Frequency response function) i.e spectrum or graphs were obtained.

Data for the free vibration characteristics has been taken for first mode i.e. fundamental mode at the free end of the cantilever setup. The damping ratio was also calculated with the help of half power bandwidth method.

4.1 NOTATIONS USED FOR SPECIMENS

The notations used for specimens of different dimensions are following:-

SPECIMEN	DIMENSION	NOTATION
1) MILD STEEL	690mm X 25.4mm X 6mm	MS1
	690mm X 25.4mm X 3mm	MS2
	500mm X 25.4mm X 6mm	MS3
	500mm X 25.4mm X 3mm	MS4
2) ALUMINIUM	690mm X 25.4mm X 6mm	AL1
	690mm X 25.4mm X 3mm	AL2
	500mm X 25.4mm X 6mm	AL3
	500mm X 25.4mm X 3mm	AL4
3) BRASS	690mm X 25.4mm X 6mm	BR1
	690mm X 25.4mm X 3mm	BR2

	500mm X 25.4mm X 6mm	BR3
	500mm X 25.4mm X 3mm	BR4

Table 5.2 Notations used for different specimens

The graphs have been plotted between excitation frequency and amplitude in terms of displacement are as given below:-

4.2 VIBRATION CHARACTERSTICS OF BRASS BEAMS BY VARYING LENGTH AND THICKNESS

The Vibration characteristics (natural frequency & damping ratio) of brass beam of length (690mm, 500mm) and thickness (6mm, 3mm) and constant width (25.4mm) have been determined by spectrum graphs between amplitude in terms of displacement and excitation frequency. Damping ratio is calculated with the help of half power bandwidth method.

The FRFs obtained using omnitrend software for brass beam of same length i.e. 690 mm and different thickness i.e. 6 mm & 3 mm using vibscanner has been shown in the figures 4.1 to 4.4.

This shows that variation of amplitude in terms of displacement with respect to excitation frequency for brass beam in vertical direction by analyzing the displacement signals from aligned accelerometer.

The peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. resonant frequency.

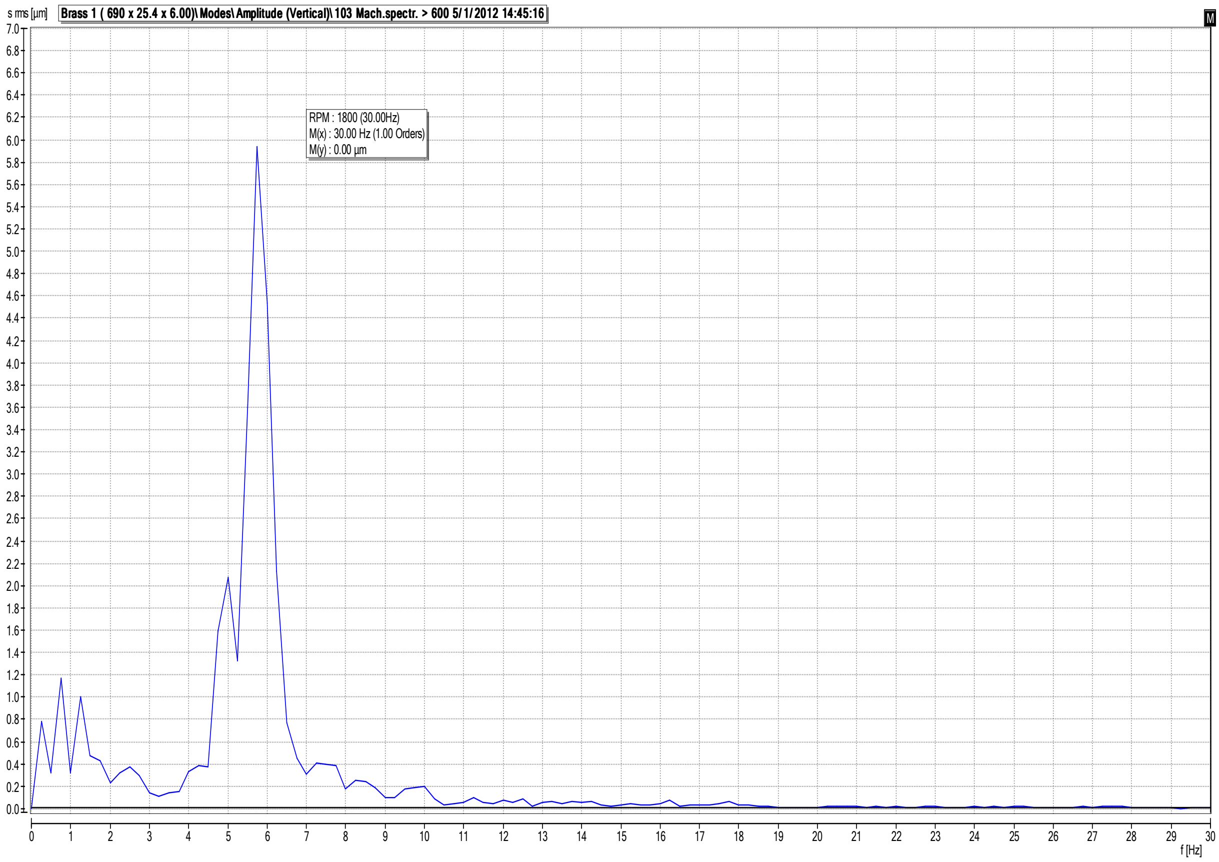

Fig.4.1 Vibration Characteristics of BR1 beam

From fig 4.1 the natural frequency corresponds to the peak response can be seen to be 5.9 Hz. The half power points where the response amplitude is equal to .707 times the peak response can be identified as ω_1 = 6 Hz and ω_2 = 5.5 Hz.

From analysis point of view resonant frequency response amplitude is very important. Since, the output can be viewed as frequency response function .Hence output response can be considered for analysis of damping for first mode. Hence the damping of specimen made up of brass corresponding to natural frequency 5.9 Hz is 0.042.

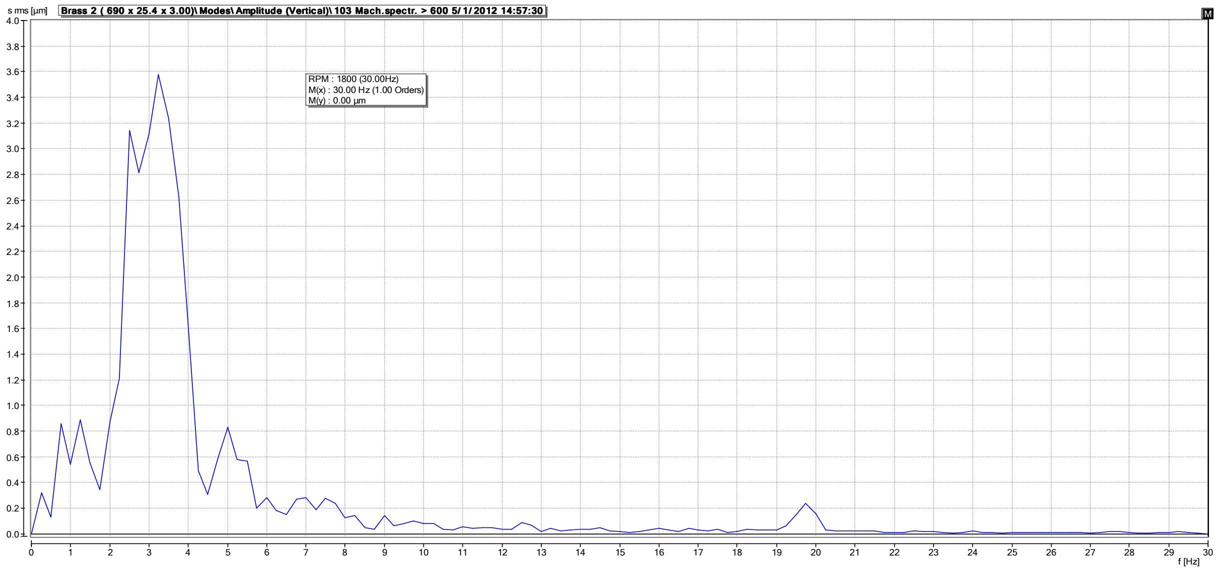

Fig.4.2 Vibration Characteristics of BR2 beam

The half power points where the response amplitude is equal to .707 times the peak response can be identified as ω_1 = 2.4 Hz and ω_2 = 3.8 Hz as shown in fig 4.2

The natural frequency corresponds to the peak response can be seen to be 3.3 Hz. Hence the damping of specimen made up of brass corresponding to natural frequency 3.3 Hz is 0.111.

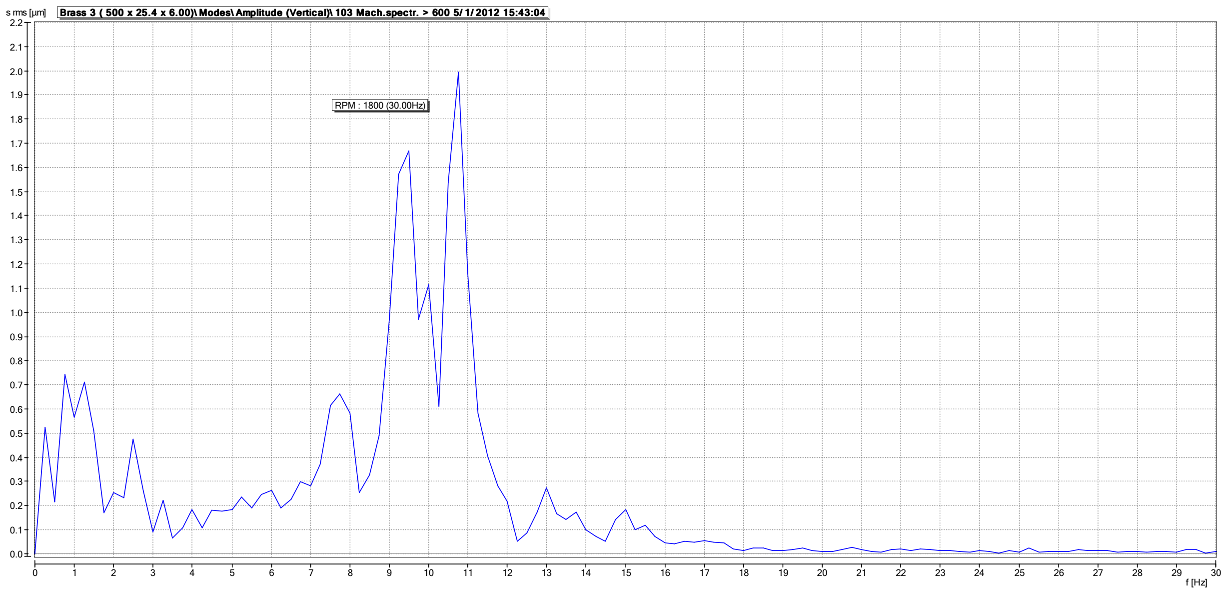

Fig.4.3 Vibration Characteristics of BR3 beam

In fig 4.3 the natural frequency corresponds to the peak response can be seen to be 10.8 Hz. The peak response can be identified as ω_1 = 2.4 Hz and ω_2 = 3.8 Hz. Hence the damping of specimen made up of brass is 0.018.

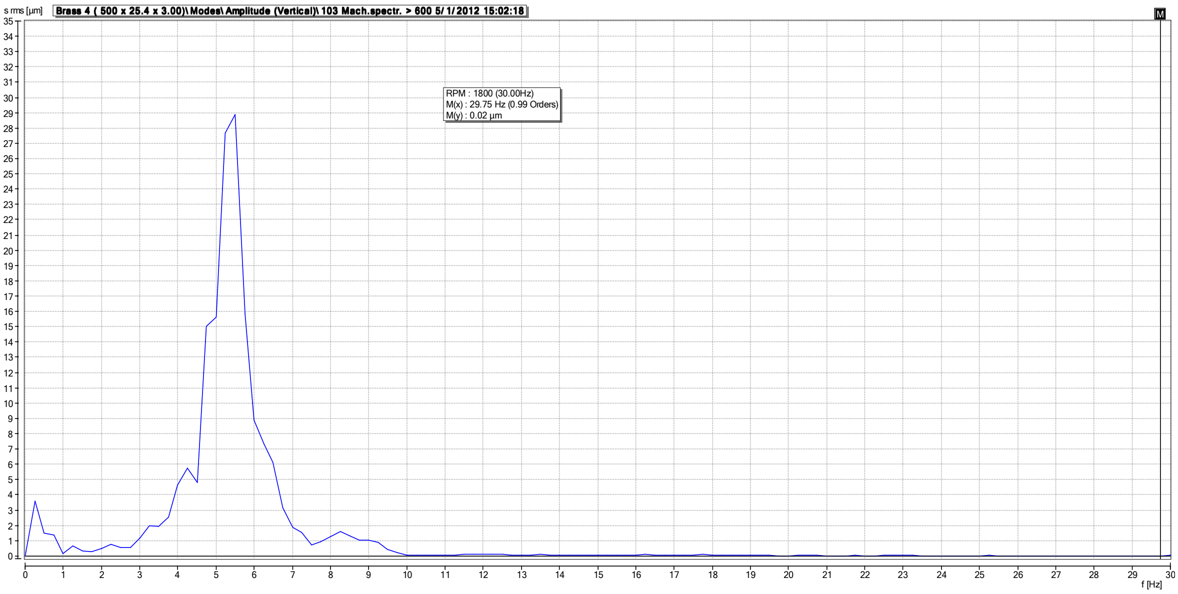

Fig.4.4 Vibration Characteristics of BR4 beam

The peak response can be identified as ω_1 = 5.2 Hz and ω_2 = 5.8 Hz as shown in fig 4.4. The natural frequency corresponds to the peak response can be seen to be 5.5 Hz. Hence the damping of specimen made up of brass corresponding to natural frequency 5.5 Hz is 0.054.

4.3 VIBRATION CHARACTERSTICS OF ALUMINIUM BEAMS BY VARYING LENGTH AND THICKNESS

The Vibration characteristics (natural frequency & damping ratio) of aluminium beam of length (690mm, 500mm) and thickness (6mm, 3mm) and constant width(25.4mm) have been determined by spectrum graphs between amplitude in terms of displacement and excitation frequency. Damping ratio is calculated with the help of half power bandwidth method.

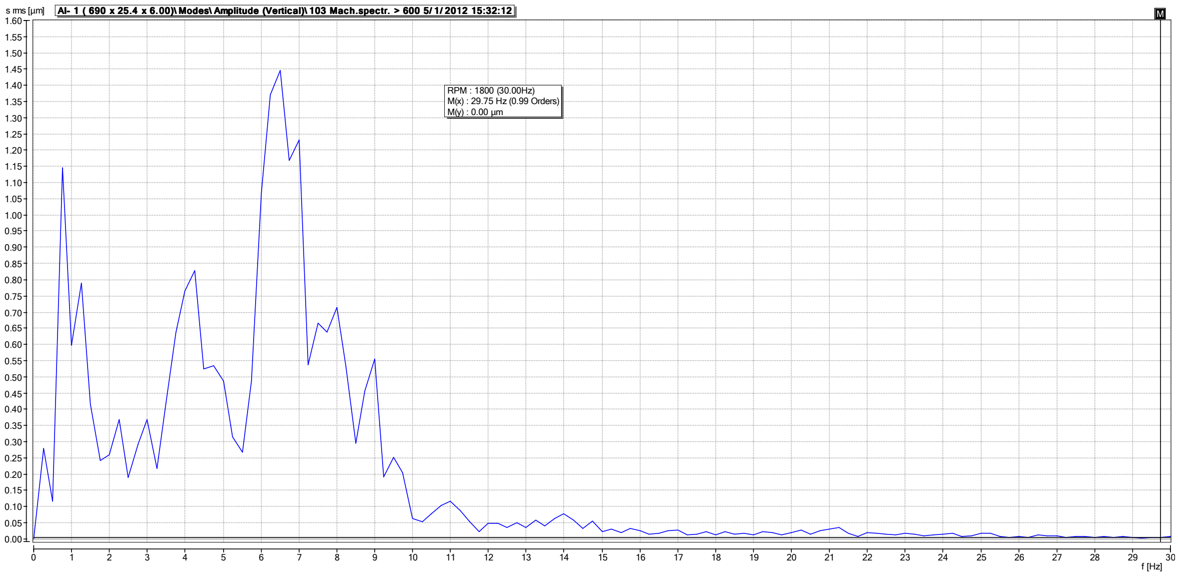

Fig.4.5 Vibration Characteristics of AL1 beam

The peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 6.5 Hz. The half power points where the response amplitude is equal to .707 times the peak response can be identified as $\omega_1 = 6.0$ Hz and $\omega_2 = 6.9$ Hz as shown in fig 4.5. Hence the damping of specimen made up of aluminium corresponding to natural frequency 6.5 Hz is 0.039.

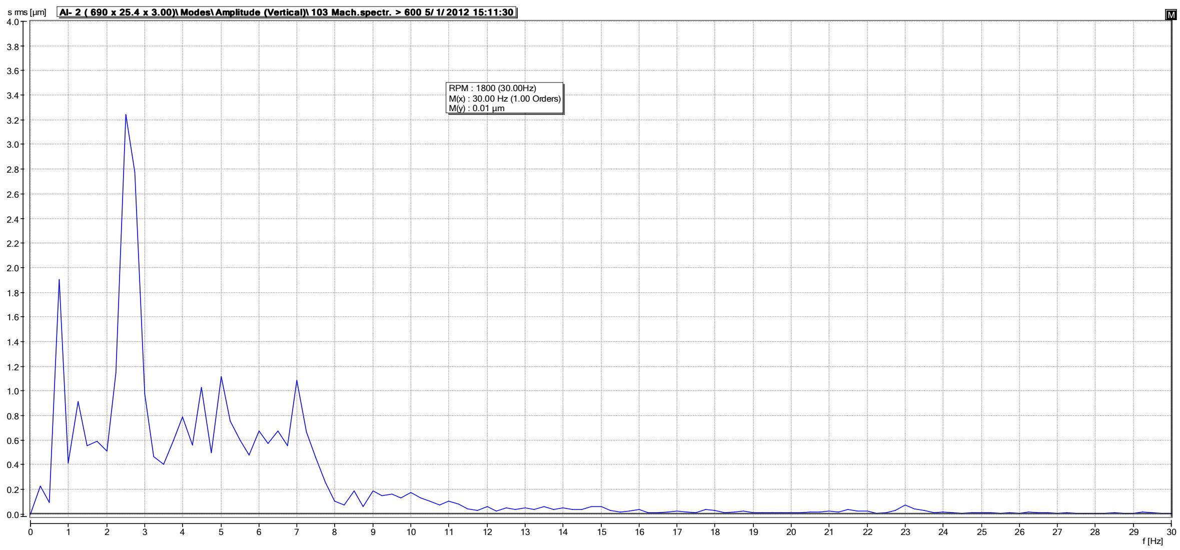

Fig.4.6 Vibration Characteristics of AL2 beam

In fig 4.6 the peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 2.5 Hz. The peak response can be identified as $\omega_1 = 2.4$ Hz and $\omega_2 = 2.9$ Hz. Hence the damping of specimen made up of aluminium is 0.08.

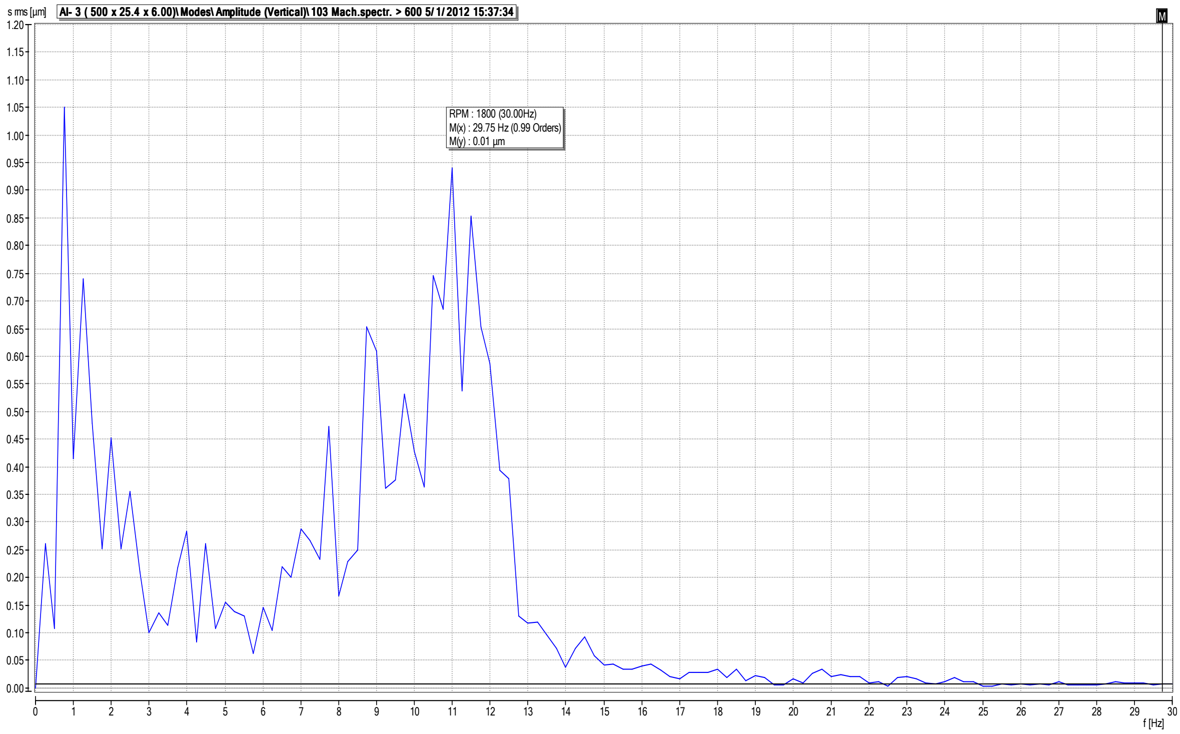

Fig.4.7 Vibration Characteristics of AL3 beam

The half power points where the response amplitude is equal to .707 times the peak response can be identified as $\omega_1 = 10.8$ Hz and $\omega_2 = 11.2$ Hz as shown in fig 4.7.

From analysis point of view resonant frequency response amplitude is very important. Since, the output can be viewed as frequency response function .Hence output response can be considered for analysis of damping for first mode. Hence the damping of specimen made up of aluminium corresponding to natural frequency 11 Hz is 0.017.

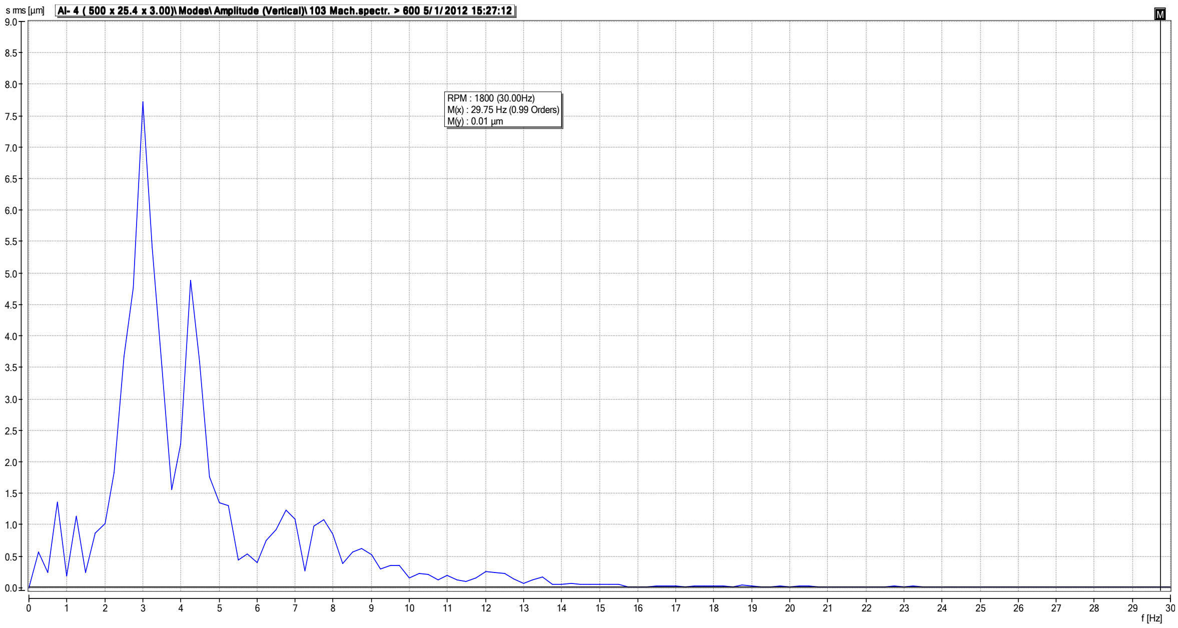

Fig.4.8 Vibration Characteristics of AL4 beam

In fig 4.8 the peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 3 Hz. From analysis point of view resonant frequency response amplitude is very important. Since, the output can be viewed as frequency response function .Hence output response can be considered for analysis of damping for first mode. The peak response can be identified as ω_1 = 2.4 Hz and ω_2 = 2.9 Hz. Hence the damping of specimen made up of aluminium corresponding to natural frequency 3 Hz is 0.033.

4.4 VIBRATION CHARACTERSTICS OF MILD STEEL BEAMS BY VARYING LENGTH AND THICKNESS

The Vibration characteristics (natural frequency & damping ratio) of mild steel beam of length (690mm, 500mm) and thickness (6mm, 3mm) and constant width(25.4mm) have been determined by spectrum graphs between amplitude in terms of displacement and excitation frequency.

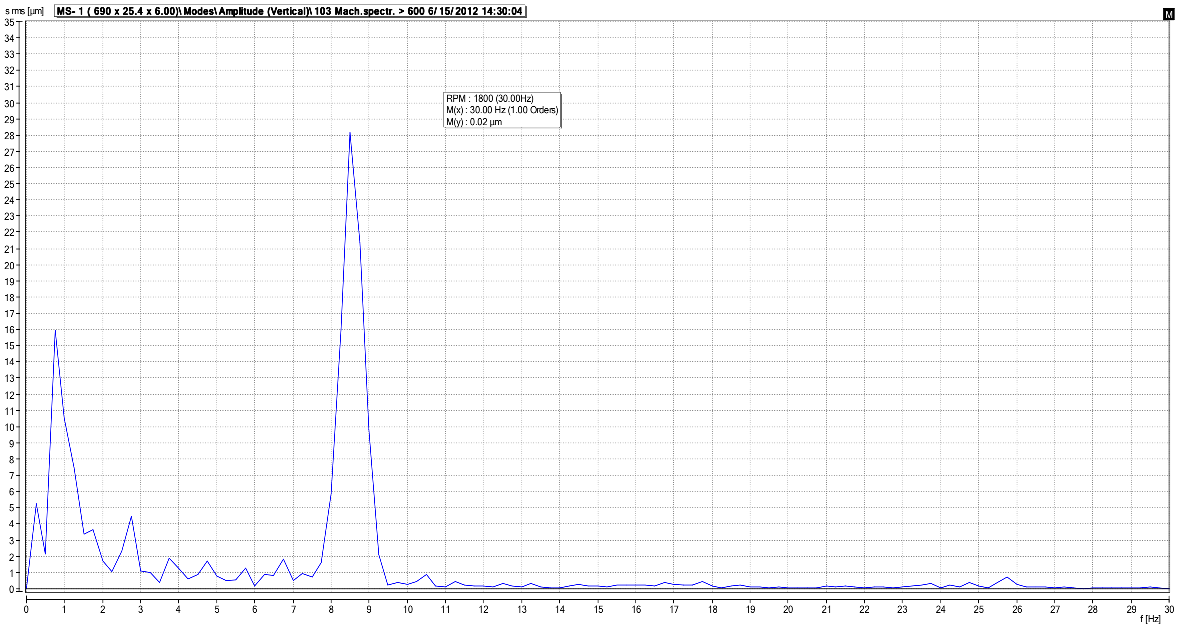

Fig.4.9 Vibration Characteristics of MS1

In fig 4.9 the peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 8.5 Hz. The peak response can be identified as ω_1 = 8.3 Hz and ω_2 = 8.9 Hz. Hence the damping of specimen made up of mild steel is 0.069

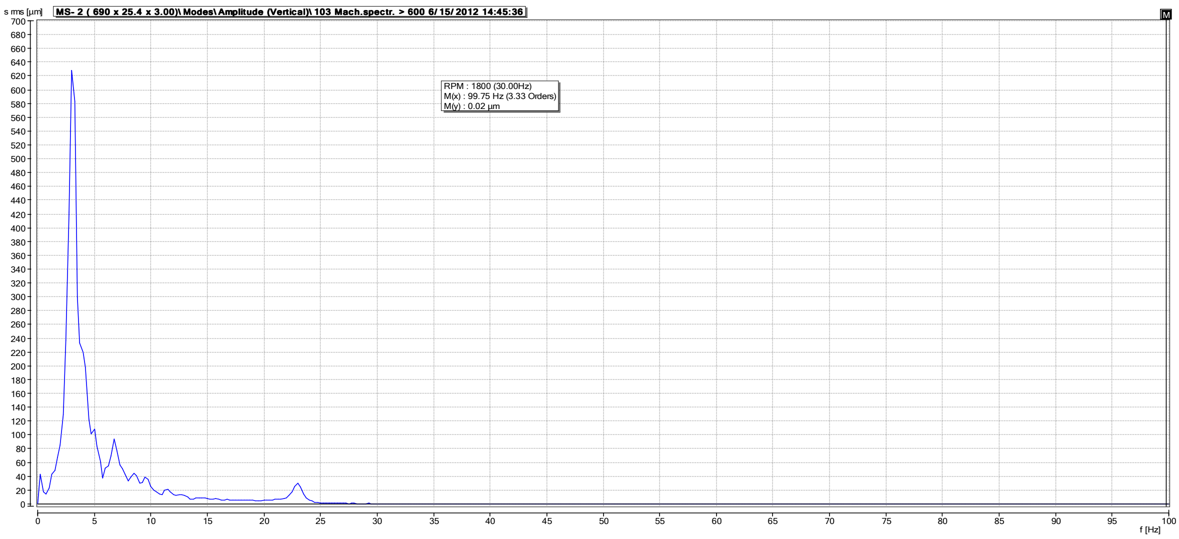

Fig.4.10 Vibration Characteristics of MS2

In fig 4.10 the peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 2.7 Hz and the output can be viewed as frequency response function .Hence output response can be considered for analysis of damping for first mode. The peak response can be identified as ω_1 = 2.5 Hz and ω_2 = 3 Hz. Hence the damping of specimen made up of mild steel corresponding to natural frequency 2.7 Hz is 0.212.

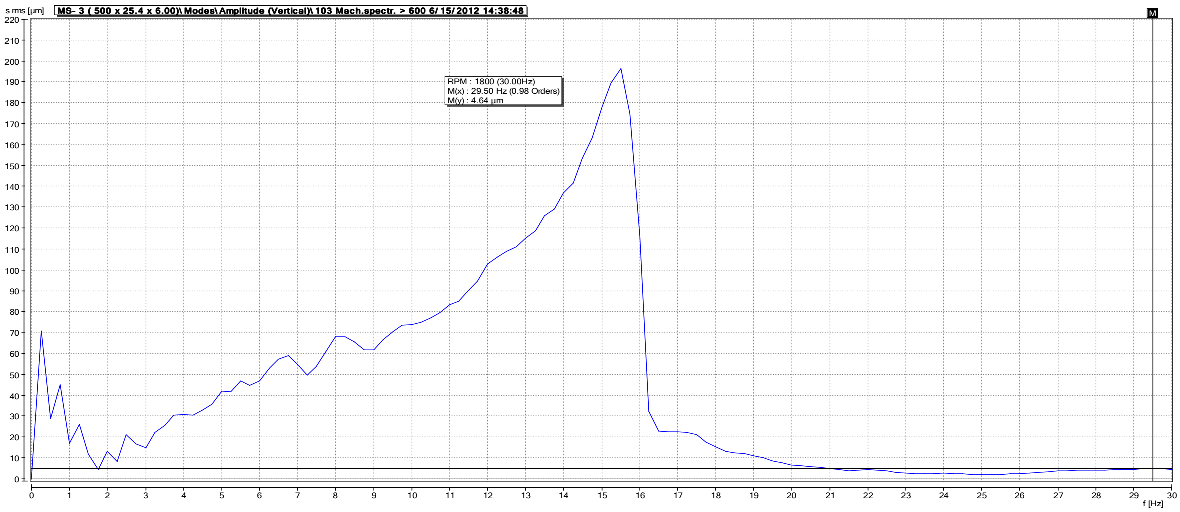

Fig.4.11 Vibration Characteristics of MS3

The peak value gives maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 15.5 Hz as shown in fig 4.11. This graph is not giving the exact readings due to experimental limitations. The peak response can be identified as ω_1 = 14 Hz and ω_2 = 16 Hz. Hence the damping of specimen made up of mild steel is 0.064.

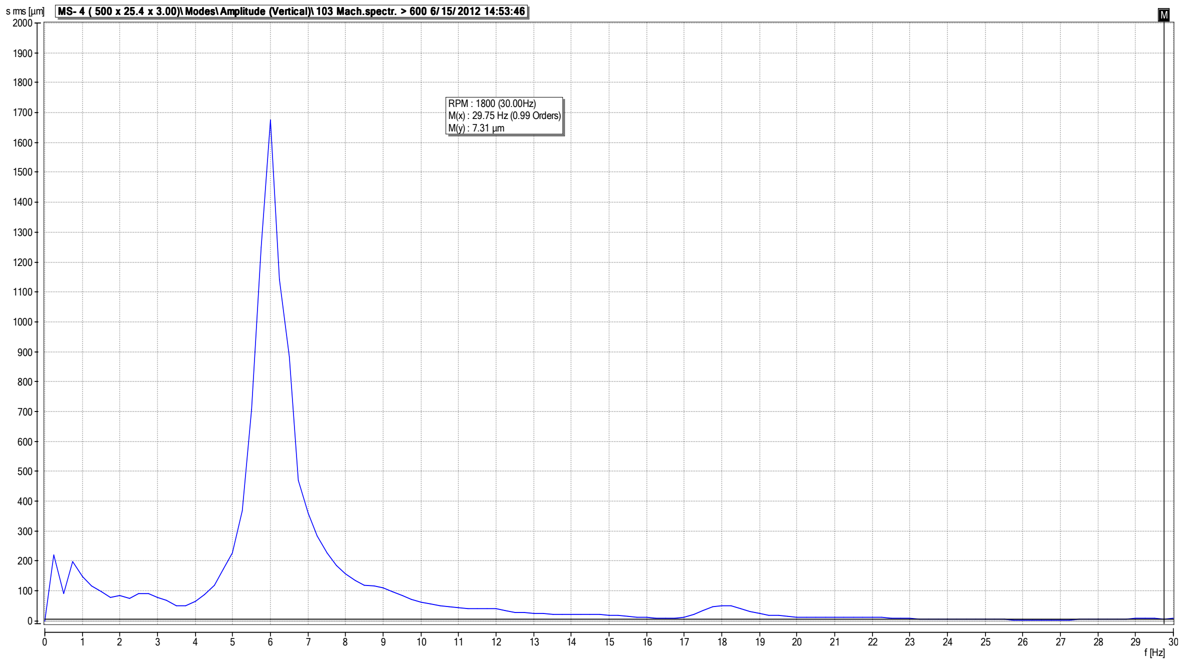

Fig.4.12 Vibration Characteristics of MS4

In fig 4.12 the peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. 6 Hz. From analysis point of view resonant frequency response amplitude is very important. Since, the output can be viewed as frequency response function. Hence output response can be considered for analysis of damping for first mode. The peak response can be identified as ω_1 = 5.8 Hz and ω_2 = 6.2 Hz. Hence the damping of specimen made up of mild steel corresponding to natural frequency 6 Hz is 0.083.

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4.5 CALCULATION OF DAMPING RATIO (ζ)

The Damping ratio is a parameter, usually denoted by ζ (zeta) provides a mathematical means of expressing the level of damping in a system relative to critical damping.

Damping of specimens made up of different materials (brass,aluminium and mildsteel) was calculated with the help of half power bandwidth method .

4.5.1 HALF POWER BANDWIDTH METHOD

The damping value is calculated as:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} \tag{4.4}$$

where ω_2 and ω_1 are the frequencies corresponding to the half power points which are defined at which the response amplitude is 0.707 times the resonant response amplitude and ω_n is the resonant frequency. The damping of specimens made up of different materials calculated by this methods are as following:-

S. NO.	Specimens	Experimental (ω_n) Hz	ζ (Half power bandwidth)
1.	MS1	8.5	0.069
2.	MS2	3.5	0.212
3.	MS3	15.5	0.064
4.	MS4	6	0.083
5.	AL1	6.5	0.039
6.	AL2	2.5	0.08
7.	AL3	11	0.017
8.	AL4	3	0.033
9.	BR1	5.9	0.042
10.	BR2	3.3	0.111
11.	BR3	10.8	0.018
12.	BR4	5.5	0.054

Table 4.2 Evaluation of damping ratio

CHAPTER 5

HARMONIC ANALYSIS IN ANSYS AND THEORETICAL CALCULATIONS

5.1 INTRODUCTION

ANSYS is engineering simulation software based on the finite element method and is capable of performing static (stress) analysis, thermal analysis, modal analysis, frequency response analysis, transient simulation and also coupled field analysis. The ANSYS multiphysics can couple various physical domains such as structural, thermal and electromagnetic.

5.2 ELEMENT TYPE

A beam 2 node 188 is used to model the specimen. Two nodes are required for this element. Each node has three degrees of freedom-translations in the nodal x and y direction and rotation about the nodal z axis.

5.3 MATERIAL PROPERTIES

SPECIMENS	**DENSITY(kg/m^3)**	**MODULUS OF ELASTICITY(Pa)**	**POISSON's RATIO**
Mild Steel	7850	$2e^{11}$	0.3
Aluminium	2700	$7e^{10}$	0.33
Brass	7037	$1.03e^{11}$	0.34

Table: 5.1 Material properties of specimens

5.4 GEOMETRY

For the load deflection curve the beam was analysed as deformation controlled analysis. Value of displacement in y direction at support nodes were used to create the support. Fig.5.1 shows the boundary condition for a typical model.

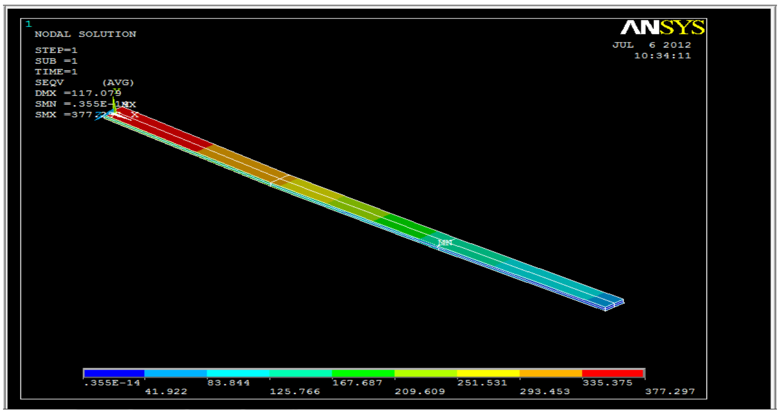

Fig.5.1 Analysis of a beam

5.5 HARMONIC ANALYSIS

The harmonic analysis is performed in ANSYS to find the natural frequency of first mode and to plot the graph between frequency and displacement. There are three steps involved in full harmonic response analysis of a structure. These steps are:

- Building the model
- Applying loads and constraints
- Reviewing the results

After building model the following steps are involved in harmonic analysis:-

1. APPLYING LOADS AND CONSTRAINTS

- Open up the Solution Menu, and select New Analysis...-> Harmonic .Click on Analysis Options... The following window will open showing the methods available for harmonic analysis. Select Full.

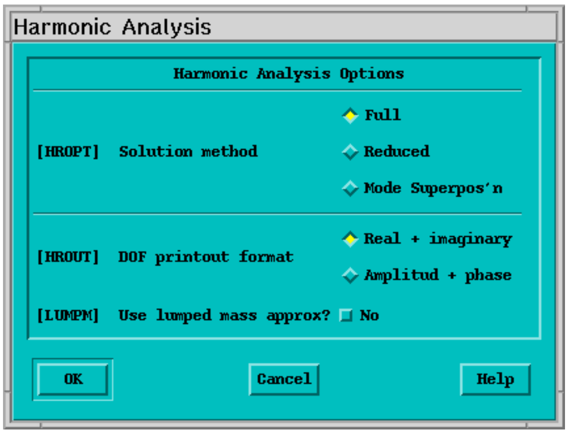

Fig.5.2 Harmonic analysis options

- Applying constraints: Click Apply -> Displacement -> On Nodes and click on the node at x=0. Constrain all the degrees of freedom at this node.

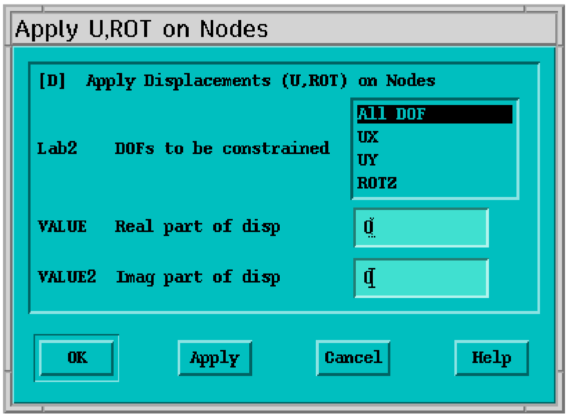

Fig.5.3 Applying U, ROT on nodes

- Applying Loads: From the Apply menu, click on Force/Moment -> On Nodes and click the node at x=1. Apply a load in the Y direction with a real value of 100 and an imaginary value of 0.
- For harmonic analysis we can observe the structural response to a load over a range of frequencies. To do this, select Time/Frequency below the -Load Step Opts- (Solution Menu). Select Freq & Substeps... and specify a frequency range of 0 - 100Hz with 100 substeps.

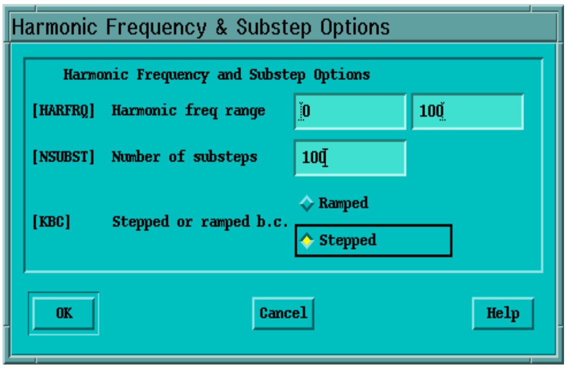

Fig. 5.4 Harmonic Frequency and Substep Options

By doing this the beam is subjecting to loads at 1 Hz, 2 Hz, 3 Hz, ... 100 Hz. A stepped boundary condition (SBC) is specified as this will ensure that the same amplitude (100 N) will be applied for each of the frequencies.

The ramped option, on the other hand, would ramp up the amplitude where at 1 Hz the amplitude would be 1 N and at 100 Hz the amplitude would be 100 N.

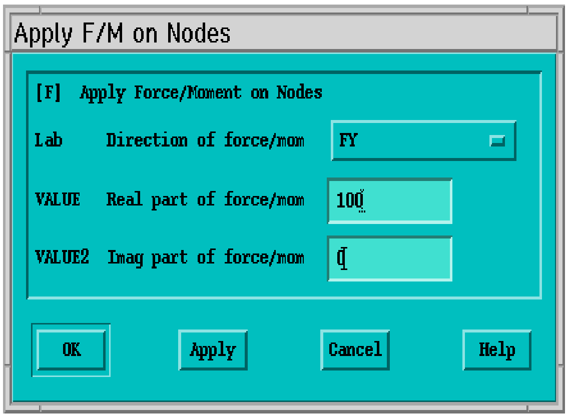

Fig.5.5 Applying F/M on nodes

2. REVIEWING THE RESULTS

- Use Time History Post Processing (POST26). POST26 is mainly used to observe certain variables as a function of either time or frequency.

- Enter POST26 by selecting TimeHistory Postprocessing from the ANSYS Main Menu.

By default, Variable 1 is assigned either Time or Frequency. In our case it will be assigned frequency . The displacement UY at the node at x=1, which is node #2. (To get a list of nodes and their attributes, select List -> nodes from the Utility Menu).

- Select Define Variables... and the following window should pop up.

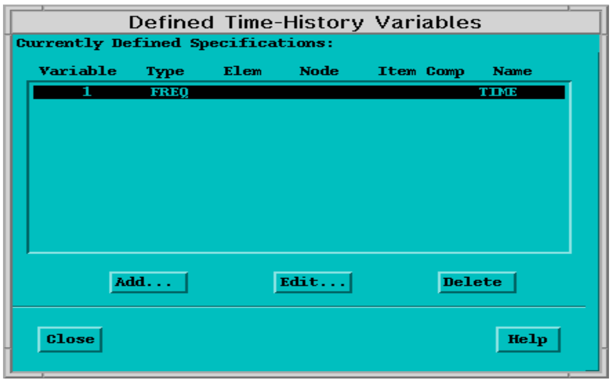

Fig.5.6 Time-history variables

- Select Add.. from this window and the following window should appear

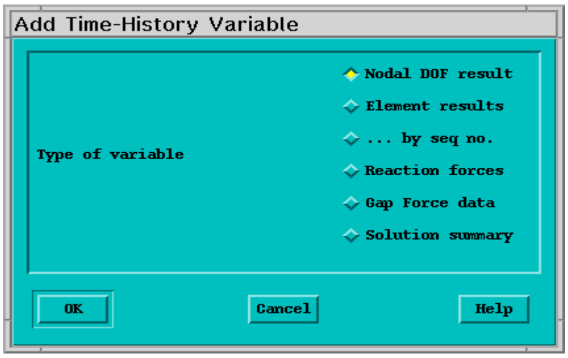

Fig.5.7 Time history variables

- The following dialog box will open up. Enter values as shown below (note UY has been selected).

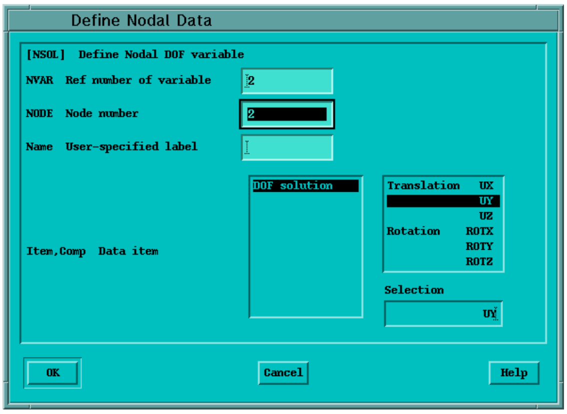

Fig. 5.8 Nodal data

- Note that by default ANSYS names variable number 1 as Time, whether the variable is time or frequency.

- Now plot UY vs. frequency. To do this select Graph Variables.. and select variable 2 as the first variable to plot.

A harmonic analysis yields solutions of time-dependent equations of motion associated with linear structures undergoing steady-state vibration. To this end , all displacements are assumed to vary with different frequencies. The analysis is performed on beams of different materials (mild steel, aluminium, brass) by varying their length and thickness. The following graphs were obtained from harmonic analysis as shown in fig 5.9-5.20

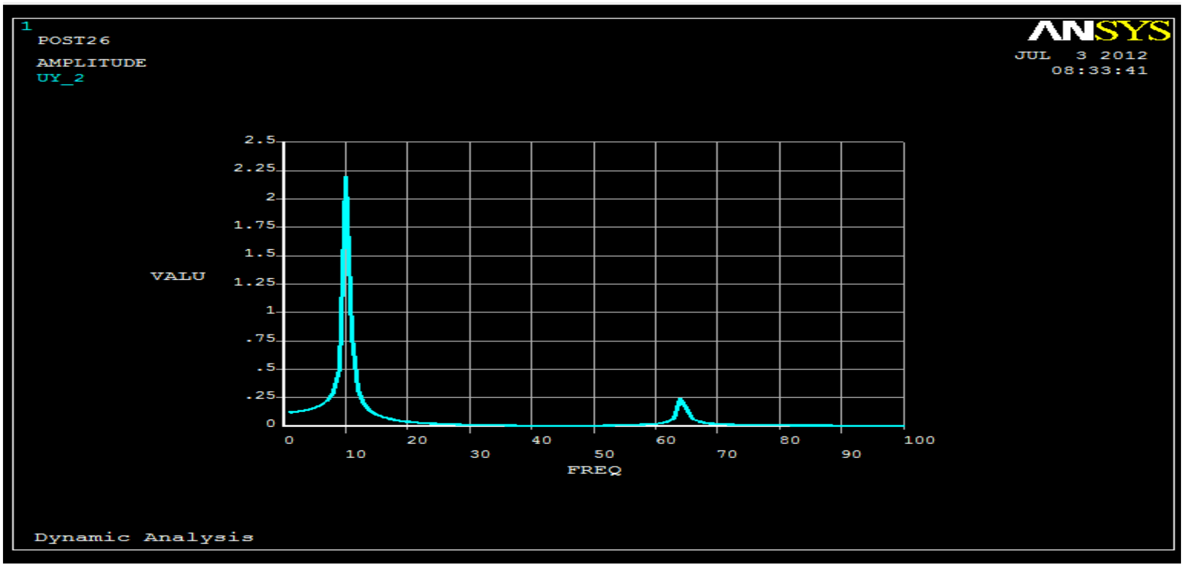

Fig.5.9 FRF graph of MS1

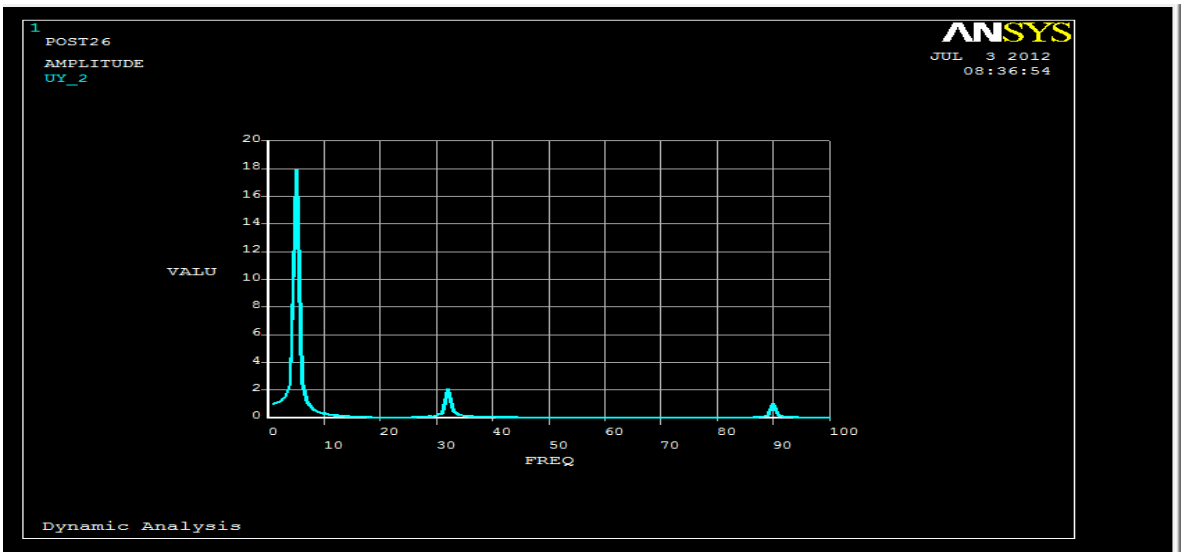

Fig.5.10 FRF graph of MS2

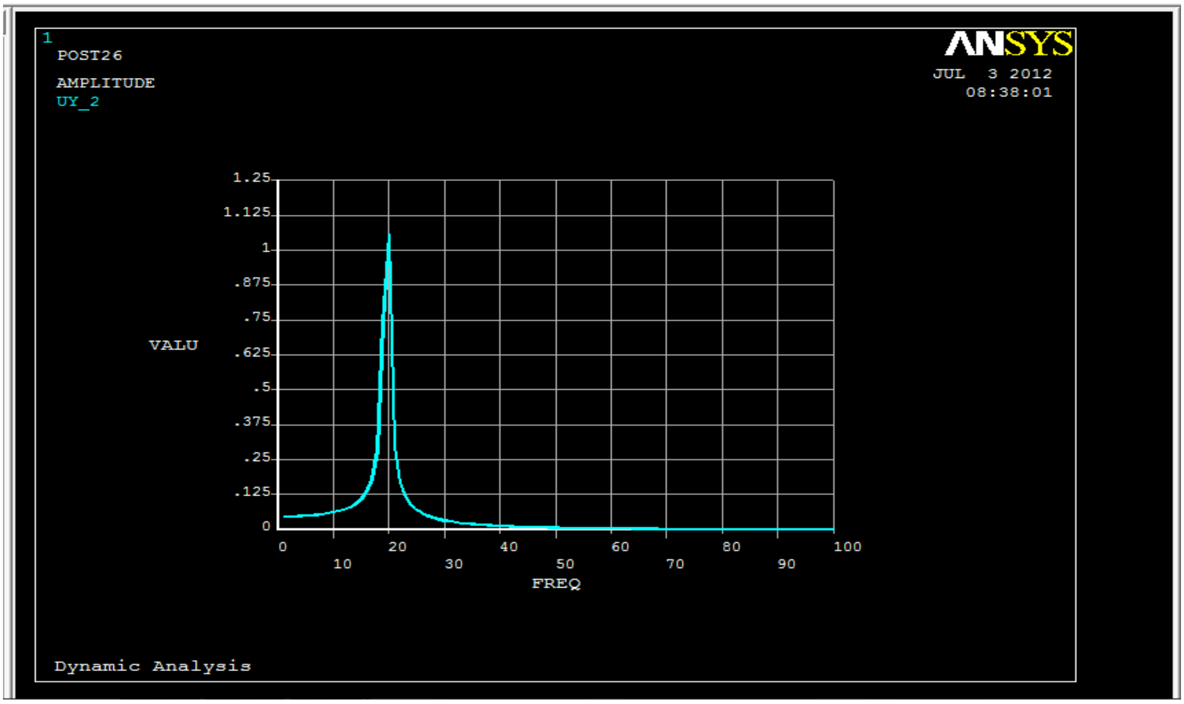

Fig.5.11 FRF graph of MS3

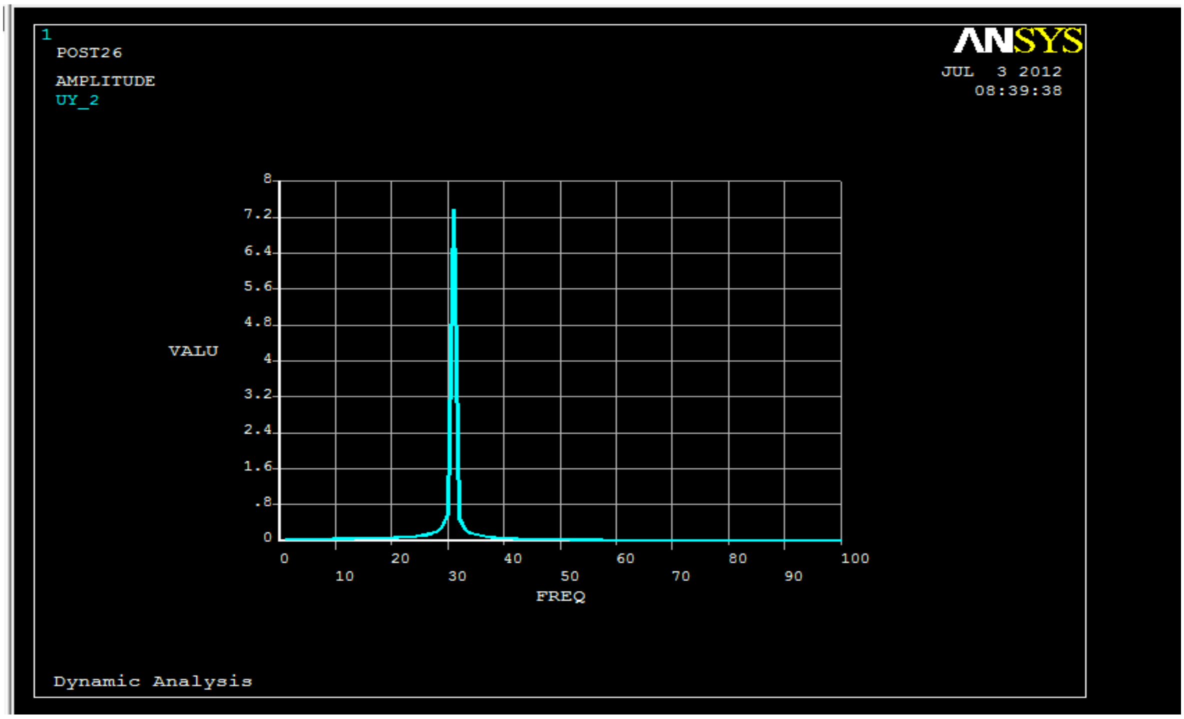

Fig.5.12 FRF graph of MS4

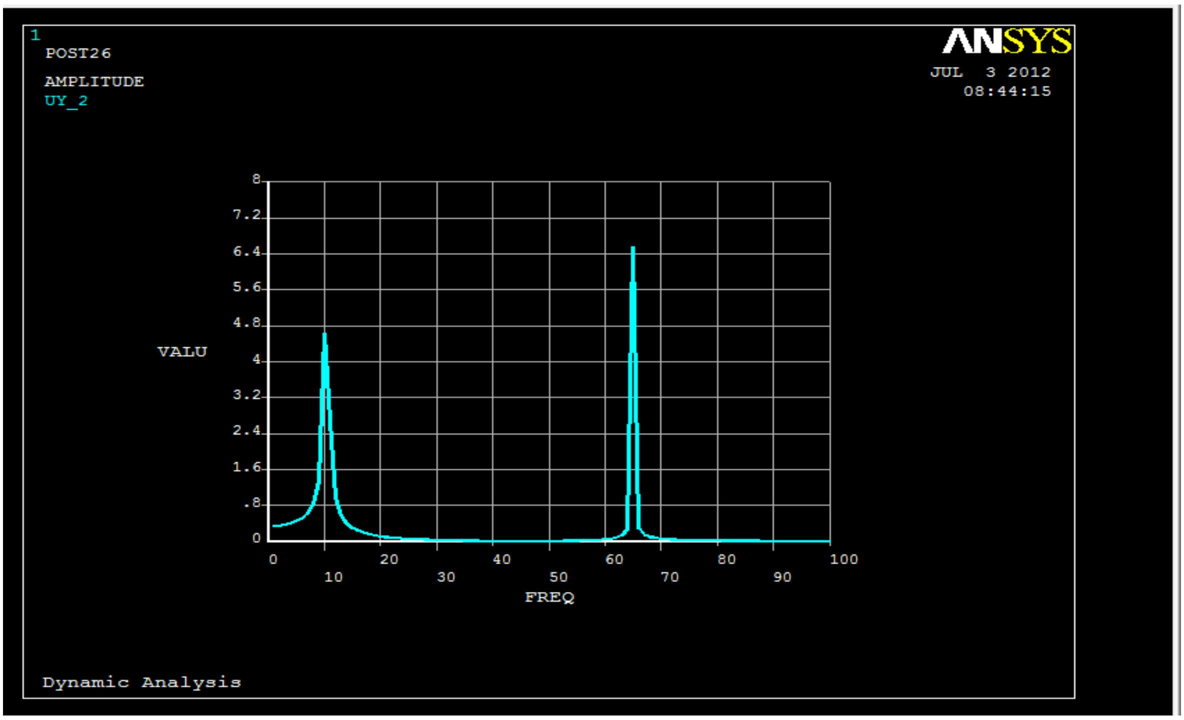

Fig.5.13 FRF graph of AL1

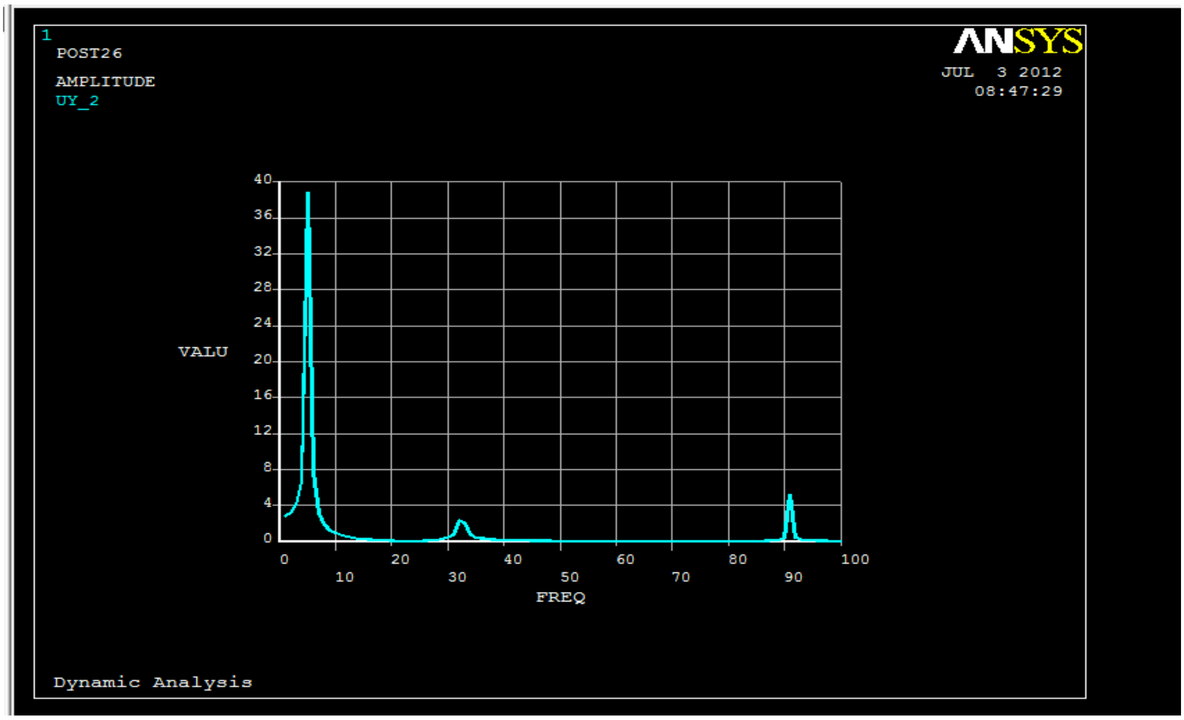

Fig.5.14 FRF graph of AL2

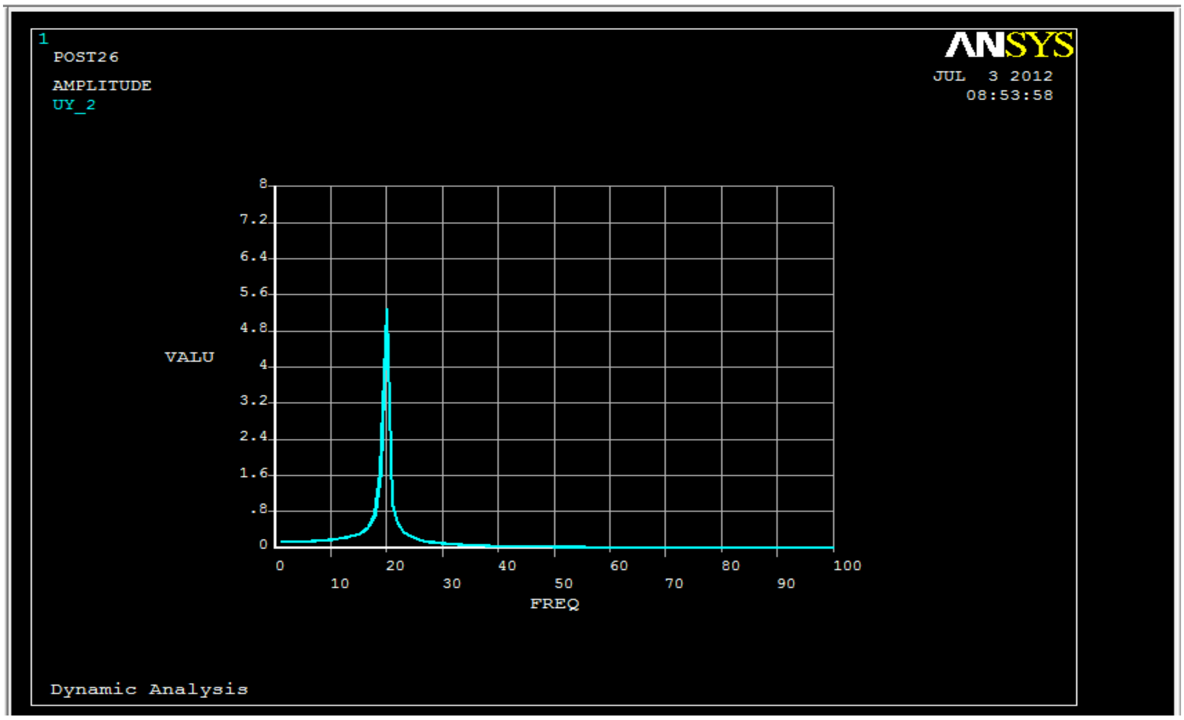

Fig.5.15 FRF graph of AL3

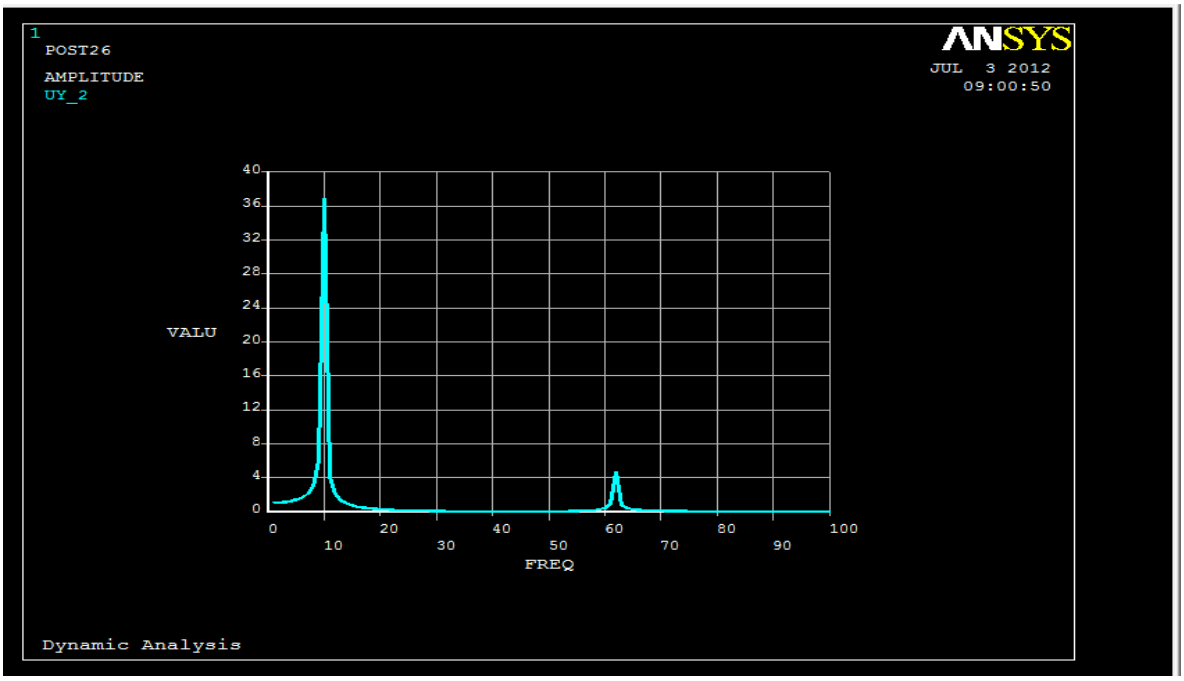

Fig.5.16 FRF graph of AL4

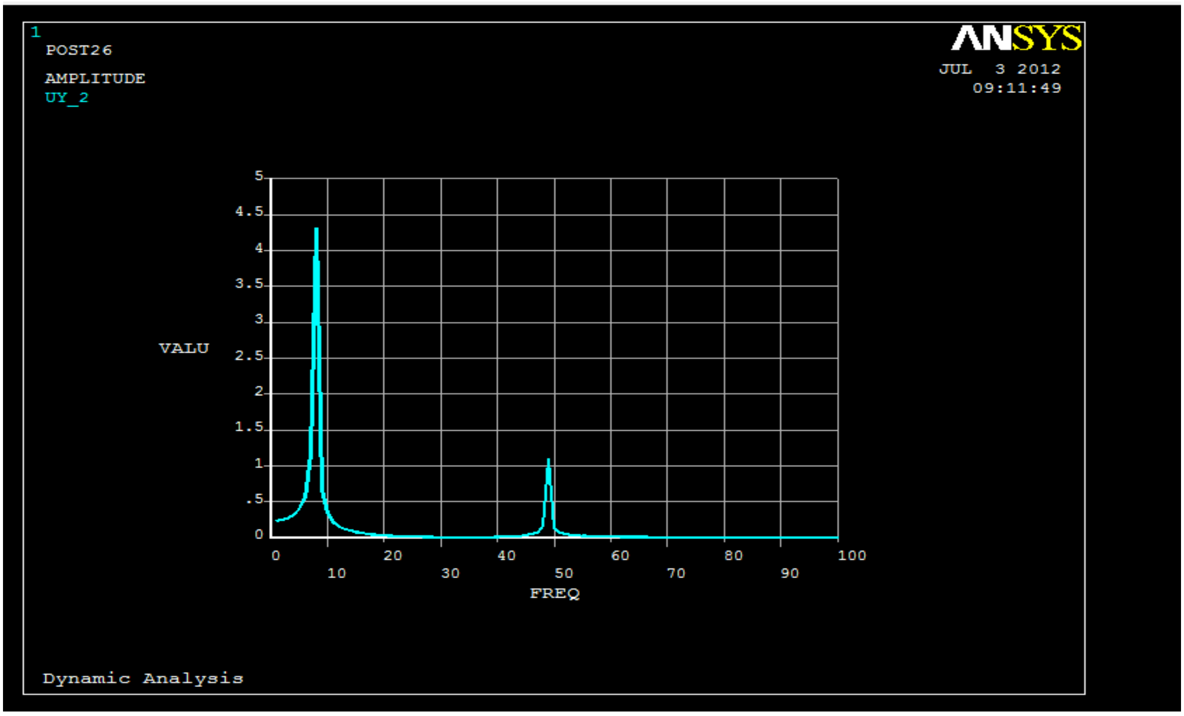

Fig.5.17 FRF graph of BR1

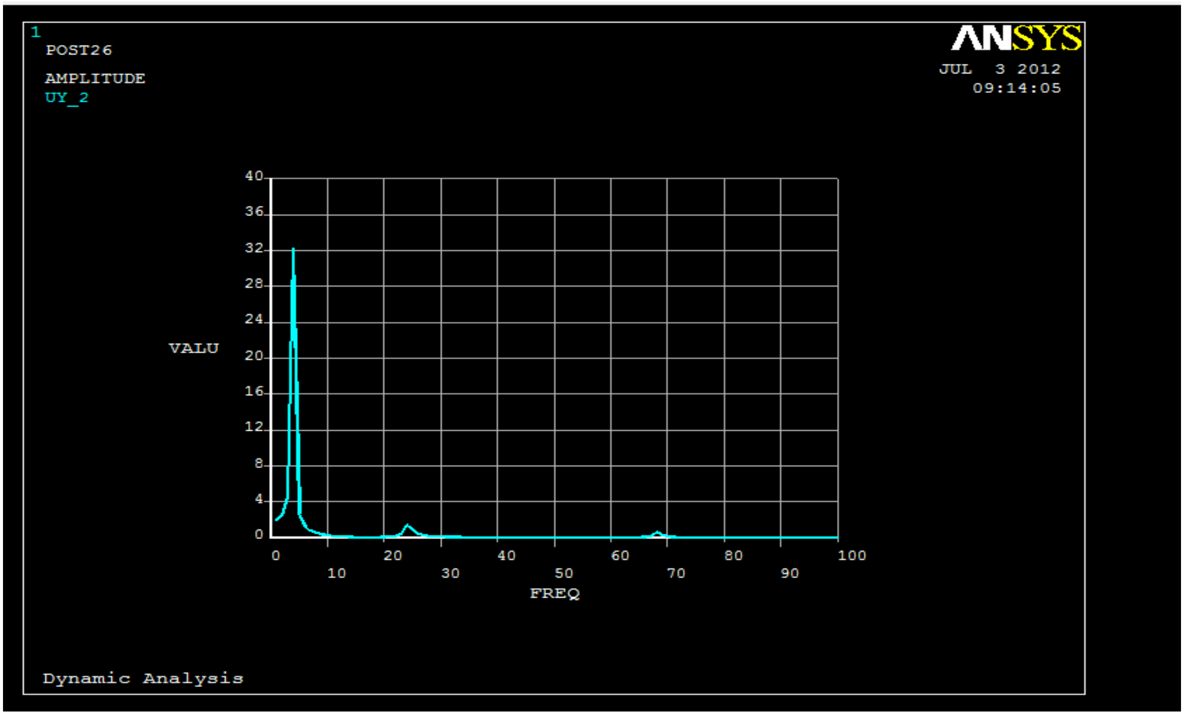

Fig.5.18 FRF graph of BR2

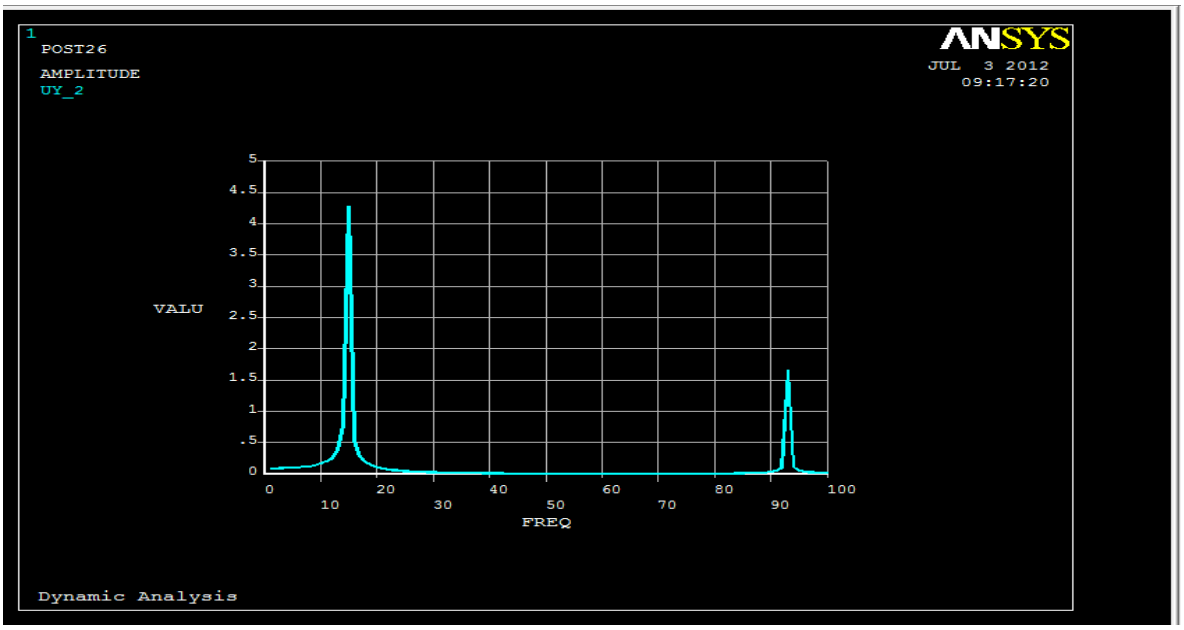

Fig.5.19 FRF graph of BR3

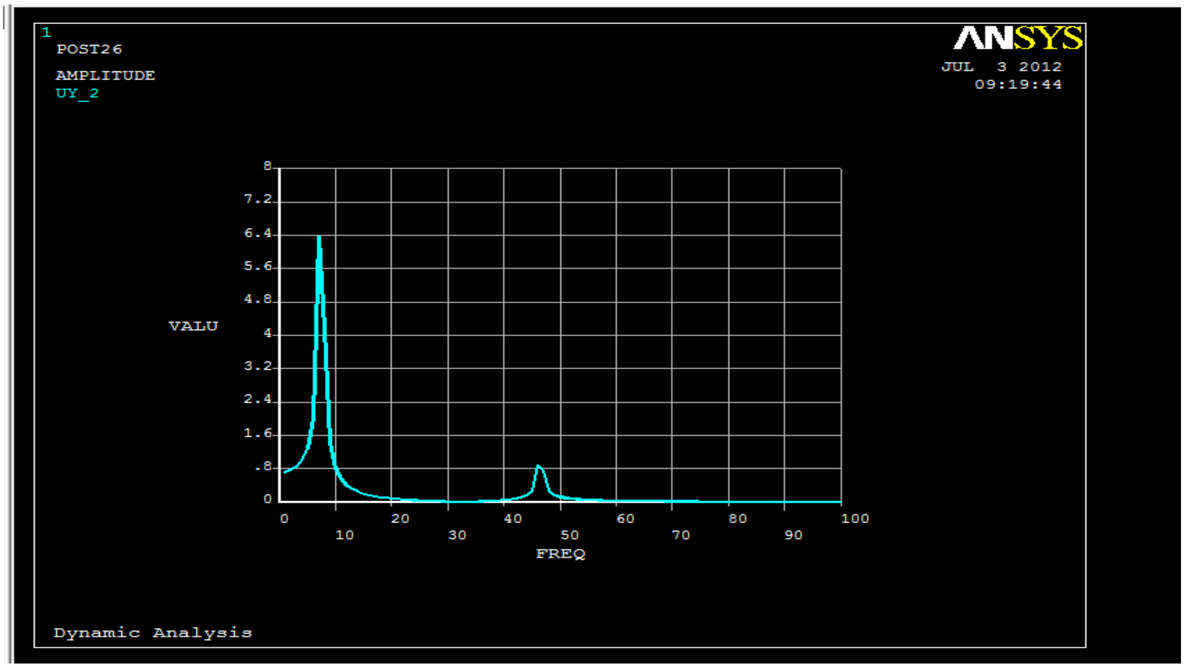

Fig.5.20 FRF graph of BR4

Fig. 5.9-5.20 shows the FRF (Frequency Response Function) plots between displacement and frequency of different specimens with different dimensions. These graphs are plotted by harmonic analysis done in ANSYS. From these graphs natural frequency was calculated. This is basically theoretical modal analysis done in ANSYS.

5.6 THEORETICAL CALCULATIONS

All the calculations used to find the natural frequency and damping ratio are as following:-

5.6.1 Natural frequency

For calculating natural frequency, first of all calculate stiffness i.e. K which is given by:-

$$K = \frac{3EI}{L^3} \qquad (5.1)$$

After this calculate the mass of beam from its dimensions. This is calculated by a factor which is multiplied with the dimensions of a beam. This multiplying factor i.e. density ($\frac{g}{cm^3}$) is different for different materials as given in the table below:-

S.NO.	SPECIMENS	MULTIPLYING FACTOR
1.	MILD STEEL	7.8
2.	ALUMINIUM	2.6
3.	BRASS	8.5

Table 5.2 Multiplying factor of different specimens to calculate mass

Thus calculate mass of the beams with different dimensions. Then calculate the natural frequency with the formula:-

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \text{ Hz} \qquad (5.2)$$

The natural frequencies calculated by this method and by harmonic analysis are given below in the table:-

S.NO	SPECIMENS	m = lXbXtXρ	$K = \frac{3EI}{L^3}$	THEORETICAL FREQUENCY(Hz) $\omega_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$	UNDAMPED FREQUENCY (HARMONIC ANALYSIS) (Hz)
1.	MS1	0.82	835.0	5.08	10.00
2.	MS2	0.41	104.3	2.53	5.00
3.	MS3	0.59	2194.56	9.71	20.00
4.	AL1	0.273	292.2	5.20	10.00
5.	AL2	0.136	36.53	2.60	4.00
6.	AL3	0.198	768.04	10.12	20.00
7.	AL4	0.09	96.01	5.20	10.00
8.	BR1	0.89	431.71	3.50	8.00
9.	BR2	0.44	53.96	1.76	3.00
10.	BR3	0.64	1134.5	6.70	13.00
11.	BR4	0.32	141.8	3.35	7.00

Table 5.3 Natural frequency (theoretical and harmonic analysis) of different specimens

CHAPTER 6

RESULTS AND DISCUSSION

The objective of the present work is to compare the natural frequencies and damping of different structural materials i.e. mild steel, aluminium and brass. In the present study the observations have been taken to calculate damping ratio for different materials by varying the length and thickness of materials and constant width at frequency ranges from 0 to 100 (Hz).

Data for the free vibration characteristics has been taken for first mode i.e. fundamental mode at the free end of the cantilever setup. The graphs have been plotted between excitation frequency and amplitude in terms of displacement.

The natural frequencies and damping ratio are determined based on frequency response spectrum obtained by analyzing the acceleration signals from the beam as the response data.

The natural frequencies (only for fundamental mode) of the specimens are identified as the frequencies corresponding to peaks present in the FFT spectrum.

6.1 NATURAL FREQUENCY

The natural frequencies of different specimens (brass, aluminium and mild steel) of different dimensions by varying their length and thickness which were calculated experimentally and theoretically are compared with each other and results are discussed and shown with the help of bar charts as following:

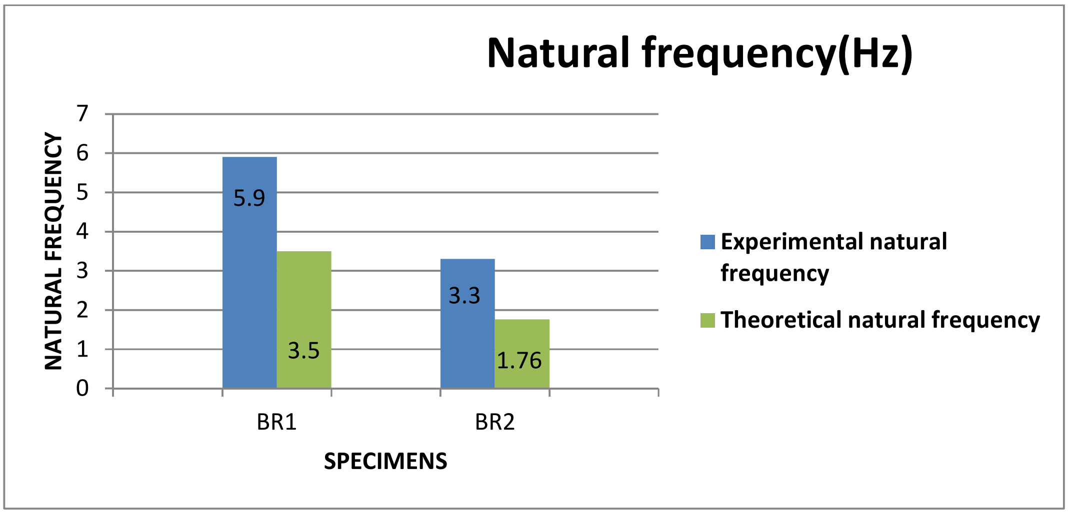

Fig.6.1 Comparison of experimental and theoretical natural frequency of BR1 and BR2

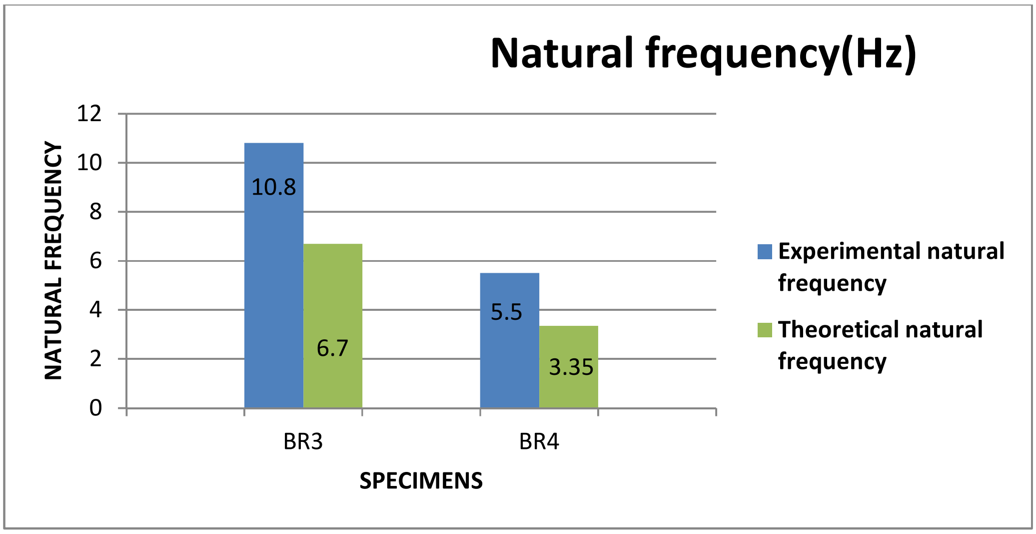

Fig.6.2 Comparison of experimental and theoretical natural frequency of BR3 and BR4

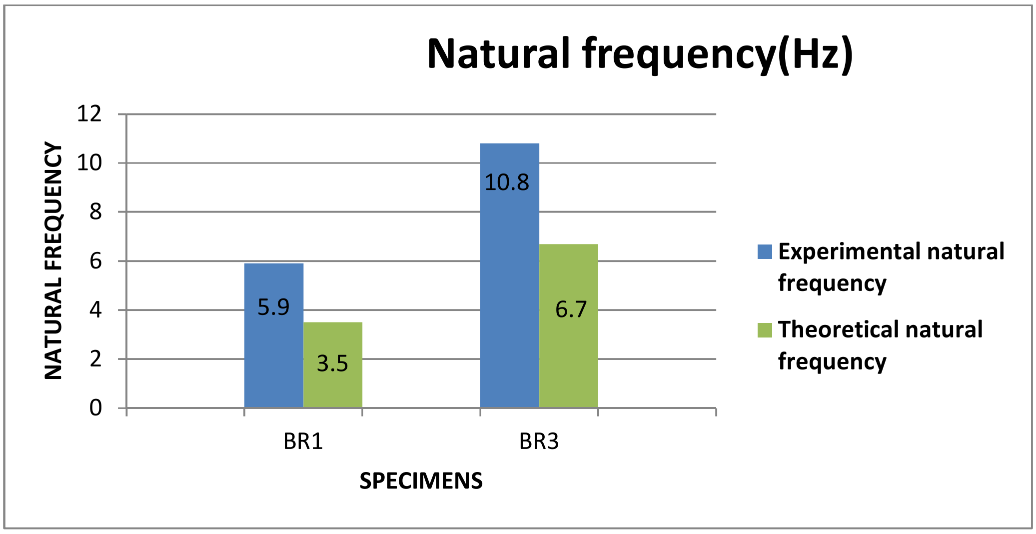

Fig.6.3 Comparison of experimental and theoretical natural frequency of BR1 and BR3

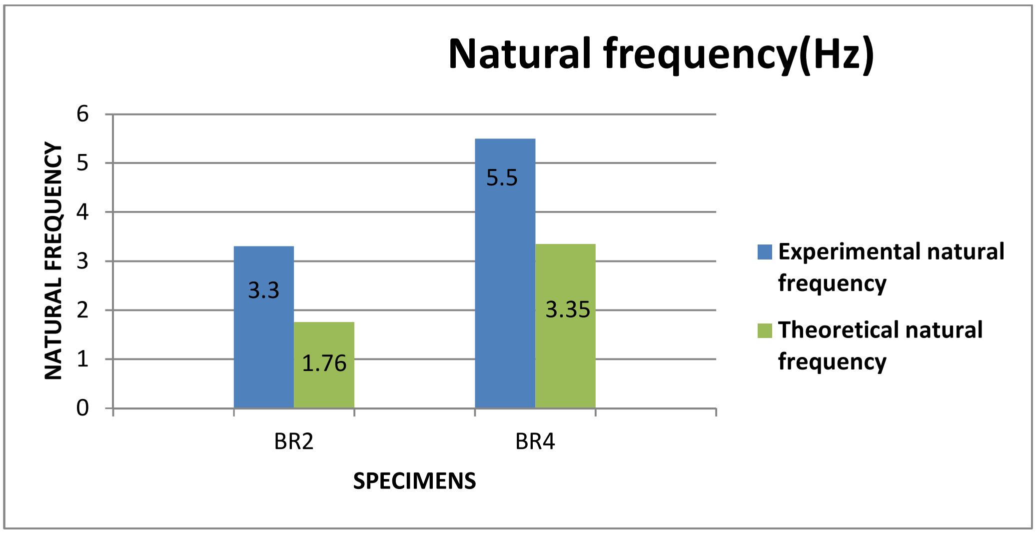

Fig.6.4 Comparison of experimental and theoretical natural frequency of BR2 and BR4

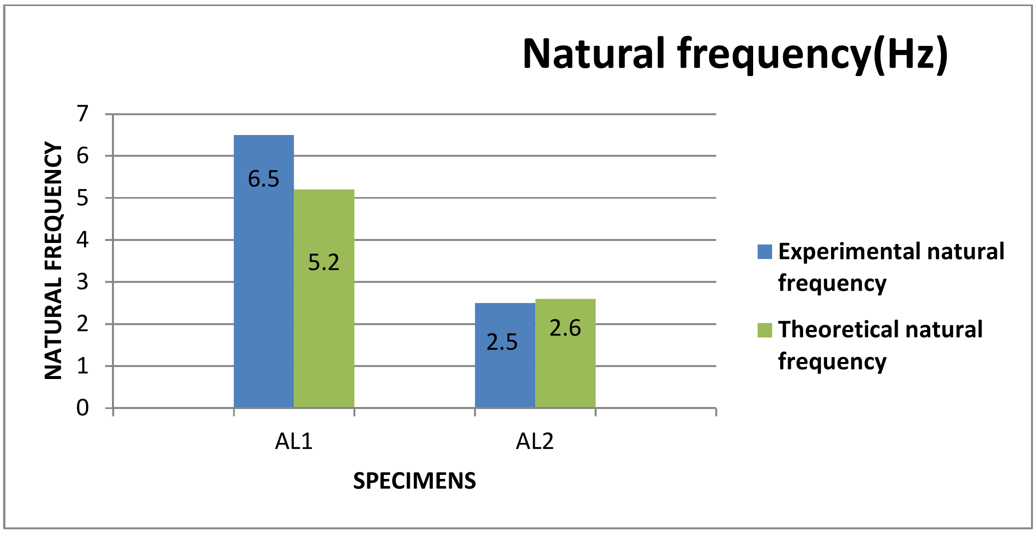

Fig.6.5 Comparison of experimental and theoretical natural frequency of AL1 and AL2

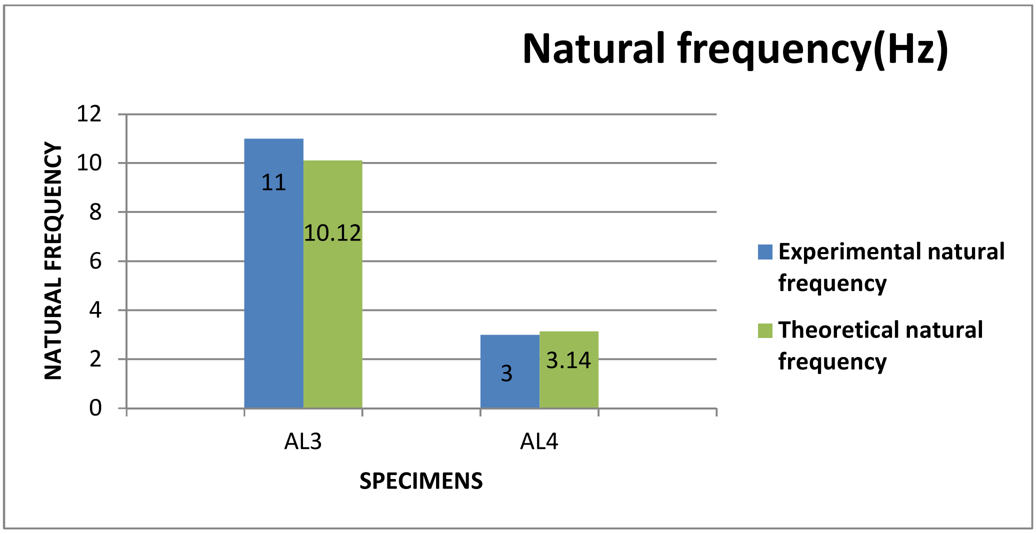

Fig.6.6 Comparison of experimental and theoretical natural frequency AL3 and AL4

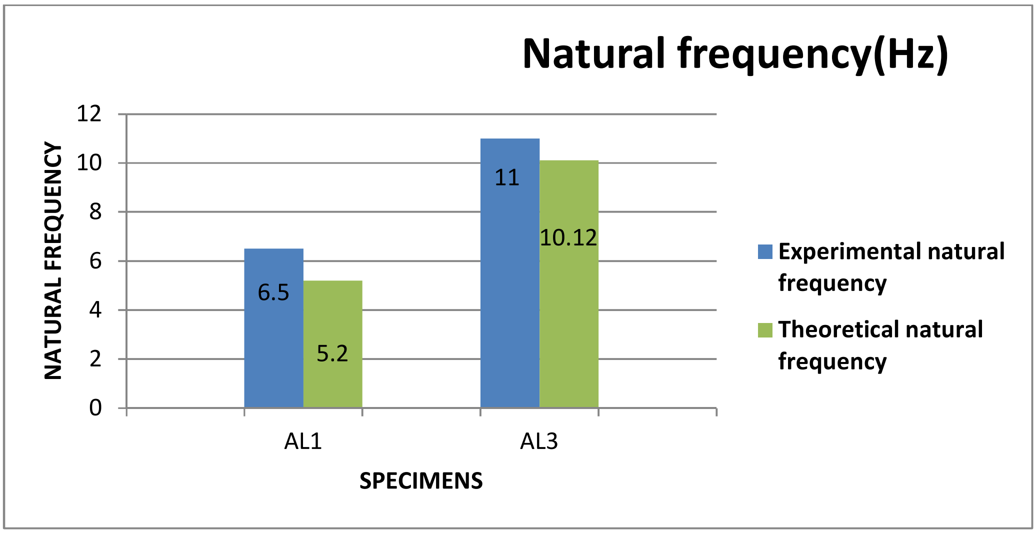

Fig.6.7 Comparison of experimental and theoretical natural frequency of AL1 and AL3

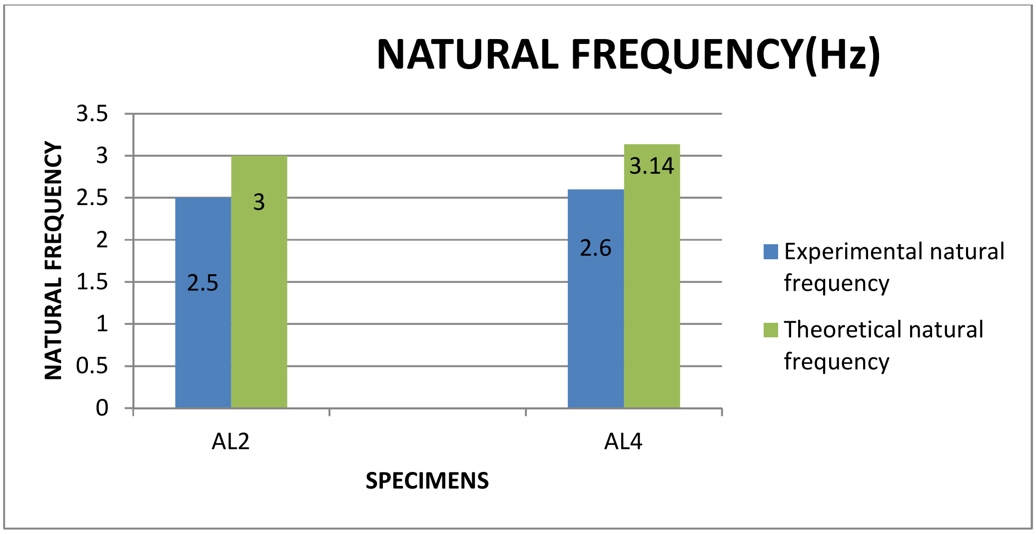

Fig.6.8 Comparison of experimental and theoretical natural frequency of AL2 and AL4

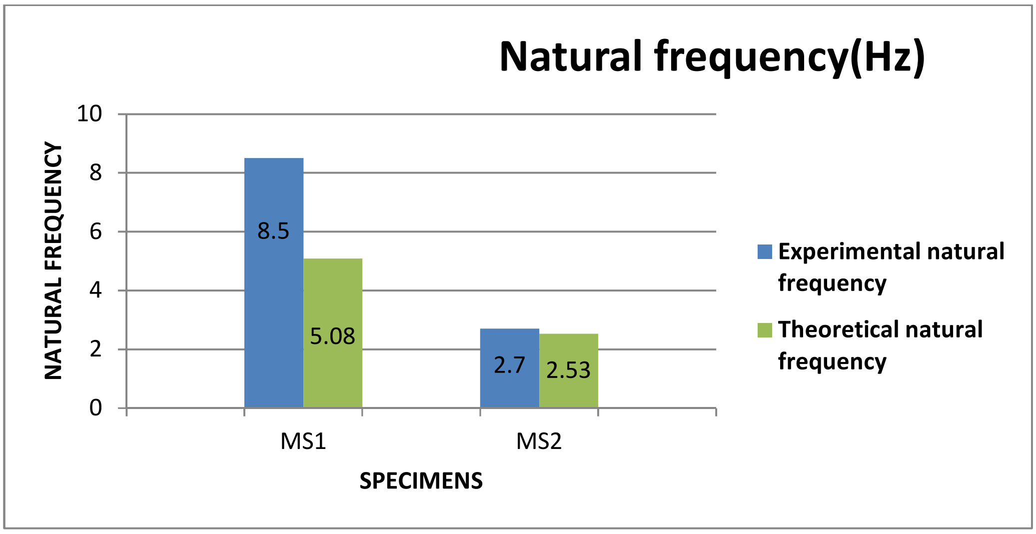

Fig.6.9 Comparison of experimental and theoretical natural frequency of MS1 and MS2

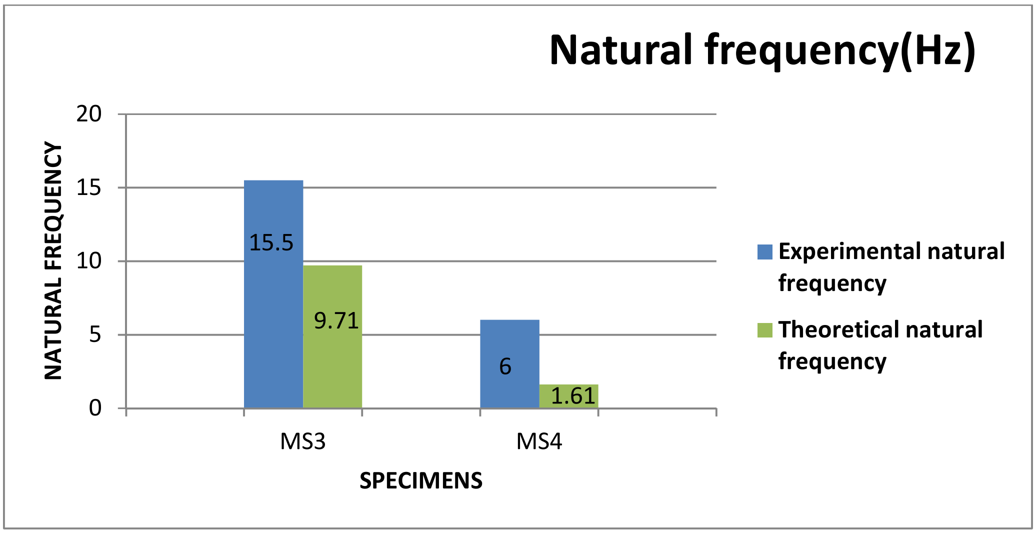

Fig.6.10 Comparison of experimental and theoretical natural frequency of MS3 and MS4

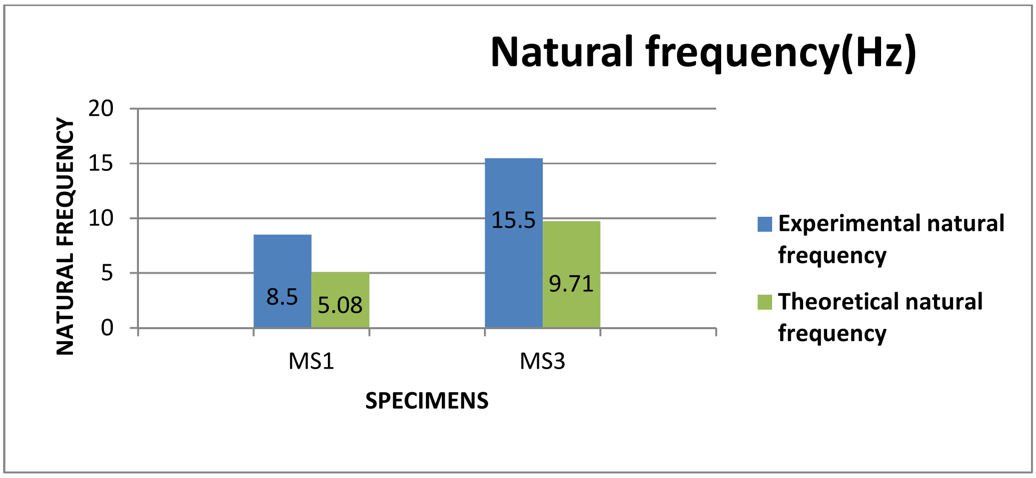

Fig.6.11 Comparison of experimental and theoretical natural frequency of MS1 and MS3

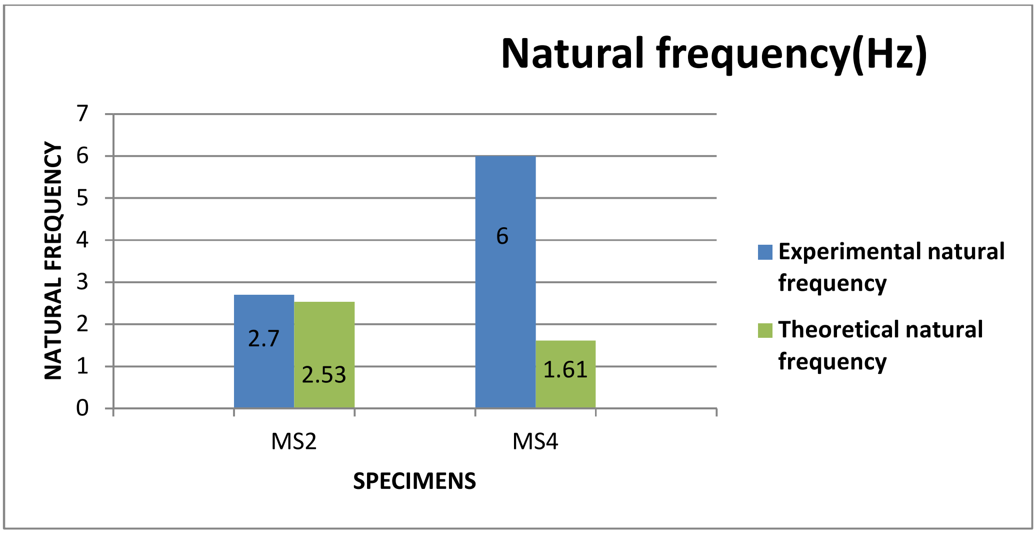

Fig.6.12 Comparison of experimental and theoretical natural frequency of MS2 and MS4

As seen from the experimental investigation and theoretical analysis the natural frequency decreases as the decrease in thickness for the same length of beams of different materials. But decrease in length increases the natural frequency of the beams.

6.1.1 COMPARISON OF EXPERIMENTAL AND THEORETICAL FUNDAMENTAL NATURAL FREQUENCIES OF BEAMS OF DIFFERENT MATERIALS

The modal frequencies (Hz) for different beam specifications are compared as below:

Material	**Beam dimensions**	**Frequency(by theoretical method)**	**Frequency(by experimental method)**	**Percentage of error**
Aluminium	690mm x 25.4mm x 6mm	5.20	6.5	20%
Brass	690mm x 25.4mm x 6mm	3.50	5.9	40.67%
Mild steel	690mm x 25.4mm x 6mm	5.08	8.5	40.23%

Table: 6.1 Comparison of fundamental frequencies of beams of thickness 6mm

Material	**Beam dimensions**	**Frequency(by theoretical method)**	**Frequency(by experimental method)**	**Percentage of error**
Aluminium	690mm x 25.4mm x 3mm	2.6	2.5	3.84%
Brass	690mm x 25.4mm x 3mm	1.76	3.3	46.66%
Mild steel	690mm x 25.4mm x 3mm	2.53	3.5	27.71%

Table: 6.2 Comparison of fundamental frequencies of beams of thickness 3mm

Material	Beam dimensions	Frequency(by theoretical method)	Frequency(by experimental method)	Percentage of error
Aluminium	500mm x 25.4mm x 6mm	10.12	11	8%
Brass	500mm x 25.4mm x 6mm	6.70	10.8	37.96%
Mild steel	500mm x 25.4mm x 6mm	9.71	15.5	37.35%

Table: 6.3 Comparison of fundamental frequencies of beams of length 500mm & thickness 6mm

Material	Beam dimensions	Frequency(by theoretical method)	Frequency(by experimental method)	Percentage of error
Aluminium	500mm x 25.4mm x 3mm	3.14	3	4.45%
Brass	500mm x 25.4mm x 3mm	3.35	5.5	39.09%
Mild steel	500mm x 25.4mm x 3mm	1.61	6	73.16%

Table: 6.4 Comparison of fundamental frequencies of beams of length 500mm & thickness 3mm

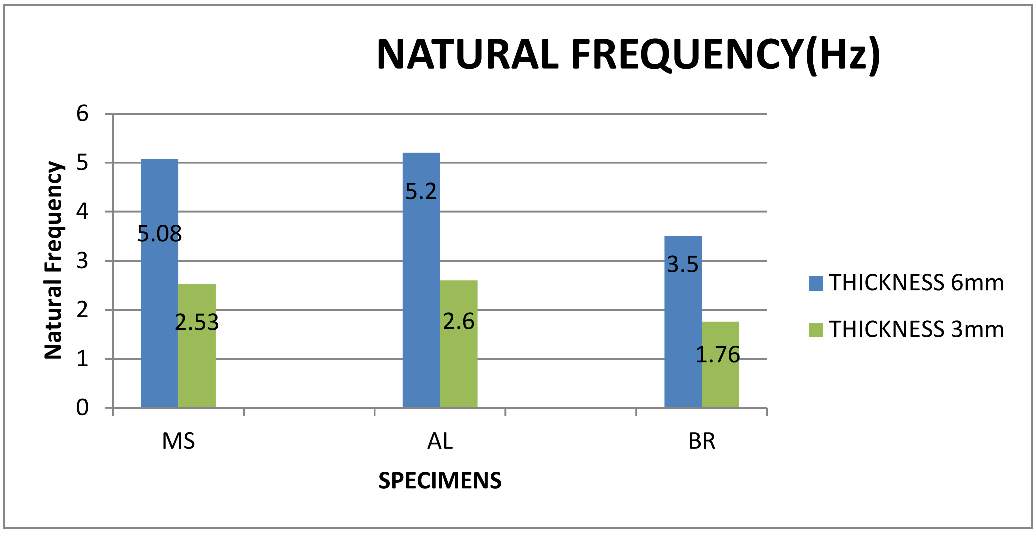

Fig.6.13 Comparison of theoretical determined natural frequencies of three specimens

(690mm X 25.4mm X 6mm) (690mm X 25.4mm X 3mm)

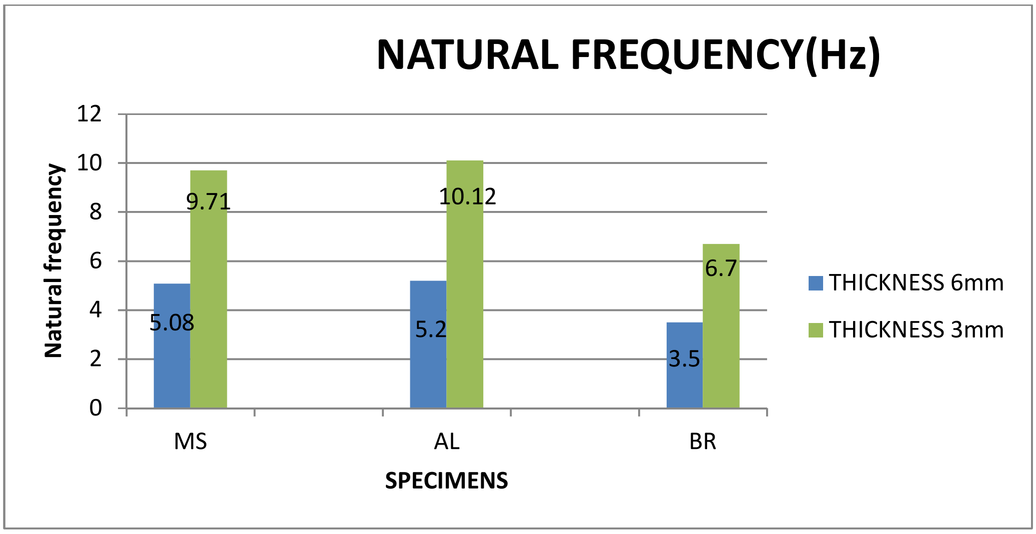

Fig.6.14 Comparison of experimentally determined natural frequencies of three specimens

(690mm X 25.4mm X 6mm) (690mm X 25.4mm X 3mm)

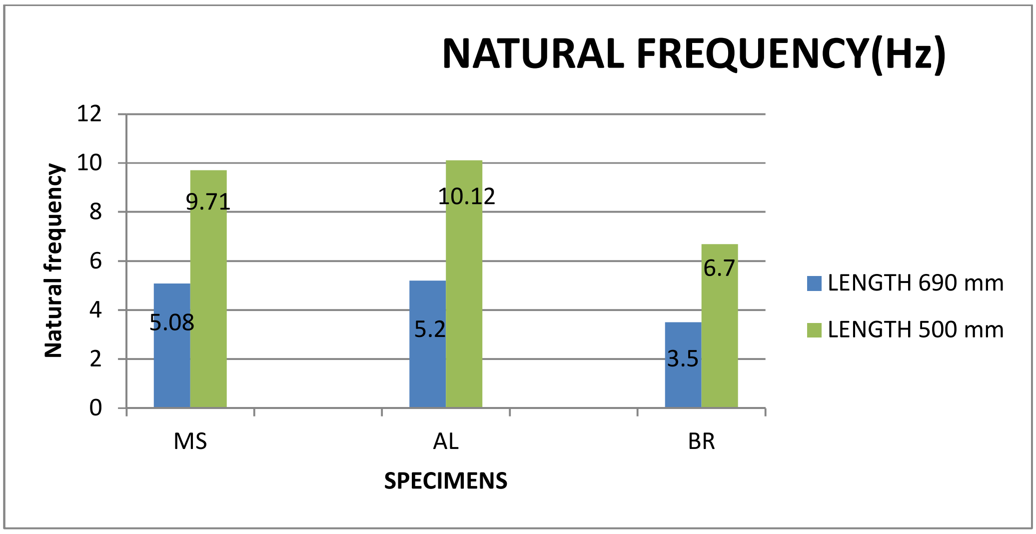

Fig.6.15 Comparison of theoretical determined natural frequencies of three specimens (690mm X 25.4mm X 6mm) (500mm X 25.4mm X 6mm)

From fig.6.14 it can be seen from the experiment that steel and aluminium has higher natural frequency in comparison with brass. The reasons could be of higher stiffness hence in turn higher elastic modulus. Fig.6.13-6.14 compares natural frequency of three specimens of same length and same width but different thickness. As seen from the theoretical and experimental values of natural frequencies for the first fundamental mode in the above tables, the effect of thickness variates the natural frequency. In case of beam of thickness 3mm, the natural frequency reduces for all the three materials in comparison with beam of thickness 6mm.The can be accounted to be lower moment of inertia which results in lower stiffness value.

Fig.6.15 compares natural frequency of three specimens of same width and same thickness but different length. As seen from the theoretical and experimental values of natural frequencies for the first fundamental mode in the above tables, the effect of length also variates the natural frequency. In case of beam of length 500mm, the natural frequency increases for all the three material in comparison with beam of thickness 690mm.

6.2 DAMPING RATIO

The damping of specimens of different materials are compared with each other and results are discussed.

6.2.1 COMPARISON OF DAMPING OF SPECIMENS FOR DIFFERENT MATERIALS BEAMS BY VARYING LENGTH AND THICKNESS

The damping of specimens of different materials ‘ ζ ‘ are compared as below in tables:-

Dimensions 690mm X 25.4mm X 6mm

Material	**Fundamental mode(Hz)**	**Damping ratio(ζ)**
Mild steel	8.5	0.069
Brass	5.9	0.042
Aluminium	6.5	0.039

Table: 6.5 Estimated damping by half power bandwidth method

Dimensions 690mm X 25.4mm X 3mm

Material	**Fundamental mode(Hz)**	**Damping ratio(ζ)**
Mild steel	2.7	0.212
Brass	3.3	0.111
Aluminium	2.5	0.08

Table: 6.6 Estimated damping ratio by half power bandwidth method

Dimensions 500mm X 25.4mm X 6mm

Material	**Fundamental mode(Hz)**	**Damping ratio(ζ)**
Mild steel	15.5	0.064
Brass	10.8	0.018
Aluminium	11	0.017

Table: 6.7 Estimated damping ratio by half power bandwidth method

Dimensions 500mm X 25.4mm X 3mm

Material	Fundamental mode(Hz)	Damping ratio(ζ)
Mild steel	6	0.083
Brass	5.5	0.054
Aluminium	3	0.033

Table: 6.8 Estimated damping ratio by half power bandwidth method

From Tables 6.5-6.8, it is evident that material damping for the first mode is higher for steel in comparison with brass and aluminium of same length and width by varying thickness. Also of same width and thickness by varying length. Fig.6.16-6.18 compares material damping of three specimens by changing parameters i.e. thickness and length.

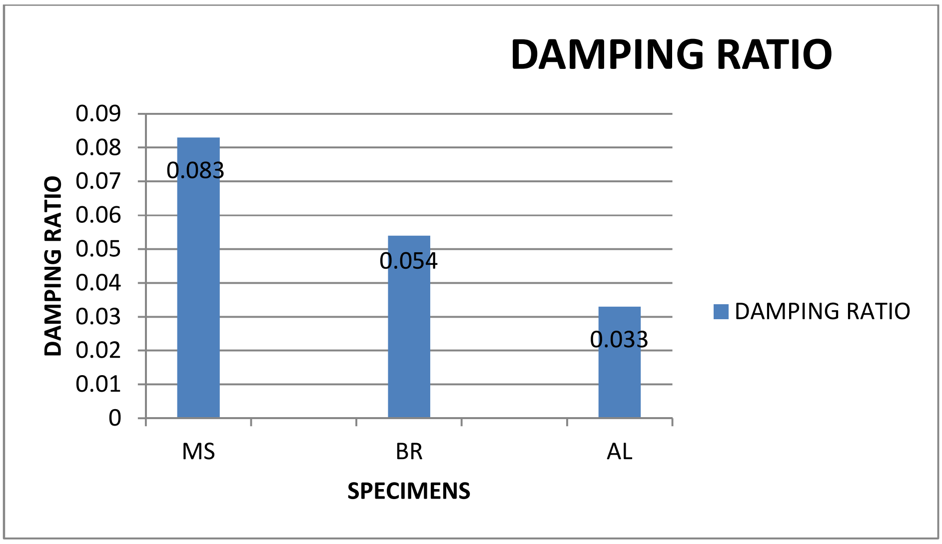

Fig.6.16 Comparison of damping ratio of three specimens of dimensions (690mm X 25.4mm X 6mm)

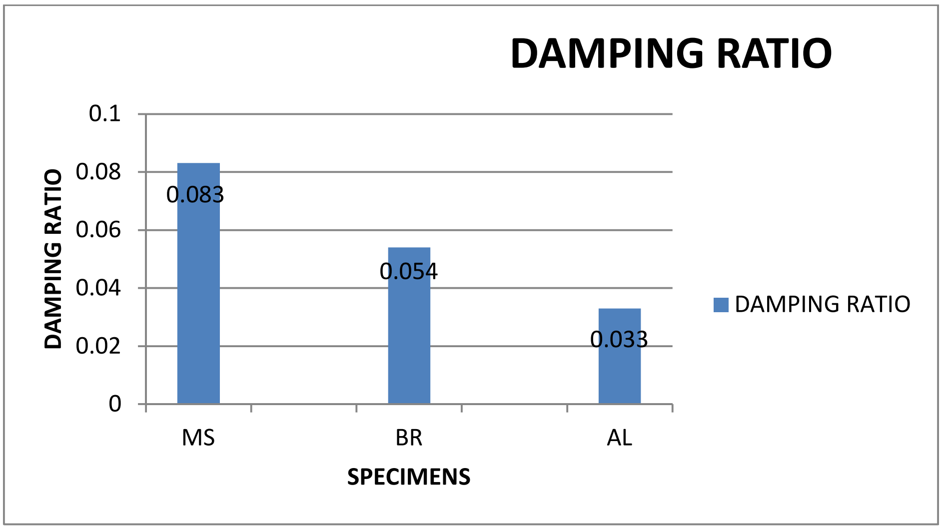

Fig.6.16 Comparison of damping ratios of three specimens of dimensions (690mm X 25.4mm X 3mm)

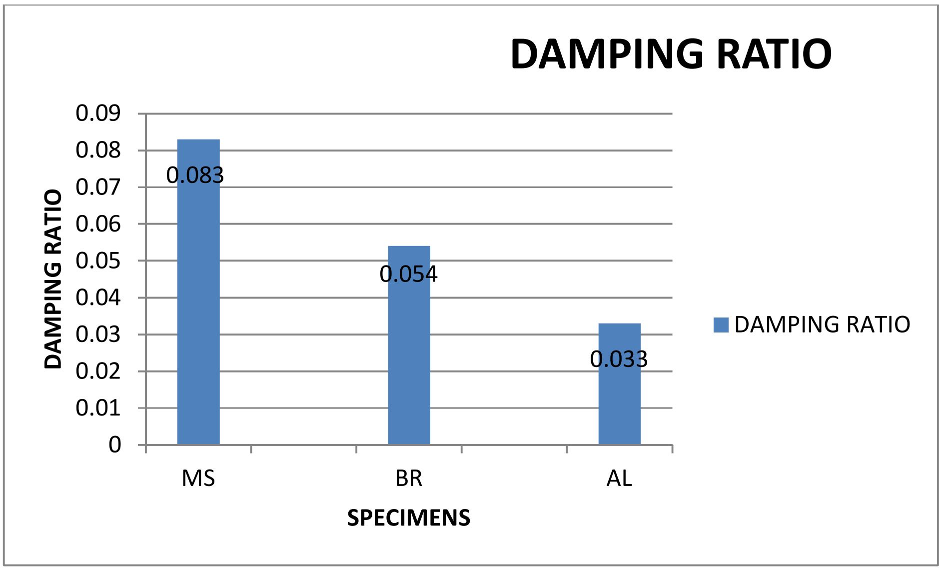

Fig.6.17 Comparison of damping ratios of three specimens of dimensions (500mm X 25.4mm X 6mm)

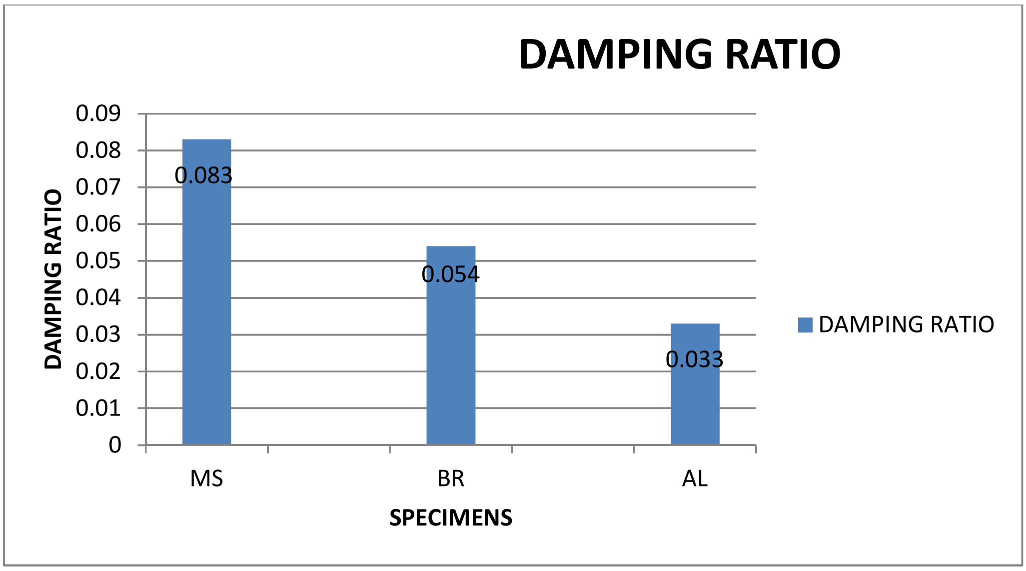

Fig.6.18 Comparison of damping ratios of three specimens of dimensions (500mm X 25.4mm X 3mm)

The effect of thickness and length also variates the material damping. The different material beam of thickness 6mm has found to be higher damping ratio as compared to thickness 3mm. Brass has second highest damping value after steel. Aluminium damping was found to be lowest.

6.3 SUMMARIZED RESULTS AND DISCUSSIONS

1. The natural frequencies obtained experimentally at the first mode of vibration of all the three specimens have been compared with the theoretical values.
2. The harmonic analysis also carried out in ANSYS to plot the FRF's.
3. There is reasonable agreement of the theoretical calculated natural frequency with the experimental one.
4. The discrepancy may be due to loading effect in experimentation.
5. Enharmonics occurred and graphs are not cleared due to experimental limitations.
6. Results could be better if more samples taken.

CHAPTER 7

CONCLUSION AND SCOPE FOR FUTURE WORK

7.1 CONCLUSION

The main objective of the present work is to study the vibration damping characteristics of three materials i.e. steel, brass and aluminium. To collect the data based on excitation frequency using free vibration technique and compare it theoretical results. The cantilever beams have been subjected to impact hammer test and damping ratio has been computed using half power band width method.

On the basis of present study following conclusions are drawn:
From the experiment it is evident that material damping is higher for steel in comparison with brass and aluminum.

- The increase in material damping could be correlated to the stiffness of materials.
- The damping ratio increases with decrease in thickness for each material.
- The natural frequency decreases with decreases in thickness for each material. But it is vice-versa in case of length.
- The damping of specimen made up of aluminium was found to be lowest than either steel or brass.
- The theoretical result obtained by the method proposed in this work and experiment result vibration are in fair matching in terms of natural frequency.

7.2 SCOPE FOR FUTURE WORK

- The present work may be extended in one of the following ways. Direction in which the work can be further carried out in future are:
- Use of tapered beams, variable thickness beams.
- There are other composite materials i.e. metal matrix composite (MMC) in the use today.

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