PRODUCTION PLANNING IN AN UNCERTAIN ENVIRONMENT

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Certificate

I hereby certify that the dissertation entitled, "**Production Planning in an uncertain Environment**" in the partial fulfillment of the requirement for the award of degree of Master of Science in the School of Mathematics, Thapar Institute of Engineering and Technology(Deemed to be University),comprises of my own research work which is carried out under the supervision and the guidance of Dr. Jolly Puri, Assistant Professor, School of Mathematics Patiala from the period january 2019 to July 2019. The part of the work presented in this dissertation has not been submitted either in part or in full to this or to any other University/Institute for the award of degree.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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(Chandanpreet Kaur)

ABSTRACT

The decisions play an integral part in the life of humans. Human beings in a conscious state of mind or in a subconscious state of mind take numerous decisions in their day to day life, but unfortunately study reveals that many people are not so profound in taking good decisions. Also many times in group decision making the obtained decision is not suitable. This is so because different people have different alternatives according to their thinking and every decision maker has a different view regarding which alternative is better or not and out of several alternatives a single alternative is selected but it is very important to obtain good decisions especially when solving Production Planning Problem. The process of making plans in any corporation is referred to as the production planning.

The Multi-Stage and Multi-Objective Production Planning problem refers to the problem of production planning having multiple stages and also having more than one objective function/goal. Such Production Planning problems are complex to handle due to the presence of multiple stages and requirement of either one machine or more than one machine on each stage. However if there is not a balance in the relation of the input-output or if the machine fails to function (machine breakdown) at any of the stage then the production target is disturbed to a great extent and the desired target of the production is not achieved.

In the present work, a Production Planning problem is solved in an uncertain environment. The objectives of the production planning problem are fuzzy in nature. The given data consists of the production rate/capacity, New machines installation cost, Inventory cost per 100 units in each stage that are fuzzy in nature and they are converted into crisp values. Lastly, LINGO 17 is used to solve this mixed integer programming production problem.

The outline of the thesis is summarized below:

Chapter 1 is the introductory chapter in which firstly the terms production, planning are defined and then process of production planning is demonstrated. Various production planning plans such as strategic plans, tactical plans and operational plans are illustrated in this chapter. Afterwards different techniques that are used in production planning are explained and the use of decision making in the process of production planning is

demonstrated. A brief introduction to various approaches is given that are used to get the solution of the production planning problems.

Chapter 2 consists of the preliminaries chapter which deals with the fuzzy set theory**.** There are many tools that are developed in mathematics in order to solve various problems of operation research. But no new tools and techniques are developed for the problems that are cumbersome in nature and possess no proper structure. In order to solve such problems Zadeh introduced the notion of fuzziness in 1965. Sometimes it becomes very difficult for the decision maker to determine the objectives and the constraints in an exact/precise manner. Hence such objectives/constraints are specified in fuzzy terms and fuzzy linear programming is used to solve such problems. Some of the literature of the fuzzy set theory is given in this chapter. The basic definitions that are widely used in the fuzzy set theory are also explained. Defuzzification aims in determining a real value which corresponds to a fuzzy number. The various methods of defuzzification are presented. Sometimes it is a possibility that the decision making problems consists of both linguistic as well as the numerical information. Further, to handle linguistic data, a 2-tuple fuzzy linguistic representation model (Herrera and Martinez, 2000) has been presented.

Chapter 3 deals with the various ordered weighted average (OWA) aggregation operators to aggregate goals such as induced OWA and linguistic OWA etc**.** These operators are used in the decision making process in order to develop the aggregation method for combining the information. Some theorems of the OWA aggregation operator are also discussed in this chapter. Next the use of the linguistic quantifiers and the minimax disparity approach in identifying the OWA operators weights are discussed.

Chapter 4 is the review of multi-stage multi-objective production planning problem of Gupta and Mohanty (2015) which is a mixed-integer programming problem. Further, to deal with subjective preference of decision maker, we have proposed the multi-stage multi-objective production planning problem at different orness levels. The objectives that are taken in the production planning problem are fuzzy in nature. The various complications faced by the firm and the objectives/goals are defined.

Chapter 5 discusses the membership functions of all the objectives and combines them to obtain the Objective function. The solution of the resultant mixed-integer programming problem is solved using LINGO 17.

Chapter 6 consists of the bibliography.

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CHAPTER 1

Introduction

1.1 Introduction

The term production planning refers to the process of making plans in any corporation. In it all facilities are arranged which are required for the future purposes. In the process of production planning various raw materials are used and people are acquired so as to produce the output within time limits. A very large organization which is involved in the planning of production will aim to maximize its profit to a great extent. So with the help of production planning the management can predetermine money, equipments and the number of people working for the production process.

Production: The term production refers to transform the unprocessed material, thoughts, facts into finished products.

Planning: Planning in an organization defines the objective and determines various methods to achieve the output or finished product. In order to meet the target various plans are developed.

Types of Planning

i. **Strategic Plans** These plans are designed keeping in mind the whole organization. The managers at the highest post are responsible for the execution of the strategic plan. These plans look to the future. The strategic plans constitute the supporting structure for the lower planning. For example, where the firm will be after six years. Such types of answers are generated with the help of strategic plans.

ii. **Tactical Plans** The tactics that are required in achieving the desired result which the strategic plan has demonstrated are termed as the tactical plans. For example, if the strategy is to be the leader of the market then the tactical plan can be to twice the amount which is spent on advertisement.

iii. **Operational Plans** These plans reside at the lowest of the three plans. Such plans are formed by the managers at the lower level. These plans provide the work to the people which are involved in the organization along with the objectives of the strategic plan. These plans basically implements the strategies of the strategic plan. For example, giving the manufacturers of the organization as what to prepare as the desired output.

iv. **1.1.1 Techniques of production planning**

The various techniques that are used in the process of production planning are:

i. **Graphical technique** This method demonstrates the production process with the help of the graphs. For example, consider the supply of the twelve months taken on the X axis and correspondingly the level of the demand taken on the Y axis. The production is analysed graphically in this method.

ii. **Research technique** In the research technique the market area is visited by the employees in the production process. Demands of various products are seen in the market and the employees look whether the demand of the product will increase or decrease in the future. So this is a practical technique in which the market is surveyed. For example, a product is manufactured by the firm and the employee's of the firm visits the market in order to see what is the demand of the product in the market.

iii. **Mathematical technique** The mathematical techniques tells about the previous survey on the profit, loss, demand, supply. It also demonstrates in which area the demand and the sales are increased. So the production is done by the existing mathematical data. It can be done by linear programming, integer programming, goal programming. For example, it can be possible that a product manufactured by the firm is extensively used in one area and is negligibly used in the other area.

1.2 Decision Making

A decision is an action which is chosen from a set of various alternatives in order to achieve various objectives of a firm. The process of decision making is a crucial part of any business organisation or a firm. Most of the time decisions are made by the individuals at some stage of their lives. However according to various statistics a lot of individuals are below average to make the decisions.

The decision making is an integral aspect of planning process. Good decisions made by the individuals enhances the performance of the firm and the profit of the firm increases to a great extent, however bad decisions ruin the performance of the firm and the firm undergo various losses. The corporate decision making is an essential process in any firm. An alternative is chosen from the set of various alternatives in the process of decision making. Sometimes it becomes cumbersome to make decisions in a firm. This is so because group of various individuals are there in a firm. Naturally every individual will have his/her idea, perception and it is possible that alternative given by one individual completely differs from the alternative given by the other individuals which leads to the conflict in the process of decision making. And it becomes very difficult to arrive at a single decision.

Examples of decision making

i. To choose a college for study from a list of various colleges.
ii. To choose a tourist destination from the various set of alternatives of destination.
iii. To choose which election party to vote.

Agarwal et al. (2010) explains the methods of high level in Operation Research that are useful to make good decisions. The main aim is to reduce the rivalry that occurs between different individuals in the process of decision making and at last an optimal solution is reached. The manager of a firm either in a conscious state of mind or in a subconscious state of mind take plenty of decisions.

1.2.1 Decision making in Production Planning

Planning is the foremost and the most significant function in order to meet the desired objectives/ goals. But it is also important to deal with decision making in planning. If the significant methods of the decision making are not used in the production firm then it leads to various complications and the customers too will not get satisfied from the product obtained. The objective of the production process is not only to produce the product but to produce that kind of the product that would be successful in the market and would also return profit to the firm.

The managers of the production firm should make decisions daily without worrying about the consequences of the decisions that whether the decision would be successful in the future or not. In the process of decision making for producing the product the managers should predict the future conditions while keeping in mind the present and the past conditions. The experiences will improve the decision making of the managers.

Some of the firm tries to meet the demands of all its customers. The firm accepts the order of every customer and produces the desired output, while in some cases the firm choose some of the customers out of the lot and then produces the desired product for them and the rest of the customers are left for the competitors of the firm. Hence, both the decision making and production planning has a lot of impact in the flexibility of the company so as to meet the demands of its customers. They both go hand in hand. Hence the decisions related to the production capacities are equally important as that of the planning.

1.3 Literature Review

An extensive work has been done by eminent researchers in the field of production planning. Lee and Chen (2000) discussed the issue of arrangement of set of activities on set of parallel machines**.** Golany et al.(2001) and Geyer et al.(2007) gave the various algorithms for aggregate production planning. The order coordination was investigated by Nakashima et al.(2004) in which various types of inventories were discussed. Inderfurth (2004) explained that sometime it may happen that in a firm there are lacks of products that have high demand so in order to deal with such crisis a very good quality of remanufactured products were supplied. Quld-Loudy and Dolgui(2004) gave the formulation used in measuring the average cost and also gave a model that was used in solving the multi-component and multi-period planning problems. Mula (2004) gave a linear programming model for the medium term production planning. Uncertainty in the system of manufacturing has advanced a lot in the recent years. It is seen that the production planning models where the uncertainty is not recognised may give rise to planning decisions that are poor in quality while the production planning model that recognises and considers uncertainty are expected to give good planning decisions. Mula et al(2005) illustrated that it is not possible to remove uncertainty completely. Teunter et al.(2006) discussed the lot sizing problem in his research paper. Depuy et al.(2007) and Ferguson et al.(2009) explained the methods of master production scheduling. The sales and operational planning is the highest level of the hierarchical planning. Olhager et al(2001),Olhager and Rudberg(2002) Olhager and Selldin(2007) demonstrated the sales and operational planning as the long-term planning in the context of sales, production, demand. The significance of the scheduling of production and maintaining the planning is admitted by various scientists. Aghezzaf and Najid(2008) considered the integration of the production planning .There was a lot of complexity in the scheduling of production and maintenance of planning. Berrichi et al.(2010) demonstrated that scheduling of production and maintenance of planning were carried out independently for so many tears due to their tedious nature. Mitra et al.(2012) in the master production planning investigated the energy consumption. Latifoglu et al.(2013) was able to develop a very fast planning apprach. Lin et al.(2015) discussed a multi-objective optimization algorithm. Gupta and Mohanty (2015) presented multi-stage multi-objective production planning using linguistic and numeric data and proposed a fuzzy integer programming model. In this thesis, we review the work done by Gupta and Mohanty (2015) and present an algorithm based upon minimax disparity approach.

1.3.1 Approaches to solve various production planning problems

The manufacturers have to deal with so much of competition nowadays, so it becomes very important to get desired output from production planning process. The manufacturers of the firm have to make resources in such a way so as to fulfil the demands of its clients. Hence, the methods that are used to find the solution to the production process should be such that optimal solution is reached in lesser time. Various methods to solve the production planning problems are:

1) **Linear Programming Approach** The approach of linear programming is used to solve the production planning problems. The aim of the firm/company that uses linear programming in order to solve the production planning problem is to maximize profit or to minimize the cost. Both of these objectives cannot be fulfilled together in the production planning problem. As most of the production planning problems are real life problems so the approach of linear programming fails to solve these problems.
2) **Goal Programming Approach** Goal programming was developed by Charnes and Cooper. There are some production planning problems in which more than one objective is there. Hence such type of production planning problems with more than one objective are solved with the help of goal programming. In goal programming problem the objectives are ranked in accordance of their significance. The decision maker will also give an aspiration level which will reflect his desire.
3) **Mixed Integer Programming Approach** Some of the production planning problems have various resources, products, setup times, periods and setup costs. Mixed integer programming method is used to formulate such type of the production planning problems. Such types of the problem are the NP-hard problems. For these problems no algorithm known is there for which the problem can be solved in the polynomial time. Hence it is impossible to obtain the optimal solution of such type of the production planning problems. Therefore the mixed integer programming production planning problem becomes a very hard task to solve as compared to the linear production planning problems.
4) **Bilevel Programming Problems** Many of the production planning problems are hierarchical in nature. Hence these type of the production planning problems are solved as the bilevel programming problem. The bilevel optimization problem has a nested

inner optimization problem which acts as a constraint to the outer optimization problem. In such type of the problem for the upper level optimization problem firstly the lower level optimization problem is solved which would give the optimal response to the upper level optimization problem. This type of the problems is NP hard. Hence the time taken to find the result of the problem increases exponentially as the size of the problem increases.

5) **Genetic Algorithm** The genetic algorithms are a part of artificial intelligence that are used in solving various production planning problems. In order to solve the NP-hard problems such algorithms have been proven successful. The problems of real life are solved with the help of the genetic algorithms. In the method of genetic algorithms various methods/solutions that are used to solve the production planning problem have a given structure identical to a chromosome. Every chromosome contains genes. The gene of the chromosomes represents the attributes of the production planning problems. As the complexity of the production planning problem increases the genes too increases which leads to the increase in the size of the chromosome. The total number of chromosomes that depicts the potential solution of the production planning problems is referred to as the population.

CHAPTER 2

Preliminaries

2.1 Fuzzy set theory

Many of the conventional tools that are used to solve a mathematical problem are deterministic, crisp as well as have a precise nature. In crisp set theory, an element either belongs to a set or does not belongs to a set. There is certainly no doubt about the occurrence of the values in the crisp set theory. Unfortunately there are some real life problems which are not precise in nature, and hence certain tools needs to be developed in order to handle the impreciseness and the uncertain nature of the data. Zadeh introduced the notion of "fuzziness" in 1965 to handle imprecise situations and extended the classical crisp set theory to fuzzy set theory.

In this section, the definitions of fuzzy set, convex fuzzy set, support of a fuzzy set, normal fuzzy set, Height of a fuzzy set, α – cut, strong α – cut, fuzzy number, triangular fuzzy number are presented.

There are tremendous amount of applications of the fuzzy sets. The very first paper on the fuzzy set theory was published by Lofti Asker Zadeh in 1965. From the year 1965 to the year 1975 Zadeh also published some papers on the fuzzy theory and decision making. Pedrycz (2009) described various logic operators in the fuzzy set theory. Liu, et al. (2009) were able to develop the fuzzy rough sets. Garcia-Horado and Trillas (2011) described the linguistic roots of the fuzzy sets. Kerre (2011) illustrated what is the impact of the fuzzy set theory in the field of mathematics (contemporary). Pedrycz, et al. (2012) described the granular computing. Deschrijver (2013) presented the neither conjunctive nor disjunctive in nature uninorms in the fuzzy set theory. Gorjanac Ranitoic' and Petojevic'(2014) represented the interval value fuzzy sets as lattices. Salazar and Soriano (2015) described the application of the convex combination in the theory of interval-valued fuzzy sets as well as the fuzzy sets. Acampora, et al. (2016) gave type-2 fuzzy framework.

A plenty of tools in mathematics are developed for solving various problems in operation research. After the modelling of the problem the optimal solution is found with the help of these mathematical tools. However there is no development of the new tools for the solutions of such problems that have no proper structure and which are cumbersome to solve with the help of the classical tools in mathematics. Rather than the classical mathematics the fuzzy theory is adaptable to various problem structures and is used in various decision making process. The term “Fuzzy” refers to vagueness or impreciseness for which there are no fixed boundaries. For example “Old” is a fuzzy word. There is not a single numeric value that could define the term “Old”. Some people may consider age 50 as Old while other may consider age 60 as Old. Hence for the concept “Old” there is no fixed boundary.

The fuzzy concept is the one in which the boundaries varies in excess amount and are not fixed. Hence a fuzzy concept refers to a vague concept and lacks preciseness. Although there is a clear meaning of the fuzzy concept but the exact meaning of the fuzzy concept can only be illustrated by elaborating the concept.

The main aim of a linear programming problem having a linear objective function and linear constraints is either to minimize the linear objective function or to maximize the linear objective function. However this becomes sometimes very difficult for the decision maker to determine the objectives and the constraints in an exact/precise manner. Hence such objectives/constraints are specified in fuzzy terms and a fuzzy linear programming is used to solve such problems.

2.1.1 Basic definitions of fuzzy set theory

Definition 1 (**Crisp Set)** A set say “A” is called a crisp set if for any element “x”, either x belongs the set A or x does not belong to the set A. Hence an element x will assume a Boolean value and either x will be the member of the set A or x will not be member of the set A.

Definition 2 (Characteristic Function) The characteristic function of a crisp set “A” is defined as

$$\mu_A(x) = \begin{cases} 1, if\ x \in A \\ 0, if\ x \notin A \end{cases}$$

Definition 3 (Fuzzy Set) Let X be the universe of discourse then a fuzzy set (Zimmermann 1996) $\widetilde{A}$ that is defined on universe of discourse X is defined as:

$$\tilde{A}=\{(x,\ \mu_{\tilde{A}}(x)), x \in X)\}$$

where x refers to the member of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}: X \rightarrow [\,0, 1\,]$ is the membership function that represents the membership degree of x being in $\widetilde{A}$.

The membership functions are used to describe the fuzziness and they also tell the extent to which an element belongs to the given set. There are different types of membership functions like triangular, trapezoidal, Singleton, Gaussian, and Piecewise Linear. Our main concern is the triangular membership functions.

Definition 4 (Convex fuzzy set) A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be a convex fuzzy set (Zimmermann, 1996) if and only if for x_1 and x_2 in X, $0 \leq \lambda \leq 1$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ where $\mu_{\tilde{A}}(x_1)$ and $\mu_{\tilde{A}}(x_2)$ are the membership functions of x_1 and x_2 respectively.

In other words for a convex function the membership function of the convex combination i.e $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2)$ should be greater than minimum value that comes out of the two membership functions $\mu_{\tilde{A}}(x_1)$ and $\mu_{\tilde{A}}(x_2)$.

Definition 5 (Height of a fuzzy set) The height of a fuzzy set is the maximum value of the membership function.

Mathematically $H(\tilde{A}) = \max(\mu_{\tilde{A}}(x))$.

Definition 6 (Normal fuzzy set) A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be a normal fuzzy set (Zimmermann 1996) if there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 7 (Support of fuzzy set) The support of the fuzzy set $\tilde{A}$ consists of all those elements for which the value of the membership function is greater than zero. So Support of a fuzzy set will be a crisp set.

Mathematically $\overline{S}(\tilde{A}) = \{x / (\mu_{\tilde{A}}(x) > 0)\}$.

Definition 8 (α-cut or α-level set) If $\tilde{A}$ is a fuzzy set in the universe of discourse X, then the α-cut (Zimmermann 1996) of $\tilde{A}$ is defined by $(\breve{A})_\alpha = \{x / (\mu_{\tilde{A}}(x) \geq \alpha)\}$, $\alpha \in (0, 1]$

Hence the α-cut or α-level set consists of all those elements for which the membership function is greater than or equal to α, where α is an arbitrary value in (0, 1]. So α-cut or α-level set will be a crisp set.

Definition 9 (Strong α-cut set) The strong α-cut set consists of all those elements for which the membership function is greater than α,

Mathematically $\tilde{A}_{\alpha'} = \{x / (\mu_{\tilde{A}}(x) > \alpha)\}$

Definition 10 (Fuzzy number) (Puri and Yadav, 2015) A fuzzy number (FN) $\tilde{A}$ is a fuzzy subset in the universe of discourse X that is convex, normal and piecewise continuous.

Definition 11 (Triangular Fuzzy Number) (Zimmermann 1996) A Triangular fuzzy number (TFN) $\tilde{A} = (a, b, c)$ is a fuzzy number with the membership function $\mu_{\tilde{A}}$ is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a < x \le b, \\ (x-c)/(b-c), & b \le x < c, \\ 0, & \text{otherwise} \end{cases}$$

A triangular fuzzy number $\tilde{A}$ is depicted as:

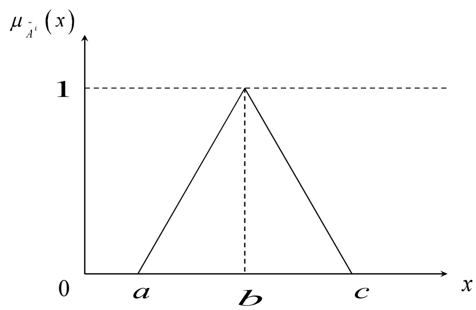

Figure 1 Triangular fuzzy number $\tilde{A} = (a, b, c)$.

A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be non-negative iff $a \ge 0$.

Definition 12 (Interval number) The interval number $\hat{x}$ is represented by $\hat{x} \in [X_L, X_U]$ where $X_L \le X_U$. For example $[1, 5]$ is an interval and 3 as the interval number such that 3 $\in [1, 5]$ with 1 as the left value or the lower value and 5 as the right value and $1 < 5$.

Example to illustrate support, α-cut, Strong α-cut of a fuzzy set Let X be the universal set that represents age where X= $\{3,6,9,12,15,18,24\}$.Let the fuzzy sets be Toddler, Teenager and Youth. The ages and the various membership functions for the fuzzy sets are given as:

Table 1: Membership values of different fuzzy sets corresponding to age

Age(X)	Membership values of fuzzy sets		
	Toddler	Teenager	Youth
3	0	0	0
6	0	0	0
9	0	0.2	0
12	0	1	0.3
15	0	0.9	0.5
18	0	0.4	0.6
21	0	0	1
24	0	0	1

1) Support of teenager is $\overline{S}(\overline{Tennager})$= {9, 12, 15, 18}
2) Support of youth is $\overline{S}(\overline{Youth})$= {12, 15, 18, 21, 24}

3) Support of toddler is ϕ

4) α-cut of the fuzzy sets will be:

$teenager_{0.2}$={9,12,15,18}, $youth_{0.3}$={12,15,18,21,24} , $youth_{0.6}$={18,21,24}

5) Strong α-cut of the fuzzy sets will be

$$youth_{0.3'}=\{15,18,21,24\}$$

2.1.2 Defuzzification of a fuzzy number

Defuzzification can be defined as the reverse of fuzzification. Defuzzification aims in determining a real value which corresponds to a fuzzy number.

Let $\tilde{A}$ is a fuzzy number which is defined on □ then $h(\tilde{A})$ denotes the height of a fuzzy number which is defined as $h(\tilde{A})=sup_{x \in □}\ \mu_{\tilde{A}}(x)$. Let $M\ (\tilde{A}) = \{x\ \in □ \mid \mu_{\tilde{A}}(x) = □(\widetilde{A})\}$ defines a set of all the points for which the membership value is equal to height of$(\widetilde{A})$. Then the various defuzzification methods (Puri and Yadav, 2013) are:

1) **Middle / Mean of maximum (MOM) method:** The Mean of Maximum methods give the mean of all those points where the membership value is maximum.

   Mathematically the defuzzified value $d_{MOM}(\tilde{A})$ of $\tilde{A}$ is given by:

$$d_{MOM}(\tilde{A}) = \frac{\sum_{x \in M\ (\tilde{A})} x}{|M\ (\tilde{A})|}$$

   where |.| stands for the cardinality of the set.

2) **Largest of maximum (LOM) method:** The largest of maximum method results into the largest of all those points where the membership value is maximum

   Mathematically the defuzzified value $d_{LOM}(\tilde{A})$ of $\tilde{A}$ is given by:

$$d_{LOM}(\tilde{A}) = \max\{x \mid x \in M\ (\tilde{A})\}$$

3) **Smallest of maximum method (SOM) method:** The smallest of maximum method gives the minimum of all those points where the membership value is maximum.

   Mathematically the defuzzified value $d_{SOM}(\tilde{A})$ of $\tilde{A}$ is given by:

$$d_{SOM}(\tilde{A}) = \min\{x \mid x \in M\ (\tilde{A})\}$$

4) **Bisector / Centre of area (COA) method:** The bisector method gives the value that divides the region into two sub-regions of equal area

   Mathematically the defuzzified value $d_{COA}(\tilde{A})$ of $\tilde{A}$ is given

$$\int_{x_{min}}^{d_{COA}(\tilde{A})} \mu_{\tilde{A}(x)} dx = \int_{d_{COA}(\tilde{A})}^{x_{max}} \mu_{\tilde{A}(x)} dx$$

5) **Centroid/Centre of gravity (COG) method:** The centre of gravity method determines the centre of gravity of a fuzzy number.

   Mathematically the defuzzified value $d_{COG}(\tilde{A})$ of $\tilde{A}$ is given by:

$$d_{COG}(\tilde{A}) = \frac{\int x \, \mu_{\tilde{A}(x)} \, dx}{\int \mu_{\tilde{A}(x)} \, dx}$$

2.2 Fuzzy Linguistic Approach

The process of deciding and getting a result afterwards can be defined as the decision making process. In the process of decision making various alternatives are suggested and different people can have different views on which alternative is better than the other and at the last step of the decision making various methods are adopted to reach to the solution. In today's era most of the problems are qualitative and quantitative in nature .The assessment of the quantitative aspects are done with the numerical values and the assessment of qualitative aspects are done with the help of fuzzy linguistic approach.

Sometimes the decision making problems have linguistic and numeric information. This is so because there are several persons in the decision making problem each one having his/her alternative to solve the problem. The Multi-attribute decision making problems(MADM) problems comprises of both qualitative and quantitative attributes and deals with linguistic information as well as the numeric information.

2.2.1 2-tuple fuzzy linguistic representation model

Consider a set S that consists of the seven terms. The seven terms in the set S are defined as follows: S=$\{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$

Here the value of *i* ranges from 0, 1, 2, 3, 4, 5, 6 and *g* corresponds to the total number of the terms in the set .So here *g* = 6.

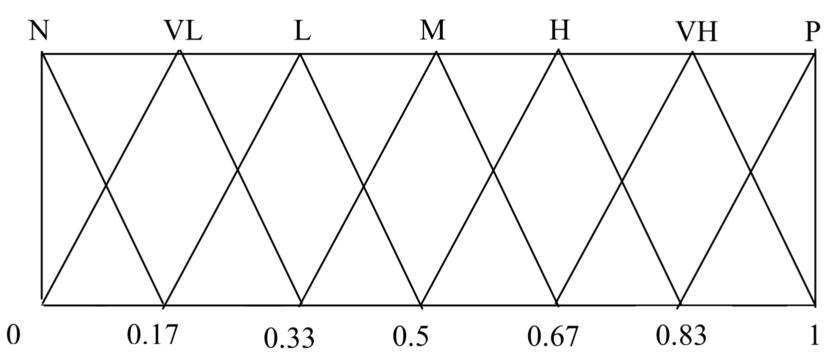

Figure 2 A set of seven terms with its semantics (Herrera and Martinez, 2000)

where N = None, VL = Very-Low, L = Low, M = Medium, H = High, VH = Very-High, P = Perfect.

The linguistic term set need to satisfy the following:

1. The negation operator *Neg* (s_i) =s_j where $j = g - i$. The cardinality of the set is g + 1 that is (6 + 1 = 7) as there are seven terms in the set S. For example,
   (a) Let $i = 3$, then $j = g - i = 3$ that is $j = 6 - 3 = 3$.Hence we conclude that in the linguistic term set S the negation of s_3 *is* s_3 which means that the negation of Medium is Medium.
   (b) Let us take i = 4, then $j = g - i$ that is $j = 6 - 4 = 2$. Hence in the linguistic term set S the negation of s_4 is s_2. Thus, the negation of s_4 = High is s_2 = Low.
2. There is the existence of the maximization and the minimization operator in the linguistic term set S that is $s_i \leq s_j$ *iff* $i \leq j$. For example, $s_3 \leq s_4$ *iff* $3 \leq 4$.

2.2.1.1 Basic Definitions

This section includes some basic definitions (Herrera and Martinez, 2000).

Definition 13 (Herrera and Martinez, 2000) The symbolic translation of a linguistic term $s_i \in S = \{s_0, s_1, s_2, ... s_g\}$ consists of a numerical value $\alpha_i \in$ [-0.5, 0.5) that supports the "difference of information" between a counting of information β assessed in [0, g] obtained after a symbolic aggregation operation (acting on the order index of the labels) and the closest value in $\{0, ..., g\}$that indicates the index of the closest linguistic term in S (s_i).

Hence a linguistic representation model is developed which represents the linguistic information with the help of 2 – tuples (r_i, α_i), $r_i \in$ S and $\alpha_i \in$ [-.5, .5)

Definition 14 (Herrera and Martinez, 2000) Let $s_i \in S$ be a linguistic term, then its equivalent 2 – tuple representation is obtained by means of the function θ as:

$$\theta : \text{S->(S} \times [-0.5, 0.5))$$

$$\theta : (s_i) = (s_i, 0) \text{ for all } s_i \in S$$

For example, the 2-tuple of the linguistic term High, i.e., H will be (H, 0) with the help of the function θ.

Definition 15 (Herrera and Martinez, 2000) Let S=$\{s_0, s_1, s_2 ... s_g\}$ be a linguistic term set and also let β ∈ [0, g] represents the value supporting the result of the symbolic aggregation operator. The 2-tuple that expresses the equivalent information to β is obtained with the following function:

Δ: [0, g] -> S × [-0.5, 0.5)

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

Here "round" refers to the rounding operation. s_i has the closest index label to β and α is the value of the symbolic translation.

For example, Let β = 4.3. Then, i = round (4.3) = 4 and $\alpha = \beta - i$ = 4.3 − 4 = 0.3.Therefore the 2-tuple that represents the equivalent information to β = 4.3 is (H, 0.3).

Definition 16 (Herrera and Martinez, 2000) Let the linguistic term set be defined as S=$\{s_0, s_1, s_2 \dots s_g\}$ and the 2-tuple be defined as (s_i, α). A function Δ^{-1} is there such that, from a 2-tuple it returns its equivalent value $\beta \in$ [0, g] that is the subset of ℝ. Consider Δ^{-1}: S × $[-.5, .5)$-> [0, g] defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$.

Hence in order to get numerical value β from the 2-tuple function Δ^{-1} is used.

For example**,** From the previous example only the linguistic 2-tuple is taken as (H, 0.3).Substituting the value in above equation we get $\Delta^{-1}(s_4, \alpha) = i + \alpha = 4 + 0.3 = 4.3 = \beta$.

Remark: With the help of lexicographic order the comparison of (s_i, α_1) and (s_j, α_2) can be given as follows:

1. When $i < j$ then(s_i, α_1) will be less than (s_j, α_2).
2. When i and j are equal then under the following three cases we have
   i. When both α_1 and α_2 are equal then the same information is represented by the two tuples (s_i, α_1) and (s_j, α_2).
   ii. When α_1 is less than α_2 the two tuple (s_i, α_1) will be smaller than the two tuple (s_j, α_2).
   iii. When α_1 is greater than α_2 the two tuple (s_i, α_1) will be larger than the two tuple (s_j, α_2).

Definition 17 (Herrera and Martinez, 2000) **(Negation Operator)** $Neg(s_i, \alpha) = \Delta(g - (\Delta^{-1}((s_i, \alpha)))$.

For example, let 2-tuple be (H, 0.3). Then,

$$Neg(H, 0.3) = \Delta(6 - (\Delta^{-1}((s_4, 0.3))) = \Delta(6-4.3) = \Delta(2.7) = (M, -0.3).$$

Definition 18 (Herrera and Martinez, 2000) Consider the set of the 2-tuples which is defined as $\{(r_1, \alpha_1), (r_2, \alpha_2) \dots (r_m, \alpha_m)\}$.Then the computation of the arithmetic mean is done as $\overline{x} = \Delta\left(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n} \sum_{i=1}^{n} \beta_i\right)$

For example, Consider the linguistic term set S where S=$\{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$. $\{s_2 = L,\ s_4 = H,\ s_5 = VH,\ s_6 = P\}$ are the terms that are aggregated with the help of the aggregation operator $\overline{x}$. Firstly the 2-tuple representation of the linguistic terms of the set S with help of function θ is obtained. As θ (s_i) = (s_i, 0) for all s_i ∈S. Applying θ function to $\{s_2 = L,\ s_4 = H,$ $s_5 = VH, s_6 = P\}$ the obtained set is $\{(P, 0), (L, 0), (H, 0), (VH, 0)\}$. Now,

$$\overline{x} = \Delta\left(\frac{1}{4}(6 + 2 + 4 + 1)\right) = \Delta\left(\frac{13}{4}\right) = \Delta\,(3.25)$$

$$\Delta\,(3.25) = \begin{cases} s_3 & i = round(3.25) = 3 \\ \alpha = 0.25 & \alpha \in [-.5, .5) \end{cases} = (s_3, \alpha) = (M, 0.25)$$

Definition 19 (Herrera and Martinez, 2000) Let $v \in$ [0, 1] and $S = \{s_{0,}\ s_1, s_2 \dots, s_g\}$ be a numerical value and a linguistic term set respectively. v is transformed into a fuzzy set in S with the function τ defined by

τ : [0,1] -> F(S) where τ (v)=$\{(s_0,\ w_0), (s_1,\ w_1), \dots, (s_g,\ w_g)\}$, where $s_i \in$ S and $w_i \in$ [0,1] such that

$$\alpha_i = \mu_{s_i}(v) = \begin{cases} 0, & if\ v \notin support\ \mu_{s_i}(x) \\ \frac{(v-a_i)}{(b_i-a_i)} & if\ a_i \le v \le b_i \\ 1 & if\ b_i \le v \le d_i \\ \frac{(c_i-v)}{(c_i-d_i)} & if\ d_i \le v \le c_i \end{cases}$$

Definition 20 (Herrera and Martinez, 2000) Let τ(v) = $\{(s_0,\ w_0), (s_1,\ w_1), \dots, (s_g,\ w_g)\}$ be a fuzzy set which is representing the numerical value v $\in [0, 1]$ over S $= \{s_{0,}\ s_1, s_2 \dots,\ s_g\}$ which is the linguistic term set. A numerical value is obtained that represents the information from fuzzy set which is assessed in [0, g] with the help of function χ.

$$\chi : F\,(S_T) \to [0, g]$$

$$\chi\,(\tau\,(v)) = \chi\,(\{(s_j, w_j)\,, j=0,.., g\}) = \frac{\sum_{j=0}^{g} j w_j}{\sum_{j=0}^{g} w_j} = \beta$$

Then with the help of Δ, β is transformed into linguistic 2-tuple.

Definition 21 (Herrera and Martinez, 2000) Let the linguistic 2-tuple be $(s_{k\,,}\alpha)$ which is based on the symbolic translation where $s_k \in$ S=$\{s_{0,}\ s_1,\ s_2 \dots,\ s_g\}$ and α ∈ [-.5, .5) and its equivalent value in numerical form is $\Delta^{-1}((s_{k\,,}\alpha))$ = β with β ∈ [0, g].The function δ gives two 2-tuples which are based on membership degree from initial linguistic 2-tuple which support same counting of information.

$$\delta : [0, g] \text{->} \{s_T \times [0,1]\} \times \{s_T \times [0,1]\} \text{ defined by}$$

$$\delta(\beta)=\{(s_{\square}, 1-\gamma)\}, (s_{\square+1}, \gamma)\}$$

where h = *trunc* (β) *and* $\gamma = \beta - h$. Here *trunc* refers to the truncation operation.

Definition22 (Herrera and Martinez, 2000) Let ($s_{\square}$, 1 - γ) and ($s_{\square+1}$, γ) be two 2-tuples based on the degree of membership their equivalent numerical value that is assessed in [0, 1] is obtained by using k as:

$$\mathrm{K} : \{s_T \times [0,1]\} \times \{s_T \times [0,1]\} \text{->} [0, 1] \text{ defined by}$$

$$\mathrm{K}\ ((s_{\square}, 1-\gamma), (s_{\square+1}, \gamma)) = CV\ (s_{\square})\ (1 - \gamma) + CV\ (s_{\square+1})\ (\gamma), \text{where CV}$$

(.) is a function providing a characteristic value and it is defined by any of defuzzification methods (Puri and Yadav, 2013).

2.2.1.2 The 2-tuple fuzzy linguistic representation model to transform [0, 1] into linguistic 2-tuple

Consider a numerical value v ∈ [0, 1] and a linguistic term set S=$\{s_{0,}\ s_{1,}\ s_2 \ldots, s_g\}$. The schematic view of the transformation is given by Figure 3.

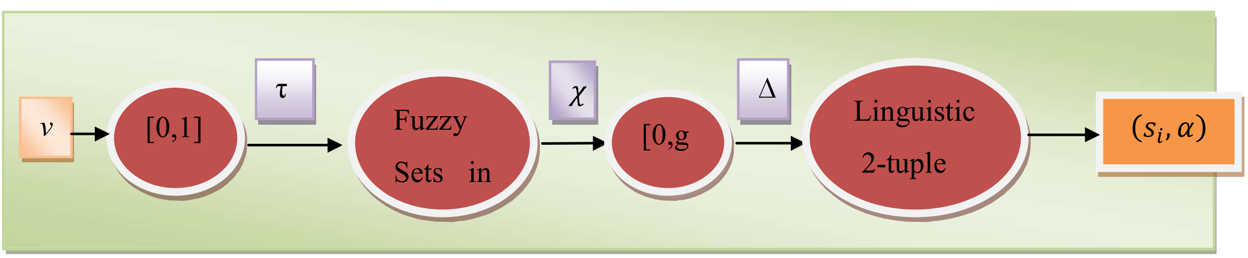

Figure 3 Transformation of value in [0,1] into linguistic 2-tuple.(Herrera and Martinez, 2000)

The procedure to transform v into linguistic 2-tuple (s_i, α) is given below:

Step 1 Convert the exact value of v into a fuzzy set in S, by using the function τ given in the Definition 19. Here v ∈ [0, 1] is transformed into a fuzzy set in F(S) by computing the membership values of v in the function associated with the linguistic terms of S.

Step 2 Transform a fuzzy set in S (obtained in Step 1) into a linguistic 2-tuple assessed in S. For this transformation we use a function χ defined in the Definition 20 that computes the central numerical value (β) from the membership degrees of the fuzzy set (τ (v)) by using a weighted average function.

The central value is evaluated in the interval [0, g]. Further the value β can easily be transformed into a linguistic 2-tuple using the function Δ defined in the Definition 15. For example, Suppose v = 7.8.Now there is a need to transform v into a fuzzy set. Here five linguistic terms set is taken and then five fuzzy sets of v are computed.

Table 2 Linguistic term set

	S_1
L	(0,0,0.3)
VL	(0,0.3,0.5)
M	(0.2,0.5,0.8)
H	(0.5,0.7,1)
VH	(0.7,1,1)

Source: Herrera and Martinez (2000)

Subsequently we get:

$\tau_{S_1}(.78)=\{(VL, 0), (L, 0.0), (M, 0.06), (H, 0.73), (VH, 0.26)\}$

The above values are obtained by the formula

$$\alpha_i = \mu_{s_i}(v) = \begin{Bmatrix} 0, & if\ V\ \notin support\ \mu_{s_i}(x) \\ \frac{(v-a_i)}{(b_i-a_i)} & if\ a_i \leq v \leq b_i \\ 1 & if\ b_i \leq v \leq d_i \\ \frac{(c_i-v)}{(c_i-d_i)} & if\ d_i \leq v \leq c_i \end{Bmatrix}$$

For example,

Step1 Transformation of *v* into a fuzzy set in S.

Consider the set S_1 and in it consider Medium (*M)* having semantics as $(0.2, 0.5, 0.8)$.

Here a = 0.2, b = 0.5, d = 0.5 and c = 0.8. By the above formula, it is obvious that the value of v lies between d and c .So by using the formula

$$\frac{(c_i - v)}{(c_i - d_i)} if\ d_i \leq v \leq c_i$$

we get $\frac{0.8-0.78}{0.8-0.5}$=0.0666 when $d_i \leq v \leq c_i$

Hence the value of v has been transformed into a fuzzy set in S.

Step 2 Transformation of the Fuzzy set in S into linguistic 2-tuples assessed in S

The function χ gives central value from the membership degrees of τ (v) that is the fuzzy set with help of weighted average function. The central value obtained is assessed in [0, g] which is the interval of granularity.

For example, with the help of Definition 20**,** the function χ is used firstly to obtain β

$\chi(\tau_{S_1}(.78))=\chi\{(VL,0),(L,0.0),(M,0.06),(H,0.73),(VH,0.26)\}=3.19$

The 2-tuple are obtained by applying function

$$\Delta\,(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5,.5) \end{cases} \text{ as:}$$

$\Delta(3.19)=(H,0.19)=S_1$.

2.2.1.3 The 2-tuple fuzzy linguistic representation model to transform linguistic 2-tuple into [0, 1] value

Let (s_i,α) be a linguistic 2-tuple which is to be transformed into a value $v \in [0, 1]$.The graphical representation of the transformation is as follows.

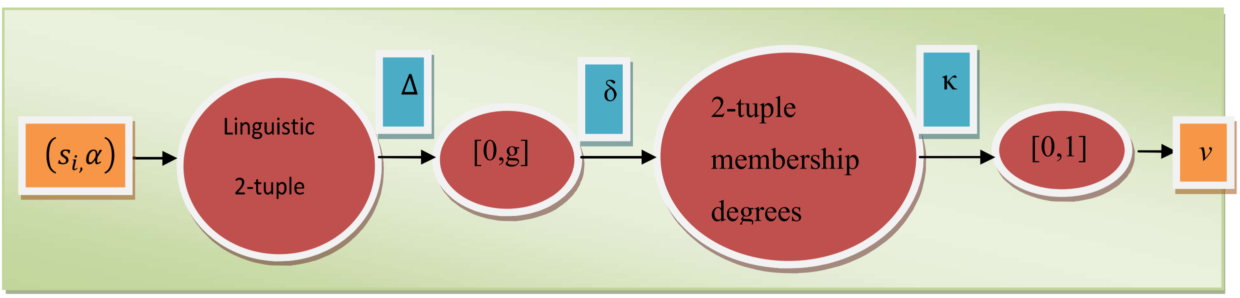

Figure 4 Transformation of linguistic 2-tuple into [0,1] value (Herrera and Martinez, 2000)

Now, the procedure of the transformation is given below.

Step 1 Transform of linguistic 2-tuple (s_i,α) into two 2-tuples $\{(s_k,\ w_k)\},(s_{k+1},\ w_{k+1})\}$ using the function δ defined in the Definition 21. Furthur these two 2-tuples based on the membership degree will be used to evaluate the numerical value in [0, 1] by using the function κ defined in the Definition 22.

For Example, Consider the 2-tuple (*H*, 0.19) which is to be transformed into a numeric value *v* in [0, 1].

Firstly the function Δ^{-1} is used as:$\Delta^{-1}(s_i\,,\alpha)$ = *i* + *α* = *β,* hence Δ^{-1}*(H,0.19)*= 4 + 0.19= 4.19 = *β*

Then the function δ is used in order to transform the value of *β* = *4.19* into two 2-tuples as*:*

δ (β) = $\{(s_h,\ 1-\gamma)\},(s_{h+1},\ \gamma)\}$ where h=trunc(β) and *γ* = *β* - *h*

Hence h = trunc(4.19) = 4, and *γ* = *4.19 – 4* = *0.19.*

Therefore by the formula *of* δ we get δ (4.19) = $\{(s_4,\ 1-0.19)\},(s_{4+1},\ 0.19)\}$

$= \{(s_4,\ 0.81)\},(s_5,\ 0.19)\}$

$= \{(H,\ 0.81)\}, (VH,\ 0.19)\}$

After the two 2-tuples are obtained their equivalent numerical value is obtained with the help of the function κ as κ (($s_h,\ 1-\gamma$), (s_{h+1}, γ)) =CV (s_h) (1 - γ) +CV (s_{h+1}) (γ)

Here CV represents the characteristic value.

Hence we obtain κ $\{(H,\ 0.81)\}, (VH,\ 0.19)\}$ = CV(s_4)(1-0.19)+CV(s_5)(0.19)

= *(0.67)(0.81) + (0.83)(0.19)*

= 0.7004 which is giving the numerical value of the 2-tuple *(H, 0.19*).

2.2.1.4 Conditions for the transformation of the values in [0, 1] into the linguistic 2-tuples having no loss of information

Some conditions can be imposed on the set S consisting of the linguistic terms such that no information is lost after performing the desired transformations.

Theorem 1 (Herrera and Martinez, 2000) Consider the linguistic term set $S = \{s_0,\ s_1,\ s_2\ ...\ s_g\}$.The necessary and the sufficient conditions for the exact transformation between the linguistic 2-tuples and the [0, 1] values and between the [0, 1] values and the linguistic 2-tuples are as follows:

1) S is a fuzzy partition. A finite family $S = \{s_0,\ s_1,\ s_2\ ...\ s_g\}$ of the fuzzy subsets in the universe of discourse X where X = [0, 1] is a fuzzy partition if $\sum_{i=0}^{g} \mu_{s_i}(x) = 1$, for all $x \in$ X.
2) There should be triangular membership functions.
3) The CV (s_i) = $x/\mu_{s_i}(x)$ = 1 that is the characteristic value function used returns the value with the maximum membership degree b_i.

   If all the above conditions are verified then there will be no loss in the information of *v* after performing the desired transformation

$$κ\ (\delta(\chi((τ\ (v)))) = v$$

CHAPTER 3

Aggregation Operators

3.1 Ordered Weighted Average Operator

Ordered Weighted Averaging (OWA) operator was presented by Yager (1988). In order for the determination of the weights of the OWA operator O'Hagan (1988) was able to introduce the maximum entropy by using the degree of orness. There are various families of the OWA operators which includes the S-OWA, Maximum entropy and may more that were introduced by Yager in 1993. Yager was also able to introduce the intention of the aggregate dependent weights. Fodor et al.(1995) classified the aggregation operators as neutral and monotonic. Mitchell and Estrakh (1996) gave a proposal of the modified OWA operator. In the modified OWA operator by Mitchell and Estrakh the input arguments were rearranged according to their estimated relative values and not according to their actual relative value. Mitchell and Estrakh also gave the application of the modified OWA operator. Yager and Filev (1999) were able to present the Induced OWA operators which were further used for presenting various kinds of the aggregation models. Fuller and Majlender (2001) gave a method for the determination of the weights. The method of the Langrange multipliers was used in order to solve the constrained optimization problem.

Xu and Da (2002) inquired the uncertain OWA operators where every input argument is presented as an interval that has numeric values and the value ranges could be obtained.Pereira and Riberio (2003) were able to present the weighted aggregation operators in (MADM) i.e multiple attribute decision making with an objective to investigate various ways in which weights could depend upon the degree of satisfaction Xu (2006) gave various new aggregation operators that included uncertain linguistic weighted geometric mean (ULWGM) operator, uncertain linguistic geometric mean(ULGM) operator , induced uncertain linguistic ordered weighted geometric (IULOWG) operator and uncertain linguistic ordered weighted geometric (ULOWG) operator. Merig and Lafuente (2009) gave the (IGOWA) Operator i.e the induced generalised ordered weighted averaging operator. The main features of induced OWA and OWA were included in IGOWA. Merig and Lafuente also dicussed about the Quasi-IOWA operator. Merig and Casanovas(2011) dicussed about the (IOWAD) operator or in other words induced ordered weighted averaging distance operator. (IOWAD) operator is having a lot of merits. A new model using induced ordered weighted averaging (IOWA) operator and the weighted average (WA) were investigated by Merigo(2011).The intuitionistic linguistic generalized dependent hybrid weighted aggregation (ILGDHWA) operator and the intuitionistic linguistic generalized dependent ordered weighted average (ILGDOWA) operator were given by Liu(2013). He et al.(2015) gave an Ordered weighted averaging operator for a social network.

Definition 23 (Ordered Weighted Averaging Operator) (Verma and Puri, 2017) The OWA operator that has a dimension n defines a map F: $R^n \rightarrow$ R with associated weighting vetor W = $(w_1, w_2, \dots, w_n)^T$ such that

$$\sum_{i=1}^{n} w_i = 1 \text{ and } 0 \leq w_i \leq 1 \text{ i} = 1, 2, 3, \dots, n.$$

and F$(a_1, a_2, \dots, a_n) = \sum_{i=1}^{n} w_i b_i$ where b_i is the *ith* largest of $a_1, a_2, a_3, \dots, a_n$

For example (Yager, 1988) Assume F is an ordered weighted averaging operator of size n=4 with the weighting vector, W $= (0.2, 0.3, 0.1, 0.4)^T$. Then the aggregation of $(0.6, 1, 0.3, 0.5)^T$ is done in the following manner:

The ordered argument vector B for the given argument vector $(0.6, 1, 0.3, 0.5)^T$ is B = $(1, 0.6, 0.5, 0.3)^T$ and the weighted vector W is given by [0.2, 0.3, 0.1, 0.4] .By applying the OWA aggregation operator (0.2) ×(1) + (0.3) × (0.6) + (0.1) × (0.5) + (0.3) × (0.4) = 0.55

Definition 24 (Induced Ordered Weighted Average Operator) (Merig and Gil – Lafuente, 2009) An induced OWA (IOWA) of dimension n is a mapping $F_I : R^n \rightarrow$ R defined by an associated weighting vector W = $(w_1, w_2, w_3, \dots w_n)^T$ of dimension n such that $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$ and a set of ordered-inducing variables u_i by the formula of the following form:

$F_I(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{i=1}^{n} w_i b_i$

where $(b_1, b_2, b_3, \dots, b_n)$ is simply $(a_1, a_2, a, \dots, a_n)$ reordered in decreasing order of the values of u_i, u_i is the order-inducing variable and a_i is the argument variable.

Definition 25 (Degree of Orness) (Verma and Puri, 2017) Let F be an OWA operator.

The orness is the value that lies in an interval [0, 1].The degree of orness denoted by α, is defined as: α where α = $\frac{1}{n-1} \sum_{i=1}^{n}((n-i)\, w_i)$

3.1.1 Properties of OWA aggregation operator

The OWA aggregation operators possess the properties like commutativity, monotonicity, idempotency and boundedness. The properties of the OWA aggregation operators are discussed in the following theorems.

Theorem 2 (Verma and Puri, 2017) Assume that F is an OWA operator. Let A= $[a_1, a_2, a_3, \dots, a_n]^T$ be an ordered argument vector. Let B = $[b_1, b, b_3, \dots, b_n]^T$ be a second ordered argument vector such that for each j, $a_j \geq b_j$ then $F(A) \geq F(B)$.

Proof since $F(A) = W^T A$ and $F(B) = W^T B$. The result follows directly from the property that $a_j \geq b_j$.

Theorem 3 (Verma and Puri, 2017) Assume that F is an OWA operator.

Then $F(a_1, a_2, a_3, \dots, a_n)$=$F(a'_1, a'_2, a'_3, \dots, a'_n)$ where $(a_1{}', a_2{}', \dots a_n{}')$ is any permutation of the element in $(a_1, a_2, a_3, \dots, a_n)$.

Proof If B and B' are the ordered argument vectors of $(a_1, a_2, a_3, \dots, a_n)$ and $(a'_1, a'_2, a'_3, \dots, a'_n)$ then $B = B'$. Therefore, $F(B) = F(B')$.

Theorem 4 (Verma and Puri, 2017) All the OWA operators are idempotent in the sense that if $a_j = a$, for all j = 1, 2, 3,..., n, then $F(a_1, a_2, a_3, \dots, a_n) = a$.

Proof From the definition,

$$F(a_1, a_2, a_3, \dots, a_n) = \sum_{j=1}^{n} w_j b_j$$

As $a_j = a$ then also $b_j = a$

$$\Rightarrow F(a_1, a_2, a_3, \dots, a_n) = \sum_{j=1}^{n} w_j a$$

As $\sum_{i=1}^{n} w_i = 1$,

$$\Rightarrow F(a_1, a_2, a_3, \dots, a_n) = a.$$

Theorem 5 (Verma and Puri, 2017) Assume that $(a_1, a_2, a_3, \dots, a_n)$ is the collection of numbers each lying in the unit interval, then

(a)$F_*(a_1, a_2, a_3, \dots, a_n) = Min_j(a_j)$

(b)$F^*(a_1, a_2, a_3, \dots, a_n) = Max_j(a_j)$

Proof (a) The weighting vector W_* defined such that

$w_n = 1$ and $w_i = 0$ for i $\neq$ n gives the aggregation $F_*(a_1, a_2, a_3, \dots, a_n) = Min_j[a_j]$

(b) The weighting vector W^*defined such that $w_1 = 1$and $w_j = 0$ for all $j \neq i$ gives the aggregation $F^*(a_1, a_2, a_3, \dots, a_n) = Max_i[a_i]$

Therefore W^* provides the largest aggregation of the arguments.

Theorem 6 (Verma and Puri, 2017) Assume B is an arbitrary ordered input vector. Then for any weighted vector W,

$(W_*)^T B \leq W^T B \leq (W^*)^T B$

Proof $(W_*)^T B \leq W^T B$

$(W_*)^T B = b_n$

$$W^T B = \sum_n w_j b_j = b_n w_n + \sum_{j=1}^{n-1} w_j b_j$$

Since B is an ordered input vector then $b_j \geq b_k$ for $K > J$. In particular, $b_n < b_j$ for *j=1, 2, 3,. .., n -1,* hence

$$W^T B \geq b_n w_n + b_n \sum_{j=2} w_j$$

However,

$$\sum_{j=1}^{n-1} w_j = 1 - w_n$$

$$W^T B \geq b_n w_n + (1 - w_n) b_n \geq b_n \geq (W_*)^T \mathrm{B}$$

(b) $(W^*)^T B \geq W^T B$

$(W^*)^T B = b_1$

$$W^T B = \sum_n w_j b_j = b_1 w_1 + \sum_{j=2}^{n} w_j b_j$$

Since $b_i \geq b_j$ then

$$W^T B \leq b_1 w_1 + b_1 \sum_{j=2}^{n} w_j \leq b_1$$

3.1.2 Computation of weights of OWA aggregation operator

3.1.2.1 Weight determination using linguistic quantifiers

Quantifier can be defined as a word that is used to depict the amount or the extent of it that is being considered. There are a diverse range of quantifiers like "a few "Most", "At least half", "As many as possible". For representing various quantifiers Zadeh in 1983 introduced linguistic quantifiers and distinguished two types of linguistic quantifiers:

(a) Absolute Quantifiers: The absolute quantifiers represent the quantities that are absolute in nature.

Example: Less than 7, More than 5

(b) Proportional Quantifiers: In proportional quantifiers the quantities are represented that are relative in nature.

Example: Most, At least, At most.

Definition 26 A nondecreasing proportional quantifier satisfies for all a, b if $a > b$ then $Q(a) \geq Q(b)$. The membership function of a nondecreasing proportional quantifier is:

$$Q(r) = \begin{cases} 0 & if\ r < a \\ \frac{r-a}{b-a} & if\ a \leq r \leq b \\ 1 & if\ r \geq b \end{cases}$$

The linguistic quantifiers are used to compute the weights of the OWA aggregation operator. For the nondecreasing proportional quantifier Q the formula to compute the weight is given by:

$$w_i = Q\left({}^{i}/_{n}\right) - Q\left({}^{(i-1)}/_{n}\right),\ i = 1,2,..n$$

3.1.2.2 Weight determination using Minimax disparity approach

The main problem in the ordered weighted average operator is the determination of the weights. The Minimum Variance approach and the Maximum Entropy approach makes the weights closer to one another when the degress of orness is considered. The minimax disparity approach(Wang and Parkhan,2005) also aims to make the disparities as small as they could be between the two adjacent weights.

Model-1

$$\text{Minimize}\{Max_{i \in \{1,2,3....,n-1\}} |\ w_i - w_{i+1}|\}$$

$$\text{Subject to orness (W)} = \alpha = \frac{1}{n-1} \sum_{i=1}^{n} (n-i)\, w_i,\ 0 \leq \alpha \leq 1,$$

$$\sum_{i=1}^{n} w_i = 1,\ 0 \leq w_i \leq 1,\ i=1,2,3...n$$

Suppose that the maximum disparity that is between the adjacent OWA weights is given by: $\delta = Max_{i \in \{1,2,3....,n-1\}} |\ w_i - w_{i+1}$

It implies that

$$|\ w_i - w_{i+1}| \leq \delta, \quad i=1,2,3,...n-1.$$

$$\Rightarrow -\delta \leq w_i - w_{i+1} \leq \delta, \quad i=1,2,3,...n-1.$$

$$\Rightarrow w_i - w_{i+1} - \delta \leq 0, \quad i=1,2,3,...n-1.$$

Also, $w_i - w_{i+1} - \delta \geq 0, \quad i=1,2,3,...n-1.$

Hence the minimax disparity Model-1 can be reduced to the below form as:

Model-2

Minimize δ

With the Orness given by Orness(W) $= \alpha = \frac{1}{n-1} \sum_{i=1}^{n} (n-i)\, w_i,\ 0 \leq \alpha \leq 1,$

$$\sum_{i=1}^{n} w_i = 1,\ 0 \leq w_i \leq 1,\ i=1,2,3...n$$

$$w_i - w_{i+1} - \delta \leq 0 \qquad i=1,2,3,...n-1.$$

$$w_i - w_{i+1} - \delta \geq 0 \qquad i=1,2,3,...n-1.$$

$$w_i \geq 0, i=1.2.3....n.$$

The following are some characteristics features obtained after using the minimax disparity approach explained with the help of a theorem.

Theorem 7 For an OWA operator weight vector W = $(W_1, W_2, W_3, \ldots W_n)^T$ that is determined in the model 2,

I. If orness(W) = 1, then W = $(1,0,0, \ldots 0)^T$.

II. If orness(W) = 0, then W = $(0,0,0, \ldots 1)^T$.

III. If orness(W) = 0.5, then W =$\left(1/n, 1/n, 1/n, \ldots 1/n\right)^T$.

CHAPTER 4

Multi-stage Multi-objective Production Planning in Uncertain Environment

4.1 Introduction

In this chapter a methodology is introduced in order to solve a Multi-stage production planning problem having multiple objectives. There are several stages of the production process and every stage has either one machine or may have more than one machine. The objectives in the production planning problem are non commensurable, fuzzy and are not compatible with one another. In each of the processing stage either work in process goods are produced or the semi-finished goods are produced which acts as the output to the stage at which they are produced. These goods that are the output to the previous stage become the input to the next stage. However if there is not a balance in the relation of the input-output or if the machine fails to function (machine breakdown) at any of the stage occurs then the production target is disturbed to a great extent and the desired target of the production is not achieved.

Hence in order to achieve the desired production target a methodology is provided so that a balance is maintained in the I/O relation at every stage of the production. This is accomplished with the help of WIP i.e the Work-in-process inventory and by installing new machines at the required stage. The goals here are represented with the help of the fuzzy sets. For the aggregation of the accordance objectives in the order of their priorities the Induced Ordered Weighted Averaging (IOWA) operator has been used here. The formulation of the production process is done with the help of mixed integer programming problem.

4.1.1 Complications in the production process faced by the firm

A lot of complications in the production process are faced by the firm some of which are listed below:

1. If there is not a balance in the Input-Output between the machines at various stages then it may interrupt the arrangement of the machines and the flow of the operations.

2. There are multiple objectives of the production process that are not compatible with one another.
3. The capacity of the machine will not be determined if the machines do not function properly hence the machine suffers due to the idle time and the production target is adversely affected.
4. It is a very hard task to determine the WIP i.e work-in –process inventory and also the requirement of the new machines at any stage of the production planning.

The above problems are solved with the help of goal programming.

4.1.2 Objectives/Goals to be kept in mind in order to have an efficient production.

1. To maximize the production.
2. To maintain the work in process inventory.
3. To minimize the cost of the work-in-process inventory.
4. To minimize the installation cost of the new machines.

All the objectives that are listed above are expressed in fuzzy terms. This is so because:

1. It is cumbersome to know the exact capacity of the machine if the machine does not function properly. Hence it is expressed in the fuzzy terms.
2. It becomes very difficult to give an exact amount of production target. Hence the production target is expressed with the help of the linguistic terms that are fuzzy in nature.
3. The deviation from the production target may occur due to the increase in the cost of the things that are used in the production. Hence the target values are expressed as the vague terms.

The below are the four objectives/goals that are given by the firm. The first goal represents the highest priority and consequently the last goal represents the lowest priority. The words that are represented in italics below are fuzzy in nature.

1. 10746 should be the amount of the production target. A *small* amount of variation can be there.
2. There should be 10 units of WIP i.e the work-in-process inventory. A *little* variation of 10 *units* can be there however the variation should not exceed 20 units.
3. The cost of the Inventory holding should *be around half of the current value* of the inventory holding cost (6846).
4. The cost of the installation of the new machine *shouldn't be higher than 747.*

Here the fuzzy set S has nine linguistic terms which are as follows:

S=$\{s_0 = IMP,\ s_1 = NL,\ s_2 = VL,\ s_3 = L, s_4 = M, s_5 = H, s_6 = VH, s_7 = SH, s_8 = EH\}$

For Example, Now the first goal is that the target of production should be 10746, however a *small deviation* is possible. This means that there should not be so much deviation from the production target of 10746 to be produced. Hence the linguistic term Very Low fits exactly in this context which is written as $(VL, -0.1)$.This means that the deviation is 10% lower than the linguistic term VL.

By using (Herrera and Martinez, 2000) the linguistic 2-tuple $(VL, -0.1)$ is transformed into [0,1] values and the result comes out to be 0.238.The 2-tuple $(VL, -0.1)$ or its numerical value 0.238 is representing the deviation that is allowed for the production target of 10746.Hence the least level of production that is acceptable to the firm is $(1 - 0.238)\times$ 10746 = 8188.

The membership function given below defines the acceptance of the production level 'P'.

$$\mu_p(P) = \begin{cases} 0, & if\ P\ < 8188 \\ \frac{P-8188}{2556}, & if\ 8188\ \leq P\ \leq 10746 \\ 1, & if\ P\ \geq 10746 \end{cases}$$

The membership function defined above means

1. If the production target is 10746 then there will be full satisfaction of the firm.
2. If the production target is less than 10746 then there will be decrease in the satisfaction of the firm.
3. The satisfaction level will be 0 if the production target is either 8188 or is less than 8188.

For the other goals in a similar manner firstly the 2-tuple is constructed and then the 2-tuple is transformed into [0, 1] values and then the membership function of the objectives/goals can be defined.

4.1.3 Multi-Stage Production Planning

Let *s* represents the number of the stages in the multi-stage production.

In the first stage the raw materials are given as an input to the machine(s) and the finished product is manufactured at the concluding stage .In order to obtain the desired production target of 10746 and to maintain the I/O relation the installation of new machines at the required stages is of great importance.

Notations

$\boldsymbol{s_i}$ = Combined monthly operational days of machine in the ith stage.

$\boldsymbol{y_i}$ = Bulk of inventory at the ith stage

$\boldsymbol{m_i}$ = Whether there is the installation of the new machine at the ith stage or there is not the installation of the new machine at the ith stage.

$\boldsymbol{i_i}$ = Whether to buy the inventory at the ith stage or not to buy the inventory at the ith stage.

$\boldsymbol{z_i}$ = The utilization of the capacity of the new machine at the stage i.

$\boldsymbol{p_{n_i}}$ = The rate of monthly production at the ith stage of new machines.

$\widetilde{\boldsymbol{p_{ij}}}$= The rate of daily production of the existing machine (j) at the ith stage.

$\widetilde{\boldsymbol{c_i}}$= The cost of holding the inventory at the ith stage

$\widetilde{\boldsymbol{c_{n_i}}}$= The new machine installation cost at the ith stage.

$\boldsymbol{h_i}$ = Cardinality of the existing machine at the ith stage**.**

$\boldsymbol{a_{ij}}$ = The effective production capacity of the machine j at the stage i is the number of the operational days in a month when the machine do not function properly.

Here $\widetilde{p_{ij}}$, $\widetilde{c_i}$ and $\widetilde{c_{n_i}}$ are expressed as the fuzzy numbers.

4.1.3.1 Resource constraints in the production planning problem

(a) Machine Capacity Constraints

Let h_i be the number of the existing machines at the ith stage.

Auxiliary Capacity: The minimum of the capacities of the existing machines is defined as the auxiliary capacity and it is represented by a_i.

Auxiliary capacity = $a_i = min_{j=1,2,\dots h_i}(a_{ij})$

Auxiliary production Rate: The sum of the equivalent production rates of the machines corresponding to the auxiliary machine capacity is termed as auxiliary production rate. It is represented by p_i.

Auxiliary production Rate = $p_i = \sum_{j=1}^{h_i} (p_{ij} \times a_{ij}/a_i)$

For example, Consider three machines at a production stage and their daily production rate, capacity is given by (240.66 / 22), (160.66 / 23), (90.66 / 24). Now the Auxiliary Capacity and the Auxiliary production Rate will be

Auxiliary capacity = $a_i = min_{j=1,2,\dots h_i}(a_{ij})$ = min (22, 23, 24) = 22 days.

Auxiliary production Rate =$p_i = \sum_{j=1}^{h_i} (p_{ij} \times a_{ij}/a_i) = \frac{240.66 \times 22}{22} + \frac{160.66 \times 23}{22} + \frac{90.66 \times 24}{22} = 507.52$

Hence the machine capacity constraint for every stage is $s_i \leq a_i$ (i = 1, 2,..s)

(b) New machines installation constraints

$\sum_{i=1}^{s} m_i \leq$ New machines

(c) Maximum allowable inventory

$\sum_{i=1}^{s-1} y_i \leq$ Allowable Inventory

4.1.3.2 Operational constraints

If the capacities of the existing machine are exhausted then only the inventory or new machine should be bought. This is ensured by the operational constraint.

Let t_i be the unutilised capacity which is given by $t_i = a_i - s_i$ where a_i is the machine capacity constraint and s_i is denoting the capacities that are utilised at the ith stage.

(a) Decision whether to buy or not to buy the new machine

The new machine should only be bought if the capacities of the existing machine are exhausted. Hence the production firm should not bring the new machine ($m_i = 0$) if some unutilised capacity ($t_i > 0$) is still there.

$$t_i \leq l \times (1 - m_i)$$

Here $$m_i = \begin{cases} 1, & \text{if the new machine is bought} \\ 0, & \text{if the new machine is not bought} \end{cases}$$

And l represents a large number.

Now if $m_i = 1$ then $t_i \leq l \times (1 - m_i) \leq l \times (1 - 1) \leq 0$.

So when $m_i = 1$ then $t_i \leq 0$, which means that new machine should be bought if there is no unutilised capacity.

If $m_i = 0$ then $t_i \leq l \times (1 - 0) \leq l$

So when $m_i = 0$ then $t_i \leq l$, which means that the new machine should not be bought if there is some unutilised capacity.

(b) Decision to buy or not to buy the inventory

The inventory should only be bought at a stage when there is full utilization of the existing capacity. So the production firm should not bring inventory ($i_i = 0$) when there is unutilized capacity ($t_i > 0$).

$$t_i \leq l \times (1 - i_i)$$

Here

$$i_i = \begin{cases} 1, & if\ the\ inventory\ is\ bought\ at\ the\ ith\ stage \\ 0, & if\ the\ inventory\ is\ not\ bought\ at\ the\ ith\ stage \end{cases}$$

and *l* represents a large number.

Now if $m_i = 1$ then $t_i \leq l \times (1 - i_i) \leq l \times (1 - 1) \leq 0$.

So when $m_i = 1$ then $t_i \leq 0$, which means that the inventory is bought as there is no unutilised capacity.

If $m_i = 0$ then $t_i \leq l \times (1 - 0) \leq l$

So when $m_i = 0$ then $t_i \leq l$, which means that the inventory should not be bought as there is some unutilised capacity.

4.1.3.3 The objective functions of the Production Process

(a) The value of the Production target is given by:

$p_{ns} \times z_s + p_s \times s_s = p$, where p refers to the production target.

(b) The value of the intermediate work-in-process inventory i_{bi} which is not able to be processed with the help of the machines is given by:

$(y_i + p_{n_i} \times z_i + p_i \times s_i) - (p_{n_{i-1}} \times z_{i+1} + p_{i+1} \times s_{i+1}) = i_{b_i}$

In the above equation the first half is representing the semi-finished item which is at the *i* th stage and the second half of the equation is representing the units which are consumed by the new machine as well as by the existing machine at the stage i+1.

(c) The value of the Work In Process Inventory buying cost is given by:

$$\sum_{i=1}^{s}(c_i \times y_i) = ic$$

(d) The value of the New machine installation Cost is given by:

$$\sum_{i=1}^{s}(c_{ni} \times m_i) = nmc$$

Production target, inventory balance at each stage, Cost of carrying the inventory, Cost of installing the new machine represents the goals. Also the amount of the inventory y_i, the number of the operational days s_i at the stage i where *i=1,2,3,...s* and the proportional usage of the new machines z_i represents the decision variables.

CHAPTER 5

Solution Procedure to Multi-stage Multi-objective Production Planning in Uncertain Environment

The goals in the production planning process are fuzzy in nature. The goals are aggregated with the help of the induced ordered weighted average operator by taking their priorities into the consideration. The Process of the Production is formed as the Mixed Integer type of the problem.

The Solution Procedure is obtained as:

5.1 Deriving membership functions with the help of the goal statements.

As it is already illustrated that the goals taken are fuzzy in nature hence it is not necessary that the precise value of the target would be obtained. The target could deviate to some extent. The tolerable amount of the flexibility in all the objectives/goals is determined. Firstly the tolerable amount of the flexibility is determined with the help of the linguistic terms and then these linguistic terms are converted into the numerical form with the help of (Herrera and Martinez, 2000).The conversion of the linguistic term into its equivalent numerical form is illustrated in the chapter 2 of our work.

For example, Consider the production target to be "approximately" p units then the production target could be interpreted as (p, ∈) where ∈ is used in order to represent "approximately" p units. Here ∈ is the linguistic term that is taken from the linguistic term set. The goal deviations that are defined linguistically could be demonstrated with the help of the linguistic term set S where $S = (IMP, NLG, VL. L, M, H, VH, SH, EH)$.

For example, Let the target is represented by n_g where g represents the goal. Let l_g represents the flexibility where l_g = (s, α), s ∈ S and α ∈ [-0.5, 0.5).Then with the help of (Herrera and Martinez, 2000) let us suppose that d_g is representing the numeric equivalent of l_g where d_g belongs to the set [0, 1].The membership function of the goal g is defined with the help of the goal deviation d_g.

5.1.1 Membership Values

(a) The production goal's membership value having the target n_p is given by:

$$m_p(p) = \begin{cases} 0, & if\ p < (n_p \times (1 - d_p)) \\ \frac{p - n_p \times (1 - d_p)}{n_p \times d_p}, & if\ n_p \times (1 - d_p) \le p \le n_p \\ 1, & if\ p > n_p \end{cases}$$

(b) The inventory balance goal's membership value with the target n_{ib_i} is given by:

$$m_{ib_i}(ib_i) = \begin{cases} 0, & if\ ib_i \ge n_{ib_i} \times (1 + d_{ib_i}) \\ \frac{n_{ib_i} \times (1 + d_{ib_i}) - ib_i}{n_{ib_i} \times d_{ib_i}}, & if\ n_{ib_i} \times (1 + d_{ib_i}) > ib_i > n_{ib_i} \\ 1, & if\ ib_i = n_{ib_i} \\ \frac{ib_i - n_{ib_i} \times (1 - d_{ib_i})}{n_{ib_i} \times d_{ib_i}}, & if\ n_{ib_i} > ib_i > n_{ib_i} \times (1 - d_{ib_i}) \\ 0, & if\ ib_i \le n_{ib_i} \times (1 - d_{ib_i}) \end{cases}$$

(c) The inventory carrying cost's membership function with goal as n_{ic} is given by:

$$m_{ic}(ic) = \begin{cases} 0, & if\ ic \ge n_{ic} \times (1 + d_{ic}) \\ \frac{n_{ic} \times (1 + d_{ic}) - ic}{n_{ic} \times d_{ic}}, & if\ n_{ic} \times (1 + d_{ic}) > ic > n_{ic} \\ 1, & if\ ic \le n_{ic} \end{cases}$$

(d) The new machine set up cost's membership function with goal as n_{nmc} is given by:

$$m_{nmc}(nmc) = \begin{cases} 0, & if\ nmc \ge n_{mc} \times (1 + d_{nmc}) \\ \frac{n_{nmc} \times (1 + d_{nmc}) - nmc}{n_{nmc} \times d_{nmc}}, & if\ n_{nmc} \times (1 + d_{nmc}) > nmc > n_{nmc} \\ 1, & if\ nmc \le n_{nmc} \end{cases}$$

5.2 Aggregation of the objectives of the Production Process:

All the membership functions that are defined above needs to be aggregated so that a single objective function is obtained. The aggregation of the membership functions is done according to the level of the priority of each membership function**.** Gupta and Mohanty (2015) presented a mixed integer programming problem for production planning in which the induced ordered weighted average operator is used to combine the objectives/goals according to the priority of the goal.

The objective function of the production planning problem is hence formulated:

Max(IOWA:($< u_{p,} m_p >$,$< u_{ib,} m_{ib} >$,$< u_{ic,} m_{ic} >$,$< u_{nmc,} m_{nmc} >$)

5.2.1 Proposed Approach

In the proposed approach to deal with the subjective preferences of the decision maker, a Minimax Disparity Approach has been used to find the OWA operators weights (w_1, w_2, w_3, w_4) at different orness level. Therefore the objective function is obtained as at degree of orness (α) is given by

$$max_\alpha \quad w_1 * m_p + \; w_2 * m_{ib} + w_3 * m_{ic} + w_4 * m_{nmc}$$

5.3Application in an automobile industry

A multi-stage production-planning problem is taken into consideration in real life which is in an automobile industry. The production process consists of the 6 stages and the production process has either one or more than one machine at every stage of the 6 stages. Here in the process of the production like in any other process firstly the raw materials are processed in the starting stage and at the subsequent stages the semi-finished products are produced and the final product is produced at the (6) *th* stage. As no production process is perfect hence sometimes the machines may not function properly. Hence in order to produce the desired product the additional standby machines could be needed at any of the 6 stages of the production process.

The data obtained from the firm is as follows:

(1) How many number of stages are there in total and what number of the machines are used at the each stage.
(2) The existing machine’s daily production rate that are fuzzy in nature and the capacities that are in terms of the operational days.
(3) The cost of the installation of the new machines and the cost of the inventory both taken in fuzzy terms at all the 6 stages of the production.
(4) The existing operational as well as the logical constraints.
(5) The objectives of the company which are defined in linguistic terms and the level of the priority of the objectives.

The data for (1),(2),(3) is given in the table as follows:

Table 3 The Rate of Production, The Capacity, The cost of installation of the new machine, The inventory cost per 100

Goals	Target Value	Deviation in linguistic terms from the current value	Deviation in numeric terms
Production	10746	(NLG)	0.125
Inventory – balance	10	(EH)	1
Inventory buying cost	6846	(M)	0.5
New machine installation cost	747	(VL)	0.25

The firm has some restrictions on the system of production as follows:

1. The new machines setups should not exceed five in number.
2. The firm can buy atmost 4000 units of the work in process inventory during the whole process of production.

   Now, with the help of the defuzzification method the fuzzy values of the production rates of the machines, cost of the new machine and the cost of buying the inventory are converted into numeric form. The table below gives the crisp values of the the production rates of the machines, cost of the new machine and the cost of buying the inventory.

. **Table 4** The crisp daily production rate (existing machines) and the monthly production rate of new machines.

Stages	Production rate/Capacity of the existing machine	Combined production rate/capacity	New machines installation cost	Monthly production rate of new machine	Inventory cost per 100 units
1	533.33/22,503.33/24	1102.42/22	251.66	13833.33	56.67
2	253.33/22,148.33/23	408.409/22	101.66	6333.33	80.67
3	400/24.8	400/24.8	100	10,000	100.67
4	498.33/25	498.33/25	150.33	12458.33	75.33
5	251.66/23,81.66/22.5,101.66/24	447.37/22.5	200.33	6291.66	160.00
6	293.33/23	293.33/23	511.66	7333.33	190.00

Also with the help of the Auxiliary capacity and Auxiliary production rate the combined capacities and the combined production rate are obtained.

The mixed integer programming problem is formulated as:

The value of the objective function is given by:

Max = 0.5 × m_p + 0.207 × m_{ib} + 0.158 × m_{ic} + 0.1339 × m_{nmc}

Where m_p, m_{ib}, m_{ic}, m_{nmc} are the factors that determine the priority order of the goals.

(1) The value of the production function is given by:

p=6375 × z_6 + 631.9583 × s_6

(2) The new machine installation cost function is given by:

$mc = 251.66 \times m_1 + 101.66 \times m_2 + 100 \times m_3 + 15033 \times m_4 + 200.33 \times m_5 + 511.66 \times m_6$

(3) The work in process inventory cost is given by:

$ic = 0.01 \times (56.66 \times y_1 + 80.66 \times y_2 + 100.66 \times y_3 + 75.33 \times y_4 + 160 \times y_5 + 190 \times y_6$

(4) The capacity constraints(Stage Wise) are given by:

$s_1 \leq 22$

$s_2 \leq 22$

$s_3 \leq 24.8$

$s_4 \leq 25$

$s_5 \leq 22.5$

$s_6 \leq 23$

(5) The number of the Machine constraints are given by:

$m_1 + m_2 + m_3 + m_4 + m_5 + m_6 \leq 5$

(6) The number of the inventory units constraints are given by:

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \leq 4000$

(7) The Stage wise residual capacity is given by:

$t_1 = 22 - s_1$

$t_2 = 22 - s_2$

$t_3 = 24.8 - s_3$

$t_4 = 25 - s_4$

$t_5 = 22.5 - s_5$

$t_6 = 23 - s_6$

(8) In the production of an item the New machines are brought when the existing capacity of the machine gets exhausted.

$t_i \leq 1000 \times (1\text{-}m_i)$ where i=1, 2, 3, 4, 5, 6

(9) The proportion that the new machine is used or not will completely depend on whether the new machine is bought or the new machine is not bought

$z_i \leq 1000 \times (1\text{-}m_i)$ where i=1, 2, 3, 4, 5, 6

(10)Any new inventory can be bought when the existing capacity is fully exhausted:

$$t_i \leq 1000 \times (1\text{-}i_i) \text{ where } i\text{=1, 2, 3,4,5,6}$$

(11)The quantity of the inventory completely depends on whether or not the inventory is bought

$$y_i \leq 4000 \times i_i \text{ where } i\text{=1, 2, 3,4,5,6}$$

(12)At every stage either a new machine is bought or an inventory is bought

$$m_i + i_i \leq 4000 \text{ where } i\text{=1, 2, 3, 4, 5}$$

(13) The input at the every should be greater than equal to the production capacity of the subsequent stage:.

$ib_1 = y_1 + 13833.33 \times z_1 + 1102.424 \times s_1 - 6333.33 \times z_2 - 408.4091 \times s_2$

$ib_2 = y_2 + 6333.33 \times z_2 + 408.4091 \times s_2 - 10000 \times z_3 - 400 \times s_3$

$ib_3 = y_3 + 10000 \times z_3 + 400 \times s_3 - 12458.33 \times z_4 - 408.33 \times s_2$

$ib_4 = y_4 + 12458.33 \times z_4 + 498.33 \times s_4 - 6291.66 \times z_5 - 447.37 \times s_5$

$ib_5 = y_5 + 6291.66 \times z_5 + 447.37 \times s_5 - 293.33 \times s_6$

(14) $m_p \leq 0.000745 \times \text{p} - 7.00149$

(15) $m_{nmc} \leq 3.994652 - 0.00535 \times mc$

(16) $m_{ic} \leq 2.666926 - 0.00039 \times ic$

(17) $m_{ib_i} \leq 2 - 0.1 \times ib_i$ where *i=1, 2,3,4,5.*

(18) $m_{ib_i} \leq 0.1 \times ib_i$ where *i=1, 2, 3, 4, 5.*

(19) $m_{ib} = \sum_{i=1}^{24} {m_{ib_i}}/{5}$

(20) $s_i, y_i, z_i, ib_i, m_p, m_{ic}, m_{nmc}, m_{ib_i} \geq 0$

(21) $m_p, m_{ic}, m_{nmc}, m_{ib_i}, z_i \leq 1$

(22) $m_i, i_i \in \{0,1\}$

Table 5 Results using LINGO 17

Variables	**Gupta & Mohanty Approach**	**Proposed approach**										
		α=0	**α=0.1**	**α=0.2**	**α=0.3**	**α=0.4**	**α=0.5**	**α=0.6**	**α=0.7**	**α=0.8**	**α=0.9**	**α=1**
Mp	1	0	0	0	1	1	1	1	1	1	1	1
Mib	0.20833	0	0	0.2083	0.2083	0.2083	0.2083	0.2083	0.2083	0.2083	0.2083	0
Mic	1	0	1	1	1	1	1	1	1	1	0	0
Mnmc	1	1	1	1	1	1	1	1	1	0	0	0
P	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92	14079.92
Mc	511.66	511.66	511.66	511.66	511.66	511.66	511.66	511.66	511.66	511.66	511.66	511.66
m1	0	0	0	0	0	0	0	0	0	0	0	0
m2	0	0	0	0	0	0	0	0	0	0	0	0
m3	0	0	0	0	0	0	0	0	0	0	0	0
m4	0	0	0	0	0	0	0	0	0	0	0	0
m5	0	0	0	0	0	0	0	0	0	0	0	0
m6	1	1	1	1	1	1	1	1	1	1	1	1
Ic	0	0	0	0	0	0	0	0	0	0	0	0
y1	0	0	0	0	0	0	0	0	0	0	0	0
y2	0	0	0	0	0	0	0	0	0	0	0	0
y3	0	0	0	0	0	0	0	0	0	0	0	0
y4	0	0	0	0	0	0	0	0	0	0	0	0
y5	0	0	0	0	0	0	0	0	0	0	0	0
y6	0	0	0	0	0	0	0	0	0	0	0	0
s1	6.1651	6.2105	6.2105	6.1651	6.1651	6.1651	6.1651	6.1651	6.1651	6.1651	6.1651	6.2105
s2	16.6171	16.7151	16.7151	16.6171	16.6171	16.6171	16.6171	16.6171	16.6171	16.6171	16.6171	16.7151
s3	16.9415	17.0165	17.0165	16.9415	16.9415	16.9415	16.9415	16.9415	16.9415	16.9415	16.9415	17.0165
s4	13.5785	13.6187	13.6187	13.5785	13.5785	13.5785	13.5785	13.5785	13.5785	13.5785	13.5785	13.6187
s5	15.1029	15.1253	15.1253	15.1029	15.1029	15.1029	15.1029	15.1029	15.1029	15.1029	15.1029	15.1253
s6	23	23	23	23	23	23	23	23	23	23	23	23

t1	15.8349	15.7895	15.7895	15.8348	15.8348	15.8348	15.8348	15.8348	15.8348	15.8348	15.8348	15.7895
t2	5.3828	5.2849	5.2849	5.3828	5.3828	5.3828	5.3828	5.3828	5.3828	5.3828	5.3828	5.2849
t3	7.8585	7.7835	7.7835	7.8585	7.8585	7.8585	7.8585	7.8585	7.8585	7.8585	7.8585	7.7835
t4	11.4215	11.3813	11.3813	11.4214	11.4214	11.4214	11.4214	11.4214	11.4214	11.4214	11.4214	11.3813
t5	7.3971	7.3747	7.3747	7.3971	7.3971	7.3971	7.3971	7.3971	7.3971	7.3971	7.3971	7.3747
t6	0	0	0	0	0	0	0	0	0	0	0	0
z1	0	0	0	0	0	0	0	0	0	0	0	0
z2	0	0	0	0	0	0	0	0	0	0	0	0
z3	0	0	0	0	0	0	0	0	0	0	0	0
z4	0	0	0	0	0	0	0	0	0	0	0	0
z5	0	0	0	0	0	0	0	0	0	0	0	0
z6	1	1	1	1	1	1	1	1	1	1	1	1
i1	0	0	0	0	0	0	0	0	0	0	0	0
i2	0	0	0	0	0	0	0	0	0	0	0	0
i3	0	0	0	0	0	0	0	0	0	0	0	0
i4	0	0	0	0	0	0	0	0	0	0	0	0
i5	0	0	0	0	0	0	0	0	0	0	0	0
i6	1	1	1	1	1	1	1	1	1	1	1	1
ib1	10	20	20	10	10	10	10	10	10	10	10	20
ib2	10	20	20	10	10	10	10	10	10	10	10	20
ib3	10	20	20	10	10	10	10	10	10	10	10	20
ib4	10	20	20	10	10	10	10	10	10	10	10	20
ib5	10	20	20	10	10	10	10	10	10	10	10	20
ib6	0	0	0	0	0	0	0	0	0	0	0	0
mib1	1	0	0	1	1	1	1	1	1	1	1	0
mib2	1	0	0	1	1	1	1	1	1	1	1	0
mib3	1	0	0	1	1	1	1	1	1	1	1	0
mib4	1	0	0	1	1	1	1	1	1	1	1	0
mib5	1	0	0	1	1	1	1	1	1	1	1	0
mib6	0	0	0	0	0	0	0	0	0	0	0	0

5.4Conclusion

In this work, the review of multi-stage multi-objective production planning problem of Gupta and Mohanty (2015) has been done which is a mixed-integer programming problem. Further, to deal with subjective preference of decision maker, we have proposed the multi-stage multi-objective production planning problem at different orness levels. The objectives that are taken in the production planning problem are fuzzy in nature. In order to achieve the desired production target a methodology is provided so that a balance is maintained in the I/O relation at every stage of the production process. The various complications faced by the firm and the objectives/goals are defined. The aggregation of the objectives are done using OWA operator. The weights corresponding to the OWA operator has been evaluated using minimax disparity approach. Further, the solution of the resultant mixed-integer programming problem is obtained using LINGO 17. The comparison of the proposed approach has been done with Gupta and Mohanty (2015) approach. It shows that the results at some orness level are similar with the proposed approach and at some orness level, the results are different which shows the use of proposed approach according to the subjective preference of the decision maker.

CHAPTER 6

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