

**CAVITY SOLITON IN VCSEL USING
GRAPHENE AS SATURABLE ABSORBER**

A thesis submitted in partial fulfillment of the requirements for the award of

degree of

Master of Science

in

Physics

Submitted by:

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July 2017

I dedicate this work to my parents, my brother “Yattanjit Singh” and my friends who encouraged and supported me.

CERTIFICATE

I hereby declare that the work which has been presented in this thesis entitled "**Cavity Soliton in VCSEL using graphene as saturable absorber**" is an authentic record of my own work carried out for the partial fulfillment of the requirement for the award of the degree of Masters of Science in Physics at Thapar University, Patiala (Punjab), under the guidance of **Dr. Soumendu Jana**, Associate Professor, School of Physics and Materials Science and refers other researcher's work which are duly listed in the reference section. The intellectual content of this thesis is the product of my own work and contains no material which to a substantial extent has been accepted for the award of any other degree at this or any other educational institution, except where due acknowledgment is made in the thesis.

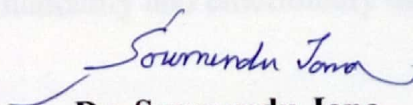
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It is to certify that the above statement made by the candidate is correct and true to the best of my knowledge and belief.



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ACKNOWLEDGEMENT

Work on this thesis would not have been possible without the encouragement and the support of many people. I would like to express my sincere gratitude especially to the following:

The first and greatest thank to my supervisor **Dr. Soumendu Jana** for his continuous support and supervision, for his excellent guidance, patience, motivation and enthusiasm. I could not have imagined having a better advisor and mentor than him for my thesis.

I would like to thank **Dr. Manoj Kumar Sharma** for providing me with an excellent atmosphere and all the necessary facilities for my research. Besides, my supervisor, I would like to thank from the bottom of my heart, **Ms. Baldeep Kaur** and **Mr. Gurkirpal Singh**, Research Scholar, for sharing their valuable experiences. I would not have been able to complete my dissertation without their cooperation.

I would like to acknowledge my lab mate and friend **Harleen Kaur** who supported me at each step.

I would like to thank my friends **Amanpreet, Aaditi, Navneet and Raveena** being their always with me.

Last but not the least, I would like to thank my parents and my younger brother for their unconditional support, both financially and emotionally throughout my degree.

Above all I render my gratitude to the almighty who bestowed upon me the strength and vision to walk on the path of truth.

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ABSTRACT

In this thesis we show the generation and dynamics of cavity soliton in VCSEL with saturable absorber. For first time, we replace semiconductor saturable mirror by graphene saturable absorber (GSA) in the context of cavity soliton formation. GSA provides significantly higher nonlinear saturation with different strength of GSA coefficient. Cavity soliton has been generated the spontaneous dynamics of the generated cavity soliton are presented.

INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION:

Cavity soliton(CS) is great area of research because of its large applications in the field of memory storage, in future for delay lines and optical sensors [1]. CS is dissipative optical solitons that is localized structure inside a lossy cavity. Cavity soliton is generated in a dissipative cavity, like Vertical cavity surface emitting laser (VCSEL). A broad area device like VCSEL is ideal not only to generate CS or cluster of CS but also to trace dynamics of CS. There are various models to obtain cavity soliton like VCSEL with holding beam (HB), VCSEL with Bragg reflector, VCSEL with saturable absorber and many more. Out of these VCSEL with saturable absorber is simple and efficient model [2]. Not only in the formation of cavity soliton vertical cavity surface emitting lasers (VCSEL) are efficient work in Q switching, mode locking with the use of saturable absorbers (SA) [3]. Some of SA are semiconductor saturable absorber mirrors (SESAM)[4], lead sulfide suspended in glasses [5], carbon nano tubes (CNT) [6,7,8], Gallium arsenide and many more. These SA are used according to different requirement of system. After the discovery of ‘magic material’ graphene, its interesting properties become the center of attraction. Its zero band gap structure, high conductivity, great absorbing power makes it a useful broad band saturable absorber [9,10,11]. Graphene is used in VCSEL as saturable absorber and provides advantage like flexible design of the parameters and low saturable losses. Graphene saturable absorber (GSA) provides 2-4 times faster recovery time, approximately double absorption power, higher saturation

fluency than the traditionally used saturable absorbers in VCSELs [4,2]. However no work is done using graphene as SA in VCSEL for the formation of CS. It will be quite interesting to observe the dynamics of CS using first discovered 2-D material as saturable absorber [9]. How the opto-electric properties, electrical efficiency, broad band saturation makes the work of CS formation easier and what challenges have to face to overcome its drawbacks like very high nonlinearity [11].

1.2 LITERATURE REVIEW:

Soliton is a beam of light which propagates without change of form leading to robust structures in which nonlinearity counter balance diffraction or dispersion [1]. In soliton study, one of the most exciting areas of research is soliton interaction. Soliton have the property of recovering their shape after collision [12]. Self-localized states of light have the ability to beat diffraction through nonlinearities [13]. Localized structure generated in a cavity filled with a non-linear medium called cavity soliton [14]. Cavity soliton form in a homogenous background of a radiation are bright intensity peaks over dark homogenous background [15]. Cavity soliton become topic of great interest because of its possible applications in all optical information processing and switching. These seemed to be suited for photonic applications in quite fast, compact and robust systems such as semiconductor micro cavities [13]. Cavity soliton are bistable which means these can be absent or present under the same external conditions and are independent from the system's boundaries [1].

In last decade, cavity soliton in semiconductor become area of interest because of its small size, medium that provide immediate response and giving the localized structures

which bistable and robust in nature. However conditions needed to achieve that kind of soliton is very difficult to achieve. But these difficulties could be managed using cavity soliton laser (CSL), the laser that gives soliton without using any external holding beam (HB). These lasers instead of HB use mechanical feedback and detuning between external frequency selecting element and resonator in a VSCSEL cavity or use saturable absorber in VSCSEL cavity. One of this kind model use two mutually coupled micro resonators where acts as saturable absorber and other serves the purpose of amplifier [14]. Another effective method to produce CSL is using highly reflective mirrors providing frequency selective feedback. In this model optical parametric oscillators, absorption cavities and semiconductor amplifier support CS. One use FSF to selectively lower the system losses. But in this case our input beam should be intense so it can stabilize the cavity soliton until the feedback get provide because the feedback process is little slow [16]. This theory of using saturable absorber instead of holding beam (HB) in VSCSEL for the formation of CS is proposed by Rasanov and co-worker. In this type of laser with saturable absorber (LSA) it is possible to achieve such conditions below threshold where contrast between CS and homogenous background is maximum. This type of laser is suitable for both active and passive medium. One can use photorefractive material as active medium and organic material acts as passive material. Using LSA it is also possible to excite one peak at a time [17].

VECSELs have been passively mode locked with semiconductor saturable absorber mirrors. But graphene has several advantages over SESAM, thus considered as a better saturable absorber. It offers an unlimited bandwidth and a fast recovery time. Including fiber and solid state bulk lasers a number of different laser types and gain materials have

been mode locked with graphene [18]. In laser research ultrafast laser become one of most active fields because of the wide and significant applications in science, industry and military. They are obtained from the mode-locked laser by semiconductor saturable mirror [10]. In this context VCSEL coupled with GSA has huge potential.

1.3 MOTIVATION:

Cavity Soliton has been widely studied in VCSEL coupled with semiconductor saturable absorber mirror. However graphene saturable absorber has some more interesting attributes, like, zero band gap and great absorption power. But no initiative is made to use graphene as saturable absorber in VCSEL for the formation of cavity soliton. Thus it opens a new gate for research on cavity soliton formation, in VCSEL coupled with graphene saturable absorber.

1.4 OBJECTIVE:

Our objectives of investigation are

- To form cavity soliton using graphene as saturable absorber in VCSEL
- To study the dynamics of cavity soliton formed in VCSEL coupled graphene saturable absorber.

CHAPTER 2

INTRODUCTION TO GRAPHENE

Mostly, the task of obtaining ultrafast pulses is done by mode locking lasers with semiconductor saturable mirror (SESAM). But there are several disadvantage of this method like SESAM require epitaxial method of fabrication which is very expensive, complex. Also intra cavity non linearity and Bragg mirror's reflectivity limits the band of saturable absorption. The problem of limited saturable absorption band is solved by using single walled carbon nanotubes (SWNTs). In this approach of obtaining wide saturable absorption band different tube diameters is mixed with SWNTs, which cause very high intersection loss [10]. But the discovery of graphene in 2004 by Andre Geim and his colleague Kostya Novoselovhe at Manchester University announces the new era of revolution. Now the days, graphene takes the position of a reliable saturable absorber. Its gapless band structure in which Dirac electrons are linearly dispersed made it wide range saturable absorber [18,10].

Before we further precede let us mention some key components which are used earlier or will be used in later discussion.

2.1 SOME DEFINATIONS

2.11 SOLITON:

Consider a light wave or water wave after some time it starts diminishing, but if a wave travels a long distance not even showing minute difference from its initial shape and size? It seems to be impossible but it is possible by compensating the losses during the propagation of wave. In a nonlinear medium this compensation is done by nonlinearity, this nonlinearity beats the diffraction/dispersion and sets a proper balance between self-focusing and diffraction. This type of unchanged structures which does not losses their identity even after the collision with other same kind of structure is named as soliton [19]. It is first proposed by Jhon Scott Russell in 1834 [20].

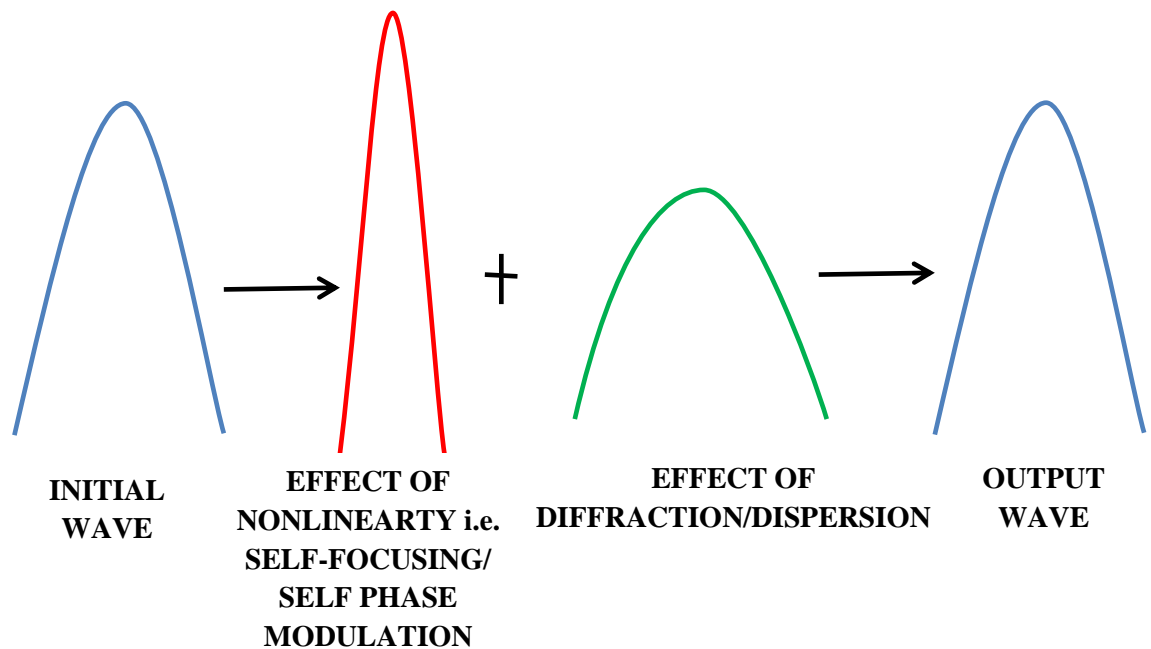


Figure 2.1: The proper balance between nonlinearity and diffraction/dispersion and soliton formation.

2.12 CAVITY SOLITON:

Cavity soliton (CS) is self-localized light states obtained in the transverse plane of cavity as dark spot on bright background or bright spot on dark background [21,22]. Cavity soliton exhibit following properties (a) cavity soliton can be found in several different locations in a given cavity and can be erased or written independently; (b) cavity soliton does not depend on excitation that is responsible for their birth and also independent of system boundaries;(c) cavity soliton can move spontaneously if the system is homogenous hence can feel the presence of any kind of gradient. Above properties leads to various applications of CS like its switching property gives idea to use CS in memory devices also its position controllability benefits in optical memory devices. Its other possible applications are in soliton force microscopy and all optical delay line [22,2].

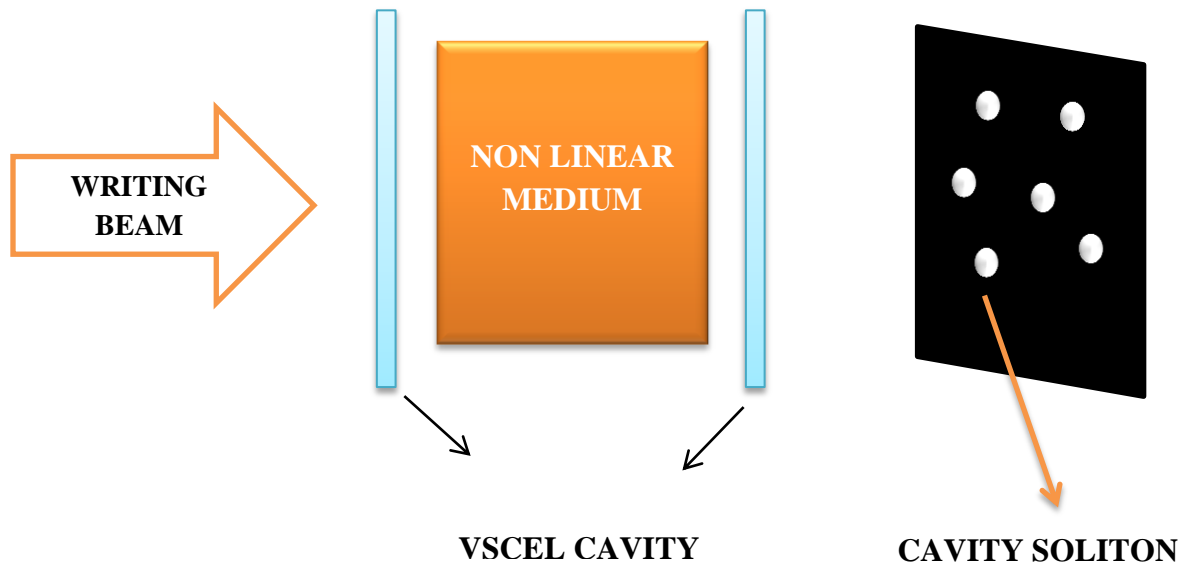


Figure 2.2: Scheme diagram of cavity soliton formation.

2.13 VERTICAL CAVITY SURFACE EMITTING LASER (VCSEL)

VCSEL is second largest produced laser among all types of semiconductor lasers after Fabry-Perot type edge emitting lasers. Its research begins in 1977 with the suggestion of Prof. Kenji Iga. It stole the attention of scientists because of its various advantages like large output powers, coherent emission over large areas with associated small beam divergences, array formation [23]. It has combined advantages of semiconductor lasers like compact footprint and solid state lasers like high average, low timing jitter, very good beam quality and peak power [24,25]. It is ideal laser for the study of spatio-temporal dynamics. Its active medium lengths which are order of $\lambda/4$, where λ is wave length, large cavity length of $10\mu\text{m}$, planar mirrors, more than $150\mu\text{m}$ diameter is helpful to observe structures like cavity soliton with short correlate length [22].

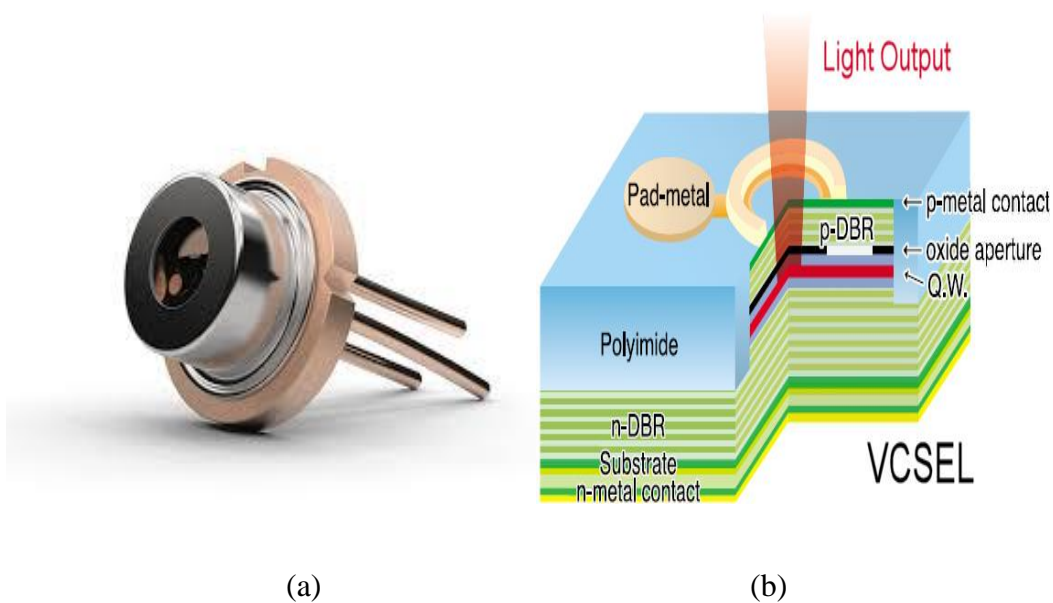


Figure 2.3: (a) Typical Vertical cavity surface emitting laser. (b) Structure of VCSEL (image courtesy:-Wikipedia)

2.14 SATURABLE ABSORBER (SA):

Saturable absorber an optical component having some optical loss but its loss starts reducing with increasing optical intensities. The relation of absorption coefficient α on the intensity of incident laser I is given as follows

$$\alpha = \frac{\alpha_o}{1 + I/I_S}$$

Where, I_S is saturation intensity, α_o is low intensity absorption coefficient.

The phenomenon of saturable absorption is responsible for bistability [26]. Its major applications are in Q- switching in lasers, passive mode locking, optical signal processing and non-linear filtering in resonators. Saturable absorber is of various types and accordingly used for various purposes. Some examples of saturable absorber are semiconductor saturable mirrors used in passive Q switching at low pulse energies, Gallium arsenide (GaAs) used in $1\mu\text{m}$ lasers for passive Q switching, very thin layers of single wall carbon nanotubes (CNT) serves the purpose of mode locking [7,8]. For $1\mu\text{m}$ lasers $\text{Cr}^{4+}:\text{YAG}$ is used as Q switcher[27], for $1.3\mu\text{m}$ spectral region $\text{V}^{3+}:\text{YAG}$ [28]and for $1.5\mu\text{m}$ lasers $\text{Co}^{2+}:\text{MgAl}_2\text{O}_4$ serves the purpose of Q switching, graphene which our topic of concern is a broadband saturable absorber [10].

Using saturable absorber in laser (LSA) it is possible to obtain cavity soliton without using holding beam below threshold conditions. Thus it is step towards cavity soliton laser [2].

Since our interest is on GSA it is wise to give a brief review on graphene itself.

2.15 GRAPHENE:

Graphene is first discovered two dimensional material. It is allotrope of carbon with atomic scale hexagonal structure in which atom forms each vertex [29]. Carbon atoms at vertex are sp^2 hybridized [30]. It is strongest material ever exist, it is 200 times stronger than steel. It efficiently conducts electricity and is nearly transparent. It shows a large nonlinear diamagnetism [9]. Nonlinear refractive index of graphene $\eta_2=10^{-7}cm^2W^{-1}$ [33]. Electrons in graphene follow a linear dispersion relation also these electrons acts as massless relativistic particles [9]. Optical and electronic properties of graphene are explained in terms of massless Dirac fermions with dispersion near Fermi energy which is linear in nature [30]. Graphene has various revolutionary properties it is very light material so can be used in aero-plane parts, it can used in filtering sea water, in display of electronic devices as it is flexible conductor, it will be used future batteries and other electronic devices [31,32].

Graphene can be used as saturable absorber because its absorption is 2.3% per single pass. Recovery time is also very fast it is only tens of femtoseconds, its saturation fluency is very close to material damage threshold thus it avoids the problem of low damage in threshold. Its gapless structure provides the facility of broadband saturable absorber [4].

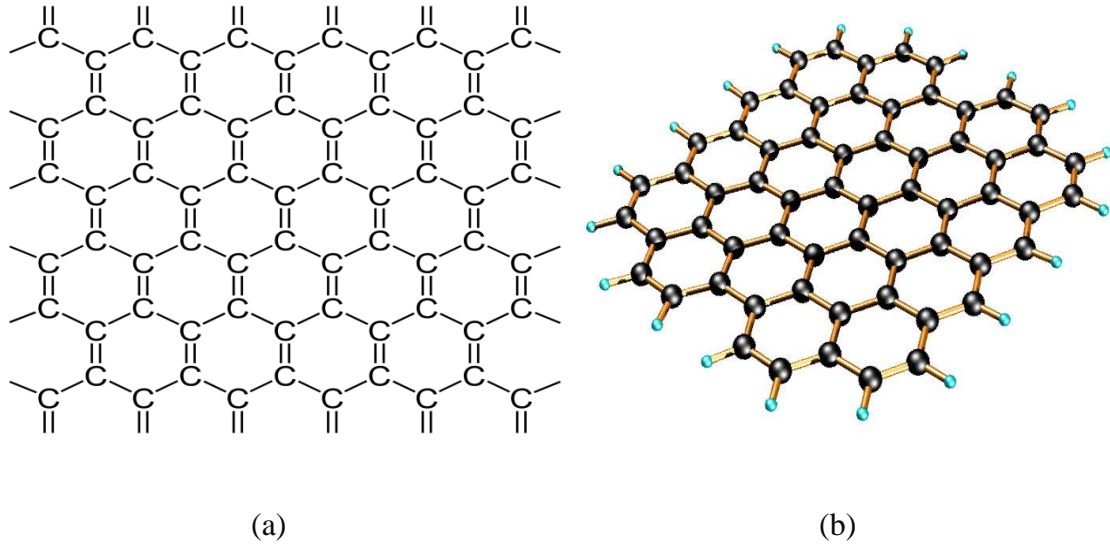


Figure 2.4: (a) chemical structure of single layer graphene. (b) Model of single layer graphene (image courtesy: Wikipedia)

After studying these keywords let us come to our topic of concern i.e. how one can use graphene as saturable absorber in VCSEL for obtaining cavity soliton? However graphene is already used in VCSEL, but cavity soliton is not obtained using this model. Thus first of all we review that how graphene become suitable for VCSEL and what are advantages and disadvantages using graphene as saturable absorber in VCSEL devices.

2.2 GRAPHENE IN VCSEL DEVICES AS SATURABLE ABSORBER:

As explained earlier various types of saturable absorbers are available for different purposes. But here we want to use graphene in VCSEL, thus saturable absorber used in VCSEL should have fast recovery time, high damage threshold, large ratio of saturable to non-saturable losses, large operation band width, high modulation depth [24,25]. Hence

we can find properties of graphene benefits VCSEL and which properties are needed to change in graphene to make it beneficial to use as saturable absorber in VCSEL.

2.21 ADVANTAGES OF GSA

As mentioned earlier graphene has zero band gap structure which made it wide range saturable absorber. Hence graphene can be saturated by visible to infrared region. Hence this problem to select fixed frequency beam in order to saturate absorber is solved [30]. Graphene has ultrafast recovery time of less than 100fs, thus our absorber is ready for next pulse in just few fs. Also single layered graphene (SLG) is very easy to integrate and easily grown. Its assembly and fabrication is cheap as compare to SESAM [25]. Single layered graphene have pulse generation of order of femtoseconds [18].

2.22 CHALLENGES FOR GSA IN VCSEL:

For single layer graphene non-saturated loss is approximately 4.6% in double pass per round. But in VCSEL required non-saturated is about 3%. However SESAM provides the less than 3% of such kind of loss. Single layered graphene have high nonlinearity because of this it is difficult to form stable localized pulse. Saturation fluency of graphene is very high, it is very close to damage threshold, it is major disadvantage of using graphene because very high intensity is needed to saturate absorber which can also damage the absorber [4,25].

2.23 HOW GRAPHENE MADE SUITABLE FOR VCSEL WITHOUT DESTROYING ITS BENEFICIAL PROPERTIES:

To reduce the non saturable loss from 2.3% to 1.5% per single various methods are used like gating, doping and by controlling intensity of electric field in graphene mirror on highly reflective mirror. However grating and doping are very challenging methods. Because it is tough to control required high doping and grating makes system more complex because of the use of unnecessary electrical drivers and contacts. Thus SLG changing the absorption by making control on intensity of electric field is good method. Using this model adjustable loss ranging from 0% to 10% can be obtained without losing high modulation depth of 5%. We use $\lambda/8$ GSAM because of its required less than 3% non-linear loss as well as large modulation depth which is greater than 0.9% in this case [25]. Thin layer of graphene is responsible for destroying the stability of pulse formed by mode locking but if one use thick layer of graphene then pulse will face less nonlinearity and thus there is possibility to stable pulses. It is experimentally found by comparing 7,11,14,21 layered graphene, that graphene with 21 layers thickness is more likely give stable mode locked pulses with pulse width 432.47fs and band width 0.323nm [34].

2.3 WHY WE ARE INTERESTED IN OBTAINING CAVITY SOLITON IN VCSEL WITH GRAPHENE AS SATURABLE ABSORBER? :

The major reason of using graphene as SA for obtaining CS is its zero band gap structure which gives the liberty of choosing the wide range of frequency as writing beam in order to saturate absorber.

Second big reason is optoelectronic properties of graphene. It provides advancement to VCSEL because it can be electrically controlled. As we know graphene has honeycomb, 2-D structure. In which each carbon atom makes covalent bonds with 3 carbon atoms and one electron in carbon atom remains free. These free electrons show very uncommon energy bands. When one plots energy versus momentum graph of these electrons it shows the shape of two cones connected to their extremities called 'Dirac cones'. There is zero band gap between conduction band (CB) and valence band (VB). Thus electrons from valence band could be excited by any photon ranging from visible to infrared region. After this excitation electrons in CB thermalize within the time period of 10-50fs to give Fermi-Dirac distribution. Due to this distribution electron hole pairs get created. These electron hole pairs oppose some of inter-band optical transitions and thus reduce absorption of photons. It is for low intensity photons. If intensity of photons increases carrier concentration also increases. Because of this edge of VB and CB get filled and after that no more absorption takes place i.e. graphene gets saturated [30].

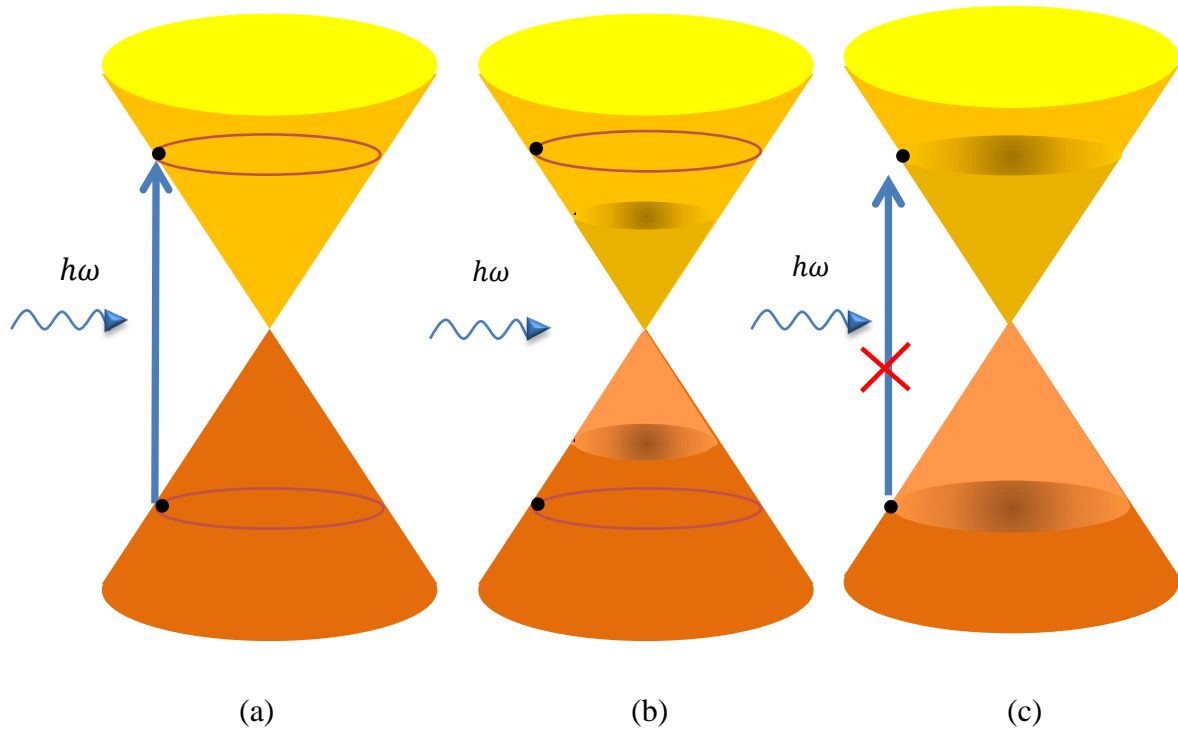
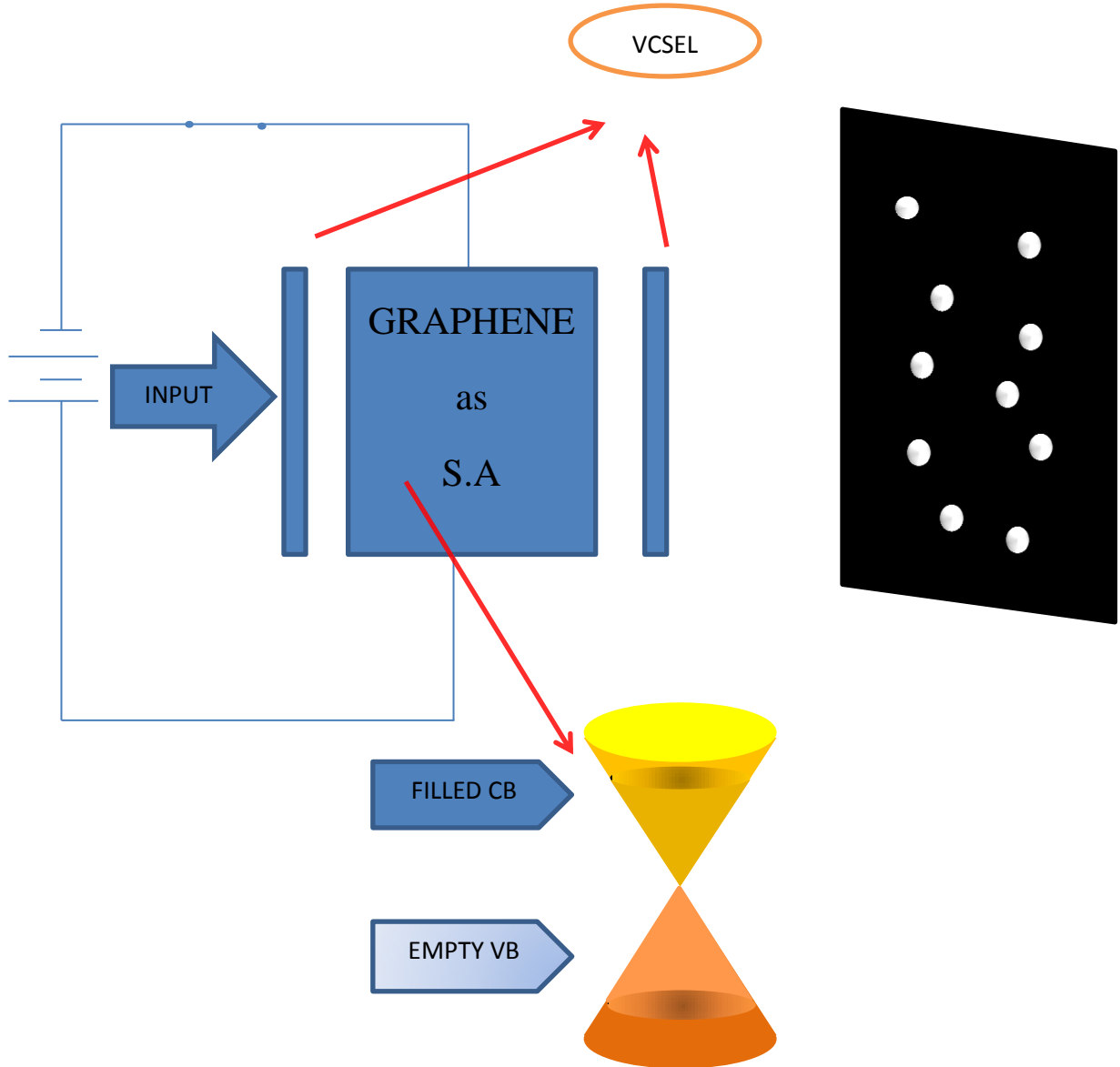


Figure 2.6: Absorption of light in graphene (a) Schematic excitation process responsible for absorption of light in graphene. (b) Due to high intensity light electrons from valance band start filling conduction band. (c) When states near the edge of conduction band get filled no absorption takes place, graphene gets saturated. (Image concept:- Atomic-Layer Graphene as a SA for Ultrafast pulsed lasers)

Graphene acts as a very good conductor. Thus electrons from valance band to conduction band get excited using electric field. With this method CB get populated and thus no more electrons can go for transition from VB to CB. Now graphene becomes saturated. On the other hand saturation in semiconductor is done by high intensity photon beam of particular frequency according to semiconductor material. Also one can de-saturate graphene electrically by switching off the electric field according to requirement of system. Whereas, when semiconductor is used as saturable absorber in VCSEL cavity

optical beam is required to produce soliton and optical beam of opposite phase erase the soliton. But in the case of graphene as saturable absorber there is no need of giving beam of opposite phase, soliton can be deleted by simply cut off the electric field because of which graphene acts as saturable absorber. Thus graphene as saturable absorber gives the facility to control its saturation properties either electrically or optically [11].



(a)

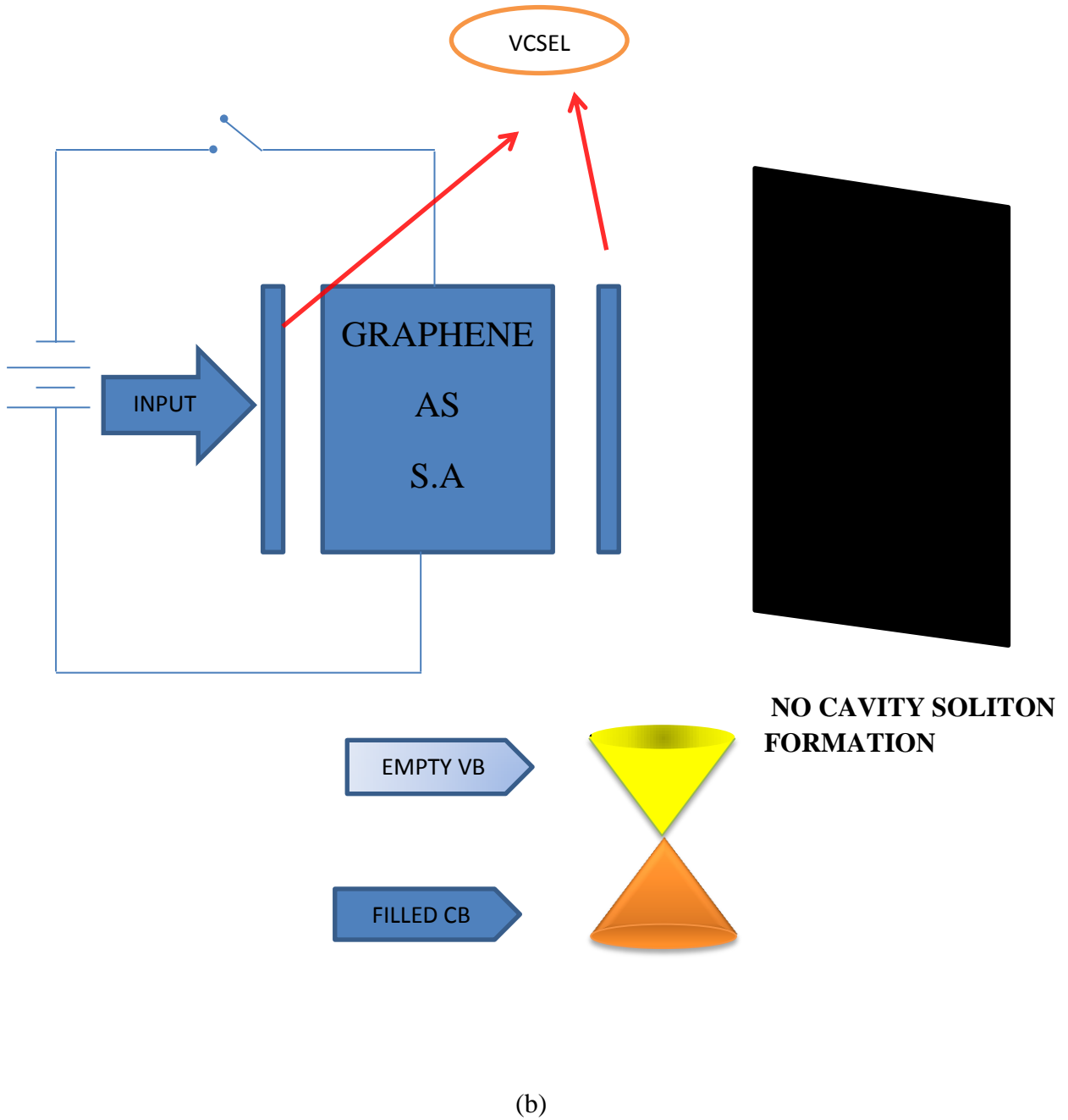


Figure 2.7: Switching the cavity CSs ON/OFF by GSA (a) Graphene gets saturated electrically thus formation of C.S takes place. (b) Graphene does not saturate because electrical supply is off thus no C.S formed. (Image concept:- Optimizing and Applying Graphene as a Saturable)

2.4 METHODOLOGY:

The following CGLE describing our system consists of VCSEL coupled with frequency selective feedback and graphene as saturable absorber;

$$\frac{\partial E}{\partial t} = \left[-(1 - i\theta) + \frac{\mu(1-i\alpha)}{1+g|E|^2} - \frac{\gamma(1-i\beta)}{1+s|E|^2+\alpha_{ns}} + i\Delta \right] E + F \quad (2.4.1)$$

Where,

E is electric field of photon; μ , γ are pumping parameter; α, β are line width enhancement factor; s is saturation parameter; σ is feedback strength; λ is band width of filter reflector; F is feedback field; α_{ns} is non saturable absorption loss; g represent a coefficient of saturation of GSA [35].

This equation is too complex to solve analytically because of the non-linearity in the partial differential equation. This equation can be solved by numerical method so that one can understand the effects caused by non-linearity in VCSEL. However this equation can be solved by either of numerical method that are pseudo spectral method or finite difference method. Here we choose fast method, i.e., pseudo spectral method. To solve the pulse propagation problem in non-linear dispersive media the method used to solve is split step Fourier method (SESAM) which is pseudo-spectral numerical method [36].

2.41 SPLIT STEP FOURIER METHOD:

Step 1: SESAM starts by introducing linear and non-linear operator. We write the non-linear equation in terms of diffraction operator \hat{D} and nonlinear operator \hat{N}

$$\frac{\partial E}{\partial T} = [\hat{D} + \hat{N}]E \quad (2.4.2)$$

$$\text{Where } \widehat{D} = -(1 - i\theta) + i\Delta + F \quad (2.4.3 \text{ (a)})$$

$$\widehat{N} = \frac{\mu(1-i\alpha)}{1+g|E|^2} - \frac{\gamma(1-i\beta)}{1+sg|E|^2+\alpha_{ns}} \quad (2.4.3 \text{ (b)})$$

\widehat{D} is linear operator and \widehat{N} is non-linear operator.

Step 2: SSFM is an approximate method thus is supposed that non-linear and diffraction acts independently in VCSEL cavity after the small time ‘ h ’ of propagation. This small time step is further divided in two equal steps of $h/2$.

In first Step of $h/2$, diffraction acts alone, thus $\widehat{N}=0$ and we can write

$$\frac{\delta E}{\delta T} = \widehat{D}E \quad (2.4.4)$$

By integrating from limit $E(z, T)$ to $E(z, T + h)$

$$\int_{E(z,T)}^{E(z,T+h)} \frac{\delta E}{E} = \widehat{D} \int_T^{T+h} \delta T \quad (2.4.5)$$

$$E(z, T + h) = E(z, T) \exp(h\widehat{D}) \quad (2.4.6)$$

In second Step of $h/2$ nonlinearity acts alone, thus $\widehat{D}=0$ and we can write

$$\frac{\delta E}{\delta T} = \widehat{N}E \quad (2.4.7)$$

By integrating from limit $E(z, T)$ to $E(z, T + h)$

$$\int_{E(z,T)}^{E(z,T+h)} \frac{\delta E}{E} = \widehat{N} \int_T^{T+h} \delta T \quad (2.4.8)$$

$$E(z, T + h) = E(z, T) \exp(h\widehat{N}) \quad (2.4.9)$$

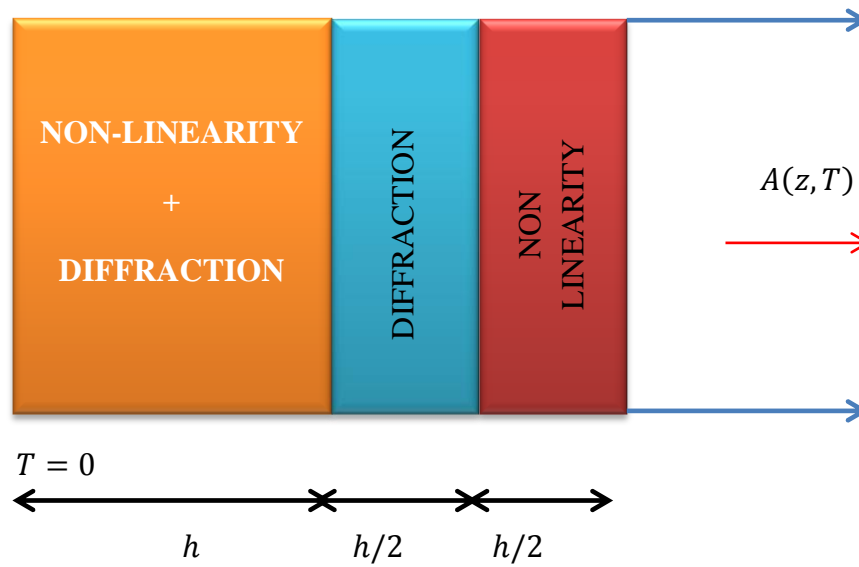


Figure 2.8: Schematic illustration of the symmetrized split-step Fourier method. In the VCSEL cavity after time h diffraction and non-linearity act independently. In the first time interval $h/2$ only diffraction play its role and second time interval of $h/2$ non-linearity acts solely. (Image concept:- Nonlinear fibre optics 4th edition, Govind P Agarwal, page 43)

Step 3: We can mathematically write by combining equation (2.4.6) and equation (2.4.9)

$$E(z, T + h) = \exp(h\widehat{D})\exp(h\widehat{N})A(z, T) \quad (2.4.10)$$

In Fourier domain, $\exp(h\widehat{D})$ expressed as;

$$\exp(h\widehat{D})B(z, T) = F_T^{-1}\exp[h\widehat{D}(-i\omega)]F_TB(z, T) \quad (2.4.11)$$

In Fourier domain, ω is the frequency, F_T is operator used for Fourier transformation.

$\widehat{D}(i\omega)$ is taken from equation 2.4.3(a) by replacing Δ by $i\omega$.

$\widehat{D}(-i\omega)$ is only a number in Fourier space. Thus equation 2.4.11 can be easily calculated. Hence the use of FFT algorithm makes numerical evaluation of equation 2.4.2 relatively fast. The exact solution of equation 2.4.2 is given by;

$$E(z, T + h) = \exp[h(\widehat{D} + \widehat{N})]E(z, T) \quad (2.4.12)$$

If \widehat{N} is supposed to be not dependent on T .

For two non-commuting operators \widehat{a} and \widehat{b} it is useful to recall the Baker-Hausdorff formula;

$$\exp(\widehat{a}) \exp(\widehat{b}) = \exp\left(\widehat{a} + \widehat{b} + \frac{1}{2}[\widehat{a}, \widehat{b}] + \frac{1}{12}[\widehat{a} - \widehat{b}, [\widehat{a}, \widehat{b}]] + \dots\right) \quad (2.4.13)$$

$[\widehat{a}, \widehat{b}] = ab - ba$. The above equation is given by using $\widehat{a} = h\widehat{D}$ and $\widehat{b} = h\widehat{N}$. If \widehat{D} and \widehat{N} are two non-commuting operator then $\frac{1}{2}h^2[\widehat{D}\widehat{N}]$ is found to be dominant error term. Hence SSFM neglects only the nature of non-commutating in operators. Because of this approximation SSFM is accurate only to second order in the step size h . The accuracy can be improved, if equation (2.4.10) can be replaced by;

$$A(z, T + h) = \exp\left(\frac{h}{2}\widehat{D}\right)\exp\left(\int_T^{T+h} N(T')dT'\right)\exp\left(\frac{h}{2}\widehat{D}\right)A(z, T) \quad (2.4.14)$$

The Split Step Fourier method has been applied to a wide variety of optical problems. This method has been used extensively because of its fast execution compared with most finite difference schemes [25].

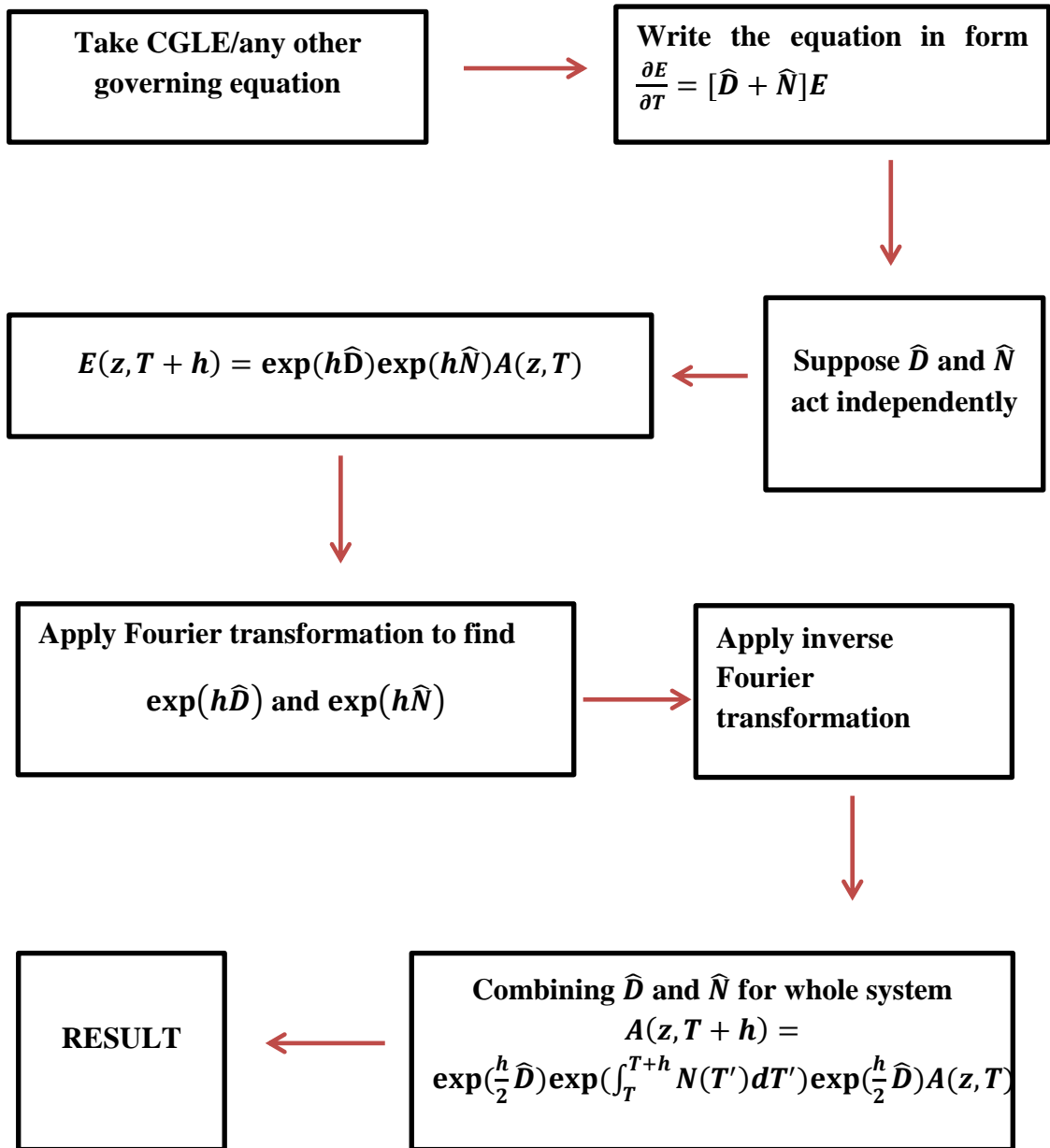


Figure 2.9: SSFM flow chart representation

3.1 MODEL:

Our system consists of VCSEL, frequency selective feedback and saturable absorber. Here we are using graphene as saturable absorber instead of semiconductor saturable absorber mirror.

The equation describing our system is;

$$\frac{\partial E}{\partial t} = \left[-(1 - i\theta) + \frac{\mu(1-i\alpha)}{1+g|E|^2} - \frac{\gamma(1-i\beta)}{1+sg|E|^2+\alpha_{ns}} + i\Delta \right] E + F \quad (3.1.1)$$

$$\frac{\partial F}{\partial t} = -(\lambda + i\Omega)F + \sigma\lambda E \quad (3.1.2)$$

$$E(x, t) = A(\operatorname{sech}(kx)) \quad (3.1.3)$$

Where,

E is cavity field; θ is detuning parameter; μ, γ are pumping parameter for active and passive material respectively; α, β are line width enhancement factor parameter for active and passive material respectively; s is saturation parameter; σ is feedback strength; λ is bandwidth of filter reflector; Ω is frequency of feedback field; α_{ns} is non saturable absorption loss; g represent a number by which saturable absorption of graphene increased.

In equation 3.1.1, $(1 - i\theta)$ represents linear loss or gain; $\frac{\mu(1-i\alpha)}{1+g|E|^2}$ shows saturable absorption for active medium; $\frac{\gamma(1-i\beta)}{1+sg|E|^2+\alpha_{ns}}$ shows saturable absorption for passive medium; $i\Delta$ is diffraction term; and F is feedback.

In our system, equation is little different from the system equation of the model where SESAM is used as saturable absorber. In this system equation we introduce new parameters and modify some pre-existing parameters. In our present discussion we introduce a new parameter ' g ', it tells the strength of saturable absorption of graphene w.r.t. semiconductor saturable absorber mirror. There also exist another new parameter α_{ns} , which is used for non saturable absorption. It is zero in SESAM. We also adjust the value of feedback parameter sigma σ according to ' g '. Now we will discuss the effect of these newly introduced parameters to show the effects of GSA.

3.2 RESULT AND DISCUSSION

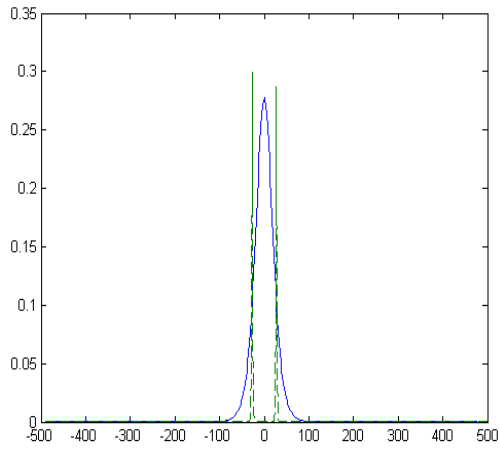
3.21 VARIATION IN NON SATURABLE ABSORPTION PARAMETER α_{ns}

In GSA, absorption coefficient α depends upon intensity of writing beam I , saturation intensity I_s , saturable absorption component α_s , non saturable absorption component α_{ns} as follows [30];

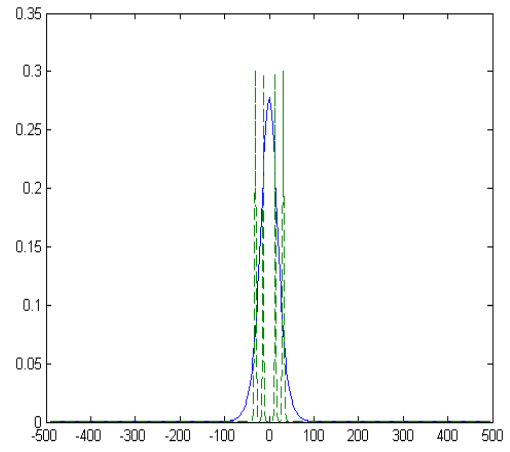
$$\alpha(I) = \frac{\alpha_s}{1+\frac{I}{I_s}} + \alpha_{ns} \quad (3.1.4)$$

Firstly, we introduce α_{ns} and vary its value in order to get stable soliton. For the whole investigation the values of parameters, unless mentioned otherwise, are as follows

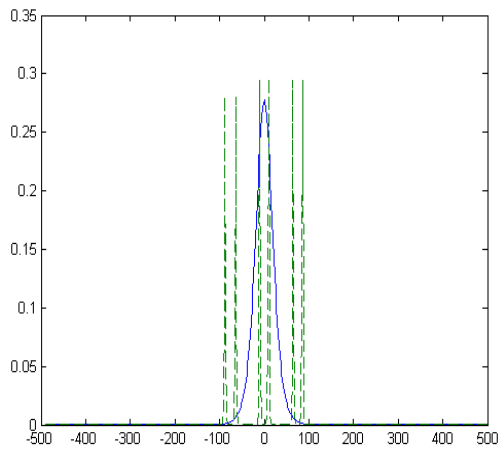
$\theta=1.3$, $\mu=1.37$, $\alpha=2.7$, $\Omega=1.7$, $\beta=0$, $\gamma=0.5$, $\lambda=0.5$ and $s=10$



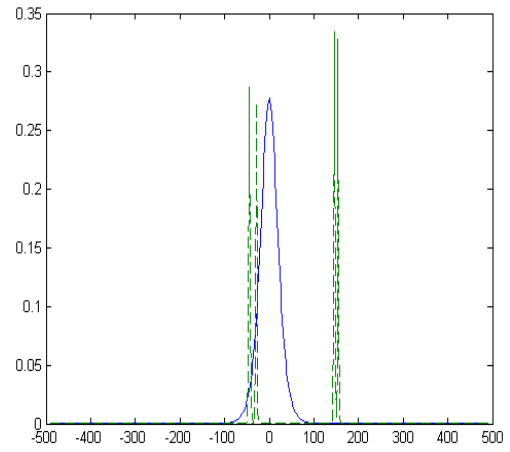
(a)
 $\alpha_{ns} = 0$



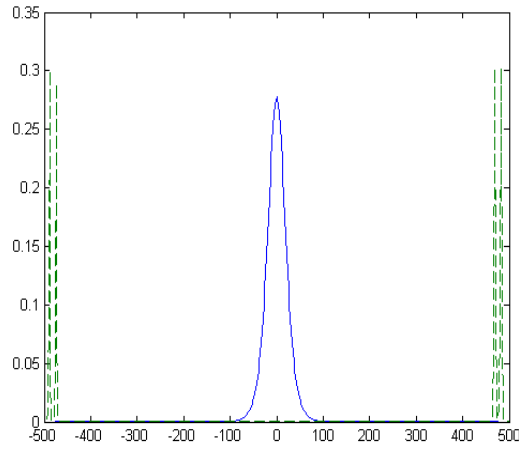
(b)
 $\alpha_{ns} = 1.984 \times 10^{-4}$



(c)
 $\alpha_{ns} = 1.5 \times 10^{-4}$

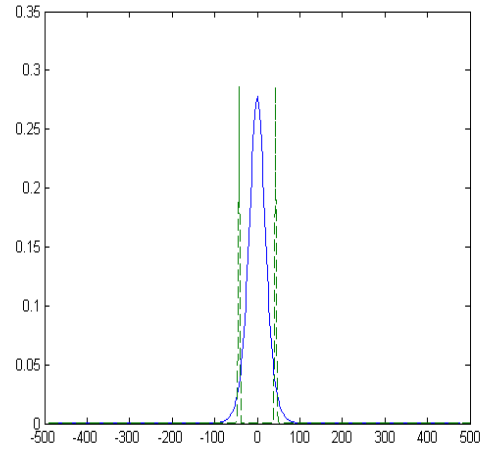


(d)
 $\alpha_{ns} = 6.0 \times 10^{-5}$



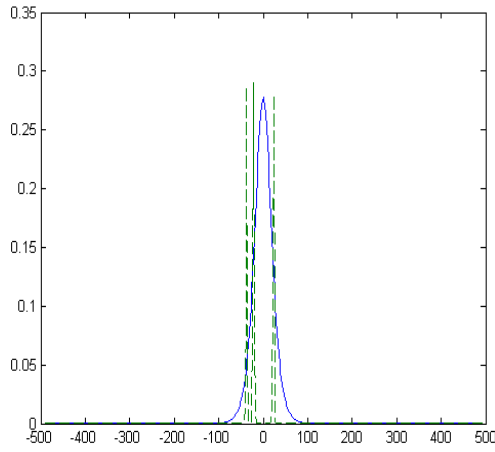
(e)

$$\alpha_{ns} = 4.0 \times 10^{-5}$$



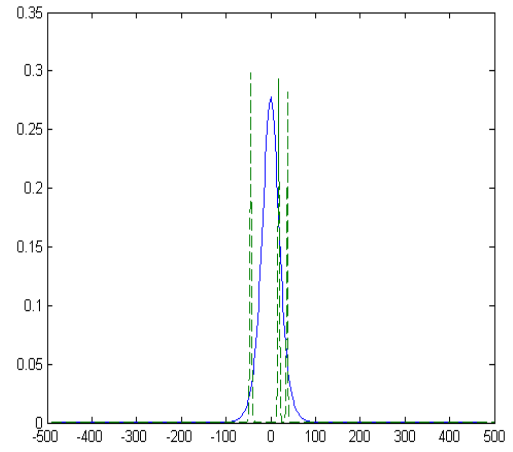
(f)

$$\alpha_{ns} = 9.7 \times 10^{-6}$$



(g)

$$\alpha_{ns} = 9.7 \times 10^{-6}$$



(h)

$$\alpha_{ns} = 6.0 \times 10^{-6}$$

Figure 3.1: Cavity soliton formation with different strength of non-saturable absorption parameter (α_{ns}). (a) $\alpha_{ns} = 0$; (b) $\alpha_{ns} = 1.984 \times 10^{-4}$, (c) $\alpha_{ns} = 1.5 \times 10^{-4}$; (d) $\alpha_{ns} = 6.0 \times 10^{-5}$; (e) $\alpha_{ns} = 4.0 \times 10^{-5}$; (f) $\alpha_{ns} = 9.7 \times 10^{-6}$; (g) $\alpha_{ns} = 9.7 \times 10^{-6}$; and (h) $\alpha_{ns} = 6.0 \times 10^{-6}$. Also $\theta=1.3$, $\mu=1.37$, $\alpha=2.7$, $\Omega=1.7$, $\beta=0$ and $\gamma=0.5$,

Figures 3.1(a) to 3.1(f) show the generation of CS and CS clusters for different zero and non-zero α_{ns} . We have varied the range of α_{ns} from 1×10^{-8} to 1.984×10^{-4} . It is found that maximum allowed value of $\alpha_{ns} = 1.984 \times 10^{-4}$, above this value no stable soliton is obtained. During this process lots of variations in number of soliton peaks, gap between soliton peaks are observed. The following graph will show variation in number of peaks of cavity soliton versus non saturable absorption coefficient α_{ns} .

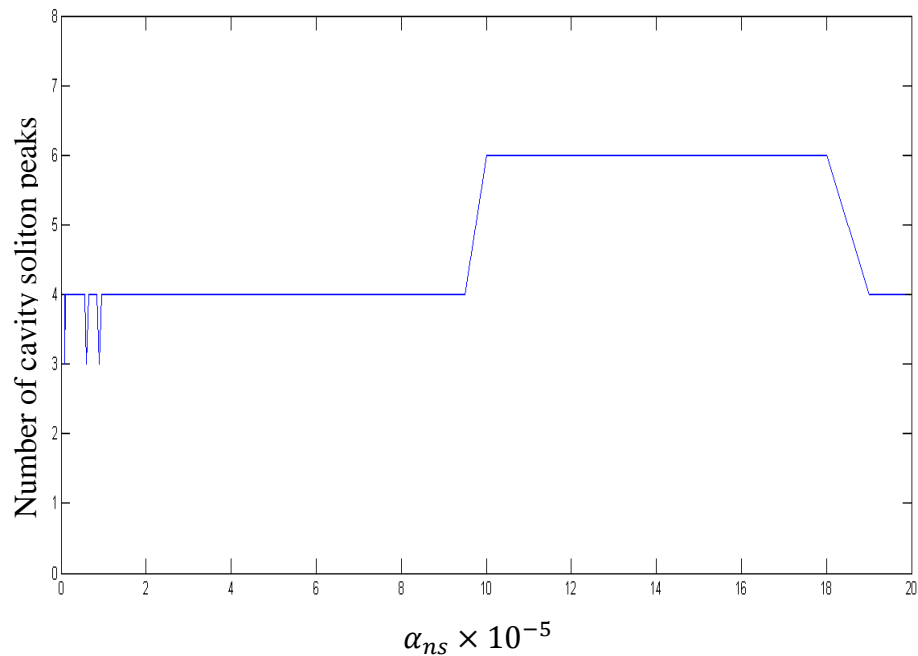


Figure 3.2: Variation in number of CS peaks with non saturable absorption coefficient (α_{ns})

For the above graph it is clear that no trend is found between number of peaks and non saturable loss α_{ns} . Mostly four or six soliton peaks are observed. The significant finding is that we now predict the number of cavity soliton peaks from the above set graph for different values of α_{ns} . Now we find how gap between first and last CS peak vary with

the change in α_{ns} .

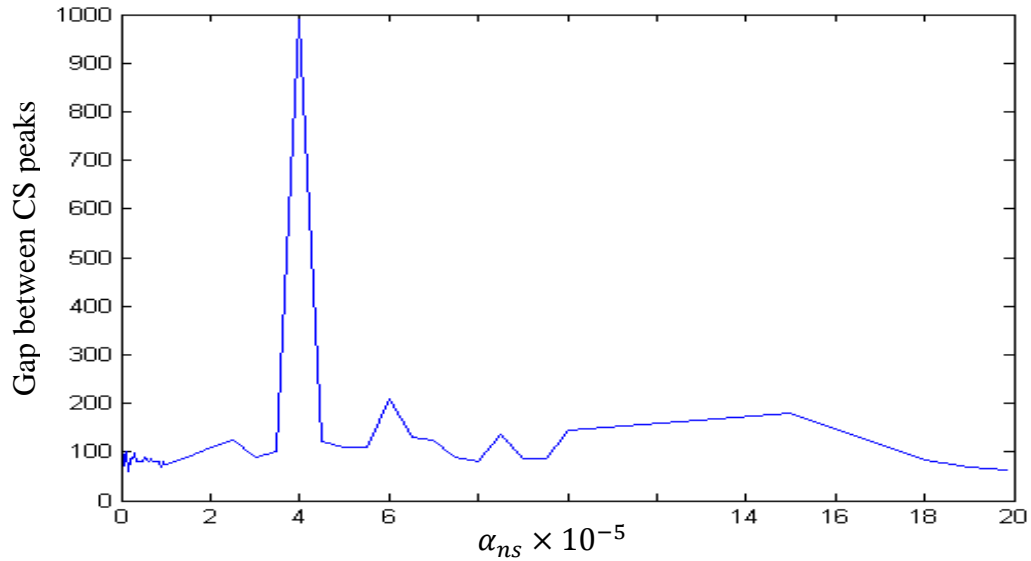
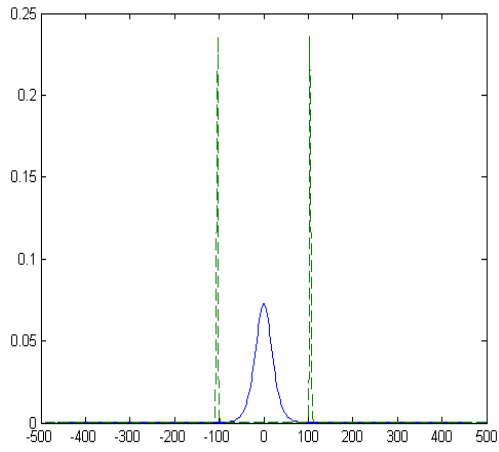


Figure 3.3: Variation of gap between CS peaks with non saturable absorption coefficient (α_{ns})

As observed in figure 3.3, α_{ns} versus number of peaks here also no trend is observed, however in certain cases width increase very sharply.

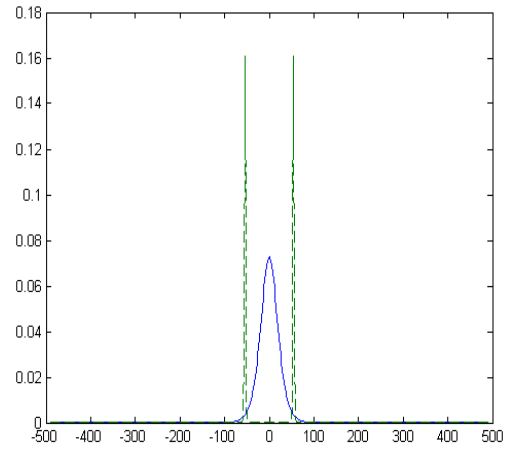
3.22 CHANGES IN FEEDBACK (σ) DUE TO VARIATION IN ‘ g ’:

It is mentioned earlier that saturable absorption of graphene is several times more than saturable absorption due to SESAM, hence the factor ‘ g ’ (which tells how many times GSA has more saturable absorption w.r.t SESAM) is varied from 1 to 10. Here α_{ns} is kept constant which is 8×10^{-5} .



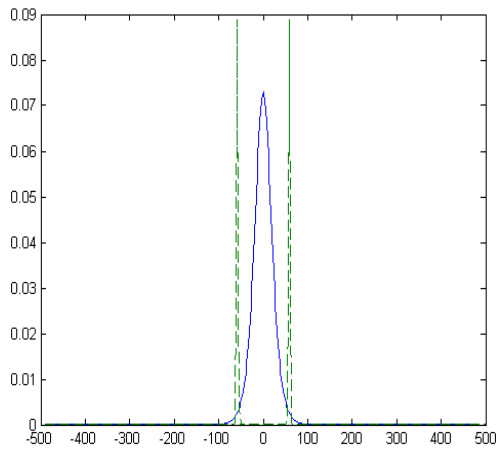
(a)

$\sigma=0.33$ for $g=1.0$



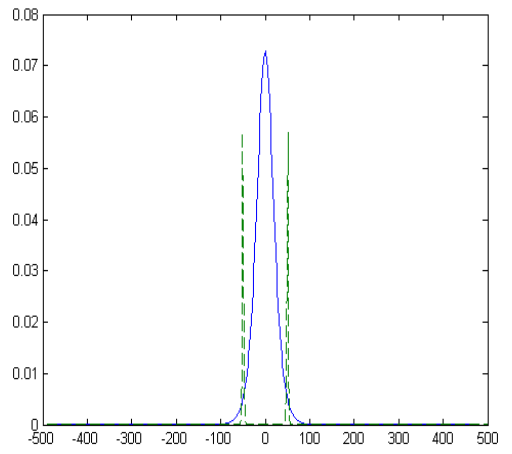
(b)

$\sigma=0.37$ for $g=2.0$



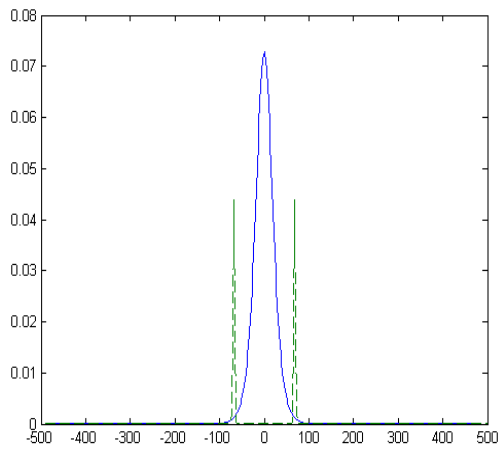
(c)

$\sigma=0.266$ for $g=3.0$

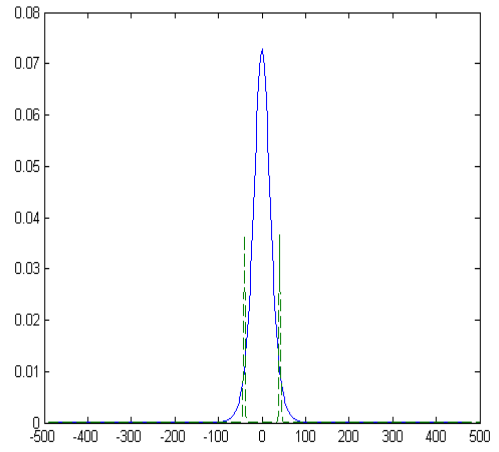


(d)

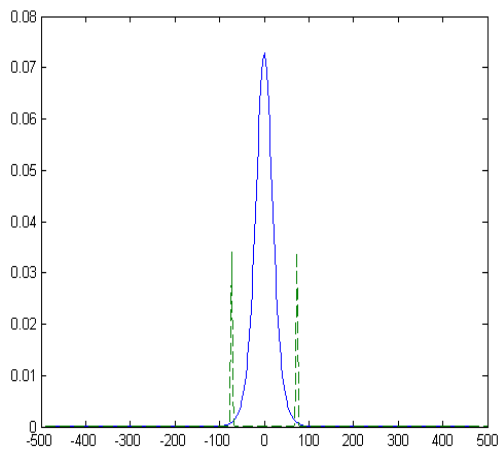
$\sigma=0.265$ for $g=4.0$



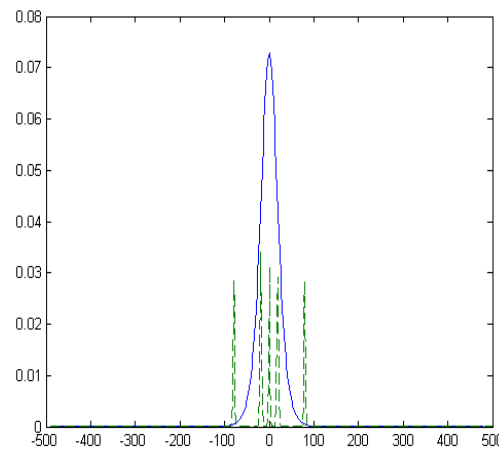
(e)
 $\sigma=0.261$ for $g=5.0$



(f)
 $\sigma=0.261$ for $g=6.0$



(g)
 $\sigma=0.30$ for $g=7.0$



(h)
 $\sigma=0.30$ for $g=8.0$

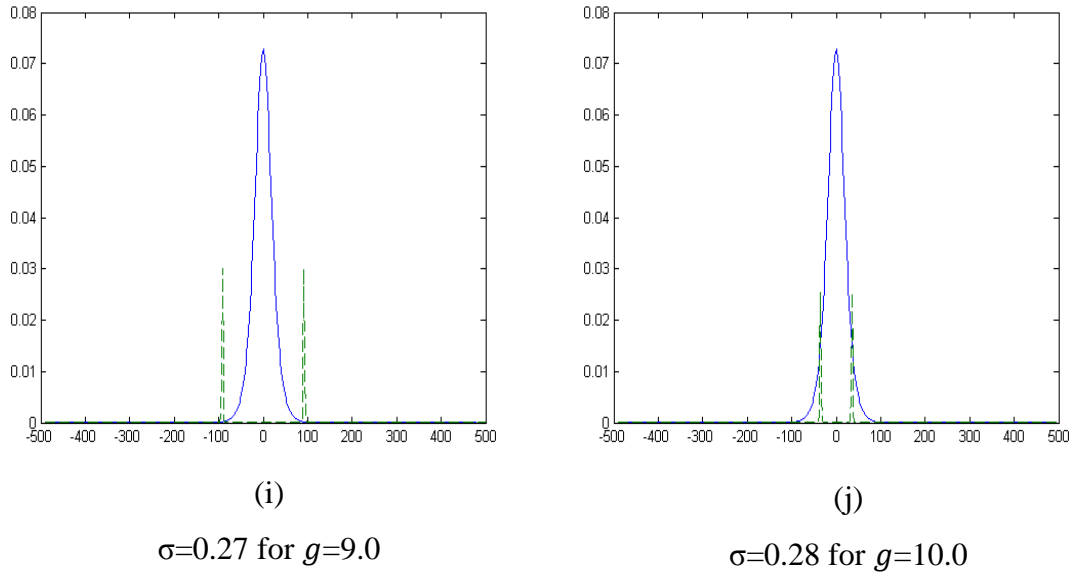


Figure 3.4: Generation of CS for different value of GSA strength ‘ g ’. (a) $g=1.0$, $\sigma=0.3$; (b) $g=2.0$, $\sigma=0.37$; (c) $g=3.0$, $\sigma=0.266$; (d) $g=4.0$, $\sigma=0.265$; (e) $g=5.0$, $\sigma=0.261$; (f) $g=6.0$, $\sigma=0.261$; (g) $g=7.0$, $\sigma=0.30$; (h) $g=8.0$, $\sigma=0.30$; (i) $g=9.0$, $\sigma=0.27$; and (j) $g=10$, $\sigma=0.28$. Also $\theta=1.3$, $\mu=1.37$, $\alpha=2.7$, $\Omega=1.7$, $\beta=0$, $\gamma=0.5$, $\lambda=0.5$, $s=10$ and $\alpha_{ns} = 8.0 \times 10^{-5}$

Figure 3.4(a) to 3.4(j) show the generation of CS clusters for different strength of GSA. For each case different feedback strength (σ) is required for stabilizing CS. Above figures also show how much feedback σ is required if ‘ g ’ is increased. However these figures do not show any pattern or trend w.r.t. increase in ‘ g ’ but show the formation of soliton in such a high saturable absorption with certain feedback. Figure shows the variation of feedback σ with ‘ g ’ is given below.

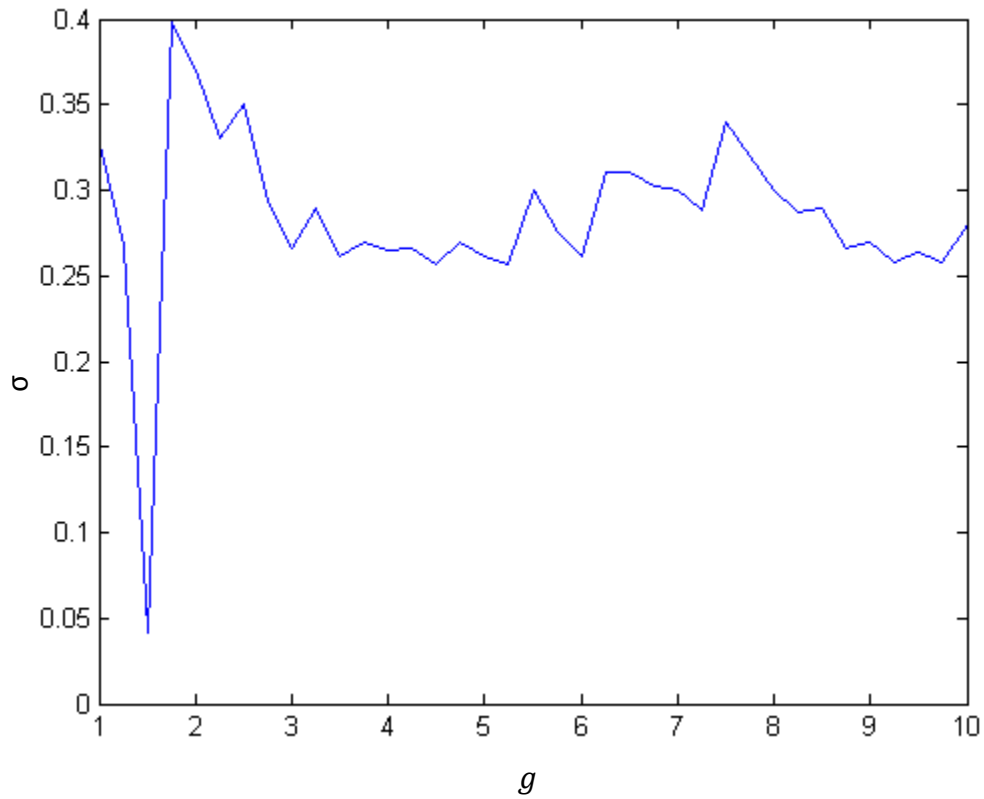
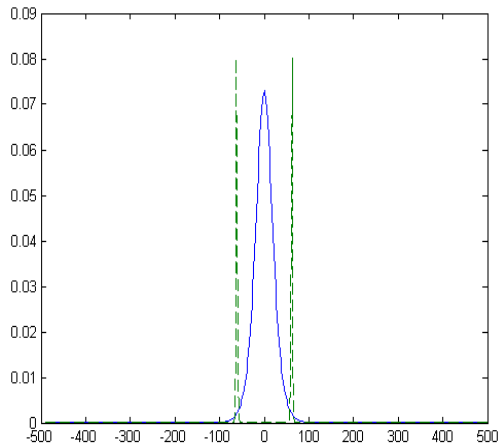


Figure 3.5: Variation of σ w.r.t. g

Above graph shows that feedback factor changes because of factor ‘ g ’. But not the general pattern of variation in σ due to g could be found. Figure 3.5 will significantly assists the experimentalist to generate CS in VSCEL with GSA.

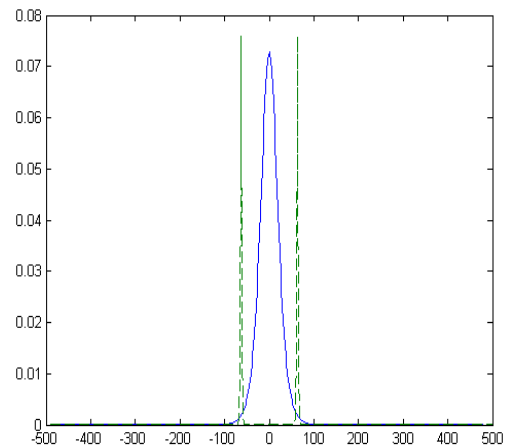
3.23 CAVITY SOLITON IN VISIBLE REGION:

Till date CS are obtained in infra-red region. Our CSs, obtained till now, are also lying in that region(wave length of $981nm$). Now we venture for CS in visible region. Since GSA can be saturated by light of visible to infra-red region. Our motive of getting CS in visible region is feasible.



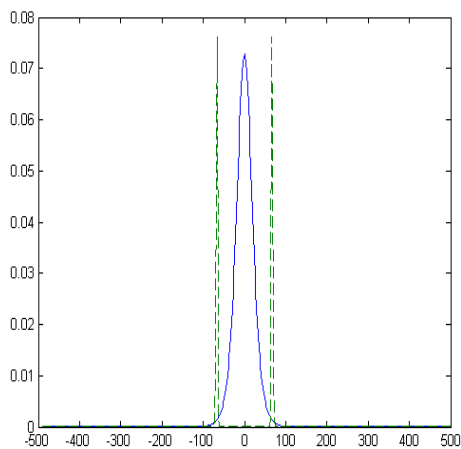
(a)

Wave length= $400nm$



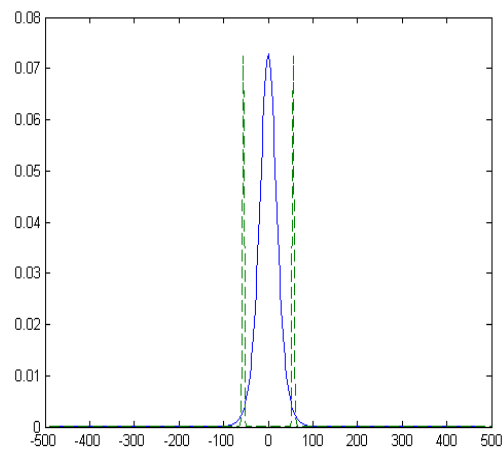
(b)

Wave length= $450nm$



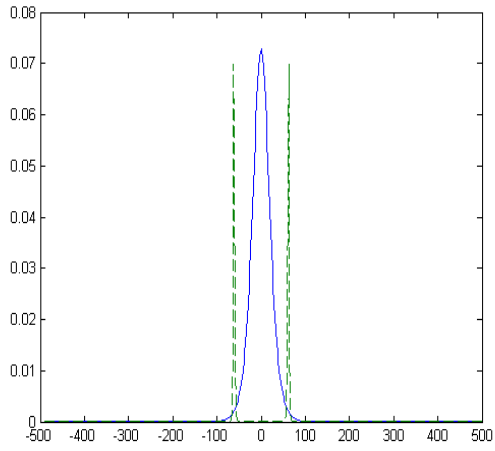
(c)

Wave length= $500nm$



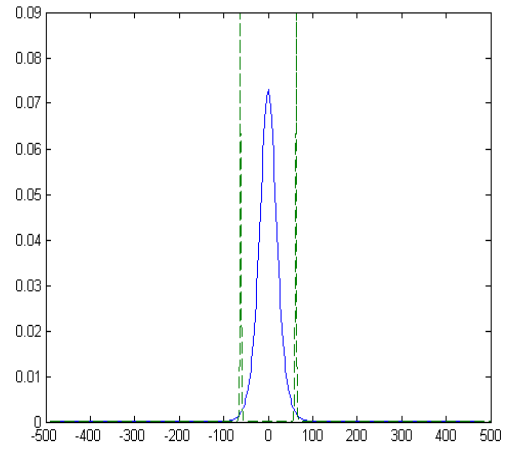
(d)

Wave length= $550nm$



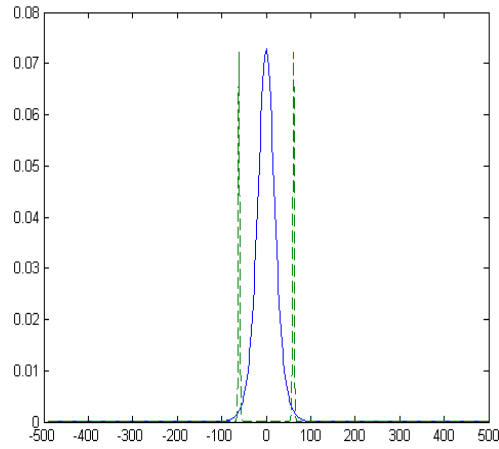
(e)

Wave length=600nm



(f)

Wave length=650nm



(g)

Wave length=700nm

Figure 3.6: Generation of CS cluster at different wave lengths. (a) Wave length=400nm; (b) Wave length=450nm; (c) Wave length=500nm; (d) Wave length=550nm; (e) Wave length=600nm; (f) Wave length=650nm; and (g) Wave length=700nm. Also $\theta=1.3$, $\mu=1.37$, $\alpha=2.7$, $\Omega=1.7$, $\beta=0$, $\gamma=0.5$, $\lambda=0.5$, $s=10$, $\alpha_{ns} = 8.0 \times 10^{-5}$ and $g=3$

Figure 3.6 shows the CS clusters for a wide range of wave length starting for $400nm$ - $700nm$. This is a big step forward in view of applications of CS.

3.24 CAVITY SOLITON IN OPTICAL COMMUNICATION WAVE LENGTHS:

In view of practical application (fiber optical link) we generate CS at $133nm$ and $155nm$.

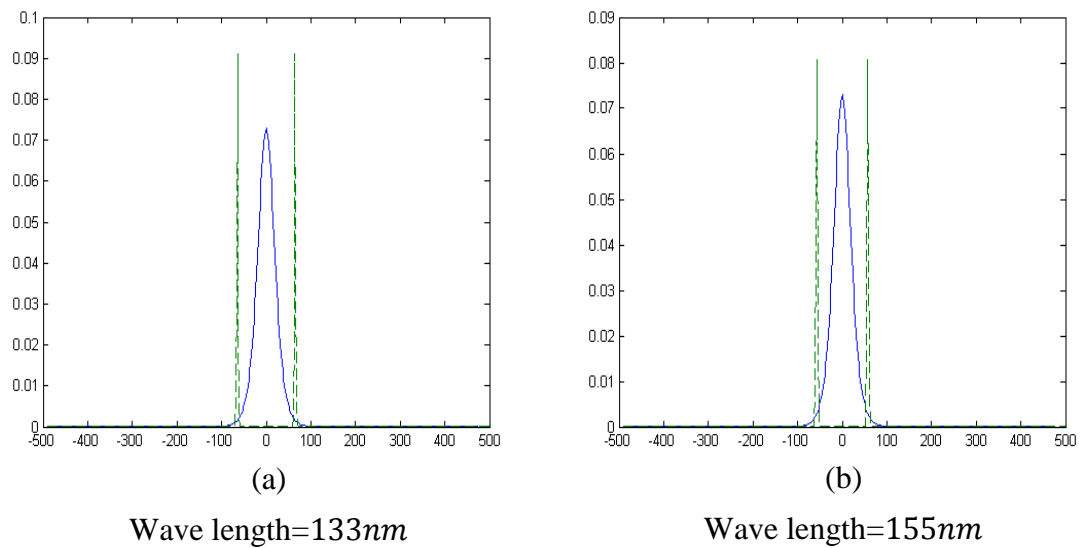


Figure 3.7: Generation of CS cluster at wave lengths compatible for fiber optical communication. (a) Wave length= $133nm$; (b) Wave length= $155nm$. Also $\theta=1.3$, $\mu=1.37$, $\alpha=2.7$, $\Omega=1.7$, $\beta=0$, $\gamma=0.5$, $\lambda=0.5$, $s=10$, $\alpha_{ns} = 8.0 \times 10^{-5}$; and $g=3$.

Figure 3.7 shows the CS at those two wave lengths of $133nm$ and $155nm$.

3.2 CONCLUSION:

We found CS and CS clusters in VCSEL coupled with FSF and GSA as saturable layer.

To the best of our knowledge, this is the first ever observation of CS in GSA embedded

in VCSEL. The effects of different saturation strength of GSA on the generation of CS are demonstrated. Effect of non saturable absorption coefficient α_{ns} has also been studied. Taking the advantages of GSA for first time CSs in visible region is obtained. Also we generate CS at the wave length widows compatible for fiber optical communication. The results presented in this thesis may open a new horizon for CS research and application; provided some challenges inherent with GSA. The electrically controlled absorption properties of GSA may be studied in future in the context of CS generation.

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