System of Roche Coordinates of Rotationally and Tidally Distorted Stars in Presence of Coriolis Force

Thesis Submitted in Partial fulfillment of the requirements for

The award of the degree of

Master of Science

In

Mathematics and Computing

Submitted by

Navjot Kaur

(Roll. No. 301203008)

Under the supervision of

Dr. A.K. Lal

August 2014

School of Mathematics and Computer Applications

Thapar University, Patiala

Punjab,India

CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled " **System of Roche Coordinates of Rotationally and Tidally Distorted Stars in Presence of Coriolis Force** *" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. A. K. Lal.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

Navjot Kaur

(Navjot Kaur)
Reg. No. 301203008

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

(Dr. A. K. Lal)
Associate Professor,
School of Mathematics and Computer Applications,
Thapar University,Patiala.
Punjab, India.

Countersigned by:

Dr.Rajesh Kumar
(Associate Professor & Head)
School of Mathematics & Computer Applications
Thapar University, Patiala.

Dr. S.K.Mohapatra
Dean of Academic Affairs
Thapar University
Patiala.

Acknowledgements

The key elements concentration,dedications,hard work and applications are not the essential factors for achieving the desired goals but also guidelines, assistance and cooperation of people is necessary.

I would like to express my deep and sincere gratitude to my supervisor **Dr. A. K. Lal** *Associate professor,School of Mathematics and Computer Applications (SMCA), Thapar University, Patiala. His wide knowledge and logical way of thinking have been of great value for me. His understanding and personal guidance have provided a good basis for the present thesis. I greatly appreciate his support and guidance not only in research but also in my day to day life.*

I owe my sincere thanks to **Dr. Rajesh Kumar**, *Associate professor and Head of School of Mathematics and Computer Applications (SMCA), Thapar university, Patiala for his encouragement and providing necessary facilities needed for this work.*

I was very fortunate to have unconditional support from my family. I thank my parents,who gave me the courage to get my education, supported me in all achievements throughout my life. Without their encouragement and suggestion,this work would indeed have been very difficult for me to tackle. Last but not least, thanks to god for giving me inner peace and strength. May your name be exalted, honoured and glorified.

18-07-2014 Navjot Kaur

Dated **Navjot Kaur**

Contents

Abstract

Besides the gravitational and centrifugal forces, coriolis force also plays an important role while studying the equilibrium structures and the periods of oscillations of rotating stars and stars in binary system. Kopal (6) introduced a new system of co-ordinates called Roche-coordinates and imphasized how this system of co-ordinates could be used to study the problems of vibrations of rotationally and tidally distorted stellar models. Some of his co-workers (Kitamura(33), Ali(34)) also investigated some mathematical properties of the system of Roche Coordinates. Since then several authors such as Mohan and Singh (36), Singh(35), Singh and Gupta(37), Kumar(38) etc. have extensively discussed the system of Roche coordinates of equipotential surfaces and applied these methodologies to analyse the vibrations of certain types of distorted stars. The expressions for Roche-coordinates of distorted stellar models developed by them only incorporate the effect of gravitational and centrifugal forces. However, the contribution of coriolis force on the Roche coordinates has not been taken into account while formulating Roche equipotential surface.

Pathania (11) modified the expression for Roche equipotentials of rotationally and tidally distorted stellar models by taking into account of coriolis force. He has studied the equilibrium structure and periods of oscillations of rotationally and tidally distorted stars under the influence of gravitational, centrifugal and coriolis forces. But, he has not discussed at all about its Roche coordinates and its related properties .

In the present thesis an attempt has been made to develop expressions for the Roche-coordinates and related parameters for Roche equipotential surfaces of stars distorted by rotation and tidal forces earlier discussed by Pathania (11).

Chapter wise summary of the work presented in the subsequent chapters of this thesis is as follows:

First chapter is introductory in nautre. A brief literature available on this topic is discussed

in this chapter. Introduction of general curvilinear coordinate system as well as Roche coordintes have also been briefly discussed in this chapter.

In chapter 2, we have obtained the explicit expressions of Roche-coordinates for Roche equipotential surfaces of rotationally and tidally distorted stellar models in which effect of coriolis force has been taken into account in addition to gravitational and centrifugal force. We have also discussed in this chapter how the Roche coordinates of earlier authors can be obtained as a special case who have not considered the effect of force.

Chapter 3 deals with the formulation of metric coefficients h_1, h_2 *and* h_3 *which are associated with the Roche coordinates* ξ, η *and* ζ *developed in chapter 2. Certain conclusions based on the present study have finally been drawn in this chapter.*

Chapter 1

INTRODUCTION

This chapter is introductory in nature. In section 1.1, a brief survey of literature on the subject is presented. Basics of general curvilinear co-ordinates system is presented in section 1.2. Section 1.3 deals with the concept of Roche equipotentials which plays a vital role in the present study of Roche coordinates. In section 1.4, the system of Roche coordinates is introduced. A brief summary of work presented in succeeding chapters is finally presented in section 1.5.

1.1 BRIEF SURVEY OF THE LITERATURE

The star, supposed to be gaseous sphere in hydrostatic and thermal equilibrium is radiating huge amount of energy generated by thermonuclear reactions. Most of the theoretical studies about the equilibrium structures of stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. (Extensive literature is now available on this subject Chandrasekhar (2), Rosseland (17), Schwarzschild (20), Eddington (3), Mentzel et al.(12) cox and Guili (24), Kippenhahn and Weigert (25), Clement (26), kopal (6), Abhyankar (1)).

Most of the stars are observed to be rotating about their own axis. This rotation can be a solid body uniform rotation as well as differential rotation in which different parts of the stars are rotating with different angular velocities. Equilibrium structure of rotating stars get distorted by tidal effects alone if the star is not rotating and by the combined effects of tidal and rotational effects if it is a rotating star and stars in binary system. Kippenhahn and Thomas (27) suggested a practical way of analysing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Mohan and Sexena (15,16) use the Kippenhahn and Thomas (27) averaging technique in conjuction with Kopal's results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures and oscillations of the polytropic model of stars. This approach was presented in Sexena (13) and further used by Mohan and

Aggarwal(23) to study the effects of rotation and tidal distortions on the structure and periods of small adiabatic oscillations of prasad and composite model of stars. The technique was formalized by Mohan et al.(41) and used to study the problems of equilibrium structures and oscillations of rotationally and tidally distorted main sequence stars. Lal (28) studied in detail the equilibrium structures and periods of oscillations of differentially rotating stellar models. Later on Singh and Sharma (29) also studied the oscillations of differentially rotating stars in binary system. Lal et al.(10) applied this technique to study the equilibrium structures of differentially rotating and tidally distorted white dwarf model of stars. Equilibrium structures of these type of white dwarf stars which is assumed to be primary components of binary system, are being influenced by the combined effects of differential rotation as well as the gravitational effects of the companion star causing tidal distortions.

Certain comments have generally been made that the approaches which are based on Roche equipotential for computing the equilibrium structure of stars in binary system, developed in fixed frame of reference and hence do not account for coriolis force which is expected to arise in such cases when rotating frame of reference are used. Jain and Sharma (30) have studied the effect of rotation and tidal distortions on the equilibrium structures of gaseous spheres in the presence of the coriolis force. Such a study has practical relevance in astrophysics where it is expected to help in better understanding the problems of stellar stability and stellar variability of rotating stars as well as stars in binary and multiple system. Later, Pathania (31), studied the effect of coriolis force besides the centrifugal and gravitational forces. He developed expression for the Roche equipotential of rotating stars in a binary system in a rotating frame of reference to explicitly include the effect of coriolis force besides the centrifugal and gravitational forces. The methodology developed is next used to determine the effects of coriolis force on the shapes of Roche equipotential surfaces and position of Roche limit for different types of binary stars. Pathania et al. (11) also studied the effects of coriolis forces on the equilibrium structures of rotationally and tidally distorted polytropic models of stars.

Kopal (6) introduced a system of coordinates, known as Roche coordinates, to study the problem of rotating stars in binary system. Further, Kopal and Ali (34) discussed the intgerability of Roche coordinates. Kopal (6) also emphasized applicability of Roche coordinates and explained how the system of Roche coordinates could be used to study the problems of periods of rotationally and tidally distorted stars. Later, Mohan and Singh (36) used Roche coordinate system in the problems of small oscillations of rotationally and tidally distorted stellar models separately. Recently, Kumar (38) discussed the Roche coordinates and Roche harmonics for stars possessing uniform and differential rotation as well as rotation influenced by tidal forces. While analysing the properties of Roche equipotentials, Mohan et. al(41) utilized the results of Kopal (6) on Roche coordinates and then developed series expansion for some of the coordinates instead of closed form mathematical expansion. Later, Lal et al.(39) investigated the validity of series expansion for certain parameters used in the system of Roche coordinates.

However, these authors discussed the properties of Roche coordinates, Roche harmonics and their use by assuming Roche equipotential surfaces which is influenced by the gravitational and centrifugal forces only. The Roche coordinates of the distorted stars in the presence of coriolis force, in addition to the gravitational and centrifugal forces, has not been satisfactory tackled in the literature. In the present thesis an attempt has been made to obtain explicit expressions for Roche coordinates of Roche equipotential surfaces experiencing gravitational, centrifugal and coriolis forces.

1.2 BASICS OF GENERAL CURVILINEAR COORDINATE SYSTEM

Many operators have particularly simple forms in cartesian coordinate system and are easy to remember. One can easily evaluate by using cartesian coordinate system. However, when problems deal with highly symmetric systems it is often helpful to use coordinate systems which exploit the symmetry. In such a case, it is useful to have a general method for expressing operators in non-cartesian forms. Curvilinear coordinate system is such a coordinate system for euclidean space in which the coordinate lines may be curved. The curvilinear coordinates firstly introduced by the French mathematician Lamé.

This was developed from the fact that the coordinate surfaces of the curvilinear systems are curved. These coordinates may be derived from a set of cartesian coordinates by using a transformation, that is, locally invertible (a one-to-one map) at each point. In fact, one can convert a point given in a cartesian coordinate system to its curvilinear coordinates and back. For example, let (x,y,z) and (q_1,q_2,q_3) be cartesian and curvilinear coordinates respectively. Then the relation between these are given by:

$$x = x(q_1,q_2,q_3); \quad y = y(q_1,q_2,q_3); \quad z = z(q_1,q_2,q_3) \tag{1.1}$$

(q_1,q_2,q_3) can be obtained by using of following inverse transformation as:

$$q_1 = q_1(x,y,z) \quad q_2 = q_2(x,y,z) \quad q_3 = q_3(x,y,z) \tag{1.2}$$

Taking the differentials of equation(1.2.1), we get,

$$dx = \frac{\partial x}{\partial q_1}dq_1 + \frac{\partial x}{\partial q_2}dq_2 + \frac{\partial x}{\partial q_3}dq_3 \tag{1.3}$$

Now, it is useful to know about the measure of distance or metric in a given coordinate system. In cartesian coordinate system, the distance between two points whose coordinates differ by dx, dy, dz is ds. ds is also known as arc length which is given by:

$$ds^2 = (dx)^2 + (dy)^2 + (dz)^2 \tag{1.4}$$

In curvilinear coordinates (shown in fig.1), if we change all three coordinates q_i by infinitesimal quantity dq_i (i=1,2,3), then from eqn(1.2.3) we get,

$$dx = h_1e_1dq_1 + h_2e_2dq_2 + h_3e_3dq_3 \tag{1.5}$$

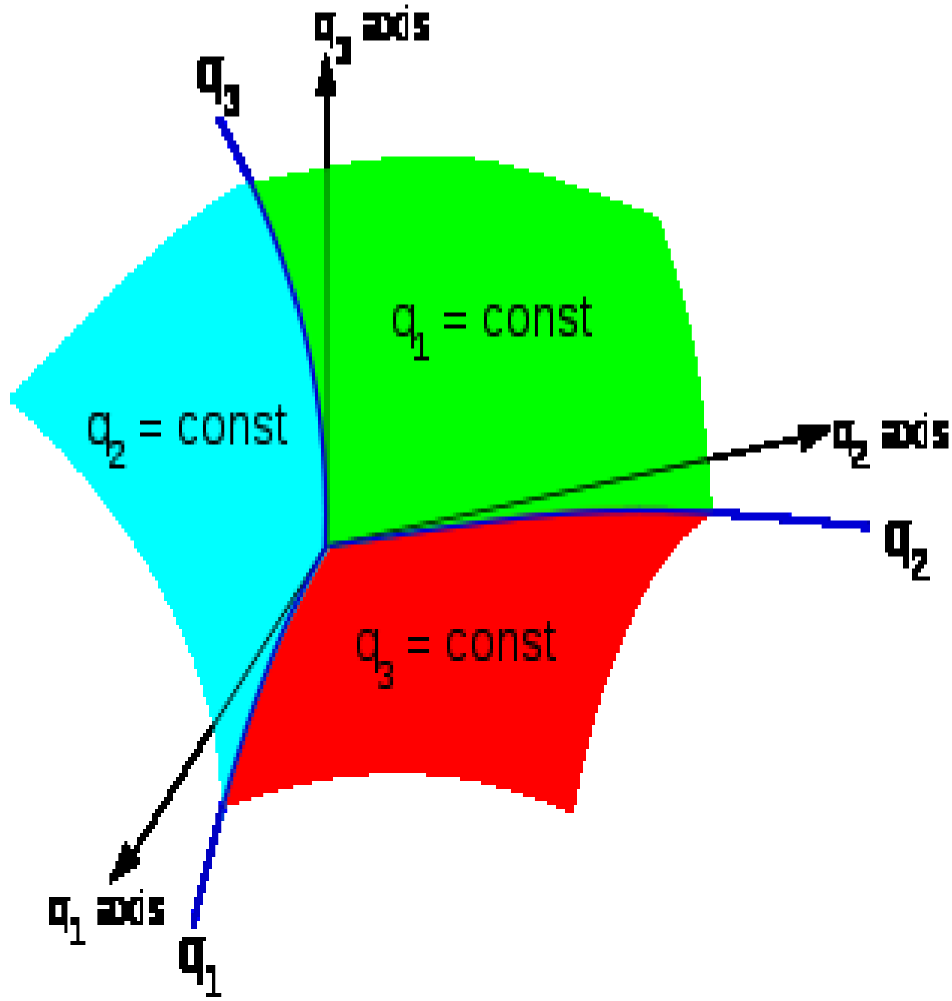

Fig.1.1: Curvilinear coordinate system

In the case of orthogonal curvilinear coordinates, suppose any arbitrary point P has position $r = r(q_1, q_2, q_3)$. If q_1 is increased by a small amount dq_1 then r moves consequently to position $(r + dr)$. The explicit expression for r is given by:

$$dr = \frac{\partial r}{\partial q_1} dq_1 \equiv h_1 e_1 dq_1 \tag{1.6}$$

where the unit vectors e_i and scale vectors h_i (i=1,2,3) are defined as:

$$\boxed{h_i = |\frac{\partial r}{\partial q_i}| \text{ and } e_i = \frac{1}{h_i}\frac{\partial r}{\partial q_i}}$$

The vector e_i is always directed in the increasing direction of q_i and the scale factor h_i gives the magnitude of dr when q_i is increased by small amount dq_i. Thus for small change in q_i, we have

$$|dr| = h_1 dq_1$$

The Values of h_1, h_2, h_3 for for some standard coordinate system are shown in the following table:

Coordinate system	h_1	h_2	h_3
Cartesian	1	1	1
Cylindrical polar	1	r	1
Spherical polar	1	r	$r sin\theta$

Fig 1.2: Values of scale factors (h_i; i=1,2,3) for various coordinate systems

1.3 THE CONCEPT OF ROCHE EQUIPOTENTIAL

In order to introduce the concept of Roche-equipotentials, we assume two components of close binary system which are known as primary and secondry components, in which primary component is supposed to be more massive than the secondry which acts as a point mass causing the tidal effects on the more massive primary component. Both the components of binary system are assumed to be rotating about their axis as well as common center of gravity. Kopal(6) and Mohan and Singh(21), Mohan, Lal and Singh(21) etc. have extensively studied the Roche equipotentials and their use in distorted stars. Certain results on Roche-equipotentials which are of practical interest to the present study are summarized below.

As shown in fig.(1.2), suppose M_0 and M_1 are the masses of two components of a closed binary system separted by a distane D. The primary component of this system of mass M_0 is supposed to be much larger then it's companion star of mass M_1 $(M_0 > M_1)$. Suppose that the position of the two components is reffered to a rectangular system of cartesian-coordinates with the origin at the center of gravity of mass M_0, the axis along the line joining the center of masses of two components and z-axis perpendicular o the plane of the orbit of the two components(see Fig(1.2). Then the total potential ψ of the gravitational and disturbing force acting at an arbitrary point P(x,y,z) is given by:

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_1} + \frac{\omega^2}{2}\{(x - \frac{M_1 D}{M_0 + M_1})^2 + y^2\} \tag{1.7}$$

where $r^2 = x^2 + y^2 + z^2$ and ${r_1}^2 = (D-x)^2 + y^2 + z^2$ represents the squares of distance of P from the center of gravity of the two components, ω denotes the angular velocity of rotation of the system about an axis perpendicular to the xy plane and passing through the center of gravity of the system and G is the constant of gravitation.

The first term on the right hand side of above equation represents the potential arising due to the mass of primary component of mass M_0, the second represents the disturbing potential of secondry component of mass M_1 and the third term denotes the potential arising due to centrifugal force. In non dimensional form above equation can be represented as:

$$\psi^* = \frac{1}{r^*} + q\{\sqrt{1 - 2\lambda r^* + r^{*2}} - \lambda r^*\} + nr^{*2}(1 - \nu^2) \tag{1.8}$$

where $\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$ is the non-dimensional form of total potential ψ and $r^* = \frac{r}{D}$ is the non-dimensional form of r, $\lambda = r\cos\theta\sin\phi$, $\mu = r\sin\theta\sin\phi$ and $\nu = r\cos\theta$, ((r,θ,ϕ) being the spherical coordinates at the point P). Also $q = \frac{M_1}{M_0}$ is the non-dimensional parameter representing the ratio of two masses of the secondry over primary components 2n represents the square of normalized angular velocity ω.

The surface generated by setting ψ = constant on the left hand side of equation(1.3.2), are referred to as Roche-equipotential surfaces. The form of the Roche-equipotential surface depends entirely on the value ψ. If ψ is large then corresponding equipotentials surfaces will consist of two separate ovals close around each of the two mass points. The right hand side of equation(1.3.2) can be large only if r or r_1 becomes small. Therefore, large value of ψ corresponds to equipotentials which differ, but little from spheres. With decreasing value of ψ the ovals defined by equation(1.3.2) become increasingly elongated in the direction of center of gravity of the system until for a certain critical value of ψ both ovals will unite in a single point on the x-axis to form a dumbbell like configuration. These limiting value of ψ are Roche-limits. For certain mass ratio Kopal (6) computed the numerical values of Roche-limits in the case of synchronous binary stars for a number of q ranging from zero to one(cf. table, pp. , Kopal(6)).

1.4 THE SYSTEM OF ROCHE-COORDINATES

We introduce a new system of curvilinear coordinates known as Roche Coordinates given by kopal(6) and some of his co-workers (kitamura(33), Ali(34)). In this system, spheres of constant radius in spherical polars are replaced by the surfaces of constant potential of a rotating gravitational dipole while the angular coordinates remains orthogonal to the equipotentials. Kopal(6) also studied some of the mathematical properties of the system of Roche coordinates and indicated that how this system of coordinates could be used to study the problems of vibrations of rotationally and tidally distorted steller models.

In case of actual stars, greater part of their mass is concentrated very near to the center.

Therefore, their structure comes much closer to the Roche model (by the Roche model we mean a model in which whole mass of the star is suuposed to be concentrated at the center and this point mass is surrounded by an evanescent envelop in which density is assumed to vary inversely as some positive power of the distance from the center). On the basis of some extensive numerical investigations, Chandrasekhar(2) has shown that for stars whose central density bears to the mean density a ratio of 100 or more, the rotating Roche model of a rotating configuration represents the actual form of the Roche equipotential surfaces of a rotating star within an error of less than one percent.

In the system of Roche coordinates the equipotential surface of a distorted Roche model are chosen to represent the equipotential surfaces of an actual stellar model distorted by rotational and tidal forces. Choosing the equipotential as one coordinate, the other two are chosen in the form of the triple orthogonal system. The coordinates (ξ, η, ζ) represents the three equipotential surfaces in the system of Roche coordinates. We take the ξ-coordinate to be an equipotential surface of the form:

$$\xi = \frac{1}{r} + q[\frac{1}{\sqrt{1-2\lambda r + r^2}} - \lambda r] + nr^2(1-\nu^2) = constant$$

and choose the other two coordinates η and ζ in such a way as to satisfy the conditions of mutual orthogonality with respect to ξ as well to as to each other.

Kopal(6) has shown that second and third coordinates are given by:

$$\eta = cos^{-1}\lambda - \frac{q}{\sqrt{1-\lambda^2}} \sum_{j=2}^{4} \frac{r^{j+1}}{j+1} P_j^{'}\lambda \tag{1.9}$$

and

$$\zeta = cos^{-1} \frac{\nu}{\sqrt{1-\lambda^2}} \tag{1.10}$$

respectively. In these relations,a prime denotes differentiation with respect to λ.

The research work of kopal and his co-workers shows that it is not possible, in general, to derive the expressions for η and ζ in closed analytic form.

Kopal(6) investigated the particular cases of this problem in detail. In the first q is taken to be zero, and in second case, ω^2 is taken to be zero.The first corresponds to the Roche-coordinates of a star distorted by rotational forces alone and the second corresponds to the Roche-coordinates of a non-rotating star distorted by the tidal effects of a companion star.

Mohan and singh (22) used the system of Roche-coordinates to obtain the explicit forms of equations of small radial oscillations of rotationally distorted and tidally distorted stars assuming Roche-model for the stars and used these to numerically compute certain eigen-frequencies of oscillations of such distorted models. Their works shows that the system of Roche-coordinates can be used with advantage to study the problem of small oscillations of rotating stars as well as tidally distorted stars. The main advantage of the technique of studying small oscillations of rotating stellar models through the use of Roche coordinates is that we are able to account for the effects of distortion caused by rotation and tidal effects automatically while studying the problems of small oscillations of these models in usual way. One limitation of the present technique, however, is that it must be applied with care when studying the vibrations of stellar models which have unusually large angular velocities of rotation. We do not get the expression for the third Roche-coordinates in closed analytical form and, instead, have to express it as an infinite series in ascending power of the angular velocity of rotation.

However, it was observed that use of this approach for determining the combined effect of the rotation and tidal distortions on the equilibrium structure and periods of small oscillations of binary stars, in which the rotational and tidal effects have to be considered jointly, is not convenient. Moreover, the method could not be conveniently used when more realistic models in place of Roche-model are to be used for the inner structure of star.

1.5 THE PRESENT WORK

Besides the gravitational and centrifugal forces, coriolis force also plays an important role while studying the equilibrium structures and the periods of oscillations of rotating stars and stars in binary system. Kopal (6) introduced a new system of co-ordinates called Roche-coordinates and imphasized how this system of co-ordinates could be used to study the problems of vibrations of rotationally and tidally distorted stellar models. Some of his co-workers (Kitamura(33), Ali(34)) also investigated some mathematical properties of the system of Roche Coordinates. Since then several authors such as Mohan and Singh (36), Singh(35), Singh and Gupta(37), Kumar(38) etc. have extensively discussed the system of Roche coordinates of equipotential surfaces and applied these methodologies to analyse the vibrations of certain types of distorted stars. The expressions for Roche-coordinates of distorted stellar models developed by them only incorporate the effect of gravitational and centrifugal forces. However, the contribution of Coriolis force on the Roche coordinates has not been taken into account while formulating Roche equipotential surface.

Pathania (11) modified the expression for Roche equipotentials of rotationally and tidally distorted stellar models by taking into account of coriolis force. He has studied the equilibrium structure and periods of oscillations of rotationally and tidally distorted stars under the influence of gravitational, centrifugal and coriolis forces. But, he has not discussed at all about its Roche coordinates and its related properties .

In the present thesis an attempt has been made to develop expressions for the Roche-coordinates and related parameters for Roche equipotential surfaces of stars distorted by rotation and tidal forces earlier discussed by Pathania (11).

Chapter wise summary of the work presented in the subsequent chapters of this thesis is as follows:

First chapter is introductory in nautre. A brief literature available on this topic is discussed in this chapter. Introduction of general curvilinear coordinate system as well as Roche coordintes have also been briefly discussed in this chapter.

In chapter 2 , we have obtained the explicit expressions of Roche-co-ordinates for Roche equipotential surfaces of rotationally and tidally distorted stellar models in which effect of coriolis force has been taken into account in addition to gravitational and centrifugal force. We have also discussed in this chapter how the Roche coordinates of earlier authors can be obtained as a special case who have not considered the effect of force.

Chapter 3 deals with the formulation of metric coefficients h_1, h_2 and h_3 which are associated with the Roche coordinates ξ, η and ζ developed in chapter 2. Certain conclusions based on the present study have finally been drawn in this chapter.

Chapter 2

ROCHE-COORDINATES FOR ROCHE EQUIPOTENTIAL SURFACES OF RO-TATIONALLY AND TIDALLY DISTO-RTED STELLAR MODELS IN PRESE-NCE OF CORIOLIS FORCE

The approaches based on Roche-equipotentials for computing the equilibrium structures of stars in binary systems carry out in the studies in fixed frame of reference and do not account for coriolis force which is expected to arise in such cases when rotating frame of reference is used. Pathania(11) studied the effect of coriolis force besides the centrifugal and gravitational forces. He developed the expression for the Roche-equipotential of rotating star in binary system in a rotating frame of reference to explicitly include the effect of coriolis force besides the centrifugal and gravitational forces. In the present chapter, Roche equipotentials of rotatianally and tidally distorted stellar models in the presence coriolis force is discussed in section 2.1. In section 2.2, the explicit expression of Roche coordinates for modified Roche equipotential surfaces distorted by rotational and tidal force is obtained. Certain conclusions based on the present studies made in section 2.3.

2.1 ROCHE EQUIPOTENTILAS OF ROTATING AND TIDALLY DISTORTED STELLAR MODELS IN THE PRESENCE OF CORIOILS FORCE

Kopal (6) introduced the concept of Roche equipotentials to analyze the problem of rotating stars and binary stars. then several authors such as kopal (6), Kopal and song(40) Eggleton(3), Mohan and Singh(22), Mohan and Saxena(16), Mohan et al.(41), Lal et al. (10) have used this concept to analyze the problem of rotationally and/or tidally distorted stars. In this approach Roche approximation for the inner structure of a star is used to obtain an expression for the potential of rotating star and stars in binary system. This approach accounts for the effect of gravitational and centrifugal forces but does not take into the account the effect of coriolis force except in case of synchronous binaries. The coriolis force has been generally neglected as it not only complicates the analysis but it's effect is also expected to be small as compared to the effects of other two forces.

An expression for the Roche-equipotential surface, which incorporates the effect of coriolis force in addition to the centrifugal and gravitational force as obtained by Pathania (11), is summarized below:

According to the classical dynamics for the system described in fig(2.1), the potential at a point P inside the primary component, which experiences the effect of coriolis force besides the gravitational and centrifugal forces, is given by:

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}(\Omega \times r).(\Omega \times r) + V.(\Omega \times r) \tag{2.1}$$

Here V is the velocity of particle of unit mass at point P(x,y,z) w.r.t a rotating frame of reference which is rotating with the uniform angular velocity of Ω.

the first two terms in equation(2.1.1) represents the gravitational potential arises due to the secondry components of binary system and third is potential arises due to centrifugal force. First three terms are same as earlier obtained by Kopal (6) in his studies on the problems of Roche-model and it's applications to close binary system. The fourth term $V.(\Omega \times r)$ represents the contribution of the coriolis force to the potential at point P, where V is the tangential of velocity of this particle in the rotating frame of reference. Points inside the rotating star will be subjected to coriolis force even when such a point is not having any external velocity because of the differences in the velocity of the rotation of the primary and angular velocity of revolution of the non-synchronous binary system.

In the earlier studies carried out on Roche equipotentials by Kopal (6) and Mohan and Saxena (16), the contribution of this last term $V.(\Omega \times r)$, which arises on the account of coriolis force, has been neglected assuming it's effect to be small compared to the other forces in the non-synchronous case. But it was observed a contrary results which studying the effects of rotational and tidal distortion on the equilibrium structures and eigen frequencies of radial and non radial oscillations of rotating stars and stars in binary system by using Mohan and Sexena(15,16) and Mohan et. al. (41) approach. The approach which do not accounts for the effect of coriolis force. It was noticed that the values of eigen frequencies of g-modes of non radial oscillations decrease in presence of rotation using this approach whereas results of other are contrary to it. Keeping in this view Pathania (11) analysed the effects of coriolis force in addition to the gravitational and centrifugal forces, on the equilibrium structure and periods of oscillations of rotating stars and stars in the binary systems using a rotating frame of reference.

Pathania(11) considered the contribution of the last two terms $V.(\Omega \times r)$ and developed a modified expression for potential at a point P inside the star. This expression in cartesian form is given as:

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x-d_1)^2 + y^2] + (\Omega\Omega_1 - \Omega^2)(x^2 + y^2 - xd_1) \qquad (2.2)$$

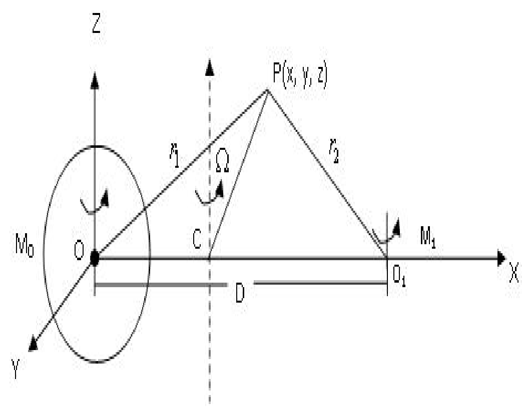

Fig 1.1: Axis of reference for a binary system

Where the symbol M_0, M_1, r_1, r_2, d_1, Ω, and Ω_1 have their usual meanings. The equation(2.1.1) can be expressed in non-dimensional form as:

$$\xi = \frac{1}{r_1^*} + \frac{q}{\sqrt{1 - 2\lambda r_1^* + r_1^{*2}}} + \frac{\Omega^{\bullet 2}}{2}[r_1^{\bullet 2}(1-\nu^2) + d_1^{\bullet 2} - 2\lambda r_1{}^* d_1^{\bullet}] + (\Omega_P^{\bullet 2} - \Omega^{\bullet 2})[r_1^{\bullet 2}(1-\nu^2) - \lambda r_1^* d_1^{\bullet}] \qquad (2.3)$$

where $\xi = \frac{D\psi}{GM_0}$; $\Omega^{\bullet 2} = \frac{\Omega^2 D^3}{GM_0}$; $\Omega_P^{\bullet 2} = \frac{D^3(\Omega\Omega_1)}{GM_0}$; and $d_1^{\bullet} = \frac{M_1}{M_0+M_1}$

In the above expressions $r_1^* = \frac{r_1}{d}$ is a non-dimensional form of r_1, q a non-dimensional parameter representing the ratio of the mass of the secondary component over primary component(assume $q << 1$) and $\lambda = sin\theta cos\phi$, $\mu = sin\theta sin\phi$, $\nu = cos\theta$.

Equation(2.1.3) is the most general equation of the potential at point P (inside primary component of binary system) which incorporates the effect of coriolis force in addition to centrifugal force and gravitational forces. If we assume that the angular velocity Ω is identical with the Keplerian angular velocity (Ω_K) where $\Omega_K^2 = \frac{G(M_0+M_1)}{d^3}$ then in terms of the non-dimensional variables used by us,a relation $\Omega^2 = 2n = (q+1)$ is obtained. Using this relation, equation(1.4.9) becomes more simplified as:

$$\psi^{\bullet} = \frac{1}{r_1^*} + q[\frac{1}{\sqrt{1-2\lambda r_1^* + r_1^{*2}}} - \alpha\lambda r_1^*] + \beta n r_1^{*2}(1-\nu^2) \tag{2.4}$$

where $\alpha = \frac{2n_1}{(1+q)}, \quad \beta = (\frac{2n_1}{n} - 1), \quad \text{and} \quad \Omega_P^{\bullet 2} = 2n_1$

For binary system rotating synchronously, the angular velocity due to rotation and revolution will be same, that is, $\Omega_1 = \Omega$ and the terms containing coriolis force will not appear explicitly. Therefore equation(2.1.4) reduces to:

$$\psi^{\bullet} = \frac{1}{r_1^*} + q[\frac{1}{\sqrt{1-2\lambda r_1^* + r_1^{*2}}} - \lambda r_1^*] + n r_1^{*2}(1-\nu^2) \tag{2.5}$$

where

$$\psi^{\bullet} = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$$

Expression(2.1.5) is same as earlier obtained by Kopal(6).
In pure tidal case there is no coriolis force and therefore for $\psi^{\bullet}$ is identical to it's earlier expression obtained by Kopal (6) and can be derived from (2.1.5) by putting $n = 0$.

In case of pure rotation also, coriolis force is not generated because there is no revolution on center of the star and therefore no rotating frame of reference. In such case the expression for Roche-equipotential for a purely rotating star which is not subjected to tidal effects of the companion star becomes:

$$\psi^{\bullet} = \frac{1}{r_1^{\bullet}} + nr_1^2(1-\nu^2) \tag{2.6}$$

Thus on explicit inclusion of coriolis force expression for Roche-equipotentials gets modified from the earlier one obtained by Kopal (6) only in the case of of non-synchronous binaries. In case of synchronous binaries, purely rotating and purely tidally distorted stars, there is no change.

Following an approach earlier used by Kopal(6) and Mohan et al (41) Pathania (11) described e new non-dimensional variable r_0 given by the relation

$$r_0 = \frac{1}{\psi^{**} - q} \tag{2.7}$$

Following Kopal (6) ,Pathania(11) developed an explicit expression of relation connecting the variables (r,θ,φ) on the surface of modified Roche-equipotentials (2.1.5). this relation gives the shape of Roche equipotential surface.

$$\begin{aligned} r =& r_0[1+\lambda qtr_0^2+a_0r_0^3+(qP_3+2\lambda^2t^2)r_0^4+(qP_4+5a_0\lambda qt)r_0^5+(qP_5+3a_0^5+6\lambda q^2tP_3)r_06+ \\ & (qP_6+7a_0qP_3+7\lambda q^2t^4P_4)r_0^7+(qP_7+8a_0qP_4+8\lambda q^2tP_5+4q^2P_3^2)r_0^8+(qP_8+9a_0qP_5+ \\ & 9\lambda q^2tP_6+9q^2P_3P_4)r_0^9+(qP_9+10a_0qP_6+10\lambda q^2tP_7+5q^2P_4^2+2P_3P_5)....] \end{aligned} \tag{2.8}$$

where $a_0 = qP_2 + \beta n(1-\nu^2)$, $t = 1-\alpha$ and $P_j = P_j(\lambda)$ denote the Legendre polynomial. As discussed earlier, terms upto second order of smallness in n, n_1, and q, and terms upto $r_0^1 0$ in r_0 are retained in (2.1.8). Relation (2.1.8) incorporates the effect of coriolis force and can be further used to obtain the shapes of various roche equipotential surfaces ψ^{**}=constant.

Once the explicit expression for total potential ξ and r which includes the effect of gravitational, centrifugal and coriolis forces are known we are now ready to study the Roche coordinates of such distorted stars.

2.2 EXPLICIT EXPRESSION OF ROCHE COORDINATES FOR MODIFIED ROCHE EQUIPOTENTIAL SURFACES DISTORTED BY ROTATIONAL AND TIDAL FORCE

We introduce the system of coordinates ξ, η and ζ in which ξ coordinate is defined by the Roche equipotential suraface in closed form given by expression(2.1.5) while the coordinates η and ζ are defined by their that they are orthoganal to ξ as well as with respesct to each other.

The conditions that must be satisfied by any system of orthogonal coordinates are same as discussed by Kopal (6) which are:

$$\xi_x\eta_x + \xi_y\eta_y + \xi_z\eta_z = 0 \quad \xi_x\zeta_x + \xi_y\zeta_y + \xi_z\zeta_z = 0 \quad and \quad \eta_x\zeta_x + \eta_y\zeta_y + \eta_z\zeta_z = 0 \tag{2.9}$$

Differentiating partially equation (2.1.5) with respect to r , λ and ν , we get

$$\xi_r = -\frac{1}{r^2} + q[\frac{(\lambda - r)}{(1 - 2\lambda r + r^2)^{3/2}} - \alpha\lambda] + 2\beta nr(1 - \nu^2) \tag{2.10}$$

$$\xi_\lambda = q[\frac{r}{(1 - 2\lambda r + r^2)^{3/2}} - \alpha r] \tag{2.11}$$

$$\xi_\nu = -2\beta nr^2\nu \tag{2.12}$$

By simple transformation of coordinates,it follows that

$$\xi_x = \lambda\xi_r - \frac{(1 - \lambda^2)}{r}\xi_\lambda - \frac{\lambda\nu}{r}\xi_\nu \tag{2.13}$$

$$\xi_y = \mu\xi_r - \frac{\lambda\mu}{r}\xi_\lambda - \frac{\mu\nu}{r}\xi_\nu \tag{2.14}$$

$$\xi_z = \nu\xi_r - \frac{\lambda\nu}{r}\xi_\lambda - \frac{(1 - \nu^2)}{r}\xi_\nu \tag{2.15}$$

Using (2.2.2 - 2.2.4) into (2.2.5 - 2.2.7), we get

$$\xi_x = -\frac{\lambda}{r^2} + \frac{2q\lambda^2 - q\lambda r - q}{(1-2\lambda r + r^2)^{3/2}} - \alpha q(\lambda+1) + 2\beta n r\lambda \tag{2.16}$$

$$\xi_y = -\frac{\mu}{r^2} - \frac{q\mu r}{(1-2\lambda r + r^2)^{3/2}} + 2\beta n r\mu \tag{2.17}$$

$$\xi_z = -\frac{\nu}{r^2} - \frac{q\nu r}{(1-2\lambda r + r^2)^{3/2}} + 4\beta n r(1-\nu^2) \tag{2.18}$$

Hence, we have

$$(\xi_x^2+\xi_y^2+\xi_z^2)^{-1/2} = r^2 1 + \lambda q(2\lambda^2 - \alpha\lambda - \alpha - 1)r^2 - [q\nu - 4\beta n\nu(1-\nu^2) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2)]r^3 - \frac{r^4}{2}[-30\lambda^5 + \lambda^4 q^2 7\lambda^3 q^2 - \frac{5}{2}\lambda^2 q^2 - 3q\lambda + q^2 + 6\lambda q (\mu^2+\nu^2) + 6\lambda^6 q^2] \tag{2.19}$$

Following Kopal (6) the differential equations generating the equipotentials surfaces are of the form :

$$\xi_x d_x + \xi_y d_y + \xi_z d_z = 0 \tag{2.20}$$

While the equations generating the lines that are ortoganal to the equipotentials are given by:

$$\frac{dx}{\xi_x} = \frac{dy}{\xi_y} = \frac{dz}{\xi_z} \tag{2.21}$$

and the Roche coordinates η and ζ corresponding to the angular coordinates of spherical polars will be obtained as the integration constants of above equation(2.2.13). As equation (2.2.13) is not in a proper form to integrate, we divide them by normal elements ds, we get

$$\frac{(dy/ds)}{dx/ds} = \frac{\xi_y}{\xi_x} \qquad and \qquad \frac{(dz/ds)}{(dx/ds)} = \frac{\xi_z}{\xi_x} \tag{2.22}$$

Since by defination the direction cosines of vector normal to the surface ξ=constant, must satisfy the relation

$$(dx/ds)^2 + (dy/ds)^2 + (dz/ds)^2 = 1 \tag{2.23}$$

Thus, equation(2.2.14 - 2.2.15) can be solved to obtain:

$$\frac{dx}{ds} = \frac{-\xi_x}{\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}} \tag{2.24}$$

$$\frac{dy}{ds} = \frac{-\xi_y}{\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}} \tag{2.25}$$

$$\frac{dz}{ds} = \frac{-\xi_z}{\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}} \tag{2.26}$$

since $\xi(x,y,z)$ is a diminishing function of s, so we have taken the negative sign of the square root. Using the spherical polar coordinates,

$$x = rsin\theta cos\phi \quad y = rsin\theta sin\phi \quad and \quad z = rcos\theta \tag{2.27}$$

we can obtain

$$\frac{dx}{dr} = r\frac{d\lambda}{dr} + \lambda \tag{2.28}$$

$$\frac{dy}{dr} = r\frac{d\mu}{dr} + \mu \tag{2.29}$$

$$\frac{dz}{dr} = r\frac{d\nu}{dr} + \nu \tag{2.30}$$

Since ds is very small, so it can be replaced by line element dr with our scheme of first order approximation

$$\begin{aligned}\frac{dx}{dr} =& \lambda + qr^2(2\lambda^2 - \alpha\lambda - \alpha - 1)(\lambda^2 - 1) + r^3\lambda[\lambda^2(14q - 4\beta n) - q(4+\nu) + 2\beta n(1+\nu(1-\nu^2)) \\ &+ \mu^2(q - 2\beta n)] + r^4[15\lambda^6 q - 3\lambda^7 q^2 - \frac{9}{2}\lambda^5 q^2 + \lambda^4 q(15 - \frac{27}{2}q) + \frac{13}{4}\lambda^3 q^2 + q\lambda^2(-12+ \\ &(\mu^2+\nu^2)) - \frac{\lambda}{2}q^2 + \frac{3}{2}q] + r^5[2\lambda^2 q^2(\nu + \mu^2 + 2) - q^2(\nu^2 - \mu^2) - 8\lambda q^2] + r^6[-\lambda^7 \\ &(30q^2+36)\lambda^5 q(\frac{3}{2}q + 24 - 12\beta n) + 48q^2\lambda^4 + 3\lambda^3 q^2(-1 + 4\mu^2 + 2\nu^2 + 2\nu) - 32\lambda^2 q^2 + \\ &\lambda q(\frac{3}{2}q - 7\lambda q\mu^2 - 3q\nu^2 + 2\beta n\mu^2)]\end{aligned} \tag{2.31}$$

$$\frac{dy}{dr} = \mu + \mu\lambda q(2\lambda^2 - \alpha\lambda - \alpha - 1)r^2 + r^3[(q\mu - 2\beta n\mu) - \mu(q\nu - 4\beta n\nu(1-\nu^2)) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2)] + r^4[3\lambda\mu q - \frac{\mu}{2}(-30\lambda^5 q + \lambda^4 q^2 + 27\lambda^3 + 2q^2 - \frac{5}{2}\lambda^2 q^2 - 3q\lambda + 6\lambda q(\mu^2 + \nu^2) + 6\lambda^6 q^2)] + \lambda q[(2\lambda^2 - \alpha\lambda - \alpha - 1)(q\mu - 2\beta n\mu)]r^5 - r^6[(q\mu - 2\beta n\mu) - \mu(q\nu - 4\beta n\nu(1-\nu^2)) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2) + 3q^2\lambda^2\mu(2\lambda^2 - \alpha\lambda - \alpha - 1)] \tag{2.32}$$

$$\frac{dz}{dr} = \nu + \lambda\nu q(2\lambda^2 - \alpha\lambda - \alpha - 1)r^2 + r^3[q\nu - 4\beta n(1-\nu^2) - \nu(q\nu - 4\beta n\nu(1-\nu^2) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2))] - r^4[3q\lambda\nu + \frac{\nu}{2}(-30\lambda^5 q + \lambda^4 q^2 + 27\lambda^3 q^2 - \frac{5}{2}\lambda^4 q^2 - 3q\lambda + q^2 + 6\lambda q(\mu^2 + \nu^2) + 6\lambda^6 q^2)] + r^5\lambda q[q\nu - 4\beta n(1-\nu^2)][2\lambda^2 - \alpha\lambda - \alpha - 1] - r^6[q\nu - 4\beta n(1-\nu^2) + 3q^2\lambda^2\nu(2\lambda^2 - \alpha\lambda - \alpha - 1)] \tag{2.33}$$

Comparing equations(2.2.11 - 2.2.12) and (2.2.14 - 2.2.16),we get

$$r\frac{d\lambda}{dr} + \lambda = \lambda + qr^2(2\lambda^2 - \alpha\lambda - \alpha - 1)(\lambda^2 - 1) + r^3\lambda[\lambda^2(14q - 4\beta n) - q(4+\nu) + 2\beta n(1 + \nu(1-\nu^2)) + \mu^2(q - 2\beta n)] + r^4[15\lambda^6 q - 3\lambda^7 q^2 - \frac{9}{2}\lambda^5 q^2 + \lambda^4 q(15 - \frac{27}{2}q) + \frac{13}{4}\lambda^3 q^2 + q\lambda^2(-12 + (\mu^2 + \nu^2)) - \frac{\lambda}{2}q^2 + \frac{3}{2}q] + r^5[2\lambda^2 q^2(\nu + \mu^2 + 2) - q^2(\nu^2 - \mu^2) - 8\lambda q^2] + r^6[-\lambda^7(30q^2 + 36) \lambda^5 q(\frac{3}{2}q + 24 - 12\beta n) + 48q^2\lambda^4 + 3\lambda^3 q^2(-1 + 4\mu^2 + 2\nu^2 + 2\nu) - 32\lambda^2 q^2 + \lambda q(\frac{3}{2}q - 7\lambda q\mu^2 - 3q\nu^2 + 2\beta n\mu^2)] \tag{2.34}$$

$$r\frac{d\mu}{dr} + \mu = \mu + \mu\lambda q(2\lambda^2 - \alpha\lambda - \alpha - 1)r^2 + r^3[(q\mu - 2\beta n\mu) - \mu(q\nu - 4\beta n\nu(1-\nu^2)) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2)] + r^4[3\lambda\mu q - \frac{\mu}{2}(-30\lambda^5 q + \lambda^4 q^2 + 27\lambda^3 + 2q^2 - \frac{5}{2}\lambda^2 q^2 - 3q\lambda + 6\lambda q(\mu^2 + \nu^2) + 6\lambda^6 q^2)] + \lambda q[(2\lambda^2 - \alpha\lambda - \alpha - 1)(q\mu - 2\beta n\mu)]r^5 - r^6[(q\mu - 2\beta n\mu) - \mu(q\nu - 4\beta n\nu(1-\nu^2)) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2) + 3q^2\lambda^2\mu(2\lambda^2 - \alpha\lambda - \alpha - 1)] \tag{2.35}$$

$$r\frac{d\nu}{dr} + \nu = \nu + \lambda\nu q(2\lambda^2 - \alpha\lambda - \alpha - 1)r^2 + r^3[q\nu - 4\beta n(1-\nu^2) - \nu(q\nu - 4\beta n\nu(1-\nu^2) - \lambda(6\lambda^3 q - 8\lambda q + 4\beta n\lambda) + (q\mu^2 - 2\beta n\mu^2))] - r^4[3q\lambda\nu + \frac{\nu}{2}(-30\lambda^5 q + \lambda^4 q^2 + 27\lambda^3 q^2 - \frac{5}{2}\lambda^4 q^2 - 3q\lambda + q^2 + 6\lambda q(\mu^2 + \nu^2) + 6\lambda^6 q^2)] + r^5\lambda q[q\nu - 4\beta n(1-\nu^2)][2\lambda^2 - \alpha\lambda - \alpha - 1] - r^6[q\nu - 4\beta n(1-\nu^2) + 3q^2\lambda^2\nu(2\lambda^2 - \alpha\lambda - \alpha - 1)] \tag{2.36}$$

On integrating above set of equations(2.2.21 - 2.2.28) by picard's method of succesive approximation up to first order, we get

$$\lambda_1 =\{q\frac{r^2}{2}(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)(\lambda_0^2-1)+\frac{r^3}{3}\lambda_0[\lambda_0^2(14q-4\beta n)-q(4+\nu_0)+2\beta n(1+\nu_0(1-\nu_0^2)) +\mu_0^2(q-2\beta n)]+\frac{r^4}{4}[15\lambda_0^6 q-3\lambda_0^7 q^2-\frac{9}{2}\lambda_0^5 q^2+\lambda_0^4 q(15-\frac{27}{2}q)+\frac{13}{4}\lambda_0^3 q^2+q\lambda_0^2(-12+(\mu_0^2+\nu_0^2)) -\frac{\lambda_0}{2}q^2+\frac{3}{2}q]+\frac{r^5}{5}[2\lambda_0^2 q^2(\nu_0+\mu_0^2+2)-q^2(\nu_0^2-\mu_0^2)-8\lambda_0 q^2]+\frac{r^6}{6}][-{\lambda_0}^7(30q^2+36)\lambda_0^5 q (\frac{3}{2}q+24-12\beta n)+48q^2{\lambda_0}^4+3{\lambda_0}^3 q^2(-1+4\mu_0^2+2\nu_0^2+2\nu_0)-32{\lambda_0}^2 q^2+\lambda_0 q(\frac{3}{2}q -7\lambda_0 q{\mu_0}^2-3q{\nu_0}^2+2\beta n{\mu_0}^2)]\}+C_1 \tag{2.37}$$

$$\mu_1 =\{\mu\lambda_0 q(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)\frac{r^2}{2}+\frac{r^3}{3}[(q\mu_0-2\beta n\mu_0)-\mu_0(q\nu_0-4\beta n\nu_0(1-\nu_0^2)) -\lambda_0(6\lambda_0^3 q-8\lambda_0 q+4\beta n\lambda_0)+(q\mu_0^2-2\beta n{\mu_0}^2)]+\frac{r^4}{4}[3\lambda_0\mu_0 q-\frac{\mu_0}{2}(-30{\lambda_0}^5 q+\lambda_0^4 q^2 +27\lambda_0^3+2q^2-\frac{5}{2}\lambda_0^2 q^2-3q\lambda_0+6\lambda_0 q(\mu_0^2+\nu_0^2)+6\lambda_0^6 q^2)]+\lambda_0 q[(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)(q\mu_0 -2\beta n\mu_0)]\frac{r^5}{5}-\frac{r^6}{6}[(q\mu_0-2\beta n\mu_0)-\mu_0(q\nu_0-4\beta n\nu_0(1-\nu_0^2))-\lambda_0(6\lambda_0^3 q-8\lambda_0 q+4\beta n\lambda_0) +(q\mu_0^2-2\beta n{\mu_0}^2)+3q^2{\lambda_0}^2\mu_0(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)]\}+C_2 \tag{2.38}$$

$$\nu_1 =\{\lambda_0\nu_0 q(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)\frac{r^2}{2}+\frac{r^3}{3}[q\nu_0-4\beta n(1-\nu_0^2)-\nu(q\nu_0-4\beta n\nu_0(1-\nu_0^2) -\lambda_0(6{\lambda_0}^3 q-8\lambda_0 q+4\beta n\lambda_0)+(q\mu^2-2\beta n\mu_0^2))]-\frac{r^4}{4}[3q\lambda_0\nu_0+\frac{\nu_0}{2}(-30{\lambda_0}^5 q+\lambda_0^4 q^2+27{\lambda_0}^3 q^2 -\frac{5}{2}{\lambda_0}^4 q^2-3q\lambda_0+q^2+6\lambda_0 q(\mu_0^2+\nu_0^2)+6\lambda_0^6 q^2)]+\frac{r^5}{5}\lambda_0 q[q\nu_0-4\beta n(1-\nu_0^2)][2{\lambda_0}^2-\alpha\lambda_0 -\alpha-1]-\frac{r^6}{6}[q\nu_0-4\beta n(1-\nu_0^2)+3q^2{\lambda_0}^2\nu(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)]\}+C_3 \tag{2.39}$$

where C_1, C_2 and C_3 are constants of integration which can be obtained by enforcing the condition

$$\lambda_1^2+\mu_1^2+\nu_1^2=1 \tag{2.40}$$

It follows that $C_1=\lambda_0$, $C_2=\mu_0$ and $C_3=\nu_0$, where zero subscript refers to zeroth order approximation. In terms of spherical polar coordinates, λ_0, μ_0 and ν_0 are given by:

$$\lambda_0 = cos\theta sin\phi \quad \mu_0 = sin\theta sin\phi \quad and \quad \nu_0 = cos\theta \tag{2.41}$$

with $\lambda_0^2+\mu_0^2+\nu_0^2=1$, for zeroth order approximation. So, using dependence of λ_0 , μ_0 and ν_0 on η and ζ , we propose to use the following substitutions in the case of rotational distortion as

$$\lambda_1 = cos\eta sin\zeta, \qquad \mu_1 = sin\eta sin\zeta \qquad and \qquad \nu_1 = cos\zeta \tag{2.42}$$

Using equation (2.2.20 - 2.2.22) and (2.2.23) , we get

$$cos\eta = \frac{\lambda_1}{\sqrt{1-\nu^2}} \tag{2.43}$$

and

$$\begin{aligned}\cos\zeta =&\nu_0-\{\lambda_0\nu_0 q(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)\frac{r^2}{2}+\frac{r^3}{3}[q\nu_0-4\beta n(1-\nu_0^2)-\nu(q\nu_0-4\beta n\nu_0(1-\nu_0^2)\\&-\lambda_0(6{\lambda_0}^3q-8\lambda_0 q+4\beta n\lambda_0)+(q\mu^2-2\beta n\mu_0^2))]-\frac{r^4}{4}[3q\lambda_0\nu_0+\frac{\nu_0}{2}(-30{\lambda_0}^5q\\&+\lambda_0^4q^2+27{\lambda_0}^3q^2-\frac{5}{2}{\lambda_0}^4q^2-3q\lambda_0+q^2+6\lambda_0 q(\mu_0^2+\nu_0^2)+6\lambda_0^6q^2)]+\frac{r^5}{5}\lambda_0 q[q\nu_0\\&-4\beta n(1-\nu_0^2)][2{\lambda_0}^2-\alpha\lambda_0-\alpha-1]-\frac{r^6}{6}[q\nu_0-4\beta n(1-\nu_0^2)+3q^2{\lambda_0}^2\nu(2{\lambda_0}^2-\alpha\lambda_0-\alpha-1)]\}\end{aligned} \tag{2.44}$$

It is important to note that, the expression for η given in equation (2.2.35) is exact, however the expression of ζ obtained in equation(2.2.36) is correct upto second order terms in n, n_1 and q.

2.3 CONCLUSIONS

The Roche coordinates ξ, η and ζ given by equations (2.2.5), (2.2.35) and (2.2.36) respectively includes the effect of coriolis force in addition to gravitational and centrifugal force. In a special cases when $\alpha = 1$ and $\beta = 1$, these reduce to the Roche coordinates of equipotential surfaces which does not take into account coriolis force.

Chapter 3

THE METRIC COEFFICIENTS OF ROCHE COORDINATES

In this chapter, we present the explicit expressions of metric coefficients h_1, h_2 and h_3 associated with the Roche coordinates of ξ, η and ζ discussed in chapter 2. This chapter is organised as follows: In section 3.1 we discuss the general equations of orthogonality. The metric coefficient of Roche coordinates is determined in section 3.2. Certain conclusions based on the present studies are finally presented in section 3.3.

3.1 GENERAL EQUATIONS OF ORTHOGONALITY

In the system of coordinates (ξ, η, ζ) if we take ξ=constant then choose η and ζ in such a way that they will satisfy the conditions of mutual orthogonality with respect to ξ as well as to each other.

The general equations of orthogonality that must be satisfied by our curvilinear system of coordinates are the same as discussed in equation(2.2.1).

If we consider in general

$$x = x(\xi, \eta, \zeta); \quad y = y(\xi, \eta, \zeta) \quad and \quad z = z(\xi, \eta, \zeta) \tag{3.1}$$

in which the surface ξ=constant, η=constant and ζ=constant are to form an orthogonal set, the metric coefficients h_1, h_2 and h_3 is three dimensional transformation

$(dx)^2 + (dy)^2 + (dz)^2 = h_1^2(d\xi)^2 + h_2^2(d\eta)^2 + h_3^2(d\zeta)^2$,

will be given by the following equations:

$$h_1^{-2} = \xi_x^2 + \xi_y^2 + \xi_z^2 \tag{3.2}$$

$$h_2^{-2} = \eta_x^2 + \eta_y^2 + \eta_z^2 \tag{3.3}$$

$$h_3^{-2} = \zeta_x^2 + \zeta_y^2 + \zeta_z^2 \tag{3.4}$$

where suffixes denote partial differentiation with respect to x,y,z.

3.2 DETERMINATION OF METRIC COEFFICIENTS

In this section, we present the explicit expressions of metric coefficients h_1, h_2 and h_3 associated with the Roche-coordinates (ξ, η, ζ) for the Roche-equipotentials of rotatianally and tidally distorted stellar models in the presence of coriolis force. The metric coefficients h_1, h_2 and h_3 as defined by (3.1.2 - 3.1.4) by take the following form:

$$h_1(\xi,\zeta) = r_0^2 + r_0^4[(2\lambda_0 qt) + \lambda_0 q(2\lambda_0^2 - \alpha\lambda_0 - \alpha - 1)] + r_0^5[2a_0 - (q\nu_0 - 4\beta n\nu_0(1-\nu_0^2) - \lambda_0(6\lambda_0^3 q - 8\lambda_0 q + 4\beta n\lambda_0) + (q\mu_0^2 - 2\beta n\mu^2))] + r_0^6[\lambda_0^2 + q^2 t + 4\lambda_0^2 q^2 t(2{\lambda_0}^2 - \alpha\lambda_0 - \alpha - 1) + 30\lambda_0^5 q - \lambda_0^4 q^2 - 27\lambda_0^3 q^2 + \frac{5}{2}\lambda_0^2 q^2 + 3q\lambda_0 - 6\mu_0 q(\mu^2+\nu^2)] + r_0^7[2\lambda qt] + r_0^8[a_0^2 + 2\lambda qt (qP_3 + 2\lambda_0^2 q^2 t^2) + 6\mu_0^2 q^2 t a_0(2\lambda_0^2 - \alpha\lambda_0 - \alpha - 1)] \tag{3.5}$$

Proceeding in the like manner consistent with our scheme of approximation and using (2.2.35) and (2.2.36), it may be shown that

$$\eta_x = -\frac{\mu}{r(1-\nu^2)}, \qquad \eta_y = \frac{\lambda}{r(1-\nu^2)}, \qquad \textit{and} \qquad \eta_z = 0 \tag{3.6}$$

From equation (3.2.2) , it can be shown that

$$h_2 = r\sqrt{(1-\nu^2)} \tag{3.7}$$

On using the values of μ, ν and r, we get:

$$h_2(\xi,\zeta) = r_0 sin(\zeta)[1 + \lambda qt r_0^2 + a_0 r_0^3 + (qP_3 + 2\lambda^2 t^2) r_0^4 + (qP_4 + 5a_0\lambda qt) r_0^5 + (qP_5 + 3a_0^5 + 6\lambda q^2 tP_3) r_0 6 + (qP_6 + 7a_0 qP_3 + 7\lambda q^2 t^4 P_4) r_0^7 + (qP_7 + 8a_0 qP_4 + 8\lambda q^2 tP_5 + 4q^2 P_3^2) r_0^8 + (qP_8 + 9a_0 qP_5 + 9\lambda q^2 tP_6 + 9q^2 P_3 P_4) r_0^9 + (qP_9 + 10a_0 qP_6 + 10\lambda q^2 tP_7 + 5q^2\{P_4^2 + 2P_3P_5\}) r_0^1 0 + \ldots\ldots] \tag{3.8}$$

And finally explicit expressions for h_3 can be obtained by using the following relations:

$$h_3 = (\zeta_x^2 + \zeta_y^2 + \zeta_z^2)^{-1/2} \tag{3.9}$$

Thus, equations (3.2.1), (3.2.4) and (3.2.5) represents explicit expressions for metric coefficients h_1, h_2 and h_3 respectively associated with system of Roche coordinates ξ, η, ζ for the distorted strars in the presence of coriolis force.

3.3 CONCLUSION

In the present thesis we have developed an explicit expression for Roche coordinates ξ, η and ζ as well as metric coefficients h_1, h_2 and h_3 which are associated with ξ, η and ζ. These expressions incorporate the effect of coriolis force in addition to the centrifugal and gravitational forces. In a special cases these expressions can be reduced to the previous one where the effects of coriolis force has been neglected.

The Roche coordinates and other related parameters thus developed in this thesis which can further be used to study the period of oscillations of rotationally and tidally distorted stars subjected to the coriolis force. We are always interested in such studies where coriolis force plays significant role in the rotianally and tidally distorted stars. It may be noted that we have considered the Roche equipotential surfaces by assuming primary component of binary to be a Roche model of the star. The present methodology can further be extended to more realistic stars instead of Roche model of star for their inner structure.

Bibliography

[1] Abhyankar, K.D., Astrophysics Stars and Galaxies, Tata McGraw Hill Publishing Company Ltd., 1992.

[2] Chandrasekhar, S. and Lebovitz, N. R., On the oscillations and the stability of rotating gaseous masses III. The distorted polytropes, Astrophysical Journal, 136, 1082 - 1104, 1962.

[3] Eddington, A.S., The internal constituion of the star, Dover Publication, Newyork, 1959.

[4] Kopal, Z., Roche Model and Its Application to Close Binary Systems, Advances in Astronomy and Astrophysics, 9, 1 - 65, 1972.

[5] Kopal, Z., Astrophys. Space Sci., ”Gravity-darkening in eclipsing binaries. II: Early main-sequence systems”, Vol. 2, pp. 166-170, 1968.

[6] Kopal, Z., ”Dynamics of close binary system”, D. reidel publ. Co., Dordrecht, Holland, 1978.

[7] Kopal, Z., ”Clairaut’s coordinates and the vibrational stability of distorted stars”, Astrophys. Space Sci., Vol. 70, p. 407, 1980.

[8] Kopal, Z., ”Vibrational stability of rotating stars”, Astrophys. Space Sci., Vol. 76, p. 187, 1981.

[9] Kopal, Z., ”Effects of rotation on Internal Structure of Stars”, Astrophys.Space Sci., Vol. 93, p. 149, 1983.

[10] Lal, A.K., Mohan, C. and Singh, V.P., ”Equilibrium structure of differentially rotating and tidally distorted white dwarf models of stars Astrophys Space Sci., Vol. 301, p. 51, 2006.

[11] Lal, A.K.,Pathania,A and Mohan, C., ”Effect of coriolis force on the equilibrium structures of rotating stars and stars in binary system”, Astrophys. Space Sci., Vol. 315, pp.157-165, 2008.

[12] Mentzel, D.H., Bhatnagar, P.L. And Sen, H.K., Stellar Interiors, Chapman and Hall Ltd., Londan, 1963.

[13] Mohan, C. and Saxena, R. M., Effects of rotation and tidal distortions on the structure of polytropic stars, Astrophysics and Space Science, 95, 369 - 381, 1983.

[14] Mohan, C. and Saxena, R. M., Effects of rotation and tidal distortions on the periods of small adiabatic oscillations of the polytropic models of the stars, Astrophysics and Space Science, 113, 155 - 169, 1985

[15] Mohan, C., Saxena, R. M. and Agarwal, S. R., Equilibrium structures of rotationally and tidally distorted stellar models, Astrophysics and Space Science, 163, 23 - 39, 1990.

[16] Mohan, C., Saxena, R. M. and Agarwal, S. R., Eigenfrequencies of small adiabatic barotropic models of oscillations of rotationally and tidally - distorted stars, Astrophysics and Space Science, 178, .89 - 105, 1991.

[17] Rosseland, S., The pulsation Theory of Variable Stars, Oxford, Clarendon Press, 1949

[18] Singh, V.P., ”Systems of Roche-Coordinates for stellar model disorted by differential rotation”, Indian J.pure appl.Math, 22, 769-779, 1991.

[19] Singh, V.P. and Gupta, V., ”Roche-harmonics for stellar model distorted by differential rotation”, IndianJ.pure an appl. Math., 20, 284-291,1989.

[20] Schwarzchild, M., Structure and Evolution of the Stars, Princeton University Press, Princeton, 1958.

[21] Mohan, C. and Singh, V. P., Use of Roche coordinates in the problems of small oscillations of rotationally distorted stellar models, Astrophysics and Space Science, 54, 293 - 304, 1978

[22] Mohan, C. and Singh, V. P., Use of Roche Coordinates in the problems of small oscillations of tidally distorted stellar models, Astrophysics and Space Science, 56, 109 - 117, 1978.

[23] Mohan, C. And Aggarwal, S.R., Effects of rotation and tidal distortions on the Periods of adiabatic oscillations of composite model of stars, Astrophysics And space science, 129, 73-94, 1987.

[24] Cox, J. P. And Guili, R.T., Principle of stellar structure, Vol. I and II, Gordon And Breach, New York, 1968.

[25] Kippenhahn, R. And Weigert, A., Stellar Structure And Evolution, Springer-Verkag publ., 1990.

[26] Clement, M .J., In A. slettebak (ed.), Stellar Rotation, D. Reidal Publ. co., Dordercht, Holland, 1970.

[27] Kippenhahn, R. And Thomas, H. C., A simple method for the solution of the Stellar structure equation including Rotation and Tidal forces, Stellar Rotation, Proceeding of IAU Colloq 4, Ohio State Univ, Coloumbs, ed. Slettebak, A., D., Gordon and Breach Science Publishers, 1970.

[28] Lal, A. K., Structure oscillations and stability of differentially rotating and tidally distorted stellar models, PhD Thesis, Roorkee University, Roorkee, India, 1993.

[29] Singh, V. P. and Sharma, M.k., Effect of differential rotation on the periods of small adiabatic oscillations on Stellar Models, India J pure and appl. Math., 26(1), 69-79, 1995.

[30] Jain, N. and Sharma, M. K., Effects of Coroilis Force on the Equilibrium structures of Rotionally and Tidally Distorted stellar Models, Electronics Thesis and Dissertations, Thapar University, Patiala, 2009.

[31] Pathania, A., Analytic and Computational studies of structures and oscillations of rotating stars and stars in binary system, PhD thesis, Thapar University, Patiala, India, 2009.

[32] Lal, A. K., Pathania, A. And Mohan, C., Structures and Oscillations of Rotationally and Tidally Distorted Polytropic Model Of Stars In The Presence of Coriolis Force, Online Published, 2011.

[33] Kopal, Z. and Kitamura, M., ”adv. in Astron. and Astrophys” , Academic Press New York, Vol. 6, p. 125-172, 1968.

[34] Kopal, Z. and A.K.M., Sekender Ali, ”On the inerrability of Roche-coordinates” , Astrophys. Space sci., Vol. 11, p. 423-429, 1971.

[35] Singh, V.P., ”System of Roche coordinates for stellar model distorted by differential rotation” , Indian J. pure ans appl. Math., Vol. 22, p.769-779, 1991.

[36] Mohan, C. and Singh, V.P., ”Use of coordinates in the problems of small oscillations of rotationally distorted stellar models” , Astrophys. Space. Sci., Vol. 54, p. 293, 1978.

[37] Singh, V.P. and Gupta, V., ”Roche-harmonics for stellar model distorted by differential rotation”, Indian J pure and appl. Math., Vol., 20, p.769-779, 1991.

[38] Kumar Sunil, Vibrational stability of differentially and rotating gaseous spheres, PhD Thesis, Choudhary Charan Singh university, Meerut, India, 2010.

[39] Lal, A.k., Saini, S., Mohan, C. and Singh, V.P., ”on the validity of series expansion being used for the position of a point on the Roche-equipotential” , Astrophys. Space Sci., Vol. 311, p.317-320, 2007.

[40] Kopal, Z., Song, G.X.: Astrophys. Space Sci. 96, 381-404(1983).

[41] i) Mohan, C., Saxena , R.M. , Agarwal, S.R.: Astrophys. Space sci. 163, 23-39 (1990).
ii) Mohan, C., Lal, A.k., Singh, V.P.: Astrophys. Space Sci., 193, 69-85 (1992).
iii) Mohan, C., Lal, A.K., Singh, V.P.: Astropyhs. Space Sci. 254, 97-109(1997).