Symbol Error Rate Performance of Convolution Coded 4X4 MIMO Systems using LDC and Precoding

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Submitted by

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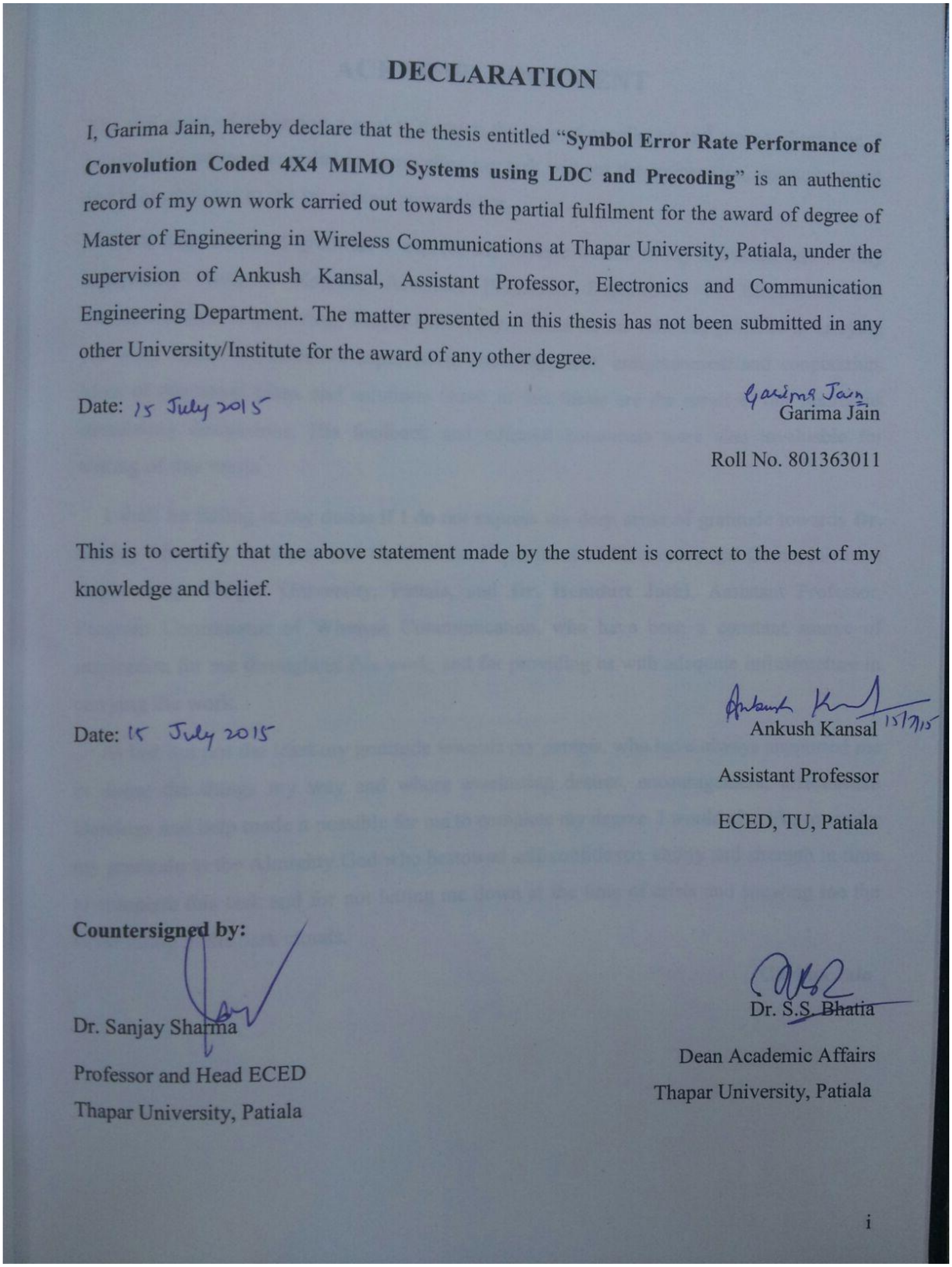

DECLARATION

I, Garima Jain, hereby declare that the thesis entitled "**Symbol Error Rate Performance of Convolution Coded 4X4 MIMO Systems using LDC and Precoding**" is an authentic record of my own work carried out towards the partial fulfilment for the award of degree of Master of Engineering in Wireless Communications at Thapar University, Patiala, under the supervision of Ankush Kansal, Assistant Professor, Electronics and Communication Engineering Department. The matter presented in this thesis has not been submitted in any other University/Institute for the award of any other degree.

Date: 15 July 2015

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This is to certify that the above statement made by the student is correct to the best of my knowledge and belief.

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Garima Jain

ABSTRACT

In today's wireless world, the demand for high speed data is increasing at fast pace as the number of wireless based applications are increasing day by day. Next generation wireless communication systems have to achieve very high throughputs and improved reliability in order to fulfil the desire for faster and reliable communications. But present generation systems cannot achieve higher data rates because of scarcity in band limited spectrum for wireless communication.

MIMO's are the most promising techniques for providing outstanding reliability and exceptionally high data rates. LTE-A has introduced 4x4 MIMO in the uplink. LTE networks based on 4x4 MIMO system targets at the peak data rate of 500 Mbit/s for uplink. But design challenge puts hindrance to LTE's movement to 4x4 MIMO. Next generation wireless systems has to develop efficient coding and signal processing techniques to easily switch to 4x4 MIMO systems with increased data rate and high spectral efficiency. Linear Dispersion Codes (LDC) outperforms many space time coding techniques and BLAST technique. LDC provides high data rate and good error performance for any arbitrary configuration of transmit and receive antennas.

In this dissertation work LDC systems of 4x4 MIMO configurations has been used. LDC outperformed spatially multiplexed systems by 6dB at Symbol Error Rate (SER) of $10^{-2.9}$. However, the said system was suffering from burst errors. In order to combat burst errors, channel encoding has been used along with LDC systems.

Further, in this dissertation work, precoding has been used in the convolution concatenated 4x4 MIMO systems using LDC. By precoding, Convolution coded system shows SER of 10^{-3} at 16dB while LDC system shows same SER at SNR of 24dB for Rayleigh channel. The proposed system has achieved SNR of 13dB at $10^{-2.9}$ SER, while traditional systems were achieving a maximum of 21dB of SNR at same SER for Rician channel. Henceforth, the proposed system achieved an improvement of 8dB as compared to conventional systems in both fading models.

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LIST OF ABBREVIATIONS

SISO	Single Input Single Output
LDC	Linear Dispersive Codes
BER	Bit Error Rate
CC	Convolution Code
MIMO	Multiple Input Multiple Output
PEP	Pairwise Error Probability
ZF	Zero Forcing
SNR	Signal to Noise Ratio
MMSE	Minimum Mean Squared Error
SM	Spatially Multiplexing
STC	Space Time Coding
BLAST	Bell Labs Layered Space Time
CSIT	Channel State Information at Transmitter
CSIR	Channel State Information at Receiver
SIMO	Multiple Input Single Output
MISO	Multiple Input Single output
LTE-A	Long Term Evolution
OSTBC	Orthogonal Space Time Block Codes
SER	Symbol Error Rate
ML	Maximum Likelihood
AWGN	Additive White Gaussian Noise

ARR	Automatic Repeat Process
FEC	Forward Error Correction
QAM	Quadrature Amplitude Modulation

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CHAPTER 1

INTRODUCTION

This chapter introduces MIMO systems and various space time coding techniques. An overview of channel encoding for increasing reliability of the communication system has also been presented.

1.1 Evolution of MIMO Systems

The earlier wireless systems before MIMO were subject to low network capacity which affects channel quality and coverage. Considering the transposal over multiple channels, the problem can be understood. The communication channel in wireless communication is subject to multipath generation due to scattering on various obstacles [1]. The main problem in communication systems is multipath propagation along with time spread (frequency selectivity) and time fading. The channel path is fading which resulted in SNR variations for time variation. In case of time spread, it becomes necessity for adequate frequency selectivity. The signal reaches the receiver through various directions. In their journey to the receiver, the signals got distracted from various obstacles like trees, houses, buildings etc. But with the use of MIMO technology, the problem gets resolved as a special signal processing technique/algorithm being used at the receiving end that sorts out one signal from the multiple signals that carries the originally transmitted data. MIMO uses the space dimension which provides better wireless system capacity, range and reliability [2]. Considerable enhancement in data throughput and link range can be obtained by using MIMO without increasing additional bandwidth or transmitting power.

Channel capacity increases with the increase in number of antenna element. MIMO system has the capacity of linear increasing of the channel capacity as the no. of antennas increased [3]. On the other hand, in SIMO and MISO system there is logarithmic-increasing of channel capacity.

Various receivers for decoding of codes used in MIMO systems were designed such as Zero-Forcing (ZF), Minimum mean squared error (MMSE), etc. [4]. The introduced receivers were simple and effective for decoding purposes having less complexity.

Spatially multiplexed systems in 1994 came into picture and gave the idea of splitting the high data rate into streams of low rate. This achieved tremendous capacity gain. The Bell Labs Layered Space Time (BLAST) technology given by Foschini achieved tremendous increase in capacity and improved spectral efficiency [5]. BLAST decoding can be done by ZF, MMSE, etc. The developments of the MIMO systems over the period of time have been shown in the Table 1.1.

Table 1.1: Evolution of MIMO Systems [1]

1970	First Description of MIMO Channels
1974	Capacity Analysis
1976	Receiver Structures
1994	Concept of Spatial Multiplexing
1995	MIMO Channel Capacity
1998	BLAST Technology (BELL Laboratory)
2001	Practical Development

The Table 1.1 summarises the growth made in MIMO systems year wise. Table briefs about the definition of MIMO, analysis in terms of capacity and various coding techniques with their years.

1.2 MIMO Reliability: Improvement in Error Performance

The multipath fading deteriorates signal strength resulting in decrease of throughput. One way to get high data rates in rich scattering environment is MIMO [6].MIMO offers remarkable data rates; range and enhanced reliability without demanding additional transmit power or bandwidth. These systems generate multiple separate channels for sending several data streams.

In wireless link, transmitted signals reach at the receiver through many different paths known as multipaths [7]. There are fluctauations in the received signal and becomes function of time.

These fluctuations in the received signal are called fading and degrade the strength. But MIMO systems have converted these multipaths into a boon for wireless communication. The main arguments for this technique are:

1.2.1 Array gain

Array gain is the average increment in signal to noise ratio attained by combining of multiple antenna signals either at transmitter side, reciever side or at both the sides [7]. The average increase in strength of signal power and the number of antennas at the receiving end varies proportionally. Array gain needs CSIT if multiple antennas are present at the transmitter.

1.2.2 Interference reduction

Co-channel interference is the main cause of the noise present in the wireless system and degrades the system performance [8]. To overcome the effect of the interfering signals, multiple antennas must be used at the transmitter side or at the receiver or at both the sides, which leads to improved system capacity. Reduction of Interference requires knowledge of the desired signal. CSI at the transmitter and receiver end is not necessary.

1.2.3 Diversity gain

Diversity plays an important role in reducing multipath fading. There are three types of diversity techniques i.e. time, frequency and space diversity [8]. In wireless communication space and antenna diversity plays very crucial role. Receive and transmit diversity makes use of multiple antennas for reception and transmission.

1.2.4 Receive Diversity

This type of diversity is based on the assumption that signals fade independently. Signals are being combined and processing of signals is done at the receiver's end. Fading effect i.e. random fluctuations are reduced by the use of this diversity [9]. Number of receiver fading branches defines the receive diversity. There are four main types of signal combining methods at the receiver: switched combining, equal-gain combining maximum ratio combining (MRC) and selection combining.

1.2.5 Transmit Diversity

Winters introduced the idea of transmit diversity. Processing of information is done at the transmitter end and multiple antennas disperse the streams in channel medium [10]. The system performance depends on the received signal copies in fading conditions.

1.3 Various Multiple Antenna Configurations:

Wireless systems, which consist of a radio channel, a transmitter and a receiver, are categorized by their number of outputs and inputs. Single antenna mounted both sides of the wireless link, which is known as single-input/single output (SISO) system is the simplest configuration. Configuration in which uses multiple antennas on one or both sides of the wireless link are known as multiple input/multiple output (MIMO) systems [1].

The difference between a MIMO system and SISO system with receive antennas and transmit antennas is the process of mapping the stream of data symbols in the form of streams of symbols to transmit antennas [2]. The inverse operation is performed at the receiver side. Systems which have multiple antennas on the receive side only are known as single input/multiple output (SIMO) systems and systems with multiple antennas at the transmitter side only are known multiple input/single output (MISO) systems.

The MIMO system is the general case and incorporates the MISO, SIMO, SISO systems. So, the term MIMO [3] is used in general for multiple antenna systems. MIMO offers remarkable data rates; range and enhanced reliability without demanding additional transmit power or bandwidth. But MIMO systems face problem in mapping the information at the transmitter and decoding the information to achieve optimum performance. To improve the communication performance multiple antennas are used at both the transmitter and the receiver end in MIMO system.

Researchers' main focus is on the parameters like reliability, bit rate and complexity. The main goal is to design a low complex and robust wireless communication system that will provide the highest possible bit rate per unit bandwidth. Various combinations of antenna allocation are shown in Table 1.2.

Table 1.2: Multiple Antenna Systems

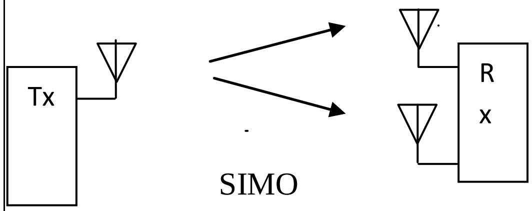
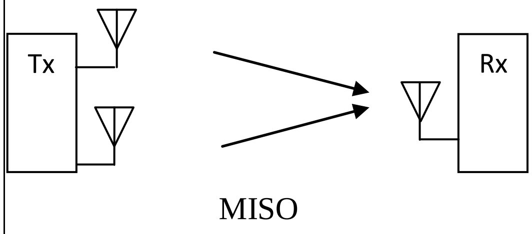
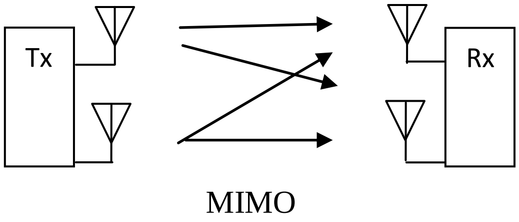
SYSTEM	FULL-FORM	SYSTEM DEFINITION	SYSTEM DIAGRAM
SISO	Single Input-Single Output	In this system, the receiver and transmitter of the radio system have only one antenna.	
SIMO	Single Input-Multiple Output	In this system, the receiver has multiple antennas while transmitter has one antennas	
MISO	Multiple Input-Single Output	In this system, the transmitter has multiple antenna while the receiver has one antennas	
MIMO	Multiple Input-Multiple Output	In this system, the both the receiver and transmitter have multiple antennas.	

Table 1.2 describes various multiple antenna configurations systems, their definitions, full forms and block diagram representation. To improve the communication performance multiple antennas are used at both the transmitter and the receiver end in MIMO systems.

In MIMO channel, the number of independent multiple channels and affiliated data streams is equivalent to the minimal no. of transmit or receive antennas [11]. Thus, configurations such as 2x2, 3x3 MIMO system can support at most 2 and 3 data streams respectively, hence 4x4 MIMO system can support 4 data streams and is a more reliable system. The emerging technology LTE-A has introduced 4x4 MIMO in the uplink [12]. In LTE networks 4x4 MIMO deliver ample benefits to mobile users, manufacturers. LTE-A uplink targets at the peak data rate of 500 Mbit/s. But design challenge puts hindrance to LTE's movement to 4x4 MIMO [13]. MIMO systems make use of multiple transmission paths that are ideally 'uncorrelated' and independent. Here, uncorrelated means that if one link is in deep fade, then another link does not get affected. For independent channels, the required antenna spacing is given by $\frac{\lambda_d}{2}$ *and* $\lambda_d = \frac{c_d}{f_{c_d}}$.

Where λ_d =wavelength of the carrier, c_d =speed of light, f_{c_d} =carrier frequency.

Most of today's mobile handsets do not even include 2x2 MIMO. For GSM systems, carrier frequency f_{c_d} is approximately 900 MHz

$$\lambda_d = \frac{3\times10^8}{900\times10^6} = 33.33cm \tag{1.1}$$

Hence, spacing between antennas $d(\lambda_d) = \frac{\lambda_d}{2} = \frac{33.33}{2} = 16.66cm$ (1.2)

Usually, mobile phone is 8-9 cm. Thus, it is not possible to place multiple antennas in GSM phones. But the emerging new LTE-A system allows fitting of 4 or more antennas into a smart phone form with sufficient isolation.

1.4 MIMO Model

MIMO is a point-to-point communication system wherein multiple antennas are available at both ends i.e. the transmitting and the receiving end. Presence of multiple antennas at both the ends improves performance. This system is better than the SIMO and MISO where either the transmitter or receiver, have more than 1 antenna. Data rates of wireless system can be increased significantly with MIMO system [1]. This can be possible even without any increment in transmit power or bandwidth. But this results in increase in cost of whole setup due to installation of multiple antennas and the space that is required for installation of these antennas.

The model is equipped with the transmitter and receiver antenna arrays. The input stream is mapped to the transmit antennas and propagates through channel medium. The receiver array collects the transmitted stream and decodes the stream to recover the original input stream. MIMO system model is shown in Figure 1.1 where s is the $N_r \times 1$ received signal vector as there are N_r antennas in receiver. H represented by $M_t \times N_r$ channel matrix, r is the $M_t \times 1$transmitted signal vector as there are M_t antennas in transmitter and v is a N_r vector of additive noise term [2].

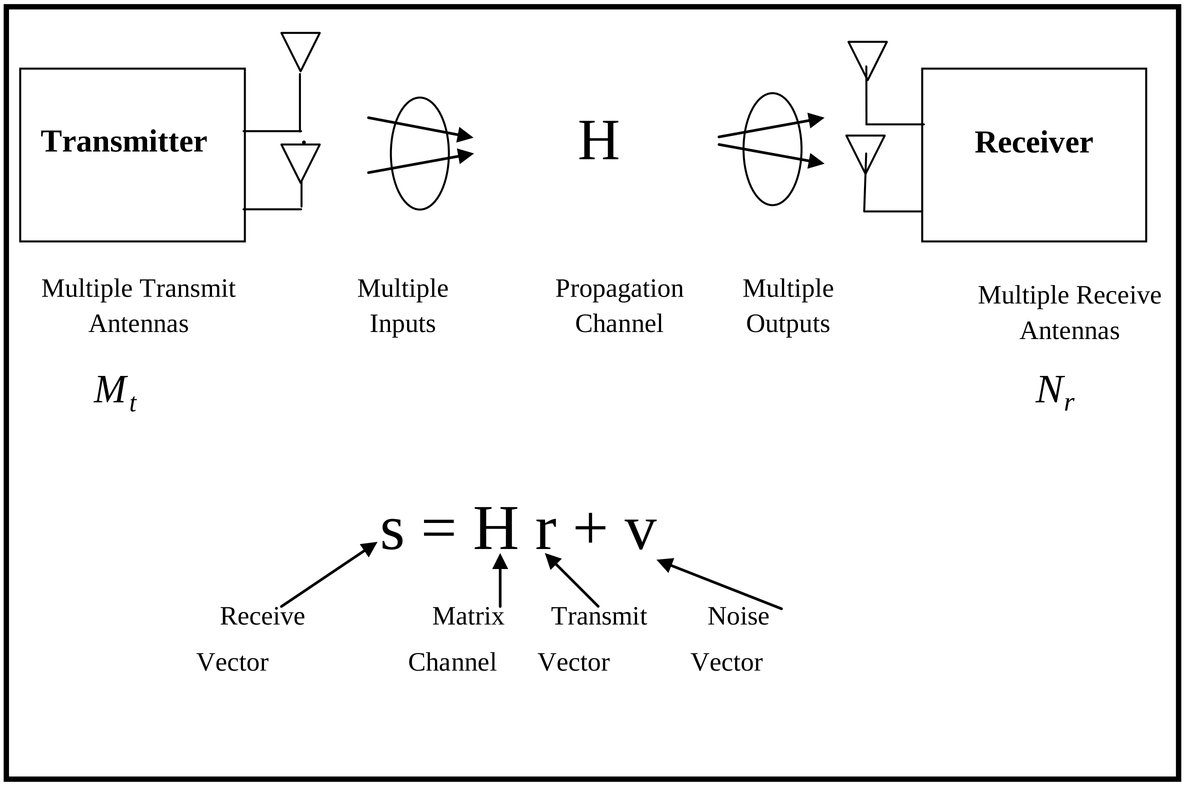


Figure 1.1 Representation of MIMO System

Figure 1.1 represents the architecture of MIMO system. The basic vector equation of MIMO system is also shown representing the transmit vector r, receive vector s and noise vector v in the presence of channel represented by H.

1.5 Diversity coding:

Diversity coding is used to enhance the system performance in presence of multipath fading conditions. In this technique, there is a transmission of a multiple copies of single streams from different antennas. This technique utilises independent fading channels to increase the diversity of the system. This Diversity coding makes use of space time coding techniques such as orthogonal space time codes (OSTBC), quasi-OSTBC, etc. for encoding purposes [11]. This technique is employed when there is no CSI at the transmitter (CSIT). As CSIT is not used, this technique lacks in providing array gain and beamforming.

In wireless link, different transmission techniques such as Space time coding (STC), spatial multiplexing (SM) are employed to transmit information. If CSIT is not known, then SM or STC can be employed for transmission [12]. Beamforming can be used for transmission in RF channel when perfect CSIT is known. Thus, different transmission coding techniques enhance the spectral efficiency and robustness of the wireless system. Choice of transmission technique depends on the parameters such as reliability, bit rate and complexity. STC provides high diversity and low decoder complexity but lacks in providing high data rate. SM promises high data rate but provides less reliability. Beamforming shows robustness against channel fading, thus providing array gain.

1.6 Spatial multiplexing:

In spatially multiplexed systems, the prime requisite is MIMO systems. Spatial multiplexing (SM) is used to increase capacity i.e. data rate over wide range of signal to noise ratio. In this technique, no diversity coding is used. Thus it does not guard against errors occurring due to multipaths and noise. In SM, decomposition of high rate signal into enormous streams of low rate is done. Each transmit antenna sends lower rate streams in the same frequency band [5].

The receiver can break these low rate streams into channels in parallel if antenna array at the receiver has different signatures spatially. The number of spatial streams depends on the MIMO configuration [13]. The streams get decreased with decrease in number of transmit and receive antennas.

Spatially multiplexed systems can work in the presence of CSIT and without CSIT also. Another application of spatially multiplexed systems is that they can be used for space division multiple accessing by allowing simultaneous transmissions.

1.7 Space Time Coding

Tarokh *et al.* [7] introduced space time codes to provide high spectral efficiency, coding gain and diversity gain. In space time codes (STC), redundancy is added into the design of the system to acquire high error performance and reliability. These codes are designed in such away so that they fulfil the optimality criteria. The effect of multipaths was lessened by the diversity schemes frequency and time diversity. Receiver diversity methods were also employed such as Maximum ratio combining, threshold combining etc. STC differs from spatially multiplexed systems in transmission of number of symbols. In multiplexed systems, transmitted symbol are more in number while Space time codes send 1 symbol per channel for reliable transmission by making use of diversity method.

It is difficult to employ enormous antennas at the receiver side at remote stations as remote units demand cost-effective, simple methods. So remote units prefer to employ schemes involving transmit diversity.

1.8 Linear Dispersion Codes

Linear dispersion Codes (LDC) are linear space time codes that provides advantages of diversity and coding gain [15]. These codes includes properties of many space time codes such as V-BLAST providing high data rate and space time block codes providing diversity and coding gain.

In this coding, information data is split into smaller streams. These streams have linearity property and is dispersed spatially and temporally [16]. Linear dispersion codes are designed in such a way that the mutual information is maximized between the transmitted signals and the received signals. Unlike V-BLAST, the configuration of transmit and receive antennas is not a constraint and can work with any configuration.

The splitting of input data into substreams in linear arrangement dispersed spatially and temporally, therefore the code is termed as Linear Dispersion Codes. In this coding, it is assumed that channel remains constant for certain interval of time i.e. T [17]. The transmitted signal matrix R of dimension $T\ X\ M_t$ is mapped onto the transmitted antennas M_t in interval T when channel is not mobile. It is assumed that the input Data sequence is split into Q substreams and R_1, R_2,…….R_Q are the modulated g-QAM symbols. The characteristics of LDC are as follows [18]:

- These codes incorporate the properties of block codes and BLAST system.
- LDC outperforms both the spatially multiplexed system and systems with diversity coding as it incorporates the properties of both techniques.
- LDC can work with any arbitrary configuration of transmit and receive antennas.
- The encoding scheme of LDC is quite simple in nature.
- LDC structure possess linearity property, as a result decoding can be done in efficacious ways such as linear receivers such as zero-forcing, BLAST decoding and sphere decoding, successive nulling and cancelling.
- LDC are termed as high rate codes fulfilling the criterion of optimality.

1.9 Precoding

In precoding of MIMO system, transmission of multi-layer signals is done. In single layer transmission, signals are assigned weights at the transmitter end and maximization of signal power is done at the receiver end. In MIMO systems, maximization of throughput is done by beamforming of multi-layered signals. This result in increase in strength of signal power and fading effect is greatly reduced. Following are the precoding classifications:

1.9.1 Precoding for single user MIMO

In single user MIMO systems, the communication takes place between the transmitter and the receiver that both have multiple antennas [24]. Almost many traditional precoding results presume narrowband and slowly fading channels, which means that channel can be presented by the single matrix for a certain length of time and it does not alter faster [25]. OFDM is the example in which such channels could be achieved. The Precoding scheme that increases the channel capacity is dependent on CSI that is present in the system.

1.9.2 Precoding for Multi User MIMO

In multi-user MIMO, the multi-antenna transmitter corresponds with multiple receivers, which have one or numerous antennas [26.] This is called as space-division multiple access (SDMA). From an implementation perspective, precoding algorithms for such systems can be further divided into linear and non linear precoding types. The algorithms that gain capacity are nonlinear, but linear precoding approaches normally gain reasonable performance and that too with much lower complexity. MMSE precoding, simplified zero-forcing (ZF) precoding are the examples of linear precoding strategies. Other precoding strategies designed for low-rate feedback of CSI are also available like random beam forming. Nonlinear precoding is structured on the basis of dirty paper coding (DPC), which highlights that any known interference at the transmitting end can be deducted without any penalty of radio resources only if optimal precoding design could be applied on the transmit signal.

1.10 Receiver Strategies:

At the transmitter side, complexity of processing of signal is less whereas complexity is more at the receiver side because receiver has to recover the transmitted symbols from the modified received symbols [27]. Various receiver strategies are as follows:

1.10.1 Maximum Likelihood (ML) Receivers

In communication world, the best receiver performance is given by maximum likelihood receiver as it provides best error rate performance and diversity can be maximised. But it has to undergo complex decoding search by computing the distance of the received signal from all the possible signals transmitted from the MIMO transmit antennas to recover the original transmitted signal [28]. Maximum likelihood receiver searches for the signal having minimum Euclidean distance between the received signal and all posiible transmitted signal. The signal having minimum Euclidean system is chosen and estimates that it is the maximum likely signal which was transmitted. This receiver gives optimum bit error performance results even in the presence of Additive White Gaussian Noise (AWGN).

The complexity of the maximum likelihood receiver increases with the increase in the modulation index of the modulation scheme [29]. Thus, it is not considered good for systems using high modulation techniques. By using higher signal modulation, this receiver idea is very complex. The complexity of the maximum likelihood is high compared to linear receivers and BLAST receivers. There are many other startegies which give the same error performance with less complexity. One such receiver is sphere decoding receiver. This receiver provides less complexity compared to maximum likelihood receiver. The codewords which are enclosed in the sphere centred on the Received signal undergoes Maximum Likelihood detection and the rest are inferred unlikely to be transmitted [30]. This method reduces the number of codewords for search thereby decreasing complexity.

1.10.2 Linear receivers

For multiple antenna systems undergoing encoding process, simple and effective linear receivers such as Zero-forcing (ZF) and Minimum Mean Square Error (MMSE) are used [31]. Zero-forcing makes use of deterministic least square error while minimum mean square error makes use of Bayesian approach taking transmitted variables random in nature. In zero-forcing, pseudo inverse of the receive signal in frequency response is taken nullifying the influence of other transmitted signals that create interference. But in this decoding, additive white Gaussian noise gets amplifies as pseudo inverse is multiplied with the AWGN resulting in poor performance of the system.

The complexity of these linear receivers is comparatively less than that that of maximum likelihood (ML) as separate decision is taken for each data stream. The MMSE receiver minimises the error and does not result in amplification of the AWGN. In this minimization of the error occurred between the transmitted and detected symbol is done resulting in data streams having residual noise [32]. After minimum mean square error equalization, the residual signal is subject to detection process. But this technique requires correct values of the noise parameter which is difficult to attain. Hence it is not a optimal solution for detection process. An improvement in terms of noise amplification is the only advantage achieved compared to zero-forcing method. As a result zero-forcing method is commonly used for decoding purposes. Zero-forcing is a simple linear estimation decoding technique. In this effective technique, inversion of the received signal in terms of frequency response is taken and signal gets restored [7]. Zero-forcing technique are designed with an assumption that CSIR i.e. information of channel is available at the receiver end. But perfect CSIR may not be present. These receivers mitigate the effect of interference caused by other transmitted signals. In case of Zero Forcing the transmitted symbol is estimated as:

$$r = pinv(H) * s \tag{1.3}$$

where $pinv$ denotes the pseudo inverse operation

$$pinv(H) = \left(H^H H\right)^{-1} H^H \tag{1.4}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{M_t} \end{bmatrix} = \left(H^H H\right)^{-1} H^H \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{N_r} \end{bmatrix} \tag{1.5}$$

r denotes the estimate of r. Thus, r is obtained by the multiplication of s with pseudo-inverse of channel matrix *H*.

1.10.3 Bell Labs Layered Space-Time (BLAST) Nulling and Cancelling

Blast is a non-linear receiver and makes use of successive nulling and cancelling for decoding. This is a Decision Feedback strategy used for multiuser detection [33]. Symbols are detected on the basis of strength of SNR in a sequential order. The strongest SNR symbols are detected first and then weaker SNR symbols in sequence.

In detection of a new symbol, previously detected symbols are cancelled and remaining symbols are treated as interference i.e. nulled. The nulling effect can be done by equalization schemes such as ZF or MMSE. Nulling without cancelling step is often implemented in one step known as equalisation. Their performance lies between the performance of linear receivers and ML receivers.

1.11 Channel Coding

In wireless communication world, reliability of the signals is a matter of concern for transmission of signals. The signals strength gets changed and corrupted due to burst noise and interference caused by the medium resulting in increase in error rate. Channel coding offers error resistance by encoding the information symbols [38]. Channel coding minimises the fluctuations and distortions caused due to channel, thereby reducing the error rate and maximising the quality. There are numerous error correcting ways in channel coding.

Automatic Repeat Request (ARQ) is the commonly used backward error correcting technique. In ARQ, the recipient keeps a check on the transmitted signal and if an error is encountered, it asks to retransmit the signal.

Forward error correction (FEC) is also used for error correction. In this information bits are encoded using as special algorithm and receiver decodes the signal. FEC and ARQ are used in hybrid way for reducing error rate to a great extent and signal gets corrected within a channel code otherwise complete transmission would take place.

FEC tries to find out the errors and correct them. However, ARQ just finds out the error and sends request to the transmitter to resend. The FEC-based methods are more complex as compared with ARQ.

In these methods, extra bits known as parity bits are added resulting in error reduction but spectral efficiency gets poor due to consumption of extra bandwidth [39]. There are block codes which are memoryless codes for error correction. Convolution codes are error correcting codes with memory. In wireless world, convolution codes are mostly used for error correction. These codes require memory. These codes do not make use of algebraic structure rather mathematical structure.

1.11.1 Convolution Encoding

In 1955, Elias gave the design of Convolution codes (CC) and can be used as an alternative to block codes [40]. Block codes are memoryless but CC are encoders with memory and the output of the system depends on the previous m input states and also on the input k states.

In convolution encoder, mapping of input bits to codeword is done. The code rate k_c / n_c is defined as the ratio of number of input bits k_c to the number of output bits n_c. Constraint length K_c gives the information about the number of shifts input bit has to undergo. With large K_c, there is an increase in coding gain thereby, the codes become powerful.

For example: the Architecture of convolution encoder with constraint length K_c=3, k_c =1 and n_c =2 is depicted in Figure 1.2. The generator matrices are: **g**1= [101], **g**2= [111]. The generators are represented conveniently in octal forms as $(5, 7)_8$.

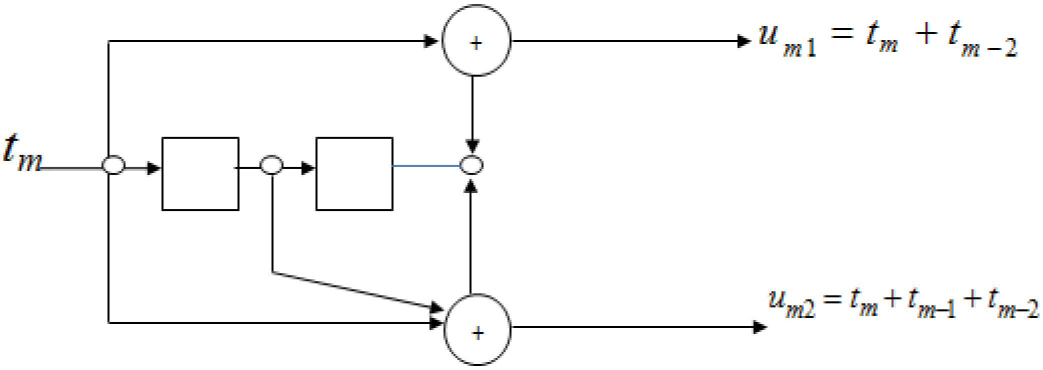


Figure 1.2: Architecture of convolution encoder with polynomial generator $(5, 7)_8$ [40]

In Figure 1.2 the input sequence t_m is fed to the convolution encoder with polynomial generator $(5, 7)_8$ producing the codewords u_{m1}, u_{m2}. The code rate of the system is 1/2 having constraint length 3.

1.11.2 Properties of Convolution Encoder In Terms of Distance Parameter:

State diagram helps in providing the information about the distance properties of the convolution encoder. Distance parameters are as follows:

Free distance d_{free} **:**

If the hamming weight between the two different encoded codewords in the trellis is the smallest, then the path having this weight is termed as free distance [41]. If a code possesses larger value of the free distance, then it is considered as a good correcting code.

Transfer Function $Tr(D_r,O_r,J_r)$ **:**

Transfer function is a tool for providing information about weights and the associated lengths of the codewords [42]. It is represented as follows:

$$Tr(D_r,O_r,J_r)=\sum_{d_r=0}^{\infty}\sum_{n_r=0}^{\infty}\sum_{j_r=0}^{\infty} g_{d_r,o_r,j_r} D_r^{d_r},O_r^{n_r},J_r^{j_r} \qquad (1.6)$$

where $\overline{g_{d_r,o_r,j_r}}$ represents the no. of trellis paths having input weight d_r, output weight o_r and length of the path j_r. For the expression $2D_rO_r^2J_r+4D_r^2O_r^3J_r^4$, value of place holders is represented in the Table 1.3:

Table 1.3: Value of Placeholders

	$\overline{g_{d_r,o_r,j_r}}$	d_r	o_r	j_r
$2D_rO_r^2J_r$	2	1	2	1
$4D_r^2O_r^3J_r^4$	4	2	3	4

In Table 1.3, $2D_rO_r^2J_r$ denotes $\overline{g_{1,2,1}}$ paths having input weight 1, output weight 2, and length 1. The expression $4D_r^2O_r^3J_r^4$ denotes $\overline{g_{2,3,4}}$ paths with input weight 2, output weight 3, and length 4.

Transfer function can be calculated from the modified state diagram of convolution encoder. Modified state diagram of convolution encoder of rate ½ with polynomial generator $(5, 7)_8$ is as follows:

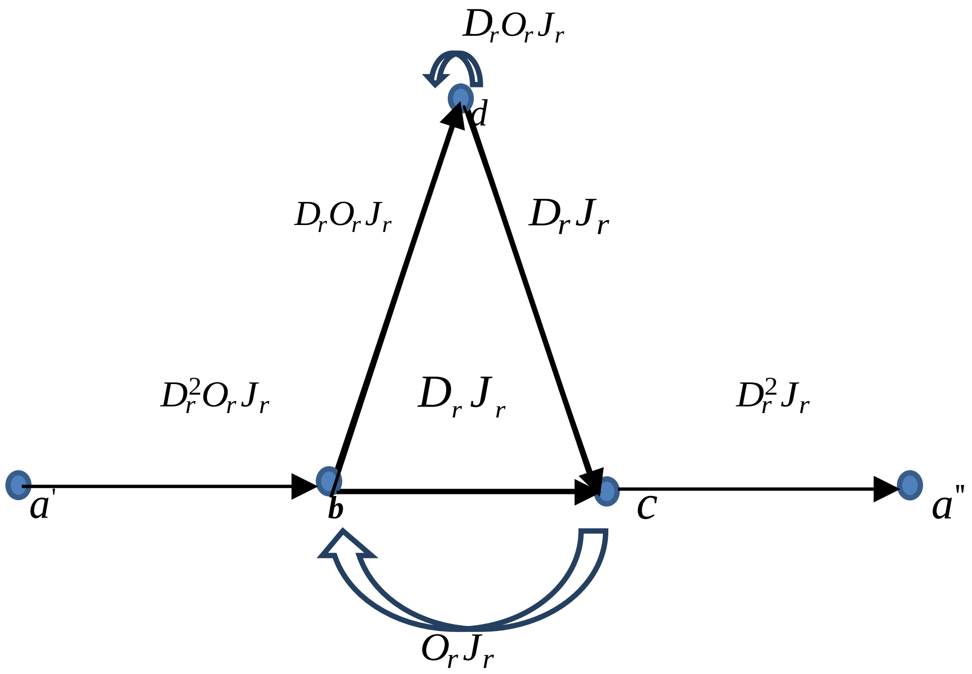


Figure 1.3: Modified state diagram of the system with generator polynomial (5, 7)$_8$ [42]

Eliminations nodes b, c and d from the modified diagram given in Figure 1.3, transfer function is given by:

$$Tr(D_r,O_r,J_r)=\frac{a''}{a'}=\frac{D_r^5 O_r J_r^3}{1-D_r,O_r,J_r-D_r,O_r,J_r^2}=D_r^5 O_r J_r^3+D_r^6 O_r^2 J_r^4+\ldots \quad (1.7)$$

Thus this transfer function is having free distance of 5.

1.11.3 Hard Decision Decoding:

The task of the demodulator is to interpret the received signal in a certain interval of time. It estimates the signal in “1” or “0” form. Thus it is known as hard estimate and decoding is termed as hard decision decoding [43]. It does not provide information about the reliability of the decision.

Decoding rule of Maximum Likelihood Decoder for hard decision is that it selects the path having minimum Hamming distance from the received codeword in the trellis.

1.11.4 Soft Decision Decoding:

If the estimates the received symbol in the form of real numerical value, then it is termed as balanced or soft decision decoding [44]. The demodulator provides side information which gives measure of confidence. Decoding rule of Maximum Likelihood Decoder for hard decision: It selects the path having minimum Euclidian distance from the received codeword in the trellis.

1.12 Dissertation Structure

This dissertation consists of 6 chapters, which are organized as below:

CHAPTER 1: INTRODUCTION, discusses about need of MIMO system, describes spatial multiplexing, various Space Time Codes, precoding and convolution encoding for reduction of error rate.

CHAPTER 2: LITERATURE SURVEY, briefs about the developments made in MIMO systems, Linear Dispersion Codes, Wishart distribution and channel encoding.

CHAPTER 3: METHODOLOGY, describes system model of the convolution coded 4x4 MIMO systems using LDC and Precoding model using Channel state information.

CHAPTER 4: PERFORMANCE ANALYSIS OF LDC AND CONVOLUTION CODED MIMO SYSTEMS, presents the derivation of pairwise error probability of LDC and upper bound for error probability for convolution coded system.

CHAPTER 5: RESULTS AND DISCUSSIONS, gives simulated results of uncoded 4x4 MIMO systems, LDC system, Convolution coded 4x4 MIMO system using LDC, precoded system.

CHAPTER 6: CONCLUSION AND FUTURE SCOPE, sums up the results concluded from the proposed technique and implementation of other techniques and discusses the possibility of future work.

CHAPTER 2

LITERATURE REVIEW

This chapter briefs about the developments made in MIMO systems, linear dispersion codes, Wishart Distribution and channel encoding. Based on the study, gaps in study and objectives have been framed.

2.1 Space time coding techniques in MIMO systems

In MIMO systems, various space time codes are employed to increase reliability and high data rates. The developments made by space time coding techniques to improve error performance in the presence of noise and fading have been presented as follows:

G. J. Foshini *et al.* [5]: explored the technology that processes both the spatial and temporal domains to improve spectral efficiency and achieve high bit rate. They examined multiple antennas among system when the channel information is not available at the transmitter side, while the signal is tracked by Receiver (Rx) perfectly in Rayleigh fading having trait of independent and identically distributed (i.i.d). This paper explored need to understand the extreme boundaries in context of delivery of bandwidth at even higher bit rates in a wireless communication and for finding out the ways that how these limits will be achieved. They examined the application of multi element array (MEA) technique which is using the spatial dimension to improve the wireless capacities in few applications. The paper highlights the basic information theory conclusions that commit more benefits of using MEAs in wireless Local Area Networks. It is found that receiver tracked channel feature even when the channel information is not available at the transmitter.

Space Time Trellis Coding (STCC) is proposed which uses Trellis Coded Modulation (TCM) and it is defined in terms of trellis tree. These codes achieved high spectral efficiencies and improvement in bit error rate. **V. Tarokh** *et al.* [6] showed that this technology performed well in frequency flat Rayleigh fading and in slowly fading atmosphere. But there is an significant increase in decoding problems with the no. of antenna. Both frequency diversity and coding gain are provided by it. By the study new category of codes were introduced known as Space time codes utilized for transmission by multiple transmit antennas over Rayleigh or Rician channels. Some other families of space time codes were also unveiled. Many subfamilies of space–time codes were also introduced. Excellent performance and decoding complexity was shown by these codes in comparison to Gaussian channel codes. These codes have feature of simple and systolic architecture which can be readily utilized in DSP and VLSI.

Work in the paper focused on digital correspondence in a Rayleigh hazy environment when the channel features is unavailable at the transmitting end, but it can be tracked at the receiving end. To stand firmly in competitive areas like indoor and fixed wireless, discovering a codec design that can give an important portion of great capacity as committed by information theory is must. **G. J. Foschini [7]** elaborated point to point communication system architecture by utilizing antenna array of equal number at transmitter and receiver side. Designing of the model is done for Rayleigh environment in the case when there is not complete channel information at the transmitting side. Thus the new framework for communication is obtained, called as layered space time structure. This new communication framework, known as layered space-time structure is the future of wireless systems, that resulted in high bit rates. This framework is also being used by Local Area network (LAN) applications, for which it commits extraordinarily high bit rates.

Multi-User MIMO (MU-MIMO) scheduling in 3GPP LTE-Advanced (3GPP LTE-A) cellular uplink was proposed. The precoded multistream communication i.e. precoded MIMO from each anticipated user is being allowed by the 3GPP LTE-A uplink. It also allows elastic multi-user (MU) scheduling in which same time-frequency resource can be given to multiple users at a single point of time. But, carrying out these features was a challenging task due to various practical constraints. **N. Prasad** *et al*. [8] proposed constant-factor polynomial-time approximation algorithms and demonstrate their superior performance through simulations. They considered the distribution of resource in the 3GPP LTE-A cellular uplink which allowed for MIMO transmission from each scheduled user as well as multi-user scheduling. In this paper, they also proposed constant-factor polynomial-time approximation algorithms.

V. Abbasi *et al.* [9]: presented a new concept of space time block code (STBC) for LTE-A system which is fully equipped with 2 antennas and 3 completely different time slots. The introduced STBC has features like, it achieves full diversity and rate one, its maximum likelihood decoding requires a combined detection of 3 real symbols, by increasing signal continuity (running) size, the MDV i.e. Minimum determinant value does not vanish and bit error rate results proved that the STBC outperforms the previous schemes for LTE-A.

A scheme that provided importance in diversity order by employing two transmit antennas and one receive antenna was proposed. Further **Alamouti** [10] extended his study to 2 transmit plus M recieve antennas which gave 2M diversity order. This scheme was largely used in distant areas as it made use of two transmit antennas at the base station in spite of 2 receive antennas at remote station.

2.2 Linear Dispersive Codes

The space time codes used in the dissertation are Linear Dispersive Codes (LDC) which provide high data rate. In the literature, advantages and disadvantages of LDC , their error performance has been done.

B. Hassibi *et al.* [15] proposed a multiple antenna system that is operating at higher rate, while utilizing efficient transmission scheme based on space time codes, which is capable of handling the real time large traffic volume at the rates of tens of bits per second per hertz. Vertical Bell Labs Layered Space Time (V-BLAST) architecture in which, independent substream of data is transmitted by the every user shows good performance and simple scheme for encoding and decoding. This architecture is incapable to work with lesser antennas at the receiver side. There is not any built in spatial coding to encounter the deep fades from transmitting side. Previously discussed space time codes had better fading resistance and easy decoding. In the proposed scheme the data sequence was transmitted in linear combinations over space and time. Designing of codes is done in such a manner that leads to improvement in mutual information between the transmitted and received signals. Their performance proved superior to previously suggested methods.

MIMO system for existing signalling technique is discussed. Main focus was multiplexing to achieve high data rate or diversity for greater link reliability. In this paper, **R. W. Health** *et al.* [16] presented MIMO model designing based on linear dispersion code for Rayleigh fading channels. The proposed design is able to give better performance in terms of ergodic capacity and error probability, since it reduce the space between the multiplexing and diversity. The design had greater significance since the earlier the schemes having well ergodic capacity did not give better performance for error probability and vice versa. Different techniques are discussed to find out better error probability performance codes.

A research of error performance linear dispersion codes is presented. A hard upper bound of average error probability at greater SNR for linear dispersion code is obtained in the presented work and checked with simulated results. **M. Gheryani** *et al.* [17] obtained an upper bound on average error probability for any signal to noise ratio, then on the basis of this bound a tight upper bound is calculated. Diversity benefits of these linear dispersion codes were achieved based on this tight upper bound. This upper bond showed the association of error probability with constellation size and space time symbol rate.

R. H. Gohary *et al.* [18]: proposed a design scheme for LD (linear dispersion) codes A design process was created for the class of linear dispersion (LD) codes. The growth begins by presenting that for systems with greater number of transmit antennas, LD codes obtained from unitary coding matrices are best asymptotically, like minimum mean square error (MMSE), reciprocal information, and mean pair wise error probability (PEP). These parameters show great affect on detection complexity, data ratio rate and error performance that could be accomplished by space time code. Employing perception generated by output of asymptotic approach, design techniques regarding LD coding matrices, which was suitable for wide range of configuration was given. The codes obtained could result in high data rates provided performances benefits on present designs when decoding is done by standard decoder. On the basis of asymptotic results, a scheme of row interleaving was suggested, and it was shown to result in notable performance enhancement.

This paper examined the diversity-multiplexing trade-off (DMT) of multi-input-multi-output (MIMO) systems in which linear dispersion (LD] codes were employed. This required dealing with distribution of a convoluted function and examining of the joint Eigen value density. **A. Kuhestani** *et al.* [20] furnished analysis in the lower values of SNR for miscellaneous MIMO system, and then did it for a 2x2 MIMO system at high SNR regime for the LD coded 2x2 MIMO systems. By employing the joint Eigen value density, the outage probability is showed in a very serried expression, which ordained DMT framework at this regime simulation. The obtained result showed that the obtained phrase for the outage probability is sufficient precise at elevated SNR regimes but less precise in the transition region. Further by increasing they derived both the outage probability and DMT framework in exact. Further by increasing the multiplexing gain, the obtained formula for the outage probability becomes more precise in the transition region ultimately, for the unique conditions of MISO systems.

O. Ordentlich *et al.* [21]: presented the additive Gaussian noise MIMO channel in an open loop scenario, where the receiver had whole channel state information whereas transmitter only knew the white-input mutual information. It was presented that by employing linear precoding at the schemes was not compassionate to a degrees of freedom mismatch. The compound channel studied in this paper included all the matrices with the same white-input mutual information. In certain scenario, such as multicasting the same message to a limit set of users whose channel matrices. This is the first scheme that assures an additive loss w.r.t. the compound capacity. Such a performance guarantee is much stronger than DMT optimality, which is at present the common benchmark for evaluating schemes. The results were free from any assumptions. They developed new upper bounds on an uncoded version of Integer Forcing (IF) equalization, which was more adequate for fast fading. While uncoded IF equalization was quite similar to lattice reduction aided decoding, the performance of the latter was never analysed at such a fine scale. The scheme did not show dependency on the receiving antenna numbers.

2.3 Wishart Distribution

Wishart distribution is used for the derivation of error probability of MIMO systems. various closed form expressions for outage probability performance has been presented in the literature.

M. Kang *et al.* [34]: proposed a method to obtain closed form expression for CDF (Cumulative density function) and PDF (Probability density function of the greatest Eigen value of non-central Wishart matrices, a san extension to the Khatri's results on the central case. The results obtained were then applied to characterize the working of MIMO MRC systems operating over Rician-fading channels. In particular, output SNR PDF and outage probability of system were derived in closed form and comparison is mode out for MIMO MRC system over Rician-fading channels. Numerical examples were provided in order to prove the analytical results and to show impact of different system parameters on the MIMO MRC system performance. It was shown by the analytical and numerical results that for a fixed number of antenna elements and under the similar scattering condition, MRC systems are equivalent to MRC systems and the minimum outage probability performance can be acquired if there is evenly distributed number of antenna elements between transmitter and receiver antenna.

P. Kuo*et al.* [35] had shown two major contributions. Firstly, they had derived the joint probability density function (PDF) for the Eigen values of the actual and the perturbed channel. Secondly, they had evaluated the time-varying characteristics of the MIMO channel. In specific, there joint PDF used in calculation of the transition probabilities between modulation states in MIMO-singular value decomposition (SVD) system and Monte-Carlo simulations was used to make calculations from the resultant Finite State Markov Chain (FSMC). MSER and ASD (average stage duration) was computed analytically and noticed that system nobility can badly influence the adaptive modulation selection performance due the time correlated Rayleigh channel changing transmitter in conjunction with integer-forcing equalization at the receiver satisfies to approach the capacity of this compound channel to within a persistent space, depending only on the no. of transmit antennas.

Analysis of MIMO systems with multi-channel beam forming is done in Rician fading channel. The results applied to a broad class of multi channels system which conveys on the Eigen modes of the MIMO channels. **S. Jin** *et al.* [36] firstly presented new closed form expressions for the marginal order Eigen value distribution of complex non central Wishart matrices. On this basis, exact SER expression had been shown. Close-form expressions for the array gain and diversity order was obtained and also for the outage probability. Global SER performance driven by sub channel equivalent to the minimum channel singular value also presented. They also showed that at low outage levels, the outage probability changes reciprocally with the Rician K- factor in case where only the transmission is most dominated subchannel. Numerical results were shown to verify the theoretical analysis.

D. Morales *et al.* [37]: proposed a method to find PDF of diagonal elements of a complex Wishart matrix. The matrix is based on multivariate square distribution. The expression for density is an infinite series representation that converts very fast and is very easily computed. By this density expression distribution of diagonal elements is possible to achieve, which lead to great help in analysis of two completely distinct MIMO systems underground conditions. The density expression makes possible the distribution of maximum of diagonal elements, which makes it possible to analyse the performance of two completely different MIMO systems under practical conditions. Primarily application of results was done on outage probability of MIMO systems. After that similar results were utilized in the analysis the outage probability of beam forming system at transmitter side under limited rate feedback.

2.4 Channel Encoding In STC

Channel encoding is used to reduce the effect of fading and increase system reliability. Channel encoding is concatenated with MIMO systems to improve the system performance. Various channel encoding techniques used in conjunction with MIMO systems and their associated error probability derivations has been done in the literature.

M. L. McCloud *et al.* [38]: highlighted the issue of formulating signal waveforms for the multiple antennas at transmit and receiving side over the wireless block-fading channel. In Rayleigh-fading channel, when there is not any information regarding fading at transmitter and receiver side. This issue had received greater attention due to works of Telatar, Gans and

Foschini. They proved that when receiver had complete side information about the state of channel, there are noticeable gains in Rayleigh-fading channel by the use of diversity antennas at both transmitter and receiver sides. The researchers had presented a completely new way-out in designing of non-coherent signals for Rayleigh fading channel of multi antenna system. The non-coherent signal design for the multi antenna Rayleigh-fading channel based on Asymptotic Union Bound (AUB) had been taken. A new solution to the signal design problem for non-coherent fading channel can be provided by adopting asymptotic bound on probability of error. The researchers had used notion of distance which is the major difference point as compared to other studies. By using this distance measure, they had surety of achieving complete diversity order of channel, which was otherwise impossible, if chordal distance was used.

The capacity expression for the transmitter having multiple antennas over Rayleigh channel had been presented. CSI is presumed to be available at both the ends. Average power is a constraint at the transmitter's end. **S. K. Jayaweera** *et al.* [39] also gave analytical expression for upper bound and concluded that with increment in the antenna elements, there was a reduction in outage probability. The researchers tested the capacity when multiple antennas were used at the transmitter's end only. Then they extended to multiple antennas at both the ends. They found cut off value for the systems through an equation. Capacity of system was also examined where CSI is available only at receiver side. In all the circumstances, evaluating cut off value was the goal. They equated approximations to the cut-off value in all the above cases. Results showed that approximations gave better capacity estimates when sufficient numbers of antennas were available. So they concluded that large capacity gains can be achieved in multiple antenna systems as compared to receiver-CSI-only systems.

The analytical expression for performance analysis of STBC system was derived. Then these expressions were used to further derive the bounds for the convolution coded (CC) STBC system. **S. Ali** *et al.* [40] provided the simulation result to verify the analysis expression for the CC STBC system. Transmission diversity in communication systems was introduced by STBC codes. The paper derived the general expression regarding performance error of STBC systems. The paper introduced convolution codes by which diversity order can be improved.

Multiple antennas were used in the communication systems due to two aims. First the diversity is provided in in communication systems by using different channel characteristics between each pair of transmitter and receiver. Second capacity of communication systems had increased. STBC provide a layout of using diversity into base stations. But it cannot be used in mobiles due to size issue. Moreover it is not practical to use a multiple antenna on a mobile because of space constraints. However STBC can overcome this difficulty by convolution codes that can provide more diversity without using multiple antennas.

The performance behaviour of serial convolution coding (CC) with the STBC coding had been derived. This concatenated scheme quantified the result of using the CC with STBC, as compared to using a STBC code only. Error bounds were obtained for STBC system with ideal channel interleaving. The research showed that these error bounds are more significant at low BER levels and have good tie-up with exact BER performance. **W. Hamouda** *et al.* [41] also showed that only SNR coding gain got affected by the use of antenna at the receiving side. Overall diversity order do not got affected. Moreover, they showed that the receiver’s performance was significantly affected by the channel interleaving. The experiment to compare the quasi-static and fast fading channels was carried, in which it was proved that whole system diversity was dominated by the time diversity order.It was also noted that disregarding the channel model used, the SNR reduction was nearly the same by the use of antenna selection results.There was a noticeable increase in space-time coding techniques in recent years. These techniques were mostly used in fading channels. These types of channels are called as the quasi-static fading channel (QSFC).Time-division multiple-access (TDMA) system is an example of such a channel system.

J. Hu *et al.* [42] presented a new approach on upper bound to improve performance of QSFC (convolutionally coded systems over a quasi satic fading channel) system. The bound used both the classical union bound and the new upper bound. The classical union bound was used when the fading channel was in high signal-to-noise ratio (SNR), whereas upper bound was used for the low signal-to-noise state. The new bounding technique was applied in BPSK convolutional and turbo codes and also in BPSK convolutional and STBC. This technique made bounds that are tighter than existing bounds (mostly 1dB tighter). The researchers on the basis of this new bound presented an upgraded design in slow fading channels for convolutionally coded systems.

N. Kim *et al.* [43] introduced the analytical expression of a bit error rate (BER) of a Multiple input and Multiple output system with MMSE i.e. minimum mean square error by making use of moment generating function (MGF) of SINR distribution. The BER performance of MIMO system can be estimated by the use of MGF of SINR at the MMSE detector output. On the basis of results, it was derived that diversity order is fully dependent on distance of convolutional code and configuration of the antenna. Higher data rate from the transmission systems is the continuous demand of new communication services. Multiple antennas at both the communication ends provide better capacity as compared to single antenna system. MIMO system has been universally accepted as the best system to increase the performance in wireless communications. But use of MIMO system along with channel coding techniques are one step further in improving the data rate in communication mechanism. For this, MIMO is categorized in space time coding and spatial multiplexing. Where STC ensures the reliability of communication system, SM focuses on higher data rate.

The approach to perform joint decoding and detection for spatial MIMO input output systems which used convolutional codes has been discussed. The bit error rate (BER) value of the discussed approach was considerably less than that of systems which used different decoding and detection blocks. **C. P. Sukumar** *et al.* [44] derived algorithms with two possible ways in this paper. For one algorithm concept of VLSI architecture is given and a unique way to diminish usage of memory was presented. Results showed that good performance with considerable amount of complexity is achieved over conventional systems. They are widely adopted by many industries such as WiMax and LTE. By the transmission of multiple data streams at the same time enhached capacity can be achieved.

2.5 Gaps

In the literature of MIMO systems, many STC techniques achieve high quality and reliable wireless communication. Many gaps have been observed which are as follows:

- The space-time codes have disadvantage of complexity and degraded performance when used with a large array of antennas and at high data rates [9].
- BLAST offer less decoding complexity with the use of all the available sub channels i.e. spatial multiplexing. The major drawback in BLAST is that there is no attempt to reduce error rates. Hence BLAST has to make trade off between capacity and diversity gain. BLAST has a major constraint that no of receive antennas should be less than the no. of transmit antennas [33].
- LD codes outperform many STC providing high data rate and good error reliability [20]. From literature, it has been found that the in design of LDCs, CSIT is not taken into consideration. Hence design of LDC lack in optimality as they do not make use of CSIT.
- Average probability of error has been derived for LDC systems but no mathematical analysis has been done for convolution concatenated LDC systems [17].

2.6 Objectives

This research accomplishes following objectives:

- To study and simulate the symbol error rate of spatially multiplexed non-coded 4x4 MIMO systems.
- To derive the average probability of error of Convolution concatenated MIMO systems using LDC and verify by Matlab simulations.
- To study and analyse the Convolution concatenated MIMO systems using LDC with and without Precoding in different fading conditions.

2.7 Methodology

The aim of wireless systems is to provide high rate codes with good reliability. As Linear Dispersive Codes provide high data rate and reliability, these are used for 4x4 MIMO systems. Channel encoding is used to mitigate the effect of burst errors caused by fading. Use of convolution encoder at the transmitter reduces outage probability, hence reduces symbol error rates. Since using channel state information at the transmitter improves the bit error performance, eigen beamforming is used in Precoded Linear Dispersion coded 4x4 MIMO Systems.

CHAPTER 3

4X4 MIMO USING PRECODED LINEAR DISPERSION CODES AND CONVOLUTION ENCODING

This chapter presents the system model of Convolution coded (CC) 4x4 MIMO system using Linear Dispersion Codes (LDC) in different fading conditions. System model of transmit precoding and receiver shaping is also presented.

3.1 System Model

The system under consideration consists of M_t transmit and N_r receive antennas over Rayleigh and Rician fading channel model. According to information theory, if channel considered is Gaussian then distribution of the transmitted signals is also Gaussian. Transmitted signals are assumed to be independent and identical distributed (i.i.d) having zero mean. Block diagram of system model is shown in Figure 3.1.

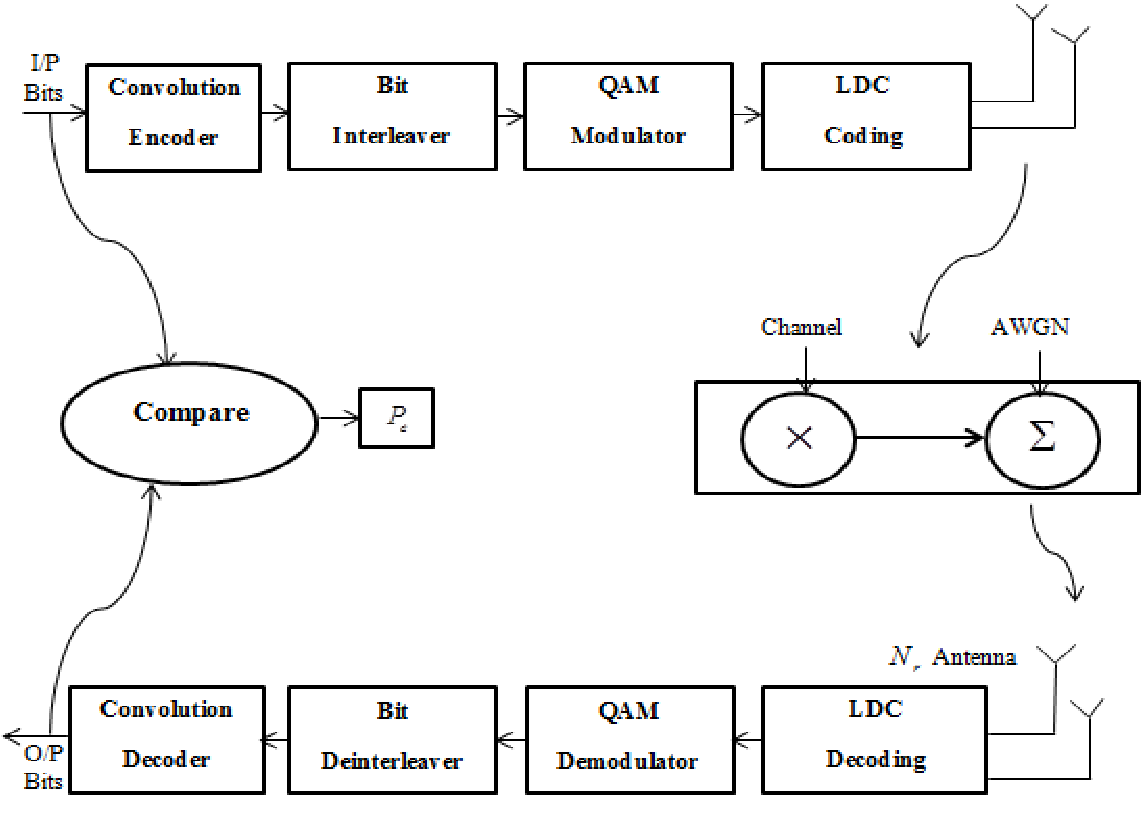


Figure 3.1: Block Diagram of M_t X N_r MIMO System Model Using Convolution Encoder

3.1.1 Transmitter Structure:

Blocks that build MIMO communication is depicted in the figure Fig.1. At first stage of fig, input data is encoded by CC, bit-interleaved and then mapped into symbols using constellation 4-QAM.Then symbols are dispersed into space and time using LDC.

3.1.1.1 Convolutional encoding

Convolution encoder is an encoder having memory which maps the bit stream to a single code. This encoding convolves the bit stream with generator sequence. This encoding is not based on algebraic technique but depends on the constructional technique. It is an error correcting code and mitigates the distortion introduced due to noise and interference [45].

In convolution encoder, mapping of input bits to codeword is done. The code rate k_c / n_c is defined as the ratio of number of input bits k_c to the number of output bits n_c. Constraint length K_c gives the information about the number of shifts input bit has to undergo.

The Performance of CC depends on the following parameters [46]:

Constraint Length K_c: With increase in constraint length,

- Coding gain of the system increases
- Code becomes more powerful
- There is an increase in the delay of the decoder.
- There is an increase in the complexity of the decoder.

Coding Rate k_c / n_c: A code having smaller code rate has following features:

- There is decrease in bandwidth efficiency.
- Extra redundancy adds strength to the code.

In this system model, hard decision decoding has been used for convolution decoding. Viterbi algorithm has been used for the decoding of Convolution codes. Viterbi algorithm is a forward error correction technique used to improve the errors caused by the burst noise and fading.

Figure 3.2 represents Convolution Encoder for generator polynomial $(171,133)_8$ of rate ½ having constraint length 7 is used. The free distance of this encoder is 10.

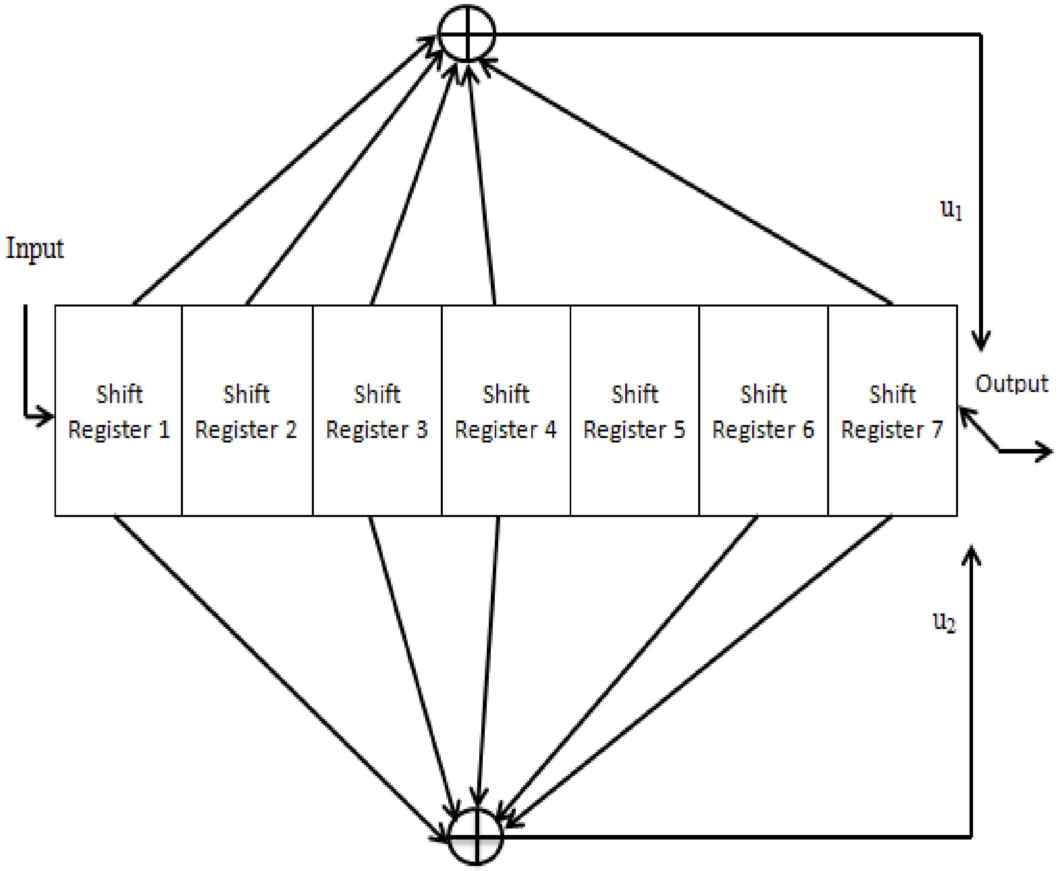


Figure 3.2 Representation of Convolution encoder for generator polynomial $(171,133)_8$ [46]

In Figure 3.2, input bits are fed to convolution encoder with generator polynomial $(171,133)_8$ and u_1 *and* u_2 are the encoded branch words obtained. Input bits pass through seven shift registers. The output word depends on the previous input bits also.

3.1.1.2 Bit-Interleaving:

Convolution codes are capable of correcting random independent errors. But in fading conditions, errors occur in burst. Thus, interleaver is used to lessen the effect of burst errors [46]. It shuffles the encoded codewords in a deterministic manner. Then de-interleaver reshuffles the codewords to get the original codeword back. Thus error correcting code can easily correct as it becomes random and independent. Interleaver fulfils the purpose of time diversity in fading conditions. It provides large hamming distance between the encoded sequences. A pseudo-random interleaver has been used in the system model.

3.1.1.3 Modulation:

Rate of the code reckons on the Q substreams, block interval of T and size of the constellation i.e. g-QAM. Rate X is given by the following formula [15]:

$$X = \frac{Q}{T}\log_2 g \ . \tag{3.1}$$

The number of matrices will be 2^{XT} resulting in exhaustive search at the decoder. But LDC develops this large constellation with ease and allows efficacious decoding.

3.1.1.4 Uncoded 4×4 MIMO System Model

Spatially multiplexed system with no diversity coding of 4x4 MIMO configuration in Rayleigh and Rician fading environment is assumed. These systems give high data rates but fail to give good error performance. It makes use of spatial diversity in splitting the symbols in scattering environment.

Consider a transmission sequence R. The symbols are sent in their respective timeslots turn wise from single transmit antenna but in spatial multiplexing grouping of symbols can be done. In each time slot, group of 4 symbols is made and transmitted from four transmit antennas and its representation is given in Table 3.1.

Table 3.1: Transmission of symbols in two different time periods in spatial multiplexing

	Tx1	Tx2	Tx3	Tx4
Time slot 1	r_1	r_2	r_3	r_4
Time slot 2	r_5	r_6	r_7	r_8

where Tx denotes the transmitter and r_i denotes the QAM modulated Symbols in Table 3.1. In each time slot, there is an involvement of total 16 channels. The impulse responses of the channel model obey flat faded Rayleigh and Rician fading distribution.

The design of the multipath fading channel is such that it does not distort the spectral characteristics but strength of the signal does not get preserved due to the fluctuations [5]. For first time slot, the system equation is represented as follows:

$$s_1 = \begin{bmatrix} h_{11} & h_{12} & \ldots\ldots & h_{14} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} + v_1 \tag{3.2}$$

Equivalently,

$$s = hr + v \tag{3.3}$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{....} & h_{....} & h_{14} \\ h_{21} & h_{22} & h_{....} & h_{....} \\ h_{....} & h_{....} & h_{33} & h_{34} \\ h_{41} & h_{....} & h_{....} & h_{44} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{3.4}$$

The equation (3.3) has been presented in matrix form in equation (3.4).

3.1.1.5 4x4 MIMO Using Linear Dispersion Codes:

LDC aspires to grant both high reliability and great spectrum efficiency by optimizing the mutual information. These codes hold both the capacity and diversity benefits. LDC does not put obligations on the number of transmit and receive antennas and can support any configuration. LDC sends data divided into substreams over space and time in linear combinations. This code encloses many STC such as orthogonal, quasi-orthogonal, V-Blast, etc. In LDC, codewords are designed from linear synthesis of input symbols and symbols' complex conjugate.

The signal R has dimension of $T \times M_t$ .Symbols are parsed into blocks of length Q. Symbols are denoted by [15]:

$$r = \begin{bmatrix} r_1, r_2 \ldots\ldots r_Q \end{bmatrix}^T \tag{3.5}$$

Data stream is divided into Q substreams $r_1, r_2 \ldots\ldots r_Q$ chosen from g-QAM constellation. The inequality $Q \le TM_t$ has to be satisfied.

$$R = \sum_{q=1}^{Q} \left(C_q r_q + D_q r*_q \right) \tag{3.6}$$

Where C_q, D_q are matrices of dimension $T \times M_t$.

$$R = \sum_{q=1}^{Q} \left(\alpha_q A_q + j\beta_q B_q \right) \tag{3.7}$$

Where A_q, B_q represent dispersion matrices for the q^{th} symbol.

Where $A_q = C + D, B_q = C - D$ (3.8)

Where the real scalars $\{\alpha_q, \beta_q\}$ are determined by:

$r_q = \alpha_q + j\beta_q \qquad q = 1, 2 ... Q$ (3.9)

Choice of parameters T, Q and $\{A_q, B_q\}$ contributes to the design of the code. Elements α_1………α_q and β_1…..…. β_q are uncorrelated and have variance half.

3.1.2 Channel Structure

This system model considers Rayleigh and Rician fading channel model. It is assumed that the CSI is known in the form of matrix H at the receiver end.

3.1.2.1 Rayleigh Fading Environment

In Rayleigh fading model, there are multiple scattering paths and no LOS is present. All the reflected signals become uncorrelated in amplitude at the receiver end and phase is uniformly distributed (0 to 2π). The in-phase and quadrature parts follow Gaussian distribution. Complex fading coefficient is defined by [45]:

$h_0 = in_0 + jqu_0$ (3.10)

Here, i_0 and q_0 represent the sum of large no. of random components having zero mean and variance 1/2.Their envelope $a = \sqrt{in_o^2 + qu_o^2}$ follows Rayleigh fading density $f(a) = 2ae^{-a^2}$ (3.11)

3.1.2.2 Rician Fading Environment

When LOS is present in multipath fading environment, then channel model is termed as Rician fading model. Consider two random variables E_1 and F_1 which are Gaussian distributed.E_1 represents LOS component while F_1 represents scatter component.E_1 has non-zero mean and variance ½. F_1 has zero mean and variance ½. Their envelope $a = \sqrt{E_1^2 + F_1^2}$ follows Rician distribution [46].

Rician factor (Υ) is defined as the ratio of power of LOS component (m_u^2) to the power of scatter component (σ_o^2).

$$\Upsilon = \frac{m_u^2}{2\sigma_u^2}\ ,\ m_u = \sqrt{\frac{\Upsilon}{\Upsilon+1}}\ \text{ and } \sigma_u = \sqrt{\frac{1}{2(\Upsilon+1)}} \tag{3.12}$$

3.1.3 Receiver Structure:

The receiver uses ML scheme for the detection of *r*. LDC possesses linearity property which gives an advantage of efficacious decoding schemes. At receiver end signal can be represented as [15]:

$$\mathrm{S}=\sqrt{\frac{\rho}{\mathrm{M_t}}}\mathrm{HR}+\mathrm{V}=\sqrt{\frac{\rho}{\mathrm{M_t}}}\sum_{q=1}^{Q}\left(\alpha_q \mathrm{A}_q + j\beta_q \mathrm{B}_q\right)\mathrm{H}+\mathrm{V} \tag{3.13}$$

V represents AWGN and is assumed to be ZMCSCG.

Decomposing the above equation into real and imaginary components [16]:

$$\mathrm{S_{real}} + j\mathrm{S_{Imag}} = \sqrt{\frac{\rho}{\mathrm{M}_t}}\sum_{q=1}^{Q}\left[\alpha_q\left(\overline{\mathrm{A}_{real,q}} + j\mathrm{A_{Imag,q}}\right) + j\beta_q\left(\overline{\mathrm{B_{real,q}}} + j\mathrm{B_{Imag,q}}\right)\right]$$

$$\times(\mathrm{H_{real}} + j\mathrm{H_{Imag}}) + (\mathrm{V_{real}} + j\mathrm{V_{Imag}}) \tag{3.14}$$

where

$$\mathrm{H_{Re}} = real(\mathrm{H})\ ;\ \mathrm{H_{Imag}} = \mathrm{Imag(H)} \tag{3.15}$$

$$\mathrm{S_{real}} = \sqrt{\frac{\rho}{\mathrm{M}_t}}\sum_{q=1}^{Q}\left[\alpha_q\left(\overline{\mathrm{A}_{real,q}}\mathrm{H_{real}} - \mathrm{A_{Imag,q}}\mathrm{H_{Imag}}\right) + \beta_q\left(-\mathrm{B_{Imag,q}}\mathrm{H_{real}} - \overline{\mathrm{B_{real,q}}}\mathrm{H_{Imag}}\right) + \mathrm{V_{real}}\right] \tag{3.16}$$

$$\mathrm{S_{Imag}} = \sqrt{\frac{\rho}{\mathrm{M}_t}}\sum_{q=1}^{Q}\left[\alpha_q\left(\mathrm{A_{Imag,q}}\mathrm{H_{real}} + \overline{\mathrm{A}_{real,q}}\mathrm{H_{Imag}}\right) + \beta_q\left(\overline{\mathrm{B_{real,q}}}\mathrm{H_{real}} - \mathrm{B_{Imag,q}}\mathrm{H_{Imag}}\right) + \mathrm{V_{Imag}}\right] \tag{3.17}$$

where

$$\mathrm{A_q} = \begin{bmatrix} \overline{\mathrm{A}_{real,q}} & -\mathrm{A_{Imag,q}} \\ \mathrm{A_{Imag,q}} & \overline{\mathrm{A}_{real,q}} \end{bmatrix} \tag{3.18}$$

$$B_q = \begin{bmatrix} -\mathrm{B_{Imag,q}} & -\overline{\mathrm{B_{real,q}}} \\ \overline{\mathrm{B_{real,q}}} & -\mathrm{B_{Imag,q}} \end{bmatrix} \tag{3.19}$$

$$\mathrm{h_n} = \begin{bmatrix} \mathrm{h_{real,n}} \\ \mathrm{h_{Imag,n}} \end{bmatrix} \tag{3.20}$$

Where $n = 1, 2 \ldots N_r$ Congregate the equations in S_{real} and S_{Imag} to structure into real set of equations:

Represent the columns of

$\mathrm{S_{real}}, \mathrm{S_{Imag}}, \mathrm{H_{real}}, \mathrm{H_{Imag}}, \mathrm{V_{real}}$, and $\mathrm{V_{Imag}}$ *by* $s_{\mathrm{real}}, s_{\mathrm{Imag}}, h_{\mathrm{real}}, h_{\mathrm{Imag}}, v_{\mathrm{real}}$ *and* v_{Imag}

$$\overline{\overline{\begin{bmatrix} s_{real,1} \\ s_{Imag,1} \\ . \\ . \\ . \\ s_{real,N_r} \\ s_{Imag,N_r}\end{bmatrix}}} = \sqrt{\frac{\rho}{M_t}}\,\overline{H}\,\overline{\begin{bmatrix}\alpha_1\\ \beta_1 \\ . \\ . \\ . \\ \alpha_Q \\ \beta_Q\end{bmatrix}} + \overline{\begin{bmatrix} v_{real,1}\\ v_{Imag,1}\\ . \\ . \\ . \\ v_{real,N_r}\\ v_{Imag,N_r}\end{bmatrix}} \tag{3.21}$$

Where the equivalent real channel matrix of dimension $2N_rT$ x $2Q$ is given by [19]:

$$\overline{H} = \begin{bmatrix} A_1h_1 & B_1h_1 & \ldots & A_Qh_1 & B_Qh_1 \\ . & . & \ldots & . & . \\ A_1h_{N_r} & B_1h_{N_r} & \ldots & A_Qh_{N_r} & B_Qh_{N_r} \end{bmatrix} \tag{3.22}$$

Receiver is having information about the channel matrix $\overline{H}$ as channel matrix H and $\{A_q, B_q\}$ parameters are already known to the receiver. Dispersion matrices $\{A_q, B_q\}$ are drawn randomly from complex Normal distribution.

LDC decoding can be done in various ways such as ZF, MMSE, sphere decoding, etc. After LDC decoding, data is then de-mapped, de-interleaved and convolution decoded to get original bit stream.

3.2 Transmit Precoding and Receiver Shaping

The design of LDCs does not consider CSI at the transmitter end. The error performance of LDC design can be improved if CSIT is available through feedback mechanism. Precoding can used with LDC which makes use of perfect CSIT and CSIR.

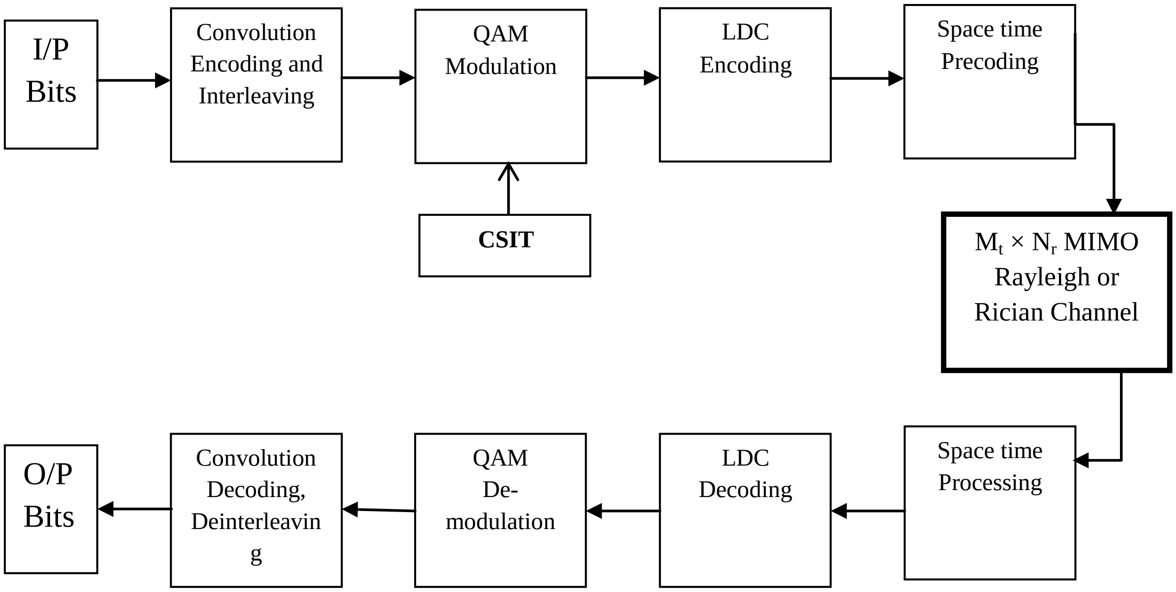


Figure 3.3: Block Diagram of $M_t \times N_r$ MIMO System Model Using Convolution Encoder and Precoder

In the block diagram, the input bits are convolution encoded and bit-interleaved. Modulation technique maps the interleaved code to symbols in the symbol mapper. These symbols are dispersed into space and time by liner dispersive codes resulting in spatial data streams. Precoding block maps these streams to M_t transmit antennas and passes through channel. Receive antennas N_r gathers the signals. Processing of signals is done, decoded, demodulated, de-interleaved and Viterbi decoded to get the output bits.

Performance gain can be achieved by applying precoding mechanism on MIMO systems. This performance gain in MIMO systems is termed as multiplexing gain [25]. The multiplexing gain is obtained by the decomposition of MIMO channel into a set of G channels independent of each other. In this system, independent data is multiplexed onto the independent channels. As a result, data rate of the MIMO system increases compared to SISO system.

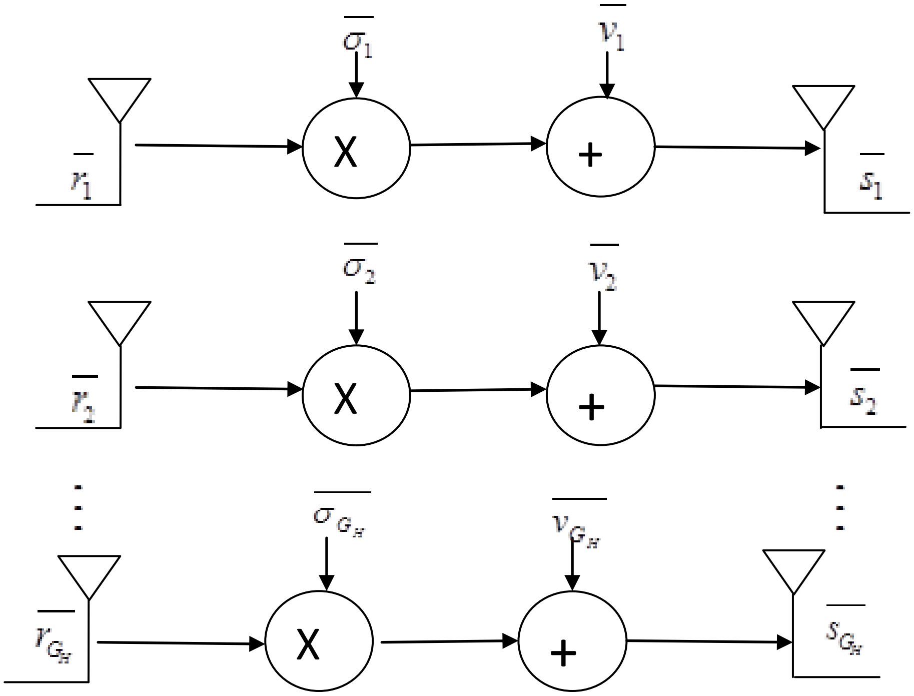


Figure 3.4: Decomposition of MIMO channel by SVD [26].

Now consider a MIMO channel with $M_t \times N_r$ with channel gain matrix H with known CSIT and

CSIR. Let G_H denotes the rank of channel matrix *H*. In dissertation, Singular Value Decomposition algorithm is used for precoding. From matrix theory, singular value decomposition (SVD) algorithm used for precoding of channel matrix *H* is given by [27]:

$$H = J\xi K^{\mathrm{H}} \tag{3.20}$$

where the matrix *J* and *K* are unitary matrices of dimensions $N_r \times N_r$ and $M_t \times M_t$ respectively.

$$J^{\mathrm{H}} J = I_{N_r} \; and \; K^{\mathrm{H}} K = I_{M_t} \tag{3.21}$$

Diagonal matrix ξ of dimension $M_t \times N_r$ denotes singular values $\overline{\sigma_i}$ of *H*.

The singular value possesses this property $\overline{\sigma_i} = \sqrt{\kappa_i}$.

where κ_i denotes i^{th} largest eigen value of HH^H.

Rank G_H of $\overline{\sigma}$ is nonzero and $G_H \le min(M_t, N_r)$.

In a rich scattering environment, rank G_H of channel matrix *H* is full and hence

$$G_H = min(M_t, N_r). \tag{3.22}$$

In other environments, rank G_H of channel matrix H may be low. High correlated channel have rank one. In transmit precoding, a linear transformation is applied on the input vector *r* to get precoded vector $\bar{r}$ as:

$$\bar{r} = Kr \tag{3.23}$$

In receive shaping; a linear transformation is applied on the output $\bar{s}$ to get the output vector s as:

$$s = J^{\mathrm{H}} \bar{s} \tag{3.24}$$

Now, the received vector s from SVD is obtained as follows:

$$\bar{s} = H\bar{r} + v \tag{3.25}$$

Substitute value of $\bar{r}$ from equation (3.23) in the equation (3.25):

$$\bar{s} = HKr + v \tag{3.26}$$

Substitute value of *H* from equation (3.20) in the equation (3.26):

$$\bar{s} = J\xi K^H Kr + v \tag{3.27}$$

$$\bar{s} = J\xi r + v \tag{3.28}$$

Now substitute the value of $\bar{s}$ in equation (3.24):

$$s = J^{\mathrm{H}}\bar{s} = J^{\mathrm{H}}(J\xi r + v) \tag{3.29}$$

$$s = J^{\mathrm{H}}J\xi r + J^{\mathrm{H}}v \tag{3.30}$$

$$s = \xi r + \bar{v} \tag{3.31}$$

where

$$\bar{v} = J^{H}v \tag{3.32}$$

Representation of transmit precoding and receiver shaping equations is shown in the figure.

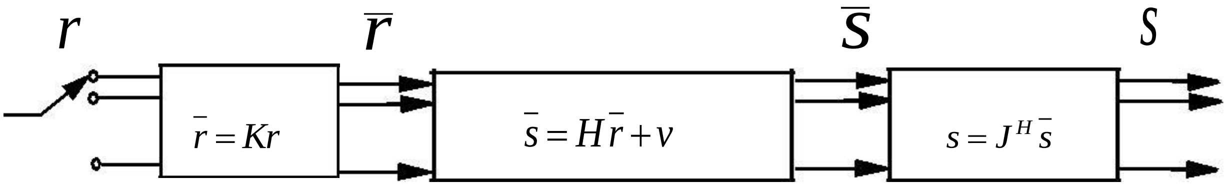


Figure 3.5: Transmit Precoding and Receiver Shaping [27]

The figure represents the precoding at the transmitter side and receiver shaping. The input vector *r* is passed through precoder giving $\bar{r}$ which is passed through channel and receiver shaping of $\bar{s}$ is done giving back the restored signal vector *s*.

CHAPTER 4

PERFORMANCE ANALYSIS OF LDC AND CONVOLUTION CODED MIMO SYSTEM

In this chapter, BER performance of LDC is initially taken into consideration and this BER result of LDC has been used to derive BER performance of Convolution coded (CC) 4x4 MIMO system using LDC.

4.1 Average Pairwise Error Probability of LDC:

Hassibi *et al.* [15] derived the upper bound for average pairwise error probability (PEP) which is function of random matrices. M. Ghreyani *et al.* [17] derived the tighter bound for average PEP demonstrating the relationship with constellation size and symbol rate and constellation size. The received signal represented in terms of transmitted signal and channel matrix in the presence of noise is given as [45]:

$$s = \sqrt{\frac{\rho}{M_t}} Hr + v \tag{4.1}$$

Conditional pairwise error probability (Pep) is composed as follows [46]:

$$Pep\left(pairwise\right) \ = E_x \ \left(Pep\left(pairwise / H\right)\right) \tag{4.2}$$

Two independent $2Q \times 1$ signal vectors r and $\tilde{r}$, are assumed to be selected with entrants from independent real Gaussian probability densities with zero mean and variance ½. The AWGN $2NT \times 1$ vector v also obeys the Gaussian probability density. Average probability conditioned on H $Pep\left(pairwise/H\right)$ that Maximum Likelihood (ML) decoder misinterprets r for $\tilde{r}$, given that r is transmitted and obeys Gaussian distribution,

$$Pep\left(pairwise/H\right) = P\left\{\left\|s-\sqrt{\frac{\rho}{M_t}}H\tilde{r}\right\| < \left\|s-\sqrt{\frac{\rho}{M_t}}Hr\right\| \times \left|r \quad transmitted\right.\right\} \qquad (4.3)$$

$\|B\|$ represents the Forbenius norm of B.

Substitute the value of s given by (4.1) in equation (4.3):

$$P\left\{\left\|\sqrt{\frac{\rho}{M_t}}Hr+v-\sqrt{\frac{\rho}{M_t}}H\tilde{r}\right\| < \left\|\sqrt{\frac{\rho}{M_t}}Hr+v-\sqrt{\frac{\rho}{M_t}}Hr\right\| \times \left|r \ transmitted\right.\right\} \qquad (4.4)$$

$$P\left\{\left\|\sqrt{\frac{\rho}{M_t}}H\left(r-\tilde{r}\right)+v\right\| < \|v\|\right\} \qquad (4.5)$$

As

$$\|B\| = \sqrt{B^t B} \qquad (4.6)$$

Therefore,

$$P\left\{\left\|\sqrt{\frac{\rho}{M_t}}H\left(r-\tilde{r}\right)+v\right\| < \|v\|\right\} = P\left\{\left(\sqrt{\frac{\rho}{M_t}}H\left(r-\tilde{r}\right)+v\right)^t\left(\sqrt{\frac{\rho}{M_t}}H\left(r-\tilde{r}\right)+v\right) < v^t v\right\} \qquad (4.7)$$

Difference of r and $\tilde{r}$ has been replaced by a single vector with twice variance as r and $\tilde{r}$ are independent Gaussian vectors [15]:

$$P\left\{\left(\sqrt{\frac{2\rho}{M_t}}Hr+v\right)^t\left(\sqrt{\frac{2\rho}{M_t}}Hr+v\right) < v^t v\right\} \qquad (4.8)$$

$$P\left\{\left(\sqrt{\frac{2\rho}{M_t}}r^t H^t + v^t\right)\left(\sqrt{\frac{2\rho}{M_t}}Hr+v\right) < v^t v\right\} \qquad (4.9)$$

$$P = \left\{\left(\frac{2\rho}{M_t}\right)r^t H^t Hr + \sqrt{\frac{2\rho}{M_t}}\left(r^t H^t v + v^t Hr\right) < 0\right\} \qquad (4.10)$$

$$Let \quad z = \left(\frac{2\rho}{M_t}\right)r^t H^t Hr + \sqrt{\frac{2\rho}{M_t}}\left(r^t H^t v + v^t Hr\right) \qquad (4.11)$$

$$z = \left[r^t v^t \right] \begin{bmatrix} \left(\frac{2\rho}{M_t} \right) H^t H & \sqrt{\frac{2\rho}{M_t}} H^t \\ \sqrt{\frac{2\rho}{M_t}} H & 0 \end{bmatrix} \begin{bmatrix} r \\ \hline v \end{bmatrix} \tag{4.12}$$

$$Let \qquad \overline{W} = \begin{bmatrix} \left(\frac{2\rho}{M_t} \right) H^t H & \sqrt{\frac{2\rho}{M_t}} H^t \\ \sqrt{\frac{2\rho}{M_t}} H & 0 \end{bmatrix} \tag{4.13}$$

The characteristic function of z [48]:

$$\varphi(w) = E_x \, e^{jwz} \tag{4.14}$$

$$\varphi(w) = \int drdv \frac{1}{(2\pi)^{Q+N_rT} \left(\frac{1}{2} \right)^{Q+N_rT}} \times \exp\left\{ -\left[r^t v^t \right] \left(I + jw\overline{W} \right) \begin{bmatrix} r \\ \hline v \end{bmatrix} \right\} \tag{4.15}$$

Use formula:

$$\int da e^{-a^t \Lambda a} = \pi^{p/2} \left(\det \Lambda \right)^{-1/2} \tag{4.16}$$

Where a is a real $p \times 1$ vector

$$\phi(w) = \det\left(I + jw\overline{W} \right)^{-1/2} \tag{4.17}$$

$$Pep\left(pairwise / H \right) = P\{ z < 0 \} \qquad (4.18) = \int_{-\infty}^{0} dzp(z)$$

$$(4.19) \; = \frac{1}{2\pi} \int_{-\infty}^{0} dz \int_{-\infty}^{0} dw \, e^{jwz} \det\left(I + jw\overline{W} \right)^{-1/2} \tag{4.20}$$

Change the order of integration and the path of integration to $w \in \left(-\infty - j/2 \;\; \infty - j/2 \right)$:

$$Pep\left(pairwise / H \right) = \frac{1}{2\pi} \int_{-\infty - j/2}^{\infty - j/2} dw \int_{-\infty}^{0} dz \, e^{jwz} \;\; \det\left(I + jw\overline{W} \right)^{-1/2} \tag{4.21}$$

$$= \frac{1}{2\pi} \int_{-\infty - j/2}^{\infty - j/2} dw \det\left(I + jw\overline{W}\right)^{-1/2} \left.\frac{e^{iwz}}{jw}\right|_{-\infty}^{0} \tag{4.22}$$

$$= \frac{1}{2\pi j} \int_{-\infty - j/2}^{\infty - j/2} dw \left(\frac{1}{w}\right) \det\left(I + jw\overline{W}\right)^{-1/2} \tag{4.23}$$

$\det\left(I + jw\overline{W}\right)$ is computed by first finding out the eigenvalues of W

$$\det\begin{bmatrix} \kappa I_{2Q} - \left(2\rho/M_t\right) H^t H & -\sqrt{2\rho/M_t}\, H^t \\ -\sqrt{\dfrac{2\rho}{M_t}} H & \kappa I_{2N_rT} \end{bmatrix} = 0 \tag{4.24}$$

as a polynomial expression in κ .Rewriting this equation by employing standard determinant identify [15]:

$$0 = l^{2N_rT} \det\left(lI_{2Q} - \left(\frac{2\rho}{M_t}\right) H^t H - \frac{2\rho}{M_t l} H^t H \right) \tag{4.25}$$

$$= l^{2N_rT-2Q} \det\left(l^2 I_{2Q} - 2\left(l+i\right)\left(\rho/M_t\right) H^t H \right) \tag{4.26}$$

$$= \left(l^{2N_rT-2Q} \prod_{q=1}^{2Q} \left(l^2 - 2\left(l+1\right)\right) U_q \right) \tag{4.27}$$

Eigen values of $\left(\frac{\rho}{M_t}\right) H^t H$ are $U_1 \ldots\ldots\ldots\ldots\ldots\ldots.. U_{2Q}$. On computation of equation gives $2N_rT - 2Q$ zero eigin values, with the remaining $4Q$ eigenvalues as following:

$$U_q \pm \sqrt{U_q^2 + 2U_q} \qquad q = 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots Q \tag{4.28}$$

Thus,

$$\det\left(I + jw\overline{w}\right) = \prod_{q=1}^{2Q} \left[1 + jw\left(U_q + \sqrt{U_q{}^2 + 2U_q}\right) \right] \times \left[1 + jw\left(U_q - \sqrt{U_q{}^2 + 2U_q}\right) \right] \tag{4.29}$$

$$=\prod_{q=1}^{2Q}\left[1+2wU_q\left(j+w\right)\right] \tag{4.30}$$

$$Pep\left(pairwise/H\right)=\frac{1}{2\pi j}\int_{-\infty-j/2}^{\infty-j/2} dw\frac{1}{w}\prod_{q=1}^{2Q}\left[1+2wU_q\left(j+w\right)\right]^{-1/2} \tag{4.31}$$

$$=\frac{1}{2\pi j}\int_{-\infty}^{\infty} dw\frac{1}{w-j/2}\times\prod_{q=1}^{2Q}\left[l+2U_q\left(w-j/2\right)\left(w+j/2\right)\right]^{-1/2} \tag{4.32}$$

$$=\frac{1}{2\pi j}\int_{-\infty}^{\infty} dw\frac{w+j/2}{w^2+1/4}\prod_{q=1}^{2Q}\left[l+2U_q\left(w^2+1/4\right)\right]^{-1/2} \tag{4.33}$$

$$=\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{1}{w^2+1/4}\prod_{q=1}^{2Q}\left[l+2U_q\left(w^2+1/4\right)\right]^{-1/2} \tag{4.34}$$

Neglecting the second appearance of w^2

$$Pep\left(pairwise/H\right)\leq\frac{1}{4\pi}\int_{-\infty}^{\infty} dw\frac{1}{w^2+1/4}\prod_{q=1}^{2Q}\left[1+U_q/2\right]^{-1/2} \tag{4.35}$$

$$=\frac{1}{2}\prod_{q=1}^{2Q}\left[1+U_q/2\right]^{-1/2} \tag{4.36}$$

$$=\frac{1}{2}\det\left[I+\frac{\rho}{2M_t}H^tH\right]^{-1/2} \tag{4.37}$$

Thus, $Pep\left(pairwise/H\right)\leq E_x\frac{1}{2}\det\left[I+\frac{\rho}{2M_t}H^tH\right]^{-1/2}$ (4.38)

Transmission rate is *X*, so constellation comprise of 2^{XT} elements; then

$$Pep\leq 2^{XT}E_x\frac{1}{2}\det\left[I+\frac{\rho}{2M_t}H^tH\right]^{-1/2} \tag{4.39}$$

$$=2^{XT-1}E_x\det\left[I+\frac{\rho}{2M_t}H^tH\right]^{-1/2} \tag{4.40}$$

Wishart distribution is represented as [47]

$$W = \begin{cases} HH^{H} ....N_r \leq M_t \\ H^{H}H ....N_r > M_t \end{cases} \tag{4.41}$$

Let’s define

$$n = max\ \left(N_r,\ M_t\right)\ and\ \ m = min\ \left(N_r,\ M_t\right) \tag{4.42}$$

The probability density of W is given by [48]

$$p(W) = \frac{etr\left(-\Sigma^{-1}W\right)\left(\det W\right)^{n-m}}{\Gamma_m\left(n\right)\left(\det\Sigma\right)^{n}} \tag{4.43}$$

Where multivariate gamma function $\Gamma_m\left(n\right)$ is given by [49]:

$$\Gamma_m\left(n\right) = \pi^{\frac{m\left(m-1\right)}{2}} \prod_{j=0}^{m-1} \Gamma\left(n-j+1\right) \tag{4.44}$$

W is a $m \times m$ non-negative complex matrix with probability distribution $p(W)$.

Let $\left\{\kappa_j, \forall j = 0,.......m-1\right\}$ represents eigen values of W.

$$\Lambda = diag\left(\kappa_0,......\kappa_{m-1}\right) \tag{4.45}$$

$W = E\Lambda E^{H}$ is the eigenvalue decomposition.

$$\Lambda = \left[\kappa_o,........\kappa_{m-1}\right]^{T} \tag{4.46}$$

Equation (4.40) can be written as

$$Pep \leq 2^{XT-1} E_x \left\{ \left[\prod_{j=0}^{m-1} \left(1+\bar{\gamma}\kappa_j\right) \right]^{-T} \right\} \tag{4.47}$$

Then joint density of eigen values $\left(\kappa_1,......\kappa_m\right)$ of W is [47]

$$p\left(\Lambda\right) = \frac{\pi^{m\left(m-1\right)}\left(\sigma^2\right)^{-nm}}{\Gamma_m\left(m\right)\Gamma_n\left(n\right)} \times \prod_{j=0}^{m-1} \kappa_j^{n-m} \prod_{j<l} \left(\kappa_j - \kappa_l\right)^2 \exp\left(-\frac{1}{\sigma^2}\sum_{j=0}^{m-1}\kappa_j\right) \tag{4.48}$$

The density in (4.48) represents ordered eigen density. Thus unordered eigen density is acquired by dividing (4.48) by $m!$ and substituting (4.44) in (4.48).

$$p\left(\Lambda\right)=\left(\frac{1}{m!}\right)\times\frac{\pi^{m\left(m-1\right)}\left(\sigma^{2}\right)^{-nm}}{\pi^{\frac{m\left(m-1\right)}{2}}\prod_{j=0}^{m-1}\Gamma_{m}\left(m-j+1\right)\pi^{\frac{m\left(m-1\right)}{2}}\prod_{j=0}^{m-1}\Gamma_{n}\left(n-j+1\right)}$$
$$\times\prod_{j=0}^{m-1}\kappa_{j}^{n-m}\prod_{j<l}\left(\kappa_{j}-\kappa_{l}\right)^{2}\exp\left(-\frac{1}{\sigma^{2}}\sum_{j=0}^{m-1}\kappa_{j}\right)\tag{4.49}$$

Substitute $\sigma^{2}=1$ in (4.49)

$$p\left(\Lambda\right)=\left(\frac{1}{m!}\right)\frac{1}{\prod_{j=0}^{m-1}\Gamma_{m}\left(m-j+1\right)\Gamma_{n}\left(n-j+1\right)}\times\prod_{j=0}^{m-1}\exp\left(-\kappa_{j}\right)\kappa_{j}^{n-m}\prod_{j<l}\left(\kappa_{j}-\kappa_{l}\right)^{2}\tag{4.50}$$

Let $c=\left(\frac{1}{m!}\right)\frac{1}{\prod_{j=0}^{m-1}\Gamma_{m}\left(m-j+1\right)\Gamma_{n}\left(n-j+1\right)}$ (4.51)

$$p\left(\Lambda\right)=c\prod_{j}e^{-\kappa_{j}}\left(\kappa_{j}\right)^{n-m}\prod_{j<l}\left(\kappa_{j}-\kappa_{l}\right)^{2}\tag{4.52}$$

Rewrite equation (4.47) in terms of eigenvalues of W [17]:

$$Pep\leq 2^{XT-1}\int^{(m)}\prod_{j=0}^{m-1}\left(1+\bar{\gamma}\kappa_{j}\right)^{-T}p\left(\Lambda\right)d^{(m)}\kappa\tag{4.53}$$

Let $P_{k}\left(\Lambda\right)=\prod_{j=0}^{m-1}\left(1+\bar{\gamma}\kappa_{j}\right)^{-T}p\left(\Lambda\right)$ (4.54)

Substituting (4.52) into (4.54)

$$P_{k}\left(\Lambda\right)=c\prod_{j<l}\left(\kappa_{j}-\kappa_{l}\right)^{2}\prod_{j=0}^{m-1}e^{-\kappa_{j}}\kappa_{j}^{n-m}\left(1+\bar{\gamma}\kappa_{j}\right)^{-T}\tag{4.55}$$

Let $D=\prod_{j<l}\left(\kappa_{j}-\kappa_{l}\right)^{2}=\left\{\det\left[\begin{pmatrix}1&\cdots&1\\\vdots&\ddots&\vdots\\\kappa_{0}^{m-1}&\cdots&\kappa_{m-1}^{m-1}\end{pmatrix}\right]\right\}^{2}$ (4.56)

$$P_k\left(\Lambda\right)=cD\left[\prod_{j=0}^{m-1}e^{-\kappa_j}\kappa_j^{n-m}\left(1+\bar{\gamma}\kappa_j\right)^{-T}\right] \tag{4.57}$$

$$=c\left\{\det\left[\begin{pmatrix} f_o\left(\kappa_o\right) & \cdots & f_o\left(\kappa_{m-1}\right) \\ \vdots & \ddots & \vdots \\ f_{\kappa-1}\left(\kappa_o\right) & \cdots & f_{m-1}\left(\kappa_{m-1}\right) \end{pmatrix}\right]\right\}^2 \tag{4.58}$$

Where $f_j\left(\kappa\right)=\left[e^{-\kappa}\kappa^{n-m}\left(1+\bar{\gamma}\kappa\right)^{-T}\right]^{1/2}\kappa^j$ (4.59)

$$P_k\left(\Lambda\right)=c\det^2\left[f_j\left(\kappa_l\right)\right]_{jl=0}^{m-1} \tag{4.60}$$

Substituting (4.60) into (4.53)

$$Pep\le 2^{XT-1}c\int^{m}\det^2\left[f_j\left(\kappa_l\right)\right]_{jl=0}^{m-1}d^{(m)}\left(\kappa\right) \tag{4.61}$$

At high SNR, the upper bound from [17] is as follows:

$$P_e\le cm!\prod_{j=0}^{m-1}\Gamma\left(n-m+j-T\right)2^{XT-1}\left(\frac{\rho}{2M_t}\right)^{-mT} \tag{4.62}$$

where ρ denotes SNR

4.2 Performance Analysis Of Convolution Coded System Using Hard Decision Decoding:

Assume that the convolution coded system is using hard decision Viterbi decoding. For this decoding, the tighter bound of average probability of error for CC is given by [45]. For d=odd

$$P\left(d\right)=\sum_{i=\frac{d+1}{2}}^{d}\binom{d}{i}p^i\left(1-p\right)^{d-i} \tag{4.63}$$

For d=even $P\left(d\right)=\sum_{i=\frac{d}{2}+1}^{d}\binom{d}{i}p^i\left(1-p\right)^{d-i}+\frac{1}{2}\binom{d}{\frac{d}{2}}p^{\frac{d}{2}}\left(1-p\right)^{\frac{d}{2}}$ (4.64)

p=probability of error for BSC

d=hamming distance

It is assumed that p approximates P_e given in (4.62) for LDC system in Rayleigh fading condition. A random interleaver is assumed to be used with the proposed system.

The bound for probability of error is given by [45]

$$P_b < \sum_{d=d_{free}}^{\infty} g_d P\left(d\right) \tag{4.65}$$

where g_d represents the coefficients present in the expression of derivative of the transfer function $Tr(D_r, O_r, J_r)$ computed at $O_r = 1$ and d_{free} represents the free distance of CC.

CHAPTER 5
RESULTS AND DISCUSSION

This chapter gives simulated results of uncoded 4x4 MIMO systems, LDC system; Convolution coded 4x4 MIMO system using LDC and Precoded system in different fading conditions.

5.1 Spatial Multiplexed 4x4 MIMO System

Spatial multiplexing is used for the reduction of symbol error rate (SER) and to increase the data rate. Spatially multiplexed system of 4x4 MIMO configuration has been assumed. At transmitter and receiver, number of antennas is assumed to be 4 for this system. There is no diversity coding used in spatial multiplexed system. Figure 5.1 shows SER performance of spatially multiplexed system with no diversity coding.

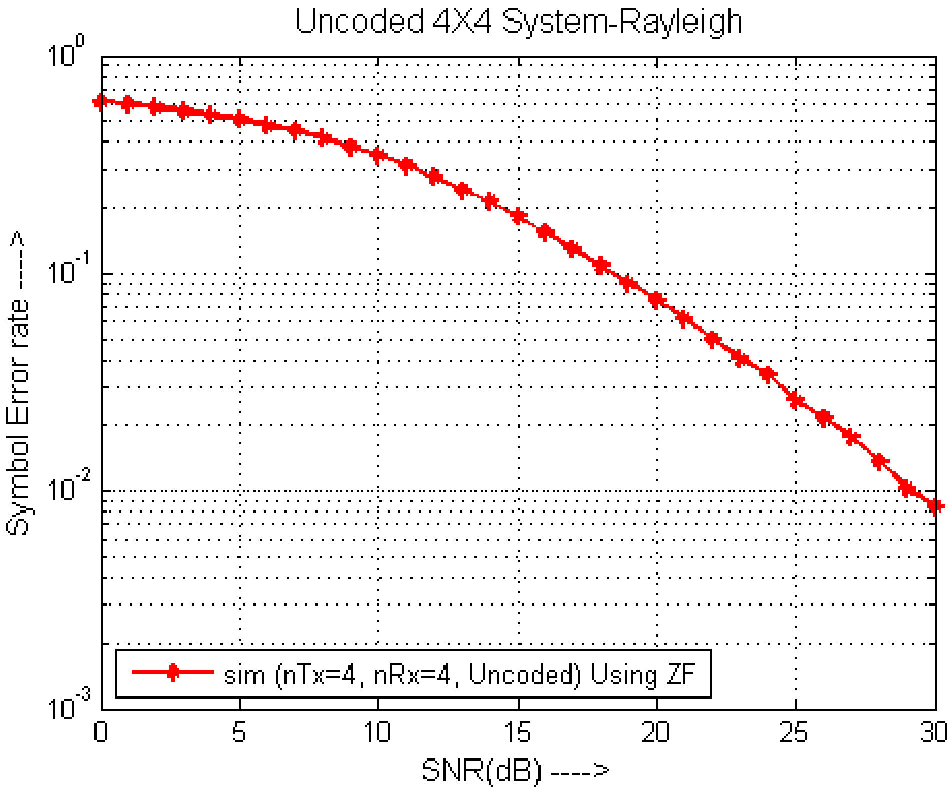


Figure 5.1: SER analysis of uncoded 4x4 MIMO System

Simulation of spatially multiplexed 4x4 MIMO systems has been done. 4-QAM modulation technique over Rayleigh channel has been assumed. Result depicts that spatial multiplexing results in higher error rates i.e. at SNR of 20Db error rate of $10^{-1.4}$ is obtained. For LDC decoding, ZF decoding has been assumed. Symbol error rate in Multiplexing (4x4) MIMO System is very high.

5.2 LDC Coded 4x4 MIMO Systems

From each antenna of MIMO system, the same copy of signal is transmitted in diversity coding. SER of the diversity coded system is less as compared to the spatially multiplexed system without coding. LDC for 4x4 MIMO configurations has been used for diversity coding and provides diversity gain. Value of time interval for which channel is assumed to be constant T is four and value of 12 has been assumed for Q. Modulation technique assumed is 4-QAM. Dispersion matrices disperse these Q substreams spatially and temporally. Figure 5.2 shows SER performance of LD coded 4x4 MIMO systems.

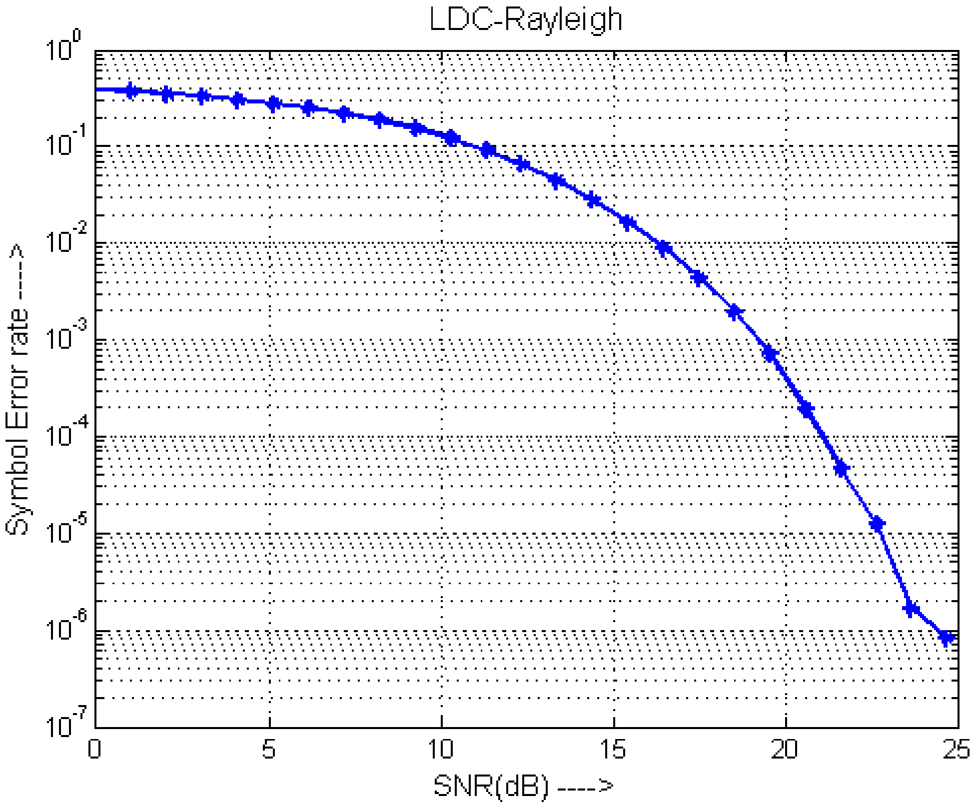


Figure 5.2: SER analysis of Linear Dispersion Coded 4x4 MIMO System

Simulation results shows that less error rates are observed using diversity coding i.e. at SNR of 20 dB error rate of $10^{-1.9}$ is observed. Zero-forcing decoding has been used for decoding of LDC. In this decoding, the received signal is multiplied with the pseudo inverse of channel matrix giving the desired result.

SER Comparison of LDC system with uncoded spatial multiplexed system for 4x4 MIMO configurations has been done. Figure 5.3 depicts performance improvement of LDC systems.

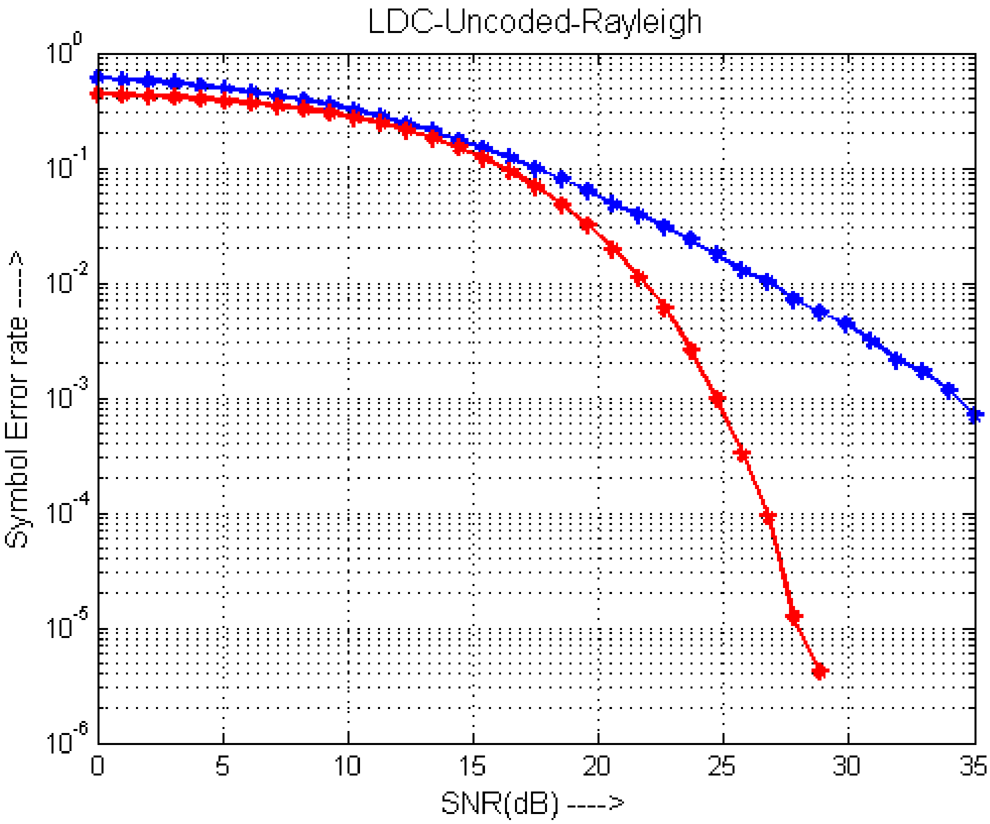


Figure 5.3 SER comparison of LDC coded and uncoded spatial multiplexed 4x4 MIMO Systems

For Rayleigh channel, LDC coded MIMO system has shown SER of 10^{-3} at SNR of 25dB whereas uncoded spatially multiplexed system has shown SER of $10^{-1.9}$ at the same SNR in Figure 5.3. Spatially multiplexed systems show higher error rate while LDC shows lesser error rate at the same SNR. Hence, LDC provides better SER performance than spatially multiplexed systems. Spatially multiplexed systems do not have the inbuilt quality of guarding the MIMO systems against fading, noise or interference. But LDCs' have the abilty to gurad against distortions introduced by the channel.

5.3 Convolution Encoded 4x4 MIMO System Using LDC

Channel encoding has been used with LDC systems to enhance system error performance and reducing the effect of burst noise. An encoder with constraint length K_c =7 have coding rate of ½. Analytically, minimum free distance $\left(d_{free}\right)$ for generator $(171,133)_8$ is assumed to be 10. And the corresponding coefficient g_d's value is taken as 36. Substitute these values in equation (4.65) to obtain the result. Simulation and analytical result for convolution coded 4x4 MIMO using LDC have been depicted in Figure 5.4. Simulation parameters $M_t = N_r$=T=4 and 4 QAM constellation are assumed.

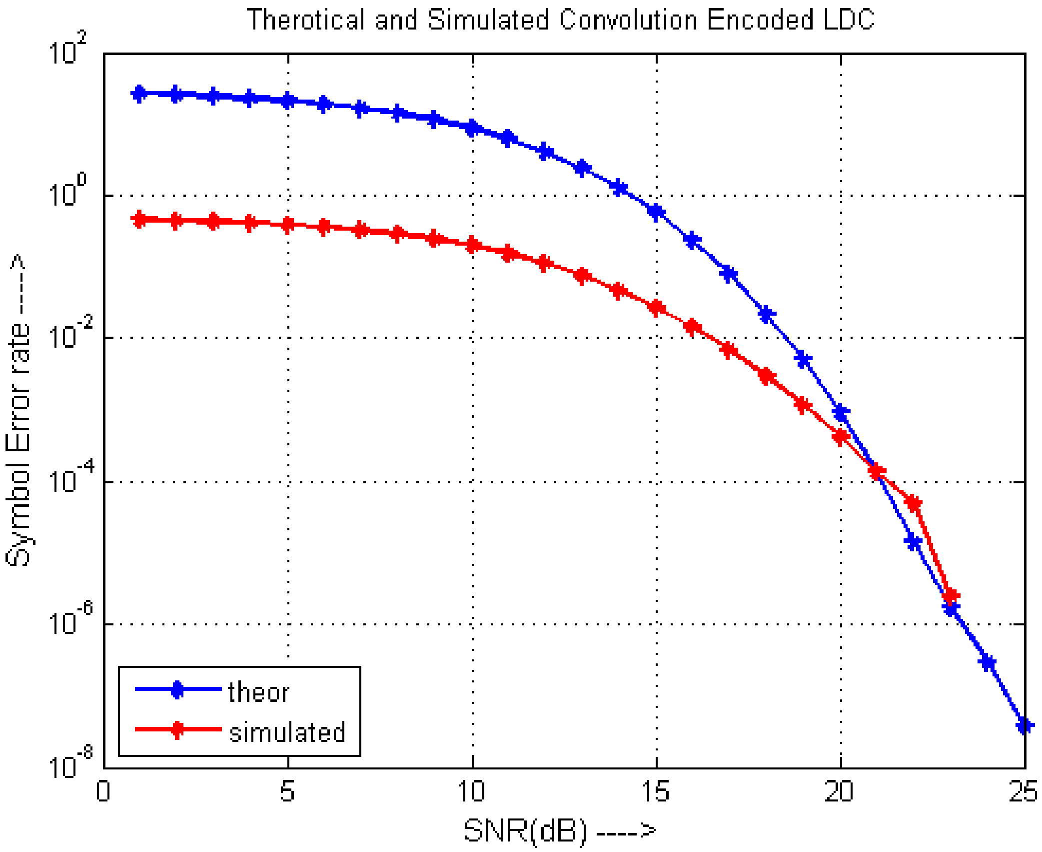


Figure 5.4 SER comparison of theoretical and simulated Convolution encoder using LDC

In Figure.5.4, convolution encoder has been used of rate ½ having free distance of 10 to mitigate the effect of burst errors so that SER performance of the system increases. In this system, hard decision decoding has been done using Viterbi algorithm. Theoretical SER of LDC has been used to derive the analytical result of error probability of convolution concatenated 4x4 MIMO systems. Equation (4.65) has been used to plot theoretical results. Analytical results match well with the simulated results at high SNRs'. Hence, analytical results have been verified.

5.4 Precoding of Convolution concatenated 4x4 MIMO system using LDC in different fading conditions

Precoding breaks the channel matrix into the equalizer matrix which decomposes the MIMO channel into independent SISO channels. Linear precoder makes use of transmit correlation and mean of the channel to improve the system performance **[7].** Precoding requires CSIT for its processing. For precoding, algorithm based on Singular Value Decomposition (SVD) has been applied on the channel matrix. Simulation parameters are mentioned in the Table 5.1

Table 5.1: **Simulation Parameters**

S.no	**Simulation Parameter**	**Parameter Value**
1	Numbers of transmit antenna	4
2	Numbers of receive antenna	4
3	Time interval (T)	4
4	No. of substreams(Q)	12
5	Modulation technique	4-QAM
6	Polonymial generator of convolution encoder	$(171,133)_8$
7	Constraint length of convolution encoder	7
8	Rate of convolution encoder	1/2
9	Rician factor (Υ)	0.1

Simulation parameters given in Table 5.1 are used for simulation of convolution coded 4x4 MIMO systems using LDC with and without precoding in Rayleigh and Rician fading environments. These simulation parameters are also used for comparison of convolution coded systems with spatially multiplexed systems and LDC systems.

5.4.1 SER comparison of Precoded system model with spatial multiplexed non-coded system in Rayleigh fading channel model

Precoding of convolution coded 4x4 MIMO system using LDC has been done in Rayleigh fading channel. Precoding is based on the SVD algorithm. In simulations, Rayleigh fading channel is assumed with zero mean uncorrelated complex Gaussian noise across the receive antennas. Simulation parameters are given in Table 5.1**.**

Comparison of Precoded system model with spatial multiplexed non-coded system in Rayleigh fading channel model has been done in Figure 5.5.

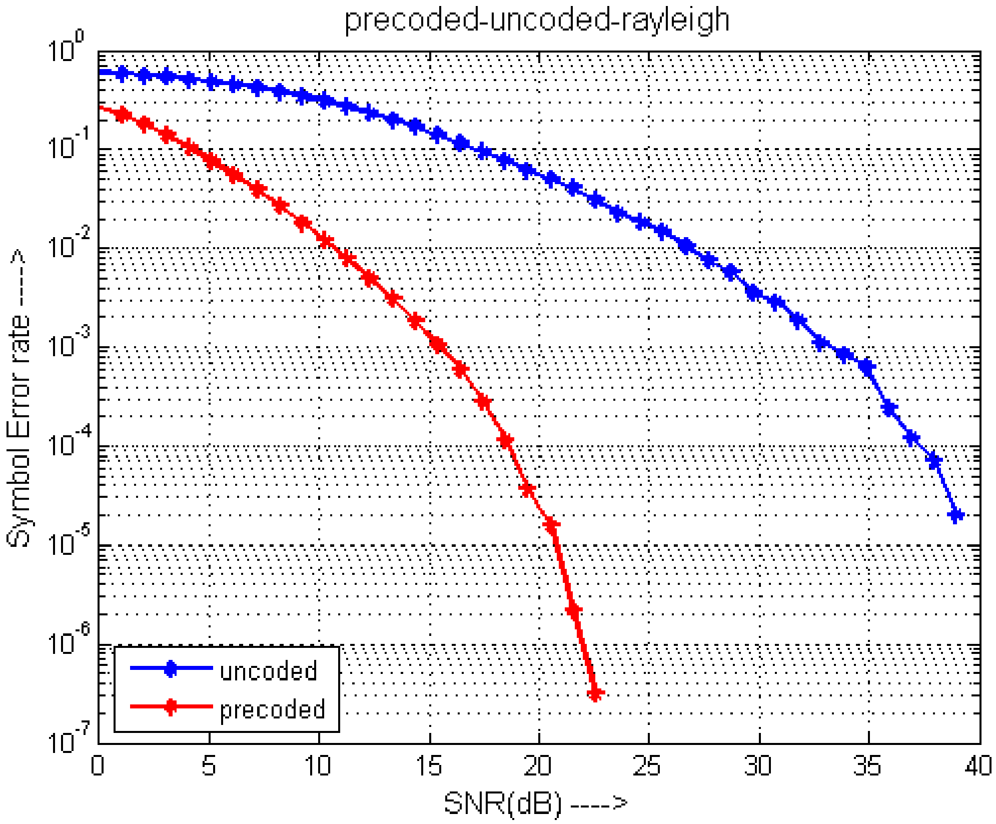


Figure 5.5: SER comparison of Precoded system with spatial multiplexed non-coded system for Rayleigh channel

Precoding based on SVD algorithm is applied on convolution concatenated 4x4 MIMO system using LDC for Rayleigh channel. Figure 5.5 shows using Precoding for Rayleigh channel, symbol error performance of $10^{-2.6}$ is obtained at SNR of 14 dB whereas spatial multiplexed non-coded system at same symbol error performance is obtained at 30 dB; hence 16 dB performance improvement is obtained.

5.4.2 SER comparison of Convolution coded system model with spatial multiplexed non-coded system in Rayleigh fading channel model

Convolution encoding of LDC coded 4x4 MIMO for Rayleigh channel has been done. Convolutional code of rate ½ having constraint length K_c=7 with generator $(133{,}171)_8$ has been used. Bit-interleaving has been used with convolution coded systems to enhance system performance.

Comparison of Convolution coded system model with spatial multiplexed non-coded system in Rayleigh fading channel model has been done by Matlab Simulations. Simulation parameters are given in Table 5.1.

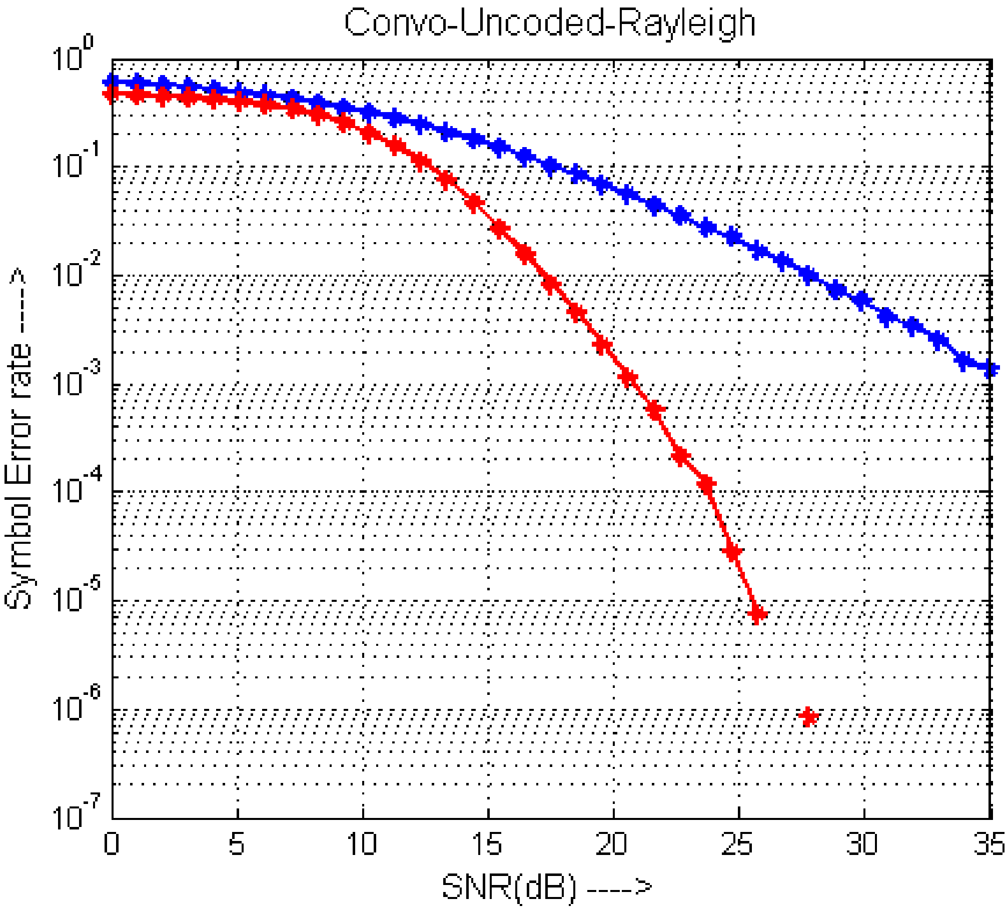


Figure 5.6: SER comparison of Convolution coded system with spatial multiplexed non-coded system for Rayleigh channel

Figure 5.6 shows using Convolution encoding without precoder for Rayleigh channel, symbol error performance of $10^{-2.6}$ is obtained at SNR of 18 dB whereas spatial multiplexed non-coded system at same symbol error performance is obtained at 30 dB; hence 12 dB performance improvements is obtained.

5.4.3 SER comparison of Precoded system model with spatial multiplexed non-coded system in Rician fading channel model:

Precoding of convolution coded 4x4 MIMO system using LDC has been done in Rayleigh fading channel. Precoding is based on the SVD algorithm. In simulations, Rician fading channel is assumed with non-zero mean uncorrelated complex Gaussian noise across the receive antennas. Rician factor Υ is assumed to be 0.1. Simulation parameters are given in Table 5.1**.**

Comparison of Precoded system model with spatial multiplexed non-coded system in Rician fading channel model has been done.

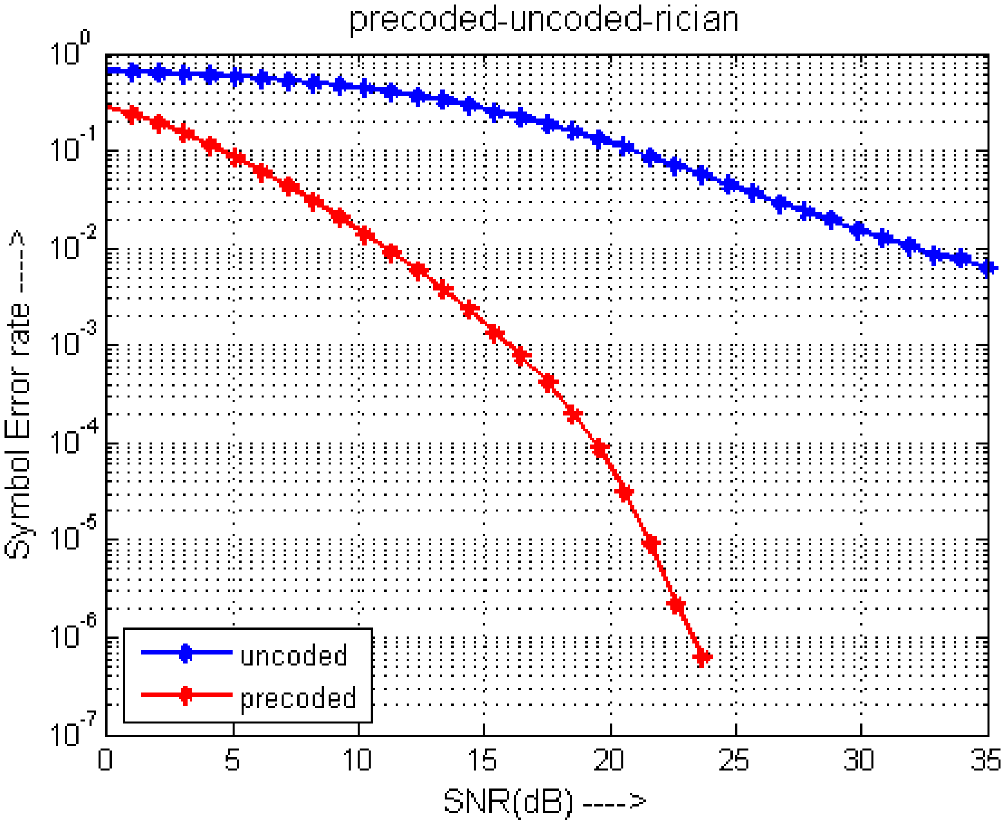


Figure 5.7: SER comparison of Precoded system with spatial multiplexed non-coded system for Rician channel

Precoding based on SVD algorithm is applied on convolution concatenated 4x4 MIMO system using LDC for Rician channel. Figure 5.7 shows using Precoding, symbol error performance of

$10^{-1.9}$ is obtained at SNR of 10Db whereas spatial multiplexed non-coded system at same symbol error performance is obtained at 28Db; hence 18Db performance improvement is obtained.

5.4.4 SER comparison of Convolution coded system model with spatial multiplexed non-coded system system in Rician fading channel model:

Convolution encoding of LDC coded 4x4 MIMO for Rician channel has been done. Convolutional code of rate ½ having constraint length K_c=7 with generator $(133,171)_8$ has been used. Bit-interleaving has been used with convolution coded systems to enhance system performance.

Comparison of Convolution coded system model with spatial multiplexed non-coded system in Rician fading channel model has been done by Matlab Simulations. Simulation parameters are given in Table 5.1**.**

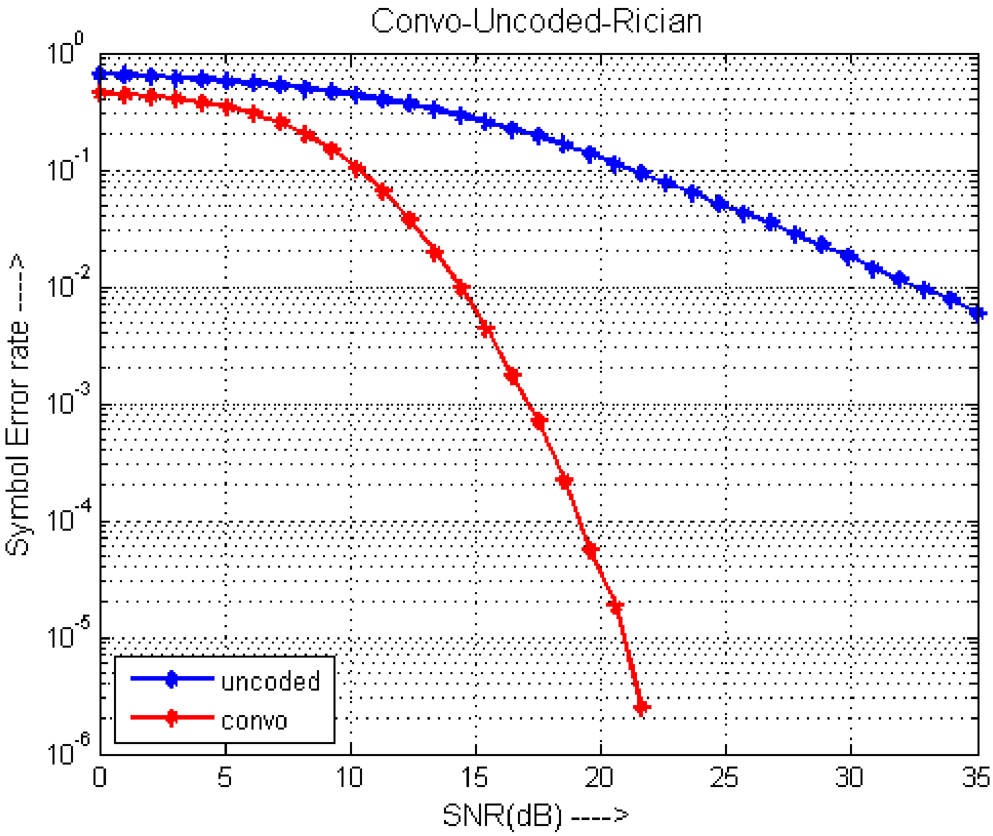


Figure 5.8: SER comparison of Convolution coded system with spatial multiplexed non-coded system for Rician channel

Figure 5.8 shows using Convolution encoding without precoder for Rician channel, symbol error performance of $10^{-1.9}$ is obtained at SNR of 14 dB whereas spatial multiplexed non-coded system at same symbol error performance is obtained at 28 dB; hence 14 dB performance improvement is obtained.

5.4.5 SER comparison of Precoded system model with LDC system in Rayleigh fading channel model

Precoding matrix can be obtained if CSIT is available. For CSIT, the channel should not be moving and remain constant for specific period of time. In case of LDC, for time interval T, channel is assumed to be constant. For time interval T, transmission of each row of codeword R takes place over the same channel in LDC design.

Symbol error rate comparison of Precoded LDC 4x4 MIMO with and without use of convolution encoder at transmitter for Rayleigh channel has been done. Simulation parameters are given in Table 5.1**.**

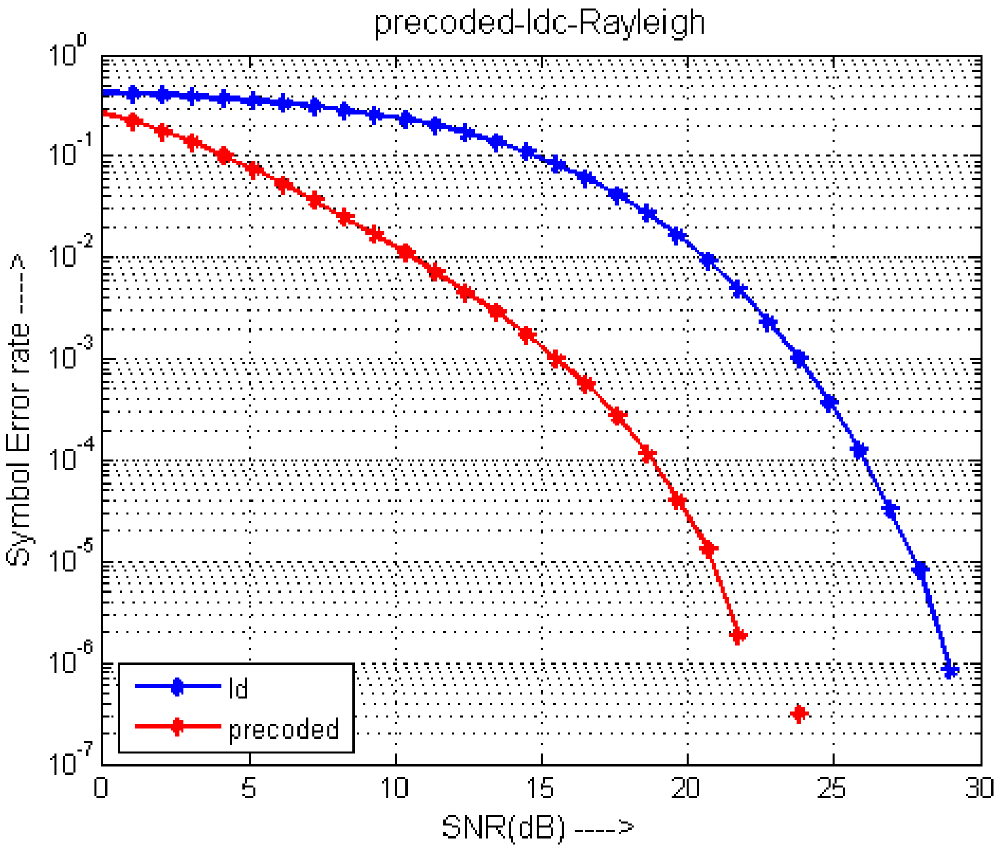


Figure 5.9: SER comparison of Precoded LDC 4x4 MIMO with LDC system for Rayleigh channel

Figure 5.9 depicts using Precoding in Rayleigh environment, symbol error performance of 10^{-3} is obtained at SNR of 16 dB whereas without precoding, for convolution coded 4x4 MIMO system at same symbol error performance is obtained at 24 dB; hence 8 dB performance improvement is obtained.

5.4.6 SER comparison of Convolution coded system model with LDC system in Rayleigh fading channel model

Convolution encoding of LDC coded 4x4 MIMO for Rayleigh channel has been done. Convolutional code of rate ½ having constraint length K_c=7 with generator $(171,133)_8$ has been used. Bit interleaving has been used along with convolution encoding to increase the hamming distance between the codewords. A pseudo-random interleaver has been used in the system. Comparison of Convolution encoded LDC system and without encoder LDC system for Rayleigh fading environment has been done by Matlab Simulations in Figure5.10. Simulation parameters are given in Table 5.1.

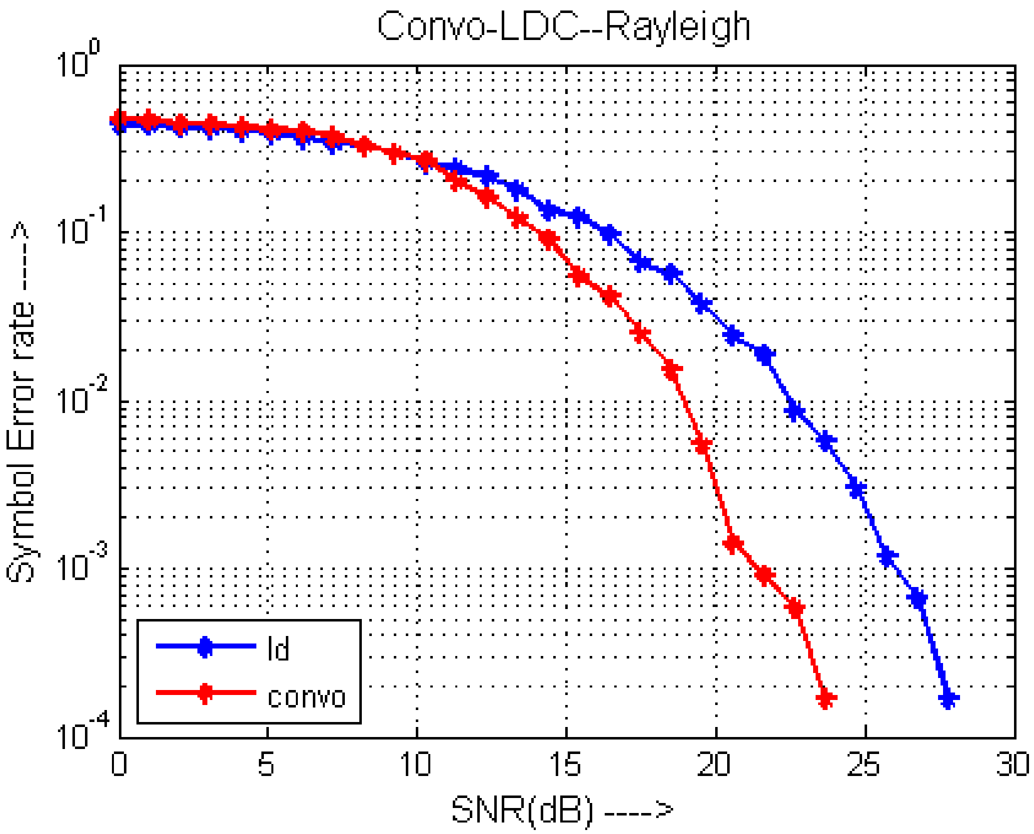


Figure 5.10: SER comparison of Convolution encoded LDC system and without encoder LDC system for Rayleigh channel.

Performance of the convolution coded system starts getting better than LDC system without encoder at higher SNR i.e. above 12 dB. Figure 5.10 shows using convolution encoding symbol error performance of 10^{-3} is obtained at SNR of 22 dB whereas without channel coding at same symbol error performance is obtained at 26 dB, hence 4 dB performance improvement is obtained.

5.4.7 SER comparison of Precoded system model with LDC system in Rician fading channel model

Precoding matrix can be obtained if CSIT is available. For CSIT, the channel should not be moving and remain constant for specific period of time. In case of LDC, for time interval T, channel is assumed to be constant. For time interval T, transmission of each row of codeword R takes place over the same channel in LDC design.

Symbol error rate comparison of Precoded LDC 4x4 MIMO with and without use of convolution encoder at transmitter for Rician channel has been done in Figure5.11. Simulation parameters are given in Table 5.1**.**

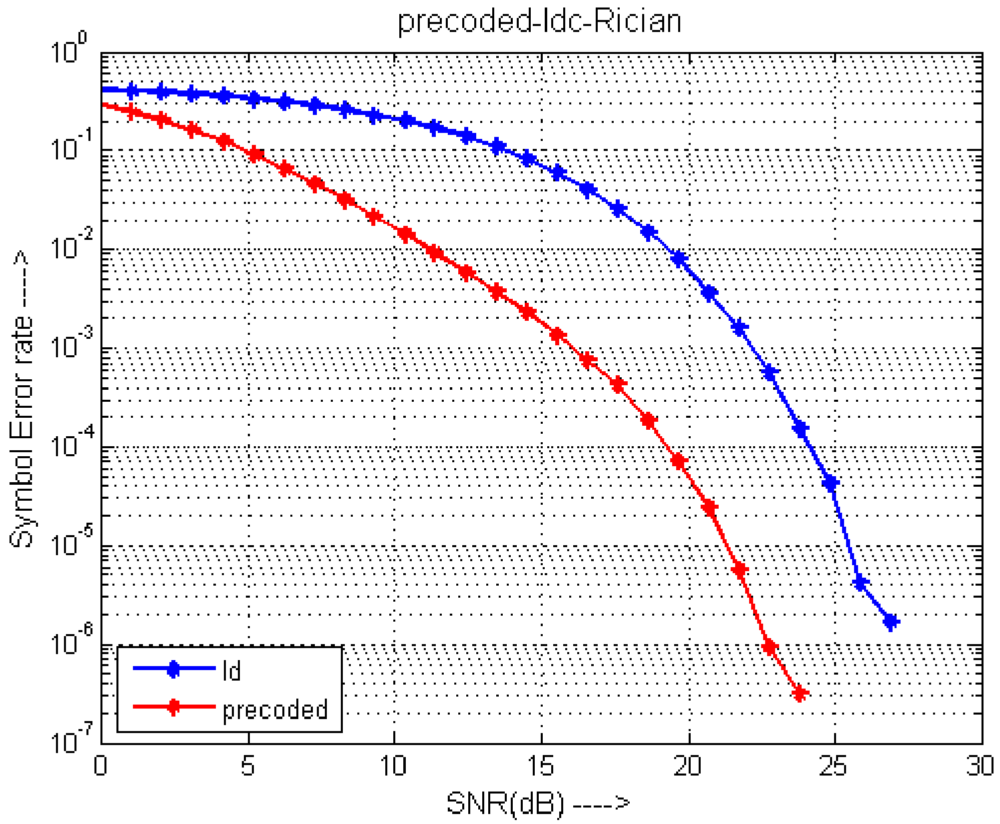


Figure 5.11: SER comparison of Precoded LDC 4x4 MIMO with LDC system for Rician channel

Figure5.11 depicts using Precoding in Rician environment, symbol error performance of $10^{-2.9}$ is obtained at SNR of 13 dB whereas without precoding, convolution coded 4x4 MIMO system at same symbol error performance is obtained at 21 dB; hence 8 dB performance improvement is obtained.

5.4.8 SER comparison of Convolution coded system model with LDC system in Rician fading channel model

Convolution encoding of LDC coded 4x4 MIMO for Rician channel has been done. Convolutional code of rate ½ having constraint length K_c=7 with generator $(133,171)_8$ has been used. Bit interleaving has been used along with convolution encoding to increase the hamming distance between the codewords. A pseudo-random interleaver has been used in the system. Comparison of Convolution encoded LDC system and without encoder LDC system for Rician fading environment has been done in Figure 5.12.

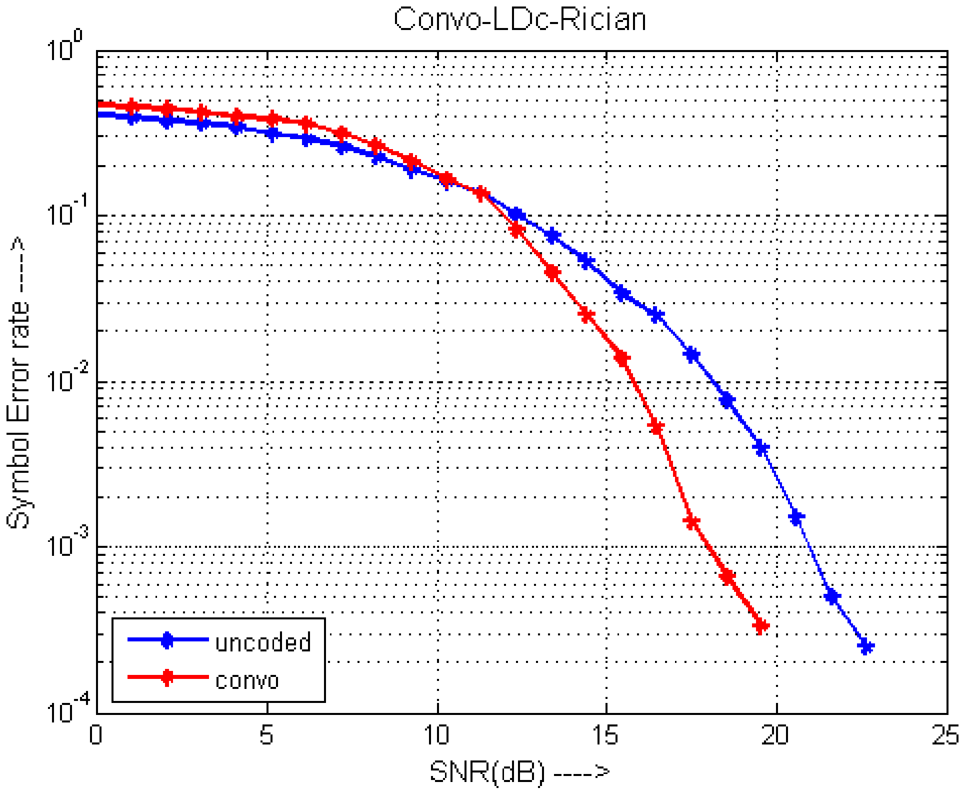


Figure 5.12: SER comparison of Convolution encoded LDC system and without encoder LDC system for Rician channel.

Performance of the convolution coded system starts getting better than LDC system without encoder at higher SNR i.e. above 12 dB. Figure 5.12 shows using convolution encoding symbol error performance of $10^{-2.9}$ is obtained at SNR of 17 dB whereas without channel coding at same symbol error performance is obtained at 21 dB, hence 4 dB performance improvements is obtained.

The comparison of the convolution coded systems with and without precoding for 4x4 MIMO configuration has been made with the spatially multiplexed systems and LDC over the Rayleigh and Rician fading environment presented in the Table 5.2.

Table 5.2: Comparison table for different techniques used in Rayleigh and Rician Fading Channel

Technique used for system 1	Technique used for system 2	Fading	SER of system 1 & 2	SNR of system 1	SNR of system 2	Gain achieved
Convolution coded system with precoding	Spatially Multiplexed system	Rayleigh	$10^{-2.6}$	14dB	30dB	16dB
Convolution coded system without precoding	Spatially Multiplexed system	Rayleigh	$10^{-2.6}$	18dB	30dB	12dB
Convolution coded system with precoding	Spatially Multiplexed system	Rician	$10^{-1.9}$	10 dB	28dB	18dB
Convolution coded system without precoding	Spatially Multiplexed system	Rician	$10^{-1.9}$	14 dB	28dB	14dB
Convolution coded system with precoding	LDC System	Rayleigh	10^{-3}	16 dB	24dB	8dB
Convolution coded system without precoding	LDC System	Rayleigh	10^{-3}	22 dB	26dB	4dB

Convolution coded system with precoding	LDC System	Rician	$10^{-2.9}$	13 dB	21dB	8dB
Convolution coded system without precoding	LDC System	Rician	$10^{-2.9}$	17 dB	21dB	4dB

From Table 5.2, it has been observed that the convolution coded systems with precoding show substantial coding gain in comparison to spatially multiplexed systems in both the fading conditions. By using precoding, convolution coded systems show an improvement of 8dB when compared with LDC systems.

CHAPTER 6
CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

MIMO systems provide high quality services with high data rate and good spectral efficiency. The multipath fading deteriorates signal strength resulting in decrease of throughput. But MIMO systems turn multipath fading into a boon for communication. Spatial multiplexed systems provide high data rate, improvement in SER and handles interference management.

By deploying 4x4 MIMO system in 4G wireless world, there is a reduction of SER and increase in the capacity of the MIMO systems. LDC make trade-off between reduction in SER and increment in capacity, thereby providing high data rate. In this research work, SER performance of 4x4 MIMO system using LDC has been carried out. Here channel encoding has been used in conjunction with LDC coded 4x4 MIMO to increase the system reliability. Performance bound of LDC has been used to derive probability of error for convolution coded system. At high SNR's, the bound for probability of error matches well with the simulated results for 4x4 MIMO systems. Hence, from the proposed technique it can be concluded that convolution coded LDC system performs better than without convolution code.

Precoding has been used to lessen the effect of channel fading. With availability of CSIT, fading effect is greatly reduced. In this dissertation, Singular value decomposition (SVD) algorithm has been used for precoding. Performance of precoded systems is better than systems making use of equalization technique without precoding. SER performances of convolution coded 4x4 MIMO systems using LDC with and without precoder for both fading models have been presented. For Precoded systems in Rayleigh environment, SER performance of 10^{-3} is obtained at SNR of 16 dB whereas without precoding, for convolution coded 4x4 MIMO system at same symbol error performance is obtained at 24 dB; hence 8 dB performance improvement is obtained. For Precoded systems in Rician environment, symbol error performance of $10^{-2.9}$ is obtained at SNR of 13 dB whereas without precoding, convolution coded 4x4 MIMO system at same symbol error performance is obtained at 21 dB; hence 8 dB performance improvement is obtained. SER comparison of precoded systems with spatially multiplexed systems has also been done. Simulations result depicts that the performance of precoded system is better than spatially multiplexed non-coded systems. There has been a gain of 16 dB and 18 dB observed in Rayleigh and Rician fading models respectively.

6.2 Future Scope

Space-time (ST) codes with large antenna array suffer decoding complexity in the implementation of high data rate MIMO systems. Decoding can be done by simplified techniques such as sphere decoding offering less complexity.

The Linear Dispersive codes technique for higher number of antennas can be combined with OFDM to enhance the performance in rich scattered frequency selective environment.

In the precoding technique, it is presumed that channel correlation properties make use of precoding vectors for decoupling at the transmitter and receiver end. P-LDC structure could be extended to channels whose transmit and receive correlations are not decoupled.

The proposed scheme can be done for Nakagami-m fading model in future.

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