EXPERIMENTAL INVESTIGATIONS AND SIMULATION OF STRAIN IN DEVELOPABLE SURFACES

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by

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Certificate

I hereby declare that the thesis entitled "**Experimental Investigation And Simulation Of Strain In Developable Surface**", is an authentic record of my work carried out as per requirements for the award of the degree of **Master of Engineering** in **Production Engineering** at **Thapar University, Patiala** under the supervision of **Dr. Vineet Srivastava and Dr Gulshan Kumar,** Assistant Professor, Mechanical Engineering Department, Thapar University, Patiala during July, 2016 to July, 2017. No part of the matter embodied in this report has been submitted to any other university or institute for the award of any degree.

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It is certified that the above statement made by the student is correct to the best of our knowledge and belief.

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Dedicated

to

myfamily

For their endless love, support and encouragement

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HITESH MITTAL

ABSTRACT

Surface modeling is a technique used for representation and shape design of complex objects such as car, ship and airplane bodies etc, which cannot be achieved through wireframe modelling. Their use in engineering and design environment can be extended beyond just geometric design and representation due to the richness in information of surface models. The main objective of the present work is to generate a developable surface and to observe the total strain induced during the deformation of the developable surface along the length, width and thickness direction. A MATLAB program has also been developed to know the nature of the surface: whether the surface is developable or non developable. The deformation in the surface has been done in two parts. A flat sheet is formed into a curved shape and then the formed curved shape is again formed to regain its original shape. Further experiment has been performed to observe the strain generation during different stages of the deformation. A 3-Dimensional FE (Finite element) model has also been developed to understand the behavior of the sheet during the bending and forming operations. A good agreement between the experiment and the simulated strain distribution was observed in this study.

Keywords: Surface modeling, Developable surface, Strain, FEM, Forming.

List of Abbreviations

u, v	variable
P (u, v)	Vector valued function of two variables
P_{ij}	Control point
$P_u(u, v)$	Tangent vector in u direction
$P_v(u, v)$	Tangent vector in v direction
n_u n_v	Unit vector of the tangent vector
P_{uv}	Twist vector
N(u,v)	Surface normal at a point
n	Unit normal vector
E, F, G	First fundamental coefficient of the surface
L, N, M	Second fundamental coefficient of the surface
K_n	Surface curvature at a point
p	Radius of curvature
K	Gaussian curvature
H	Mean curvature
K_{max}	Maximum principal curvature
K_{min}	Minimum principal curvature
ε^u	Strain along u direction
ε^v	Strain along v direction
L	Length
W	Width
T	Thickness

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Representation and shape design of complex objects such as car, ship and airplane bodies etc cannot be achieved through wireframe modeling. Surface modeling is the technique through which these objects can be modeled well. Their use in engineering and design environment can be extended beyond just geometric design and representation due to the richness in information of surface models [1].

In general surface generation requires some quantitative data, such as set of point, tangent and also some qualitative data, such as intuition for the desired surface and smoothness. These (Quantitative and Qualitative) are also known as hard and soft data. Surface formulation provides the designer with the flexibility to use both type of data in a simple form that is suitable for interactive use. Available modeling techniques interpolate and approximate the given hard data. Bezier surface, for example is a form of approximation and B-spline surface is a form of interpolation [1].

Application and the manufacturing method are responsible for the choice of surface formation. Thus there is no single solution to all the problems. Generally it is preferred to choose a surface that can be produced using three-axis machining. All surface forms are easy to differentiate to determine surface tangents, normal, and curvatures. Polynomial functions are easy to deal with. Due to the large number of coefficients, higher polynomials are avoided for surface design problems. Large numbers of coefficients make it difficult to control the resulting surface. It has been observed that for most practical surface applications, cubic polynomials are sufficient.

It is always desirable to choose a surface mathematical described or formed that is applicable to both surface design and surface representation. Surface representation involves using given data to display and view the surface. Surface design involves using key given data and making interactive changes to obtain a desired surface. Generally three dimensional spaces are required to create a surface. Sufficient points are required to describe a surface. This approach is cumbersome and cannot be used to derive any surface properties. Instead, it is common to employ some form of interpolation scheme and use fewer points. It is even more convenient if analytic form of surface exist all the time. When analytic forms are not available, surfaces are defined in patches. These patches are connected like curves in a piecewise manner.

1.2 TYPES OF SURFACES

There are two types of surfaces for engineering applications, developable surface and non developable surface. These surfaces are also called singly curved surface and doubly curved surface. Developable surfaces are those surfaces which have Gaussian curvature zero at all the points. On the other hand non developable surfaces are those surfaces which have non-zero Gaussian curvature on some of the points on the surface. Developable surfaces are appreciated for metal forming process because forming of these surfaces can be done without much tear and stretching. More or less, this is the only reason that developable surfaces are broadly used for the manufacturing of those parts whose material is not amenable to stretch. But engineering structures surfaces are more commonly fabricated as doubly curved surfaces for the fulfillment of functional requirement like hydrodynamic, aesthetic, or structural. For a given three dimensional surface of curved plate or shell, fabrication process is divided in steps. [3]

Given a three dimensional design surface, which represents a face of curved plate or shell, the first step in the fabrication process is flattening or planar development of this surface into planar shape so that the manufacturer can determine the initial shape of the flat plate and also estimate the strain distribution required to form the shape. The planar shape is formed into an approximation of the design surface by various approaches like forming by matching dies, by continuous hammering, or by line heating using a laser etc. this planar shape for doubly curved surface is usually not unique. Theoretically a large variety of initial planar shape can be deformed into a given curved surface if adequate stretching or shrinkage is allowed. A planer development corresponding to minimum stretching or shrinkage is highly desirable because it saves material and it reduces the work required to form the planer shape to the doubly curved design surface. Early surface development procedures were implemented in shipyards based on geodesic development during the last three decades, mainly for ship hull plates, whose Gaussian curvature is very small. [2]

1.3 LITERATURE SURVEY

A survey of available literature suggests that **Manning (1980) [4]** has developed a procedure for surface development based on an isometric tree. A tree of lines with a spine and branches was first drawn on the curved surface. Then these were developed isometric ally onto planar curves, using the geodesic curvature of the spine and branches on the surface as the curvature of the planar curves. The envelope of the developed pattern formed the planar developed shape. The shape of planar development is dependent on the choice of the spine and branch curves. The stretching along both spine and branch curves was zero. The major limitation of this procedure was that it might not be applicable in metal forming and also does not provide description of strain field.

T. Shimada and Y. Tada (1991) [5] has reported that approximate transformation of curved surface with arbitrary shape into a plane is possible. They have used dynamic programming approach. In designing for curved shapes, CAD systems have a prominent role: in particular methods for the transformation of arbitrary region on the curved surface into flattened forms are indispensable for manufacturing processes. Generally, until now, in the absence of systematic scheme for obtaining flattened forms, manual methods have needed a great deal of work. An objective curved surface is decomposed into region of adjacent strips. Then, each region is developed in turn into a flattened shape. Solving multistage decision processes derives the whole shape. Moreover, a two step algorithm was proposed for obtaining a good initial shape. By way of illustration, the results have been then applied to the problem of transforming duct and shoe models into flattened forms.

B.K. Hinds et al. (1991)[6] have developed patterns for simple surfaces that have been considered in term of Gaussian curvature. Patterns are derived for elliptic and hyperbolic curvature regions. Similar methods have been applied to garments pieces that have been defined as 3D mathematical models at a workstation. Approximations have been applied to patterns to reduce the complexity to level that is acceptable to the clothing industry the limitation of the method is that the developed shape depends on the choice of starting edge. If used in metal forming, it is not guaranteed that the forming process would be realizable from the planar shape to the curved shape.

Michael S. Floater and Kai Hormann (2003) [7] has provided a tutorial and survey of methods for parameterizing surfaces with a view to applications in geometric modeling and computer graphics. They have gathered various concepts from differential geometry, relevant to surface mapping and have used them to understand the strengths and weakness of the many methods for parameterizing piecewise linear surfaces and their relationship to one another.

Chao Liu and Y. Lawrence yao (2002) [8] have proposed a response surface methodology based optimization method for laser forming process design. The propagation of error technique is built into process design as an additional response to be optimized via desirability design as an additional response to be optimized via desirability functions in order to make the design robust.

E Abed et al. (2005)[9] have presented a predictive and adaptive approach to control 3-D laser forming of MS sheet into a desired developable surface. This was defined as a surface that can be unfolded onto the plane without stretching and tearing. The main point of the process was the development of a predictive model to give scan strategies based on required geometry. The forming rate and distribution of the magnitude of forming across the surface were controlled by process speed. If geometry was not formed within one pass, an incremental adaptive approach can be used for subsequent passes. It utilized the error between current and desired geometry to give a new scan strategy.

Cai et al. (2007)[10] introduced a new method of planner flattening required for the development of doubly curved surfaces with necessary geometric constraints through the minimization of edge difference of discretized elements. 3D characteristic of original surface is reduced using this method and is quite different from many other approaches used. Thickness of the sheet is also taken into account by considering the volume conservation of the element using this method. Although because of the ignorance of the hypothesis of constant spring constant and shear stiffness energy model has not been estimated in this study. Ignoring hypothesis of constant of spring constant and shear stiffness results in some error in the energy estimation.

Gan et al.(1996) [11] describes a new method known as computer method for the transformation of any arbitrary surface into flattened pattern. Developable surface is a special case of ruled surface. Gaussian curvature is zero at any point on the developable surface. On developable surfaces some curves are geodesic curves and all the points on geodesic curve are zero. The base of this flattening technique introduce in the process is preservation of length of geodesic curve and linear mapping principles. Untrimmed and trimmed both kinds of developable surfaces can be flattened using this algorithm.

Urooj et al. (2014)[12] computed and investigated the properties of aluminum and steel using the ABAQUS software. The investigated and analyzed the effect of impact loading and change of temperature for solid and hollow cylinder, solid cantilever beam and studied the effect of changing parameter. Results shows stress value is high in steel structure than aluminum. Also they found displacement with change in time is not at all is effected by the type of material. Heat flux is greater in aluminum as compared to steel.

R.Neugebauer et al.(2011) [13] studied about the most important parameter in influencing performance, cost, energy and efficiency of the work. Many of this parameter linked to the velocity parameter of the process. They have found the effect of velocity that how increase and decrease in velocity affect the parameters significantly.

Michael P.Pereira et al. (2008)[14] used finite element analysis to investigate and explained the distribution of contact pressure over die radius, throughout the duration of a channel forming process. Accurate determination of the contact pressure distribution estimates about the tool life. He did numerical analysis to predict the behavior of contact pressure and examined the wear response of an experimental stamping operation.

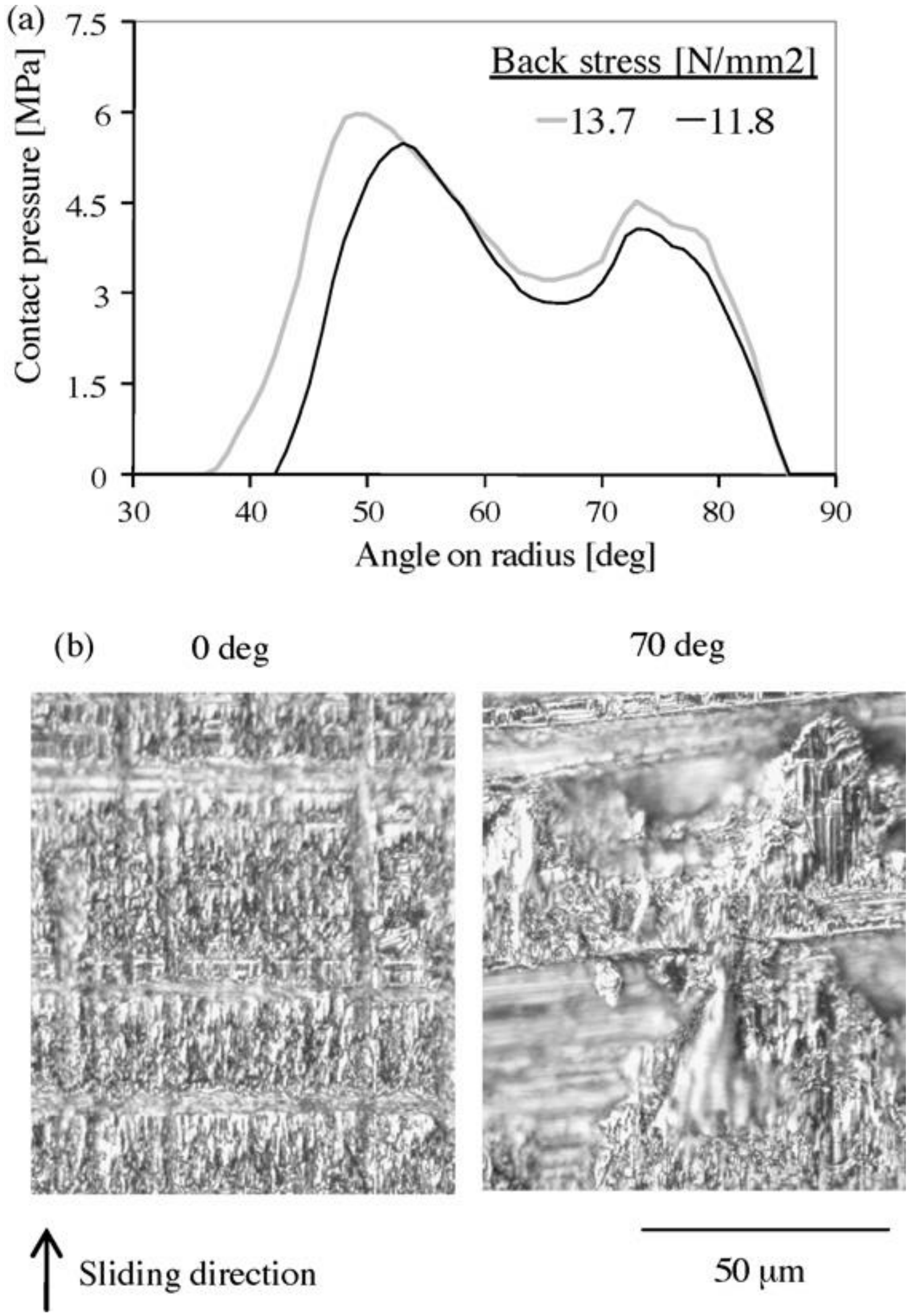

Fig. 1.Experimentally measured pressure distribution over the radius of a bending-under-tension test for 0.8mm thick mild steel strip drawn over a 20mm radius at various back tensions. (b) Optical microscope images at two locations on die radius surface (after 140 parts were formed), from laboratory based channel forming wear test.[14]

Maziar Ramezani et al. (2010) [15] done experimental and FEM study of rubber pad forming process. He used three different types of rubber as flexible punch. The influence of different type of rubber material and speed of punch on punch load and specimen thinning is studied experimental. Using FEM software ABAQUS, he investigated the distribution of stress and strain in the work piece, die and the flexible punch. In the results they found silicon rubber as flexible punch decrease the punch load drastically and polyurethane rubber as a flexible punch is best choice. Good agreement found between FE experiment and simulations results and maximum error of 5% for natural rubber and 5.1% for polyurethane and silicon rubber.

Srivastav et al. [**16]** did structural analysis of piston head to know the performance of piston in real working conditions. He carried out the work to know the distribution of stress on the top surface of piston. He found piston experience maximum stress at piston head and skirts. This maximum stress causes the deformation in the piston.

Hwang et al. (2015) [17] developed a model of developable surfaces by wrapping a planar figure around cones and cylinders. Complicated developable can be constructed by successive mappings using cones and cylinders of different sizes and shapes. They also propose an intuitive control mechanism, which allows a user to select an arbitrary point on the planar figure and move it to a new position. Numerical techniques are then used to find a cone or cylinder that produces the required mapping.

Hong Shen *et al.* (2016) [18]developed an algorithm to minimize the strain energy required for the deformation of the sheet. The total strain energy developed during the process is chosen to evaluate the optimality of a developing process, since less total strain energy developed in the process means less energy consumed in processing, which guarantees that the shape of sheet after development is close enough to the original. The total strain energy during the developing process is calculated by summing the element strain energy of all elements. Then the optimization is implemented.

1.4 RESEARCH GAP

From the review of literature, it was found analyzing the surface properties tool a lot of time as there was no algorithm present which could perform this work. Further the nature of deformation during the forming of sheet into developable surface has not been explained. Therefore in this present work, the following objectives have been defined.

1. To develop a code to determine whether the generated surface is developable or non developable.
2. To generate the desired shape through forming process.
3. To conduct experiment on the developable surface to identify the strains during the forming process.
4. To simulate the forming of developable surface to obtain the strain and analyse the forming process.

CHAPTER 2

Generation of developable surface

2.1 INTRODUCTION

There are important aspects of study in sheet metal forming of thin plate. The plate having thickness in the range of one-tenth to hundred of typical planar dimensions are called thin plates. The points that are at equal distance from both surfaces are called locus points of thin plate. The shape of thin plates is defined using locus points. The locus surface is a neutral surface where deflections are very small due to very small strain. On the contrary, if the strain is large, the analysis of strain gains importance.

An arbitrary surface is not characterized globally, but only at local points. That is only the local shape can be known and be determined by the derivatives of the surface vector. The nature of local shape is classified into four cases according to the Gaussian curvature and the coefficients of the second fundamental form viz. L, M and N at a point. The equation defining coefficients L, M and N have been presented. The commonly used surface are elliptic, hyperbolic, parabolic and planer. They are characterized below: [3]

1) Elliptical surface: it has positive Gaussian curvature.
2) Hyperbolic surface: it has negative Gaussian curvature.
3) Parabolic surface: it has zero Gaussian curvature and $L^2+M^2+N^2 \neq 0$.
4) Planar surface: it has zero Gaussian curvature and L=M=N=0

2.2 SURFACE GENERATION

The conditions in each case are to be satisfied at all focus points. A doubly curved surface has non zero Gaussian curvature, like the elliptic case and hyperbolic case. It generally requires both in plane and bending strains to form. For thin plates, in plane strain is usually much larger than the bending strain and therefore only the former is considered. A doubly curved surface has a form of Bezier surface. An orderly set of data or control points is used to build a topologically rectangular surface. Points on the Bezier surface are given by the following equation [1]

$$P(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{ij} B_{ij}(u) B_{jm}(v) \text{..................} 2.1$$

$$u, v, \varepsilon\ [0,1]$$

Where, P_{ij} is the ij^{th} control points. These point forms the vertices of the control or characteristic polyhedron of the resulting Bezier surface. $B_{in}(u) = C(n,i)u^i(1-u)^{\ n-1}$ & C (n, i) is the binomial coefficient and is given as $C(n,i) = \frac{n!}{i!\times(n-1)}$

On expanding the equation and rewriting

$$P(u,v) = P_{00}(1-u)^m + P_{11}C(n,1)C(m,1)(1-u)^{n-1}(1-v)^{m-1}uv$$

$$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+P_{(n-1)(m-1)}C(n,n-1)C(m,m-1)u_{n-1}v_{m-1}(1-u)(1-v)$$

$$+P_{mn}$$

$$+P_{10}C(n,1)u(1-u)^{n-1}(1-v)^m + P_{01}C(m,1)v(1-u)^n(1-v)^{m-1}$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+P_{(n-1)m}C(n,n-1)u^{n-1}v^m(1-u) + P_{(m-1)n}u^nv^{m-1}(1-v)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2$$

The surface interpolates the four corner points on substitution of (u, v) values of (0, 0) (0, 1) (1, 0) (1, 1) into equation 2.2. The surface is also tangents to the corner segment of the control polyhedron. In the design environment, the Bezier surface is superior to bi-cubic surface because it does not require tangent or twist vector to define a surface. Its main disadvantage is lack of local control. Changing one or more control points affects the shape of whole surface. The user cannot change the shape of part of the surface also. Therefore a bi-cubic Bezier surface has been selected to have more control over surface[1].

For a bi-cubic Bezier surface patch, sixteen points are required to determine the surface patch. The points on the surface patch can be expressed in the matrix form as

$$P(u,v) = [(1-u)^3 \quad 3u^2(1-u) \quad u^3] \times Q \times \begin{bmatrix} (1-v)^3 \\ 3v(1-v)^2 \\ 3v^2(1-v) \\ v^3 \end{bmatrix} \text{……….2.3}$$

Where Q is a matrix have (m+1) rows and (n+1) columns containing sixteen control points because in bi-cubic Bezier surface m=3 and n=3.

We have generated three surfaces as shown in the Figure 2.1, 2.2, 2.3. These are developable surfaces whose Gaussian curvature is zero at all the points on its surface or grid points. Surface shown in the Figure 2.1 has been generated against the surface equation, $f(x,y) = x^2 - y$ where, $-1<x<1$ *and* $-1<y<1$.

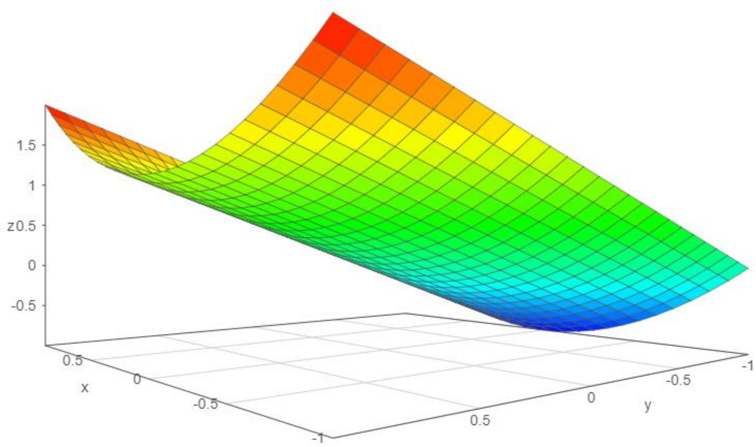

Figure 2.1 Generated Surface

CHAPTER3

SURFACE REPRESENTATION

3.1 INTRODUCTION

Surfaces can be described mathematically in three dimensional spaces by nonparametric and parametric equation. There are several methods to fit nonparametric surfaces to a given set of data points. These fall under two categories. In the first, one equation is fitted to pass through all the points while in the second the data points are used to develop a series of surface patches that connect together with position and first derivative continuity. In both categories, the equation of the surface or surface patch is given by [1]

$$P = \{x \quad y \quad z\}^T = \{x \quad y \quad f(x,y)]^T \ldots\ldots\ldots\ldots\ldots\ldots..3.1$$

Where, P is the position vector of a point on the surface. The natural form of the function f(x,y) for a surface to pass through all the given data points is polynomial, that is

$$z = f(x,y) = \sum_{i=0}^{m} \sum_{i=0}^{n} a_{ij} x^i y^j \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots3.2$$

Where the surface is described by a grid of size (m+1)*(n+1) points

The parametric representation of a surface means a continuous, vector valued function P(u, v) of the two variables, or parameter, u and v where the variables are allowed to range over some connected region of the uv plane and , as they do so, P(u, v) at certain u and v values is the point on the surface at these values. The general way to describe the parametric equation of a three dimensional curved surface in space is

$$P(u,v) = \{x \ \ y \ \ z\}^T = \{x(u,v) \ \ y(u,v) \ z(u,v)\}^T \ldots\ldots\ldots.3.3$$

$$u_{min} \leq u \leq u_{\max,}\, v_{min} \leq v \leq v_{max}$$

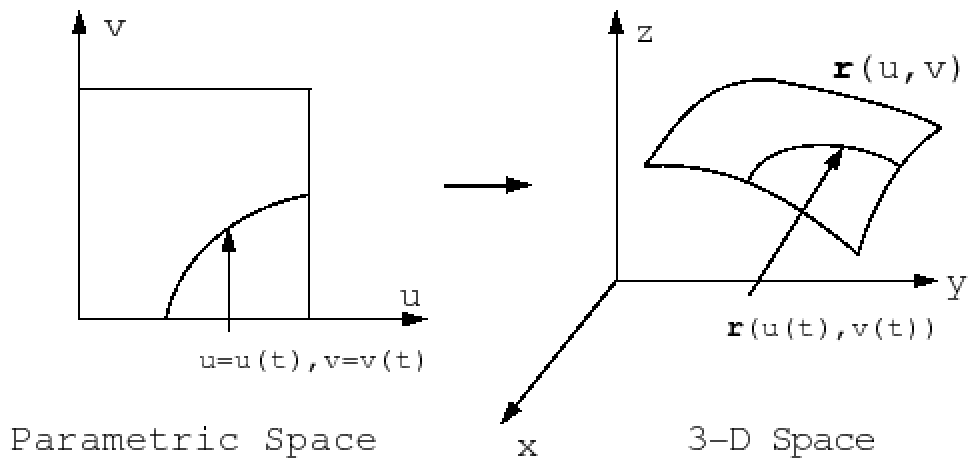

Figure 3.1 A parametric representation of a three dimension surface [2]

The above equation suggests that a general three dimensional surface can be modeled by dividing it into an assembly of topological patches. A patch is considered the basic mathematical element to model a complete surface. Some surface may consist of one patch only while other may be a few patches connected together. The topology of the patch may be rectangular or triangular.

There are two types of surface, analytic and synthetic. Analytic surface are based on wireframe entities and include the plane surface, ruled surface and surface of revolution. Synthetic surface are formed from a given set of data points or curves and include the bi-cubic, Bezier, B-spline and Coons patches. There are few methods to generate synthetic surface such as the tensor product method, rational method and blending method. The tensor product method is the most popular and is widely used in surface modeling [1]. Its widespread use is largely due to its simple separable nature involving only product of univariate basis functions, usually polynomial. It introduces no new conceptual complication due to the higher dimensionality of a surface over a curve. The tensor product formulation is a mapping of a rectangular domain described by the u and v values. Tensor product surfaces fit naturally onto rectangular patches. They have an explicit unique orientation and special parametric or coordinate directions associated with each independent parametric variable.

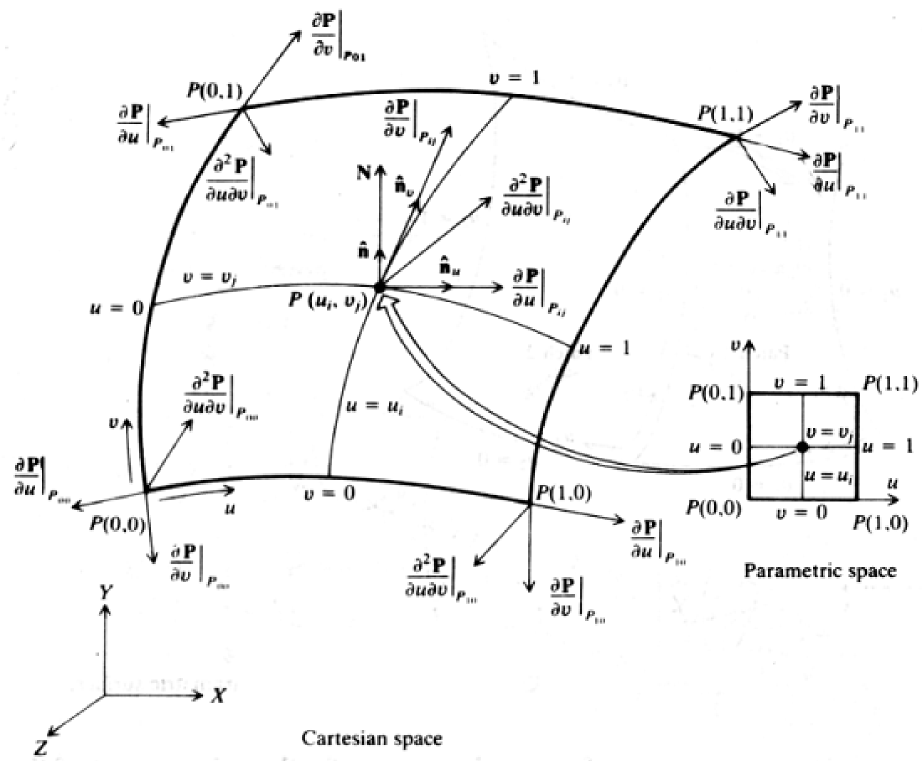

Figure 3.2 A parametric surface patch with its boundary condition [1]

There is a set of boundary conditions associated with a rectangular patch. There are sixteen vectors and four boundary curves as shown. The vectors are four position vector for the four corner points, eight tangent vectors and four twist vectors at the corner points. The four boundary curves are described by holding one parametric variable fixed at one of its limiting values and allowing the other to change freely. Geometric surface analysis is performed using principles differential geometry. The parametric surface P (u, v) is directly amenable to differential analysis. There are intrinsic differential characteristics of a surface such as the unit normal and the principal curvature and directions which are independent of parameterization.

3.2 FIRST FUNDAMETAL FORM:

The tangent vector at any point P (u, v) on the surface is obtained by keeping one parameter as a constant and differentiating position vector of the point with respect to the other. Thus there are two tangent vectors, a tangent to each of the intersecting curves passing through the point. These vectors are given by

$$P_u\ (u,v) = \frac{\partial P}{\partial u} = \frac{\partial x}{\partial u}\hat{\imath} + \frac{\partial y}{\partial u}\hat{\jmath} + \frac{\partial z}{\partial u}\hat{k} \ldots\ldots\ldots\ldots\ldots..3.4$$

$$u_{min} \le u \le u_{min}, v_{min} \le v \le v_{max}$$

Along the v = constant curve, and

$$P_v\ (u,v) = \frac{\partial P}{\partial v} = \frac{\partial x}{\partial v}\hat{\imath} + \frac{\partial y}{\partial v}\hat{\jmath} + \frac{\partial z}{\partial v}\hat{k} \ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

$$u_{min} \le u \le u_{min}, v_{min} \le v \le v_{max}$$

along the u = constant curve. The equation 3.4 and 3.5 are combined to given

$$P_u = [\frac{\partial x}{\partial u} \quad \frac{\partial y}{\partial u} \quad \frac{\partial z}{\partial u}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.6$$

$$P_v = [\frac{\partial x}{\partial v} \quad \frac{\partial y}{\partial v} \quad \frac{\partial z}{\partial v}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

Tangent vectors are useful in determining boundary conditions for patching surface together. The magnitudes and unit vectors of the tangent vectors are given by

$$|P_u| = \sqrt{\left\{\left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2\right.} \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

$$|P_v| = \sqrt{\left\{\left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right.} \ldots\ldots\ldots\ldots\ldots\ldots...3.9$$

$$n_u = \frac{P_u}{|P_u|} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.10$$

$$n_v = \frac{P_v}{|P_v|} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots 3.11$$

Twist vector at a point on the surface is a measure of twist in the surface at that point. It is the rate of change of the tangent vector P_u with respect to v or vice versa. Figure shows the geometric interpretation of the twist vector. If an increment is made in u and v by Δu and Δv and draw the tangent vectors, incremental changes in P_u and P_v at point P are obtained by translating P_u (u, v +Δv) and P_u (u + Δu, v) to P. the incremental rate of change of the two tangent vectors becomes $\Delta P_{u/}$ Δv and ΔP_v /Δu, and the infinitesimal rate of change is given by the following limits

$$\lim_{\Delta v \to 0} \frac{\Delta P_u}{\Delta v} = \frac{\partial P_u}{\partial v} = \frac{\partial^2 P}{\partial u \partial v} = P_{uv} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.12$$

$$\lim_{\Delta u \to 0} \frac{\Delta P_v}{\Delta u} = \frac{\partial P_v}{\partial u} = \frac{\partial^2 P}{\partial u \partial v} = P_{uv} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.13$$

The twist vector depends upon both the surface geometric characteristics and its parameterization. The twist vector can be written in term of its Cartesian components as

$$P_{uv} = \left(\frac{\partial^2 x}{\partial u \partial v} \quad \frac{\partial^2 y}{\partial u \partial v} \quad \frac{\partial^2 z}{\partial u \partial v} \right)^T$$

$$P_{uv} = \left(\frac{\partial^2 x}{\partial u \partial v} \hat{\imath} + \frac{\partial^2 y}{\partial u \partial v} \hat{\jmath} + \frac{\partial^2 z}{\partial u \partial v} \hat{k} \right) \ldots\ldots\ldots\ldots\ldots\ldots.3.14$$

$$u_{min} \leq u \leq u_{min}, v_{min} \leq v \leq v_{max}$$

The normal to the surface is another important analytical property. The surface normal at a point is a vector, which is perpendicular to both tangent vectors at the point, that is

$$N\ (u, v) = \left(\frac{\partial P}{\partial u} \right) \times \left(\frac{\partial P}{\partial v} \right) = P_u \times P_v \ldots..\ldots\ldots\ldots.3.15$$

And the unit normal vector is given by

$$n = \frac{N}{|N|} = \frac{(P_u \times P_v)}{|P_u \times P_v|} \ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots3.16$$

The sense of N, or n is chosen to suit the application. The surface normal is zero when P_u x P_v = 0. It occurs at points lying on cusp, ridge, or self-intersecting surfaces. It can also occur when the two derivatives P_u and P_v are parallel, or one has zero magnitude.

The calculation of the distance between two points on a curved surface becomes an important aspect for surface analysis. In general, two distinct points on a surface can be connected by many different paths, of different lengths, on the surface. The path that have minimum lengths are analogous to a straight line connecting two points in the Euclidean space and are called geodesics. The infinitesimal distance between two points (u, v) and (u +du , v + dv) on a surface is given by

$$ds^2 = P_u.P_u du^2 + 2P_u.P_v dudv + P_v.P_v dv^2 \text{.........}3.17$$

The above equation is called as the first fundamental quadratic form of the surface and is also written as

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2 \text{........................}3.18$$

$where\ E(u, v) = P_u.P_u\ , F(u, v) = P_u.P_v\ and\ G(u, v) = \ P_v.P_v$

E, F and G are the first fundamental or metric coefficient of the surface. This coefficient provides the basis for the measurement of length and areas, and the specification of direction and angles on a surface.

3.3 SECOND FUNDAMENTAL FORM:

The first fundamental form gives the distance element 'ds' which lies in the tangent plane of the surface at P (u, v). It yields no information on how surface curves away from the tangent plane at that point. To investigate the surface curvature, another distance perpendicular to the tangent plane at P (u, v) is introduced and is given by.

$$1/_2\, dh^2 = n.P_{uu} du^2 + 2n.P_{uv} dudv + n.P_{vv} dv^2 \text{...............}3.19$$

The above equation is often called the second fundamental quadratic form of the surface and is given as

$$1/_2\, dh^2 = Ldu^2 + 2Mdudv + Ndv^2 \text{...........................}3.20$$

$where\ L(u, v) = n.P_{uu}\ , M(u, v) = 2n.P_{uv}\ , N(u, v) = n.P_{uv}$

L, M and N are the second fundamental coefficient of the surface and form the basis for defining and analyzing the curvature of a surface.

3.4 CURVATURE

Curvature is the amount by which a geometric object deviates from being flat. There are generally the three types of curvatures i.e. normal curvature, principal curvature and gauss curvature. The curvature of the normal section of the surface is termed as normal curvature. Consider the intersection of the surface with a plane containing a fixed normal vector at the point. This intersection is a plane curve and has a curvature. This is the Normal curvature k_n, and varies with the normal vector. Now the normal curvature can be expressed in terms of first fundamental form coefficient and second fundamental form coefficient. The maximum and minimum values of the normal curvature at a point are called the principle curvatures, k_{max} and k_{min}, and the extreme directions are called principal directions. The Gaussian curvature is equal to the product of the principal curvatures, k_{max} and k_{min}.

3.5 GAUSSIAN CURVATURE

A surface curvature at a point (u, v) is defined as the curvature of the normal section that lies on the surface and passes by the point. A normal section curve is the intersection curve of a plane passing through the normal n at the point and the surface. The surface curvature at the point on a normal section curve given by the form (u =u(t), v= v(t) can be written as

$$K_n = \frac{Lu^2+2Muv+Nv^2}{Eu^2+2Fuv+Gv^2} \qquad \text{……………………3.21}$$

And the radius of curvature at the point is $p = 1/k_n$. The above equation gives the surface curvature in any direction at point (u, v). During analysis of curved surface two curvature viz. Gaussian curvature and mean curvature are used. The Gaussian curvature K and Mean Curvature H are defined by

$$K = \frac{(L\times N)-M^2}{(E\times G)-F^2} \qquad \text{……………………………..3.22}$$

$$H = \frac{(E\times N)+(G\times L)-(2\times F\times M)}{2\times((E\times G)-F^2)} \qquad \text{………………..3.23}$$

3.6 PRINCIPAL CURVATURE:

The values of Gaussian curvature and mean curvature are use to obtain the principal curvature, which are the upper (maximum) and lower (minimum) bounds on the curvature at the point.

$$k_{max} = H + \sqrt{(H^2 - k)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.24$$

$$k_{min} = H - \sqrt{(H^2 - k)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.25$$

Gaussian and mean curvature can be written in term of k_{max} and k_{min} as

$$K = k_{max} \times k_{min}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.26$$

$$H = {}^{1}/_{2}\,(k_{min} \times k_{max})\ldots\ldots\ldots\ldots\ldots\ldots.3.27$$

The value of Gaussian curvature can be positive, negative or zero depending upon the signs of k_{min} and k_{max}. surface that have zero Gaussian curvature over the entire surface are called developable, that is , they can be laid that flat on a plane without stretching, tearing or distorting them. Tables, give the value of coefficients of first fundamental form, coefficients of second fundamental for, Gaussian curvature, maximum and minimum principal curvature at various points on the generated surfaces.

3.7 GENERATED SURFACE

The equation for the surface is given as

$$z(x, y) = x^2 - y\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.7.1$$

$$let\ x = u, y = v, z = u^2 - v$$

Now equation of surface patch in parametric form is given by

$$P = \{u, v, u^2 - v\}^T\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.7.2$$

$$u_{min} \le u \le u_{min}, v_{min} \le v \le v_{max}$$

First fundamental form:

Tangent Vector along 'u'

$$P_u(u, v) = \frac{\partial p}{\partial u} = \frac{\partial x}{\partial u}\hat{\imath} + \frac{\partial y}{\partial u}\hat{\jmath} + \frac{\partial z}{\partial u}\hat{k}$$

$$P_u(u, v) = \hat{\imath} + 2u\hat{k}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.7.3$$

$$u_{min} \le u \le u_{max}$$

Similarly, along 'v'

$$P_v(u,v) = \frac{\partial p}{\partial v} = \frac{\partial x}{\partial v}\hat{\imath} + \frac{\partial y}{\partial v}\hat{\jmath} + \frac{\partial z}{\partial v}\hat{k}$$

$$P_v(u,v) = \hat{\jmath} - \hat{k} \quad \text{……………………………..3.7.4}$$

$$v_{min} \le v \le v_{max}$$

$$\begin{matrix} P_u \\ P_v \end{matrix} = \begin{bmatrix} 1 & 0 & 2u \\ 0 & 1 & -1 \end{bmatrix} \quad \text{…………………………...…3.7.5}$$

Tangent vector are useful in determining boundary conditions for patching surface together, the magnitudes and unit vectors of the tangent vector are given by

$$|P_u| = \sqrt{\left\{\left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2\right.}$$

$$|P_u| = \sqrt{1 + 4u^2} \quad \text{…………………………………3.7.6}$$

Similarly

$$|P_v| = \sqrt{2} \quad \text{……………………………………...3.7.7}$$

Now

$$n_u = \frac{P_u}{|P_u|}$$

$$n_v = \frac{P_v}{|P_v|}$$

$$n_u = \frac{(1+2u)}{\sqrt{1+4u^2}} \quad \text{……………………………….3.7.8}$$

$$n_v = \frac{((1+(-1))}{\sqrt{2}} \quad \text{…………………………….3.7.9}$$

$$n_v = 0.$$

Twist Vector:

Now Twist vector can be written in its Cartesian component

$$P_{uv} = \begin{pmatrix} \frac{\partial^2 x}{\partial u \partial v} & \frac{\partial^2 y}{\partial u \partial v} & \frac{\partial^2 z}{\partial u \partial v} \end{pmatrix}^T$$

$$P_{uv} = 0 \ldots\ldots 3.7.10$$

The normal to the surface is another important analytical property. The surface normal at a point is a vector, which is perpendicular to both tangent vectors at the point, that is

$$N\ (u, v) = \left(\frac{\partial P}{\partial u}\right) \times \left(\frac{\partial P}{\partial v}\right) = P_u \times P_v$$

$$N(u,v) = \begin{vmatrix} i & j & k \\ 1 & 0 & 2u \\ 0 & 1 & -1 \end{vmatrix}$$

$$N(u, v) = -2u\hat{\imath} + \hat{\jmath} + \hat{k} \ldots\ldots 3.7.11$$

Unit Normal Vector:

And the unit normal vector is given by

$$n = \frac{N}{|N|} = \frac{(P_u \times P_v)}{|P_u \times P_v|}$$

$$n = \frac{-2u\hat{\imath} + \hat{\jmath} + \hat{k}}{\sqrt{2 + 4u^2}} \ldots\ldots 3.7.12$$

Now the infinitesimal distance between two points on a surface is given by

$$ds^2 = P_u . P_u du^2 + 2P_u . P_v du dv + P_v . P_v dv^2$$

The above equation is called as the first fundamental quadratic form of the surface and is written as

$$P_u.P_u = E = \{\hat{\imath} + 2u\hat{k}\}.\{\hat{\imath} + 2u\hat{k}\}$$

$$P_u.P_u = (1 + 4u^2) \qquad \text{3.7.13}$$

$$P_u.P_v = F = \{\hat{\imath} + 2u\hat{k}\}.\{\hat{\jmath} - \hat{k}$$

$$P_u.P_v = -2u \qquad \text{3.7.14}$$

$$P_v.P_v = G = \{\hat{\jmath} - \hat{k}\}.\{\hat{\jmath} - \hat{k}$$

$$P_v.P_v = G = 2 \qquad \text{3.7.15}$$

Now the first fundamental quadratic form of surface is written as

$$ds^2 = Edu^2 + 2F\,dudv + G\,dv^2$$

$$ds^2 = (1+4u^2)du^2 + (-4u)dudv + 2\,dv^2 \qquad \text{3.7.16}$$

where $E(u,v) = P_u.P_u$, $F(u,v) = P_u.P_v$ *and* $G(u,v) = P_v.P_v$

E, F and G are the first fundamental or metric coefficient of the surface. This coefficient provides the basis for the measurement of length and areas, and the specification of direction and angles on a surface.

Second fundamental form:

To investigate the surface curvature, another distance perpendicular to the tangent plane at P (u, v) is introduce and given by

$$\tfrac{1}{2}\,dh^2 = n.P_{uu}du^2 + 2n.P_{uv}dudv + n.P_{vv}dv^2$$

The above equation is often called the second fundamental quadratic form of the surface and coefficients are calculated as:

$$P_{uu} = \begin{pmatrix} \dfrac{\partial^2 x}{\partial u^2} & \dfrac{\partial^2 y}{\partial u^2} & \dfrac{\partial^2 z}{\partial u^2} \end{pmatrix}^T$$

$$P_{uu} = [0 \quad 0 \quad 2] \qquad \text{3.7.17}$$

$$P_{uu} = (2\hat{k})$$

$$P_{vv} = \begin{pmatrix} \frac{\partial^2 x}{\partial v^2} & \frac{\partial^2 y}{\partial v^2} & \frac{\partial^2 z}{\partial v^2} \end{pmatrix}^T$$

$$P_{vv} = [0 \quad 0 \quad 0] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 3.7.18$$

$$P_{uv} = \begin{pmatrix} \frac{\partial^2 x}{\partial u \partial v} & \frac{\partial^2 y}{\partial u \partial v} & \frac{\partial^2 z}{\partial u \partial v} \end{pmatrix}^T$$

$$P_{uv} = [0 \quad 0 \quad 0]^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7.19$$

$$L = n.P_{uu} = \left\{\frac{-2u\hat{\imath} + \hat{\jmath} + \hat{k}}{\sqrt{2+4u^2}}\right\}.\{2\hat{k}\}$$

$$L = \frac{2}{\sqrt{2+4u^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7.20$$

$$M(u,v) = n.P_{uv} = \left\{\frac{-2u\hat{\imath} + \hat{\jmath} + \hat{k}}{\sqrt{2+4u^2}}\right\}.\{0\}$$

$$M(u,v) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7.21$$

$$N(u,v) = n.P_{vv} = \left\{\frac{-2u\hat{\imath} + \hat{\jmath} + \hat{k}}{\sqrt{2+4u^2}}\right\}.\{0\}$$

$$N(u,v) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7.22$$

$$\tfrac{1}{2}\, dh^2 = Ldu^2 + 2M\, du\, dv + Ndv^2$$

Now this equation comes out to be

$$1/_2\, dh^2 = \frac{2}{\sqrt{2+4u^2}} du^2 + 2\times(0)dudv + (0)dv^2 \ldots\ldots 3.7.23$$

$$where, L(u,v) = n.P_{uu}, M(u,v) = n.P_{uv}, N(u,v) = n.P_{vv}$$

L, M and N are the second fundamental coefficient of the surface and form the basis for defining and analyzing the curvature of a surface.

Gaussian Curvature is given by:

$$K = \frac{(L\times N)-M^2}{(E\times G)-F^2}$$

$$K = \frac{\frac{2}{\sqrt{2+4u^2}}\times 0-0}{\left((1+4u^2)\times 2\right)-(4)} \text{.......................3.7.24}$$

K = 0

Mean Curvature ‘H’ is given by

$$H = \frac{(E\times N) + (G\times L) - (2\times F\times M)}{2\times((E\times G) - F^2)}$$

$$H = \frac{((1+4\mathrm{u}2)\mathrm{x}(0)+2\mathrm{x}\frac{2}{\sqrt{2+4\mathrm{u}^2}} - 2\times(-2)\times 0}{2\times((1+4\mathrm{u}^2)\times 2-(-2^2)} \text{.............3.7.25}$$

$$H = \frac{\frac{1}{\sqrt{1+2u^2}}}{8u^2}$$

$$k_{max} = H + \sqrt{(}H^2 - k)$$

$$k_{min} = H - \sqrt{(}H^2 - k)$$

For singly curved or developable surface K is always zero on its surfaces at all points

For K = 0

k_{max} =H+H=2H

$$k_{min} = H - H = 0$$

Gaussian curvature can also be written as

$$K = k_{min}\times k_{max} = 0\times 2H = 0$$

Surface for the above equation (3.7.1) is shown in Figure 3.1 and similar solution of coefficient of first and second fundamental form is obtained using the surface theory.

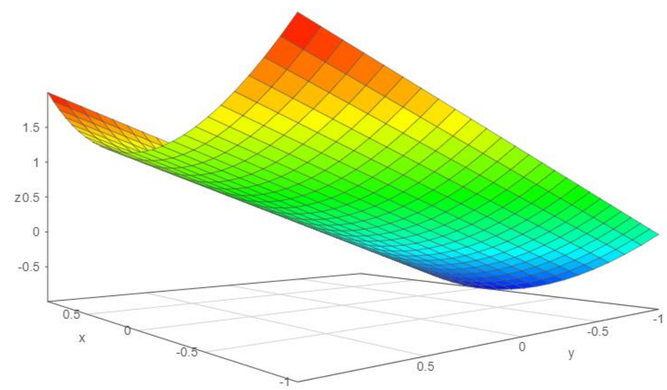

Figure 3.4 Generated Surface

Tables 3.1 give the value of coefficients of first fundamental form, coefficients of second fundamental for, Gaussian curvature, maximum and minimum principal curvature at various points on the generated surfaces of the equation Z(x,y) = x^2-y.

Table 3.1:Value of coefficients of first fundamental form, coefficients of second fundamental for, Gaussian curvature, maximum and minimum principal curvature at various points on the generated surfaces of the equation Z(x,y) = x^2-y

U	V	Coefficient of first Fundamental form			Coefficient of second Fundamental form			Mean Curvature	Max. Principal Curvature	Min. Principal Curvature	Gaussian Curvature
		E	F	G	L	M	N	H	K_{max}	K_{min}	K
0	0	1	0	2	1.4142	0	0	0	0	0	0
0.25	0.25	1.25	0.5	2	1.3333	0	0	1.88	0.65982	0	0
0.5	0.5	2	1	2	1.1547	0	0	0.41	0.3698	0	0
0.75	0.75	3.25	1.5	2	0.9701	0	0	0.15	0.41525	0	0
1	1	5	2	2	0.7559	0	0	0.072	0.09485	0	0

CHAPTER 4

SIMULATION OF FORMING PROCESS

4.1 PHYSICAL DESCRIPTION OF MODEL

A three dimensional FE model was developed to simulate a sheet forming process. There are two FE models were developed in this study. First model was created to simulate a flat sheet into curved sheet as shown in the figure and the second model was developed to simulate the sheet flattening process of the formed curved sheet from the first model as shown in the figure 4.2. The first model consists of three components: punch, die and a flat sheet with dimensions 75 X 75 mm and the second model consists of punch, die and a curved sheet. The model was constructed using Abaqus™ 3D CAE (complete abaqus environment) system. In FE model, punch and die was modeled as discrete rigid and sheet was considered as an elasto-plastic body.

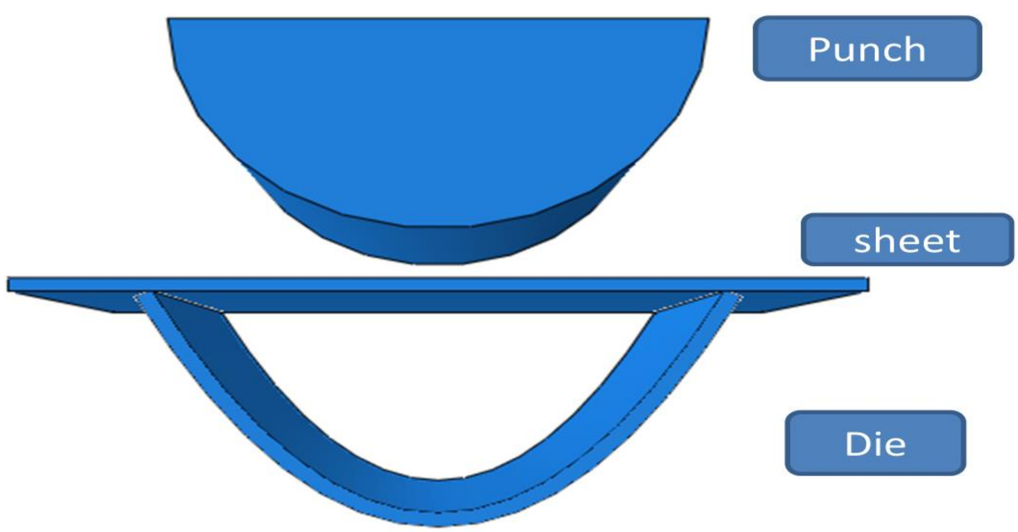

Figure 4.1: Die and Punch arrangement for curving a flat sheet

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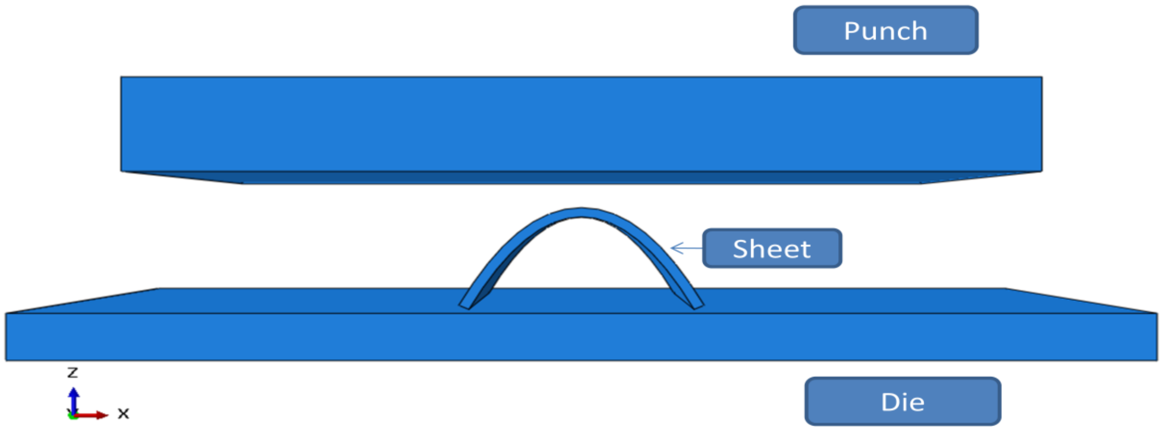

Figure 4.2: Die and Punch arrangement for flattening of the sheet

For the second model, dimension of the punch and die were 100 X 100 mm and 140 X 100 mm respectively. Abaqus™ provides both implicit and explicit techniques for simulating the present problem. It is important to note that the explicit method demands less space, less memory usage and less computational time than the implicit method. The convergence difficulty that may be present in implicit method can also be avoided with an explicit method. Thus, a dynamic explicit code has been used for determining the distribution of plastic strain.

4.2 NUMERICAL FORMULATION

The geometric model used a three dimensional continuum stress/displacement, 8-noded linear brick, reduced integration, hourglass control element (C3D8R element). Abaqus/Explicit provides three options for the formulation of C3D8R element: average strain kinematic, orthogonal and centroid formulation. The average strain kinematic formulation though takes longer time was preferred over other two techniques due to the highest level of accuracy.

A structured mesh was used in this model. The present model contains 46875 elements and 63504 nodes with 3 elements in the thickness direction as shown in Figure 4.3.

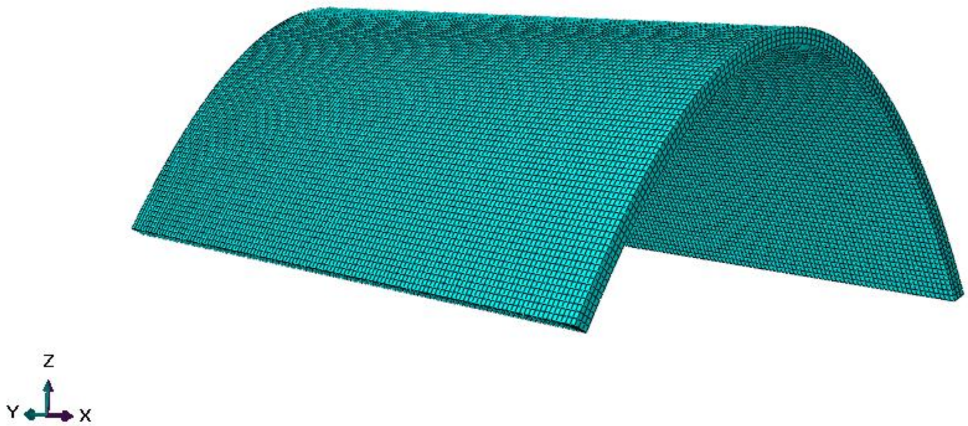

Figure 4.3: Meshing of part

4.3 MATERIAL MODEL

Abaqus™ offers an user defined material model for the purpose of finite element simulation. The elastic-plastic deformation behavior was used to define the material model for the present problem. The isotropic elastic properties include young modulus of elasticity (E) and poisson's ratio was assumed to be 207 GPa and 0.3, respectively [19]. The plastic flow stress was defined in the tabular form of yield stress and plastic strain as shown in the table1 [19].

Table 4.1: Plastic flow stress [19]

Yield stress	Plastic strain
286	0
350	0.04
450	0.14
500	0.25
550	0.35
600	0.495

4.4 BOUNDARY AND CONTACT CONDITIONS

The translation and rotational movement of the die was encastered (U1=U2=U3=UR1=UR2=UR3=0) in all directions for both first and second model. For the second model, a displacement/rotation boundary condition was created to constrain the motion of sheet. The rotational degree of freedom of both sides of the sheet was made zero in all direction, throughout the simulation. The punch was given a velocity of 20 mm/sec in downward direction (figure 4.4)

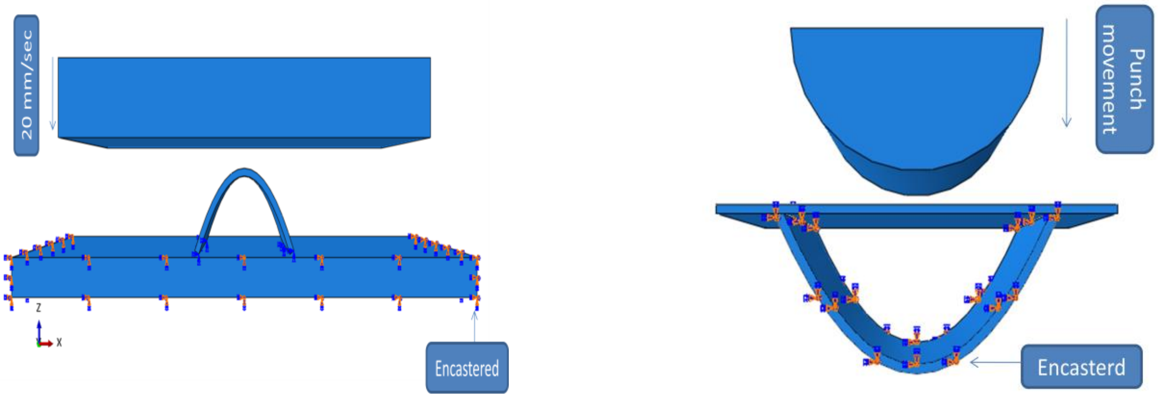

Figure 4.4 Boundary Conditions for strain simulation.

An interaction property was defined between the two contact surfaces in terms of frictional force. In finite element analysis, a proper interaction between the elements is an essential part. Thus, in FE analysis, when two surfaces come in contact with each other, an interaction between the elements and forces is transmitted across their common surface. In the mechanical contact analysis, the 'tangential behavior' was specified to define the friction between the surfaces. Friction formulation between the surfaces was selected as 'penalty' to allow some relative motion of the surface. The surface to surface contact was utilized to define the interaction. Punch and die was chosen as master surface and surface of sheet was chosen as slave surface for both the models as shown in the figure 4.5.

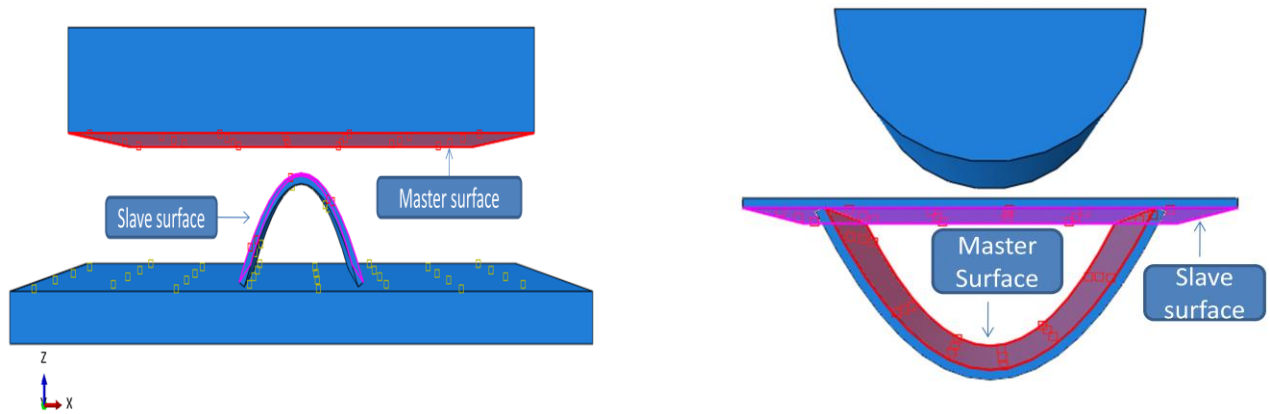

Figure 4.5 Master and slave surface

4.5 MESH SENSITIVITY

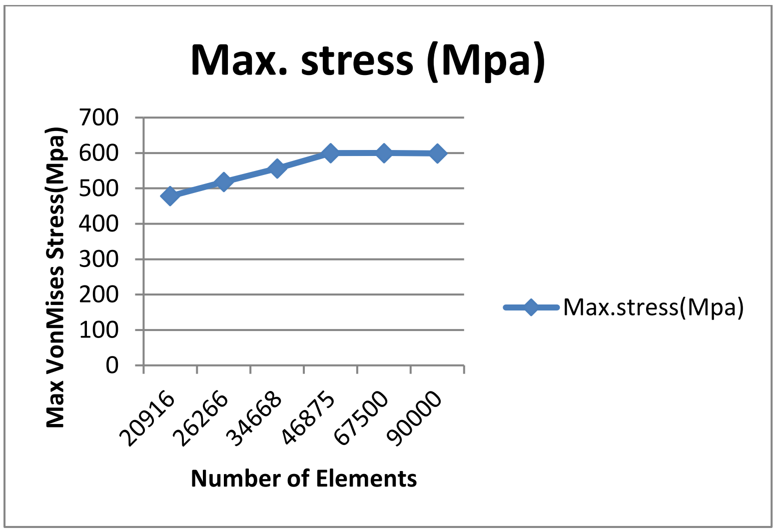

Figure 4.6 Mesh Sensitivity

Element size plays an important role in attaining accurate results. The large number of elements can increase the computational time and a very small number of elements can lead to the inaccurate results. Therefore, determining optimal element size is must for carrying out the simulation effectively. Simulations were done using different mesh sizes and the number of elements in the model ranged between 20000 and 90000. It can be seen from the figure 4.8 that the maximum stress values saturate at 46875 elements.

4.5 RESIDUAL STRESS

The elastic-plastic formulation is divided into elastic and plastic components. In this work, to get the residual stress of the deformed sheet after process is over, sufficient relaxation time was provided for the elastic recovery by giving time step of 0.7. As the sheet pressed below the punch and then punch moves up, stresses starts decreasing and reaches as saturation value. The stresses left in the deformed part after the removal of loads are known as residual stress as shown in the figure 4.6, 4.7.

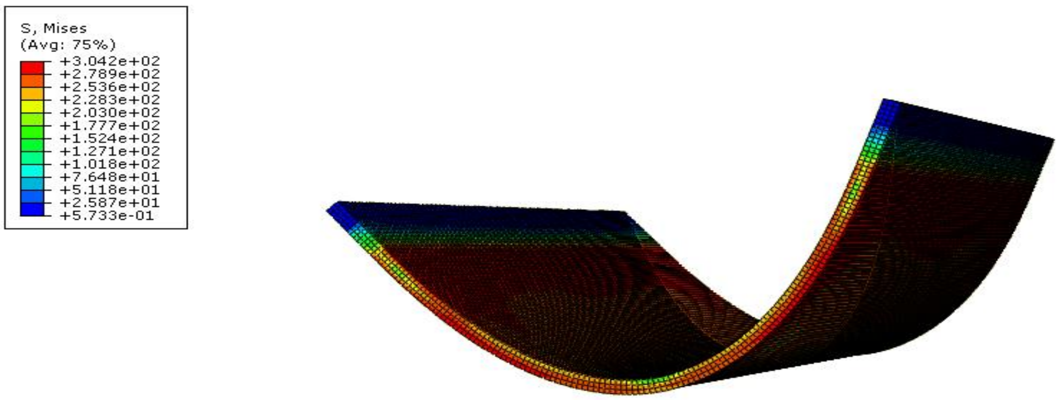

Figure 4.7 Residual stresses left after removal of load from the first model

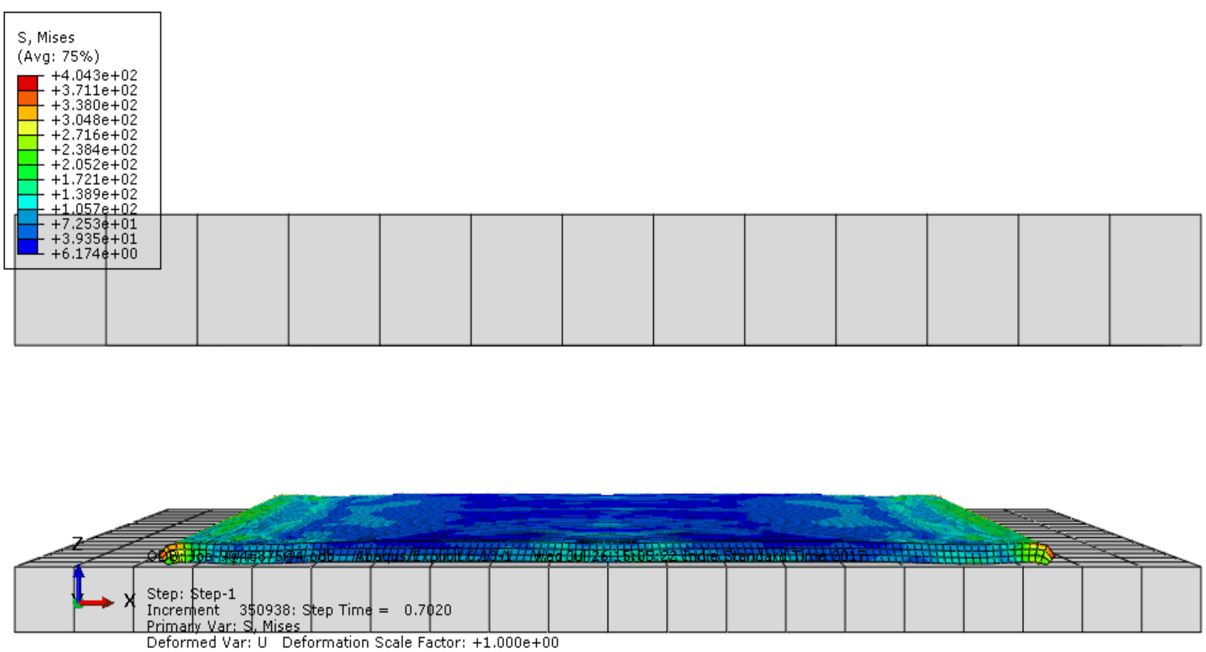

Figure 4.8 Residual stresses left after removal of load from the second model

4.7PLASTIC EQUIVALENT STRAIN

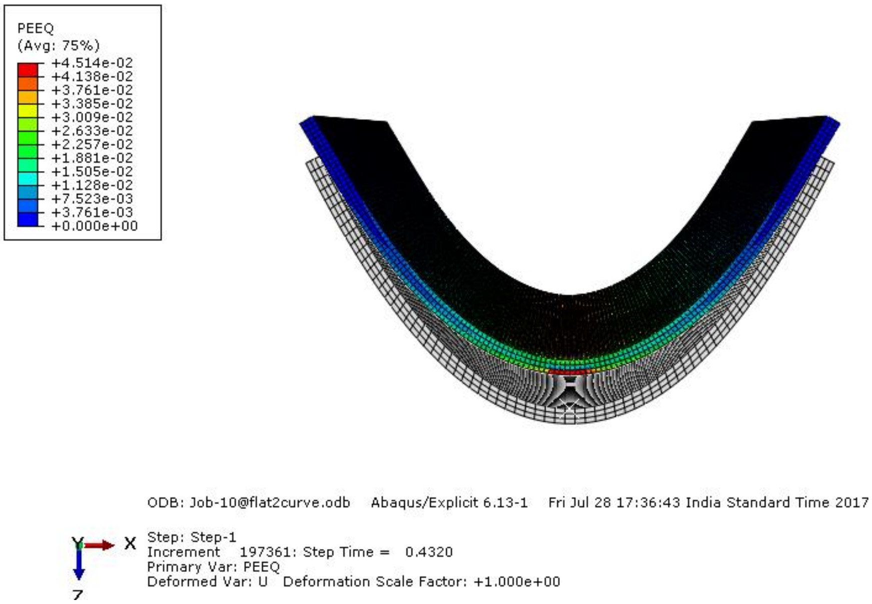

Figure 4.9 Plastic equivalent strain induced during bending

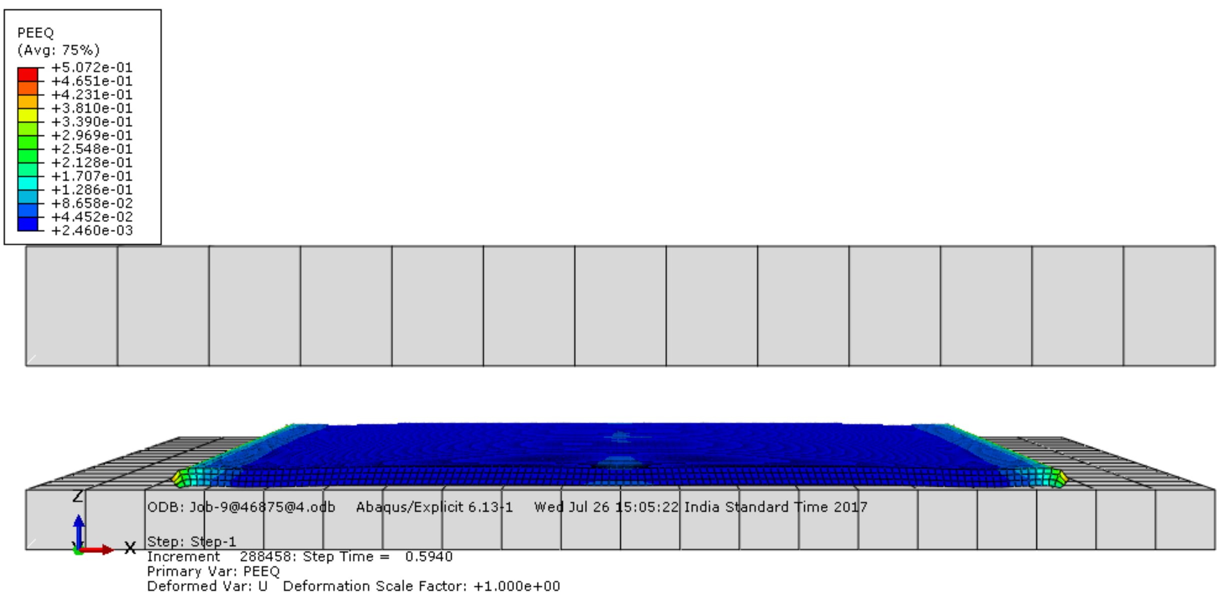

Figure 4.10 Plastic equivalent strain induced during flattening of curved sheet

From the simulation the dimensional changes after the first simulation model and second model are shown in table (4.2, 4.3, 4.4).

Table 4.2 Dimensions of blank (in mm)

L	W	T
75	75	1.5

Table 4.3 Dimensions after first deformation (in mm)

L	W	T
75.93	75.014	1.492

Table 4.4 Dimensions after second deformation (in mm)

L	W	T
76.395	75.046	1.463

CHAPTER 5

EXPERIMENTAL VALIDATION

5.1 INTRODUCTION

The experimental work consists of: (i) formation of a curved profile sheet using the flat sheet of stainless steel 304, (ii) flattening of formed curved profile sheet. This work was done using hydraulic press machine along with assembly of die and punch arrangement. Die was made of wood. The experiment begins with the die placed on the base of the hydraulic press machine and then stainless steel 304 blank (75 x 75 mm) was introduced between the die and the punch. After this, punch was pushed by the hydraulic cylinder moves down to stamp the blank into the required shape as shown in the figure 5.1 and 5.2. During the experiment, the pressure exerted by the punch on the blank and die was noted down from the pressure gauge attached with the hydraulic press machine was 1 ton/ in^2.

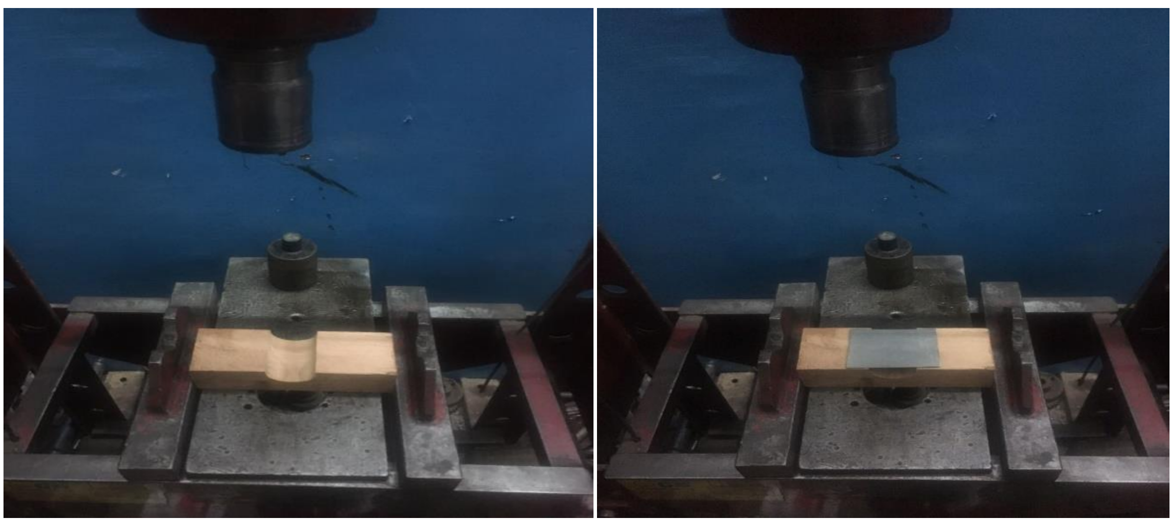

Figure 5.1 Die and flat sheet placed on die

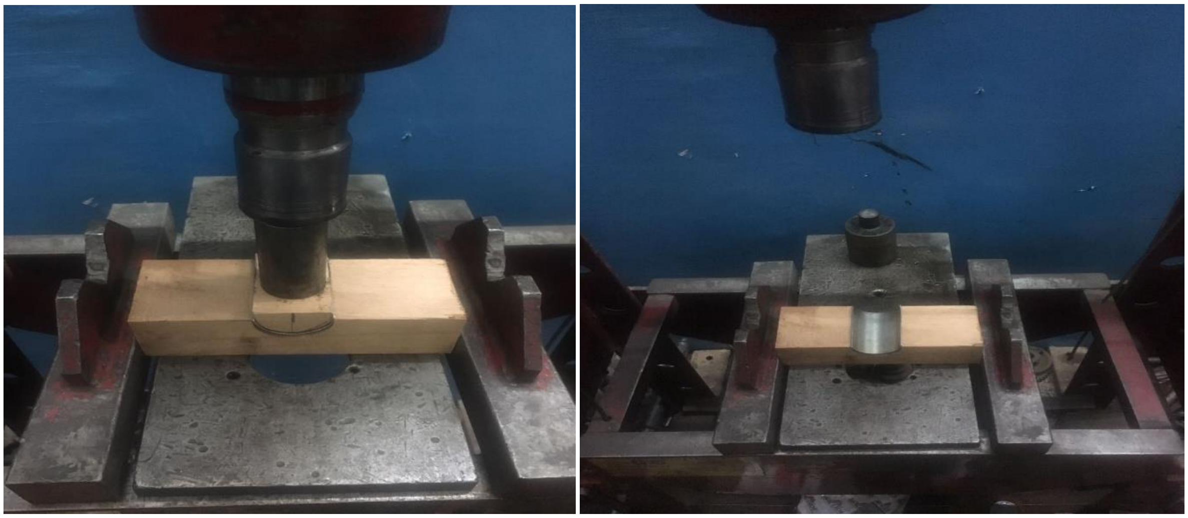

Figure 5.2 Deformation of flat sheet to form curve Shape

Secondly, formed curved (profile) sheet was flattened using flat die and flat punch arrangement. The profiled curve sheet was placed on the flat die and punch was pushed down by the hydraulic cylinder with a load of 8KN to flatten the curved profile sheet as shown in the figure 5.3 and 5.4.

Figure 5.3 Curved profile sheet

Figure 5.4 Deformation of curved sheet to flat

5.2 ANALYSIS

Sheet metal forming produces parts having large surface area to thickness ration. In sheet metal forming thickness variations are not desirable. Examples of sheet metal forming are: beverage cans, automobile body etc. presses apply gradual force of required amount to form the part. The total energy available for the process is equal to the kinetic energy of the ram plus the hydraulic pressure energy. Forging hammers provide impact loads. Gravity hammer provide the forging load by the falling weight of the ram.

The true strain of the part can be found from the formula

$$\varepsilon = ln\frac{l_0}{l_f}$$

Where l_0 is the initial length and l_f is the final deformed length f the part. If we neglect the friction at interface between the contacting parts, the force is given by: *F= YA*.

Y is the yield stress of the material and A is area in contact.

The area of contact keeps on increasing as the punch proceeds. As a result the force required increases. Flow stress also increase due to work hardening. This also leads to the application of the greater load with continued deformation.

A flat sheet of stainless steel 304 with dimension 75 x 75 x 1.5mm was deformed to a curved profile. Due to the deformation there is a change in dimension of part. The curved profile sheet again deformed to flat and again due to this deformation of flattening there is some change is the dimension of the part. The change in dimension during experiment after first deformation and second deformation is shown in table (5.1, 5.2, 5.3).

Table 5.1 Dimensions of Blank (in mm)

L	W	T
75	75	1.5

Table 5.2 Dimensions after first deformation (in mm)

L	W	T
75.77	75.02	149

Table 5.3 Dimensions after second deformation (in mm)

L	W	T
76.69	75.065	1.453

The obtained values have been analysed with respect to the values obtained from the simulation of the dimensional changes after the first simulation model and second model as shown in table (4.2, 4.3, 4.4).

5.3 COMPARISON OF EXPERIMENTAL AND SIMULATED DATA

Total strain produced during experiment and simulation was compared and shown in tabulated form.

Table 5.4 Total strain produced along the length

	L_o	L_f	$\varepsilon = ln\frac{l_f}{l_o}$
Experimental	75	75.77	0.0102143
Simulated	75	76.89	0.0111796

Table 5.5 Total strain value along the width after first deformation

	w_o	w_f	$\varepsilon = ln\frac{w_f}{w_o}$
Experimental	75	75.02	0.00026631
Simulated	75	75.014	0.00018665

Table 5.6 Total strain value along the thickness after first deformation

	t_o	t_f	$\varepsilon = ln\frac{t_f}{t_o}$
Experimental	1.5	1.49	-0.00668898
Simulated	1.5	1.492	-0.00534760

TOTAL STRAIN PRODUCED AFTER FIRST AND SECOND DEFORMATION

Table 5.7 Total strain produced along the length

	L_o	L_f	$\varepsilon = ln\frac{l_f}{l_o}$
Experimental	75	76.69	0.022283
Simulated	75	76.395	0.0184

Table 5.8 Total strain produced along width

	W_o	W_f	$\varepsilon = ln\frac{w_f}{w_o}$
Experimental	75	75.065	8.6629×10^{-04}
Simulated	75	75.046	6.13145×10^{-04}

Table 5.9 Total strain produced along thickness

	t_o	t_f	$\varepsilon = ln\frac{t_f}{t_o}$
Experimental	1.5	1.453	-0.0318
Simulated	1.5	1.463	-0.0249

5.4 ERROR CALCULATION

Difference between the experimental and simulated results of total strain after first and second deformation was calculated in the form of % error as shown in the graph. (in figure 6.1, 6.2 6.3).

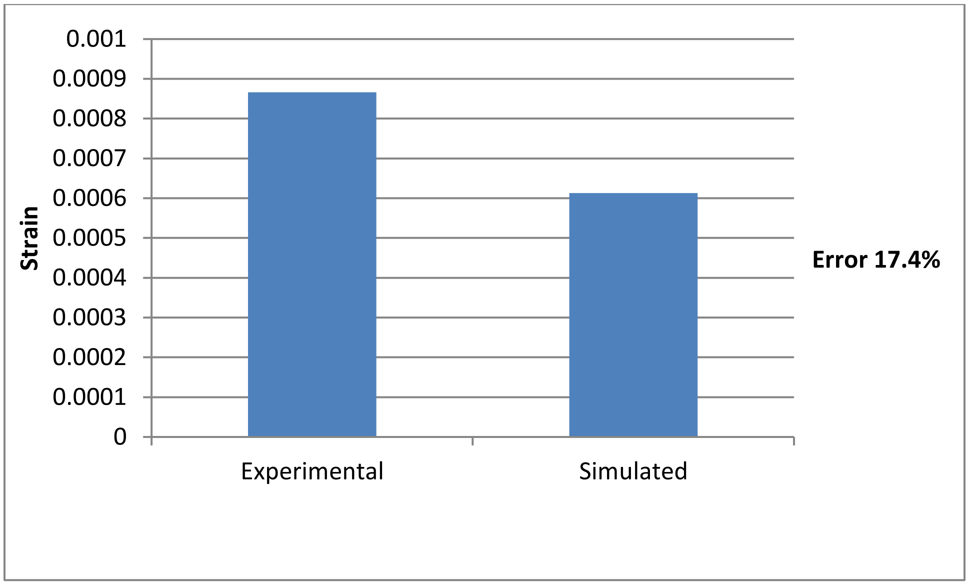

Figure 5.5 Error for total strain produced along length (x-x direction)

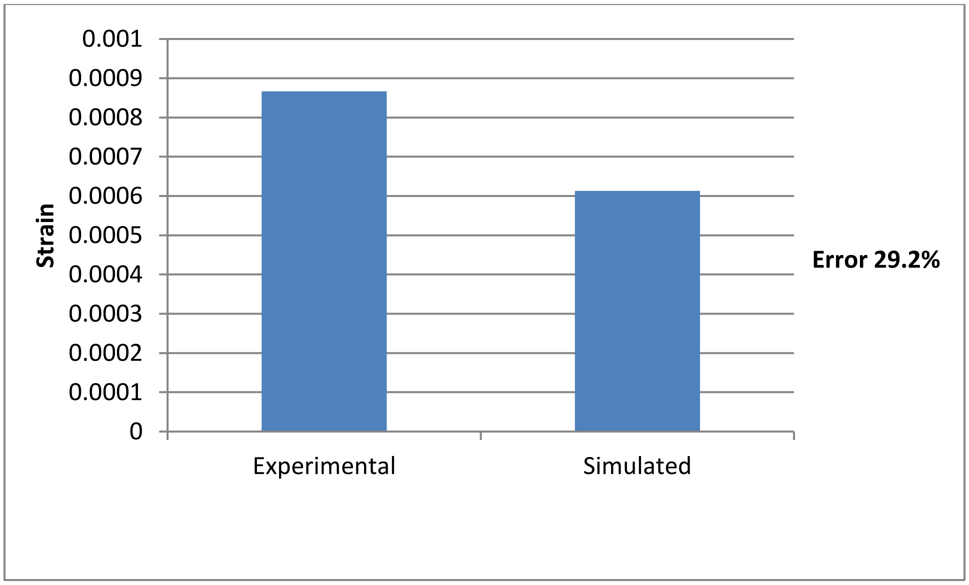

Figure 5.6 Error for total strain produced along width (Y-Y direction)

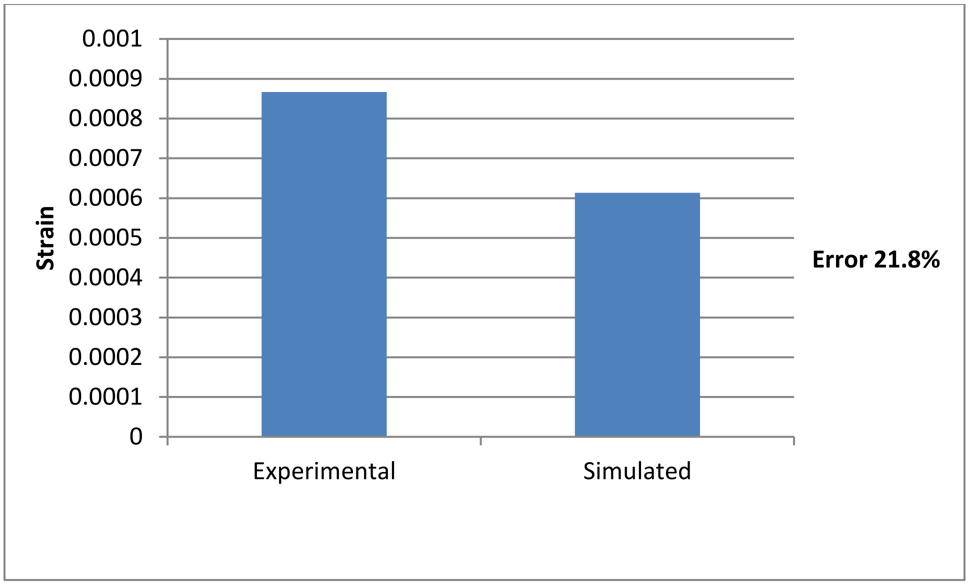

Figure 5.7 Error for Strain along thickness (Z-Z direction)

It is also shown that how the strain along the X-X (length) direction and Y-Y (width) direction changes with the time as shown in the figure 5.8, 5.9.

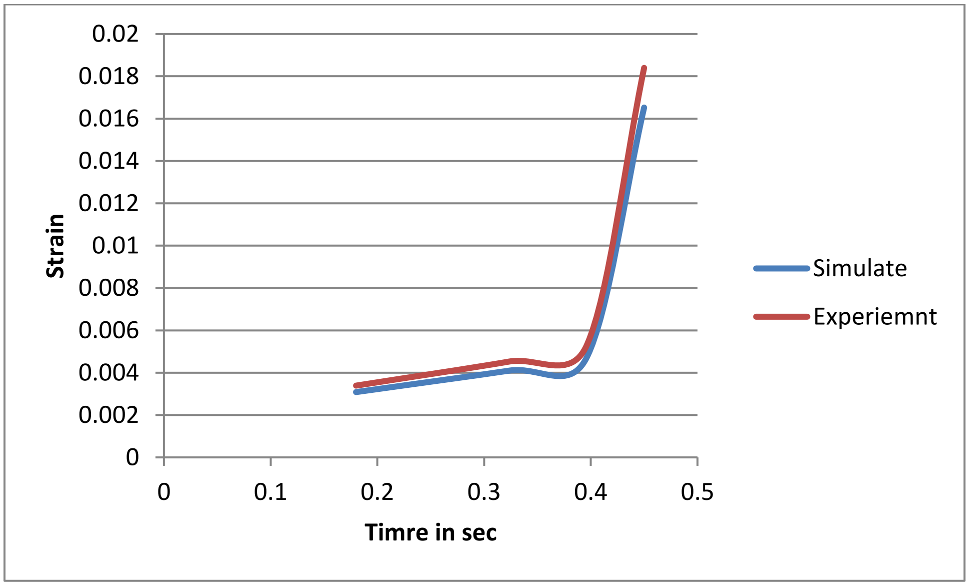

Figure 5.8 comparison of experimental and simulated strain along the length with time

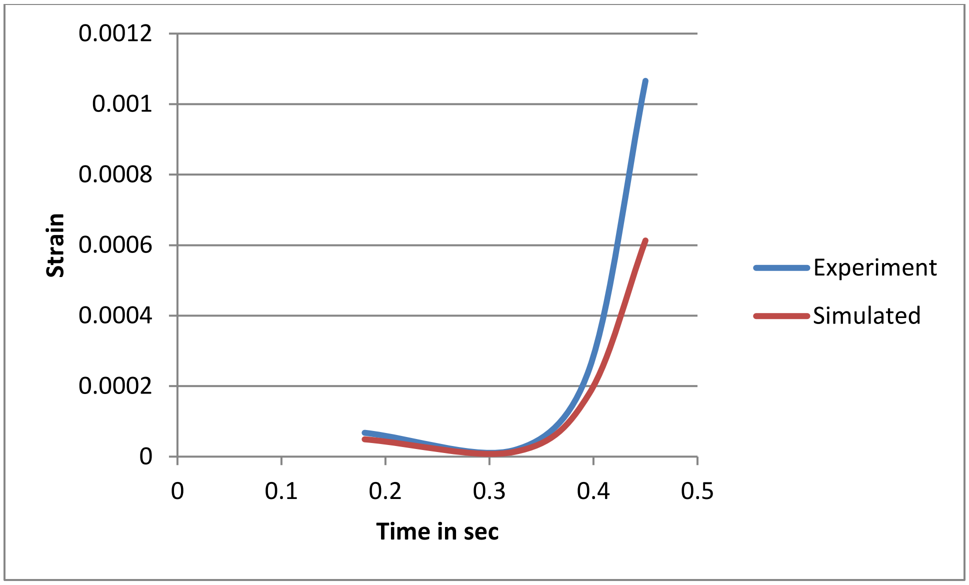

Figure 5.9 comparison of experimental and simulated strain along the width (Y-Y) with time

From the graph (In figure 5., 5.2, 5.3), it can be seen that experimental results are having more strain value than the simulated results. The error found is of 17.4%, 29.2% and 21.8% in strain along the length, thickness and width respectively.

CHAPTER 6

CONCLUSION AND FUTURE SCOPE OF THE WORK

Based on the results obtained, the following conclusion has been drawn.

6.1 CONCLUSION

1. An algorithm has been developed to calculate the value of mean curvatures and Gaussian curvature.
2. A surface has been developed which has the value of Gaussian curvature as zero (0).
3. Simulation has been successfully performed to analyse the development of a sheet when it is first deformed and then converted into a plane sheet.
4. Strain has been calculated along the length, width and height of the sheet during both the stages of development.
5. A die and a punch have been successfully fabricated to replicate the shape of the deformation.
6. Experimental study has also been performed so as to analyse and evaluate the results of simulation.
7. The variation of strain between experimental and simulated results varied from 17.4% (along the length) to 29.% (along the width).
8. The trends of strain were found to be complying when experimental and simulated results were analysed.

6.2 FUTURE SCOPE

For the purpose of further study, the following points may be explored:

1. The planer surface development of the doubly curved surface having both positive and negative values of Gaussian curvature.
2. The change in coordinate value of the thin sheet of surface development to be measured by CMM to calculate the strain.

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APPENDIX I

```
clc
clear all
syms u;syms v;
f1 = u;
ru1=diff(f1,u);rv1=diff(f1,v);
f2=v;
ru2=diff(f2,u);rv2=diff(f2,v);
f3=(u^2)-(v);
ru3=diff(f3,u);rv3=diff(f3,v);P=[f1 f2 f3];
% ***************TANGENT VECTORS****************************************
Pu=[ru1 ru2 ru3]
Pv=[rv1 rv2 rv3]
Qu=((ru1^2)+(ru2.^2)+(ru3.^2))^0.5;Qv=((rv1^2)+(rv2.^2)+(rv3.^2))^0.5;
nu=Pu/Qu;nv=Pv/Qv;
% **************TWIST VECTORS********************************************
Puv=diff(Pu,v)
Puu=diff(Pu,u)
Pvv=diff(Pv,v)
% **************NORMAL VECTOR********************************************
N1=(ru2*rv3)-(ru3*rv2);N2=(ru3*rv1)-(ru1*rv3);N3=(ru1*rv2)-(ru2*rv1);
N=[N1 N2 N3]
Q=((N1.^2)+(N2.^2)+(N3.^2))^0.5;
n=N/Q
%**********************************************************************
E=dot(Pu,Pu)
F=dot(Pu,Pv)
G=dot(Pv,Pv)
L=dot(n,Puu)
M=dot(n,Puv)
N=dot(n,Pvv)
```

APPENDIX II

```
clc
clear all
prompt = 'Enter the u value\n';
u = input (prompt);
prompt = 'Enter the v value\n';
v = input (prompt);
E = 4*u*conj(u)+1 ;
F = 2*conj(u);
G = 2;
L = 6/conj((4*u^2* + 2)^(1/2);
M = 0;
N = 0;
K=((L*N)-(M.^2))/((E*G)-(F.^2))
```